CS323: Theory of Computation

Lecture 1

Introductory Lecture

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Google classroom

- Theory of Computations (General)
  - https://classroom.google.com/c/NjYzODA2MTYzNTkw?cjc=jt6zoej
  - Code: jt6zoej
- Theory of Computations (Special)
  - https://classroom.google.com/c/NjYzODA2NDcxODY5?cjc=xq7ojfx
  - Code: xq7ojfx

Syllabus and Terminologies

- Regular Languages .. Regular Sets
  - REs (Regular Expressions)
  - FSMs (or FSA/FA) … Finite State Machines/Automata
    - DFA vs. NFA … Deterministic vs. Non-deterministic FSA
      - Comparison and conversion
    - Examples & Closure Operations
    - Pumping Lemma
- Context Free Languages
  - CFGs … Context Free Grammars
  - PDA … Push Down Automata
  - Parsing: CFG generating strings vs. PDA recognizing strings
- Turing Machine

Resources

Text Books:

- "***Introduction to Computer Theory***", 2^{nd} Edition, by *Daniel I.A. Cohen, John Wiley & Sons.*

- "***An Introduction to Formal Languages & Automata***" by Peter Linz, Jones & Bartlett Publishers, 3rd edition, Inc. January 2001; ISBN: 0763714224.

- "***Theory of Computation an Introduction***" by James L. Hein, Jones & Bartlett Publishers 1996; ISBN: 0-86720-497-4.

Course Grading Policy

Midterm Exam	20
Lab Tasks & Practice	20
Final	60
Total	100

Lab Activities

- ***Theoritical:*** *Problem Solving*
- ***Practical:*** Programming practice on:
  - REs ... Text Matching
  - FSAs ... Simulation of operations
  - PDAs ... Simulation of operations
  - Turing Machine ... Simulation of operations

Why to study ***Theory of Computation***?

- It is the most abstract course in the whole curriculum
- But a fundamental course for all CSians
- This course is not about how to develop a program or build a computer
- It helps you know a lot more about how people looked to CS *as a science* in the past 70 years

Why to study ***Theory of Computation***?

- It is about:
  - What kind of things can we really compute?
  - How fast we can do it? and
  - How much space/memory it needs to be computed?

Why to study *Theory of Computation*?

- This course is about different models of computations you can use to compute stuff
- But ***abstracted*** from the hardware/machine specifications
- We will deal with kinds of problems that takes ***inputs***, and ***process it***, then ***decide*** whether these inputs are ***acceptable*** or not
- Inputs are set of symbols formed by an alphabet (binary, English alphabet, ...)

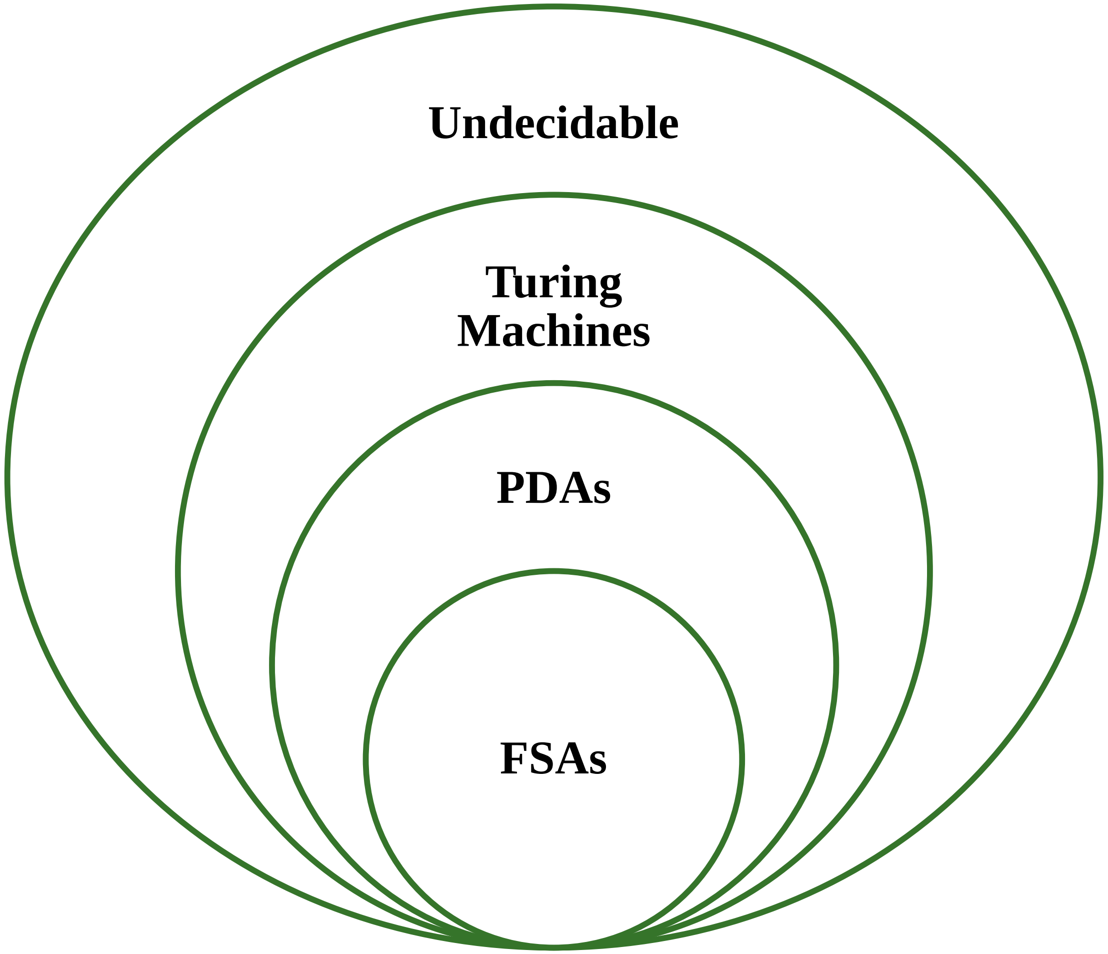
Why to study *Theory of Computation*?

