

EXERCICE 1

$L1 = \{ ww^r, w \in \{0, 1\}^* \}$	
$X = \{0, 1\}, Y = X, S = \{S_0, S_1, S_f\}, F = \{S_f\}$ et II : $\# S_0 0 \rightarrow \# 0 S_0$ $\# S_0 1 \rightarrow \# 1 S_0$ $\# S_0 \rightarrow \# S_f$ $0 S_0 0 \rightarrow 0 0 S_0$ $1 S_0 0 \rightarrow 1 0 S_0$	$0 S_0 1 \rightarrow 0 1 S_0$ $1 S_0 1 \rightarrow 1 1 S_0$ $0 S_0 0 \rightarrow S_1$ $1 S_0 1 \rightarrow S_1$ $0 S_1 0 \rightarrow S_1$ $1 S_1 1 \rightarrow S_1$ $\# S_1 \rightarrow S_f$
$L2 = \{(01)^j aa 0^{i+j}, i, j \geq 0\}$	
$X = \{0, 1, a\}, Y = X, S = \{S_0, S_1, S_3, S_f\}, F = \{S_f\}$ et II : $\# S_0 0 \rightarrow \# 0 S_1$ $0 S_1 1 \rightarrow 0 S_0$ $0 S_0 0 \rightarrow 00 S_1$ $\# S_0 a \rightarrow \# S_2$ $0 S_0 a \rightarrow 0 S_2$	$\# S_2 a \rightarrow \# S_3$ $0 S_2 a \rightarrow 0 S_3$ $0 S_3 0 \rightarrow S_3$ $\# S_3 0 \rightarrow \# S_3$ $\# S_3 \rightarrow \# S_f$

EXERCICE 2

$L = \{wcw_1w_2 / w, w_1, w_2 \in X^* \text{ avec } w_1 = \text{Pref}(w^r) \text{ et } |w_1| + |w_2| = |w| \}$

EXERCICE 3

Une grammaire est propre si elle est ε -libre, d'épouvée de symboles inutiles, sans cycle

$G < \{a, b\}, \{S, A\}, S, P > P = \{S \rightarrow Ba / A b / \varepsilon / aDb \ A \rightarrow AA a / \varepsilon / Sa \ B \rightarrow S b / \varepsilon / D \rightarrow aDb / AD\}$		
$S \rightarrow Ba / A b / \varepsilon / aDb$ $A \rightarrow AA a / \varepsilon / Sa$ $B \rightarrow S b / \varepsilon$ $D \rightarrow aDb / AD$	$S' \rightarrow S$ $S \rightarrow Ba / A b / \varepsilon / aDb$ $A \rightarrow AA a / \varepsilon / Sa$ $B \rightarrow S b / \varepsilon$ $D \rightarrow aDb / AD$	
$S' \rightarrow S$ $S \rightarrow Ba / A b / \varepsilon / aDb$ $A \rightarrow AA a / \varepsilon / Sa$ $B \rightarrow S b / \varepsilon$ $D \rightarrow aDb / AD$	$S' \rightarrow S / \varepsilon$ $S \rightarrow Ba / A b / aDb / a / b$ $A \rightarrow AA a / A a / a / Sa / a$ $B \rightarrow S b / b$ $D \rightarrow aDb / AD$	
$S' \rightarrow S / \varepsilon$ $S \rightarrow Sba / ba / A b / aDb / a / b$ $A \rightarrow AA a / A a / a / Sa / a$ $D \rightarrow aDb / AD$	forme normale de Chomsky $S' \rightarrow S / \varepsilon$ $S \rightarrow SS_1 / B'A' / A B' / a / b$ $S_1 \rightarrow B'A'$ $A \rightarrow AS_2 / AA' / SA' / a$ $S_2 \rightarrow AA'$ $A' \rightarrow a$ $B' \rightarrow b$	

