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2020

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Honestamente acho que isto vai ter que ser escrito antes da entrega

Honestly this will be written in english translated poorly from above :)

Resumo Esta parte esta em pt

Abstract

The Standard Model of particle physics has been for some time now recognized as a placeholder theory. Too many problems have been propping up over the years, such as the strong CP problem, neutrino oscillations, matter–antimatter asymmetry, the nature of dark matter and dark energy and most recently the existence of gravitational waves background ?. In response many theories have been proposed to deal with each one of these problems. However, it's important to realise that these are not independent problems and as such we must search for a way to tackle all of these. Here we propose a simple model and look into some (maybe all?) of these problems.

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1 Introduction

Although the current theory widely used in the study of particle physics is the Standard Model (SM). A theory that has been shown to be thus far the best descriptor for the experimentally observed spectra of particles and their interactions at all probable scales. The SM boasts being one of the most monumental achievements in physics as it combined three of the four fundamental forces of nature in a very well motivated framework and now that it's last fundamental component, the Higgs Boson as been discovered, we can finally attest that the mechanism of spontaneous symmetry breaking of the $SU(2)_L \times U(1)_Y$ into the electromagnetic symmetry $U(1)_{EM}$ generates all physical masses for the fermions and mediator bosons.

However, despite all these successes the SM still lacks a strong theoretical explanation for several experimental observations. Firstly, the SM can not account for one of the most important cosmological discoveries of the century observed trough gravitational lensing and many other experiments, the existence of dark matter. This is a fundamental flaw since the SM lacks a possible dark matter candidate, or dark particle.

Secondly, the SM lacks any justification for the existence of baryon asymmetry in the universe, i.e. why is the universe primarily made of matter rather than anti-matter. Although note that the Electroweak baryogenesis (EWBG) remains a theoretically possible and experimentally testable in future? scenario for explaining the cosmic baryon asymmetry, a scenario viable in the SM framework is it? what about CP violation?. As its name suggests, EWBG refers to a mechanism that produces an asymmetry in the density of baryons decaying during the electroweak phase transition. As any of this been tested by GW experiments?. This puts some requirements on the composition of the universe but would imply that all matter anti-matter asymmetry is created during the time when the Higgs field is settling into it's new vacuum expectation value (VEV).

Thirdly, the SM has particular oddities in the fermion sector where peculiar and more importantly unjustified mass and mixing hierarchies occur. This is usually refereed to as the *flavour problem* and is considered a drawback of accepting the SM as is. Specifically, we observe the top quark to be five order of magnitudes heavier the up quark, and eleven orders of magnitude than the observed neutrino masses. It is believe by many that eleven orders of magnitude is far too high of a gap to be merely it's nature, so a mechanism that would justify such gap is a desired property of most Beyond the Standard Model frameworks.

Fourth, note neutrino masses are not included by any mechanism in the SM. Although there are precise oscillation measurements that show it's masses in the eV range with precise mixing in between 3 different generations.

These are just some of the typical justifications given to explore possible BSM scenarios. The holy grail of which would be a model that include all these problems in a properly motivated framework that addresses both cosmological, gravitational, and phenomenological problems. I should mention cosmos.

However the available space for new physics gets reduced by each successful experiment, being one of the reasons the SM remains so prevalent that it has shown puzzling consistence with some measurements that were initial believed to be a possible gateway to new physics. Thus the search continues for hints at possible directions to complete the SM. I should mention flavour changing

Chief among these experiments is the Large Hadron Collider (LHC), whose large amount of collect data over these past years is setting more and more stringent bounds on viable parameter spaces of popular BSM scenarios. And as available space for new physics decreases it becomes more challenging to reveal remaining space without falling within the possibility of fine tuning our model. How to properly explain what fine tunning is?

Conventionally, BSM searches in these multi-dimensional parameter spaces have often been made in large computer-clusters with use of several weeks of computational time trough simple Monte-Carlo methods. Although this is the basis of the work presented here a effort was made to incorporate new machine learning routines trough the initial building of smaller learning sets trough conventional methods. Unfortunately this wasn't accomplished in this work.

During this work we shall do a small expedition into possible BSM scenarios. To begin we discuss possible extensions of scalar sectors that can be embedded into the SM while also examining the consequences of those addition in the other sectors. Note while the minimally structure of the Higgs sector postulated by the SM is not a immediate contradiction of measurements. It is not manifestly required by the data.

This is done as a exercise to observe if such additions are viable despite the relatively tight bounds on Higgs boson couplings to SM gauge boson and heavy fermions. how much should I mention of the 3HDM and the BLSM in the introduction?

We give a higher repute to the Higgs Sector since fermion masses and mixing patterns relate often to the specific structure of the Higgs sector. Also the addition of new scalars offer a large playground for colliders and often the inclusion of new neutrino physics. Models with more than one Higgs doublet also addresses the observed charge parity CP violation with the drawback of potentially having large FLavor Changing Neutral Currents (FCNCs). These FCNCs are undesirable at least in large number given observations and multiple Higgs Doublets could include these diagrams at tree-level, very problematic. A easier model would be a simple addition of a unitary symmetry.

Two particular multi-Higgs extensions will be presented in this work a phenomenological study of a 3 Higgs Doublet model (3HDM) with softly broken $U(1) \times Z_2$ symmetry and a simple Unitarity ($U(1)$) extension of the SM based on the apparent Baryon minus Lepton symmetry (BLSM) model.

2 The Standard Model of Particle Physics

To pave the way for our future studies we present a brief overview of the SM. Complete with a brief overview of it's mechanisms and a historical introduction. Has stated, it is hard to question the validity of the SM as a successful approximate framework with whom to describe the phenomenology of Particle Physics up to the largest energy scales probed by collider measurements so far. The SM was proposed in the sixties by Glashow, Salam and Weinberg and since it has been extensively tested. Both in contemporary direct searches for new physics and indirect probes via e.g. flavour anomalies and precise electroweak parameter measurements in proton-electron collisions the SM has been showing an increasingly consistency with real results, puzzling many researchers as more data comes in.

