FurtherQ
Problem Set 1

Public Edition

Version 1.1

September 2022

Problems

Editions

PB - Public Edition
SS - Secondary School Edition

Question Difficulty

Easy - Warm up
Intermediate - Standard
Challenging - Difficult
Schadenfreude - Interesting
Extensions - Out of syllabus

Organizing Team

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Paper Info

Total Time: 180 Mins (3 hrs)
Graded Questions: 1-8 & 10
Total Marks: 105

Attempted	/9
Marks	/105
Percentage	%

1. [**Suggested Time: 7 mins | Total Marks: 7 | Easy**]
   A function f *is defined by* $f : x \mapsto \sqrt{2}\,(\sin x + \cos x + 1)$ *for* $0 \leq x \leq 2\pi$
   Points P *and* Q *are the maximum and minimum points of* f *respectively,*
   Without the use of calculus, find the coordinates P *and* Q
   Leave your answers in exact values. [7]

2. [**Suggested Time: 5 mins | Total Marks: 5 | Easy**]

Without the use of a calculator, *Solve the following equation*

$6^{2x} + 36^{x} = 2\sqrt[3]{6\sqrt[3]{216\sqrt[3]{6}}}$

Hence find the value of $\sqrt[19]{2^{6x}}$ [5]

3. [**Suggested Time: 15 mins | Total Marks: 10 | Intermediate**]
The first three terms of the expansion $(2-x)\left(2+\frac{x^2}{4}\right)^n\left(4+4x+x^2\right)$ *is* $a+ax^2-bx^3$, *where* $n \in \mathbb{Z}^+, n \geq 2$. *Find the values of* a, b, n. [10]

4. [**Suggested Time: 23 mins | Total Marks: 15 | Intermediate**]
Find all the angles between 0 to 2π inclusive which satisfies

$$\left(\sqrt{3}\tan(\pi\theta)+2\right)^{\frac{1}{\sqrt{3}}\sin(e\theta)+\cos(e\theta)-1}=1$$

Leave your answers in exact values. [15]

5. [**Suggested Time: 25 mins | Total Marks: 16 | Intermediate**]

(i) *Prove the following:*

(a) $\cos^4\theta - \sin^4\theta = \cos 2\theta$ [2]

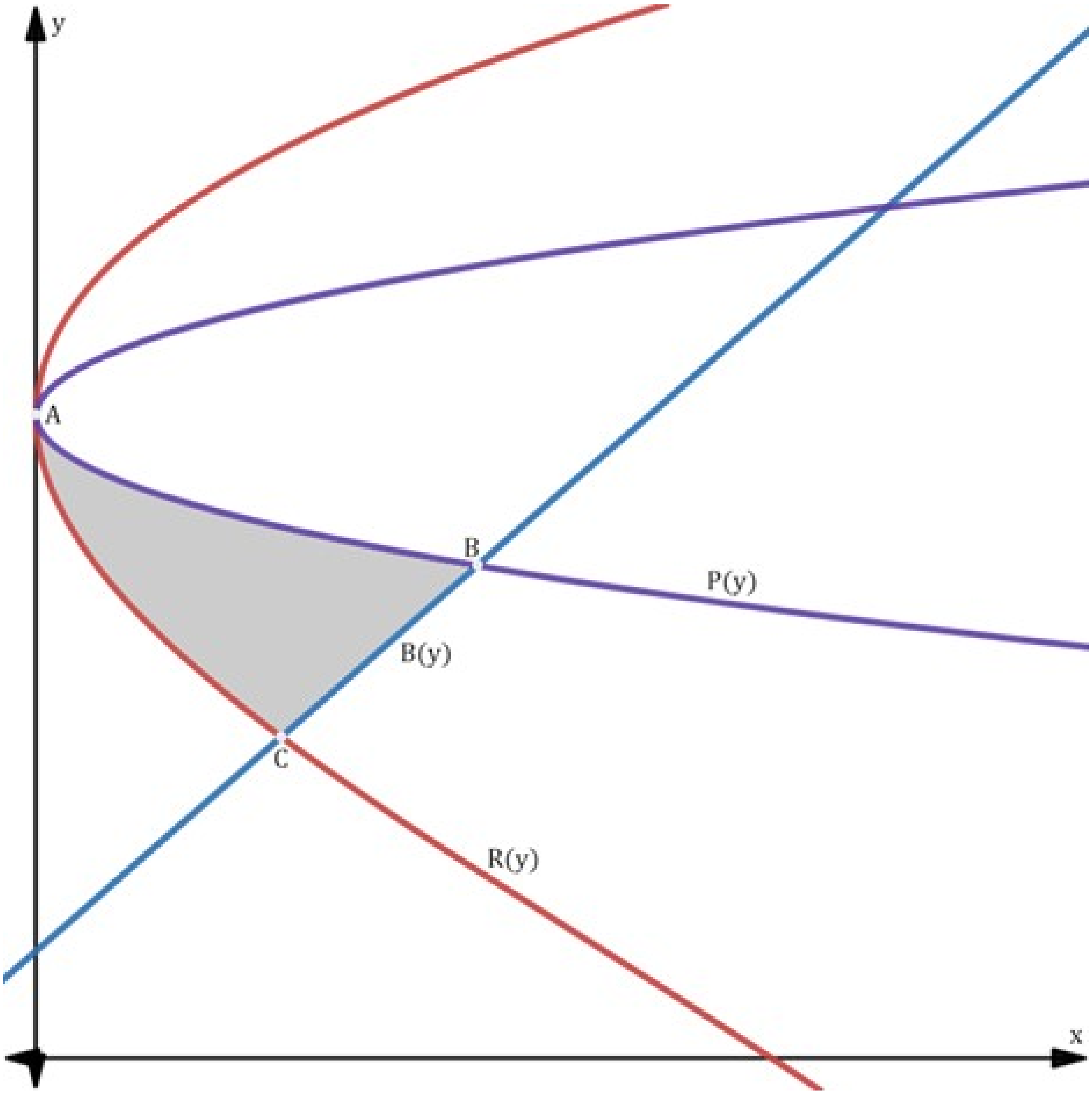
(b) $\cos^4\theta + \sin^4\theta = \frac{3}{4} + \frac{1}{4}\cos 4\theta$ [4]

