Numerical analysis

Root Finding Methods

Contribution

Name: Ahmed Osama

ID: 6078

Name: Youssef Sabry

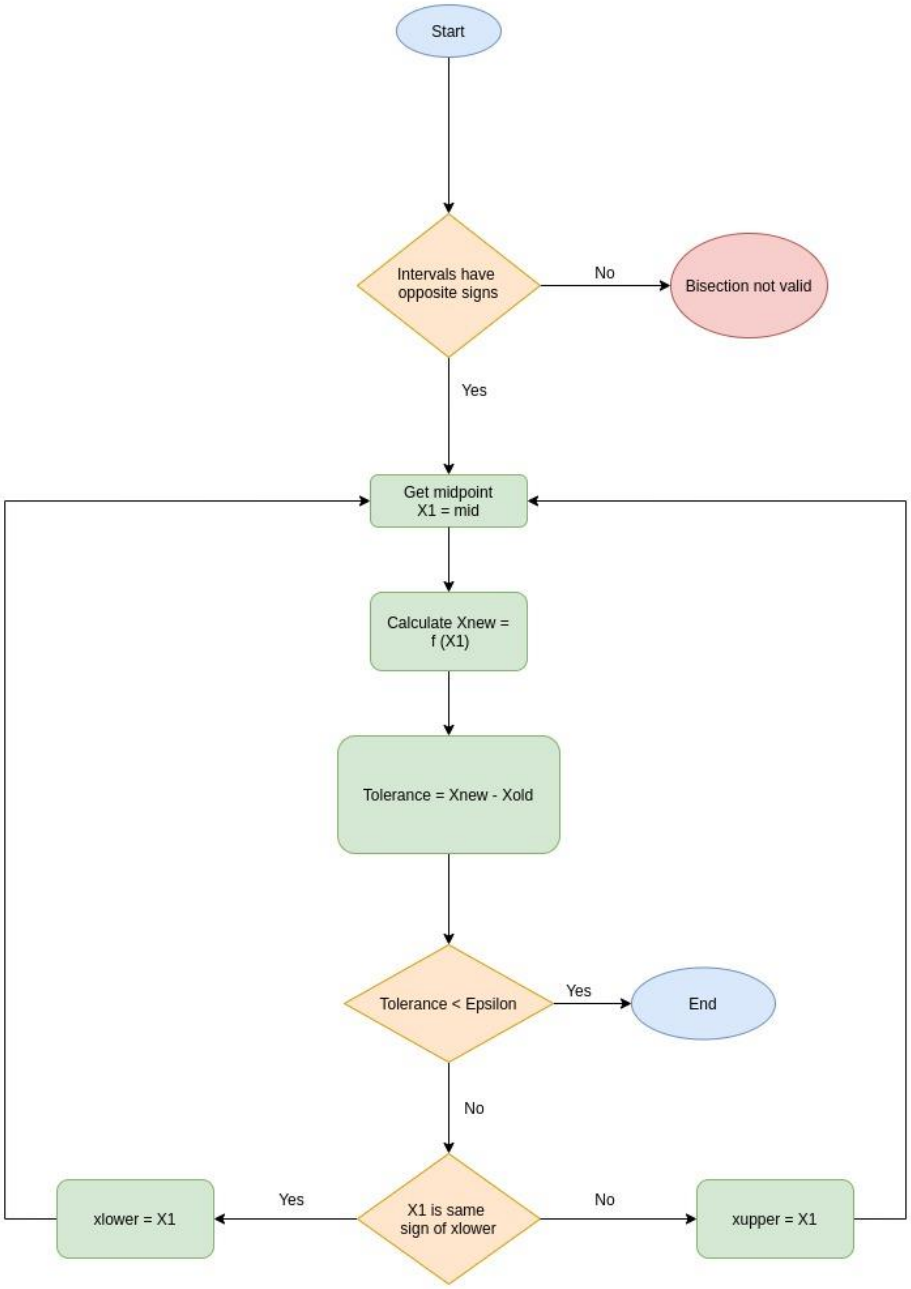
ID: 6239

Name: Nour El-Din Hazem

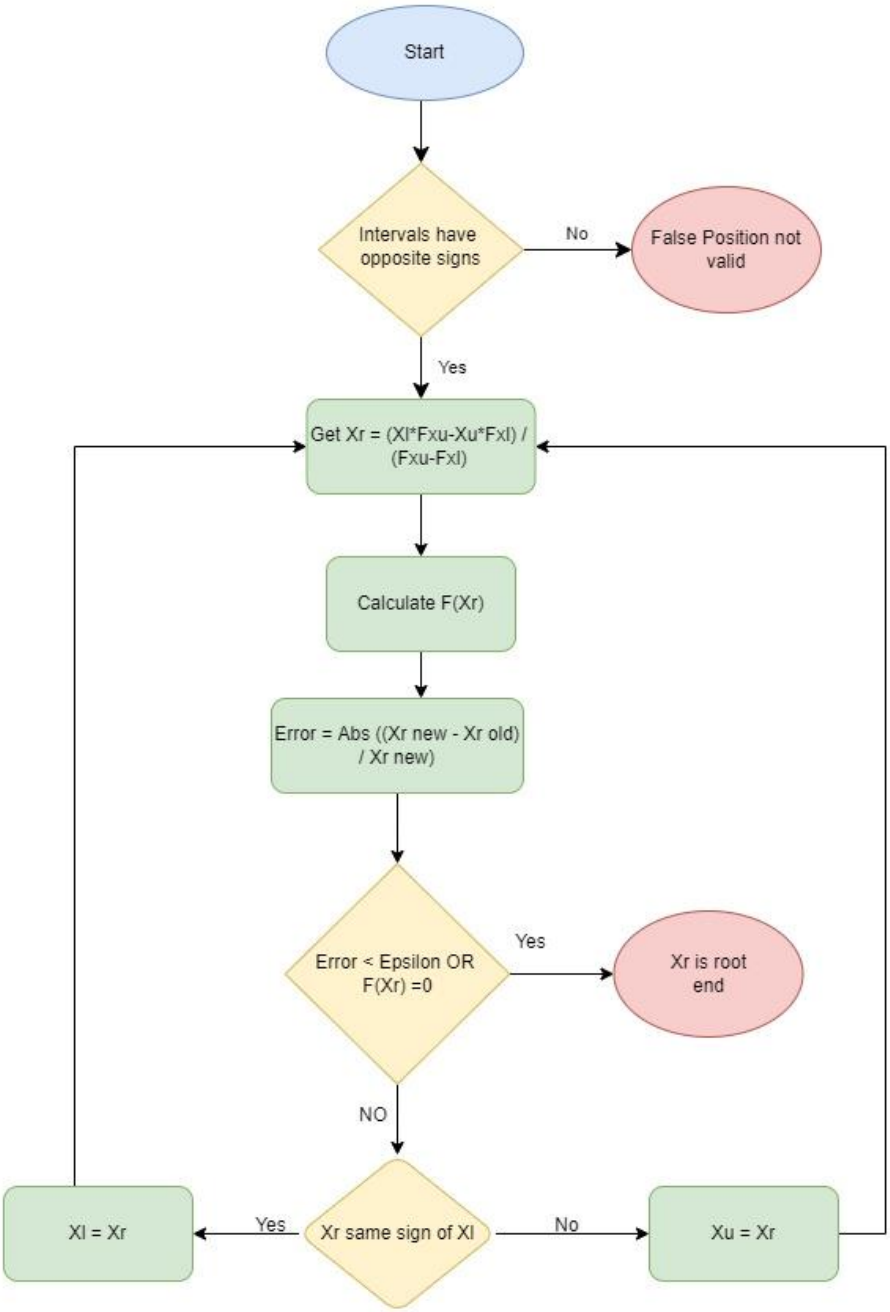
ID: 6261

Flowcharts Part

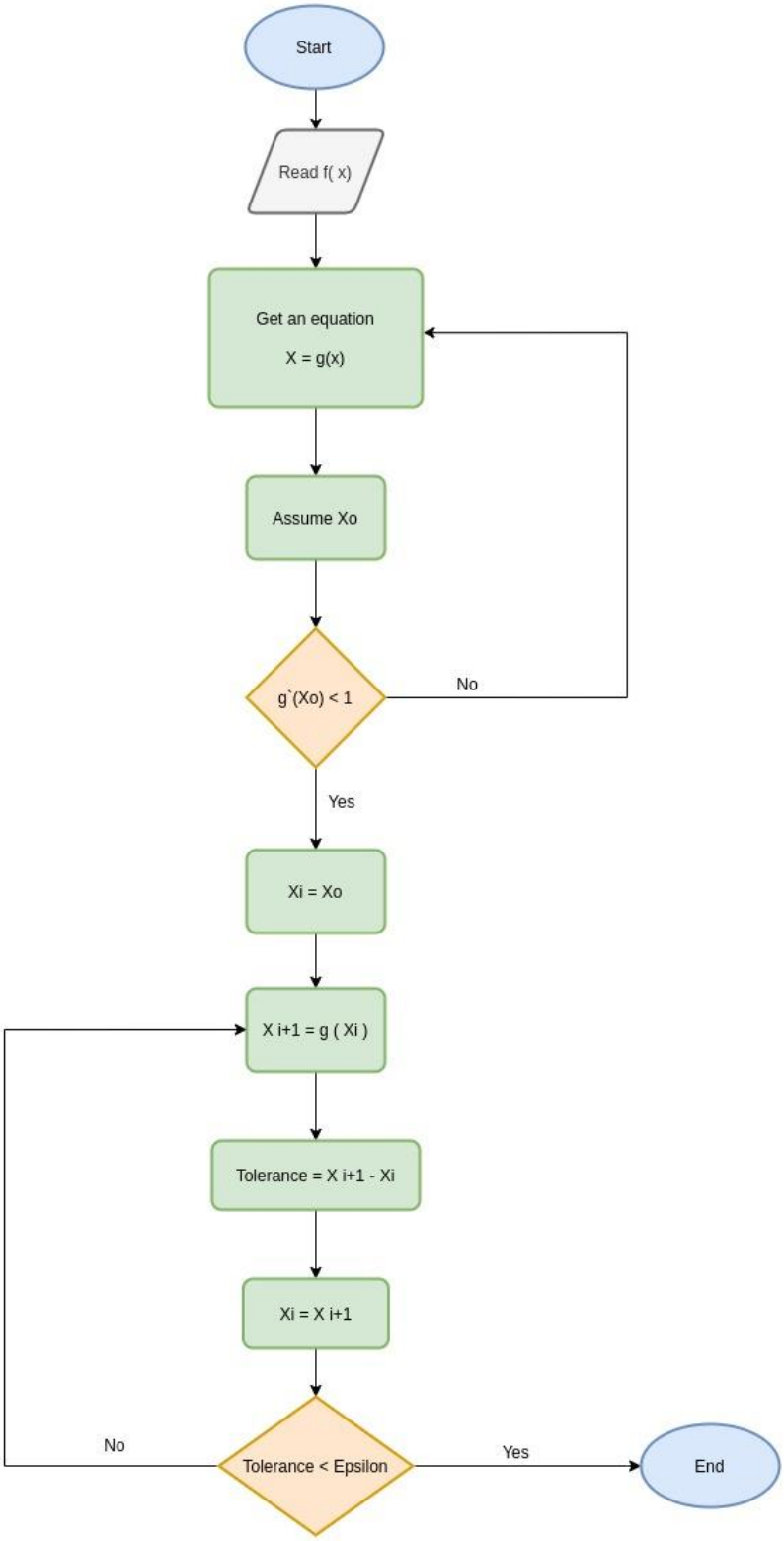
1) Bisection Method



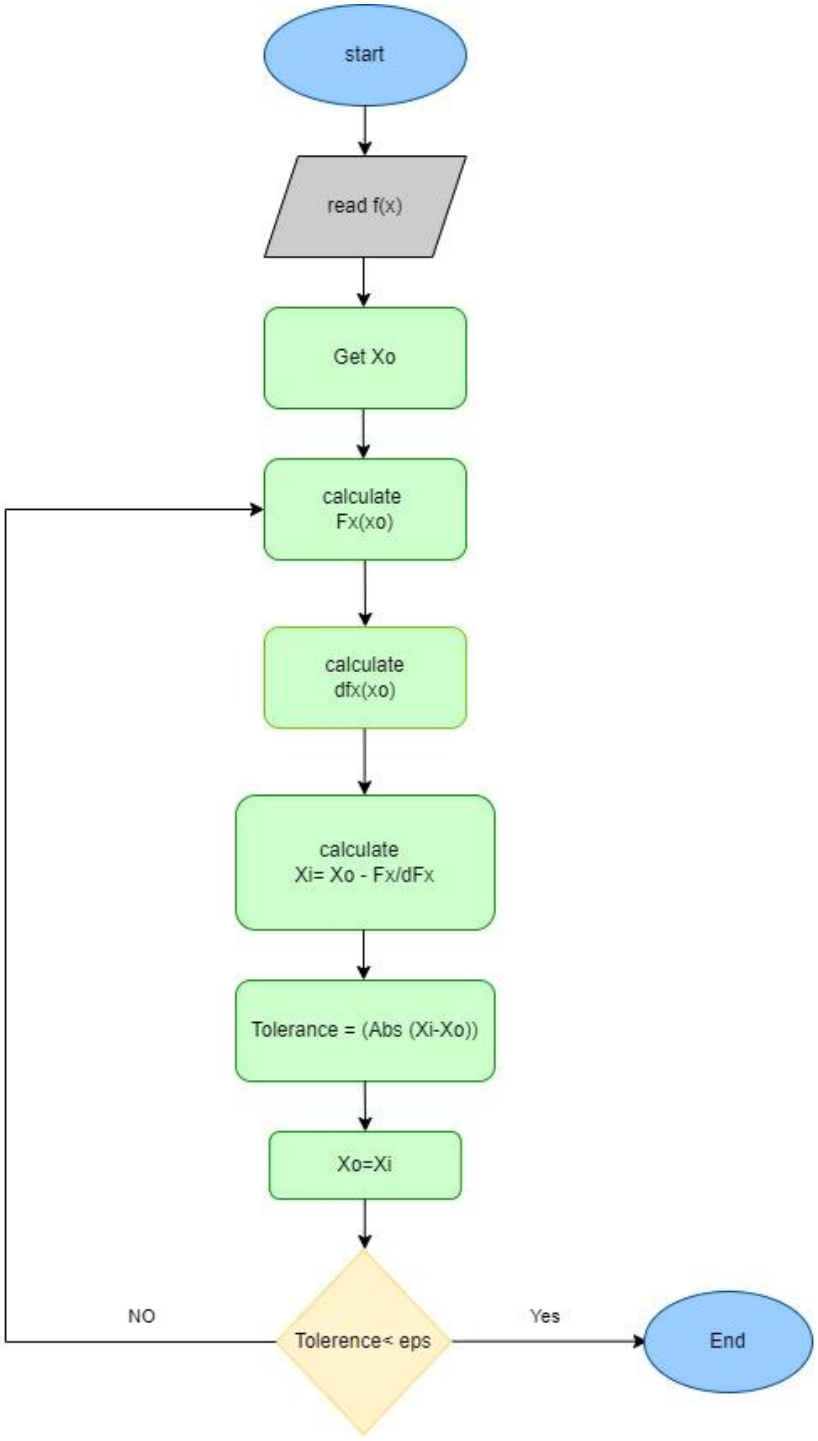
2) False-Position Method



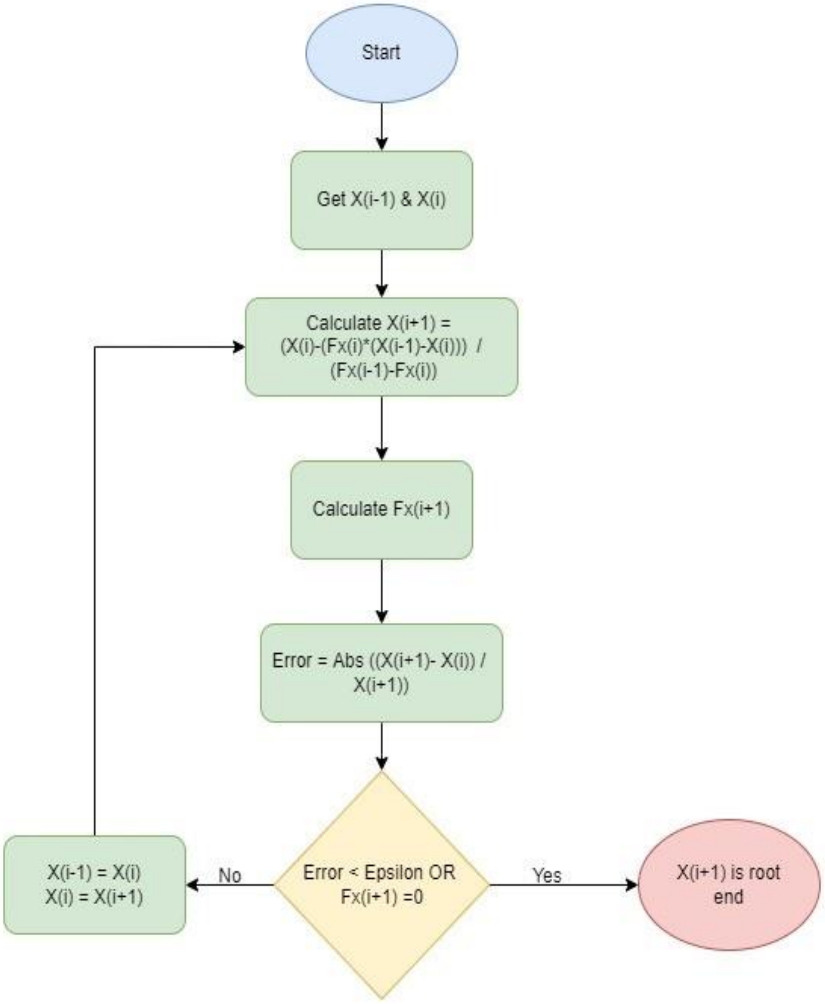
3) Fixed-Point Method



4) Newton-Raphson's Method



5) Secant Method



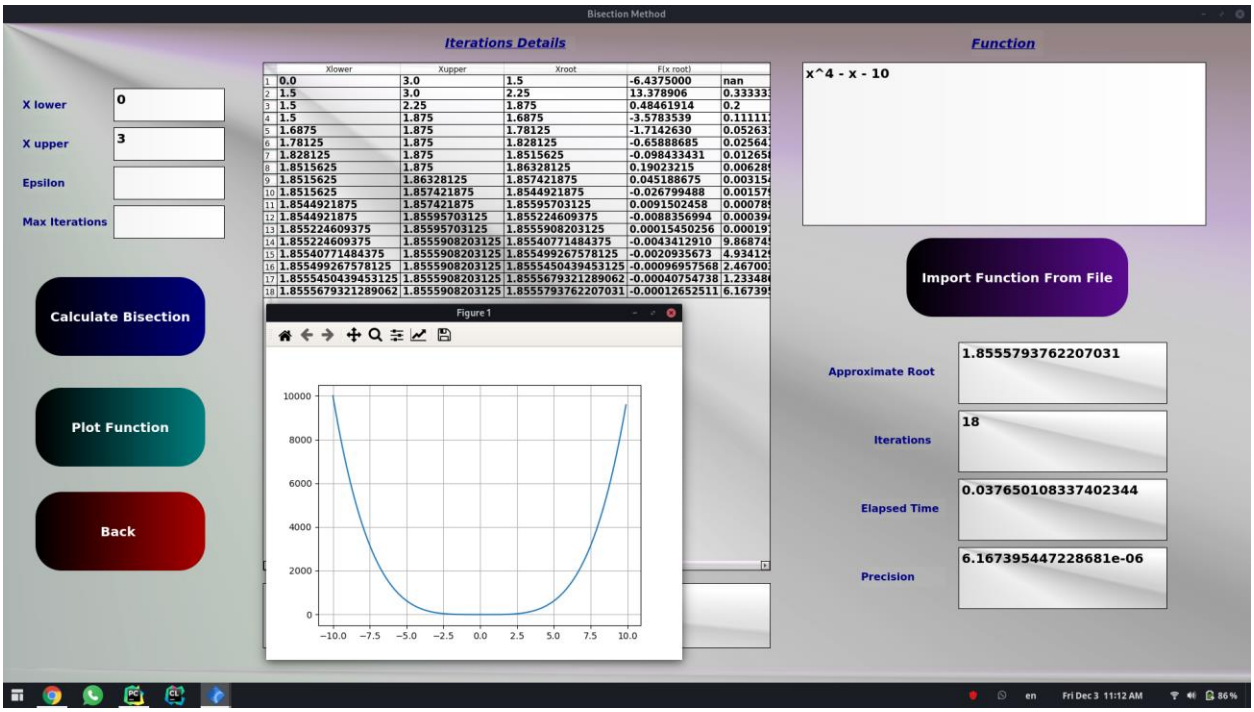
Data Structure Used

- **Nothing used except a 2D list to store the data returned from each method. This helped presented the data in a tabular view and retrieving data in an easy way using pre-known indexes of desired values.**

