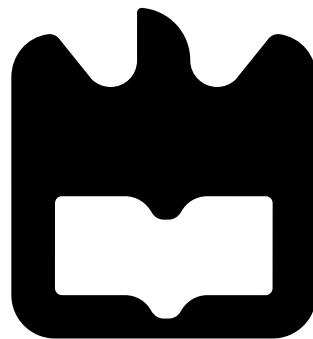João Pedro Dias
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Phenomenology of the minimal B - L extension of the Standard Model

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Resumo

Teoria quântica de campo é o nome dado à atualmente aceite teoria quântica que descreve os modelos de interações de partículas e esta é uma combinação da teoria clássica de campo, relatividade restrita e mecânica quântica. O modelo de partículas amplamente aceite para a descrição de partículas é o chamado modelo padrão que em 2012 teve sua ultima partícula descoberta. No entanto, o consenso é também que este modelo está incompleto levando a um vasto interesse em encontrar um novo modelo que explique todas as novas observações experimentais como massas não nulas para neutrinos e a existência de matéria negra. Motivados por tal, neste trabalho iremos examinar o modelo padrão e discutir as suas falhas, e feito isto, iremos estudar uma possível extensão conhecida como o modelo B-L-SM. Iremos apresentar como este modelo resolve o problema de massa de neutrinos e que outras diferenças esperamos observar em relação ao modelo padrão. Esta análise não será puramente teórica e, usando uma série de ferramentas computacionais iremos fazer um curto estudo fenomenológico do setor escalar e de gauge do modelo B-L-SM.

Abstract

Quantum field theory is the name of the currently accepted quantum theory that describes models of physical particle interactions. It is a combination of classical field theory, special relativity and quantum mechanics. The currently widely accepted model describing particle interactions is the Standard Model that, in 2012, had its last missing particle discovered. However, the Standard Model is currently thought to be incomplete due to experimental observations like non vanishing neutrino masses and the existence of dark matter. Motivated by this, in this essay, we will examine the Standard Model and discuss its shortcomings with the goal of introducing the B-L-SM model. We will discuss how this model fixes the neutrino mass problem and what other differences we expect to see in relation to the Standard Model. This analysis will not be purely theoretical since we will use an array of computer tools to also study the gauge and scalar sector to the B-L-SM model

Contents

List of Figures

List of Tables

1 Introduction

When the the Large Hadron Collider (LHC) first came online it opened the gate to new discoveries in high energy Particle Physics. Since then it has not only found the last missing particle of the Standard Model (SM), the Higgs boson, but indicated to physicists that there are possibly new horizons for particle physics beyond the SM. This theory is accurate for most cases, however the current consensus is that it is incomplete.

The experimental data is mounting and the SM is full of inconsistencies, the most damning evidence against it being the observed spectrum of neutrino masses, the existence of dark matter and the observed matter-antimatter asymmetry. Another telling bit that the SM might be incomplete is the presence an accidental symmetry, and it is now widely believed that extensions to the SM are necessary. New theories can be formulated that include this additional symmetry, allowing for the solutions to the aforementioned problems. During this essay we will be taking a close look at one of these extensions, the B-L-SM model.

Our goal is then to first introduce the Standard Model, examine its mechanics and then comparing it to the B-L-SM by exploring how and where it differs and how it can address some of the SM shortcomings. Our analysis will not be only analytical. We will be using a series of computer tools to also study how well our model matches real observed results at particle acceleration and collision experiments like ATLAS. We will follow a bottom up approach and use the $U(1)_{B-L}$ extension of the SM. The model under study will contain an exotic boson denoted as Z prime, which is associated to the addition of a new unitary gauge group. We will later see that this is an extension at the TeV scale, meaning that the new $U(1)$ gauge field will lead to a mass at the TeV scale proportional to the vacuum expectation value (VEV) af a new scalar that will spontaneously break, $U(1)_{B-L}$. This will be reflected in the mass spectrum of the exotic boson and additional particles stemming from interactions with the broken gauge field.

1.1 Basic concepts

The complexity behind the matters approached in this essay is not to be underestimated, thus we begin with a sizeable review of fundamental concepts.

We will begin by introducing Classical field theory, which can be interpreted as the application of classical dynamics to physical fields and understand how these physical fields interact [2]. After which we will introduce the famous Noether's theorem as we begin our study of the concept of symmetry in the context of system dynamics.

1.1.1 Classical Field Theory

To further elaborate on the reasons behind the study of classical field theory, we will introduce this theory as a means to familiarize ourselves with physical fields and their dynamics. Recall that, from the early quantum mechanical theories, situations like the non-relativistic or relativistic treatment of a "single particle" with the Klein-Gordon equation led to a great deal of problems such as negative energy states, a non

zero value of the propagator of a particle for a point outside the light cone allowing the particle to break causality. Quantum Field theory came to later fix these issues through the union of quantum mechanics and classical field theory, creating quantum field theory.

Moving onwards to definitions, we can conceptualise a physical field as an assignment of a physical quantity to some representation of space. As a quick example of a classical field we can think of the value of temperature in a volume, or as a vector field describing the wind direction and intensity, note how one is a normal vector field and another is a scalar field even though they are both physical fields.

The first physical theory that can be considered a prime example of classical field theory was Newtons non-relativistic theory of gravity that related a vector field of the force felt on a massive body with mass m, $\mathbf{F}(\mathbf{r})$, with the gravitational field created by a massive body with mass M, $\mathbf{g}(\mathbf{r})$, which can be written as,

$$\mathbf{F}(r) = M\hat{\mathbf{g}}(\hat{\mathbf{r}}) = -\frac{GMm}{r^2}\hat{\mathbf{r}} \quad , \tag{1}$$

where G is Newton's gravitational constant, $\hat{\mathbf{r}}$ is the unitary vector that points from either body to the other and r^2 is the square of the distance between both bodies.

1.1.2 Lagrangian Formalism

We will begin our discussion of Lagrangian dynamics applied to fields by introducing the action that is written as:

$$\mathcal{S} = \int L \, dt = \int \mathcal{L}(\Phi, \partial_\mu \Phi) \, d^4x \quad , \tag{2}$$

where the Lagrangian, L, and the *Lagrangian density*, $\mathcal{L}$ are functions of the field Φ and its spatial and temporal derivative that can be simplified in covariant notation as $\partial_\mu \Phi$. And note that it's required that the action and by extension the Lagrangian always take real values.

$\mathcal{S}$ is then defined as a functional of Φ. During this essay we will mostly try to stay faithful to the distinction of Lagrangian and Lagrangian density but these terms are usually used interchangeable and we will later, as an abuse of language use them in that fashion, this action is constrained like in classical mechanics to obey the principle of *least action*, this requires the path taken by a field between an initial and final set of coordinates to leave the action invariant, this can be expressed by:

$$\delta \mathcal{S} = 0 \quad . \tag{3}$$

This condition can be shown to lead directly to the *Euler-Lagrange* equations,

$$\partial_\mu \left(\frac{\partial \mathcal{L}(\Phi, \partial_\mu \Phi)}{\partial(\partial_\mu \Phi)} \right) - \frac{\partial \mathcal{L}(\Phi, \partial_\mu \Phi)}{\partial \Phi} = 0 \quad , \tag{4}$$

from where we can extract the equations of motion for the fields. As a example, let's consider a free scalar field. It's Lagrangian density is: [1]:

$$\mathcal{L} = \frac{1}{2} \eta^{\nu\mu} \partial_\mu \Phi^* \partial_\nu \Phi - \frac{1}{2} m^2 \Phi^* \Phi \quad , \tag{5}$$

---

[1]Note that Φ could be a complex field and the need for the action to be real makes it so that Φ must always be multiplied by its conjugate.

the Lagrangian has this form because by definition it is the difference between the kinetic energy of the field, T, and the potential energy of the field, V, mathematically written as $L = T - V$. Hence the Lagrangian must be this way to ensure the "classical" forms for these quantities.

$$T = \int \frac{1}{2} \left(\frac{\partial \Phi}{\partial t}\right)^2 d^3x \qquad V = \int \frac{1}{2} \left((\nabla \Phi)^2 + m^2 \Phi^2\right) d^3x \quad . \tag{6}$$

Using then the *Euler-Lagrange* equations we can retrieve the Klein-Gordon equation of motion for this field.

$$\begin{aligned} (\Box + m^2)\Phi &= 0 \\ (\Box + m^2)\Phi^* &= 0 \quad , \end{aligned} \tag{7}$$

where we define the d'Alembert operator as $\Box = \partial_\mu \partial^\mu$.

1.1.3 Transformations, symmetry and Noether's theorem

A symmetry can be very broadly defined as a property of a system that is preserved or remains unchanged. In physics it's used as a powerful tool that usually helps us to solve problems by allowing for systematic simplifications.

The most obvious example of symmetry in physics would probably be the discrete rotational symmetry in the shapes of regular crystalline structures, of course in our study matter we aren't really interested in symmetries of *shape*. In our study we will be rather more focused on *internal symmetries* when discussing particle physics [3].

Newton was the first to realise that the observation of symmetry in the dynamics of a system or object must not come from a particular solution of the system but by the careful examination of all the possible solutions for the dynamics of a given system, which means that observing a symmetry in the motion of a system must come from the equations of motion and not from a particular solution of those equations. As an example consider a generic transformation of a field ϕ,

$$\phi \rightarrow \phi + \delta\phi \quad , \tag{8}$$

a theory could be called symmetric when such transformation leaves the Lagrangian invariant meaning that $L(\phi) = L(\phi + \delta\phi)$. For example, the complex free scalar field Lagrangian,

$$\mathcal{L} = |\partial_\mu \phi|^2 - m^2 |\phi|^2 \quad , \tag{9}$$

it is invariant under the generic transformation where $\phi \rightarrow e^{i\alpha}\phi$ for any real α. In this complex field ϕ we have two different real degrees of freedom since $\phi = \phi_1 + i\phi_2$, but it's more convenient to treat them as ϕ and ϕ^*. The invariance of the Lagrangian preserves the equation of motion since they are directly related as we've seen before.

The deeper implications of a system having a symmetry were successfully mathematically expressed by Noether with her famous first theorem. Noether's theorem states that the existence of a natural symmetry implies a conservation law and vice-versa any conservation law implies an underlying symmetry. Let's assume we have a generic field Φ that is transformed in a infinitesimal manner,

$$\Phi(x) \longrightarrow \Phi'(x) = \Phi(x) + \alpha\delta\Phi(x) \quad , \tag{10}$$

3

where α is a constant infinitesimal parameter and $\delta\Phi$ is a deformation of the field. Now to examine any changes made to the dynamics of the system, we do this by imputing the new field into the Lagrangian Density and deriving the new *Euler-Lagrange* equations. We could then say, if the system dynamics remains the same, that this is a **symmetry** of the system since its dynamics are **invariant** under these transformations. This is equivalent to stating that the invariance of a system should leave the Lagrangian density invariant up to a 4-divergence, $\mathcal{J}$, because of how the *Euler-Lagrange* equations are formulated.

$$\mathcal{L} \rightarrow \mathcal{L} + \alpha\partial_\mu\mathcal{J}^\mu \quad , \tag{11}$$

where the variation of $\mathcal{L}$ i.e $\Delta\mathcal{L}$ can be written as,

$$\alpha\Delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\phi}(\alpha\Delta\phi) + \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right)\partial_\mu(\alpha\Delta\phi)$$

$$= \alpha\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right) + \alpha\left[\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right)\right] + \Delta\phi \quad . \tag{12}$$

We can clearly see that the second term is the Euler-Lagrange equation so that term vanishes and setting the equation equal to $\alpha\partial_\mu\mathcal{J}^\mu$ we obtain the following

$$\partial_\mu j^\mu(x) \quad \text{for} \quad j^\mu(x) = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\Delta\phi - \mathcal{J}^\mu \quad , \tag{13}$$

As a note, if we have a symmetry of several fields the first term of j_μ should be replaced by a sum over those several fields. This j_μ is the current associated with a symmetry in a system that is conserved through all transformations and for every distinct symmetry we have a different conserved current, so a system with 2 distinct symmetries will have 2 different sets of conserved 4-currents. The charge, Q_{sym}, associated to this current is given by the integral:

$$Q_{\text{sym}} = \int\limits_{\text{all of space}} j^0 d^3x \quad , \tag{14}$$

so if initially we had a charge inside a volume the only way for us to lose charge over time would be to have a flux of charge leaving the space [3].

