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Honestamente acho que isto vai ter que ser escrito antes da entrega

Honestly this will be written in english translated poorly from above :)

Resumo Esta parte esta em pt

Abstract

The Standard Model of particle physics has been for some time now recognized as a placeholder theory. Too many problems have been propping up over the years, such as the strong CP problem, neutrino oscillations, matter–antimatter asymmetry, the nature of dark matter and dark energy and most recently the existence of gravitational waves background ?. In response many theories have been proposed to deal with each one of these problems. However, it's important to realise that these are not independent problems and as such we must search for a way to tackle all of these. Here we propose a simple model and look into some (maybe all?) of these problems.

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2.1 Introduction

It is hard to question that the Standard Model (SM) describes successful approximate framework with whom to describe the phenomenology of Particle Physics up to the largest energy scales probed by collider measurements so far. Proposed in the sixties by Glashow, Salam and Weinberg it has been extensively tested and in contemporary direct searches for new physics or indirect probes via e.g. flavour anomalies and precise electroweak parameter measured in proton-electron collisions, has been showing an increasingly consistency with real results. Given this it is fair to say that the joint description of the electromagnetic and the weak interaction by a single theory certainly is one of major achievements of the physical science in this century.

However, the SM is far from perfect with several open questions that are yet to be fully understood, it is these questions that modern physicists use to justify the research made in the area of high energy physics and Phenomenology. As a example, one of such weaknesses is a missing explanation of tiny neutrino masses confirmed by flavour-oscillation experiments.

Given it's successes researchers have long been tempted to try to complete the SM somehow rather than fundamentally alter it. In fact several mechanisms have been proposed that build upon the SM rather than replace it. We'll investigate some of these in this project (BLSM 3HDM) .

2.2 Up to Gauge Theory?

It is well known that symmetry played a very important role in the development of modern physics ever since Emmy Noether's first theorem, which derives conserved quantities from symmetries. Precisely the theorem states if an action is invariant under some group of transformations (symmetry), then there exist one or more conserved quantities (constants of motion) which are associated to these transformations.

The question that the lead to the framework of the standard model was: upon imposing to a given Lagrangian the invariance under a certain symmetry, would it be possible to determine the form of the interaction among the particles? In other words, could symmetry also imply dynamics. This train of thought led to Quantum Electrodynamics (QED) the first successful prototype of quantum field theory.

In QED the existence and some of the properties of the gauge field (which we'll later identify as the photon) follow from a principle of invariance under local gauge transformations of the $U(1)$ group.

We can quote Salam and Ward:

"Our basic postulate is that it should be possible to generate strong, weak and electromagnetic interaction terms (with all their correct symmetry properties and also with clues regarding their relative strengths) by making local gauge transformations on the kinetic energy terms in the free Lagrangian for all particles."

We are glossing over a lot of complexity here, for the SM to be truly complete Noether's theorem alone wouldn't suffice and new concepts had to be introduced. In the case of weak interactions the presence of very heavy weak gauge bosons require the new concept of spontaneous breakdown of the gauge symmetry and the Higgs mechanism. Also, the concept of asymptotic freedom played a crucial role to describe perturbatively the strong interaction at short distances, making the strong gauge bosons trapped.

2.2.1 Symmetries

A symmetry can be very broadly defined as a property of a system that is preserved or remains unchanged. However for our interests we are going to look at field transformations that leave a Lagrangian system invariant. To exemplify this consider the following generic transformation of a field ϕ:

$$\phi \longrightarrow \phi' = \phi + \delta\phi \tag{1}$$

To be invariant means the langraingian will be unchanging, thus,

$$\mathcal{L}(\phi, \frac{d\phi}{dt}) = \mathcal{L}(\phi', \frac{d\phi'}{dt}) \tag{2}$$