Given it's successes researchers have long been tempted to try to complete the SM somehow rather than fundamentally alter it. In fact several mechanisms have been proposed that build upon the SM rather than replace it.

The path to the formulation of the SM came from previous principles relating to symmetries in nature, specificity symmetry in physical laws. In fact much in modern physics can be attributed to Emmy Noether. Who deduced trough her first theorem that if the action is invariant under some group of transformations (symmetry), then there exist one or more conserved quantities (constants of motion) which are associated to these transformations.

But this led to the question behind the formulation of the SM. Is it possible that upon imposing to a given Lagrangian the invariance under a certain group of symmetries to reach a given form of the dynamics. These dynamics would be particle interactions in the SM. This train of thought led to Quantum Electrodynamics (QED) the first successful prototype of quantum field theory.

We can quote Salam and Ward:

"Our basic postulate is that it should be possible to generate strong, weak and electromagnetic interaction terms (with all their correct symmetry properties and also with clues regarding their relative strengths) by making local gauge transformations on the kinetic energy terms in the free Lagrangian for all particles."

We are glossing over a lot of complexity here, and for the SM to be truly new concepts had to be introduced. In the case of weak interactions the presence of very heavy weak gauge bosons require the new concept of spontaneous breakdown of the gauge symmetry and the Higgs mechanism. Also, the concept of asymptotic freedom played a crucial role to describe perturbatively the strong interaction at short distances.

2.1 Composition of the Standard Model

The Standard Model spectra after the process of Spontaneous Symmetry breaking (SBB) is composed by, first, the weak force carriers, gauge bosons W and Z, then, the photon γ, the electromagnetic interaction messenger and finally the strong force mediators, the gluons, as well, of course, by the matter particles, the quarks and leptons.

Fermions and quarks are organized in three generations each, with 2 pairs by each generation leading to 6 different particles for each family. For quarks we have the up and down for the first generation, charm and strange for the second as well as top and bottom for the third one. Similarly, there are 6 types of leptons, the charged ones, electron, muon and tau, and the associated neutrinos. These are represented in different manners, being that the quarks are represented by the letters (u, d, c, s, t, b) while leptons as $(e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau)$.

However we have not yet explained how such states have acquired their masses and quantum numbers, such as colour and electric charge. To show this, we start by presenting the symmetry group the SM originates from,

$$SU(3)_c \times SU(2)_L \times U(1)_Y \quad . \tag{1}$$

Here we have see the $SU(3)_c$ group corresponding to Quantum chromodynamics (QCD) responsible for the strong force. We'll see this group remains unbroken. On the other hand we have the $(SU(2)_L \times U(1)_Y$ group that will be broken by the Higgs VEV. Each particle stems from a field that is charged in a particular manner on each of these groups.

Fermions are half integer spin particles most of which have electrical charge (except the neutrinos). While quarks interact via the weak, electromagnetic and strong forces, the charged leptons only feel the electromagnetic and weak forces and the neutrinos are weakly interacting.

A physical fermion is composed of a left-handed and a right-handed field. While the left transform as $SU(2)_L$ doublets and can be written as,

$$L^i = \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix} \quad \text{and} \quad Q^i = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \quad , \tag{2}$$

where the i index stands for generation, the latter are $SU(2)_L$ singlets and can be simply represented as

$$e_R^i = \{e_R, \mu_R, \tau_R\}, \quad u_R^i = \{u_R, c_R, t_R\}, \quad d_R^i = \{d_{e_R}, s_{e_R}, b_{e_R}\} \quad , \tag{3}$$

note also that the quarks form triplets of $SU(3)_C$ whereas leptons are colour singlets. The Higgs boson also emerges from an $SU(2)_L$ doublet with the form,

$$H = \begin{pmatrix} \phi^1 + \, i \, \phi^2 \\ \phi^3 + \, i \, \phi^4 \end{pmatrix} \quad , \tag{4}$$

The full set of gauge quantum numbers in the SM is given in tables 1 and 2. We can then

Table 1: Gauge bosons and Scalar fields in the SM

Fields	Spin 0 field	Spin 1 Field	$SU(3)_C \times SU(2)_L \times U(1)_Y$
Gluons	$\times$	g	(8,1,0)
A bosons	$\times$	A^i	(1,3,0)
B bosons	$\times$	B	(1,1,0)
Higgs field	$(\phi^{\pm}, \phi^0)$	$\times$	(1,2,1)

write a Lagrangian invariant under transformations of the $SU(3) \times SU(2) \times U(1)$ as.

$$\begin{aligned} \mathcal{L}_{SM} \quad = \quad & (D_\mu H)^\dagger (D^\mu H) - V(HH^\dagger) - \frac{1}{4} F^i_{\mu\nu} F^{i,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ & + \overline{L^i_L}(i\gamma^\mu D_\mu) L^i_L + \overline{Q^i_L}(i\gamma^\mu D_\mu) Q^i_L + \overline{L^i_R}(i\gamma^\mu D_\mu) L^i_R + \overline{Q^i_R}(i\gamma^\mu D_\mu) Q^i_R \\ & - [y^d_{jk} \overline{Q}^j_L d^k_R H + y^u_{jk} \overline{Q}^j_L u^k_R \tilde{H} + y^e_{jk} \overline{L}^j e^k_R H + h.c.] \quad , \end{aligned} \tag{5}$$

where $\tilde{H} = i\sigma_2 H$. We define the covariant derivative as, D_μ

$$D_\mu = \partial_\mu - i g_S \tau^a G^a_\mu - i g T^i A^i_\mu - i g' Y B_\mu \quad , \tag{6}$$