2 The Standard Model

The SM is the presently accepted quantum field theory accurately describing strong and electroweak interactions, and has been successfully tested in a number of experiments. The Standard Model is comprised by fermions which are subdivided into quarks and leptons, gauge bosons and a scalar particle, the Higgs boson.
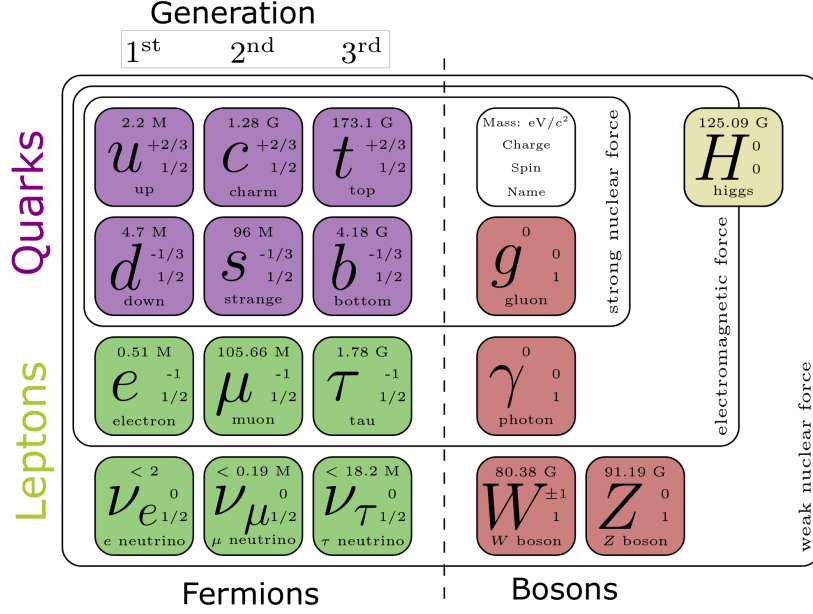
Figure 1: A diagram of the types and groups of particles that comprise the SM containing information about spin, mass and charge [1].

As shown in the diagram of Fig. 1 the Standard Model is composed by force carriers, the weak gauge bosons W and Z, the electromagnetic interaction messenger, the photon, and the strong force mediators, the gluons, as well by matter particles, the quarks and leptons. The Higgs boson is responsible for the mass generation mechanism as we will study in this section. Fermions are organized in three generations. Furthermore, there are 6 different types of quarks, up and down for the first generation, charm and strange for the second as well as top and bottom for the third one. Similarly, there are 6 types of leptons, the charged ones, electron, muon and tau, and the associated neutrinos, respectively represented by (u, d, c, s, t, b) while leptons as $(e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau)$

So far we have described the physical states that are often denoted as the building blocks of nature. However we have not yet explained how such states have acquired their masses and gauge quantum numbers, such as colour and electric charge. To see this, we start by noting the the SM is a gauge theory based on the group.

$$SU(3)_c \times SU(2)_L \times U(1)_Y \quad . \tag{15}$$

Fermions are half integer spin particles most of which have electrical charge (except the neutrinos). While quarks interact via the weak, electromagnetic and strong forces, the charged leptons only feel the electromeagnetic and weak forces and the neutrinos are solely weakly interacting. A physical fermion is composed of a left-handed and a

right-handed part. While the former transform as $SU(2)_L$ doublets and can be written as

$$L^i = \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix} \quad \text{and} \quad Q^i = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \quad , \tag{16}$$

where the i index stands for generation, the latter are $SU(2)_L$ singlets and can be simply represented as

$$e_R^i = \{e_R, \mu_R, \tau_R\}, \quad u_R^i = \{u_R, c_R, t_R\}, \quad d_R^i = \{d_{e_R}, s_{e_R}, b_{e_R}\} \quad , \tag{17}$$

note also that the quarks form triplets of $SU(3)_C$ whereas leptons are colour singlets. The Higgs boson also emerges from an $SU(2)_L$ doublet with the form,

$$H = \begin{pmatrix} \phi^1 + i\,\phi^2 \\ \phi^3 + i\,\phi^4 \end{pmatrix} \quad , \tag{18}$$

The full set of gauge quantum numbers in the SM is given in tables 1 and 2.

Table 1: Gauge bosons and Scalar fields in the SM

Fields	Spin 0 field	Spin 1 Field	$SU(3)_C \times SU(2)_L \times U(1)_Y$
Gluons	$\times$	g	$(8,1,0)$
A bosons	$\times$	A^i	$(1,3,0)$
B bosons	$\times$	B	$(1,1,0)$
Higgs field	$(\phi^\pm, \phi^0)$	$\times$	$(1,2,1)$

Table 2: Fermion field dimensions in the SM

Fields	Spin $\frac{1}{2}$ Field	$SU(3)_C \times SU(2)_L \times U(1)_Y$
Quarks (3 gen.)	$Q = (u_L, d_L)$	$(3, 2, \frac{1}{3})$
	u_R	$(3, 1, \frac{4}{3})$
	d_R	$(3, 1, -\frac{2}{3})$
Leptons (3 gen.)	$(L = (\nu_{e_L}, e_L)$	$(1, 2, -1)$
	e_R	$(1, 1, -2)$

We can then write a Lagrangian invariant under transformations of the $SU(3) \times SU(2) \times U(1)$ as.

$$\begin{aligned} \mathcal{L}_{SM} &= (D_\mu H)^\dagger (D^\mu H) - V(HH^\dagger) - \frac{1}{4} F^i_{\mu\nu} F^{i,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ &+ \overline{L_L^i}(i\gamma^\mu D_\mu) L_L^i + \overline{Q_L^i}(i\gamma^\mu D_\mu) Q_L^i + \overline{L_R^i}(i\gamma^\mu D_\mu) L_R^i + \overline{Q_R^i}(i\gamma^\mu D_\mu) Q_R^i \\ &- [y^d_{jk} \overline{Q_L^j} d_R^k H + y^u_{jk} \overline{Q_L^j} u_R^k \tilde{H} + y^e_{jk} \overline{L^j} e_R^k H + h.c.] \quad , \end{aligned} \tag{19}$$

where $\tilde{H} = i\sigma_2 H$. We define the covariant derivative as, D_μ

$$D_\mu = \partial_\mu - ig_S \tau^a G^a_\mu - igT^i A^i_\mu - ig'Y B_\mu \quad , \tag{20}$$

where $\tau^a = \frac{\lambda_a}{2}$, $(a = 1,...,8)$ are the generators of $SU(3)_c$, $T_i = \frac{\sigma_i}{2}$, $(i = 1,2,3)$ are the generators of $SU(2)_L$ and Y is the generator of $U(1)_Y$. Here the symbols λ_a and σ_i represent the Gell-Mann and Pauli matrices respectively. In the first line of (19), the first term represents the interactions of gauge bosons with the Higgs field and the second term is the scalar potential associated to the said field. The second line represents the gauge-kinetic terms and gauge boson self interactions. The third line describes the fermion kinetic terms as well as the interactions among fermions and gauge bosons. Finally, the last line shows the Yukawa interactions between the Higgs and the fermions. It is due to the Yukawa interactions that the SM fermions acquire their masses once the electro-weak (EW) symmetry is broken, as we will later see.

Symmetry plays as we will see a key-role in particle physics and given by the field properties we assigned to each field, i.e. the representations of the groups that make up the SM. The focus of our discussion of the SM is to show how stemming from this we derive the physical particle spectrum that accurately describes reality.

2.1   Spontaneous Symmetry Breaking and Goldstone's Theorem

Having dealt with the fundamentals of field theory and with the concepts of symmetry we find ourselves ready to begin our discussion of the Standard Model. However, before jumping into the full details of symmetry breaking in the context of the SM theory, let us look into a simple example, the Abelian Higgs model with a global U(1) symmetry, and stepwise introduce all necessary non-trivial concepts.

Consider the Lagrangian associated to a complex scalar field theory,

$$\mathcal{L} = (\partial_\mu \Phi)^*(\partial^\mu \Phi) - \mu^2(\Phi^*\Phi) - \lambda(\Phi^*\Phi)^2 \quad , \tag{21}$$

here λ is the quartic term is the self interaction that must be a positive value ($\lambda > 0$) to allow for a spectrum of stable bound states and μ is a real value. This Lagrangian is invariant under global unitary transformations belonging to the $U(1)$ group.

$$\phi' \rightarrow e^{i\alpha}\phi \quad , \quad \phi^{*\prime} = e^{-i\alpha}\phi^* \quad , \tag{22}$$

where α is a real value. This can be easily proven as all terms are even powers of ϕ

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi'^* \partial^\mu \Phi') - \mu^2 \Phi' \Phi'^* - \frac{1}{4}\lambda(\Phi'\Phi'^*)^2$$

$$\mathcal{L} = \frac{1}{2}e^{i(\alpha-\alpha)}(\partial_\mu \Phi^* \partial^\mu \Phi) - e^{i(\alpha-\alpha)}\mu^2 \Phi\Phi^* - \frac{1}{4}\lambda(e^{i(\alpha-\alpha)})^2(\Phi\Phi^*)^2 \tag{23}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^* \partial^\mu \Phi^*) - \mu^2 \Phi\Phi^* - \frac{1}{4}\lambda(\Phi\Phi^*)^2 \quad .$$

The next step is to examine the potential portion of the Lagrangian,

$$V(\Phi) = \frac{1}{2}\mu^2 \Phi\Phi^* + \lambda(\Phi\Phi^*)^2 \quad , \tag{24}$$

and determine its minima by solving:

$$\frac{\partial V(\Phi)}{\partial |\Phi|} = 0 \quad . \tag{25}$$

7

Analysing the lower energy states we note that in the case of $\mu > 0$ the potential is simply a parabola with its minima sitting at the origin. However were we to say μ is a negative number (25) changes to,

$$2\mu^2|\Phi| + 4\lambda|\Phi|^3 = 0 \quad , \tag{26}$$

from this equation we conclude that,

$$|\Phi_{min}|^2 = -\frac{\mu^2}{2\lambda} = v^2 \quad . \tag{27}$$

The value of the scalar field in the vacuum is non-vanishing and is denoted as vacuum expectation value, or VEV for short. We can parametrize the field at the minimum by rewriting it in radial form,

$$\Phi_{min} = |\Phi_{min}|e^{i\gamma} \quad , \tag{28}$$

which represents a set of degenerate vacuum states. Comparing both cases, for positive and negative values of μ^2, we see that for the latter case, while a phase invariance is still preserved we no longer have a directional radial invariance about the VEV. So the previously held symmetry was spontaneously broken (SSB) and a graphical representation of this can be seen in Fig. 2.

Defining $|\Phi_{min}| = v$ we can write and study the scalar field in the vacuum by performing the shift,

$$\Phi(x) \longrightarrow \Phi'(x)$$
$$\frac{1}{\sqrt{2}}\left(\phi_1(x) + i\phi_2(x)\right) \longrightarrow \frac{1}{\sqrt{2}}\left(\eta(x) + v + i\epsilon(x)\right) \quad , \tag{29}$$

Where v is equal to $\sqrt{-\frac{\mu}{\lambda}}$. Now plugging this new field into the Lagrangian must return an equivalent physical description of the dynamics of the system, now expressed as,

$$\mathcal{L} = \frac{1}{2}\partial_\mu\epsilon(x)\partial^\mu\epsilon(x) + \frac{1}{2}\partial_\mu\eta(x)\partial^\mu\eta(x) - \frac{1}{2}(2\mu^2)\eta^2 - \frac{1}{4}\lambda(\epsilon^2 + \eta^2)^2 - \lambda v(\epsilon^2 + \eta^2)\eta . \tag{30}$$

Comparing this new Lagrangian density and the former we see that while in the former we had a squared mass term with value μ^2 for both ϕ_1 and ϕ_2, now in this new redefinition we only have one mass term in η with squared mass equal to $m_\eta^2 = -2\mu^2 = 2\lambda v$ and a massless scalar field ϵ. The physical meaning of these fields is that η represents the quantum fluctuations above the constant vaccum value along the radial direction and the field ϵ represents the fluctuations along the angular directions.The fact that the potential doesn't vary along the latter direction is reflected in a massless scalar field. Note that, whenever we have a continuous symmetry that is spontaneoulsy broken, there will be as many massless scalar particles as the number of broken generators. This results is know as the *Goldstone Theorem* and such massless degrees of freedom are denoted as Goldstone bosons. For our particular case, the only single generator of the $U(1)$ symmetry is sponteneoulsy broken, which yields one massless Goldstone boson that we have identified with ϵ.

