Cairo University
Faculty of Computers & Artificial Intelligence
Theory of Computations

Lab#8

Context free Grammar & Push Down Automata

Remember:

Definition of context free grammar (CFG):

A context-free grammar is a 4-tuple **(V, Σ, R, S)**, where
1. V is a finite set called the variables,
2. Σ is a finite set, disjoint from V , called the terminals,
3. R is a finite set of rules, with each rule being a variable and a string of variables and terminals, and
4. S ∈ V is the start variable.

Definition of push down automata (PDA):

A pushdown automaton is a 6-tuple **(Q, Σ, Γ, δ, q0, F)**, where Q, Σ, Γ, and F are all finite sets, and
1. Q is the set of states,
2. Σ is the input alphabet,
3. Γ is the stack alphabet,
4. $\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \to P(Q \times \Gamma_\varepsilon)$ is the transition function,
5. $q_0 \in Q$ is the start state, and
6. $F \subseteq Q$ is the set of accept states.

Theorem 2:

A language is context free if and only if some pushdown automaton recognizes it

Lemma 2.1:

If a language is context free, then some pushdown automaton recognizes it.

Lemma 2.2:

If a pushdown automaton recognizes some language, then it is context free.

Steps converting from CFG To PDA:

The following is an informal description of P.

1. Place the marker symbol \$ and the start variable on the stack.
2. Repeat the following steps forever.
   a. If the top of stack is a variable symbol A, nondeterministically select one of the rules for A and substitute A by the string on the right-hand side of the rule.
   b. If the top of stack is a terminal symbol a, read the next symbol from the input and compare it to a. If they match, repeat. If they do not match, reject on this branch of the nondeterminism.
   c. If the top of stack is the symbol \$, enter the accept state. Doing so accepts the input if it has all been read.

Steps Converting from PDA to CFG:

PROOF Say that $P = (Q, \Sigma, \Gamma, \delta, q_0, \{q_{\text{accept}}\})$ and construct G. The variables of G are $\{A_{pq} | \; p, q \in Q\}$. The start variable is $A_{q_0, q_{\text{accept}}}$. Now we describe G's rules in three parts.

1. For each $p, q, r, s \in Q$, $u \in \Gamma$, and $a, b \in \Sigma_\varepsilon$, if $\delta(p, a, \varepsilon)$ contains (r, u) and $\delta(s, b, u)$ contains (q, ε), put the rule $A_{pq} \to aA_{rs}b$ in G.
2. For each $p, q, r \in Q$, put the rule $A_{pq} \to A_{pr}A_{rq}$ in G.
3. Finally, for each $p \in Q$, put the rule $A_{pp} \to \varepsilon$ in G.

IMPORTANT:

First, we simplify our task by modifying P slightly to give it the following three features.

1. It has a single accept state, q_{accept}.
2. It empties its stack before accepting.
3. Each transition either pushes a symbol onto the stack (a *push* move) or pops one off the stack (a *pop* move), but it does not do both at the same time.

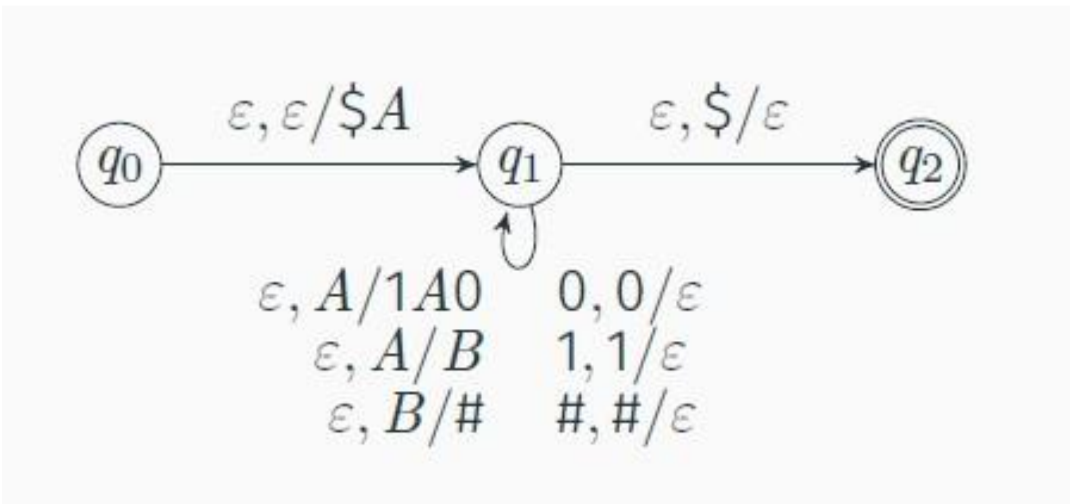
Practice:

1. Convert the following CFG to PDA:

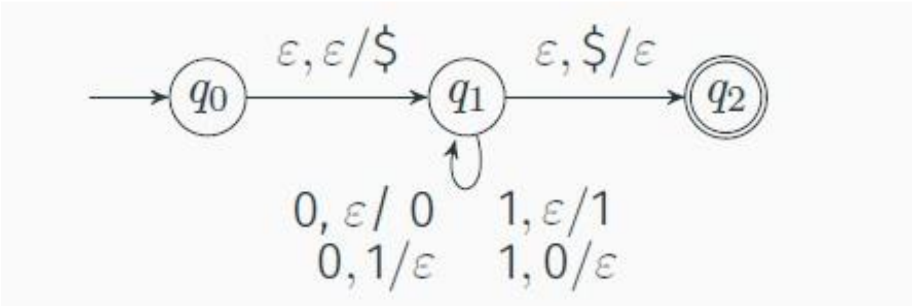
   A → 0A1 | B

   B → #

Solution:



2. Convert the following PDA to CFG:



(q_0, E, E) = (q_1, \$) → 1
(q_1, 0, E) = (q_1, 0) → 2
(q_1, 0, 1) = (q_1, E) → 3
(q_1, 1, E) = (q_1, 1) → 4
(q_1, 1, 0) = (q_1, E) → 5
(q_1, E, \$) = (q_2, E) → 6

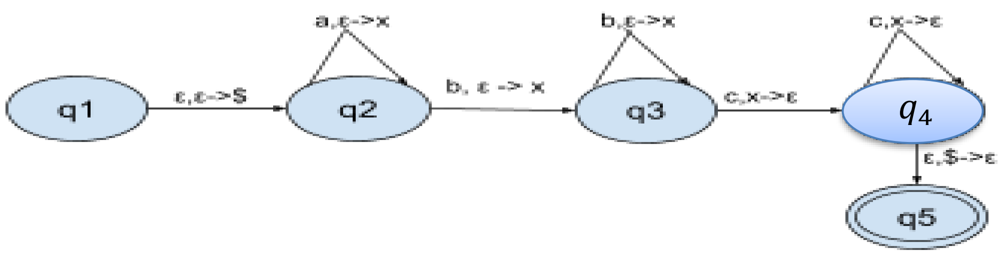
Equation	Result	Reason
Eq 1,6	A02 → E A11 E	Same Pop and push
Eq 1,2 & 1,3 & 1,4 & 1,5	A01 → A01 A11	Common r
Eq 1,6	A02 → A01 A12	Common r
Eq 2,3	A11 → A11 A11	Common r
Eq 2,4	A11 → A11 A11	Common r
E 2,5	A11 → 0 A11 1	Common push and pop
Eq 2,6	A12 → A11 A12	Common r
Eq 3,4	A11 → 1 A11 0	Common push and pop
Eq 3,5	A11 → A11 A11	Common r
Eq 3,6	A12 → A11 A12	Common r

Eq 4,5	A11 → A11 A11	Common r
Eq 4,6	A12 → A11 A12	Common r
Eq 5,6	A12 → A11 A12	Common r

Simplify

From	To	Reason
A02 → E A11 E	E	No Rule call A02
A01 → A01 A11	E	A11 goes to E
A02 → A01 A12	E	No valid rule that defines A01
A11 → A11 A11	E	A11 goes to E
A11 → A11 A11	E	
A11 → 0 A11 1	A11 → 0 A11 1	
A12 → A11 A12	E	A11 goes to E A12 → A12 meaningless
A11 → 1 A11 0	A11 → 1 A11 0	
A11 → A11 A11	E	
A12 → A11 A12	E	
A11 → A11 A11	E	
A12 → A11 A12	E	
A12 → A11 A12	E	
Therefore: S → 0 S 1 | 1 S 0 | E		

3.



