

FurtherQ

Problem Set 1

Secondary School Edition

Version 1.0

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Editions

PB - Public Edition

SS - Secondary School Edition

Question Difficulty

Easy - Warm up

Intermediate - Standard

Challenging - Interesting

Schadenfreude - Yes

Paper Info

Marked Question: 1-9

Total Marks: 105

Total Time: 180 Mins (3 hrs)

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1. [Suggested Time: 7 mins | Total Marks: 7 | Easy]

A function f is defined by $f(x) = \sqrt{2}(\sin x + \cos x + 1)$ for $0 \leq x \leq 2\pi$
Points P and Q are the maximum and minimum points of f respectively,
Without the use of calculus, find the coordinates P and Q
Leave your answers in exact values.

[7]

2. [Suggested Time: 5 mins | Total Marks: 5 | Easy]

Without the use of a calculator, Solve the following equation

$$6^{2x} + 36^x = 2\sqrt[3]{6\sqrt[3]{216\sqrt[3]{6}}}$$

Hence find the value of $\sqrt[19]{2^{6x}}$

[5]

3. [Suggested Time: 15 mins | Total Marks: 10 | Intermediate]

The first three terms of the expansion $(2 - x) \left(2 + \frac{x^2}{4}\right)^n (4 + 4x + x^2)$ is $a + ax^2 - bx^3$,
where $n \in \mathbb{Z}^+, n \geq 2$. Find the values of a, b, n . [10]

4. [Suggested Time: 23 mins | Total Marks: 15 | Intermediate]

Find all the angles between 0 to 2π inclusive which satisfies

$$\left(\sqrt{3}\tan(\pi\theta) + 2\right)\frac{1}{\sqrt{3}}\sin(e\theta) + \cos(e\theta) - 1 = 1$$

Leave your answers in exact values.

[15]

5. [Suggested Time: 25 mins | Total Marks: 16 | Intermediate]

(i) Prove the following:

$$(a) \cos^4 \theta - \sin^4 \theta = \cos 2\theta \quad [2]$$

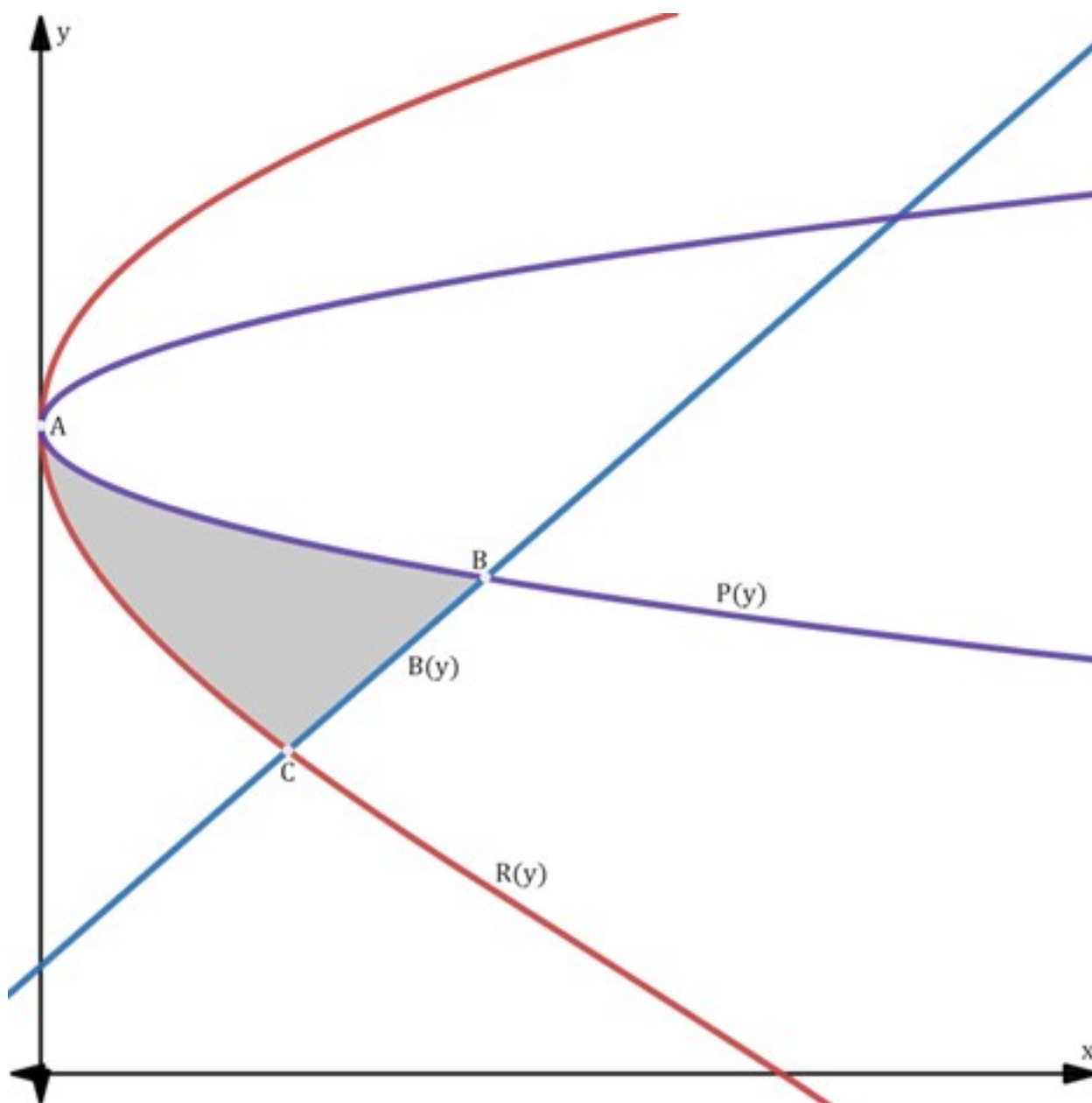
$$(b) \cos^4 \theta + \sin^4 \theta = \frac{3}{4} + \frac{1}{4} \cos 4\theta \quad [4]$$

