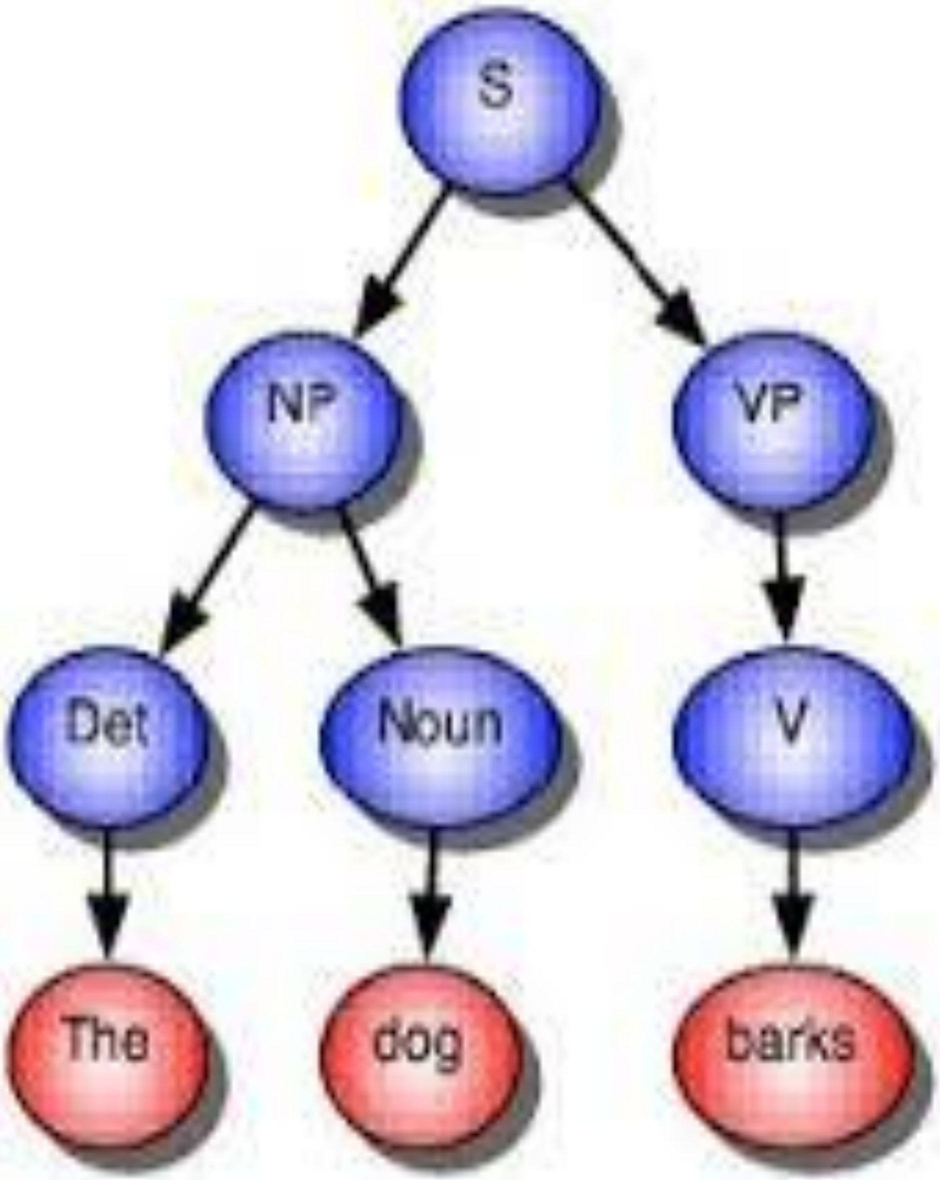
Context Free Grammar CFG



CFG and Parsing

Instructor : Dr. Hanaa Bayomi Ali
Mail : h.mobarz @ fci-cu.edu.eg

The path so far

- Originally, we treated language as a **sequence of words**

  n-gram language models

- Then, we introduced the notion of **syntactic properties of words**

  part-of-speech tags

- Now, we look at **syntactic relations** between words

  syntax trees

Introduction

- Finite Automata **accept** all regular languages and only regular languages

- Languages that require an ability to count are not regular.

