

### EXERCICE 3

Une grammaire est propre si elle est  $\varepsilon$ -libre, d'épouvree de symboles inutiles, sans cycle

$G < \{a,b\}, \{S,A\},S,P> \quad P = \{S \rightarrow Ba / A b / \varepsilon / aDb \quad A \rightarrow AA a / \varepsilon / Sa \quad B \rightarrow S b / \varepsilon$ $D \rightarrow aDb/ AD\}$		
$S \rightarrow Ba / A b / \varepsilon / aDb$ $A \rightarrow AA a / \varepsilon / Sa$ $B \rightarrow S b / \varepsilon$ $D \rightarrow aDb/ AD$	$S' \rightarrow S$ $S \rightarrow Ba / A b / \varepsilon / aDb$ $A \rightarrow AA a / \varepsilon / Sa$ $B \rightarrow S b / \varepsilon$ $D \rightarrow aDb/ AD$	
$S' \rightarrow S$ $S \rightarrow Ba / A b / \varepsilon / aDb$ $A \rightarrow AA a / \varepsilon / Sa$ $B \rightarrow S b / \varepsilon$ $D \rightarrow aDb/ AD$	$S' \rightarrow S / \varepsilon$ $S \rightarrow Ba / A b / aDb / a / b$ $A \rightarrow AA a / A a / a / Sa / a$ $B \rightarrow S b /b$ $D \rightarrow aDb/ AD$	
$S' \rightarrow S / \varepsilon$ $S \rightarrow Sba / ba / A b / aDb / a / b$ $A \rightarrow AA a / A a / a / Sa / a$ $D \rightarrow aDb/ AD$		forme normale de Chomsky $S' \rightarrow SS_1 / B'A' / A B' / a / b / \varepsilon$ $S \rightarrow SS_1 / B'A' / A B' / a / b$ $S_1 \rightarrow B'A'$ $A \rightarrow AS_2/ A A' / SA' / a$ $S_2 \rightarrow A A'$ $A' \rightarrow a$ $B' \rightarrow b$
Élimination des productions unitaires $S' \rightarrow Sba / ba / A b / a / b / \varepsilon$ $S \rightarrow Sba / ba / A b / a / b$ $A \rightarrow AA a / A a / a / Sa / a$		
$G < \{a,b\}, \{S\},S,P> \quad P= \{ S \rightarrow aSbS / bSaS / \varepsilon\}$		
$S \rightarrow aSbS / bSaS / \varepsilon$ $S' \rightarrow S / \varepsilon$ $S \rightarrow aSbS / bSaS / abS / baS / aSb / bSa / ab / ba$ Élimination des productions unitaires $S' \rightarrow aSbS / bSaS / abS / baS / aSb / bSa / ab / ba / \varepsilon$ $S \rightarrow aSbS / bSaS / abS / baS / aSb / bSa / ab / ba$		
forme normale de Chomsky $S' \rightarrow ASBS / BSAS / ABS / BAS / ASB / BSA / AB / BA$ $S \rightarrow ASBS / BSAS / ABS / BAS / ASB / BSA / AB / BA$ $A \rightarrow a$ $B \rightarrow b$ $S \rightarrow AX_1/ BX_3 / AX_2 / BX_4 / AX_5 / BX_6 / AB / BA$ $X_1 \rightarrow SX_2$ $X_2 \rightarrow BS$ $X_3 \rightarrow SX_4$ $X_4 \rightarrow AS$ $X_5 \rightarrow SB$ $X_6 \rightarrow SA$ $A \rightarrow a$ $B \rightarrow b$		