Analysis For The Behavior

Bisection:

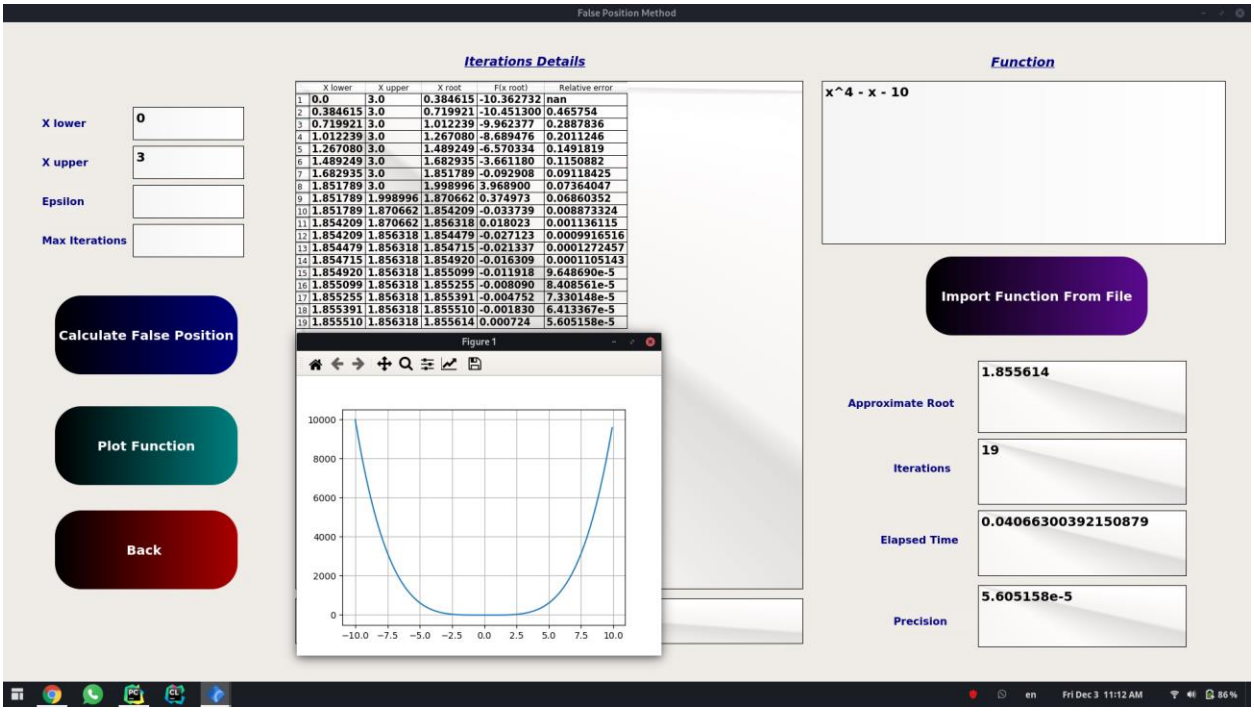
- Some functions are not bisection method applicable due to invalid intervals as the root does not lie between the intervals.
- Bisection is relatively slow because it is a bracketing method, could consume more iterations, more time.



- Took 18 iterations, 0.03 seconds for default epsilon 0.00001.

False Position:

- Some functions are not false position method applicable due to invalid intervals as the root does not lie between the intervals.
- False Position is relatively slow because it is a bracketing method, could consume more iterations, more time.
- False position is the slowest in steep functions.