- Many simple languages are non regular:

  -$\{a^n b^n : n = 0, 1, 2, \ldots\}$

  - the language of well-matched sequences of brackets. (((()))),(()())

and there is no finite automata that accepts them.

- So we'd like some more powerful means of defining languages.

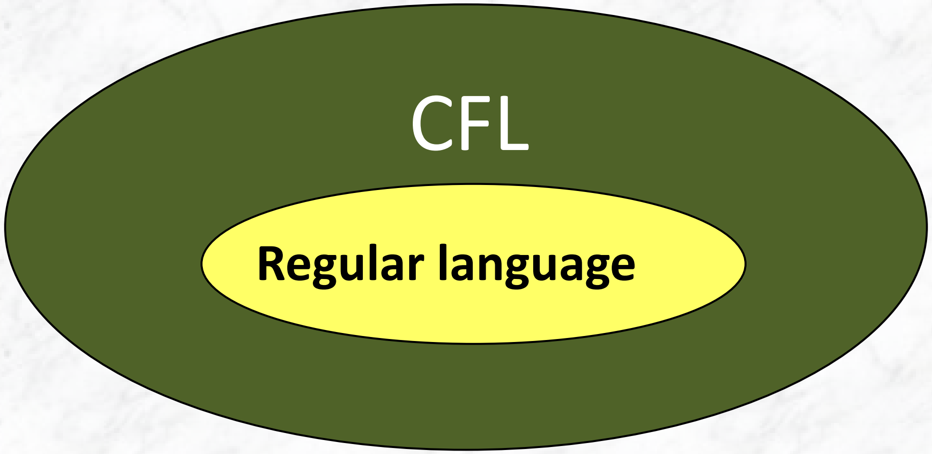
- **Generative grammars**. A language is defined by giving a set of rules capable of `generating' all the sentences of the language.

- The particular kind of generative grammars we'll consider are called **context-free grammars.**

Context-Free Grammars

- Languages that are **generated** by context-free grammars are **context-free languages**

- Context-free grammars are more expressive than finite automata: if a language L is **accepted** by a finite automata then L can be **generated** by a context-free grammar
  - Beware: The converse is NOT true



Context-Free Grammars

- A *context-free grammar* is a notation for describing languages.

- It is more powerful than finite automata or RE's, but still cannot define all possible languages.

- Useful for nested structures, e.g., parentheses in programming languages.

Example: CFG for { 0^n1^n | $n \geq 1$}

- Productions:

  S -> 01

  S -> 0S1

- Basis : 01 is in the language.
- Induction: if w is in the language, then so is 0w1.

CFG Formalism

- *Terminals* = symbols of the alphabet of the language being defined. (RHS)

S -> 01

S -> 0S1

CFG Formalism

- *Terminals* = symbols of the alphabet of the language being defined.(RHS)
- *Variables* = *nonterminals* = a finite set of other symbols, each of which represents a language.
- (mainly in LHS but may be appear in RHS)

S -> 01

S -> 0S1

CFG Formalism

- *Terminals* = symbols of the alphabet of the language being defined.(RHS)
- *Variables* = *nonterminals* = a finite set of other symbols, each of which represents a language.
- (mainly in LHS but may be appear in RHS)

S -> 01

S -> 0S1

- *Start symbol* = the variable whose language is the one being defined.

CFG Formalism

- *Terminals* = symbols of the alphabet of the language being defined.(RHS)

$S \to 01$

$S \to 0S1$

- *Variables* = *nonterminals* = a finite set of other symbols, each of which represents a language.
- (mainly in LHS but may be appear in RHS)
- *Start symbol* = the variable whose language is the one being defined.
- *A production* has the form

variable -> string of variables and terminals.