$G < \{a,b\}, \{S\}, S, P > P = \{ S \rightarrow aSbS / bSaS / \epsilon \}$	
$S \rightarrow aSbS / bSaS / \epsilon$ $S \rightarrow aSbS / bSaS / abS / baS / aSb / bSa / ab / ba$	
forme normale de Chomsky $S \rightarrow ASBS / BSAS / ABS / BAS / ASB / BSA / AB / BA$ $A \rightarrow a$ $B \rightarrow b$ $S \rightarrow AX_1 / BX_3 / AX_2 / BX_4 / AX_5 / BX_6 / AB / BA$ $X_1 \rightarrow SX_2$ $X_2 \rightarrow BS$ $X_3 \rightarrow SX_4$ $X_4 \rightarrow AS$ $X_5 \rightarrow SB$ $X_6 \rightarrow SA$ $A \rightarrow a$ $B \rightarrow b$	
$G < \{a,b\}, \{S,A,C\}, S, P > P = \{ S \rightarrow AS / bC \ A \rightarrow a / \epsilon \ C \rightarrow aC / a / \epsilon \}$	
$S \rightarrow AS / bC$ $A \rightarrow a / \epsilon$ $C \rightarrow aC / a / \epsilon$	$S \rightarrow AS / bC / b$ $A \rightarrow a$ $C \rightarrow aC / a$
forme normale de Chomsky $S \rightarrow AS / BC / b$ $C \rightarrow AC / a$ $A \rightarrow a$ $B \rightarrow b$	
$G < \{a,b,c\}, \{S,A,B,C,D,E\}, S, P > P = \{ S \rightarrow aS / BS / \epsilon \ B \rightarrow bAb / SaS \ A \rightarrow a / Sa \}$	
$S \rightarrow aS / BS / \epsilon$ $B \rightarrow bAb / SaS$ $A \rightarrow a / Sa$	$S' \rightarrow S / \epsilon$ $S \rightarrow aS / BS / B / a$ $B \rightarrow bAb / SaS / aS / Sa / a$ $A \rightarrow a / Sa$
$S' \rightarrow S / \epsilon$ $S \rightarrow aS / BS / bAb / SaS / aS / Sa / a$ $B \rightarrow bAb / SaS / aS / Sa / a$ $A \rightarrow a / Sa$	forme normale de Chomsky $S' \rightarrow S / \epsilon$ $S \rightarrow XS / BS / YAY / SXS / XS / SX / a$ $B \rightarrow YAY / SXS / XS / SX / a$ $A \rightarrow a / SX$ $X \rightarrow a$ $Y \rightarrow b$
$S' \rightarrow S / \epsilon$ $S \rightarrow XS / BS / YX_1 / SX_2 / XS / SX / a$ $B \rightarrow YX_1 / SX_2 / XS / SX / a$ $A \rightarrow a / SX$ $X \rightarrow a$ $Y \rightarrow b$	$X_1 \rightarrow AY$ $X_2 \rightarrow XS$

EXERCICE 4

$S \rightarrow aSb / aSa / bSa / bSb / aAa / bAb / c$
 $A \rightarrow aAa / bAb / c$

forme normale de Chomsky

$S \rightarrow XX_1 / XX_2 / YX_2 / YX_1 / XX_3 / YX_4 / c$
 $A \rightarrow XX_3 / YAY / c$

$X_1 \rightarrow SY$

$X_2 \rightarrow SX$

$X_3 \rightarrow AX$

$X_4 \rightarrow AY$

$X \rightarrow a$

$Y \rightarrow b$

EXERCICE 5

$G < \{a,b,c\}, \{S,A,B,C\}, S, P > P: \{ S \rightarrow AA / a / b \ A \rightarrow SS / b \ B \rightarrow BC / AB \ C \rightarrow aB / b \}$

$S \rightarrow AA / a / b$

$A \rightarrow SS / b$

$G < \{a,b,c,d\}, \{S,A\}, S, P > P: \{ S \rightarrow aSAb / bSS / d \ A \rightarrow cASaA / bcd \}$

$S \rightarrow aSAb / bSS / d$

$A \rightarrow cASaA / bcd$

$S \rightarrow XSAY / YSS / d$

$A \rightarrow ZASXA / YZT$

$X \rightarrow a$

$Y \rightarrow b$

$Z \rightarrow c$

$T \rightarrow d$

$S \rightarrow XX_1 / YX_3 / d$

$A \rightarrow ZX_4 / YX_7$

$X_1 \rightarrow SX_2$

$X_2 \rightarrow AY$

$X_3 \rightarrow SS$

$X_4 \rightarrow AX_5$

$X_5 \rightarrow SX_6$

$X_6 \rightarrow XA$

$X_7 \rightarrow ZT$

$X \rightarrow a$

$Y \rightarrow b$

$Z \rightarrow c$

$T \rightarrow d$

$G < \{a,b\}, \{S,A,B,C\}, S, P > P: \{ S \rightarrow bA / aB \ A \rightarrow bAA / aS/a \ B \rightarrow aBB / bS / b \}$

