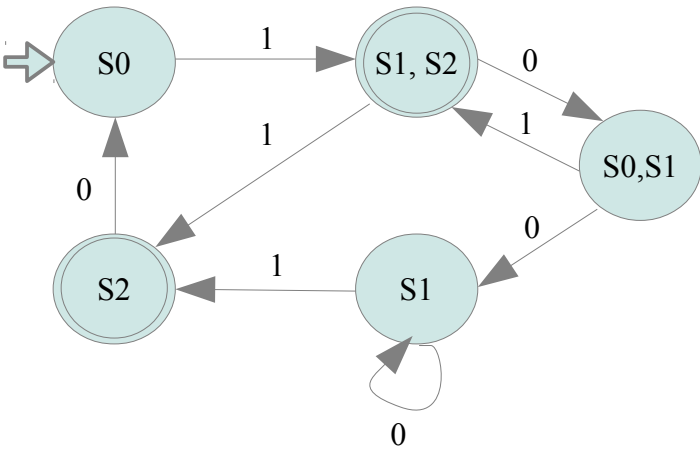


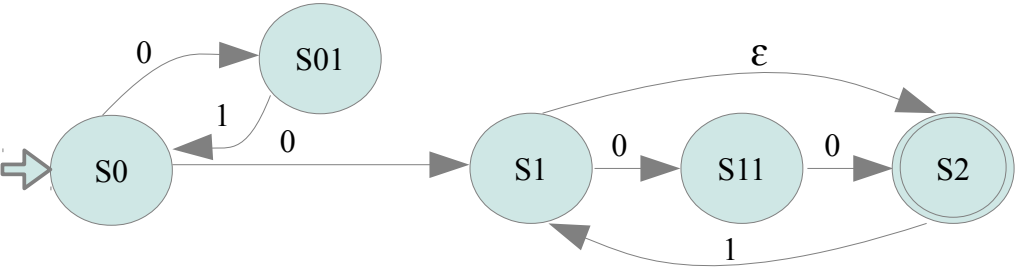
Exercise 1

	0	1
{S0}	/	{S1,S2}
{S1,S2}	{S0,S1}	{S2}
{S0,S1}	{S1}	{S1,S2}
{S2}	{S0}	/
{S1}	{S1}	{S2}



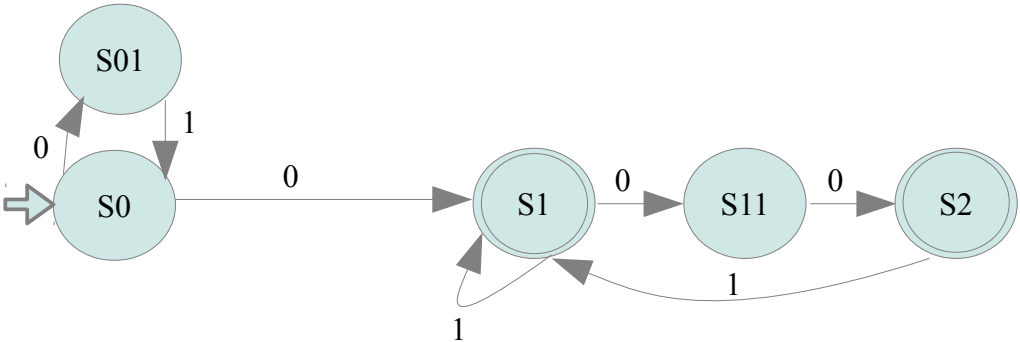
Exercise 2

1.  
Étape 1: décomposition des transitions complexes



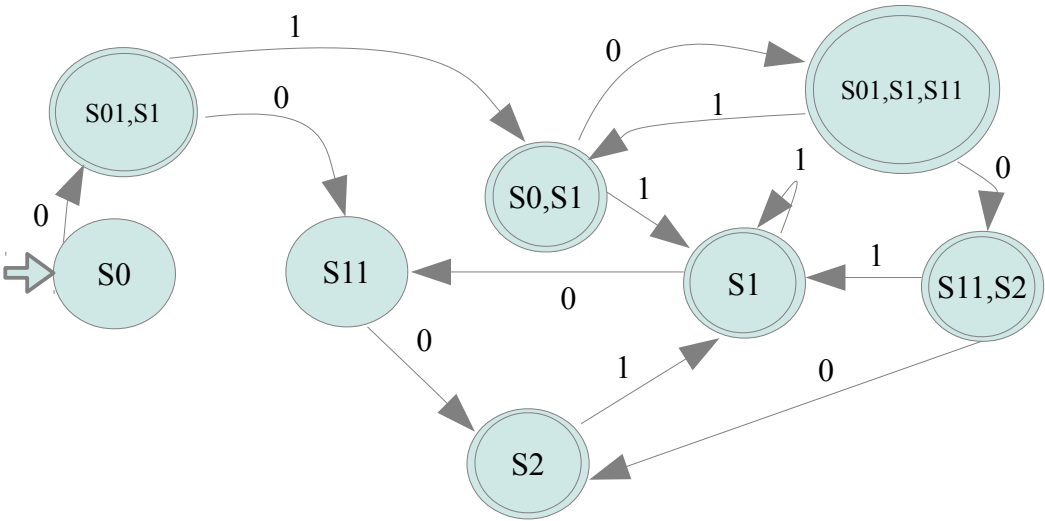
Étape 2: Élimination des  $\epsilon$  transitions  
 $\epsilon$ -fermeture(S0)={S0} ,  $\epsilon$ -fermeture(S01)={S01} ,  $\epsilon$ -fermeture(S1)={S1,S2} ,  $\epsilon$ -fermeture(S11)={S11}  
 $\epsilon$ -fermeture(S2)={S2}