- There are a lot of applications that connect to this course:
  - Theory behind writing *compilers/translators* for programming languages
  - *Computer Architecture* → model particular processes/states using finite-state-machines (FSMs)
  - *String search* algorithms, *word processing* algorithms, any kind of editors → that is a RE/FSM
  - When you do a representation of a language like XML, it is just a *grammar-writing*

Turing Machine

- Turing Machine invented by *Alan-Turing* in 1940's
- It is an abstraction of how programming languages and computers work nowadays
- He invented a *mathematical abstraction* of a complete representation of how might we can do a computation
- Any problem that can be computed by a program on a computer, it can be *computed by* (*simulated on*) a Turing Machine

Hierarchy of Machines



Turing Machine

- Turing Machine is like a *human with organs*, cutting one organ (going down into lower machine) will make it handicap, or taking higher time and resources
- The lower level is the FSM, and then add up pieces to form more powerful machines (till getting a Turing Machine)
- Then you will get into the *twilight zone* that has problems can't be computed

Definition of a language "L"

$\sum$ is the set of alphabets:

Language = set of strings can be formed by $\sum$

- If $\sum$ = Binary digits (0/1)
  - Language = all binary strings
- If $\sum$ = English characters
  - Language = English sentences
- If $\sum$ = ASCII character-set
  - Language = C++ Programs

Definition of a language "L"

- Language: is a **set** of strings.

  - *If A is an alphabet, then a language over A is a collection of all strings over A.*

  - *A* is the biggest possible language over A, and every other language over A is a **subset** of A*.*

- ***Example***

  If A={a} then the following are simple examples of languages over an alphabet A:

  {Φ}, {λ}, {a}, and **A*** *={λ, a, aa, aaa, ………}.*

Note: λ, Λ, **ε** are used interchangeably in textbooks

Example

Let the alphabet be $\sum$ = {a, b } then,

- *{Φ} = Empty Language*
- *{λ} = Empty Character*

" "

null string

- *{a} = Letter 'a'*
- *{b} = Letter 'b'*
- **a*** *={λ, a, aa, aaa, ………}*
- **$\sum$* = {λ, a, b, aa, ab, ba, bb, aaa, bbb, …}**

```
i=0;
cin>>n;
while(i<n)
{
  cout<<'a';
  i++;
}
```

- **$\sum^+ = \sum^* - \{\lambda\}$**
- **L_1 = { a, aa, bbb}** **finite**
- **$L_2 = \{a^n b^n : n >= 0\}$** **infinite**
- **$\overline{L_1} = \sum^* - L_1$**

Definition of a language "L"