(ii) Hence or otherwise, and **leaving your answers in exact values**, evaluate

(a) $\int_0^\pi \cos^4 \theta \, d\theta$ [5]

(b) $\int_0^\pi \sin^4 \theta \, d\theta$ [5]

6. [Suggested Time: 40 mins | Total Marks: 20 | Challenging]



Answers by Accurate drawings or graphical methods are not accepted.

The graphs are plotting y against x . Point A is a common stationary point of $R(y)$ and $P(y)$. $B(y)$ passes through the points $D(10, 2)$ and $E(1, \frac{1}{2})$.

$$P(y) = 33(y - 2)^2, \quad B(y) = R''(y)$$

Degree of polynomial $R(y)$ is 3.

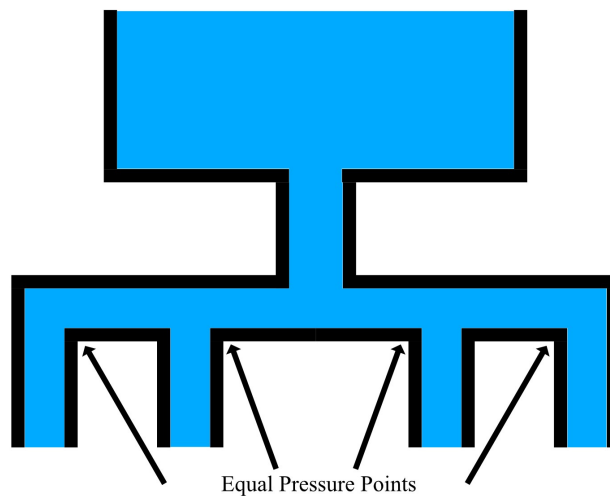
Find the shaded area.