Table 2: Fermion field dimensions in the SM

Fields	Spin $\frac{1}{2}$ Field	$SU(3)_C \times SU(2)_L \times U(1)_Y$
Quarks (3 gen.)	$Q = (u_L, d_L)$	$(3, 2, \frac{1}{3})$
	u_R	$(3, 1, \frac{4}{3})$
	d_R	$(3, 1, -\frac{2}{3})$
Leptons (3 gen.)	$(L = (\nu_{e_L}, e_L)$	$(1, 2, -1)$
	e_R	$(1, 1, -2)$

where $\tau^a = \frac{\lambda_a}{2}$, $(a = 1, ..., 8)$ are the generators of $SU(3)_c$, $T_i = \frac{\sigma_i}{2}$, $(i = 1, 2, 3)$ are the generators of $SU(2)_L$ and Y is the generator of $U(1)_Y$. Here the symbols λ_a and σ_i represent the Gell-Mann and Pauli matrices respectively. In the first line of (5), the first term represents the interactions of gauge bosons with the Higgs field and the second term is the scalar potential associated to the said field. The second line represents the gauge-kinetic terms and gauge boson self interactions. The third line describes the fermion kinetic terms as well as the interactions among fermions and gauge bosons. Finally, the last line shows the Yukawa interactions between the Higgs and the fermions. It is due to the Yukawa interactions that the SM fermions acquire their masses once the electro-weak (EW) symmetry is broken, as we will later see.

Symmetry plays as we will see a key-role in particle physics and given by the field properties we assigned to each field, i.e. the representations of the groups that make up the SM. The focus of our discussion of the SM is to show how stemming from this we derive the physical particle spectrum that accurately describes reality.

2.2 Gauging U(1): The example of Quantum Electrodynamics

In the previous section we discussed the particular case of a complex scalar field transforming under a global symmetry. Let us now consider a local transformation by replacing the phase in (**??**) by a space-time dependent phase $\alpha(x)$

$$\phi(x) \to e^{iq\alpha(x)}\phi(x) \quad , \quad \phi^*(x) \to e^{-iq\alpha(x)}\phi^*(x) \quad , \tag{7}$$

Introducing this transformation, where q is the charge associated to the symmetry, to the former Lagrangian density (**??**), we can see that all but the derivative terms are invariant. Those particular terms now take the form,

$$\partial_\mu \Phi' \to \partial_\mu \left(e^{i\alpha(x)}\Phi\right) = e^{i\alpha(x)}.\,(i\;\Phi(\partial_\mu \alpha) + \partial_\mu \Phi) \tag{8}$$

$$\partial_\mu \Phi'^{\,*} \to \partial_\mu \left(e^{-i\alpha(x)}\Phi^*\right) = e^{-i\alpha(x)}.\,(-i\;\Phi^*(\partial_\mu \alpha) + \partial_\mu \Phi^*) \quad , \tag{9}$$

meaning the Lagrangian is no longer invariant. In fact we note it changes by,

$$\delta\mathcal{L} = i\;q\Phi(\partial_\mu\alpha)\Phi^* - i\;q\partial_\mu\Phi\Phi^*(\partial_\mu\alpha) + \Phi\Phi^* q^2 \partial_\mu\alpha\partial^\mu\alpha \quad . \tag{10}$$

To reattain invariance under local gauge transformations we have to introduce a new 4-vector A_μ, denoted as the gauge field in what follows, as well as three new terms in the Lagrangian allowed by the gauge symmetry. We also note that the gauge field has to transform in such a way to cancel the effects coming from the derivative terms in 9 and has the form

$$A_\mu + \frac{1}{q}\partial_\mu \alpha(x) = A'_\mu \quad , \tag{11}$$

The new added terms are then,

$$\mathcal{L}_1 = -i\, q\left(\Phi^* \partial_\mu \Phi - \Phi \partial_\mu \Phi^*\right) A_\mu \tag{12}$$
$$\mathcal{L}_2 = q^2 A^\mu A_\mu \Phi \Phi^* \quad , \tag{13}$$

In addition to these terms we add another term to the Lagrangian, $\mathcal{L}_3$, containing kinetic terms for the gauge field and invariant under the transformation (9) and reads,

$$\mathcal{L}_3 = -\frac{1}{4}\left[\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right)\right],$$
$$\mathcal{L}_3 = \frac{1}{4} F_{\nu\mu}(x) F^{\nu\mu}(x)\ , \tag{14}$$

Puting all together we get,

$$\begin{aligned}
\mathcal{L}_{tot} =& \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 \\
\mathcal{L}_{tot} =& (\partial_\mu \Phi)(\partial^\mu \Phi^*) - iq(\Phi^* \partial^\mu \Phi - \Phi \partial^\mu \Phi^*) A_\mu + q^2 A_\mu A^\mu \Phi^* \Phi - m \Phi^* \Phi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \\
\mathcal{L}_{tot} =& (\partial_\mu \Phi + iq A_\mu \Phi)(\partial^\mu \Phi^* - iq A^\mu \Phi) - m^2 \Phi^* \Phi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \\
\mathcal{L}_{tot} =& D_\mu \Phi D^\mu \Phi^* - m^2 \Phi^* \Phi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \quad ,
\end{aligned} \tag{15}$$

which preserves the original structure but with normal derivatives replaced by what we denote as covariant derivatives. Note that it is in the terms involving the covariant derivatives that we have interactions involving the gauge and the scalar fields.

2.3 The Higgs mechanism and the mass generation of the Gauge bosons

Coming back to the SM as defined above, we can now study the mechnism of spontaneous symmetry breaking. Let us then consider the part of the Lagrangian containing the scalar covariant derivatives as defined in eq. (5) , the scalar potential and the gauge-kinetic terms:

$$\mathcal{L}_{gauge} = (D_\mu H)^\dagger (D_\mu H) - V(HH^\dagger) - \frac{1}{4} F^i_{\mu\nu} F^{i,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \quad , \tag{16}$$

The elements of this sector are defined as,

$$V(HH^\dagger) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2 \quad , \tag{17}$$

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g \epsilon_{ijk} A^j_\mu A^k_\nu \quad , \tag{18}$$

In this formulation the constants g and g' are the gauge couplings of the groups $SU(2)_L$ and $U(1)_Y$. As the potential is identical to the one dicussed in Sec. **??**, we know that we have a phase shift, namely $\mu^2 < 0$ and what kind of VEV we are expected to find, namely

$$(H^\dagger H) = \frac{-\mu^2}{2\lambda} = \frac{1}{2} v \quad , \tag{19}$$

The vacuum can be aligned in such a way that we have

$$H_{min} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad . \tag{20}$$

This vacuum will break the $SU(2)_L \times U(1)_Y$ symmetry down to $U(1)_Q$. this means that in the begining there are four generators, which are $T^{1,2,3}$ and Y, and after the breaking we are solely left with one unbroken combination that is $Q = (T^3 + 1/2)$. This means that in total we will have three broken generators, thus, from Goldstone Theorem, there will be three massless particles.

