Cairo University
Faculty of Computers & Artificial Intelligence
Theory of Computations

# Lab#8

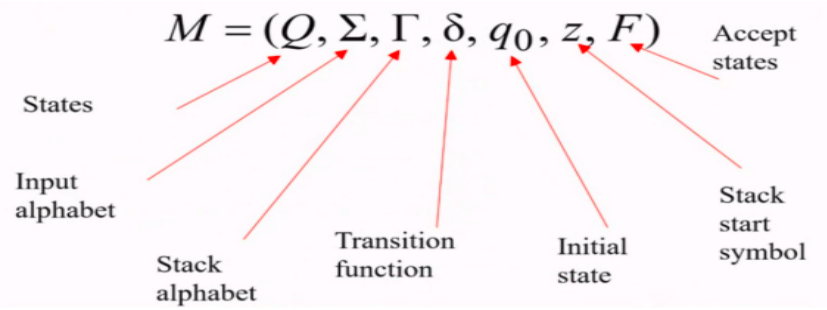
Push Down Automata

## Push Down Automata (PDA) :

It is a type of computational model that is used to recognize context-free grammar.
These automata are like nondeterministic finite automata but have an extra component called a stack.

## Formal definition of PDA:

A pushdown automaton is a 6-tuple (Q, Σ, Γ, δ, q0, F), where Q, Σ, Γ, and F are all finite sets, and
1. Q is the set of states,
2. Σ is the input alphabet,
3. Γ is the stack alphabet,
4. δ : Q × Σε × Γε-⟶P(Q × Γε) is the transition function,
5. q0 ∈ Q is the start state, and
6. F ⊆ Q is the set of accept states.




## Practice:

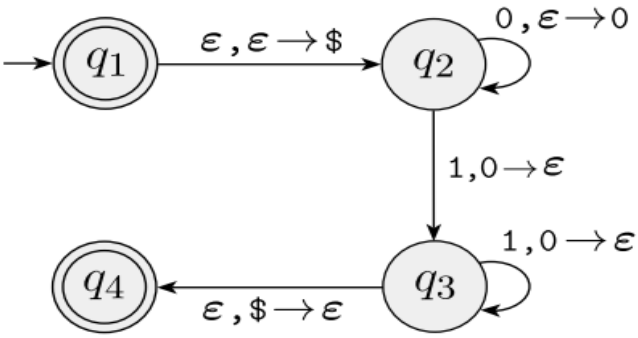
Design a PDA for accepting the following language and validate your answer using accepted and rejected string

1. $\{0^n1^n | n \geq 0\}$*
2. $\{a^nb^{2n} | n>=1\}$
3. $\{0^n1^m0^n | m, n>=1\}$
4. $\{a^ib^jc^k | i, j, k >= 0$ and i = j or i = k$\}$
5. $\{a^nb^mc^k | n,m,k>0$ && n+m=k$\}$*
6. $\{ww^R | w \in \{0,1\}^*\}$

* already covered in the lecture

## Solution:

1.



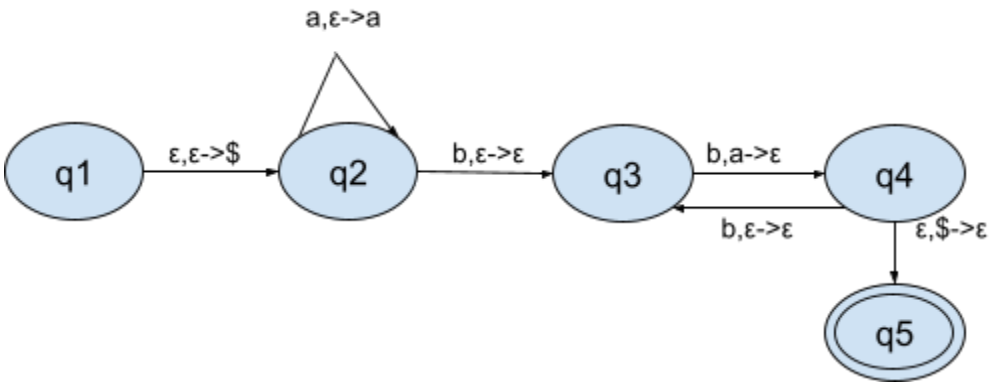

Accepted String: 01

| State | String | Stack |
|---|---|---|
| q1 | 01 | $ |
| q2 | 1 | $0 |
| q3 | ε | $ |
| q4 (Accepted State) | ε | ε |

Rejected String: 011

| State | String | Stack |
|---|---|---|
| q1 | 011 | $ |
| q2 | 11 | $0 |
| q3 | 1 | $ |

2.



