Cairo University
Faculty of Computers & Artificial Intelligence
Theory of Computations

Lab#1

Problem#1:

- If X = {a, b, c, d, e}, Y = { a, c, e, f, g, h}, Z = {c, g, m, n}
  Find:

  - $X \cup Y$ =
  - $X \cup Y \cup Z$ =
  - $Y \cup \varnothing$ =
  - $X \cap Y$ =
  - $Y \cap Z$ =
  - $Z \cap \varnothing$ =
  - X - Y =
  - Y - X =
  - Z - X =
  - X - X =
  - X x Y =
  - Z x Y =
  - |X| =
  - |Z x Y| =
  - Is $a \in X$?
  - Is $X \subseteq Y$?

Solution:

- $X \cup Y = \{a, b, c, d, e, f, g, h\}$
- $X \cup Y \cup Z = \{a, b, c, d, e, f, g, h, m, n\}$
- $Y \cup \varnothing = Y = \{ a, c, e, f, g, h\}$
- $X \cap Y = \{a, c, e\}$
- $Y \cap Z = \{c, g\}$
- $Z \cap \varnothing = \varnothing$
- $X - Y = \{b, d\}$
- $Y - X = \{f, g, h\}$
- $Z - X = \{g, m, n\}$
- $X - X = \varnothing$
- $X \times Y = \{(a,a), (a,c), (a,e), (a,f), (a,g), (a,h), (b,a), \ldots ,(e,h)\}$
- $Z \times Y = \{(c,a), (c,c), (c,e), (c,f), (c,g), (c,h), (g,a), \ldots ,(n,h) \}$
- $|X| = 5$
- $|Z \times Y| = 24$
- Is $a \in X$? True
- Is $X \subseteq Y$? False

Problem#2:

- If a=hello, b=world, c=0, d=Λ
  Find:

    - |a| =
    - |c| =
    - |d| =
    - ab =
    - bd =
    - a^R =
    - ab^R =
    - |ad| =
    - |bc| =

Solution:

- |a| = 5
- |c| = 1
- |d| = 0
- ab = helloworld
- bd = world
- a^R = olleh
- ab^R = hellodlrow
- |ad| = |a| + |d| = 5 + 0 = 5
- |bc| = |b| + |c| = 5 + 1 = 6

Notes

1. ∀ X [Px] = every x is p

   Example: “Every dog is happy”

   Correct Solution: ∀ X dog(x) → happy(x)

   Means that for every x, if x is a dog, then x is happy

   Wrong solution: ∀ X [dog(x) ^ happy(x)]

   Means that for every x, x is a dog and x is happy

2. ~for all (x) = there exist (~x)
   ~there exist(x) = for all (~x)
   ~for all (x) =/= for all (~x)
   ~there exist (x) =/= there exist (~x)

   Example: “Nobody likes taxes”

   Solution: ¬∃ X likes (X, taxes)

   Or: ∀ X dislikes(X, takes)

Problem#3:

- Define a predicate for the following:

  I. Ali is a student

  II. Mary loves flowers

  III. All volleyball players are tall

  IV. There a person who makes noise

  V. Everyone likes fries

  VI. Nobody hates fun

  VII. Carols is happier than Sue, but sadder than Fred

  VIII. James is a troublemaker when Kevin dislikes him.

  IX. John didn't study but he is lucky.

  X. All cats and dogs are animals.

Solution:

- Ali is a student
  - Student(Ali)

- Mary loves flowers
  - Loves(Mary, flowers)

- All volleyball players are tall
  - $\forall$ X play(X, volleyball) $\rightarrow$ tall(X)

- There a person who makes noise
  - ∃ X makes(X, noice)

- Everyone likes fries
  - ∀ X likes(X, fries)

- Nobody hates fun
  - ¬∃ X hates(X, fun)

- Carols is happier than Sue, but sadder than Fred

  H(x,y) = x is happier than y

  S(x,y) = x is sadder than y
  - H(carols, sue) ^ S(carlos,fred)

- James is a troublemaker when Kevin dislikes him.
  - dislikes(kevin, james) → troublemaker(james)

- John didn't study but he is lucky.
  - ¬study(john) ^ lucky(john)

- All cats and dogs are animals
  - ∀ X ∀ Y cat(X) ^ dog(Y) → animals(X) ^ animals(Y)