Higgs Vacuum Stability in $B - L$ extended Standard Model

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We study vacuum stability of $B - L$ extension of the Standard Model (SM) and its super-symmetric version. We show that the generation of non-vanishing neutrino masses through TeV inverse seesaw mechanism leads to a cutoff scale of SM Higgs potential stability of order 10^5 GeV. However, in the non-supersymmetric $B - L$ model, we find that the mixing between the SM-like Higgs and the $B - L$ Higgs plays a crucial role in alleviating the vacuum stability problem. We also provide the constraints of stabilizing the Higgs potential in the supersymmetric $B - L$ model.

I. INTRODUCTION

Recent results announced by ATLAS and CMS experimental collaborations at the Large Hadron Collider (LHC) [1, 2] confirmed the discovery of a Higgs boson with mass of order 125 GeV. Both ATLAS and CMS have performed searches for the Higgs boson in the following five decay channels: $H \to \gamma\gamma$, $H \to ZZ^{(*)} \to 4l$, and $H \to WW^{(*)} \to l\nu l\nu$, $H \to \tau^+\tau^-$ and $H \to b\bar{b}$, at integrated luminosities of 5.1 fb^{-1} at energy $\sqrt{s} = 7$ TeV and 19.6 fb^{-1} at $\sqrt{s} = 8$ TeV.

One important question is whether this scalar boson is compatible with Standard Model (SM) predictions or it is a SM-like Higgs of an extension of the SM. It is worth mentioning that the signal strength of $H \to \gamma\gamma$ seems not consistent with the SM predictions [3, 4]. It is found to be of order 1.65 by ATLAS and about 0.78 by CMS, while the corresponding SM signal strength should be exactly one. In addition, it is well known that if the SM Higgs mass is less than 130 GeV, then the quartic Higgs self-coupling runs to negative values at high energy scales, leading to vacuum instability at these scales [5–14]. In particular, for Higgs mass of order 125 GeV, one finds that the cutoff scale of stability for the SM Higgs potential is of the order $\mathcal{O}(10^{9\text{-}10})$ GeV. A natural solution for this problem is to consider a possible new physics beyond the SM that changes the running of the quartic coupling and prevents it from running into negative values [15–24]. One can also study the issue of vacuum stability in a model independent way in an effective Lagrangian framework [25]. The addition of a higher dimensional operator to the Higgs potential changes the boundary condition for the quartic coupling at the scale of vacuum stability. In this work the effect of the higher dimensional operator will be neglected and only the running of the couplings will be used to determine vacuum stability.

Non-vanishing neutrino masses are now firm evidence for an extension of the SM. One of the attractive scenarios for accommodating the neutrino masses is the inverse seesaw mechanism, which is based on the extension of the SM with TeV scale right handed neutrinos with unsuppressed couplings to the SM leptons [26–48]. In this case, one can show that the contribution of the right-handed neutrinos has a large impact on the Higgs quartic coupling and, similar to the top contribution, drives it to negative values. Therefore, the SM Higgs potential is unstable at a scale of order $\mathcal{O}(10^{5\text{-}6})$ GeV and the vacuum stability problem becomes more severe. The investigation of vacuum stability within different type of seesaw mechanisms have been explored in Refs. [49–54].

In this article, we analyze the vacuum stability problem in simple extensions of the SM. In particular, we focus on the $B - L$ extension of the SM with and without supersymmetry. The $B - L$ model is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ [55–57]. It naturally introduces three SM singlet fermions to cancel the $U(1)_{B-L}$ anomalies and account for the current experimental results of light neutrino masses and their large mixings [58]. In addition, the extra-gauge boson and the extra-Higgs, predicted in the $B - L$ model, have interesting phenomenology that can be probed at the LHC [59–63]. Within a supersymmetric context, it was emphasized that the three relevant physics scales related to the supersymmetry, electroweak and $B - L$ symmetry breaking are linked together and occur at the TeV scale [64–67]. Finally, it is worth mentioning that within $B - L$ Supersymmetric Standard Model (BLSSM) with inverse seesaw, the one-loop radiative corrections to the lightest SM-like Higgs boson mass, due to the right-handed neutrinos and sneutrinos, can be significant [68], and hence the Higgs mass can be easily of order 125 GeV without pushing the SUSY spectrum to TeV

scale as in MSSM.

We show that, in non-supersymmetric $B-L$ model with type-I seesaw or inverse seesaw mechanisms, the non-vanishing mixing between the SM and $B-L$ Higgs bosons raises the initial value of the SM-like Higgs coupling. In addition, in this case the running of the SM-like Higgs receives a positive contribution from the $(B-L)$-like heavy Higgs. Therefore, the Higgs self-coupling remains positive all the way up to the GUT scale that ensures the vacuum stability. We also analyze the vacuum stability of SM-like Higg potential in supersymmetric $B-L$ model. The conditions securing the stability of this potential in both flat and non-flat directions are derived.

The paper is organized as follows. In section 2 we reappraise the Higgs vacuum stability in the SM extended by TeV scale right-handed neutrinos with inverse seesaw mechanism. Section 3 is devoted for the Higgs vacuum stability in $B-L$ extension of the SM. We show that the mixing between the SM-like Higgs and the $B-L$ Higgs resolve the vacuum stability problem In section 4 we analyze the vacuum stability in supersymmetric theories. In particular, we consider the stability in MSSM and BLSSM. Finally, we give our conclusions in section 5.

II. VACUUM STABILITY OF SM EXTENDED WITH TEV SCALE RIGHT-NEUTRINOS

In this section, we analyze the impact of massive neutrinos on the SM vacuum stability by extending the SM by right-handed neutrinos. As known, the non-vanishing small neutrino masses can be generated through type-I seesaw mechanism or inverse seesaw mechanism. In type-I seesaw, one assumes that the SM lagrangian is extended as follows:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + Y_\nu \bar{l}\tilde{\Phi}\nu_R + M\overline{\nu}_R^c \nu_R, \tag{1}$$

where ν_R is a SM singlet fermion, called the right-handed neutrino and M is Majorana mass which is not restricted by the electroweak symmetry breaking scale, so it can take any value up to any high scale. In this case, one finds that the lightest neutrinos get the following masses $m_\nu \sim \frac{(Y_\nu v)^2}{M}$, where $v = \langle \phi \rangle$ is the electroweak VEV. Therefore, if $M \sim \mathcal{O}(1)$ TeV, the light neutrino masses can be of order electron volt, provided that $Y_\nu \sim 10^{-6}$. In this case the contribution of the right handed neutrinos to the Renormalization Group Equation (RGE) of the Higgs quartic coupling is negligible, and one ends with the SM results for the Higgs vacuum stability.

We now turn to inverse seesaw mechanism. In this case, three extra SM singlet neutral fermions S_i are required in addition to the three right-handed neutrinos ν_{R_i} and the lagrangian in this case is given by

$$\mathcal{L} = Y_\nu \bar{l}\tilde{\Phi}\nu_R + M\overline{\nu}_R^c S + \mu_s \overline{S}^c S + h.c. \tag{2}$$

Thus, the neutrino mass matrix is given by

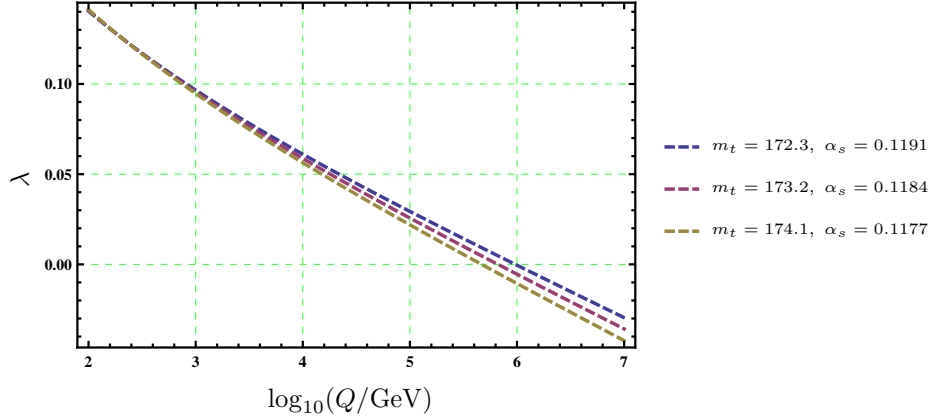$$\begin{pmatrix} 0 & vY_\nu & 0 \\ vY_\nu^T & 0 & M \\ 0 & M^T & \mu_s \end{pmatrix}. \tag{3}$$

FIG. 1. The running of the quartic Higgs coupling for Higgs mass $m_h = 125$ GeV and $Y_\nu = 0.7$ in the extended SM with right handed neutrinos and inverse seesaw mechanism.

Hence, the light neutrino masses are given by

$$m_{\nu_l} = v^2 Y_\nu M^{-1} \mu_s (M^T)^{-1} Y_\nu^T, \tag{4}$$

which can be of order eV, as required by the oscillation data, for $M \sim \mathcal{O}(1)$ TeV if μ_s is sufficiently small, namely, $\mu_s \lesssim 10^{-7}$ GeV. In this case, the Yukawa coupling Y_ν can be of order one. Hence, the right-handed neutrino's contribution to the RGE of the Higgs quartic coupling λ, which is proportional to the neutrino Yukawa coupling Y_ν [69], can be significant

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2}\left[24\lambda^2 + 4\lambda(3Y_t^2 + Y_\nu^2) - 2(Y_\nu^4 + 3Y_t^4) - 3\lambda(3g_2^2 + g_1^2) + \frac{9}{8}g_2^4 + \frac{3}{8}g_1^4 + \frac{3}{4}g_2^2 g_1^2\right]. \tag{5}$$

In addition, the RGEs of top and neutrino Yukawa couplings are given by

$$\frac{d}{dt}Y_t = \frac{Y_t}{16\pi^2}\left(\frac{9}{2}Y_t^2 + Y_\nu^2 - 8g_s^2 - \frac{9}{4}g^2 - \frac{17}{12}g_1^2\right) \tag{6}$$

$$\frac{d}{dt}Y_\nu = \frac{Y_\nu}{16\pi^2}\left(\frac{5}{2}Y_\nu^2 + 3Y_t^2 - \frac{9}{4}g^2 - \frac{3}{4}g_1^2\right)$$

In Fig. 1 we display the running of the Higss self coupling λ in the extended SM with right-handed neutrinos with inverse seesaw for Higgs mass $m_h = 125$ GeV. From this figure, it is clear that the scale of Higgs vacuum stability is reduced from $10^{9\text{-}10}$ GeV in the SM to $10^{5\text{-}6}$ GeV. This can be easily understood from the RGE (5), where the neutrino Yukawa coupling Y_ν contributes to the evolution of λ, with fourth power and negative sign, similar to the top Yukawa coupling contribution. Therefore, one can conclude that solving the puzzle of neutrino masses in the context of the SM gauge group with inverse seesaw mechanism affects the Higgs vacuum stability negatively.

III. VACUUM STABILITY IN $U(1)_{B-L}$ EXTENSION OF THE SM

TeV scale $B{-}L$ extension of the SM, which is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ is one of the most straightforward extensions of the SM. It permits to introduce naturally

three right-handed neutrinos, with $B - L$ charge $= -1$, due to the anomaly cancellation condition. In the $B - L$ model with type-I seesaw mechanism [56, 65, 70–94], the $U(1)_{B-L}$ is spontaneously broken by a SM singlet scalar χ with $B - L$ charge $= +2$ which acquires a VEV v'. Since the kinetic mixing term between the field strength tensors of $U(1)_Y$ and $U(1)_{B-L}$ is allowed by gauge symmetry, the gauge-invariant kinetic lagrangian is given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{\kappa}{2}F_{\mu\nu}F'^{\mu\nu}. \tag{7}$$

This mixing can be absorbed by a suitable transformation of the gauge fields that will modify the covariant derivatives. This can be understood as follows: from Eq.7 one can write the covariant derivative as

$$D_\mu = \partial_\mu - iQ_\phi^T G A_\mu, \tag{8}$$

where Q_ϕ is a vector containing the charges of the field ϕ with respect to the two abelian gauge groups, G is the gauge coupling matrix:

$$G = \begin{pmatrix} g_{YY} & g_{YB} \\ g_{BY} & g_{BB} \end{pmatrix}, \tag{9}$$

and A_μ is given, in terms of the $U(1)_Y$ and $U(1)_{B-L}$ gauge bosons, as

$$A_\mu = \begin{pmatrix} A_\mu^Y \\ A_\mu^{B-L} \end{pmatrix}. \tag{10}$$

One can perform an orthogonal rotation O of the gauge fields A_μ, without reintroducing the kinetic mixing, such that

$$Q_\phi^T G A = Q_\phi^T G(O^T O)A = Q_\phi^T \tilde{G} B, \tag{11}$$

where $\tilde{G} = GO^T$ and $B = OA$. If one chooses the orthogonal matrix $O = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}$ such that:

$$c_\theta = \frac{g_{BB}}{\sqrt{g_{BB}^2 + g_{BY}^2}}, \tag{12}$$

$$s_\theta = \frac{g_{YB}}{\sqrt{g_{BB}^2 + g_{BY}^2}}, \tag{13}$$

then the transformed gauge coupling matrix $\tilde{G}$ takes the form:

$$\tilde{G} = \begin{pmatrix} g_1 & 0 \\ \tilde{g} & g_1' \end{pmatrix}, \tag{14}$$

where

$$g_1 = \frac{g_{YY}g_{BB} - g_{YB}g_{BY}}{\sqrt{g_{BB}^2 + g_{BY}^2}}, \tag{15}$$

$$\tilde{g} = \frac{g_{BB}g_{YB} + g_{YY}g_{BY}}{\sqrt{g_{BB}^2 + g_{BY}^2}}, \tag{16}$$

$$g_1' = \sqrt{g_{BB}^2 + g_{BY}^2} \tag{17}$$

Therefore, the covariant derivative takes the form:

$$D_\mu = \cdots - ig_1 Y B_\mu - i(\tilde{g} Y + g_1' Y_{B-L}) B_\mu'. \tag{18}$$

The neutrino Yukawa interactions are given by

$$\mathcal{L}_Y^\nu = Y_\nu \bar{l}_L \phi \nu_R + Y_N \bar{\nu}_R^c \chi \nu_R + \text{h.c.} \tag{19}$$

As mentioned above, with $v' \simeq \mathcal{O}(1)$ TeV , the neutrino Yukawa coupling is constrained to be $\lesssim 10^{-6}$ and hence does not affect vacuum stability of the Higgs. However, in the $B-L$ extension of the SM with inverse seesaw, the $U(1)_{B-L}$ symmetry is spontaneously broken by a SM singlet scalar χ with $B-L$ charge $= -1$. Also three SM pairs of singlet fermions $S_{1,2}^i$ with $B-L$ charge $= \mp 2$, respectively, are introduced in addition to ν_{R_i} to implement the inverse seesaw mechanism. Note that the addition of the extra singlet fermions $S_{1,2}$ in pairs is necessary in order to prevent the $B-L$ triangle anomalies. In this case, the neutrino Yukawa lagrangian is given by

$$\mathcal{L}_Y^\nu = Y_\nu \bar{l}_L \phi \nu_R + Y_N \bar{\nu}_R^c \chi S_2 + \mu_s \bar{S}_2^c S_2, \tag{20}$$

Therefore, after the $B-L$ and the electroweak symmetry breaking, one finds that the neutrino mass matrix can be written as $\bar{\psi}^c \mathcal{M}_\nu \psi$ with $\psi = (\nu_L^c, \nu_R, S_2)$ and $\mathcal{M}_\nu$ given by

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_R \\ 0 & M_R^T & \mu_s \end{pmatrix}, \tag{21}$$

where $m_D = \frac{1}{\sqrt{2}} Y_\nu v$ and $M_R = \frac{1}{\sqrt{2}} Y_N v'$ and $\mu_s = \frac{v'^4}{4M^3} \lesssim 10^{-7}$ GeV may be generated from non-renormalizable terms like $\bar{S}_2^c \chi^{\dagger 4} S_2 / M^3$. Thus, the light and heavy neutrino masses are given by

$$m_{\nu_l} = m_D M_R^{-1} \mu_s (M_R^T)^{-1} m_D^T, \tag{22}$$

$$m_{\nu_H}^2 = m_{\nu_{H'}}^2 = M_R^2 + m_D^2. \tag{23}$$

Therefore, the light neutrino mass can be of order eV with a TeV scale M_R, provided that μ_s is very small. In this case, the Yukawa coupling Y_ν is no longer restricted to a very small value and it can be of order one.

In both scenarios of $B-L$ extensions of the SM, with type-I seesaw or inverse seesaw mechanism, the Higgs sector in this model consists of one complex SM scalar doublet and one complex SM scalar singlet with the following scalar potential $V(\phi, \chi)$ [58]

$$V(\phi, \chi) = m_1^2 |\phi|^2 + m_2^2 |\chi|^2 + \lambda_1 |\phi|^4 + \lambda_2 |\chi|^4 + \lambda_3 |\phi|^2 |\chi|^2. \tag{24}$$

As in the SM, in order to ensure non-vanishing vevs of the Higgs fields ϕ, χ, the squared masses m_1^2, m_2^2 are assumed to be negative. In order for this potential to be stable, the coefficient matrix of the quartic terms,

$$\begin{pmatrix} \lambda_1 & \frac{\lambda_3}{2} \\ \frac{\lambda_3}{2} & \lambda_2 \end{pmatrix}, \tag{25}$$

has to be co-positive [95]. The conditions of co-positivity of such a matrix are given by

$$\lambda_1, \lambda_2 > 0, \tag{26}$$

$$\frac{\lambda_3}{2} + \sqrt{\lambda_1 \lambda_2} > 0 . \tag{27}$$

The $U(1)_{B-L}$ and the electroweak gauge symmetry are broken by the non-zero vevs: $\langle \chi \rangle = v'/\sqrt{2}$ and $\langle \phi \rangle = v/\sqrt{2}$, where v and v' satisfy the following minimization conditions:

$$v^2 = \frac{-\lambda_2 m_1^2 + \frac{\lambda_3}{2} m_2^2}{\lambda_1 \lambda_2 - \frac{\lambda_3^2}{4}}, \qquad v'^2 = \frac{-\lambda_1 m_2^2 + \frac{\lambda_3}{2} m_1^2}{\lambda_1 \lambda_2 - \frac{\lambda_3^2}{4}} . \tag{28}$$

The mixing between the two neutral Higgs scalars leads to the mass eigenstates fields h and H, which are defined in terms of ϕ^0 and χ. The physical mass eigenstates fields h and H are given by

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \phi^0 \\ \chi \end{pmatrix}, \tag{29}$$

where the mixing angel θ is given by

$$\tan 2\theta = \frac{\lambda_3 v v'}{\lambda_1 v^2 - \lambda_2 v'^2} . \tag{30}$$

The range of the mixing angle θ can be: $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$. Also, the masses of light and heavy Higgs particles are given by

$$m_{h,H}^2 = \lambda_1 v^2 + \lambda_2 v'^2 \mp \sqrt{(\lambda_1 v^2 - \lambda_2 v'^2)^2 + (\lambda_3 v v')^2}, \tag{31}$$

From the above expressions, one can easily express the scalar potential parameters: λ_1, λ_2 and λ_3 in terms of the physical quantities: m_h, m_H and $\sin 2\theta$ as follows [96]

$$\lambda_1 = \frac{m_h^2}{4v^2}(1 + \cos 2\theta) + \frac{m_H^2}{4v^2}(1 - \cos 2\theta),$$

$$\lambda_2 = \frac{m_h^2}{4v'^2}(1 - \cos 2\theta) + \frac{m_H^2}{4v'^2}(1 + \cos 2\theta), \tag{32}$$

$$\lambda_3 = \sin 2\theta \left(\frac{m_H^2 - m_h^2}{2 v v'} \right).$$

From these equations, one notices that the initial condition of the SM-like Higgs quartic coupling, λ_1, at the electroweak scale can be different from that in the SM. This, as we will see, can have an important impact on the evolution of this coupling and Higgs vacuum stability.

The RGEs of the scalar couplings: λ_1, λ_2 and λ_3 in the context of $B - L$ extension of the SM, are given by [96]

$$\frac{d\lambda_1}{dt} = \frac{1}{16\pi^2} \left(24\lambda_1^2 + \lambda_3^2 + 4\lambda_1(3Y_t^2 + Y_\nu^2) - 2(Y_\nu^4 + 3Y_t^4) + \frac{9}{8}g_2^4 + \frac{3}{8}g_1^4 + \frac{3}{4}g_2^2 g_1^2 + \frac{3}{4}g_2^2 \tilde{g}^2 \right.$$

$$\left. + \frac{3}{4}g_1^2 \tilde{g}^2 + \frac{3}{8}\tilde{g}^4 - 9\lambda_1 g_2^2 - 3\lambda_1 g_1^2 - 3\lambda_1 \tilde{g}^2 \right), \tag{33}$$

$$\frac{d\lambda_2}{dt} = \frac{1}{8\pi^2} \left(10\lambda_2^2 + \lambda_3^2 - \frac{1}{2}Tr\left[(Y_N)^4 \right] + 48 g_1^{'4} + 4\lambda_2 Tr\left[(Y_N)^2 \right] - 24\lambda_2 g_1^{'2} \right), \tag{34}$$

$$\frac{d\lambda_3}{dt} = \frac{\lambda_3}{8\pi^2} \left(6\lambda_1 + 4\lambda_2 + 2\lambda_3 + 3Y_t^2 - \frac{9}{4}g_2^2 - \frac{3}{4}g_1^2 - \frac{3}{4}\tilde{g}^2 + 2Tr\left[(Y_N)^2 \right] - 12 g_1^{'2} + 6\frac{\tilde{g}^2 g_1^{'2}}{\lambda_3} \right), \tag{35}$$

where $\widetilde{g}$ and g_1' are the gauge couplings of the $U(1)$'s mixing and $U(1)_{B-L}$ as defined in Eq. 18. Y_N is the Yukawa coupling defined in Eq. 19. The scalar couplings λ_1, λ_2 and λ_3 are defined in Eq. 24. For completeness, we give also the RGEs of g_1' and $\widetilde{g}$, which can be written as [96]

$$\frac{dg_1'}{dt} = \frac{1}{16\pi^2}\left[12g_1'^3 + \frac{32}{3}g_1'^2\widetilde{g} + \frac{41}{6}g_1'\widetilde{g}^2\right],\tag{36}$$

$$\frac{d\widetilde{g}}{dt} = \frac{1}{16\pi^2}\left[\frac{41}{6}\widetilde{g}\left(\widetilde{g}^2 + 2g_1^2\right) + \frac{32}{3}g_1'(\widetilde{g}^2 + g_1^2) + 12g_1'^2\widetilde{g}\right].\tag{37}$$

The RGEs of the gauge couplings, g_3, g_2 and g_1 remain intact. Finally, the RGEs of the Yukawa couplings Y_t, Y_ν and Y_N are as follows [96]

$$\frac{dY_t}{dt} = \frac{Y_t}{16\pi^2}\left(\frac{9}{2}Y_t^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_1^2 - \frac{17}{12}\widetilde{g}^2 - \frac{2}{3}g_1'^2 - \frac{5}{3}\widetilde{g}g_1'\right),\tag{38}$$

$$\frac{dY_\nu}{dt} = \frac{Y_\nu}{16\pi^2}\left(\frac{5}{2}Y_\nu^2 + 3Y_t^2 - \frac{9}{4}g_2^2 - \frac{3}{4}g_1^2 - 6g_1'^2\right)\tag{39}$$

$$\frac{dY_{N_i}}{dt} = \frac{Y_{N_i}}{16\pi^2}\left(4(Y_{N_i})^2 + 2Tr\left[(Y_N)^2\right] - 6g_1'^2\right),\qquad (i = 1\ldots 3),\tag{40}$$

where, we consider the basis of real and diagonal Y_N, i.e. $Y_N \equiv \mathrm{diag}\,(Y_{N_1}, Y_{N_2}, Y_{N_3})$. It is worth noting that within inverse seesaw, the RGE of $B-L$ couplings g_1' and $\widetilde{g}$ are slightly modified, due to the impact of the two fermions $S_{1,2}$, which are charged under $B-L$. They take the form:

$$\frac{dg_1'}{dt} = \frac{1}{16\pi^2}\left[27g_1'^3 + \frac{32}{3}g_1'^2\widetilde{g} + \frac{41}{6}g_1'\widetilde{g}^2\right],\tag{41}$$

$$\frac{d\widetilde{g}}{dt} = \frac{1}{16\pi^2}\left[\frac{41}{6}\widetilde{g}\left(\widetilde{g}^2 + 2g_1^2\right) + \frac{32}{3}g_1'(\widetilde{g}^2 + g_1^2) + 27g_1'^2\widetilde{g}\right].\tag{42}$$

From Eq.(33) of the RGE of the coupling λ_1, we notice that the mixing parameter λ_3 contributes positively to the evolution of λ_1, unlike the top Yukawa and neutrino Yukawa couplings. Note that the evolution of λ_3 (and also the running of λ_1) is enhanced by the positive effect of the self-coupling of $B-L$ heavy Higgs, λ_2. Therefore, with non-negligible λ_3, the scale of Higgs vacuum stability can be pushed to higher values. In case of inverse seesaw, where $Y_\nu \sim \mathcal{O}(1)$, a larger mixing parameter is required to overcome the effects of both the top and neutrino Yukawa couplings that pull the stability scale down. Note, since the Higgs scalar is not charged under $U(1)_{B-L}$, the running of λ_1 has no dependence on g_1'. The only extra gauge contribution to $d\lambda_1/dt$ is due to the small gauge mixing $\widetilde{g}$, which leads to a negligible effect.

As emphasized, the parameter that is responsible for the scalar mixing λ_3 is expressible in terms of the physical quantities m_h, which is fixed by the detected Higgs mass 125 GeV and the heavy Higgs mass m_H and the mixing angel θ. In Fig. 2 we show the running, up to the GUT scale, for the quartic couplings λ_1 and the condition of bounded from below: $\lambda_3 + 2\sqrt{\lambda_1\lambda_2}$ in the $B-L$ extension of the SM with type-I seesaw. It is worth noting that λ_2 is unconditionally positive as can be seen from its RG equation (34). In these plots, we consider three values of the Higgs mixing angle: $\theta = 0, 0.1$, and 0.2. Also we fix the SM-like Higgs mass with 125 GeV and the heavy Higgs mass $m_H = 500$ GeV. As can be seen from this figure, at $\theta = 0$ where there is no mixing between the SM Higgs and $B-L$ Higgs, the running of λ_1 coincides with that of the SM. Hence one again finds that the Higgs potential becomes
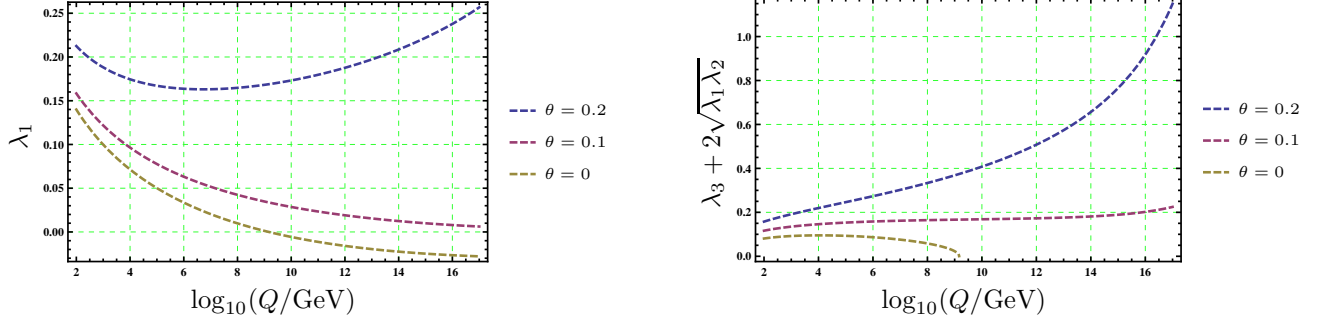
FIG. 2. (Left panel): The RG running of the quartic coupling λ_1 in the $B-L$ extension of the SM with type-I seesaw, for three values of the scalar mixing angle θ for SM-like Higgs mass $m_h = 125$ GeV. (Right panel): The evolution of the second stability condition, $\lambda_3 + 2\sqrt{\lambda_1\lambda_2}$, up to the GUT scale.
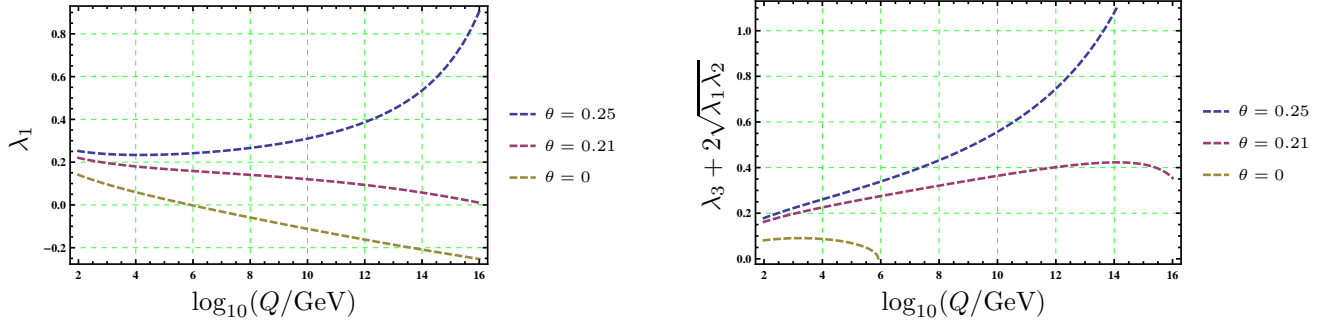

FIG. 3. (Left panel): The running of the quartic couplings λ_1 in the $B-L$ extension of the SM with inverse seesaw, for three values of scalar mixing angle θ for SM-like Higgs mass $m_h = 125$ GeV. (Right panel): The evolution of the second stability condition, $\lambda_3 + 2\sqrt{\lambda_1\lambda_2}$, up to the GUT scale.

unstable at an energy scale $\gtrsim 10^{9\text{-}10}$ GeV. With non-vanishing θ one finds that λ_1 gets initial values at electroweak scale larger than its value in the SM and also its scale dependence becomes rather different. Therefore in this case one finds that it is quite plausible, with not very large mixing, to keep λ_1 and also $\lambda_3 + 2\sqrt{\lambda_1\lambda_2}$ positive up to the GUT scale, and hence the Higgs vacuum stability is accomplished.

Similarly, in Fig. 3 we display the running of λ_1 and $\lambda_3 + 2\sqrt{\lambda_1\lambda_2}$ in $B-L$ extension of the SM with inverse seesaw, for $\theta = 0, 0.21$, and 0.25, $m_h = 125$ GeV, $m_H = 500$ GeV and $Y_\nu = 0.7$. It is clear that with $\theta = 0$, we get the non $B-L$ limit for the instability of the Higgs potential, where both λ_1 and $\lambda_3 + 2\sqrt{\lambda_1\lambda_2}$ become negative at $\sim 10^{5\text{-}6}$ GeV. Also, we find that for $\theta \gtrsim 0.21$, the Higgs vacuum stability is achieved up to the GUT scale.

IV. VACUUM STABILITY IN SUPERSYMMETRIC EXTENSIONS OF THE SM

In this section we analyze the Higgs vacuum stability in supersymmetric extensions of the SM. We start with the MSSM, which is the most widely studied SUSY model. The MSSM is based on the

same gauge group of the SM, i.e, $SU(3)_C \times SU(2)_L \times U(1)_Y$, with the following superpotential

$$W = Y_u Q_L U_L^c H_2 + Y_d Q_L D_L^c H_1 + Y_e L_L E_L^c H_1 + \mu H_1 H_2. \tag{43}$$

In MSSM, two Higgs doublet superfields are required for the Higgsino anomalies to cancel among themselves. From the superpotential one can determine the scalar potential. Thus, the potential for the neutral Higgs fields can be written

$$V(H_1, H_2) = m_1^2 \, H_1^2 + m_2^2 \, H_2^2 - 2m_3^2 \, H_1 H_2 + \frac{g^2 + g'^2}{8} \left(H_1^2 - H_2^2 \right)^2, \tag{44}$$

where the masses m_i^2 are given in terms of the soft SUSY breaking terms: $m_{H_i}^2$, B and the μ parameter as follows:

$$m_i^2 = m_{H_i}^2 + |\mu|^2, \quad m_3^2 = B\mu. \tag{45}$$

This potential is the SUSY version of the Higgs potential which induces $SU(2)_L \times U(1)_Y$ breaking in the SM, where the usual self-coupling constant is replaced by the squared gauge couplings.

In order to study the stability of the MSSM Higgs potential, one should consider the following two cases: (i) Flat direction, where $H_1 = H_2 =: H$. (ii) Non-flat directions. In the flat direction, the quartic terms vanish and the potential takes the simple form:

$$V(H) = (m_1^2 + m_2^2 - 2m_3^2)H^2, \tag{46}$$

which is stable only if the coefficient $(m_1^2 + m_2^2 - 2m_3^2)$ is non-negative. This is the well known condition for avoiding the unboundedness of MSSM potential from below.

On the other hand, on non-flat directions the quartic terms in Eq. (44) are non-vanishing and dominate the potential for large value of the scalar fields $H_{1,2}$. Thus, the stability is unconditionally guaranteed because the quartic coupling $(g^2 + g'^2)/8$ is always positive. Therefore, one concludes that the MSSM Higgs potential is identically stable at any direction except the flat one, which requires the following condition:

$$m_1^2 + m_2^2 \geq 2m_3^2, \tag{47}$$

Now we turn to the supersymmetric $B - L$ extension of the SM (BLSSM). The minimal version of BLSSM is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, with particle content that includes the following fields in addition to those of the MSSM: three chiral right-handed superfields (N_i), the vector superfield necessary to gauge the $U(1)_{B-L}$ (Z_{B-L}), and two chiral SM-singlet Higgs superfields (χ_1, χ_2 with $B - L$ charges $Y_{B-L} = -2$ and $Y_{B-L} = +2$, respectively). As in the MSSM, the introduction of a second Higgs singlet (χ_2) is necessary in order to cancel the $U(1)_{B-L}$ anomalies produced by the fermionic member of the first Higgs superfield (χ_1). The Y_{B-L} for quark and lepton superfields are assigned in the usual way.

The interactions between the Higgs and matter superfields are described by the superpotential

$$\begin{aligned} W = {} & (Y_U)_{ij} Q_i H_2 U_j^c + (Y_D)_{ij} Q_i H_1 D_j^c + (Y_L)_{ij} L_i H_1 E_j^c + (Y_\nu)_{ij} L_i H_2 N_j^c \\ & + (Y_N)_{ij} N_i^c N_j^c \chi_1 + \mu H_1 H_2 + \mu' \chi_1 \chi_2. \end{aligned} \tag{48}$$

Therefor, the BLSSM Higgs potential is given by

$$V(H_1, H_2, \chi_1, \chi_2) = m_1^2\,H_1^2 + m_2^2\,H_2^2 - 2m_3^2\,H_1H_2 + \mu_1^2\,\chi_1^2 + \mu_2^2\,\chi_2^2 - 2\mu_3^2\,\chi_1\chi_2$$
$$+ \frac{g^2 + g_{YY}^2 + g_{YB}^2}{8}\,(H_1^2 - H_2^2)^2 + \frac{g_{BB}^2 + g_{BY}^2}{2}\,(\chi_1^2 - \chi_2^2)^2$$
$$+ \frac{g_{BB}g_{YB} + g_{BY}g_{YY}}{2}\,(H_1^2 - H_2^2)\,(\chi_1^2 - \chi_2^2) \qquad (49)$$

where

$$m_i^2 = m_{H_i}^2 + |\mu|^2, \quad \mu_i^2 = m_{\chi_i}^2 + |\mu'|^2, \quad m_3^2\,(\mu_3^2) = B\mu\,(B\mu'). \qquad (50)$$

Similar to the MSSM, in order to study the stability of this potential, one should consider the two cases of flat direction, in which $H_1 = H_2 =: H$ & $\chi_1 = \chi_2 =: \chi$, and the other non-flat directions. In the flat direction, all the quartic terms vanish, and the potential turns to the simple form:

$$V(H, \chi) = (m_1^2 + m_2^2 - 2m_3^2)H^2 + (\mu_1^2 + \mu_2^2 - 2\mu_3^2)\chi^2, \qquad (51)$$

which is stable under the conditions

$$m_1^2 + m_2^2 \geq 2m_3^2, \qquad (52)$$
$$\mu_1^2 + \mu_2^2 \geq 2\mu_3^2. \qquad (53)$$

On the other hand, the quartic terms are non-vanishing in the other directions and they dominate the quadratic terms. Thus, the stability is guaranteed only if the matrix of quartic terms,

$$\begin{pmatrix} \frac{g^2 + g_{YB}^2 + g_{YY}^2}{8} & -\frac{g^2 + g_{YB}^2 + g_{YY}^2}{8} & \frac{g_{BB}g_{YB} + g_{BY}g_{YY}}{4} & -\frac{g_{BB}g_{YB} + g_{BY}g_{YY}}{4} \\ -\frac{g^2 + g_{YB}^2 + g_{YY}^2}{8} & \frac{g^2 + g_{YB}^2 + g_{YY}^2}{8} & -\frac{g_{BB}g_{YB} + g_{BY}g_{YY}}{4} & \frac{g_{BB}g_{YB} + g_{BY}g_{YY}}{4} \\ \frac{g_{BB}g_{YB} + g_{BY}g_{YY}}{4} & -\frac{g_{BB}g_{YB} + g_{BY}g_{YY}}{4} & \frac{g_{BB}^2 + g_{BY}^2}{2} & -\frac{g_{BB}^2 + g_{BY}^2}{2} \\ -\frac{g_{BB}g_{YB} + g_{BY}g_{YY}}{4} & \frac{g_{BB}g_{YB} + g_{BY}g_{YY}}{4} & -\frac{g_{BB}^2 + g_{BY}^2}{2} & \frac{g_{BB}^2 + g_{BY}^2}{2} \end{pmatrix} \qquad (54)$$

is co-positive. Applying the co-positivity criteria of a 4×4 matrix [97] (See appendix A for brief review) implies that the condition:

$$g^2(g_{BB}^2 + g_{BY}^2) + g_{YY}^2 g_{BB}^2 + g_{YB}^2 g_{BY}^2 \geq 2g_{YY}g_{BB}g_{YB}g_{BY} \qquad (55)$$

should be satisfied in order for the potential in Eq. (49) to be stable in the non-flat direction. It is worth noting that, in the case of no gauge mixing ($g_{YB} = 0 = g_{BY}$), the condition (55) is automatically satisfied. In this regard, the BLSSM Higgs potential is stable if and only if the conditions in Eqs. (52), (53) and (55) are satisfied.

In Fig. 4, we present the running of the BLSSM stability indicator $R \equiv g^2(g_{BB}^2 + g_{BY}^2) + g_{YY}^2 g_{BB}^2 + g_{YB}^2 g_{BY}^2 - 2g_{YY}g_{BB}g_{YB}g_{BY}$ fixing the values of the MSSM gauge coupling at the EW-scale by its known values, and fixing the mixing parameters g_{YB} & g_{BY} to be zero at the EW-scale and varying the values of the free g_{BB}. It is clear that the stability indicator R is always positive for any value of g_{BB} which means that no theoretical bounds can be put on the g_{BB} from the stability conditions. It is worth mentioning that the situation does not change when we relax the conditions on the mixing gauge couplings, $g_{YB}(EW) = 0 = g_{BY}(EW)$, by allowing nonzero values less than 10^{-3}[98].
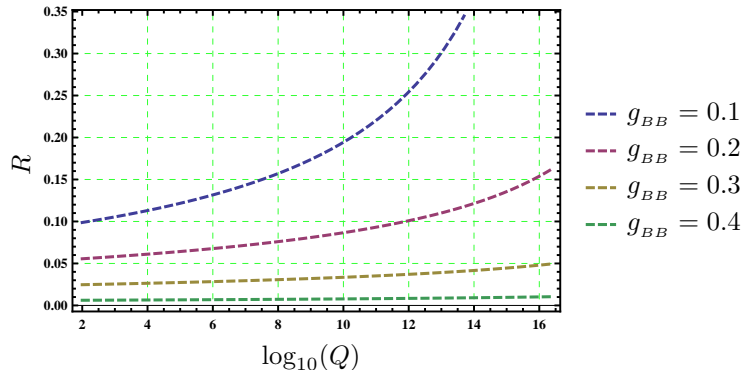
FIG. 4. Running of the BLSSM condition $R \equiv g^2(g_{BB}^2 + g_{BY}^2) + g_{YY}^2 g_{BB}^2 + g_{YB}^2 g_{BY}^2 - 2g_{YY} g_{BB} g_{YB} g_{BY}$ for different initial values of g_{BB} at the EW-scale, fixing the initial mixing parameters g_{YB} & g_{BY} to be zero at the EW-scale.

CONCLUSIONS

In this paper we have analyzed the Higgs vacuum stability problem in the $B-L$ extension of the SM and also in the MSSM. We have shown that within the context of the inverse seesaw mechanism, which is an elegant TeV scale mechanism for generating the neutrino masses, the Higgs vacuum stability is affected negatively, and the cutoff scale for vacuum instability is reduced from 10^{10} GeV in the SM to 10^5 GeV. We emphasized that the mixing between the SM-like Higgs and the $(B-L)$-like Higgs resolves this problem due to the following reasons: (i) Possible enhancement of the initial value of the SM-like Higgs self-coupling. (ii) The positive contribution of the $(B-L)$ Higgs coupling to the running of the SM-like Higgs self coupling.

We also studied the stability conditions in the supersymmetric $B-L$ model. We showed, similar to the MSSM in Higgs flat directions, the requirement of vacuum stability imposed constraints on the Higgs masses. In the non-flat directions, the stability of the Higgs potential lead to a constraint on the gauge couplings, which is automatically satisfied if there is no kinetic mixing between $U(1)_Y$ and $U(1)_{B-L}$.

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Appendix A: Co-positivity of Order Four Matrices

The co-positivity of a square matrix can be tested through some conditions that depends only on the dimension of the matrix as well as the signs of its elements. Such a subject is too lengthy to be presented here as a whole. Thus, we shall present the co-positivity conditions of only one class of 4×4 matrices to-which the matrix in Eq. (54) belongs.

Consider a symmetric 4×4 matrix

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33} & a_{34} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{pmatrix}, \tag{A1}$$

such that $a_{12}, a_{14}, a_{23}, a_{34} \leq 0$. Therefore, A is co-positive only if the following conditions are satisfied:

- $a_{ii} \geq 0$, $i = 1, \ldots, 4$.

- $a_{11}a_{22} - a_{12}^2 \geq 0$.

- The symmetric matrices:

$$\begin{pmatrix} a_{33}\left(a_{22}a_{13}^2 - 2a_{12}a_{23}a_{13} + a_{11}a_{23}^2\right) & a_{33}\left(a_{13}a_{22} - a_{12}a_{23}\right) & a_{33}\left(a_{13}a_{24} - a_{14}a_{23}\right) \\ \cdot & a_{22}a_{33} - a_{23}^2 & a_{24}a_{33} - a_{23}a_{34} \\ \cdot & \cdot & a_{33}a_{44} - a_{34}^2 \end{pmatrix}, \tag{A2}$$

$$\begin{pmatrix} a_{44}\left(a_{22}a_{14}^2 - 2a_{12}a_{24}a_{14} + a_{11}a_{24}^2\right) & a_{44}\left(a_{11}a_{24} - a_{12}a_{14}\right) & a_{44}\left(a_{13}a_{24} - a_{14}a_{23}\right) \\ \cdot & a_{11}a_{44} - a_{14}^2 & a_{13}a_{44} - a_{14}a_{34} \\ \cdot & \cdot & a_{33}a_{44} - a_{34}^2 \end{pmatrix}, \tag{A3}$$

are co-positive.

Fortunately, there is no need to review the co-positivity conditions of a 3×3 matrix here, because the associated 3×3 matrices of the matrix (54) are diagonal, hence the only condition is the non-negativity of its diagonal elements.

For a complete review of the general co-positivity conditions of any squared symmetric matrix, we suggest the Refs. [97, 99].

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