Example: Formal CFG

- Here is a formal CFG for $\{ 0^n1^n \mid n \geq 1\}$.
- Terminals = {0, 1}.
- Variables = {S}.
- Start symbol = S.
- Productions =
  - S -> 01
  - S -> 0S1

Derivations – Intuition

We *derive* strings in the language of a CFG by starting with the **start symbol**, and repeatedly replacing some variable A by the right side of one of its productions.

That is, the "productions for A" are those that have A on the left side of the ->.

- **Definition.** v is **one-step derivable** from u, written $u \Rightarrow v$

- **Definition.** v is **derivable** from u, written $u \Rightarrow^* v$, if:

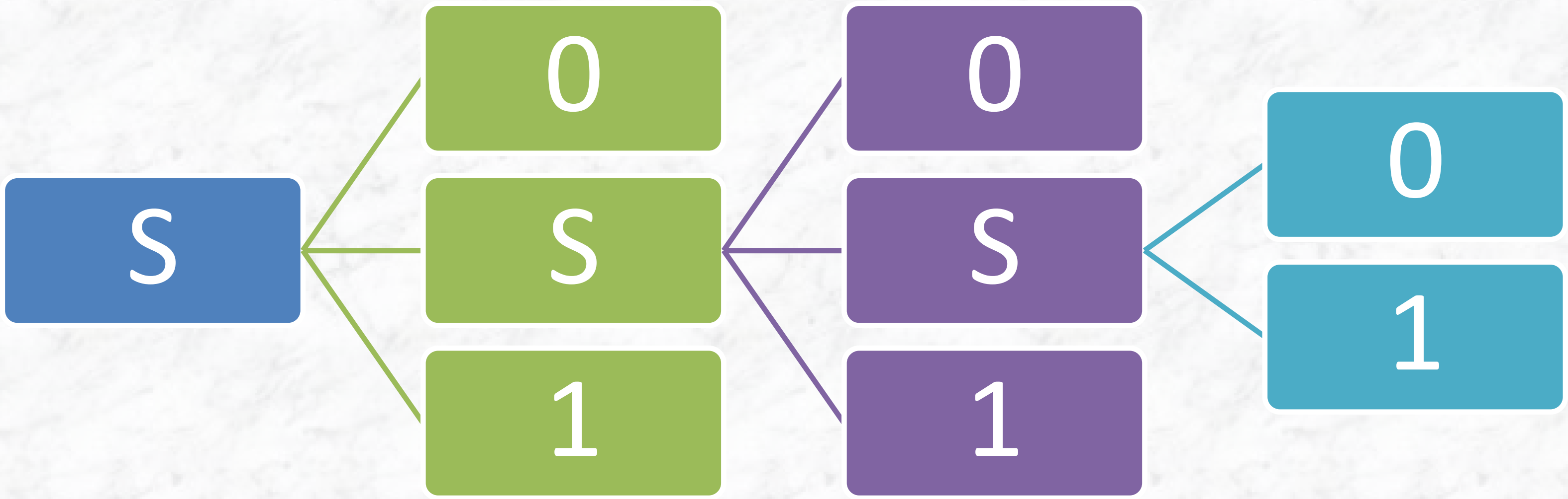
There is a chain of one-derivations of the form:

$$u \Rightarrow u_1 \Rightarrow u_2 \Rightarrow \ldots \Rightarrow v$$

Example

Example: S -> 01; S -> 0S1.

S => 0S1 => 00S11 => 000111.