	0	1
S0	S01, S0	/
S01	/	S0
S1	S11	S1
S11	S2	S1



Étape 3: : l'automate déterministe correspondant

	0	1
S0	{S01, S1}	/
{S01, S1}	S11	{S0, S1}
{S0, S1}	{S01, S1, S11}	S1
{S01, S1, S11}	{S11, S2}	{S0, S1}
S1	S11	S1
S11	S2	/
{S11, S2}	S2	S1
S2	/	S1



2ème méthode

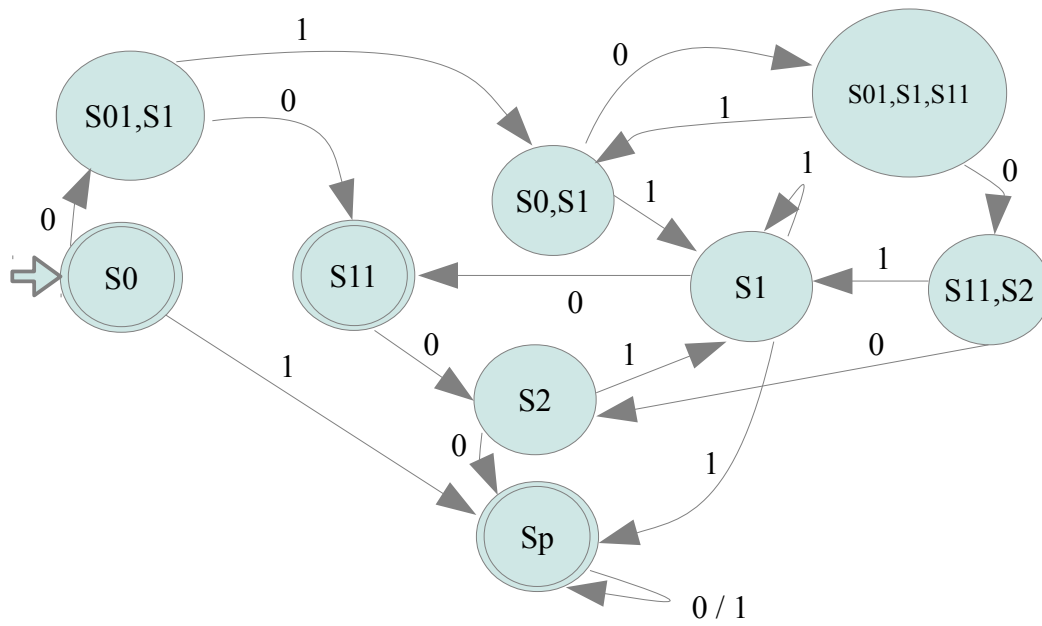
On applique les transitions ensuite la fermeture pour chaque état

	0	1
S0	{S01,S1,S2}	/
{S01,S1,S2}	S11	{S0,S1,S2}
{S0,S1,S2}	{S01,S1,S11,S2}	{S1,S2}
{S01,S1,S11,S2}	{S11,S2}	{S0,S1,S2}
{S1,S2}	S11	{S1,S2}
{S11,S2}	S2	{S1,S2}
S11	S2	/
S2	/	{S1,S2}

2.

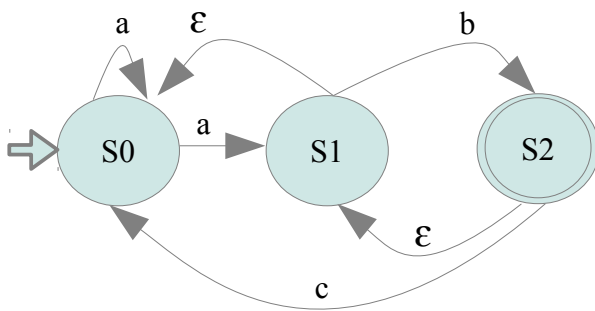
- on ajoute un état puits  $S_p$
- on ajoute les transition  $\{(S0,1,S_p), (S1,1,S_p), (S2,0,S_p)\}$

3.

