Integer Linear Programming Model for HR Optimization

Sets and Indices

- W: Set of workers, indexed by w.
- S: Set of shifts, indexed by s.
- L: Set of locations, indexed by l.
- D: Set of weekdays (Monday, Tuesday, Wednesday, Thursday, Friday), indexed by d.

Parameters

- cost$[w, s, l]$: Cost of assigning worker w to shift s at location l.
- required_skills(s, l): Set of required skills for shift s at location l.
- worker_skills(w): Set of skills possessed by worker w.
- min_workers(s, l): Minimum number of workers required for shift s at location l.
- shift_durations(s): Duration of shift s in hours.
- max_hours(w): Maximum weekly working hours for worker w.
- $daily\_shifts(d)$: set of shifts that are in that day d.

Decision Variables

- $x_{w,s,l} \in \{0, 1\}$: Binary variable equal to 1 if worker w is assigned to shift s at location l, and 0 otherwise.
- $y_{w,l} \in \{0, 1\}$: Binary variable equal to 1 if worker w is assigned to location l during the entire week, and 0 otherwise.

Objective Function

$$\text{Minimize} \quad \sum_{w \in W} \sum_{s \in S} \sum_{l \in L} \text{cost}[w, s, l] \cdot x_{w,s,l} \tag{1}$$

Constraints

Skill Matching

$$x_{w,s,l} = 0 \quad \text{if required\_skills}(s, l) \not\subseteq \text{worker\_skills}(w) \quad \forall w \in W, s \in S, l \in L \tag{2}$$

Minimum Workers Per Shift-Location

$$\sum_{w \in W} x_{w,s,l} \geq \text{min\_workers}(s,l) \quad \forall s \in S, l \in L \tag{3}$$

One Shift Per Day

$$\sum_{s \in daily\_shifts(d)} \sum_{l \in L} x_{w,s,l} \leq 1 \quad \forall w \in W, d \in D \tag{4}$$

Maximum Weekly Hours

$$\sum_{s \in S} \sum_{l \in L} \text{shift\_durations}(s) \cdot x_{w,s,l} \leq \text{max\_hours}(w) \quad \forall w \in W \tag{5}$$

Skill Coverage

$$\sum_{w \in \{w \in W | skill \in \text{worker\_skills}(w)\}} x_{w,s,l} \geq 1 \quad \forall s \in S, l \in L, skill \in \text{required\_skills}(s,l) \tag{6}$$

One Location Per Week

$$\sum_{s \in S} x_{w,s,l} \leq |S| \cdot y_{w,l} \quad \forall w \in W, l \in L \tag{7}$$

where $|S|$ represents the cardinality of S.

One Location Per Worker

$$\sum_{l \in L} y_{w,l} = 1 \quad \forall w \in W \tag{8}$$