Chain of one-derivations

Context-free grammars: examples

Q1) all strings of balanced parentheses

((((((((()))))))))

$$P \rightarrow \mathbf{()}$$

$$P \rightarrow \mathbf{(}\ P\ \mathbf{)}$$

Q2) $\{a^m\ b^n\ c^{m+n} \mid m,n \geq 0\}$

Rewrite as $\{\mathbf{a}^m\ \mathbf{b}^n\ \mathbf{c}^n\ \mathbf{c}^m \mid m,n \geq 0\}$:

$S \rightarrow S' \mid \mathbf{a}\ S\ \mathbf{c}$

$S' \rightarrow \varepsilon \mid \mathbf{b}\ S'\ \mathbf{c}$

Context-free grammars: example3

Q3) Write CFG generates simple arithmetic expressions such as

6 + 7   5*(x + 3)   x *((z* 2) + y)   8   z

Exp → Var | Num | (Exp)

Exp → Exp + Exp

Exp → Exp * Exp

Exp → Exp / Exp

Var → x |y | z

Num → 0 9

Context-free grammars: example3

```
Exp → Var | Num | ( Exp )
Exp → Exp + Exp
Exp → Exp * Exp
Exp → Exp / Exp
Var → x |y | z
Num → 0 ........9
```

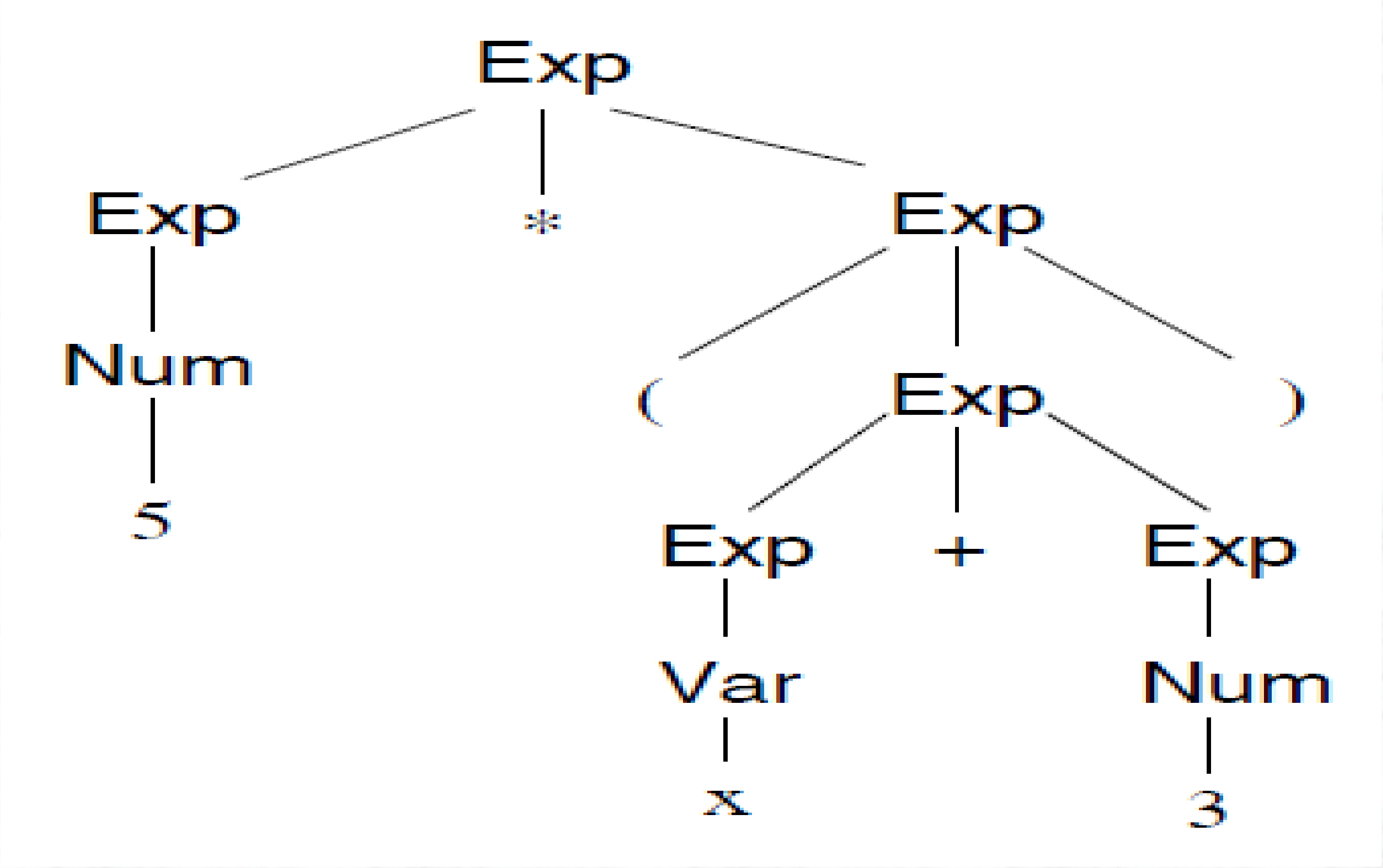
The symbols **+,*, /, (,), x, y, z, 0, ….., 9** are called terminals: these form the ultimate constituents of the phrases we generate.