As we have seen for the abelian Higgs model, the Goldstones modes can be parameterized as phases in the field space and then can be "rotated away" in the physical basis, leaving us with a single physical massive scalar, the Higgs boson. Note that, with this transformation we are removing three scalar degrees of freedom. However, they cannot just disappear from the theory and will be absorbed by the massive gauge bosons. In fact, a massless gauge boson contains only two scalar degrees of freedom (transverse polarization). Meanwhile, a massive vector boson has two transverse and a longitudinal polarization, i.e., three scalar degrees od freedom. So, as we discussed above, while before the breaking of the EW symmetry we have four massless gauge bosons, after the breaking we are left with three massive ones. This means that there are three extra scalar degrees of freedom showing up in the gauge sector. It is then commonly said that the goldstone bosons are "eaten" by the massive gauge bosons and the total number of scalar degrees of freedom in the theory is preserved. Therefore, without loss of generality, we can rewrite the Higgs doublet as

$$\begin{pmatrix} G_1 + iG_2 \\ v + h(x) + iG_3 \end{pmatrix} = H(x) \to H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \quad . \tag{21}$$

Once the Higgs doublet acquires a VEV, the Lagrangian (16) can be recast as:

$$\begin{aligned} \mathcal{L}' =& \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2}(2v^2\lambda)h^2 - \frac{1}{4}F^i_{\mu\nu}F^{i,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &+ \frac{1}{8}v^2 g^2 (A^1_\mu A^{1,\mu} + A^2_\mu A^{2,\mu}) + \frac{1}{8}v^2(g^2 A^3_\mu A^{3,\mu} + g'^2 B_\mu B^\mu - 2g^2 g'^2 A^3_\mu B^\mu) \quad , \end{aligned} \tag{22}$$

A few things become obvious first, we have a lot of mass terms most stemming from the squared gauge fields and a lonesome squared mass term belonging to the real scalar field we know to be the Higgs field. This makes the Higgs boson mass in the SM to be given by,

$$M_h = (2v^2\lambda) \quad . \tag{23}$$

To obtain masses for the gauge bosons we need to rotate the gauge fields to a basis where the mass terms are diagonal. First, it is straightforward to see that the electrically charged eigenstates are given by

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(A^{(1)}_\mu \pm iA^{(2)}_\mu) \quad , \tag{24}$$

meaning that the mass of the W bosons is,

$$M_W = \frac{1}{2}vg \quad . \tag{25}$$

The situation becomes a bit more complicated for the second term in (22) due to a mixing between A^3_μ and B_μ. In the gauge eigenbasis the mass terms read

$$\begin{pmatrix} A^3_\mu & B_\mu \end{pmatrix} \cdot \frac{1}{4}\nu^2 \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \cdot \begin{pmatrix} A^3_\mu \\ B_\mu \end{pmatrix} \quad , \tag{26}$$

which can be diagonalized to obtain

$$\begin{pmatrix} A_\mu & Z_\mu \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{2}v\sqrt{g^2+g'^2} \end{pmatrix} \begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix} \quad , \tag{27}$$

we identify the eigenvector associated to the eigenvalue 0 to the photon and the massive one, $M_Z = \frac{1}{2}v\sqrt{g^2+g'^2}$, to the Z boson. Such eigenvectors can be written as

$$A_\mu = \cos(\theta_\omega)B_\mu + \sin(\theta_\omega)A^3_\mu \quad , \tag{28}$$

$$Z_\mu = -\sin(\theta_\omega)B_\mu + \cos(\theta_\omega)A^3_\mu \quad , \tag{29}$$

where θ_w is the so called Weinberg mixing angle and is defined as

$$\cos(\theta_\omega) = \frac{g}{\sqrt{g^2+g'^2}} \quad , \tag{30}$$

thus clearly showing the massless photon along with a massive Z boson with mass $M_Z = \frac{1}{2}\nu\sqrt{g^2+g'^2}$. So we conclude our exploration of the electroweak sector with all the correct massive spectrum observed and its origin discussed.

2.4 The fermion sector on the Standard Model

In order to generate mass for the fermions we can have a closer look at the last line in eq. (5). If we replace the Higgs by the shift in Eq. (21) we get,

$$\begin{aligned}
\mathcal{L}_y = y^d \begin{pmatrix} \overline{u_L} & \overline{d_L} \end{pmatrix} d_R \begin{pmatrix} 0 \\ v+h(x) \end{pmatrix} + y^s \begin{pmatrix} \overline{c_L} & \overline{s_L} \end{pmatrix} s_R \begin{pmatrix} 0 \\ v+h(x) \end{pmatrix} \\
+ y^b \begin{pmatrix} \overline{t_L} & \overline{b_L} \end{pmatrix} b_R \begin{pmatrix} 0 \\ v+h(x) \end{pmatrix} + y^u \begin{pmatrix} \overline{u_L} & \overline{d_L} \end{pmatrix} d_R \begin{pmatrix} v+h(x) \\ 0 \end{pmatrix} \\
+y^c \begin{pmatrix} \overline{c_L} & \overline{s_L} \end{pmatrix} d_R \begin{pmatrix} v+h(x) \\ 0 \end{pmatrix} + y^t \begin{pmatrix} \overline{t_L} & \overline{b_L} \end{pmatrix} t_R \begin{pmatrix} v+h(x) \\ 0 \end{pmatrix} \\
+y^e \begin{pmatrix} \overline{\nu_{e_L}} & \overline{e_L} \end{pmatrix} e_R \begin{pmatrix} 0 \\ v+h(x) \end{pmatrix} + y^\mu \begin{pmatrix} \overline{\nu_{\mu_L}} & \overline{\mu_L} \end{pmatrix} \mu_R \begin{pmatrix} 0 \\ v+h(x) \end{pmatrix} \\
+y^\tau \begin{pmatrix} \overline{\nu_{\tau_L}} & \overline{\tau_L} \end{pmatrix} \tau_R \begin{pmatrix} 0 \\ v+h(x) \end{pmatrix} + h.c \quad ,
\end{aligned} \tag{31}$$