Noether explored this relation, noting the Lagrangian would transform itself like,

$$\mathcal{L}(\phi, \partial\phi) \longrightarrow \mathcal{L}'(\phi + \delta\phi, \partial\phi + \delta\partial\phi) \tag{3}$$

leading to the form,

$$\mathcal{L}' = \mathcal{L}(\phi, \partial\phi) + \partial\phi\frac{\partial\mathcal{L}}{\partial\phi} + \delta\frac{d\phi}{dt}\frac{\partial\mathcal{L}}{\partial\frac{d\phi}{dt}} \tag{4}$$

where assuming the equations of motion are satisfied $\left(\frac{\partial\mathcal{L}}{\partial\phi} = \frac{d}{dt}\frac{\partial\mathcal{L}}{\frac{d\phi}{dt}}\right)$ we can reach a expression for the first order change in the Lagrangian given by,

$$\mathcal{L}' = \mathcal{L} + \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\frac{d\phi}{dt}}\delta\phi\right) \tag{5}$$

Here we define, j, as the Noether Current,

$$j = \frac{\partial\mathcal{L}}{\partial\frac{d\phi}{dt}}\delta\phi \tag{6}$$

This way we can define a transformation, $\delta\phi$ that leaves the action invariant, as,

$$\delta\mathcal{S} = 0 \implies \delta\mathcal{L} = 0 \implies \frac{\partial\mathcal{L}}{\partial\frac{d\phi}{dt}}\delta\phi = 0 \implies \frac{d\,j}{dt} = 0 \tag{7}$$

This way we can say that j is constant, this means there is a conversed quantity. A simple real example of this would be the case of a projectile in Lagrangian physics. The lagrangian would be,

$$\mathcal{L} = \frac{1}{2}m\left(\frac{d\,x^2}{dt} + \frac{d\,y^2}{dt}\right) - mgy \tag{8}$$

We can see this is unchanged by moving the x axis by a quantity ϵ, translated by the x' transformation,

$$x' \longrightarrow x + \epsilon \implies \frac{d\,x'}{dt} \longrightarrow \frac{d\,x}{dt} \tag{9}$$

by checking the current,

$$j = \frac{\partial\mathcal{L}}{\partial\frac{d\phi}{dt}}\delta\phi = m\frac{d\,x}{dt} \tag{10}$$

is also conserved. We know this to be form of the momentum in the x-direction which we expected to be conserved in this problem. A more laborious exercise could show that conservation of energy comes from the invariance of an action under translations in time. And even things like conservation of charge, which are a little more complicated, come from this symmetry principle.

2.2.2 In Minkowski space

In the normal 3+1 dimensional space the form of Noether's current changes to,

$$\partial_\mu j^\mu = 0 \implies \frac{\partial j^0}{\partial t} + \nabla \cdot \mathbf{j} = 0 \tag{11}$$

where we usually call j^0 the charge density while $\mathbf{j}$ is named the current density.

2.2.3 Classical Electrodynamics

maybe if there is space

2.2.4 Gauge Transformations

2.3 Higgs Mechanism

2.4 Composition of the Standard Model

The Standard Model is composed by force carriers, the weak gauge bosons W and Z, the photon, the electromagnetic interaction messenger and the strong force mediators, the gluons, as well by matter particles, the quarks and leptons. Being that the Higgs boson is responsible for the mass generation mechanism.

Fermions are organized in three generations. Furthermore, there are 6 different types of quarks, up and down for the first generation, charm and strange for the second as well as top and bottom for the third one. Similarly, there are 6 types of leptons, the charged ones, electron, muon and tau, and the associated neutrinos, respectively represented by (u, d, c, s, t, b) while leptons as $(e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau)$

So far we have described the physical states that are often denoted as the building blocks of nature. However we have not yet explained how such states have acquired their masses and gauge quantum numbers, such as colour and electric charge. To see this, we start by noting the the SM is a gauge theory based on the group.

$$SU(3)_c \times SU(2)_L \times U(1)_Y \quad . \tag{12}$$

Fermions are half integer spin particles most of which have electrical charge (except the neutrinos). While quarks interact via the weak, electromagnetic and strong forces, the charged leptons only feel the electromagnetic and weak forces and the neutrinos are solely weakly interacting.