A Language is a set of strings

- alphabet → language
  - $\Sigma = \{x\}$ $\Sigma^* = \{\Lambda, x, xx, xxx, \ldots\}$

    or directly $\{x\}^* = \{\Lambda, x, xx, xxx, \ldots\}$
- language → language
  - $S = \{xx, xxx\}$ $S^* = \{\Lambda, xx, xxx, xxxx, \ldots\}$

    or directly $\{xx, xxx\}^* = \{\Lambda, xx, xxx, xxxx, \ldots\}$
- "letter" → language
  - $\mathbf{x^*}$ (written in bold)

    language($\mathbf{x^*}$) $= \{\Lambda, x, xx, xxx, \ldots\}$

    or informally $\mathbf{x^*} = \{\Lambda, x, xx, xxx, \ldots\}$

Regular Languages/Sets … RLs/RSs

- It is a language that:
  - Can be governed/expressed by rules called ***Regular Expressions*** (REs).
  - or
  - Has a *finite accepter/automaton/machine* that can describe/accept it.

Theorem

$$\left\{\begin{array}{l}\text{Languages} \\ \text{Generated by} \\ \text{Regular Expressions}\end{array}\right\} = \left\{\begin{array}{l}\text{Regular} \\ \text{Languages}\end{array}\right\}$$

Properties of Regular Languages

For regular languages L_1 and L_2 we have

$$\left.\begin{array}{rl} \text{Union:} & L_1 \cup L_2 \\ \text{Concatenation:} & L_1 L_2 \\ \text{Star:} & L_1 * \end{array}\right\} \text{Are regular Languages}$$

Properties of Regular Languages

For regular languages L_1 and L_2 we have

Union: $L_1 \cup L_2$

$L_4 = \{a, bc, x, yz\}$

$L1 = \{a, bc\}$
$L2 = \{x, yz\}$

Concatenation: $L_1 L_2$

$L3 = \{ax, ayz, bcx, bcyz\}$

$\{\lambda, a, ab, aa, abab, aab, \ldots\}$

Star: $L_1{}^*$

$L1 = \{a, ab\}$

Regular Expressions

- RE is a **syntax** for describing simple/regular languages and patterns.
- REs are also used in applications that involve file parsing and text matching.
- Many implementations have been made for RE.

RE Rules

1. Let $\sum = \{x\}$, then $L = \sum^*$

In RE, we write x^*

2. Let $\sum = \{x, y\}$, then $L = \sum^*$

In RE, we write $(x+y)^*$

3. Kleene's star "*" means any combination of letters of length zero or more.

$* = 0$ $\quad$ $\underline{\text{output strings}}$ $\quad \lambda$

$* = 1$ $\quad \{x\}$ or $\{y\}$

$* = 2$ $\quad \{xx, xy, yx, yy\}$

$* = 3$ $\quad \{xxx, xxy, \ldots\}$

RE Rules

Given an alphabet ∑, the set of regular expressions is defined by the following rules.

1. For every letter in ∑, the letter written in bold is a regular expression. **Λ-lamda or ε-epsilon** is a regular expression. Λ **or ε** means an empty letter.
2. If $\mathbf{r_1}$ and $\mathbf{r_2}$ are regular expressions, then so are:
   1. $\mathbf{(r_1)}$
   2. $\mathbf{r_1 r_2}$ **and**
   3. $\mathbf{r_1+r_2}$ **or**
   4. $\mathbf{r_1^*}$ **and also** $\mathbf{r_2^*}$

NOTE 1: r_1^+ is not a RE

NOTE 2: spaces are ignored as long as they are not included in ∑

3. Nothing else is a regular expression.

Recursive Definition

Primitive regular expressions: $\emptyset, \quad \lambda, \quad a$

Given regular expressions r_1 and r_2

$$\left.\begin{array}{l} r_1 + r_2 \\ r_1 \cdot r_2 \\ r_1 * \\ (r_1) \end{array}\right\} \text{Are regular expressions}$$

Examples

A regular expression: $(a+b\cdot c)^{*}\cdot(c+\varnothing)$

Not a regular expression: $(a+b+)$

Languages of Regular Expressions

- $L(r)$: language of regular expression r

$a^* = \{\lambda, a, aa, aaa, aaaa, \ldots\}$

- Example: $(a+bc).(a+bc)$
  - If $\boldsymbol{r = (a + b \,.\, c)^*}$ $= \{\lambda, a, bc, aa, abc,$

$bc.bc, bca$

$$L((a+b\cdot c)^*) = \{\lambda, a, bc, aa, abc, bca, \ldots\}$$

Regular Expressions

- Regular expressions
  - describe regular languages

- Example:

  Describe the language $(a+b\cdot c)^*$

$$\{a,bc\}^* = \{\lambda, a, bc, aa, abc, bca, ...\}$$

Definition (continued)