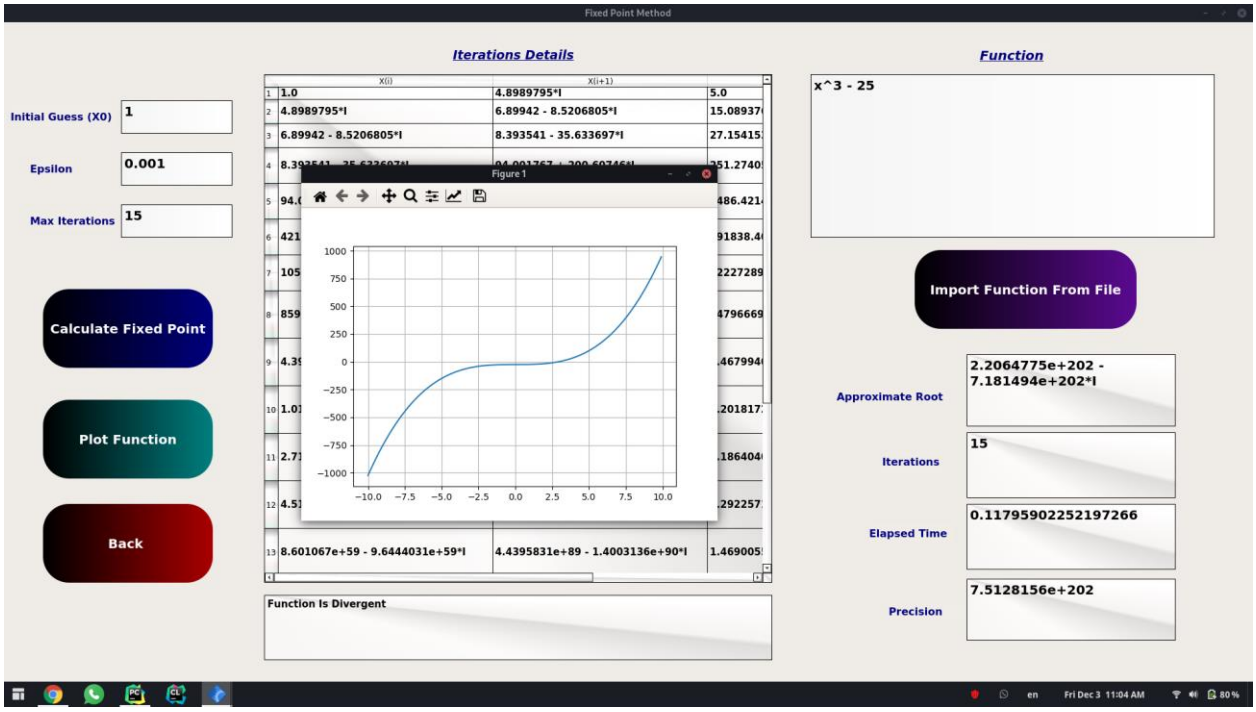


	X lower	X upper	X root	F(x root)	Relative error
1	0.0	3.0	0.384615	-10.362732	nan
2	0.384615	3.0	0.719921	-10.451300	0.465754
3	0.719921	3.0	1.012239	-9.962377	0.2887836
4	1.012239	3.0	1.267080	-8.689476	0.2011246
5	1.267080	3.0	1.489249	-6.570334	0.1491819
6	1.489249	3.0	1.682935	-3.661180	0.1150882
7	1.682935	3.0	1.851789	-0.092908	0.09118425
8	1.851789	3.0	1.998996	3.968900	0.07364047
9	1.851789	1.998996	1.870662	0.374973	0.06860352
10	1.851789	1.870662	1.854209	-0.033739	0.008873324
11	1.854209	1.870662	1.856318	0.018023	0.001136115
12	1.854209	1.856318	1.854479	-0.027123	0.0009916516
13	1.854479	1.856318	1.854715	-0.021337	0.0001272457
14	1.854715	1.856318	1.854920	-0.016309	0.0001105143
15	1.854920	1.856318	1.855099	-0.011918	9.648690e-5
16	1.855099	1.856318	1.855255	-0.008090	8.408561e-5
17	1.855255	1.856318	1.855391	-0.004752	7.330148e-5
18	1.855391	1.856318	1.855510	-0.001830	6.413367e-5
19	1.855510	1.856318	1.855614	0.000724	5.605158e-5

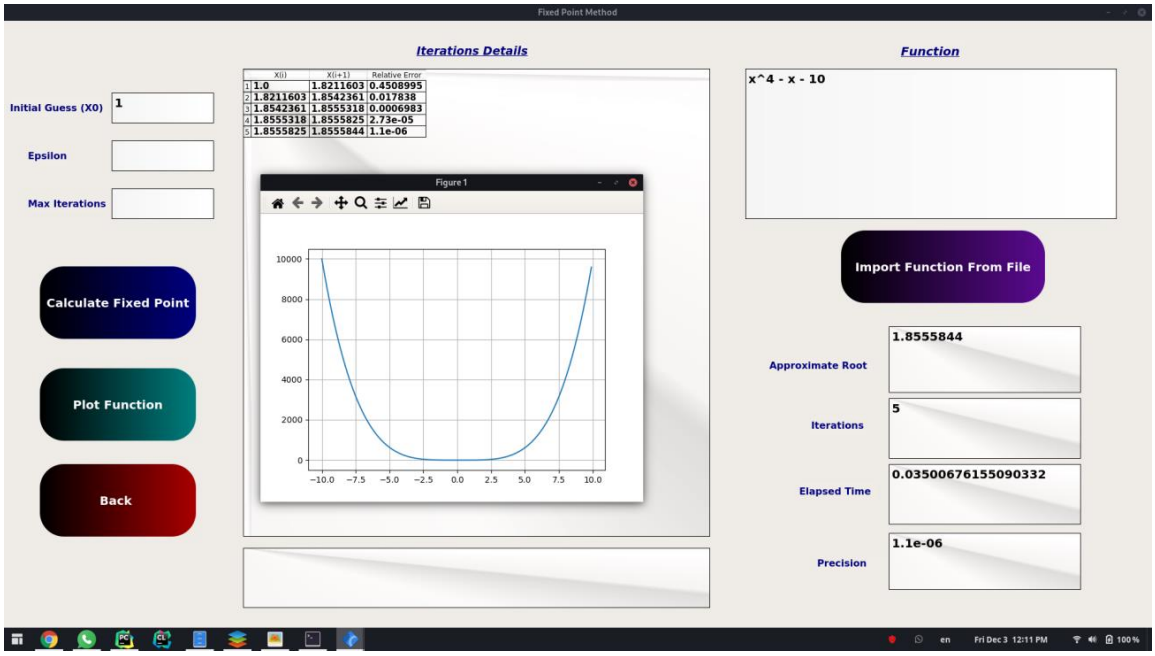
- False position took 19 iterations (max) and 0.4 seconds (max) to get the root of such a steep function.

Fixed Point:

- **Some functions diverge due to the algorithm of fixed point method so the root could be complex if a bad g(x) was generated.**

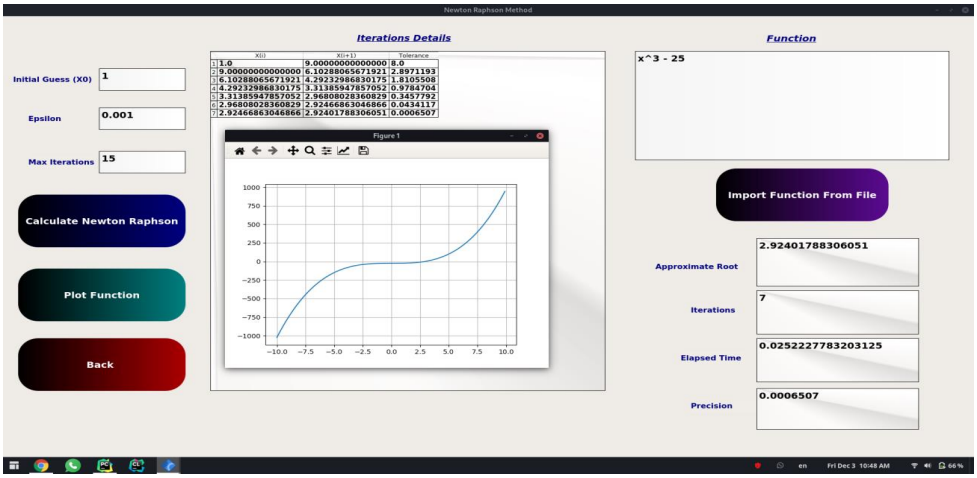


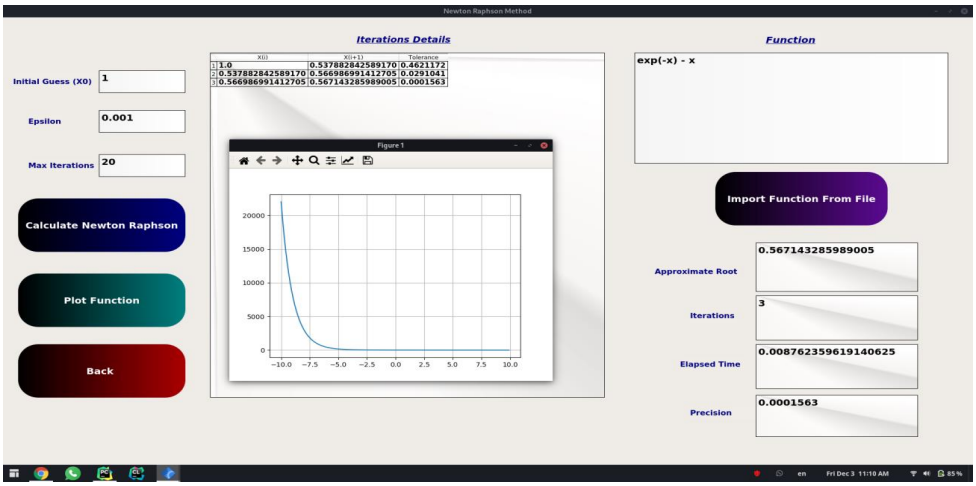
- **Otherwise it can get precise roots if the g(x) was suitable and in a relatively fast behavior.**

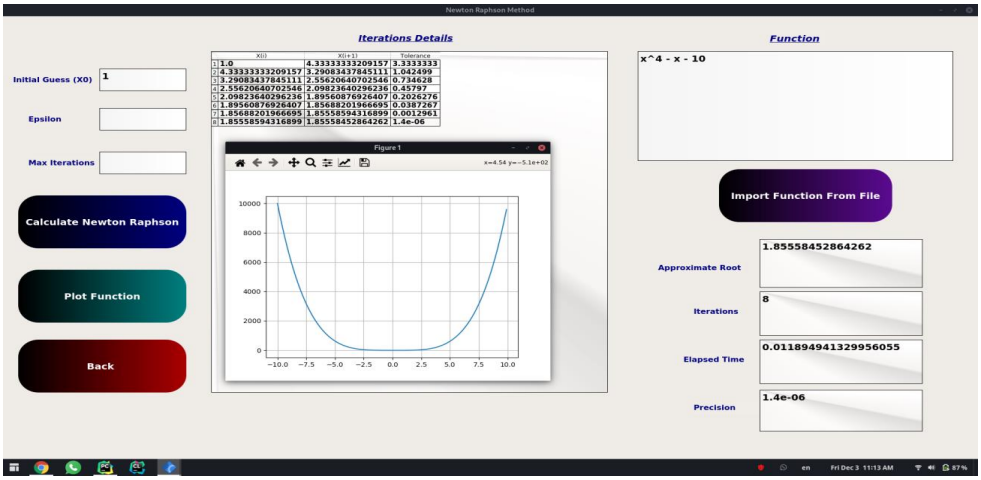

Fixed Point Method
Iterations Details
Function
Initial Guess (X0)
1
Epsilon
Max Iterations
Calculate Fixed Point
Plot Function
Back
X(i) X(i+1) Relative Error
1 1.0 1.8211603 0.4508995
2 1.8211603 1.8542361 0.017838
3 1.8542361 1.8555318 0.0006983
4 1.8555318 1.8555825 2.73e-05
5 1.8555825 1.8555844 1.1e-06
Figure 1
10000
8000
6000
4000
2000
0
−10.0 −7.5 −5.0 −2.5 0.0 2.5 5.0 7.5 10.0
x^4 - x - 10
Import Function From File
Approximate Root
1.8555844
Iterations
5
Elapsed Time
0.03500676155090332
Precision
1.1e-06
en
Fri Dec 3 12:11 PM
100 %

Newton Raphson:

- **Generally, Newton Raphson is the fastest method.**
- **It calculates the root in the shortest time and least iterations.**
- **Applicable on most of the functions if not all.**

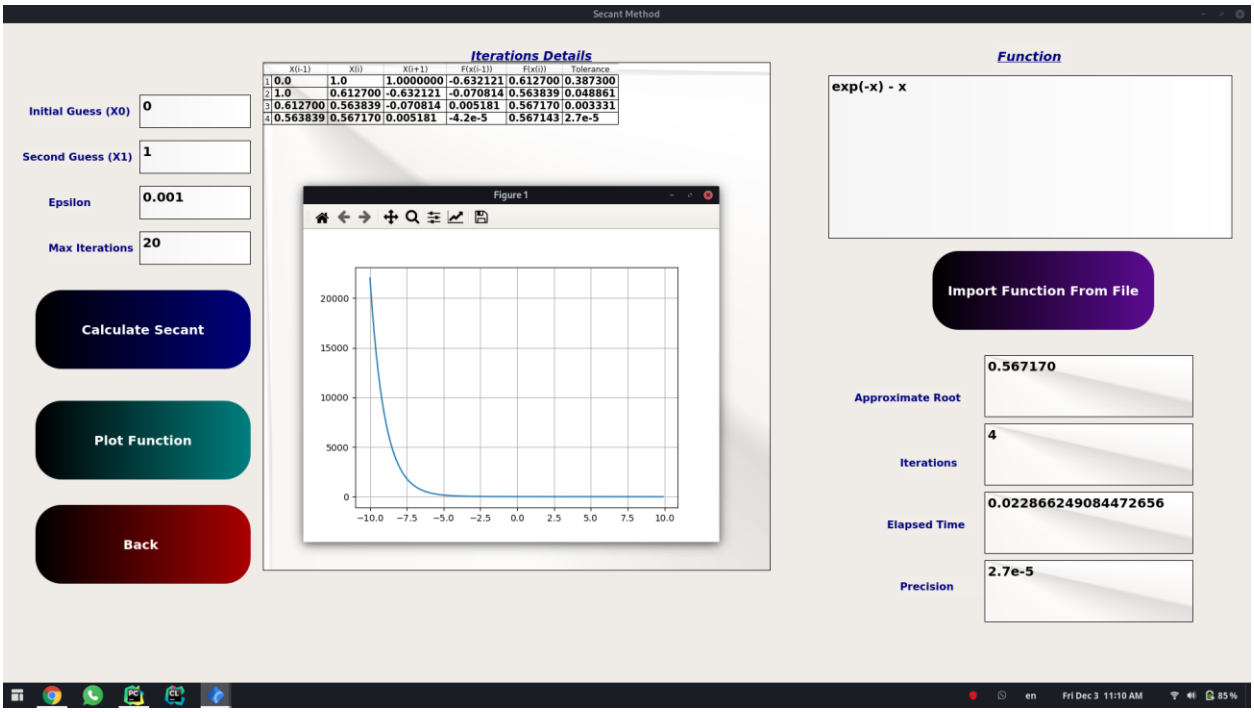






Secant:

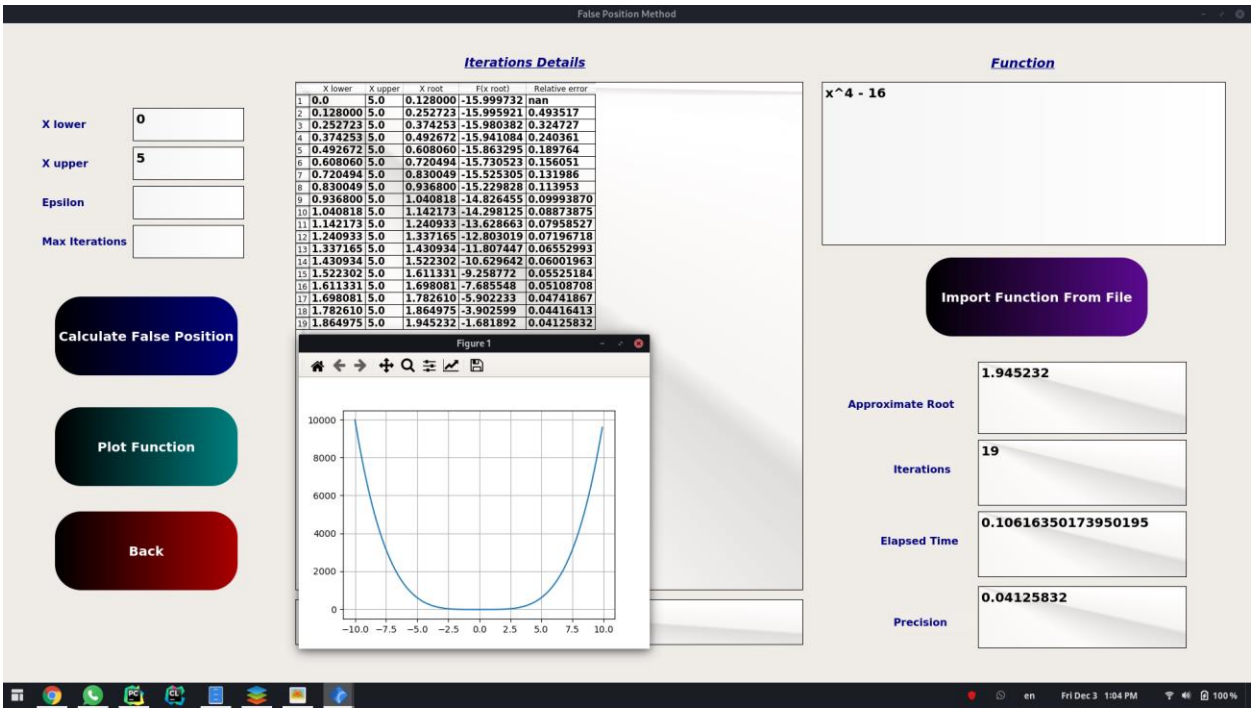
- Generally, Secant is probably the second fastest method in some problems.
- It calculates the root in a short time relatively and small number of iterations.
- Applicable on most of the functions if not all.
- Has better efficiency with exponential functions.



Problematic Functions

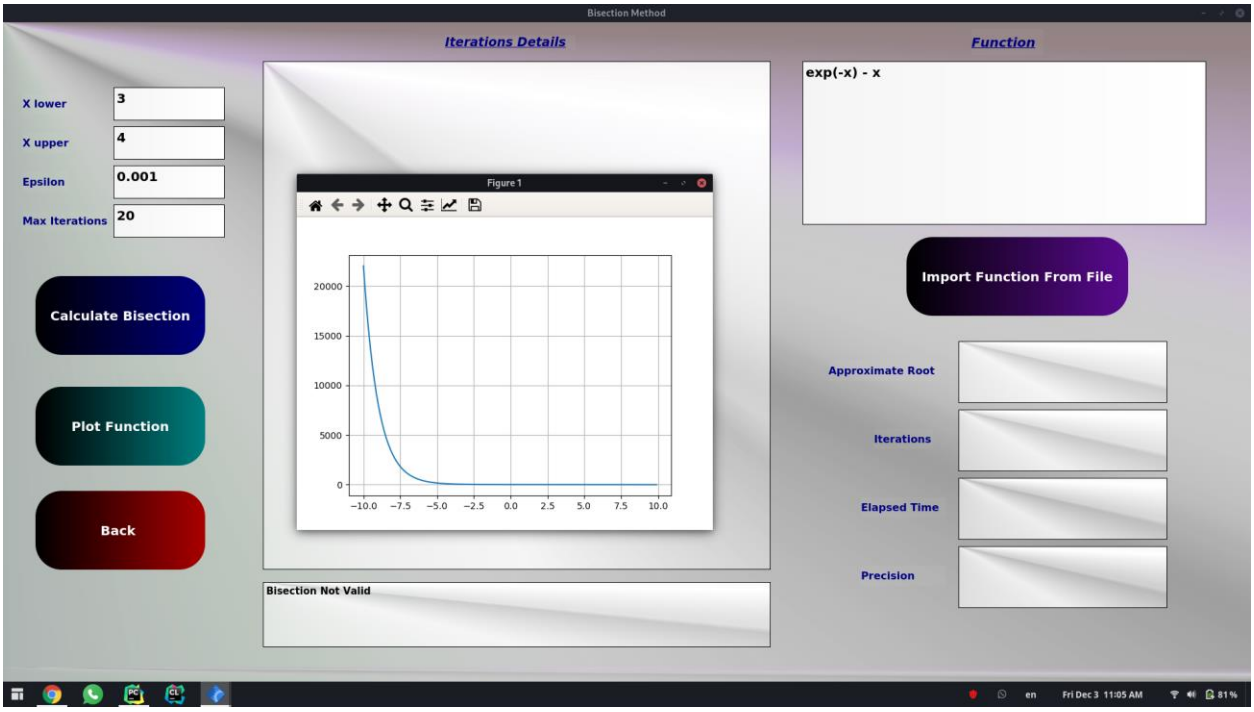
Steep Functions in False Position:

- Sometimes, in False Position methods it is too slow to calculate the root of a steep function such as x^4 - 16 or whatsoever.
- This could result in a root with very bad precision or a root after long time or iterations.
- There is no solution than just knowing that this is the behavior of the method for such functions so not using it with steep functions as it will be the slowest method out of all.
- Ex :



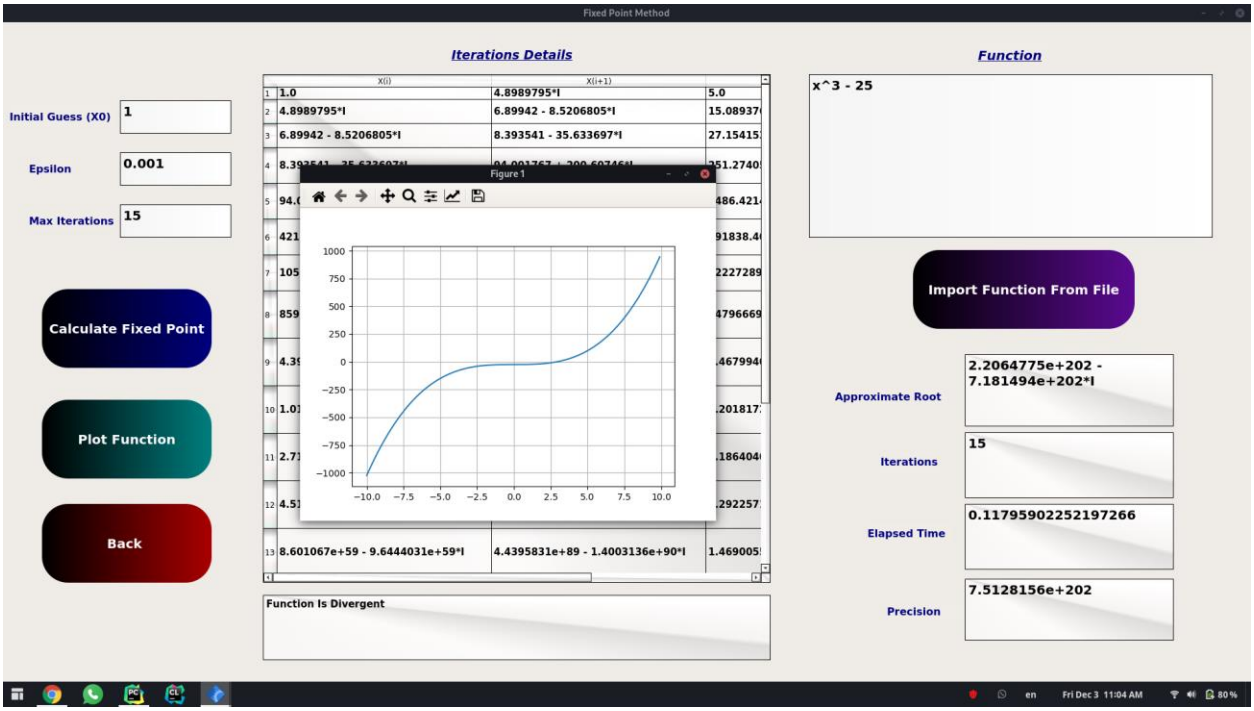
Out of interval functions:

- **Sometimes the root could lie outside the given interval in Bisection and False Position methods so both methods could not be applied.**
- **Solution is brute force as it is only by notifying the user that the method is not applicable and asking the user to enter valid intervals.**



Bad g(x) diverging Functions in Fixed Point:

- **Sometimes, the derived equation x = g(x) results in a diverging behavior that cant get an appropriate root.**
- **Solution is to take an appropriate g(x) as input from the user.**
- **Ex:**



Secant (F(Xo) = F(X1)):

- **This results in a root (X new) of infinity value as the denomenator in this case = zero.**
- **Ex:**

  F(x) = x^4 - x - 10 = 0

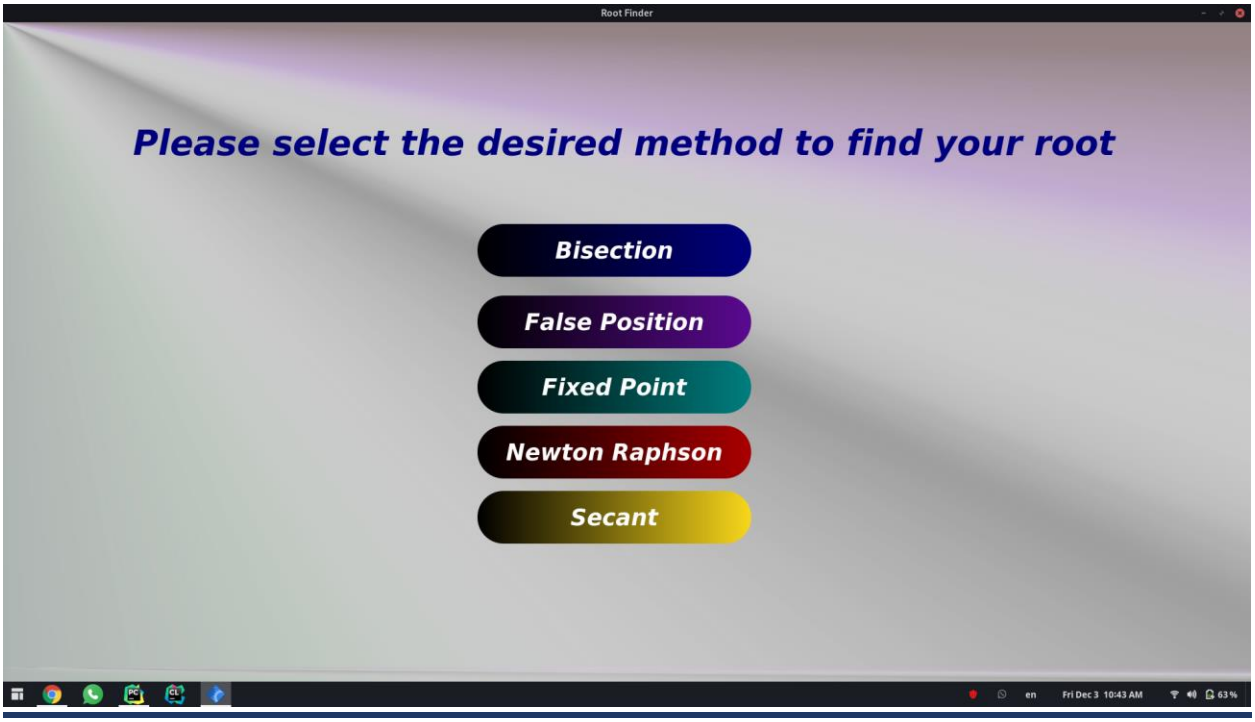
  Xo = 0

  X1 = 1

  Both F(x) = - 10.