Note that, when the sign of μ squared goes from positive to negative the system goes through a phase transition where the original symmetry is spontaneously broken in the vacuum states. This is graphically represented in Fig.2.
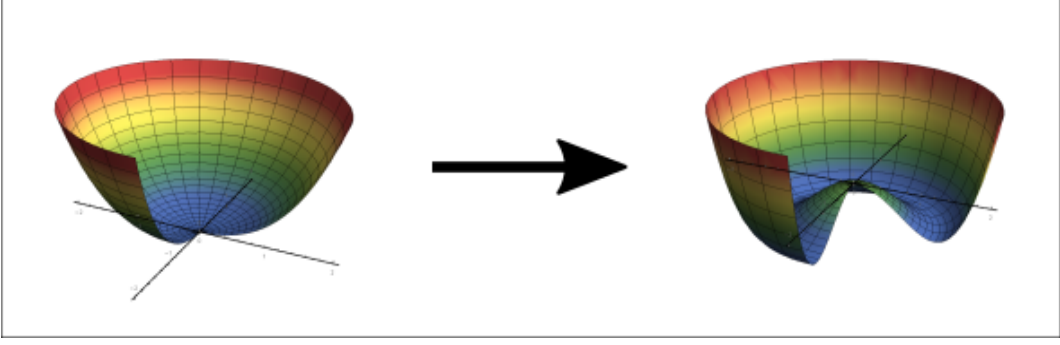
Figure 2: Phase Shift of the Potential in a Higgs Style Field

2.2 Gauging U(1): The example of Quantum Electrodynamics

In the previous section we discussed the particular case of a complex scalar field transforming under a global symmetry. Let us now consider a local transformation by replacing the phase in (22) by a space-time dependent phase $\alpha(x)$

$$\phi(x) \to e^{iq\alpha(x)}\phi(x) \quad , \quad \phi^*(x) \to e^{-iq\alpha(x)}\phi^*(x) \quad , \tag{31}$$

Introducing this transformation, where q is the charge associated to the symmetry, to the former Lagrangian density (21), we can see that all but the derivative terms are invariant. Those particular terms now take the form,

$$\partial_\mu \Phi' \to \partial_\mu \left(e^{i\alpha(x)}\Phi \right) = e^{i\alpha(x)} \cdot \left(i\ \Phi(\partial_\mu \alpha) + \partial_\mu \Phi \right) \tag{32}$$

$$\partial_\mu \Phi'^{\ *} \to \partial_\mu \left(e^{-i\alpha(x)}\Phi^* \right) = e^{-i\alpha(x)} \cdot \left(-i\ \Phi^*(\partial_\mu \alpha) + \partial_\mu \Phi^* \right) \quad , \tag{33}$$

meaning the Lagrangian is no longer invariant. In fact we note it changes by,

$$\delta \mathcal{L} = i\ q\Phi(\partial_\mu \alpha)\Phi^* - i\ q\partial_\mu \Phi \Phi^*(\partial_\mu \alpha) + \Phi\Phi^* q^2 \partial_\mu \alpha \partial^\mu \alpha \quad . \tag{34}$$

To reattain invariance under local gauge transformations we have to introduce a new 4-vector A_μ, denoted as the gauge field in what follows, as well as three new terms in the Lagrangian allowed by the gauge symmetry. We also note that the gauge field has to transform in such a way to cancel the effects coming from the derivative terms in 33 and has the form

$$A_\mu + \frac{1}{q}\partial_\mu \alpha\left(x\right) = A'_\mu \quad , \tag{35}$$

The new added terms are then,

$$\mathcal{L}_1 = -i\ q\left(\Phi^*\partial_\mu \Phi - \Phi\partial_\mu \Phi^*\right)A_\mu \tag{36}$$

$$\mathcal{L}_2 = q^2 A^\mu A_\mu \Phi\Phi^* \quad , \tag{37}$$

In addition to these terms we add another term to the Lagrangian, $\mathcal{L}_3$, containing kinetic terms for the gauge field and invariant under the transformation (33) and reads,

$$\mathcal{L}_3 = -\frac{1}{4}\left[\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right)\right],$$

$$\mathcal{L}_3 = \frac{1}{4}F_{\nu\mu}(x)F^{\nu\mu}(x) , \tag{38}$$

Puting all together we get,

$$\mathcal{L}_{tot} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3$$

$$\mathcal{L}_{tot} = (\partial_\mu \Phi)(\partial^\mu \Phi^*) - iq(\Phi^* \partial^\mu \Phi - \Phi \partial^\mu \Phi^*) A_\mu + q^2 A_\mu A^\mu \Phi^* \Phi - m\Phi^* \Phi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

$$\mathcal{L}_{tot} = (\partial_\mu \Phi + iq A_\mu \Phi)(\partial^\mu \Phi^* - iq A^\mu \Phi) - m^2 \Phi^* \Phi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

$$\mathcal{L}_{tot} = D_\mu \Phi D^\mu \Phi^* - m^2 \Phi^* \Phi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \quad , \tag{39}$$

which preserves the original structure but with normal derivatives replaced by what we denote as covariant derivatives. Note that it is in the terms involving the covariant derivatives that we have interactions involving the gauge and the scalar fields.

2.3 The Higgs mechanism and the mass generation of the Gauge bosons

Coming back to the SM as defined above, we can now study the mechnism of spontaneous symmetry breaking. Let us then consider the part of the Lagrangian containing the scalar covariant derivatives as defined in eq. (19) , the scalar potential and the gauge-kinetic terms:

$$\mathcal{L}_{gauge} = (D_\mu H)^\dagger (D_\mu H) - V(HH^\dagger) - \frac{1}{4} F^i_{\mu\nu} F^{i,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \quad , \tag{40}$$

The elements of this sector are defined as,

$$V(HH^\dagger) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2 \quad , \tag{41}$$

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\epsilon_{ijk} A^j_\mu A^k_\nu \quad , \tag{42}$$

In this formulation the constants g and g' are the gauge couplings of the groups $SU(2)_L$ and $U(1)_Y$. As the potential is identical to the one dicussed in Sec. 2.1, we know that we have a phase shift, namely $\mu^2 < 0$ and what kind of VEV we are expected to find, namely

$$(H^\dagger H) = \frac{-\mu^2}{2\lambda} = \frac{1}{2} v \quad , \tag{43}$$

The vacuum can be aligned in such a way that we have

$$H_{min} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad . \tag{44}$$

This vacuum will break the $SU(2)_L \times U(1)_Y$ symmetry down to $U(1)_Q$. this means that in the begining there are four generators, which are $T^{1,2,3}$ and Y, and after the breaking we are solely left with one unbroken combination that is $Q = (T^3 + 1/2)$. This means that in total we will have three broken generators, thus, from Goldstone Theorem, there will be three massless particles.

As we have seen for the abelian Higgs model, the Goldstones modes can be parameterized as phases in the field space and then can be "rotated away" in the physical

basis, leaving us with a single physical massive scalar, the Higgs boson. Note that, with this transformation we are removing three scalar degrees of freedom. However, they cannot just disappear from the theory and will be absorbed by the massive gauge bosons. In fact, a massless gauge boson contains only two scalar degrees of freedom (transverse polarization). Meanwhile, a massive vector boson has two transverse and a longitudinal polarization, i.e., three scalar degrees od freedom. So, as we discussed above, while before the breaking of the EW symmetry we have four massless gauge bosons, after the breaking we are left with three massive ones. This means that there are three extra scalar degrees of freedom showing up in the gauge sector. It is then commonly said that the goldstone bosons are "eaten" by the massive gauge bosons and the total number of scalar degrees of freedom in the theory is preserved. Therefore, without loss of generality, we can rewrite the Higgs doublet as

$$
\begin{pmatrix} G_1 + iG_2 \\ v + h(x) + iG_3 \end{pmatrix} = H(x) \rightarrow H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \quad . \tag{45}
$$

Once the Higgs doublet acquires a VEV, the Lagrangian (40) can be recast as:

$$
\mathcal{L}' = \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{1}{2} (2v^2 \lambda) h^2 - \frac{1}{4} F^i_{\mu\nu} F^{i,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}
$$
$$
+ \frac{1}{8} v^2 g^2 (A^1_\mu A^{1,\mu} + A^2_\mu A^{2,\mu}) + \frac{1}{8} v^2 (g^2 A^3_\mu A^{3,\mu} + g'^2 B_\mu B^\mu - 2g^2 g'^2 A^3_\mu B^\mu) \quad , \tag{46}
$$

A few things become obvious first, we have a lot of mass terms most stemming from the squared gauge fields and a lonesome squared mass term belonging to the real scalar field we know to be the Higgs field. This makes the Higgs boson mass in the SM to be given by,

$$
M_h = (2v^2 \lambda) \quad . \tag{47}
$$

To obtain masses for the gauge bosons we need to rotate the gauge fields to a basis where the mass terms are diagonal. First, it is straightforward to see that the electrically charged eigenstates are given by

$$
W^\pm_\mu = \frac{1}{\sqrt{2}} (A^{(1)}_\mu \pm i A^{(2)}_\mu) \quad , \tag{48}
$$

meaning that the mass of the W bosons is,

$$
M_W = \frac{1}{2} v g \quad . \tag{49}
$$

The situation becomes a bit more complicated for the second term in (46) due to a mixing between A^3_μ and B_μ. In the gauge eigenbasis the mass terms read

$$
\begin{pmatrix} A^3_\mu & B_\mu \end{pmatrix} \cdot \frac{1}{4} v^2 \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \cdot \begin{pmatrix} A^3_\mu \\ B_\mu \end{pmatrix} \quad , \tag{50}
$$

which can be diagonalized to obtain

$$
\begin{pmatrix} A_\mu & Z_\mu \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{2} v \sqrt{g^2 + g'^2} \end{pmatrix} \begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix} \quad , \tag{51}
$$

11

we identify the eigenvector associated to the eigenvalue 0 to the photon and the massive one, $M_Z = \frac{1}{2}v\sqrt{g^2 + g'^2}$, to the Z boson. Such eigenvectors can be written as

$$A_\mu = \cos(\theta_\omega)B_\mu + \sin(\theta_\omega)A_\mu^3 \quad , \tag{52}$$

$$Z_\mu = -\sin(\theta_\omega)B_\mu + \cos(\theta_\omega)A_\mu^3 \quad , \tag{53}$$

where θ_w is the so called Weinberg mixing angle and is defined as

$$\cos(\theta_\omega) = \frac{g}{\sqrt{g^2 + g'^2}} \quad , \tag{54}$$

thus clearly showing the massless photon along with a massive Z boson with mass $M_Z = \frac{1}{2}\nu\sqrt{g^2 + g'^2}$. So we conclude our exploration of the electroweak sector with all the correct massive spectrum observed and its origin discussed.

