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Honestamente acho que isto vai ter que ser escrito antes da entrega

Honestly this will be written in english translated poorly from above :)

Resumo

Esta parte esta em pt

Abstract

The Standard Model of particle physics has been for some time now recognized as a placeholder theory. Too many problems have been propping up over the years, such as the strong CP problem, neutrino oscillations, matter–antimatter asymmetry, the nature of dark matter and dark energy and most recently the existence of gravitational waves background ?. In response many theories have been proposed to deal with each one of these problems. However, it's important to realise that these are not independent problems and as such we must search for a way to tackle all of these. Here we propose a simple model and look into some (maybe all?) of these problems.

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2.1 Introduction

It is hard to question that the Standard Model (SM) describes successful approximate framework with whom to describe the phenomenology of Particle Physics up to the largest energy scales probed by collider measurements so far. Proposed in the sixties by Glashow, Salam and Weinberg it has been extensively tested and in contemporary direct searches for new physics or indirect probes via e.g. flavour anomalies and precise electroweak parameter measured in proton-electron collisions, has been showing an increasingly consistency with real results. Given this it is fair to say that the joint description of the electromagnetic and the weak interaction by a single theory certainly is one of major achievements of the physical science in this century.

However, the SM is far from perfect with several open questions that are yet to be fully understood, it is these questions that modern physicists use to justify the research made in the area of high energy physics and Phenomenology. As a example, one of such weaknesses is a missing explanation of tiny neutrino masses confirmed by flavour-oscillation experiments.

Given it's successes researchers have long been tempted to try to complete the SM somehow rather than fundamentally alter it. In fact several mechanisms have been proposed that build upon the SM rather than replace it. We'll investigate some of these in this project (BLSM 3HDM) .

2.2 Up to Gauge Theory?

It is well known that symmetry played a very important role in the development of modern physics ever since Emmy Noether's first theorem, which derives conserved quantities from symmetries. Precisely the theorem states if an action is invariant under some group of transformations (symmetry), then there exist one or more conserved quantities (constants of motion) which are associated to these transformations.

The question that the lead to the framework of the standard model was: upon imposing to a given Lagrangian the invariance under a certain symmetry, would it be possible to determine the form of the interaction among the particles? In other words, could symmetry also imply dynamics. This train of thought led to Quantum Electrodynamics (QED) the first successful prototype of quantum field theory.

In QED the existence and some of the properties of the gauge field (which we'll later identify as the photon) follow from a principle of invariance under local gauge transformations of the $U(1)$ group.

We can quote Salam and Ward:

"Our basic postulate is that it should be possible to generate strong, weak and electromagnetic interaction terms (with all their correct symmetry properties and also with clues regarding their relative strengths) by making local gauge transformations on the kinetic energy terms in the free Lagrangian for all particles."

We are glossing over a lot of complexity here, for the SM to be truly complete Noether's theorem alone wouldn't suffice and new concepts had to be introduced. In the case of weak interactions the presence of very heavy weak gauge bosons require the new concept of spontaneous breakdown of the gauge symmetry and the Higgs mechanism. Also, the concept of asymptotic freedom played a crucial role to describe perturbatively the strong interaction at short distances, making the strong gauge bosons trapped.

2.2.1 Symmetries

A symmetry can be very broadly defined as a property of a system that is preserved or remains unchanged. However for our interests we are going to look at field transformations that leave a Lagrangian system invariant. To exemplify this consider the following generic transformation of a field ϕ:

$$\phi \longrightarrow \phi' = \phi + \delta\phi \tag{1}$$

To be invariant means the langraingian will be unchanging, thus,

$$\mathcal{L}(\phi, \frac{d\phi}{dt}) = \mathcal{L}(\phi', \frac{d\phi'}{dt}) \tag{2}$$

Noether explored this relation, noting the Lagrangian would transform itself like,

$$\mathcal{L}(\phi, \partial\phi) \longrightarrow \mathcal{L}'(\phi + \delta\phi, \partial\phi + \delta\partial\phi) \tag{3}$$

leading to the form,

$$\mathcal{L}' = \mathcal{L}(\phi, \partial\phi) + \partial\phi\frac{\partial\mathcal{L}}{\partial\phi} + \delta\frac{d\phi}{dt}\frac{\partial\mathcal{L}}{\partial\frac{d\phi}{dt}} \tag{4}$$

where assuming the equations of motion are satisfied $\left(\frac{\partial\mathcal{L}}{\partial\phi} = \frac{d}{dt}\frac{\partial\mathcal{L}}{\frac{d\phi}{dt}}\right)$ we can reach a expression for the first order change in the Lagrangian given by,

$$\mathcal{L}' = \mathcal{L} + \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\frac{d\phi}{dt}}\delta\phi\right) \tag{5}$$

