USTHB University - Faculty of Computer Science Algiers

Module: Advanced Algorithms and Complexity - 3 Soft Eng
Lab 1: Algorithms, Programming, and Complexity **2024-2025**
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Done by:

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Part I: Sum of the First N Natural Numbers

In this part, we revisit algorithms, programming, and complexity by solving a problem involving the calculation of the sum of the first (n) natural numbers. The problem is solved using **iterative programming**.

Problem Statement

Develop an iterative algorithm and program to calculate the sum (S) of the first (n) natural numbers: [S = \sum_{i=1}^{n} i = 1 + 2 + 3 + \dots + n] where (n) is a natural integer read as input ((n \geq 1)).

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1. Iterative Algorithm (Sum_1)

The iterative algorithm, denoted **Sum_1**, uses a `while` loop to calculate the sum (S).

Algorithm Steps:

1. Read the input value (n) (ensure (n \geq 1)).
2. Initialize a variable (S) to 0 to store the sum.
3. Initialize a counter variable (i) to 1.
4. Use a `while` loop to iterate from (i = 1) to (i = n):
   - Add (i) to (S).
   - Increment (i) by 1.
5. Return the value of (S).

Pseudocode:

```
Sum_1(n):
    S = 0
    i = 1
    while i <= n:
        S = S + i
        i = i + 1
    return S
```

---

2. Time Complexity of the Algorithm

The time complexity of the algorithm **Sum_1** is determined by the number of iterations in the `while` loop.

- The loop runs (n) times (from (i = 1) to (i = n)).
- Each iteration performs a constant number of operations (addition and increment).

Thus, the **time complexity** is: [O(n)]

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3. Space Complexity of the Algorithm

The space complexity is determined by the amount of memory used by the algorithm.

- The algorithm uses a constant amount of memory to store the variables (S), (i), and (n).
- No additional data structures are used.

Thus, the **space complexity** is: [O(1)]

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4. Iterative Program in C (PSum_1)

Below is the corresponding iterative program written in C. To handle large values of (n), the `double` type is used for the sum (S).

C Code:

```c
#include <stdio.h>

double PSum_1(int n) {
    double S = 0.0;
    int i = 1;
    while (i <= n) {
        S += i;
        i++;
    }
    return S;
}

int main() {
    int n;
    printf("Enter a natural number n (n >= 1): ");
    scanf("%d", &n);

    if (n < 1) {
        printf("Error: n must be greater than or equal to 1.\n");
        return 1;
    }

    double result = PSum_1(n);
    printf("The sum of the first %d natural numbers is: %.0f\n", n, result);
```

```
    return 0;
}
```

Explanation:

1. The function `PSum_1` implements the iterative algorithm using a `while` loop.
2. The `main` function reads the input (n), validates it, and calls `PSum_1` to compute the sum.
3. The result is printed as a double, but formatted to display as an integer using `%.0f`.

Here's the rewritten **Part II** of your report, including the table and plot description:

Part II: Measuring Execution Time

In this part, we learn how to measure the execution time of a program using the time management functions provided by the C standard library (`time.h`). We modify the previous program to measure the execution time for different values of (n) and analyze the results.

C Code:

```c
#include <stdio.h>
#include <time.h>

double PSum_2(long long int n) {
    double S = 0.0;
    long long int i = 1;
    while (i <= n) {
        S += i;
        i++;
    }
    return S;
}

int main() {
    double n_values[] = {1e6, 2e6, 1e7, 2e7, 1e8, 2e8, 1e9, 2e9, 1e10, 2e10, 1e11,
2e11, 1e12, 2e12};
    int num_values = sizeof(n_values) / sizeof(n_values[0]);

    // Open a CSV file for writing
    FILE *file = fopen("data.csv", "w");
    if (file == NULL) {
        printf("Error opening file!\n");
        return 1;
    }

    // Write the header to the CSV file
    fprintf(file, "n,Execution Time (seconds)\n");
```

```c
    printf("n\t\tExecution Time (seconds)\n");
    printf("--------------------------------\n");

    for (int i = 0; i < num_values; i++) {
        long long int n = (long long int)n_values[i]; // Cast to long long int
        clock_t start = clock(); // Start time
        double result = PSum_2(n); // Call the function
        clock_t end = clock(); // End time

        double execution_time = (double)(end - start) / CLOCKS_PER_SEC;

        // Print to console
        printf("%lld\t\t%.6f\n", n, execution_time);

        // Write to CSV file
        fprintf(file, "%lld,%.6f\n", n, execution_time);
    }

    // Close the CSV file
    fclose(file);

    return 0;
}
```

1. Measuring Execution Time (PSum_2)

The program `PSum_2` calculates the sum of the first (n) natural numbers and measures the execution time for a range of (n) values. The results are recorded in the table below.