$G < \{a,b\}, \{S,A, C\}, S, P > P = \{ S \rightarrow AS / bC \quad A \rightarrow a / \varepsilon \quad C \rightarrow a C / a / \varepsilon \}$	
$S \rightarrow AS / bC$ $A \rightarrow a / \varepsilon$ $C \rightarrow a C / a / \varepsilon$	$S \rightarrow AS / bC / b$ $A \rightarrow a$ $C \rightarrow a C / a$
forme normale de Chomsky $S \rightarrow AS / BC / b$ $C \rightarrow AC / a$ $A \rightarrow a$ $B \rightarrow b$	
$G < \{a,b,c\}, \{S,A,B,C,D,E\}, S, P > P = \{ S \rightarrow aS / BS / \varepsilon \quad B \rightarrow bAb / SaS \quad A \rightarrow a / Sa \}$	
$S \rightarrow aS / BS / \varepsilon$ $B \rightarrow bAb / SaS$ $A \rightarrow a / Sa$	$S' \rightarrow S / \varepsilon$ $S \rightarrow aS / BS / B / a$ $B \rightarrow bAb / SaS / aS / Sa / a$ $A \rightarrow a / Sa$
Élimination des productions unitaires (S et B) $S' \rightarrow aS / BS / bAb / SaS / aS / Sa / a / \varepsilon$ $S \rightarrow aS / BS / bAb / SaS / aS / Sa / a$ $B \rightarrow bAb / SaS / aS / Sa / a$ $A \rightarrow a / Sa$	forme normale de Chomsky $S' \rightarrow XS / BS / YAY / SXS / XS / SX / a / \varepsilon$ $S \rightarrow XS / BS / YAY / SXS / XS / SX / a$ $B \rightarrow YAY / SXS / XS / SX / a$ $A \rightarrow a / SX$ $X \rightarrow a$ $Y \rightarrow b$
$S' \rightarrow XS / BS / YX_1 / SX_2 / XS / SX / a / \varepsilon$ $S \rightarrow XS / BS / YX_1 / SX_2 / XS / SX / a$ $B \rightarrow YX_1 / SX_2 / XS / SX / a$ $A \rightarrow a / SX$ $X \rightarrow a$ $Y \rightarrow b$	$X_1 \rightarrow AY$ $X_2 \rightarrow XS$

#### EXERCICE 4

$S \rightarrow aSb / aSa / bSa / bSb / aAa / bAb / c$ $A \rightarrow aAa / bAb / c$
forme normale de Chomsky $S \rightarrow XX_1 / XX_2 / YX_2 / YX_1 / XX_3 / YX_4 / c$ $A \rightarrow XX_3 / YAY / c$  $X_1 \rightarrow SY$ $X_2 \rightarrow SX$  $X_3 \rightarrow AX$ $X_4 \rightarrow AY$  $X \rightarrow a$ $Y \rightarrow b$

## EXERCICE 5

$G < \{a,b,c\}, \{S,A,B,C\}, S, P > \quad P: \{ S \rightarrow AA / a / b \quad A \rightarrow SS / b \quad B \rightarrow BC / AB \quad C \rightarrow aB / b \}$	
$S \rightarrow AA / a / b$ $A \rightarrow SS / b$	
$G < \{a,b,c,d\}, \{S,A\}, S, P > \quad P: \{ S \rightarrow aSAb / bSS / d \quad A \rightarrow cASaA / bcd \}$	
$S \rightarrow aSAb / bSS / d$ $A \rightarrow cASaA / bcd$	$S \rightarrow XSAY / YSS / d$ $A \rightarrow ZASXA / YZT$  $X \rightarrow a$ $Y \rightarrow b$ $Z \rightarrow c$ $T \rightarrow d$
$S \rightarrow XX_1 / YX_3 / d$ $A \rightarrow ZX_4 / YX_7$  $X \rightarrow a$ $Y \rightarrow b$ $Z \rightarrow c$ $T \rightarrow d$	$X_1 \rightarrow SX_2$ $X_2 \rightarrow AY$ $X_3 \rightarrow SS$ $X_4 \rightarrow AX_5$ $X_5 \rightarrow SX_6$ $X_6 \rightarrow XA$ $X_7 \rightarrow ZT$
$G < \{a,b\}, \{S,A,B,C\}, S, P > \quad P: \{ S \rightarrow bA / aB \quad A \rightarrow bAA / aS/a \quad B \rightarrow aBB / bS / b \}$	
$S \rightarrow YA / XB$ $A \rightarrow YAA / XS/a$ $B \rightarrow XBB / YS / b$	$X \rightarrow a$ $Y \rightarrow b$