A physical fermion is composed of a left-handed and a right-handed part. While the former transform as $SU(2)_L$ doublets and can be written as,

$$L^i = \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix} \quad \text{and} \quad Q^i = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \quad , \tag{13}$$

2.5 Quantum fields

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3 B-L-SM Model

Having discussed the Standard Model, we are ready to look at what might lie beyond it. In this chapter we introduce the minimal $U(1)_{B-L}$ extension of the Standard Model named, B-L-SM. This is a model trough which we can explain neutrino mass generation via a simple see-saw mechanism as well as, by virtue of the model containing two new physical particle states, specifically a new Higgs like boson H' and a Z' gauge Boson, other small deviations in electro-weak measurements, namely the $(g-2)_\mu$ anomaly. This refers to the discrepancy between the measured anomalous magnetic moment of the muon.

Both the additional bosons are given mass mostly trough the spontaneous breaking of the $U(1)_{B-L}$ symmetry that gives it's name to the Model. This group originates from the promotion of a accidental symmetry present in the SM, the Baryon number (B) minus the Lepton number (L) to a fundamental Abelian symmetry group. This origin for the mass of the referenced bosons means model is already very heavily constricted due to long-standing direct searches in the Large Hadron Collide (LHC).

Trough this model we can also address the metastability of the electroweak (EW) vacuum in the SM trough the addition of the new scalar. Allowing for Higgs stabilization up to the plank scale with a the new Higgs starting from few hundred of GeVs.

Lastly, the presence of the complex SM-singlet χ interacting with a Higgs doublet typically enhances the strength of EW phase transition potentially converting it into a strong first-order one. Although not covered in this work this analysis is of utmost importance given that it could provide a way to detect new physics and confirm the model without the need for a larger particle collider. This could be pointed to as future work. Verify if the BLSM can be seen in LISA or stuff like that. This part needs new citations

One of the goals of this project was to investigate precisely the phenomenological status of the B-L-SM by confronting the new physics predictions with the LHC and electroweak precision data.

As a note this model is easily embedded into higher order symmetry groups like for example the $SO(10)$ or or E_6 , giving this model the ability to be used for the study of Grand Unified Theories.

The presence of three generations of right-handed neutrinos instead of a arbitrary number of neutrinos also ensures a framework free of anomalies with their mass scale developed once the $U(1)_{B-L}$ is broken by the VEV, x, of a complex SM-singlet scalar field, χ, simultaneously giving mass to the corresponding Z' boson and H'.

The cosmological consequences of the B-L-SM formulation are also worth mentioning. First, the presence of an extended neutrino sector implies the existence of a sterile state that can play the role of Dark Matter candidate. These can be completely sterile if stabilized with a $\mathbb{Z}_2$ parity symmetry. Note that the existence of sterile neutrinos can be used to explain the baryon asymmetry via the leptogenesis mechanism.

3.1 Formulating the model

Essentially, the minimal B-L-SM is a Beyond the Standard Model (BSM) framework containing three new ingredients:

- A new gauge interaction
- Three generations of right handed neutrinos

- A complex scalar SM-singlet.

The first one is well motivated in various GUT scenarios. However note that, if a family-universal symmetry such as $U(1)_{B-L}$ were introduced without changing the SM fermion content, chiral anomalies, which is a non conservative charged current on some channels, involving the $U(1)_{B-L}$ would be generated. These aren't completely undesired by themselves, since their result would be charge conjugation parity symmetry violation, or CP-symmetry violation, a observed missing feature of the SM, but this inclusion would result in far too much of these phenomena. (but how do I justify that there would be too much CP-violation?? is this even correct?)