2.4 The fermion sector on the Standard Model

In order to generate mass for the fermions we can have a closer look at the last line in eq. (19). If we replace the Higgs by the shift in Eq. (45) we get,

$$\mathcal{L}_y = y^d \begin{pmatrix} \overline{u_L} & \overline{d_L} \end{pmatrix} d_R \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} + y^s \begin{pmatrix} \overline{c_L} & \overline{s_L} \end{pmatrix} s_R \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} + y^b \begin{pmatrix} \overline{t_L} & \overline{b_L} \end{pmatrix} b_R \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

$$+ y^u \begin{pmatrix} \overline{u_L} & \overline{d_L} \end{pmatrix} d_R \begin{pmatrix} v + h(x) \\ 0 \end{pmatrix} + y^c \begin{pmatrix} \overline{c_L} & \overline{s_L} \end{pmatrix} d_R \begin{pmatrix} v + h(x) \\ 0 \end{pmatrix} + y^t \begin{pmatrix} \overline{t_L} & \overline{b_L} \end{pmatrix} t_R \begin{pmatrix} v + h(x) \\ 0 \end{pmatrix}$$

$$\tag{55}$$

$$+ y^e \begin{pmatrix} \overline{\nu_{e_L}} & \overline{e_L} \end{pmatrix} e_R \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} + y^\mu \begin{pmatrix} \overline{\nu_{\mu_L}} & \overline{\mu_L} \end{pmatrix} \mu_R \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} + y^\tau \begin{pmatrix} \overline{\nu_{\tau_L}} & \overline{\tau_L} \end{pmatrix} \tau_R \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} + h$$

Further expansion of these terms would result in terms like *e.g.* in the electron's case,

$$\mathcal{L}_{ye} = y^e \, v \, (\overline{e_L}e_R + \overline{e_R}e_L) + y^e h(x) \, (\overline{e_L}e_R + \overline{e_R}e_L) \quad , \tag{56}$$

where since the electron field is written as,

$$e = \begin{pmatrix} e_L \\ e_R \end{pmatrix} \quad , \tag{57}$$

meaning the first terms in (56) equate to electron mass terms as, $m_e\overline{e}e$ and the second terms represent interaction between the electron and the Higgs boson. This is how the Higgs mechanism generates the mass to all of the fermionic sector except neutrinos due to the SM not containing right handed neutrinos. The absence of neutrino masses contradicts experimental observations. Note also that the mass term for every lepton depends directly on each of the respective and different Yukawa term and that the SM is a theory where the Yukawa matrices are diagonal by nature not having inter generational terms.

3 The B-L-SM model

As we have just seen, one of the problems in the SM is the absence of neutrino masses. Furthermore, the SM exhibits an accidental symmetry of unknown origin where the baryon number B minus the lepton number L, meaning, B-L, is conserved. In this chapter we will introduce a model that aims at explaining the mass of neutrinos, and the B-L accidental symmetry will be promoted to a new gauge interaction denoted as $U(1)_{B-L}$. We will then study the phenomenological consequences of this model, which is named the B-L-SM.

In the relevant literature this model is referred to as a triply-minimal model due to it adding only one unitary gauge group $U(1)_{B-L}$ related to the baryon minus lepton conserved number, a single complex scalar singlet and, in the fermion sector a single right handed neutrino per generation as a means to fix quantum anomalies created by adding the new gauge field. This type of model is refereed to as a SM extension since it is built on the SM's foundations.

This added scalar field we will denote as χ, is only charged under the the $U(1)_{B-L}$ and will acquire a VEV at, or beyond, the TeV scale. The VEV of this complex singlet will spontaneously break $U(1)_{B-L}$ generating a new massive gauge boson, the Z', as well as a new physical scalar, the h'. Furthermore, the new allowed Yukawa interactions in combination with Majorana mass terms for neutrinos will allow the generation of three light and three heavy neutrinos. We will see that the B-L-SM model follows a similar invariant Lagrangian structure to the SM based on the gauge symmetry of the group, $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. This model can be decomposed into sectors in an analogous way to the SM.

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_s + \mathcal{L}_f + \mathcal{L}_Y \tag{58}$$

The quantum numbers in the B-L-SM are the same as in the SM for the strong and electroweak sectors, while the new B-L charges are written in the last line of Tab. 3. Note that the Higgs boson is neutral, i.e. non-interacting, under the new symmetry, which means that Higgs VEVs do not break $U(1)_{B-L}$. Conversely, χ is neutral under the SM gauge group, which means that its VEV leaves the EW symmetry unbroken.

Group $\setminus$ ϕ	q_L	u_R	d_R	l_L	e_R	ν_R	H	χ
$SU(3)_c$	3	3	3	1	1	1	1	1
$SU(2)_L$	2	1	1	2	1	1	2	1
$U(1)_Y$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{-1}{3}$	$\frac{-1}{2}$	-1	0	$\frac{1}{2}$	0
$U(1)_{B-L}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	-1	-1	-1	0	2

Table 3: Charges of the B-L-SM model

The covariant derivative of the B-L-SM reads,

$$D_\mu = \partial_\mu + igT^\alpha G_\mu^\alpha + igT^\alpha W_\mu^a + ig_1 Y B_\mu + i(\tilde{g}Y + g'Y_{B-L})B'_\mu \quad , \tag{59}$$

where $\tilde{g}$ emerges due to the kinetic mixing between $U(1)_Y$ and $U(1)_{B}-L$. This because

the gauge symmetry allows for non-diagonal terms of the form,

$$\mathcal{L}_g = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\,\mu\nu} - \frac{1}{2}\sin(\beta)F_{\mathrm{i}\,<\mu\nu}F^{\mathrm{i}\,\mu\nu} \quad, \tag{60}$$

where we define the tensors $F_{\mu\nu}$ and $F'_{\mu\nu}$ as,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad, \tag{61}$$

$$F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu \quad, \tag{62}$$

These terms leave a set of arbitrary free parameters therefore we redefine the gauge fields in such a way that the kinetic terms acquire the canonical form using:

$$\begin{pmatrix} A_\mu \\ A'_\mu \end{pmatrix} = \begin{pmatrix} 1 & -\tan(\beta) \\ 0 & \frac{1}{\cos(\beta)} \end{pmatrix} \begin{pmatrix} B_\mu \\ B'_\mu \end{pmatrix} \tag{63}$$

making the covariant derivative in the mixed basis,

$$D_\mu = \partial_\mu + i\,g_1 Y A_\mu + i\,g'_1 Y_{B-L} B'_\mu$$

$$D_\mu = \partial_\mu + i\,g_1 Y (B_\mu - \tan(\beta)B'_\mu) + i\,g'_1 Y_{B-L}\frac{1}{\cos(\beta)}B'_\mu$$

$$D_\mu = \partial_\mu + i\,g_1 Y B_\mu + i\,(g_1 \tan(\beta) + g'_1 Y_{B-L}\frac{1}{\cos(\beta)})B'_\mu$$

$$D_\mu = \partial_\mu + ig_1 Y B_\mu + i\,(\tilde{g}Y + g'Y_{B-L})B'_\mu \tag{64}$$

making

$$\tilde{g} = -g_1 \tan(\beta) \quad \text{and} \quad g' = \frac{g'_1}{\cos(\beta)} \tag{65}$$

This is how in the physical basis, even though B_μ is not charged in $U(1)_{B-L}$ there exists a interaction term in the covariant derivative.

3.1 The scalar sector

The Lagrangian for the scalar-gauge interactions is given by

$$\mathcal{L}_s = (D^\mu H)^\dagger (D_\mu H) + (D^\mu \chi)^\dagger (D_\mu \chi) - V(H, \chi) \quad, \tag{66}$$

where the potential, $V(H, \chi)$ is,

$$V(H, \chi) = m^2 H^\dagger H + \mu^2 |\chi|^2 + \begin{pmatrix} H^\dagger H & |\chi|^2 \end{pmatrix} \begin{pmatrix} \lambda_1 & \frac{1}{2}\lambda_3 \\ \frac{1}{2}\lambda_3 & \lambda_2 \end{pmatrix} \begin{pmatrix} H^\dagger H \\ |\chi|^2 \end{pmatrix} \quad, \tag{67}$$

Here, the Higgs mechanism works in the same way as in the SM. However, the singlet VEV should be beyond the TeV scale in order not to generate a light Z' boson, which would contradict current experimental limits on its mass [4].

Taking the same steps as in the SM we must first ensure the potential is bounded from bellow, considering the potential in Eq. (67) in the B-L-SM we need to impose the following conditions

$$4\lambda_1\lambda_2 - \lambda_3^2 > 0 \quad , \quad \lambda_1, \lambda_2 > 0 \quad . \tag{68}$$

in order to ensure boundedness from below. Now if the conditions (68) are fulfilled we can move on to the minimization of the potential in the proper Higgs field parametrization, were the Higgs doublet and singlet will acquire each a VEV and result in a case of SSB whose transition can seen here,

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \overset{Electroweak\ SSB}{\Longrightarrow} \begin{pmatrix} w^1 - iw^2 \\ v + (h+z) \end{pmatrix} \tag{69}$$

$$\chi = \frac{1}{\sqrt{2}}\left(\chi_1 + i\chi_2\right) \overset{\chi\text{'s } SSB}{\Longrightarrow} \left(\tfrac{1}{\sqrt{2}}(x + h' + z')\right) \tag{70}$$

where w and z and z' are the Goldstone bosons that will be absorbed by the $W^{\pm}$, Z and Z'. Expanding the potential around the expected minimum value of these fields,

$$\langle H \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{and} \quad \langle \chi \rangle = \frac{x}{\sqrt{2}} \tag{71}$$

with v and x being positive real numbers.

$$\begin{cases} \frac{\partial(V(H,\chi))}{\partial\chi} = 0 \\ \frac{\partial(V(H,\chi))}{\partial H} = 0 \end{cases} \implies \begin{cases} \frac{\mu^2\chi}{2} + \lambda_2\chi^3 + \frac{1}{2}\lambda_3 HH^\dagger\chi = 0 \\ \frac{m^2 H}{2} + \lambda_2\chi^3 + \frac{1}{2}\lambda_3 H\chi^2 = 0 \end{cases} \implies \begin{cases} \mu^2 = -\frac{\lambda_2 x^2}{2} - \frac{\lambda_3 v^2}{4} \\ m^2 = -\frac{2\lambda_1 v^2}{2} - \frac{\lambda_3 x^2}{4} \end{cases} \tag{72}$$

using these equations for simplification we reach the mass matrix

$$\mathbf{M}_{ij} = \begin{pmatrix} \frac{\partial^2(V(H,\chi))}{\partial\chi^2} & \frac{\partial^2(V(H,\chi))}{\partial\chi\partial H} \\ \frac{\partial^2(V(H,\chi))}{\partial\chi\partial H} & \frac{\partial^2(V(H,\chi))}{\partial H^2} \end{pmatrix} \overset{\text{shifting to the VEV's}}{\Longrightarrow} \begin{pmatrix} 4\lambda_2 x^2 & \lambda_3 vx \\ \lambda_3 vx & 4\lambda_1 v^2 \end{pmatrix} \tag{73}$$

through the use of Eq. (72) we could also impose the condition that VEVs must be cancel out, leading us to,

$$v^2 = \frac{-\lambda_2 m^2 + \frac{\lambda_3\mu^2}{2}}{\lambda_1\lambda_2 - \frac{\lambda^2}{4}} \tag{74}$$

$$x^2 = \frac{-\lambda_1\mu^2 + \frac{\lambda_3 m^2}{2}}{\lambda_1\lambda_2 - \frac{\lambda^2}{4}} \tag{75}$$

meaning that physical solutions must have x^2 and v^2 different from 0, implying through the combination of Eqs. (75) and (74) with the conditions set by Eq. (68) that,

$$\lambda_2 m^2 < \frac{\lambda_3}{2}\mu^2 \quad \text{and} \quad \lambda_1\mu^2 < \frac{\lambda_3}{2}m^2 \tag{76}$$

Interestingly here we can conclude from this that no solutions exist for $\mu^2 > 0$, $m^2 < 0$ or $\mu^2 < 0$, $m^2 > 0$. The minimization conditions now discussed we can now examine

15

the mass matrix seen in Eq. (73) we then find its eigenstates after the field shift to be given by the following expressions,

$$m_{h_1}^2 = \lambda_1 v^2 + \lambda_2 x^2 - \sqrt{(\lambda_1 v^2 - \lambda_2 x^2)^2 + (\lambda_3 x v)^2} \tag{77}$$

$$m_{h_2}^2 = \lambda_1 v^2 + \lambda_2 x^2 + \sqrt{(\lambda_1 v^2 - \lambda_2 x^2)^2 + (\lambda_3 x v)^2} \tag{78}$$

were h_1 then would represent the SM's Higgs boson and h_2 the Higgs prime boson. This Higgs prime is always heavier than it's SM counter-part.