## EXERCICE 6

$G = \langle X, V, P, S \rangle$ où $X = \{a, b\}$ , $V = \{S, A\}$ et $P = \{ S \rightarrow aS / Aa / \epsilon \quad A \rightarrow Aa / b \}$	
$S'_1 \rightarrow S_2 / \epsilon$ $S_2 \rightarrow aS_2 / A_3a / \epsilon$ $A_3 \rightarrow A_3a / b$	$S'_1 \rightarrow S_2 / \epsilon$ $S_2 \rightarrow aS_2 / A_3a / a$ $A_3 \rightarrow A_3a / b$
$S'_1 \rightarrow S_2 / \epsilon$ $S_2 \rightarrow aS_2 / A_3a / a$ $A_3 \rightarrow A_3a / b$	$S_1 \rightarrow S_2 / \epsilon$ $S_2 \rightarrow aS_2 / A_3a / a$ $A_3 \rightarrow b / bT$ $T \rightarrow aT/a$
$S'_1 \rightarrow aS_2/ba / bTa / a / \epsilon$ $S_2 \rightarrow aS_2/ba / bTa / a$ $A_3 \rightarrow b / bT$ $T \rightarrow aT / a$	
$G \langle X, V, P, S \rangle \quad P = \{ S \rightarrow AB \quad A \rightarrow BS / b \quad B \rightarrow SA / a \}$	
$S_1 \rightarrow A_2B_3$ $A_2 \rightarrow B_3S_1 / b$ $B_3 \rightarrow S_1A_2 / a$	$S_1 \rightarrow A_2B_3$ $A_2 \rightarrow B_3S_1 / b$ $B_3 \rightarrow B_3S_1B_3A_2 / bB_3A_2 / a$
$S_1 \rightarrow A_2B_3$ $A_2 \rightarrow B_3S_1 / b$ $B_3 \rightarrow bB_3A_2 / a / bB_3A_2T / aT$ $T \rightarrow B_3S_1B_3A_2T / B_3S_1B_3A_2$	$S_1 \rightarrow A_2B_3$ $A_2 \rightarrow bB_3A_2S_1 / aS_1 / bB_3A_2TS_1 / aTS_1 / b$ $B_3 \rightarrow bB_3A_2 / a / bB_3A_2T / aT$ $T \rightarrow B_3S_1B_3A_2T / B_3S_1B_3A_2$

$S_1 \rightarrow bB_3A_2S_1B_3 / aS_1B_3 / bB_3A_2TS_1B_3 / aTS_1B_3 / bB_3$ $A_2 \rightarrow bB_3A_2S_1 / aS_1 / bB_3A_2TS_1 / aTS_1 / b$ $B_3 \rightarrow bB_3A_2 / a / bB_3A_2T / aT$ $T \rightarrow B_3S_1B_3A_2T / B_3S_1B_3A_2$
$S_1 \rightarrow bB_3A_2S_1B_3 / aS_1B_3 / bB_3A_2TS_1B_3 / aTS_1B_3 / bB_3$ $A_2 \rightarrow bB_3A_2S_1 / aS_1 / bB_3A_2TS_1 / aTS_1 / b$ $B_3 \rightarrow bB_3A_2 / a / bB_3A_2T / aT$ $T \rightarrow bB_3A_2S_1B_3A_2T / aS_1B_3A_2T / bB_3A_2TS_1B_3A_2T / aTS_1B_3A_2T /$ $bB_3A_2S_1B_3A_2 / aS_1B_3A_2 / bB_3A_2TS_1B_3A_2 / aTS_1B_3A_2$

## EXERCICE 8

2.