Further expansion of these terms would result in terms like *e.g.* in the electron's case,

$$\mathcal{L}_{y_e} = y^e \, v \, (\overline{e_L}e_R + \overline{e_R}e_L) + y^e h(x) \, (\overline{e_L}e_R + \overline{e_R}e_L) \quad , \tag{32}$$

where since the electron field is written as,

$$e = \begin{pmatrix} e_L \\ e_R \end{pmatrix} \quad , \tag{33}$$

meaning the first terms in (32) equate to electron mass terms as, $m_e\bar{e}e$ and the second terms represent interaction between the electron and the Higgs boson. This is how the Higgs mechanism generates the mass to all of the fermionic sector except neutrinos due to the SM not containing right handed neutrinos. The absence of neutrino masses contradicts experimental observations. Note also that the mass term for every lepton depends directly on each of the respective and different Yukawa term and that the SM is a theory where the Yukawa matrices are diagonal by nature not having inter generational terms.

3 B-L-SM Model

Having discussed the Standard Model, we are ready to look at what might lie beyond it. In this chapter we introduce the minimal $U(1)_{B-L}$ extension of the Standard Model named, B-L-SM. This is a model trough which we can explain neutrino mass generation via a simple see-saw mechanism as well as, by virtue of the model containing two new physical particle states, specifically a new Higgs like boson H' and a Z' gauge Boson, other small deviations in electro-weak measurements, namely the $(g-2)_\mu$ anomaly. This refers to the discrepancy between the measured anomalous magnetic moment of the muon.

Both the additional bosons are given mass mostly trough the spontaneous breaking of the $U(1)_{B-L}$ symmetry that gives it's name to the Model. This group originates from the promotion of a accidental symmetry present in the SM, the Baryon number (B) minus the Lepton number (L) to a fundamental Abelian symmetry group. This origin for the mass of the referenced bosons means model is already very heavily constricted due to long-standing direct searches in the Large Hadron Collide (LHC).

Trough this model we can also address the metastability of the electroweak (EW) vacuum in the SM trough the addition of the new scalar. Allowing for Higgs stabilization up to the plank scale with a the new Higgs starting from few hundred of GeVs.

Lastly, the presence of the complex SM-singlet χ interacting with a Higgs doublet typically enhances the strength of EW phase transition potentially converting it into a strong first-order one. Although not covered in this work this analysis is of utmost importance given that it could provide a way to detect new physics and confirm the model without the need for a larger particle collider. This could be pointed to as future work. Verify if the BLSM can be seen in LISA or stuff like that. This part needs new citations

One of the goals of this project was to investigate precisely the phenomenological status of the B-L-SM by confronting the new physics predictions with the LHC and electroweak precision data.

As a note this model is easily embedded into higher order symmetry groups like for example the $SO(10)$ or or E_6 , giving this model the ability to be used for the study of Grand Unified Theories.

The presence of three generations of right-handed neutrinos instead of a arbitrary number of neutrinos also ensures a framework free of anomalies with their mass scale developed once the $U(1)_{B-L}$ is broken by the VEV, x, of a complex SM-singlet scalar field, χ, simultaneously giving mass to the corresponding Z' boson and H'.

The cosmological consequences of the B-L-SM formulation are also worth mentioning. First, the presence of an extended neutrino sector implies the existence of a sterile state that can play the role of Dark Matter candidate. These can be completely sterile if stabilized with a $\mathbb{Z}_2$ parity symmetry. Note that the existence of sterile neutrinos can be used to explain the baryon asymmetry via the leptogenesis mechanism.

3.1 Formulating the model

Essentially, the minimal B-L-SM is a Beyond the Standard Model (BSM) framework containing three new ingredients:

- A new gauge interaction
- Three generations of right handed neutrinos

- A complex scalar SM-singlet.

The first one is well motivated in various GUT scenarios. However note that, if a family-universal symmetry such as $U(1)_{B-L}$ were introduced without changing the SM fermion content, chiral anomalies, which is a non conservative charged current on some channels, involving the $U(1)_{B-L}$ would be generated. These aren't completely undesired by themselves, since their result would be charge conjugation parity symmetry violation, or CP-symmetry violation, a observed missing feature of the SM, but this inclusion would result in far too much of these phenomena. (but how do I justify that there would be too much CP-violation?? is this even correct?)

Secondly, a new sector of additional three $U(1)_{B-L}$ charged Majorana neutrinos is essential for anomaly cancellation.

Finally, the SM-like Higgs doublet, H, does not carry neither baryon nor lepton number, this way it does not participate in the breaking of $U(1)_{B-L}$. It is then necessary to introduce a new scalar singlet, χ, solely charged under $U(1)_{B-L}$, whose VEV breaks the $B-L$ symmetry. It is also this breaking scale that generates masses for heavy neutrinos. As mentioned this breaking occurs before the EW breaking.

