

A Renormalisation Group Equation Study of the Scalar Sector of the Minimal $B - L$ Extension of the Standard Model

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Abstract

We present the complete set of Renormalisation Group Equations (RGEs) at one loop for the non-exotic minimal $U(1)$ extension of the Standard Model (SM). It includes all models that are anomaly-free with the SM fermion content augmented by one Right-Handed (RH) neutrino per generation. We then pursue the numerical study of the pure $B - L$ model, deriving the triviality and vacuum stability bounds on an enlarged scalar sector comprising one additional Higgs singlet field with respect to the SM.

I. INTRODUCTION

The Large Hadron Collider has essentially been built to confirm or disprove the existence of one or more Higgs bosons. A lot of effort has therefore been put into studying models that can accommodate the Higgs mechanism of Electroweak Symmetry Breaking (EWSB), of which the Higgs (pseudo)scalar particles are remnants. Among these, the most studied one is the Standard Model (SM). Unfortunately, the SM is flawed. There is now experimental evidence of new phenomena that cannot be explained by the SM, notably (very small) neutrino masses. At the same time, it should be noted that the accidental $U(1)_{B-L}$ global symmetry (where $B(L)$ is the Barion(Lepton) number) is not anomalous in the SM with massless neutrinos, but its origin is not understood. It thus becomes appealing to extend the SM to explain simultaneously the existence of both neutrino masses and the $B - L$ global symmetry by gauging the $U(1)_{B-L}$ group and subject it to spontaneous EWSB induced by the Higgs mechanism (therefore generating a massive Z' state on the same footing as massive W^\pm and Z states are generated from the breaking of the $SU(2)_L \times U(1)_Y$ gauge symmetry of the SM). Consequently, this requires that the fermion and scalar spectra are enlarged to account for gauge anomaly cancellations in such a way as to evade direct searches. Minimally, this requires the addition of a scalar singlet and three (massive) right-handed neutrinos, one per generation [1–4], the latter entering the see-saw mechanism to explain the smallness of the detected (SM-like) neutrino masses [5].

Generally, $U(1)$ gauge factors mix and the mixing is controlled by further gauge couplings [6]. These extra parameters can then be reabsorbed in an effective parameterisation [7]. In the model we are considering, two Abelian groups are present, and just one extra coupling is therefore needed to account for the mixing, which can effectively be reabsorbed. The arising model is a minimal (i.e., one-dimensional) $U(1)$ extension of the SM, spanning over several benchmark models among which the “pure” $B - L$ model [3, 4, 8] is a particularly simple example, as we will describe in the next Section.

Unsurprisingly, because of the simple nature of such a model, following the experimental results on neutrino masses, a plethora of papers have been published studying the phenomenology of the $B - L$ model at colliders. They have dealt with the detectability of the Z' boson (see [1] for earlier studies on generic hadron colliders) at the LHC [8–12] and at a future Linear Collider (LC) [13–15], some analyses concentrating on the Z' decaying via

heavy neutrinos, in particular into three [8] and four [16] leptons in the final state, with distinctive displaced vertices due to long lived neutrinos, a clear signature of physics beyond the SM. Also, the testability at the LHC of the see-saw mechanism in this model has been evaluated in detail [17].

In comparison to the gauge and neutrino sectors, the Higgs part of this model has undergone much less scrutiny. Apart from the benchmark study of [9] dating back a few years, only recently a systematic analysis of the Higgs sector of the $B - L$ model has started, in the attempt to define the boundaries of the associated parameter space. Ref. [18] dealt with the limits stemming from the imposition of perturbative unitarity on the model. Here, we intend to pursue further into this attempt, by investigating the triviality and vacuum stability conditions ensuing in the $B - L$ model, through a RGE analysis aiming at defining the physical values of the masses and couplings of the two Higgs states emerging in the model after EWSB, the latter depending upon the maximum energy scale after which also such a scenario ceases to be valid and further new physics dynamics ought to be invoked. We believe that, with the LHC now on line, it is of paramount importance to theoretically constrain the Higgs sector of a new physics scenario that, while incorporating the SM, it remedies its major flaw without leading to a proliferation of new particles and/or interactions, thereby retaining much of the predictivity and testability of the SM.

In the past and yet recent years, a lot of effort has been spent for similar studies. For reviews on the SM and on some of its extensions, see Refs. [19, 20] and references therein. Concerning the study presented here, earlier works focusing on extra singlet scalars or E_6 -inspired $U(1)$ augmented gauge groups in non-supersymmetric [21, 22] and supersymmetric [23, 24] extensions of the SM, respectively, have already been considered.

The plan of the paper is as follows. In section II we describe the model under study. In section III we describe our computational techniques. The following section IV presents our numerical results while we conclude in section V. We also have an appendix, where we list the RGEs of the model that we have dealt with.

II. THE PARAMETERISATION

The model under study is the minimal $U(1)_{B-L}$ extension of the SM (see ref. [4] for conventions and references), in which the SM gauge group is augmented by a $U(1)$ factor,

related to the Baryon minus Lepton ($B - L$) gauged number. In the complete model, the classical gauge invariant Lagrangian, obeying the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ gauge symmetry, can be decomposed as:

$$\mathcal{L} = \mathcal{L}_s + \mathcal{L}_{YM} + \mathcal{L}_f + \mathcal{L}_Y. \quad (1)$$

The scalar Lagrangian is:

$$\mathcal{L}_s = (D^\mu H)^\dagger D_\mu H + (D^\mu \chi)^\dagger D_\mu \chi - V(H, \chi), \quad (2)$$

with the scalar potential given by

$$\begin{aligned} V(H, \chi) &= m^2 H^\dagger H + \mu^2 |\chi|^2 + \left(H^\dagger H |\chi|^2 \right) \begin{pmatrix} \lambda_1 & \frac{\lambda_3}{2} \\ \frac{\lambda_3}{2} & \lambda_2 \end{pmatrix} \begin{pmatrix} H^\dagger H \\ |\chi|^2 \end{pmatrix} \\ &= m^2 H^\dagger H + \mu^2 |\chi|^2 + \lambda_1 (H^\dagger H)^2 + \lambda_2 |\chi|^4 + \lambda_3 H^\dagger H |\chi|^2, \end{aligned} \quad (3)$$

where H and χ are the complex scalar Higgs doublet and singlet fields, respectively.

We generalise the SM discussion of spontaneous EWSB to the more complicated classical potential of eq. (3). To determine the condition for $V(H, \chi)$ to be bounded from below, it is sufficient to study its behaviour for large field values, controlled by the matrix in the first line of eq. (3). Requiring such a matrix to be positive-definite, we obtain the conditions:

$$4\lambda_1\lambda_2 - \lambda_3^2 > 0, \quad (4)$$

$$\lambda_1, \lambda_2 > 0. \quad (5)$$

If the above conditions are satisfied, we can proceed to the minimisation of V as a function of constant Vacuum Expectation Values (VEVs) for the two Higgs fields. Making use of gauge invariance, it is not restrictive to assume:

$$\langle H \rangle \equiv \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \quad \langle \chi \rangle \equiv \frac{x}{\sqrt{2}}, \quad (6)$$

with v and x real and non-negative. The physically most interesting solutions to the minimisation of eq. (3) are obtained for v and x both non-vanishing:

$$v^2 = \frac{-\lambda_2 m^2 + \frac{\lambda_3}{2} \mu^2}{\lambda_1 \lambda_2 - \frac{\lambda_3^2}{4}}, \quad (7)$$

$$x^2 = \frac{-\lambda_1 \mu^2 + \frac{\lambda_3}{2} m^2}{\lambda_1 \lambda_2 - \frac{\lambda_3^2}{4}}. \quad (8)$$

To compute the scalar masses, we must expand the potential in eq. (3) around the minima in eqs. (7) and (8). We denote by h_1 and h_2 the scalar fields of definite masses, m_{h_1} and m_{h_2} respectively, and we conventionally choose $m_{h_1}^2 < m_{h_2}^2$. After standard manipulations, the explicit expressions for the scalar mass eigenvalues and eigenvectors are:

$$m_{h_1}^2 = \lambda_1 v^2 + \lambda_2 x^2 - \sqrt{(\lambda_1 v^2 - \lambda_2 x^2)^2 + (\lambda_3 x v)^2}, \quad (9)$$

$$m_{h_2}^2 = \lambda_1 v^2 + \lambda_2 x^2 + \sqrt{(\lambda_1 v^2 - \lambda_2 x^2)^2 + (\lambda_3 x v)^2}, \quad (10)$$

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} h \\ h' \end{pmatrix}, \quad (11)$$

where $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$ fulfils¹:

$$\sin 2\alpha = \frac{\lambda_3 x v}{\sqrt{(\lambda_1 v^2 - \lambda_2 x^2)^2 + (\lambda_3 x v)^2}}, \quad (12)$$

$$\cos 2\alpha = \frac{\lambda_1 v^2 - \lambda_2 x^2}{\sqrt{(\lambda_1 v^2 - \lambda_2 x^2)^2 + (\lambda_3 x v)^2}}. \quad (13)$$

For our numerical study of the extended Higgs sector, it is useful to invert eqs. (9), (10) and (12), to extract the parameters in the Lagrangian in terms of the physical quantities m_{h_1} , m_{h_2} and $\sin 2\alpha$:

$$\begin{aligned} \lambda_1 &= \frac{m_{h_2}^2}{4v^2}(1 - \cos 2\alpha) + \frac{m_{h_1}^2}{4v^2}(1 + \cos 2\alpha), \\ \lambda_2 &= \frac{m_{h_1}^2}{4x^2}(1 - \cos 2\alpha) + \frac{m_{h_2}^2}{4x^2}(1 + \cos 2\alpha), \\ \lambda_3 &= \sin 2\alpha \left(\frac{m_{h_2}^2 - m_{h_1}^2}{2xv} \right). \end{aligned} \quad (14)$$

Moving to the \mathcal{L}_{YM} , the non-Abelian field strengths therein are the same as in the SM whereas the Abelian ones can be written as follows:

$$\mathcal{L}_{YM}^{\text{Abel}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}F'^{\mu\nu}F'_{\mu\nu}, \quad (15)$$

where

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad (16)$$

$$F'_{\mu\nu} = \partial_\mu B'_\nu - \partial_\nu B'_\mu. \quad (17)$$

¹ In all generality, the whole interval $0 \leq \alpha < 2\pi$ is halved because an orthogonal transformation is invariant under $\alpha \rightarrow \alpha + \pi$. We could re-halve the interval by noting that it is invariant also under $\alpha \rightarrow -\alpha$ if we permit the eigenvalues inversion, but this is forbidden by our convention $m_{h_1}^2 < m_{h_2}^2$. Thus α and $-\alpha$ are independent solutions.

In this field basis, the covariant derivative is:

$$D_\mu \equiv \partial_\mu + ig_S T^\alpha G_\mu^\alpha + ig T^a W_\mu^a + ig_1 Y B_\mu + i(\tilde{g}Y + g'_1 Y_{B-L})B'_\mu. \quad (18)$$

To determine the gauge boson spectrum, we have to expand the scalar kinetic terms as for the SM. We expect that there exists a massless gauge boson, the photon, whilst the other gauge bosons become massive. The extension we are studying is in the Abelian sector of the SM gauge group, so that the charged gauge bosons W^\pm will have masses given by their SM expressions, being related to the $SU(2)_L$ factor only. Using the unitary-gauge parameterisation, the kinetic terms in eq. (2) become:

$$\begin{aligned} (D^\mu H)^\dagger D_\mu H &= \frac{1}{2} \partial^\mu h \partial_\mu h + \frac{1}{8} (h+v)^2 (0 \ 1) \left[g W_a^\mu \sigma_a + g_1 B^\mu + \tilde{g} B'^\mu \right]^2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ &= \frac{1}{2} \partial^\mu h \partial_\mu h + \frac{1}{8} (h+v)^2 \left[g^2 |W_1^\mu - i W_2^\mu|^2 \right. \\ &\quad \left. + (g W_3^\mu - g_1 B^\mu - \tilde{g} B'^\mu)^2 \right], \end{aligned} \quad (19)$$

and

$$(D^\mu \chi)^\dagger D_\mu \chi = \frac{1}{2} \partial^\mu h' \partial_\mu h' + \frac{1}{2} (h' + x)^2 (g'_1 2 B'^\mu)^2, \quad (20)$$

where we have taken $Y_\chi^{B-L} = 2$ in order to guarantee the gauge invariance of the Yukawa terms (see eq. (25)). In eq. (19) we can recognise immediately the SM charged gauge bosons W^\pm , with $M_W = gv/2$ as in the SM. The other gauge boson masses are not so simple to identify, because of mixing. In fact, in analogy with the SM, the fields of definite mass are linear combinations of B^μ , W_3^μ and B'^μ . The explicit expressions are:

$$\begin{pmatrix} B^\mu \\ W_3^\mu \\ B'^\mu \end{pmatrix} = \begin{pmatrix} \cos \vartheta_w & -\sin \vartheta_w \cos \vartheta' & \sin \vartheta_w \sin \vartheta' \\ \sin \vartheta_w & \cos \vartheta_w \cos \vartheta' & -\cos \vartheta_w \sin \vartheta' \\ 0 & \sin \vartheta' & \cos \vartheta' \end{pmatrix} \begin{pmatrix} A^\mu \\ Z^\mu \\ Z'^\mu \end{pmatrix}, \quad (21)$$

with $-\frac{\pi}{4} \leq \vartheta' \leq \frac{\pi}{4}$, such that:

$$\tan 2\vartheta' = \frac{2\tilde{g}\sqrt{g^2 + g_1^2}}{\tilde{g}^2 + 16(\frac{x}{v})^2 g_1'^2 - g^2 - g_1^2} \quad (22)$$

and

$$\begin{aligned} M_A &= 0, \\ M_{Z,Z'} &= \sqrt{g^2 + g_1^2} \cdot \frac{v}{2} \left[\frac{1}{2} \left(\frac{\tilde{g}^2 + 16(\frac{x}{v})^2 g_1'^2}{g^2 + g_1^2} + 1 \right) \mp \frac{\tilde{g}}{\sin 2\vartheta' \sqrt{g^2 + g_1^2}} \right]^{\frac{1}{2}}, \end{aligned} \quad (23)$$

where

$$\sin 2\vartheta' = \frac{2\tilde{g}\sqrt{g^2 + g_1^2}}{\sqrt{(\tilde{g}^2 + 16(\frac{x}{v})^2 g_1'^2 - g^2 - g_1^2)^2 + (2\tilde{g})^2(g^2 + g_1^2)}}.$$

LEP experiments [25] constrain $|\vartheta'| \lesssim 10^{-3}$. Present constraints on the VEV x (see section IV B) allow a generous range of \tilde{g} .

The fermionic Lagrangian (where k is the generation index) is given by

$$\begin{aligned} \mathcal{L}_f = \sum_{k=1}^3 & \left(i\overline{q_{kL}}\gamma_\mu D^\mu q_{kL} + i\overline{u_{kR}}\gamma_\mu D^\mu u_{kR} + i\overline{d_{kR}}\gamma_\mu D^\mu d_{kR} + \right. \\ & \left. + i\overline{l_{kL}}\gamma_\mu D^\mu l_{kL} + i\overline{e_{kR}}\gamma_\mu D^\mu e_{kR} + i\overline{\nu_{kR}}\gamma_\mu D^\mu \nu_{kR} \right), \end{aligned} \quad (24)$$

where the fields' charges are the usual SM and $B - L$ ones (in particular, $B - L = 1/3$ for quarks and -1 for leptons with no distinction between generations, hence ensuring universality). The $B - L$ charge assignments of the fields as well as the introduction of new fermionic RH heavy neutrinos (ν_R 's) and a scalar Higgs field (χ , charged $+2$ under $B - L$) are designed to eliminate the triangular $B - L$ gauge anomalies and to ensure the gauge invariance of the theory, respectively. Therefore, the $B - L$ gauge extension of the SM gauge group broken at the Electro-Weak (EW) scale does necessarily require at least one new scalar field and three new fermionic fields which are charged with respect to the $B - L$ group.

Finally, the Yukawa interactions are:

$$\begin{aligned} \mathcal{L}_Y = & -y_{jk}^d \overline{q_{jL}} d_{kR} H - y_{jk}^u \overline{q_{jL}} u_{kR} \tilde{H} - y_{jk}^e \overline{l_{jL}} e_{kR} H \\ & - y_{jk}^\nu \overline{l_{jL}} \nu_{kR} \tilde{H} - y_{jk}^M \overline{(\nu_R)_j^c} \nu_{kR} \chi + \text{h.c.}, \end{aligned} \quad (25)$$

where $\tilde{H} = i\sigma^2 H^*$ and i, j, k take the values 1 to 3, where the last term is the Majorana contribution and the others the usual Dirac ones.

Neutrino mass eigenstates, obtained after applying the see-saw mechanism, will be called ν_l (with l standing for light) and ν_h (with h standing for heavy), where the first ones are the SM-like ones.

A. Realistic models

The generic model that has been previously introduced spans over a continuous set of minimal $U(1)$ extensions of the SM, that can be labelled by the properties of the charge

assignments to the particle content. Notice that such models are, by construction, all and only those that are not anomalous with the SM fermion content augmented by one RH neutrino per generation. Therefore, many E_6 -inspired $U(1)$ scenarios, such as $U(1)_\psi$ or $U(1)_\eta$, are not included in our generic model.

Free parameters in our parameterisation are those appearing in the covariant derivative of eq. (18). We recall the Abelian part only:

$$D_\mu \equiv \cdots + ig_1 Y B_\mu + i(\tilde{g}Y + g'_1 Y_{B-L})B'_\mu.$$

This form of the covariant derivative can be re-written defining an effective coupling Y^E and an effective charge g_E :

$$g_E Y^E = \tilde{g}Y + g'_1 Y_{B-L}. \quad (26)$$

As any other parameter in the Lagrangian, \tilde{g} and g'_1 are running parameters [6, 7], therefore their values ought to be defined at some scale. A discrete set of popular Z' models (see, e.g., Refs. [26, 27]) can be recovered by a suitable definition of both \tilde{g} and g'_1 .

We will focus our numerical analysis on the scalar sector of the “pure” $B-L$ model, that is defined by the condition $\tilde{g}(Q_{EW}) = 0$, i.e., we nullify it at the EW scale. This implies no mixing at the tree-level between the $B-L$ Z' and SM Z gauge bosons. Other benchmark models of our general parameterisation are for example the Sequential SM (SSM), defined by $Y^E = Y$ (that in our notation corresponds to the condition $g'_1 = 0$ at the EW scale) and the $U(1)_R$ model, for which RH fermion charges vanish (that is recovered here by the condition $\tilde{g} = -2g'_1$ at the EW scale).

It is important to note that none of the models described so far is orthogonal to the $U(1)_Y$ of the SM, therefore the RGE running of the fundamental parameters, \tilde{g} and g'_1 , will modify the relations above. The only orthogonal $U(1)$ extension of the SM is the “ $SO(10)$ -inspired” $U(1)_\chi$ model, that in our notation reads $\tilde{g} = -\frac{4}{5}g'_1$. Although the \tilde{g} and g'_1 couplings run with a different behaviour, the EW relation $\tilde{g}/g'_1 = -4/5$ is preserved (at one-loop) at any scale.

Nonetheless, as indeed true for the SM, the gauge sector affects marginally the scalar sector in its running, so the analysis we are going to show is effectively independent of the specific charge assignation. However, we might expect differences regarding the interplay between the gauge sector and the neutrino evolution, that impinge on the vacuum stability studies of the scalar sector as the top quark does for the SM Higgs sector. We will report

separately on the study of the differences in the RGE study of the specific benchmark models in our generic parameterisation.

III. COMPUTATIONAL DETAILS

The complete set of RGEs for the generic model are derived for the parameters in the Lagrangian and are collected in appendix A. For their numerical study, we put boundary conditions at the EW scale on the physical observables: m_{h_1} , m_{h_2} , α , v , $M_{Z'}$, g'_1 , \tilde{g} , $m_{\nu_h}^{1,2,3}$, that we trade for m , μ , λ_1 , λ_2 , λ_3 , x , $y_{1,2,3}^M$ using, for the relevant parameters therein, eq. (14). Where stated in the text, we impose boundary conditions on some parameters of the Lagrangian rather than on the physical observables. This is done for consistency of those studies.

For the pure $B - L$ model, object of the numerical analysis in this work, the definition $\tilde{g} = 0$ holds, and as a consequence, we also have that the $B - L$ breaking VEV x can be easily related to the new Z' boson mass by $x = \frac{M_{Z'}}{2g'_1}$, where we fixed $g'_1 = 0.1$. Regarding the neutrinos, for simplicity we consider them degenerate and we fix their masses to $m_{\nu_h}^{1,2,3} \equiv m_{\nu_h} = 200$ GeV (whenever not specified otherwise), a value that can lead to some interesting phenomenology [8]. The free parameters in our study are then m_{h_1} , m_{h_2} , α and x . The general philosophy is to fix in turn some of the free parameters and scan over the other ones, individuating the allowed regions fulfilling the following set of conditions.

We first define a parameter to be “perturbative” for values less than unity. This is a conservative definition, as we could relax it by an order of magnitude and still get values of the parameters for which the perturbative series will converge². RGE evolution can then constrain the parameter space of the scalar sector in two complementary ways. From one side, the couplings must be perturbative. This condition reads:

$$0 < \lambda_{1,2,3}(Q') < 1 \quad \forall Q' \leq Q, \quad (27)$$

and it is usually referred to as the “triviality” condition. On the other side, the vacuum of the theory must be well-defined at any scale, that is, to guarantee the validity of eqs. (4)

² Notice that, in analogy with QED, the parameters upon which the perturbative expansion is performed are usually of the form $\sqrt{\alpha} = g/\sqrt{4\pi}$, rather than being g itself.

and (5) at any scale $Q' \leq Q$:

$$0 < \lambda_{1,2,3}(Q') \quad \text{and} \quad 4\lambda_1(Q')\lambda_2(Q') - \lambda_3^2(Q') > 0 \quad \forall Q' \leq Q. \quad (28)$$

Eq. (28) is usually referred to as the “vacuum stability” condition. In contrast to the SM, in which it is sufficient the Higgs self-coupling λ be positive, in the case of this model the vacuum stability condition (and especially the second part of eq. (28)) can be violated even for positive $\lambda_{1,2,3}$.

One should notice that our conventional choice $m_{h_1} < m_{h_2}$, as noted previously, let us consider α and $-\alpha$ as two independent solutions, although the theory is manifestly invariant under the symmetry $\alpha \rightarrow -\alpha$. These two solutions are complementary, meaning that the region excluded by the choice $m_{h_1} < m_{h_2}$ at a certain value of the angle α is precisely the allowed one for the complementary angle $\pi/2 - \alpha$. The special case $\alpha = \pi/4$ is symmetric, and corresponds to maximal mixing between the scalars. $\alpha = 0$ corresponds to a SM scalar sector totally decoupled from the extended one, and h_1 is the usual SM Higgs boson. $\alpha = \pi/2$ is the specular case, in which h_2 plays the role of the SM Higgs boson.

Notice also that, again in contrast to the SM in which the gauge couplings have a marginal effect, in our case the RH neutrinos play for the extra scalar singlet the role of the top quark for the SM Higgs in the vacuum stability condition³. Their RGE are then controlled by the Yukawa coupling with a negative contribution coming from g'_1 (see eq. (A9)). Therefore, in some regions of the parameter space, the impact of the gauge sector is not marginal and can effectively stabilise the otherwise divergent evolution of the Majorana Yukawa couplings for the RH neutrinos. We will report on the effect of RH neutrinos in our analysis in section IV D.

A final remark is in order about eq. (A33), the evolution of λ_3 , the mixing parameter of the scalar potential (see eq. (3)). This RGE is almost proportional to λ_3 itself, so a vanishing boundary condition is almost stable⁴. Non-proportional terms arise from the new gauge couplings (\tilde{g} and g'_1), i.e., deviations from the vanishing boundary conditions are of the order of the gauge coupling, hence quite small. They are particularly negligible in the

³ Also notice that we have three RH neutrinos, as we have three colours for the top quark. However, they are Majorana particles rather than Dirac ones, so they carry half (independent) degrees of freedom than the top quark.

⁴ From the last line of eq. (14), setting $\lambda_3 = 0$ corresponds to $\alpha = 0$, but not vice versa.

pure $B - L$ model, as also \tilde{g} has a vanishing boundary condition, with a weak departure from it due to the mixing in the gauge coupling sector [4]. Nonetheless, other benchmark models in our general parameterisation could show different behaviours.

IV. RESULTS

We present here our results for the pure $B - L$ model, the chosen benchmark of our general parameterisation. We will first present a brief analysis of the gauge sector, followed by a quick review of the present experimental constraints on the Higgs boson masses. Finally, we will fully describe the scalar sector analysis, argument of this paper⁵.

A. Gauge sector

Before starting the analysis of the scalar sector, we can briefly look at the gauge sector, where the RGE evolution gives us indications for the validity of the model concerning the gauge couplings. In particular, their evolution must stay perturbative up to some particular scale. In the $B - L$ model, the conditions that the free parameters in the gauge sector must fulfil are:

$$g'_1(Q') < 1 \quad \forall Q' \leq Q \quad \text{and} \quad \tilde{g}(Q_{EW}) = 0, \quad (29)$$

where the second condition in eq. (29) defines the pure $B - L$ model.

Varying the scale Q , the maximum scale up to which we want the model to be well-defined, we get an upper bound on $g'_1(Q_{EW})$ as a function of Q , as shown in figure 1. Typical results are summarised in table I.

$Log_{10}(Q/\text{GeV})$	3	5	7	10	15	19
$g'_1(Q_{EW})$	0.860	0.693	0.593	0.497	0.397	0.342

TABLE I: *Maximum allowed values by eq. (29) for $g'_1(Q_{EW})$ in the $B - L$ model for selected values of the scale Q .*

⁵ Notice that we study the gauge sector of the model (and, in particular, the Abelian part of it) independently of any other sector as the corresponding RGEs fully decouple.

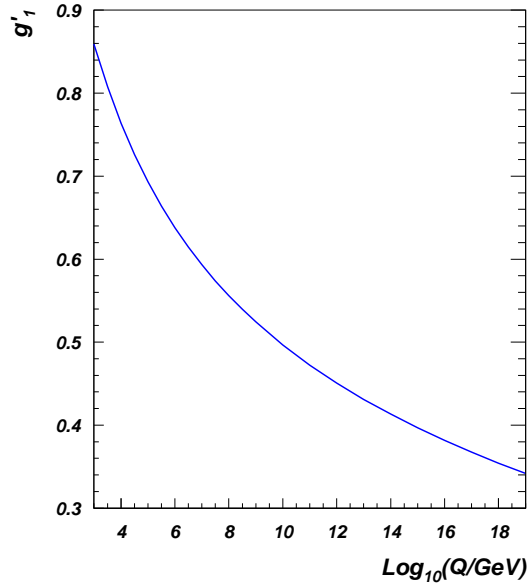


FIG. 1: *Maximum allowed values by eq. (29) for $g'_1(Q_{EW})$ in the $B - L$ model as a function of the scale Q .*

B. Experimental limit

Past and current experiments have set limits on the scalar sector parameters in the SM as well as in various extensions of it, see for example Ref. [28] for LEP and Ref. [29] for Tevatron. For the model discussed here, the relevant analysis is summarised in figure 2, in which a generic overall factor ξ has been introduced. Such parameter is defined as the coupling(s) to the Z boson of the Higgs particle(s) in the considered extension normalised to the SM:

$$\xi \equiv \frac{g_{HZZ}}{g_{HZZ}^{SM}}, \quad (30)$$

hence it parametrises the deviations of the new model with respect to the SM.

In the minimal $U(1)$ extension of the SM, argument of this paper, two scalar eigenstates exist: the one coming from the Higgs singlet, required to break the extra $U(1)_{B-L}$ gauge factor (and therefore giving the Z' gauge boson a mass), and the one coming from the Higgs doublet, required to break the SM gauge symmetry to give masses to the W and Z bosons. With reference to eq. (11), we called h_1 the lightest of such eigenstates, that couples to the Z boson proportionally to $\cos \alpha$, and with h_2 we referred to the heaviest scalar, that couples to the Z boson proportionally to $\sin \alpha$. Hence, the LEP lower bounds on the scalar masses of the $U(1)_{B-L}$ extension here considered are read straightforwardly from figure 2

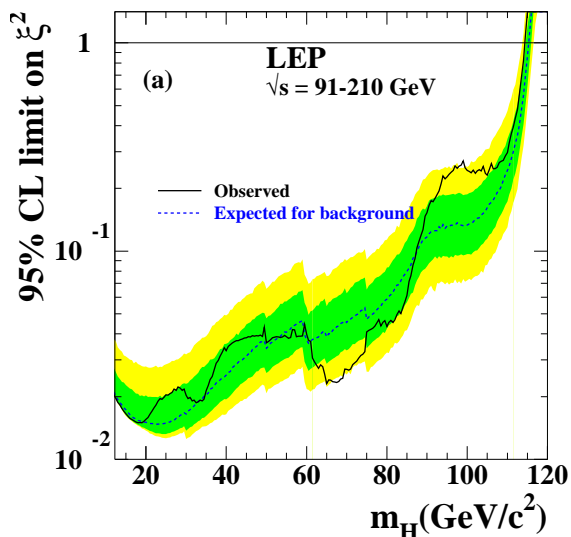


FIG. 2: The 95% C.L. upper bound on $\xi = g_{HZZ}/g_{HZZ}^{SM}$ [28]. In the $B-L$ model, $\xi = \cos \alpha (\sin \alpha)$ for $H = h_1 (h_2)$.

by considering:

$$\begin{cases} \xi = \cos \alpha & \text{for } H = h_1, \\ \xi = \sin \alpha & \text{for } H = h_2, \end{cases} \quad (31)$$

i.e., the limit for $h_1 (h_2)$ are extracted by considering ξ as the cosine(sine) of the mixing angle in the scalar sector (see eq. (11) and the following ones).

Figure 2 shows the lower bound on the Higgs mass as a function of ξ . The SM Higgs is recovered by the condition $\xi = 1$. We see that we can have significant deviation from the SM Higgs mass limit, $m_h > 114.4$ GeV, only for values of the angle $\alpha > \pi/4$, for the lightest state h_1 . For example, for $\alpha = \pi/3$, the LEP limit on the lightest Higgs state reads as $m_{h_1} > 100$ GeV. That is, in this model, a light Higgs with mass smaller than the SM limit can exist only if it is highly mixed, i.e., the light Higgs is mostly the singlet state. For the same value of the angle, the limit for m_{h_2} is more stringent than the condition $m_{h_2} > m_{h_1}$, in fact for $\alpha = \pi/3$, $m_{h_2} \gtrsim 114$ GeV must be fulfilled.

The LEP experiments are also able to provide a lower bound for the $B-L$ breaking VEV x . In fact, the LEP bound on the $B-L$ Z' mass [30],

$$\frac{M_{Z'}}{g'_1} \geq 7 \text{ TeV} \quad (32)$$

can be rewritten as a lower bound for the VEV:

$$x \geq 3.5 \text{ TeV}, \quad (33)$$

since $M_{Z'} = 2xg'_1$ in the pure $B - L$ model.

C. Scalar sector

Given the simplicity of the scalar sector in the SM, the triviality and vacuum stability conditions can be studied independently and they both constrain the Higgs boson masses, providing an upper bound and a lower bound, respectively. In more complicated models as the one considered here, it might be more convenient to study the overall effect of eqs. (27)-(28), since there are regions of the parameter space in which the constraints are evaded simultaneously. This is the strategy we decided to follow.

Figure 3 shows the allowed region in the parameter space m_{h_1} - m_{h_2} for increasing values of the mixing angle α , for fixed VEV $x = 7.5$ TeV and heavy neutrino masses $m_{\nu_h} = 200$ GeV, corresponding to Yukawa couplings whose effect on the RGE running can be considered negligible. For $\alpha = 0$, the allowed values for m_{h_1} are the SM ones and the extended scalar sector is completely decoupled. The allowed space is therefore the simple direct product of the two, as we can see in figure 3a. When there is no mixing, the bounds we get for the new heavy scalar are quite loose, allowing a several TeV range for m_{h_2} , depending on the scale of validity of the theory. We observe no significant lower bounds (i.e., $m_{h_2} > 0.5$ GeV), as the RH Majorana neutrino Yukawa couplings are negligible.

As we increase the value for the angle, the allowed space deforms towards smaller values of m_{h_1} . If for very small scales Q of validity of the theory such masses have already been excluded by LEP, for big enough values of Q , at a small angle as $\alpha = 0.1$, the presence of a heavier boson allows the model to survive up to higher scales for smaller h_1 masses if compared to the SM (in which just h_1 would exist). Correspondingly, the constraints on m_{h_2} become tighter. Moving to bigger values of the angle, the mixing between h_1 and h_2 grows up to its maximum, at $\alpha = \pi/4$, where h_1 and h_2 both contain an equal amount of doublet and singlet scalars. The situation is therefore perfectly symmetric, as one can see from figure 3c. Finally, in figure 3d, we see that the bounds on m_{h_2} are getting tighter, approaching the SM ones, and those for m_{h_1} are relaxing. That is, for values of the angle $\pi/4 < \alpha < \pi/2$, the situation is qualitatively not changed, but now h_2 is the SM-like Higgs boson. Visually, one can get the allowed regions at a given angle $\pi/2 - \alpha$ by simply taking the transposed about the $m_{h_1} = m_{h_2}$ line of the plot for the given angle α .

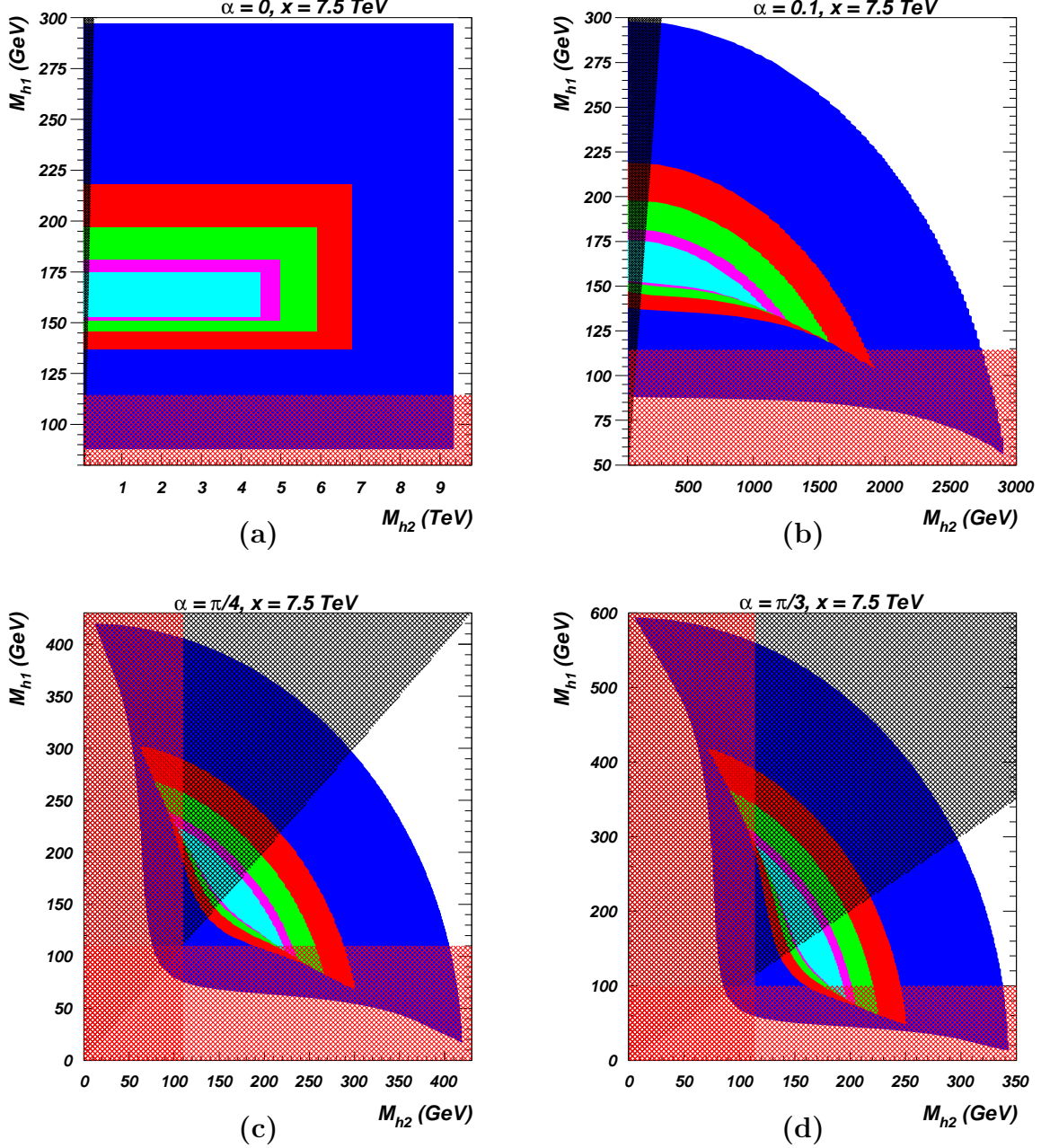


FIG. 3: Allowed values in the m_{h_1} vs. m_{h_2} space in the $B-L$ model by eqs. (27) and (28), for (3a) $\alpha = 0$, (3b) $\alpha = 0.1$, (3c) $\alpha = \pi/4$ and (3d) $\alpha = \pi/3$. Colours refer to different values of Q/GeV : blue (10^3), red (10^7), green (10^{10}), purple (10^{15}) and cyan (10^{19}). The shaded black region is forbidden by our convention $m_{h_2} > m_{h_1}$, while the shaded red region refers to the values of the scalar masses forbidden by LEP. Here: $x = 7.5 \text{ TeV}$, $m_{\nu_h} = 200 \text{ GeV}$.

Per each value of the angle, we can then fix the lighter Higgs mass m_{h_1} to some benchmark

values (allowed by LEP for the SM Higgs) and plot the allowed mass for the heavier Higgs as a function of the scale Q . This is done in figure 4, where the allowed masses are those contained between the same colour lines. Notice that here the VEV x is fixed to a different value, $x = 3.5$ TeV. The effects of changing the VEV x will be described in section IV E.

As previously noticed, the allowed range in m_{h_2} gets smaller as we increase the angle. Apart from the case $\alpha = 0$ where there is no dependency at all from m_{h_1} , there is a strong effect from m_{h_1} on the bounds on m_{h_2} . Not all the allowed regions at a fixed h_1 mass are contained in the region for a smaller m_{h_1} . This is true only for $m_{h_1} > 160$ GeV. For smaller m_{h_1} 's, the distortion in the allowed region constraints tightly m_{h_2} for the survival of the model to big scales Q . This is because such distortion is just towards smaller h_1 masses, see figure 3.

Complementary to the previous study, we can now fix the light Higgs mass at specific, experimentally interesting⁶, values, i.e., $m_{h_1} = 100, 120, 160$ and 180 GeV, and show the allowed region in the m_{h_2} vs. α plane. This is done in figure 5.

From this figures it is clear the transition of h_2 from the new extra scalar to the SM-like Higgs boson as we scan on the angle. As we increase m_{h_1} (up to $m_{h_1} = 160$ GeV), a bigger region in m_{h_2} is allowed for the model to be valid up to the Plank scale (the most inner regions, in cyan). Nonetheless, such a region exists also for a value of the light Higgs mass excluded by LEP for the SM, $m_{h_1} = 100$ GeV, but only for big values of the mixing angle. No new regions (with respect to the SM) in which the model can survive up to the Plank scale open for $m_{h_1} > 160$ GeV, as the allowed space deforms towards smaller values of m_{h_1} .

D. Heavy neutrino mass influence

As stated in section III, the RH neutrinos play for the extra scalar singlet the role of the top quark for the SM Higgs. This is particularly true for the vacuum stability condition, as the fermions in general provide the negative term that can drive the scalar couplings towards negative values. Figure 6 shows how the allowed regions in the m_{h_1} - m_{h_2} plane change for a RH Majorana neutrino Yukawa coupling $y^M = 0.2$ (that for $x = 3.5$ TeV correspond to $m_{\nu_h} = 1$ TeV), not negligible anymore. For $y^M = 0.4$, the changes are even more drastic,

⁶ The chosen values maximise the probability for the decays $h_1 \rightarrow b\bar{b}$, $h_1 \rightarrow \gamma\gamma$, $h_1 \rightarrow W^+W^-$ and $h_1 \rightarrow ZZ$, respectively.

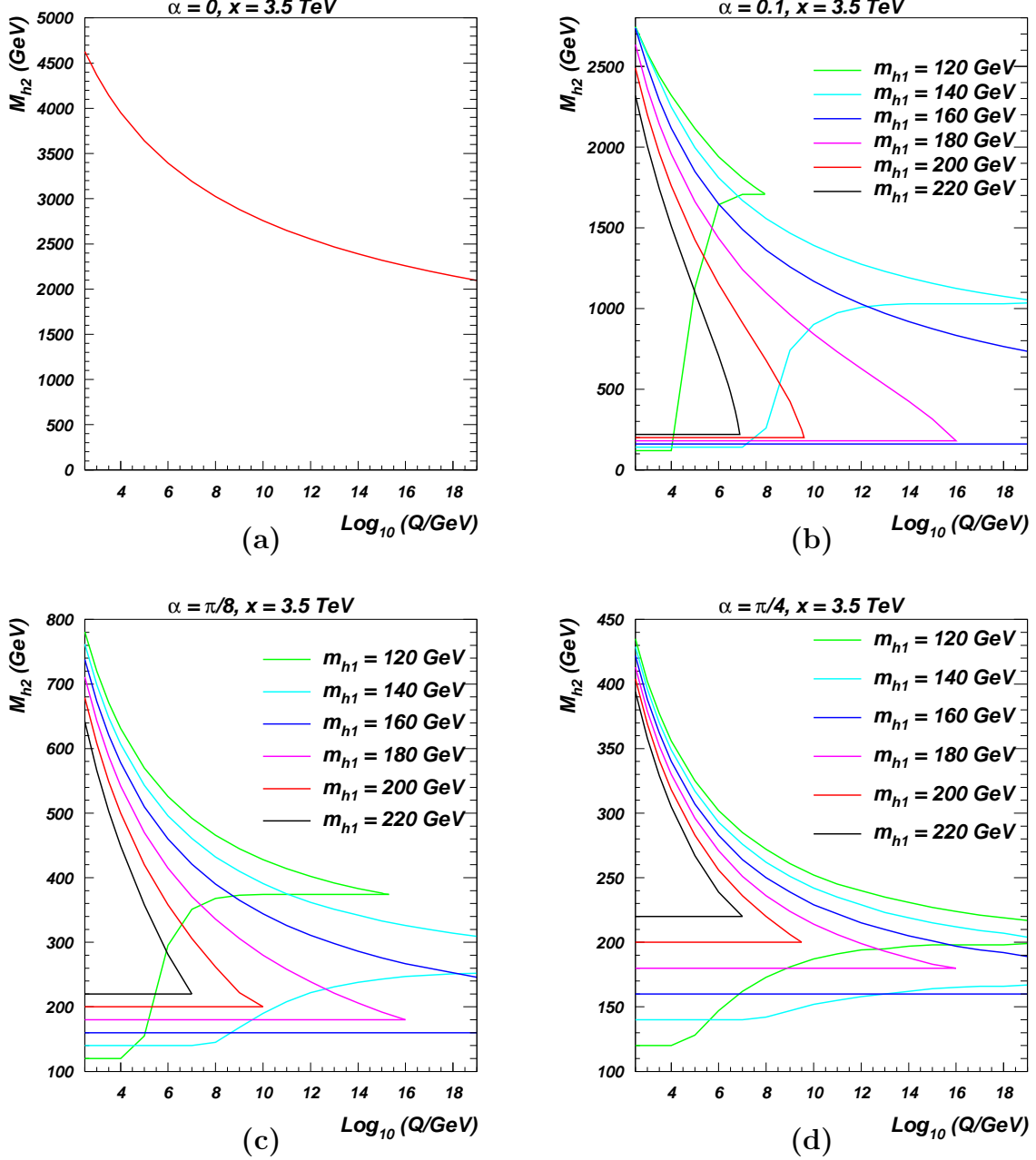


FIG. 4: Allowed values (that are those between the same colour lines) for m_{h2} as a function of the scale Q in the $B - L$ model by eqs. (27) and (28), for several values of m_{h1} and (4a) $\alpha = 0$, (4b) $\alpha = 0.1$, (4c) $\alpha = \pi/8$ and (4d) $\alpha = \pi/4$. Also, $x = 3.5 \text{ TeV}$ and $m_{\nu_h} = 200 \text{ GeV}$. Only the allowed values by our convention $m_{h2} > m_{h1}$ are shown.

shrinking the allowed region even further.

The effect of having non negligible y^M couplings is evident if we compare figure 6 to figure 3. Notice that also the VEV x is changed (from 7.5 TeV to 3.5 TeV), but this is only

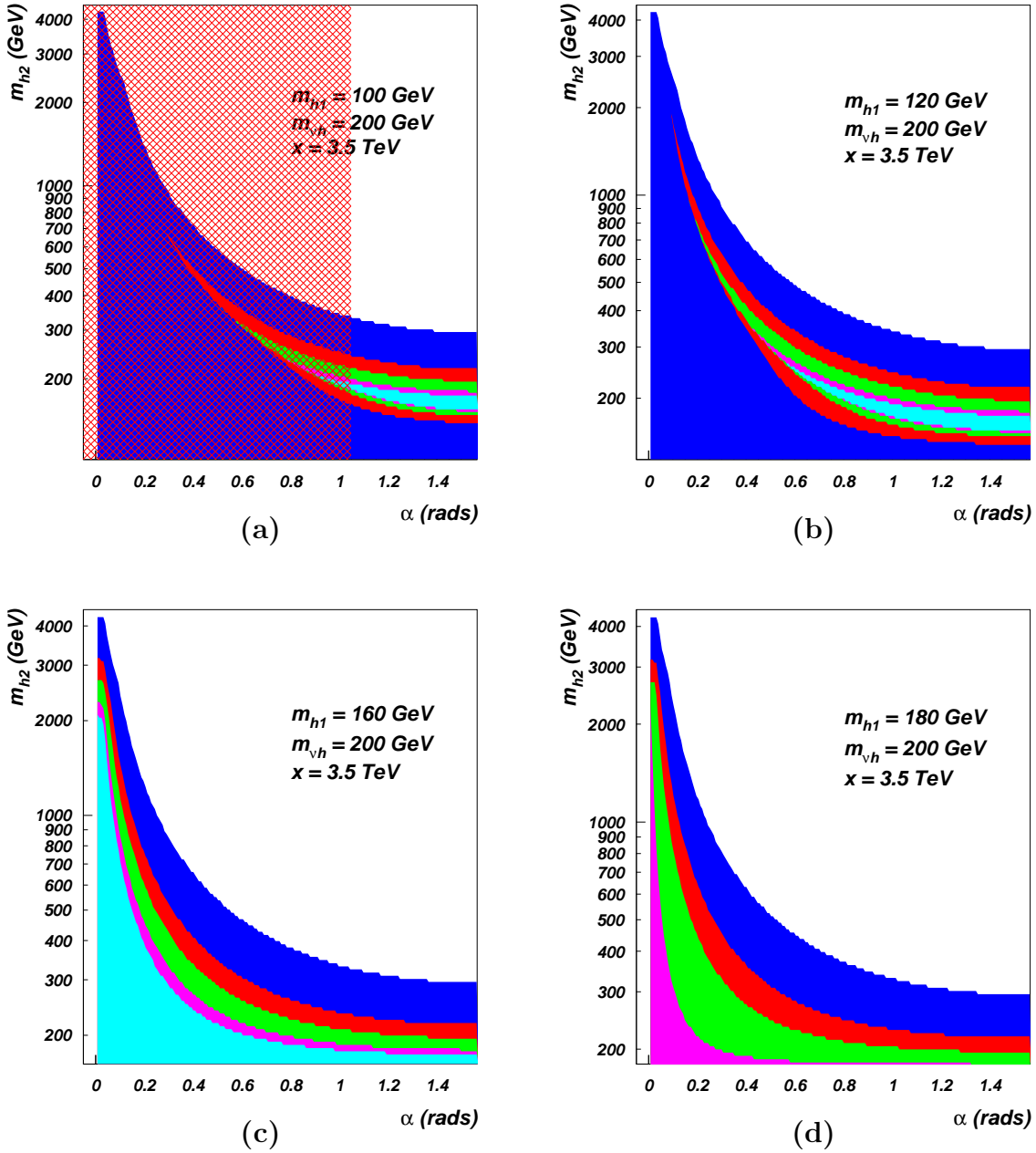


FIG. 5: Allowed values in the m_{h_2} vs. α space in the $B - L$ model by eqs. (27) and (28), for (5a) $m_{h_1} = 100$ GeV, (5a) $m_{h_1} = 120$ GeV, (5a) $m_{h_1} = 160$ GeV and (5a) $m_{h_1} = 180$ GeV. Colours refer to different values of Q/GeV : blue (10^3), red (10^7), green (10^{10}), purple (10^{15}) and cyan (10^{19}). The plots already encode our convention $m_{h_2} > m_{h_1}$ and the shaded red region refers to the values of α forbidden by LEP. Here: $x = 3.5$ TeV, $m_{\nu_h} = 200$ GeV.

responsible for the smaller upper bounds of m_{h_2} in figures 6a and 6b. For small values of α it is evident our analogy between the top quark and the RH neutrinos, as now m_{h_2} has

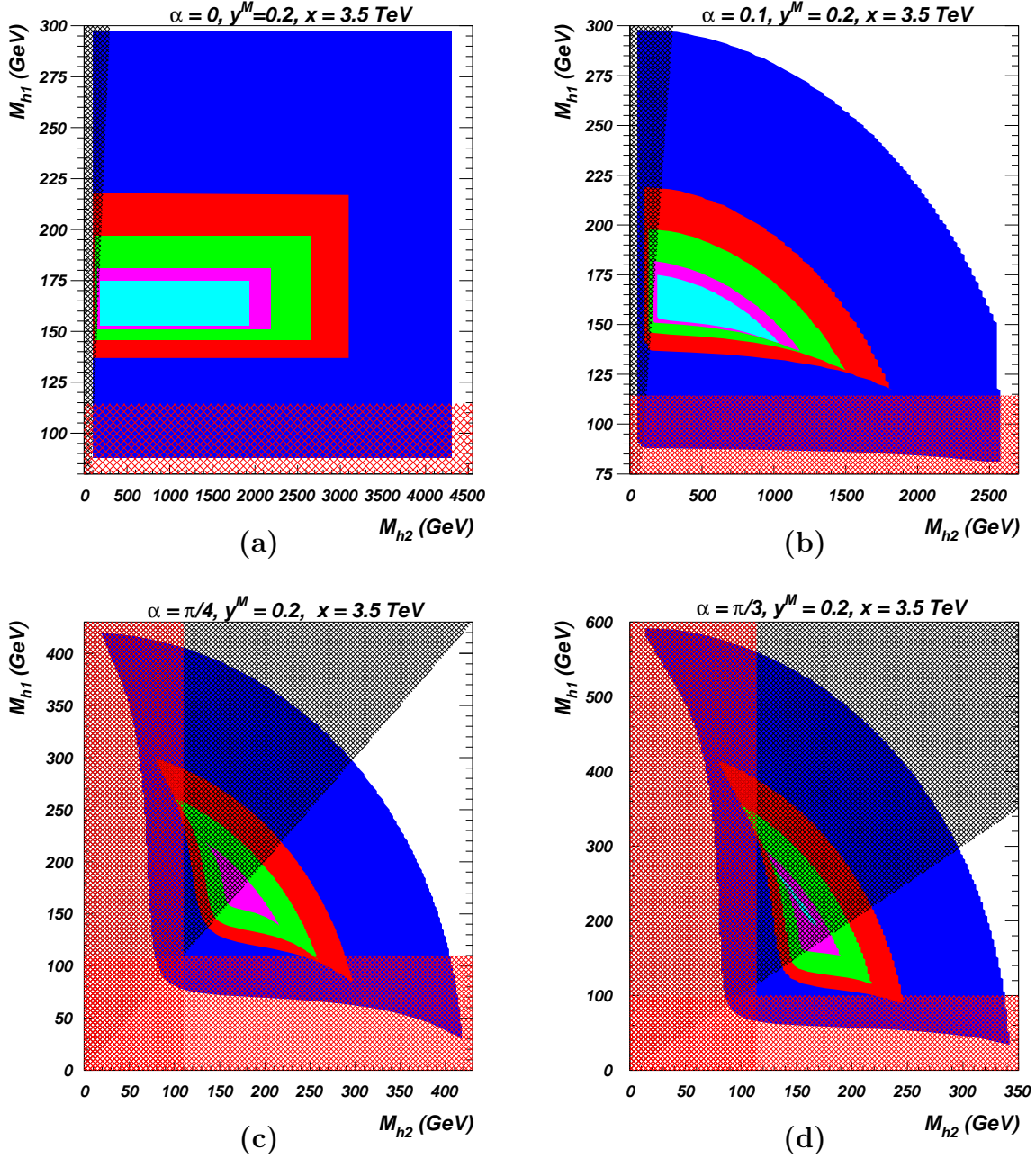


FIG. 6: Allowed values in the m_{h1} vs. m_{h2} space by eqs. (27) and (28), for (6a) $\alpha = 0$ and (6b) $\alpha = 0.1$, (6c) $\alpha = \pi/4$ and (6d) $\alpha = \pi/3$, for $m_{\nu_h} = 1 \text{ TeV}$ and $x = 3.5 \text{ TeV}$. Colours refer to different values of Q/GeV : blue (10^3), red (10^7), green (10^{10}), purple (10^{15}) and cyan (10^{19}). The shaded black region is forbidden by our convention $m_{h2} > m_{h1}$, while the shaded red region refers to the values of the scalar masses forbidden by LEP.

a sensible lower bound too. The analogy holds also for bigger values of the angle, as the allowed region of masses is shrunk from below as we increase the RH Majorana neutrino

Yukawa coupling, while the upper bound stays unaffected. The effect is even more evident for big values of the scale Q , with the Plank scale precluded now for whatever Higgs boson masses at $\alpha = \pi/4$ and tightly constraining the allowed ones at $\alpha = \pi/3$.

Moving to the m_{h_2} - α scan at fixed m_{h_1} values, figure 7 shows the effect of the heavy neutrinos in this case, to be compared to figure 5. It is evident that this model can survive until very large scales Q with massive heavy neutrinos (for which, $y^M > 0.2$) only for the light Higgs boson masses allowed in the case of the SM, that is, $m_{h_1} \sim 160$ GeV. The mixing angle must also be small, $\alpha < \pi/5$, providing a tight constraint on m_{h_2} . For smaller h_1 masses, the effect of a large y^M is to preclude scales $Q \gtrsim 10^7$ GeV almost completely, with for example just a tiny strip for $m_{h_1} = 120$ GeV for which there exists a combination of m_{h_2} and α such that the model is consistent up to $Q = 10^{10}$ GeV. Finally, figure 7d is not visibly different from figure 5d just because we are showing only the $m_{h_2} > m_{h_1}$ region, the shrunk region being below.

E. VEV effect

The last effect to evaluate comes from changing the values for the $B - L$ breaking VEV x . Figure 8a shows the allowed regions in the m_{h_2} vs. α plane for fixed $m_{h_1} = 160$ GeV and $y^M = 0.2$ (that is, a particular case that shows all the interesting effects at once). As expected, since λ_2 is a function of m_{h_2}/x (see for instance eq. (14)), at $\alpha = 0$ the bounds on m_{h_2} simply scale linearly with the VEV. Regarding the upper bound, increasing the VEV x naively increases the allowed region of the heavy Higgs masses, but it is remarkable that the effects are present only for small angles, $\alpha < 0.1$ radians, being the bigger angles unaffected. Concerning the lower bound, or the vacuum stability of the model, at fixed y^M , increasing the VEV x requires to increase m_{h_2} to keep λ_2 constant at the EW scale. This explains why, with non negligible y^M , the allowed heavy Higgs masses are shrinking from below when we increase the VEV x , as one can see in figure 8a and comparing figure 8b with figure 6c, both for $\alpha = \pi/4$ and $y^M = 0.2$, but for $x = 3.5$ and $x = 7.5$ TeV, respectively.

In general, for the model to survive up to very large scales $Q \sim M_{\text{Plank}}$, it is preferred the heavy neutrinos to be light with respect to the VEV x , in such a way that their Yukawa couplings are negligible in the RGE evolution of the scalar sector.

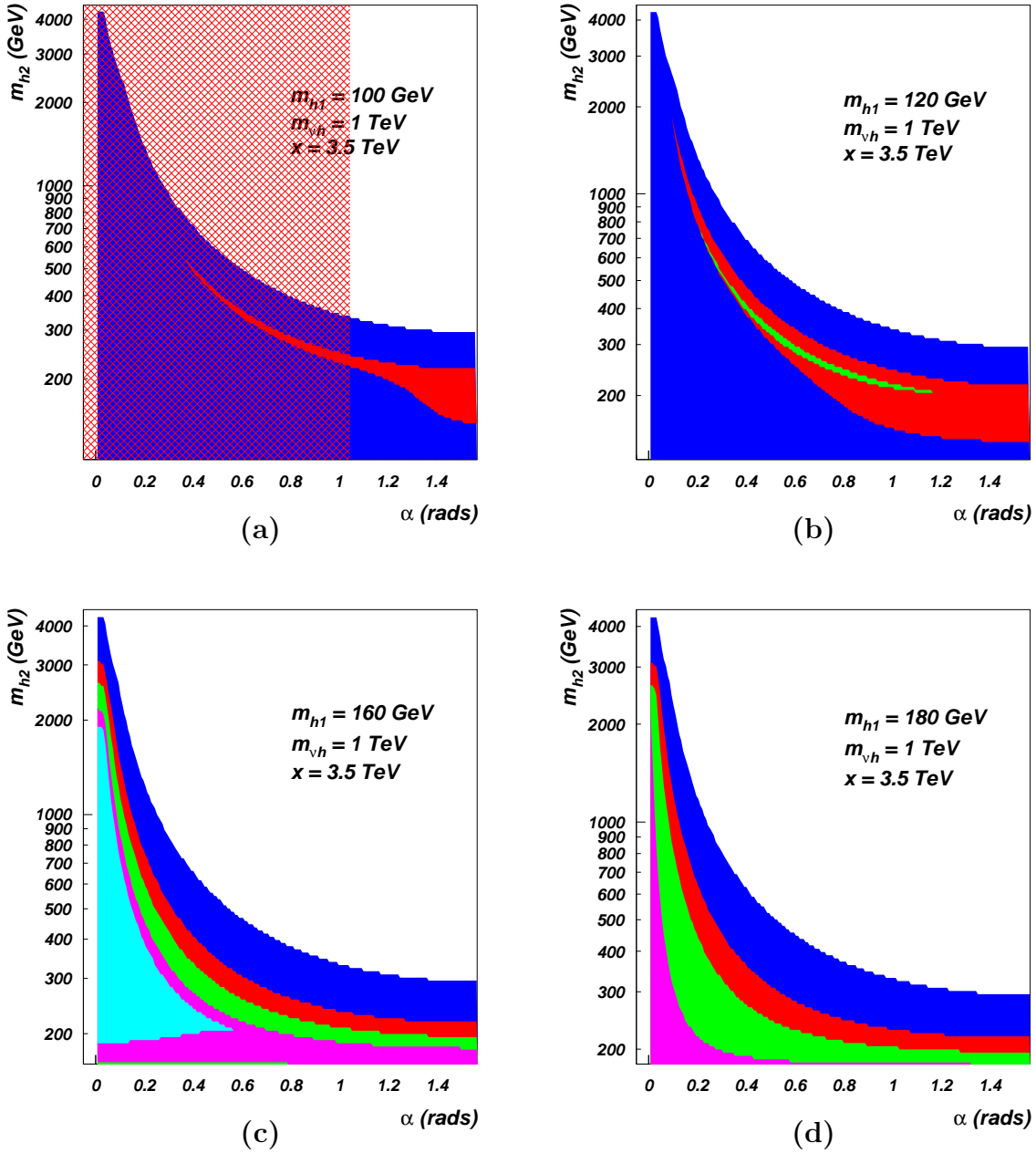


FIG. 7: Allowed values in the m_{h_2} vs. α space in the $B-L$ model by eqs. (27) and (28), for (7a) $m_{h_1} = 100$ GeV, (7b) $m_{h_1} = 120$ GeV, (7c) $m_{h_1} = 160$ GeV and (7d) $m_{h_1} = 180$ GeV. Colours refer to different values of Q/GeV : blue (10^3), red (10^7), green (10^{10}), purple (10^{15}) and cyan (10^{19}). The plots already encode our convention $m_{h_2} > m_{h_1}$ and the shaded red region refers to the values of α forbidden by LEP. Here: $x = 3.5$ TeV, $m_{\nu h} = 1$ TeV.

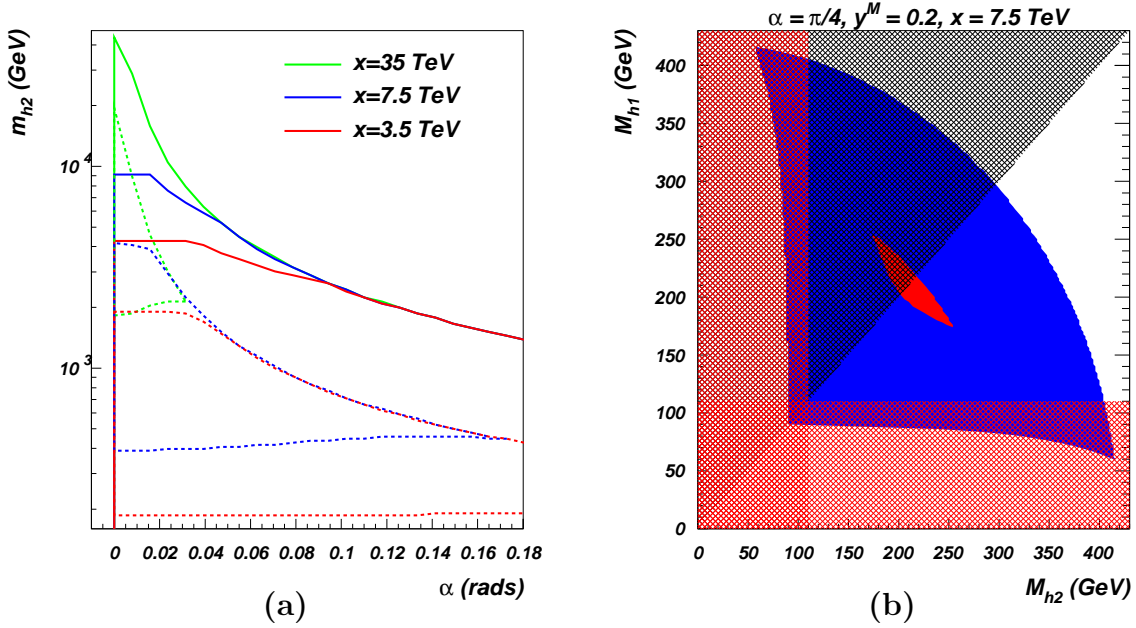


FIG. 8: Allowed values by eqs. (27) and (28) (8a) in the m_{h_2} vs. α space for $m_{h_1} = 160$ GeV and $y^M = 0.2$, for $Q = 10^3$ GeV (straight line) and $Q = 10^{19}$ GeV (dashed line) for several $B - L$ breaking VEV values ($x = 3.5, 7.5$ and 35 TeV, giving $m_{\nu_h} = 1, 2$ and 10 TeV, respectively), and (8b) in the m_{h_1} vs. m_{h_2} space, for $\alpha = \pi/4$, $x = 7.5$ TeV and $y^M = 0.2$, where colours refer to different values of Q/GeV : blue (10^3), red (10^7). The plots already encode our convention $m_{h_2} > m_{h_1}$ and the shaded red region refers to the values of α forbidden by LEP.

V. SUMMARY AND CONCLUSIONS

We have investigated the triviality and vacuum stability conditions of the minimal (or pure) $B - L$ model with a particular view to define the phenomenologically viable regions of the parameter space of the scalar sector, by computing all relevant RGEs (gauge, scalar and fermionic) at the one-loop level in presence of all available experimental constraints. The RGE dependence on the Higgs masses and couplings (including mixings) has been studied in detail for selected heavy neutrino masses and couplings as well as discrete choices of the singlet Higgs field VEV.

Altogether, we have found that there exist configurations of the model for which its validity is guaranteed up to energy scales well beyond those reachable at the LHC while at the same time enabling the CERN hadron collider to probe its scalar sector in Higgs mass

and coupling regions completely different from those accessible to the SM. Furthermore, we have shown that investigations of the Higgs sector of this extended scenario may also lead to constraints on other areas, such as the (heavy) neutrino and Z' sectors (the latter indirectly, through the VEV of the singlet Higgs state directly intervening in the scalar RGEs).

Combining the results of this paper on triviality and vacuum stability with those on unitarity of Ref. [18], we are now in a position to investigate the production and decay phenomenology of both Higgs states of the minimal $B - L$ model at present and future accelerators [31].

Appendix A: RGEs

In this appendix we present the complete set of one-loop RGEs for the minimal $U(1)_{B-L}$ extension of the SM. For some parameters, the equations will be equal to those of the SM, as no extra contribution arises at one-loop level.

1. Gauge RGEs

The RGEs for the $SU(3)_C$ and $SU(2)_L$ gauge couplings g_S and g are [32]:

$$\frac{d}{dt}g_S = \frac{g_S^3}{16\pi^2} \left[-11 + \frac{4}{3}n_g \right] = \frac{g_S^3}{16\pi^2} (-7) , \quad (\text{A1})$$

$$\frac{d}{dt}g = \frac{g^3}{16\pi^2} \left[-\frac{22}{3} + \frac{4}{3}n_g + \frac{1}{6} \right] = \frac{g^3}{16\pi^2} \left(-\frac{19}{6} \right) , \quad (\text{A2})$$

where $n_g = 3$ is the number of generations.

Following standard techniques, we obtain for the Abelian couplings [6, 7]:

$$\frac{d}{dt}g_1 = \frac{1}{16\pi^2} [A^{YY}g_1^3] , \quad (\text{A3})$$

$$\frac{d}{dt}g'_1 = \frac{1}{16\pi^2} [A^{XX}g_1'^3 + 2A^{XY}g_1'^2\tilde{g} + A^{YY}g_1'\tilde{g}^2] , \quad (\text{A4})$$

$$\frac{d}{dt}\tilde{g} = \frac{1}{16\pi^2} [A^{YY}\tilde{g}(\tilde{g}^2 + 2g_1^2) + 2A^{XY}g_1'(\tilde{g}^2 + g_1^2) + A^{XX}g_1'^2\tilde{g}] , \quad (\text{A5})$$

with

$$A^{ab} = A^{ba} = \frac{2}{3} \sum_f Q_f^a Q_f^b + \frac{1}{3} \sum_s Q_s^a Q_s^b , \quad (a, b = Y, X) , \quad (\text{A6})$$

where the first sum is over the left-handed two-component fermions and the second one is over the complex scalars. For the model we are discussing (Y is the SM weak hypercharge,

$X = B - L$ is the $B - L$ number), the coefficients of eq. (A6) are, respectively:

$$A^{YY} = 41/6, \quad A^{XX} = 12, \quad A^{YX} = 16/3. \quad (\text{A7})$$

2. Fermion RGEs

From straightforward calculations we obtain:

$$\frac{d}{dt}y_t = \frac{y_t}{16\pi^2} \left(\frac{9}{2}y_t^2 - 8g_S^2 - \frac{9}{4}g^2 - \frac{17}{12}g_1^2 - \frac{17}{12}\tilde{g}^2 - \frac{2}{3}g_1'^2 - \frac{5}{3}\tilde{g}g_1' \right). \quad (\text{A8})$$

For the right-handed neutrinos, it is not restrictive to consider the basis in which the Majorana matrix of couplings is real, diagonal and positive: $y^M \equiv \text{diag}(y_1^M, y_2^M, y_3^M)$. Then we get [33, 34]⁷:

$$\frac{d}{dt}y_i^M = \frac{y_i^M}{16\pi^2} \left(4(y_i^M)^2 + 2\text{Tr}[(y^M)^2] - 6g_1'^2 \right), \quad (i = 1 \dots 3). \quad (\text{A9})$$

3. Scalar RGEs

A very straightforward way to find the one-loop RGEs for the parameters of the scalar potential is to compute the one-loop effective potential and to impose its independence from the renormalisation scale. To one-loop level, the scalar potential V reads:

$$V = V^{(0)} + \Delta V^{(1)}, \quad (\text{A10})$$

where $V^{(0)}$ is the tree-level potential and $\Delta V^{(1)}$ indicates the one-loop correction to it. To compute the latter it is useful to re-write the tree-level potential

$$V^{(0)}(H, \chi) = m^2 H^\dagger H + \mu^2 |\chi|^2 + \lambda_1 (H^\dagger H)^2 + \lambda_2 |\chi|^4 + \lambda_3 H^\dagger H |\chi|^2 \quad (\text{A11})$$

in terms of the real scalar fields:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad \chi = \frac{1}{\sqrt{2}} (\phi_5 + i\phi_6). \quad (\text{A12})$$

⁷ Notice the we get a difference of a factor 3 in the third term in the RHS of the last expression in eq. (14) contained in Ref. [34]. The authors of Ref. [34] acknowledged the difference and will correct their paper.

The only combinations of fields that are involved are $\phi^2 = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2$ and $\eta^2 \equiv \phi_5^2 + \phi_6^2$, so that eq. (A11) becomes:

$$V^{(0)}(\phi, \eta) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}\mu^2\eta^2 + \frac{1}{4}\lambda_1\phi^4 + \frac{1}{4}\lambda_2\eta^4 + \frac{1}{4}\lambda_3\phi^2\eta^2. \quad (\text{A13})$$

The one-loop correction to the tree-level potential (A13) is, in the Landau gauge,

$$\Delta V^{(1)}(\phi, \eta) = \frac{1}{64\pi^2} \sum_i (-1)^{2s_i} (2s_i + 1) M_i^4(\phi^2, \eta^2) \left[\ln \frac{M_i^2(\phi^2, \eta^2)}{\mu^2} - c_i \right], \quad (\text{A14})$$

where c_i are constants that depend on the renormalisation scheme (for example, in the $\overline{\text{MS}}$ scheme, it is $c_i = 3/2$ for scalars and fermions, $c_i = 5/6$ for vectors). Expanding eq. (A14) and keeping the contributions of the scalar fields (Higgs and Goldstone bosons), of the top-quark, of the gauge bosons and of the RH neutrinos only, we obtain

$$\begin{aligned} \Delta V^{(1)} = & \frac{1}{64\pi^2} \left\{ 3G_1^2 \left[\ln \frac{G_1}{\mu^2} - \frac{3}{2} \right] + G_2^2 \left[\ln \frac{G_2}{\mu^2} - \frac{3}{2} \right] + \text{Tr} \left(H^2 \left[\ln \frac{H}{\mu^2} - \frac{3}{2} \right] \right) \right. \\ & \left. - 12T^2 \left[\ln \frac{T}{\mu^2} - \frac{3}{2} \right] + 3\text{Tr} \left(M_G^2 \left[\ln \frac{M_G}{\mu^2} - \frac{5}{6} \right] \right) - 2 \sum_{i=1}^3 N_i^2 \left[\ln \frac{N_i}{\mu^2} - \frac{3}{2} \right] \right\}, \end{aligned}$$

where the field-dependent squared masses are, in a self-explanatory notation:

$$G_1(\phi, \eta) = m^2 + \lambda_1\phi^2 + \frac{\lambda_3}{2}\eta^2, \quad (\text{A15})$$

$$G_2(\phi, \eta) = \mu^2 + \lambda_2\eta^2 + \frac{\lambda_3}{2}\phi^2, \quad (\text{A16})$$

$$H(\phi, \eta) = \begin{pmatrix} m^2 + 3\lambda_1\phi^2 + \frac{\lambda_3}{2}\eta^2 & \lambda_3\phi\eta \\ \lambda_3\phi\eta & \mu^2 + 3\lambda_2\eta^2 + \frac{\lambda_3}{2}\phi^2 \end{pmatrix}, \quad (\text{A17})$$

$$T(\phi, \eta) = \frac{1}{2}(y_t\phi)^2, \quad (\text{A18})$$

$$M_G(\phi, \eta) = \frac{1}{4} \begin{pmatrix} g_1^2\phi^2 & -gg_1\phi^2 & g_1\tilde{g}\phi^2 \\ -gg_1\phi^2 & g^2\phi^2 & -g\tilde{g}\phi^2 \\ g_1\tilde{g}\phi^2 & -g\tilde{g}\phi^2 & \tilde{g}^2\phi^2 + 16\eta^2 g_1'^2 \end{pmatrix}, \quad (\text{A19})$$

$$N_i(\phi, \eta) = \frac{1}{2}(y_i^M\eta)^2. \quad (\text{A20})$$

As usual, we define the beta functions β_i ($i = 1 \dots 3$) for the quartic couplings, the gamma functions $\gamma_{m,\mu}$ for the scalar masses and the scalar anomalous dimensions $\gamma_{\phi,\eta}$ as follows

($t = \ln Q$):

$$\frac{d\lambda_i}{dt} = \beta_i, \quad (\text{A21})$$

$$\frac{dm^2}{dt} = \gamma_m m^2, \quad (\text{A22})$$

$$\frac{d\mu^2}{dt} = \gamma_\mu \mu^2, \quad (\text{A23})$$

$$\frac{d\phi^2}{dt} = 2\gamma_\phi \phi^2, \quad (\text{A24})$$

$$\frac{d\eta^2}{dt} = 2\gamma_\eta \eta^2. \quad (\text{A25})$$

Now we can extract the RGEs for the parameters of the scalar potential just by requiring that the first derivative of the effective potential with respect to the scale t vanishes:

$$\frac{d}{dt}V^{(1)} \equiv \frac{d}{dt}(V^{(0)} + \Delta V^{(1)}) \equiv 0, \quad (\text{A26})$$

keeping only the one-loop terms. Reorganising it in a more convenient way, we see that eq. (A26) implies the following equations:

$$\begin{aligned} \frac{m^2 \phi^2}{2} \left[\gamma_m + 2\gamma_\phi - \frac{1}{16\pi^2} \left(12\lambda_1 + 2\frac{\mu^2}{m^2} \lambda_3 \right) \right] &= 0, \\ \frac{\mu^2 \eta^2}{2} \left[\gamma_\mu + 2\gamma_\eta - \frac{1}{16\pi^2} \left(8\lambda_2 + 4\frac{m^2}{\mu^2} \lambda_3 \right) \right] &= 0, \\ \frac{\phi^4}{4} \left[\beta_1 + 4\lambda_1 \gamma_\phi - \frac{1}{16\pi^2} \left(24\lambda_1^2 + \lambda_3^2 - 6y_t^4 + \frac{9}{8}g^4 + \frac{3}{8}g_1^4 + \frac{3}{4}g^2 g_1^2 \right. \right. \\ &\quad \left. \left. + \frac{3}{4}g^2 \tilde{g}^2 + \frac{3}{4}g_1^2 \tilde{g}^2 + \frac{3}{8}\tilde{g}^4 \right) \right] = 0, \\ \frac{\eta^4}{4} \left[\beta_2 + 4\lambda_2 \gamma_\eta - \frac{1}{8\pi^2} \left(10\lambda_2^2 + \lambda_3^2 - \frac{1}{2}Tr[(y^M)^4] + 48g_1'^4 \right) \right] &= 0, \\ \frac{\phi^2 \eta^2}{4} \left[\beta_3 + 2\lambda_3(\gamma_\phi + \gamma_\eta) - \frac{1}{8\pi^2} \left(6\lambda_1 \lambda_3 + 4\lambda_2 \lambda_3 + 2\lambda_3^2 + 6\tilde{g}^2 g_1'^2 \right) \right] &= 0. \end{aligned}$$

Imposing that each term between squared brackets vanishes, we can obtain the RGEs for the parameters of the scalar potential after inserting the explicit expression of the scalar anomalous dimensions γ_ϕ and γ_η . The latter are easily computed and read [19, 33, 34]:

$$\gamma_\phi = -\frac{1}{16\pi^2} \left(3y_t^2 - \frac{9}{4}g^2 - \frac{3}{4}g_1^2 - \frac{3}{4}\tilde{g}^2 \right), \quad (\text{A27})$$

$$\gamma_\eta = -\frac{1}{16\pi^2} \left(2Tr[(y^M)^2] - 12g_1'^2 \right). \quad (\text{A28})$$

Inserting eqs. (A27) and (A28) into the RGEs, we finally obtain the RGEs for the five parameters in the scalar potential:

$$\gamma_m \equiv \frac{1}{m^2} \frac{dm^2}{dt} = \frac{1}{16\pi^2} \left(12\lambda_1 + 6y_t^2 + 2\frac{\mu^2}{m^2}\lambda_3 - \frac{9}{2}g^2 - \frac{3}{2}g_1^2 - \frac{3}{2}\tilde{g}^2 \right), \quad (\text{A29})$$

$$\gamma_\mu \equiv \frac{1}{\mu^2} \frac{d\mu^2}{dt} = \frac{1}{16\pi^2} \left(8\lambda_2 + 4\text{Tr}[(y^M)^2] + 4\frac{m^2}{\mu^2}\lambda_3 - 24g_1'^2 \right), \quad (\text{A30})$$

$$\beta_1 \equiv \frac{d\lambda_1}{dt} = \frac{1}{16\pi^2} \left(24\lambda_1^2 + \lambda_3^2 - 6y_t^4 + \frac{9}{8}g^4 + \frac{3}{8}g_1^4 + \frac{3}{4}g^2g_1^2 + \frac{3}{4}g^2\tilde{g}^2 \right. \\ \left. + \frac{3}{4}g_1^2\tilde{g}^2 + \frac{3}{8}\tilde{g}^4 + 12\lambda_1y_t^2 - 9\lambda_1g^2 - 3\lambda_1g_1^2 - 3\lambda_1\tilde{g}^2 \right), \quad (\text{A31})$$

$$\beta_2 \equiv \frac{d\lambda_2}{dt} = \frac{1}{8\pi^2} \left(10\lambda_2^2 + \lambda_3^2 - \frac{1}{2}\text{Tr}[(y^M)^4] + 48g_1'^4 + 4\lambda_2\text{Tr}[(y^M)^2] \right. \\ \left. - 24\lambda_2g_1'^2 \right), \quad (\text{A32})$$

$$\beta_3 \equiv \frac{d\lambda_3}{dt} = \frac{\lambda_3}{8\pi^2} \left(6\lambda_1 + 4\lambda_2 + 2\lambda_3 + 3y_t^2 - \frac{9}{4}g^2 - \frac{3}{4}g_1^2 - \frac{3}{4}\tilde{g}^2 \right. \\ \left. + 2\text{Tr}[(y^M)^2] - 12g_1'^2 + 6\frac{\tilde{g}^2g_1'^2}{\lambda_3} \right). \quad (\text{A33})$$

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