$(q_1$, E, E) = $(q_2$, $) → 1
$(q_2$, a, E) = $(q_2$, x) → 2
$(q_2$, b, E) = $(q_3$, x) → 3
$(q_3$, b, E) = $(q_3$, x) → 4
$(q_3$, c, x) = $(q_4$, E) → 5
$(q_4$, c, x) = $(q_4$, E) → 6
$(q_4$, E, $) = $(q_5$, E) → 7

Equation	Result	Reason
Eq 1, 2	A12 → A12 A22	Common r
Eq 1, 3	A13 → A12 A23	Common r
Eq 1, 4	E	
Eq 1, 5	E	
Eq 1, 6	E	
Eq 1, 7	A15 → E A24 E	Common push and pop
Eq 2,3	A23 → A22 A23	Common r
Eq 2,4	E	
Eq 2,5	A24 → a A23 c	Common push and pop
Eq 2,6	A24 → a A24 c	Common push and pop
Eq 2,7	E	
Eq 3,4	A23 → A23 A33	Common r
Eq 3,5	A24 → A23 A34	Common r
Eq 3,5	A24 → b A33 c	Common push and pop

Eq 3,6	A24 → b A34 c	Common push and pop
Eq 3,7	E	
Eq 4,5	A34 → b A33 c	Common push and pop
Eq 4,6	A34 → b A34 c	
Eq 4,7	E	
Eq 5,6	A34 → A34 A44	Common r
Eq 5,7	A35 → A34 A45	Common r
Eq 6,7	A45 → A44 A45	Common r

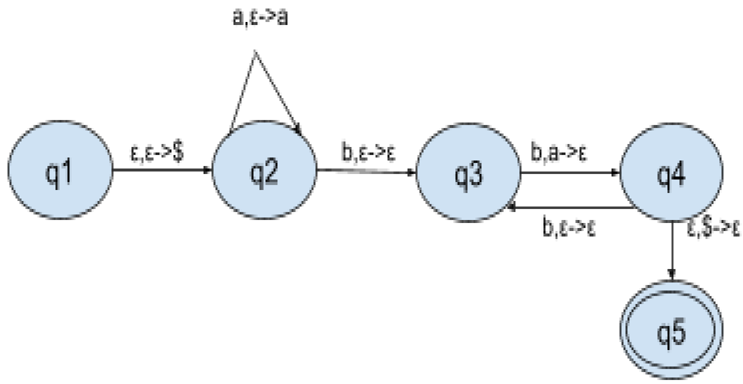
Simplify

From	To	Reason
A12 → A12 A22	E	No rule calls A12
A13 → A12 A23	E	No rule calls A13
A15 → E A24 E	E	No rule calls A15
A23 → A22 A23	E	A22 → E
A24 → a A23 c	E	No valid rule that defines A23
A24 → a A24 c	A24 → a A24 c	
A23 → A23 A33	E	A33 → E
A24 → A23 A34	E	No valid rule that defines A23
A24 → b A33 c	A24 → b c	A33 → E
A24 → b A34 c	A24 → b A34 c	
A34 → b A33 c	A34 → b c	A33 → E
A34 → b A34 c	A34 → b A34 c	
A34 → A34 A44	E	
A35 → A34 A45	E	No Rule call A35

<table>
<tr><td>A45 → A44 A45</td><td>E</td><td>A44 → E</td></tr>
<tr><td colspan="3">Therefore:
$$S \rightarrow a\,S\,c \mid b\,X\,c \mid bc$$
$$X \rightarrow b\,X\,c \mid bc$$</td></tr>
</table>

- Note the simplification is done on multiple steps
- Revisit the lab 8, you can find the vice verse of these examples

4.



Solved by Student as practice

$\{a^n b^{2n} | n>=1\}$