3.1.1 Gauge boson masses

To determine the gauge boson spectrum we expand the scalar kinetic terms as for the SM.

We expect to reproduce the mass spectrum of the SM with an extra massive boson. To see this let us consider the scalar kinetic terms and expand them:

$$(D^\mu H)^\dagger (D_\mu H) + (D^\mu \chi)^\dagger (D_\mu \chi) = \frac{1}{2} \partial^\mu h \partial_\mu h + \frac{1}{2} \partial^\mu h' \partial_\mu h'$$
$$+ \frac{1}{8}(h+v)^2 \left[g^2 [W_1^\mu - iW_2^\mu]^2 + (gW_3^\mu - g_1 B^\mu - \tilde{g} B'^{,\mu})^2 \right]$$

$$\tag{79}$$

With no surprise we retrieve the same expressions for the W masses as in the SM, $M_W = \frac{gv}{2}$, however, the same no longer happens to the Z due to the mixing in the neutral sector. This then requires us to find the eigenvalues and eigenvectors, which can be parametrized by two mixing angles, γ_ω that represents the Weiberg angle and γ', as follows

$$\begin{pmatrix} B^\mu \\ W_3^\mu \\ B'^{,\mu} \end{pmatrix} = \begin{pmatrix} \cos(\gamma_\omega) & -\sin(\gamma_\omega)\cos(\gamma') & \cos(\gamma_\omega)\sin(\gamma') \\ \sin(\gamma_\omega) & \cos(\gamma_\omega)\cos(\gamma') & -\cos(\gamma_\omega)\sin(\gamma') \\ 0 & \sin\gamma') & \cos(\gamma') \end{pmatrix} \begin{pmatrix} A^\mu \\ Z^\mu \\ Z'^{,\mu} \end{pmatrix} \tag{80}$$

The new mixing angle lies in the range $-\pi/4 < \gamma' < \pi/4$ and is defined by

$$\sin(2\gamma') = \frac{2\tilde{g}\sqrt{g^2 + g_1^2}}{\sqrt{(\tilde{g}^2 + 16(\frac{x}{y})^2 g'^2 - g^2 - g_1^2)^2 + 2\tilde{g}^2(g^2 + g_1^2)}} \tag{81}$$

The eigenvalues of Eq. (80) are then,

$$M_A = 0$$

$$M_{Z,Z'} = \sqrt{g^2 + g_1^2} \cdot \frac{v}{2} \left[\frac{1}{2} \left(\frac{\tilde{g}^2 + 16(\frac{x}{v})^2 g_1'^2}{g^2 + g_1^2} + 1 \right) \mp \frac{\tilde{g}}{\sin(2\gamma')\sqrt{g^2 + g_1^2}} \right] \tag{82}$$

Note that if we set $\tilde{g} = 0$, the kinetic mixing is switched off the massive eigenstates completely decouple from each other and the Z' mass becomes independent of the Z boson mass.

$$M_Z = \sqrt{g^2 + g_1^2} \cdot \frac{v}{2} \tag{83}$$

$$M_{Z'} = 2g_1' x \tag{84}$$

from where we see that the Z' mass is solely dependent on g_1' and x. In fact, phenomenological constrains force us to decouple M_Z from M_Z' which implies that $x \gg v$. Even for the case with kinetic mixing, the contributions $16(\frac{x}{v})^2 \ g_1'$ in (82) dominate which tells us the Z' mass is essentially sensitive to x and g_1' provided that g_1' is not small enough to compensate the $16(\frac{x}{v})^2$ pre-factor.

3.2 Fermion sector and mass generation for the neutrinos

The fermionic Lagrangian in this model is

$$\mathcal{L}_f = \sum_{k=1}^{3} (i \, \overline{Q_{kL}} \gamma_\mu D^\mu q_L^k + i \overline{u_{kR}} \gamma_\mu D^\mu u_R^k + i \, \overline{d_{kR}} \gamma_\mu D^\mu d_R^k \tag{85}$$

$$+ i \, \overline{L_{kL}} \gamma_\mu D^\mu L_L^k + i \, \overline{e_{kR}} \gamma_\mu D^\mu e_R^k + i \, \overline{\nu_{kR}} \gamma_\mu D^\mu \nu_R^k) \tag{86}$$

This sector interactions should be mostly the same as in the Standard Model however we will see that in the Yukawa section we have the addition of a Yukawa interaction involving lepton doublets and right-handed neutrinos interacting with the Higgs, as well as another Yukawa term that couples right-handed neutrinos to the complex singlet. While the former gives rise to Dirac neutrino masses, the latter generates Majorana masses.

$$\mathcal{L}_Y = - y_{jk}^d \bar{q}_{jL} d_{kR} H - y_{jk}^u \bar{q}_{jL} u_{kR} \tilde{H} - y_{jk}^e \bar{l}_j e_{kR} H \tag{87}$$

$$- y_{jk}^\nu \bar{l}_{jk} \nu_{kR} \tilde{H} - y_{jk}^M \overline{\nu_{Rj}^c} v_{kR} \chi + h.c. \tag{88}$$

Here we have $\tilde{H} = i\sigma^2 H^*$ and i, j and k are values ranging from 1 to 3. These are the only allowed gauge invariant terms.

The neutrino masses are then going to be generated via a see-saw mechanism. In particular, the neutrino mass matrix is given by

$$\mathcal{M} = \begin{bmatrix} 0 & m_D \\ m_D & M \end{bmatrix} \tag{89}$$

where

$$m_D = \frac{y^\nu}{\sqrt{2}} v \tag{90}$$

$$M = \sqrt{2} y^M x \tag{91}$$

Note that, for simplicity, we have ignored inter-generation mixing. Thus, ν_L and ν_R can be written as the following linear combination of Majorana mass eigenstates

$$\begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} = \begin{pmatrix} \cos(\alpha_\nu) & -\sin(\alpha_\nu) \\ \sin(\alpha_\nu) & \cos(\alpha_\nu) \end{pmatrix} = \begin{pmatrix} \nu_l \\ \nu_h \end{pmatrix} \tag{92}$$

where we have defined the mixing angles to be

$$\tan(\alpha_\nu) = -\frac{2m_D}{M} \tag{93}$$

This makes the real masses for light and heavy neutrinos to be approximately given by,

$$m_{\nu_l} \approx \frac{m_D^2}{M}, \tag{94}$$

$$m_{\nu_h} \approx M \tag{95}$$

from where we see that the smallness of the light neutrinos is explained by a large suppression given by M in the denominator.

4 Data analysis

4.1 Software

During the course of this essay we avoided the discussion of any in-depth quantum field theory or quantum phenomena. However given our goal to perform a phenomenological study of the B-L-SM we must find a way to take them into account. To be able to correctly check the viability of our model we then used a series of computer tools that take in account all the related phenomena. We will avoid an extensive discussion of quantum field theory and merely describe very shallowly what sort of calculations are going on inside the programs. The computer tools used were SARAH, SPheno, HiggsBounds, HiggsSignals and SSP, which we shortly discuss in this section in the context of our analysis.

The first package to be used in the software chain was SARAH. It is a Mathematica package created for the development of particle models. In essence, it receives as input, a Lagrangian and all associated symmetries and then creates all analytical expressions such as, e.g., renormalization group equations, mass matrices and vertices. SARAH also checks for possible inconsistencies of the definition of the model such as charge conservation and gauge anomalies. SARAH can provide these analytical outputs in a series of formats, some of these outputs are designed to serve as input for other programs. We used one of such SARAH output format, the LesHouches accord format, used as model file for SPheno.

SPheno stands for Super-symmetric Phenomenology, it is a Fortran90 code designed to numerically calculate all parameters associated to a given particle model, from decay rates to low energy observables all the while taking in consideration configurable quantum corrections. We will use this code to study the dependency and behaviour of our model with respect to the theory parameters. SPheno returns the physical spectrum and all vertices in the mass basis which is then used by HiggsSignals and HiggsBounds to confront against collider searches. In essence, HiggsSignals tells how well the Higgs boson is explained for each of the parameter points while HiggsBounds will look into new scalar states and confront them with LHC limits in order to decide whether such a point is allowed or excluded by data at a 95 % confidence level. In Fig. 3 we show the workflow of the used packages specifying some of the internal built-in routines. [5] [6] [7] [8]
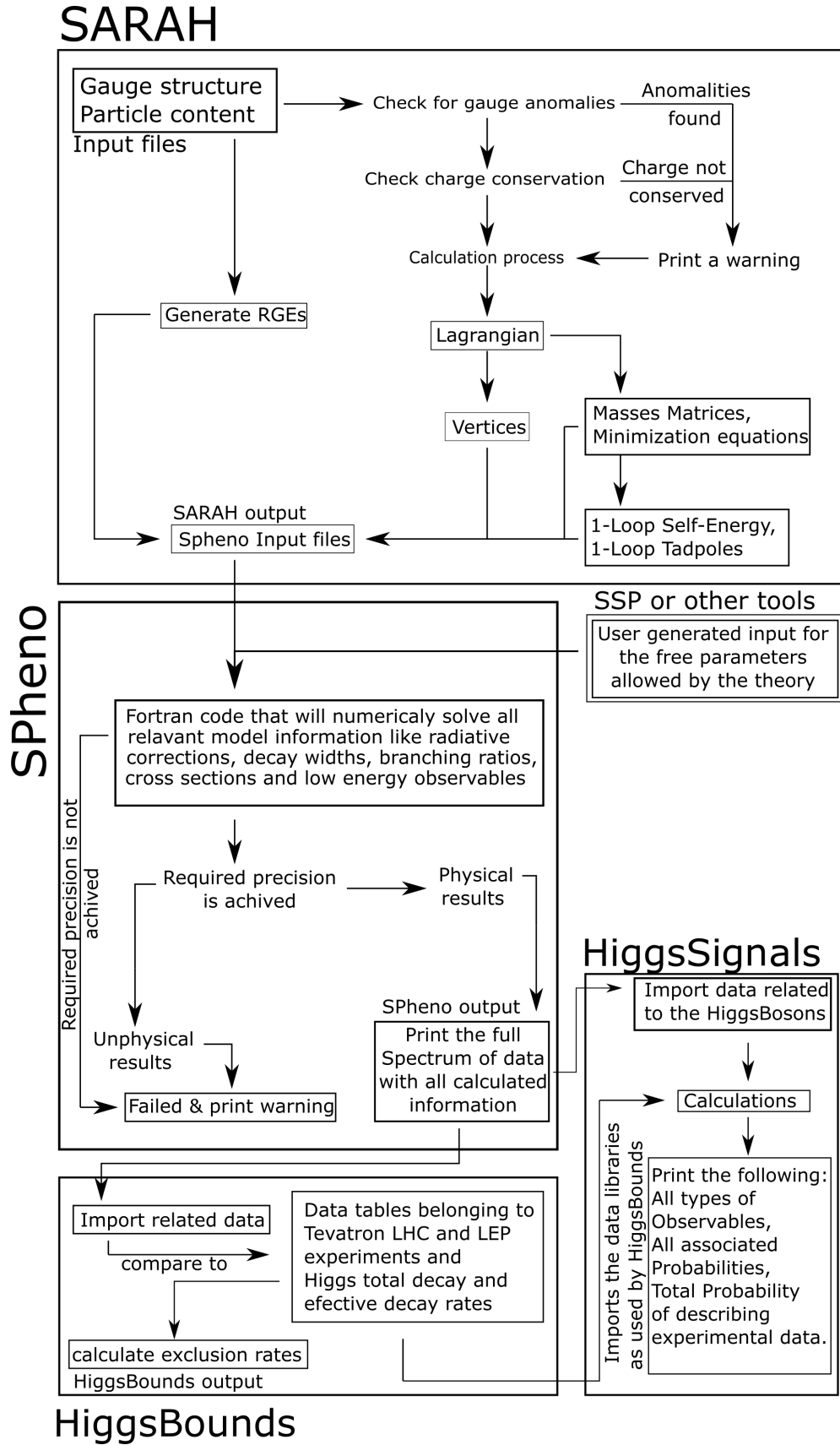
SARAH

Gauge structure
Particle content
Input files

Check for gauge anomalies — Anomalities found

Check charge conservation — Charge not conserved

Calculation process ← Print a warning

Generate RGEs

Lagrangian

Vertices

Masses Matrices,
Minimization equations

SARAH output
Spheno Input files

1-Loop Self-Energy,
1-Loop Tadpoles

SPheno

SSP or other tools

User generated input for
the free parameters
allowed by the theory

Fortran code that will numericaly solve all
relavant model information like radiative
corrections, decay widths, branching ratios,
cross sections and low energy observables

Required precision is not achived

Required precision
is achived → Physical results

Unphysical
results

Failed & print warning

SPheno output
Print the full
Spectrum of data
with all calculated
information

HiggsSignals

Import data related
to the HiggsBosons

Calculations

Print the following:
All types of
Observables,
All associated
Probabilities,
Total Probability
of describing
experimental data.