- For regular expressions r_1 and r_2

$$L(r_1 + r_2) = L(r_1) \cup L(r_2)$$

$$L(r_1 \cdot r_2) = L(r_1)\, L(r_2)$$

$$L(r_1 *) = (L(r_1))*$$

$$L((r_1)) = L(r_1)$$

Example

- Regular expression: $(a+b)\cdot a*$

$$
\begin{aligned}
L\left(\underbrace{(a+b)}_{r_1}\cdot \underbrace{a*}_{r_2}\right) &= L((a+b))\,L(a*)\\
&= L(a+b)\,L(a*)\\
&= (L(a)\cup L(b))\,(L(a))*\\
&= (\{a\}\cup\{b\})\,(\{a\})*\\
&= \{a,b\}\{\lambda,a,aa,aaa,\ldots\}\\
&= \{a,aa,aaa,\ldots,b,ba,baa,\ldots\}
\end{aligned}
$$

Example

- Regular expression $r = (aa)^*(bb)^*b$

- What is the regular languages that r describes?

$$L(r) = \{\underline{a^{2n}b^{2m}b} : \quad n, m \geq 0\}$$

$\cancel{(ab)^*(ba)^*}$

abab...

$(aabb)^*$

aabb aabb.....

Equivalent Regular Expressions

- Definition:

Regular expressions r_1 and r_2

are **equivalent** if

$$L(r_1) = L(r_2)$$

Example

L = {all strings with no two consecutive 0's }

$$r_1 = (1 + 01)^*(0 + \lambda)$$

$$r_2 = (1^*011^*)^*(0 + \lambda) + 1^*(0 + \lambda)$$

$$L(r_1) = L(r_2) = L \Longrightarrow$$

r_1 and r_2 are equivalent regular expr.

Example

- Regular expression $r = (0+1)^*00(0+1)^*$

$L(r)$ = { all strings with at least two consecutive 0's }

RE Examples

RE-1

- Example 1

$\sum = \{a, b\}$

- Formally describe all words with <u>a</u> followed by any number of <u>b</u>'s

$L = a\, b^* = ab^*$

- Give examples for words in L

{a ab abb abbb …..}

RE-1

- Example 1

$\sum = \{a, b\}$

- Formally describe all words with a followed by any number of b's

$L = a\ b^* = ab^*$

- Give examples for words in L

{a ab abb abbb …..}

RE-2

- Example

$\sum = \{a, b\}$

- Formally describe all words with a followed by one or more b's

$L = \cancel{a\,b}^{+} = abb^{*}$

- Give examples for words in L

{ab abb abbb …..}

RE-3

- Example

$\sum$ ={a, b, c}

- Formally describe all words that start with an <u>a</u> followed by any number of <u>b</u>'s and then end with <u>c</u>.

$L = a\, b^{*} c$

- Give examples for words in L

{ac abc abbc abbbc …..}

RE-4

- Example

$\sum$ ={a, b}

- Formally describe the language that contains nothing and contains words where any <u>a</u> must be followed by one or more <u>b</u>’s

L = b*(abb*)* OR (b+ab)*

- Give examples for words in L

{Λ ab abb abababb ….. b bbb…}

RE-5

- Example

$\sum = \{a, b\}$

- Formally describe all words where a's if any come before b's if any.

$L = a^* b^*$

- Give examples for words in L

$\{\Lambda\ a\ b\ aa\ ab\ bb\ aaa\ abb\ abbb\ \ bbb\ldots..\}$

NOTE: $a^* b^* \neq (ab)^*$

because first language does not contain abab but second language has.

Once single b is detected then no a's can be added

RE-6

- Example

$\sum = \{a\}$

- Formally describe all words where count of a is odd.

L = $a(aa)^*$ OR $(aa)^*a$

- Give examples for words in L

{a aaa aaaaa …..}

RE-7.1

- Example

$\sum = \{a, b, c\}$

- Formally describe all words where single <u>a</u> or <u>c</u> comes in the start then odd number of <u>b</u>'s.

$L = (a+c)b(bb)^*$

- Give examples for words in L

{ab cb abbb cbbb …..}

RE-7.2

$a(bb)^*b + c(bb)^*$

- Example

$\sum$ ={a, b, c}

- Formally describe all words where single a or c comes in the start then odd number of b's in case of a and zero or even number of b's in case of c.

L = $ab(bb)^*+c(bb)^*$

- Give examples for words in L

{ab c abbb cbb abbbbb …..}