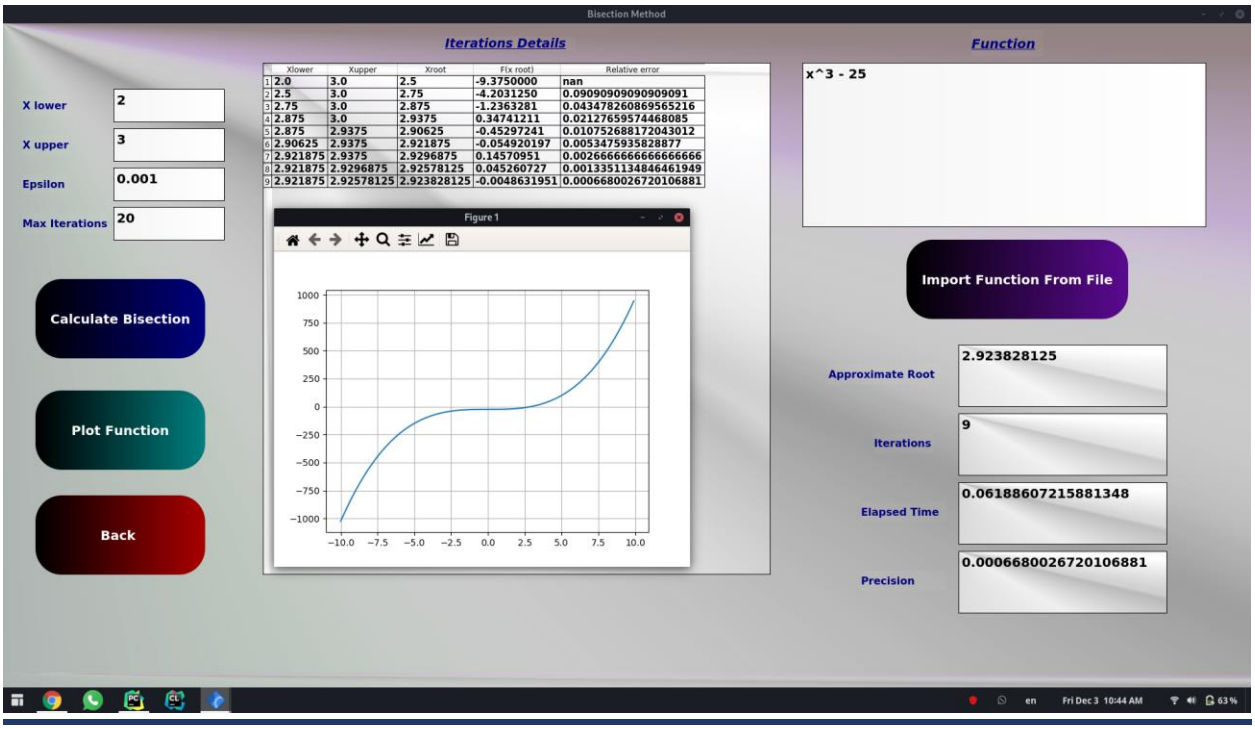
Sample runs & Snapshots

Interface:



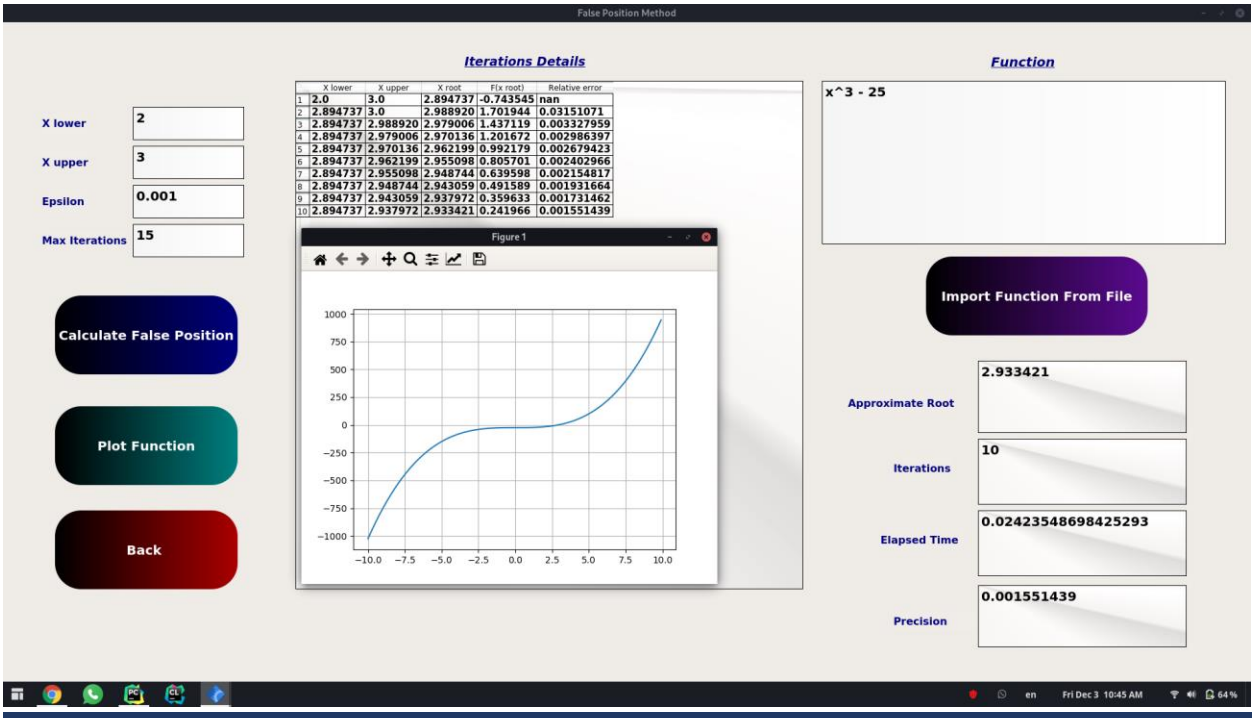
Function 1:

Bisection:



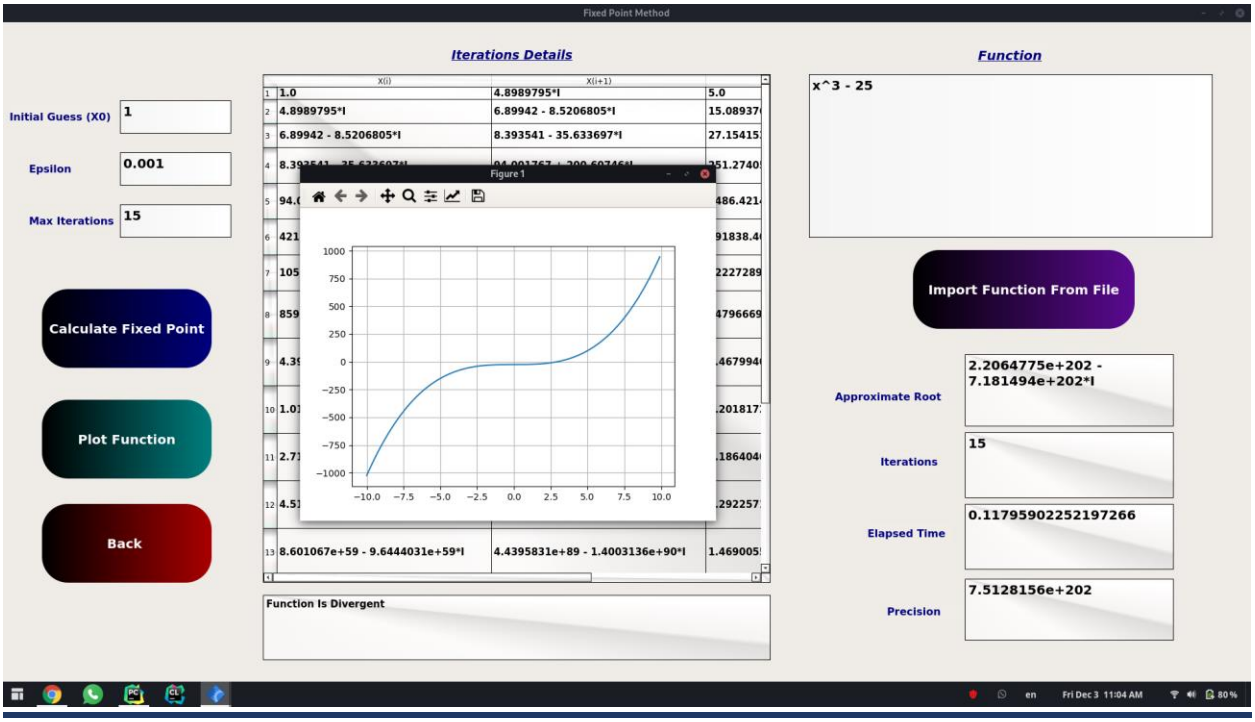
	Xlower	Xupper	Xroot	F(x root)	Relative error
1	2.0	3.0	2.5	-9.3750000	nan
2	2.5	3.0	2.75	-4.2031250	0.09090909090909091
3	2.75	3.0	2.875	-1.2363281	0.043478260869565216
4	2.875	3.0	2.9375	0.34741211	0.02127659574468085
5	2.875	2.9375	2.90625	-0.45297241	0.010752688172043012
6	2.90625	2.9375	2.921875	-0.054920197	0.005347593582887
7	2.921875	2.9375	2.9296875	0.14570951	0.0026666666666666666
8	2.921875	2.9296875	2.92578125	0.045260727	0.0013351134846461949
9	2.921875	2.92578125	2.923828125	-0.0048631951	0.0006680026720106881

False Position:

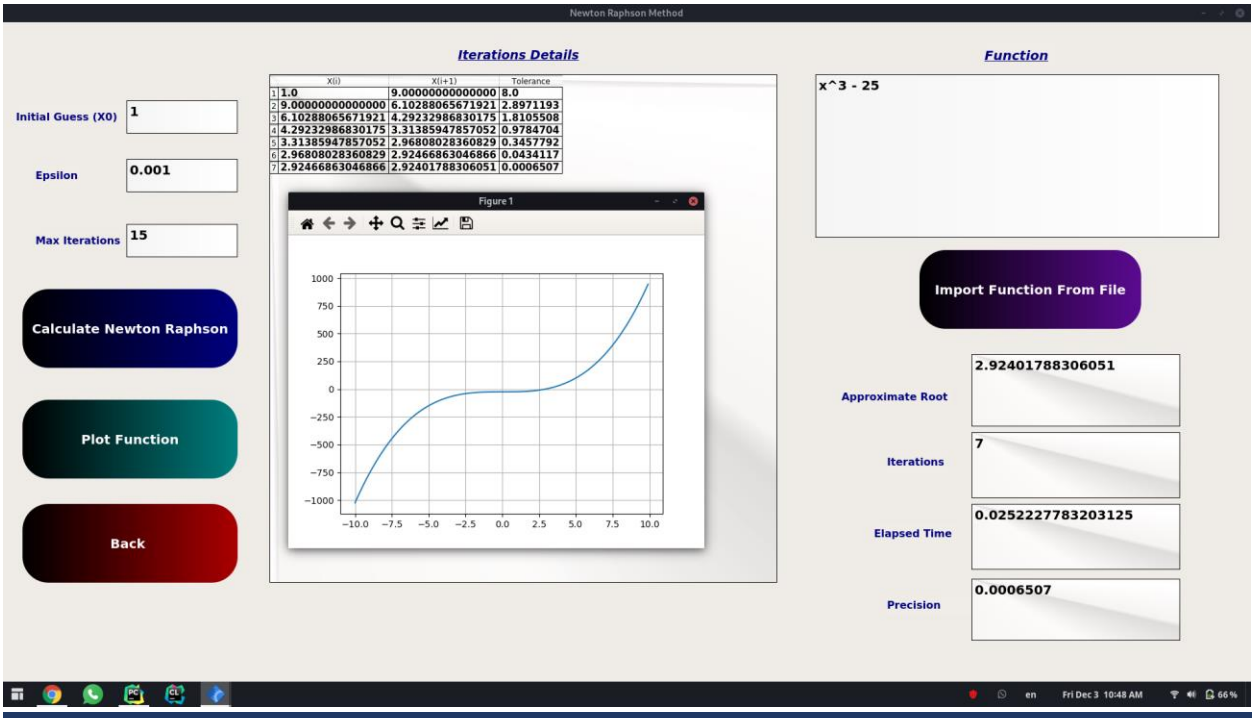


	X lower	X upper	X root	F(x root)	Relative error
1	2.0	3.0	2.894737	-0.743545	nan
2	2.894737	3.0	2.988920	1.701944	0.03151071
3	2.894737	2.988920	2.979006	1.437119	0.003327959
4	2.894737	2.979006	2.970136	1.201672	0.002986397
5	2.894737	2.970136	2.962199	0.992179	0.002679423
6	2.894737	2.962199	2.955098	0.805701	0.002402966
7	2.894737	2.955098	2.948744	0.639598	0.002154817
8	2.894737	2.948744	2.943059	0.491589	0.001931664
9	2.894737	2.943059	2.937972	0.359633	0.001731462
10	2.894737	2.937972	2.933421	0.241966	0.001551439