Here we define, j, as the Noether Current,

$$j = \frac{\partial\mathcal{L}}{\partial\frac{d\phi}{dt}}\delta\phi \tag{6}$$

This way we can define a transformation, $\delta\phi$ that leaves the action invariant, as,

$$\delta\mathcal{S} = 0 \implies \delta\mathcal{L} = 0 \implies \frac{\partial\mathcal{L}}{\partial\frac{d\phi}{dt}}\delta\phi = 0 \implies \frac{d\,j}{dt} = 0 \tag{7}$$

This way we can say that j is constant, this means there is a conversed quantity. A simple real example of this would be the case of a projectile in Lagrangian physics. The lagrangian would be,

$$\mathcal{L} = \frac{1}{2}m\left(\frac{d\,x^2}{dt} + \frac{d\,y^2}{dt}\right) - mgy \tag{8}$$

We can see this is unchanged by moving the x axis by a quantity ϵ, translated by the x' transformation,

$$x' \longrightarrow x + \epsilon \implies \frac{d\,x'}{dt} \longrightarrow \frac{d\,x}{dt} \tag{9}$$

by checking the current,

$$j = \frac{\partial\mathcal{L}}{\partial\frac{d\phi}{dt}}\delta\phi = m\frac{d\,x}{dt} \tag{10}$$

is also conserved. We know this to be form of the momentum in the x-direction which we expected to be conserved in this problem. A more laborious exercise could show that conservation of energy comes from the invariance of an action under translations in time. And even things like conservation of charge, which are a little more complicated, come from this symmetry principle.

2.2.2 In Minkowski space

In the normal 3+1 dimensional space the form of Noether's current changes to,

$$\partial_\mu j^\mu = 0 \implies \frac{\partial j^0}{\partial t} + \nabla \cdot \mathbf{j} = 0 \tag{11}$$

where we usually call j^0 the charge density while $\mathbf{j}$ is named the current density.

2.2.3 Classical Electrodynamics

maybe if there is space

2.2.4 Gauge Transformations

2.3 Higgs Mechanism

2.4 Composition of the Standard Model

The Standard Model is composed by force carriers, the weak gauge bosons W and Z, the photon, the electromagnetic interaction messenger and the strong force mediators, the gluons, as well by matter particles, the quarks and leptons. Being that the Higgs boson is responsible for the mass generation mechanism.

Fermions are organized in three generations. Furthermore, there are 6 different types of quarks, up and down for the first generation, charm and strange for the second as well as top and bottom for the third one. Similarly, there are 6 types of leptons, the charged ones, electron, muon and tau, and the associated neutrinos, respectively represented by (u, d, c, s, t, b) while leptons as $(e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau)$

So far we have described the physical states that are often denoted as the building blocks of nature. However we have not yet explained how such states have acquired their masses and gauge quantum numbers, such as colour and electric charge. To see this, we start by noting the the SM is a gauge theory based on the group.

$$SU(3)_c \times SU(2)_L \times U(1)_Y \quad . \tag{12}$$

Fermions are half integer spin particles most of which have electrical charge (except the neutrinos). While quarks interact via the weak, electromagnetic and strong forces, the charged leptons only feel the electromagnetic and weak forces and the neutrinos are solely weakly interacting.

A physical fermion is composed of a left-handed and a right-handed part. While the former transform as $SU(2)_L$ doublets and can be written as,

$$L^i = \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix} \quad \text{and} \quad Q^i = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \quad , \tag{13}$$

2.5 Quantum fields

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3 B-L-SM Model

Now having discussed the Standard Model we can begin to look at what might lie beyond it. Here we introduce the minimal $U(1)_{B-L}$ extension of the Standard Model named, B-L-SM. This is a model trough which we can explain neutrino mass generation via a see-saw mechanism as well as by virtue of the model containing two new physical particle states, specifically a new Higgs like boson H' and a Z' gauge Boson, other small deviations in electroweak measurements, namely the $(g-2)_\mu$ anomaly. This refers to the discrepancy between the measured anomalous magnetic moment of the muon. We can also address the metastability of the electroweak (EW) vacuum in the SM trough the addition of the new scalar allowing for Higgs stabilization up to the plank scale with a the new Higgs starting from few hundred of GeVs. Last, but not least, the presence of the complex SM-singlet χ interacting with a Higgs doublet typically enhances the strength of EW phase transition potentially converting it into a strong first-order one. Although not covered in this work this analysis is of utmost importance given that it could provide a way to detect new physics and confirm the model without the need for a larger particle collider. This could be pointed to as future work.