Import related data

compare to

Data tables belonging to
Tevatron LHC and LEP
experiments and
Higgs total decay and
efective decay rates

Imports the data libraries
as used by HiggsBounds

calculate exclusion rates
HiggsBounds output

HiggsBounds

Figure 3: A box diagram showing in schematic form the procedure and for the computer calculation of model observable and the program chain

4.2   Results

4.2.1   Higgs Sector Search

Now that we've seen and discussed how mass is generated in both the SM and in the B-L-SM model, we find ourselves ready to investigate the viability of our model using the computer tools we just approached in the discussed fashion.

We begin by re-examining the mass eigenstates seen in the equations (77) and (78) we could define and then study a region where the Higgs boson mass would be equal to 125.09 GeV, the observed value. First, note that the masses of both physical scalars is dependent on 5 parameters, say, $m_{h_1}(\lambda_1, \lambda_2, \lambda_3, v, x)$ One of these parameters, the Higgs VEV, is fixed to $v = 175\sqrt{2} \approx 247$ GeV, then, if we further fix $x = 1000$ GeV we can visually study the behaviour of the scalar masses against the quartic couplings λ_i. However, note that this is only a tree-level expression and numerical deviations are expected once quantum corrections, built into SPheno, are accounted. In fact this is observed in Fig. (5). While on the left panel, Fig. (4), we represent a surface of $m_{h_1} = 125$ GeV against $\lambda_{1,2,3}$, scanning this surface with Spheno we see on the right panel significant deviations from this value coming just from 1-loop corrections on the mass of the SM like Higgs boson of this model. Note that, in this qualitative analysis, quantum corrections tend to be larger for λ_1 values without mixing and low λ_2 values.
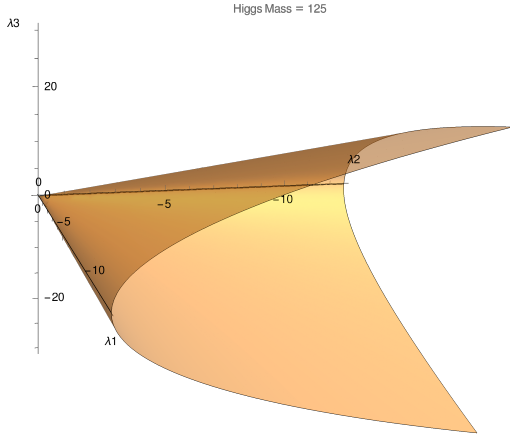

Figure 4:   A theoretical parametrization of fold where at tree level the higgs mass would be 125.
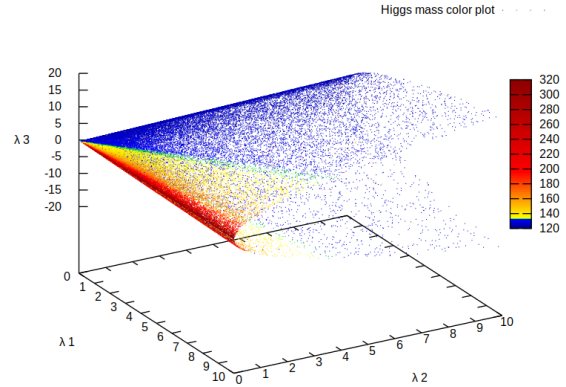
Figure 5: 1-loop corrections for a tree-level 125 GeV Higgs mass.

It can also be seen, on fig. 5, that there is a green (dim) line where all points are inside the correct Higgs mass range, i.e. 125.09 ± 0.21, meaning that the tree-level contributions are not dominant in that region or somehow cancel out. For the Higgs prime mass we note that its mass grows with λ_2 indicating that this is the dominant parameter defining the mass of the new scalar Higgs Prime. In fact, as a first approach, we can look again to the respective tree-level expressions Eq. (78) and observe that, for $v \ll x$, we have

$$m_{h_2}^2 \approx x^2 \lambda_2^2 + \sqrt{(-x)^2 \lambda_2} + \sqrt{xv\lambda_3} \approx 2x^2 \lambda_2^2 \tag{96}$$

which explains why the new scalar mass is mostly sensitive to λ_2 Both these conclusions are taken from the following, now logarithmic scale plots seen in fig. 6 and 7, here figure 6 is equivalent to figure 5 in a different scale.
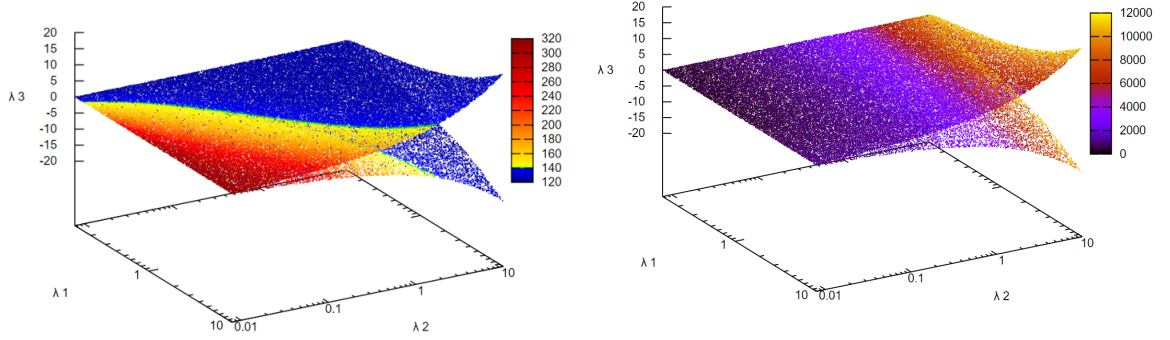

Figure 6: A logarithmic representation of the formely Higgs Boson Scan trough a pallete color code in GeVs.

Figure 7: The corresponding Higgs Prime boson mass along the same parametrization as seen in fig. 6 in GeVs.

Looking closer into these millions of points we find a plane of correct mass region where the Higgs boson mass is within the 125.09 ± 3 GeV. We show such points in Fig. 8. This seemingly indicates that the SM like Higgs Boson is not heavily influenced by quantum corrections up to a cut off point.
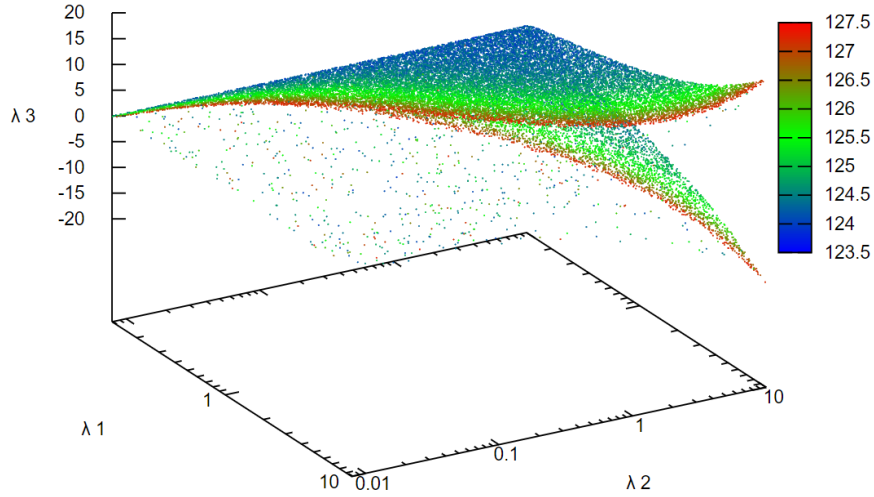
Figure 8: A selection of points where the Higgs Boson Mass was found to be within a 3 GeV interval of the observed 125.09 result.

The next step was dedicated to increase the accuracy in the calculation of the Higgs mass by including 2-loop corrections and other miscellaneous corrections. We used the ARGUS computing power and run over several days in order to maximize the amount of data.
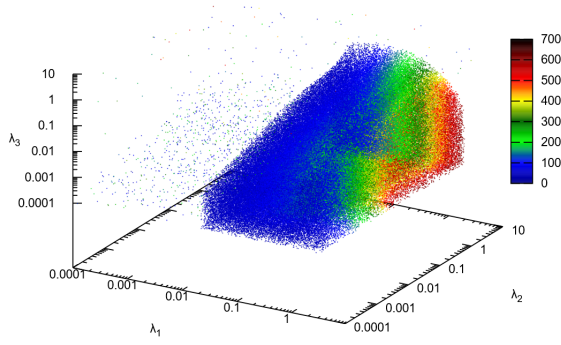
22

Figure 9: A random scan palette plot of the SM Higgs boson Mass where a viable spectrum was outputed by SPheno with 2 loop corrections.
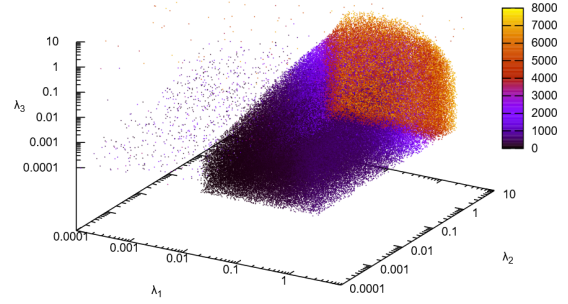
Figure 10: A random scan palette plot of the new Higgs Prime boson Mass with 2 loop corrections.

The following scans were made under a uniform random logarithmic distribution for the λ_i couplings between the values of 10^{-4} up to 5 in fact, we know that loop corrections are formally a perturbation expansion on the couplings, which is only well defined around small values of the couplings. It is then possible that 2-loop contributions diverge for larger values of λ_i. However, a full understanding of this behaviour requires a good knowledge of quantum field theory, which is well beyond the scope of this essay. Our unfiltered after turning on 2-loop correction our random scan results are shown in fig. 10 and 9. The same observations as for the 1-loop case hold but now within a range of values for the quartic couplings limited from above by $\lambda_i \sim 1$ to always be approximately in between the 0.1 and 1 scale. Once again, we select those points that lie within some deviation from the central value for the Higgs boson mass and show them in Fig. 11
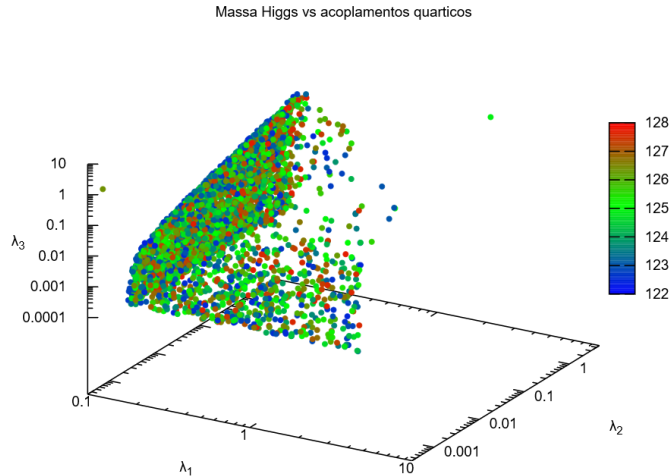

Figure 11: A palette plot of a selection of points where the SM Higgs Boson mass was found to be within a range of 3 sigma between the values of 122.5 and 127.5 GeVs.