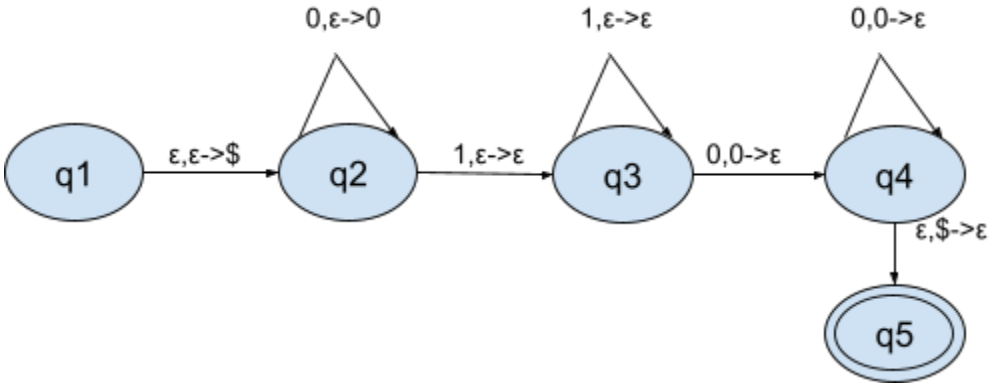

Accepted String:abb

| State | String | Stack |
|---|---|---|
| q1 | abb | $ |
| q2 | bb | $a |
| q3 | b | $a |
| q4 | ε | $ |
| q5 (Accepted State) | ε | ε |

Rejected String: ab

| State | String | Stack |
|---|---|---|
| q1 | ab | $ |
| q2 | b | $a |
| q3 | ε | $a |

3.