The symbols **Exp, Var, Num** are called non-terminals: they name various kinds of `sub-phrases'.

We designate Exp the start symbol.

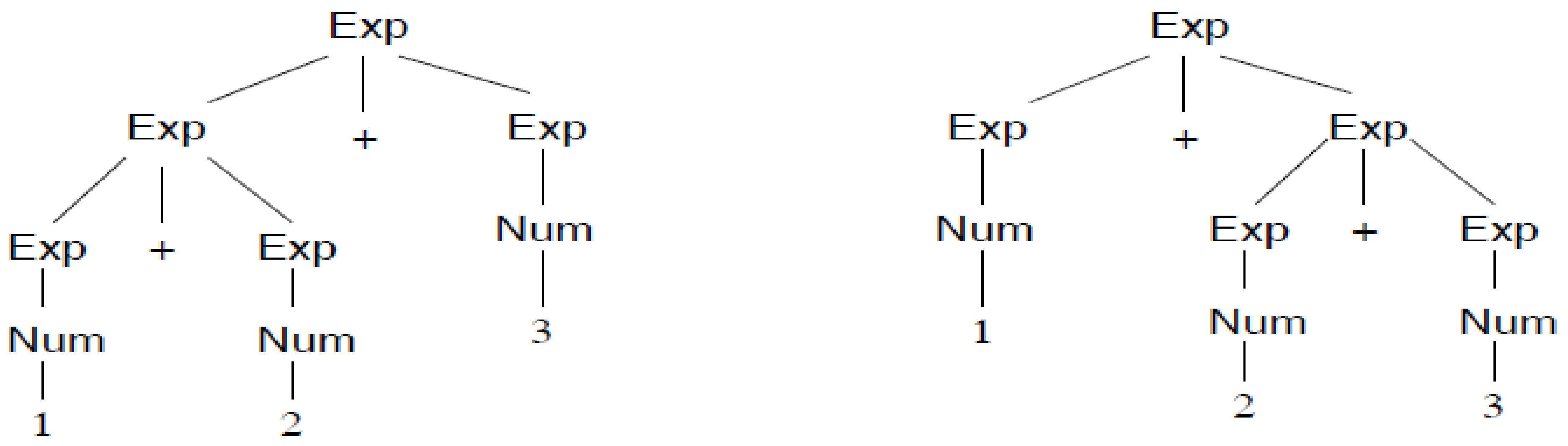
Syntax tree

We grow syntax trees by repeatedly **expanding non-terminal** symbols using these rules. E.g.: 5*(x +3)



The language defined by a grammar

- By choosing different rules to apply, we can generate infinitely many strings from this grammar.
- The **language** generated by the grammar is, by definition, the set of all strings of terminals that can be derived from the start symbol via such a syntax tree.
- Note that strings such as 1+2+3 may be generated by more than one tree (structural ambiguity):



Challenge question

How many possible syntax trees are there for the string below

1+2+3+4

Exp → Var | Num | (Exp)
Exp → Exp + Exp
Exp → Exp * Exp
Exp → Exp / Exp
Var → x |y | z
Num → 0 9

Derivations

As a more `machine-oriented' alternative to syntax trees, we can think in terms of derivations involving (mixed) strings of terminals and non-terminals. E.g.

