

MIDTERM EXAMINATION

Instructors: Dr. Manar El-Kady

Winter 2023 – CLOSED Book Exam –Total marks: 20 – Duration: 60 mins

This exam comes in **four** pages.

NAME _____ ID _____ Group _____

Question 1 [3 marks] Obtain the regular expressions for the following sets:

(a) The set of all strings over {a, b} beginning and ending with 'a'.

$a(a+b)^*a$

(b) $\{b^2, b^5, b^8, \dots\}$

$bb(bbb)^*$

(c) $\{a^{2n+1} \mid n \geq 0\}$

$aaa(aa)^* \text{ OR } a(aa)^*$

Question 2 [4 marks]

For each of the following languages, give two strings that are members and two strings that are *not* members—a total of four strings for each part. Assume the alphabet $\Sigma = \{a,b\}$ in all parts.

a. $a(ba)^*b$

b. $a^* \cup b^*$

c. $\Sigma^* a \Sigma^* b \Sigma^* a \Sigma^*$

d. $(a \cup ba \cup bb) \Sigma^*$

Answer: Any string matched each part.

Question 3 [3 marks]

Define the language that can be defined using the alphabet $\Sigma = \{0, 1\}$ and can be represented using the following regular expressions

a. $\Sigma^* 1 \Sigma^*$

b. $(\Sigma \Sigma \Sigma)^*$

c. $1^*(01^+)^*$

Answer

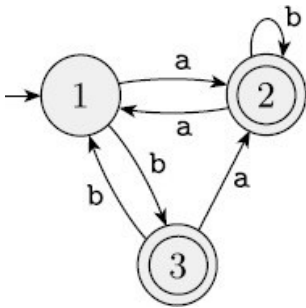
$$\Sigma^*1\Sigma^* = \{w \mid w \text{ has at least one } 1\}.$$

$$(\Sigma\Sigma\Sigma)^* = \{w \mid \text{the length of } w \text{ is a multiple of } 3\}.$$

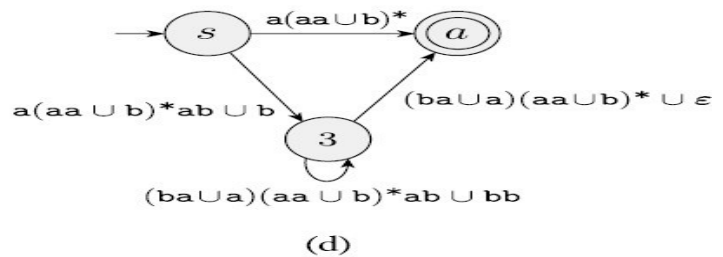
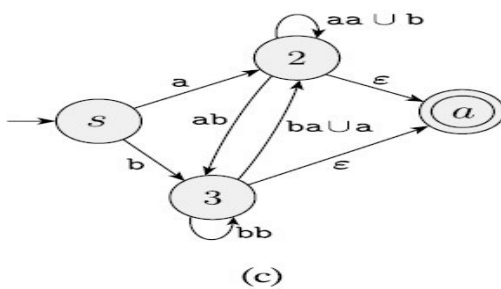
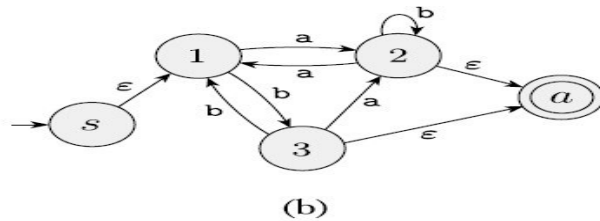
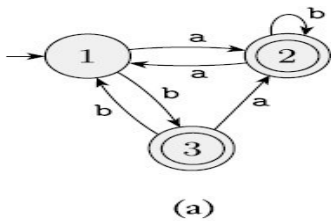
$$1^*(01^+)^* = \{w \mid \text{every } 0 \text{ in } w \text{ is followed by at least one } 1\}.$$

Question 4 [6 marks]

a. Convert the following FA into regular expressions. [2 marks]



Answer :