The particle content and related charges of the minimal $U(1)_{B-L}$ extension of the SM are shown in the table. Note these are similar to the SM as to be expected.

	q_L	u_R	d_R	l_L	e_R	ν_R	H	χ
$SU(3)_c$	3	3	3	1	1	1	1	1
$SU(2)_L$	2	1	1	2	1	1	2	1
$U(1)_Y$	$\frac{1}{6}$	$\frac{2}{3}$	-$\frac{1}{3}$	-$\frac{1}{2}$	-1	0	$\frac{1}{2}$	0
$U(1)_{B-L}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	-1	-1	-1	0	2

Table 3: Quantum fields and their respective quantum numbers in the minimal B-L-SM extension. The last two lines represent the weak and $B-L$ hypercharges

3.1.1 Scalar sector

Given the information we now posses we can begin examining the new Lagrangian terms. Starting by the scalar potential,

$$V(H,\chi) = \mu_1^2 H^\dagger H + \mu_2^2 \chi^* \chi + \lambda_1 (H^\dagger H)^2 + \lambda_2 (\chi^* \chi)^2 + \lambda_3 \chi^* \chi H^\dagger H \tag{34}$$

For the scalar Potential to be bounded from below (BFB). BFB conditions exist fundamentally to ensure there is a single global minima. Studying the potential 34 we deduce the conditions,

$$4\lambda_1\lambda_2 - \lambda_3^2 > 0 \quad , \quad \lambda_1, \lambda_2 > 0 \tag{35}$$

Should I explain what is bound from bellow? Basically we must ensure there is a single globla minima. Where the full components of the scalar fields are given by,

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} -i\,(\omega_1 - i\omega_2) \\ v + (h + iz) \end{pmatrix} \qquad \chi = \frac{1}{\sqrt{2}} \left(x + \left(h' + iz'\right)\right) \tag{36}$$

In these equations we can see h and h' representing the radial quantum fluctuations around the minimum of the potential. These will constitute the physical degrees of freedom associated to the H and H'. There are also four Goldstone directions denoted as ω_1, ω_2, z and z' which are absorbed into longitudinal modes of the $W^\pm$, Z and Z' gauge bosons once spontaneous symmetry breaking (SSB) takes place. After SSB the associated VEVs take the form,

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \langle \chi \rangle = \frac{x}{\sqrt{2}} \tag{37}$$

here, recall v and x are the associated VEVs to each field. From here we can solve the tadpole equations in relation to each of the VEVs as to ensure non-zero minima, we arrive at,

$$v^2 = \frac{-\lambda_2\mu_1^2 + \frac{\lambda_3}{2}\mu_2^2}{\lambda_1\lambda_2 - \frac{1}{4}\lambda_3^2} > 0 \qquad \text{and} \qquad x^2 = \frac{-\lambda_1\mu_2^2 + \frac{\lambda_3}{2}\mu_1^2}{\lambda_1\lambda_2 - \frac{1}{4}\lambda_3^2} > 0 \tag{38}$$

which, when simplified with the bound from bellow conditions yield a simpler set of equations,

$$\lambda_2\mu_1^2 < \tfrac{\lambda_3}{2}\mu_2^2 \qquad \text{and} \qquad \lambda_1\mu_2^2 < \tfrac{\lambda_3}{2}\mu_1^2 \tag{39}$$

Note that although λ_1 and λ_2 must be positive to ensure the correct potential shape, no such conditions exist for the sign of λ_3 , μ_1 and μ_2. However observing equation 39 we can infer that some combinations of signs are impossible, For our studies we decided to leave the

	$\mu_2^2 > 0$ $\mu_1^2 > 0$	$\mu_2^2 > 0$ $\mu_1^2 < 0$	$\mu_2^2 < 0$ $\mu_1^2 > 0$	$\mu_2^2 < 0$ $\mu_1^2 < 0$
$\lambda_3 < 0$	✗	✓	✓	✓
$\lambda_3 > 0$	✗	✗	✗	✓

Table 4: Possible Signs of the potential parameters in (34). While the ✓ symbol indicates the existence of solutions for tadpole conditions (39), the ✗ indicates unstable configurations.

sign of λ_3 positive, choosing a configuration where both μ parameters are negative. This doesn't directly translate to any real physical consequence. These conditions now established we proceed to investigate the physical states of B-L-SM scalar sector. By first, taking the Hessian matrix evaluated at the vacuum value,

$$\mathbf{M}^2 = \begin{pmatrix} 4\lambda_2 x^2 & \lambda_3 vx \\ \lambda_3 vx & 4\lambda_1 v^2 \end{pmatrix} , \tag{40}$$

Moving this matrix to it's physical mass eigenbase, we obtain the following eigenvalues,

$$m_{h_{1,2}}^2 = \lambda_1 v^2 + \lambda_2 x^2 \mp \sqrt{(\lambda_1 v^2 - \lambda_2 x^2)^2 + (\lambda_3 xv)^2}\,, \tag{41}$$

The physical basis vectors h_1 and h_2 can then be related to the original fields of gauge eigenbasis h and h' trough a simple rotation matrix:

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \mathbf{O} \begin{pmatrix} h \\ h' \end{pmatrix} . \tag{42}$$

The rotation matrix being written as,

$$\mathbf{O} = \begin{pmatrix} \cos\alpha_h & -\sin\alpha_h \\ \sin\alpha_h & \cos\alpha_h \end{pmatrix} . \tag{43}$$

Recall that due to the SSB order $x > v$. And here the mixing angle is represented simply by,

$$\tan 2\alpha_h = \frac{|\lambda_3|\, vv'}{\lambda_1 v^2 - \lambda_2 {v'}^2} \tag{44}$$

It is worth presenting the case of approximate decoupling where, $v/x \ll 1$. In this case scalar masses and mixing angle become particularly simple,

$$\sin\alpha_h \approx \frac{1}{2}\frac{\lambda_3}{\lambda_2}\frac{v}{x} \qquad m_{h_1}^2 \approx 2\lambda_1 v^2 \qquad m_{h_2}^2 \approx 2\lambda_2 x^2 \tag{45}$$

Given the mass scale of our results, these equations serve as a good approximation for most of the phenomenologically consistent points in our numerical analysis discussed below.