Secondly, a new sector of additional three $U(1)_{B-L}$ charged Majorana neutrinos is essential for anomaly cancellation.

Finally, the SM-like Higgs doublet, H, does not carry neither baryon nor lepton number, this way it does not participate in the breaking of $U(1)_{B-L}$. It is then necessary to introduce a new scalar singlet, χ, solely charged under $U(1)_{B-L}$, whose VEV breaks the $B-L$ symmetry. It is also this breaking scale that generates masses for heavy neutrinos. As mentioned this breaking occurs before the EW breaking.

The particle content and related charges of the minimal $U(1)_{B-L}$ extension of the SM are shown in the table. Note these are similar to the SM as to be expected.

	q_L	u_R	d_R	l_L	e_R	ν_R	H	χ
$SU(3)_c$	3	3	3	1	1	1	1	1
$SU(2)_L$	2	1	1	2	1	1	2	1
$U(1)_Y$	$\frac{1}{6}$	$\frac{2}{3}$	-$\frac{1}{3}$	-$\frac{1}{2}$	-1	0	$\frac{1}{2}$	0
$U(1)_{B-L}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	-1	-1	-1	0	2

Table 1: Quantum fields and their respective quantum numbers in the minimal B-L-SM extension. The last two lines represent the weak and $B-L$ hypercharges

3.1.1 Scalar sector

Given the information we now posses we can begin examining the new Lagrangian terms. Starting by the scalar potential,

$$V(H,\chi) = \mu_1^2 H^\dagger H + \mu_2^2 \chi^* \chi + \lambda_1 (H^\dagger H)^2 + \lambda_2 (\chi^* \chi)^2 + \lambda_3 \chi^* \chi H^\dagger H \tag{14}$$

For the scalar Potential to be bounded from below (BFB). BFB conditions exist fundamentally to ensure there is a single global minima. Studying the potential 14 we deduce the conditions,

$$4\lambda_1\lambda_2 - \lambda_3^2 > 0 \quad , \quad \lambda_1, \lambda_2 > 0 \tag{15}$$

Should I explain what is bound from bellow? Basically we must ensure there is a single globla minima. Where the full components of the scalar fields are given by,

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} -i\,(\omega_1 - i\omega_2) \\ v + (h + iz) \end{pmatrix} \qquad \chi = \frac{1}{\sqrt{2}} \left(x + \left(h' + iz'\right)\right) \tag{16}$$

In these equations we can see h and h' representing the radial quantum fluctuations around the minimum of the potential. These will constitute the physical degrees of freedom associated to the H and H'. There are also four Goldstone directions denoted as ω_1, ω_2, z and z' which are absorbed into longitudinal modes of the $W^{\pm}$, Z and Z' gauge bosons once spontaneous symmetry breaking (SSB) takes place. After SSB the associated VEVs take the form,

$$\langle H\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} \qquad \langle\chi\rangle = \frac{x}{\sqrt{2}} \tag{17}$$

here, recall v and x are the associated VEVs to each field. From here we can solve the tadpole equations in relation to each of the VEVs as to ensure non-zero minima, we arrive at,

$$v^2 = \frac{-\lambda_2\mu_1^2+\frac{\lambda_3}{2}\mu_2^2}{\lambda_1\lambda_2-\frac{1}{4}\lambda_3^2} > 0 \qquad \text{and} \qquad x^2 = \frac{-\lambda_1\mu_2^2+\frac{\lambda_3}{2}\mu_1^2}{\lambda_1\lambda_2-\frac{1}{4}\lambda_3^2} > 0 \tag{18}$$

which, when simplified with the bound from bellow conditions yield a simpler set of equations,

$$\lambda_2\mu_1^2 < \tfrac{\lambda_3}{2}\mu_2^2 \qquad \text{and} \qquad \lambda_1\mu_2^2 < \tfrac{\lambda_3}{2}\mu_1^2 \tag{19}$$