$G \langle X=\{a, b\}, V=\{S, A, B, D, F\}, P, S \rangle \quad P = \{ S \rightarrow SaB / bB / aDB / F \quad A \rightarrow aAB / aA / \epsilon \quad B \rightarrow aS / aSB / BaB / \epsilon \quad D \rightarrow aD / Da \quad F \rightarrow \epsilon \}$	
$S \rightarrow SaB / bB / aDB / F$ $A \rightarrow aAB / aA / \epsilon$ $B \rightarrow aS / aSB / BaB / \epsilon$ $D \rightarrow aD / Da$ $F \rightarrow \epsilon$	$S \rightarrow SaB / bB / F$ $B \rightarrow aS / aSB / BaB / \epsilon$ $F \rightarrow \epsilon$
$S' \rightarrow S / \epsilon$ $S \rightarrow SaB / Sa / aB / bB / a / b$ $B \rightarrow aS / aSB / BaB / Ba / aB / a$	$S' \rightarrow SXB / SX / XB / YB / a / b / \epsilon$ $S \rightarrow SXB / SX / XB / YB / a / b$ $B \rightarrow XS / XSB / BXB / BX / XB / a$
$S' \rightarrow SaB / Sa / aB / bB / a / b / \epsilon$ $S \rightarrow SaB / Sa / aB / bB / a / b$ $B \rightarrow aS / aSB / BaB / Ba / aB / a$	$X \rightarrow a$ $Y \rightarrow b$
$S' \rightarrow SX_1 / SX / XB / YB / a / b / \epsilon$ $S \rightarrow SX_1 / SX / XB / YB / a / b$ $B \rightarrow XS / XX_2 / BX_1 / BX / XB / a$ $X \rightarrow a$ $Y \rightarrow b$ $X_1 \rightarrow XB$ $X_2 \rightarrow SB$	

3.

$S_1' \rightarrow S_2 / \epsilon$ $S_2 \rightarrow S_2aB_3 / S_2a / aB_3 / bB_3 / a / b$ $B_3 \rightarrow aS_2 / aS_2B / B_3aB_3 / B_3a / aB_3 / a$
$S_1' \rightarrow aB_3 / bB_3 / a / b / aB_3X / bB_3X / aX / bX / \epsilon$ $S_2 \rightarrow aB_3 / bB_3 / a / b / aB_3X / bB_3X / aX / bX$ $X \rightarrow S_2aB_3 / S_2a / S_2aB_3X / S_2aX$ $B_3 \rightarrow aS_2 / aS_2B_3 / aB_3 / a / aS_2Y / aS_2B_3Y / aB_3Y / aY$ $Y \rightarrow B_3aB_3 / B_3a / B_3aB_3Y / B_3aY$
$X \rightarrow aB_3aB_3 / bB_3aB_3 / aaB_3 / baB_3 / aB_3XaB_3 / bB_3XaB_3 / aX / bXaB_3 /$ $aB_3a / bB_3a / aa / ba / aB_3Xa / bB_3Xa / aXa / bXa /$ $aB_3aB_3X / bB_3aB_3X / aaB_3X / baB_3X / aB_3XaB_3X / bB_3XaB_3X / aXaB_3X / bXaB_3X /$ $aB_3aX / bB_3aX / aaX / baX / aB_3XaX / bB_3XaX / aXaX / bXaX$

$Y \rightarrow aS_2aB_3 / aS_2B_3aB_3 / aB_3aB_3 / aaB_3 / aS_2YaB_3 / aS_2BYaB_3 / aB_3YaB_3 / aYaB_3 /$   
 $aS_2a / aS_2B_3a / aB_3a / aa / aS_2Ya / aS_2B_3Ya / aB_3Ya / aYa /$   
 $aS_2aB_3Y / aS_2B_3aB_3Y / aB_3aB_3Y / aaB_3Y / aS_2YaB_3Y / aS_2BYaB_3Y / aB_3YaB_3Y / aYaB_3Y /$   
 $aS_2aY / aS_2B_3aY / aB_3aY / aaY / aS_2YaY / aS_2B_3YaY / aB_3YaY / aYaY$