3.1.2 Gauge Sector

Moving onto the gauge boson and Higgs kinetic terms in the B-L-SM, consider the following portion of the Lagrangian,

$$\mathcal{L}_{U(1)'s} = |D_\mu H|^2 + |D_\mu \chi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{1}{2}\kappa F_{\mu\nu}F'^{\mu\nu} \tag{46}$$

where $F^{\mu\nu}$ and $F'^{\mu\nu}$ are the standard field strength tensors, respectively for the hypercharge $U(1)_Y$ and B minus L $U(1)_{B-L}$,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \text{and} \qquad F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu \,. \tag{47}$$

written in terms of the gauge fields A_μ and A'_μ, respectively. Given this is a model with two Unitary groups we must consider the possible mixing in between these groups. This shall be parametrized trough a κ factor.

The Abelian part of the covariant derivative in equation 46 is given by,

$$D_\mu \supset i g_1 Y A_\mu + i g'_1 Y_{\mathrm{B-L}} A'_\mu \,, \tag{48}$$

with g_1 and g'_1 being the $U(1)_Y$ and $U(1)_{B-L}$ the gauge couplings with the Y and $B-L$ charges are specified in Tab. 3. However it is convenient to rewrite the gauge kinetic terms in the canonical form, i.e.

$$F_{\mu\nu}F^{\mu\nu} + F'_{\mu\nu}F'^{\mu\nu} + 2\kappa F_{\mu\nu}F'^{\mu\nu} \to B_{\mu\nu}B^{\mu\nu} + B'_{\mu\nu}B'^{\mu\nu} \,. \tag{49}$$

A generic orthogonal transformation in the field space does not eliminate the kinetic mixing term. So, in order to satisfy Eq. (49) an extra non-orthogonal transformation should be imposed such that Eq. (49) is realized. Taking $\kappa = \sin\alpha$, a suitable redefinition of fields $\{A_\mu, A'_\mu\}$ into $\{B_\mu, B'_\mu\}$ that eliminates κ-term according to Eq. (46) can be cast as

$$\begin{pmatrix} A_\mu \\ A'_\mu \end{pmatrix} = \begin{pmatrix} 1 & -\tan\alpha \\ 0 & \sec\alpha \end{pmatrix} \begin{pmatrix} B_\mu \\ B'_\mu \end{pmatrix} , \tag{50}$$

Note there is a limit without kinitic mixing where $\alpha = 0$. Note that this transformation is generic and valid for any basis in the field space. The transformation (50) results in a modification of the covariant derivative that acquires two additional terms encoding the details of the kinetic mixing, i.e.

$$D_\mu \supset \partial_\mu + i\,(g_Y\, Y + g_B Y\, Y_{B-L})\, B_\mu + i\,(g_{B-L}\, Y_{B-L} + g_{YB}\, Y)\, B'_\mu\,, \tag{51}$$

where the gauge couplings take the form

$$\begin{cases} g_Y = g_1 \\ g_{B-L} = g'_1 \sec\alpha \\ g_{YB} = -g_1 \tan\alpha \\ g_{BY} = 0 \end{cases}, \tag{52}$$

which is the standard convention in the literature. The resulting mixing between the neutral gauge fields including Z' can be represented as follows

$$\begin{pmatrix} \gamma_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W & 0 \\ -\sin\theta_W \cos\theta'_W & \cos\theta_W \cos\theta'_W & \sin\theta'_W \\ \sin\theta_W \sin\theta'_W & -\cos\theta'_W \sin\theta'_W & \cos\theta'_W \end{pmatrix} \begin{pmatrix} B_\mu \\ A^3_\mu \\ B'_\mu \end{pmatrix} \tag{53}$$

where θ_W is the weak mixing angle and θ'_W is defined as

$$\sin(2\theta'_W) = \frac{2 g_{YB}\sqrt{g^2 + g_Y^2}}{\sqrt{(g_{YB}^2 + 16(\frac{x}{v})^2 g_{B-L}^2 - g^2 - g_Y^2)^2 + 4 g_{YB}^2 (g^2 + g_Y^2)}}\,, \tag{54}$$

in terms of g and g_Y being the $SU(2)_L$ and U_Y gauge couplings, respectively. In the physically relevant limit, $v/x \ll 1$, the above expression greatly simplifies leading to

$$\sin\theta'_W \approx \frac{1}{16}\frac{g_{YB}}{g_{B-L}}\left(\frac{v}{x}\right)^2 \sqrt{g^2 + g_Y^2}\,, \tag{55}$$

up to $(v/x)^3$ corrections. In the limit of no kinetic mixing, i.e. $g_{YB} \to 0$, there is no mixture of Z' and SM gauge bosons.

Note, the kinetic mixing parameter θ'_W has rather stringent constraints from Z pole experiments both at the Large Electron-Positron Collider (LEP) and the Stanford Linear Collider (SLC), restricting its value to be smaller than 10^{-3} approximately, which we set as an upper bound in our numerical analysis. Expanding the kinetic terms $|D_\mu H|^2 + |D_\mu \chi|^2$ around the vacuum one can extract the following mass matrix for vector bosons

$$m_V^2 = \frac{v^2}{4}\begin{pmatrix} g^2 & 0 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 & 0 \\ 0 & 0 & g^2 & -g g_Y & -g g_{YB} \\ 0 & 0 & -g g_Y & g_Y^2 & g_Y g_{YB} \\ 0 & 0 & -g g_{YB} & g_Y g_{YB} & g_{YB}^2 + 16\left(\frac{x}{v}\right)^2 g_{B-L}^2 \end{pmatrix} \tag{56}$$

whose eigenvalues read

$$m_A = 0\,, \qquad m_W = \tfrac{1}{2} v g \tag{57}$$

corresponding to physical photon and $W^{\pm}$ bosons as well as

$$m_{Z,Z'} = \sqrt{g^2 + g_Y^2} \cdot \frac{v}{2} \sqrt{\frac{1}{2}\left(\frac{g_{YB}^2 + 16(\frac{x}{v})^2 g_{\rm BL}^2}{g^2 + g_{\rm Y}^2} + 1\right) \mp \frac{g_{YB}}{\sin(2\theta'_W)\sqrt{g^2 + g_{\rm Y}^2}}}\,. \tag{58}$$

for two neutral massive vector bosons, with one of them, not necessarily the lightest, representing the SM-like Z boson. It follows from LEP and SLC constraints on θ'_W, that Eq. (55) also implies that either g_{YB} or the ratio $\frac{v}{x}$ are small. In this limit, Eq. (58) simplifies to

$$m_Z \approx \tfrac{1}{2} v \sqrt{g^2 + g_Y^2} \qquad \text{and} \qquad m_{Z'} \approx 2 g_{B-L} x\,, \tag{59}$$

where the $m_{Z'}$ depends only on the SM-singlet VEV ,x and on the $U(1)_{B-L}$ gauge coupling and will be attributed to a heavy Z' state, while the light Z-boson mass corresponds to its SM value.