(ii) *Hence or otherwise, and* ***leaving your answers in exact values****, evaluate*

(a) $\displaystyle\int_0^{\pi} \cos^4\theta \, d\theta$ [5]

(b) $\displaystyle\int_0^{\pi} \sin^4\theta \, d\theta$ [5]

6. [**Suggested Time: 40 mins | Total Marks: 20 | Challenging**]



Answers by Accurate drawings or graphical methods are not accepted.

The graphs are plotting y against x. Point A is a common stationary point of $R(y)$ and $P(y)$. $B(y)$ passes through the points $D(10, 2)$ and $E(1, \frac{1}{2})$.

$P(y) = 33(y-2)^2$, $B(y) = R''(y)$

Degree of polynomial $R(y)$ is 3.

Find the shaded area.
Leave your answers in exact values, *in the form* $a + b\sqrt{339}$ *, where* $a, b \in \mathbb{Q}$ [20]

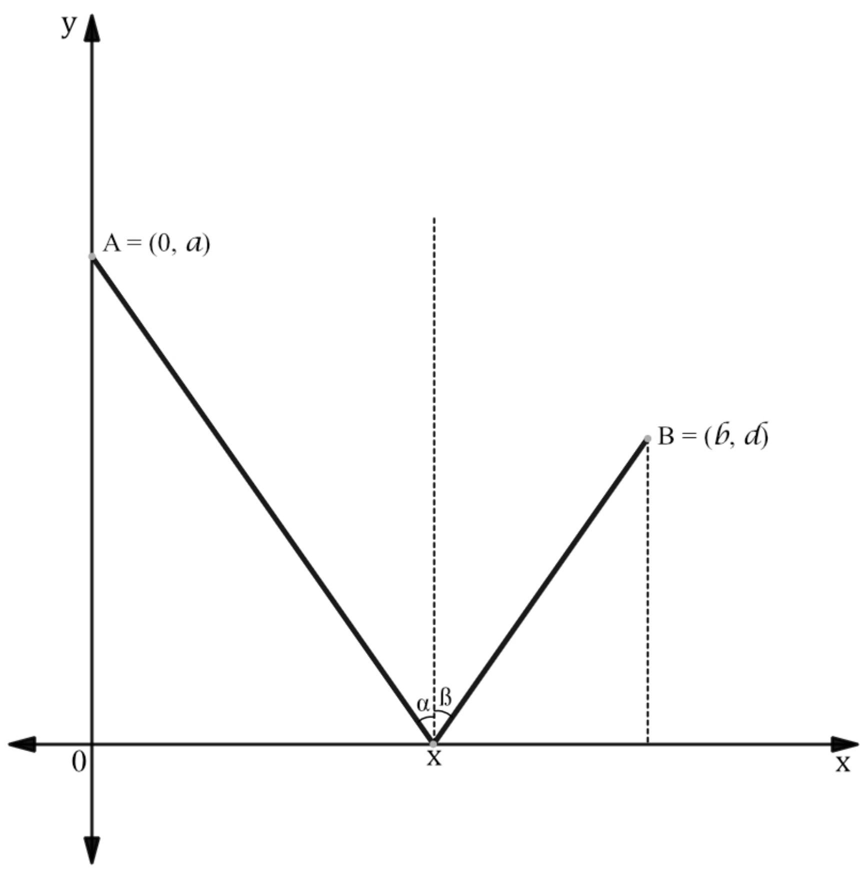
7. [**Suggested Time: 20 mins | Total Marks: 10 | Challenging**]

Fermat's Principle of Least Time

Fermat's Principle of Least Time states that out of all neighbouring paths available, light travels between two points along the path that requires the least time.

