Literature review of Age and Block replacement Policies

Supervised by Mr. Mehdi RADHOUI
B.Industrial Engineering - 2021
The National Engineering School of Carthage
Course: Reliability of production systems

***Abstract*— One of the most common maintenance strategies are age and block replacements, which are based on taking preventive measures to avoid a system's failure. For age replacement policy, a system is replaced when it fails or when it reaches a predefined replacement age, whichever comes first. For block replacement policy, all units are replaced periodically at times kT (k= 1, 2, ...) independent of the ages of units in use. The age of a system was once considered to be a random variable, but in the past decades, different replacement policies were proposed and also developed by many researchers. In this paper we will focus on age and block replacement policies. Two resources have been reviewed titled as follows: -**

1) **Optimum preventive maintenance policies by Barlow, R. and Hunter.**
2) **Mathematical theory of reliability, by R.Barlow, AND F.Proschan,**

The following text describes a brief overview of what these papers have reviewed for this topic.

I. Optimum preventive maintenance policies by R.Barlow and L.Hunter

A. Authors

(Richard Barlow and Larry Hunter, Operations Research 1960).

B. Overview

This research proposed two preventive maintenance policies, one of which is best for maintaining simple equipment and the other for sustaining large, complicated systems. Those policies were respectively named Policy I and Policy II. In this work, a policy is shown to be optimal if it maximizes optimum limiting efficiency. The following key terms areas formed the main interest of this paper:

- Eff_T: expected fractional amount of time system is on during [0,T].
- Eff: The limiting Efficiency of the system: used to evaluate the policy, with $Eff_\infty = lim_T - > \infty \ Eff_T$.

1) Policy I: Preventive maintenance is performed after t_0 hours of continuous counting operation. If the system fails before t_0, an emergency replacement is undertaken at the time of failure, and preventive maintenance is rescheduled for that time.

- Assumptions: This policy has been conducted upon the following assumptions:
  - t_0 can be infinite, that is to say no preventive maintenance is scheduled.
  - The system is assumed to be as good as new after a replacement is performed.
- Replacement Solution:
  - t_0 exists only if the system considered possesses an increasing failure rate, therefore this policy implies, that the system is more prone to failure with age increasing. As a result, even if a unit is still functional, it is reasonable to replace it after a determined period of time.
  - However, the optimal t_0 for this policy is shown to be depending only on the ratio T_e/T_s.
    - ∗ With T_e is the expected time to perform emergency maintenance.
    - ∗ With T_s is the expected time to perform scheduled maintenance.
- Cost Solution : T_e and T_s are then considered to be mean cost to perform emergency maintenance and scheduled maintenance respectively called C_e and C_s
  - To be noted that the same formulas for average times when replaced by costs to

repair, give the minimum cost solution.
  - Therefore Age replacement is only significant if $C_e > C_s$.

2) Policy II: Preventive maintenance is performed once the system has been operational for a total of t^* hours, regardless of the number of failures.

- Assumptions: This policy has been conducted upon the following assumptions:
  - Only minor repairs are made after each failure.
  - The system's failure rate is unchanged after minor repairs.
  - The system is prone to failure after replacement as before failure, due to the aging of other components.
  - The system is assumed to be as good as new after after a replacement is performed.
- Replacement solution:
  - t^* exists only if the system considered possesses an increasing failure rate, therefore this policy implies, that the system is more prone to failure with age increasing. As a result, even if a unit is still functional, it is reasonable to replace it after a determined period of time.
  - However the optimal t^* for this policy is shown to be depending only on the ratio T_s/T_m.
    * With T_m is the expected time to perform minimal repair.
    * With T_s is the expected time to perform scheduled maintenance.
- Cost solution : T_m and T_s are then considered to be mean cost to perform minimal repair and scheduled maintenance respectively called C_m and C_s.
  - To be noted that the same formulas for average times when replaced by costs to repair, give minimum cost solution.

C. Policy Comparison:

Finally the paper compared the two policies and showed that under reasonable restrictions these policies have unique solutions that can be evaluated and therefore comparing their efficiencies, to do so a knowledge of these policies parameters is required.

II. Mathematical theory of reliability, by R.Barlow, AND F.Proschan,

A. Authors

(Richard E. Barlow, Frank Proschan, 1965).