3.2 The Yukawa sector

One of the key features of the B-L-SM is the presence of non-zero neutrino masses. In its minimal version, such masses are generated via a type-I seesaw mechanism. The Yukawa Lagrangian of the model reads

$$\mathcal{L}_f = -Y_u^{ij} \overline{q_{\rm Li}} u_{\rm Rj} \widetilde{H} - Y_d^{ij} \overline{q_{\rm Li}} d_{\rm Rj} H - Y_e^{ij} \overline{\ell_{\rm Li}} e_{\rm Rj} H - Y_\nu^{ij} \overline{\ell_{\rm Li}} \nu_{\rm Rj} \widetilde{H} - \frac{1}{2} Y_\chi^{ij} \overline{\nu_{\rm Ri}^c} \nu_{\rm Rj} \chi + \text{c.c.} \tag{60}$$

Notice that Majorana neutrino mass terms of the form $M \overline{\nu_R^c} \nu_R$ would explicitly violate the $U(1)_{B-L}$ symmetry and are therefore not present. In Eq. (60), Y_u, Y_d and Y_e are the 3×3 Yukawa matrices that reproduce the quark and charged lepton sector of the SM, while Y_ν and Y_χ are the new Yukawa matrices responsible for the generation of neutrino masses and mixing. In particular, one can write

$$\mathbf{m}_{\nu_l}^{Type-I} = \frac{1}{\sqrt{2}} \frac{v^2}{x} \mathbf{Y}_\nu^t \mathbf{Y}_\chi^{-1} \mathbf{Y}_\nu\,, \tag{61}$$

for light ν_l neutrino masses, whereas the heavy ν_h ones are given by

$$\mathbf{m}_{\nu_h}^{Type-I} \approx \frac{1}{\sqrt{2}} \mathbf{Y}_\chi x\,, \tag{62}$$

where we have assumed a flavour diagonal basis. Note that the smallness of light neutrino masses imply that either the x VEV is very large or (if we fix it to be at the $\mathcal{O}\,(TeV)$ scale and $\mathbf{Y}_\chi \sim \mathcal{O}\,(1)$) the corresponding Yukawa coupling should be tiny, $\mathbf{Y}_\nu < 10^{-6}$. It is clear that the low scale character of the type-I seesaw mechanism in the minimal B-L-SM is *faked* by small Yukawa couplings to the Higgs boson. A more elegant description was proposed in Ref. [**?**] where small SM neutrino masses naturally result from an inverse seesaw mechanism. In this work, however, we will not study the neutrino sector and thus, for an improved efficiency of our numerical analysis of Z' observables, it will be sufficient to fix the Yukawa couplings to $\mathbf{Y}_\chi = 10^{-1}$ and $\mathbf{Y}_\nu = 10^{-7}$ values such that the three lightest neutrinos lie in the sub-eV domain.

3.3 Phenomenological analysis

The numerical portion of the study that was developed during the course of this thesis had the goal to manufacture a wide scan across all the possible parameter space, testing all the limits of the model against modern constraints at the LHC.

For this a great deal of extensive set of calculations had to be made to reach a low scale model spectrum. This spectrum can be used to calculate two and three body decays including the Higgs bosons. Along with this we also need to calculate new physics contributions to the anomalous magnetic moment of lepton particles.

The 2-loop renormalization group equations as well as the one-loop finite corrections a la Bagger, Matchev, Pierce and Zhang are included. In addition the two-loop corrections to the neutral Higgs boson masses

The SPheno code is writen in Fortran 90 with an emphasis on easy generalisability.

During this thesis I developed a set of automatic tools that verify a large number of constraints.

4 3HDM

Let us now consider an extended version of the SM, with an enlarged Higgs sector that contains three generations of scalar-doublets. These Higgs will be named ϕ^i with $i = 1, 2, 3$.

5 Conclusions and Future Work

6 Appendix

6.1 Gamma Matrices

The γ matrices are defined as,

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} I \tag{63}$$

where,

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{64}$$

and if $\gamma_\mu = (\gamma^0, \gamma)$ then it is usual to require for the hermitian conjugate matrices,

$$\gamma^{0\dagger} = \gamma^0 \quad \text{and} \quad \gamma^\dagger = -\gamma \tag{65}$$

6.2 Lagrangian Dynamics

In Lagrangian dynamics we define the action S has,

$$\mathcal{S} = \int L \, dt = \int \mathcal{L}(\phi, \partial\phi) \, d^4x \tag{66}$$

where L is the Lagrangian, and the $\mathcal{L}$ is designated as the *Lagrangian density*, note these terms are usually used interchangeable. Here $\mathcal{L}$ is a function of the field ϕ and it's spatial derivatives.

The action S is constrained by the principle of least action, this requires the "path" taken by a field between an initial and final set of coordinates to leave the action invariant, this can be expressed by,

$$\partial\mathcal{S} = 0 \tag{67}$$

from here one can deduce the *Euler-Lagrange* equations,

$$\partial_\mu \left(\frac{\partial\mathcal{L}(\phi, \partial\phi)}{\partial(\partial_\mu)} \right) - \frac{\partial\mathcal{L}(\phi, \partial\phi)}{\partial\phi} = 0 \tag{68}$$