Note that although λ_1 and λ_2 must be positive to ensure the correct potential shape, no such conditions exist for the sign of λ_3 , μ_1 and μ_2. However observing equation 19 we can infer that some combinations of signs are impossible, For our studies we decided to leave the

	$\mu_2^2 > 0$ $\mu_1^2 > 0$	$\mu_2^2 > 0$ $\mu_1^2 < 0$	$\mu_2^2 < 0$ $\mu_1^2 > 0$	$\mu_2^2 < 0$ $\mu_1^2 < 0$
$\lambda_3 < 0$	✗	✓	✓	✓
$\lambda_3 > 0$	✗	✗	✗	✓

Table 2: Possible Signs of the potential parameters in (14). While the ✓ symbol indicates the existence of solutions for tadpole conditions (19), the ✗ indicates unstable configurations.

sign of λ_3 positive, choosing a configuration where both μ parameters are negative. This doesn't directly translate to any real physical consequence. These conditions now established we proceed to investigate the physical states of B-L-SM scalar sector. By first, taking the Hessian matrix evaluated at the vacuum value,

$$\mathbf{M}^2 = \begin{pmatrix}4\lambda_2 x^2 & \lambda_3 vx\\ \lambda_3 vx & 4\lambda_1 v^2\end{pmatrix}, \tag{20}$$

Moving this matrix to it's physical mass eigenbase, we obtain the following eigenvalues,

$$m_{h_{1,2}}^2 = \lambda_1 v^2 + \lambda_2 x^2 \mp \sqrt{(\lambda_1 v^2 - \lambda_2 x^2)^2 + (\lambda_3 x v)^2}\,, \tag{21}$$

The physical basis vectors h_1 and h_2 can then be related to the original fields of gauge eigenbasis h and h' trough a simple rotation matrix:

$$\begin{pmatrix}h_1\\ h_2\end{pmatrix} = \mathbf{O}\begin{pmatrix}h\\ h'\end{pmatrix}. \tag{22}$$

The rotation matrix being written as,

$$\mathbf{O} = \begin{pmatrix} \cos\alpha_h & -\sin\alpha_h \\ \sin\alpha_h & \cos\alpha_h \end{pmatrix} . \tag{23}$$

Recall that due to the SSB order $x > v$. And here the mixing angle is represented simply by,

$$\tan 2\alpha_h = \frac{|\lambda_3|\, vv'}{\lambda_1 v^2 - \lambda_2 {v'}^2} \tag{24}$$

It is worth presenting the case of approximate decoupling where, $v/x \ll 1$. In this case scalar masses and mixing angle become particularly simple,

$$\sin\alpha_h \approx \frac{1}{2}\frac{\lambda_3}{\lambda_2}\frac{v}{x} \qquad m_{h_1}^2 \approx 2\lambda_1 v^2 \qquad m_{h_2}^2 \approx 2\lambda_2 x^2 \tag{25}$$

Given the mass scale of our results, these equations serve as a good approximation for most of the phenomenologically consistent points in our numerical analysis discussed below.

3.1.2 Gauge Sector

Moving onto the gauge boson and Higgs kinetic terms in the B-L-SM, consider the following portion of the Lagrangian,

$$\mathcal{L}_{U(1)'s} = |D_\mu H|^2 + |D_\mu \chi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{1}{2}\kappa F_{\mu\nu}F'^{\mu\nu} \tag{26}$$

where $F^{\mu\nu}$ and $F'^{\mu\nu}$ are the standard field strength tensors, respectively for the hypercharge $U(1)_Y$ and B minus L $U(1)_{B-L}$,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \text{and} \qquad F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu . \tag{27}$$

written in terms of the gauge fields A_μ and A'_μ, respectively. Given this is a model with two Unitary groups we must consider the possible mixing in between these groups. This shall be parametrized trough a κ factor.