Leave your answers in exact values, in the form $a + b\sqrt{339}$, where $a, b \in \mathbb{Q}$ [20]

7. [Suggested Time: 20 mins | Total Marks: 10 | Challenging]

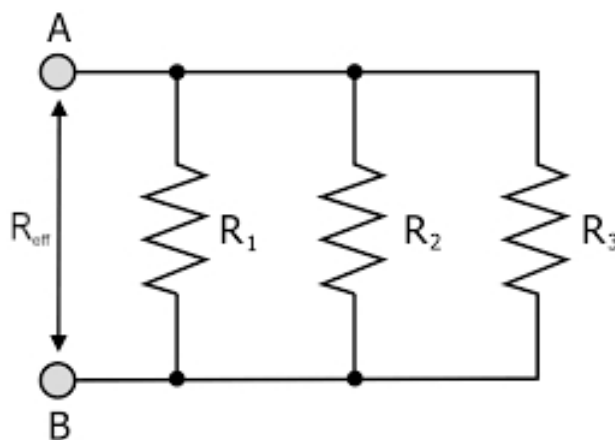
Circuit Theory

The workings of a parallel circuit can be explained with a water tank, where the water tank is like a battery in the circuit, current I is the water flowing in the pipes, voltage V can be seen as the pressure which the water experiences at a certain point and resistance R is a measure of how much pressure is needed to move a certain quantity of water from a point to another

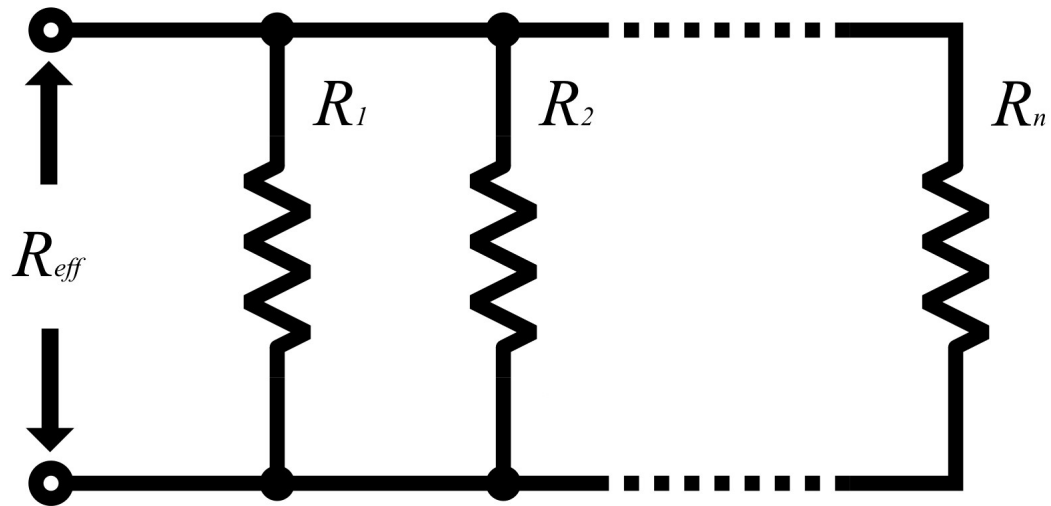


In the diagram shown above, it is known that the pressure at all 4 points which the arrows are pointing at are equivalent, and the total volume of water that flows out of the pipes is equal to the volume of water that flows into the pipes from the water tank.

It is also given that ohmic conductors follow ohm's law, $V = IR$



- (i) Using the information given above, deduce the effective resistance, R_{eff} , in the circuit shown above, and consequently, find an expression for a parallel circuit with n resistors. **Leave your answers in terms of V, I and R** [5]



(ii)(a) Hence, using (i) or otherwise, find the effective resistance of the circuit shown above.

Where $R_1 = \sqrt{1} + \sqrt{2}$, $R_2 = \sqrt{2} + \sqrt{3}$, $R_3 = \sqrt{3} + \sqrt{4} \dots$

Leave your answers in exact values, and in terms of n .