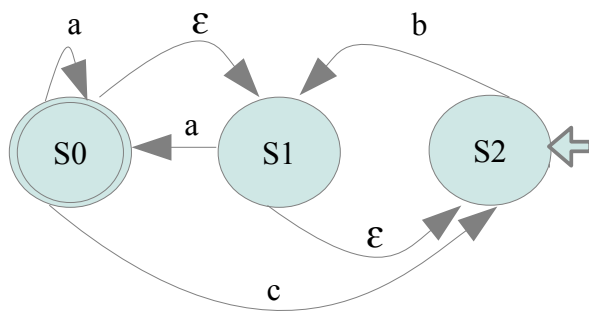


### Exercise 3

1. Automate A

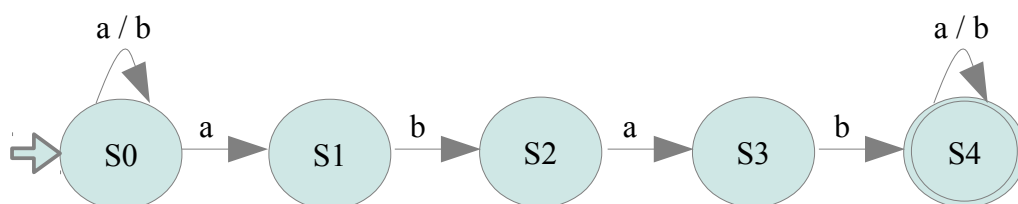


2. Automate B

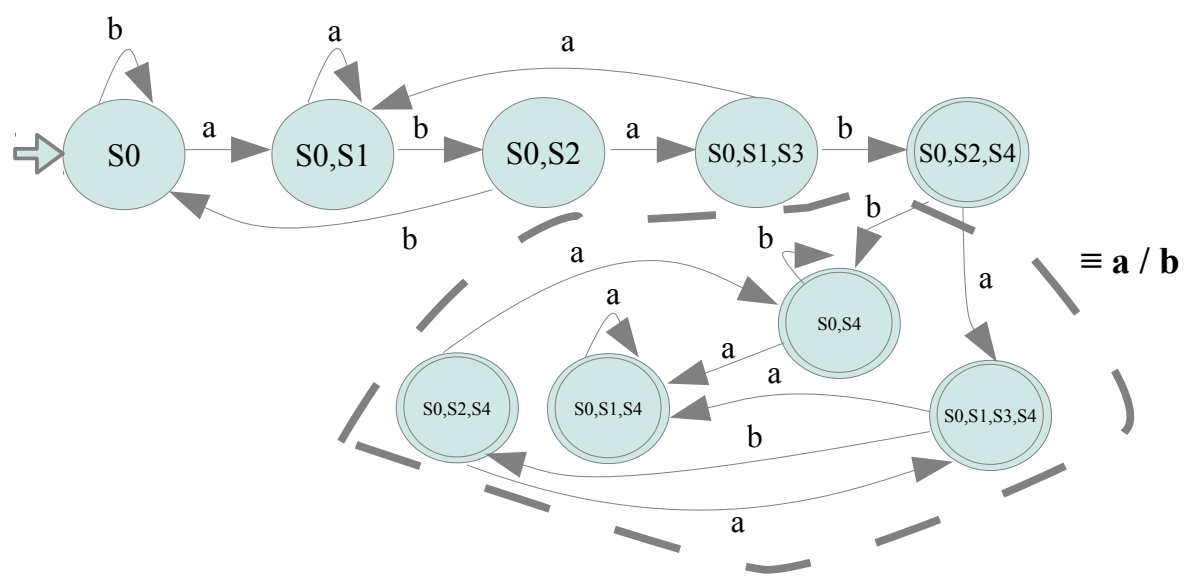


### Exercise 4

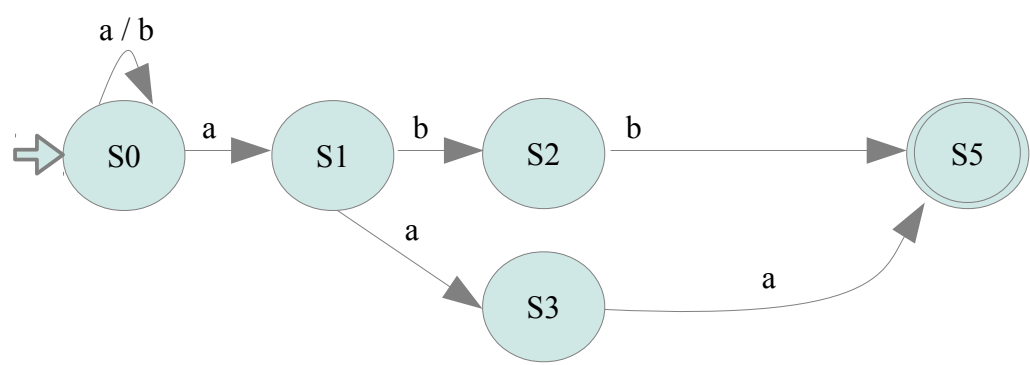
1.