The Abelian part of the covariant derivative in equation 26 is given by,

$$D_\mu \supset i g_1 Y A_\mu + i g'_1 Y_{\mathrm{B-L}} A'_\mu , \tag{28}$$

with g_1 and g'_1 being the $U(1)_Y$ and $U(1)_{B-L}$ the gauge couplings with the Y and $B-L$ charges are specified in Tab. 1. However it is convenient to rewrite the gauge kinetic terms in the canonical form, i.e.

$$F_{\mu\nu}F^{\mu\nu} + F'_{\mu\nu}F'^{\mu\nu} + 2\kappa F_{\mu\nu}F'^{\mu\nu} \to B_{\mu\nu}B^{\mu\nu} + B'_{\mu\nu}B'^{\mu\nu} . \tag{29}$$

A generic orthogonal transformation in the field space does not eliminate the kinetic mixing term. So, in order to satisfy Eq. (29) an extra non-orthogonal transformation should be imposed such that Eq. (29) is realized. Taking $\kappa = \sin\alpha$, a suitable redefinition of fields $\{A_\mu, A'_\mu\}$ into $\{B_\mu, B'_\mu\}$ that eliminates κ-term according to Eq. (26) can be cast as

$$\begin{pmatrix} A_\mu \\ A'_\mu \end{pmatrix} = \begin{pmatrix} 1 & -\tan\alpha \\ 0 & \sec\alpha \end{pmatrix} \begin{pmatrix} B_\mu \\ B'_\mu \end{pmatrix} , \tag{30}$$

Note there is a limit without kinitic mixing where $\alpha = 0$. Note that this transformation is generic and valid for any basis in the field space. The transformation (30) results in a modification of the covariant derivative that acquires two additional terms encoding the details of the kinetic mixing, i.e.

$$D_\mu \supset \partial_\mu + i\,(g_Y\,Y + g_B Y\,Y_{B-L})\,B_\mu + i\,(g_{B-L}\,Y_{B-L} + g_{YB}\,Y)\,B'_\mu\,, \tag{31}$$

where the gauge couplings take the form

$$\begin{cases} g_Y = g_1 \\ g_{B-L} = g'_1 \sec\alpha \\ g_{YB} = -g_1 \tan\alpha \\ g_{BY} = 0 \end{cases}, \tag{32}$$

which is the standard convention in the literature. The resulting mixing between the neutral gauge fields including Z' can be represented as follows

$$\begin{pmatrix} \gamma_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W & 0 \\ -\sin\theta_W\cos\theta'_W & \cos\theta_W\cos\theta'_W & \sin\theta'_W \\ \sin\theta_W\sin\theta'_W & -\cos\theta'_W\sin\theta'_W & \cos\theta'_W \end{pmatrix} \begin{pmatrix} B_\mu \\ A^3_\mu \\ B'_\mu \end{pmatrix} \tag{33}$$

where θ_W is the weak mixing angle and θ'_W is defined as

$$\sin(2\theta'_W) = \frac{2g_{YB}\sqrt{g^2+g_Y^2}}{\sqrt{(g_{YB}^2 + 16(\frac{x}{v})^2 g_{B-L}^2 - g^2 - g_Y^2)^2 + 4g_{YB}^2(g^2+g_Y^2)}}\,, \tag{34}$$

in terms of g and g_Y being the $SU(2)_L$ and U_Y gauge couplings, respectively. In the physically relevant limit, $v/x \ll 1$, the above expression greatly simplifies leading to

$$\sin\theta'_W \approx \frac{1}{16}\frac{g_{YB}}{g_{B-L}}\left(\frac{v}{x}\right)^2\sqrt{g^2+g_Y^2}\,, \tag{35}$$

up to $(v/x)^3$ corrections. In the limit of no kinetic mixing, i.e. $g_{YB} \to 0$, there is no mixture of Z' and SM gauge bosons.