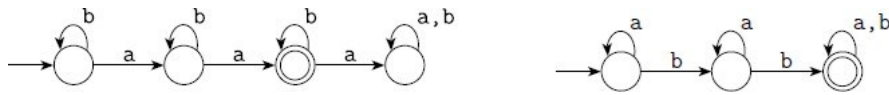


b. Construct a DFA for the following languages [4 marks]

i. $\{w \mid w \text{ has exactly two a's}\}$

ii. $\{w|w \text{ has at least two b's}\}$

Answer



Formal Definition

$M1 = \{\{q0, q1, q2, q3\}, \{a, b\}, T1, q0, \{q2\}\}$

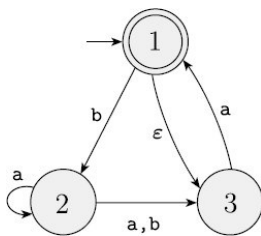
$M2 = \{\{q0, q1, q2\}, \{a, b\}, T2, q0, \{q2\}\}$

OR can define each item in a separate row

T1	A	B
q0	q1	q0
q1	q2	q1
q2	q3	q2
q3	q3	q3

T2	A	b
q0	q0	q1
q1	q1	q2
q2	q2	q2

Question 5 [4 marks] Given the following NFA N construct the equivalent DFA D . You must illustrate all the steps in clear way.



Answer:

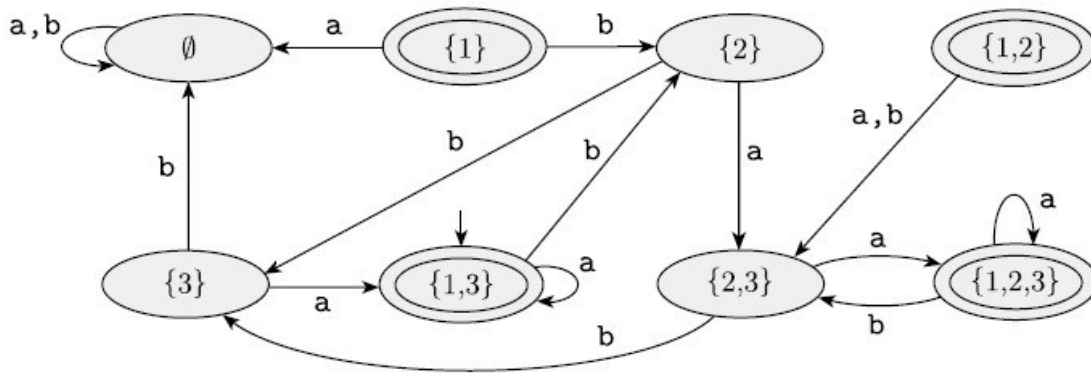


FIGURE 1.43
A DFA D that is equivalent to the NFA N_4

We may simplify this machine by observing that no arrows point at states $\{1\}$ and $\{1,2\}$, so they may be removed without affecting the performance of the machine. Doing so yields the following figure.

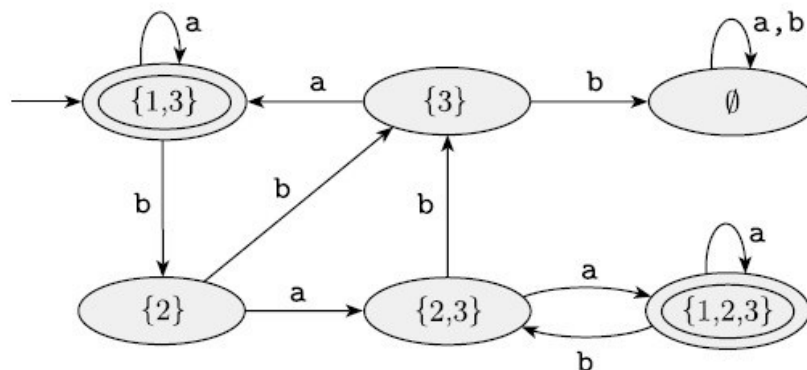


FIGURE 1.44
DFA D after removing unnecessary states

End of exam