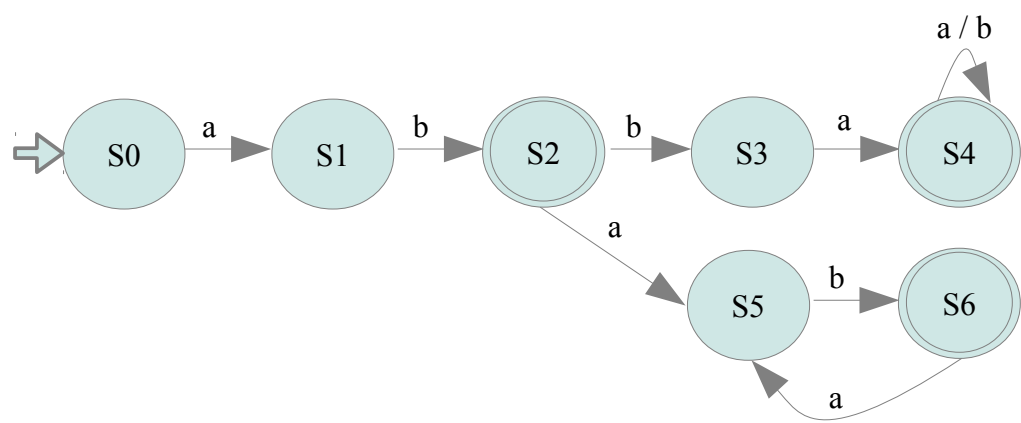
	a	b
S0	{S0,S1}	S0
{S0,S1}	{S0,S1}	{S0,S2}
{S0,S2}	{S0,S1,S3}	S0
{S0,S1,S3}	{S0,S1}	{S0,S2,S4}
{S0,S2,S4}	{S0,S1,S3,S4}	{S0,S4}
{S0,S4}	{S0,S1,S4}	{S0,S4}
{S0,S1,S3,S4}	{S0,S1,S4}	{S0,S2,S4}
{S0,S1,S4}	{S0,S1,S4}	{S0,S2,S4}



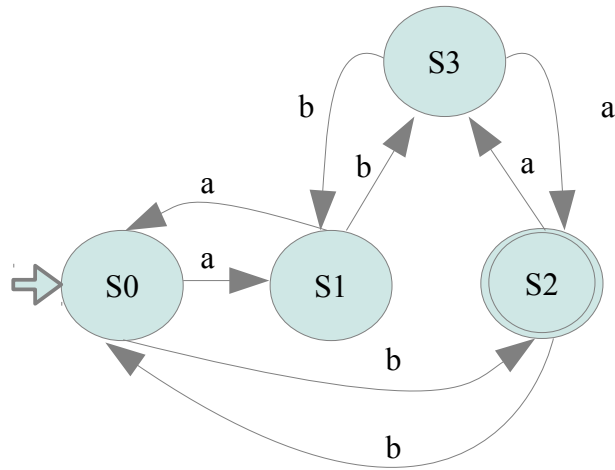
2.



3.

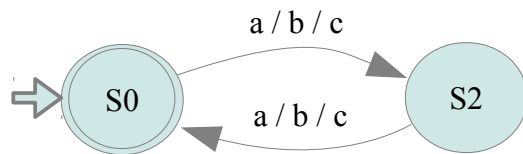


4.



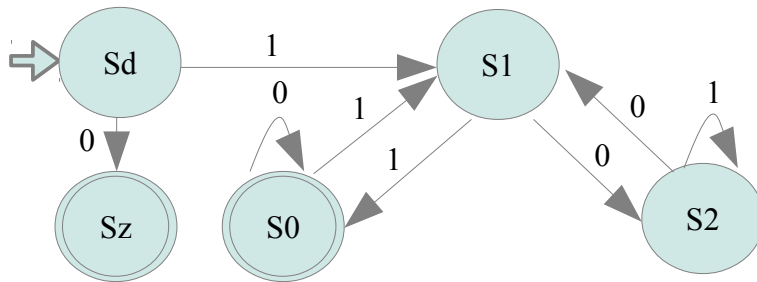
5.