Here we see that again quantum corrections seem to keep the Higgs mass around

the tree-level value up to a point seemingly again mainly defined by λ_2 where they start having a massive impact.

We can now test the phenomenological viability of these points by running the HiggsBounds and HiggsSignals packages on each of them.

In fig 12 we show the Higgs boson mass versus the exclusion ratio given by HiggsBounds. In Fig. 13 we show a different projection where the observation ratio is plotted against the new Higgs prime mass. In simple terms, if the exclusion ratio is larger than 1 the point is excluded by experimental data. In fact, it means that such a parameter point is predicting a signal larger than the observed limits. On the contrary, if this ratio is smaller than 1, the point is viable. In these two figures, we see that all viable points correspond to an observation ratio larger than 0.6 and, most of them, above 0.85. This means that such points are still valid but either close to rejection or to the discovery of a new scalar particle. It is also interesting to note that, as expected, if we move away from the central Higgs mass, a smooth line shows that the model is no longer as viable. However, there is no clear correlation observed for the Higgs prime mass.
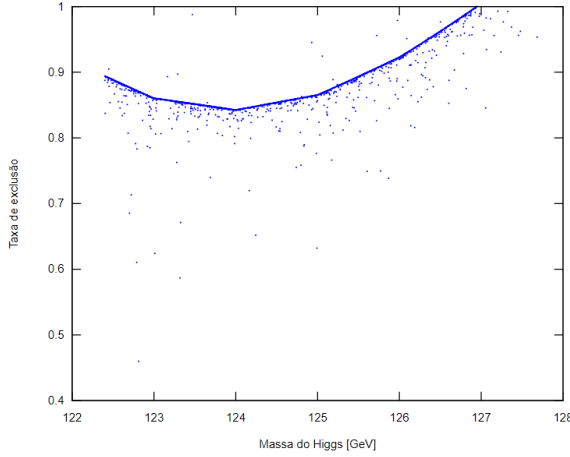

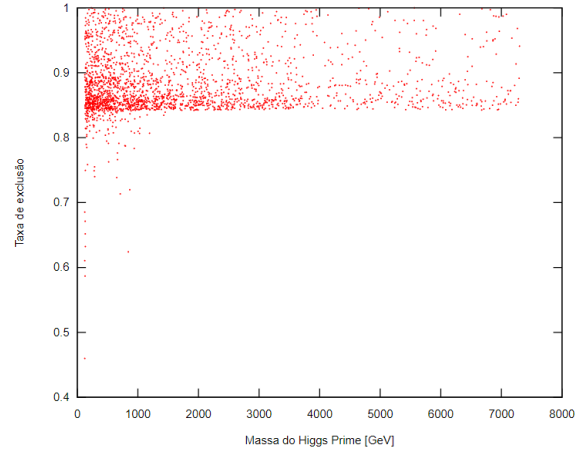Figure 12: A plot of a selection of non excluded SM Higgs boson mass vs the outputed HiggsBounds exclusion ratio.

Figure 13: A plot of a selection of non excluded Higgs prime boson mass vs the outputed HiggsBounds exclusion ratio.

In Figs. 14 and 15 we show what is denoted the Pvalue against the Higgs and Higgs prime masses respectively. The Pvalue is a parameter calculated by HiggsSignals that tells how well the Higgs boson is reconstructed in each point. It is then without surprise that we see a Pvalue close to 1 at about 125.40 GeV and quickly dropping below 124 GeV or above 126.5 GeV. Note also that, as expected, the Higgs prime mass does not have any impact on this value.
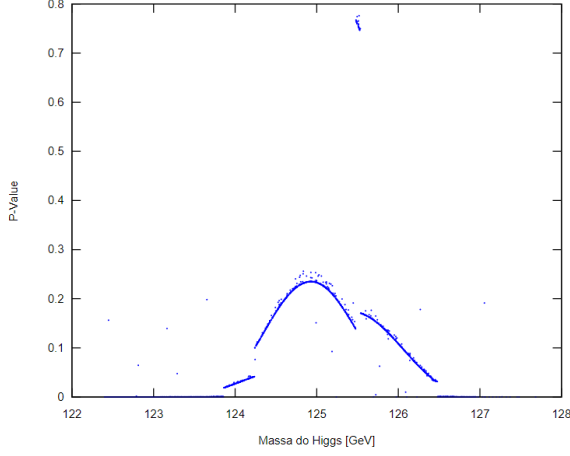
Figure 14: Exclusion rate vs Higgs mass for the allowed points.
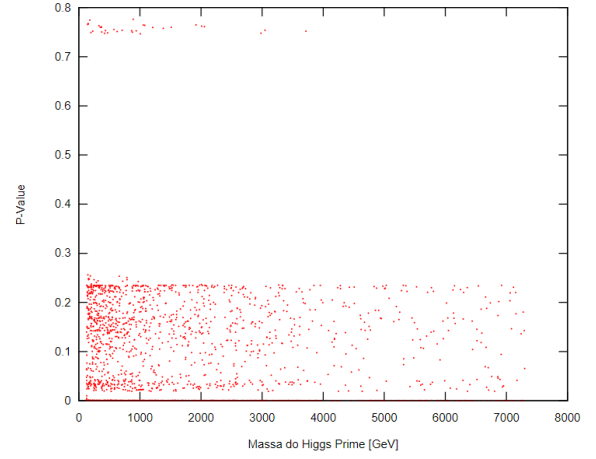

Figure 15: Exclusion rate vs Higgs prime mass for the allowed points.

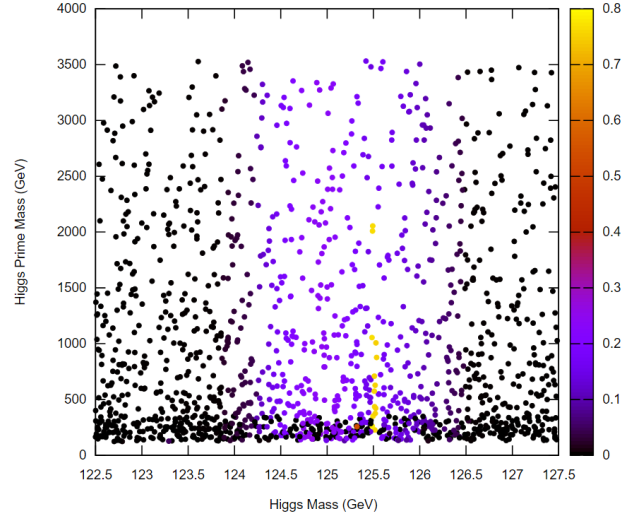combining these plots in a color palette plot we see,

Figure 16: A pallete plot of the probabilty factor vs the Higgs boson and Higgs prime boson.

The points that survived the HiggsBounds exclusion for a viable new scalar and that explain the Higgs boson signal up to to 3 standard deviations are shown in Fig. 17. We see that there is a viability region where all points with small λ_2 are rejected but only points with small λ_1 (from 0.12 up to 0.13) are accepted.
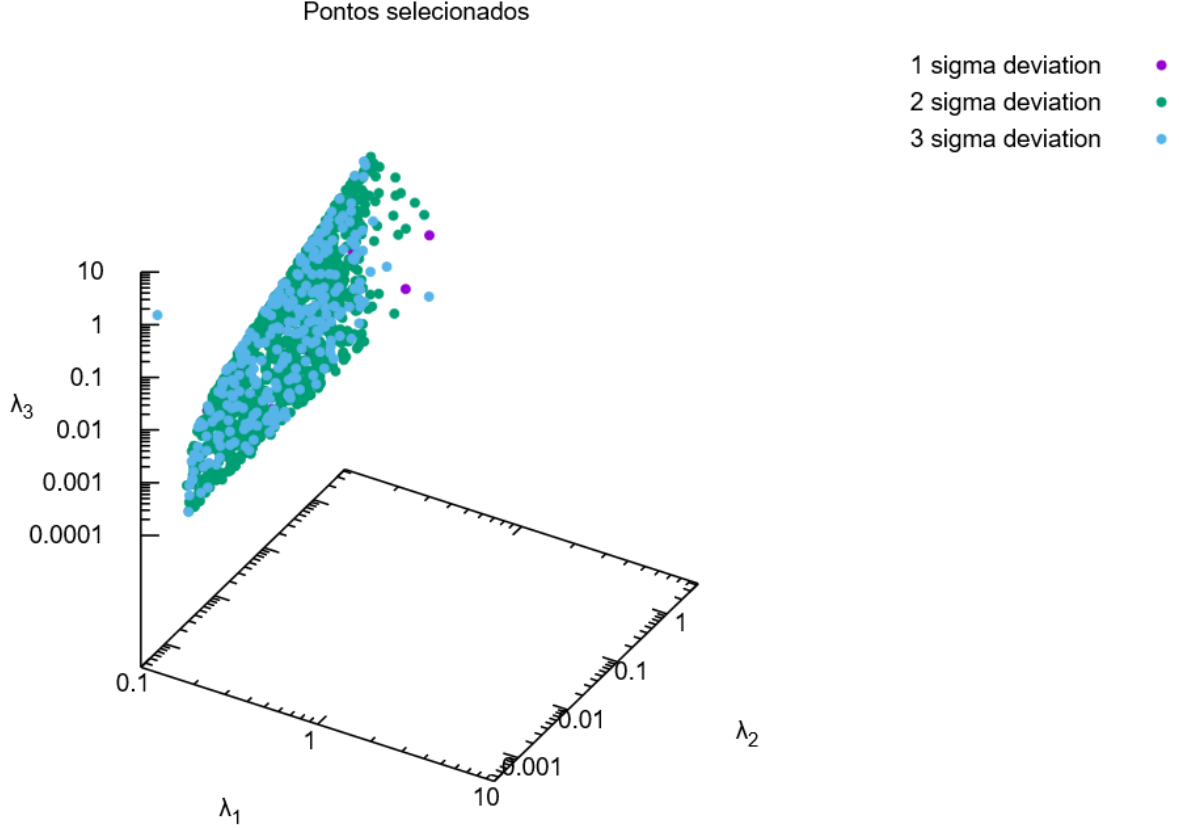
Figure 17: The quartic couplings points that led to a probability value within a deviation of 1 to 3 σ factor.

Projections planes of this data along the auxiliary planes can be done for better analysis of the exclusions behaviour. Note that there is not a clear observable pattern, perhaps due to the the lack of points, in relation to the sigma deviations so all the conclusions we'll take from these projections are in regards to the distribution of accepted points.