Both these bosons are given mass trough the spontaneous breaking of the $U(1)_{B-L}$ symmetry that gives it's name to the Model. This group originates from the promotion of a accidental symmetry present in the SM, the Baryon number (B) minus the Lepton number (L) to a fundamental Abelian symmetry group. This origin for the mass of the referenced bosons means model is already very heavily constricted due to direct searches in the Large Hadron Collide (LHC).

One of the goals of this project was to investigate precisely the phenomenological status of the B-L-SM by confronting the new physics predictions with the LHC and electroweak precision data.

As a note this model is easily embedded into higher order symmetry groups like for example the $SO(10)$ or or E_6 , giving this model the ability to be used for the study of Grand Unified Theories.

The presence of three generations of right-handed neutrinos instead of a arbitrary number of neutrinos also ensures a framework free of anomalies with their mass scale developed once the $U(1)_{B-L}$ is broken by the VEV, x, of a complex SM-singlet scalar field, χ, simultaneously giving mass to the corresponding Z' boson and H'.

The cosmological consequences of the B-L-SM formulation are also worth mentioning. First, the presence of an extended neutrino sector implies the existence of a sterile state that can play a role of Dark Matter candidate. That can be completely sterile if stabilized with a $\mathbb{Z}_2$ parity symmetry. Note that the existence of sterile neutrinos can be used to explain the baryon asymmetry via the leptogenesis mechanism.

3.1 Formulating the model

Essentially, the minimal B-L-SM is a Beyond the Standard Model (BSM) framework containing three new ingredients:

- A new gauge interaction
- Three generations of right handed neutrinos
- A complex scalar SM-singlet.

The first one is well motivated in various GUT scenarios. However note that, if a family-universal symmetry such as $U(1)_{B-L}$ were introduced without changing the SM fermion content, chiral anomalies, which is a non conservative charged current on some channels, involving the $U(1)_{B-L}$ would be generated. These aren't completely undesired by themselves, since their result would be charge conjugation parity symmetry violation, or CP-symmetry violation, a observed missing feature of the SM, but this inclusion would result in far too much of these phenomena. (but how do I justify that there would be too much CP-violation?? is this even correct?)

Secondly, a new sector of additional three $U(1)_{B-L}$ charged Majorana neutrinos is essential for anomaly cancellation.

Finally is also required that the SM-like Higgs doublet, H, does not carry neither baryon nor lepton number, this way it does not participate in the breaking of $U(1)_{B-L}$. It is then necessary to introduce a new scalar singlet, χ, solely charged under $U(1)_{B-L}$, whose VEV breaks the $B-L$ symmetry at a scale higher than the electro-weak breaking scale. It is also this breaking scale that generates masses for heavy neutrinos.

The particle content and related charges of the minimal $U(1)_{B-L}$ extension of the SM are shown in the table .

	q_L	u_R	d_R	l_L	e_R	ν_R	H	χ
$SU(3)_c$	3	3	3	1	1	1	1	1
$SU(2)_L$	2	1	1	2	1	1	2	1
$U(1)_Y$	$\frac{1}{6}$	$\frac{2}{3}$	-$\frac{1}{3}$	-$\frac{1}{2}$	-1	0	$\frac{1}{2}$	0
$U(1)_{B-L}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	-1	-1	-1	0	2

Table 1: Quantum fields and their respective quantum numbers in the minimal B-L-SM extension. The last two lines represent the weak and $B-L$ hypercharges

Given these, we can write the scalar potential of the Lagrangian as,

$$V(H,\chi) = \mu_1^2 H^\dagger H + \mu_2^2 \chi^*\chi + \lambda_1 (H^\dagger H)^2 + \lambda_2 (\chi^*\chi)^2 + \lambda_3 \chi^*\chi H^\dagger H \tag{14}$$

For the scalar Potential to be bounded from below (BFB)) we deduce the conditions,

$$4\lambda_1\lambda_2 - \lambda_3^2 > 0 \quad , \quad \lambda_1, \lambda_2 > 0 \tag{15}$$

In this potential, 14, the full components of the scalar fields are given by,

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} -i\,(\omega_1 - i\omega_2) \\ v + (h + iz) \end{pmatrix} \qquad \chi = \frac{1}{\sqrt{2}} \left(x + (h' + iz')\right) \tag{16}$$