	0≡[2]      A	1≡[2]      B
A		a,b,c
B	a,b,c	



### Exercise 5

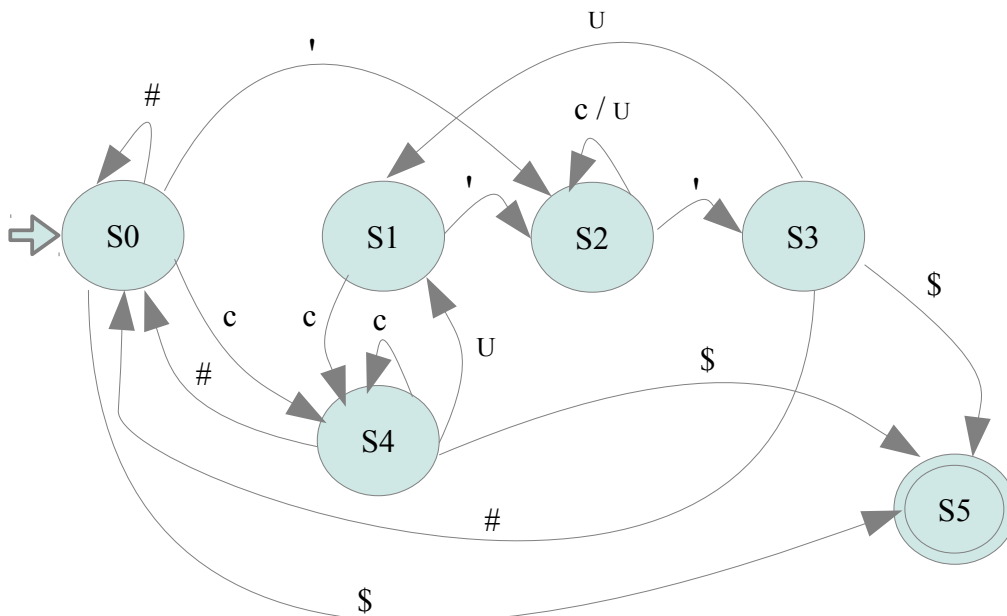
1.



2.  $A < X, (S_0, \dots, S_{p-1}, S_d, S_z), S_d, (S_0, S_z), \Pi >$

$\Pi = \{ (S_d, 0, S_z) ; (S_d, 1, S_1) ; (S_i, 0, S_{(2i \equiv [p])}) ; (S_i, 1, S_{(2i+1 \equiv [p])}) \}$  avec  $p > i \geq 0$

### Exercise 6



## Exercice 7

### Notation des états

'H' pour Homme, 'L' pour Loup, 'C' pour Chèvre et 'S' pour Salade

'/' pour déterminer la position sur les rives

### Notation des conditions

'h' lorsque l'homme traverse seul,

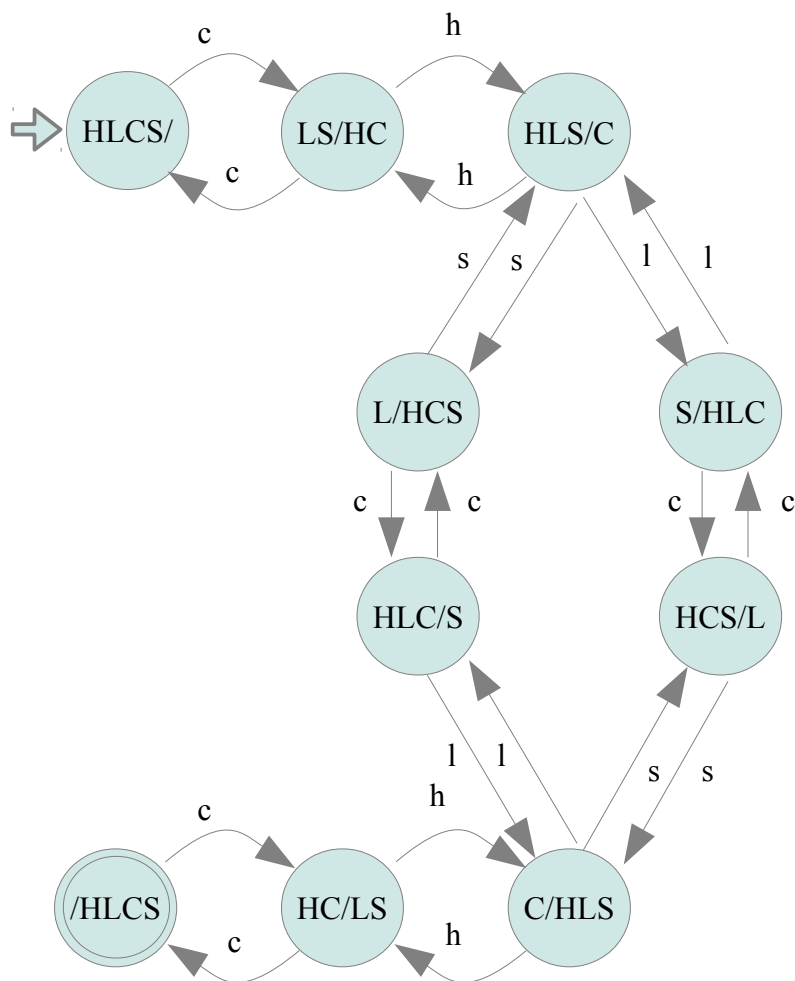
'l' lorsque l'homme et le loup traversent

'c' lorsque l'homme et la chèvre traversent

's' lorsque l'homme et la salade traversent

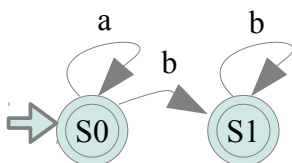
**10 états autorisés :** HLCS/, LS/HC, HLS/C, S/HLC, L/HCS, HCS/L, HLC/S, C/HLS, HC/LS et /HLCS

**6 états interdits :** LC/HS, HS/LC, HL/CS, CS/HL, CLS/H et H/CLS



## Exercice 8

1.