B. Overview

This book presents a survey of mathematical models and discussed how life distributions are used to determine maintenance policies.The following key sections formed the main interest of this book:

- Operating Characteristics of Maintenance Policies.
- Comparison of replacement policies.
- Optimum Replacement Policies.

1) Operating Characteristics of Maintenance Policies:

- The following metrics were proposed to evaluate the replacements policies:
  - The distribution of the number of failures.
  - The distribution of the total number of removals.
  - The expected time to an in-service failure.

  This book presented a summary of the most relevant aspects in Renewal theory and highlighted its most important results.

2) Comparison of Age and Block replacement policies:

- It was shown that assuming an IFR unit failure distribution, the number of failures in [0, t] is stochastically larger under an age policy than under a block policy. Furthermore, the number of planned replacements and the overall number of removals is always stochastically smaller under an age policy than under a block policy. If failure distribution is IFR:

$$\mathrm{P}[\mathrm{N(t)} \geq n] \geq P[\ \mathrm{N}_A\text{*(t)} \geq n] \geq P[\mathrm{N}_B\text{* (t)} \geq n]$$

- Noting that the number of renewals in [0, t] when replacement occurs only at failure is N(t).

- The number of failures in [0, t] under a block policy is N_B*(t) and under an age policy is N_A*(t), both having replacement interval T.

$$E[N_B(t)] \geq E[N_A(t)]$$

- Noting that the number of renewals in [0, t] under Block respectively under age policy are N_B(t) and N_A(t).

Whereas Block replacement doesn't require keeping records of components use, it is more wasteful since it was shown that more unfailed components are being replaced under this policy.

3) Optimum Replacement Policies:

- In order to specify a replacement policy which balances the cost of failures of units alongside the cost of planned replacement, different replacement policies were compared.
- Let the expected cost C(t) be :

  $$C(t)=C_1.\ EN_B(t)+\ C_2\ .EN_A(t)$$

  Noting that:
  - N_1(t): Number of failures during [0,t]
  - N_2(t): Number of exchanges of non failed items during [o,t].
  - C1 and C2 denote respectively the cost for each failed item replaced and the cost of each non failed item exchanged.
- For age replacement policy: The main idea was to determine the replacement interval minimizing the expected cost per unit of time. Therefore the following assumptions were made for all replacement models:
  - The completion times of all interventions are neglected.
  - All interventions are perfectly executed.
  - The equipment is assumed to be as good as new after a replacement is performed.
  - For each intervention all necessary resources are always available.
  - All costs related to each intervention are assumed to be fixed and constant.
- Let A(G) be the average cost per unit of time over a long interval:

  $$C_1F(t)+\ C_2\ R(t)\ /\ \int_0^T x.f(x)\,dx \qquad (1)$$

  Noting that:
  - R(t) represents the probability that the equipment is still operating at time t
  - F(t) represents the probability that the equipment will fail before time t.
  - Since this quotient is considered continuous for $x > 0$, the minimum of this function exists and if the distribution F is having an Increasing Failure Rate, this minimum is unique.
  - The following results were demonstrated as-well:
    * Optimum age replacement policy is nonrandom.
    * Optimum replacement intervals are finite.
    * If the Failure distribution is continuous, there exists a minimum cost age replacement policy for any finite time span during [0,t].
  - For Block replacement policy, let the expected cost B(t) per unit of time following a block replacement policy at interval T over an infinite time given by:

    $$C_1M(t)+\ C_2\ \ /\ T \qquad (2)$$

    Noting that:
    * M(t) is the renewal function (the expected number of renewals) given by:

      $$M(t) = \sum_{i=1}^{\infty} F^i(T)$$

  - The following results were demonstrated:
    * If the Failure distribution is continuous, there exists a minimum cost for block replacement policy for any finite time span during [0,t].

III. CONCLUSIONS

The purpose of this review was to define the different replacement policies and the different assumptions taken into account to model the Cost function and to represent the optimal solution. Along with this, different figure of merits were proposed to evaluate the replacement policies such as: the expected number of failures, the expected number of planned replacements and the probability of non failure. As the trade-off between productivity and cost for preventive maintenance planning is still persisting, the right decision of Corrective vs. preventive planning is still being debated. To conclude, these results paved the way for renewal theory to determine optimal maintenance policies and to constitute different modeling approaches.

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