In these equations we can see h and h' representing the radial quantum fluctuations around the minimum of the potential that will constitute the physical degrees of freedom associated to the H and H'. There are also four Goldstone directions denoted as ω_1, ω_2, z and z' which are absorbed into longitudinal modes of the $W^\pm$, Z and Z' gauge bosons once spontaneous symmetry breaking (SSB) takes place. After SSB the fields take the form,

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \langle \chi \rangle = \frac{x}{\sqrt{2}} \tag{17}$$

where recall v and x are the associated VEVs to each field. From here solving the tadpole equations in relation to each of the VEVs, we arrive at,

$$v^2 = \frac{-\lambda_2\mu_1^2+\frac{\lambda_3}{2}\mu_2^2}{\lambda_1\lambda_2-\frac{1}{4}\lambda_3^2} > 0 \qquad \text{and} \qquad x^2 = \frac{-\lambda_1\mu_2^2+\frac{\lambda_3}{2}\mu_1^2}{\lambda_1\lambda_2-\frac{1}{4}\lambda_3^2} > 0 \tag{18}$$

which, when simplified with the bound from bellow conditions yield a simpler set of equations,

$$\lambda_2\mu_1^2 < \tfrac{\lambda_3}{2}\mu_2^2 \qquad \text{and} \qquad \lambda_1\mu_2^2 < \tfrac{\lambda_3}{2}\mu_1^2 \tag{19}$$

Note that although λ_1 and λ_2 must be positive to ensure the correct potential shape initially no such conditions exist for the sign of λ_3 , μ_1 and μ_2. However observing equation 19 we can infer the following,

	$\mu_2^2 > 0$ $\mu_1^2 > 0$	$\mu_2^2 > 0$ $\mu_1^2 < 0$	$\mu_2^2 < 0$ $\mu_1^2 > 0$	$\mu_2^2 < 0$ $\mu_1^2 < 0$
$\lambda_3 < 0$	✗	✓	✓	✓
$\lambda_3 > 0$	✗	✗	✗	✓

Table 2: Possible Signs of the potential parameters in (14). While the ✓ symbol indicates the existence of solutions for tadpole conditions (19), the ✗ indicates unstable configurations.

For our studies we decided to leave the sign of λ_3 unconstrained, choosing a configuration where both μ parameters are negative.

Taking the Hessian matrix evaluated at the vaccum value,

$$M^2 = \begin{pmatrix} 4\lambda_2 x^2 & \lambda_3 vx \\ \lambda_3 vx & 4\lambda_1 v^2 \end{pmatrix}, \tag{20}$$

3.1.1 Neutrino masses

As mentioned briefly during the course of this dissertation the SM suffers from lacking a way to explain the observed neutrino masses by default. The minimal way of addressing this problem is by adding heavy Majorana type neutrinos in order to realise a seesaw mechanism. In this chapter we hope to explain how by perform the addition we could generate light neutrino states and how this addition is justified as part of a larger theory.

3.2 Electro-Weak searches

3.2.1 Oblique parameter analysis

3.2.2 The $(g-2)_\mu$ anomaly

4 3HDM

5 Conclusions and Future Work

6 Appendix

6.1 Gamma Matrices

The γ matrices are defined as,

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} I \tag{21}$$

where,

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{22}$$

and if $\gamma_\mu = (\gamma^0, \boldsymbol{\gamma})$ then it is usual to require for the hermitian conjugate matrices,

$$\gamma^{0\dagger} = \gamma^0 \quad \text{and} \quad \boldsymbol{\gamma}^\dagger = -\boldsymbol{\gamma} \tag{23}$$

6.2 Lagrangian Dynamics

In Lagrangian dynamics we define the action S has,

$$\mathcal{S} = \int L\, dt = \int \mathcal{L}(\phi, \partial\phi)\, d^4x \tag{24}$$

where L is the Lagrangian, and the $\mathcal{L}$ is designated as the *Lagrangian density*, note these terms are usually used interchangeable. Here $\mathcal{L}$ is a function of the field ϕ and it's spatial derivatives.

The action S is constrained by the principle of least action, this requires the "path" taken by a field between an initial and final set of coordinates to leave the action invariant, this can be expressed by,

$$\partial\mathcal{S} = 0 \tag{25}$$

from here one can deduce the *Euler-Lagrange* equations,

$$\partial_\mu \left(\frac{\partial\mathcal{L}(\phi, \partial\phi)}{\partial(\partial_\mu)} \right) - \frac{\partial\mathcal{L}(\phi, \partial\phi)}{\partial\phi} = 0 \tag{26}$$