Exp ->Exp * Exp
->Num * Exp
-> Num* (Exp)
-> Num * (Exp + Exp)
-> 5 * (Exp + Exp)
-> 5 * (Exp + Num)
-> 5 * (Var + Exp)
->5 * (x + Exp)
->5 * (x + 3)

Exp → Var | Num | (Exp)
Exp → Exp + Exp
Exp → Exp * Exp
Exp → Exp / Exp
Var → x |y | z
Num → 0 9

At each stage, we choose one non-terminal and expand it using a suitable rule. When there are only terminals left, we can stop!

Multiple derivations

Clearly, any derivation can be turned into a syntax tree.
However, even when there's only one syntax tree, there might be many derivations for it:

$$
\begin{aligned}
\mathrm{Exp} &\Rightarrow \mathrm{Exp} + \mathrm{Exp} \\
&\Rightarrow \mathrm{Num} + \mathrm{Exp} \\
&\Rightarrow 1 + \mathrm{Exp} \\
&\Rightarrow 1 + \mathrm{Num} \\
&\Rightarrow 1 + 2
\end{aligned}
$$

(. . . a leftmost derivation)

$$
\begin{aligned}
\mathrm{Exp} &\Rightarrow \mathrm{Exp} + \mathrm{Exp} \\
&\Rightarrow \mathrm{Exp} + \mathrm{Num} \\
&\Rightarrow \mathrm{Exp} + 2 \\
&\Rightarrow \mathrm{Num} + 2 \\
&\Rightarrow 1 + 2
\end{aligned}
$$

(. . . a rightmost derivation)

In the end, it's the syntax tree that matters | we don't normally care about the difference between various derivations for it.

However, derivations - especially leftmost and rightmost ones-will play a significant role when we consider parsing algorithms.

Quiz

State whether the grammar *CAN* or *CAN'T* produce each part-of-speech tag sequence below.

$$
\begin{aligned}
NP &\rightarrow \text{art } NP1\\
NP &\rightarrow \text{ppro } NP1\\
NP1 &\rightarrow \text{num } NP1\\
NP1 &\rightarrow NP2\\
NP2 &\rightarrow \text{adj } NP2\\
NP2 &\rightarrow \text{adj } NP3\\
NP3 &\rightarrow \text{noun } NP3\\
NP3 &\rightarrow \text{noun}
\end{aligned}
$$

1. art num noun
2. art num num noun
3. art adj noun
4. art noun
5. art num adj noun
6. art num num adj adj noun noun
7. art adj num noun
8. art adj adj noun noun
9. art ppro num noun
10. ppro num num adj noun noun
11. ppro noun noun noun
12. ppro adj adj adj

Answer

State whether the grammar *CAN* or *CAN'T* produce each part-of-speech tag sequence below.

$$
\begin{aligned}
NP &\rightarrow \text{art } NP1\\
NP &\rightarrow \text{ppro } NP1\\
NP1 &\rightarrow \text{num } NP1\\
NP1 &\rightarrow NP2\\
NP2 &\rightarrow \text{adj } NP2\\
NP2 &\rightarrow \text{adj } NP3\\
NP3 &\rightarrow \text{noun } NP3\\
NP3 &\rightarrow \text{noun}
\end{aligned}
$$

1. art num noun
CANNOT
2. art num num noun
CANNOT
3. art adj noun
CAN
4. art noun
CANNOT
5. art num adj noun
CAN
6. art num num adj adj noun noun
CAN

7. art adj num noun
CANNOT
8. art adj adj noun noun
CAN
9. art ppro num noun
CANNOT
10. ppro num num adj noun noun
CAN
11. ppro noun noun noun
CANNOT
12. ppro adj adj adj
CANNOT