Consider a light ray from a source which strikes a mirror and is reflected. Let A be a point on the ray before it strikes the mirror and B be the point on the ray after reflection. v m/s is the speed of light.

A coordinate system is placed in a plane such that the x-axis runs along the mirror's surface and the point A lies on the y-axis.



Prove that the angle of incidence α is equal to the angle of reflection β.
(Proof that T is minimum is not required) [10]

8. [**Suggested Time: 25 mins | Total Marks: 12 | Challenging**]
A soft drink company wants to design a 500ml soda can using the least amount of material for their new Moon Shine soda. Assume that the soda can is a perfect cylinder. Using the information given below, find the cheapest material cost to produce 1000 cans. ***Leave your answers in USD*** [12]

Information List	
Price of Aluminium	$2515\,USD/Metric\,Tonne$
Density of Aluminium	$2.7g/cm^3$
1 Atmosphere (Pressure)	101325 Pa
Room Temperature	$T_c = 28°c$
Temperature (Kelvin)	$T_k = T_c + 273$ K
Gas Constant (R)	$R = 8.314\,m^3\,Pa/mol\,K$

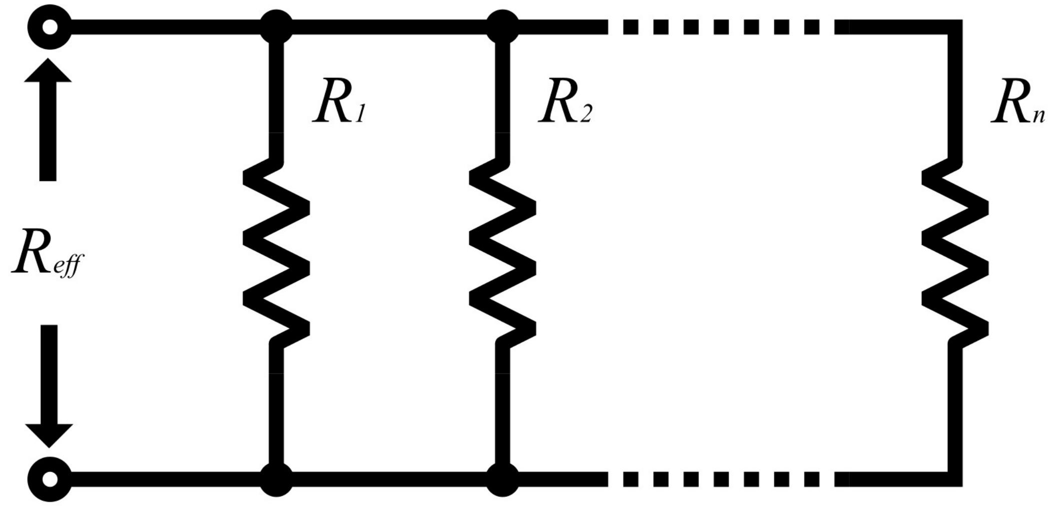
Ideal Gas Law
$\boldsymbol{PV = nRT}$
$P - Pressure$
$V - Volume$
$n - Amount\ of\ Substance$
$R - Gas\ Constant$
$T - Temperature$

The Soda can is able to tolerate up to 5 atm of pressure, and has a uniform thickness of 0.01 *cm. The soda releases up to* $\frac{x}{10}\ cm^3$ *of gas for every* x *ml of soda under room temperature and pressure.*

9. [**Suggested Time: N.A. | Total Marks: N.A. | Schadenfreude**]
Given that if x satisfies $x^n + a_{n-1}x^{n-1} + \cdots + a_0 = 0$ for some integers $a_{n-1}, \cdots, a_0$, then x is irrational unless x is an integer.
Prove that $\sqrt{2} + \sqrt[3]{2}$ is irrational.

10. [**Suggested Time: 25 mins | Total Marks: 10 | Extensions**]

(i) *Deduce that the effective resistance,* R_{eff}*, of* n *parallel resistors is* $R_{eff} = \left(\sum_{i=1}^{n} \frac{1}{R_i} \right)^{-1}$ [5]



(ii)(a) *Hence or otherwise, find the effective resistance of the circuit shown above.*
Where $R_1 = \sqrt{1} + \sqrt{2}, R_2 = \sqrt{2} + \sqrt{3}, R_3 = \sqrt{3} + \sqrt{4}\ \cdots$
Leave your answers in exact values*, and in terms of* n. [4]

(ii)(b) *Explain, with relevant workings, if the effective resistance of the circuit will approach a unique value as more resistors are added into the circuit.* [1]