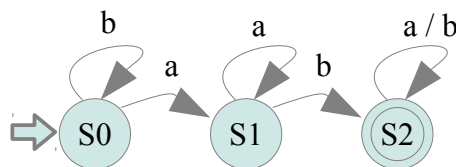
$$a^*b^* \parallel a = a^*b^*$$

$$a^*b^* \parallel b = b^*$$

$$b^* \parallel a = \emptyset$$

$$b^* \parallel b = b^*$$

2.



$$X^*abX^* \parallel a = X^*abX^* \cup bX^*$$

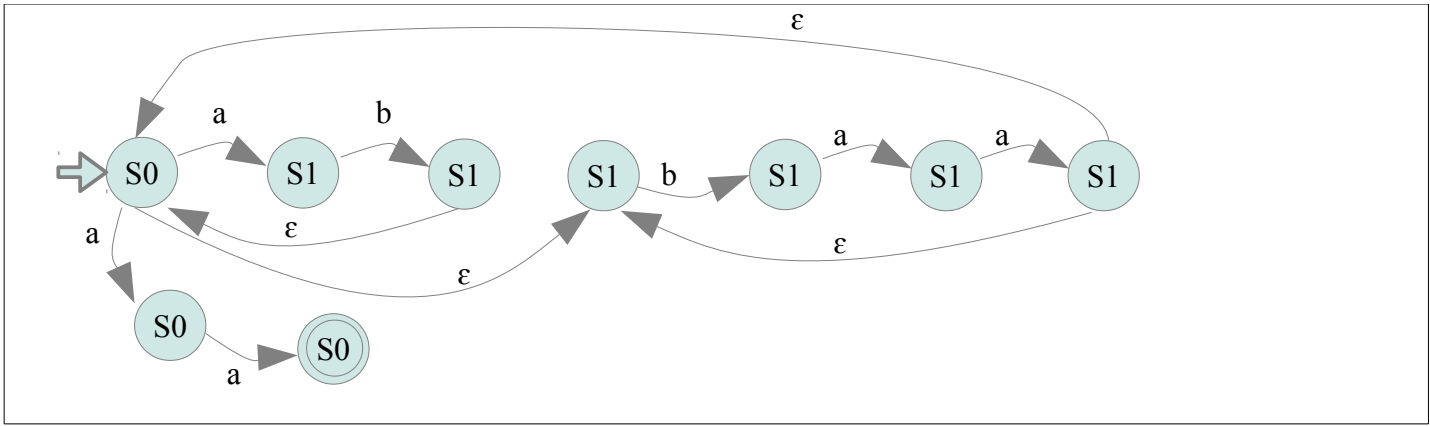
$$X^*abX^* \parallel b = X^*abX^*$$

$$(X^*abX^* \cup bX^*) \parallel a = X^*abX^* \cup bX^*$$

$$(X^*abX^* \cup bX^*) \parallel b = X^*$$

$$X^* \parallel a = X^* \quad X^* \parallel b = X^*$$

3. 1<sup>ère</sup> méthode



2<sup>ème</sup> méthode

$((ab)^*(baa)^*)^*aa$

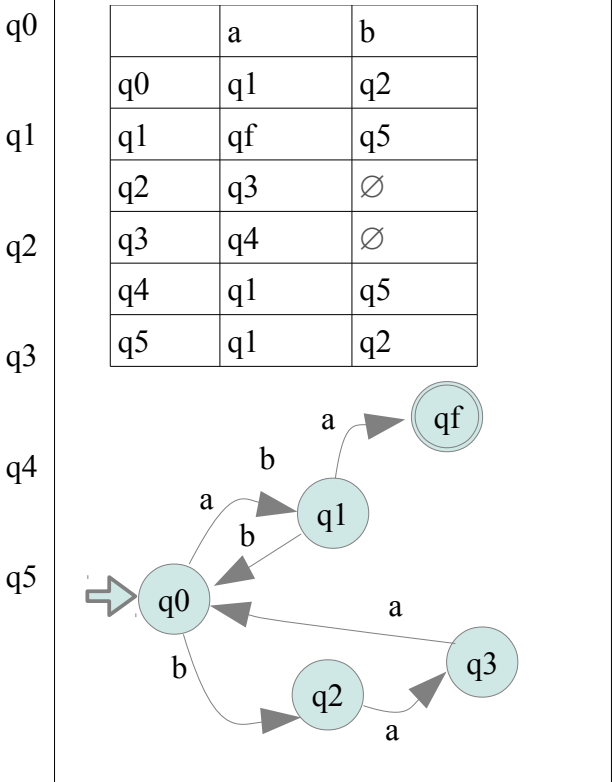
$((a_1b_2)^*(b_3a_4a_5)^*)^*a_6a_7$

begin={a<sub>1</sub>,b<sub>3</sub>,a<sub>6</sub>} end={a<sub>7</sub>}

	a	b
0	{a <sub>1</sub> , a <sub>6</sub> }	{b <sub>3</sub> }
1	∅	{b <sub>2</sub> }
2	{a <sub>1</sub> , a <sub>6</sub> }	{b <sub>3</sub> }
3	{a <sub>4</sub> }	∅
4	{a <sub>5</sub> }	∅
5	{a <sub>1</sub> , a <sub>6</sub> }	{b <sub>3</sub> }
6	{a <sub>7</sub> }	∅
7	∅	∅

3<sup>ème</sup> méthode

$((ab)^*(baa)^*)^*aa \parallel a = b(ab)^*(baa)^*((ab)^*(baa)^*)^*aa \mid a$   
 $((ab)^*(baa)^*)^*aa \parallel b = aa(baa)^*((ab)^*(baa)^*)^*aa$   
 $b((ab)^*(baa)^*)^+aa \mid a \parallel a = \epsilon$   
 $b((ab)^*(baa)^*)^+aa \mid a \parallel b = ((ab)^*(baa)^*)^+aa$   
 $aa(baa)^*((ab)^*(baa)^*)^*aa \parallel a = a(baa)^*((ab)^*(baa)^*)^*aa$   
 $aa(baa)^*((ab)^*(baa)^*)^*aa \parallel b = \emptyset$   
 $a(baa)^*((ab)^*(baa)^*)^*aa \parallel a = (baa)^*((ab)^*(baa)^*)^*aa$   
 $a(baa)^*((ab)^*(baa)^*)^*aa \parallel b = \emptyset$   
 $(baa)^*((ab)^*(baa)^*)^*aa \parallel a = b((ab)^*(baa)^*)^+aa \mid a$   
 $(baa)^*((ab)^*(baa)^*)^*aa \parallel b = aa(baa)^*((ab)^*(baa)^*)^*aa$   
 $((ab)^*(baa)^*)^+aa \parallel a = b((ab)^*(baa)^*)^+aa \mid a$   
 $((ab)^*(baa)^*)^+aa \parallel b = aa(baa)^*((ab)^*(baa)^*)^*aa$   
 $((ab)^*(baa)^*)^+aa \equiv ((ab)^*(baa)^*)^*aa$   
 $(baa)^*((ab)^*(baa)^*)^*aa \equiv ((ab)^*(baa)^*)^*aa$   
 remplace q<sub>5</sub> et q<sub>4</sub> avec q<sub>0</sub> dans le tableau



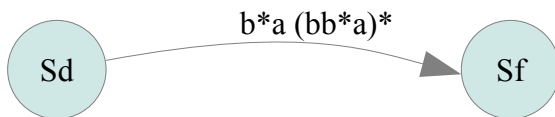
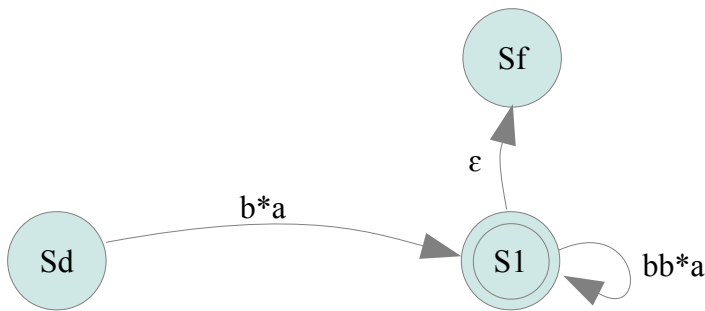
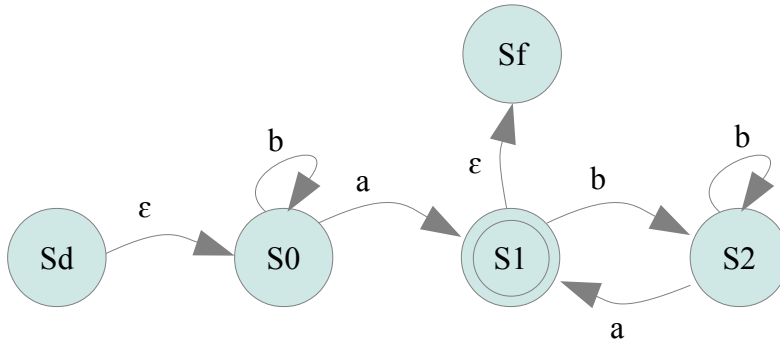
## Exercise 9

a.

1<sup>ère</sup> méthode

$L0 = bL0 \cup aL1$ $L1 = bL2 \cup \epsilon$ $L2 = bL2 \cup aL1$	$L2 = bL2 \cup a(bL2 \cup \epsilon)$ $L2 = bL2 \cup abL2 \cup a$ $L2 = (b \cup ab)L2 \cup a$ $L2 = (b \cup ab)^*a$	$L1 = bL2 \cup \epsilon$ $L1 = b((b \cup ab)^*a) \cup \epsilon$ $L0 = bL0 \cup a(b((b \cup ab)^*a) \cup \epsilon)$	$L0 = b^*a(b((b \cup ab)^*a) \cup \epsilon)$ $L0 = b^*a b(b \cup ab)^*a \cup b^*a$
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2<sup>ème</sup> méthode



b.

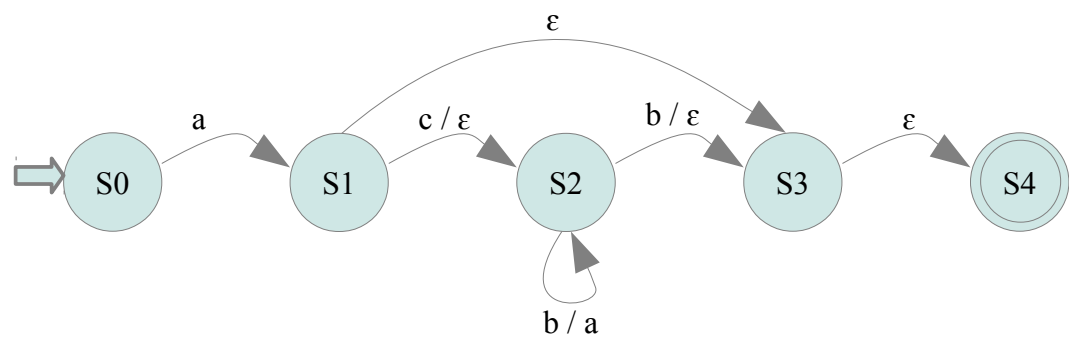
$L0 = cL0 \cup aL1$ $L1 = (a \cup b)L1 \cup bL0 \cup cL2$ $L2 = aL3 \cup \epsilon$ $L3 = bL1$	$L0 = c^*aL1$ $L1 = (a \cup b)L1 \cup bc^*aL1 \cup cL2$ $L1 = (a \cup b \cup bc^*a)L1 \cup cL2$ $L1 = (a \cup b \cup bc^*a)^*cL2$	$L2 = aL3 \cup \epsilon$ $L3 = b(a \cup b \cup bc^*a)^*cL2$ $L3 = b(a \cup b \cup bc^*a)^*c(aL3 \cup \epsilon)$ $L3 = b(a \cup b \cup bc^*a)^*c(aL3 \cup \epsilon)$ $L3 = b(a \cup b \cup bc^*a)^*caL3 \cup b(a \cup b \cup bc^*a)^*c$ $L3 = (b(a \cup b \cup bc^*a)^*ca)^*b(a \cup b \cup bc^*a)^*c$
$L3 = (b(a \cup b \cup bc^*a)^*ca)^*b(a \cup b \cup bc^*a)^*c$ $L2 = aL3 = a((b(a \cup b \cup bc^*a)^*ca)^*b(a \cup b \cup bc^*a)^*c)$ $L1 = (a \cup b \cup bc^*a)^* \cup cL2 = (a \cup b \cup bc^*a)^* \cup ca((b(a \cup b \cup bc^*a)^*ca)^*b(a \cup b \cup bc^*a)^*c)$ $L0 = c^*a((a \cup b \cup bc^*a)^* \cup ca((b(a \cup b \cup bc^*a)^*ca)^*b(a \cup b \cup bc^*a)^*c))$		

c.

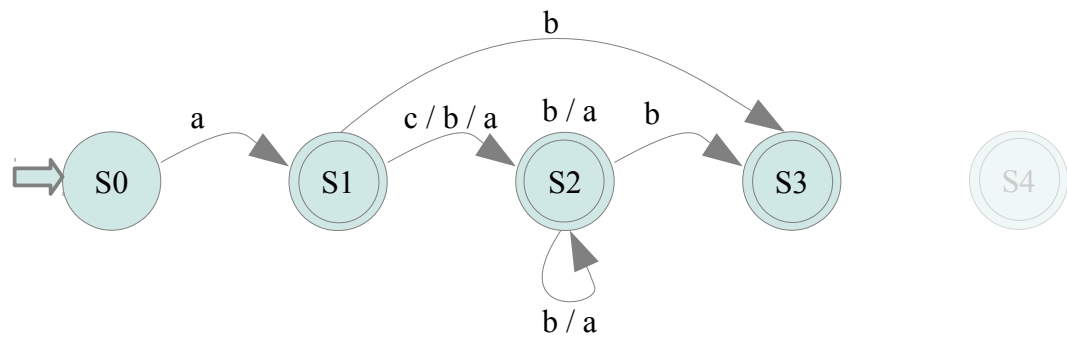
$L0 = aL1 \cup bL2$ $L1 = bL2$ $L2 = aL1 \cup \epsilon$	$L0 = aL1 \cup bL2$ $L1 = bL2$ $L2 = abL2 \cup \epsilon$	$L2 = (ab)^*$ $L1 = b(ab)^*$ $L0 = ab(ab)^* \cup b(ab)^* = (ab \cup b)(ab)^*$
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Exercice : élimination transitions spontanées



élimination transitions spontanées arrière en utilisant  $\epsilon$ -fermeture



élimination transitions spontanées avant