Starting by the (λ_1,λ_2) plane seen in fig. 18, the top view, we can see that there appears to be a triangular shape with most points clustered around a value of λ_1.
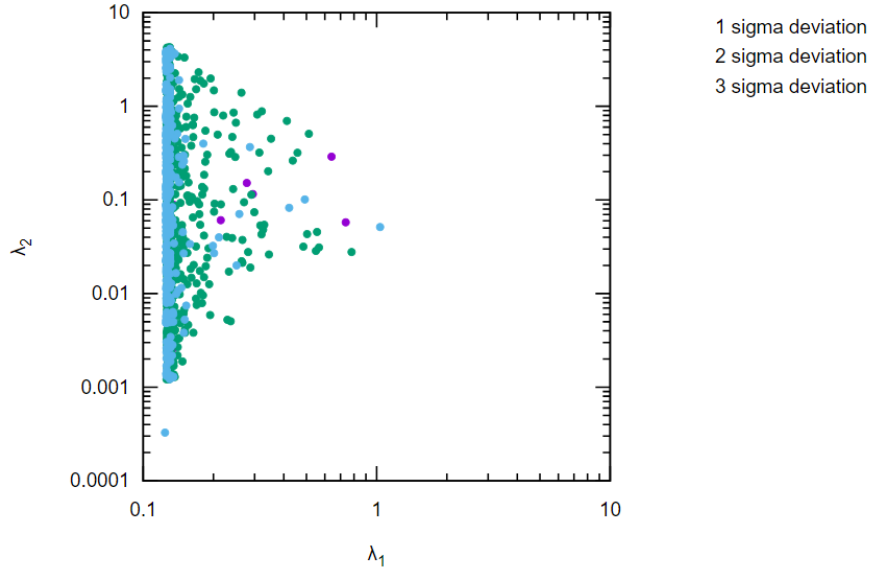
Figure 18: The quartic couplings points that led to a probability value within a deviation of 1 to 3 σ factor projected onto the auxilary plane (λ_1,λ_2).

The projection on the plane (λ_2,λ_3) plane seen in fig. 19, the "side" view, is more interesting since we can see the curvature of the fold of accepted points and here we can see it's stepper for lower values of λ_2.
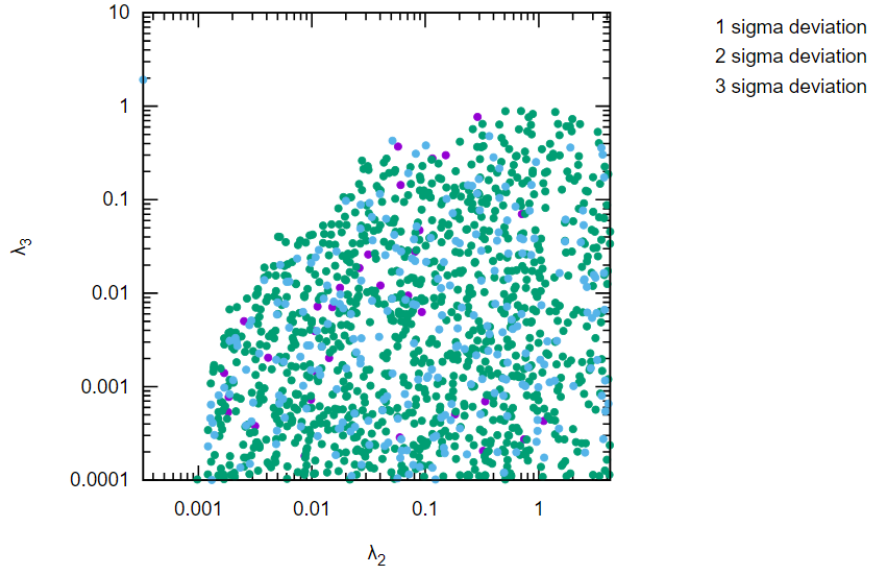

Figure 19: The quartic couplings points that led to a probability value within a deviation of 1 to 3 σ factor projected onto the auxilary plane (λ_2,λ_3).

The projection on the plane (λ_1,λ_3) plane seen in fig. 20, the "front" view, reveals the points are tightly constraint for λ_1 unless for reasonable values of λ_3.
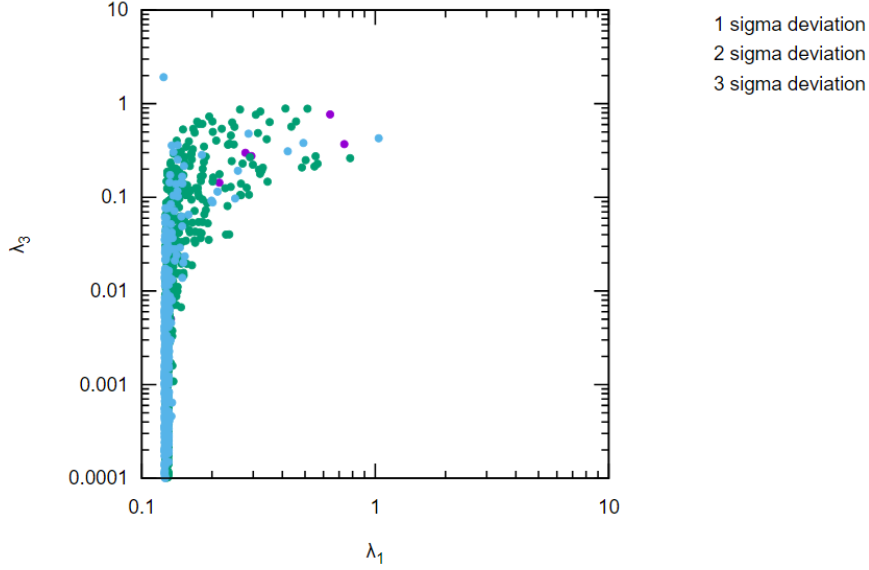
Figure 20: The quartic couplings points that led to a probability value within a deviation of 1 to 3 σ factor projected onto the auxilary plane (λ_1, λ_3).

4.2.2 The Z' boson

As it was already discussed, the requirement of a large $x \gg v$ implies that the Z' mass is decoupled from the SM Z boson and essentially depends on g'_1 and x. Searches for Z' bosons in the B-L-SM were already performed by ATLAS and are reported in

Model	Lower limits on					
	ee		$\mu\mu$		$l\,l$	
Z'_{B-L}	obs	exp	obs	exp	obs	exp
	4.0	4.0	3.6	3.6	4.2	4.1

Table 4: Experimental limits on the Z prime boson

In particular, the most stringent limit is coming from the decay into electrons, $Z' \to ee$, where the Lower limit in the Z' mass is 4.0 TeV at 95 % confidence level as seen in table 4. This means that $M_{Z'\text{min}} \simeq 2g'_1 x \Rightarrow x_{\text{min}} \simeq \frac{4}{2g'_1}$ TeV $\Rightarrow x_{\text{min}} \simeq \frac{2}{g'_1}$ TeV. Then, for our choice of x in the previous section, recall $x = 1$ TeV, it implies that the gauge coupling should be $g'_1 = 2$ for our model points not being excluded. We also see that if we were to choose $g'_1 = 1$, 0.5 and 0.1, the minimum values of x would be 2 TeV, 4 TeV and 20 TeV respectively. As a future study it would be interesting to study the impact of larger VEVs in the masses and observation ratios of the new scalar h' [4].

4.3 Conclusion

We began this essay by studying the SM and saw just how effective it can be at describing particle interactions up to the electroweak energy scale and how in the SM the mechanism behind mass generation stems from the breaking of a symmetry $SU(2) \times U(1)$ symmetry. We noted its inability to produce mass for neutrinos and this led us to explore the B-L-SM model, a extension of the SM that allowed to solve these issues. We used a minimal extension by means of a new Abelian gauge $U(1)_{B-L}$ symmetry and the introduction of right handed neutrinos. To break the $U(1)_{B-L}$ we also added a complex singlet whose VEV is responsible for generating mass to a new Z' gauge boson, a new scalar particle as well as three light and three heavy neutrinos. for right handed neutrinos and massive left handed neutrinos. We saw this model added a new Z' boson and new scalar particle Higgs particle.

We explored computer tools that allow for a study of the viability for our model. We gave the new gauge field χ a VEV of 1 TeV we discovered it to be far too low for a non-perturbative theory like quantum field theory to likely explain the Z prime boson, however for it we found a possibly viable range for the self interaction and cross interaction couplings of the scalar sector of the B-L-SM model.

We have thoroughly studied the scalar sector and its phenomenological viability in terms of accurately reproducing the Higgs mass and predicting a new scalar. We have studied the impact of one and two-loop corrections to the Higgs sector and verified that in the later case the quartic couplings in the scalar potential become limited from above with the maximum value $\lambda_i = 1$. We have also required that for all viable model points that we studied the Z' mass was 4 TeV and hence the g'_1 coupling had to be equal to 2.

As a future work, it would be interesting to let the singlet VEV x vary and, while imposing an allowed mass for the Z' gauge boson, study the effect of x on the viability region found in Fig. 17. It would also be interesting to perform phenomenological studies on the three heavy neutrinos and understand whether they could be observable or not at collider experiments. This model would also allow for a mass study of the neutrino mass spectrum based on the variation of yukawa couplings.

5 Appendix

5.0.1 Notation, Relativity and Lorentz transformations

Since the focus of this essay approaches the analysis of quantum theories that are tied to particle physics at the electroweak scale and beyond, we must ensure our study must be consistent with the theory of special relativity. Therefore we must revisit the definitions, the notation and the representations used in relativistic theories. From here onwards we will use natural units, so consider $\hbar = c = 1$. This gives all quantities dimensions of mass to some power.

Starting off shallowly touching on some basic concepts of relativity. Starting with the notion of interval between two events, this notion will replace our previous understanding of distance in a frame of motion.

For a generic set of space-time coordinates the interval between them is the quantity,s and is defined by $s^2 = c^2 dt^2 - (dx^2 + dy^2 + dz^2)$. This configuration of signs is stipulated by convenience, it would still be equivalent to have the plus and minus signs swapped but we will use the mostly negative representation during this essay.

$$x^\mu = (x^0, x^1, x^2, x^3) = (-ct, x, y, z) \tag{97}$$

$$x_\mu = (x_0, x_1, x_2, x_3) = (ct, -x, -y, -z) \tag{98}$$

We also define the covariant differential operator as a vector in this notation:

$$\partial_\mu = g_{\mu\nu}\partial^\nu = (\partial_0, \partial_1, \partial_2, \partial_3) = (\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}) = (\frac{1}{c}\frac{\partial}{\partial t}, \nabla) = \frac{\partial}{\partial x^\mu} \tag{99}$$

In this notation a vector with a upper index is called a covariant index and a vector with a lower index is called a contravariant vector. The inner product of these vectors would as we mention be a Lorentz invariant scalar. To further simplify the notation we will be combining this with the summation convention so a "contracted" index will automatically be summed from 0 to 3 as shown

$$\sum_{\mu=0}^{3} V_\mu V^\mu \longrightarrow V_\mu V^\mu \tag{100}$$

The relation relation between covariant and contravariant vectors is expressed with auxiliary to the space metric, $\eta_{\mu\nu}$. [9]

$$x_\mu = \eta_{\mu\nu} x^\nu \tag{101}$$

$$x_\mu = \eta_{\mu 0} x^0 + g_{\mu 1} x^1 + g_{\mu 2} x^2 + g_{\mu 3} x^3 \tag{102}$$

References

[1] Wikimedia Commons. File:standard model of elementary particles.svg — wikimedia commons, the free media repository, 2018. [Online; accessed 3-August-2018].

[2] Matthew D Schwartz. *Quantum field theory and the standard model*. Cambridge University Press, 2014.

[3] Giovanni Costa and Gianluigi Fogli. *Symmetries and group theory in particle physics: an introduction to space-time and internal symmetries*, volume 823. Springer Science & Business Media, 2012.

[4] Morad Aaboud et al. Search for new high-mass phenomena in the dilepton final state using 36 fb^{-1} of proton-proton collision data at $\sqrt{s} = 13$ TeV with the ATLAS detector. *JHEP*, 10:182, 2017.

[5] Werner Porod. Spheno, a program for calculating supersymmetric spectra, susy particle decays and susy particle production at e+ e- colliders. *Computer Physics Communications*, 153(2):275–315, 2003.

[6] Werner Porod and Florian Staub. Spheno 3.1: Extensions including flavour, cp-phases and models beyond the mssm. *Computer Physics Communications*, 183(11):2458–2469, 2012.

[7] Avelino Vicente. Computer tools in particle physics. *arXiv preprint arXiv:1507.06349*, 2015.

[8] Florian Staub, Thorsten Ohl, Werner Porod, and Christian Speckner. A tool box for implementing supersymmetric models. *Computer Physics Communications*, 183(10):2165–2206, 2012.

[9] Lewis H Ryder. *Quantum field theory*. Cambridge university press, 1996.