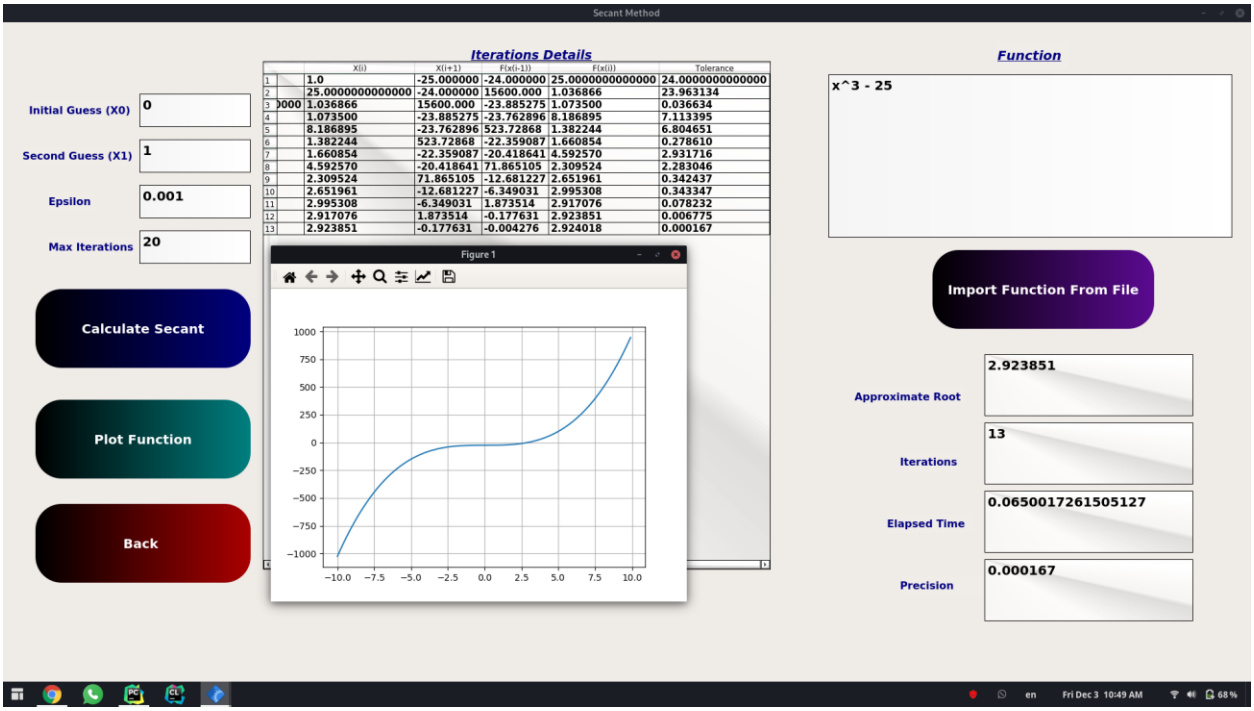
Fixed Point:


Fixed Point Method
Iterations Details
Function
Initial Guess (X0)
1
Epsilon
0.001
Max Iterations
15
Calculate Fixed Point
Plot Function
Back
x^3 - 25
Import Function From File
Figure 1
Approximate Root
2.2064775e+202 - 7.181494e+202*I
Iterations
15
Elapsed Time
0.11795902252197266
Precision
7.5128156e+202
Function Is Divergent
Fri Dec 3 11:04 AM

Newton Raphson:



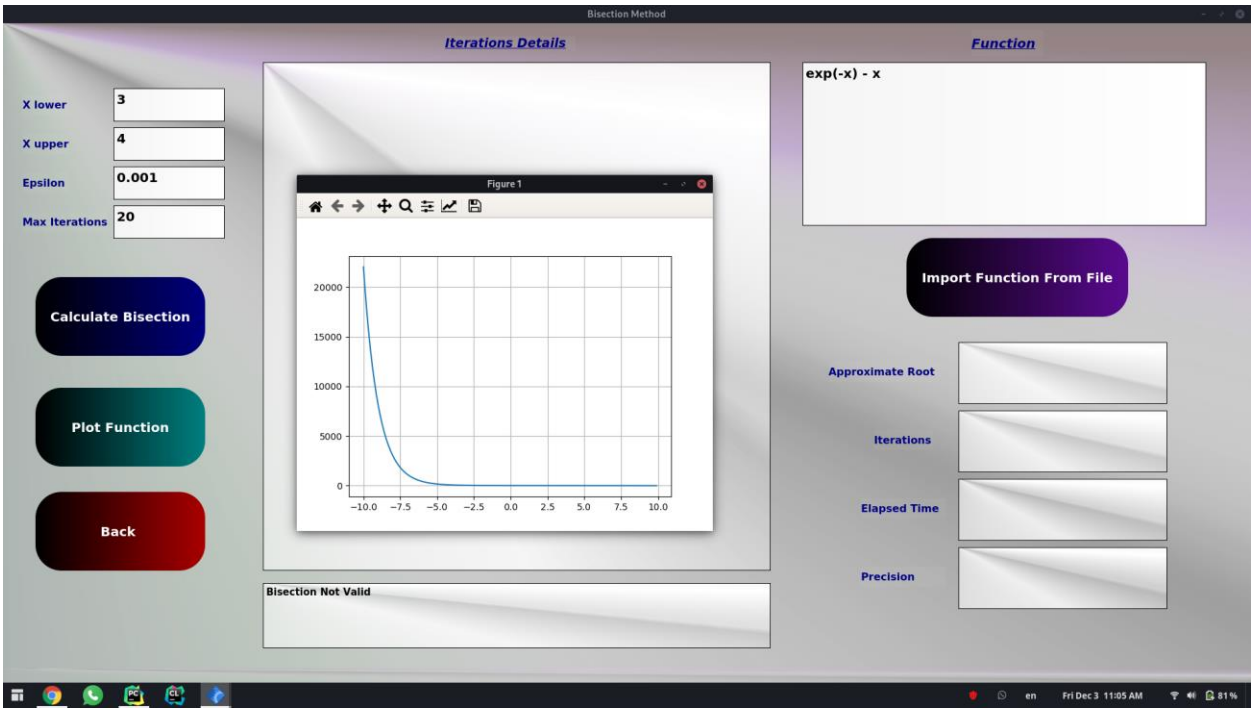
Secant:



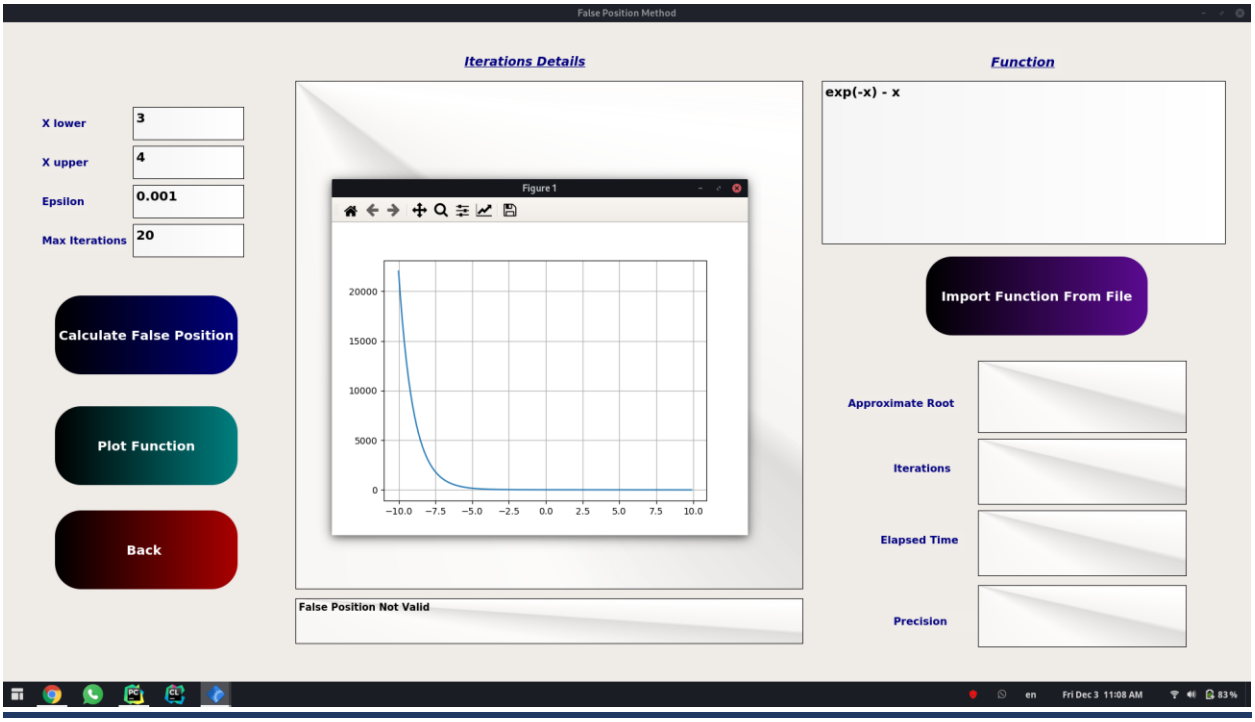
	X(i)	X(i+1)	F(x(i-1))	F(x(i))	Tolerance
1	1.0	-25.000000	-24.000000	25.0000000000000	24.0000000000000
2	25.0000000000000	-24.000000	15600.000	1.036866	23.963134
3	0000 1.036866	15600.000	-23.885275	1.073500	0.036634
4	1.073500	-23.885275	-23.762896	8.186895	7.113395
5	8.186895	-23.762896	523.72868	1.382244	6.804651
6	1.382244	523.72868	-22.359087	1.660854	0.278610
7	1.660854	-22.359087	-20.418641	4.592570	2.931716
8	4.592570	-20.418641	71.865105	2.309524	2.283046
9	2.309524	71.865105	-12.681227	2.651961	0.342437
10	2.651961	-12.681227	-6.349031	2.995308	0.343347
11	2.995308	-6.349031	1.873514	2.917076	0.078232
12	2.917076	1.873514	-0.177631	2.923851	0.006775
13	2.923851	-0.177631	-0.004276	2.924018	0.000167

Function 2:

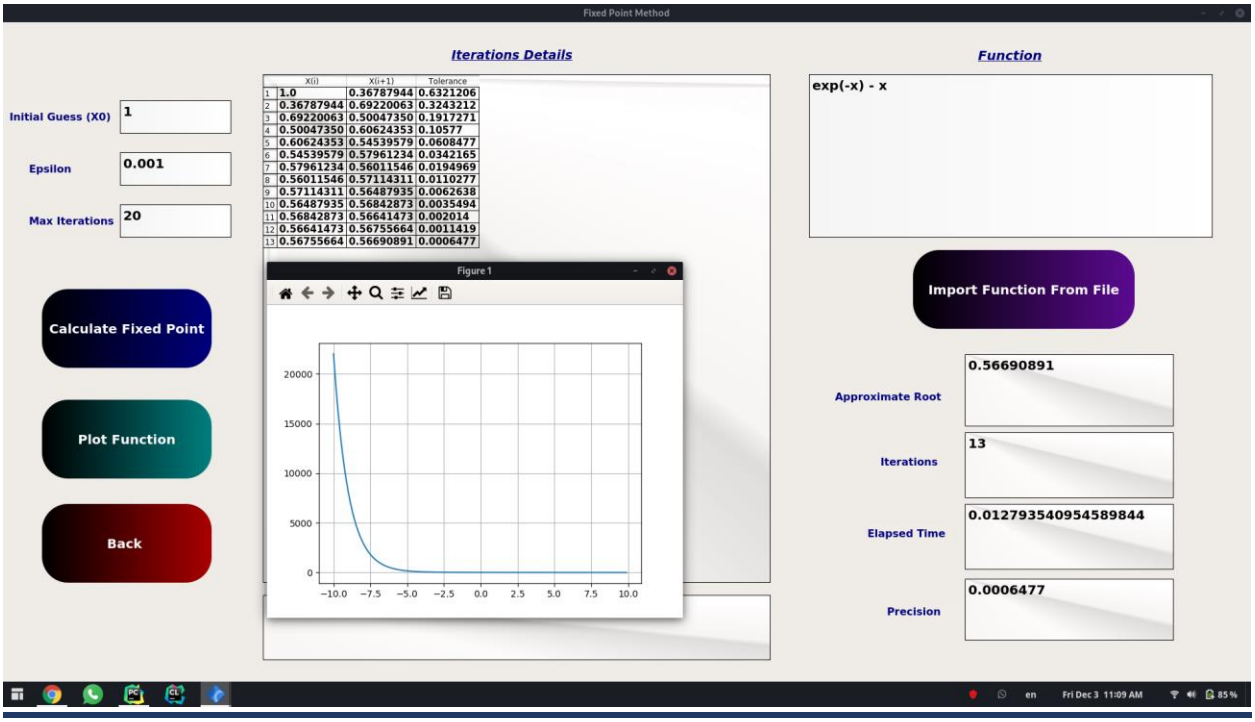
Bisection:



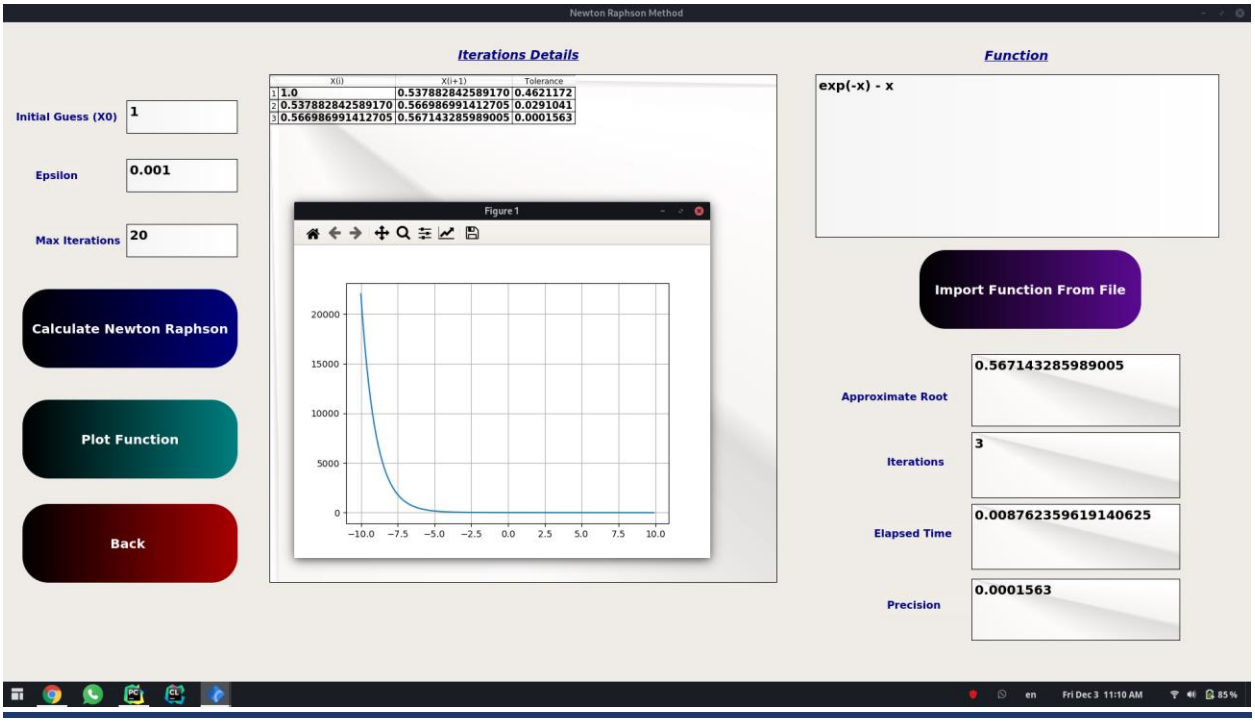
False Position:



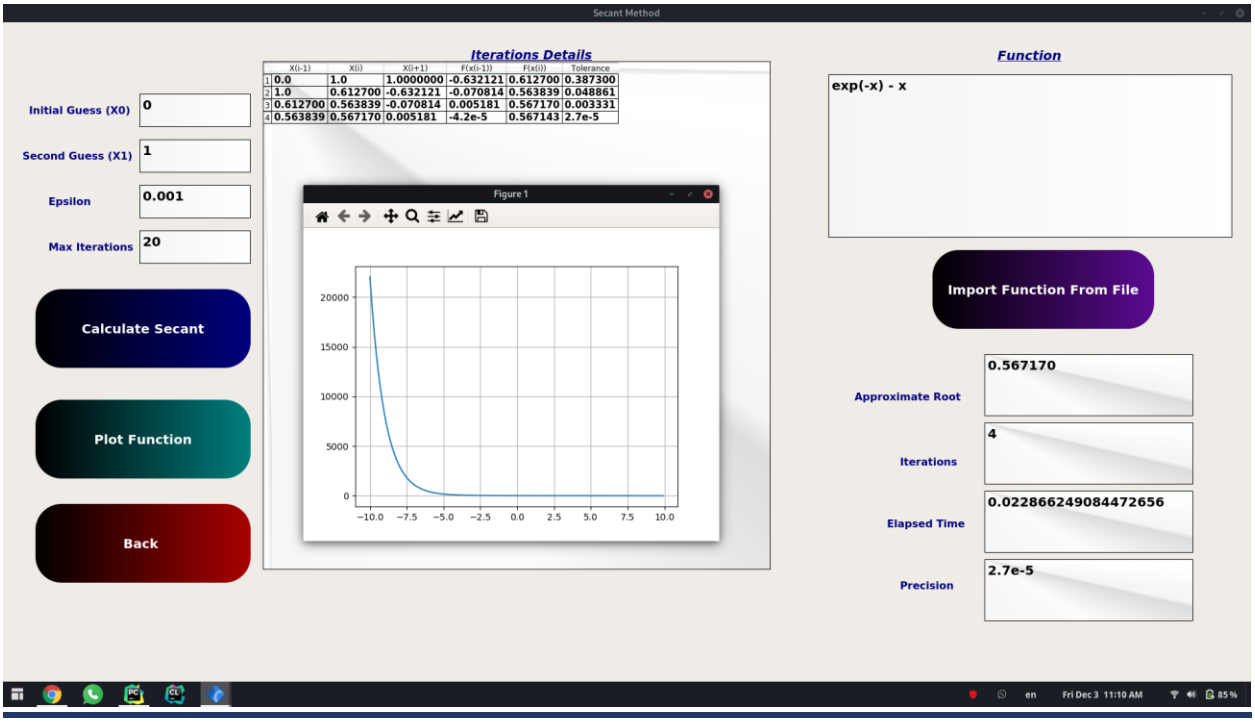
Fixed Point:


Fixed Point Method
Iterations Details
Function
Initial Guess (X0)
1
Epsilon
0.001
Max Iterations
20
Calculate Fixed Point
Plot Function
Back
X(i) X(i+1) Tolerance
1 1.0 0.36787944 0.6321206
2 0.36787944 0.69220063 0.3243212
3 0.69220063 0.50047350 0.1917271
4 0.50047350 0.60624353 0.10577
5 0.60624353 0.54539579 0.0608477
6 0.54539579 0.57961234 0.0342165
7 0.57961234 0.56011546 0.0194969
8 0.56011546 0.57114311 0.0110277
9 0.57114311 0.56487935 0.0062638
10 0.56487935 0.56842873 0.0035494
11 0.56842873 0.56641473 0.002014
12 0.56641473 0.56755664 0.0011419
13 0.56755664 0.56690891 0.0006477
Figure 1
20000
15000
10000
5000
0
−10.0 −7.5 −5.0 −2.5 0.0 2.5 5.0 7.5 10.0
exp(-x) - x
Import Function From File
Approximate Root
0.56690891
Iterations
13
Elapsed Time
0.012793540954589844
Precision
0.0006477
en
Fri Dec 3 11:09 AM
85 %

Newton Raphson:

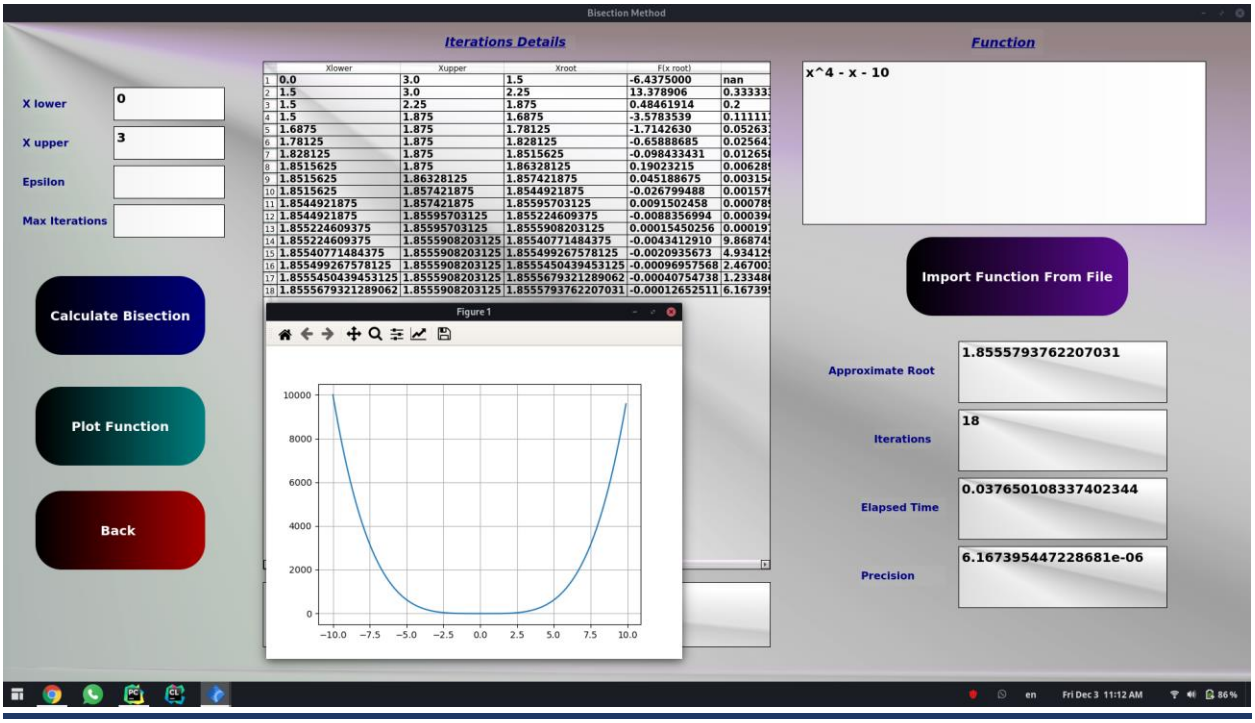


Secant:

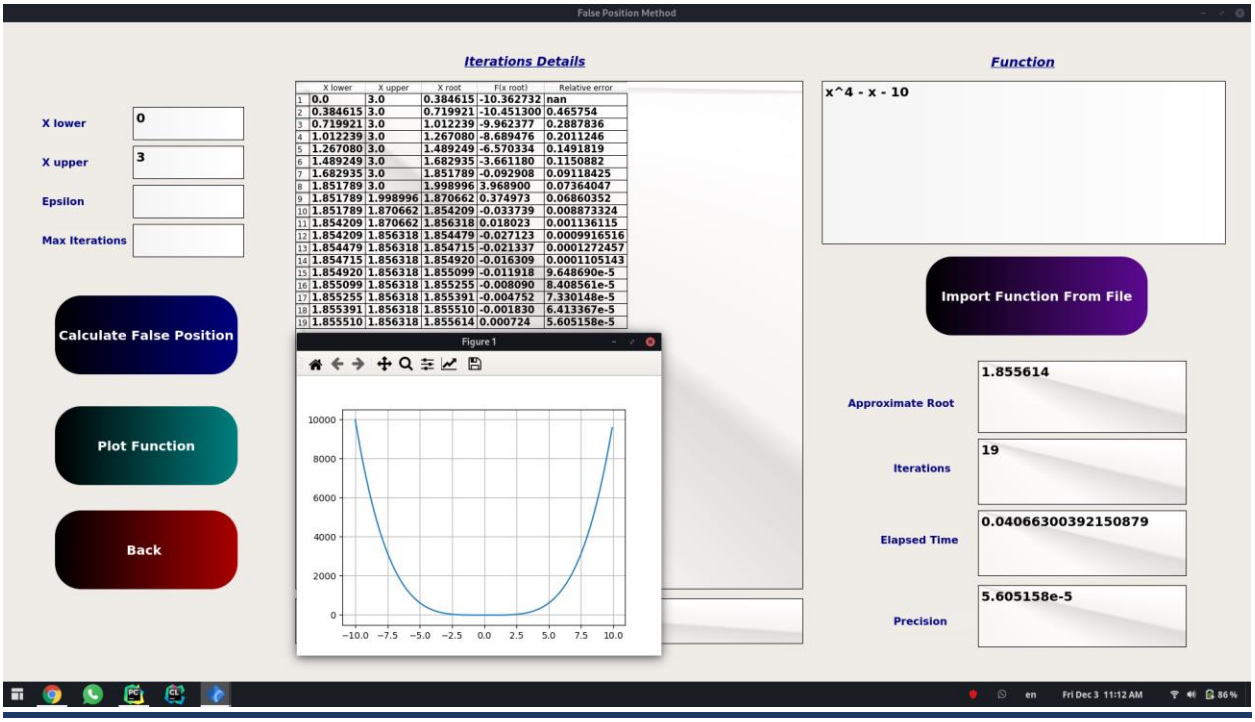

Secant Method
Iterations Details
X(i-1) X(i) X(i+1) F(xi-1)) F(xi)) Tolerance
1 0.0 1.0 1.0000000 -0.632121 0.612700 0.387300
2 1.0 0.612700 -0.632121 -0.070814 0.563839 0.048861
3 0.612700 0.563839 -0.070814 0.005181 0.567170 0.003331
4 0.563839 0.567170 0.005181 -4.2e-5 0.567143 2.7e-5
Initial Guess (X0)
0
Second Guess (X1)
1
Epsilon
0.001
Max Iterations
20
Calculate Secant
Plot Function
Back
Figure 1
20000
15000
10000
5000
0
−10.0 −7.5 −5.0 −2.5 0.0 2.5 5.0 7.5 10.0
Function
exp(-x) - x
Import Function From File
Approximate Root
0.567170
Iterations
4
Elapsed Time
0.022866249084472656
Precision
2.7e-5
en
Fri Dec 3 11:10 AM
85%

