

# Introduction to the Symmetry Breaking Sector\*

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## Abstract

The basic ingredients of the Spontaneous Symmetry Breaking Phenomenon and of the Higgs Mechanism are reviewed in these lectures of pedagogical character. Some relevant topics related with the breaking  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{em}}$  are selected and discussed here. A brief survey of the experimental Higgs particle searches and the theoretical limits on  $M_H$  are also presented. The main features of the most popular models of symmetry breaking beyond the Standard Model are briefly considered. It includes a short summary of the Higgs Sector in the Minimal SUSY Model, the basic ideas of Technicolor models and a brief introduction to Strongly Interacting Scalar sectors and to the Effective Chiral Lagrangian Approach to the Electroweak Theory.

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# 1 Introduction

One of the key ingredients of the Standard Model of electroweak interactions (SM) [1] is the concept of Spontaneous Symmetry Breaking (SSB) [2], giving rise to Goldstone-excitations which in turn can be related to gauge boson mass terms. This procedure, usually called Higgs Mechanism [3], is necessary in order to describe the short ranged weak interactions by a gauge theory without spoiling gauge invariance. The discovery of the  $W^\pm$  and  $Z$  gauge bosons at CERN in 1983 may be considered as the first experimental evidence of the Spontaneous Symmetry Breaking Phenomenon in electroweak interactions [4, 5]. In present and future experiments one hopes to get insight into the nature of this Symmetry Breaking Sector ( $\mathcal{SBS}$ ) and this is one of the main motivations for constructing the next generation of accelerators. In particular, it is the most exciting challenge for the recently approved LHC collider being built at CERN.

In the SM, the symmetry breaking is realized linearly by a scalar field which acquires a non-zero vacuum expectation value. The resulting physical spectrum contains not only the massive intermediate vector bosons and fermionic matter fields but also the Higgs particle, a neutral scalar field which has successfully escaped experimental detection until now. The main advantage of the Standard Model picture of symmetry breaking lies in the fact that an explicit and consistent formulation exists, and any observable can be calculated perturbatively in the Higgs self-coupling constant. However, the fact that one can compute in a model doesn't mean at all that this is the right one.

The concept of spontaneous electroweak symmetry breaking is more general than the way it is usually implemented in the SM. Any alternative  $\mathcal{SBS}$  has a chance to replace the standard Higgs sector, provided it meets the following basic requirements: 1) Electromagnetism remains unbroken; 2) The full symmetry contains the electroweak gauge symmetry; 3) The symmetry breaking occurs at about the energy scale  $v = (\sqrt{2}G_F)^{-\frac{1}{2}} = 246 \text{ GeV}$  with  $G_F$  being the Fermi coupling constant.

In these lectures I will review all these basic ingredients of the Symmetry Breaking Phenomenon in the Electroweak Theory, and I will discuss some relevant topics related with this breaking. The lectures aim to be of pedagogical character and they are essentially addressed to young particle physicists without too much theoretical background in Quantum Field Theory. The lectures include a survey of experimental Higgs particle searches and the present status of theoretical bounds on the Higgs mass. Part of these lectures is devoted to present selected  $\mathcal{SBS}$  beyond the SM Higgs sector. It includes short introductions to: The Higgs sector of the Minimal SUSY Model (MSSM), Technicolor models, Strongly Interacting Scalar sectors and the Electroweak Chiral Lagrangian (EChL). These Lectures do not pretend to treat exhaustively the subject of Electroweak Symmetry Breaking nor to provide a complete set of references. I appologize for possible (most probably) missing references.

## 2 The Phenomenon of Spontaneous Symmetry Breaking

A simple definition of the phenomenon of spontaneous symmetry breaking (SSB)[4] is as follows:

*A physical system has a symmetry that is spontaneously broken if the interactions governing the dynamics of the system possess such a symmetry but the ground state of this system does not.*

An illustrative example of this phenomenon is the infinitely extended ferromagnet. For this purpose, let us consider the system near the Curie temperature  $T_C$ . It is described by an infinite set of elementary spins whose interactions are rotationally invariant, but its ground state presents two different situations depending on the value of the temperature  $T$ .

**Situation I:**  $T > T_C$

The spins of the system are randomly oriented and as a consequence the average magnetization vanishes:  $\vec{M}_{\text{average}} = 0$ . The ground state with these disoriented spins is clearly rotationally invariant.

**Situation II:**  $T < T_C$

The spins of the system are all oriented parallelly to some particular but arbitrary direction and the average magnetization gets a non-zero value:  $\vec{M}_{\text{average}} \neq 0$  (*Spontaneous Magnetization*). Since the direction of the spins is arbitrary there are infinite possible ground states, each one corresponding to one possible direction and all having the same (minimal) energy. Furthermore, none of these states is rotationally invariant since there is a privileged direction. This is, therefore, a clear example of spontaneous symmetry breaking since the interactions among the spins are rotationally invariant but the ground state is not. More specifically, it is the fact that the system 'chooses' one among the infinite possible non-invariant ground states what produces the phenomenon of spontaneous symmetry breaking.

On the theoretical side, and irrespectively of what could be the origin of such a physical phenomenon at a more fundamental level, one can parametrize this behaviour by means of a simple mathematical model. In the case of the infinitely extended ferromagnet one of these models is provided by the Theory of Ginzburg-Landau [6]. We present in the following the basic ingredients of this model.

For  $T$  near  $T_C$ ,  $\vec{M}$  is small and the free energy density  $u(\vec{M})$  can be approached by (here higher powers of  $\vec{M}$  are neglected):

$$\begin{aligned} u(\vec{M}) &= (\partial_i \vec{M})(\partial_i \vec{M}) + V(\vec{M}) ; i = 1, 2, 3 \\ V(\vec{M}) &= \alpha_1(T - T_C)(\vec{M} \cdot \vec{M}) + \alpha_2(\vec{M} \cdot \vec{M})^2 ; \alpha_1, \alpha_2 > 0 \end{aligned} \quad (1)$$

The magnetization of the ground state is obtained from the condition of extremum:

$$\frac{\delta V(\vec{M})}{\delta M_i} = 0 \Rightarrow \vec{M} \cdot [\alpha_1(T - T_C) + 2\alpha_2(\vec{M} \cdot \vec{M})] = 0 \quad (2)$$

There are two solutions for  $\vec{M}$ , depending on the value of  $T$ :

**Solution I:**

$$\text{If } T > T_C \Rightarrow [\alpha_1(T - T_C) + 2\alpha_2(\vec{M} \cdot \vec{M})] > 0 \Rightarrow \vec{M} = 0$$

The solution for  $\vec{M}$  is the trivial one and corresponds to the situation I described before where the ground state is rotational invariant. The potential  $V(\vec{M})$  has a symmetric shape with a unique minimum at the origin  $\vec{M} = 0$  where  $V(0) = 0$ . This is represented in Fig.1 for the simplified bidimensional case,  $\vec{M} = (M_X, M_Y)$ .

**Fig.1** The potential  $V(\vec{M})$  in the symmetric phase

**Solution II:**

If  $T < T_C \Rightarrow \vec{M} = 0$  is a local maximum and eq.(2) requires:

$$\alpha_1(T - T_C) + 2\alpha_2(\vec{M} \cdot \vec{M}) = 0 \Rightarrow |\vec{M}| = \sqrt{\frac{\alpha_1(T_C - T)}{2\alpha_2}}$$

Namely, there are an infinite absolute minima having all the same  $|\vec{M}|$  above, but different direction of  $\vec{M}$ . This corresponds to the situation II where the system has infinite possible degenerate ground states which are not rotationally invariant. The potential  $V(\vec{M})$  has a 'mexican hat shape' as represented in Fig.2 for the bidimensional case.

**Fig.2** The potential  $V(\vec{M})$  in the spontaneously broken phase

Notice that it is the choice of the particular ground state what produces, for  $T < T_C$ , the spontaneous breaking of the rotational symmetry.

### 3 Spontaneous Symmetry Breaking in Quantum Field Theory: QCD as an example

In the language of Quantum Field Theory, *a system is said to possess a symmetry that is spontaneously broken if the Lagrangian describing the dynamics of the system is invariant under these symmetry transformations, but the vacuum of the theory is not.* Here the vacuum  $|0\rangle$  is the state where the Hamiltonian expectation value  $\langle 0|H|0\rangle$  is minimum.

For illustrative purposes we present in the following the particular case of Quantum Chromodynamics (QCD) where there is a symmetry, the chiral symmetry, that is spontaneously broken [7]. For simplicity let us consider QCD with just two flavours. The Lagrangian is given by:

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2}\text{Tr}G^{\mu\nu}G_{\mu\nu} + \sum_{u,d}(i\bar{q}D_{\mu}\gamma^{\mu}q - m_q\bar{q}q) \quad (3)$$

where,

$$G_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} - ig_s[A_{\mu}, A_{\nu}]$$

$$\begin{aligned}
D_\mu q &= (\partial_\mu - ig_s A_\mu)q \\
A_\mu &= \sum_{a=1}^8 \frac{1}{2} A_\mu^a \lambda_a
\end{aligned} \tag{4}$$

It is easy to check that for  $m_{u,d} = 0$ ,  $\mathcal{L}_{QCD}$  has (apart from the  $SU(3)_C$  gauge symmetry) a global symmetry  $SU(2)_L \times SU(2)_R$ , called chiral symmetry, that is defined by the following transformations:

$$\begin{aligned}
\Psi_L &\rightarrow \Psi'_L = U_L \Psi_L \\
\Psi_R &\rightarrow \Psi'_R = U_R \Psi_R
\end{aligned}$$

where,

$$\begin{aligned}
\Psi &= \begin{pmatrix} u \\ d \end{pmatrix} ; \Psi_L = \frac{1}{2}(1 - \gamma_5)\Psi ; \Psi_R = \frac{1}{2}(1 + \gamma_5)\Psi \\
U_L &\in SU(2)_L ; U_R \in SU(2)_R
\end{aligned}$$

$U_L$  and  $U_R$  can be written in terms of the 2x2 matrices  $T_L^a$  and  $T_R^a$  ( $a = 1, 2, 3$ ) corresponding to the generators  $Q_L^a$  and  $Q_R^a$  of  $SU(2)_L$  and  $SU(2)_R$  respectively:

$$U_L = \exp(-i\alpha_L^a T_L^a) ; U_R = \exp(-i\alpha_R^a T_R^a)$$

It turns out that the physical vacuum of QCD is not invariant under the full chiral  $SU(2)_L \times SU(2)_R$  group but just under the subgroup  $SU(2)_V = SU(2)_{L+R}$  that is the well known isospin symmetry group. The transformations given by the axial subgroup,  $SU(2)_A$ , do not leave the QCD vacuum invariant. Therefore, QCD with  $m_{u,d} = 0$  has a chiral symmetry which is spontaneously broken down to the isospin symmetry:

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$$

The fact that in nature  $m_{u,d} \neq 0$  introduces an extra explicit breaking of this chiral symmetry. Since the fermion masses are small this explicit breaking is soft. The chiral symmetry is not an exact but approximate symmetry of QCD.

One important question is still to be clarified. How do we know from experiment that, in fact, the QCD vacuum is not  $SU(2)_L \times SU(2)_R$  symmetric?. Let us assume for the moment that it is chiral invariant. We will see that this assumption leads to a contradiction with experiment.

If  $|0\rangle$  is chiral invariant  $\Rightarrow$

$$U_L|0\rangle = |0\rangle ; U_R|0\rangle = |0\rangle \Rightarrow T_L^a|0\rangle = 0 ; T_R^a|0\rangle = 0 \Rightarrow Q_L^a|0\rangle = 0 ; Q_R^a|0\rangle = 0$$

In addition, if  $|\Psi\rangle$  is an eigenstate of the Hamiltonian and parity operator such that:

$$H|\Psi\rangle = E|\Psi\rangle ; P|\Psi\rangle = |\Psi\rangle$$

then,

$$\exists |\Psi' \rangle = \frac{1}{\sqrt{2}}(Q_R^a - Q_L^a)|\Psi \rangle \quad / \quad H|\Psi' \rangle = E|\Psi' \rangle ; \quad P|\Psi' \rangle = -|\Psi' \rangle$$

In summary, if the QCD vacuum is chiral invariant there must exist pairs of degenerate states in the spectrum, the so-called parity doublets as  $|\Psi \rangle$  and  $|\Psi' \rangle$ , which are related by a chiral transformation and have opposite parities. The absence of such parity doublets in the hadronic spectrum indicates that the chiral symmetry must be spontaneously broken. Namely, there must exist some generators  $Q^a$  of the chiral group such that  $Q^a|0 \rangle \neq 0$ . More specifically, it can be shown that these generators are the three  $Q_5^a$  ( $a = 1, 2, 3$ ) of the axial group,  $SU(2)_A$ . In conclusion, the chiral symmetry breaking pattern in QCD is  $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$  as announced.

## 4 Goldstone Theorem

One of the physical implications of the spontaneous symmetry breaking phenomenon is the appearance of massless modes. For instance, in the case of the infinitely extended ferromagnet and below the Curie temperature there appear modes connecting the different possible ground states, the so-called spin waves.

The general situation in Quantum Field Theory is described by the Goldstone Theorem [2]:

*If a Theory has a global symmetry of the Lagrangian which is not a symmetry of the vacuum then there must exist one massless boson, scalar or pseudoscalar, associated to each generator which does not annihilate the vacuum and having its same quantum numbers. These modes are referred to as Nambu-Goldstone bosons or simply as Goldstone bosons.*

Let us return to the example of QCD. The breaking of the chiral symmetry is characterized by  $Q_5^a|0 \rangle \neq 0$  ( $a = 1, 2, 3$ ). Therefore, according to Goldstone Theorem, there must exist three massless Goldstone bosons,  $\pi^a(x)$   $a = 1, 2, 3$ , which are pseudoscalars. These bosons are identified with the three physical pions.

The fact that pions have  $m_\pi \neq 0$  is a consequence of the soft explicit breaking in  $\mathcal{L}_{QCD}$  given by  $m_q \neq 0$ . The fact that  $m_\pi$  is small and that there is a large gap between this mass and the rest of the hadron masses can be seen as another manifestation of the spontaneous chiral symmetry breaking with the pions being the pseudo-Goldstone bosons of this breaking.

## 5 Dynamical Symmetry Breaking

In the previous sections we have seen the equivalence between the condition  $Q^a|0 \rangle \neq 0$  and the non-invariance of the vacuum under the symmetry transformations generated by the  $Q^a$

generators:

$$U|0\rangle \neq |0\rangle ; U = \exp(i\epsilon^a Q^a)$$

In Quantum Field Theory, it can be shown that an alternative way of characterizing the phenomenon of SSB is by certain field operators that have non-vanishing vacuum expectation values (v.e.v.).

$$\text{SSB} \iff \exists \Phi_j / \langle 0 | \Phi_j | 0 \rangle \neq 0$$

This non-vanishing v.e.v. plays the role of the order parameter signaling the existence of a phase where the symmetry of the vacuum is broken.

There are several possibilities for the nature of this field operator. In particular, when it is a composite operator which represents a composite state being produced from a strong underlying dynamics, the corresponding SSB is said to be a dynamical symmetry breaking. The chiral symmetry breaking in QCD is one example of this type of breaking. The non-vanishing chiral condensate made up of a quark and an anti-quark is the order parameter in this case:

$$\langle 0 | \bar{q}q | 0 \rangle \neq 0 \Rightarrow SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$$

The strong interactions of  $SU(3)_C$  are the responsible for creating these  $\bar{q}q$  pairs from the vacuum and, therefore, the value of the condensate  $\langle 0 | \bar{q}q | 0 \rangle$  should, in principle, be calculable from QCD.

It is interesting to mention that this type of symmetry breaking can happen similarly in more general  $SU(N)$  gauge theories. The corresponding gauge couplings become sufficiently strong at large distances and allow for spontaneous breaking of their additional chiral-like symmetries. The corresponding order parameter is also a chiral condensate:  $\langle 0 | \bar{\Psi}\Psi | 0 \rangle \neq 0$ .

## 6 The Higgs Mechanism

The Goldstone Theorem is for theories with spontaneously broken global symmetries but does not hold for gauge theories. When a spontaneous symmetry breaking takes place in a gauge theory the so-called Higgs Mechanism operates [3]:

*The would-be Goldstone bosons associated to the global symmetry breaking do not manifest explicitly in the physical spectrum but instead they 'combine' with the massless gauge bosons and as result, once the spectrum of the theory is built up on the asymmetrical vacuum, there appear massive vector particles. The number of vector bosons that acquire a mass is precisely equal to the number of these would-be-Goldstone bosons.*

There are three important properties of the Higgs Mechanism for 'mass generation' that are worth mentioning:

- 1.- It respects the gauge symmetry of the Lagrangian.



- 2.- It preserves the total number of polarization degrees.
- 3.- It does not spoil the good high energy properties nor the renormalizability of the massless gauge theories [8].

We now turn to the case of the Standard Model (SM) of Electroweak Interactions [1, 5]. We will see in the following how the Higgs Mechanism is implemented in the  $SU(2)_L \times U(1)_Y$  Gauge Theory in order to generate a mass for the weak gauge bosons,  $W^\pm$  and  $Z$ .

The following facts must be considered:

- 1.- The Lagrangian of the SM is gauge  $SU(2)_L \times U(1)_Y$  symmetric. Therefore, anything we wish to add must preserve this symmetry.
- 2.- We wish to generate masses for the three gauge bosons  $W^\pm$  and  $Z$  but not for the photon,  $\gamma$ . Therefore, we need three would-be-Goldstone bosons,  $\phi^+$ ,  $\phi^-$  and  $\chi$ , which will combine with the three massless gauge bosons of the  $SU(2)_L \times U(1)_Y$  symmetry.
- 3.- Since  $U(1)_{\text{em}}$  is a symmetry of the physical spectrum, it must be a symmetry of the vacuum of the Electroweak Theory.

From the above considerations we conclude that in order to implement the Higgs Mechanism in the Electroweak Theory we need to introduce 'ad hoc' an additional system that interacts with the gauge sector in a  $SU(2)_L \times U(1)_Y$  gauge invariant manner and whose self-interactions, being also introduced 'ad hoc', must produce the wanted breaking,  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{em}}$ , with the three associated would-be-Goldstone bosons  $\phi^+$ ,  $\phi^-$  and  $\chi$ . This system is the so-called Symmetry Breaking Sector ( $\mathcal{SBS}$ ).

## 7 The Symmetry Breaking Sector of the Electroweak Theory

In this section we introduce and justify the simplest choice for the  $\mathcal{SBS}$  of the Electroweak Theory.

Let  $\Phi$  be the additional system providing the  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{em}}$  breaking.  $\Phi$  must fulfil the following conditions:

- 1.- It must be a scalar field so that the above breaking preserves Lorentz invariance.
- 2.- It must be a complex field so that the Hamiltonian is hermitian.
- 3.- It must have non-vanishing weak isospin and hypercharge in order to break  $SU(2)_L$  and  $U(1)_Y$ . The assignment of quantum numbers and the choice of representation of  $\Phi$  can be done in many ways. Some possibilities are:

- Choice of a non-linear representation:  $\Phi$  transforms non-linearly under  $SU(2)_L \times U(1)_Y$ .
  - Choice of a linear representation:  $\Phi$  transforms linearly under  $SU(2)_L \times U(1)_Y$ . The simplest linear representation is a complex doublet. Alternative choices are: complex triplets, more than one doublet, etc. In particular, one may choose two complex doublets  $H_1$  and  $H_2$  as in the Minimal Supersymmetric Standard Model.
- 4.- Only the neutral components of  $\Phi$  are allowed to acquire a non-vanishing v.e.v. in order to preserve the  $U(1)_{\text{em}}$  symmetry of the vacuum.
  - 5.- The interactions of  $\Phi$  with the gauge and fermionic sectors must be introduced in a gauge invariant way.
  - 6.- The self-interactions of  $\Phi$  given by the potential  $V(\Phi)$  must produce the wanted breaking which is characterized in this case by  $\langle 0|\Phi|0 \rangle \neq 0$ .  $\Phi$  can be, in principle, a fundamental or a composite field.
  - 7.- If we want to be predictive from low energies to very high energies the interactions in  $V(\Phi)$  must be renormalizable. If instead one renounces to the predictivity at such high energies there is an alternative possibility: The SM could be considered as an effective theory of some other fundamental theory which operates at much higher energies. In that case the emerging effective potential  $V_{\text{eff}}(\Phi)$  could be non-renormalizable and its 'predictivity' must be restricted just to low energies.

By taking into account the above seven points one is led to the following simplest choice for the system  $\Phi$  and the Lagrangian of the **SBS** of the Electroweak Theory:

$$\begin{aligned}\mathcal{L}_{\text{SBS}} &= (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \\ V(\Phi) &= -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 ; \lambda > 0\end{aligned}\tag{5}$$

where,

$$\begin{aligned}\Phi &= \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix} \\ D_\mu \Phi &= (\partial_\mu - \frac{1}{2}ig\vec{\tau} \cdot \vec{W}_\mu - \frac{1}{2}ig'B_\mu)\Phi.\end{aligned}\tag{6}$$

Here  $\Phi$  is a fundamental complex doublet with hypercharge  $Y(\Phi) = 1$  and  $V(\Phi)$  is the simplest renormalizable potential.  $\vec{W}_\mu$  and  $B_\mu$  are the gauge fields of  $SU(2)_L$  and  $U(1)_Y$  respectively and  $g$  and  $g'$  are the corresponding gauge couplings.

It is interesting to notice the similarities with the Ginzburg-Landau Theory. Depending on the sign of the mass parameter  $(-\mu^2)$ , there are two possibilities for the v.e.v.  $\langle 0|\Phi|0 \rangle$  that minimizes the potential  $V(\Phi)$ ,

1)  $(-\mu^2) > 0$ : The minimum is at:

$$\langle 0|\Phi|0 \rangle = 0$$

The vacuum is  $SU(2)_L \times U(1)_Y$  symmetric and therefore no symmetry breaking occurs.

2)  $(-\mu^2) < 0$ : The minimum is at:

$$|\langle 0|\Phi|0 \rangle| = \left( \frac{0}{\frac{v}{\sqrt{2}}} \right) ; \text{ arbitrary } \arg \Phi ; v \equiv \sqrt{\frac{\mu^2}{\lambda}}$$

Therefore, there are infinite degenerate vacua corresponding to infinite possible values of  $\arg \Phi$ . Either of these vacua is  $SU(2)_L \times U(1)_Y$  non-symmetric and  $U(1)_{\text{em}}$  symmetric. The breaking  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{em}}$  occurs once a particular vacuum is chosen. As usual, the simplest choice is taken:

$$\langle 0|\Phi|0 \rangle \equiv \left( \frac{0}{\frac{v}{\sqrt{2}}} \right) ; \arg \Phi \equiv 0 ; v \equiv \sqrt{\frac{\mu^2}{\lambda}}$$

The two above symmetric and non-symmetric phases of the Electroweak Theory are clearly similar to the two phases of the ferromagnet that we have described within the Ginzburg Landau Theory context. In the SM, the field  $\Phi$  replaces the magnetization  $\vec{M}$  and the potential  $V(\Phi)$  replaces  $V(\vec{M})$ . The SM order parameter is, consequently,  $\langle 0|\Phi|0 \rangle$ . In the symmetric phase  $V(\Phi)$  is as in Fig.1, whereas in the non-symmetric phase it is as in Fig.2.

Another interesting aspect of the Higgs Mechanism, as we have already mentioned, is that it preserves the total number of polarization degrees. Let us make the counting in detail:

### 1) Before SSB

4 massless gauge bosons:  $W_{1,2,3}^\mu, B^\mu$

4 massless scalars: The 4 real components of  $\Phi$ ,  $(\phi_1, \phi_2, \phi_3, \phi_4)$

Total number of polarization degrees =  $4 \times 2 + 4 = 12$

### 2) After SSB

3 massive gauge bosons:  $W^\pm, Z$

1 massless gauge boson:  $\gamma$

1 massive scalar:  $H$

Total number of polarization degrees:  $3 \times 3 + 1 \times 2 + 1 = 12$

Furthermore, it is important to realize that one more degree than needed is introduced into the theory from the beginning. Three of the real components of  $\Phi$ , or similarly  $\phi^\pm \equiv \frac{1}{\sqrt{2}}(\phi_1 \mp i\phi_2)$  and  $\chi = \phi_3$ , are the needed would-be Goldstone bosons and the fourth one  $\phi_4$  is introduced just to complete the complex doublet. After the symmetry breaking, this extra degree translates into the apparition in the spectrum of an extra massive scalar particle, the Higgs boson particle  $H$ .

Before ending this section, we would like to address the following important question: Is it possible the Higgs Mechanism without the Higgs particle?. From the previous counting we see that, strictly speaking, it is not needed to implement the symmetry breaking. The Higgs Mechanism does require just the three would-be Goldstone bosons in order to generate the masses of the  $W^\pm$  and  $Z$  gauge bosons. Two more questions then arise. How must be introduced these three scalars? and, what are the consequences of eliminating the Higgs particle?. It turns out that the only possibility of introducing the minimal number of scalars, that is three, is by means of a non-linear representation. One example is:

$$U \equiv \exp \left( i \frac{\vec{\tau} \cdot \vec{\phi}}{v} \right), \quad v = 246 \text{ GeV}, \quad \vec{\phi} = (\phi^1, \phi^2, \phi^3)$$

$U$  is a  $2 \times 2$  unitary matrix and transforms linearly under  $SU(2)_L \times U(1)_Y$ :

$$U \rightarrow g_L U g_R^\dagger; \quad g_L \in SU(2)_L, \quad g_R \in U(1)_Y$$

However, the would-be Goldstone boson fields transform non-linearly:

$$\vec{\Phi} \rightarrow F(\vec{\Phi})$$

with  $F$  a non-linear function.

Moreover, one may build a potential in terms of the field  $U$  and its derivatives,  $V(U, \partial_\mu U)$ , such that it produces the wanted breaking  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{em}}$ . It can be shown that the model in which one replaces the standard  $V(\Phi)$  by this  $V(U, \partial_\mu U)$  fulfils the requirements 1 to 6 presented before. Of course, The derivatives in the later should be replaced by the corresponding covariant derivatives in order to get gauge invariance of the full Lagrangian.

The drawback of this model is that it fails in condition 7. In contrast to the standard  $V(\Phi)$ ,  $V(U, \partial_\mu U)$  is non-renormalizable and therefore it cannot be predictive to very high energies. In conclusion, if we want to implement the Higgs Mechanism without the Higgs particle we must renounce to the renormalizability of the  $\mathcal{SBS}$  and we need to build a sensible low energy effective theory and define a way to deal with non-renormalizable interactions.

## 8 The particle spectra of the Electroweak Theory

In order to get the particle spectra and the particle masses we first write down the full SM Lagrangian which is  $SU(2)_L \times U(1)_Y$  gauge invariant:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_\Psi + \mathcal{L}_{\text{SBS}} + \mathcal{L}_{\text{YW}} \quad (7)$$

Here,  $\mathcal{L}_{\text{YM}}$ ,  $\mathcal{L}_\Psi$ ,  $\mathcal{L}_{\text{SBS}}$  and  $\mathcal{L}_{\text{YW}}$  are the Lagrangians of the Yang Mills fields, the fermionic fields, the  $\mathcal{SBS}$  and the Yukawa interactions respectively:

$$\mathcal{L}_{\text{YM}} = \frac{1}{2} \text{Tr} (\mathcal{W}_{\mu\nu} \mathcal{W}^{\mu\nu} + \mathcal{B}_{\mu\nu} \mathcal{B}^{\mu\nu}) + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}} \quad (8)$$

$$\mathcal{L}_\Psi = \sum_\Psi i\bar{\Psi}\gamma^\mu D_\mu \Psi \quad (9)$$

$$\mathcal{L}_{\text{SBS}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 \quad (10)$$

$$\mathcal{L}_{\text{YW}} = \lambda_e \bar{l}_L \Phi e_R + \lambda_u \bar{q} \tilde{\Phi} u_R + \lambda_d \bar{q}_L \Phi d_R + h.c. + 2^{nd} \text{ and } 3^{rd} \text{ families} \quad (11)$$

where,  $\mathcal{L}_{\text{GF}}$  and  $\mathcal{L}_{\text{FP}}$  are the gauge fixing and Faddeev-Popov terms respectively that we omit here for brevity.

The field strength tensors are,

$$\begin{aligned} \mathcal{W}_{\mu\nu} &\equiv \partial_\mu \mathcal{W}_\nu - \partial_\nu \mathcal{W}_\mu - g[\mathcal{W}_\mu, \mathcal{W}_\nu], \\ \mathcal{B}_{\mu\nu} &\equiv \partial_\mu \mathcal{B}_\nu - \partial_\nu \mathcal{B}_\mu \end{aligned} \quad (12)$$

and the fields are given by,

$$\begin{aligned} \mathcal{W}_\mu &\equiv \frac{-i}{2} \vec{W}_\mu \cdot \vec{\tau}; \quad \mathcal{B}_\mu \equiv \frac{-i}{2} B_\mu \tau^3 \\ l_L &= \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}; \quad q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \\ \Phi &= \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}; \quad \tilde{\Phi} = i\tau_2 \Phi^* = \begin{pmatrix} \phi_0^* \\ -\phi^- \end{pmatrix} \end{aligned} \quad (13)$$

The following steps summarize the procedure to get the spectrum from  $\mathcal{L}_{\text{SM}}$ :

1.- A non-symmetric vacuum must be fixed. Let us choose, for instance,

$$\langle 0|\Phi|0 \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

2.- The physical spectrum is built by performing 'small oscillations' around this vacuum. These are parametrized by,

$$\Phi(x) = \exp \left( i \frac{\vec{\xi}(x) \vec{\tau}}{v} \right) \begin{pmatrix} 0 \\ \frac{v+H(x)}{\sqrt{2}} \end{pmatrix}$$

where  $\vec{\xi}(x)$  and  $H(x)$  are 'small' fields.

3.- In order to eliminate the unphysical fields we make the following gauge transformations:

$$\begin{aligned} \Phi' &= U(\xi) \Phi = \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}; \quad U(\xi) = \exp \left( -i \frac{\vec{\xi} \vec{\tau}}{v} \right) \\ l'_L &= U(\xi) l_L; \quad e'_R = e_R; \quad q'_L = U(\xi) q_L; \quad u'_R = u_R; \quad d'_R = d_R \\ \left( \frac{\vec{\tau} \cdot \vec{W}'_\mu}{2} \right) &= U(\xi) \left( \frac{\vec{\tau} \cdot \vec{W}_\mu}{2} \right) U^{-1}(\xi) - \frac{i}{g} (\partial_\mu U(\xi)) U^{-1}(\xi); \quad B'_\mu = B_\mu \end{aligned} \quad (14)$$

- 4.- Finally, the weak eigenstates are rotated to the mass eigenstates which define the physical gauge boson fields:

$$\begin{aligned} W_\mu^\pm &= \frac{W_\mu'^1 \mp iW_\mu'^2}{\sqrt{2}}, \\ Z_\mu &= c W_\mu'^3 - s B'_\mu, \\ A_\mu &= s W_\mu'^3 + c B'_\mu, \end{aligned} \quad (15)$$

where,  $c \equiv \cos \theta_W$ ,  $s \equiv \sin \theta_W$  and  $\theta_W$  is the weak angle defined by  $\tan \theta_W \equiv \frac{g'}{g}$ .

It is now straightforward to read the masses from the following terms of  $\mathcal{L}_{\text{SM}}$ :

$$\begin{aligned} (D_\mu \Phi')^\dagger (D^\mu \Phi') &= \left( \frac{g^2 v^2}{4} \right) W_\mu^+ W^{\mu-} + \frac{1}{2} \left( \frac{(g^2 + g'^2) v^2}{4} \right) Z_\mu Z^\mu + \dots \\ V(\Phi') &= \frac{1}{2} (2\mu^2) H^2 + \dots \\ \mathcal{L}_{\text{YW}} &= \left( \lambda_e \frac{v}{\sqrt{2}} \right) \bar{e}'_R e'_R + \left( \lambda_u \frac{v}{\sqrt{2}} \right) \bar{u}'_R u'_R + \left( \lambda_d \frac{v}{\sqrt{2}} \right) \bar{d}'_R d'_R + \dots \end{aligned} \quad (16)$$

and get finally the tree level predictions:

$$\begin{aligned} M_W &= \frac{gv}{2}; \quad M_Z = \frac{\sqrt{g^2 + g'^2} v}{2} \\ M_H &= \sqrt{2} \mu \\ m_e &= \lambda_e \frac{v}{\sqrt{2}}; \quad m_u = \lambda_u \frac{v}{\sqrt{2}}; \quad m_d = \lambda_d \frac{v}{\sqrt{2}}; \dots \end{aligned} \quad (17)$$

where,

$$v = \sqrt{\frac{\mu^2}{\lambda}}; \quad g = \frac{e}{s}; \quad g' = \frac{e}{c}$$

Some comments are in order.

- All masses are given in terms of a unique mass parameter  $v$  and the couplings  $g$ ,  $g'$ ,  $\lambda$ ,  $\lambda_e$ , etc..
- The interactions of  $H$  with fermions and with gauge bosons are proportional to the gauge couplings and to the corresponding particle masses:

$$f \bar{f} H : -i \frac{g}{2} \frac{m_f}{M_W}; \quad W_\mu^+ W_\nu^- H : ig M_W g_{\mu\nu}; \quad Z_\mu Z_\nu H : \frac{ig}{c} M_Z g_{\mu\nu}$$

- The v.e.v.  $v$  is determined experimentally from  $\mu$ -decay. By identifying the predictions of the partial width  $\Gamma(\mu \rightarrow \nu_\mu \bar{\nu}_e e)$  in the SM to low energies ( $q^2 \ll M_W^2$ ) and in the V-A Theory one gets,

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2}$$

where,

$$G_F = (1.16639 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2}$$

And from here,

$$v = (\sqrt{2}G_F)^{-\frac{1}{2}} = 246 \text{ GeV}$$

- The values of  $M_W$  and  $M_Z$  were anticipated successfully quite before they were measured in experiment. The input parameters were  $\theta_W$ , the fine structure constant  $\alpha$  and  $G_F$ . Before LEP these were the best measured electroweak parameters.
- In contrast to the gauge boson sector, the Higgs boson mass  $M_H$  and the Higgs self-coupling  $\lambda$  are completely undetermined in the SM.
- The hierarchy in the fermion masses is also completely undetermined in the SM.

## 9 The $\rho$ parameter and the custodial symmetry

In this section we comment on the relevance of the  $\rho$  parameter and the custodial symmetry for the study of the  $\mathcal{SBS}$  of the Electroweak Theory.

The  $\rho$  parameter is defined as the ratio of neutral to charged current interactions at low energies:

$$\rho \equiv \frac{T_{NC}(q^2 \ll M_Z^2)}{T_{CC}(q^2 \ll M_W^2)} \quad (18)$$

and is known from  $\nu$ -scattering experiments to be very close to one:  $\rho_{\text{exp}} \approx 1$ .

The SM prediction at tree level is given by:

$$\rho_{\text{tree}}^{\text{SM}} = \frac{M_W^2}{M_Z^2 c^2} \quad (19)$$

From this equation and by using the tree level expressions of eq.(17) one gets the well known result,

$$\rho_{\text{tree}}^{\text{SM}} = 1$$

At one loop and by keeping just the so-called 'oblique' corrections, namely, the self-energies but not the vertex and box corrections, one gets:

$$\rho = \frac{\rho_{\text{tree}}}{1 - \Delta\rho} \quad (20)$$

where  $\Delta\rho$  can be written in terms of the renormalized gauge bosons self-energies as follows:

$$\Delta\rho = \frac{\Sigma_Z^{\text{R}}(0)}{M_Z^2} - \frac{\Sigma_W^{\text{R}}(0)}{M_W^2} \quad (21)$$

or, alternatively, in terms of the unrenormalized self-energies:

$$\Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2} - \frac{2s}{c} \frac{\Sigma_{\gamma Z}(0)}{M_Z^2} \quad (22)$$

where,  $M_W$ ,  $M_Z$ ,  $s$  and  $c$  are the renormalized parameters in the on-shell scheme [9].

In the SM,  $\Delta\rho$  receives contributions from both the bosonic and fermionic loops and they are of electroweak strength, meaning  $\Delta\rho \sim O(10^{-2})$ .

The fact that  $\rho$  is very close to one can be understood from the theory point of view as a consequence of an additional approximate global symmetry of  $\mathcal{L}_{\text{SM}}$ . This is the named custodial symmetry,  $SU(2)_C$  [10] and it would be an exact symmetry if the masses of the fermions in each fermionic weak doublet (or, equivalently, their corresponding Yukawa couplings) were equal,  $m_{f_1} = m_{f_2}$ , and if  $g' = 0$ . The existence of mass splittings and interactions mediated by the hypercharge boson produce some explicit  $SU(2)_C$  breaking terms. However, they are in general small except for the top-bottom mass splitting effect.

It is interesting to realize that if one isolates the pure scalar sector of the SM it turns out to be exactly  $SU(2)_C$  symmetric. In order to show this let us write the Lagrangian of the scalar sector, eq.(5) in terms of a different parametrization:

$$\mathcal{L}_{\text{SBS}} = \frac{1}{4} \text{Tr} [(\partial_\mu M)^\dagger (\partial^\mu M)] - V(M) ; \quad (23)$$

$$V(M) = \frac{1}{4} \lambda \left[ \frac{1}{2} \text{Tr}(M^\dagger M) + \frac{\mu^2}{\lambda} \right]^2 \quad (24)$$

where  $M$  is a  $2 \times 2$  matrix containing the four real scalar fields of  $\Phi$ :

$$\begin{aligned} M &\equiv \sqrt{2}(\tilde{\Phi}\Phi) = \sqrt{2} \begin{pmatrix} \phi_0^* & \phi^+ \\ -\phi^- & \phi_0 \end{pmatrix} ; \\ \Phi &= \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix} ; \\ \tilde{\Phi} &= i\tau_2 \Phi^* = \begin{pmatrix} \phi_0^* \\ -\phi^- \end{pmatrix} \end{aligned} \quad (25)$$

It is easy to check that  $\mathcal{L}_{\text{SBS}}$  is invariant under the transformations:

$$M \rightarrow g_L M g_R^\dagger ; \quad g_L \in SU(2)_L ; \quad g_R \in SU(2)_R$$

This global symmetry  $SU(2)_L \times SU(2)_R$  is called the 'chiral' symmetry of the scalar sector of the SM because of its analogy with the chiral symmetry of QCD. Furthermore, if one studies the vacuum state of the theory defined by this pure scalar sector one finds out that it is not 'chiral' invariant but just invariant under a lower symmetry given by the diagonal subgroup  $SU(2)_{L+R}$  which is identified with the custodial symmetry group,  $SU(2)_C$ . This is the analogous to the isospin symmetry group of QCD. In summary, the **SBS** of the SM has a spontaneously broken global chiral symmetry:

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_C$$



Once the subgroup  $SU(2)_L \times U(1)_Y$  is gauged, the complete  $\mathcal{L}_{\text{SM}}$  is not anymore  $SU(2)_L \times SU(2)_R$  nor custodial invariant. The explicit breaking in the bosonic sector is produced by the hypercharge boson mediated interactions and is small since it is proportional to  $g'$ . Some observables as  $\Delta\rho$  measure precisely these custodial breaking terms and, therefore, they vanish in the limit  $g' \approx 0$ , that is  $\Delta\rho_{\text{bosonic}} \approx 0$ . One crucial point is that this result is true even in the hypothetical case that the  $\mathcal{SBS}$  be strongly interacting. The prediction of  $\rho \approx 1$ , at tree level and beyond tree level, is protected against potentially large corrections from this sector due to the approximate custodial symmetry of the Electroweak Theory. It is, therefore, a reasonable choice in model building beyond the SM to assume that this custodial symmetry is indeed a symmetry of its corresponding  $\mathcal{SBS}$ .

## 10 Experimental bounds on $M_H$

The search of the Higgs particle at present  $e^+e^-$  and  $\bar{p}p$  colliders is very difficult due the smallness of the cross-sections for Higgs production [12] which, in turn, is explained in terms of the small couplings of the Higgs particle to light fermions:  $H\bar{f}f \leftrightarrow -i\frac{g}{2}\frac{m_f}{M_W}$ . On the other hand, at present available energies, the dominant decay channel is  $H \rightarrow b\bar{b}$  (see Fig.3) which is not easy to study due to the complexity of the final state and the presence of large backgrounds.

**Fig.3** Higgs decay branching ratios (ref.[21])

## I.- Higgs search at $e^+e^-$ colliders (LEP, SLC)

The Higgs search at present  $e^+e^-$  colliders is done mainly by analysing the process [15, 13]:

$$e^+e^- \rightarrow Z \rightarrow Z^*H$$

with the virtual  $Z^*$  decaying as  $Z^* \rightarrow l^+l^-$  or  $Z^* \rightarrow \nu\bar{\nu}$  and the Higgs particle decaying as  $H \rightarrow b\bar{b}$ .

At LEPI with a center-of-mass-energy adjusted to the  $Z$  mass,  $\sqrt{s} \sim M_Z$ , a very high statistic has been reached and a systematic search of the Higgs particle for all kinematically allowed  $M_H$  values has been possible. The absence of any experimental signal from the Higgs particle implies a lower bound on  $M_H$ . The most recent reported bound from LEP is [11, 13, 15]:

$$M_H > 65.1 \text{ GeV (95\%C.L.)}$$

In the second phase of LEP, LEP II, a center-of-mass-energy of up to  $\sqrt{s} \sim 175 \text{ GeV}$  is expected to be reached. The relevant process for Higgs searches will be:

$$e^+e^- \rightarrow Z^* \rightarrow ZH$$

where now the intermediate  $Z$  boson is virtual and the final  $Z$  is on its mass shell. The analyses of the various relevant  $Z$  and  $H$  decays will explore the following mass values:

$$M_H < \sqrt{s} - M_Z \sim 80 \text{ GeV}$$

In addition to the direct bounds on  $M_H$  from LEP data, a great effort is being done also in the search of indirect Higgs signals from its contribution to electroweak quantum corrections. In fact, there have been already the first attempts to extract experimental bounds on  $M_H$  from the measurement of observables as  $\Delta\rho$  whose prediction in the SM is well known. It is interesting to mention that neither the Higgs particle nor the top quark decouple from these low energy observables. It means that the quantum effects of a virtual  $H$  or  $t$  do not vanish in the limit of infinitely large  $M_H$  or  $m_t$  respectively. For instance, the leading corrections to  $\Delta\rho$  in these limits are [17, 18]:

$$\begin{aligned} (\Delta\rho)_t &= \frac{g^2}{64\pi^2} N_C \frac{m_t^2}{M_W^2} + \dots \\ (\Delta\rho)_H &= -\frac{g^2}{64\pi^2} 3 \tan^2 \theta_W \log \frac{M_H^2}{M_W^2} + \dots \end{aligned} \quad (26)$$

Whereas the top corrections grow with the mass as  $m_t^2$ , the Higgs corrections are milder growing as  $\log M_H^2$ . It means that the top non-decoupling effects at LEP are important. In fact they have been crucial in the search of the top quark and have provided one of the first indirect indications of the 'preference of data' for large  $m_t$  values. This has been finally confirmed with the discovery of the top quark at TEVATRON and the measurement of its mass [16], resulting in a weighted average value of  $m_t = 180 \pm 12 \text{ GeV}$ .

The fact that the Higgs non-decoupling effects are soft was announced a long time ago by T.Veltman in the so-called *Screening Theorem* [17]. This theorem states that, *at one-loop, the*

dominant quantum corrections from a heavy Higgs particle to electroweak observables grow, at most, as  $\log M_H$ . The Higgs corrections are of the generic form:

$$g^2(\log \frac{M_H^2}{M_W^2} + g^2 \frac{M_H^2}{M_W^2} + \dots)$$

and the potentially large effects proportional to  $M_H^2$  are 'screened' by additional small  $g^2$  factors.

Although some analyses performed at LEP show a slight 'preference of data' for a light Higgs ( $M_H < 600 \text{ GeV}$ , 95% C.L., with  $m_t$  fixed to the TEVATRON value [13]) there are still many uncertainties in this interpretation and the conclusion is highly dependent on the assumed input values of  $m_t$ ,  $\alpha_s(M_Z^2)$  and  $\alpha(M_Z^2)$ . In fact, there are some parallel works [14] where the opposite preference for a heavy Higgs is claimed. At present, it is therefore too premature to reach a definite conclusion and we should wait till the uncertainties in the input parameters be considerably reduced.

## II.- Higgs search at hadronic colliders

The relevant subprocesses for Higgs production at hadronic  $pp$  and  $p\bar{p}$  colliders are shown in Fig.4.

**Fig.4** Higgs production mechanisms at hadronic colliders

At present available energies the dominant subprocess is  $gg$ -fusion. This can be seen in Fig.5 where the cross section for the various Higgs production channels at the present collider TEVATRON with  $\sqrt{s} = 2 \text{ TeV}$  are shown.

**Fig.5** Higgs production rates at TEVATRON (ref.[19])

Unfortunately, TEVATRON is not an efficient experiment for Higgs searches, mainly because of lack of statistics, even for the case of a light Higgs where the cross section is maximum. For instance, for the energy and luminosity values of  $\sqrt{s} = 1.8 \text{ TeV}$  and  $\mathcal{L} = 10^{-31} \text{ cm}^{-2} \text{ sec}^{-1}$  a Higgs particle with  $M_H = 60 \text{ GeV}$  would produce just about 400 events per year. Furthermore, in order to detect a light Higgs at hadron colliders one must look at the cleanest decays as, for instance, the  $H \rightarrow \gamma\gamma$  channel which has a too small branching ratio,  $BR(H \rightarrow \gamma\gamma) \sim 10^{-3}$ .

The Higgs search at the recently approved LHC collider being built at CERN, is fortunately more promising [20]. The cross section of Higgs production in the various modes at LHC are shown in Fig.6.

**Fig.6** Higgs production rates at LHC (ref.[21])

For  $M_H < 800 \text{ GeV}$  and  $m_t \sim 180 \text{ GeV}$  the dominant channel is still  $gg$ -fusion. However, the  $WW$  and  $ZZ$  fusion channels become also relevant in the large  $M_H$  region and, in particular, they can provide valuable information on the Higgs system if it is strongly interacting.

The various exhaustive studies done so far indicate that if the LHC nominal parameters of  $\sqrt{s} = 14 \text{ TeV}$  and one-year-integrated luminosity of  $L = 10^5 \text{ pb}^{-1}$  are reached, the whole missing mass range of  $80 \text{ GeV} \leq M_H \leq 1 \text{ TeV}$  can be covered [21].

## 11 $WW$ scattering and the Effective $W$ approximation

For a very heavy Higgs particle with  $M_H \sim O(1 \text{ TeV})$  the Higgs width is comparable with its mass and to consider vector boson fusion channels as in Fig.4 is not anymore a good approximation. In this case, all diagrams that participate in the  $VV$  scattering subprocess ( $V = W^\pm, Z$ ) must be included. For instance, the diagrams contributing to  $W^+W^-$  production from  $W^+W^-$  fusion are shown in Fig.7.

**Fig.7** Contributing diagrams to  $qq \rightarrow qqWW$  through  $WW$  scattering

The computation of all these diagrams is quite lengthy. For simplicity, it is convenient to use the so-called Effective  $W$  Approximation [22] which is very similar to the well known Effective Photon Approximation [23]. In the  $W$  Effective Approximation, the process  $qq \rightarrow qqVV$  with  $V = W^\pm, Z$  is factorized out into the production of 'quasireal'  $V$ 's being radiated from the initial quarks and the posterior rescattering of these bosons which are assumed to be on-shell. It works well because the cross section of the full process  $qq \rightarrow qqVV$  is known to be dominated by the kinematical region where the intermediate  $V$ 's in this process are emitted close to their mass shell and with small scattering angles with respect to the outgoing quark. From the computational point of view, the  $V$ 's are considered as partons with certain probability distributions and the cross section of the full process is obtained by making the convolution of these functions with the cross section of the corresponding  $VV$  scattering subprocess. For instance, for the  $WW$  case one

writes:

$$\sigma(pp \rightarrow (WW \rightarrow WW)X) = \int_{\tau_{\min}}^1 d\tau \left( \frac{d\mathcal{L}}{d\tau} \right)_{pp/WW} \sigma(WW \rightarrow WW) \quad (27)$$

where the luminosity of  $W$ 's from the protons is given by

$$\left( \frac{d\mathcal{L}}{d\tau} \right)_{pp/WW} = \sum_{ij} \int_{\tau}^1 \frac{d\tau'}{\tau'} \int_{\tau'}^1 \frac{dx}{x} q_i(x) q_j\left(\frac{\tau'}{x}\right) \left( \frac{d\mathcal{L}}{d\zeta} \right)_{q_i q_j / WW} \quad (28)$$

and the corresponding one from the quarks is

$$\left( \frac{d\mathcal{L}}{d\zeta} \right)_{q_i q_j / WW} = \int_{\zeta}^1 \frac{dy}{y} f_{q/W}(y) f_{q/W}\left(\frac{\zeta}{y}\right); \quad \zeta = \frac{\tau}{\tau'} \quad (29)$$

Here,  $q_i(x)$  are the quark distribution functions in the proton and  $f_{q/W}$  is the  $W$  distribution function in the quark  $q$ . The simplest version of these functions for longitudinal and transverse  $W$ 's are the following,

$$f_{q/W}^L = \frac{g^2}{16\pi^2} \left( \frac{1-x}{x} \right); \quad f_{q/W}^T(x) = \frac{g^2}{64\pi^2} \log \left( \frac{4E^2}{M_W^2} \right) \left( \frac{1+(1-x)^2}{x} \right) \quad (30)$$

Here  $x$  is the momentum fraction of the quark  $q$  that is carried by the emitted  $W$  and  $2E$  is the total energy of the  $qq$  system. We have averaged over the two transverse polarizations. Similar equations are provided for the case of  $Z$  gauge bosons.

The Effective  $W$  Approximation is particularly useful in the case where the  $\mathcal{SBS}$  is strongly interacting and more generally for the kind of models that predict different  $V_L V_L \rightarrow V_L V_L$  scattering amplitudes than those of the SM.

## 12 The Equivalence Theorem

This theorem states the following [24]:

*The scattering amplitudes of longitudinal gauge bosons  $V_L$  ( $V = W^\pm, Z$ ), at high energies,  $\sqrt{s} \gg M_V$ , are equivalent to the scattering amplitudes of their corresponding would-be Goldstone bosons  $w$ ,*

$$T(V_L^1 V_L^2 \dots V_L^N \rightarrow V_L^1 V_L^2 \dots V_L^{N'}) \approx i^N (-i)^{N'} T(w_1 w_2 \dots w_N \rightarrow w_1 w_2 \dots w_{N'}) \quad (31)$$

It can be seen as the reflect that in a gauge theory with spontaneous symmetry breaking, the needed longitudinal polarization degrees of the massive vector bosons are originated by the Higgs Mechanism precisely from the corresponding would-be Goldstone bosons.

The Equivalence Theorem works at tree level and beyond tree level and it is very useful in simplifying a number of involved computations. For instance, it can be applied to compute the

partial widths of a heavy Higgs into longitudinal  $W$  and  $Z$  bosons. In this case,

$$\begin{aligned}\Gamma(H \rightarrow W_L^+ W_L^-) &= \Gamma(H \rightarrow w^+ w^-) + O\left(\frac{M_W}{M_H}\right) \\ \Gamma(H \rightarrow Z_L Z_L) &= \Gamma(H \rightarrow z z) + O\left(\frac{M_Z}{M_H}\right)\end{aligned}\quad (32)$$

where  $w^\pm$  and  $z$  are the three would-be Goldstone bosons of the Electroweak Theory. At tree level it gives,

$$\Gamma(H \rightarrow W_L^+ W_L^-) \simeq 2\Gamma(H \rightarrow Z_L Z_L) \simeq \frac{g^2}{64\pi} \frac{M_H^3}{M_W^2} \quad (33)$$

It has been computed also to one loop [25] and recently to two loops [26].

One of the most important applications of the Equivalence Theorem is in the study of  $VV$  scattering at the future LHC collider. Let us see, for instance, how it works in the case of  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$  scattering at tree level.

The contributing diagrams, in the SM, to the amplitudes with external  $W_L$ 's are the same as in the ring of Fig.7. The diagrams for scattering with external  $w$ 's are shown in Fig.8.

**Fig.8** Contributing diagrams to  $w^+ w^- \rightarrow w^+ w^-$  scattering

Here the polarization vector of an external  $W_L$  with momentum  $k$  is given by

$$\epsilon_L^\mu = \frac{1}{M_W}(|\vec{k}|, 0, 0, k_0)$$

After the computation of these diagrams one gets the following results:

$$T(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = -\frac{1}{v^2} \left\{ -s - t + \frac{s^2}{s - M_H^2} + \frac{t^2}{t - M_H^2} + 2M_Z^2 + \right.$$



$$\frac{2M_Z^2 s}{t - M_Z^2} + \frac{2t}{s}(M_Z^2 - 4M_W^2) - \frac{8s_W^2 M_W^2 M_Z^2 s}{t(t - M_Z^2)} \quad (34)$$

$$T(w^+ w^- \rightarrow w^+ w^-) = -\frac{M_H^2}{v^2} \left\{ \frac{s}{s - M_H^2} + \frac{t}{t - M_H^2} \right\} \quad (35)$$

By studying the above expressions in the large energy limit,  $\sqrt{s} \gg M_W, M_Z$ , and by keeping just the leading term, one can check the validity of the Equivalence Theorem which in this case reads,

$$T(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = T(w^+ w^- \rightarrow w^+ w^-) + O\left(\frac{M^2}{s}\right) \quad (36)$$

and it holds for any value of  $M_H$ . The above amplitudes have been computed up to one loop [45] and the Equivalence Theorem seems to work well.

## 13 Theoretical bounds on $M_H$

In this section we summarize the present bounds on  $M_H$  from the requirement of consistency of the theory.

### I.- Upper bound on $M_H$ from Unitarity

Unitarity of the scattering matrix together with the elastic approximation for the total cross-section and the Optical Theorem imply certain elastic unitarity conditions for the partial wave amplitudes. These, in turn, when applied in the SM to scattering processes involving the Higgs particle, imply an upper limit on the Higgs mass. Let us see this in more detail for the simplest case of scattering of massless scalar particles:  $1 + 2 \rightarrow 1 + 2$ .

The decomposition of the amplitude in terms of partial waves is given by:

$$T(s, \cos \theta) = 16\pi \sum_{J=0}^{\infty} (2J+1) a_J(s) P_J(\cos \theta) \quad (37)$$

where  $P_J$  are the Legendre polynomials.

The corresponding differential cross-section is given by:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |T|^2 \quad (38)$$

Thus, the elastic cross-section is written in terms of partial waves as:

$$\sigma_{\text{el}} = \frac{16\pi}{s} \sum_{J=0}^{\infty} (2J+1) |a_J(s)|^2 \quad (39)$$

On the other hand, the Optical Theorem relates the total cross-section with the forward elastic scattering amplitude:

$$\sigma_{\text{tot}}(1 + 2 \rightarrow \text{anything}) = \frac{1}{s} \text{Im } T(s, \cos \theta = 1) \quad (40)$$

In the elastic approximation for  $\sigma_{\text{tot}}$  one gets  $\sigma_{\text{tot}} \approx \sigma_{\text{el}}$ . From this and by identifying eqs.(39) and (40) one finally finds,

$$\text{Im } a_J(s) = |a_J(s)|^2 ; \forall J \quad (41)$$

This is called the elastic unitarity condition for partial wave amplitudes. It is easy to get from this eq.(41) the following inequalities:

$$|a_J|^2 \leq 1 ; 0 \leq \text{Im } a_J \leq 1 ; |\text{Re } a_J| \leq \frac{1}{2} ; \forall J \quad (42)$$

These are necessary but not sufficient conditions for elastic unitarity. It implies that if any of them is not fulfilled then the elastic unitarity condition of eq.(41) also fails, in which case the unitarity of the theory is said to be violated.

Let us now study the particular case of  $W_L^+ W_L^-$  scattering in the SM and find its unitarity conditions. The  $J = 0$  partial wave can be computed from:

$$a_0(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = \frac{1}{32\pi} \int_{-1}^1 T(s, \cos \theta) d(\cos \theta) \quad (43)$$

where the amplitude  $T(s, \cos \theta)$  is given in eq.(34). By studying the large energy limit of  $a_0$  one finds,

$$|a_0| \xrightarrow{s \gg M_H^2, M_V^2} \frac{M_H^2}{8\pi v^2} \quad (44)$$

Finally, by imposing the unitarity condition  $|\text{Re } a_0| \leq \frac{1}{2}$  one gets the following upper bound on the Higgs mass:

$$M_H < 860 \text{ GeV} \quad (45)$$

One can repeat the same reasoning for different channels and find similar or even tighter bounds than this one.

At this point, it should be mentioned that these upper bounds based on perturbative unitarity do not mean that the Higgs particle cannot be heavier than these values. The conclusion should be, instead, that for those large  $M_H$  values a perturbative approach is not valid and non-perturbative techniques are required.

## II.- Upper bound on $M_H$ from Triviality

Triviality in  $\lambda\Phi^4$  theories [27] (as, for instance, the scalar sector of the SM) means that the particular value of the renormalized coupling of  $\lambda_R = 0$  is the unique fixed point of the theory. A theory with  $\lambda_R = 0$  contains non-interacting particles and therefore it is trivial. This behaviour can already be seen in the renormalized coupling at one-loop level:

$$\lambda_R(Q) = \frac{\lambda_0}{1 - \frac{3}{2\pi^2} \lambda_0 \log(\frac{Q}{\Lambda})} ; \lambda_0 \equiv \lambda_R(Q = \Lambda) \quad (46)$$

As we attempt to remove the cut-off  $\Lambda$  by taking the limit  $\Lambda \rightarrow \infty$  while  $\lambda_0$  is kept fixed to an arbitrary but finite value, we find out that  $\lambda_R(Q) \rightarrow 0$  at any finite energy value  $Q$ . This, on the other hand, can be seen as a consequence of the existence of the well known Landau pole of  $\lambda\Phi^4$  theories.

The triviality of the  $\mathcal{SBS}$  of the SM is cumbersome since we need a self-interacting scalar system to generate  $M_W$  and  $M_Z$  by the Higgs Mechanism. The way out from this apparent problem is to assume that the Higgs potential  $V(\Phi)$  is valid just below certain 'physical' cut-off  $\Lambda_{\text{phys}}$ . Then,  $V(\Phi)$  describes an effective low energy theory which emerges from some (so far unknown) fundamental physics with  $\Lambda_{\text{phys}}$  being its characteristics energy scale. We are going to see next that this assumption implies an upper bound on  $M_H$  [28].

Let us assume some concrete renormalization of the SM parameters. The conclusion does not depend on this particular choice. Let us define, for instance, the renormalized Higgs mass parameter as:

$$M_H^2 = 2\lambda_R(v)v^2 \quad (47)$$

where,

$$\lambda_R(v) = \frac{\lambda_0}{1 - \frac{3}{2\pi^2}\lambda_0 \log(\frac{v}{\Lambda_{\text{phys}}})} \quad (48)$$

Now, if we want  $V(\Phi)$  to be a sensible effective theory, we must keep all the renormalized masses below the cut-off and, in particular,  $M_H < \Lambda_{\text{phys}}$ . However, from eqs.(47) and (48) we can see that for arbitrary values of  $\Lambda_{\text{phys}}$  it is not always possible. By increasing the value of  $\Lambda_{\text{phys}}$ ,  $M_H$  decreases and the other way around, by lowering  $\Lambda_{\text{phys}}$ ,  $M_H$  grows. There is a crossing point where  $M_H \approx \Lambda_{\text{phys}}$  which happens to be around an energy scale of approximately 1 *TeV*. Since we want to keep the Higgs mass below the physical cut-off, it implies finally the announced upper bound,

$$M_H^{1\text{-loop}} < 1 \text{ TeV}$$

Of course, this should be taken just as a perturbative estimate of the true triviality bound. A more realistic limit must come from a non-perturbative treatment. In particular, the analyses performed on the lattice [29] confirm this behaviour and place even tighter limits. The following bound is found,

$$M_H^{\text{Lattice}} < 640 \text{ GeV}$$

Finally, a different but related perturbative upper limit on  $M_H$  can be found by analysing the renormalization group equations in the SM to one-loop. Here one includes, the scalar sector, the gauge boson sector and restricts the fermionic sector to the third generation. By requiring the theory to be perturbative (i.e. all the couplings be sufficiently small) at all the energy scales below some fixed high energy, one finds a maximum allowed  $M_H$  value [30]. For instance, by fixing this energy scale to  $10^{16} \text{ GeV}$  and for  $m_t = 170 \text{ GeV}$  one gets:

$$M_H^{\text{RGE}} < 170 \text{ GeV}$$

Of course to believe in perturbativity up to very high energies could be just a theoretical prejudice. The existence of a non-perturbative regime for the scalar sector of the SM is still a possibility and one should be open to new proposals in this concern.

### III.- Lower bound on $M_H$ from Vacuum Stability

Once the asymmetric vacuum of the  $SU(2)_L \times U(1)_Y$  theory has been fixed, one must require this vacuum to be stable under quantum corrections. In principle, quantum corrections could destabilize the asymmetric vacuum and change it to the symmetric one where the spontaneous symmetry breaking does not take place. This phenomenon can be better explained in terms of the effective potential with quantum corrections included on it. Let us take, for instance, the effective potential of the Electroweak Theory to one loop in the small  $\lambda$  limit:

$$V_{\text{eff}}^{1\text{-loop}}(\Phi) \simeq -\mu^2 \Phi^\dagger \Phi + \lambda_R(Q_0)(\Phi^\dagger \Phi)^2 + \beta_\lambda (\Phi^\dagger \Phi)^2 \log \left( \frac{\Phi^\dagger \Phi}{Q_0^2} \right) \quad (49)$$

where,  $\beta_\lambda \equiv \frac{d\lambda}{dt} \simeq \frac{1}{16\pi^2} \left[ -3\lambda_t^4 + \frac{3}{16}(2g^4 + (g^2 + g'^2)^2) \right]$ .

The condition of extremum is:

$$\frac{\delta V_{\text{eff}}^{1\text{-loop}}}{\delta \Phi} = 0 \quad (50)$$

which leads to two possible solutions: a) The trivial vacuum with  $\Phi = 0$ ; and b) The non-trivial vacuum with  $\Phi = \Phi_{\text{vac}} \neq 0$ . If we want the true vacuum to be the non-trivial one we must have:

$$V_{\text{eff}}^{1\text{-loop}}(\Phi_{\text{vac}}) < V_{\text{eff}}^{1\text{-loop}}(0) \quad (51)$$

However, the value of the potential at the minimum depends on the size of its second derivative:

$$M_H^2 \equiv \frac{1}{2} \left\{ \frac{\delta^2 V}{\delta \Phi^2} \right\}_{\Phi=\Phi_{\text{vac}}} \quad (52)$$

and, it turns out that for too low values of  $M_H^2$  the condition above, eq.(51), turns over. That is,  $V(0) < V(\Phi_{\text{vac}})$  and the true vacuum changes to the trivial one. The condition for vacuum stability then implies a lower bound on  $M_H$  [31]. More precisely,

$$M_H^2 > \frac{3}{16\pi^2 v^2} (2M_W^4 + M_Z^4 - 4m_t^4) \quad (53)$$

Surprisingly, for  $m_t > 78 \text{ GeV}$  this bound disappears and, moreover,  $V_{\text{eff}}^{1\text{-loop}}$  becomes unbounded from below!. Apparently it seems a disaster since the top mass is known at present and is certainly larger than this value. The solution to this problem relies in the fact that for such input values, the 1-loop approach becomes unrealistic and a 2-loop analysis of the effective potential is needed. Recent studies indicate that by requiring vacuum stability at 2-loop level and up to very large energies of the order of  $10^{16} \text{ GeV}$ , the following lower bound is found [32]:

$$M_H^{\text{v.stab.}} > 132 \text{ GeV} \quad (54)$$

This is for  $m_t = 170 \text{ GeV}$  and  $\alpha_s = 0.117$  and there is an uncertainty in this bound of 5 to 10  $\text{GeV}$  from the uncertainty in the  $m_t$  and  $\alpha_s$  values.

## 14 The naturalness Problem

In order to show the so-called naturalness problem, let us compute first the renormalized Higgs mass,  $M_H^R$ , to one-loop in the SM. Since the SM is a renormalizable theory we can apply the renormalization program as usual. Let us choose, for instance, the on-shell scheme where  $M_H^R$  coincides with the physical mass  $M_H$  and it is related with the bare (unphysical) mass  $M_H^0$  by:

$$\begin{aligned} (M_H^R)^2 &= (M_H^0)^2 + \delta M_H^2 \\ \delta M_H^2 &= \text{Re } \Sigma_H [(M_H^R)^2] \end{aligned} \quad (55)$$

where  $-i\Sigma_H [q^2]$  is given by the sum of the 1-loop diagrams contributing to the Higgs self-energy. Some of these diagrams are shown in Fig.9.

**Fig.9** Some contributing diagrams to the Higgs self-energy

Some of these diagrams are quadratically divergent and some others are logarithmically divergent. This can be easily shown by computing the integrals with a cut-off  $\Lambda$  in the ultraviolet region. For instance,

$$\begin{aligned} [-i\Sigma_H^a]_{\text{div}} &\sim \frac{3}{16\pi^2} \lambda_R \Lambda^2 \\ [-i\Sigma_H^b]_{\text{div}} &\sim \frac{\lambda_R^2 v^2}{16\pi^2} \log\left(\frac{\Lambda}{M_W}\right) \end{aligned} \quad (56)$$

The relation between the renormalized and the bare masses is, therefore, of the following generic form at on-loop:

$$(M_H^R)^2 = (M_H^0(\Lambda))^2 + [C_1 \Lambda^2 + C_2 \log \Lambda + C_3] ; C_i = O\left(\frac{1}{16\pi^2}\right) \quad (57)$$

The renormalization program tells us how the unphysical  $M_H^0(\Lambda)$  must be fixed in order to absorb all the divergences and to get a finite  $M_H^R$  in the  $\Lambda \rightarrow \infty$  limit. The cut-off here is unphysical

and must be removed at the end from physical quantities. So far, there is nothing unnatural. It is just the standard renormalization procedure.

The problem arises if (and only if) one wants to interpret the SM as a low energy effective theory of some fundamental theory which operates at very high energies, say  $M_{\text{GUT}} \sim 10^{16} \text{ GeV}$  or  $M_{\text{Planck}} \sim 10^{19} \text{ GeV}$ , etc.. In this case, the cut-off becomes a physical quantity and must be related somehow to the energy at which the new physics of the fundamental theory manifests. Besides,  $M_H^0(\Lambda)$  must be predictable from this underlying theory. For high cut-off values, the 1-loop corrections in eq.(57) tend to push  $M_H^R$  to high values as well, so that if one wishes to keep  $M_H^R$  within a reasonable low energy range,  $M_H^0(\Lambda)$  must be adjusted accordingly. Sometimes this adjustment is critical. For instance, for  $\Lambda = M_{\text{Planck}}$  and if we require  $M_H^R < 1 \text{ TeV}$ ,  $M_H^0$  must be fine-tuned up to 30 decimals!. This extreme fine-tuning is what is considered unnatural.

There are two most popular proposed solutions to the naturalness problem, one is based on Supersymmetry and the other one on Technicolor.

### I.- Supersymmetry

The SM is assumed to be the low energy effective theory of some fundamental theory which is supersymmetric and operates at very high energy, say  $\Lambda_{\text{SUSY}} \sim O(M_{\text{Planck}})$ .

The new symmetry between bosons and fermions, the Supersymmetry (SUSY), [33] implies an extension of the SM spectrum. In particular, for each scalar boson particle with mass  $m_{\text{particle}}$  there must exist a fermionic superpartner with its same mass,  $m_{\text{sparticle}} = m_{\text{particle}}$ . Thus, if this Supersymmetry is exact there is an exact cancelation between each 1-loop diagram with a scalar particle flowing in the loop and the corresponding diagram with its fermionic superpartner in the loop. As a consequence the quadratic divergences vanish and only softer logarithmic divergences remain:

$$(M_H^R)^2 = (M_H^0(\Lambda_{\text{SUSY}}))^2 + [\hat{C}_2 \log \Lambda_{\text{SUSY}} + \hat{C}_3] \quad (58)$$

Therefore, no unnatural fine-tuning is needed in the exact SUSY case.

However, the absence of scalar particles in the spectrum with the same mass as the known fermions indicates that the Supersymmetry must be broken at low energies (i.e, energies available at present experiments). If SUSY is not exact the particles and their superpartners are not degenerated anymore and  $m_{\text{sparticle}}$  must be larger than  $m_{\text{particle}}$ . However, it cannot be too large if we want the naturalness problem not to come back. That is to say, if  $m_{\text{sparticle}} > m_{\text{particle}}$  the quadratic divergences reappear and contribute to  $(M_H^R)^2$  as:

$$(m_{\text{sparticle}}^2 - m_{\text{particle}}^2) \frac{\Lambda_{\text{SUSY}}^2}{m_{\text{sparticle}}^2}$$

Therefore, to keep the fine-tuning controlled at less than a few percent level, the sparticle spectrum must appear below about  $1 \text{ TeV}$ :

$$m_{\text{sparticle}} \leq O(1 \text{ TeV})$$

It announces an interesting phenomenology for sparticle searches at present and future colliders [34]. In particular the LHC collider seems very promising.

## II.- Technicolor

In this class of theories, the  $\mathcal{SBS}$  does not contain an elementary Higgs particle and the symmetry breaking,  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ , is produced dynamically by new gauge interactions of a  $SU(N_{TC})$  gauge theory [35]. It is a confining theory at large distances and has strong interactions similar to  $SU(3)_C$  of QCD. Because of this analogy with QCD,  $SU(N_{TC})$  is called Technicolor Theory. The fundamental fields are the techniquarks  $q_{TC}$  and technigluons  $G_{TC}$ , and  $N_{TC}$  is the total number of technicolors.

The absence of an elementary Higgs boson in Technicolor theories automatically avoids the naturalness problem. The Higgs Mechanism is implemented by a techniquark condensate in analogy to the quark condensate of QCD:

$$\langle 0 | \bar{q}_{TC} q_{TC} | 0 \rangle \neq 0 \quad (59)$$

This condensate must have non-vanishing  $SU(2)_L$  and  $U(1)_Y$  charges in order to produce the wanted breaking. On the other hand, the strong interactions of  $SU(N_{TC})$  are assumed to be the responsible for producing these condensates.

## 15 The Spectrum of Technicolor

In Technicolor Theory  $SU(N_{TC})$  there is an additional symmetry,  $SU(2)_L \times SU(2)_R$ , and it happens to be the same global symmetry of the  $\mathcal{SBS}$  of the SM. (In more complex technicolor models this symmetry can be even larger). Moreover, this symmetry is spontaneously broken by the condensate eq.(59) to the diagonal subgroup:

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$$

By virtue of the Goldstone Theorem, this breaking leads to the existence of three Goldstone bosons, the so-called technipions  $\pi_{TC}^\pm$  and  $\pi_{TC}^0$ . When the subgroup  $SU(2)_L \times U(1)_Y$  is gauged the Higgs Mechanism takes place: The three would-be-Goldstone bosons disappear from the spectrum and they are replaced by the longitudinal gauge bosons,  $W_L^\pm$  and  $Z_L$ .

The coupling of the technipions to the weak current is given by:

$$\langle 0 | J_L^{+\mu} | \pi_{TC}^-(p) \rangle = \frac{i F_\pi^{TC}}{\sqrt{2}} p^\mu \quad (60)$$

where the technipion decay constant is:

$$F_\pi^{TC} = v = 246 \text{ GeV}$$

which is obviously the analogous to  $f_\pi$  of QCD.

The spectrum of Technicolor is a copy of the spectrum of QCD as well: Technipions ( $\pi_{TC}^\pm, \pi_{TC}^0$ ), Technirhos ( $\rho_{TC}^\pm, \rho_{TC}^0$ ), etc...An estimate of their masses and widths can be obtained by rescaling

the corresponding values in QCD with an appropriate factor. This factor can be written as:

$$\frac{F_\pi^{TC}}{f_\pi} \cdot f(N_{TC}, N_C)$$

where,

$$\frac{F_\pi^{TC}}{f_\pi} = \frac{246 \text{ GeV}}{0.094 \text{ GeV}} \sim 2700$$

and  $f(N_{TC}, N_C)$  is a function of the number of technicolors and the number of colors. A naive estimate of this function can be got by using the large  $N_{TC}$  approximation in  $SU(N_{TC})$  and, similarly, large  $N_C$  in  $SU(N_C)$  [37]. Thus, for instance, by knowing the behaviour at large  $N_{TC}$  and  $N_C$  of the technimeson and meson parameters respectively,

$$\begin{aligned} m_{\text{meson}} &\sim O(1) ; & f_\pi &\sim \sqrt{N_C} \\ m_{\text{Tmeson}} &\sim O(1) ; & F_\pi^{TC} &\sim \sqrt{N_{TC}} \end{aligned} \quad (61)$$

one finds out,

$$\frac{m_{\text{Tmeson}}}{m_{\text{meson}}} \sim \frac{F_\pi^{TC}}{f_\pi} \cdot \sqrt{\frac{N_C}{N_{TC}}} \quad (62)$$

and, therefore, the first expected resonance is the technirho with a mass of

$$m_{\rho_{TC}} = \frac{F_\pi^{TC}}{f_\pi} \cdot \sqrt{\frac{N_C}{N_{TC}}} m_\rho \quad (63)$$

For instance, for  $N_C = 3$ ,  $N_{TC} = 4$  and  $m_\rho = 760 \text{ MeV}$  one gets,

$$m_{\rho_{TC}} = 1.8 \text{ TeV}$$

This, on the other hand, gives the order of magnitude of the effective cut-off of Technicolor Theory where the new physics sets in:

$$\Lambda_{TC}^{\text{eff}} \sim O(1 \text{ TeV})$$

Similar arguments can be applied to get the technimeson widths,

$$\begin{aligned} \Gamma_{\text{meson}} &\sim O\left(\frac{1}{N_C}\right) ; & \Gamma_{\text{Tmeson}} &\sim O\left(\frac{1}{N_{TC}}\right) \\ \Rightarrow & & \frac{\Gamma_{\text{Tmeson}}}{\Gamma_{\text{meson}}} &\sim \frac{N_C}{N_{TC}} \frac{m_{\text{Tmeson}}}{m_{\text{meson}}} \\ \Rightarrow & & \Gamma_{\rho_{TC}} &= \frac{N_C}{N_{TC}} \frac{m_{\rho_{TC}}}{m_\rho} \Gamma_\rho \end{aligned} \quad (64)$$

For instance, for  $N_C = 3$ ,  $N_{TC} = 4$  and  $\Gamma_\rho = 151 \text{ MeV}$  one gets:

$$\Gamma_{\rho_{TC}} = 260 \text{ GeV}$$

The prediction of spectrum at the  $\text{TeV}$  energies in Technicolor Theories opens new possibilities for particle searches at future colliders as LHC.



At this point, one should mention about the drawbacks of Technicolor Theories. It is known that in order to generate the fermion masses one needs more complex models as the so-called Extended Technicolor Theories [38] which are not free of problems. For instance, it is very difficult to avoid flavor changing neutral currents in these models. There are various versions of Technicolor Theories dealing with these problems, but we are not going to discuss them here. For more information, we address the reader to ref.[36].

## 16 The Higgs Sector in MSSM

The Minimal Supersymmetric Standard Model is the simplest extension of the SM,  $SU(3)_C \times SU(2)_L \times U(1)_Y$ , that is supersymmetric and contains the minimal particle spectrum.

The following are some of the assumptions done to build up this model:

- The MSSM is considered as a low energy effective theory that should be used just at energies below the effective scale of Supersymmetry,  $\Lambda_{\text{SUSY}}^{\text{eff}} \sim O(1 \text{ TeV})$ .
- The MSSM comes from a more fundamental Supersymmetric Theory (It could be, Supergravity, Superstring Theory, etc...) which operates at much higher energy scales, say  $\Lambda_{\text{SUSY}} \sim O(M_{\text{Planck}})$ , and where SUSY is an exact symmetry.
- In MSSM it is not needed to specify the particular fundamental theory, but whatever it might result, it must be the responsible for generating the so-called soft-SUSY-breaking terms which must be included in the Lagrangian of MSSM. These terms are needed to break SUSY at low energies and to explain  $m_{\text{sparticle}} > m_{\text{particle}}$ .
- The breaking of SUSY in MSSM must be soft to guarantee that no quadratic divergences reappear in the scalar selfenergies and, thus, to avoid the naturalness problem could emerge again.
- The soft terms of MSSM are fixed in a way that can produce as well the wanted breaking:  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{em}}$ .

In the following we present the Higgs sector in MSSM and tell how the breaking of  $SU(2)_L \times U(1)_Y$  occurs. For the remaining spectrum and more information on MSSM we address the reader to Haber's Lectures in ref.[33] where most of this section has been borrowed from.

In addition to the complex scalar doublet of the SM  $\Phi$  with  $Y(\Phi) = 1$ , we need in SUSY theories a second complex scalar doublet with opposite hypercharge. Let  $H_1$  and  $H_2$  be these two doublets:

$$\begin{aligned} H_1 &= \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \quad ; \quad Y(H_1) = -1 \\ H_2 &= \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \quad ; \quad Y(H_2) = +1 \end{aligned} \tag{65}$$

Because of the Supersymmetry, in addition to these scalars, there are two associated fermionic superpartners with their same quantum numbers:

$$\begin{aligned}\widetilde{H}_1 &= \begin{pmatrix} \widetilde{H}_1^0 \\ \widetilde{H}_1^- \end{pmatrix} ; \quad Y(\widetilde{H}_1) = -1 \\ \widetilde{H}_2 &= \begin{pmatrix} \widetilde{H}_2^+ \\ \widetilde{H}_2^0 \end{pmatrix} ; \quad Y(\widetilde{H}_2) = +1\end{aligned}\tag{66}$$

The reason to introduce two scalar doublets instead of one as in the SM is two fold: 1) Only by including the fermionic doublets in pairs it is possible to cancel their contribution to the gauge anomaly; 2) A second scalar doublet  $H_1$  with  $Y = -1$  is needed to generate the masses of the  $u$  type quarks. The complex conjugate  $H_2^*$  cannot play this role, as in the SM does, since the requirement of SUSY implies the Superpotential must be an analytic function and therefore it does not allow for complex conjugate scalar fields.

The counting of polarization degrees in the electroweak symmetry breaking within the MSSM is different than in the SM. It goes as follows,

1) **Before SSB**

4 massless gauge bosons:  $W_{1,2,3}^\mu, B^\mu$

8 massless scalars: The 4+4 real components of the complex doublets  $H_1$  and  $H_2$

Total number of polarization degrees =  $4 \times 2 + 8 = 16$

2) **After SSB**

3 massive gauge bosons:  $W^\pm, Z$

1 massless gauge boson:  $\gamma$

5 massive scalar bosons:  $H^\pm, A^0, H^0, h^0$

Total number of polarization degrees:  $3 \times 3 + 1 \times 2 + 5 = 16$

As in the SM, the total number of polarization degrees is preserved in the breaking but now it is larger than in SM. Apart from the three needed would-be-Goldstone bosons, there have been introduced 'ad hoc' five more polarization degrees (instead of one as in the SM) which, after the breaking, emerge as the five physical massive Higgs bosons of the MSSM:

- two charged scalar bosons,  $H^+$  and  $H^-$
- one CP-odd neutral scalar boson,  $A^0$
- two CP-even neutral scalar bosons,  $H^0$  and  $h^0$

The simplest potential in terms of  $H_1$  and  $H_2$  that is supersymmetric is given by:

$$V(H_1, H_2) = |\mu|^2(|H_1|^2 + |H_2|^2) + \frac{1}{8}(g^2 + g'^2)(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2}g^2|H_1^*H_2|^2 \quad (67)$$

This should be compared with the Higgs potential of the SM:

$$V(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 \quad (68)$$

The following observations are in order:

- In the SUSY potential there is the same coefficient for the  $|H_1|^2$  and  $|H_2|^2$  terms. This will lead to some mass relations.
- In the SUSY potential the scalar self-coupling is not an independent parameter but it is given in terms of the gauge coupling constants  $g^2$  and  $g'^2$ . Therefore, the Higgs sector in the MSSM is always weakly interacting and contains, at least, one light Higgs boson with  $m_{h^0} \sim O(100 \text{ GeV})$ .
- The SUSY potential cannot produce the wanted electroweak symmetry breaking since it is definite positive,  $V(H_1, H_2) \geq 0 \quad \forall H_{1,2}$ , and its minimum is at the trivial vacuum,  $H_1 = H_2 = 0$ . In order to generate a non-trivial asymmetric vacuum some additional terms are needed in the potential. These are the above mentioned soft-breaking terms whose role is two fold: To break the Supersymmetry and to break  $SU(2)_L \times U(1)_Y$ .

The simplest potential with soft breaking terms included is given by,

$$V_{\text{MSSM}} = m_{1H}^2|H_1|^2 + m_{2H}^2|H_2|^2 - m_{12}^2(\epsilon_{ij}H_1^iH_2^j + h.c.) + \frac{1}{8}(g^2 + g'^2)(|H_1|^2 - |H_2|^2)^2 + \frac{1}{8}g^2|H_1^*H_2|^2 \quad (69)$$

where,

$$m_{iH}^2 \equiv |\mu|^2 + m_i^2 ; i = 1, 2 \quad (70)$$

and  $m_1^2$ ,  $m_2^2$  and  $m_{12}^2$  are the soft SUSY breaking parameters.

One can check that the following are the necessary conditions for  $SU(2)_L \times U(1)_Y$  breaking:

$$1.- |m_{12}^2|^2 > m_{1H}^2 m_{2H}^2$$

This condition insures the existence of infinite degenerate vacua with  $\langle H_1^0 \rangle \neq 0$  and  $\langle H_2^0 \rangle \neq 0$ .

$$2.- m_{1H}^2 + m_{2H}^2 \geq 2|m_{12}^2|^2$$

This condition is needed to insure vacuum stability.

Once these conditions are imposed, the next step is to choose one out of the infinite degenerate vacua. The usual asymmetric vacuum is the simplest one which is defined by the following configuration:

$$\begin{aligned}
\langle H_1^0 \rangle &= v_1 ; \langle H_2^0 \rangle = v_2 \\
v_1 \text{ and } v_2 &\text{ are real and positive} \\
m_{12}^2 &\text{ is real and positive}
\end{aligned}$$

Furthermore,  $v_1$  and  $v_2$  are not completely free. They must fulfil the following additional constraint:

$$m_W^2 = \frac{1}{2}g^2(v_1^2 + v_2^2) \Rightarrow v_1^2 + v_2^2 = (246 \text{ GeV})^2 \quad (71)$$

One can use  $m_{1H}^2$ ,  $m_{2H}^2$  and  $m_{12}^2$  as input parameters to characterize the breaking or some three alternative parameters. For instance,  $v_1$ ,  $v_2$  and the mass of the CP-odd scalar boson  $m_{A^0}$ . If we consider, in addition, the constraint of eq.(71) we are left with just two independent parameters. It is customary to choose as input parameters:  $m_{A^0}$  and  $\tan\beta \equiv \frac{v_2}{v_1}$ .

After some algebra one finds out the Higgs masses in terms of these parameters:

$$\begin{aligned}
m_{H^\pm}^2 &= m_W^2 + m_{A^0}^2 \\
m_{H^0, h^0}^2 &= \frac{1}{2} \left[ m_{A^0}^2 + m_Z^2 \pm \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4m_Z^2 m_{A^0}^2 \cos^2 2\beta} \right]
\end{aligned} \quad (72)$$

as well as the Higgs spectrum:

$$\begin{aligned}
H^+ &; H^- \\
H^0 &= (\sqrt{2} \text{Re } H_1^0 - v_1) \cos \alpha + (\sqrt{2} \text{Re } H_2^0 - v_2) \sin \alpha \\
h^0 &= -(\sqrt{2} \text{Re } H_1^0 - v_1) \sin \alpha + (\sqrt{2} \text{Re } H_2^0 - v_2) \cos \alpha
\end{aligned} \quad (73)$$

where,

$$\cos 2\alpha \equiv -\cos 2\beta \left( \frac{m_{A^0}^2 - m_Z^2}{m_{H^0}^2 - m_{h^0}^2} \right) ; \sin 2\alpha \equiv -\sin 2\beta \left( \frac{m_{H^0}^2 + m_{h^0}^2}{m_{H^0}^2 - m_{h^0}^2} \right) \quad (74)$$

The result in eq.(72) indicates that the following inequalities hold in the MSSM at tree level,

$$m_{h^0} \leq m_Z ; m_{H^0} \geq m_Z ; m_{H^\pm} \geq m_W \quad (75)$$

Interestingly, there is a neutral Higgs boson  $h_0$  lighter than the  $Z$  boson. This fact, when it was noticed, seemed to announce a possible discovery of  $h_0$  at LEP. However, it was realized later that, beyond tree level, these inequalities in eq.(75) do not hold anylonger. In particular,  $m_{h^0}$  gets large corrections from top and stop loops and one finds out to one loop that  $m_{h^0} > m_Z$ . By scanning the whole parameter space, recent studies indicate, however, that the values obtained for  $m_{h^0}$ , including the complete one-loop corrections, never exceed certain value. In ref.[40] the following absolute upper limit is found:

$$m_{h^0} < 140 \text{ GeV}$$

On the other hand, since these scalar particles have not been seen at present experiments one can extract experimental lower mass limits. From the absence of any Higgs signal at the LEP experiment one finds [41],

$$m_{h^0}^{\text{exp}} > 52 \text{ GeV} ; m_{A^0}^{\text{exp}} > 54 \text{ GeV} ; m_{H^\pm}^{\text{exp}} > 44 \text{ GeV} \quad (95\%C.L.) \quad (76)$$

## 17 Strongly Interacting $\mathcal{SBS}$

The strongly interacting hypothesis in the SM refers to the possibility that the scalar self-coupling  $\lambda$  be large and a perturbative approach in powers of this coupling is not anylonger valid. Since in the SM at tree level there is a direct relation between  $\lambda$  and  $M_H$  given by  $\lambda = \frac{g^2 M_H^2}{8M_W^2}$ , a large value of  $\lambda$  implies a large value of  $M_H$ . Thus, for instance, for a very heavy Higgs with  $M_H \sim 1 \text{ TeV}$  one gets a non perturbative coupling of  $\lambda \sim 7$ .

Given the SM potential of eq.(5), a large value of  $\lambda$  implies that the interactions among the three would-be-Goldstone bosons and the Higgs particle are strong. Since, by virtue of the Equivalence Theorem of eq.(31), there is a relation between the Goldstone bosons and the longitudinal gauge bosons scattering amplitudes,

$$T(V_L^1 V_L^2 \rightarrow V_L^3 V_L^4) = T(w^1 w^2 \rightarrow w^3 w^4) + O\left(\frac{M_V^2}{s}\right); \sqrt{s} \gg M_V; V^i = W^\pm, Z \quad (77)$$

it in turn implies that, at high enough energies, the  $W_L^\pm$  and  $Z_L$  gauge bosons become strongly interacting too [42, 43]. The amplitudes of longitudinal gauge bosons are expected to show the typical features of a strong interaction as, for instance, the appearance of multiple resonances with sizeable widths in the TeV energy region etc. [43]. In the case of the SM, the first resonance would be the Higgs particle itself with a large mass and a large width [17].

A common feature to all models of strongly interacting  $\mathcal{SBS}$  is that the size of the cross-section for production of longitudinal gauge bosons pairs at these high energies,  $\sqrt{s} \sim O(1 \text{ TeV})$ , is expected to be larger than in weakly interacting theories as, for instance, the SM with a light Higgs boson. Although, there are several interesting possibilities to look for strongly interacting signals, the most obvious and, therefore, the most studied one is precisely  $V_L V_L$  production at the future collider LHC in the various possible channels,  $V = W^\pm, Z$  [43]. In the large energy region it comes mainly from the so-called gauge boson fusion processes (see Fig.7). Several studies indicate that an enhancement in  $V_L V_L$  production over the expected background could be observed at LHC in the mass invariant region of  $M_{VV} \sim O(1 \text{ TeV})$  [43, 44].

Some comments are in order. There are some intrinsic problems connected to the above definition of strongly interacting  $\mathcal{SBS}$ . One is that for a too heavy Higgs particle, the width is comparable with the mass and an interpretation of  $H$  as a particle or as a resonance makes no sense. Another one is that for such a heavy boson, the elastic unitarity condition is violated in  $V_L V_L$  scattering, what indicates the failure of perturbation theory. In practice, some unitarization procedure must, therefore, be implemented to cure this bad behaviour. In a more ambitious program, a non-perturbative treatment of this strongly interacting system should be performed. The lattice, would be obviously one possibility for future estimates.

Finally it is worth mentioning that one can also postulate the hypothesis of a strongly interacting  $\mathcal{SBS}$  beyond the SM. There are several proposals and all have as a common assumption that either the Higgs particle does not exist or it is not a fundamental particle. Some examples are: Technicolor Models [35, 36], models where the Higgs boson is a top-antitop condensate [46], the BESS model [47] etc..

## 18 Low Energy Theorems

These theorems are a consequence of the additional global symmetry,

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R} \quad (78)$$

which is present in the  $\mathcal{SBS}$  of the SM and we wish to be present as well in any alternative Higgs sector in view of the successful prediction of  $\rho = 1$  based on this symmetry pattern.

The Low Energy Theorems are universal since they rely just on symmetry arguments and, therefore, they must hold in any possible scenario for the  $\mathcal{SBS}$ . They state the following:

*The Goldstone boson scattering amplitudes that are imposed by the symmetry of eq.(78) are given at low energies by the following simple expressions [48]:*

$$\begin{aligned} T(w^+ w^- \rightarrow w^+ w^-) &= -\frac{u}{v^2} \\ T(w^+ w^- \rightarrow zz) &= \frac{s}{v^2} \\ T(zz \rightarrow zz) &= 0 \end{aligned} \quad (79)$$

where,  $v = 246 \text{ GeV}$ ,  $(s, t, u)$  are the Mandelstam variables and low energies here refer to energies well below any possible emerging resonance.

It is interesting to notice the similarities with the Low Energy Theorems for  $\pi\pi$  scattering which are associated with the Chiral symmetry of QCD [49]:

$$\begin{aligned} T(\pi^+ \pi^- \rightarrow \pi^+ \pi^-) &= -\frac{u}{f_\pi^2} \\ T(\pi^+ \pi^- \rightarrow \pi^0 \pi^0) &= \frac{s}{f_\pi^2} \\ T(\pi^0 \pi^0 \rightarrow \pi^0 \pi^0) &= 0 \end{aligned} \quad (80)$$

where,  $f_\pi = 94 \text{ MeV}$  and low energies here means  $\sqrt{s} \ll m_\rho$ .

Finally, by using the Equivalence Theorem, the above expressions in eq.(79) are translated into Low Energy Theorems for the scattering of longitudinal gauge bosons:

$$\begin{aligned} T(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) &= -\frac{u}{v^2} \\ T(W_L^+ W_L^- \rightarrow Z_L Z_L) &= \frac{s}{v^2} \\ T(Z_L Z_L \rightarrow Z_L Z_L) &= 0 \end{aligned} \quad (81)$$

They are universal as well and hold for any particular  $\mathcal{SBS}$ . The energy must be in the range of applicability of both the Equivalence Theorem and the Low Energy Theorems. It means an energy larger than the  $W^\pm$  and  $Z$  masses but lower than the first possible resonance. In the particular case of the SM, the above expressions should hold in the energy range  $M_{W,Z} \ll \sqrt{s} \ll M_H$ . This is indeed what results from the exact tree level expressions of eq.(34) when this energy limit is considered.

## 19 Effective Lagrangian Approach to Electroweak Theory

The electroweak interactions can be described at low energies by means of an effective Lagrangian which is  $SU(2)_L \times U(1)_Y$  gauge invariant and is written in terms of just the light fields [50]. In particular, the effective Lagrangian which does not contain explicitly the Higgs field in its formulation has been named Electroweak Chiral Lagrangian and has some interesting applications to electroweak phenomenology [44, 51, 52]. In this approach the Higgs particle is assumed either very heavy, say  $M_H \sim O(1 \text{ TeV})$ , or nonexistent.

In the bosonic sector, the EChL is a non-linear theory which is built in terms of a field  $U$  that parametrizes the three would-be-Goldstone bosons, its covariant derivative  $D_\mu U$  and the  $SU(2)_L \times U(1)_Y$  gauge boson fields [53]:

$$\begin{aligned} U &\equiv \exp\left(i \frac{\vec{\tau} \cdot \vec{w}}{v}\right), \quad v = 246 \text{ GeV}, \quad \vec{w} = (w^1, w^2, w^3) \\ D_\mu U &\equiv \partial_\mu U - g \mathcal{W}_\mu U + g' U \mathcal{B}_\mu \end{aligned} \quad (82)$$

where,  $\mathcal{W}_\mu$  and  $\mathcal{B}_\mu$  are defined in eq.(13).

It is a non-linear theory since the would-be-Goldstone bosons transforms non-linearly under  $SU(2)_L \times U(1)_Y$  and it is a consequence of the lack of the Higgs field in this theory that could complete together with the  $w$ 's a linear multiplet. The  $U$  field, however, transforms linearly under  $SU(2)_L \times U(1)_Y$ :

$$U(x) \rightarrow g_L U(x) g_Y^\dagger; \quad g_L \in SU(2)_L; \quad g_Y \in U(1)_Y \quad (83)$$

The EChL has the following generic form:

$$\mathcal{L}_{\text{EChL}} = \mathcal{L}_{\text{NL}} + \sum_{i=0}^{13} \mathcal{L}_i. \quad (84)$$

where,

$$\mathcal{L}_{\text{NL}} = \frac{v^2}{4} \text{Tr} [D_\mu U^\dagger D^\mu U] + \mathcal{L}_{\text{YM}} \quad (85)$$

is the Lagrangian of the well known gauged non-linear sigma model and  $\mathcal{L}_{\text{YM}}$  is the Yang-Mills Lagrangian containing the kinetic, the gauge fixing and the Faddeev Popov terms.

The  $\mathcal{L}_i$ 's in eq.(84) are the so-called chiral effective operators. They are the complete set of operators with dimension up to four (notice that the field  $U$  is dimensionless) that can be built up in terms of the light bosonic fields,  $U$ ,  $W_\mu^\pm$ ,  $Z_\mu$  and  $\gamma_\mu$ , and that are  $SU(2)_L \times U(1)_Y$  and CP invariant. The building blocks to implement gauge invariance are, therefore, the covariant derivative  $D_\mu U$  and the field strength tensors  $\mathcal{W}_{\mu\nu}$  and  $\mathcal{B}_{\mu\nu}$  of eq.(12).

For completeness, we include here the list of operators [53]<sup>1</sup>:

$$\mathcal{L}_0 = a_0 g'^2 \frac{v^2}{4} [\text{Tr} (T V_\mu)]^2$$

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<sup>1</sup> The relation with Longhitano's notation in ref.[53] is the following:  $a_0 = \frac{g'^2}{g^2} \beta_1$ ;  $a_1 = \frac{g}{g'} \alpha_1$ ;  $a_2 = \frac{g}{g'} \alpha_2$ ;  $a_3 = -\alpha_3$ ;  $a_i = \alpha_i$ ,  $i = 4, 5, 6, 7$ ;  $a_8 = -\alpha_8$ ;  $a_9 = -\alpha_9$ ;  $a_{10} = \alpha_{10}/2$ ;  $a_{11} = \alpha_{11}$ ;  $a_{12} = \alpha_{12}/2$ ;  $a_{13} = \alpha_{13}$ .

$$\begin{aligned}
\mathcal{L}_1 &= a_1 \frac{igg'}{2} B_{\mu\nu} \text{Tr}(T\mathcal{W}^{\mu\nu}) \\
\mathcal{L}_2 &= a_2 \frac{ig'}{2} B_{\mu\nu} \text{Tr}(T[V^\mu, V^\nu]) \\
\mathcal{L}_3 &= a_3 g \text{Tr}(\mathcal{W}_{\mu\nu}[V^\mu, V^\nu]) \\
\mathcal{L}_4 &= a_4 [\text{Tr}(V_\mu V_\nu)]^2 \\
\mathcal{L}_5 &= a_5 [\text{Tr}(V_\mu V^\mu)]^2 \\
\mathcal{L}_6 &= a_6 \text{Tr}(V_\mu V_\nu) \text{Tr}(TV^\mu) \text{Tr}(TV^\nu) \\
\mathcal{L}_7 &= a_7 \text{Tr}(V_\mu V^\mu) [\text{Tr}(TV^\nu)]^2 \\
\mathcal{L}_8 &= a_8 \frac{g^2}{4} [\text{Tr}(T\mathcal{W}_{\mu\nu})]^2 \\
\mathcal{L}_9 &= a_9 \frac{g}{2} \text{Tr}(T\mathcal{W}_{\mu\nu}) \text{Tr}(T[V^\mu, V^\nu]) \\
\mathcal{L}_{10} &= a_{10} [\text{Tr}(TV_\mu) \text{Tr}(TV_\nu)]^2 \\
\mathcal{L}_{11} &= a_{11} \text{Tr}((D_\mu V^\mu)^2) \\
\mathcal{L}_{12} &= a_{12} \text{Tr}(TD_\mu D_\nu V^\nu) \text{Tr}(TV^\mu) \\
\mathcal{L}_{13} &= a_{13} \frac{1}{2} [\text{Tr}(TD_\mu V_\nu)]^2
\end{aligned} \tag{86}$$

where,

$$T \equiv U\tau^3 U^\dagger, \quad V_\mu \equiv (D_\mu U)U^\dagger. \tag{87}$$

By reading from  $\mathcal{L}_{\text{NL}}$  the quadratic terms in the gauge fields one finds out the gauge boson masses:

$$M_W^2 = \frac{g^2 v^2}{4}; \quad M_Z^2 = \frac{(g'^2 + g^2)v^2}{4}; \quad M_\gamma^2 = 0 \tag{88}$$

Therefore, the EChL describes as well a  $SU(2)_L \times U(1)_Y$  gauge theory with spontaneous symmetry breaking to  $U(1)_{\text{em}}$ . Furthermore, one can check that the scalar sector of EChL has the additional global symmetry  $SU(2)_L \times SU(2)_R$  and it is spontaneously broken down to the custodial symmetry group  $SU(2)_{L+R}$ . This is precisely the origin of including the name 'chiral' in the EChL.

This approach to Electroweak Theory is inspired in the well known Chiral Lagrangian approach to QCD at low energies and the Chiral Perturbation Theory [54]. In particular, the predictions in this theory for the would-be-Goldstone boson scattering amplitudes (in the approximation of neglecting the gauge interactions versus the scalar self-interactions) are similar to the pion scattering amplitudes of Chiral Perturbation Theory [51]. Furthermore, by virtue of the Equivalence Theorem [55] it implies predictions for the scattering amplitudes of the longitudinal gauge bosons.

Finally, the coefficients  $a_i$  in front of the effective operators are called the chiral parameters, and are very important since they encode the information on the particular underlying physics



which is the responsible of generating this effective Lagrangian to low energies.  $\mathcal{L}_{\text{NL}}$  and the effective operators are universal whereas the values of the chiral parameters do depend on the underlying assumed fundamental theory. Therefore it is crucial to measure in experiment the  $a_i$  values in order to be able to discriminate among the different possible scenarios. On the other hand, it is also important to compute these coefficients from the various possible theories. Several works have been done along these two lines. Some of the  $a_i$ 's can already be bounded from present experiments as LEP (Holdon et al., Dobado et al., Golden et al. in ref.[52]; [61]). In fact, one can find relations between some of these parameters and the S, T and U parameters of ref.[59] or, equivalently, the  $\epsilon_i$  variables of ref.[60] (see, for instance, ref.[57]) which have been object of many studies in the last years.

The most challenging experiment for the study of the  $\mathcal{SBS}$  with Effective Lagrangians will be LHC. Recent studies indicate that the chiral parameters will be measured (or bounded) mainly by analysing gauge boson pair production processes in the high mass invariant region [56]. On the other hand, there are already available the computations of the chiral parameters in the two most typical scenarios: The SM with a heavy Higgs particle [57, 58], and Technicolor [62].

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