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$U(1)$ EXTENSIONS OF THE STANDARD MODEL

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Introduction

The Standard Model (SM) of strong and electroweak interactions has collected an outstandingly vast series of successes since its formulation, becoming the most precisely tested theory ever built. Still, hints that this cannot be the ultimate theory exist, both on the theoretical and on the experimental side.

Theoretically, it is obvious that a theory is needed, which incorporates gravity in a consistent quantum framework. However, it may be that manifestations of this larger and deeper theory do not appear until energy scales close to the Planck scale, $M_P \approx 10^{19}$ GeV. On the other hand, the 'hierarchy problem' of the SM strongly suggests than an enlargement of the present theory is in order already at much lower scales, possibly around the TeV. Essentially, the hierarchy problem has to do with the stability of the weak scale, $G_F^{-1/2} \approx 250\,\mathrm{GeV}$, with respect to enormous energy scales, such as a possible Grand Unification scale, $M_U \approx 10^{16}$ GeV, or the Planck scale. As will be discussed in the final part of this thesis, supersymmetry is a plausible candidate for solving this difficulty, but other candidates exist, all requiring new physics at the TeV scale.

On the experimental side, the most compelling evidence that we must go beyond the minimal SM is provided by the observation of neutrino oscillations, which proved that neutrinos are massive. Also, an explanation of the evidence for Dark Matter calls for new particles at the TeV scale, or below. In addition, precision tests provide some clues for new physics, even though in a less compelling and non-unique sense: perhaps the most famous examples are the difference between the indirect and direct mass determinations of the Higgs boson (the SM would 'prefer' a Higgs mass below the direct LEP2 limit), and the muon anomalous magnetic moment.

A possible strategy in the study of extensions of the SM is to assume a bottom-up approach, studying the simplest possibilities for new physics, and computing the constraints that available data put on the additional parameters introduced. This attitude seems more than ever appropriate in this historical moment, with the Large Hadron Collider (LHC) at CERN nearly ready to start its quest in the TeV range, where hopefully evidence for new phenomena will be found. In this thesis we follow this route, studying one of the simplest extensions of the SM, namely a *minimal Z' model*, where Z' stands for a second massive neutral gauge boson besides the observed Z boson of the SM. Models with additional gauge bosons have been extensively studied in the literature, theoretical motivations for their possible presence being given by Grand Unified Theories (GUTs), by various models of compactification in superstring theory, and by other constructions addressing the hierarchy problem. We introduce some general aspects of Z' models in Chapter 1.

Then, in Chapter 2, we concentrate on the minimal Z' model, where the minimality of our choice can be summarized as follows:

(i) a single additional $U(1)$ factor in the gauge group;

(ii) no exotic fermions, except for the introduction of one right-handed neutrino per family;

(iii) no exotic scalars, meaning that the physical scalars associated with the additional Higgs fields that break the extra $U(1)$ are sufficiently heavy or sufficiently decoupled from the SM states that they can be neglected, at least in our 'discovery study'.

Automatic anomaly cancellation is obtained, without introducing any other fermions or calling for generalized anomaly cancellation mechanisms (such as the Green-Schwarz mechanism), if we choose the new Abelian symmetry to be associated with a linear combination of the SM hypercharge Y and $B - L$, namely baryon-minus-lepton number. Within this minimal framework, only three parameters are introduced in addition to those of the SM, namely the Z' mass, $M_{Z'}$, and two effective coupling constants. One additional attractive feature of our minimal extension is the possibility to generate a realistic pattern for neutrino masses via renormalizable operators, invoking a see-saw mechanism.

The detailed phenomenological study of this minimal model, performed in Chapter 2, is for the most part original, and is contained in an article [1] that will be soon submitted for publication. After introducing our parameterization, which fully accounts for both mass and kinetic mixing, we compute the indirect bounds that electroweak precision tests set on the parameter space of the model, as well as the limits coming from direct searches performed at the Fermilab Tevatron collider. We also compute the region of parameter space that would be naturally preferred by GUTs. Next, we move on to analyze in detail the discovery reach of the LHC, focusing on its early running phase. At the moment, the schedule for the first year of LHC operations consists in a very first run at low energy ($\sqrt{s} = 7\,\mathrm{TeV}$) and low luminosity ($< 100\,\mathrm{pb}^{-1}$), followed by a second run at higher energy ($\sqrt{s} = 10\,\mathrm{TeV}$) and up to $300\,\mathrm{pb}^{-1}$ of data. We find that the LHC will first enter a (narrow) unexplored region of parameter space at masses around 700 GeV, corresponding to a rather weakly coupled Z'. At 10 TeV, on the other hand, the GUT-preferred region will be under reach, however a larger luminosity ($O(1)\,\mathrm{fb}^{-1}$) than the one currently foreseen would be needed to explore it quite thoroughly. Our study proves that the discovery prospects at each value of energy and luminosity depend in a nontrivial way on the present bounds, thus suggesting the use of model-independent parameterizations such as the one proposed here. In fact, relying on specific models, as is often done in the literature, may focus on regions of parameter space already ruled out by the present bounds.

A natural extension of minimal Z' models would include supersymmetry, even though such an extension requires the introduction of many free parameters associated with the additional fields. In Chapter 3 we review some minimal supersymmetric Z' models that have been proposed. We show that, in contrast with an assumption frequently made, kinetic mixing is not negligible, and must be included in the effective low energy theory. In fact, we show that even if we start from a 'pure $B - L$' model (i.e., no kinetic mixing) at the unification scale, the mixing effects in the renormalization group equations destabilize this choice, reintroducing significant kinetic mixing at the weak scale. We then conclude by studying some supersymmetric models with an extra $U(1)$ that do not require the introduction of additional Higgs fields beyond those of the Minimal Supersymmetric Standard Model, since the extra $U(1)$ is spontaneously broken by non-vanishing vacuum expectation values of the scalar partners of right-handed neutrinos.

1. General aspects of $U(1)$ extensions of the Standard Model

In this chapter we introduce a simple class of extensions of the Standard Model (SM), namely those including an additional Abelian factor in the gauge group. Emphasis is on general aspects, rather than on specific models. Before entering the discussion, we recall in the first section some features of the SM that will play a role in the following and are useful to establish our notation and conventions (for more details on the latter, see Appendix A). In the second section, we discuss some extensions of the minimal SM that allow to account for massive neutrinos, as required by the experimental observation of neutrino oscillations. We describe, in particular, the see-saw mechanism, which can be realized in the context of the models studied in the following chapters. In the third section, we summarize the precision tests of the SM, and discuss how they can be used to constrain models of new physics, such as the Z' models studied in this thesis. The fourth section finally contains an introduction to Z' models, including the discussion of anomaly cancellation. In the fifth section we review, for completeness, models where the additional $U(1)$ is anomalous, even if they will not be discussed further in this thesis.

1.1. The Standard Model

The SM is the presently accepted quantum field theory describing strong and electroweak interactions, and has been successfully tested in a number of experiments, even though one of the particles in its physical spectrum, the Higgs boson, has not been found yet. The SM is a gauge theory, based on the group

$$SU(3)_c \times SU(2)_L \times U(1)_Y \,, \tag{1.1}$$

which completely determines the vector boson content. The fermion and scalar field content (three fermion families and one complex Higgs doublet) are summarized in Table 1.1, in a self-explanatory notation. Notice that, for the moment, we are discussing the minimal version of the SM, which does not account for neutrino masses: the required modifications will be discussed in the following section. The generic covariant derivative is

$$D_\mu = \partial_\mu - ig_S \mathcal{T}^a G^a_\mu - igT^i W^i_\mu - ig' Y B_\mu \,, \tag{1.2}$$

where $\mathcal{T}^a = \lambda^a/2$, $(a = 1, \ldots, 8)$ are the generators of $SU(3)_c$, $T^i = \sigma^i/2$, $(i = 1, 2, 3)$ are the generators of $SU(2)_L$ and Y is the generator of $U(1)_Y$. G^a_μ are the eight gluons, the mediators of strong interactions, whereas (W^i_μ, B_μ) are the four bosons mediating electroweak

	$SU(3)_c \times SU(2)_L \times U(1)_Y$
$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$(\mathbf{3}, \mathbf{2}, 1/6)$
u_R	$(\overline{\mathbf{3}}, \mathbf{1}, 2/3)$
d_R	$(\overline{\mathbf{3}}, \mathbf{1}, -1/3)$
$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, -1/2)$
e_R	$(\mathbf{1}, \mathbf{1}, -1)$
$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, +1/2)$

Table 1.1.: Fermion and scalar field content of the SM. Family indices are understood for the fermion fields.

interactions. The symbols λ^a and σ^i denote the Gell-Mann and Pauli matrices, respectively, satisfying the normalization conditions

$$Tr(\lambda^a \lambda^b) = 2\delta^{ab}\,, \qquad Tr(\sigma^i \sigma^j) = 2\delta^{ij}\,. \tag{1.3}$$

The normalization chosen for the generator of $U(1)_Y$ is such that

$$Q = T_{3L} + Y\,, \tag{1.4}$$

where Q is the electric charge and T_{3L} is the third component of the weak isospin (i.e., the eigenvalue of T^3). Therefore the generalized kinetic terms for the fermion and Higgs fields, including their gauge interactions, read

$$\mathcal{L}_f + \mathcal{L}_H = \sum_{\Psi^j} i\overline{\Psi}^j \not{D} \Psi^j + (D^\mu H)^\dagger (D_\mu H)\,, \tag{1.5}$$

where $\Psi^j = q_L, u_R, d_R, l_L, e_R$, and we have introduced the Feynman notation $\not{D} \equiv \gamma^\mu D_\mu$. The pure Yang-Mills part of the Lagrangian is

$$\mathcal{L}_{YM} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\,a} - \frac{1}{4} W^i_{\mu\nu} W^{\mu\nu\,i} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}\,, \tag{1.6}$$

where

$$G^{\mu\nu\,a} = \partial^\mu G^{\nu\,a} - \partial^\nu G^{\mu\,a} + g_S f^{abc} G^{\mu\,b} G^{\nu\,c}\,, \tag{1.7}$$

$$W^{\mu\nu\,i} = \partial^\mu W^{\nu\,i} - \partial^\nu W^{\mu\,i} + g\epsilon^{ijk} W^{\mu\,j} W^{\nu\,k}\,, \tag{1.8}$$

$$B^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu\,. \tag{1.9}$$

In eqs. (1.7) and (1.8), f^{abc} are the structure constants of $SU(3)_c$, while ϵ^{ijk} are those of $SU(2)_L$: they are defined by

$$[\mathcal{T}^a, \mathcal{T}^b] = i f^{abc} \mathcal{T}^c\,, \tag{1.10}$$

$$[T^i, T^j] = i\,\epsilon^{ijk} T^k\,. \tag{1.11}$$

ϵ^{ijk} is the completely antisymmetric tensor of rank 3 ($\epsilon^{123} = +1$), whereas the explicit form of the constants f^{abc} can be found, e.g., in [2].

1.1.1. Scalar potential and electroweak symmetry breaking

Gauge invariance and renormalizability restrict the form of the scalar potential of the SM to

$$V(H) = m^2|H|^2 + \lambda|H|^4\,, \tag{1.12}$$

and we take $m^2 < 0$, $\lambda > 0$. The latter condition is necessary for V to be bounded from below; the former is imposed because we do not want $\langle H\rangle = 0$ to be a minimum, as this would prevent spontaneous symmetry breaking to occur. To extremize the action, we can look for constant field configurations which are minima of V, i.e. satisfy

$$|H|^2 = -\frac{m^2}{2\lambda} \equiv \frac{v^2}{2}\,, \tag{1.13}$$

and by a suitable gauge transformation, we are free to choose:

$$\langle H\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}\,, \tag{1.14}$$

with v real and positive. Thus the Higgs vacuum expectation value (VEV) (1.14), being invariant under $U(1)_{em}$, realizes the spontaneous breaking

$$SU(2)_L \times U(1)_Y \to U(1)_{em}\,. \tag{1.15}$$

We expand H around its minimum (1.14), choosing the unitarity gauge

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}\,, \tag{1.16}$$

obtaining (with obvious contractions of Lorentz indices in the squares)

$$\mathcal{L}_H = (D^\mu H)^\dagger(D_\mu H) = \frac{1}{2}\partial^\mu h\partial_\mu h + \frac{(v+h)^2}{8}\left[g^2|W^{1\,\mu} + iW^{2\,\mu}|^2 + (g' B^\mu - g\,W^{3\,\mu})^2\right]. \tag{1.17}$$

We now introduce the mass eigenstates,

$$W^\pm_\mu = \frac{W^1_\mu \mp iW^2_\mu}{\sqrt{2}}\,, \tag{1.18}$$

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix}\begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix}\,, \tag{1.19}$$

where the weak mixing angle is defined by

$$\tan\theta_w = \frac{g'}{g}\,. \tag{1.20}$$

The field A^μ is the photon, which remains massless, being $U(1)_{em}$ unbroken. Equation (1.17) can be written in the mass eigenstate basis as follows

$$\mathcal{L}_{gH} = \frac{1}{2}\partial^\mu h\partial_\mu h + \frac{(v+h)^2}{4}g^2 W^{+\,\mu}W^-_\mu + \frac{(v+h)^2}{8}(g^2 + g'^2)Z_\mu Z^\mu\,, \tag{1.21}$$

from which we can read out the masses of the gauge bosons:

$$M_W^2 = \frac{1}{4} g^2 v^2 \,, \tag{1.22}$$

$$M_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2 \,. \tag{1.23}$$

In the unitarity gauge, the scalar potential reads (apart from an irrelevant constant term)

$$V = \frac{1}{2}(2\lambda v^2) h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4 \,, \tag{1.24}$$

from which the value of the Higgs mass is obtained,

$$m_h^2 = 2\lambda v^2 = -2m^2 \,. \tag{1.25}$$

Recalling the definition of the Fermi constant,

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8\, M_W^2} \,, \tag{1.26}$$

and making use of (1.22), we obtain

$$v = \sqrt{\frac{1}{\sqrt{2} G_F}} \,; \tag{1.27}$$

thus from the measurement of G_F in muon decay we extract $v = 246.22\,\mathrm{GeV}$. On the contrary, the parameter λ (and as a consequence, the Higgs mass) is not fixed by present experimental data. In the mass eigenstate basis for vectors, but still in the interaction eigenstate basis for fermions, the interactions between gauge bosons and fermions read

$$\mathcal{L}_{cc} + \mathcal{L}_{nc} + \mathcal{L}_{qg} \,, \tag{1.28}$$

where the charged current terms are (flavour and colour indices are suppressed)

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} W_\mu^+ (\overline{\nu_L} \gamma^\mu e_L + \overline{u_L} \gamma^\mu d_L) + h.c. \,, \tag{1.29}$$

whereas the neutral current terms read

$$\mathcal{L}_{nc} = \sum_{\psi^i} \left[e A_\mu Q^i \overline{\psi}^i \gamma^\mu \psi^i + \frac{g}{\cos\theta_w} Z_\mu \left(T_{3L}^i - \sin^2\theta_w Q^i \right) \overline{\psi}^i \gamma^\mu \psi^i \right] \tag{1.30}$$

with $\psi^i = \{u_L, d_L, u_R, d_R, \nu_L, e_L, e_R\}$, and

$$e = g' \cos\theta_w = g \,\sin\theta_w \tag{1.31}$$

is the unit electric charge. Finally,

$$\mathcal{L}_{qg} = g_S \, \left(\overline{u} \gamma^\mu T^a u + \overline{d} \gamma^\mu T^a d \right) \, G_\mu^a \,, \tag{1.32}$$

where u and d are the full Dirac spinors, including both left-handed and right-handed components. Also the pure Yang-Mills Lagrangian (1.6) can be written in terms of mass eigenstates, obtaining canonical kinetic terms for $A^\mu, Z^\mu, W^{\pm\,\mu}$ and three- and four-gauge boson interactions. We write here only the terms containing the Z boson, because in all the Z' models we will consider in this thesis only Z-Z' mixing is present, whereas the fields $W^{\pm\,\mu}$ and A^μ are unaffected and remain as in the SM. We get:

$$\mathcal{L}^Z_{YM} = -\frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + ig\cos\theta_w \left(W^+_{\mu\nu} W^{-\,\mu} Z^\nu - W^-_{\mu\nu} W^{+\,\mu} Z^\nu + Z_{\mu\nu} W^{+\,\mu} W^{-\,\nu} \right)$$
$$- \frac{g^2}{2} \left(2 g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}\right) \left[W^+_\mu W^-_\nu \left(2 A_\rho Z_\sigma \sin\theta_w \cos\theta_w + Z_\rho Z_\sigma \cos^2\theta_w \right) \right] \,, \quad (1.33)$$

where

$$Z^{\mu\nu} \equiv \partial^\mu Z^\nu - \partial^\nu Z^\mu \,,$$
$$W^{\pm\,\mu\nu} \equiv \partial^\mu W^{\pm\,\nu} - \partial^\nu W^{\pm\,\mu} \,.$$

Since the remaining terms of $\mathcal{L}_{YM}$ will not be crucial for what follows and can be found in textbooks (see, e.g., [3]), they will be omitted here for brevity.

1.1.2. Fermion masses and mixing

Because of the chiral fermion charge assignments under $SU(2)_L \times U(1)_Y$ (see Table 1.1), direct mass terms for fermions are forbidden as they would not respect gauge invariance. On the other hand, we can introduce the Yukawa terms

$$\mathcal{L}_Y = -(\lambda_u)_{mn} \overline{q_{L}}_m \widetilde{H} u_{Rn} - (\lambda_d)_{mn} \overline{q_{L}}_m H d_{Rn} - (\lambda_e)_{mn} \overline{l_{L}}_m H e_{Rn} + h.c. \,, \quad (1.34)$$

where $\widetilde{H} \equiv i\sigma^2 H^*$ transforms as a doublet under $SU(2)_L$ but has hypercharge $Y = -1/2$, and $(\lambda_u)_{mn}, (\lambda_d)_{mn}, (\lambda_e)_{mn}$ are complex 3×3 matrices in flavour space. In the unitarity gauge, eq. (1.34) reads

$$-(\lambda_u)_{mn} \overline{u_{L}}_m \left(\frac{v+h}{\sqrt{2}}\right) u_{Rn} - (\lambda_d)_{mn} \overline{d_{L}}_m \left(\frac{v+h}{\sqrt{2}}\right) d_{Rn} - (\lambda_e)_{mn} \overline{e_{L}}_m \left(\frac{v+h}{\sqrt{2}}\right) e_{Rn} + h.c. \,, \quad (1.35)$$

which shows how fermion mass terms are generated: for instance, the mass terms for charged leptons are

$$-(\lambda_e)_{mn} \frac{v}{\sqrt{2}} \overline{e_{L}}_m e_{Rn} + h.c. \,. \quad (1.36)$$

The Yukawa matrices can be diagonalized by means of a bi-unitary transformation:

$$V_f^\dagger \lambda_f U_f = \lambda_f^d \,, \quad (1.37)$$

($f = u, d, e$), where λ_f^d is diagonal and has real and positive eigenvalues. The diagonalization is obtained by means of the transformations

$$f_L = V_f f'_L \,, \quad (1.38)$$
$$f_R = U_f f'_R \,, \quad (1.39)$$

where flavour indices are understood, which yield for the Yukawa terms

$$-\left(\frac{h+v}{\sqrt{2}}\right) \overline{f'_L} \lambda_f^d f'_R \,. \quad (1.40)$$

The transformations (1.38) and (1.39) leave fermion kinetic terms and neutral current interactions unchanged, however they affect the charged currents: the Lagrangian (1.29) becomes

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} W^+_\mu (\overline{\nu_L} \gamma^\mu V_e e'_L + \overline{u'_L} \gamma^\mu V_u^\dagger V_d d'_L) + h.c.\,. \tag{1.41}$$

At this point, we are free to redefine the neutrino fields as

$$\nu_L = V_e \nu'_L\,, \tag{1.42}$$

yielding finally

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} W^+_\mu (\overline{\nu'_L} \gamma^\mu e'_L + \overline{u'_L} \gamma^\mu V_{CKM} d'_L) + h.c.\,, \tag{1.43}$$

where $V_{CKM} = V_u^\dagger V_d$ is the Cabibbo-Kobayashi-Maskawa unitary matrix. A $N \times N$ unitary matrix has N^2 real parameters, amongst these $\frac{1}{2}N(N-1)$ are angles and the remaining $\frac{N}{2}(N+1)$ are complex phases. However, $2N-1$ phases can be eliminated by redefining the phases of N up-type quarks and of N down-type quarks (we get $2N-1$ because rotating N down quarks eliminates the phases in a row, and rotating N up type quarks eliminates the phases in a column, but the phase in the entry at the intersection of the row and the column cannot be eliminated twice). In conclusion, the physical parameters are $\frac{1}{2}N(N-1)$ angles and $\frac{1}{2}(N-1)(N-2)$ phases, which for $N=3$ amount to 3 angles and 1 phase.

1.2. Neutrino masses and right-handed neutrinos

As we discussed in the previous section, the SM does not leave room for neutrino masses. However, measurements of neutrino oscillations have established that at least two neutrino masses must be nonzero, therefore we need to extend the model to incorporate this property. A customary way to do this is to introduce three right-handed (RH) neutrinos, which are taken to be singlets under the SM gauge group (*sterile* neutrinos). In the same notation of Table 1.1:

$$\nu_R \sim (\mathbf{1}, \mathbf{1}, 0)\,. \tag{1.44}$$

1.2.1. Dirac masses

If RH neutrinos are introduced, then additional Yukawa terms are allowed by gauge invariance:

$$-(\lambda_\nu)_{mn} \overline{l_{Lm}} \widetilde{H} \nu_{Rn} + h.c.\,, \tag{1.45}$$

which after electroweak symmetry breaking read

$$-(\lambda_\nu)_{mn} \overline{\nu_{Lm}} \left(\frac{v+h}{\sqrt{2}} \right) \nu_{Rn} + h.c.\ . \tag{1.46}$$

If conservation of the total lepton number L is imposed, these are the only allowed renormalizable terms in the Lagrangian that can originate neutrino masses. We can diagonalize λ_ν in the same way as we did for the SM Yukawa couplings, by means of the transformations

$$\nu_L = V_\nu \nu'_L\,, \tag{1.47}$$
$$\nu_R = U_\nu \nu'_R\,, \tag{1.48}$$

which yield

$$V_\nu^\dagger \lambda_\nu U_\nu = \lambda_\nu^d \,. \tag{1.49}$$

The effects of (1.47) and (1.48) on the neutral current and kinetic terms cancel out, while the charged current terms (1.29) become

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} W_\mu^+ (\overline{\nu'_L} \gamma^\mu V_\nu^\dagger V_e e'_L + \overline{u'_L} \gamma^\mu V_u^\dagger V_d d'_L) + h.c. \,, \tag{1.50}$$

and now we are not free to redefine neutrino fields, as we did in eq. (1.42), therefore we have also in the leptonic sector a physical mixing matrix $U_{PMNS} = V_\nu^\dagger V_e$. A counting of the parameters contained in U_{PMNS} can be done exactly in the same way as for V_{CKM}, giving 3 angles and 1 phase as independent real parameters.

1.2.2. Majorana masses and the see-saw mechanism

When RH neutrinos are introduced, also Majorana mass terms $\sim \nu_R \nu_R$ are allowed by gauge invariance, precisely:

$$-\frac{M_{mn}}{2} \overline{\nu^c_{Lm}} \nu_{Rn} + h.c. \,, \tag{1.51}$$

where

$$\nu^c_{L,R} \equiv (\nu^c)_{L,R} = P_{L,R} \nu^c = P_{L,R} C \overline{\nu}^T = C \overline{\nu_{R,L}}^T = (\nu_{R,L})^c \,. \tag{1.52}$$

ψ^c is the charge conjugate of ψ, see Appendix A. To be general, we must include the new terms (1.51) in the Lagrangian. Then the full neutrino mass Lagrangian reads (by suitably redefining the RH neutrinos, it is always possible to write M_{mn} in diagonal form, with real and positive eigenvalues):

$$-\mathcal{L}_{\nu\ mass} = \frac{1}{2} M_{mn} (\overline{\nu^c_{Lm}} \nu_{Rn} + \overline{\nu_{Rm}} \nu^c_{Ln}) + (m_{Dmn} \overline{\nu_{Lm}} \nu_{Rn} + h.c.) \,, \tag{1.53}$$

which can be rewritten as

$$-\mathcal{L}_{\nu\ mass} = \frac{1}{2} \begin{pmatrix} \overline{\nu_L} & \overline{\nu^c_L} \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix} \begin{pmatrix} \nu^c_R \\ \nu_R \end{pmatrix} + h.c. \,, \tag{1.54}$$

where we have suppressed flavour indices. We focus first on the simplest case of only one generation: then we can diagonalize the matrix in (1.54) by means of the rotation [4, 5]

$$\begin{pmatrix} \nu_L \\ \nu^c_L \end{pmatrix} = R_\varphi \begin{pmatrix} i\nu_{1L} \\ \nu^c_{2L} \end{pmatrix} \,, \qquad \begin{pmatrix} \nu^c_R \\ \nu_R \end{pmatrix} = R_\varphi \begin{pmatrix} i\nu^c_{1R} \\ \nu_{2R} \end{pmatrix} \tag{1.55}$$

with

$$R_\varphi = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \,, \qquad \tan 2\varphi = \frac{2m_D}{M} \,. \tag{1.56}$$

The rotated Lagrangian reads

$$-\mathcal{L}_{\nu\ mass} = \frac{1}{2} (-m_1 \overline{\nu_{1L}} \nu^c_{1R} + m_2 \overline{\nu^c_{2L}} \nu_{2R} + h.c.) = -\frac{1}{2} m_1 \overline{N}_1 N_1 + \frac{1}{2} m_2 \overline{N}_2 N_2 \,, \tag{1.57}$$

where

$$N_1 = \nu_{1R}^c + \nu_{1L} \,, \tag{1.58}$$

$$N_2 = \nu_{2L}^c + \nu_{2R} \tag{1.59}$$

are the mass eigenstates, and the mass eigenvalues read

$$m_{1,2} = \frac{M}{2}\left(1 \mp \sqrt{1 + \frac{4m_D^2}{M^2}}\right) . \tag{1.60}$$

We see that m_1 is negative: this is why we have introduced the i in (1.55), so that $\widetilde{m}_1 = -m_1$ is the physical mass of N_1. Notice that iN_1 and N_2 are Majorana spinors, in the sense that $(iN_1)^c = -iN_1$ and $N_2^c = N_2$. Now taking $M >> m_D$, we realize the so-called see-saw mechanism [6]: the angle φ is small, and the masses become

$$\widetilde{m}_1 \approx \frac{m_D^2}{M} \,, \qquad m_2 \approx M \,, \tag{1.61}$$

whereas the eigenstates read

$$iN_1 \approx \nu_L + \nu_R^c \,, \qquad N_2 \approx \nu_R + \nu_L^c \,. \tag{1.62}$$

Thus we have obtained a light Majorana neutrino N_1 which is essentially ν_L, and a heavy Majorana neutrino N_2 which is essentially ν_R. We can now estimate the order of magnitude of M needed to obtain viable light neutrino masses: requiring $\widetilde{m}_1 \leq 1\,\text{eV}$, as approximately suggested by tritium β-decay and cosmology (see [7] for a recent review of neutrino physics, containing an extensive list of references), and setting $m_D = O(100\,\text{GeV})$ (i.e. $\lambda_\nu = O(1)$), we get

$$M \geq 10^{13}\,\text{GeV} \,. \tag{1.63}$$

However, λ_ν could be smaller than 1, thus lowering significantly the scale of M: suppose for instance that $\lambda_\nu = O(10^{-5})$ (strength similar to that of λ_e), then the lower bound on M becomes

$$M \geq 10^{3}\,\text{GeV} \,. \tag{1.64}$$

In the case of 3 generations, the diagonalization of the mass matrix in (1.54) is obviously more complicated; however, if we assume $M >> m_D$, the situation again simplifies greatly: we get

$$-\mathcal{L}_{\nu\ mass} = \frac{1}{2}\overline{\nu} M_l \nu + \frac{1}{2}\overline{N} M_h N \,, \tag{1.65}$$

where ν and N are respectively three light and three heavy Majorana neutrinos, and the mass matrices are given by

$$M_l \approx m_D M^{-1} m_D^T \,, \tag{1.66}$$

$$M_h \approx M \,. \tag{1.67}$$

Analogously to what happens for one generation, the heavy neutrino states are mostly right-handed, while the light states are mostly left-handed.

Denoting by $\nu_L = T_L \nu'_L$ the unitary transformation on ν_L needed to diagonalize the see-saw mass matrix, the lepton charged current sector becomes

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} W^+_\mu (\overline{\nu'_L} \gamma^\mu T^\dagger_L V_e e'_L) + h.c. . \tag{1.68}$$

The mixing matrix $U_{PMNS} = T^\dagger_L V_e$ has 3 angles and 6 phases as real parameters; now only three phases can be reabsorbed in a redefinition of the charged lepton fields, while the neutrino phases cannot be redefined, because the Majorana mass term is not $U(1)$-invariant. Therefore $6 - 3 = 3$ physical phases remain.

1.2.3. Neutrino masses from higher-dimensional operators

We have to remark that other mechanisms exist, which can account for massive neutrinos without including ν_R. The most important example is the introduction of a dimension-5 operator (which violates total lepton number L by two units)

$$\mathcal{L}_5 = -\frac{Z^\nu_{mn}}{\Lambda} \left(\overline{l_{Lm}} \widetilde{H}\right) \left(\widetilde{H}^T l^c_{L\,n}\right) + h.c. , \tag{1.69}$$

where Λ is a large mass scale. In the unitarity gauge, equation (1.69) becomes

$$\mathcal{L}_5 = -\frac{Z^\nu_{mn}}{\Lambda} \left(\frac{v+h}{\sqrt{2}}\right)^2 \overline{\nu_{Lm}} \nu^c_{R\,n} + h.c. , \tag{1.70}$$

containing a Majorana mass term for ν_L:

$$(M_\nu)_{mn} = Z^\nu_{mn} \frac{v^2}{\Lambda} . \tag{1.71}$$

We see that in this case, neutrino masses have a natural suppression of a factor v/Λ with respect to the masses of all other fermions. To estimate the approximate value of Λ needed to obtain viable neutrino masses (i.e. $m_\nu \leq 1\,\mathrm{eV}$), we can take the one-family case, finding

$$M_\nu = \frac{v^2}{\Lambda} Z \leq 1\,\mathrm{eV} \Rightarrow \Lambda \geq 10^{13}\,\mathrm{GeV} , \tag{1.72}$$

where we have assumed $Z = O(1)$ and $v = O(100\,\mathrm{GeV})$. Going back to three generations, when the mass matrix $(M_\nu)_{mn}$ is diagonalized, we obtain a unitary lepton mixing matrix U_{PMNS}, and only three phases can be reabsorbed in the charged lepton fields, because the Majorana mass term for ν_L is not invariant under a phase transformation. In conclusion, we are left with 3 angles and 3 physical phases, exactly as it happens in the case of the see-saw mechanism with ν_R.

1.3. Precision tests of the Standard Model

The number of precision tests to which the SM has been exposed is extremely large, and it would be impossible to list them all here. Therefore, we will mention only a subset of these measurements, addressing the reader to [8] for a more detailed discussion. Among the most precise measurements performed to test the SM, we recall:

- the fine structure constant α, obtained from the electron magnetic moment;
- G_F, measured in muon decay;
- M_Z, measured at LEP1;
- M_W, measured at LEP2 and at the Tevatron;
- atomic parity violation in atoms. The most precise results have been obtained with Cesium atoms, see [9, 10] for the most recent results; the measured quantity is the *weak charge* of the atom, given by

$$Q_W = -2\left(C_{1u}(2Z+N) + C_{1d}(Z+2N)\right) \ , \tag{1.73}$$

  where N is the number of neutrons and Z the number of protons in the nucleus, and C_{1q} , $q = u, d$ are obtained from the parity-violating component of the effective Z exchange between electron and quark:

$$\mathcal{L}_{PV}^{eff} = \frac{G_F}{\sqrt{2}} \sum_{q=u,d} \left[C_{1q}(\bar{e}\gamma^\mu\gamma_5 e)(\bar{q}\gamma_\mu q) + C_{2q}(\bar{e}\gamma^\mu e)(\bar{q}\gamma_\mu\gamma_5 q)\right] \ ; \tag{1.74}$$

- the forward-backward asymmetry A_{FB} in $e^+e^- \to \overline{f}f$;
- the left-right asymmetry A_{LR} in $e^+e^- \to \overline{f}f$;
- the total width of the Z, Γ_Z;
- the Z branching ratios;
- $\sigma(e^+e^- \to \overline{f}f)$ above the Z pole, $\sqrt{s} > m_Z$, as measured at LEP2 at energies up to $\sqrt{s} = 209\,\text{GeV}$, and the corresponding angular distributions.

Low-energy data were collected at a number of experimental facilities, whereas Z-pole data are from LEP1 and SLD.

Within the framework of the SM, a global fit to all precision data can be used to constrain the only free parameter, namely the Higgs mass. Two types of fit can be performed: the so-called *standard* fit, which does not include results from direct Higgs searches, and the *complete* fit, which takes into account also direct searches. A recent fit to the SM is contained in [11], where in the complete fit were included not only the results of Higgs searches at LEP2 (which excluded at 95% CL a Higgs mass lower than 114.4 GeV, [12]), but also preliminary results from the Tevatron Higgs analysis. We report the results of [11] in Table 1.2.

1.3.1. Constraining new physics through precision tests

If a broader perspective is assumed, precision measurements can be used to constrain effects of physics beyond the Standard Model. In general, the effects of new physics (which is assumed to be heavy) can be described by adding to the SM Lagrangian a set of dimension-6 operators containing the SM fields, which are collected in $\mathcal{L}_{BSM}$. It can be proved that, if lepton and quark universality are assumed, a set of 18 independent operators is needed to describe all

Fit	χ^2_{min}/d , $P_d(\chi^2 > \chi^2_{min})$	m_h(GeV)	2σ (GeV)	3σ (GeV)
Standard	16.4/13, 23%	80^{+30}_{-23}	$[39, 155]$	$[26, 209]$
Complete	18.0/14, 21%	$116.4^{+18.3}_{-1.3}$	$[114, 145]$	$[113, 168] \cup [180, 225]$

Table 1.2.: Results of fits to the SM Higgs mass, as reported in [11]. d is the number of degrees of freedom. The last two columns show the 2σ and 3σ allowed intervals respectively.

electroweak precision measurements. Their explicit expressions can be found in Appendix A of [13]. Thus in general, we have to compute all the 18 dimension-6 operators in the model under study, and perform a global fit to the data, obtaining in this way electroweak precision constraints on the model parameters. However, not all the operators in $\mathcal{L}_{BSM}$ are strongly constrained by precision data, therefore it is often useful to select a subset of parameters, dropping the less important ones. Following [13], in order to perform this task we proceed in the following way: given the full set of operators describing new physics, we use the equations of motion of the gauge bosons to eliminate the 3 currents involving charged leptons:

$$\overline{e_L}\gamma^\mu e_L \ , \ \overline{e_R}\gamma^\mu e_R \ , \ \overline{e_L}\gamma^\mu \nu_L + h.c.. \tag{1.75}$$

This choice is justified, because most of the precision measurements were done at e^+e^- colliders, therefore operators containing charged leptons are very strongly constrained. $\mathcal{L}_{BSM}$ can be written in the following way:

$$\mathcal{L}_{BSM} = \mathcal{L}_{oblique} + \mathcal{L}_{couplings} \ , \tag{1.76}$$

where $\mathcal{L}_{oblique}$ contains all the effects which can be absorbed into the self energies of the gauge bosons, and $\mathcal{L}_{couplings}$ contains corrections to the couplings of neutrinos and quarks to γ and Z. Notice that $\mathcal{L}_{couplings}$ cannot contain any charged lepton currents, so all the observables involving charged leptons are described by $\mathcal{L}_{oblique}$ in the chosen formalism. We have the effective Lagrangian in momentum space

$$-\mathcal{L}_{oblique} = \frac{1}{2}W^3_\mu \Pi_{33}(p^2) W^{3\,\mu} + \frac{1}{2}B_\mu \Pi_{00}(p^2) B^\mu + W^3_\mu \Pi_{30}(p^2) B^\mu + W^+_\mu \Pi_{WW}(p^2) W^{-\,\mu} \ , \tag{1.77}$$

which can be expanded in powers of p^2, giving

$$\Pi(p^2) = \Pi(0) + p^2\Pi'(0) + \frac{p^4}{2}\Pi''(0) + \dots \ , \tag{1.78}$$

where we have neglected terms which would correspond to operators of dimension higher than 6. The expansion expressed by (1.78) contains 12 parameters. Among these, 3 can be traded for the parameters g, g', v, while two additional conditions are imposed by the masslessness of the photon: $\Pi_{\gamma\gamma} = 0$ and $\Pi_{\gamma Z} = 0$ [14]. As a result, only $12 - 5 = 7$ parameters are independent. Their explicit expressions read

$$\widehat{S} = \frac{g}{g'}\Pi'_{30}\,, \;\; \widehat{T} = \frac{\Pi_{33} - \Pi_{WW}}{M_W^2}\,, \;\; W = \frac{M_W^2}{2}\Pi''_{33}\,, \;\; Y = \frac{M_W^2}{2}\Pi''_{00}\,, \tag{1.79}$$

$$\widehat{U} = \Pi'_{WW} - \Pi'_{33}\,, \;\; V = \frac{M_W^2}{2}(\Pi''_{33} - \Pi''_{WW})\,, \;\; X = \frac{M_W^2}{2}\Pi''_{30}\,, \tag{1.80}$$

where all Πs are computed at $p^2 = 0$. $\mathcal{L}_{couplings}$ is defined as follows, again expanding in powers of p^2:

$$\mathcal{L}_{couplings} = \sum_f (\overline{f}\gamma^\mu f)\left[eA_\mu \frac{C_f^A}{M_W^2}p^2 + \sqrt{g^2 + g'^2}Z_\mu\left(\frac{C_f^Z}{M_W^2}(p^2 - M_W^2) + \delta g_f\right)\right]\,, \tag{1.81}$$

where $f = u_L, d_L, u_R, d_R, \nu_L$; obviously $C_{\nu_L}^A = 0$. It is important to note that the only neutrino parameter that has been measured is δg_{ν_L}, which enters the invisible decay width of the Z measured at LEP1. Furthermore, δg_{ν_L} can be written as a combination of the oblique parameters $V, \hat{U}, X$, therefore all measurements involving only leptons are fully described by $\mathcal{L}_{oblique}$. Focusing on the quarks, we see that (1.81) apparently contains 12 quark parameters (4 C^A, 4 C^Z and 4 δg). However, only 11 of them are independent, as shown in [13]. To sum up, we have a total of $7 + 11 = 18$ parameters describing new physics. Notice that no approximation has been made in addition to lepton and quark universality, up to this point. Now, we recall that leptonic final states are measured with greater precision than hadronic ones, suggesting that the 7 oblique parameters $\widehat{S}, \ldots$ should be the most relevant. This was proved by the detailed analysis performed in [13]. In addition to the oblique parameters, a minimal subset of quark parameters is added, namely the combinations

$$\delta\epsilon_q = \delta g_{u_L} - \delta g_{d_L}\,, \tag{1.82}$$

$$\delta C_q = C_{u_L}^Z - C_{d_L}^Z\,, \tag{1.83}$$

leading to a 9-parameter approximation which was shown in [13] to be quite accurate with respect to the global analysis. We will make use of this approximation when computing electroweak precision constraints on our model in the next chapter. The global fit to the data performed in [13] gives

$$\widetilde{R}\begin{pmatrix} \widehat{S} \\ \widehat{T} \\ \widehat{U} \\ V \\ W \\ X \\ Y \\ \delta C_q \\ \delta\epsilon_q \end{pmatrix} = \begin{pmatrix} -0.04 + 0.54l \pm 0.21 \\ 0.13 + 0.08l \pm 0.43 \\ 0.41 + 0.21l \pm 0.50 \\ 0.16 + 0.72l \pm 0.54 \\ -0.36 - 0.33l \pm 0.75 \\ 0 + 0.16l \pm 1.2 \\ -0.9 - 0.12l \pm 1.5 \\ -5.6 - 0.31l \pm 2.0 \\ -0.9 - 0.12l \pm 1.5 \\ -26 + 0.66l \pm 18 \end{pmatrix}\,, \tag{1.84}$$

with $l = \log(m_h/M_Z)$, and

$$\widetilde{R} = \begin{pmatrix} -404 & 353 & -133 & 173 & 137 & -753 & 276 & 4 & 45 \\ -245 & -19 & 492 & -747 & 30 & -37 & 280 & 15 & -275 \\ -16 & 208 & 146 & -152 & -724 & -224 & -407 & 319 & 293 \\ -222 & 691 & -76 & 5 & -120 & 550 & 285 & -129 & 271 \\ -17 & -330 & 177 & -36 & 114 & -31 & 273 & -12 & 877 \\ 3 & 232 & -7 & -283 & 303 & -118 & -589 & -581 & 34 \\ -42 & -68 & 132 & 31 & -44 & -37 & -66 & -288 & 906 \\ -203 & -200 & 350 & 375 & -445 & -9 & 126 & -587 & -406 \\ -642 & -381 & -575 & -219 & -161 & 147 & -112 & -41 & 20 \\ 519 & 0 & -458 & -341 & -329 & -199 & 376 & -337 & 1 \end{pmatrix} . \tag{1.85}$$

The linear combinations given in (1.84) are statistically uncorrelated.

1.4. $U(1)$ extensions of the SM

Extensions of the SM containing one (or more) additional $U(1)$ Abelian factor in the gauge group have been extensively studied, and they are one of the simplest and best motivated possibilities for physics beyond the SM. Motivations for this kind of extensions are provided by Grand Unified Theories based on groups larger than the $SU(5)$ of the original formulation [15], in particular $SO(10)$ and E_6, which having rank larger than four could break to $G_{SM} \times U(1)^n$, $n \geq 1$. Also, the presence of additional $U(1)$ gauge symmetries is frequent in string compactifications (see, e.g., [16, 17]). In addition to the discovery of a new gauge boson, these extensions may provide other interesting features, such as an enlarged Higgs sector to break the extra $U(1)$ (however, this is not necessary, see the next chapter) and additional fermion fields with respect to the minimal SM, required to cancel anomalies introduced by the new $U(1)$. Also, significant changes in the possibilities for neutrino masses could be produced.

1.4.1. General framework

In general, the neutral electroweak Lagrangian of a $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$ model would be

$$\mathcal{L} = -\frac{1}{4} F_Y^{\mu\nu} F_{Y\,\mu\nu} - \frac{1}{4} F_X^{\mu\nu} F_{X\,\mu\nu} - \frac{k}{2} F_{Y\,\mu\nu} F_X^{\mu\nu} - \frac{1}{4} W^{3\,\mu\nu} W^3_{\mu\nu} + \frac{1}{2} M^2_{AB} A^A A^B + g_A A^A J_A + \dots , \tag{1.86}$$

with $A, B = Y, T_{3L}, X$, where the dots stand for terms that are not relevant for the following discussion. The mass matrix M^2_{AB} depends on the vacuum expectation values (VEVs) of the Higgs fields:

$$\frac{1}{2} M^2_{AB} = \sum_{\phi^i} Q_A^{\phi^i} Q_B^{\phi^i} \left| \langle \phi^i \rangle \right|^2 , \tag{1.87}$$

where the sum is over all scalar fields. We will not assume a specific Higgs sector, apart from the SM doublet H, which is needed to break $SU(2)_L \times U(1)_Y \to U(1)_{em}$. The currents read

$$J_A^\mu = \sum_{\psi^j} Q_A^{\psi^j} \overline{\psi}^j \gamma^\mu \psi^j + \sum_{\phi^i} i\, Q_A^{\phi^i} \phi^{i\,\dagger} \overleftrightarrow{\partial^\mu} \phi^i \,, \tag{1.88}$$

where the symbol $\overleftrightarrow{\partial^\mu}$ is defined by $\phi^\dagger \overleftrightarrow{\partial^\mu} \phi = \phi^\dagger(\partial^\mu \phi) - (\partial^\mu \phi^\dagger)\phi$. ψ^j runs over fermions (possibly including exotics) and ϕ^i runs over complex scalars, including H. In this thesis we will focus on the case of only one additional $U(1)$, since including two or more extra Abelian factors would make the mixing patterns very complicated.

The parameters introduced in the Lagrangian are the coupling g_X, the kinetic mixing parameter k, the Higgs VEVs contained in M^2_{AB} (which determine the scale at which $U(1)_X$ breaks), the charges under $U(1)_X$ of all fermions and scalars, and the SM charges of exotic fields. Clearly, many different choices are possible, leading to a large variety of models. These range from minimal assignments, including only right-handed neutrinos as additional fermion fields, to extreme scenarios such as family-dependent charges, models with large exotic sectors, GeV-scale Z', and many others. No attempt will be made to review the existing literature on Z' models; the reader is referred to [18].

1.4.2. Anomaly cancellation

If a classically conserved current is not conserved anymore at the quantum level, the associated symmetry (which is not respected by the quantum effective action) is said to be anomalous. In a gauge theory, the gauge symmetry is crucial to preserve unitarity: we must therefore require that its anomaly vanishes. The anomaly is related to triangle diagrams such as that in Fig. 1.1, whose contribution to the anomaly is proportional to

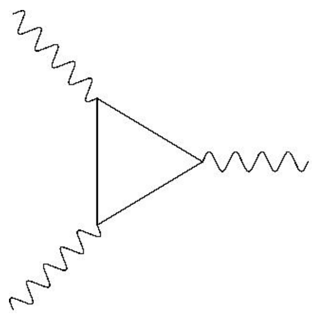

Figure 1.1.: Triangle diagram associated with the anomaly.

$$\mathcal{D}^{\alpha\beta\gamma} = Tr\left[\left\{T^\alpha, T^\beta\right\} T^\gamma\right] \,, \tag{1.89}$$

where $T^\alpha, T^\beta, T^\gamma$ are generators of the gauge group, and the trace is taken over all *left-handed* fermion and antifermion fields. If we want all gauge anomalies to be zero (i.e., if we want all gauge currents to be conserved at the quantum level), we must therefore require that $\mathcal{D}^{\alpha\beta\gamma}$ vanishes for every combination of the generators.

The condition $\mathcal{D}^{\alpha\beta\gamma} = 0$ is satisfied by the SM in a non-trivial way, thanks to the interplay between quark and lepton contributions, as we will prove in a moment. This fundamental property can also be understood by noticing (see, e.g., [19]) that the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ is a subgroup of $SO(10)$, and that one generation of the SM fermions (with all fields taken left-handed) plus one SM singlet fill exactly the **16** representation of $SO(10)$. Being all the representations of $SO(10)$ real, that is, they satisfy

$$T^{\alpha\, T} = -U T^{\alpha} U^{-1} \tag{1.90}$$

with U unitary, they are all anomaly-free, because (1.90) substituted into (1.89) gives $\mathcal{D}^{\alpha\beta\gamma} = -\mathcal{D}^{\alpha\beta\gamma}$. Therefore, it follows that the SM is anomaly-free too (the additional singlet obviously does not contribute to the anomaly).

A direct proof of anomaly cancellation in the SM proceeds as follows: diagrams $[SU(2)]^3$, $[SU(2)][Y]^2$, $[SU(3)]^3$, $[SU(3)][Y]^2$, $[SU(3)]^2[SU(2)]$ and $[SU(3)][SU(2)]^2$ vanish due to the tracelessness of the generators $\mathcal{T}^a$ and T^i (that is, of the Gell-Mann and Pauli matrices), while $[Y][SU(2)]^2$ and $[Y]^3$ are proportional to $Tr[Q_d]$, the trace over all $SU(2)$ doublets of the electric charge. Finally, $[Y][SU(3)]^2$ is proportional to $Tr[Q_t]$, the trace of Q over all $SU(3)$ triplets. Thus the SM is anomaly-free if $Tr[Q_d] = Tr[Q_t] = 0$. We have:

$$Tr[Q_d] = 3\left[\frac{2}{3}\cdot 3 - \frac{1}{3}\cdot 3 - 1\right] = 0\,, \tag{1.91}$$

$$Tr[Q_t] = 3\left[\frac{2}{3}\cdot 3 - \frac{1}{3}\cdot 3 - \frac{2}{3}\cdot 3 + \frac{1}{3}\cdot 3\right] = 0\,, \tag{1.92}$$

where the fields are ordered as in Table 1.1. One additional condition that must be satisfied is the cancellation of the so-called gauge-gravity mixed anomaly, corresponding to a triangle graph having on the external lines two gravitons and a gauge boson. This diagram is proportional to

$$Tr\left[T^{\alpha}\right]\,, \tag{1.93}$$

and must vanish for every T^{α}. Indeed, the generators of any non-Abelian gauge group are traceless, therefore one needs to check whether (1.93) vanishes only for $U(1)$ generators. In the SM, we have

$$Tr\left[Y\right] = 3\left[\frac{1}{6}\cdot 3\cdot 2 - \frac{2}{3}\cdot 3 + \frac{1}{3}\cdot 3 - \frac{1}{2}\cdot 2 + 1\right] = 0\,. \tag{1.94}$$

This completes the proof that the SM is anomaly-free.

In what follows, we will focus on anomaly-free $U(1)'$ models, for which the condition $\mathcal{D}^{\alpha\beta\gamma} = 0$ is satisfied for all the generators of the gauge group. We mention, however, that generalized anomaly cancellation mechanisms exist, for instance we can add gauge-variant terms to the effective action, which are able to cancel the anomaly. Actually, it is possible to consider $U(1)$ extensions of the SM where the additional Abelian factor is anomalous; the anomaly in the low energy theory must be cancelled in a generalized way, via a so-called *Green-Schwarz mechanism*, because the underlying complete field theory or string theory is anomaly-free. A brief description of the Green-Schwarz mechanism, which requires the introduction of non-renormalizable operators, is given in Section 1.5. For an introduction to anomalies, see, e.g., [20].

1.4.3. Anomaly cancellation in $U(1)$ extensions of the SM

When another Abelian factor $U(1)_X$ is added to the gauge group, cancellation of the anomalies introduced by $U(1)_X$ imposes constraints on the X charges of the fermion fields, namely the following identities must hold:

$$[SU(2)]^2[U(1)_X] \qquad Tr\left[\left\{T^i, T^j\right\} X\right] = 0\,, \tag{1.95}$$

$$[SU(3)]^2[U(1)_X] \qquad Tr\left[\left\{\mathcal{T}^a, \mathcal{T}^b\right\} X\right] = 0\,, \tag{1.96}$$

$$[U(1)_Y]^2[U(1)_X] \qquad Tr\left[Y^2 X\right] = 0\,, \tag{1.97}$$

$$[U(1)_Y][U(1)_X]^2 \qquad Tr\left[Y X^2\right] = 0\,, \tag{1.98}$$

$$[U(1)_X]^3 \qquad Tr\left[X^3\right] = 0\,. \tag{1.99}$$

In addition, the gauge-gravity anomaly condition for $U(1)_X$ needs to be satisfied:

$$Tr\,[X] = 0\,. \tag{1.100}$$

If exotic fermions are introduced, their charges must also respect the anomaly cancellation conditions involving only the generators of the SM gauge group that we have discussed in the previous paragraph. Considering only the minimal SM fermions and taking family-independent $U(1)_X$ charges (models with family-dependent charges have to face strong constraints from the observed severe suppression of flavour changing neutral currents (FCNC); there are, however, models of this kind which generate sufficiently suppressed FCNC effects, by suitable choices of the charges: see [21]), the only allowed solution is $X \propto Y$, that is, a (heavy) replica of the SM hypercharge. Therefore, most models include exotic matter, with possibilities that range from the minimal choice (three right-handed neutrinos) to more elaborate scenarios. Usually, extra fermions are assumed to be nonchiral under $SU(3)_c \times SU(2)_L \times U(1)_Y$, in order to evade electroweak precision constraints [8] and to preserve the anomaly cancellation of the SM. In conclusion, the discussion of anomaly cancellation is extremely model-dependent.

1.4.4. Kinetic mixing

In addition to the kinetic terms for Y and X, we have included in (1.86) a 'kinetic mixing' term [22]:

$$\mathcal{L}_{\substack{Abelian\\kinetic}} = -\frac{1}{4}F_Y^{\mu\nu}F_{Y\,\mu\nu} - \frac{1}{4}F_X^{\mu\nu}F_{X\,\mu\nu} - \frac{k}{2}F_{Y\,\mu\nu}F_X^{\mu\nu}\,, \tag{1.101}$$

which is gauge invariant, because the Abelian field strengths $F_{Y,X}^{\mu\nu} = \partial^\mu A_{Y,X}^\nu - \partial^\nu A_{Y,X}^\mu$ are themselves gauge invariant (this term has no obvious non-Abelian counterpart). The mixing term in (1.101) must be in general included in the Lagrangian; the parameter k must satisfy the condition $|k| < 1$ in order to ensure positivity of the kinetic energy. $\mathcal{L}_{\substack{Abelian\\kinetic}}$ can be recast in canonical form by means of the transformation $A^A = C^A{}_a A^a$, $a = \widetilde{Y}, 3L, \widetilde{X}$, explicitly

$$\begin{pmatrix} A^Y \\ A^{3L} \\ A^X \end{pmatrix} = C \begin{pmatrix} A^{\widetilde{Y}} \\ A^{3L} \\ A^{\widetilde{X}} \end{pmatrix} \tag{1.102}$$

with

$$C = \begin{pmatrix} 1 & 0 & -\frac{k}{\sqrt{1-k^2}} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\sqrt{1-k^2}} \end{pmatrix} , \tag{1.103}$$

which has the following effect on the couplings of currents to gauge bosons:

$$g\, J_{3L} A^{3L} + g' J_Y A^Y + g_X J_X A^X \rightarrow g\, J_{3L} A^{3L} + g' J_Y A^{\widetilde{Y}} + \left(g'_X J_X + g_Y J_Y\right) A^{\widetilde{X}} , \tag{1.104}$$

where we have defined

$$g'_X = \frac{g_X}{\sqrt{1-k^2}} , \tag{1.105}$$

$$g_Y = -\frac{k}{\sqrt{1-k^2}} g' . \tag{1.106}$$

The couplings to $A^{\widetilde{Y}}$ therefore are the same as those to A^Y, while $A^{\widetilde{X}}$ now couples to a linear combination of the hypercharge current J_Y and of J_X. We can also define an overall coupling constant and new charges as follows: $\hat{g}\hat{Q} \equiv g_Y Y + g'_X X$. If the mixing parameter k is small, then (1.105) and (1.106) become

$$g'_X \cong g_X , \tag{1.107}$$

$$g_Y \cong -kg' , \tag{1.108}$$

where we have neglected terms of $O(k^2)$.

1.4.5. Gauge boson masses

The effect of (1.102) on the mass term is

$$\frac{1}{2} A^A M^2_{AB} A^B \rightarrow \frac{1}{2} A^a M^2_{ab} A^b , \tag{1.109}$$

where $M^2_{ab} = C^A{}_a M^2_{AB} C^B{}_b$. Next, we isolate the photon, which must remain massless, by means of the rotation

$$A^a = A^i R_i{}^a , \tag{1.110}$$

$$R_i{}^a = \begin{pmatrix} \cos\theta_w & \sin\theta_w & 0 \\ -\sin\theta_w & \cos\theta_w & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{1.111}$$

which yields a mass matrix of the generic form

$$M_{ij} = R_i{}^a M^2_{ab} R_j{}^b = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M^2_{\widetilde{Z}} & \Delta^2 \\ 0 & \Delta^2 & M^2_{\widetilde{Z}'} \end{pmatrix} , \qquad (i, j = \gamma, \widetilde{Z}, \widetilde{Z}'). \tag{1.112}$$

After (1.110), the couplings of the gauge bosons to currents read

$$eJ_{em} A^\gamma + \frac{g}{\cos\theta_w} \left(J_{3L} - \sin^2\theta_w\right) A^{\widetilde{Z}} + \left(g'_X J_X + g_Y J_Y\right) A^{\widetilde{Z}'} , \tag{1.113}$$

where $J_{em} = J_Y + J_{3L}$ is the electromagnetic current. We see that in this basis $(\gamma, \widetilde{Z})$ couple to the same currents as (γ, Z) do in the SM. However, there remains to diagonalize the mass mixing between $\widetilde{Z}$ and $\widetilde{Z}'$. This is done by writing $A^i = A^\alpha U_\alpha{}^i$ $(\alpha = \gamma, Z, Z')$, explicitly

$$\begin{pmatrix} \gamma \\ Z \\ Z' \end{pmatrix} = U \begin{pmatrix} \gamma \\ \widetilde{Z} \\ \widetilde{Z}' \end{pmatrix} \tag{1.114}$$

with

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\hat{\theta} & \sin\hat{\theta} \\ 0 & -\sin\hat{\theta} & \cos\hat{\theta} \end{pmatrix} , \tag{1.115}$$

$$\tan(2\hat{\theta}) = \frac{2\Delta^2}{M^2_{\widetilde{Z}} - M^2_{\widetilde{Z}'}} . \tag{1.116}$$

The resulting mass matrix is

$$U_\alpha{}^i M^2_{ij} U_\beta{}^j = U M^2_{ij} U^T = diag[0, M^2_Z, M^2_{Z'}] , \tag{1.117}$$

with mass eigenvalues (assuming $M_{Z'} > M_Z$)

$$M^2_{Z',Z} = \frac{1}{2}\left[M^2_{\widetilde{Z}} + M^2_{\widetilde{Z}'} \pm \sqrt{\left(M^2_{\widetilde{Z}} - M^2_{\widetilde{Z}'} \right)^2 + 4\Delta^4} \right] . \tag{1.118}$$

The Z-pole precision measurements made at LEP1 strongly constrain the Z-Z' mixing.

1.4.6. The sequential Z'_{SM}

The 'sequential' Z', denoted by Z'_{SM}, is defined to have the same couplings *to fermions* as the SM Z boson, but no couplings to the SM vector bosons. Even though this Z' does not have a precise theoretical motivation, it is often used as a reference model when specifying experimental bounds. To give the reader a rough idea of the present experimental reach in Z' direct searches, we mention the most recent lower bounds on the mass of Z'_{SM}, as set by the CDF and D0 experiments: $M_{Z'_{SM}} \geq 961\,\mathrm{GeV}$ from CDF $Z' \to e^+e^-$ searches [23], $M_{Z'_{SM}} \geq 1030\,\mathrm{GeV}$ from CDF $Z' \to \mu^+\mu^-$ searches [24], and $M_{Z'_{SM}} \geq 950\,\mathrm{GeV}$ from D0 dielectron searches [25]. We mention these numbers only to set orders of magnitude; indeed, the actual lower limit on the mass of a Z' is highly model dependent, and can be significantly lower or higher than these figures.

1.5. Anomalous $U(1)'$s

In this section we briefly introduce the Lagrangian for anomalous $U(1)'$s, following the arguments of [26]. As we will see, cancellation of the anomalies given by triangle graphs requires the introduction of additional non-renormalizable, gauge variant terms in the Lagrangian. The gauge variance of these terms is tuned to cancel the contributions from triangle diagrams, so that the full low-energy effective Lagrangian is gauge invariant. Some of the anomalous Abelian gauge bosons acquire mass through the Stückelberg mechanism, which we shall now describe.

1.5.1. The Stückelberg mechanism

Consider the following classical Lagrangian for an Abelian vector field:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(mA_\mu + \partial_\mu\sigma)^2 + gA_\mu J^\mu\,. \tag{1.119}$$

J^μ is a conserved current, and σ is an axionic scalar field. Axions are light pseudoscalar particles, which couple weakly to ordinary matter and radiation. Historically, an axion was first introduced as an attempt to solve the so-called *strong CP problem*, a difficulty which arises from the CP-violating term

$$\frac{\Theta g_S^2}{64\pi^2}\epsilon_{\mu\nu\rho\sigma}G^{a\,\mu\nu}G^{a\,\rho\sigma}\,, \tag{1.120}$$

where $\epsilon_{\mu\nu\rho\sigma}$ is the completely antisymmetric tensor of rank 4 and G^a is the gluon field strength, which must be in general included in the QCD Lagrangian. Measurements of the neutron electric dipole moment imply an unnaturally strong bound on the parameter Θ: $|\Theta| \leq 10^{-10}$. To solve this problem, a global $U(1)$ symmetry was introduced: the Goldstone boson associated to the spontaneous breaking of this Abelian global symmetry is the axion (actually, the symmetry is explicitly broken by a small amount, so the Goldstone boson acquires a mass). The anomaly of the global symmetry gives rise to a quantum Lagrangian of the form

$$\frac{g_S^2}{64\pi^2}\left(\Theta - \frac{\sigma}{f}\right)\epsilon_{\mu\nu\rho\sigma}G^{a\,\mu\nu}G^{a\,\nu\sigma}\,, \tag{1.121}$$

where σ is the axion field, and f is a constant of mass dimension 1. The potential for σ has a minimum at $\sigma = \Theta f$, thus canceling the Θ-term and solving the difficulty (see [27] and references therein).

Now going back to the Stückelberg mechanism, $\mathcal{L}$ is invariant under the following gauge transformation:

$$A_\mu \to A_\mu + \partial_\mu\epsilon\,, \qquad \sigma \to \sigma - m\epsilon\,. \tag{1.122}$$

Among the various possible sources of Stückelberg couplings, there is the familiar spontaneous symmetry breaking via Higgs VEVs, in which case the axion is the phase of the Higgs field. Another possibility, in the context of string theories, is to have Ramond-Ramond axions, originated by the extra-dimensional components of antisymmetric tensor fields.

1.5.2. Low-energy effective theory for anomalous $U(1)$s

In this paragraph we describe the anomaly-related terms in the effective action for a set of anomalous $U(1)$s. We also introduce non-Abelian gauge bosons in the theory. The kinetic terms read

$$\mathcal{L}_{kin} = -\frac{1}{2}\sum_\alpha Tr\left[G^\alpha_{\mu\nu}G^{\alpha\,\mu\nu}\right] - \sum_i \frac{1}{4}F^i_{\mu\nu}F^{i\,\mu\nu} + \frac{1}{2}\sum_I \left(\partial_\mu a^I + M^I_i A^i_\mu\right)^2\,, \tag{1.123}$$

where $\alpha = 1, \ldots, N_{YM}$ runs over the simple factors of the non-Abelian group, $i = 1, \ldots, N_V$ runs over Abelian factors and $I = 1, \ldots, N_a$ runs over axions. The field strengths are given by

$$G^{\alpha}_{\mu\nu} = \partial_\mu B^{\alpha}_{\nu} - \partial_\nu B^{\alpha}_{\mu} - i g_\alpha [B^{\alpha}_{\mu}, B^{\alpha}_{\nu}]\,, \tag{1.124}$$

$$F^{i}_{\mu\nu} = \partial_\mu A^{i}_{\nu} - \partial_\nu A^{i}_{\mu}\,. \tag{1.125}$$

It is important to remark that we are assuming that kinetic mixing among $U(1)$ factors has been removed, by suitably redefining the coefficients M^I_i. Next, we separate massive Abelian fields from massless ones, obtaining [1] [26]

$$\mathcal{L}_{kin} = -\frac{1}{2} Tr\left[G^{\alpha}_{\mu\nu} G^{\alpha\,\mu\nu}\right] - \frac{1}{4} F^{a}_{\mu\nu} F^{a\,\mu\nu} - \frac{1}{4} F^{m}_{\mu\nu} F^{m\,\mu\nu} - \frac{1}{2}\partial_\mu b^u \partial^\mu b^u + \frac{1}{2} M_a^2 \left(\partial_\mu b^a + Q^a_\mu\right)^2\,, \tag{1.126}$$

where Q^a, $a = 1, \ldots, N_M$ are the massive $U(1)$ vectors with b^a their associated axions, while Y^m, $m = 1, \ldots, N_0$ are the massless Abelian gauge bosons. b^u, $u = 1, \ldots, N_{inv}$ are the gauge-invariant axions ($N_{inv} \equiv N_a - N_M$). The Abelian gauge transformations read

$$Q^a \to Q^a + \partial\epsilon^a \;\;,\;\; b^a \to b^a - \epsilon^a \;\;,\;\; Y^m \to Y^m + \partial\epsilon^m \;\;,\;\; b^u \to b^u\,. \tag{1.127}$$

Under these transformations, as well as under the non-Abelian ones:

$$B^{\alpha}_{\mu} \to B^{\alpha}_{\mu} + D_\mu \epsilon^\alpha\,, \qquad D_\mu\epsilon^\alpha = \partial_\mu \epsilon^\alpha - i g_\alpha [B^{\alpha}_{\mu}, \epsilon^\alpha]\,, \tag{1.128}$$

$\mathcal{L}_{kin}$ is invariant.

We now introduce gauge variant terms, which will be needed to cancel the anomalous contributions of triangle diagrams. Two types of terms must be considered. The first are the Peccei-Quinn terms, which read

$$\mathcal{L}_{PQ} = \frac{b^u}{24\pi^2}({C^u}_{ab} F^a \wedge F^b + \ldots + {D^u}_{\alpha} Tr[G^\alpha \wedge G^\alpha]) + \frac{b^a}{24\pi^2}({C^a}_{bc} F^b \wedge F^c + \ldots + {D^a}_{\alpha} Tr[G^\alpha \wedge G^\alpha])\,, \tag{1.129}$$

(the full expressions can be found in [26]; our aim here is just to sketch the method) where ${C^u}_{ab}$, ${D^u}_{\alpha}$, ... are arbitrary coefficients. We have used form notation,

$$F^q = \frac{1}{2} F^{q}_{\mu\nu} dx^\mu dx^\nu\,, \tag{1.130}$$

$$F^q \wedge F^t = \frac{1}{4} F^{q}_{\mu\nu} F^{t}_{\rho\sigma} dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma = \frac{1}{4}\epsilon^{\mu\nu\rho\sigma} F^{q}_{\mu\nu} F^{t}_{\rho\sigma} d^4x\,. \tag{1.131}$$

Under a gauge transformation, the variation of the Peccei-Quinn terms reads

$$\delta\mathcal{L}_{PQ} = -\frac{\epsilon^a}{24\pi^2}({C^a}_{bc} F^b \wedge F^c + \ldots + {D^a}_{\alpha} Tr[G^\alpha \wedge G^\alpha])\,. \tag{1.132}$$

The second class of terms are the generalized Chern-Simons (GCS) terms. Two types of GCS terms exist, Abelian and mixed Abelian-non Abelian. The Abelian ones read

$$S^{ijk} = \frac{1}{48\pi^2}\int \epsilon^{\mu\nu\rho\sigma} A^i_\mu A^j_\nu F^k_{\rho\sigma} d^4x\,, \tag{1.133}$$

[1] In the following, the fields b^a are dimensionless, as a consequence of a convenient redefinition. However, we stress that Peccei-Quinn terms are non-renormalizable, their expression in terms of the axions a^I reading $(1/24\pi^2) C^I_{ij} \int a^i F^i \wedge F^j$, where the coefficients C^I_{ij} have mass dimension -1.

which, under Abelian gauge transformations, change according to

$$\delta S^{ijk} = \frac{1}{24\pi^2}\int \left(\epsilon^j F^i \wedge F^k - \epsilon^i F^j \wedge F^k\right) . \tag{1.134}$$

To define the mixed terms, we introduce the non-Abelian CS form

$$\Omega_{\mu\nu\rho} = \frac{1}{3} Tr\left[B_\mu (G_{\nu\rho} + \frac{1}{3} ig[B_\nu, B_\rho]) + cyclic\right] , \tag{1.135}$$

which satisfies

$$\frac{1}{2}\partial_\mu \Omega_{\nu\rho\sigma} dx^\mu dx^\nu dx^\rho dx^\sigma = Tr[G \wedge G] , \tag{1.136}$$

and transforms under a gauge transformation in the following way

$$\delta\Omega_{\mu\nu\rho} = \frac{1}{3} Tr[\partial_\mu \epsilon(\partial_\nu B_\rho - \partial_\rho B_\nu) + cyclic] . \tag{1.137}$$

The mixed GCS terms read

$$S^{i,\alpha} = \frac{1}{48\pi^2}\int \epsilon^{\mu\nu\rho\sigma} A^i_\mu \Omega^\alpha_{\nu\rho\sigma} d^4x , \tag{1.138}$$

and their change under a gauge transformation is found to be, using (1.136) and (1.137):

$$\delta S^{i,\alpha} = \frac{1}{24\pi^2}\int F^i \wedge Tr[\epsilon \widetilde{G}^\alpha] - \epsilon^i Tr[G^\alpha \wedge G^\alpha] , \tag{1.139}$$

where $\widetilde{G}^\alpha \equiv \partial_\mu G^\alpha_\nu - \partial_\nu G^\alpha_\mu$. The most general expression for the GCS terms reads

$$\mathcal{L}_{GCS} = \frac{1}{48\pi^2}\epsilon^{\mu\nu\rho\sigma}\left[E_{mnr} Y^m_\mu Y^n_\nu F^r_{\rho\sigma} + \ldots + (Z^m{}_\alpha Y^m_\mu + Z^a{}_\alpha Q^a_\mu)\Omega^\alpha_{\nu\rho\sigma}\right] , \tag{1.140}$$

where E_{mnr}, $Z^m{}_\alpha$, $Z^a{}_\alpha$,… are coefficients.

The anomalous gauge variation due to triangle diagrams can be computed to be

$$\begin{aligned}\delta\mathcal{L}_{triangle} = -\frac{1}{24\pi^2}&\left\{\epsilon^a\left[t_{abc}F^b \wedge F^c + \ldots + T_{a\alpha} Tr[G^\alpha \wedge G^\alpha]\right] + \right.\\ &\left.\epsilon^m\left[t_{mnr}F^n \wedge F^r + \ldots + T_{m\alpha} Tr[G^\alpha \wedge G^\alpha]\right] + 2T_{a\alpha} Tr[\epsilon\widetilde{G}^\alpha] \wedge F^a + \ldots\right\} ,\end{aligned} \tag{1.141}$$

where t and T are traces over fermions of cubic combinations of generators, namely

$$t_{abc} = Tr[\mathcal{Q}_a \mathcal{Q}_b \mathcal{Q}_c] \ , \ T_{a\alpha} = Tr[\mathcal{Q}_a (TT)_\alpha] , \qquad \ldots , \tag{1.142}$$

with $\mathcal{Q}_a$ Abelian generators, and where $(TT)_\alpha$ is the quadratic Casimir of the α-th simple group. In conclusion, the full effective Lagrangian reads

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{PQ} + \mathcal{L}_{GCS} + \mathcal{L}_{triangle} , \tag{1.143}$$

which must be gauge invariant. From the definitions of the gauge-variant terms we made above, it should be clear that the strategy is to impose gauge invariance of $\mathcal{L}$, requiring that PQ and GCS terms cancel $\delta\mathcal{L}_{triangle}$. This forces definite relations between the coefficients

C, D, E, Z and the traces t and T (which can be computed explicitly using the fermion spectrum), thus fixing the full effective Lagrangian[2].

It is interesting to note that this generalized anomaly cancellation mechanism, known as Green-Schwarz mechanism from the names of its discoverers [28], can be produced both in the effective theory descending from a string theory, and in the framework of a quantum field theory, where an anomalous set of heavy fermions is integrated out, leaving a low-energy effective theory involving light fermions [29, 30]. In this case too, the anomalies given by triangle graphs are cancelled by additional gauge variant terms. Finally, one interesting phenomenological consequence of anomalous $U(1)'$ models is the appearance, at tree-level, of anomalous three-gauge-boson couplings, such as the $\gamma - Z - Z'$ vertex, leading to $Z' \to Z\,\gamma$ decay. These couplings are present also in non-anomalous models (such as the minimal model described in Chapter 2), due to one-loop diagrams with gauge bosons in the loops. A detailed phenomenological study may allow to distinguish non-anomalous models from models where anomalies are cancelled by the generalized mechanism introduced in this section.

[2]Actually, an ambiguity remains, due to the presence of certain combinations of axionic and GCS terms which are gauge-invariant, see [26].

2. A minimal $U(1)$ extension

In this chapter we analyze a 'minimal' extension of the Standard Model, which includes three generations of right-handed (RH) neutrinos and an additional $U(1)$ factor in the gauge group, associated with $B-L$, i.e. baryon minus total lepton number. Automatic anomaly cancellation is therefore obtained. Our parameterization, described in Section 2.1, fully accounts for both kinetic and mass mixing, and requires the introduction of only three parameters in addition to the SM ones, namely the Z' mass, $M_{Z'}$, and two effective coupling constants.

The second section contains the analysis of the bounds set by electroweak precision tests (EWPTs) on the parameter space, performed applying the results of [13]. In the third section, we present a discussion of the constraints imposed by a recent re-analysis of atomic parity violation (APV) in Cesium atoms, showing that APV bounds are always weaker than those given by previous EWPTs. In the fourth section, we study the bounds imposed by direct searches performed at the Tevatron collider, and compare the latter with EWPTs and APV. We show that Tevatron searches set more stringent bounds for low Z' masses ($M_{Z'} < 700\,\mathrm{GeV}$ approximately), whereas for larger masses EWPTs impose the strongest constraints on the model. We also comment on the recent observation, by the CDF collaboration, of an excess in the e^+e^- channel, and find that the region of parameter space which could produce such an excess is not presently ruled out by EWPTs.

In the fifth section, we analyze the discovery potential of the LHC in view of the schedule of its early running phase, which foresees energies in the $7 \div 10\,\mathrm{TeV}$ range, and integrated luminosity from 100 pb^{-1} to a few hundred pb^{-1}. We find that at 7 TeV and with a luminosity of $100\,\mathrm{pb}^{-1}$, the LHC will enter a quite narrow region of unexplored parameter space. At a later stage, the upgrade in energy to 10 TeV will open up (for a luminosity of $200\,\mathrm{pb}^{-1}$) regions of parameter space in the mass range from 400 to 1500 GeV, with EWPTs still unsurpassed for larger masses, and Tevatron searches still more stringent for lower masses. Finally, in the last section, we compute the region of parameter space that is preferred by GUTs. Using the results of the previous section, we find that to search for GUT-preferred Z's, which are constrained by EWPTs to have a mass larger than a TeV approximately, the LHC will need (at 10 TeV) a luminosity roughly of the order of 1 fb^{-1}, thus larger than the one presently foreseen for 2010, which is $< 300\,\mathrm{pb}^{-1}$.

2.1. General parameterization

We enlarge the SM gauge group by adding an Abelian factor, which we take to be $U(1)_{B-L}$, and in addition to the fermion content of the minimal SM, we consider three RH neutrinos. As for the scalars, in addition to the SM Higgs doublet H, we consider also an additional complex scalar field ϕ, neutral under the SM gauge group, but charged under $B-L$. We stress that this is not the only choice at hand, since we could introduce a direct mass term to break $U(1)_{B-L}$ (this would not spoil renormalizability), assuming a Stückelberg mechanism (see Section 1.5). In what follows we choose to introduce the complex scalar ϕ, but in our phenomenological analysis we neglect the associated physical scalar h', assuming that it is either sufficiently heavy or sufficiently decoupled from the SM fields, including the SM Higgs.

The neutral gauge part of the Lagrangian reads (throughout this chapter we omit QCD terms, unless otherwise stated):

$$\mathcal{L}_{gauge} = -\frac{1}{4} h_{AB} F^{A}_{\mu\nu} F^{B\,\mu\nu} + \frac{1}{2} M^2_{AB} A^{A\mu} A^{B}_{\mu} + A^{A}_{\mu} J^{\mu}_{A}\,, \tag{2.1}$$

with $A, B = Y, T_{3L}, B-L$. The coefficients of the kinetic terms are given by

$$h_{AB} = \begin{pmatrix} \frac{1}{g'^2} & 0 & \frac{k}{g' G_{BL}} \\ 0 & \frac{1}{g^2} & 0 \\ \frac{k}{g' G_{BL}} & 0 & \frac{1}{G^2_{BL}} \end{pmatrix}, \tag{2.2}$$

where g, g', G_{BL} are the $SU(2)_L$, $U(1)_Y$, $U(1)_{B-L}$ coupling constants respectively, and k is the kinetic mixing parameter introduced in Chapter 1. The mass matrix has the following expression:

$$M^2_{AB} = \frac{1}{4} \begin{pmatrix} v^2 & -v^2 & 0 \\ -v^2 & v^2 & 0 \\ 0 & 0 & 4 Q^{\phi\,2}_{BL} v^2_{BL} \end{pmatrix}, \tag{2.3}$$

where we have assumed that the Higgs fields H and ϕ have the following transformation properties under $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$:

$$H \sim (\mathbf{2}, \frac{1}{2}, 0)\,, \qquad \phi \sim (\mathbf{1}, 0, Q^{\phi}_{BL})\,, \tag{2.4}$$

and that the VEVs of the Higgs fields are

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \qquad \langle \phi \rangle = \frac{v_{BL}}{\sqrt{2}}\,. \tag{2.5}$$

As evident from equation (2.4), we assume $Q^{\phi}_{Y} = 0$: there is no loss of generality in doing this, as discussed in [31]. The argument goes as follows: we take as a starting point a basis $U(1)_1 \times U(1)_2$, where hypercharge is not identified with either $U(1)_1$ or $U(1)_2$. Then, performing a $SO(2)$ rotation on the gauge fields (A^1, A^2), we can always choose a basis $U(1)'_1 \times U(1)'_2$ where the charge of ϕ under one of the two factors, say $U(1)'_1$, is zero. At this point, we identify $U(1)'_1$ with the SM hypercharge, and suitably redefining the associated coupling constant we can take $Y(H) = 1/2$. $U(1)'_2$ is identified with $B-L$, and since the SM Higgs H has vanishing $B-L$, we obtain (2.4).

The charges of the fields are summarized in Table 2.1: the normalization of the hypercharge is such that the electric charge is, for a generic fermion or scalar field Φ, $Q^{\Phi} = T^{\Phi}_{3L} + Y^{\Phi}$. The currents J^{μ}_{A} are given in eq. (2.6)[1]

$$J^{\mu}_{A} = \sum_i Q^{i}_{A} \overline{\psi}_i \gamma^{\mu} \psi_i + i\, Q^{H}_{A} H^{\dagger} \overleftrightarrow{\partial^{\mu}} H + i\, Q^{\phi}_{A} \phi^{\dagger} \overleftrightarrow{\partial^{\mu}} \phi\,, \tag{2.6}$$

where $i = u_L, d_L, \nu_L, e_L, u_R, d_R, \nu_R, e_R$.

[1] The neutral part of the generic covariant derivative is $D_{\mu}\Phi = \partial_{\mu}\Phi - i Q^{\Phi}_{A} A^{A}_{\mu} \Phi$.

Name	T_{3L}	Y	$B-L$
$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$	$\frac{1}{6}$	$\frac{1}{3}$
u_R	0	$\frac{2}{3}$	$\frac{1}{3}$
d_R	0	$-\frac{1}{3}$	$\frac{1}{3}$
$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$	$-\frac{1}{2}$	-1
ν_R	0	0	-1
e_R	0	-1	-1
$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$	$\frac{1}{2}$	0
ϕ	0	0	Q^ϕ_{BL}

Table 2.1.: Field content of the minimal $U(1)_{B-L}$ SM extension.

2.1.1. Anomaly cancellation

In this paragraph we check whether the minimal model satisfies the anomaly cancellation conditions introduced in Chapter 1. Keeping our discussion general, suppose we have the Lagrangian (2.1) with a generic additional Abelian factor $U(1)_X$ in place of $U(1)_{B-L}$, with corresponding fermion charges X_i, $i = q, l, u^c, d^c, \nu^c, e^c$ (we recall that the trace in (1.89) must be computed over all *left-handed* fermions and antifermions, so we take the charge conjugates of the right-handed fields: obviously $X_{f^c} = -Z_{f_R}$). Anomaly cancellation then imposes the following constraints [31]:

$$[SU(2)]^2[U(1)_X] \qquad X_l + 3X_q = 0\,, \tag{2.7}$$

$$[SU(3)]^2[U(1)_X] \qquad X_{d^c} + X_{u^c} + 2X_q = 0\,, \tag{2.8}$$

$$[U(1)_Y]^2[U(1)_X] \qquad \frac{2}{3}X_q + 2X_l + \frac{16}{3}X_{u^c} + \frac{4}{3}X_{d^c} + 4X_{e^c} = 0\,, \tag{2.9}$$

$$[U(1)_Y][U(1)_X]^2 \qquad 2X_q^2 - 2X_l^2 - 4X_{u^c}^2 + 2X_{d^c}^2 + 2X_{e^c}^2 = 0\,. \tag{2.10}$$

Other conditions that must be satisfied are:

$$[U(1)_X]^3 \qquad 3(6X_q^3 + 2X_l^3 + 3X_{u^c}^3 + 3z_{d^c}^3 + z_{e^c}^3) + 3X_{\nu^c}^3 = 0\,, \tag{2.11}$$

$$[U(1)_X] \qquad 3(6X_q + 2X_l + 3X_{u^c} + 3X_{d^c} + X_{e^c}) + 3X_{\nu^c} = 0\,. \tag{2.12}$$

Equation (2.12) is the mixed gauge-gravity anomaly cancellation condition. We proceed as follows:

(1) Making use of (2.7),(2.8),(2.9), equation (2.10) is automatically satisfied.

(2) From (2.12), by means of (2.7),(2.8),(2.9) we find

$$3(4X_q + X_{u^c}) = 3X_{\nu^c}\,, \tag{2.13}$$

Then, use of (2.7),(2.8),(2.9) in (2.11) gives

$$3(4X_q + X_{u^c})^3 = 3X_{\nu^c}^3\,, \tag{2.14}$$

and now using (2.13) we obtain

$$X_{\nu^c}^3 = X_{\nu^c}^3 \,, \tag{2.15}$$

which is of course an identity.

Thus we have 6 equations, but only four are independent: we are left with 6 parameters and 4 independent equations, and we can express the 6 fermion charges as functions of only two independent parameters, say for example X_{u^c} and X_q. The result is straightforwardly seen to be:

$$X_l = -3X_q \,, \tag{2.16}$$

$$X_{d^c} = -2X_q - X_{u^c} \,, \tag{2.17}$$

$$X_{e^c} = 2X_q - X_{u^c} \,, \tag{2.18}$$

$$X_{\nu^c} = X_{u^c} + 4X_q \,. \tag{2.19}$$

The most general solution of this set of equations is $X_i = a\,Y_i + b\,(B-L)_i$ with a, b arbitrary parameters, as the reader can verify by making use of the charges in Table 1. Thus we have proved that the minimal model, which corresponds to the case $a = 0\,,\; b = 1$, is anomaly-free. Actually, it can be shown (see Appendix B) that there is no loss of generality in choosing $a = 0$: stated differently, considering both $Y-X$ kinetic mixing and an X linear combination of Y and $B-L$ is a redundant parameterization. It follows that the one introduced in this section is the most general anomaly-free Z' model containing the SM fields plus three RH neutrinos.

2.1.2. Removing kinetic mixing

We want to write the Lagrangian (2.1) in a basis where the kinetic terms are canonical. This can be done by writing

$$h_{AB} = e_A{}^a e_B{}^a \,, \tag{2.20}$$

($a = Y, T_{3L}, Z'^0$) with $e_A{}^a$ a triangular 'vielbein' matrix, and defining (we drop from now on Lorentz indices)

$$A^a = A^A e_A{}^a \,. \tag{2.21}$$

The inverse of (2.21) is

$$A^A = A^a e_a{}^A \,, \tag{2.22}$$

so the following identities hold:

$$e_B{}^a e_a{}^A = \delta_B^A \,, \tag{2.23}$$

$$e_a{}^A e_A{}^b = \delta_a^b \,. \tag{2.24}$$

In the new basis, the kinetic matrix is the identity:

$$-\frac{1}{4} h_{AB} F^A F^B = -\frac{1}{4} F^a F^a \,. \tag{2.25}$$

The explicit expression for ${e_a}^A$ is

$$ {e_a}^A = \begin{pmatrix} g' & 0 & 0 \\ 0 & g & 0 \\ g_Y & 0 & g_{BL} \end{pmatrix} , \tag{2.26} $$

where the parameters g_Y and g_{BL} are defined by the following relations [2]:

$$ k = \frac{-g_Y}{\sqrt{g_Y^2 + g'^2}} , \tag{2.27} $$

$$ G_{BL} = \frac{g' g_{BL}}{\sqrt{g_Y^2 + g'^2}} . \tag{2.28} $$

The inverse of (2.26) is given by

$$ {e_A}^a = \begin{pmatrix} \frac{1}{g'} & 0 & 0 \\ 0 & \frac{1}{g} & 0 \\ -\frac{g_Y}{g' g_{BL}} & 0 & \frac{1}{g_{BL}} \end{pmatrix} . \tag{2.29} $$

The effect of (2.22) on the mass and interaction terms is

$$ \frac{1}{2} M^2_{AB} A^A A^B + A^A J_A = \frac{1}{2} M^2_{ab} A^a A^b + A^a J_a , \tag{2.30} $$

where $M^2_{ab} = {e_a}^A M^2_{AB} {e_b}^B$ and $J_a = {e_a}^A J_A$. The explicit expression of M^2_{ab} is

$$ M^2_{ab} = {e_a}^A M^2_{AB} {e_b}^B = \frac{1}{4} \begin{pmatrix} g'^2 v^2 & -g' g v^2 & g_Y g' v^2 \\ -g' g v^2 & g^2 v^2 & -g_Y g v^2 \\ g_Y g' v^2 & -g_Y g v^2 & g_Y^2 v^2 + 4 g_{BL}^2 Q_{BL}^{\phi\,2} v_{BL}^2 \end{pmatrix} , \tag{2.31} $$

while the currents in the new basis read

$$ J_a = {e_a}^A J_A = \begin{pmatrix} g' & 0 & 0 \\ 0 & g & 0 \\ g_Y & 0 & g_{BL} \end{pmatrix} \begin{pmatrix} J_Y \\ J_{3L} \\ J_{BL} \end{pmatrix} = \begin{pmatrix} g' J_Y \\ g J_{3L} \\ g_Y J_Y + g_{BL} J_{BL} \end{pmatrix} . \tag{2.32} $$

We define new charges as follows:

$$ g_Z Z^\Phi \equiv g_Y Y^\Phi + g_{BL} Q^\Phi_{BL} , \tag{2.33} $$

for a generic field Φ, where $g_Z \equiv \sqrt{g^2 + g'^2}$. The charges can be written as $Z^\Phi = \widetilde{g}_Y Y^\Phi + \widetilde{g}_{BL} Q^\Phi_{BL}$, where we have introduced the parameters

$$ \widetilde{g}_Y = \frac{g_Y}{g_Z} , \tag{2.34} $$

$$ \widetilde{g}_{BL} = \frac{g_{BL}}{g_Z} . \tag{2.35} $$

These two parameters, together with the Z' mass, $M_{Z'}$, describe the model under our minimal assumptions, and will be constantly used in what follows. From eq. (2.32) we see that in the basis where kinetic terms are canonical, the additional gauge boson Z'^0 couples to a linear combination of the hypercharge current and of the $B-L$ current.

[2] These are equivalent to the redefinitions introduced in Chapter 1, eqs. (1.105) and (1.106): we simply have to replace g_X with G_{BL} and g'_X with g_{BL}.

2.1.3. Mass eigenstates

We now proceed to diagonalize the mass matrix in eq. (2.31). Our starting point is

$$\mathcal{L} = -\frac{1}{4}F^a F^a + \frac{1}{2}M^2_{ab}A^a A^b + A^a J_a \,, \tag{2.36}$$

where $a, b = Y, T_{3L}, Z'^0$, and we write

$$A^a = A^i D_i{}^a \,, \tag{2.37}$$

where $i = \gamma, Z, Z'$, and $D_i{}^a$ is an orthogonal matrix. The mass terms become

$$\frac{1}{2}M^2_{ij}A^i A^j \,, \tag{2.38}$$

with $M^2_{ij} = D_i{}^a M^2_{ab} D_j{}^b = diag[0, M^2_Z, M^2_{Z'}]$. M_Z, $M_{Z'}$ are the physical masses of the Z and Z' bosons respectively. The explicit expression of $D_i{}^a$ reads

$$D_i{}^a = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta' & \sin\theta' \\ 0 & -\sin\theta' & \cos\theta' \end{pmatrix} \begin{pmatrix} \cos\theta_w & \sin\theta_w & 0 \\ -\sin\theta_w & \cos\theta_w & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{2.39}$$

where θ_w is the weak mixing angle, and θ' is the Z-Z' mixing angle: its explicit expression is

$$\tan 2\theta' = \frac{2g_Y\sqrt{g^2+g'^2}}{g_Y^2 + 4Q^{\phi\,2}_{BL}g^2_{BL}(\frac{v_{BL}}{v})^2 - g^2 - g'^2} \,. \tag{2.40}$$

The mass eigenvalues are $M^2_\gamma = 0$, and

$$\begin{aligned} M^2_{Z',Z} = \frac{v^2(g^2+g'^2)}{4}\Bigg[\frac{1}{2}\left(1 + \frac{g_Y^2 + g^2_{BL}(\frac{v_{BL}}{v})^2 4Q^{\phi\,2}_{BL}}{g^2+g_Y^2}\right) \\ \pm \frac{1}{2(g^2+g'^2)}\sqrt{\left(g^2+g'^2 - g_Y - g^2_{BL}\left(\frac{v^2_{BL}}{v^2}\right)4Q^{\phi\,2}_{BL}\right)^2 + (2g_Y)^2(g^2+g'^2)}\,\Bigg] \,. \end{aligned} \tag{2.41}$$

More manageable expressions are obtained by using the parameters $\widetilde{g}_Y$, $\widetilde{g}_{BL}$ and

$$M^{0\,2}_{Z'} \equiv \frac{1}{4}\left[g_Y^2 v^2 + 4g^2_{BL}Q^{\phi\,2}_{BL}v^2_{BL}\right] \,. \tag{2.42}$$

$M^{0\,2}_{Z'}$ is the (3,3) element of the mass matrix in the basis where the kinetic terms are canonical, see (2.31). It coincides with the physical squared mass $M^2_{Z'}$ if $\theta' = 0$, which can happen rigorously only for $g_Y = 0$ (absence of kinetic mixing), but is approximately well verified by experimentally allowed models, as we will see later. Making use of these parameters, we can write

$$\tan 2\theta' = 2\widetilde{g}_Y \frac{M^{0\,2}_Z}{M^{0\,2}_{Z'} - M^{0\,2}_Z} \,, \tag{2.43}$$

and

$$M^2_{Z',Z} = M^{0\,2}_Z \frac{1}{2}\left[\left(1 + \frac{M^{0\,2}_{Z'}}{M^{0\,2}_Z}\right) \pm \sqrt{\left(1 - \frac{M^{0\,2}_{Z'}}{M^{0\,2}_Z}\right)^2 + 4\widetilde{g}^2_Y}\,\right] \,, \tag{2.44}$$

where $M_Z^{0\,2}$ is the squared mass of the Z without kinetic mixing, i.e. $M_Z^{0\,2} = \frac{v^2}{4} g_Z^2$. We will see that electroweak precision tests (indeed, mainly LEP1 measurements at the Z pole) strongly constrain θ', requiring $|\theta'| \leq 10^{-3}$ approximately (see Section 2.2). From (2.43), we see that this condition can be satisfied provided either $\widetilde{g}_Y$ is small or $M_{Z'}^{0\,2}$ is large. Notice, however, that it is not necessary that *both* conditions hold.

Under (2.37), the kinetic terms remain diagonal, since ${D_i}^a$ is orthogonal. The currents transform in the following way:

$$A^a J_a = A^i J_i \,, \tag{2.45}$$

with

$$J_i = {D_i}^a J_a \,. \tag{2.46}$$

Explicitly:

$$\begin{pmatrix} J_\gamma \\ J_Z \\ J_{Z'} \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w & 0 \\ -\sin\theta_w\cos\theta' & \cos\theta_w\cos\theta' & \sin\theta' \\ \sin\theta_w\sin\theta' & -\sin\theta'\cos\theta_w & \cos\theta' \end{pmatrix} \begin{pmatrix} g' J_Y \\ g J_{T_{3L}} \\ g_Y J_Y + g_{BL} J_{BL} \end{pmatrix} \,, \tag{2.47}$$

therefore the interactions of the mass-eigenstate vector bosons with the currents can be rewritten in the following way:

$$e A^\gamma J_Q + A^Z(\cos\theta' J_{Z^0} + \sin\theta' J_{Z'^0}) + A^{Z'}(\cos\theta' J_{Z'^0} - \sin\theta' J_{Z^0}) \,, \tag{2.48}$$

where

$$J_Q = J_Y + J_{3L} = \sum_i Q^i \overline{\psi}_i \gamma^\mu \psi_i + i Q^H H^\dagger \overleftrightarrow{\partial^\mu} H \,, \tag{2.49}$$

$i = u_L, d_L, \nu_L, e_L, u_R, d_R, \nu_R, e_R$, is the electromagnetic current (Q^i are the electric charges), whereas

$$J_{Z'^0} = g_Z(\widetilde{g}_Y J_Y + \widetilde{g}_{BL} J_{BL}) \,, \tag{2.50}$$

and finally

$$J_{Z^0} = g\cos\theta_w J_{3L} - g'\sin\theta_w J_Y = \frac{g}{\cos\theta_w}(J_{3L} - \sin^2\theta_w J_Q) \tag{2.51}$$

is the current to which the Z couples in the SM. We name $(A^\gamma, A^{Z^0}, A^{Z'^0})$ the basis where the kinetic terms are canonical, the weak rotation θ_w has been performed, but there is still in general mass mixing between A^{Z^0} and $A^{Z'^0}$.

2.1.4. Fermion masses

With the introduction of RH neutrinos, the Yukawa couplings read (flavour indices are understood)

$$\mathcal{L}_Y = -\lambda_u \overline{q_L} \widetilde{H} u_R - \lambda_d \overline{q_L} H d_R - \lambda_e \overline{l_L} H e_R - \lambda_\nu \overline{l_L} \widetilde{H} \nu_R + h.c. \,, \tag{2.52}$$

where we recall that $\widetilde{H} \equiv i\sigma^2 H^*$. It is a straightforward exercise to verify that all the terms in (2.52) are invariant under both Y and $B-L$, thus they are allowed for the most generic non-anomalous $U(1)_X$ charge assignment, i.e. for $X_i = aY_i + b(B-L)_i$. Alternatively, we can use directly the anomaly cancellation conditions (2.16)-(2.19) to prove the invariance of $\mathcal{L}_Y$ under $U(1)_X$. Therefore, Dirac masses for all fermions, including ν_R, are generated.

Neutrino Masses

After electroweak symmetry breaking, the Yukawa sector generates Dirac mass terms for neutrinos:

$$-(\lambda_\nu)_{mn}\frac{v}{\sqrt{2}}\overline{\nu_{Lm}}\nu_{Rn} + h.c. = -(m_D)_{mn}\overline{\nu_{Lm}}\nu_{Rn} + h.c. \; . \tag{2.53}$$

In principle, this could be enough to account for neutrino masses; however, the extremely small values of the entries of λ_ν required to obtain $m_\nu \leq 1\,\mathrm{eV}$, in agreement with experimental data, suggest us to consider an alternative. Provided $Q^\phi_{BL} = 2$, the following additional Yukawa terms are gauge invariant and thus allowed:

$$\hat{\mathcal{L}}_Y = -(\lambda_M)_{mn}\phi\overline{\nu^c_{Lm}}\nu_{Rn} + h.c. \, , \tag{2.54}$$

which generate, after ϕ gets a VEV, Majorana mass terms for ν_R:

$$\hat{\mathcal{L}}_Y = -(\lambda_M)_{mn}\frac{v_{BL}}{\sqrt{2}}\overline{\nu^c_{Lm}}\nu_{Rn} + h.c. = -\frac{M_{mn}}{2}\overline{\nu^c_{Lm}}\nu_{Rn} + h.c. \; . \tag{2.55}$$

As a result, a see-saw mechanism is produced, which yields, as described in Chapter 1, three light and three heavy neutrinos, with masses respectively given by

$$M_l \approx m_D M^{-1} m_D^T \, , \tag{2.56}$$

$$M_h \approx M \, . \tag{2.57}$$

To estimate the order of magnitude of M needed to obtain viable neutrino masses, we can take the one-family case. Requiring $M_l \leq 1\,\mathrm{eV}$, and setting $m_D = O(100\,\mathrm{GeV})$ (i.e. $\lambda_\nu = O(1)$), we get

$$M \geq 10^{13}\,\mathrm{GeV} \; . \tag{2.58}$$

However, as already noticed in in Sect. 1.2, λ_ν could be significantly smaller than 1, thus lowering the scale of M: in particular, if we take $\lambda_\nu = O(10^{-5})$ (strength similar to that of the electron Yukawa coupling λ_e), then the lower bound on M becomes

$$M \geq 10^{3}\,\mathrm{GeV} \; . \tag{2.59}$$

Since $M = \sqrt{2}\lambda_M v_{BL}$, taking the natural scale for v_{BL}, namely $v_{BL} \sim 1\,\mathrm{TeV}$, leads to conclude that λ_M can be of $O(1)$ in this scenario (see [32] for a detailed discussion of see-saw in a supersymmetric version of this model).

2.1.5. Higgs sector

Making use of the unitarity gauge, i.e.

$$H = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix} , \qquad \phi = \frac{v_{BL} + h'(x)}{\sqrt{2}} \, , \tag{2.60}$$

with h, h' real scalar fields, we get the different interactions containing the Higgs fields.

Gauge interactions

In the basis where kinetic mixing is still present, we obtain

$$
\begin{aligned}
(D^\mu H)^\dagger(D_\mu H)+(D^\mu\phi)^\dagger(D_\mu\phi) = {}& \frac{1}{2}\partial_\mu h\partial^\mu h+\frac{1}{8}(h+v)^2\left[|A^{1L}+iA^{2L}|^2+(A^{3L}-A^Y)^2\right] \\
&+\frac{1}{2}\partial_\mu h'\partial^\mu h'+\frac{1}{2}(h'+v_{BL})^2 Q_{BL}^{\phi\,2}A^{BL\,2}\,.
\end{aligned}
$$

Dropping terms containing A^{1L} and A^{2L}, which remain exactly as in the SM, we make the transformation (2.22), obtaining in the basis where kinetic terms for gauge bosons are canonical:

$$
\frac{1}{2}\partial_\mu h\partial^\mu h+\frac{1}{8}(h+v)^2\left[gA^{3L}-g'A^Y-g_YA^{Z'0}\right]^2+\frac{1}{2}\partial_\mu h'\partial^\mu h'+\frac{1}{2}(h'+v_{BL})^2 g_{BL}^2 Q_{BL}^{\phi\,2}A^{Z'0\,2}\,.
$$

Finally, the transformation (2.37) allows us to write in the basis of mass eigenstates for vectors:

$$
\begin{aligned}
&\frac{1}{2}\partial_\mu h\partial^\mu h+\frac{1}{8}(h+v)^2\left[A^Z(c_{\theta'}g_Z-g_Ys_{\theta'})-A^{Z'}(s_{\theta'}g_Z+g_Yc_{\theta'})\right]^2 \\
&\qquad+\frac{1}{2}\partial_\mu h'\partial^\mu h'+\frac{1}{2}(h'+v_{BL})^2 g_{BL}^2 Q_{BL}^{\phi\,2}\left[s_{\theta'}A^Z+c_{\theta'}A^{Z'}\right]^2\,,
\end{aligned}
\tag{2.61}
$$

($c_{\theta'}\equiv\cos\theta', s_{\theta'}\equiv\sin\theta'$). Notice that the non-diagonal Z-Z' mass terms in (2.61) must vanish, because we are in the mass eigenstates basis: it is an easy exercise to show that indeed

$$
-\frac{1}{4}v^2(c_{\theta'}g_Z-g_Ys_{\theta'})(s_{\theta'}g_Z+g_Yc_{\theta'})+g_{BL}^2v_{BL}^2Q_{BL}^{\phi\,2}s_{\theta'}c_{\theta'}=0\,,
\tag{2.62}
$$

thus cross-checking the correctness of the computation.

Some remarks are in order. First, we see that the photon does not couple to the Higgses, as it must be. Next, we have trilinear interactions $h\,Z\,Z'$ and $h'\,Z\,Z'$, as well as quartic vertices $h^2Z\,Z'$ and $h'^2Z\,Z'$. All these interactions could lead to tree level decays of the Z', provided the corresponding processes are kinematically allowed. We will assume that h' is very heavy, so its interactions and mixing with the other fields can be neglected. Nevertheless, the terms involving h remain, therefore the decays $Z'\to Z\,h$ and $Z'\to Z\,h\,h$ should be taken into account when computing the width of the Z'. We will see in Section 2.4 that the decay $Z'\to Z\,h$, even though suppressed by the small Z-Z' mixing, has a branching ratio of the order of 1 percent (the actual value obviously depends on the values of the parameters $\widetilde{g}_Y,\widetilde{g}_{BL},M_{Z'}$).

Yukawa interactions

The full Yukawa part of the Lagrangian reads (assuming $Q_{BL}^\phi=2$):

$$
\mathcal{L}_Y+\hat{\mathcal{L}}_Y=-\lambda_u\overline{q_L}\widetilde{H}u_R-\lambda_d\overline{q_L}Hd_R-\lambda_e\overline{l_L}He_R-\lambda_\nu\overline{l_L}\widetilde{H}\nu_R-\lambda_M\phi\overline{\nu_L^c}\nu_R+h.c.\,,
\tag{2.63}
$$

which, in the unitarity gauge, becomes

$$\mathcal{L}_Y + \hat{\mathcal{L}}_Y = -\lambda_u \overline{u_L}\left(\frac{v+h}{\sqrt{2}}\right) u_R - \lambda_d \overline{d_L}\left(\frac{v+h}{\sqrt{2}}\right) d_R - \lambda_e \overline{e_L}\left(\frac{v+h}{\sqrt{2}}\right) e_R \\ - \lambda_\nu \overline{\nu_L}\left(\frac{v+h}{\sqrt{2}}\right) \nu_R - \lambda_M \overline{\nu_L^c}\left(\frac{v_{BL}+h'}{\sqrt{2}}\right) \nu_R + h.c.\,, \quad (2.64)$$

where all flavour indices are understood.

Scalar Potential

To be general, we do not assume a specific form for the scalar potential. We only require that its minimization realizes the spontaneous breakings of $SU(2)\times U(1)_Y \to U(1)_{em}$ at the electroweak scale v, and of $U(1)_{B-L}$ at the scale v_{BL}.

2.2. Constraints from electroweak precision data

In this section we compute the constraints on the parameter space of our model put by electroweak precision data, by means of the procedure proposed in [13], and summarized in Section 1.3. We need to compute the 9 pseudo-observables

$$\widehat{S},\, \widehat{T}\,, \widehat{U},\, W,\, Y\,, V\,, X\,, \delta\epsilon_q,\, \delta C_q \quad (2.65)$$

in our model. In the basis where the kinetic terms are diagonal, but there is mass mixing among the fields A^a, $(a = Y, T_{3L}, Z'^0)$, we can write [13]:

$$\widehat{S} = \frac{2M_W^2 g_Z^2}{g^2 g'^2 M_{Z'}^{0\,2}} \left(Z^{e^c} - Z^H + Z^l\right)\left(g^2 Z^{e^c} + g'^2 (Z^{e^c} + 2Z^l)\right)\,, \quad (2.66)$$

$$\widehat{T} = \frac{4M_W^2 g_Z^2}{g^2 M_{Z'}^{0\,2}} \left(Z^{e^c} - Z^H + Z^l\right)^2\,, \quad (2.67)$$

$$\widehat{U} = \frac{4M_W^2 g_Z^2}{g^2 M_{Z'}^{0\,2}} \left(Z^{e^c} - Z^H + Z^l\right)\left(Z^{e^c} + 2Z^l\right)\,, \quad (2.68)$$

$$W = \frac{M_W^2 g_Z^2}{g^2 M_{Z'}^{0\,2}} \left(Z^{e^c} + 2Z^l\right)^2\,, \quad (2.69)$$

$$Y = \frac{M_W^2 g_Z^2}{g'^2 M_{Z'}^{0\,2}} \left(Z^{e^c}\right)^2\,, \quad (2.70)$$

$$V = \frac{M_W^2 g_Z^2}{g^2 M_{Z'}^{0\,2}} \left(Z^{e^c} + 2Z^l\right)^2 = W\,, \quad (2.71)$$

$$X = -\frac{M_W^2 g_Z^2}{g' g M_{Z'}^{0\,2}} Z^{e^c}\left(Z^{e^c} + 2Z^l\right)\,, \quad (2.72)$$

$$\delta\epsilon_q = \frac{2M_W^2 g_Z^2}{g^2 M_{Z'}^{0\,2}} Z^H \left(Z^{e^c} + 2Z^l\right)\,, \quad (2.73)$$

$$\delta C_q = \frac{2M_W^2 g_Z^2}{(g^2 + g'^2) M_{Z'}^{0\,2}} \left(Z^{e^c} + 2Z^l\right)\left(Z^{e^c} + Z^l\right)\,, \quad (2.74)$$

where $M_W^2 = g^2v^2/2$, and we recall the definition of $M_{Z'}^{0\,2}$ given in eq. (2.42). Expressing eqs. (2.66) - (2.74) in terms of the parameters $\widetilde{g}_Y$, $\widetilde{g}_{BL}$, $M_{Z'}^{0\,2}$ gives

$$\widehat{S} = 0\,, \tag{2.75}$$

$$\widehat{T} = 0\,, \tag{2.76}$$

$$\widehat{U} = 0\,, \tag{2.77}$$

$$W = \frac{M_W^2}{M_{Z'}^{0\,2}}\frac{g_Z^2}{g^2}\widetilde{g}_{BL}^2\,, \tag{2.78}$$

$$Y = \frac{M_W^2}{M_{Z'}^{0\,2}}\frac{g_Z^2}{g'^2}(\widetilde{g}_Y + \widetilde{g}_{BL})^2\,, \tag{2.79}$$

$$V = \frac{M_W^2}{M_{Z'}^{0\,2}}\frac{g_Z^2}{g^2}\widetilde{g}_{BL}^2 = W\,, \tag{2.80}$$

$$X = \frac{M_W^2}{M_{Z'}^{0\,2}}\frac{g_Z^2}{gg'}\widetilde{g}_{BL}(\widetilde{g}_Y + \widetilde{g}_{BL})\,, \tag{2.81}$$

$$\delta\epsilon_q = -\frac{M_W^2}{M_{Z'}^{0\,2}}\frac{g_Z^2}{g^2}\widetilde{g}_Y\,\widetilde{g}_{BL}\,, \tag{2.82}$$

$$\delta C_q = -\frac{M_W^2}{M_{Z'}^{0\,2}}\widetilde{g}_Y\,\widetilde{g}_{BL}\,. \tag{2.83}$$

Notice the symmetry under the simultaneous reflection $(\widetilde{g}_Y, \widetilde{g}_{BL}) \to (-\widetilde{g}_Y, -\widetilde{g}_{BL})$. Our model has then four parameters: three describing the Z', namely $\widetilde{g}_Y$, $\widetilde{g}_{BL}$, $M_{Z'}^{0\,2}$, and the Higgs mass m_h. To extract bounds on these parameters, we perform a least χ^2 fit: eq. (1.84) can be written as

$$\widetilde{R}_{ij}P_j = H_i + x_i \pm \sigma_i \tag{2.84}$$

where $\vec{P}^{\,T} = (\widehat{S}(M_{Z'}^{0\,2}, \widetilde{g}_Y, \widetilde{g}_{BL}), \dots, \delta\epsilon_q(M_{Z'}^{0\,2}, \widetilde{g}_Y, \widetilde{g}_{BL}))$, $\vec{H}^{\,T} = (0.54\,l, \dots, 0.66\,l)$, $\vec{x}^{\,T} = (-0.04, \dots, -26)$ and $\vec{\sigma}^{\,T} = (0.21, \dots, 18)$. We recall that $l = \log(m_h/M_Z)$ encodes the approximate dependence on the Higgs mass. Therefore, we form the χ^2

$$\chi^2 = \sum_{k=1}^{10}\frac{[\widetilde{R}_{kj}P_j - H_k - x_k]^2}{\sigma_k^2}\,, \tag{2.85}$$

and minimize it with respect to the model parameters.

2.2.1. χ^2 minimization

First of all, we notice that eqs. (2.75) - (2.83) depend on only three parameters, namely m_h, $\widetilde{R}_Y = \widetilde{g}_Y/M_{Z'}^0$ and $\widetilde{R}_{BL} = \widetilde{g}_{BL}/M_{Z'}^0$, as noticed also in [33]. Then we proceed as follows: we set $m_h = 120\,\mathrm{GeV}$ (slightly above the lower LEP bound), and perform fits with the remaining two parameters left free, for $10-2=8$ degrees of freedom. The results are as follows: the minimum of χ^2 is $\chi^2_{min} = 12.49$, with an associated probability $P_8(\chi^2 > 12.49) \approx 13\%$, while the minimizing values of the parameters are $\widetilde{R}_{Y\,min} = -1.1\cdot 10^{-4}\,\mathrm{GeV}^{-1}$, $\widetilde{R}_{BL\,min} = 0.58\cdot 10^{-4}\,\mathrm{GeV}^{-1}$.

We choose some reference values of $M_{Z'}^0$, and draw isocontours in the $(\widetilde{g}_Y, \widetilde{g}_{BL})$ plane for each of these values, making use of the well-known correspondence between $\Delta\chi^2$ and

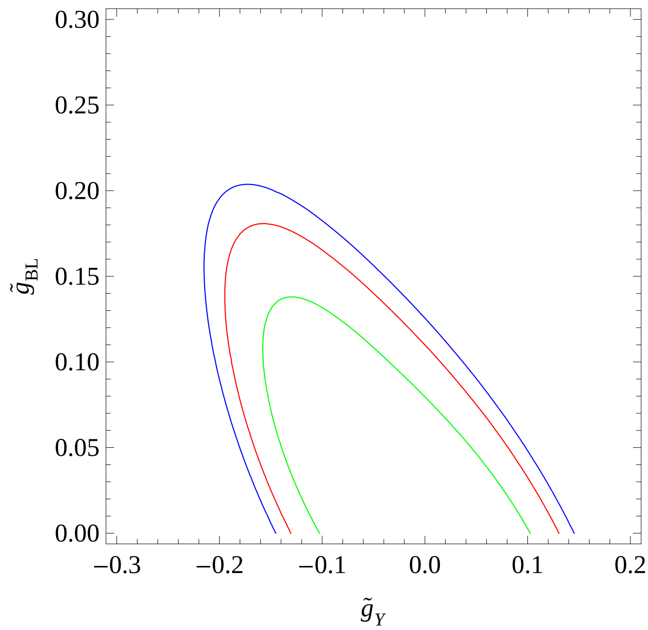

Figure 2.1.: Exclusion contours at 68.27, 95, 99% CL in the $(\widetilde{g}_Y, \widetilde{g}_{BL})$ plane, for $M_{Z'} = 600$ GeV and $m_h = 120$ GeV. The excluded regions lie outside each curve.

probability, see Table 2.2. The contours are symmetric under simultaneous reflection of $\widetilde{g}_Y$ and $\widetilde{g}_{BL}$, as expected (but not shown in the plots, which are for $\widetilde{g}_{BL} > 0$ only). Fig. 2.1 shows, as an example, 68.27, 95 and 99% confidence level contours for $M^0_{Z'} = 600$ GeV, while in Fig. 2.2 we report 95% CL bounds for a set of values of $M^0_{Z'}$.

$CL \backslash N$	1	2	3
68.27%	1.00	2.30	3.53
84%	1.97	3.67	5.17
95%	3.84	5.99	7.82
95.45%	4.00	6.17	8.02
99%	6.63	9.21	11.34
99.73%	9.00	11.83	14.16

Table 2.2.: Values of $\Delta\chi^2$ corresponding to various values of the number of parameters N (columns) and different confidence levels (rows).

2.2.2. Constraints on the mixing angle

We recall the expression of the Z-Z' mixing angle:

$$\tan 2\theta' = 2\widetilde{g}_Y \frac{M_Z^{0\,2}}{M_{Z'}^{0\,2} - M_Z^{0\,2}}\,.$$

From Fig. 2.2, we can read off upper bounds on $|\theta'|$: they are collected in Table 2.3. The strong

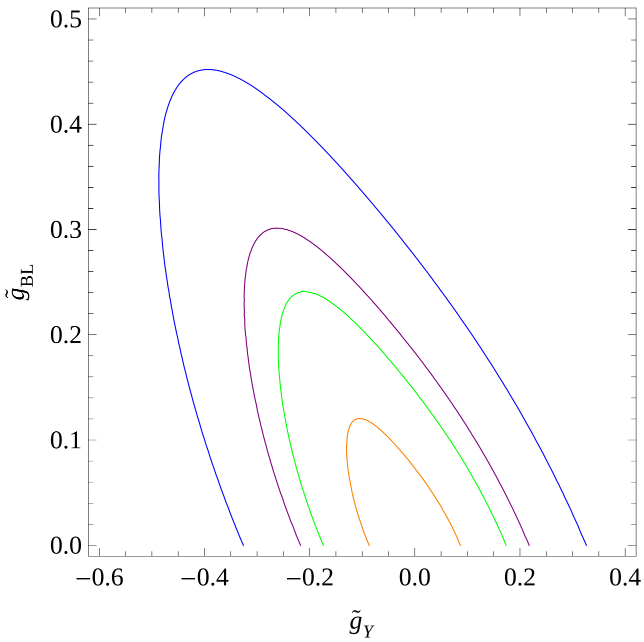

Figure 2.2.: Exclusion contours at 95% CL in the $(\widetilde{g}_Y, \widetilde{g}_{BL})$ plane for $M_{Z'} = 400, 800, 1000, 1500$ GeV, and $m_h = 120$ GeV. The excluded regions lie outside each curve.

$M^0_{Z'}$(GeV)	$\lvert\theta'\rvert \leq$
400	$7.1 \cdot 10^{-3}$
800	$3.4 \cdot 10^{-3}$
1000	$2.7 \cdot 10^{-3}$
1500	$1.8 \cdot 10^{-3}$

Table 2.3.: Upper bound on the absolute value of the Z-Z' mixing angle θ', for different values of $M^0_{Z'}$, obtained from the 95% CL bounds in the $(\widetilde{g}_Y, \widetilde{g}_{BL})$ plane of Fig. 2.2.

constraints obtained on θ' are a relevant result, showing that the approximation $M^0_{Z'} \approx M_{Z'}$ can be safely adopted.

2.2.3. Constraints on Z'_χ

Z'_χ is a very well known GUT Z', which occurs in the breaking $SO(10) \to SU(5)\times U(1)_\chi$. The generator of $U(1)_\chi$ is a combination of Y and $B-L$, therefore the model can be described in our framework; actually, the generator of $U(1)_\chi$ is orthogonal to the hypercharge generator on the **16** representation of $SO(10)$. In a basis where the kinetic terms are canonical (but there is in general Z-Z' mass mixing, see [18]), the couplings to the Z' are given by $g_{Z'}Q_\chi$, where

$$Q_\chi = \frac{1}{2\sqrt{10}}(4Y - 5Q_{BL})\,. \tag{2.86}$$

We perform a fit with one free parameter, $g_{Z'}/M^0_{Z'}$, obtaining the excluded region shown in Fig. 2.3. The overall coupling constant is often normalized as follows in the literature:

$$g_{Z'} = \sqrt{\frac{5}{3}}g'\,, \tag{2.87}$$

therefore from

$$\sqrt{\frac{5}{3}}g'\,Q_\chi = g_Y Y + g_{BL}Q_{BL}, \tag{2.88}$$

we extract

$$g_{BL} = -\frac{5}{2\sqrt{10}}\sqrt{\frac{5}{3}}g' = -0.364 \quad , \quad g_Y = \frac{2}{\sqrt{10}}\sqrt{\frac{5}{3}}g' = 0.291\,. \tag{2.89}$$

From Fig. 2.3 we can read out the lower bound on $M^0_{Z'}$ for this special value of the coupling $g_{Z'}$: we get $M^0_{Z'} \geq 1.85\,\text{TeV}$ at 95% CL. We recall that $M^0_{Z'}$ is not the mass eigenvalue; we can check however that the mixing angle θ' is small: from (2.43) we obtain at 95% CL

$$|\theta'| \leq 9.5 \cdot 10^{-4}\,, \tag{2.90}$$

which proves that the bound on $M^0_{Z'}$ discussed above is actually a bound on the physical mass of the Z'_χ.

2.2.4. Constraints on the pure $B-L$ model

Setting $g_Y = 0$, we have the *pure* $B-L$ model, where kinetic mixing is absent. Performing a fit with R_{BL} as a free parameter gives at 99% CL

$$\frac{M_{Z'_{B-L}}}{g_{BL}} > 7.1\,\text{TeV}\,, \tag{2.91}$$

in agreement with the result of [13]. In Fig. 2.4 we plot the corresponding bound on g_{BL}/g' as a function of $M_{Z'_{B-L}}$.

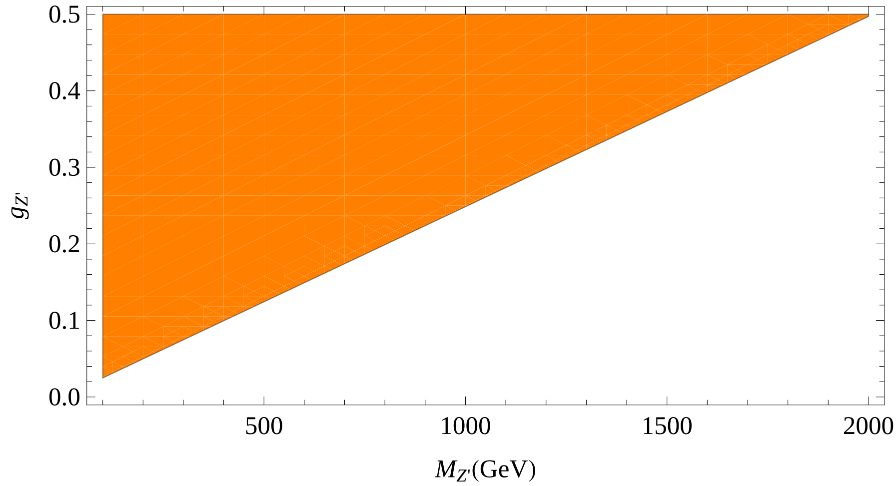

Figure 2.3.: Region of the parameter space of Z_χ excluded at 95% CL by electroweak precision tests.

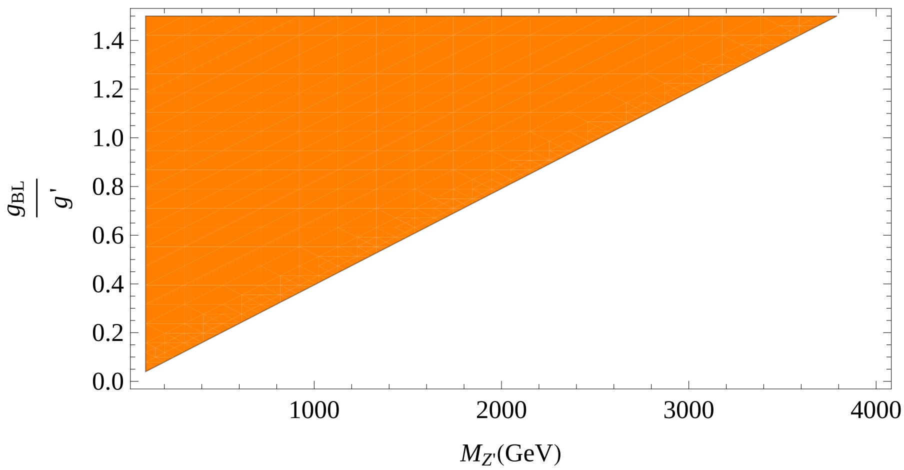

Figure 2.4.: Upper bound at 99% CL on g_{BL}/g' vs $M_{Z'_{B-L}}$ for the pure $B-L$ model. Comparison with a similar plot contained in [34] shows that our bound is stronger. We believe this could be due to the fact that in [34] a two-parameter fit $(g_{BL}, M^0_{Z'})$ was performed, which gives a less stringent limit.

2.2.5. Consistency checks

SM limit

We can check the consistency of our results by taking the limit $\widetilde{g}_Y, \widetilde{g}_{BL} \to 0, M^{0\,2}_{Z'} \to \infty$ and fitting the observables in (2.84) using only m_h as a free parameter. We obtain, for a minimizing value $m_h = 82.0$ GeV, $\chi^2_{min} = 11.31$, with an associated probability, for 9 degrees of freedom, $P_9(\chi^2 > 11.31) \approx 25\%$. The $\Delta\chi^2 = 1$ limits are ($\Delta\chi^2 = 1$ corresponds to 1σ for 1 parameter, see Table 2.2)

$$m_h = 82.0^{+32.8}_{-23.4}\,\text{GeV} \,. \tag{2.92}$$

We see that our result is in good agreement with the 'standard' fit to the SM performed in [11], which we discussed in Section 1.3.

Top mass

The results of the fit depend on the value of the top mass: the value used in [13] is $m_t = 172.5 \pm 2.3\,\text{GeV}$ (see [35]), while the present best value is $m_t = 173.1 \pm 1.3\,\text{GeV}$ [36]. This small difference is expected to have a marginal effect on our results.

Dependence on the Higgs mass

Throughout our computation of bounds from electroweak precision tests, we have assumed a Higgs mass $m_h = 120\,\text{GeV}$, just above the experimental bound set by LEP2. It may therefore be interesting to check how much our results depend on m_h. To this aim, we collect in Table 2.4 the results of the fit for a Higgs mass ranging from 120 to 400 GeV. Fig. 2.5 displays

m_h(GeV)	χ^2_{min}	$P_8(\chi^2 > \chi^2_{min})$	$\widetilde{R}_{BL\,min}(10^{-4}\,\text{GeV}^{-1})$	$\widetilde{R}_Y(10^{-4}\,\text{GeV}^{-1})$
120	12.49	~ 0.13	0.58	-1.1
140	13.40	~ 0.10	0.76	-1.5
165	14.56	~ 0.07	0.90	-1.9
200	16.17	~ 0.04	1.0	-2.2
300	20.43	~ 0.01	1.2	-2.8
400	24.16	~ 0.004	1.4	-3.2

Table 2.4.: Dependence of the results of χ^2 minimization on the Higgs mass.

95% CL contours in the $(\widetilde{R}_Y, \widetilde{R}_{BL})$ plane, for m_h = 120, 140, 165, 200, 300, 400 GeV. For aesthetical reasons only, we display the full plane instead of the upper half $\widetilde{R}_{BL} > 0$ (the contours are invariant under $(\widetilde{R}_Y, \widetilde{R}_{BL}) \to (-\widetilde{R}_Y, -\widetilde{R}_{BL})$, as already mentioned). We see that varying m_h in the range allowed by bounds from EWPTs ($m_h < 200\,\text{GeV}$ approximately) only produces a small shift in the contours. This is expected, due to the logarithmic dependence on m_h of eq. (2.84).

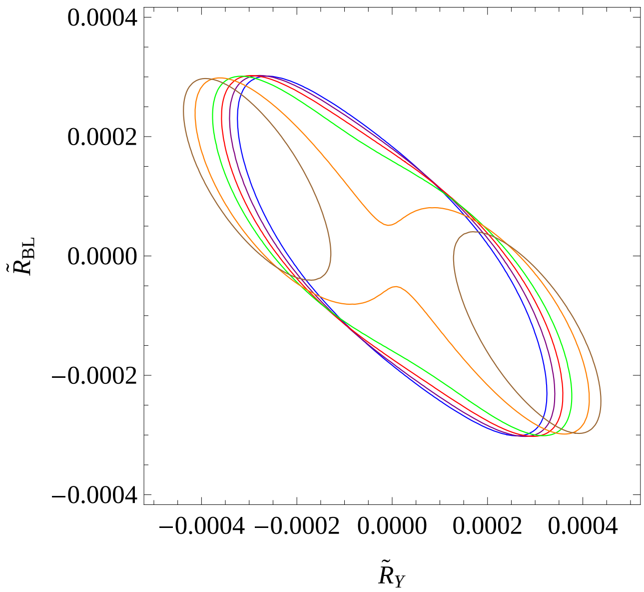

Figure 2.5.: 95% CL contours in the $(\widetilde{R}_Y, \widetilde{R}_{BL})$ plane, for $m_H = 120 \div 400\,\mathrm{GeV}$.

2.3. More constraints from precision data: atomic parity violation

In Chapter 1 we mentioned the measurement of the weak charge Q_W of ^{133}Cs atoms as one of the precision tests of the SM. Indeed, Q_W was included in the data set used for the fit performed in [13]. Recently, however, a re-analysis [10] of the most precise measurement [9] of the weak charge of Cs has been performed, reducing the theoretical uncertainty (in fact, the value of Q_W is not directly measured, rather it is extracted through a theoretical computation from the data). The authors of [10] obtained the result $Q_W(Cs) = -73.16 \pm 0.29\,(exp) \pm 0.20\,(th)$: the theoretical uncertainty is halved with respect to previous computations. The SM value is $Q_W(Cs) = -73.16 \pm 0.03$, thus perfect agreement exists between theory and experiment. To compute bounds from this measurement on our minimal model, we make use of the results of [37], where the additional contribution to Q_W due to the presence of a Z' was computed to be

$$\delta Q_W = Q_W - Q_W^{SM} = 16\left\{\frac{1}{16}\left[\left(1 + 4\frac{\sin^4\theta_w}{\cos(2\theta_w)}\right)Z - N\right]\Delta\rho_M - \left[(2Z+N)(a_e v'_u + a'_e v_u)\right.\right.$$
$$\left.\left. +(Z+2N)(a_e v'_d + a'_e v_d)\right]\theta' + \left[(2Z+N)a'_e v'_u + (Z+2N)a'_e v'_d\right]\frac{M_Z^2}{M_{Z'}^2}\right\}\,, \quad (2.93)$$

where

$$\Delta\rho_M = \sin^2\theta'\left(\frac{M_{Z'}^2}{M_Z^2} - 1\right)\,, \quad (2.94)$$

and (v_f, a_f) and (v'_f, a'_f), $(f = e, u, d)$ are the vector and axial couplings of the Dirac fermion f to the Z^0 and Z'^0 respectively, defined by the interaction terms in the $(A^\gamma, A^{Z^0}, A^{Z'^0})$ basis[3]

$$A_\mu^{Z^0} \frac{g}{\cos\theta_w} \sum_{f=e,u,d} (v_f \overline{f}\gamma^\mu f + a_f \overline{f}\gamma^\mu\gamma_5 f)\,, \tag{2.95}$$

$$A_\mu^{Z'^0} \frac{g'}{\sin\theta_w} \sum_{f=e,u,d} (v'_f \overline{f}\gamma^\mu f + a'_f \overline{f}\gamma^\mu\gamma_5 f)\,. \tag{2.96}$$

Eq. (2.93) is obtained keeping only terms of first order in θ' and $M_Z^2/M_{Z'}^2$. This formula allows us to compute the region in the $(\widetilde{g}_Y, \widetilde{g}_{BL})$ plane which is not ruled out by the precise determination of Q_W made in [9, 10]. Fig. 2.11 compares the bounds from EWPTs, atomic parity violation (APV) and Tevatron direct searches (discussed in the next section) for three different Z' masses, namely 400, 800 and 1000 GeV. We see that bounds from APV can be stronger than Tevatron bounds, but are always weaker than limits from EWPTs. Still, a future fit to electroweak precision data will have to consider this new determination of Q_W, including its effects in the framework we used in Section 2.2.

To conclude, we note that the authors of [10] have set a lower bound on the mass of the GUT-motivated Z'_χ: $M_{Z'_\chi} > 1.3\,\text{TeV}$ at 84% CL. However, we believe they actually consider Z'_χ in absence of mass mixing (that is, setting $\theta' = 0$: then, only the last term in eq. (2.93) survives, giving $\delta Q_W \approx 84(M_W^2/M_{Z'}^2)$ for the χ model), therefore the correct limit should be $M_{Z'_\chi} > 1.05\,\text{TeV}$ at 84% CL (and $M_{Z'_\chi} > 0.89\,\text{TeV}$ at 95 % CL, in agreement with the result quoted in [38]). If mixing is included, we find $M_{Z'_\chi} > 1.42\,\text{TeV}$ at 84% CL, and $M_{Z'_\chi} > 1.20\,\text{TeV}$ at 95% CL. As can be seen from Fig. 2.11, the pure $B - L$ model is not constrained at all by APV, because $g_Y = 0$ implies $\theta' = 0$, so only the last term in (2.93) remains, but this term vanishes too, because if $g_Y = 0$ then the Z' has only vector-like interactions, therefore $a'_e = 0$ and the predicted δQ_W is zero.

2.4. Constraints from Tevatron direct searches

Direct searches performed at the Tevatron in the e^+e^- and $\mu^+\mu^-$ channels set upper bounds on $\sigma(Z') \times Br(Z' \to l^+l^-)$, with $l = e$, μ (see [23, 24, 25] for the most recent results). The CDF and D0 collaborations focus mainly on the above leptonic channels, as these are much 'cleaner' than $\tau^+\tau^-$ and the hadronic ones, due to low backgrounds and to good momentum resolution.

2.4.1. Cross section parameterization

Following [39], if the Z' is narrow enough, the interference with the photon and with the Z can be neglected, thus obtaining the cross section for Drell-Yan production of a Z' and its subsequent decay into a pair of charged leptons:

$$\sigma(p\overline{p} \to Z'X \to l^+l^-) = \frac{\pi}{6s}\left[c_u\, w_u(s, M_{Z'}^2) + c_d\, w_d(s, M_{Z'}^2)\right]\,, \tag{2.97}$$

[3]For the sake of clarity, we recall that in this basis the kinetic terms are canonical, the photon has been 'decoupled' via the weak rotation, and there is still in general mass mixing between Z^0 and Z'^0. The factorization of $g'/\sin\theta_w$ in (2.96) is done to agree with the notation of [37].

at next to leading order (NLO) in QCD [4]. In eq. (2.97), $c_{u,d}$ are given by

$$c_{u,d} = (A^{u_L, d_L\,2} + A^{u_R, d_R\,2})\, Br(Z' \to l^+ l^-)\,, \tag{2.98}$$

where l is a charged lepton, $l = e, \mu, \tau$, and the couplings A^f read

$$A^f = \cos\theta' g_Z(\widetilde{g}_Y Y^f + \widetilde{g}_{BL} Q^f_{BL}) - \sin\theta' \frac{g}{\cos\theta_w}(T^f_{3L} - \sin^2\theta_w Q^f)\,. \tag{2.99}$$

The structure functions $w_{u,d}(s, M^2_{Z'})$ read, at NLO in the $\overline{MS}$ scheme:

$$\begin{aligned} w_{u(d)} = \sum_{Q=u,c(d,s,b)} \int_0^1 dx_1 \int_0^1 dx_2 \int_\tau^1 dz \Big\{ & f_{Q/P}(x_1, M^2_{Z'}) f_{\overline{Q}/\overline{P}}(x_2, M^2_{Z'}) \Delta_{qq}(z, M^2_{Z'}) + \\ & f_{g/P}(x_1, M^2_{Z'}) \left[f_{Q/\overline{P}}(x_2, M^2_{Z'}) + f_{\overline{Q}/\overline{P}}(x_2, M^2_{Z'}) \right] \Delta_{gq}(z, M^2_{Z'}) \\ & + (x_1 \leftrightarrow x_2, P \leftrightarrow \overline{P}) \Big\} \delta\,(\tau - z x_1 x_2)\,, \end{aligned} \tag{2.100}$$

where $\tau = M^2_{Z'}/s$, and $f_{Q/P}\,, f_{Q/\overline{P}}$ are the parton distribution functions (PDFs) of quark Q in the proton and in the antiproton, respectively. The renormalization scale for the PDFs is set to $M_{Z'}$. The explicit expressions for Δ_{qq} and Δ_{gq} read

$$\begin{aligned} \Delta_{qq}(z, M^2_{Z'}) = \delta(1-z) + \frac{4}{3}\frac{\alpha_S(M^2_{Z'})}{\pi} \Bigg[& \delta(1-z)\left(\frac{\pi^2}{3} - 4\right) + 4\left(\frac{\log(1-z)}{1-z}\right)_+ \\ & - 2(1+z)\log(1-z) - \frac{1+z^2}{1-z}\log z \Bigg]\,, \end{aligned} \tag{2.101}$$

$$\Delta_{gq}(z, M^2_{Z'}) = \frac{\alpha_S(M^2_{Z'})}{4\pi} \left[(1 - 2z + 2z^2)\log\frac{(1-z)^2}{z} + \frac{1}{2} + 3z - \frac{7}{2}z^2 \right] \tag{2.102}$$

(see Appendix C for details on the + prescription that appears in eq. (2.101)). The Feynman diagrams contributing to the process $\overline{p}p \to Z'$, up to NLO, are shown in Fig. 2.6 (see [41]).

Our analysis includes the following steps:

(1) We choose a value for $M_{Z'}$, and compute w_u, w_d. We make use of the most recent set of MSTW PDFs at NLO [42]. In this way we obtain the theoretical cross section (2.97) as a function of the parameters c_u, c_d.

(2) The CDF and D0 collaborations have set upper limits on $\sigma \times Br(Z' \to l^+ l^-)$, $(l = e, \mu)$. Making use of these limits, we draw excluded regions in the (c_d, c_u) plane. This is a model-independent result, because all the details of the model are contained in $c_{u,d}$. We compare our plots in the (c_d, c_u) plane with those obtained by the CDF collaboration in [43] and with those in [39] (both used preliminary results of Tevatron Run II), finding agreement. We update these plots, including the most recent analyses [23, 24, 25].

[4] Equation (2.97) differs from (3.8) of [39] for a factor 8 in the denominator; in [39] in fact, a factor 8 is reabsorbed in the definition of the PDFs, as pointed out in [40], page 13.

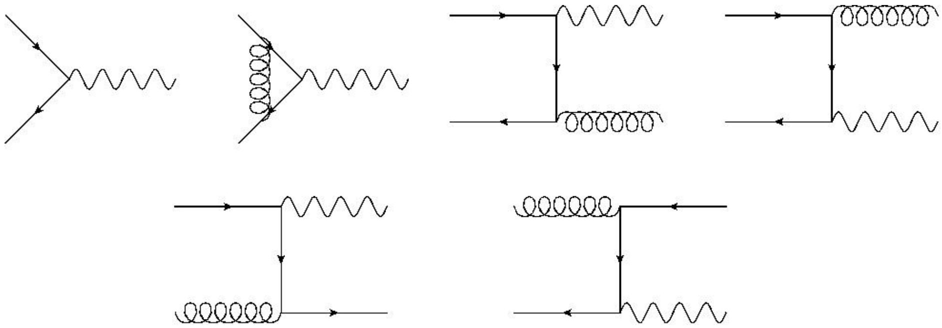

Figure 2.6.: Diagrams contributing to $\overline{p}p\,(pp) \to Z'$ at NLO in QCD.

(3) The dependence of the parameters c_u, c_d on $\widetilde{g}_Y, \widetilde{g}_{BL}$ (with $M_{Z'}$ fixed) can be easily computed in our model, therefore we are able to translate the bounds in the (c_d, c_u) plane into excluded regions in the $(\widetilde{g}_Y, \widetilde{g}_{BL})$ plane.

The next paragraph will be devoted to the computation of the coefficients $c_{u,\,d}$ in terms of $\widetilde{g}_Y, \widetilde{g}_{BL}$ and $M_{Z'}$.

2.4.2. Computation of $c_{u,\,d}$

The only nontrivial dependence of $c_{u,d}$ on $\widetilde{g}_Y, \widetilde{g}_{BL}, M_{Z'}$ is contained in $Br(Z' \to l^+l^-)$. Obviously,

$$Br(Z' \to l^+l^-) = \frac{\Gamma(Z' \to l^+l^-)}{\Gamma_{Z'}}\,, \tag{2.103}$$

where $\Gamma_{Z'}$ is the total width of the Z'. To compute the tree level total width of the Z', $\Gamma_{Z'}$, we must consider three contributions:

(i) decays into SM fermions;

(ii) decays into (mostly) RH neutrinos;

(iii) other decays, such as the two-body decays $Z' \to Z\,h$ and $Z' \to W^+W^-$, and the three-body decays $Z' \to Z\,h\,h$, $Z' \to Z\,W^+W^-$ and $Z' \to \gamma\,W^+W^-$ [5].

In what follows, we will neglect the decays in (ii), assuming that the decay channel $Z' \to \overline{\nu_R}\nu_R$ is kinematically closed.

It is a tedious exercise to verify that the tree level partial width for decay into a fermion-antifermion pair is

$$\Gamma(Z' \to \overline{f}f) = \frac{M_{Z'}}{12\pi} C_f \sqrt{1 - \frac{4m_f^2}{M_{Z'}^2}}\left[(v_f^2 + a_f^2) + 2(v_f^2 - 2a_f^2)\frac{m_f^2}{M_{Z'}^2}\right] \tag{2.104}$$

$(f = u, d, \nu, e)$, where $f = f_L + f_R$ is a Dirac fermion, C_f is a colour factor, i.e. $C_f = 3$ if f is a quark and $C_f = 1$ if f is a lepton, and v_f, a_f are defined by the interaction vertex

$$\overline{f}\gamma^\mu(v_f + a_f\gamma_5)f\,Z'_\mu\,. \tag{2.105}$$

[5] The vertices $Z\,W^+W^-$, $Z'\,Z\,W^+W^-$ and $Z'\,\gamma\,W^+W^-$ are an effect of the Z-Z' mixing: eq. (1.33) holds in the $(A^\gamma, A^{Z^0}, A^{Z'^0})$ basis, and the rotation $A^{Z^0} = \cos\theta' A^Z - \sin\theta' A^{Z'}$ then generates these interactions.

The vector and axial couplings are given by

$$v_f = \frac{1}{2}(A^{f_L} + A^{f_R})\,, \tag{2.106}$$

$$a_f = \frac{1}{2}(A^{f_R} - A^{f_L})\,, \tag{2.107}$$

where A^f, the coupling of fermion f to the Z' mass eigenstate, has been defined in (2.99). If mass mixing is negligible ($\theta' = 0$), then $A^f = g_Z Z^f = g_Z(\widetilde{g}_Y Y^f + \widetilde{g}_{BL} Q^f_{BL})$. Equation (2.104) can be rewritten, in terms of A^{f_L}, A^{f_R}, in the following way:

$$\Gamma(Z' \to \overline{f}f) = \frac{M_{Z'} C_f}{24\pi} \sqrt{1 - \frac{4m_f^2}{M_{Z'}^2}} \left[(A^{f_L\,2} + A^{f_R\,2}) \left(1 - \frac{m_f^2}{M_{Z'}^2}\right) + 6\frac{m_f^2}{M_{Z'}^2} A^{f_L} A^{f_R} \right]. \tag{2.108}$$

For all fermions but the top quark and, possibly, RH neutrinos, the fermion mass can be neglected with respect to $M_{Z'}$: in this limit, (2.108) becomes

$$\Gamma(Z' \to \overline{f}f) = \frac{M_{Z'}}{24\pi} C_f \left(A^{f_L\,2} + A^{f_R\,2}\right). \tag{2.109}$$

Inclusion of the leading QCD corrections modifies the decay width into quarks in the following way:

$$\Gamma(Z' \to \overline{Q}Q) = \frac{M_{Z'}}{8\pi} \sqrt{1 - \frac{4m_Q^2}{M_{Z'}^2}} \left[(A^{Q_L\,2} + A^{Q_R\,2}) \left(1 - \frac{m_Q^2}{M_{Z'}^2}\right) + 6\frac{m_Q^2}{M_{Z'}^2} A^{Q_L} A^{Q_R} \right] \left(1 + \frac{\alpha_S}{\pi}\right). \tag{2.110}$$

where Q is a generic quark. The partial width for decay into a top-antitop pair is therefore (if $M_{Z'} > 2m_t$)

$$\Gamma(Z' \to \overline{t}t) = \frac{M_{Z'}}{8\pi} \sqrt{1 - \frac{4m_t^2}{M_{Z'}^2}} \left[(A^{u_L\,2} + A^{u_R\,2}) \left(1 - \frac{m_t^2}{M_{Z'}^2}\right) + 6\frac{m_t^2}{M_{Z'}^2} A^{u_L} A^{u_R} \right] \left(1 + \frac{\alpha_S}{\pi}\right), \tag{2.111}$$

which slightly differs from (2.13) of [39]. The decay width into a (mostly) LH neutrino-antineutrino pair reads, neglecting their mass:

$$\Gamma(Z' \to \overline{\nu_L}\nu_L) = \frac{M_{Z'}}{24\pi} A^{\nu_L\,2}\,, \tag{2.112}$$

while for decay into (mostly) RH neutrinos we get

$$\Gamma(Z' \to \overline{\nu_R}\nu_R) = \frac{M_{Z'}}{24\pi} \sqrt{1 - \frac{4m_{\nu_R}^2}{M_{Z'}^2}}\, A^{\nu_R\,2} \left(1 - \frac{m_{\nu_R}^2}{M_{Z'}^2}\right), \tag{2.113}$$

where m_{ν_R} is the mass of the RH neutrino. In the present analysis we assume that $m_{\nu_R} > M_{Z'}/2$, so decay into RH neutrinos is kinematically forbidden.

The partial decay width into $Z\,h$ is given by [44]

$$\Gamma(Z' \to Z\,h) = \frac{C^2 v^2}{96\pi M_{Z'}^2} p \left(3 + \frac{p^2}{M_Z^2}\right), \tag{2.114}$$

where

$$C = \left(\cos\theta' g_Z - g_Y \sin\theta'\right)\left(\sin\theta' g_Z + g_Y \cos\theta'\right) \,, \tag{2.115}$$

$$p^2 = \frac{1}{4M_{Z'}^2}\left(M_{Z'}^4 + M_Z^4 + m_h^4 - 2M_{Z'}^2 M_Z^2 - 2M_{Z'}^2 m_h^2 - 2M_Z^2 m_h^2\right) \,. \tag{2.116}$$

To obtain (2.114), the mixing between the two physical Higgses h, h' has been assumed to be negligible. In the heavy Z' limit, $M_{Z'} >> M_Z$, m_h (and as a consequence $\theta' \approx 0$), eq. (2.114) becomes

$$\Gamma(Z' \to Z\, h) \approx \frac{g_Y^2}{192\pi} M_{Z'} \,, \tag{2.117}$$

which proves that, even though suppressed by a factor $192\pi \approx 600$, if g_Y is not too small the partial width can be of $O(1\,\mathrm{GeV})$ for large $M_{Z'}$.

The partial decay width for the decay into a pair of W bosons reads [44, 45]

$$\Gamma(Z' \to W^+W^-) = \frac{g_Z^2}{192\pi}\sin^2\theta' M_{Z'}\left(\frac{M_{Z'}}{M_Z}\right)^4\left(1 - \frac{4\,M_W^2}{M_{Z'}^2}\right)^{3/2}\left(1 + 20\frac{M_W^2}{M_{Z'}^2} + 12\frac{M_W^4}{M_{Z'}^4}\right) \,, \tag{2.118}$$

which for $M_{Z'} >> M_Z$ becomes

$$\Gamma(Z' \to W^+W^-) \approx \frac{g_Z^2}{192\pi}\sin^2\theta' M_{Z'}\left(\frac{M_{Z'}}{M_Z}\right)^4 \,. \tag{2.119}$$

Also the three-body decays $Z' \to Z\,h\,h$, $W^+W^-\,Z$, $W^+W^-\,\gamma$ could be kinematically open: we neglect these channels in the present analysis, since they are likely to be strongly suppressed by three-body phase space. Now summing the partial widths, the total width $\Gamma_{Z'}$ is obtained, and the branching ratio $Br(Z' \to l^+l^-)$ is explicitly written in terms of the parameters $\widetilde{g}_Y, \widetilde{g}_{BL}, M_{Z'}$. We collect in Fig. 2.8 the branching ratios of the Z' corresponding to two representative cases, namely the roughly experimentally and GUT-'favoured' (see later) $g_Y = -g_{BL}$, and the 'disfavoured' case $g_Y = g_{BL}$, for Z' masses in the range $200\,\mathrm{GeV} \div 2\,\mathrm{TeV}$. Notice that the branching ratios depend to a good approximation only on the angle formed by g_Y and g_{BL} in the $(Y, B-L)$ plane, and not on the value of $\sqrt{g_Y^2 + g_{BL}^2}$.

2.4.3. Phenomenological analysis

In Fig. 2.9 we report the bounds on the (c_d, c_u) plane, as obtained from up-to-date CDF and D0 results. We find the strongest constraints to be given by CDF dielectron searches, contained in [23]. Having computed the dependence of $c_{u,d}$ on the parameters $\widetilde{g}_Y, \widetilde{g}_{BL}$ in the previous paragraph, we can easily translate the bounds on (c_d, c_u) into excluded regions in the $(\widetilde{g}_Y, \widetilde{g}_{BL})$ plane. Fig. 2.11 displays Tevatron, electroweak and APV bounds, for different Z' masses. The comparison between EWPTs and Tevatron bounds is easier if we plot the dependence of the two bounds on $M_{Z'}$ for a specific model: this is done in Fig. 2.10 for the χ model. Similar plots appeared previously in [34, 46]. We see that for roughly $M_{Z'} > 700\,\mathrm{GeV}$, direct searches performed until now are not competitive with electroweak precision tests: to explore this higher mass range, we must wait for the LHC to start off. This is also why we did not compute Tevatron bounds on a 1.5 TeV Z': they would be irrelevant with respect to the electroweak bound.

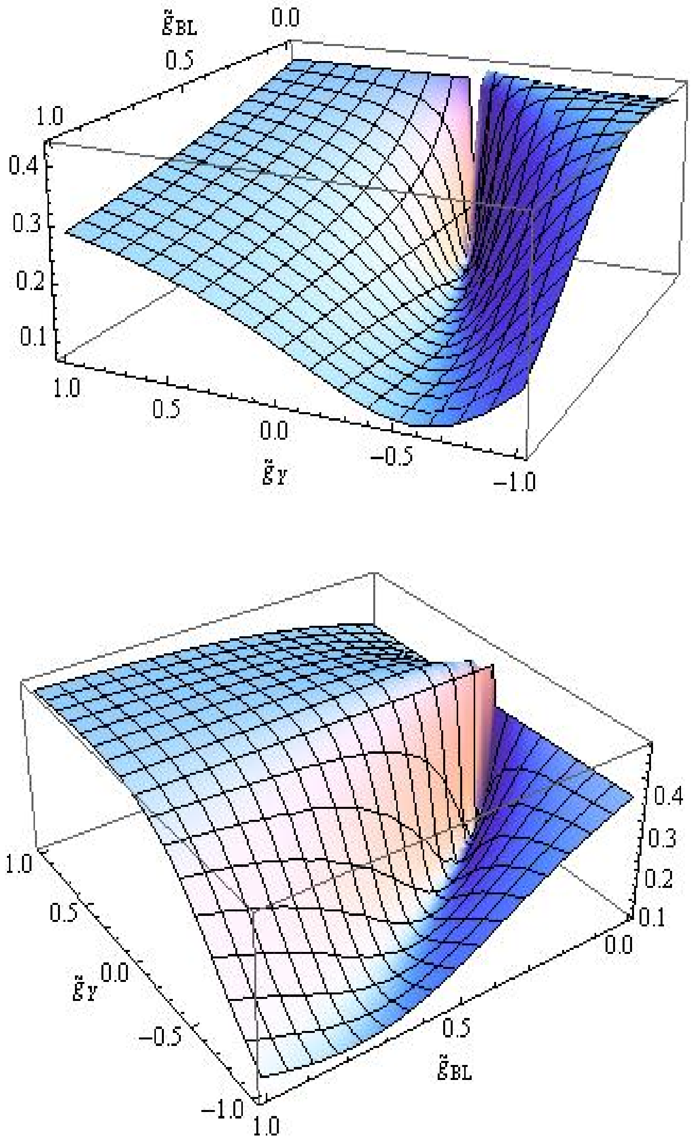

Figure 2.7.: Branching ratios of the Z' into three families of up quarks (upper plot) and three families of charged leptons (lower plot), as a function of $\widetilde{g}_Y$ and $\widetilde{g}_{BL}$, for a mass $M_{Z'} = 800\,\mathrm{GeV}$. Z' decay into RH neutrinos is assumed to be forbidden.

$M_{Z'}$	w_u	w_d	$\sigma \times Br(Z' \to e^+e^-) \leq$, [23]
400 GeV	7.981	1.57	$8fb$
700 GeV	0.384	0.037	$5fb$
800 GeV	0.137	$9.88 \cdot 10^{-3}$	$3.5fb$
1000 GeV	$1.48 \cdot 10^{-2}$	$5.35 \cdot 10^{-4}$	$3.5fb$

Table 2.5.: Numerical values used in the analysis of Tevatron bounds. The contributions to $w_{u,d}$ from c, s, b, t quarks are very small and have been neglected.

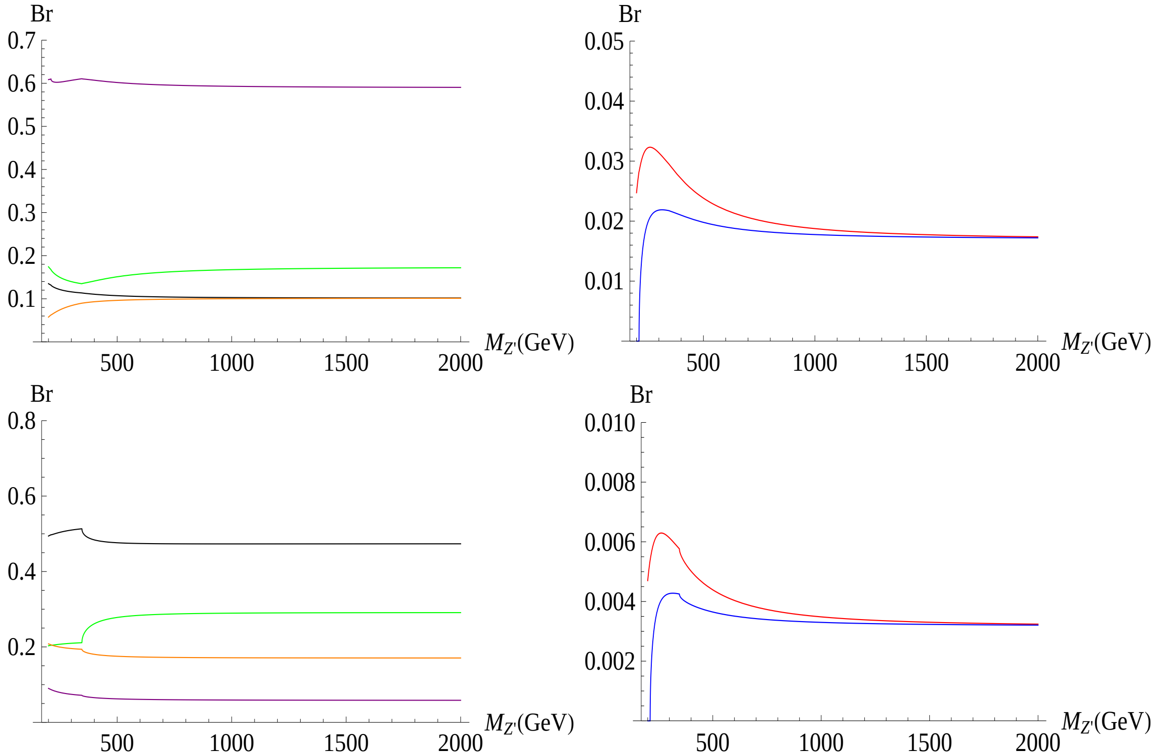

Figure 2.8.: Branching ratios of the Z' for two relevant choices of the parameters g_Y, g_{BL}. The first row is for $g_Y = -g_{BL}$, while the second row is for $g_Y = g_{BL}$. The colours green, black, purple, red, blue and orange are associated to the branching ratio into up quarks, charged leptons, down quarks, WW, Zh, and LH neutrinos respectively. In the mass range $M_{Z'} > 2\,\text{TeV}$, the branching ratios remain constant at the value they assume for $2\,\text{TeV}$.

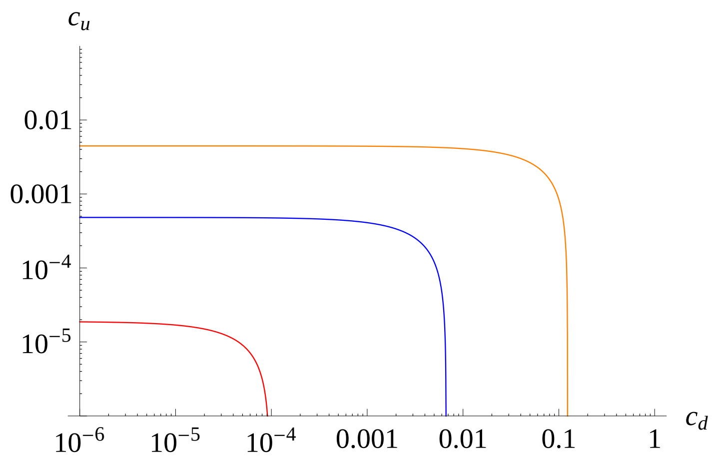

Figure 2.9.: Up-to-date 95% CL excluded regions in the (c_d, c_u) plane, obtained from [23]. The excluded regions lie above the curves. The red curve is for $M_{Z'} = 400$ GeV, the blue one for $M_{Z'} = 800\,\text{GeV}$, and the orange one for $M_{Z'} = 1\,\text{TeV}$.

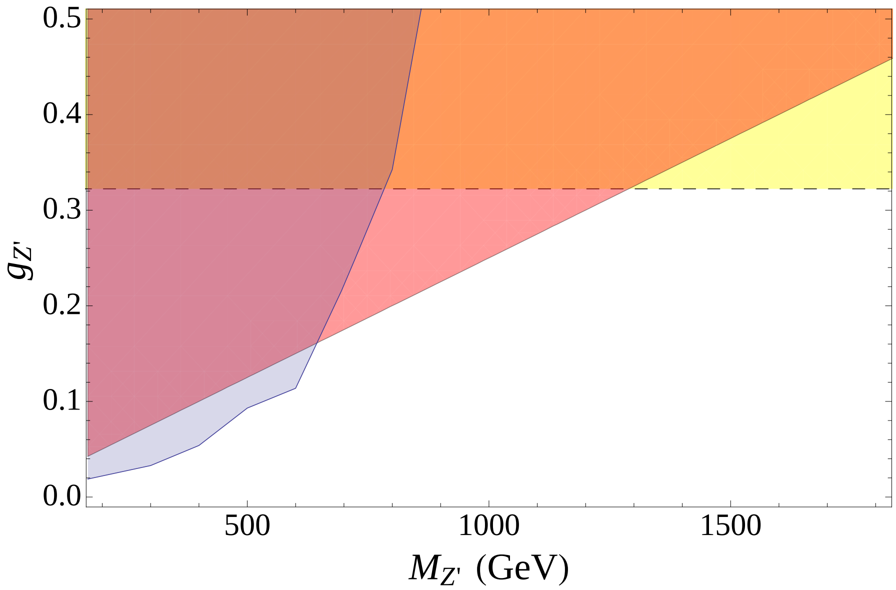

Figure 2.10.: Comparison of electroweak and Tevatron 95% CL limits for the χ model [1]. The yellow band anticipates the GUT-preferred region, see Section 2.6.

2.4.4. Comparison with previous analyses

To check the correctness of our computations, we make use of the results of CDF dilepton (ee and $\mu\mu$) preliminary searches, contained in the paper [43], with approximately 200 pb^{-1} of data at $\sqrt{s} = 1.96\,\mathrm{TeV}$. We choose three different values of the Z' mass, namely $M_{Z'} = 400, 600, 800\,\mathrm{GeV}$. Fig. 2.12 shows our results, which are in agreement with similar plots in [43] and also in [39].

2.4.5. The CDF e^+e^- excess

The CDF collaboration has recently observed [23] an excess of e^+e^- events around 240 GeV, corresponding to a $2.5\,\sigma$ fluctuation above background. This excess was however not confirmed by CDF dimuon data [24], while D0 [25] has not collected yet a sufficient amount of data to confirm or exclude such an excess. Even though our minimal model cannot explain the mismatch between electron and muon data, nevertheless it is interesting to check whether the existing bounds from EWPTs are already so strong to rule out an explanation of the excess by appealing to a model with non-universal couplings to leptons. Fig. 2.13 shows that this is not the case: given the number of events in excess, ~ 35, we let the selection efficiency vary generously between 20% and 40%, obtaining the two green contours. Since the efficiency was computed in [23] to be $27 \div 30\%$ for Z'_{SM} couplings in the mass range of interest, the loose bounds used should roughly take into account the model-dependence of the efficiency. We see from Fig. 2.13 that the values of the effective couplings $(\widetilde{g}_Y, \widetilde{g}_{BL})$ which may give rise to an excess such as the one observed by CDF are not excluded by EWPTs. We stress anyway that more data is needed to establish a non-standard origin for the excess. Should it

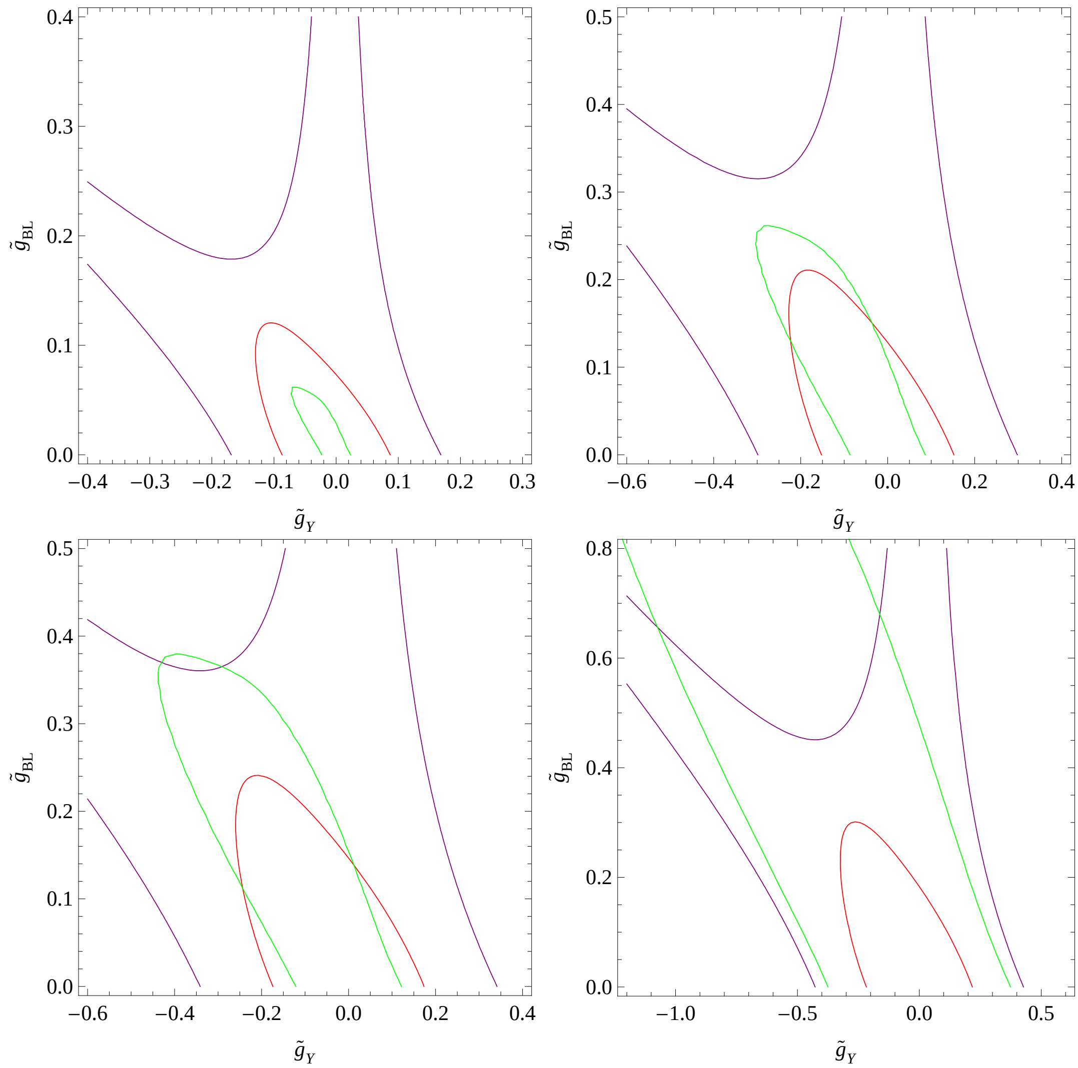

Figure 2.11.: Comparison of 95% CL limits from EWPTs, APV and Tevatron direct searches, in the $(\widetilde{g}_Y, \widetilde{g}_{BL})$ plane, for $M_{Z'} = 400$, 700, 800, 1000 GeV. The green curves are the Tevatron limits, the red ones are constraints from electroweak precision data, and the purple ones represent bounds from APV. The region outside each curve is excluded by the corresponding measurements.

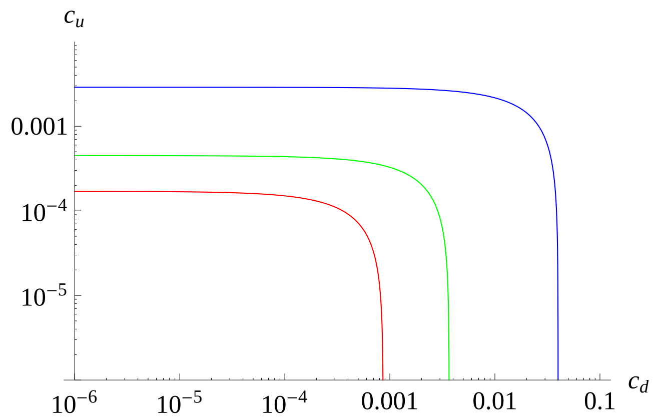

Figure 2.12.: 95% CL exclusions obtained from early Tevatron Run II data, contained in [43]. The excluded regions lie above the curves. The red curve is for $M_{Z'} = 400$ GeV, the green one for $M_{Z'} = 600\,\text{GeV}$, and the blue one for $M_{Z'} = 800\,\text{GeV}$.

be confirmed, it would be interesting to investigate models with non-universal couplings to leptons [21], to check whether such an excess is compatible with other constraints, essentially those coming from FCNC.

2.5. Early LHC prospects

We now analyze the possibility for the LHC to explore the parameter space of the minimal model, focusing on the early running scenario, i.e. $\sqrt{s}$ from 7 to 10 TeV, and an integrated luminosity ranging from $100\,\text{pb}^{-1}$ to $200\,\text{pb}^{-1}$ (at the time of writing this thesis, the schedule for the first year of LHC running [47] foresees a first run at $\sqrt{s} = 7\,\text{TeV}$, with luminosity $< 100\,\text{pb}^{-1}$, and later an upgrade in energy ($\sqrt{s} \leq 10\,\text{TeV}$), with up to $300\,\text{pb}^{-1}$ of collected luminosity.). We obtain accessible regions in the $(\widetilde{g}_Y, \widetilde{g}_{BL})$ plane, by using again the method of [39], which is based on equation (2.97). We set $M_{Z'}$ to a given value and compute the parameters w_u, w_d, paying attention to the fact that now the contributions from charm, strange and bottom quarks are not negligible, in contrast with the Tevatron case. Then, recalling that

$$\mathcal{L} \cdot \sigma = N\,, \tag{2.120}$$

where $\mathcal{L}$ is the integrated luminosity, and N is the number of events produced, we set reference values for N, namely $N = 15$ for $M_{Z'} = 400\,\text{GeV}$, $N = 5$ for $M_{Z'} = 700$, $800\,\text{GeV}$, and $N = 4$ for $M_{Z'} = 1\,\text{TeV}$, $1.5\,\text{TeV}$, and draw the corresponding contours in the $(\widetilde{g}_Y, \widetilde{g}_{BL})$ plane. We focus on $pp \to Z' \to e^+e^-$ events only. The reference values for N are chosen based on estimation of the background (mainly given by the SM Drell-Yan process, $pp \to Z/\gamma \to e^+e^-$) and of acceptance, see [1], where more accurate 5σ discovery limits are given. Two different scenarios are proposed: $\sqrt{s} = 7\,\text{TeV}$ with luminosity $\mathcal{L} = 100\,\text{pb}^{-1}$, and $\sqrt{s} = 10\,\text{TeV}$ with $\mathcal{L} = 200\,\text{pb}^{-1}$.

Figs. 2.14, 2.15 and 2.16 compare the region of parameter space accessible to the LHC with the exclusions set by electroweak precision tests and Tevatron searches, for $M_{Z'}$ equal

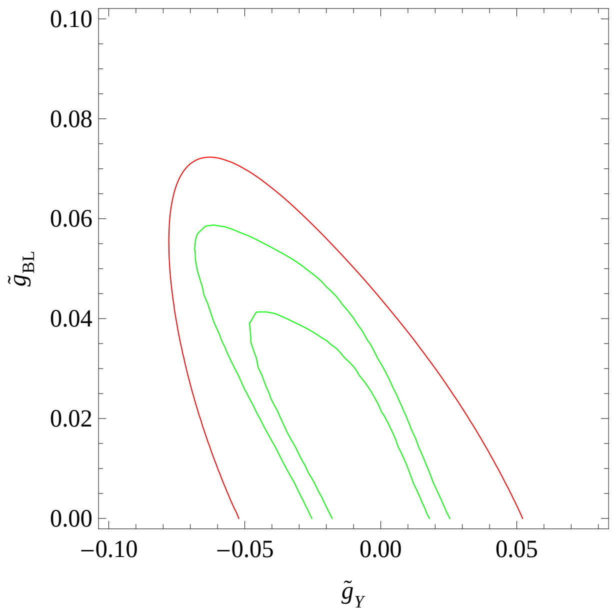

Figure 2.13.: Comparison of EWPTs with the region of the $(\widetilde{g}_Y, \widetilde{g}_{BL})$ plane which could produce an excess at 240 GeV such as that observed by CDF in [23]. We stress, however, that the excess was not found in CDF $\mu^+\mu^-$ data [24].

to 400, 700, 800, 1000 and 1500 GeV. In all the figures also the GUT-preferred region is also plotted in yellow, see Section 2.6 for details.

From the present analysis, two conclusions can be drawn: first, with $100\,\mathrm{pb}^{-1}$ of data at $\sqrt{s} = 7\,\mathrm{TeV}$, only an extremely narrow region of virgin parameter space opens up, in a mass window centered at $M_{Z'} \sim 700\,\mathrm{GeV}$ and with $\widetilde{g}_{BL} \sim 0.15 \div 0.20$, $\widetilde{g}_Y \sim -0.2 \div 0$ (this can be seen from the lower plot in Fig. 2.14). Second, in a later stage at 10 TeV, the LHC will start exploring virgin land in parameter space, for masses in the range $400 - 1500\,\mathrm{GeV}$. For $M_{Z'} > 1.5\,\mathrm{TeV}$ approximately, electroweak limits will not be trespassed with an integrated luminosity of $200\,\mathrm{pb}^{-1}$, whereas for light masses ($M_{Z'} < 400\,\mathrm{GeV}$), $200\,\mathrm{pb}^{-1}$ will not be enough for the LHC to overcome the existing Tevatron bounds. As for the GUT-preferred models, the very first steps in GUT-land will be possible at 10 TeV, however narrow regions of parameter space will be touched. To make a systematic scan of the models compatible with grand unification, luminosities of the order of $1\,\mathrm{fb}^{-1}$ will be needed.

2.6. Constraints from grand unification

The requirement of having a GUT at the unification scale M_U can constrain the allowed range of low-energy Z' parameters. Because GUTs are one of the motivations for considering Z's, in this section we compute the region in the plane $(\widetilde{g}_Y, \widetilde{g}_{BL})$ that is favoured by grand unification. We begin by recalling the one-loop RGE which are obeyed by the elements of the 2×2 submatrix $\hat{h}_{AB}$ $(A, B = Y, B - L)$ of the kinetic matrix h_{AB}:

$$\frac{d}{dt}\hat{h}_{AB} = -\frac{1}{8\pi^2} b_{AB}\,, \tag{2.121}$$

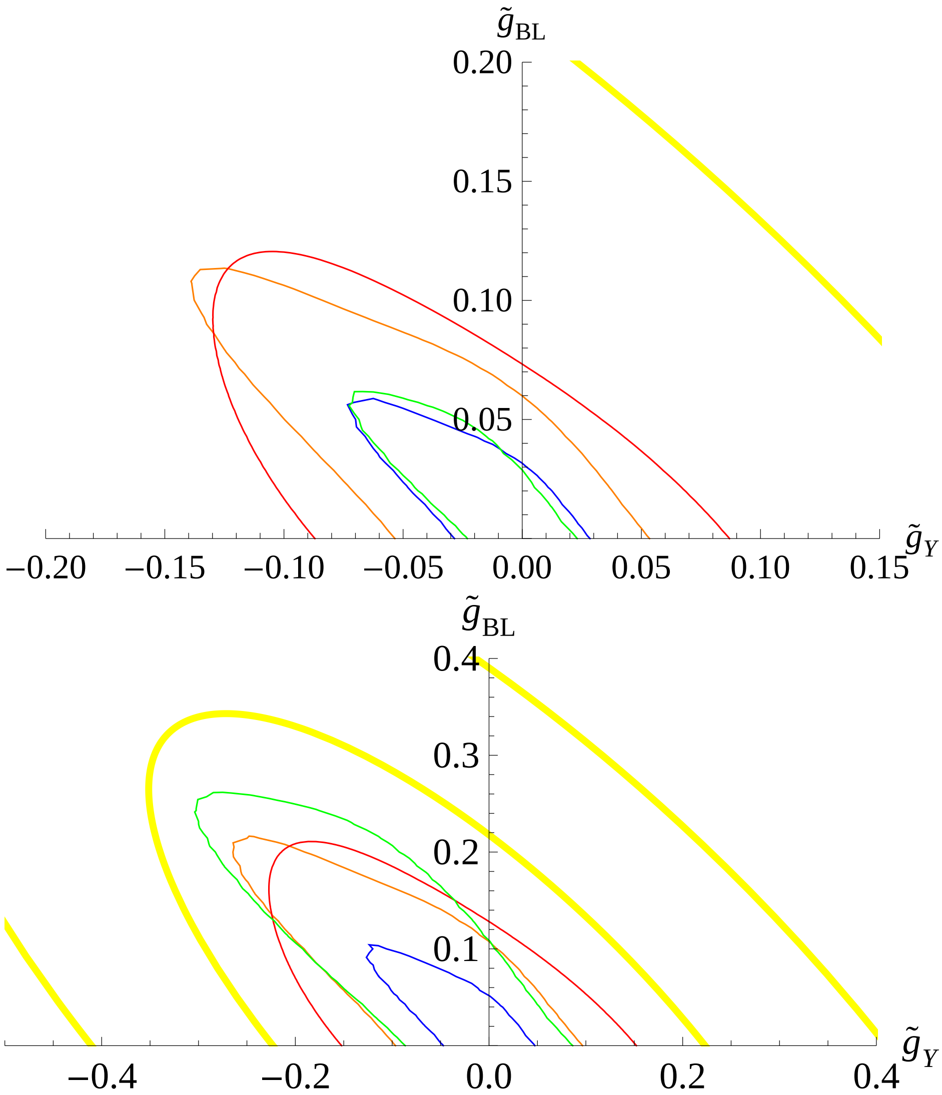

Figure 2.14.: Comparison of the region accessible to the LHC in its early running phase with electroweak and Tevatron bounds, for Z' masses of 400 and 700 GeV. Red and green curves show electroweak and Tevatron constraints, respectively. The orange curve marks the region accessible to the LHC with $\sqrt{s} = 7\,\mathrm{TeV}$ and $\mathcal{L} = 100\,\mathrm{pb}^{-1}$, while the blue one denotes the region testable with $\sqrt{s} = 10\,\mathrm{TeV}$ and $\mathcal{L} = 200\,\mathrm{pb}^{-1}$. The region allowed by present bounds and accessible to the LHC at $\sqrt{s} = 7\,\mathrm{TeV}$ or $\sqrt{s} = 10\,\mathrm{TeV}$, if present, lays *inside* both the red and green curves, and *outside* the orange and blue curve respectively. The region between the yellow curves is preferred by grand unification.

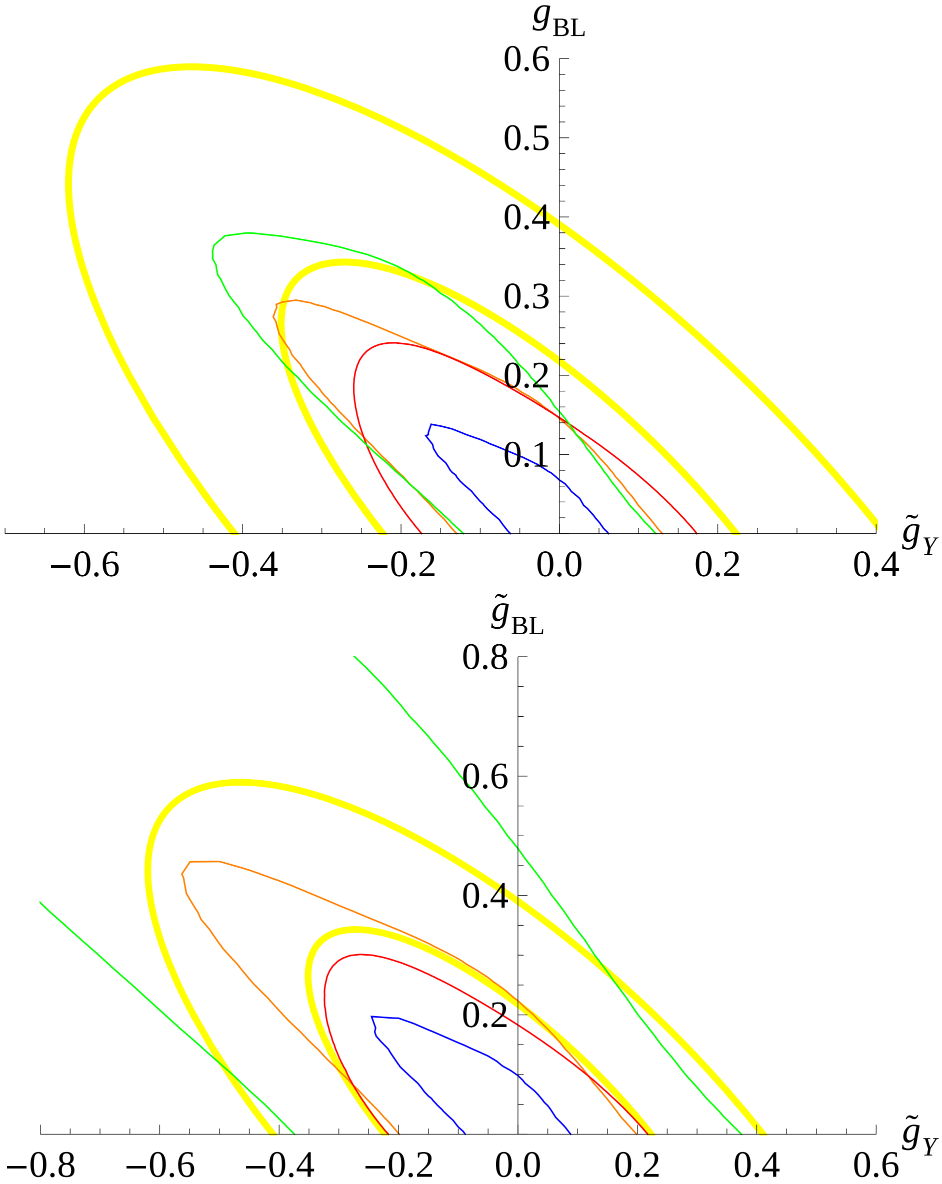

Figure 2.15.: Plots for $M_{Z'} = 800\,\text{GeV}$ and $1\,\text{TeV}$; colours as in the previous figures.

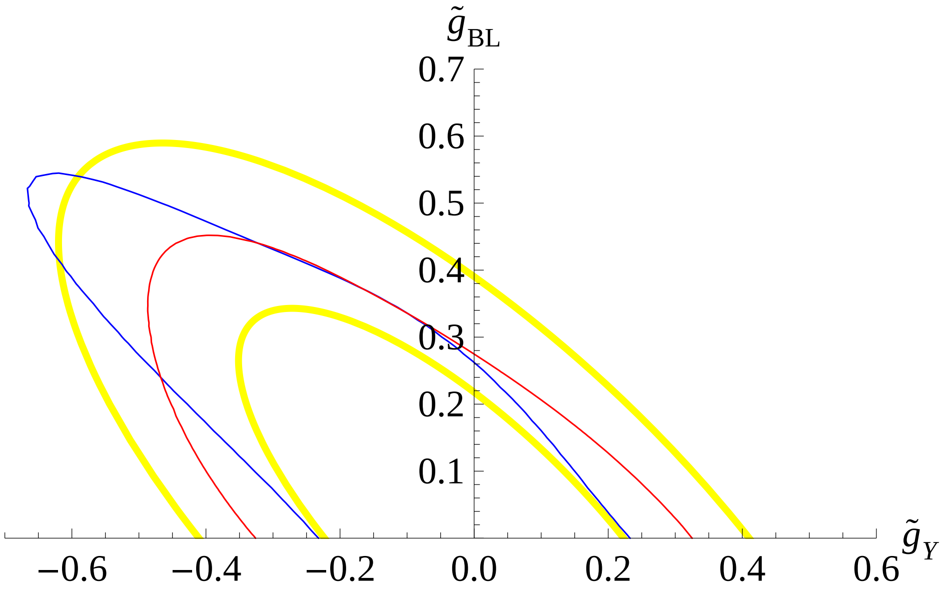

Figure 2.16.: Plot for $M_{Z'} = 1.5\,\mathrm{TeV}$; colours as in the previous figures. The Tevatron bound and LHC prospect with $\sqrt{s} = 7\,\mathrm{TeV}$ and $100\,\mathrm{pb}^{-1}$ are not shown, because they are not competititive with EWPTs.

where $t = \log(Q/Q_0)$ with Q_0 a reference scale, and

$$b_{AB} = \frac{2}{3} \sum_f Q_f^A Q_f^B + \frac{1}{3} \sum_s Q_s^A Q_s^B \,, \tag{2.122}$$

where f and s are the two-component fermions and complex scalars of the theory, respectively. Table 2.6 collects some representative values of the coefficients b_{AB}: in each case we include all the fermions in Table 2.1, and in addition (the numbers in brackets denote the representations of $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$):

(i) only the SM Higgs field;

(ii) the SM Higgs field and the SM singlet Higgs $\phi \sim (\mathbf{1}, 0, +2)$;

(iii) scalar partners for all the fermions in Table 2.1, plus two Higgs doublets $H_1 \sim (\mathbf{2}, -1/2, 0)$ and $H_2 \sim (\mathbf{2}, +1/2, 0)$. This is the field content of the Minimal Supersymmetric Standard Model, which we will introduce in the next chapter;

(iv) the same as in (iii), plus two Higgs SM singlets $\chi_1 \sim (\mathbf{1}, 0, -2)$ and $\chi_2 \sim (\mathbf{1}, 0, +2)$.

Eqs. (2.121) can be solved analytically, yielding

$$\hat{h}_{AB}(m_Z) = \hat{h}_{AB}(M_U) - \frac{b_{AB}}{8\pi^2} \log\left(\frac{M_Z}{M_U}\right) . \tag{2.123}$$

	(i)	(ii)	(iii)	(iv)
b_{YY}	41/6	41/6	11	11
$b_{Y(B-L)}$	16/3	16/3	8	8
$b_{(B-L)(B-L)}$	32/3	12	16	24

Table 2.6.: Values of b_{AB} for the different cases described in the text.

The relation between the matrix $\hat{h}_{AB}$ and the parameters g', g_Y, g_{BL} is [6]

$$\hat{h}_{AB} = \begin{pmatrix} \frac{1}{g'^2} & -\frac{g_Y}{g_{BL}}\frac{1}{g'^2} \\ \\ -\frac{g_Y}{g_{BL}}\frac{1}{g'^2} & \frac{1}{g_{BL}^2} + \frac{g_Y^2}{g_{BL}^2}\frac{1}{g'^2} \end{pmatrix}, \tag{2.124}$$

with inverse (notice that $(g_Y, g_{BL}) \to -(g_Y, g_{BL})$ leaves $\hat{h}_{AB}$ unchanged, so we choose $g_{BL} > 0$ and consequently we restrict the range of the angle α between 0 and π, see below):

$$g_{BL} = \sqrt{\frac{\hat{h}_{11}}{\hat{h}_{11}\hat{h}_{22} - \hat{h}_{12}^2}}, \tag{2.125}$$

$$g_Y = -\frac{\hat{h}_{12}}{\hat{h}_{11}}\sqrt{\frac{\hat{h}_{11}}{\hat{h}_{11}\hat{h}_{22} - \hat{h}_{12}^2}}. \tag{2.126}$$

We choose $M_U = 10^{16}$ GeV, and set the boundary conditions on the parameters g', g_Y, g_{BL} as follows: from the measured value at M_Z, and using the SM RGE, we compute $g'(M_U)$. Then we normalize the extra $U(1)$ generator as in $SO(10)$, explicitly

$$g_U^2 2 = Tr[g_U \widehat{X}]^2 = Tr[g_Y(M_U)Y + g_{BL}(M_U)(B-L)]^2, \tag{2.127}$$

where $\widehat{X}$ is the correctly normalized generator, and g_U is the GUT coupling constant. Writing

$$g_Y(M_U) = g_R \cos\alpha, \qquad g_{BL}(M_U) = g_R \sin\alpha \tag{2.128}$$

($\alpha \in (0, \pi)$), from (2.127) we obtain the value of g_R as a function of α. We allow $\alpha_U = g_U^2/4\pi$ to vary in the interval

$$\frac{1}{100} < \alpha_U < \frac{1}{20}, \tag{2.129}$$

thus roughly taking into account threshold effects, possible unknown particles, and other model-dependent effects. Using the parameters b_{AB} as in case (i), we obtain the yellow band shown in Fig. 2.17, which displays the region preferred by grand unification. In the same figure also two curves are displayed, corresponding to a continuous class of SUSY-GUT models. In these models, we assume that the GUT group $SO(10)$ is broken at M_U to $G_{SM} \times U(1)$, where G_{SM} is the Standard Model gauge group. We obtain the values $M_U \approx 2.7 \cdot 10^{16}$ GeV and $\alpha_U \approx 1/24$, and use two different running scenarios, namely (iii) and (iv). The results are the two curves shown in Fig. 2.17. Some reference models which belong to this class, and are commonly found in the literature, are represented by full dots. These are:

[6] The reader can immediately verify this by substituting eqs. (2.27) into eq. (2.2).

(a) the χ model, with canonically normalized generator $\widehat{X} = (1/2\sqrt{10})[4Y - 5(B-L)]$;

(b) the 'pure $B-L$' model, $\widehat{X} = \sqrt{3/8}(B-L)$;

(c) the T_{3R} model, $\widehat{X} = Y - (1/2)(B-L)$.

For each of these three models, two full dots are shown, representing the low-energy effective parameters obtained if the Higgs singlets χ_1, χ_2 are included (iv) or not (iii). In Fig. 2.17 also three empty dots are drawn, corresponding, from left to right, to the T_{3R}, χ, and pure $B-L$ benchmark models often used in the literature (see Table 2.7), with overall coupling constant $g_{Z'}$ normalized as follows: $g_{Z'}(M_Z) = \sqrt{5/3}\, g'(M_Z)$. For each empty dot, a dashed line is drawn, obtained by letting $g_{Z'}$ free to vary.

	Z_{B-L}	Z_χ	Z_{3R}
g_Y	0	$\frac{4}{2\sqrt{10}} g_{Z'}$	$g_{Z'}$
g_{BL}	$\sqrt{\frac{3}{8}} g_{Z'}$	$-\frac{5}{2\sqrt{10}} g_{Z'}$	$-\frac{1}{2} g_{Z'}$

Table 2.7.: Couplings of three specific models often encountered in the literature that are described by our framework. A 'GUT-inspired' normalization, $g_{Z'} = \sqrt{5/3}\, g'$, is often chosen. The χ model has already been introduced in eq. (2.86).

We notice that the effects of the running are sizable: e.g., even if we start with a pure $B-L$ model at M_U, the RG evolution reintroduces significant kinetic mixing at the weak scale. On the contrary, the direction corresponding to the χ model is quite stable under RG evolution. The reason for this is that the generator of $U(1)_\chi$ is orthogonal to the hypercharge generator on the **16** representation of $SO(10)$, therefore only the Higgs fields contribute to the mixing.

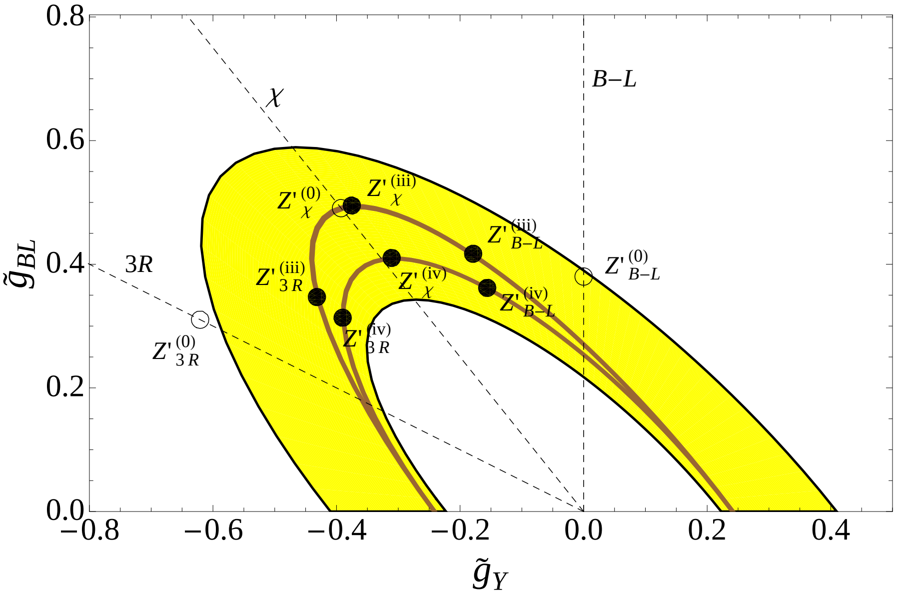

Figure 2.17.: GUT-favoured region and representative SUSY-GUT models [1]. The region between the yellow curves is generally preferred by grand unification. The two curves correspond to SUSY-GUT models, with RG running performed according to cases (iii) and (iv). The full dots indicate some specific SUSY-GUT models, whereas empty dots and dashed lines refer to benchmark models often encountered in the literature (see the text for details).

3. Supersymmetric $U(1)$ extensions of the SM

In this chapter we study some $U(1)$ extensions of the Minimal Supersymmetric Standard Model (MSSM) that have been recently proposed. In the first two sections, we briefly introduce supersymmetry (SUSY) and one of the main motivations for considering it, the so-called 'hierarchy problem'. The third section is devoted to a review of the MSSM, to set the stage for the extensions that will follow. In the fourth section, we describe kinetic mixing in SUSY gauge theories with two $U(1)$ factors, such as the minimal model we describe in the following section. After summarizing the properties of the latter model, we perform an analysis of the renormalization group equations (RGEs), showing that the assumption that kinetic mixing is negligible, made in the original formulation of the model, is not natural. In fact, we show that this choice is not stable under RG evolution: if we start from a 'pure $B-L$' model at the unification scale, the mixing effects in the RGEs generate sizable corrections to the effective weak-scale couplings, reintroducing significant kinetic mixing. We also compute the extra D-terms that arise in the scalar potential due to the presence of kinetic mixing. Finally, in the last section, we study some $U(1)$ extensions of the MSSM where the spontaneous breaking of the additional Abelian symmetry is realized by non-vanishing vacuum expectation values of the supersymmetric partners of right-handed neutrinos. In these models, also the discrete symmetry known as R-parity is spontaneously broken, therefore mixing between 'standard' particles and SUSY particles becomes possible.

Prelude: the hierarchy problem

We begin recalling one of the main motivations for considering supersymmetry, a difficulty known as the hierarchy problem of the SM. As we discussed in Chapter 1, in the SM the Higgs field acquires a VEV $\langle |H| \rangle = \sqrt{-m^2/2\lambda} = v/\sqrt{2} \approx 174\,\mathrm{GeV}$ as a result of the minimization of the scalar potential

$$V(H) = m^2 |H|^2 + \lambda |H|^4 \,, \tag{3.1}$$

where $m^2 < 0$ and $\lambda > 0$. Since λ is bounded from above by various consistency conditions (such as perturbative unitarity), it follows that it should be roughly $-m^2 \sim (100\,\mathrm{GeV})^2$. However, m^2 is expected to receive large radiative corrections from all the particles that couple to the Higgs field, either in a direct or in an indirect way. The simplest example is that of a loop containing a Dirac fermion (diagram on the left in Fig. 3.1), which gives a leading contribution, if we suppose to have a Yukawa term $-\lambda_f H \overline{f} f$ in the Lagrangian,

$$\Delta_f m^2 = -\frac{|\lambda_f|^2}{8\pi^2} \Lambda^2 \,, \tag{3.2}$$

where Λ is a cutoff scale, which can be interpreted as the scale where new physics effects become significant. If we take the cutoff to be approximately $M_P \approx 10^{19}\,\mathrm{GeV}$, the corrections

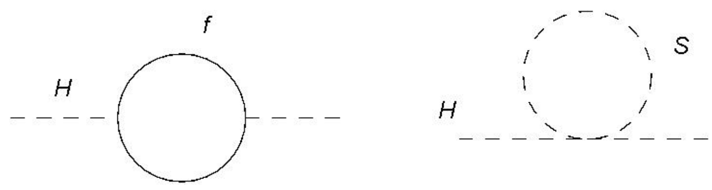

Figure 3.1.: Corrections to the Higgs mass given by a fermion loop (left) and a scalar loop (right).

due to fermion loops are enormous with respect to the weak scale. This difficulty is known as the *hierarchy problem.* It affects only the Higgs mass in a direct way, because quadratic divergences do not appear in the mass terms for fermions and vectors; however, we recall that even fermions and vector particles acquire mass via electroweak symmetry breaking, so they are indirectly affected by the hierarchy problem. Another type of contribution comes from scalar loops: suppose we have a term $-\lambda_S|H|^2|S|^2$ in the Lagrangian, with S a scalar; the diagram on the right of Fig. 3.1 will give a leading contribution to m^2 of the form

$$\Delta_s m^2 = \frac{\lambda_S}{16\pi^2}\Lambda^2\,. \tag{3.3}$$

In addition, the hierarchy problem arises even if we assume that no heavy particle couples directly to the Higgs boson, because large corrections to m^2 can be generated through loops containing gauge bosons of any interaction the heavy particles and the Higgs share. In conclusion, unless no heavy particle at all couples, either directly or indirectly, to the Higgs (a rather extreme hypothesis), the radiative corrections should drive $-m^2$ to an energy scale much higher than the observed one. Actually, another possibility is that a powerful cancellation mechanism exists, which is able to save m^2 from large contributions. Indeed, this mechanism has been discovered, and the symmetry which hides behind it is called *supersymmetry.* Supersymmetry connects fermionic and bosonic states, as can be inferred by looking at eqs. (3.2) and (3.3): suppose we have a fermion f and two complex scalars, each with $\lambda_S = |\lambda_f|^2$: in this case, the quadratically divergent correction to m^2 exactly vanishes. Thus we see that a relation between fermionic and bosonic couplings can make the cancellation to occur.

In passing, it is interesting to note that, from an historical point of view, supersymmetry was not conceived as a solution to the hierarchy problem, but was actually developed in totally different contexts [48, 49, 50, 51].

3.1. Supersymmetry

As anticipated in the introduction to this chapter, a symmetry relating fermions and bosons could provide a solution to the hierarchy problem of the SM. More generally, we may try to find an extension of the Poincaré group, introducing new generators Q^i having nontrivial transformation properties under the Lorentz group:

$$\left[Q^i\,,\,M^{\mu\nu}\right] \neq 0\,, \tag{3.4}$$

meaning that they have spin different from zero. *A priori*, we would be interested both in additional fermionic and bosonic generators Q^i. However, because what we are looking for is a possible symmetry of an interacting quantum field theory, bosonic symmetries are ruled out by the Coleman-Mandula theorem [52]. In fact, this theorem forbids the existence, in an interacting quantum field theory with a discrete spectrum of massive one-particle states, of any non-scalar conserved charges, which do not belong to the Poincaré group. This theorem applies to Lie algebras (which are spanned by bosonic generators, obeying commutation rules), but not to Lie *superalgebras*, associated with fermionic generators Q^i obeying anti-commutation rules (we denote anticommutators by curly brackets, e.g. $\{Q^i\,,\,Q^j\}$). As a result, new bosonic symmetries are excluded, and we are left to consider only fermionic operators. Furthermore, the only relevant case turns out to be that of spin 1/2: the generators of supersymmetry Q^i, $(i = 1\,\ldots, N)$ are such that

$$Q^i|\text{boson, spin } s>=|\text{fermion, spin } s \pm 1/2>\,, \tag{3.5}$$

$$Q^i|\text{fermion, spin } s>=|\text{boson, spin } s \pm 1/2>\;. \tag{3.6}$$

They satisfy the following relations, which define the *supersymmetry algebra*:

$$\begin{aligned}
[M^{\mu\nu}\,,\,M^{\rho\sigma}] &= -\,i(\eta^{\mu\rho}M^{\nu\sigma}+\eta^{\nu\sigma}M^{\mu\rho}-\eta^{\mu\sigma}M^{\nu\rho}-\eta^{\nu\rho}M^{\mu\sigma})\,, && (3.7)\\
[M^{\mu\nu}\,,\,P^{\rho}] &= -\,i(\eta^{\mu\rho}P^{\nu}-\eta^{\nu\rho}P^{\mu})\,, && (3.8)\\
[P^{\mu}\,,\,P^{\nu}] &= 0\,, && (3.9)\\
[M^{\mu\nu}\,,\,Q_\alpha] &= \frac{i}{4}(\sigma^\mu\overline{\sigma}^\nu-\sigma^\nu\overline{\sigma}^\mu)_\alpha{}^\beta Q_\beta\,, && (3.10)\\
[M^{\mu\nu}\,,\,\overline{Q}^{\dot\alpha}] &= \frac{i}{4}(\overline{\sigma}^\mu\sigma^\nu-\overline{\sigma}^\nu\sigma^\mu)^{\dot\alpha}{}_{\dot\beta}\overline{Q}^{\dot\beta}\,, && (3.11)\\
[P^{\mu}\,,\,Q_\alpha] &= 0\,, && (3.12)\\
[P^{\mu}\,,\,\overline{Q}^{\dot\alpha}] &= 0\,, && (3.13)\\
\{Q_\alpha, Q_\beta\} &= 0\,, && (3.14)\\
\{\overline{Q}^{\dot\alpha}, \overline{Q}^{\dot\beta}\} &= 0\,, && (3.15)\\
\{Q_\alpha, \overline{Q}_{\dot\beta}\} &= 2P_\mu(\sigma^\mu)_{\alpha\dot\beta}\,, && (3.16)
\end{aligned}$$

where we have introduced two-component notation for spinors (we mainly follow the notation of [53], see Appendix A), writing the SUSY charge Q as a Majorana spinor,

$$Q=\begin{pmatrix} Q_\alpha \\ \overline{Q}^{\dot\alpha}\end{pmatrix}\,. \tag{3.17}$$

The three relations (3.7) - (3.10) define the familiar Poincaré algebra. We have set $N = 1$, i.e. we do not consider *extended supersymmetry*, where N copies of SUSY generators ($N > 1$) are present. This choice is made because, while theories with extended SUSY are extremely appealing and widely studied, on the other hand they do not offer phenomenological prospects in four dimensions (if $N > 1$ and $d = 4$, fermions are not allowed to transform chirally under the gauge group), therefore their study is beyond the scope of this thesis. Haag, Lopuzanski and Sohnius [54] have proved that the supersymmetry algebra is in fact the most general superalgebra admissible as a symmetry of a four-dimensional interacting quantum field theory[1].

An important property of any supersymmetric theory is that the energy is always positive, $P^0 > 0$ [2]. The single-particle states of a supersymmetric theory are grouped in irreducible representations of the SUSY algebra, called supermultiplets. Each supermultiplet contains both fermions and bosons; three important properties of supermultiplets are:

I. For unbroken SUSY, all particles belonging to the same supermultiplet have the same mass. This is a consequence of $P^2 = P_\mu P^\mu$ being a Casimir operator of the SUSY algebra, i.e. P^2 commutes with all the elements of the SUSY algebra.

II. Each supermultiplet contains an equal number of bosonic and fermionic degrees of freedom. The proof of this statement is simple, and can be found, e.g., in [53].

III. In a supersymmetric gauge theory, because the generators of the gauge group G commute with the supersymmetry generators Q, the particles belonging to each supermultiplet must be in the same representation of G.

The simplest combinations satisfying the requirements above are:

(i) a *chiral* supermultiplet, whose propagating degrees of freedom are a Weyl fermion and a complex scalar;

(ii) a *vector* supermultiplet, whose propagating degrees of freedom are a massless spin-1 particle and a Weyl fermion.

Because in the supersymmetric theories we will study in this chapter, we will not be concerned with the description of particles of spin higher than 1, these are the building blocks we need.

3.2. SUSY gauge theories

The aim of this section is to introduce the generic form of a supersymmetric gauge theory. The easiest and most elegant way to build such theories is to employ the superfield formalism, which we use here, but only stating the results. The reader is addressed to the many excellent treatises on the subject, e.g. [53] and [55] (the notation used here is essentially that of [53], however we collect our conventions in Appendix A for clarity). The basic idea is the following. Consider first the case of translations: the effect of an infinitesimal translation with parameter Δ^μ on a Lagrangian $\mathcal{L}(x^\mu)$ reads

$$\delta\mathcal{L} = \Delta^\mu \partial_\mu \mathcal{L}\,, \tag{3.18}$$

which is a total derivative; therefore, invariance under translations is always manifest. It would be very desirable to exploit invariance under supersymmetry transformations in a similar way, by writing SUSY transformations as (generalized) translations, and therefore, having SUSY generators acting with derivatives. Indeed, this can be realized, but at the

[1] This result does not hold only for $N = 1$, but in general.

[2] In theories where SUSY is made local, i.e. in supergravity, such a statement would require several specifications that go beyond the scope of the present thesis.

price of extending the 4-dimensional spacetime to *superspace*: the coordinates of a point in superspace can be written $(x^\mu, \theta^\alpha, \overline{\theta}^{\dot\alpha})$, where θ^α, $\overline{\theta}^{\dot\alpha}$ are anticommuting two-component spinors. The general expansion of a superfield F reads

$$\begin{aligned} F(x,\theta,\overline{\theta}) \;=\; & f(x) + \theta\phi(x) + \overline{\theta}\overline{\chi}(x) + \theta\theta m(x) + \overline{\theta\theta}n(x) + \theta\sigma^\mu\overline{\theta}A_\mu(x) \\ & + \theta\theta\overline{\theta}\overline{\lambda}(x) + \overline{\theta\theta}\theta\psi(x) + \theta\theta\overline{\theta\theta}d(x)\,, \end{aligned} \tag{3.19}$$

where f, m, n, d are complex scalars, ϕ_α, $\overline{\chi}^{\dot\alpha}$, $\overline{\lambda}^{\dot\alpha}$, ψ_α are Weyl spinors, and A_μ is a complex vector field. Notice that eq. (3.19) is the most general expression that can be written: in fact, $\theta^3 = \overline{\theta}^3 = 0$, because θ and $\overline{\theta}$ anticommute. A counting of the parameters contained in F gives 16 bosonic and 16 fermionic degrees of freedom: thus, constraints must be imposed to reduce the number of components and obtain the building blocks of the theory, namely chiral and vector superfields.

A left-handed chiral superfield ϕ satisfies $\overline{D}_{\dot\alpha}\,\phi = 0$, where the covariant derivative reads

$$\overline{D}_{\dot\alpha} = \frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} - i\theta^\alpha(\sigma^\mu)_{\alpha\dot\alpha}\partial_\mu\,. \tag{3.20}$$

The general expression for a left-handed chiral multiplet reads:

$$\begin{aligned} \phi(x,\theta,\overline{\theta}) \;=\; & z(x) + \sqrt{2}\theta\psi(x) - \theta\theta f(x) - i(\theta\sigma^\mu\overline{\theta})\partial_\mu z(x) \\ & + \frac{i}{\sqrt{2}}\theta\theta(\partial_\mu\psi(x)\sigma^\mu\overline{\theta}) - \frac{1}{4}\theta\theta\overline{\theta\theta}\,\Box\, z(x)\,, \end{aligned} \tag{3.21}$$

where z, ψ, f are a complex scalar, a Weyl spinor, and an auxiliary scalar field (of mass dimension 2) respectively. A vector multiplet V satisfies the reality condition $V^\dagger = V$: its expansion contains in general 8 bosonic and 8 fermionic degrees of freedom. Performing a gauge transformation, $V \to V + \phi + \phi^\dagger$, we can choose a particular gauge where V can be written in the following way:

$$V \;=\; \theta\sigma^\mu\overline{\theta}V_\mu(x) + i\theta\theta\overline{\theta}\overline{\lambda}(x) - i\overline{\theta\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\overline{\theta\theta}D(x)\,, \tag{3.22}$$

where V_μ, λ, D are a real vector field, a Weyl spinor, and an auxiliary real scalar field of mass dimension 2 respectively. This gauge choice is known as the *Wess-Zumino gauge*: from eq. (3.22) follows that $V^n = 0$ for $n \geq 3$, therefore we can write $e^V = 1 + V + V^2/2$. As a consequence, in eq. (3.26) below, only products of up to four fields appear, making renormalizability manifest.

The auxiliary fields f, D are needed in order to render equal the number of off-shell bosonic and fermionic degrees of freedom contained in each chiral or vector superfield. In fact, we stated in Section 3.1 that a chiral supermultiplet contains only a Weyl fermion and a complex scalar. However, a Weyl fermion has 2 real degrees of freedom only when it is on-shell (that is, when it satisfies its equation of motion); in general, it has 4 degrees of freedom, so adding the 2 degrees of freedom contained in f balances the total. In a similar way, if a vector superfield contained only V^μ and λ, the number of bosonic and fermionic degrees of freedom would be equal only on-shell.

We now consider a theory describing a set of left-handed chiral superfields ϕ^i, transforming under the gauge group G, and a set of vector superfields V^a , $a = 1, \ldots, dim\, G$, transforming in the adjoint representation of G. The transformation law of the chiral multiplets reads

$$\phi'^i = [\exp\{i\Lambda^a(T^a)\}]^i{}_j\, \phi^j\,, \tag{3.23}$$

where the T^a are the generators of G, and Λ^a are left-handed chiral superfields. We assume that the generators of G satisfy the following relations:

$$\begin{aligned} Tr\,(T^a\, T^b) &= C\,\delta^{ab}\,, &(3.24)\\ [T^a\,,\, T^b] &= i\, f^{abc} T^c\,, &(3.25)\end{aligned}$$

where C is a constant, and f^{abc} are the structure constants of G. The supersymmetric Lagrangian of the theory reads:

$$\mathcal{L}_{SUSY} \;=\; \left[W(\phi) + \frac{1}{16C\,g^2} Tr\,(\mathcal{W}^\alpha \mathcal{W}_\alpha)\right]_{\theta\theta} + \,h.c.\, + \left[\phi^\dagger e^V\phi + \xi^a V^a\right]_{\theta\theta\overline{\theta}\overline{\theta}}\,, \tag{3.26}$$

where $V = V^a T^a$, and the generalized field strengths are given by

$$\mathcal{W}_\alpha = -\frac{1}{4}\overline{DD} e^{-V} D_\alpha e^V\,. \tag{3.27}$$

The superpotential reads

$$W(\phi) \;=\; a_i\phi^i + \frac{1}{2} m^2_{ij}\phi^i\phi^j + \frac{1}{3}\lambda_{ijk}\phi^i\phi^j\phi^k\,, \tag{3.28}$$

and must be invariant under transformations of the gauge group G. In (3.26), the Fayet-Iliopoulos constants ξ^a can be nonzero only for Abelian subgroups of G.

After making the redefinition $V \to 2gV$, the component Lagrangian reads

$$\begin{aligned}\mathcal{L}_{SUSY} =& (D_\mu z)^\dagger_i (D^\mu z)^i + \frac{i}{2}\psi^i\sigma^\mu(D_\mu\overline{\psi})_i - \frac{i}{2}(D_\mu\psi)^i\sigma^\mu\psi_i - \frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \frac{i}{2}\lambda^a\sigma^\mu(D_\mu\overline{\lambda})^a\\ &-\frac{i}{2}(D_\mu\lambda)^a\sigma^\mu\overline{\lambda}^a + f^\dagger_i f^i + \frac{1}{2}D^a D^a + i\sqrt{2}g_a(\overline{\psi_i\lambda}^a)T^{a\,i}{}_j\, z^j - i\sqrt{2}g_a z^\dagger_i T^{a\,i}{}_j(\lambda^a\psi^j)\\ &-\frac{1}{2}\left(\frac{d^2W}{dz^i dz^j}\psi^i\psi^j + h.c.\right) + g_a D^a\left[z_i T^{a\,i}{}_j z^j + \xi^a\right] - \left(\frac{dW}{dz^i}f^i + h.c.\right)\,,\end{aligned} \tag{3.29}$$

where the covariant derivatives read

$$\begin{aligned} (D_\mu z)^i &= \partial_\mu z^i - i g_a V^a_\mu (T^a)^i{}_j z^j\,, &(3.30)\\ (D_\mu\psi)^i &= \partial_\mu\psi^i - i g_a V^a_\mu (T^a)^i{}_j\psi^j\,, &(3.31)\\ (D_\mu\lambda)^a &= \partial_\mu\lambda^a + g_a f^{abc} V^b_\mu\lambda^c\,, &(3.32)\end{aligned}$$

and g_a is the coupling constant associated to the generator T^a. Solving the equations of motion for the auxiliary fields f^i and D^a and substituting, we finally get

$$\begin{aligned}\mathcal{L}_{SUSY} \;=&\; (D_\mu z)^\dagger_i (D^\mu z)^i + \frac{i}{2}\psi^i\sigma^\mu(D_\mu\overline{\psi})_i - \frac{i}{2}(D_\mu\psi)^i\sigma^\mu\psi_i - \frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu}\\ &+\frac{i}{2}\lambda^a\sigma^\mu(D_\mu\overline{\lambda})^a - \frac{i}{2}(D_\mu\lambda)^a\sigma^\mu\overline{\lambda}^a + i\sqrt{2}g_a(\overline{\psi_i\lambda}^a)T^{a\,i}{}_j\, z^j - i\sqrt{2}g_a z^\dagger_i T^{a\,i}{}_j(\lambda^a\psi^j)\\ &-\frac{1}{2}\left(\frac{d^2W}{dz^i dz^j}\psi^i\psi^j + h.c.\right) - V(\phi,\,\overline{\phi})\,,\end{aligned} \tag{3.33}$$

where the scalar potential reads

$$V(\phi,\,\overline{\phi}) \;=\; \sum_i \left|\frac{dW}{dz^i}\right|^2 + \frac{1}{2} g_a^2 \sum_a \left(z_i^\dagger T^{a\,i}{}_j z^j + \xi^a \right)^2 . \tag{3.34}$$

3.3. The MSSM

In this section we describe the Minimal Supersymmetric Standard Model (MSSM), the most economical supersymmetric theory which includes the SM. For a complete treatment, we refer the reader to [56].

3.3.1. Field content

The chiral supermultiplets are assigned the transformation properties shown in Table 3.1 under the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$, where we have followed the convention of

Name	Spin 0	Spin 1/2	$SU(3)_c \times SU(2)_L \times U(1)_Y$
Q	$(\widetilde{u}_L\ \widetilde{d}_L)$	$(u_L\ d_L)$	$(\mathbf{3}, \mathbf{2}, 1/6)$
U^c	$\widetilde{u}_R^*$	$u_R^\dagger$	$(\overline{\mathbf{3}}, \mathbf{1}, -2/3)$
D^c	$\widetilde{d}_R^*$	$d_R^\dagger$	$(\overline{\mathbf{3}}, \mathbf{1}, 1/3)$
L	$(\widetilde{\nu}_L\ \widetilde{e}_L)$	$(\nu_L\ e_L)$	$(\mathbf{1}, \mathbf{2}, -1/2)$
E^c	$\widetilde{e}_R^*$	$e_R^\dagger$	$(\mathbf{1}, \mathbf{1}, 1)$
H_u	$(H_u^+\ H_u^0)$	$(\widetilde{H}_u^+\ \widetilde{H}_u^0)$	$(\mathbf{1}, \mathbf{2}, +1/2)$
H_d	$(H_d^0\ H_d^-)$	$(\widetilde{H}_d^0\ \widetilde{H}_d^-)$	$(\mathbf{1}, \mathbf{2}, -1/2)$

Table 3.1.: Chiral supermultiplets in the MSSM. Family indices are understood for squarks/quarks and sleptons/leptons.

defining all chiral supermultiplets in terms of left-handed Weyl spinors. Two Higgs doublets $H_u\,, H_d$ are introduced, with three motivations. The first is to avoid anomalies generated by Higgsino fermions; second, two Higgs doublets are necessary in order to allow all the Yukawa couplings in the superpotential needed to give masses both to up-type quarks and to down-type quarks and leptons. Third, the electrically charged fermions in the gauge and Higgs sector must be massive to evade experimental bounds, and Dirac masses need an even number of 2-component spinors. The vector supermultiplets of the theory are displayed in Table 3.2.

Name	Spin 1/2	Spin 1	$SU(3)_c \times SU(2)_L \times U(1)_Y$
gluino, gluon	$\widetilde{g}^a$	g^a	$(\mathbf{8}, \mathbf{1}, 0)$
winos, W	$\widetilde{W}^i$	W^i	$(\mathbf{1}, \mathbf{3}, 0)$
bino, B	$\widetilde{B}$	B	$(\mathbf{1}, \mathbf{1}, 0)$

Table 3.2.: Vector supermultiplets in the MSSM. Indices $a = 1, \ldots, 8$ and $i = 1, 2, 3$ run over the adjoint representations of $SU(3)_c$ and $SU(2)_L$ respectively.

3.3.2. The superpotential and R-parity

The superpotential is given by:

$$W_{MSSM} = (Y_U)_{mn} Q_m H_u U_n^c + (Y_D)_{mn} Q_m H_d D_n^c + (Y_E)_{mn} L_m H_d E_n^c + \mu H_u H_d \,, \qquad (3.35)$$

where $m, n = 1, 2, 3$ are family indices, and gauge-invariant combinations are understood (for instance, $\mu H_u H_d \equiv \mu H_{u\alpha} (i\sigma^2)^{\alpha\beta} H_{d\beta}$; the reader should avoid confusion between $SU(2)_L$ indices and spinorial indices). Equation (3.35) shows why two different Higgs doublets with opposite hypercharges need to be introduced. Notice that we cannot introduce only one Higgs doublet H in the theory and use both H and H^* to give masses to all types of fermions, because the superpotential must be an analytic function of the chiral superfields, treated as complex variables, thus it cannot contain H^*.

Not all gauge invariant terms have been included in W_{MSSM}: the most general renormalizable superpotential compatible with the field content specified in Table 3.1 would contain, in addition to the terms which appear in eq. (3.35), also:

$$W_{RPV} = \lambda^{mnp} L_m L_n E_p^c + \lambda'^{mnp} L_m Q_n D_p^c + \mu'^m L_m H_u + \lambda''^{mnp} U_m^c D_n^c D_p^c \,. \qquad (3.36)$$

The first three terms in equation (3.36) contain lepton number violating interactions, while the last term violates baryon number. The presence of these terms is potentially dangerous, leading for instance to a lifetime for proton much shorter than the present experimental limits, unless a mechanism providing an enormous suppression for λ'^{mnp} and λ''^{mnp} exists. Therefore, one usually imposes on the theory a new symmetry, called R-parity, which forbids all the terms in W_{RPV}, but allows those which appear in W_{MSSM}. R-parity is defined for each particle as follows:

$$P_R = (-1)^{3(B-L)} (-1)^{2s} = P_M (-1)^{2s} \,, \qquad (3.37)$$

where $P_M = (-1)^{3(B-L)}$ is called matter parity. We require every term appearing in the Lagrangian of the MSSM (and therefore every term in the superpotential) to be such that the product of P_R for all the fields in it be $+1$. This achieves our goal of forbidding all B- and L- violating terms in (3.36): due to angular momentum conservation, the product of $(-1)^{2s}$ for every interaction vertex is always 1, so we are left with the product of matter parities. It is then immediate to check that all lepton and quark supermultiplets have $P_M = -1$, while Higgs supermultiplets have $P_M = +1$. It follows that all the terms in (3.36) violate R-parity.

We take R-parity as conserved *by definition* in the MSSM. From (3.37), it is immediate to realize that all the SM fermions and the Higgs scalars have $P_R = +1$, whereas all squarks, sleptons, gauginos and Higgsinos have $P_R = -1$. R-even fields are collectively called ‘particles’, while R-odd particles are known as ‘supersymmetric particles’ or ‘sparticles’. Conservation of R-parity has important consequences on the phenomenology of the MSSM: first, no mixing between particles and sparticles is allowed. We will introduce later in this thesis models in which R-parity is spontaneously broken, and we will see in detail the important effects of particle/sparticle mixing. Another consequence of R-parity conservation is that every interaction term in the Lagrangian of the MSSM must contain an even number of sparticles. This implies that:

(i) sparticles must appear in even numbers at every interaction vertex, with obvious consequences for sparticle production (only pair production, no single production) and decay;

(ii) the Lightest Supersymmetric Particle, or LSP, is absolutely stable, since its decay would violate R-parity. This makes the LSP a natural candidate for Cold Dark Matter [3].

Because of the large observed mass difference between third family fermions t, b, τ and those belonging to the first two families, sometimes the following approximation is made:

$$Y_{U,\,D,\,E} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t, y_d, y_\tau \end{pmatrix} \tag{3.38}$$

In this limit, only the third family matter chiral superfields appear in the superpotential.

3.3.3. Soft supersymmetry breaking

We have seen that exact supersymmetry forces all the particles belonging to a given supermultiplet to have the same mass. Since no supersymmetric particle has been observed yet, we know that supersymmetry must be broken. From a theoretical point of view, it would be desirable to build a satisfactory theory of spontaneous SUSY breaking. Unfortunately, such a theory does not exist yet, so for phenomenological purposes the standard choice is to introduce supersymmetry breaking *explicitly* in the effective low energy MSSM Lagrangian. We must be very careful in breaking supersymmetry, however: it is essential that, even in presence of broken SUSY, the relations between dimensionless coupling constants that SUSY imposes still hold. Otherwise, in fact, the miraculous cancellation between scalar and fermion loops would disappear, and the hierarchy problem would be restored. Due to this argument, SUSY breaking terms are chosen to be soft, i.e. containing only parameters of *positive* mass dimension. In this way, the new SUSY breaking terms only introduce logarithmic corrections to the Higgs mass, leaving it free of quadratic divergences. If we denote by m_{soft} the scale of the soft terms, the logarithmic corrections will be of the form

$$\Delta m^2 = m_{soft}^2 \left[\frac{\lambda}{16\pi^2} \log\left(\frac{\Lambda}{m_{soft}} \right) \right] , \tag{3.39}$$

where λ is a generic dimensionless coupling. Eq. (3.39) suggests one important remark: the value of m_{soft} cannot be too large, otherwise the solution to the hierarchy problem would be lost, due to the large $\sim m_{soft}^2$ contributions to the Higgs mass. In particular, setting $\lambda = O(1)$ and $\Lambda \approx M_P$, it follows that it should be $m_{soft} \sim 1\,\text{TeV}$ at most, to avoid requiring unnaturally large cancellations between the log terms. This, together with the observation that the mass difference between particles and sparticles is essentially tuned by m_{soft}, is often used as an argument in favour of the discovery of SUSY particles at the LHC.

[3] If the MSSM is embedded in a theory with spontaneously broken supergravity, other R-odd candidates for the LSP may appear in the gravitational sector (gravitino), or in the so-called 'hidden sector' that breaks supersymmetry.

The soft SUSY breaking terms are:

$$-\mathcal{L}_{soft} = \frac{1}{2}(M_3\widetilde{g}\widetilde{g}+M_2\widetilde{W}\widetilde{W}+M_1\widetilde{B}\widetilde{B}+h.c.)+(A_U\widetilde{Q}\widetilde{U}^cH_u+A_D\widetilde{Q}\widetilde{D}^cH_d+A_E\widetilde{L}\widetilde{E}^cH_d+h.c.)$$
$$+(bH_uH_d+h.c.)+(\widetilde{Q}^*M_Q^2\widetilde{Q}+\widetilde{L}^*M_L^2\widetilde{L}+\widetilde{U}M_U^2\widetilde{U}^c+\widetilde{D}M_D^2\widetilde{D}^c+\widetilde{E}M_E^2\widetilde{E}^c+H_u^*m_{H_u}^2H_u+H_d^*m_{H_d}^2H_d)\,, \tag{3.40}$$

where the matrices $M_Q^2,\ldots,M_E^2$ must be hermitian, $m_{H_u}^2$ and $m_{H_d}^2$ must be real, and all gauge and family indices are understood. The soft Lagrangian (3.40) contains the following parameters:

$$\begin{gathered} M_1,\, M_2,\, M_3\,, \\ A_U,\, A_D,\, A_E\,, \\ M_Q^2,\, M_L^2,\, M_U^2,\, M_D^2,\, M_E^2\,, \\ m_{H_u}^2,\, m_{H_d}^2,\, b\,. \end{gathered}$$

All these parameters should be of order m_{soft} or m_{soft}^2 depending on their dimension, where m_{soft} is roughly of order 1 TeV. Thus it appears that SUSY breaking introduces a large number of arbitrary parameters in the theory. Fortunately, from experiments testing flavour mixing and CP-violating processes, we can extract a great deal of information on these parameters, reducing the above arbitrariness in a significant way. We will now describe a *universal limit* of the soft parameters, which highly simplifies the squark and slepton mixings. Let the squark and slepton matrices be proportional to the identity in family space:

$$\begin{gathered}(M_Q^2)_{mn} = m_Q^2\delta_{mn}\,, \qquad (M_L^2)_{mn} = m_L^2\delta_{mn}\,, \qquad (M_U^2)_{mn} = m_U^2\delta_{mn}\,, \\ (M_D^2)_{mn} = m_D^2\delta_{mn}\,, \qquad (M_E^2)_{mn} = m_E^2\delta_{mn}\,,\end{gathered} \tag{3.41}$$

and let the $A_{U,\,D,\,E}$ matrices be proportional to the Yukawa couplings that appear in the superpotential,

$$(A_U)_{mn} = a_U(Y_U)_{mn}\,, \qquad (A_D)_{mn} = a_D(Y_D)_{mn}\,, \qquad (A_E)_{mn} = a_E(Y_E)_{mn}\,. \tag{3.42}$$

Then, the mass matrices $M_Q^2,\ldots,M_E^2$ do not give rise to squark and slepton mixing, and A_U, A_D, A_E generate such mixing in an appreciable way only for the third family. This happens because of the much smaller size of the Yukawa couplings for the first two families in comparison with those for the third family, and becomes evident if one takes the limit expressed by (3.38). Furthermore, if we assume that

$$a_U\,,\, a_D\,,\, a_E\,,\, M_1\,, M_2\,,\, M_3 \tag{3.43}$$

are real, the only CP-violating phase in the whole MSSM Lagrangian is contained in the usual Cabibbo-Kobayashi-Maskawa V_{CKM} matrix. It is very important to remark that equations (3.41) - (3.43) should be interpreted as holding at a very high energy scale Q_0, i.e. they are boundary conditions for the renormalization group equations (RGEs). Once we evolve all the parameters of the theory down to the electroweak scale, the above expressions are not exactly true anymore. However, the resulting CP-violating and flavour mixing effects at the EW scale are still compatible with current experimental results.

Another simplification concerns gaugino masses, which are usually assumed to unify at a scale $M_U \approx 2\times 10^{16}$ GeV. In order to justify this assumption, we need to write renormalization group equations for both gaugino masses and gauge coupling constants. The one-loop RGEs for gauge coupling constants read:

$$\frac{dg_i}{dt} = \frac{1}{16\pi^2} b_i g_i^3 \,, \tag{3.44}$$

where $t \equiv Q/Q_0$, with Q the renormalization scale and Q_0 a reference scale. The coupling constants are defined in the following way: $g_3 = g_S$, $g_2 = g$, $g_1 = \sqrt{5/3}\, g'$ [4]. The b_i coefficients are given by

$$(b_1,\, b_2,\, b_3) = (33/5,\, 1,\, -3)\,. \tag{3.45}$$

The RGEs for gaugino masses on the other hand are written as follows:

$$\frac{dM_i}{dt} = \frac{1}{8\pi^2} b_i g_i^2 M_i \,, \tag{3.46}$$

with the same b_i displayed in (3.45). M_3, M_2, M_1 are $SU(3)_c$, $SU(2)_L$, $U(1)_Y$ gaugino masses, respectively. Then, it easily follows, using (3.44) and (3.46), that

$$\frac{d}{dt}\left(\frac{M_i}{g_i^2}\right) = \frac{1}{g_i^2}\frac{dM_i}{dt} - 2\frac{M_i}{g_i^3}\frac{dg_i}{dt} = 0\,. \tag{3.47}$$

Therefore, for each $i = 1, 2, 3$, M_i/g_i^2 is independent of the renormalization scale. It is a very famous prediction of the MSSM that the gauge couplings unify (at a value g_U) at the scale $M_U \approx 2 \times 10^{16}$, so also the masses of the gauginos are usually assumed to unify at M_U:

$$M_1 = M_2 = M_3 = m_{1/2}\,. \tag{3.48}$$

This argument is supported by grand unified theories, in which unification of gaugino masses at M_U can be naturally implemented.

Minimal supergravity scenario for soft terms

In the context of Planck-scale-mediated supersymmetry breaking models, the minimal supergravity scenario leads to an extremely simple structure of soft SUSY-breaking terms: at a scale near to the Planck mass, $Q \approx M_P$, it predicts

$$M_1 = M_2 = M_3 = m_{1/2}\,, \tag{3.49a}$$
$$M_Q^2 = M_U^2 = M_D^2 = M_L^2 = M_E^2 = \mathbf{1} m_0^2\,, \tag{3.49b}$$
$$m_{H_u}^2 = m_{H_d}^2 = m_0^2\,, \tag{3.49c}$$
$$A_U = A_0 Y_U,\; A_D = A_0 Y_D,\; A_E = A_0 Y_E\,, \tag{3.49d}$$
$$b = B_0 \mu\,. \tag{3.49e}$$

In (3.49), $m_{1/2}, B_0, m_0, A_0$ are parameters of order $\sim \frac{\langle F\rangle}{M_P}$, where $\sqrt{\langle F\rangle} \approx 10^{11}$ GeV is the VEV responsible for SUSY breaking. Often, the above conditions are assumed to hold at

[4] This rescaling of g' is necessary in order for the three couplings to have the same $SU(5)$ or $SO(10)$ normalization.

M_U rather than at M_P, so that renormalization group running starts at M_U with (3.49) as boundary conditions. Even though in principle this choice neglects some threshold corrections in the results of the RG flow, nevertheless it is used because gauge coupling unification tells us we can be confident about the RG behavior up to M_U, whereas beyond that scale no clue is presently available.

3.3.4. The scalar potential and its minimization

If we write the full scalar potential V_{MSSM} of the MSSM, we can easily verify that the conditions

$$\frac{\partial V_{MSSM}}{\partial \widetilde{Q}} = \frac{\partial V_{MSSM}}{\partial \widetilde{U}^c} = \frac{\partial V_{MSSM}}{\partial \widetilde{D}^c} = \frac{\partial V_{MSSM}}{\partial \widetilde{L}} = \frac{\partial V_{MSSM}}{\partial \widetilde{E}^c} = 0$$

are satisfied when all sleptons and squarks have zero vacuum expectation values (VEVs), with arbitrary H_u, H_d. Actually, we should also verify that all mass-squared eigenvalues are positive; in the following, we shall assume that this condition is satisfied. Therefore, we can limit our attention to the terms in V_{MSSM} which contain only Higgs scalars. These are:

$$V_{Higgs} = V_D + V_F + V_{soft}\,, \tag{3.50}$$

where

$$V_D = \frac{1}{2} g^2 \sum_a (H_u^* \frac{\sigma^a}{2} H_u + H_d^* \frac{\sigma^a}{2} H_d)^2 + \frac{1}{8} g'^2 (|H_u|^2 - |H_d|^2)^2\,, \tag{3.51}$$

$$V_F = \mu^2 (|H_u|^2 + |H_d|^2)\,, \tag{3.52}$$

$$V_{soft} = (b H_u H_d + h.c.) + m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2\,, \tag{3.53}$$

are respectively the D-term, F-term and soft contributions. Notice that we have taken μ real, as it is usually done, in order to avoid large CP-violating effects. We are interested in vacua which leave $U(1)_{em}$ unbroken, so only neutral scalar fields can get VEVs. Then, because V_{Higgs} does not contain any terms linear in the charged components of H_u, H_d, it is enough to retain terms involving H_u^0, H_d^0 only:

$$V = \frac{1}{8}(g^2+g'^2)(|H_u^0|^2-|H_d^0|^2)^2+(\mu^2+m_{H_u}^2)|H_u^0|^2+(\mu^2+m_{H_d}^2)|H_d^0|^2-(bH_u^0 H_d^0+h.c.)\,. \tag{3.54}$$

Now, by a redefinition of the phase of H_u^0 or H_d^0, we can take b real and positive. Therefore, also $H_u^0 H_d^0$ must be real and positive at the minimum, so $\langle H_u^0 \rangle$ and $\langle H_d^0 \rangle$ must have opposite phases. Then by means of a $U(1)_Y$ gauge transformation we can make $\langle H_u^0 \rangle$ and $\langle H_d^0 \rangle$ both real and positive, since the Higgs supermultiplets have opposite weak hypercharges. In conclusion, $\langle H_u^0 \rangle \equiv v_u/\sqrt{2}$ and $\langle H_d^0 \rangle \equiv v_d/\sqrt{2}$ can be taken both real and positive without any further restrictions.

For the scalar potential to be bounded from below along the so called 'D-flat directions', that is where $v_u = v_d$, the following relation must hold:

$$2\mu^2 + m_{H_u}^2 + m_{H_d}^2 > 2b\,. \tag{3.55}$$

Another condition that must be satisfied is

$$b^2 > (\mu^2 + m_{H_u}^2)(\mu^2 + m_{H_d}^2)\,, \tag{3.56}$$

otherwise, $v_u = v_d = 0$ is a minimum of the potential and the breaking $SU(2)_L \times U(1)_Y \to U(1)_{em}$ does not occur. In fact, when the Higgs scalars acquire VEVs, the quadratic part of V can be written in matrix form as:

$$\frac{1}{2}\begin{pmatrix} v_u & v_d \end{pmatrix}\begin{pmatrix} \mu^2 + m_{H_u}^2 & -b \\ -b & \mu^2 + m_{H_u}^2 \end{pmatrix}\begin{pmatrix} v_u \\ v_d \end{pmatrix}\,, \tag{3.57}$$

and the condition (3.56) corresponds to having a negative eigenvalue for the matrix in (3.57). We are now ready to minimize the potential: the equations

$$\frac{\partial V}{\partial H_u^0} = \frac{\partial V}{\partial H_d^0} = 0$$

read

$$m_{H_u}^2 + \mu^2 - b\cot\beta - \frac{1}{2}M_Z^2\cos(2\beta) = 0\,, \tag{3.58a}$$

$$m_{H_u}^2 + \mu^2 - b\tan\beta + \frac{1}{2}M_Z^2\cos(2\beta) = 0\,. \tag{3.58b}$$

These can be rewritten as

$$\sin(2\beta) = \frac{2b}{m_{H_u}^2 + m_{H_d}^2 + 2\mu^2}\,, \tag{3.59a}$$

$$M_Z^2 = \frac{m_{H_u}^2 - m_{H_d}^2}{\cos(2\beta)} - (m_{H_u}^2 + m_{H_d}^2 + 2\mu^2)\,, \tag{3.59b}$$

where the following definitions have been adopted:

$$\tan\beta = \frac{v_u}{v_d}\,, \tag{3.60}$$

$$v^2 = v_u^2 + v_d^2\,, \tag{3.61}$$

$$M_Z^2 = \frac{1}{4}v^2(g^2 + g'^2)\,. \tag{3.62}$$

Usually, it is assumed that $m_{H_u}^2 = m_{H_d}^2$ holds at tree level at the very high input scale Q_0. Then RG evolution separates the two Higgs mass parameters, pushing $m_{H_u}^2$ down to a negative value at the EW scale, while $m_{H_d}^2$ remains positive. In this way, electroweak symmetry breaking is obtained with the help of radiative corrections, a mechanism known as *radiative electroweak symmetry breaking*.

3.3.5. Mass spectrum

In this paragraph we briefly review the essential properties of the mass spectrum of the MSSM.

Gauge bosons

The Standard Model expressions relating gauge and mass eigenstates for vector bosons after EW symmetry breaking are still valid in the MSSM:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2)\,, \tag{3.63a}$$

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}\,, \tag{3.63b}$$

where θ_w is the weak mixing angle, defined by

$$\tan\theta_w \equiv \frac{g'}{g}\,.$$

The photon A_μ is massless, while

$$M_Z^2 = \frac{1}{4} v^2 (g^2 + g'^2)\,,$$
$$M_W^2 = \frac{1}{4} v^2 g^2\,,$$

where we recall that $v^2 = v_u^2 + v_d^2$. The electric charge is identified by

$$g \sin\theta_w = g' \cos\theta_w = e\ .$$

The Higgs sector

The total number of real degrees of freedom contained in the two Higgs $SU(2)_L$-doublets is 8 (4 complex scalar fields). Among these, 3 CP-odd scalars G^0, $G^\pm$ are 'eaten' by the gauge bosons Z, $W^\pm$ via the Higgs mechanism when the electroweak symmetry breaks. The remaining 5 degrees of freedom form 5 mass eigenstates as follows: a couple of charged scalars $H^\pm$, and three neutral scalars, namely the CP-even h^0 and H^0, and the CP-odd A^0. Interaction and mass eigenstates are related by:

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}\,,$$

$$\begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} = \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}\,,$$

so that the quadratic part of the scalar potential V in the new (mass) basis is diagonal:

$$V_{quadratic} = \frac{1}{2} m_{h^0}^2 (h^0)^2 + \frac{1}{2} m_{H^0}^2 (H^0)^2 + \frac{1}{2} m_{A^0}^2 (A^0)^2 + m_{H^\pm}^2 |H^+|^2\,.$$

We recall that β was defined in (3.60), while α is defined by

$$\frac{\sin 2\alpha}{\sin 2\beta} = -\frac{m_{H^0}^2 + m_{h^0}^2}{m_{H^0}^2 + m_{h^0}^2}\,.$$

The masses of the Higgs scalars are, at the classical level:

$$m^2_{A^0} = \frac{2b}{\sin(2\beta)}\,, \tag{3.64a}$$

$$m^2_{h^0,H^0} = \frac{1}{2}\left(m^2_{A_0} + M^2_Z \mp \sqrt{(m^2_{A_0} + M^2_Z)^2 - 4M^2_Z m^2_{A^0}\cos^2(2\beta)}\right)\,, \tag{3.64b}$$

$$m^2_{H^\pm} = m^2_{A^0} + m^2_W\,. \tag{3.64c}$$

By using (3.64), we can easily verify the following inequality:

$$m^2_{h^0} \leq M^2_Z \cos^2(2\beta)\,, \tag{3.65}$$

that is, the tree-level mass of the lightest Higgs must be less than approximately 91.2 GeV, for any β. This inequality may look like a disaster, but fortunately this is not the whole story. In fact, the h^0 mass receives large 1-loop radiative corrections from top and stop (top squark) loops, so the LEP constraint $m_{Higgs} \geq 114.4$ GeV can indeed be satisfied.

Neutralinos and charginos

Because the $SU(3)_c \times SU(2)_L \times U(1)_Y$ symmetry is broken to $SU(3)_c \times U(1)_{em}$, mixing is possible between any fields of the same spin transforming in the same representation of $SU(3)_c \times U(1)_{em}$. Another restriction is imposed by R-parity: two fields can mix only if they are both even or both odd under R-parity. The effects of mixing in the MSSM are described in this paragraph for Higgsinos and gauginos, and in the following for squarks and sleptons.

The neutral Higgsinos $\widetilde{H}^0_u$ and $\widetilde{H}^0_d$ mix with the neutral gauginos $\widetilde{W}^3$ and $\widetilde{B}$ to yield four mass eigenstates called neutralinos, $\widetilde{N}_i$, $i = 1, \ldots, 4$. The mass term is given in the interaction eigenstate basis by

$$\mathcal{L}_{\widetilde{N}mass} = -\frac{1}{2}\begin{pmatrix}\widetilde{B} & \widetilde{W}^3 & \widetilde{H}^0_d & \widetilde{H}^0_u\end{pmatrix} M_{\widetilde{N}} \begin{pmatrix}\widetilde{B} \\ \widetilde{W}^3 \\ \widetilde{H}^0_d \\ \widetilde{H}^0_u\end{pmatrix} + h.c.\,, \tag{3.66}$$

where

$$M_{\widetilde{N}} = \begin{pmatrix} M_1 & 0 & -g'v_d/2 & g'v_u/2 \\ 0 & M_2 & gv_d/2 & -gv_u/2 \\ -g'v_d/2 & gv_d/2 & 0 & -\mu \\ g'v_u/2 & -gv_u/2 & -\mu & 0 \end{pmatrix}\,. \tag{3.67}$$

In (3.67), gaugino-gaugino terms come from the soft part of the Lagrangian; Higgsino-Higgsino entries originate from the $\mu H_u H_d$ term in the superpotential; finally, gaugino-Higgsino terms come from [gaugino][Higgsino][Higgs] interactions (derived from the trilinear terms in the superpotential) after the EW symmetry is broken and the Higgs scalars acquire VEVs. We recall that we have chosen v_u, v_d (as well as b) both real and positive, without imposing any further restrictions; in addition, we have (arbitrarily) taken μ real, to avoid large CP-violating effects. In $M_{\widetilde{N}}$, two *a priori* complex quantities appear, the soft masses M_1 and M_2; however, their phases can be absorbed by a redefinition of the fields $\widetilde{W}^3$ and $\widetilde{B}$.

In this way, all the entries of $M_{\widetilde{N}}$ are real. Diagonalization of $M_{\widetilde{N}}$ gives the four neutralino mass eigenstates. Within the framework of the MSSM (with R-parity exactly conserved) the lightest neutralino, which we conventionally name $\widetilde{N}_1$, is usually the lightest supersymmetric particle.

Let us now turn our attention to the charged winos $\widetilde{W}^{\pm}$ and Higgsinos $\widetilde{H}_u^+$,$\widetilde{H}_d^-$, which mix forming four charged mass eigenstates, the charginos $\widetilde{C}_i^{\pm}$, $i = 1, 2$. The mass term in the interaction basis is

$$\mathcal{L}_{\widetilde{C}mass} = -\frac{1}{2}\begin{pmatrix}\widetilde{W}^+ & \widetilde{H}_u^+ & \widetilde{W}^- & \widetilde{H}_d^-\end{pmatrix} M_{\widetilde{C}} \begin{pmatrix}\widetilde{W}^+ \\ \widetilde{H}_u^+ \\ \widetilde{W}^- \\ \widetilde{H}_d^-\end{pmatrix} + h.c. \,, \tag{3.68}$$

where the mass matrix is

$$M_{\widetilde{C}} = \begin{pmatrix} 0 & 0 & M_2 & gv_d/\sqrt{2} \\ 0 & 0 & gv_u/\sqrt{2} & \mu \\ M_2 & gv_u/\sqrt{2} & 0 & 0 \\ gv_d/\sqrt{2} & \mu & 0 & 0 \end{pmatrix} . \tag{3.69}$$

Analogously to the neutralino case, the entries of $M_{\widetilde{C}}$ come from soft terms, the $\mu H_u H_d$ term in the superpotential, and trilinear [Higgsino][wino][Higgs] interactions after EW symmetry breaking.

Squarks and sleptons

In principle, to find the mass eigenstates we should diagonalize a 3×3 matrix for sneutrinos (we recall that the MSSM does not include right-handed neutrinos) and three 6×6 matrices, one each for up squarks, down squarks and sleptons. However, mixing effects are much more important for third-family squarks and sleptons, because of the contributions to RGEs coming from the large Yukawa couplings and from soft trilinear couplings (we are assuming that the soft trilinear coupling matrices are proportional to Yukawa couplings, see (3.42)). Considering as an example the up-type squarks, and taking the limit expressed by (3.38), if we start from $(M_Q^2)_{mn} = m_Q^2 \delta_{mn}$ at the large input scale, we get at low energy

$$M_Q^2 \approx \begin{pmatrix} m_{Q\,1}^2 & 0 & 0 \\ 0 & m_{Q\,1}^2 & 0 \\ 0 & 0 & m_{Q\,3}^2 \end{pmatrix} , \tag{3.70}$$

i.e. the soft masses of the squarks of the first two families remain approximately degenerate, while that of the third-family squark gets renormalized differently, due to non-negligible Yukawa couplings. The same happens for M_U^2, and also for down-type squarks and for sleptons. Therefore, the scalars of the first two families have negligible mixing, thus giving rise to seven approximately degenerate pairs:

$$(\widetilde{e}_L, \widetilde{\mu}_L),\ (\widetilde{e}_R, \widetilde{\mu}_R),\ (\widetilde{u}_L, \widetilde{c}_L),\ (\widetilde{u}_R, \widetilde{c}_R),\ (\widetilde{d}_L, \widetilde{s}_L),\ (\widetilde{d}_R, \widetilde{s}_R),\ (\widetilde{\nu}_e, \widetilde{\nu}_\mu)\,.$$

Notice that the masses of the pairs $(\widetilde{u}_L, \widetilde{c}_L)$ and $(\widetilde{d}_L, \widetilde{s}_L)$ are different, even though both share the same soft mass matrix (M_Q^2). This difference originates from D-term quartic $[\text{squark}]^2[\text{Higgs}]^2$ interactions after Higgs scalars acquire VEVs. The same happens for sleptons, for which quartic $[\text{slepton}]^2[\text{Higgs}]^2$ terms lead to different masses for the pairs $(\widetilde{e}_L, \widetilde{\mu}_L)$ and $(\widetilde{\nu}_e, \widetilde{\nu}_\mu)$, even though they share the same soft mass matrix M_L^2. The general form for these quartic 'mass fine splittings' is, for a generic squark or slepton φ:

$$\Delta m_\varphi^2 = \frac{1}{4}(v_d^2 - v_u^2)[g^2 T_{3L}(\varphi) - g'^2 Y(\varphi)]\,, \tag{3.71}$$

where $T_{3L}(\varphi)$, $Y(\varphi)$ are the eigenvalue of the third generator of $SU(2)_L$ and the hypercharge of the left-handed chiral supermultiplet to which the scalar φ belongs, respectively. It is clear that this contribution is different for $\widetilde{u}_L$ and $\widetilde{d}_L$, and it is also different for $\widetilde{e}_L$ and $\widetilde{\nu}_e$. The alert reader will be aware that other contributions to squark and slepton masses exist, in addition to the soft mass terms and to the quartic D-terms in (3.71). Among these are F-term $[\text{squark}]^2[\text{Higgs}]^2$ and $[\text{squark}]^2[\text{Higgs}]$ interactions, and the analogous for sleptons. However, these mass terms are proportional to Y^2 and Y respectively, where Y stands for the generic Yukawa coupling matrix element. We already know that Yukawa couplings are negligible for the first two families, so these contributions can be omitted. Finally, there is one last mass term for squarks and sleptons, coming from $[\text{squark}, \text{slepton}]^2[\text{Higgs}]$ soft terms, but again these are proportional to the Yukawa couplings due to equation (3.42), and are therefore negligible. In conclusion, for the first two families only soft masses and D-term $[\text{squark}, \text{slepton}]^2[\text{Higgs}]^2$ masses are relevant.

For third-family squarks and sleptons instead, all the above contributions must be taken into account, and they result in significant mixing. Let us analyze in detail the stop mass matrix, as an instructive example. The stop mass terms can be written in the following way:

$$\mathcal{L}_{\widetilde{t}\,mass} = -\begin{pmatrix} \widetilde{t}_L^* & \widetilde{t}_R^* \end{pmatrix} M_{\widetilde{t}}^2 \begin{pmatrix} \widetilde{t}_L \\ \widetilde{t}_R \end{pmatrix}\,. \tag{3.72}$$

Our task is to compute $M_{\widetilde{t}}^2$. We will analyze separately diagonal and off-diagonal terms.

- *Diagonal terms* Three contributions are present: the first is given by soft masses, for which we assume that M_Q^2 is given by (3.70), whereas

  $$M_U^2 \approx \begin{pmatrix} m_{U\,1}^2 & 0 & 0 \\ 0 & m_{U\,1}^2 & 0 \\ 0 & 0 & m_{U\,3}^2 \end{pmatrix}\,.$$

  In the case under consideration, only the (3,3) entries contribute. Next, there are D-terms of the type (3.71), which we have already discussed. Thirdly, we have F-terms: the relevant part of the superpotential is

  $$\delta W_{diag} = (Y_U)_{ij} Q_i H_u U_j^c = y_t(\widetilde{t}_L H_u^0 - H_u^+ \widetilde{b}_L)\widetilde{t}_R^*$$

  where we have assumed that (3.38) holds. Among the terms generated in the Lagrangian there is, after Higgs fields acquire VEVs:

  $$-\widetilde{t}_L^* \widetilde{t}_L \frac{v_u^2 y_t}{2} = -\widetilde{t}_L^* \widetilde{t}_L m_t^2$$

(m_t is the top mass).

- *Off-diagonal terms* Two contributions must be taken into account. First, we have trilinear soft terms:

$$\mathcal{L}_{3\,soft} = -(A_U)_{ij}\widetilde{Q}_i\widetilde{U}^c_j H_u + h.c. = -a_U y_t \frac{v_u}{\sqrt{2}}\widetilde{t}_L\widetilde{t}^*_R + h.c.$$

where equations (3.42) and (3.38) have been used. The second mass term is a trilinear F-term, obtained from the following part of the superpotential:

$$\delta W_{off\,diag} = (Y_U)_{ij}Q_i H_u U^c_j + \mu H_u H_d = y_t(\widetilde{t}_L H^0_u - H^+_u\widetilde{b}_L)\widetilde{t}^*_R + \mu(H^+_u H^-_d - H^0_u H^0_d)\,.$$

From this we get in the Lagrangian, after EW symmetry breaking:

$$\mu\frac{v_d}{\sqrt{2}}y_t\widetilde{t}^*_L\widetilde{t}_R + h.c.$$

(notice the sign, which is different from that of all the other terms computed above).

To summarize, the stop mass matrix reads

$$M^2_{\widetilde{t}} = \begin{pmatrix} m^2_{Q\,3} + \Delta_{\widetilde{t}_L} + m^2_t & \frac{y_t}{\sqrt{2}}(a_U v_u - \mu v_d) \\ \frac{y_t}{\sqrt{2}}(a_U v_u - \mu v_d) & m^2_{U\,3} + \Delta_{\widetilde{t}_R} + m^2_t \end{pmatrix}. \tag{3.73}$$

The mass eigenstates $\widetilde{t}_1$, $\widetilde{t}_2$ are found by diagonalizing (3.73),

$$\begin{pmatrix} \widetilde{t}_1 \\ \widetilde{t}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\widetilde{t}} & -\sin\theta_{\widetilde{t}} \\ \sin\theta_{\widetilde{t}} & \cos\theta_{\widetilde{t}} \end{pmatrix} \begin{pmatrix} \widetilde{t}_L \\ \widetilde{t}_R \end{pmatrix}, \tag{3.74}$$

with $\theta_{\widetilde{t}}$ a mixing angle. Sbottom and stau mixing can be treated similarly.

3.4. Kinetic mixing in $U(1)$ extensions of the MSSM

In this section we introduce kinetic mixing between two Abelian factors in a supersymmetric gauge theory, applying the results of [57]. We will see that, because of the relations between particles of different spins imposed by SUSY, other effects appear, besides those described in the non-supersymmetric case. The usual form of the Abelian kinetic terms, including kinetic mixing

$$-\frac{1}{4}F^{\mu\nu}_a F_{a\,\mu\nu} - \frac{1}{4}F^{\mu\nu}_b F_{b\,\mu\nu} + \frac{k}{2}F^{\mu\nu}_a F_{b\,\mu\nu}\,, \tag{3.75}$$

generalizes to

$$\mathcal{L}_{gauge} = \frac{1}{2}\int d^2\theta\,\{\mathcal{W}_a\mathcal{W}_a + \mathcal{W}_b\mathcal{W}_b - 2k\mathcal{W}_a\mathcal{W}_b\}\,, \tag{3.76}$$

where $\mathcal{W}_\alpha = -\frac{1}{4}\overline{DD}D_\alpha V$, and a, b are group indices, labeling the two Abelian factors $U(1)_a \times U(1)_b$. The mixing term $\mathcal{W}_a\,\mathcal{W}_b$ can be introduced, because each $\mathcal{W}$ is gauge invariant in the Abelian case. [5] The integral over Grassmann numbers in (3.76) is normalized in a way such

[5]Notice that in general the field strength has the expression (3.27), which reduces to $-\frac{1}{4}\overline{DD}D_\alpha V$ in the Abelian case.

that $\int d^2\theta(\theta\theta) = 1$, explictly $d^2\theta = -\frac{1}{4}d\theta^\alpha d\theta^\beta \epsilon_{\alpha\beta}$. The rules for integration over Grassmann numbers can be found, e.g., in [55], or in [58]. For definiteness, the reader can think of $U(1)_a$ as the SM hypercharge, and of $U(1)_b$ as an additional $U(1)$ symmetry (e.g., $B-L$). Kinetic mixing is removed by the redefinition [6]

$$\begin{pmatrix} V_a' \\ V_b' \end{pmatrix} = \begin{pmatrix} 1 & -k \\ 0 & \sqrt{1-k^2} \end{pmatrix} \begin{pmatrix} V_a \\ V_b \end{pmatrix} , \tag{3.77}$$

where V_a, V_b are the vector superfields associated to $U(1)_a$ and $U(1)_b$ respectively. In the new basis, the gauge Lagrangian reads

$$\mathcal{L}_{gauge} = \frac{1}{2} \int d^2\theta \left\{ \mathcal{W}_a' \mathcal{W}_a' + \mathcal{W}_b' \mathcal{W}_b' \right\} , \tag{3.78}$$

while the interaction part transforms in the following way

$$\begin{aligned} \mathcal{L}_{int} &= \int d^4\theta \left\{ \phi_i^\dagger e^{2g_a V_a + 2g_b V_b} \phi_i \right\} \\ &= \int d^4\theta \left\{ \phi_i^\dagger e^{2g_a V_a' + 2g_a \widetilde{k} V_b' + 2g_b' V_b'} \phi_i \right\} , \end{aligned} \tag{3.79}$$

where $d^4\theta = d^2\theta\, d^2\overline{\theta}$, and the following definitions have been adopted:

$$\widetilde{k} = \frac{k}{\sqrt{1-k^2}} , \qquad g_b' = \frac{g_b}{\sqrt{1-k^2}} . \tag{3.80}$$

ϕ_i ($i = 1, \ldots N$) are the chiral superfields of the theory. We now work out explicitly the consequences of (3.79).

(i) In the basis where kinetic terms are canonical, the covariant derivative has the form:

$$D^\mu \phi_i = \left(\partial^\mu - i g_a Q_a^i V_a'^\mu - i(g_b' Q_b^i + g_a \widetilde{k} Q_a^i) V_b'^\mu \right) \phi_i , \tag{3.81}$$

an effect we studied in detail in the non-SUSY case.

(ii) Transforming the gauginos gives rise to a term

$$-i\sqrt{2} g_a \widetilde{k} z_i^\dagger Q_a^i \psi_i \lambda_b' + h.c. , \tag{3.82}$$

where z_i, ψ_i are the scalar and fermion components of the chiral superfield ϕ_i, respectively. Thus, the $U(1)_b$ gaugino couples to fields charged under $U(1)_a$, with coupling proportional to the mixing parameter $\widetilde{k}$.

(iii) The part of the Lagrangian containing D-terms reads

$$\frac{1}{2} D_a' D_a' + g_a D_a' \sum_i Q_a^i |z_i|^2 + \widetilde{k} g_a D_b' \sum_i Q_a^i |z_i|^2 + \frac{1}{2} D_b' D_b' + g_b' D_b' \sum_i Q_b^i |z_i|^2 , \tag{3.83}$$

[6] Differently from [57], we do not work at leading order in k.

from which the equations of motion are extracted:

$$D'_a = -\, g_a \sum_i Q_a^i |z_i|^2 \,, \tag{3.84}$$

$$D'_b = -\, g'_b \sum_i Q_b^i |z_i|^2 - \widetilde{k} g_a \sum_i Q_a^i |z_i|^2 \,. \tag{3.85}$$

The D-term scalar potential reads

$$V_D = \frac{1}{2} D'_a D'_a + \frac{1}{2} D'_b D'_b \,. \tag{3.86}$$

Among the extra-MSSM terms contained in V_D, there are mass terms for the MSSM scalars. In Section 3.6, we will analyze the effects of these additional scalar masses in the context of a minimal SUSY $U(1)'$ model, where the additional Abelian symmetry is given by $B - L$, which was proposed in [59] (see also [60] for an earlier, very similar model).

3.5. The Khalil-Masiero model

In this model, an additional $U(1)_{B-L}$ factor is added to the gauge group, and the Higgs sector is expanded to include two SM-singlet, $(B - L)$-charged fields $\chi_1\,, \chi_2$, which realize the spontaneous breaking of $(B - L)$ via their VEVs. In addition to the MSSM fermions, three generations of RH neutrinos are considered. The $B - L$ symmetry is *radiatively* broken, in a way very similar to the radiative electroweak symmetry breaking we mentioned in the introduction to the MSSM: the RGEs act differently on the masses of the two new Higgses, in such a way that one of them remains positive at the TeV scale, while the other one is turned to negative values by the RG evolution, thus helping to break $U(1)_{B-L}$. Notice that, similarly to what happens with the MSSM Higgs doublets, when χ_1 is added to the Lagrangian we are automatically forced to include another singlet Higgs field χ_2, with opposite $(B - L)$ charge, to cancel the anomalies introduced by Higgsinos.

We take $\chi_1 \sim (\mathbf{1}, 0, -2)$ and $\chi_2 \sim (\mathbf{1}, 0, +2)$ under $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, and we assume the following form for the superpotential:

$$W = W_{MSSM} + (Y_\nu) L H_u N^c + \mu' \chi_1 \chi_2 + (Y_N) N^c N^c \chi_1 \,.$$

The last term would not respect gauge invariance, if we chose a different value for the $(B-L)$ charges of $\chi_{1,2}$. We see immediately that both Dirac and Majorana masses are generated in this way, leading to a see-saw mechanism. A remark is in order here. Because what turns the value of $m^2_{\chi_1}$ from positive values at the GUT scale to negative values at the TeV scale is actually the Yukawa coupling Y_N, it follows that the latter parameter cannot be too small, i.e. it should be $Y_N = O(1)$. As a consequence, to obtain viable neutrino masses a value $Y_\nu \approx 10^{-5}$ is needed [7] (not very far from the electron Yukawa coupling in the SM). A detailed and recent discussion of the see-saw mechanism in this framework is contained in [32].

[7] Notice that this would also imply TeV-scale RH neutrinos, enriching the phenomenology potentially under reach of the LHC.

We now turn our attention to the scalar potential. It can be written as

$$V_{H_u,H_d} + V_{\chi_1,\chi_2}\,, \tag{3.87}$$

i.e., no mixing terms appear between the Higgs doublets and singlets. Therefore, the minimization of V_{H_u,H_d} proceeds exactly as in the MSSM, while for the singlets we have

$$V_{\chi_1,\chi_2} = 2g_{BL}^2(|\chi_2|^2-|\chi_1|^2)^2+\mu'^2(|\chi_1|^2+|\chi_2|^2)+m_{\chi_1}^2|\chi_1|^2+m_{\chi_2}^2|\chi_2|^2-(b'\chi_1\chi_2+h.c.) \tag{3.88}$$

with $b' > 0$, and where g_{BL} is the $U(1)_{B-L}$ coupling constant. The following conditions must be satisfied:

$$(\mu'^2 + m_{\chi_1}^2) + (\mu'^2 + m_{\chi_2}^2) > 2b' \tag{3.89}$$

and

$$(\mu'^2 + m_{\chi_1}^2)(\mu'^2 + m_{\chi_2}^2) < b'^2\,. \tag{3.90}$$

Minimization gives

$$\sin(2\beta') = \frac{2b'}{2\mu'^2 + m_{\chi_1}^2 + m_{\chi_2}^2}\,, \qquad M_{Z'}^2 = \frac{m_{\chi_1}^2 - m_{\chi_2}^2}{\cos(2\beta')} - (m_{\chi_1}^2 + m_{\chi_2}^2 + 2\mu'^2)\,, \tag{3.91}$$

where the following definitions have been adopted:

$$\langle\chi_1\rangle = \frac{v_1}{\sqrt{2}}\,, \qquad \langle\chi_2\rangle = \frac{v_2}{\sqrt{2}}\,, \qquad M_{Z'}^2 = 4v_{BL}^2 g_{BL}^2\,, \qquad \tan\beta' = \frac{v_1}{v_2}\,, \tag{3.92}$$

with $v_{BL}^2 = v_1^2+v_2^2$. $M_{Z'}$ is the physical mass of the Z': its expression is easily obtained in the framework of Chapter 2, after noting that, following [59], we are neglecting kinetic mixing. In [59] it was shown that with the given field content and superpotential, radiative breaking of $B-L$ really occurs, through the RG evolution of $m_{\chi_1}^2$. The two complex scalar fields χ_1, χ_2 contain 4 real degrees of freedom, one of which is absorbed via the Higgs mechanism to give a mass to the Z'. The remaining 3 degrees of freedom are physical: they form a CP-odd scalar A' and a couple of CP-even scalars H', h'. Their tree level masses are given by expressions similar to those of the MSSM neutral Higgs scalars (see [59]). Similarly to that case, the following inequality holds:

$$m_{h'}^2 \leq M_{Z'}^2 \cos^2(2\beta')\,,$$

but radiative corrections (mainly RH neutrino and RH sneutrino loops) can overcome this bound, leading to $m_{h'} > M_{Z'}$.

3.6. Kinetic mixing in the KM model

We now introduce kinetic mixing in the Khalil-Masiero model discussed in the previous section, focusing first on the running of coupling constants, and then on the extra contributions to scalar masses arising from D-terms.

3.6.1. Running of coupling constants

In Section 2.6, we discussed the region of the plane $(\widetilde{g}_Y, \widetilde{g}_{BL})$ preferred by grand unification in the minimal model: it is immediate to realize that the scenario (iv) described there corresponds to the Khalil-Masiero model in the presence of kinetic mixing. Looking at Fig. 2.17, we see that if we start from a pure $B-L$ model at M_U, the mixing effects in the RGEs generate sizable corrections to the effective weak-scale couplings. Therefore, the Khalil-Masiero model is not stable under RG evolution, and the assumption of negligible kinetic mixing, while technically possible, is not natural. This is in contrast to what happens for the χ model: the corresponding direction in the $(\widetilde{g}_Y, \widetilde{g}_{BL})$ plane is quite stable with respect to the RGEs, because being $U(1)_\chi$ and $U(1)_Y$ orthogonal over the **16** representation of $SO(10)$ (which includes all the fermions of the minimal SM plus one RH neutrino), the only contributions to the mixing come from the Higgs superfields. On the contrary, in the case of $(B-L)$ the matter fields do contribute, generating significant mixing and destabilizing the direction in the $(\widetilde{g}_Y, \widetilde{g}_{BL})$ plane.

We remark that, because gauge bosons and gauginos share the same kinetic matrix, the effects of kinetic mixing on the mass matrix of gauge bosons that we extensively discussed in Chapter 2 also apply to the gaugino mass matrix. This may have effects on the spectrum of the neutralinos and therefore on the phenomenology of the model (we recall that 7 neutralino states are present, due to the introduction of the $B-L$ gaugino and of the Higgsinos in χ_1 and χ_2).

3.6.2. Scalar Masses

We now focus on the effects of kinetic mixing on the scalar sector: in the Khalil-Masiero model, the D-term scalar potential for the Abelian factors (3.86) takes the form (we make the identifications $a = Y$ and $b = (B-L)$):

$$V_D = \frac{1}{2} g'^2 (1+\widetilde{k}^2) \sum_{ij} Y^i Y^j |z^i|^2 |z^j|^2 + \frac{1}{2} g'^2_{BL} \sum_{ij} Q^i_{BL} Q^j_{BL} |z^i|^2 |z^j|^2 + g'_{BL} g' \widetilde{k} \sum_{ij} Q^j_{BL} Y^i |z^i|^2 |z^j|^2 , \quad (3.93)$$

in a self-explanatory notation (i, j run over all the complex scalars of the model). We discuss two effects of eq.(3.93): first the contribution to the scalar potential for neutral Higgses and the consequences on its minimization, and second the new mass terms for the MSSM scalars.

Higgs scalar potential

The terms containing Higgs scalars in (3.93) read

$$\frac{1}{8} g'^2 (1+\widetilde{k}^2)(|H_u|^2 - |H_d|^2)^2 + 2 g'_{BL}(|\chi_2|^2 - |\chi_1|^2)^2 + g'_{BL} g' \widetilde{k} (|H_u|^2 - |H_d|^2)(|\chi_2|^2 - |\chi_1|^2) , \quad (3.94)$$

therefore the full scalar potential for neutral Higgses has the following expression:

$$\begin{aligned} V \; = \; & \frac{1}{8}[g^2+g'^2(1+\widetilde{k}^2)](|H_u^0|^2-|H_d^0|^2)^2+(\mu^2+m_{H_u}^2)|H_u^0|^2+(\mu^2+m_{H_d}^2)|H_d^0|^2-(bH_u^0H_d^0+h.c.) \\ & + 2{g'_{BL}}^2(|\chi_2|^2-|\chi_1|^2)^2+(\mu'^2+m_{\chi_1}^2)|\chi_1|^2+(\mu'^2+m_{\chi_2}^2)|\chi_2|^2-(b'\chi_1\chi_2+h.c.) \\ & + g'_{BL}g'\widetilde{k}(|H_u^0|^2-|H_d^0|^2)(|\chi_2|^2-|\chi_1|^2)\,. \end{aligned} \tag{3.95}$$

We see that now mixing terms arise between doublets and singlets: the minimization of V is therefore different from that performed in [59]. The consistency conditions (3.89) and (3.90) are still valid. Minimization yields:

$$\sin(2\beta) = \frac{2b}{2\mu^2+m_{H_u}^2+m_{H_d}^2}\,, \tag{3.96}$$

$$\sin(2\beta') = \frac{2b'}{2\mu'^2+m_{\chi_1}^2+m_{\chi_2}^2}\,, \tag{3.97}$$

$${M_Z^0}^2 = \frac{m_{H_u}^2-m_{H_d}^2}{\cos(2\beta)} - \frac{2b}{\sin(2\beta)} + g'_{BL}g'\widetilde{k}v_{BL}^2\frac{\cos(2\beta')}{\cos(2\beta)} - \frac{1}{4}v^2g'^2\widetilde{k}^2\,, \tag{3.98}$$

$${M_{Z'}^0}^2 = \frac{m_{\chi_1}^2-m_{\chi_2}^2}{\cos(2\beta')} - \frac{2b'}{\sin(2\beta')} - g'_{BL}g'v^2\widetilde{k}\frac{\cos(2\beta)}{\cos(2\beta')}\,, \tag{3.99}$$

where we have made the definitions

$${M_Z^0}^2 = \frac{1}{4}v^2(g^2+g'^2)\,, \qquad {M_{Z'}^0}^2 = 4v_{BL}^2{g'}_{BL}^2\,. \tag{3.100}$$

$M_{Z'}^0$ is the Z' mass in the pure $(B-L)$ model, where $\widetilde{k}=0$. The first two minimization conditions are not affected by kinetic mixing, being identical to those found in the previous section for $\widetilde{k}=0$.

D-term contributions to squark and slepton masses

We now analyze how the masses of MSSM scalars are affected by extra-MSSM D-terms in eq. (3.93). The first term in (3.93), in the $\widetilde{k}\to 0$ limit, is the MSSM D-term scalar potential, which together with the analogous term for $SU(2)_L$ forms the mass term for scalars in eq. (3.71). Kinetic mixing gives a correction of order $\widetilde{k}^2$ to this term.

The second term is the $(B-L)$ D-term. The VEVs of $\chi_{1,2}$ generate masses for squarks and sleptons, which are charged under $B-L$. Because $v_{1,2}=O(\text{TeV})$, this is an important contribution to the scalar masses of the MSSM fields.

The last term is linear in the kinetic mixing parameter $\widetilde{k}$. The VEVs of the Higgs singlets contribute to the masses of the scalars that have a nonzero Y charge. If the mixing is not small, then this term can be of the same strength as the $(B-L)$ D-term. Also, the VEVs of H_u, H_d contribute to the masses of the $(B-L)$-charged scalars.

As an example, we write the selectron mass matrix:

$$M_{\widetilde{e}}^2 = \begin{pmatrix} m_{L\,1}^2+\Delta m_{\widetilde{e}_L}^2(\widetilde{k})+m_{eD}^2(\widetilde{e}_L) & 0 \\ 0 & m_{E\,1}^2+\Delta m_{\widetilde{e}_R}^2(\widetilde{k})+m_{eD}^2(\widetilde{e}_R) \end{pmatrix}\,, \tag{3.101}$$

where $m^2_{L\,1}$, $m^2_{E\,1}$ are soft masses, while $\Delta m^2_{\widetilde{e}_L}(\widetilde{k})$, $\Delta m^2_{\widetilde{e}_R}(\widetilde{k})$ are given by (3.71) with $Y(\varphi) \to (1+\widetilde{k}^2)Y(\varphi)$; finally

$$m^2_{eD}(\widetilde{e}_L) = g'^2_{BL}(v_1^2 - v_2^2) + \frac{1}{2} g'_{BL}\widetilde{k}g'(v_1^2 - v_2^2) + \frac{1}{4} g'_{BL}g'\widetilde{k}(v_d^2 - v_u^2)\,, \tag{3.102}$$

$$m^2_{eD}(\widetilde{e}_R) = g'^2_{BL}(v_2^2 - v_1^2) + g'_{BL}\widetilde{k}g'(v_2^2 - v_1^2) + \frac{1}{4} g'_{BL}g'\widetilde{k}(v_u^2 - v_d^2)\,. \tag{3.103}$$

In eq. (3.101), we have neglected all terms containing Yukawa couplings, because for the first two generations these are very small. This example shows how scalar masses are significantly affected by D-terms introduced by the new Abelian symmetry $B-L$ and by kinetic mixing between Y and $B-L$. As a consequence, the requirement of having a realistic scalar spectrum (to begin with, the squared masses of physical scalars must be positive) can impose additional constraints on the parameters of the model (here g'_{BL}, $\widetilde{k}$ and the VEVs v_1, v_2).

In conclusion, we have shown that kinetic mixing is relevant, and it would be interesting to study its effects on the model, e.g. checking whether radiative breaking of $U(1)_{B-L}$ is still achieved, and analyzing in detail the full scalar sector. This is left for future research.

3.7. $U(1)'$ models with spontaneous R-parity breaking

In this section we study some $U(1)$ extensions of the MSSM, which are characterized by the fact that the extra Abelian symmetry is broken by a VEV acquired by RH scalar neutrinos, therefore also R-parity is spontaneously broken. This has relevant consequences on the predicted phenomenology, because large mixing patterns between particles and sparticles arise. Besides this, when R-parity is broken, the LSP is not a good candidate for CDM anymore, so models in this class have to rely on the gravitino to obtain acceptable dark matter scenarios.

3.7.1. $(B-L)$ model

In this model [61, 62], no enlargement of the Higgs sector is performed: we have still only the two doublets H_u , H_d, as in the MSSM. Three RH neutrino chiral superfields are introduced: they are singlets under the SM, but charged under the new Abelian symmetry $U(1)_X$. X is chosen to be a linear combination of hypercharge and $(B-L)$: $X = aY + b(B-L)$. This parameterization takes into account kinetic mixing: we assume to have already performed the change of basis which makes Abelian kinetic terms canonical. The $U(1)_X$ symmetry is broken by VEVs acquired by RH scalar neutrinos, which are neutral under the SM but charged under $B-L$. In addition, the presence of superpotential couplings including Higgs, LH-neutrino and RH-neutrino superfields forces also the LH sneutrinos to acquire a VEV. Thus, also R-parity is spontaneously broken, because $P_R = -1$ for sleptons, as we have seen. The breaking of R-parity has some distinctive consequences, which we will describe later in this paragraph.

To obtain the full Lagrangian of the model, we need to specify the superpotential:

$$W = W_{MSSM} + Y_\nu L_\alpha (i\sigma^2)^{\alpha\beta} H_{u\,\beta} N^c\,, \tag{3.104}$$

whereas the extra-MSSM soft terms are

$$V_{soft}^{new} = M_N^2 |\widetilde{N}^c|^2 + (\frac{1}{2} M_{BL} \widetilde{B}' \widetilde{B}' + A_\nu \widetilde{L}\, H_u \widetilde{N}^c + h.c.)\,. \tag{3.105}$$

After the Higgses and one generation of sneutrinos acquire VEVs as follows:

$$\langle H_u^0 \rangle = \frac{v_u}{\sqrt{2}}\ ,\ \langle H_d^0 \rangle = \frac{v_d}{\sqrt{2}}\ ,\ \langle \widetilde{\nu} \rangle = \frac{v_L}{\sqrt{2}}\ ,\ \langle \widetilde{\nu}^c \rangle = \frac{v_R}{\sqrt{2}}\,,$$

the minimization of the scalar potential can be performed in a general way. However, the results turn out to be very simple if we make the approximation v_R, v_u, $v_d >> v_L$ and in addition we take Y_ν small (both are clearly reasonable approximations): in this case, we get

$$v_R^2 \approx \frac{-8M_N^2 - abg_X^2(v_u^2 - v_d^2)}{g_X^2 b^2}\,, \tag{3.106}$$

$$v_L = \frac{B_\nu v_R}{M_{\widetilde{L}}^2 - \frac{1}{8} g_X^2 (a+b) b v_R^2 - D_{ew}^2}\,, \tag{3.107}$$

where the following definitions have been made:

$$B_\nu = \frac{1}{\sqrt{2}}(Y_\nu \mu v_d - A_\nu v_u)\,, \tag{3.108}$$

$$D_{ew}^2 = \frac{1}{8}(g^2 + g'^2 + a(a+b) g_X^2)(v_u^2 - v_d^2)\,. \tag{3.109}$$

The minimum conditions for v_u, v_d remain the same as in the MSSM. Notice that eq. (3.107) implies that if $v_R \neq 0$, then also $v_L \neq 0$, as we anticipated. We see that v_R is tied to the scale of the (negative) soft mass for RH sneutrinos, and therefore to the scale of SUSY breaking, while v_L is small indeed, because the Yukawa couplings Y_ν are small (and we can write as usual $A_\nu = a_\nu Y_\nu$ at some large renormalization scale).

After the breaking of $U(1)_X$, bilinear mixing terms which violate R-parity are generated: e.g., [lepton][Higgsino] terms (and their 'SUSY conjugates' terms [slepton][Higgs]) from the superpotential, and [lepton][gaugino] terms from the kinetic terms of the lepton superfields. The large mixing between fermionic states is a distinctive feature of R-parity violating models, and the neutral fermion sector is particularly relevant, because of the strong requirements imposed by the experimental smallness of neutrino masses. In the present model, mixing is generated between neutrinos (6 states) and neutralinos (5 states, namely the new $\widetilde{B}'$ and 4 MSSM fields, 2 neutral Higgsinos and 2 neutral gauginos). As a consequence, a 11×11 mass matrix is produced. Notice also that, because the Higgs sector does not include any additional fields, a large Majorana mass term for RH neutrinos cannot be generated, at least by renormalizable operators. In addition, mixing between charged leptons and charginos arises, potentially leading to corrections to the masses of charged leptons.

3.7.2. $3R \times (B-L)$ model

To conclude our study of minimal SUSY $U(1)'$ models, we mention another model with spontaneous R-parity breaking that has been recently proposed [63]. This model is based on the electroweak gauge group $SU(2)_L \times U(1)_{T_{3R}} \times U(1)_{B-L}$, where T_{3R} is right-handed isospin, related to hypercharge by $T_{3R} = Y - (1/2)(B - L)$, in the normalization convention we have adopted throughout this thesis. Analogously to the $U(1)_Y \times U(1)_X$ ($X = aY + b(B - L)$) model discussed above, no singlet Higgses are introduced; the only enlargement with respect to the field content of the MSSM is the inclusion of one RH neutrino supermultiplet per family.

In [63], the kinetic terms are assumed to be canonical in the basis $(T_{3R}, T_{3L}, B - L)$, where we introduce the coupling constants g_R, g_{BL}, in a self-explanatory notation. Performing a 'triangular' transformation, we can move to a basis where the interaction terms read $A^a J_a$, $a = Y, 3L, (B - L)$: in the latter basis, kinetic mixing is reintroduced, the kinetic mixing matrix h_{ab} containing the parameters g, g_R and g_{BL}. However, g_R and g_{BL} are not independent, because identification of the SM $U(1)_Y$ coupling g' imposes a relation between the two: therefore h_{ab} is fully determined by g, g' and, say, g_{BL}. In conclusion, this model corresponds to a supersymmetric version of the minimal model studied in Chapter 2, but with a definite relation which gives g_Y as a function of g_{BL} (and of course with a different mechanism for spontaneous breaking of the extra Abelian symmetry, see below).

The spontaneous breaking $U(1)_{T_{3R}} \times U(1)_{B-L} \to U(1)_Y$ is achieved by means of a VEV of RH sneutrinos, which are neutral under Y. Also, left-handed sneutrinos are forced (in a way similar to eq. (3.107)) to obtain a VEV, and the breaking of R-parity implies large mixing patterns, which are qualitatively similar to those described in the previous paragraph. However, the mass spectrum is quantitatively different from that of the $B - L$ model, so measuring the masses of sparticles at the LHC (in particular, the authors of [63] focus on slepton masses) could make it possible to discriminate between different models.

Conclusions and outlook

In this thesis we studied minimal Z' extensions of the SM, characterised by the inclusion of a single extra Abelian factor in the gauge group. The choice of the additional symmetry was connected with the inclusion of three right-handed neutrinos in the fermion spectrum: in this way anomaly cancellation is ensured, without the need for any other exotic fermions, if the charge associated to the extra $U(1)$ is a linear combination of the weak hypercharge Y and baryon-minus-lepton number $B-L$. Furthermore, this choice allows the generation of realistic neutrino masses.

Chapter 1 reviewed some general aspects of $U(1)$ extensions of the SM, to prepare the ground for the dedicated and mostly original study of the following chapters. In Chapter 2 we started by introducing our parameterization for the minimal Z' model, which fully accounts for kinetic and mass mixing. Thanks to the minimality assumption, only three new parameters are required, namely the mass of the additional neutral gauge boson, $M_{Z'}$, and two effective coupling constants. We performed a detailed study of the constraints that electroweak precision tests and direct searches at the Tevatron impose on the parameter space. We found that for masses below $600 \div 700\,\mathrm{GeV}$, Tevatron bounds are stronger, whereas for larger masses EWPTs put the most strict constraints. We also analyzed the additional bounds imposed by a recent re-analysis of Atomic Parity Violation in Cesium, finding that they can be competitive with Tevatron limits, but not with previous EWPTs. We also computed the region of parameter space that is broadly preferred by grand unification, by employing the renormalization group equations of the model. We found that kinetic mixing is relevant, as the running from the unification scale M_U to the weak scale can drastically modify the values of the effective coupling constants defining the model. We also briefly commented on an excess recently observed by CDF in the e^+e^- channel, but not supported by the $\mu^+\mu^-$ spectrum. Even though our model does not account for non-universal Z' couplings to leptons, we found that the region of parameter space which could produce such an excess is not ruled out by EWPTs.

Then, we moved on to analyze the discovery potential of the LHC, focusing on its early schedule, which at the time of writing foresees a very first run at $\sqrt{s} = 7\,\mathrm{TeV}$, with low luminosity ($< 100\,\mathrm{pb}^{-1}$), followed by an upgrade both in energy, approaching $\sqrt{s} = 10\,\mathrm{TeV}$, and in luminosity (up to $300\,\mathrm{pb}^{-1}$) in 2010. We found that at $7\,\mathrm{TeV}$, the LHC will breach a very narrow region in parameter space, for masses $M_{Z'} \approx 700\,\mathrm{GeV}$. When energy will be increased to $10\,\mathrm{TeV}$, on the other hand, the range in $M_{Z'}$ from 400 to 1500 GeV will open up, with the Tevatron and EWPTs still beating the LHC for smaller and larger masses respectively. The upgrade in energy will therefore allow the LHC to start exploring the parameter space of GUT-preferred models, which are constrained by present data to masses larger than approximately $1\,\mathrm{TeV}$. However, only a tasting of GUT-models will be possible with a luminosity as low as $200 - 300\,\mathrm{pb}^{-1}$, whereas a larger amount of data (roughly of

$O(1)\,\mathrm{fb}^{-1}$) will be needed to scan their parameter space in a significant way. The study of the minimal model performed in Chapter 2 is mostly original, and is contained in an article that will be soon submitted for publication [1].

In Chapter 3, we studied some supersymmetric $U(1)$ extensions, focusing on a minimal choice which can be seen as the supersymmetric version of the minimal Z' model discussed in Chapter 2. We performed an analysis of the renormalization group equations of the model, showing that kinetic mixing, which was not considered in the original formulation of the model, is indeed a relevant parameter, because if a 'pure $B - L$' model is assumed at the unification scale, the running of coupling constants reintroduces non-negligible kinetic mixing at the weak scale. We therefore computed the scalar potential in the presence of kinetic mixing, and performed its minimization; we also outlined the D-term contributions to the masses of scalars. A more detailed study of the model is left for future research. The analysis of minimal supersymmetric Z' models is a field with several problems left open for further research, another possibility being given by models with spontaneous R-parity breaking, also introduced in Chapter 3. These models are characterised by large particle/sparticle mixing patterns, and the actual possibility to generate a realistic particle spectrum, taking all possible mixings into account, has not been studied in detail yet. We also mention, as a possible future research direction, the study of models with non-universal couplings to leptons, which require to take into account the strong constraints imposed by the observed suppression of flavour-changing neutral currents, but could be interesting if the previously mentioned CDF excess in the e^+e^- channel will be confirmed by future data.

A. Notation and conventions

We collect here our notation and conventions.

The Minkowski metric is

$$g_{\mu\nu} = diag\,(+1, -1, -1, -1)\,. \tag{A.1}$$

The Dirac matrices are defined by $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$. We make use of the chiral representation,

$$\gamma^0 = \begin{pmatrix} 0 & 1_2 \\ 1_2 & 0 \end{pmatrix}\,, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}\,, \tag{A.2}$$

where $\sigma^i, (i = 1, 2, 3)$ are the Pauli matrices, defined by

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\,, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\,, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\,. \tag{A.3}$$

A fifth matrix is introduced, $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, which anticommutes with all four Dirac matrices: $\left\{\gamma^5, \gamma^\mu\right\} = 0$. In the chiral representation,

$$\gamma_5 = \begin{pmatrix} -1_2 & 0 \\ 0 & 1_2 \end{pmatrix}\,. \tag{A.4}$$

The chiral projectors are

$$P_L = \frac{1-\gamma_5}{2}\,, \qquad P_R = \frac{1+\gamma_5}{2}\,. \tag{A.5}$$

A Dirac four-component spinor reads

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}\,. \tag{A.6}$$

The Dirac conjugate is defined by $\overline{\psi} = \psi^\dagger\gamma^0$. Charge conjugation is introduced in the following way:

$$\psi^c = C\,\overline{\psi}^T\,, \tag{A.7}$$

where the matrix C has the explicit expression $C = i\gamma^2\gamma^0$.

In two-component notation, which we employ in Chapter 3, a four-component Majorana spinor reads

$$\psi = \begin{pmatrix} \psi_\alpha \\ \overline{\psi}^{\dot\alpha} \end{pmatrix}\,, \tag{A.8}$$

where $\alpha = 1, 2$ and $\dot\alpha = \dot 1, \dot 2$.

The antisymmetric tensors $\epsilon^{\alpha\beta}$ ($\epsilon^{12} = +1$) and $\epsilon_{\alpha\beta}$ ($\epsilon_{12} = -1$) are used to raise and lower indices, respectively:

$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta\,, \qquad \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta\,. \tag{A.9}$$

Analogously, $\epsilon^{\dot\alpha\dot\beta}$ ($\epsilon^{\dot1\dot2} = +1$) and $\epsilon_{\dot\alpha\dot\beta}$ ($\epsilon_{\dot1\dot2} = -1$) raise and lower dotted indices:

$$\overline{\psi}_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\overline{\psi}^{\dot\beta}\,, \qquad \overline{\psi}^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\overline{\psi}_{\dot\beta}\,. \tag{A.10}$$

The following conventions are understood:

$$\psi_1\psi_2 \equiv \psi_1^\alpha\psi_{2\,\alpha}\,, \tag{A.11}$$

$$\overline{\psi}_1\overline{\psi}_2 \equiv \overline{\psi}_{1\,\dot\alpha}\overline{\psi}_2^{\dot\alpha}\,. \tag{A.12}$$

We define

$$\sigma^\mu = (1_2, -\sigma^i)\,, \qquad \overline{\sigma}^\mu = (1_2, \sigma^i)\,, \tag{A.13}$$

where σ^i are the Pauli matrices defined above.

Unless otherwise stated, experimental data and fundamental constants are taken from [64].

The notation used in Chapter 2 agrees with the one employed in [1], except for the convention on the sign of the mixing angle θ': $\theta'([1]) = -\theta'(\text{Chapter 2})$.

B. Non-$(B-L)$ models

We now consider the possibility of having an extra $U(1)$ with associated charges $\alpha Y + \beta(B-L)$. The starting point is

$$\mathcal{L}_{gauge} = -\frac{1}{4} h_{\overline{A}\overline{B}} F^{\overline{A}}_{\mu\nu} F^{\overline{B}\,\mu\nu} + \frac{1}{2} M^2_{\overline{A}\overline{B}} A^{\overline{A}\mu} A^{\overline{B}}_{\mu} + A^{\overline{A}}_{\mu} J^{\mu}_{\overline{A}}, \tag{B.1}$$

where $\overline{A}, \overline{B} = Y, T_{3L}, V$, and $V = \alpha Y + \beta(B-L)$. The matrix $h_{\overline{A}\overline{B}}$ reads

$$h_{\overline{A}\overline{B}} = \begin{pmatrix} \frac{1}{g'^2} & 0 & \frac{k}{g' G_V} \\ 0 & \frac{1}{g^2} & 0 \\ \frac{k}{g' G_V} & 0 & \frac{1}{G_V^2} \end{pmatrix}, \tag{B.2}$$

where G_V is the $U(1)_V$ coupling constant, and k ($|k| < 1$) is the usual kinetic mixing parameter. The mass matrix reads

$$M^2_{\overline{A}\overline{B}} = \frac{1}{4} \begin{pmatrix} v^2 & -v^2 & v^2\alpha \\ -v^2 & v^2 & -v^2\alpha \\ v^2\alpha & -v^2\alpha & \alpha^2 v^2 + 4 v_{BL}^2 \beta^2 Q^{\phi\,2}_{BL} \end{pmatrix}, \tag{B.3}$$

where we have assumed, as in Section 2.1, $Q^H_{BL} = 0$ and $Y^\phi = 0$. The transformation

$$A^{\overline{A}} = O_A{}^{\overline{A}} A^A, \tag{B.4}$$

with $A = Y, T_{3L}, (B-L)$, and where

$$O_A{}^{\overline{A}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{\alpha}{\beta} & 0 & \frac{1}{\beta} \end{pmatrix}, \tag{B.5}$$

yields the new currents

$$J_A \equiv O_A{}^{\overline{A}} J_{\overline{A}} = \begin{pmatrix} J_Y \\ J_{3L} \\ J_{BL} \end{pmatrix}, \tag{B.6}$$

where we have used $J_V = \alpha J_Y + \beta J_{BL}$. The new kinetic terms read

$$h_{AB} = O_A{}^{\overline{A}} h_{\overline{A}\overline{B}} O_B{}^{\overline{B}} = \begin{pmatrix} \frac{1}{g'^2} & 0 & -\frac{\alpha}{\beta g'^2} + \frac{k}{\beta g' G_V} \\ 0 & \frac{1}{g^2} & 0 \\ -\frac{\alpha}{\beta g'^2} + \frac{k}{\beta g' G_V} & 0 & \frac{\alpha^2}{\beta^2 g'^2} - \frac{2k\alpha}{\beta^2 g' G_V} + \frac{1}{\beta^2 G_V^2} \end{pmatrix}. \tag{B.7}$$

Now we can define new parameters g_Y, g_{BL} such that

$$-\frac{\alpha}{\beta g'^2} + \frac{k}{\beta g' G_V} = \frac{-g_Y}{g'^2 g_{BL}} \,, \tag{B.8}$$

$$\frac{\alpha^2}{\beta^2 g'^2} - \frac{2k\alpha}{\beta^2 g' G_V} + \frac{1}{\beta^2 G_V^2} = \frac{g_Y^2 + g'^2}{g_{BL}^2 g'^2} \,, \tag{B.9}$$

and in this way (B.7) acquires the same form as in the $V = (B-L)$ case, see (2.124). It is easy to check that the transformations (B.8) and (B.9) are possible for any values of α and β, provided $|k| < 1$ (which must indeed be satisfied, to ensure positivity of the kinetic energy). It remains to see what happens to the mass term: we have

$$M^2_{AB} = O_A{}^{\overline{A}} M^2_{\overline{AB}} O_B{}^{\overline{B}} = \frac{1}{4} \begin{pmatrix} v^2 & -v^2 & 0 \\ -v^2 & v^2 & 0 \\ 0 & 0 & 4v_{BL}^2 Q^{\phi\,2}_{BL} \end{pmatrix} . \tag{B.10}$$

This is the same mass matrix as in the $V = (B-L)$ case. Thus we have shown that a $V = \alpha Y + \beta(B-L)$ charge can be fully absorbed into a redefinition of the parameters g_Y, g_{BL}: as a consequence, there is no loss in generality in choosing $V = (B-L)$.

C. Use of the + prescription

Because the reader may be unfamiliar with the $(\ )_+$ prescription encountered in (2.101), we give some details in this appendix. The contribution given from the + term to, e.g., w_u, is (we consider only the up quark part for simplicity, and the renormalization scale is understood to be $M_{Z'}$):

$$\frac{16}{3}\frac{\alpha_S}{\pi}\int_0^1 dx_1 \int_0^1 dx_2 \int_\tau^1 dz \left\{ f_{u/P}(x_1) f_{\overline{u}/\overline{P}}(x_2) + f_{u/\overline{P}}(x_2) f_{\overline{u}/P}(x_1) \right\} \times \left(\frac{\log(1-z)}{1-z}\right)_+ \frac{1}{x_1 x_2}\delta\left(z - \frac{\tau}{x_1 x_2}\right), \tag{C.1}$$

where the plus prescription is defined for a generic distribution $F(z)$ which is singular in $z = 1$, by (see, e.g., [65] or [3])

$$\int_0^1 dz\,(F(z))_+\, f(z) \equiv \int_0^1 dz F(z)[f(z) - f(1)]\,, \tag{C.2}$$

for any sufficiently smooth function $f(z)$. In order to apply this definition, we write

$$\begin{aligned} &\int_0^1 dx_1 \int_0^1 dx_2 \left\{ f_{u/P}(x_1) f_{\overline{u}/\overline{P}}(x_2) + f_{u/\overline{P}}(x_2) f_{\overline{u}/P}(x_1) \right\} \frac{1}{x_1 x_2}\delta\left(z - \frac{\tau}{x_1 x_2}\right) \\ = \ & \frac{1}{z}\int_{\tau/z}^1 \frac{dx_1}{x_1} \left\{ f_{u/P}(x_1) f_{\overline{u}/\overline{P}}\left(\frac{\tau}{z x_1}\right) + f_{u/\overline{P}}\left(\frac{\tau}{z x_1}\right) f_{\overline{u}/P}(x_1) \right\} \\ \equiv \ & \frac{1}{z} L\left(\frac{\tau}{z}\right). \end{aligned} \tag{C.3}$$

With this definition, (C.1) becomes

$$\frac{16\alpha_S}{3\pi}\int_\tau^1 \frac{dz}{z} L\left(\frac{\tau}{z}\right)\left(\frac{\log(1-z)}{1-z}\right)_+ . \tag{C.4}$$

Now adding and subtracting $L(\tau)$,

$$\frac{16\alpha_S}{3\pi}\int_\tau^1 dz \left[\frac{1}{z} L\left(\frac{\tau}{z}\right) - L(\tau)\right]\left(\frac{\log(1-z)}{1-z}\right)_+ + \frac{16\alpha_S}{3\pi} L(\tau)\int_\tau^1 dz \left(\frac{\log(1-z)}{1-z}\right)_+ , \tag{C.5}$$

and finally

$$\begin{aligned} &= \frac{16\alpha_S}{3\pi}\int_\tau^1 dz \left[\frac{1}{z} L\left(\frac{\tau}{z}\right) - L(\tau)\right]\frac{\log(1-z)}{1-z} - \frac{16\alpha_S}{3\pi} L(\tau)\int_0^\tau dz\, \frac{\log(1-z)}{1-z} \\ &= \frac{16\alpha_S}{3\pi}\int_\tau^1 dz \left[\frac{1}{z} L\left(\frac{\tau}{z}\right) - L(\tau)\right]\frac{\log(1-z)}{1-z} + \frac{8\alpha_S}{3\pi} L(\tau)\log^2(1-\tau). \end{aligned} \tag{C.6}$$

To obtain (C.6) from (C.5), we have used the following properties, which hold for a generic $(F(z))_+$ distribution:

C. Use of the + prescription

(i)

$$\int_0^1 dz\ (F(z))_+ = 0\,; \tag{C.7}$$

(ii) if $[a,b]$ does not contain $z=1$, then

$$\int_a^b dz\,(F(z))_+\,f(z) = \int_a^b dz F(z) f(z)\,. \tag{C.8}$$

These properties follow immediately from the definition of the plus prescription.

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