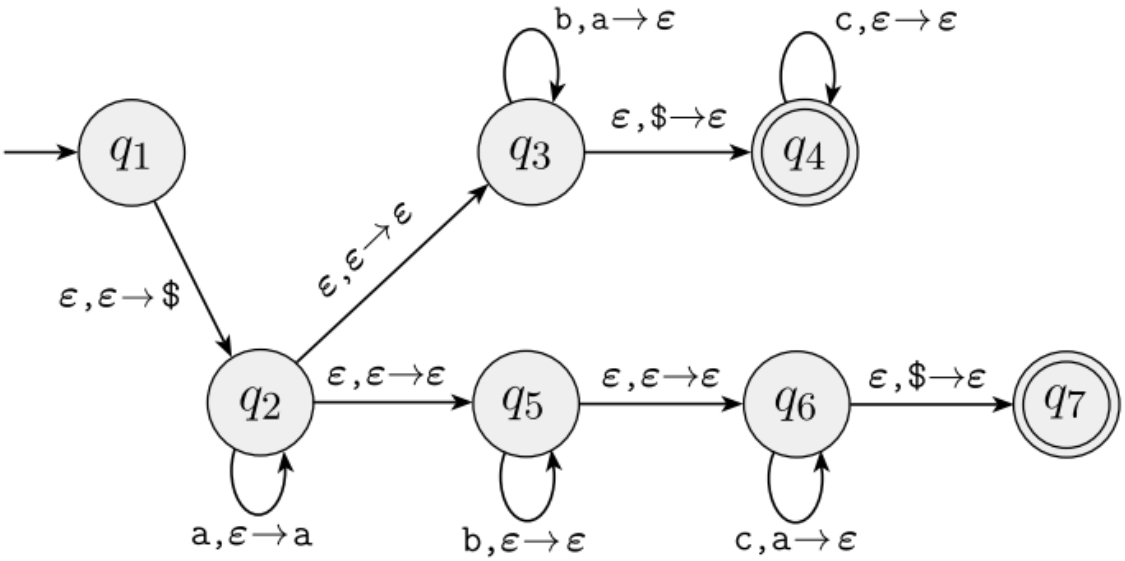

Accepted String: 010

| State | String | Stack |
|---|---|---|
| q1 | 010 | $ |
| q2 | 10 | $0 |
| q3 | 0 | $0 |
| q4 | ε | $ |
| q5 | ε | ε |

Rejected String: 011

| State | String | Stack |
|---|---|---|
| q1 | 011 | $ |
| q2 | 11 | $0 |
| q3 | 1 | $0 |
| q3 | ε | $0 |

4.



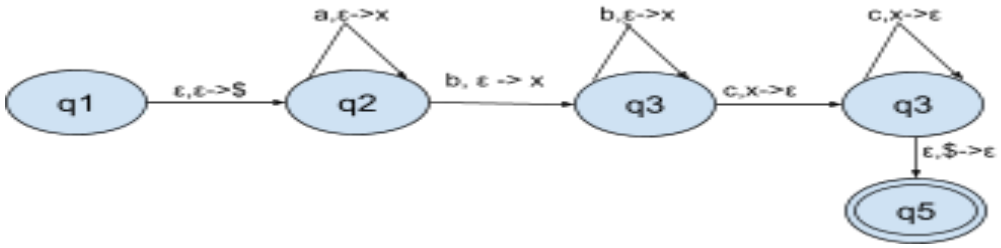

Accepted String:abc

| State | String | Stack |
|---|---|---|
| q1 | abc | $ |
| q2 | bc | $a |
| q3 | bc | $a |
| q3 | c | $ |
| q4 | c | ε |
| q4 | ε | ε |

Rejected String: aac

| State | String | Stack |
|---|---|---|
| q1 | acc | $ |
| q2 | cc | $a |
| q5 | cc | $a |
| q6 | cc | $a |
| q6 | c | $ |

5.



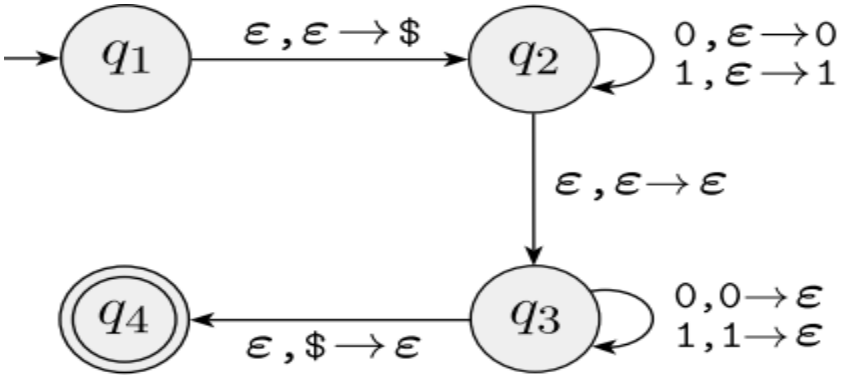

Accepted String:abcc

| State | String | Stack |
|---|---|---|
| q1 | abcc | $ |
| q2 | bcc | $x |
| q2 | cc | $xx |
| q3 | c | $x |
| q3 | ε | $ |
| q4 (Accepted state) | ε | ε |

Rejected String: abc

| State | String | Stack |
|---|---|---|
| q1 | abc | $ |
| q2 | bc | $x |
| q2 | c | $xx |
| q3 | ε | $x |

6.