$S \rightarrow YA / XB$

$A \rightarrow YAA / XS/a$

$B \rightarrow XBB / YS / b$

EXERCICE 6

$G = \langle X, V, P, S \rangle$ où $X = \{a, b\}$, $V = \{S, A\}$ et $P = \{ S \rightarrow a S / A a / \varepsilon \quad A \rightarrow A a / b \}$	
$S \rightarrow S_1 / \varepsilon$ $S_1 \rightarrow a S_1 / A_2 a / \varepsilon$ $A_2 \rightarrow A_2 a / b$	$S \rightarrow S_1 / \varepsilon$ $S_1 \rightarrow a S_1 / A_2 a / a$ $A_2 \rightarrow A_2 a / b$
$S \rightarrow S_1 / \varepsilon$ $S_1 \rightarrow a S_1 / A_2 a / a$ $A_2 \rightarrow A_2 a / b$	$S \rightarrow S_1 / \varepsilon$ $S_1 \rightarrow a S_1 / A_2 a / a$ $A_2 \rightarrow b / bT$ $T \rightarrow aT/a$
$S \rightarrow S_1 / \varepsilon$ $S_1 \rightarrow a S_1 / ba / bTa / a$ $A_2 \rightarrow b / bT$ $T \rightarrow aT / a$	
$G \langle X, V, P, S \rangle \quad P = \{ S \rightarrow A B \quad A \rightarrow B S / b \quad B \rightarrow S A / a \}$	
$S_1 \rightarrow A_2 B_3$ $A_2 \rightarrow B_3 S_1 / b$ $B_3 \rightarrow S_1 A_2 / a$	$S_1 \rightarrow A_2 B_3$ $A_2 \rightarrow B_3 S_1 / b$ $B_3 \rightarrow B_3 S_1 B_3 A_2 / b B_3 A_2 / a$
$S_1 \rightarrow A_2 B_3$ $A_2 \rightarrow B_3 S_1 / b$ $B_3 \rightarrow b B_3 A_2 / a / b B_3 A_2 T / a T$ $T \rightarrow B_3 S_1 B_3 A_2 T / B_3 S_1 B_3 A_2$	$S_1 \rightarrow A_2 B_3$ $A_2 \rightarrow b B_3 A_2 S_1 / a S_1 / b B_3 A_2 T S_1 / a T S_1 / b$ $B_3 \rightarrow b B_3 A_2 / a / b B_3 A_2 T / a T$ $T \rightarrow B_3 S_1 B_3 A_2 T / B_3 S_1 B_3 A_2$
$S_1 \rightarrow b B_3 A_2 S_1 B_3 / a S_1 B_3 / b B_3 A_2 T S_1 B_3 / a T S_1 B_3 / b B_3$ $A_2 \rightarrow b B_3 A_2 S_1 / a S_1 / b B_3 A_2 T S_1 / a T S_1 / b$ $B_3 \rightarrow b B_3 A_2 / a / b B_3 A_2 T / a T$ $T \rightarrow B_3 S_1 B_3 A_2 T / B_3 S_1 B_3 A_2$	
$S_1 \rightarrow b B_3 A_2 S_1 B_3 / a S_1 B_3 / b B_3 A_2 T S_1 B_3 / a T S_1 B_3 / b B_3$ $A_2 \rightarrow b B_3 A_2 S_1 / a S_1 / b B_3 A_2 T S_1 / a T S_1 / b$ $B_3 \rightarrow b B_3 A_2 / a / b B_3 A_2 T / a T$ $T \rightarrow b B_3 A_2 S_1 B_3 A_2 T / a S_1 B_3 A_2 T / b B_3 A_2 T S_1 B_3 A_2 T / a T S_1 B_3 A_2 T /$ $b B_3 A_2 S_1 B_3 A_2 / a S_1 B_3 A_2 / b B_3 A_2 T S_1 B_3 A_2 / a T S_1 B_3 A_2$	

EXERCICE 7

$L = \{a^i b^j c^k \text{ tq } i \geq 2j + 3k\}$	
$X = \{a, b, c\}$, $Y = \{c\}$, $S = \{S_0, S_1, S_2, S_3, S_4, S_5, S_f\}$, $F = \{S_f\}$ et II : $\# S_0 a \rightarrow \# a S_0$ $a S_0 a \rightarrow a a S_0$ $\# S_0 \rightarrow \# S_f$ $a S_0 \rightarrow a S_f$	$a S_1 b \rightarrow S_2$ $a S_2 b \rightarrow S_1$ $a S_2 \rightarrow S_f$ $\# S_2 \rightarrow \# S_f$ $a S_2 \rightarrow S_3$ $a S_3 c \rightarrow S_4$ $a S_4 c \rightarrow S_5$ $a S_5 c \rightarrow S_3$ $a S_3 \rightarrow S_f$ $\# S_3 \rightarrow \# S_f$

EXERCICE 8

1.