Function 3:

Bisection:

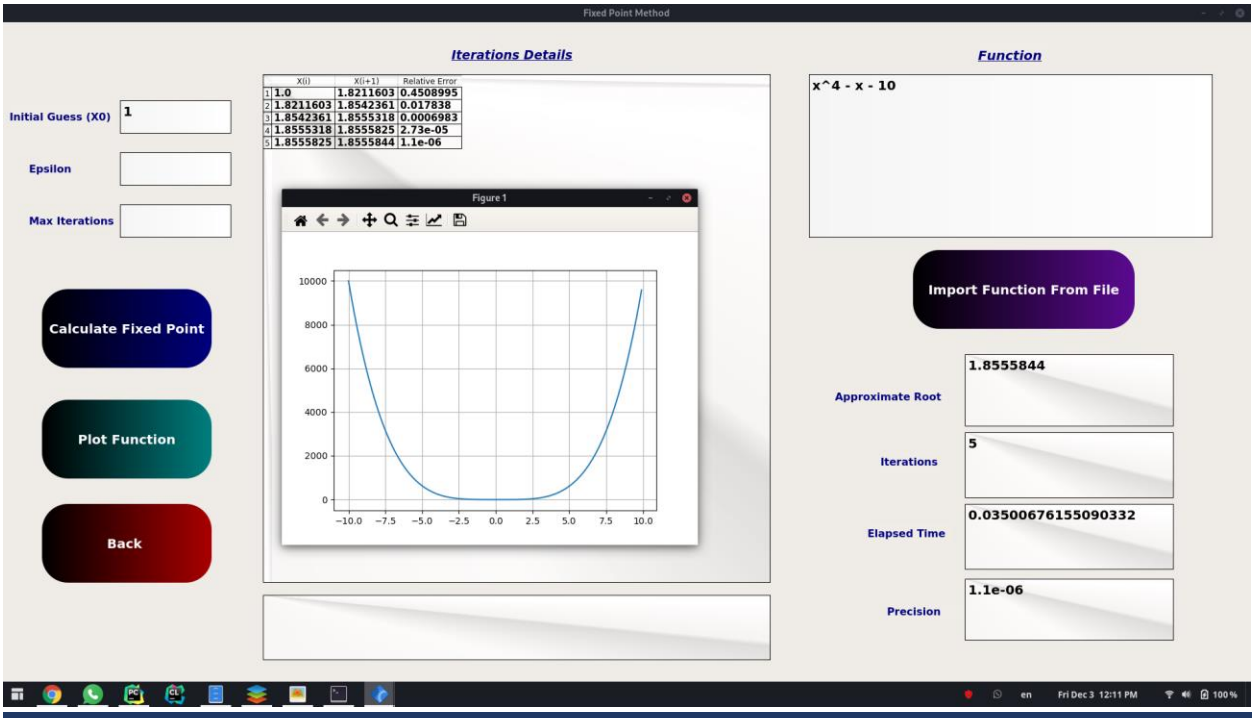


False Position:

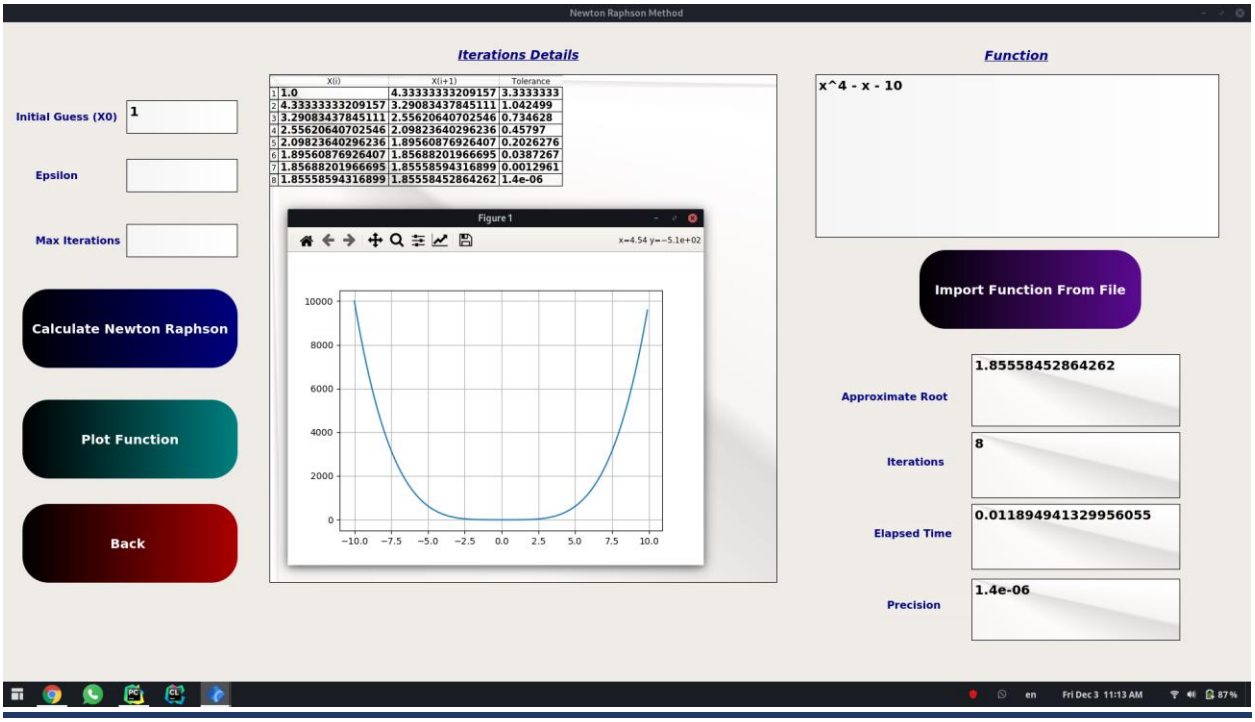


	X lower	X upper	X root	F(x root)	Relative error
1	0.0	3.0	0.384615	-10.362732	nan
2	0.384615	3.0	0.719921	-10.451300	0.465754
3	0.719921	3.0	1.012239	-9.962377	0.2887836
4	1.012239	3.0	1.267080	-8.689476	0.2011246
5	1.267080	3.0	1.489249	-6.570334	0.1491819
6	1.489249	3.0	1.682935	-3.661180	0.1150882
7	1.682935	3.0	1.851789	-0.092908	0.09118425
8	1.851789	3.0	1.998996	3.968900	0.07364047
9	1.851789	1.998996	1.870662	0.374973	0.06860352
10	1.851789	1.870662	1.854209	-0.033739	0.008873324
11	1.854209	1.870662	1.856318	0.018023	0.001136115
12	1.854209	1.856318	1.854479	-0.027123	0.0009916516
13	1.854479	1.856318	1.854715	-0.021337	0.0001272457
14	1.854715	1.856318	1.854920	-0.016309	0.0001105143
15	1.854920	1.856318	1.855099	-0.011918	9.648690e-5
16	1.855099	1.856318	1.855255	-0.008090	8.408561e-5
17	1.855255	1.856318	1.855391	-0.004752	7.330148e-5
18	1.855391	1.856318	1.855510	-0.001830	6.413367e-5
19	1.855510	1.856318	1.855614	0.000724	5.605158e-5

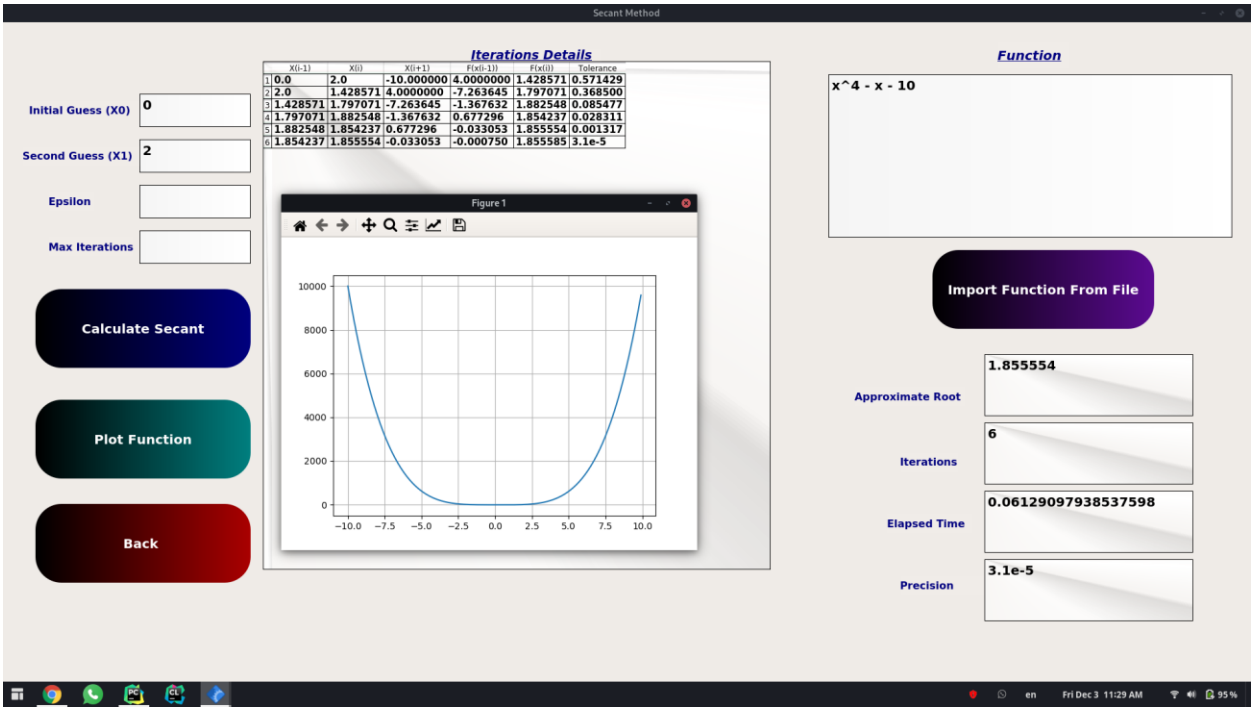
Fixed Point:


Fixed Point Method
Iterations Details
Function
Initial Guess (X0)
1
Epsilon
Max Iterations
Calculate Fixed Point
Plot Function
Back
X(i) X(i+1) Relative Error
1 1.0 1.8211603 0.4508995
2 1.8211603 1.8542361 0.017838
3 1.8542361 1.8555318 0.0006983
4 1.8555318 1.8555825 2.73e-05
5 1.8555825 1.8555844 1.1e-06
Figure 1
x^4 - x - 10
Import Function From File
Approximate Root
1.8555844
Iterations
5
Elapsed Time
0.03500676155090332
Precision
1.1e-06
en
Fri Dec 3 12:11 PM
100 %

Newton Raphson:



Secant:


Secant Method
Iterations Details
X(i-1) X(i) X(i+1) F(xi-1)) F(x(i)) Tolerance
1 0.0 2.0 -10.000000 4.0000000 1.428571 0.571429
2 2.0 1.428571 4.0000000 -7.263645 1.797071 0.368500
3 1.428571 1.797071 -7.263645 -1.367632 1.882548 0.085477
4 1.797071 1.882548 -1.367632 0.677296 1.854237 0.028311
5 1.882548 1.854237 0.677296 -0.033053 1.855554 0.001317
6 1.854237 1.855554 -0.033053 -0.000750 1.855585 3.1e-5
Function
x^4 - x - 10
Initial Guess (X0)
0
Second Guess (X1)
2
Epsilon
Max Iterations
Calculate Secant
Plot Function
Back
Figure 1
Import Function From File
Approximate Root
1.855554
Iterations
6
Elapsed Time
0.06129097938537598
Precision
3.1e-5
en
Fri Dec 3 11:29 AM
95 %

Functions could be imported from files: