Phenomenological analysis in beyond the Standard Model theories: the cases of the minimal B-L-SM and of a BGL-like 3HDM

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1. Modern Particle physics and the Standard Model
2. Flavour processes in the Standard model
3. The B-L-SM model.
4. The Three Higgs Doublet Model (3HDM) with a BGL-like flavour symmetry

- The conventional framework to describe all subatomic processes is the Standard Model **(SM)**.

- **However**, **unanswered-questions** have been revealed by modern experiments (LHC, Belle, Atlas, etc.).

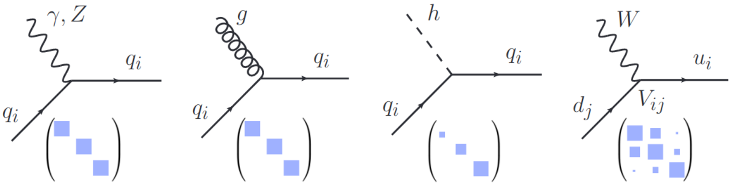
Some of these are:

- Deviations from it's theoretical predictions, e.g. **flavour deviations**, magnetic **moment anomalies in leptons** and others.
- Others are conceptual in nature, **dark matter** and **neutrino masses** cannot be explained by the SM.

These problems lead scientists to look at the SM as a effective field theory and presents us with the possibility of **New Physics**!

The exploration of New Physics beyond the standard model is the quintessential work of theoretical physicists.

The $W^{\pm}$ can mediate charge currents to change the quark flavour!
These are called Flavour Changing Charged Currents (**FCCCs**)



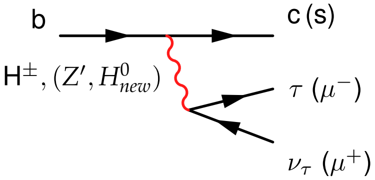
$W^{\pm}$ interactions with massive left-handed quarks fields follow from the terms,

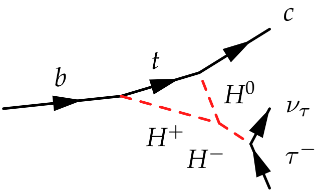
$$\mathcal{L}_{kin} \supset \frac{1}{\sqrt{2}} g \overline{u}'_L \gamma^{\mu} V_{CKM} d'_L W^+_{\mu} + \text{H.c.} \ ,$$

Controlled by the CKM matrix which is defined as, $V_{CKM} = U^u_L U^{d^\dagger}_L$.

However, in **higher order** processes, it is possible to change flavour in neutral processes. These are called Flavour Changing Neutral Currents (**FCNCs**).

These processes can be easily affected by new physics!





FCNCs can be very easily modified by new physics. Here we show, $b \to c\tau\nu$ and $b \to s\mu\mu$ transition. These processes show over 4σ deviations from their SM expected value.

- The SM contains an accidental symmetry that conserves **baryon minus lepton number**.
- In the B-L-SM it is promoted to a symmetry and the SM gains a Abelian unitary group $\mathrm{U}(1)_{\mathrm{B-L}}$.

Simple but deep! $\longrightarrow$ A great testing ground for computational tools

Motivations for $\mathrm{B-L}$ (Baryon number minus Lepton number) symmetry:

- $\mathrm{B-L}$ symmetry relevant for baryogenesis through leptogenesis,
- Grand Unified Theories, e.g. $\mathrm{SO}(10)$, E_6, $\mathrm{E}_8, \ldots$ contain gauged $\mathrm{U}(1)_{\mathrm{B-L}}$,
- Enhanced vacuum stability compared to the SM.

New Physics the B-L-SM offer:

Three generations of right-handed neutrinos $\rightarrow$ **no gauge anomalies**

- Lightest is sterile and can be keV to TeV dark matter candidate.
- Type I See-saw mechanism as to address light neutrino masses.

Model contains a complex-singlet scalar χ whose VEV breaks $\mathrm{U}(1)_{\mathrm{B-L}}$.

- New **neutral scalar** h_2.

	q_L	u_R	d_R	l_L	e_R	ν_R	H	χ
$\mathrm{SU}(3)_{\mathrm{C}}$	**3**	**3**	**3**	**1**	**1**	**1**	**1**	**1**
$\mathrm{SU}(2)_{\mathrm{L}}$	**2**	**1**	**1**	**2**	**1**	**1**	**2**	**1**
$\mathrm{U}(1)_{\mathrm{Y}}$	1/6	2/3	-1/3	-1/2	-1	0	1/2	0
$\mathrm{U}(1)_{\mathrm{B-L}}$	1/3	1/3	1/3	-1	-1	-1	0	2

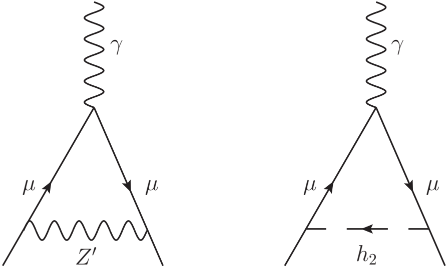
From $\mathrm{U}(1)_{\mathrm{B-L}}$ an extra Z' gauge boson is added.

- $\mathrm{U}(1)_{\mathrm{Y}} \times \mathrm{U}(1)_{\mathrm{B-L}}$ gauge kinetic-mixing parameter

Study the mutual viability of the Scalar and Gauge sector

Not studied in the B-L SM for heavy Z'

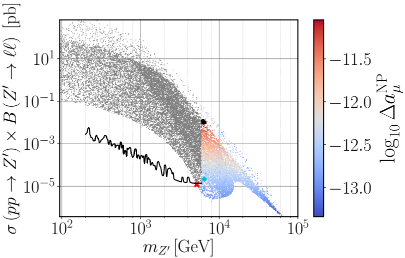
BSM vector bosons and scalars contribute to $(g-2)_\mu$ anomaly

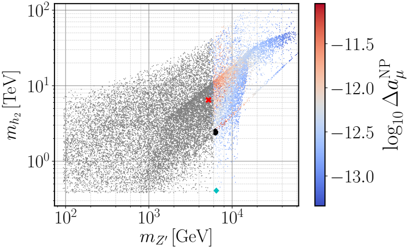


Not studied in the B-L SM

λ_1	$\lambda_{2,3}$	g_{BL}	g_{YB}	x [TeV]
$[10^{-2},\ 10^{0.5}]$	$[10^{-8},\ 10]$	$[10^{-8},\ 3]$	$[10^{-8},\ 3]$	$[0.5,\ 20.5]$

1. Model file: `SARAH-4.12.3`
2. Spectrum generator: `SPheno-4.0.3`
   - Unitarity
   - One-loop mass spectrum and two-loop Higgs mass
   - Mixing angles
   - EW precision observables STU
   - $(g-2)_\ell$ at one loop-level.
   - Decay widths and Branching Fractions
3. Generated points with $m_{h_1} \approx 125$ GeV input to `HiggsBounds-4.3.1` and `HiggsSingnals-1.4.0`
4. **Surviving** points passed to `MadGraph5_amC5@NLO`



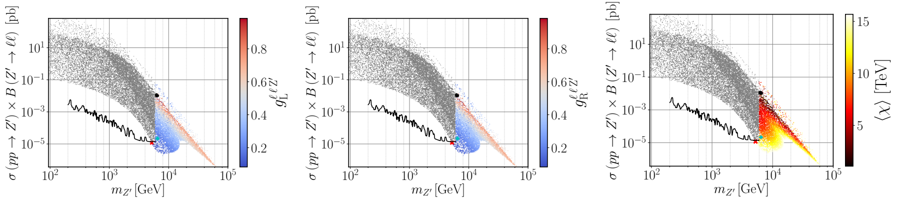


- Applied LEP constraints from 4 fermion contact interactions
- **Model shows a maximum $\Delta a_\mu^{\mathrm{NP}}$ of 8.9×10^{-12} representing a marginal of the SM value for $6.3\ \mathrm{TeV} \lesssim m_{Z'} \lesssim 6.5\ \mathrm{TeV}$ (black dots).**
- Red cross highlights a benchmark point with $m_{Z'} \approx 5.2\ \mathrm{TeV}$ and $m_{h_2} \approx 6.4\ \mathrm{TeV}$ regarded as an early-discovery (or early-exclusion) scenario in future LHC runs.
- Magenta diamond corresponds to the lightest BSM Higgs found, $m_{h_2} \approx 410\ \mathrm{GeV}$

- When $\frac{m_\mu}{m_{Z'}} \ll 1$ the Z' contribution reads

$$\Delta a_\mu^{Z'} \approx -\frac{1}{3\pi^2}\frac{m_\mu^2}{m_{Z'}^2}\left[6 g_{\mathrm{L}}^{\mu\mu Z'} g_{\mathrm{R}}^{\mu\mu Z'} - 4\left({g_{\mathrm{L}}^{\mu\mu Z'}}^2 + {g_{\mathrm{R}}^{\mu\mu Z'}}^2\right)\right]$$

$$g_{\mathrm{L}}^{\ell\ell Z'} \simeq g_{\mathrm{B-L}} + \frac{1}{2} g_{\mathrm{YB}}\,, \qquad g_{\mathrm{R}}^{\ell\ell Z'} \simeq g_{\mathrm{B-L}} + g_{\mathrm{YB}}\,.$$



- Note strong correlation between v/x and $\Delta a_\mu^{Z'}$ except for the sparser upper edge!
- Strong correlation between the physical coupling to left and right handed fermions.

$m_{Z'}$	m_{h_2}	x	$\log_{10}\Delta a_\mu^{\mathrm{NP}}$	σB	θ'_W	$\log_{10}\alpha_h$	$g_{\mathrm{B-L}}$	g_{YB}	$g_{\mathrm{L}}^{\ell\ell Z'}$
5.199	6.41	15.4	-13.01	1.16E$-$5	≈ 0	-5.18	0.17	2.0E$-$5	0.08
6.478	0.41	9.77	-12.57	2.15E$-$5	3.22E$-$7	-5.85	0.34	1.7E$-$3	0.17
6.371	2.34	1.08	**-11.05**	0.01	1.05E$-$6	-7.31	1.97	2.1E$-$3	0.98
6.260	2.31	1.15	**-11.07**	0.01	5.87E$-$5	-2.79	1.87	0.125	0.94
6.477	2.40	1.14	**-11.08**	0.01	2.75E$-$5	-4.29	1.93	0.06	0.97
6.252	2.53	1.28	**-11.08**	0.01	≈ 0	-8.65	1.86	1.6E$-$5	0.93

- **First line:** Early discovery/exclusion scenario with the lightest Z' found in the scan,
- **Second line:** Lightest new scalar found in the scan,
- **Third to fourth lines:** Four best $(g-2)_\mu$ points.

The $(g-2)_\mu$ anomaly:

- A heavy Z' between 6 and $8\ \mathrm{TeV}$ cannot fully explain the anomaly
- There is still the need to consider electron anomalous magnetic moment.
- Small remaining parameter space, model likely completely excluded by the new LHC III upgrade.

New physics searches

- We have identified 6 benchmark points to test the B-L SM at future LHC searches:
  - For a **relatively light** new scalar, $m_{h_2} \approx 420\ \mathrm{GeV}$
  - For an early discovery/exclusion Z' boson $m_{Z'} \approx 5.2\ \mathrm{TeV}$
  - For maximal contribution to the muon $(g-2)_\mu$ anomaly

Motivation for a Multi-Higgs doublet model, in particular the 3HDM

- Open the door to some of the unexplored characteristics of the SM while providing **rich** but still tractable phenomenology.
  - Charged scalars $H_i^{\pm}$
  - Pseudo-scalars A_i (CP-odd) and additional neutral scalars H_i (CP-even).
  - Additional sources of FCNCs mediated by the Scalars.
  - A family of Higgs doublets in analogy to the fermion and lepton sector.
- (Not studied here) Allow for CP-violation to be easily tuned through complex couplings.
- (Not studied here) Allows for Dark matter if stabilized with a symmetry.

However! Generally Multiple Higgs Doublet Models suffer from FCNCs at tree-level! which contradict observations

The solution for this is the implementation of a global flavor symmetry acting in the space of fermion and Higgs generations $\mathrm{U}(1) \times \mathbb{Z}_2$ flavour symmetry.

$$\mathrm{U}(1): \quad Q_{L_3} \to e^{i\alpha} Q_{L_3}, \quad p_{R_3} \to e^{2i\alpha} p_{R_3}, \quad \phi_1 \to e^{i\alpha}\phi_1, \quad \Psi_{L_1} \to e^{i\alpha}\Psi_{L_1}, \quad \phi_3 \to e^{i\alpha}\phi_3$$

$$\mathbb{Z}_2: \quad Q_{L_3} \to -Q_{L_3}, \quad p_{R_3} \to -p_{R_3}, \quad \phi_1 \to -\phi_1, \quad \Psi_{L_1} \to -\Psi_{L_1}, \quad \phi_3 \to -\phi_3$$

Unlike in the B-L-SM, for this model we scan over the physical parameters:

$$v_1, v_2, v_3 \to v, \psi_1, \psi_2.$$

$$\mu_{1,2,3}, \lambda_{1,\cdots 10}, \mu_{13}, \mu_{23}, \mu_{21} \to m_{A_{1,2}}, m_{H_{1,2}}, m_{H^{\pm}_{1,2,3}}, \gamma_{1,2}, \alpha_{1,2,3}$$

Given the three Higgs Doublets gauge interactions:

$$\mathcal{L}_{\text{gauge}} \supset \frac{1}{4}\left(v_1^2 + v_2^2 + v_3^2\right) g^2 W_\mu^+ W^{-\,\mu} + \frac{1}{8}\left(v_1^2 + v_2^2 + v_3^2\right)\left(g'^{\,2} + g^2\right) Z_\mu Z^\mu.$$

To reproduce the correct gauge boson masses we must ensure that,

$$v = \sum_{k=1}^{3} v_k^2 \approx 246^2\ (\text{GeV})^2.$$

Additionally we impose the alignment limit,

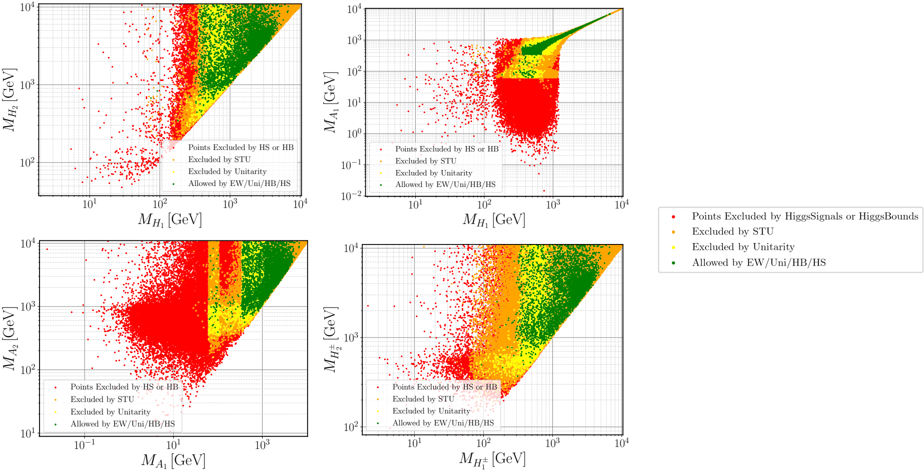
$$\begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\end{pmatrix} = \mathcal{O}\begin{pmatrix}h\\ H_1'\\ H_2'\end{pmatrix}, \quad m_h = 125.09\ \text{GeV},$$

And ensure a positively defined scalar potential.

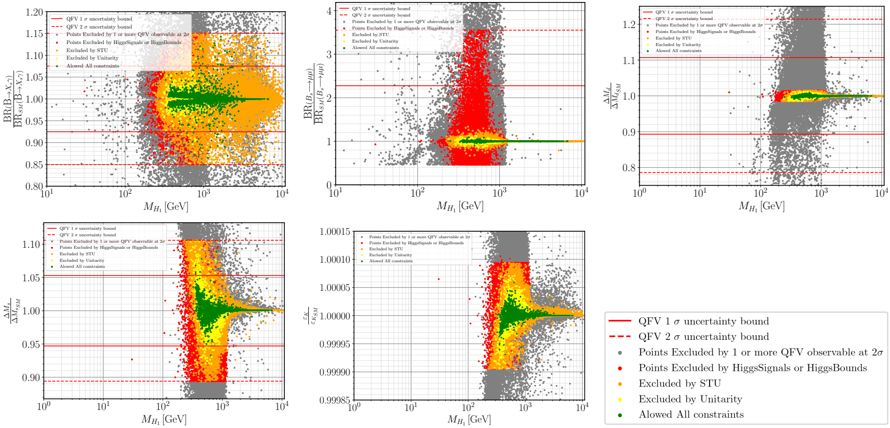
$\psi_{1,2}$; $\gamma_{1,2}$; $\alpha_{1,2,3}$	$\lVert\mu_{23}\rVert$, $\lVert\mu_{21}\rVert$, $\lVert\mu_{13}\rVert$	$m_{A_{1,2}}$,$m_{H^{\pm}_{1,2}}$,$m_{H_{1,2}}$
$0 - 2\pi$	1 GeV – 10 TeV	1 GeV – 11 TeV

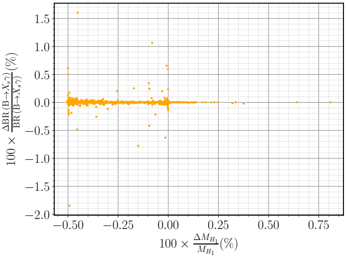
1. Model file: `SARAH-4.12.3`
2. (**added!**) Inverse spectrum parameter generator
3. Spectrum generator: `SPheno-4.0.3`
   - Unitarity
   - Tree-level mass spectrum.
   - Mixing angles
   - EW precision observables STU
   - $(g-2)_\ell$ at one loop-level.
   - Decay widths and Branching Fractions
   - Wilson coefficients
4. Generated points with $m_{h_1} \approx 125\,\mathrm{GeV}$ input to `HiggsBounds-4.3.1` and `HiggsSingnals-1.4.0`
5. (**added!**) `flavio` calculation of deviations in flavour production through Wilson coefficients.
6. **Surviving** points passed to `MadGraph5_amC5@NLO`
7. Fine-tuning complementary analysis.

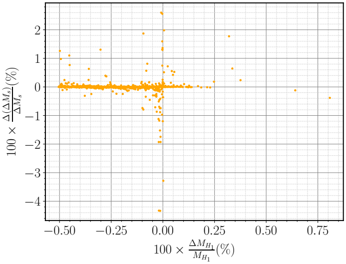
Masses

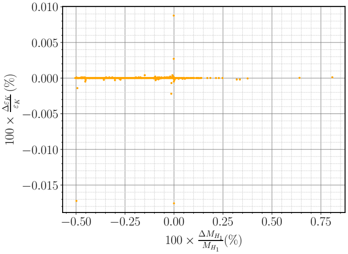

Points Excluded by HS or HB
Excluded by STU
Excluded by Unitarity
Allowed by EW/Uni/HB/HS
M_{H_1} [GeV]
M_{H_2} [GeV]
M_{A_1} [GeV]
M_{A_2} [GeV]
$M_{H_1^\pm}$ [GeV]
$M_{H_2^\pm}$ [GeV]
Points Excluded by HiggsSignals or HiggsBounds

The combination of flavour and direct measurements shows the significance of the BGL-like treatment. Direct detection creates a central region in all observables.

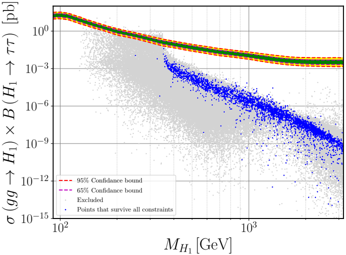


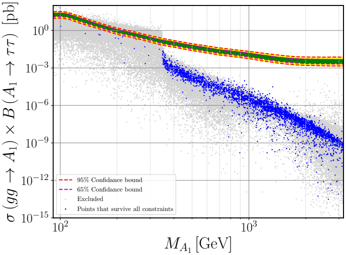


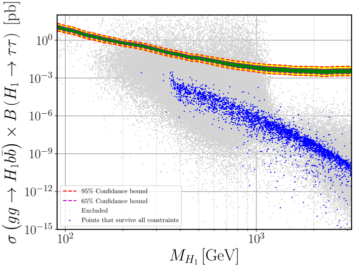




A complementary analysis was made where mass tuned through a 1% change on the soft breaking terms, μ_{13}, μ_{23} and μ_{21}.







- QFV observables lie always within allowed region of the LHC. Showing that these points can be probed in LHC III run.

	m_{H_1}	m_{H_2}	m_{A_1}	m_{A_2}	$m_{H_1^\pm}$	$m_{H_2^\pm}$
Second lightest H_1	327	1528	359	1616	229	1561
Lightest A_1	386	452	87	467	189	466
	337	446	100	395	311	374
Second Lightest $H_1^\pm$	398	1314	282	1525	173	1314
Lightest $H_1^\pm$ and H_1	251	2488	247	2616	157	2510

- A selection of five benchmark points. All masses are given in GeV.
- These correspond to the lightest scalars found that respect all QFV and scalar sector constraints.
- Coincidental lightest $H_1^\pm$ and H_1 parameter point.

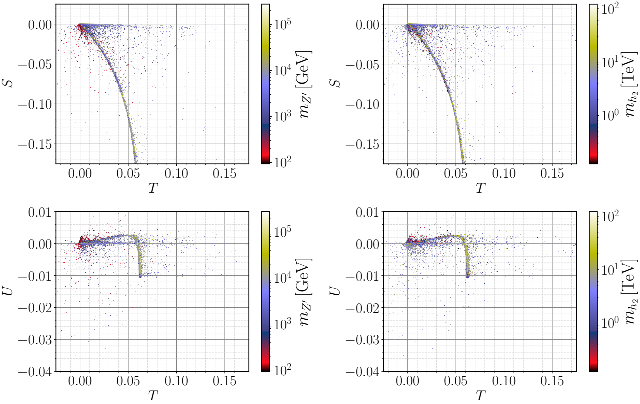
Scalar physics

- We have identified regions where both flavour and scalar sector physics are within experimental bounds and show that the BGL-like treatment can be successfully applied to a 3HDM model.
- We have also discussed the possibility of probing the model in the gluon fusion channel and shown QFV overlap with this exclusion.
- In particular we have observed that the BGL-like 3HDM offers the possibility for lighter than conventionally allowed non-standard scalars, at the reach of the LHC III.

Flavour sector

- Our analysis determined the most sensitive flavour violation channels.
- Proved our model can replicate the SM like FCNCs within 2σ deviations.

Back-up slides



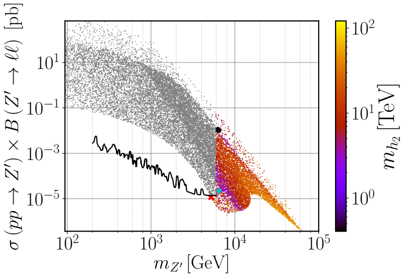
- Points shown here have both:
  - Viable scalar sector ($m_{h_1} \approx 125$ and χ^2 P-value $>$ 5%) BFB and unitarity constraint.
  - Within the 95% C.L. for STU observables
- Higher masses have a stronger effect in STU observables.

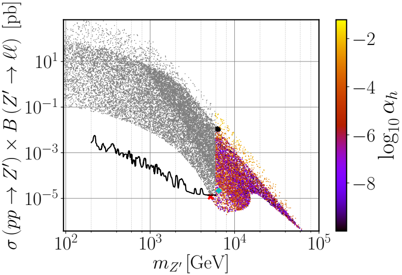
$\Delta a_\mu^{Z'}$ calculated in `SARAH` and numerically evaluated in `SPheno`

- When $\frac{m_\mu}{m_{Z'}} \ll 1$ the Z' contribution reads

$$\Delta a_\mu^{Z'} \approx -\frac{1}{3\pi^2}\frac{m_\mu^2}{m_{Z'}^2}\left[6 g_{\rm L}^{\mu\mu Z'} g_{\rm R}^{\mu\mu Z'} - 4\left({g_{\rm L}^{\mu\mu Z'}}^2 + {g_{\rm R}^{\mu\mu Z'}}^2\right)\right]$$

Contribution from h_2 is tiny: $\Delta a_\mu^{h_2} \propto \frac{m_\mu^2}{m_{h_2}^2}\left(y_\mu \sin\alpha_h\right)^2$





Suppressed by $\sin^2\alpha_h < \mathcal{O}(10^{-2})$ and $m_{h_2} > 420$ GeV

- Flavour effects need to be better understood. Generalized formulas for scalar couplings to quarks are missing.
- A Finer look at the STU dependencies with the new scalar states needs to be better understood.
- Theoretical expressions for unitarity are not written explicitly in the literature allowing us to first discovered correlations in the data.
- Computational limits exist, this model is much harder to scan than simpler models.
  - Improve the inversion process calculations by moving the C++/C, much faster.
  - Include machine learning to discover interesting regions much faster.
  - Include one-loop or two-loop corrected masses in new routines.

- The model is thoroughly examined and little parameter space remains. To provide us with more interesting physics it must be expanded.
  - The addition of a inverse seesaw-mechanism with additional neutrinos could offer more dark matter candidates.
  - The addition of new unitarity groups could allow the introduction of new mechanisms for anomalous momentum.
  - There is still the possibility of the Z' being ultra-light and accounting for neutral light-interacting dark matter.
- There is still the possibility of studying this model's phase transitions as it can provide us with cosmological evidence in the form of background gravitation waves.
  - This would be a good starting point to apply machine learning and to then expand on a more complex model.

The **SM** is gauge Quantum Field Theory (**QFT**), that is, manifestly invariant under a set of field transformations based on the gauge group,

$$\mathcal{G}_{SM} = \mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times Y\,.$$

Gauge, Fermion and Scalar fields and quantum numbers in the SM.

Fields	Spin 0 field	Spin 1 Field	$\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$
Gluons	$\times$	G^a	(**8**,**1**,0)
A bosons	$\times$	A^a	(**1**,**3**,0)
B bosons	$\times$	B	(**1**,**1**,0)
Higgs field	$(\phi^\pm, \phi^0)$	$\times$	(**1**,**2**,1)

Fields	Spin $\frac{1}{2}$ Field	$\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$
Quarks (3 gen.)	$Q = (u_L, d_L)$	$(\mathbf{3}, \mathbf{2}, 1/3)$
	u_R	$(\mathbf{3}, \mathbf{1}, 4/3)$
	d_R	$(\mathbf{3}, \mathbf{1}, -2/3)$
Leptons (3 gen.)	$L = (\nu_{e_L}, e_L)$	$(\mathbf{1}, \mathbf{2}, -1)$
	e_R	$(\mathbf{1}, \mathbf{1}, -2)$

Given the field content of the SM we can construct the Lagrangian as,

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\phi},$$

these components are written as,

$$\begin{aligned}\mathcal{L}_{\text{kin}} = & -\frac{1}{4}G_a^{\mu\nu}G_{a\,\mu\nu} - \frac{1}{4}A_a^{\mu\nu}A_{a\,\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \\ & - i\bar{Q}_{L_i}\not{D}Q_{L_i} - i\bar{u}_{R_i}\not{D}u_{R_i} - i\bar{d}_{R_i}\not{D}d_{R_i} - i\bar{L}_{L_i}\not{D}L_{L_i} - i\bar{e}_{R_i}\not{D}e_{R_i} \\ & - (D_\mu H)^\dagger(D^\mu H),\end{aligned}$$

$$\mathcal{L}_{\phi} = -\mu^2 HH^\dagger + \lambda(HH^\dagger)^2,$$

$$\mathcal{L}_{\text{Yuk}} = Y_{ij}^u \bar{Q}^i u_R^j \tilde{H} + Y_{ij}^d \bar{Q}^i d_R^j H + Y_i^e j \bar{L}^i e_{R_i} H + \text{H.c.}.$$

No explicit mass terms but for the scalar. **All** masses are generated by the Higgs mechanism.

The Higgs mechanism is characterized by the Higgs field taking on a non-zero vacuum value called the electroweak Vacuum Expectation Value (**VEV**),

$$(H^\dagger H)^2 = \frac{-\mu^2}{2\lambda} \equiv v^2.$$

This sets off the process of Spontaneous Symmetry Breaking (**SSB**). And breaks 3 of the SMs gauge group and it is modified to,

$$\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \to \mathrm{U}(1)_\mathrm{Q}.$$

The generated 3 Goldstone bosons are rotated into the gauge fields, so we can write,

$$H = \begin{pmatrix} \phi_1 + i\phi_2 \\ v + h + i\phi_3 \end{pmatrix} \to \langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}.$$

We can show how **square** terms now appear in the Lagrangian, all dependent on v. First for the gauge bosons and the Higgs,

$$
\begin{aligned}
\mathcal{L}'_{\text{Gauge}} =& \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2}(2v^2\lambda)h^2 - \frac{1}{4}A_a^{\mu\nu}A_{a\,\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \\
&+ \frac{1}{8}v^2 g^2 (A^1_\mu A^{1,\mu} + A^2_\mu A^{2,\mu}) \\
&+ \frac{1}{8}v^2 (g^2 A^3_\mu A^{3,\mu} + g'^2 B_\mu B^\mu - 2g^2 g'^2 A^3_\mu B^\mu) + \cdots \ ,
\end{aligned}
$$

From these terms we can reach the charge and mass eigenstates for the fields,

$$
W^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \pm i A^2_\mu), Z_\mu = -\sin(\theta_W)B_\mu + \cos(\theta_W)A^3_\mu,
$$

$$
A_\mu = \cos(\theta_W)B_\mu + \sin(\theta_W)A^3_\mu.
$$

From this we can see, the Higgs mass, and the following square terms,

$$
M_h = (2v^2\lambda), \quad m^2_{\text{Gauge}} = \frac{v^2}{4}\begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg_Y \end{pmatrix},
$$

The same can be done for fermions, where,

$$\begin{aligned}\mathcal{L}_{\text{lep.}} &= Y_e^{ij}\Phi_L^i H e_R^i + \text{H.c.}\\ &= \frac{y_e v}{\sqrt{2}} e_L e_R + \frac{y_\mu v}{\sqrt{2}} \mu_L \mu_R + \frac{y_\tau v}{\sqrt{2}} \tau_L \tau_R + \left(\text{Interactions with } h\right) + \text{H.c.}\,,\end{aligned}$$

given, Y_e^{ij}, is **diagonal** we can show,

$$m_e = \frac{y_e v}{\sqrt{2}}\,,\; m_\mu = \frac{y_\mu v}{\sqrt{2}}\,,\;\; m_\tau = \frac{y_\tau v}{\sqrt{2}}\,.$$

However, **for quarks**, where Y_u^{ij} is not diagonal.

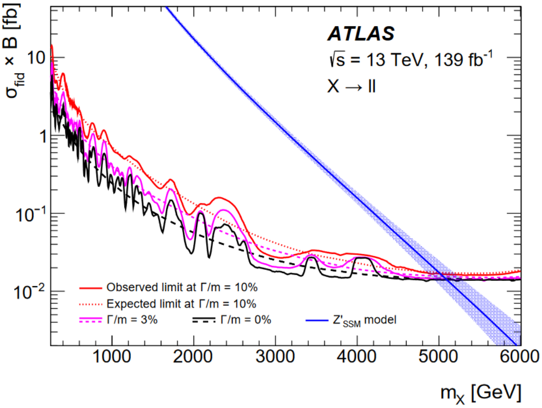
$$\mathcal{L}_{\text{qrk.}} = Y_d^{ij}\bar{Q}_L^i H d_R^j + Y_u^{ij}\bar{Q}_L^i \tilde{H} u_R^j + \text{H.c.}$$

$$\mathcal{L}_{\text{qrk.}} = \frac{Y_d^{ij} v}{\sqrt{2}} \bar{d}_L d_R + \frac{Y_u^{ij} v}{\sqrt{2}} \bar{u}_L u_R + \left(\text{Interactions with } h\right) + \text{H.c.}$$

To diagonalize the mass eigenstates we must perform a set of **unitary transformations** on the quark fields. Thus through the relations,

$$\bar{d}'_L = \bar{d}_L U_L^d \quad , \quad d'_R = U_R^{d\,\dagger} d_R \quad , \quad \bar{u}'_L = \bar{u}_L U_L^u \quad , \quad u'_R = U_R^{u\,\dagger} u_R\,.$$

Direct Z' searches exclude masses below $m_{Z'} \approx 6\ \mathrm{TeV}$



- Can the minimal B-L SM still address the muon $(g-2)_\mu$ anomaly and how well?

$$V(H,\chi) = m^2 H^\dagger H + \mu^2 \chi^* \chi + \lambda_1 (H^\dagger H)^2 + \lambda_2 \left(\chi^*\chi\right)^2 + \lambda_3 \chi^* \chi H^\dagger H$$

- Boundedness from below: $4\lambda_1\lambda_2 - \lambda_3^2 > 0$ and $\lambda_1, \lambda_2 > 0$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} -i\left(\omega_1 - i\omega_2\right) \\ v + (h + iz) \end{pmatrix} \qquad \chi = \frac{1}{\sqrt{2}} \left[x + (h' + iz')\right]$$

- $\omega^\pm = \omega_1 \mp i\omega_2$, z and z' are Goldstone bosons eaten by $W^\pm$, Z and Z'

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \langle \chi \rangle = \frac{x}{\sqrt{2}} \qquad \Rightarrow \qquad \begin{cases} v^2 = \dfrac{-\lambda_2 m^2 + \frac{\lambda_3}{2}\mu^2}{\lambda_1\lambda_2 - \frac{1}{4}\lambda_3^2} > 0 \\ x^2 = \dfrac{-\lambda_1 \mu^2 + \frac{\lambda_3}{2} m^2}{\lambda_1\lambda_2 - \frac{1}{4}\lambda_3^2} > 0 \end{cases}$$

$$\begin{cases} \lambda_2 m^2 < \frac{\lambda_3}{2}\mu^2 \\ \lambda_1 \mu^2 < \frac{\lambda_3}{2} m^2 \\ 4\lambda_1\lambda_2 - \lambda_3^2 > 0 \\ \lambda_1, \lambda_2 > 0 \end{cases}$$

✗: There is no solution

✓: There is solution

	$\mu^2 > 0$ $m^2 > 0$	$\mu^2 > 0$ $m^2 < 0$	$\mu^2 < 0$ $m^2 > 0$	$\mu^2 < 0$ $m^2 < 0$
$\lambda_3 < 0$	✗	✓	✓	✓
$\lambda_3 > 0$	✗	✗	✗	✓

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha_h & -\sin\alpha_h \\ \sin\alpha_h & \cos\alpha_h \end{pmatrix} \begin{pmatrix} h \\ h' \end{pmatrix}$$

Heavy Z' implies that $x \gg v$ for most of the parameters points:

$$\sin\alpha_h \approx \frac{1}{2}\frac{\lambda_3}{\lambda_2}\frac{v}{x} \qquad m_{h_1}^2 \approx 2\lambda_1 v^2 \qquad m_{h_2}^2 \approx 2\lambda_2 x^2$$

$$\mathcal{L}_{\text{bosons}} = |D_\mu H|^2 + |D_\mu \chi|^2 - V(H,\chi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{1}{2}\kappa F_{\mu\nu}F'^{\mu\nu}$$

- κ is a $\mathrm{U(1)_Y} \times \mathrm{U(1)_{B-L}}$ gauge kinetic-mixing parameter
- Field strength tensors $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu$
- Redefine $\kappa = \sin\alpha$ and gauge fields as (convenient basis choice)

$$\begin{pmatrix} A_\mu \\ A'_\mu \end{pmatrix} = \begin{pmatrix} 1 & -\tan\alpha \\ 0 & \sec\alpha \end{pmatrix} \begin{pmatrix} B_\mu \\ B'_\mu \end{pmatrix},$$

- Kinetic terms acquire canonical form

$$\mathcal{L}_{\text{kinetic}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}B'_{\mu\nu}B'^{\mu\nu}$$

Redefined covariant derivative absorbs the kinetic mixing information:

$$D_\mu = \partial_\mu + i\left(g_Y\, Y + g_{BY}\, Y_{B-L}\right) B_\mu + i\left(g_{BL}\, Y_{B-L} + g_{YB}\, Y\right) B'_\mu$$

- g_1 and g'_1 are $\mathrm{U}(1)_\mathrm{Y}$ and $\mathrm{U}(1)_\mathrm{B-L}$ gauge couplings
- g_{YB} and g_{BY} result from the kinetic mixing
- With our basis choice

$$\begin{cases} g_Y = g_1 \\ g_{BL} = g'_1 \sec\alpha \\ g_{YB} = -g_1 \tan\alpha \\ g_{BY} = 0 \end{cases}$$

No mixing limit: $\sec\alpha = 1 \Rightarrow g_{BL} = g'_1$

Gauge kinetic-mixing induces mixing between Z', Z and γ

$$\begin{pmatrix} \gamma_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W & 0 \\ -\sin\theta_W\cos\theta'_W & \cos\theta_W\cos\theta'_W & \sin\theta'_W \\ \sin\theta_W\sin\theta'_W & -\cos\theta'_W\sin\theta'_W & \cos\theta'_W \end{pmatrix} \begin{pmatrix} B_\mu \\ A^3_\mu \\ B'_\mu \end{pmatrix}$$

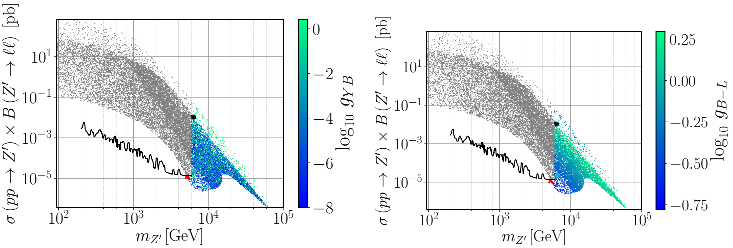
Again in the limit $x \gg v$ $$\sin\theta'_W \approx \frac{1}{8}\frac{g_{YB}}{g_{BL}}\left(\frac{v}{x}\right)^2\sqrt{g^2+g_Y^2}$$

- g is $\mathrm{SU}(2)_\mathrm{L}$ gauge coupling
- $\sin\theta'_W = 0$ for no kinetic mixing, $g_{YB} = 0$, and $Z'_\mu = B'_\mu$
  - For $g_{YB} = 0$ we have $m_Z = \frac{1}{2}v\sqrt{g^2+g_Y^2}$ and $m_{Z'} \approx 2g_{BL}x$
  - For $x \gg v$ we also have $m_{Z'} \approx 2g_{BL}x$

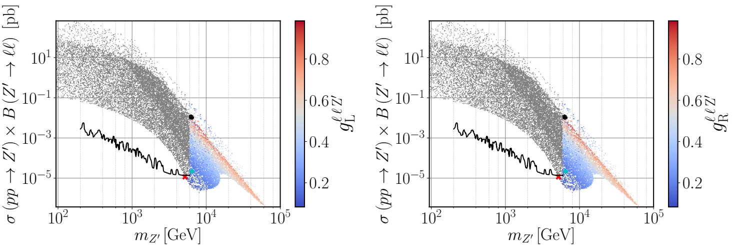
$$\mathcal{L}_{\text{Yukawa}} = - y_u^{ij}\overline{q_{\text{Li}}}u_{\text{Rj}}\widetilde{H} - y_d^{ij}\overline{q_{\text{Li}}}d_{\text{Rj}}H - y_e^{ij}\overline{\ell_{\text{Li}}}e_{\text{Rj}}H$$
$$-y_\nu^{ij}\overline{\ell_{\text{Li}}}\nu_{\text{Rj}}\widetilde{H} - \frac{1}{2}y_M^{ij}\overline{\nu_{\text{Ri}}^c}\nu_{\text{Rj}}\chi + \text{H.c.}$$

- $\widetilde{H} = i\sigma^2 H^*$
- Dirac and Majorana masses matrices: $\boldsymbol{m_D} = \frac{\boldsymbol{y_\nu}}{\sqrt{2}}v$ and $\boldsymbol{M} = \frac{\boldsymbol{y_M}}{\sqrt{2}}x$
- Neutrino masses via see-saw mechanism: $\begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \to \begin{cases} m_{\nu_l} \approx \frac{m_D^2}{M} \\ m_{\nu_h} \approx M \end{cases}$
- Small mixing angle: $\tan\alpha_\nu \approx -2\sqrt{\frac{m_{\nu_l}}{m_{\nu_h}}}$

Four-fermion contact interactions constrain $g_{\mathrm{B-L}} < 1.3$ in the B-L SM



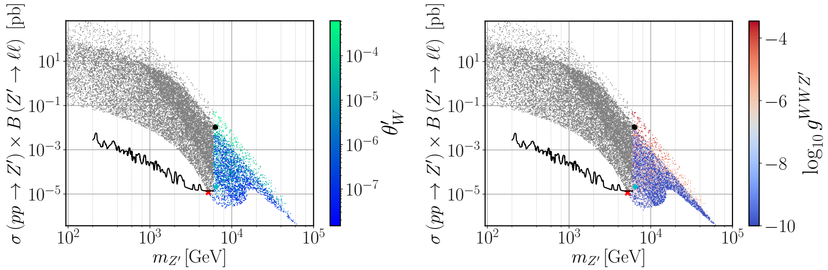
Enhancement of $\Delta a_\mu^{Z'}$ is due to sizeable g_{YB}, thus large $g_{\mathrm{L,R}}^{\ell\ell Z'}$



LEP constraints set upper bound $\sin\theta'_W \lesssim 10^{-3}$

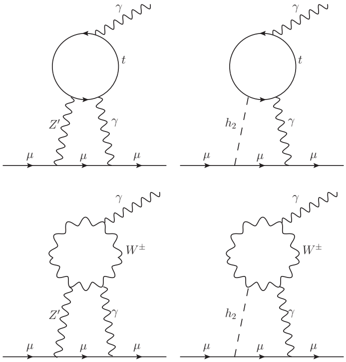
$$\sin\theta'_W \approx \frac{1}{8}\frac{g_{YB}}{g_{BL}}\left(\frac{v}{x}\right)^2\sqrt{g^2+g_Y^2}$$

which is respected even for the larger values of g_{YB}:



Small coupling of Z' to W bosons: $g^{WWZ'} \simeq \dfrac{1}{16}\dfrac{g_{\text{YB}}}{g_{\text{B-L}}}\left(\dfrac{v}{x}\right)^2$.

Two-loop Barr-Zee type contributions are subdominant



Larger contribution from the top-left diagram due to $g_{\mathrm{L,R}}^{ttZ'} \gg g^{WWZ'}, \alpha_h$, however:

$$\frac{\Delta a_\mu^{\text{Barr-Zee}}}{\Delta a_\mu^{Z'}} \simeq -\frac{1}{65536\pi^2}\frac{g^2\left(g^2+g_{\mathrm{Y}}^2\right)g_{\mathrm{YB}}^3}{\left[3g_{\mathrm{B-L}}g_{\mathrm{YB}}+2\left(g_{\mathrm{B-L}}^2+g_{\mathrm{YB}}^2\right)\right]}\left(\frac{v}{x}\right)^4 \ll 1$$

The generic scalar potential is extensively written as,

$$
\begin{aligned}
V(\phi_i) = &- \mu_1^2 \left(\phi_1^\dagger \phi_1\right) - \mu_2^2 \left(\phi_2^\dagger \phi_2\right) - \mu_3^2 \left(\phi_3^\dagger \phi_3\right) \\
&\left[-\mu_{21}^2 \left(\phi_2^\dagger \phi_1\right) - \mu_{23}^2 \left(\phi_2^\dagger \phi_3\right) - \mu_{13}^2 \left(\phi_1^\dagger \phi_3\right) + \text{H.c.}\right] \\
&+ \lambda_1 \left(\phi_1^\dagger \phi_1\right) + \lambda_2 \left(\phi_2^\dagger \phi_2\right) + \lambda_3 \left(\phi_3^\dagger \phi_3\right) \\
&+ \lambda_4 \left(\phi_1^\dagger \phi_1\right)\left(\phi_2^\dagger \phi_2\right) + \lambda_5 \left(\phi_1^\dagger \phi_1\right)\left(\phi_3^\dagger \phi_3\right) + \lambda_6 \left(\phi_2^\dagger \phi_2\right)\left(\phi_3^\dagger \phi_3\right) \\
&+ \lambda_7 \left(\phi_1^\dagger \phi_2\right)\left(\phi_1^\dagger \phi_2\right) + \lambda_8 \left(\phi_1^\dagger \phi_3\right)\left(\phi_3^\dagger \phi_1\right) + \lambda_9 \left(\phi_2^\dagger \phi_3\right)\left(\phi_3^\dagger \phi_2\right) \\
&+ \lambda_{10}\left\{ \left(\phi_1^\dagger \phi_3\right)^2 + \text{H.c.} \right\}.
\end{aligned}
$$

These fields acquire a VEV.

$$
\phi_k = \begin{pmatrix} w_k^\pm + i\, w_k^\mp \\ \frac{1}{\sqrt{2}}\left(v_k + h_k + i z_k\right) \end{pmatrix} \rightarrow \langle \phi_k \rangle = \begin{pmatrix} 0 \\ \frac{v_k}{\sqrt{2}} \end{pmatrix} \quad , \quad k = \{1, 2, 3\}.
$$

$$\mathcal{O} = \begin{pmatrix} \frac{v}{v_1} & \frac{v}{v_2} & \frac{v}{v_3} \\ \frac{v_3}{v_{13}} & 0 & \frac{-v_1}{v_{13}} \\ \frac{v_2 v_1}{v v_{13}} & \frac{v_{13}}{v} & \frac{v_2 v_3}{v v_{13}} \end{pmatrix},$$

where $v_{13} = \sqrt{v_1^2 + v_3^2}$ and $v = \sqrt{v_1^2 + v_2^2 + v_3^2}$, the total magnitude.

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \mathcal{O} \begin{pmatrix} h \\ H_1' \\ H_2' \end{pmatrix} \quad , \quad \begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \end{pmatrix} = \mathcal{O} \begin{pmatrix} z \\ A_1' \\ A_2' \end{pmatrix} \quad , \quad \begin{pmatrix} W_1^\pm \\ W_2^\pm \\ W_3^\pm \end{pmatrix} = \mathcal{O} \begin{pmatrix} \omega^\pm \\ H_1^{\pm\prime} \\ H_2^{\pm\prime} \end{pmatrix}.$$

These VEVs can also be parameterized through their mixing.

$$v_1 = v\cos(\psi_1)\cos(\psi_2) \quad , \quad v_2 = v\sin(\psi_1)\cos(\psi_2) \quad , \quad v_3 = v\sin(\psi_2),$$

Unlike in the SM, for this model we scan over the physical parameters

- This implies inverting all Lagrangian parameters and re-write the degrees of freedom as physical parameters.
- Trough this we set more than just the Higgs mass and VEV conditions but also apriori set boundess from bellow and ensure a positive mass spectrum.

To reach those equations we write the mass eigenstates in the Higgs basis. Presenting the case of the scalars,

$$V \supset \begin{pmatrix} h_1 & h_2 & h_3 \end{pmatrix} \frac{M_S^2}{2} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix},$$

The elements of this matrix are written as,

$$M_S^2 = \begin{pmatrix} \frac{2\lambda_1 v_1^3 + \mu_{21}^2 v_2 + \mu_{13}^2 v_3}{v_1} & v_1 v_2(\lambda_4 + \lambda_7) - \mu_{21}^2 & v_1 v_3(2\lambda_{10} + \lambda_5 + \lambda_8) - \mu_{13}^2 \\ v_1 v_2(\lambda_4 + \lambda_7) - \mu_{21}^2 & \frac{\mu_{21}^2 v_1 + 2\lambda_2 v_2^3 + \mu_{23}^2 v_3}{v_2} & v_2 v_3(\lambda_6 + \lambda_9) - \mu_{23}^2 \\ v_1 v_3(2\lambda_{10} + \lambda_5 + \lambda_8) - \mu_{13}^2 & v_2 v_3(\lambda_6 + \lambda_9) - \mu_{23}^2 & \frac{\mu_{13}^2 v_1 + \mu_{23}^2 v_2 + 2\lambda_3 v_3^3}{v_3} \end{pmatrix}.$$

In the 3HDM the Yukawa interactions are given by,

$$\mathcal{L}_Y = -\sum_{k=1}^{3}\left[\overline{Q}_{L_a}\left(\Gamma_k\right)_{ab}\phi_k n_{R_b} + \overline{Q}_{L_a}\left(\Delta_k\right)_{ab}\tilde{\phi}_k p_{R_b} + \text{H.c.}\right] + \left(\Psi_{L_a}\left(Y_1^e\right)_{ab}\phi_1 e_{R_b} + \text{H.c}\right).$$

The imposed symmetry, $\mathrm{U}(1)\times\mathbb{Z}_2$, forces the Yukawa textures,

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & 0 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix},$$

$$\Delta_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Delta_2 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Delta_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}.$$

The shape of the Yukawa couplings affect the mass eigenstates, we can write the following relations between the Higgs basis and physical states.

$$m_u^{\text{diag}} = V_L^n M_n V_R^n \equiv V_L^n \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix} V_R^n \quad , \quad m_d^{\text{diag}} = U_L^p M_p U_R^p \equiv U_L^p \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{pmatrix} U_R^p .$$

Where we define the unitary matrices as to include the complex phase in there,

$$V_L^n = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad , \quad V_R^n = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & e^{i\alpha} \end{pmatrix} \quad , \quad U_{L,R}^p = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix} .$$

Let us then look at the rotated Yukawa interactions for the neutral Higgs.

$$\mathcal{L}_Y^{H_1', H_2'} = \frac{H_1'}{v}\overline{n}_L N_{d\,1} n_R + \frac{H_2'}{v}\overline{n}_L N_{d\,2} n_R + \text{H.c.}\ .$$

$$\begin{aligned} N_{d\,1} =& \frac{v}{\sqrt{2}v_{13}} V_L^{n\,\dagger}\left(\Gamma_1 v_3 - \Gamma_3 v_1\right) V_R^n\ , \\ N_{d\,2} =& V_L^{n\dagger}\left[\frac{v_2}{v_{13}}\frac{1}{\sqrt{2}}\left(\gamma_1 v_1 + \gamma_3 v_3\right) - \frac{v_{13}}{v_2}\frac{1}{\sqrt{2}}\Gamma_2 v_2\right] V_R^n\ . \end{aligned}$$

$$\Gamma_3 = (\Gamma_3)_{33} P \quad , \quad \frac{1}{\sqrt{2}}\left(\Gamma_1 v_3 - \Gamma_3 v_1\right) = P M_d\ .$$

Hence,

$$\begin{aligned} (N_{d\,1})_{ij} =& \frac{v v_3}{v_1 v_{13}} (V^*_{\text{CKM}})_{3i} V_{\text{CKM}}^{3j} (M_d)_{jj} - \frac{1}{\sqrt{2}}\frac{v v_{13}}{v_1} (\Gamma_3)_{33} (V^*_{CKM})_{3i} (V_R^n)_{3j}, \\ (N_{d\,2})_{ij} =& \frac{v_{13}}{v_2}(M_d)_{jj}\delta_{ij} + \left(\frac{v_{13}}{v_2} + \frac{v_2}{v_{13}}\right)(V^*_{\text{CKM}})_{3i} V_{\text{CKM}}^{3j} (M_d)_{jj}. \end{aligned}$$