$S \rightarrow SaB / bB / \varepsilon$

$B \rightarrow aS / aSB / BaB / \varepsilon$

$L(G) = \{w / w \in X^* \text{ avec } |\text{pref}(w)|_a + 1 \geq |\text{pref}(w)|_b\}$

2.

$G \langle X = \{a, b\}, V = \{S, A, B, D, F\}, P, S \rangle$ $P = \{ S \rightarrow SaB / bB / aDB / F \quad A \rightarrow aAB / aA / \varepsilon \quad B \rightarrow aS / aSB / BaB / \varepsilon \quad D \rightarrow aD / Da \quad F \rightarrow \varepsilon \}$	
$S \rightarrow SaB / bB / aDB / F$ $A \rightarrow aAB / aA / \varepsilon$ $B \rightarrow aS / aSB / BaB / \varepsilon$ $D \rightarrow aD / Da$ $F \rightarrow \varepsilon$	$S \rightarrow SaB / bB / F$ $B \rightarrow aS / aSB / BaB / \varepsilon$ $F \rightarrow \varepsilon$
$S' \rightarrow S / \varepsilon$ $S \rightarrow SaB / Sa / aB / bB / a / b$ $B \rightarrow aS / aSB / BaB / Ba / aB / a$	$S' \rightarrow S / \varepsilon$ $S \rightarrow SXB / SX / XB / YB / a / b$ $B \rightarrow XS / XSB / BXB / BX / XB / a$ $X \rightarrow a$ $Y \rightarrow b$
$S' \rightarrow S / \varepsilon$ $S \rightarrow SX_1 / SX / XB / YB / a / b$ $B \rightarrow XS / XX_2 / BX_1 / BX / XB / a$ $X \rightarrow a$ $Y \rightarrow b$ $X_1 \rightarrow XB$ $X_2 \rightarrow SB$	

3.

$S' \rightarrow S / \varepsilon$ $S_1 \rightarrow S_1aB_2 / S_1a / aB_2 / bB_2 / a / b$ $B_2 \rightarrow aS_1 / aS_1B / B_2aB_2 / B_2a / aB_2 / a$	
$S' \rightarrow S / \varepsilon$ $S_1 \rightarrow aB_2 / bB_2 / a / b / aB_2X / bB_2X / aX / bX$ $X \rightarrow S_1aB_2 / S_1a / S_1aB_2X / S_1aX$ $B_2 \rightarrow aS_1 / aS_1B / aB_2 / a / aS_1Y / aS_1BY / aB_2Y / aY$ $Y \rightarrow B_2aB_2 / B_2a / B_2aB_2Y / B_2aY$	
$S_1 \rightarrow S_1aB_2 / S_1a / S_1aB_2X / S_1aX$ $X \rightarrow aB_2aB_2 / bB_2aB_2 / aaB_2 / baB_2 / aB_2XaB_2 / bB_2XaB_2 / aX / bXaB_2 /$ $aB_2a / bB_2a / aa / ba / aB_2Xa / bB_2Xa / aXa / bXa /$ $aB_2aB_2X / bB_2aB_2X / aaB_2X / baB_2X / aB_2XaB_2X / bB_2XaB_2X / aXaB_2X / bXaB_2X /$ $aB_2aX / bB_2aX / aaX / baX / aB_2XaX / bB_2XaX / aXaX / bXaX$	
$B_2 \rightarrow aS_1 / aS_1B / aB_2 / a / aS_1Y / aS_1BY / aB_2Y / aY$ $Y \rightarrow aS_1aB_2 / aS_1BaB_2 / aB_2aB_2 / aaB_2 / aS_1YaB_2 / aS_1BYaB_2 / aB_2YaB_2 / aYaB_2 /$ $aS_1a / aS_1Ba / aB_2a / aa / aS_1Ya / aS_1BYa / aB_2Ya / aYa /$ $aS_1aB_2Y / aS_1BaB_2Y / aB_2aB_2Y / aaB_2Y / aS_1YaB_2Y / aS_1BYaB_2Y / aB_2YaB_2Y / aYaB_2Y /$ $aS_1aY / aS_1BaY / aB_2aY / aaY / aS_1YaY / aS_1BYaY / aB_2YaY / aYaY$	