Accepted String:0110

| State | String | Stack |
|---|---|---|
| q1 | 0110 | $ |
| q2 | 110 | $0 |
| q2 | 10 | $01 |
| q3 | 10 | $01 |
| q3 | 0 | $0 |
| q3 | ε | $ |
| q4 (Accepted state) | ε | ε |

Rejected String: 01

| State | String | Stack |
|---|---|---|
| q1 | 01 | $ |
| q2 | 1 | $0 |
| q2 | ε | $01 |

# Context Free Grammar to Push Down Automata:

Lemma 2.21:

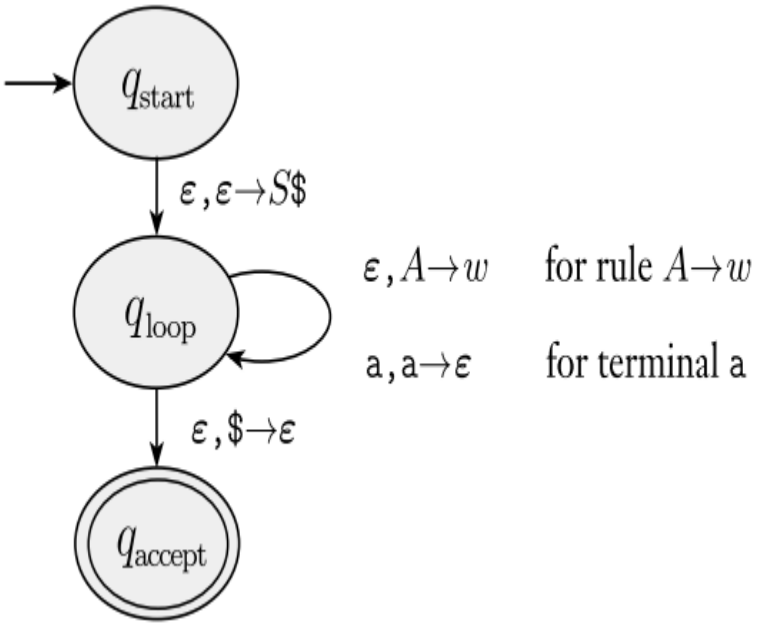
If a language is context free, then some pushdown automaton recognizes it.

Steps:

The following is an informal description of $P$.

1. Place the marker symbol \$ and the start variable on the stack.
2. Repeat the following steps forever.
   a. If the top of stack is a variable symbol $A$, nondeterministically select one of the rules for $A$ and substitute $A$ by the string on the right-hand side of the rule.
   b. If the top of stack is a terminal symbol $a$, read the next symbol from the input and compare it to $a$. If they match, repeat. If they do not match, reject on this branch of the nondeterminism.
   c. If the top of stack is the symbol \$, enter the accept state. Doing so accepts the input if it has all been read.

Illustrated in:



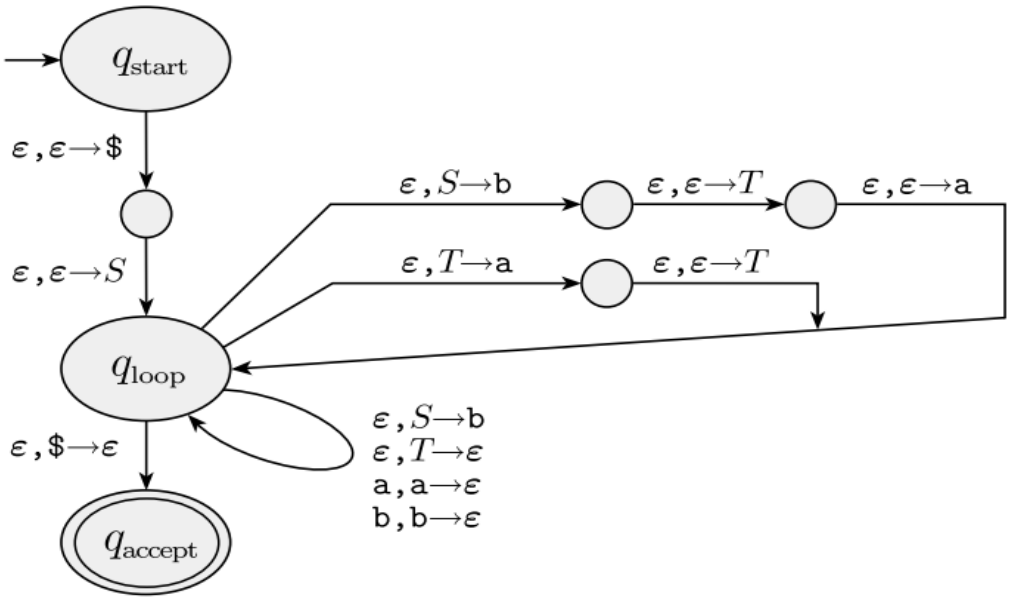
## Practice:

Convert the following CFG to PDA:

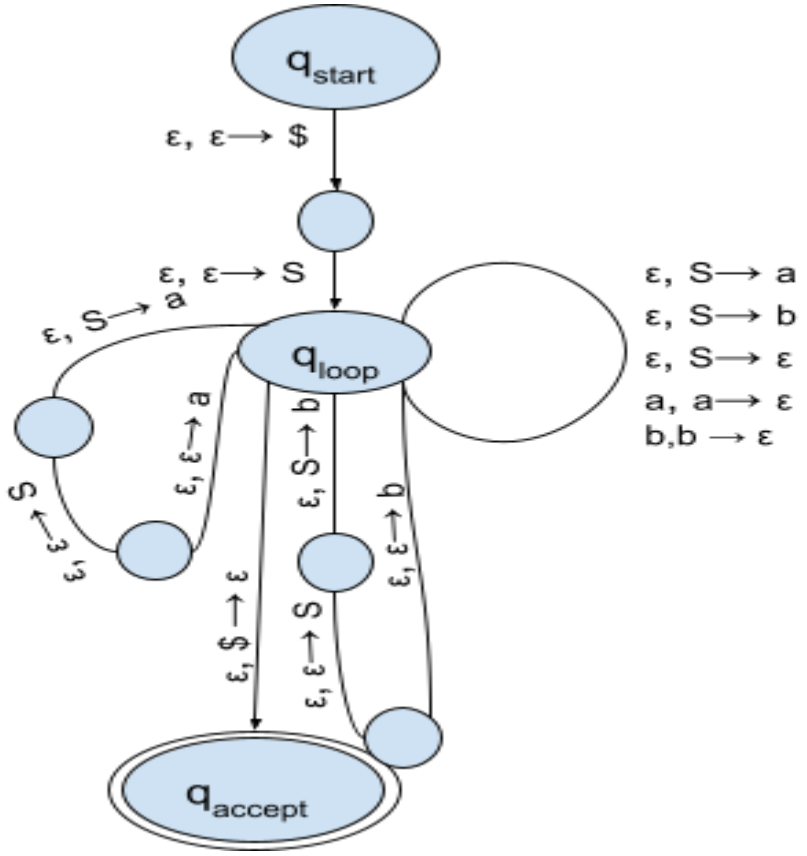
1. S → aT b | b
   T → T a | ε
2. S → aSa | bSb | a | b | ε

## Solution:

1.


$q_{start}$
$\varepsilon, \varepsilon \rightarrow \$$
$\varepsilon, \varepsilon \rightarrow S$
$\varepsilon, S \rightarrow \mathtt{b}$
$\varepsilon, \varepsilon \rightarrow T$
$\varepsilon, \varepsilon \rightarrow \mathtt{a}$
$\varepsilon, T \rightarrow \mathtt{a}$
$\varepsilon, \varepsilon \rightarrow T$
$q_{loop}$
$\varepsilon, \$ \rightarrow \varepsilon$
$\varepsilon, S \rightarrow \mathtt{b}$
$\varepsilon, T \rightarrow \varepsilon$
$\mathtt{a}, \mathtt{a} \rightarrow \varepsilon$
$\mathtt{b}, \mathtt{b} \rightarrow \varepsilon$
$q_{accept}$

2.


$q_{start}$
ε, ε → $
ε, ε → S
ε, S → a
ε, S → b
ε, S → ε
a, a → ε
b,b → ε
$q_{loop}$
a → ε, ε
b → S, ε
S → ε, ε
ε → $, ε
S → ε, ε
$q_{accept}$