[4]

- (ii)(b) Explain, with relevant workings, if the effective resistance of the circuit will approach a unique value as more resistors are added into the circuit. [1]*

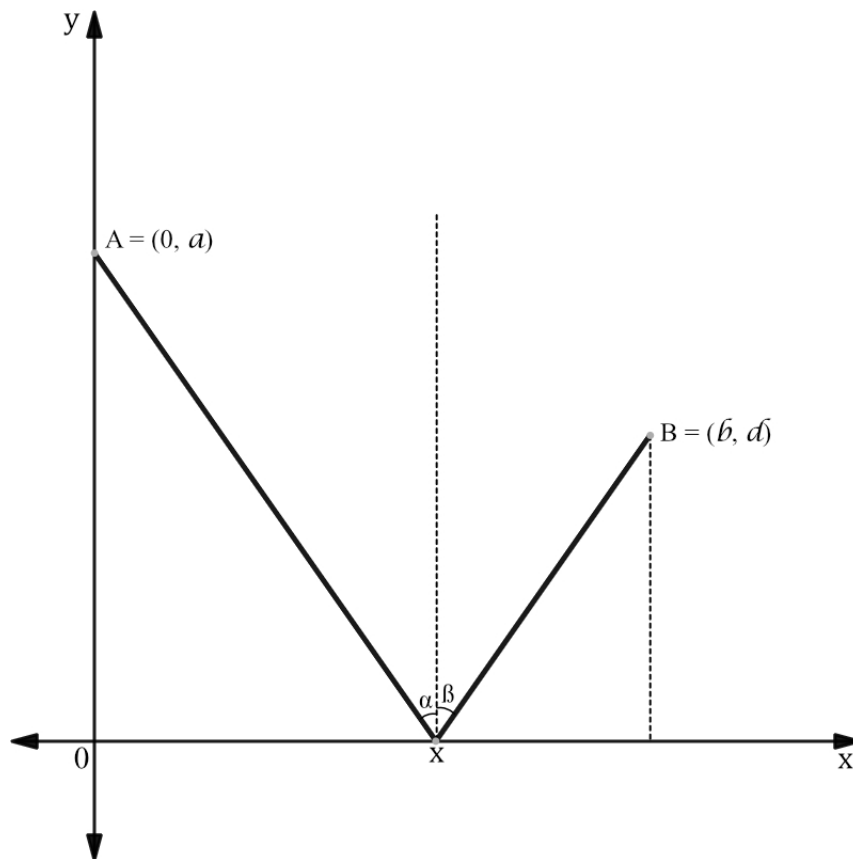
8. [Suggested Time: 20 mins | Total Marks: 10 | Challenging]

Fermat's Principle of Least Time

Fermat's Principle of Least Time states that out of all neighbouring paths available, light travels between two points along the path that requires the least time.

Consider a light ray from a source which strikes a mirror and is reflected. Let A be a point on the ray before it strikes the mirror and B be the point on the ray after reflection. v m/s is the speed of light.

A coordinate system is placed in a plane such that the x -axis runs along the mirror's surface and the point A lies on the y -axis.



Prove that the angle of incidence α is equal to the angle of reflection β .

(Proof that T is minimum is not required)

[10]

9. [Suggested Time: 25 mins | Total Marks: 12 | Challenging]

A soft drink company wants to design a 500ml soda can using the least amount of material for their new Moon Shine soda. Assume that the soda can is a perfect cylinder. Using the information given below, find the cheapest material cost to produce 1000 cans. **Leave your answers in USD**

[12]

Information List	
Price of Aluminium	2515 USD/Metric Tonne
Density of Aluminium	2.7 g/cm^3
1 Atmosphere (Pressure)	101325 Pa
Room Temperature	$T_c = 28^\circ \text{C}$
Temperature (Kelvin)	$T_k = T_c + 273 \text{ K}$
Gas Constant (R)	$R = 8.314 \text{ m}^3 \text{ Pa/mol K}$

Ideal Gas Law
$PV = nRT$ P – Pressure V – Volume n – Amount of Substance R – Gas Constant T – Temperature

The Soda can is able to tolerate up to 5 atm of pressure, and has a uniform thickness of 0.01 cm. The soda releases up to $\frac{x}{10} \text{ cm}^3$ of gas for every $x \text{ ml}$ of soda under room temperature and pressure.

10. [Suggested Time: N.A. | Total Marks: N.A. | Schadenfreude]

Given that if x satisfies $x^n + a_{n-1}x^{n-1} + \cdots + a_0 = 0$ for some integers a_{n-1}, \dots, a_0 , then x is irrational unless x is an integer.

Prove that $\sqrt{2} + \sqrt[3]{2}$ is irrational.