Table: Execution Times for Different Values of (n)

(n)	Execution Time (seconds)
1,000,000	0.00
2,000,000	0.01
10,000,000	0.03
20,000,000	0.06
100,000,000	0.30
200,000,000	0.58
1,000,000,000	2.84
2,000,000,000	5.77
10,000,000,000	29.13
20,000,000,000	58.99

(n)	Execution Time (seconds)
100,000,000,000	287.81
200,000,000,000	595.20
1,000,000,000,000	2,878.99
2,000,000,000,000	5,902.05

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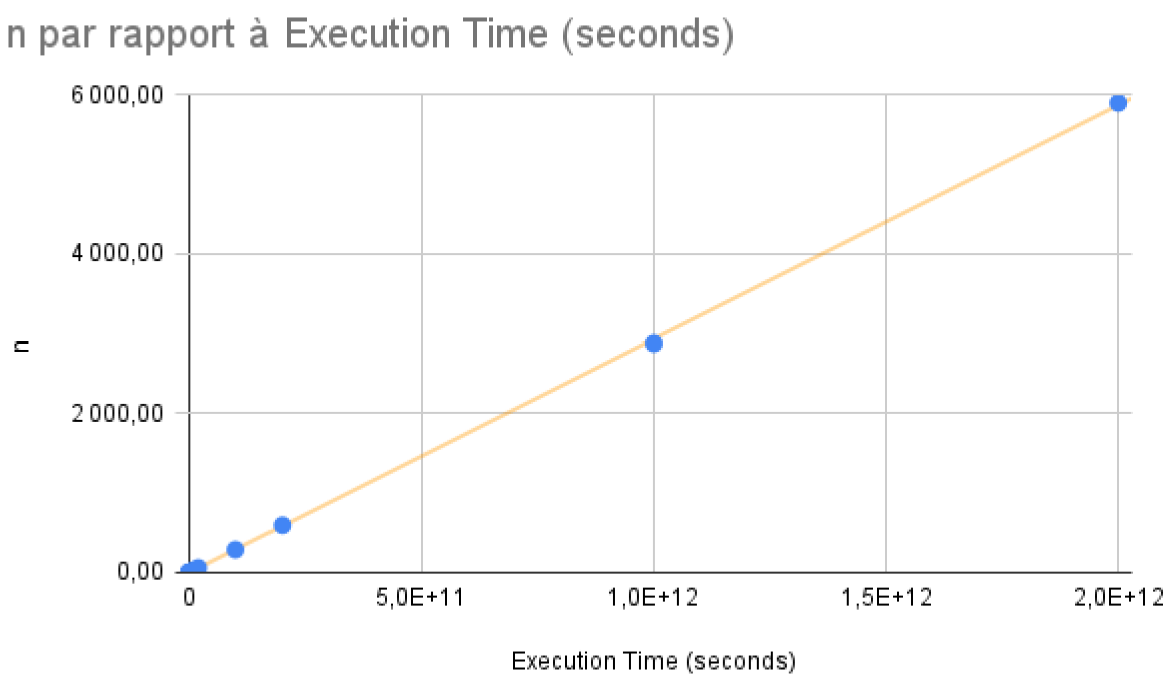
2. Plotting the Curves



3. Comparison and Interpretation of Results

Theoretical Execution Time:

- The theoretical time complexity of the algorithm is (O(n)), meaning the execution time should increase linearly with (n).

Experimental Execution Time:

- The experimental results align with the theoretical (O(n)) complexity for small to moderate values of (n).

Conclusion

- The experimental results confirm the theoretical (O(n)) time complexity for small to moderate input sizes.
- This exercise demonstrates the importance of both theoretical analysis and empirical measurement in understanding algorithm performance.