Note, the kinetic mixing parameter θ'_W has rather stringent constraints from Z pole experiments both at the Large Electron-Positron Collider (LEP) and the Stanford Linear Collider (SLC), restricting its value to be smaller than 10^{-3} approximately, which we set as an upper bound in our numerical analysis. Expanding the kinetic terms $|D_\mu H|^2 + |D_\mu \chi|^2$ around the vacuum one can extract the following mass matrix for vector bosons

$$m_V^2 = \frac{v^2}{4}\begin{pmatrix} g^2 & 0 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 & 0 \\ 0 & 0 & g^2 & -gg_Y & -gg_{YB} \\ 0 & 0 & -gg_Y & g_Y^2 & g_Y g_{YB} \\ 0 & 0 & -gg_{YB} & g_Y g_{YB} & g_{YB}^2 + 16\left(\frac{x}{v}\right)^2 g_{B-L}^2 \end{pmatrix} \tag{36}$$

whose eigenvalues read

$$m_A = 0\,, \qquad m_W = \tfrac{1}{2} v g \tag{37}$$

corresponding to physical photon and $W^\pm$ bosons as well as

$$m_{Z,Z'} = \sqrt{g^2 + g_Y^2} \cdot \frac{v}{2} \sqrt{\frac{1}{2}\left(\frac{g_{YB}^2 + 16(\frac{x}{v})^2 g_{\rm BL}^2}{g^2 + g_{\rm Y}^2} + 1\right) \mp \frac{g_{YB}}{\sin(2\theta_W') \sqrt{g^2 + g_{\rm Y}^2}}}\,. \tag{38}$$

for two neutral massive vector bosons, with one of them, not necessarily the lightest, representing the SM-like Z boson. It follows from LEP and SLC constraints on θ_W', that Eq. (35) also implies that either g_{YB} or the ratio $\frac{v}{x}$ are small. In this limit, Eq. (38) simplifies to

$$m_Z \approx \tfrac{1}{2} v \sqrt{g^2 + g_Y^2} \qquad \text{and} \qquad m_{Z'} \approx 2 g_{B-L} x\,, \tag{39}$$

where the $m_{Z'}$ depends only on the SM-singlet VEV ,x and on the $U(1)_{B-L}$ gauge coupling and will be attributed to a heavy Z' state, while the light Z-boson mass corresponds to its SM value.

3.2 The Yukawa sector

One of the key features of the B-L-SM is the presence of non-zero neutrino masses. In its minimal version, such masses are generated via a type-I seesaw mechanism. The Yukawa Lagrangian of the model reads

$$\mathcal{L}_f = -Y_u^{ij} \overline{q_{\rm Li}} u_{\rm Rj} \widetilde{H} - Y_d^{ij} \overline{q_{\rm Li}} d_{\rm Rj} H - Y_e^{ij} \overline{\ell_{\rm Li}} e_{\rm Rj} H - Y_\nu^{ij} \overline{\ell_{\rm Li}} \nu_{\rm Rj} \widetilde{H} - \frac{1}{2} Y_\chi^{ij} \overline{\nu_{\rm Ri}^c} \nu_{\rm Rj} \chi + \text{c.c.} \tag{40}$$

Notice that Majorana neutrino mass terms of the form $M \overline{\nu_R^c} \nu_R$ would explicitly violate the $U(1)_{B-L}$ symmetry and are therefore not present. In Eq. (40), Y_u, Y_d and Y_e are the 3×3 Yukawa matrices that reproduce the quark and charged lepton sector of the SM, while Y_ν and Y_χ are the new Yukawa matrices responsible for the generation of neutrino masses and mixing. In particular, one can write

$$\mathbf{m}_{\nu_l}^{Type-I} = \frac{1}{\sqrt{2}} \frac{v^2}{x} \mathbf{Y}_\nu^t \mathbf{Y}_\chi^{-1} \mathbf{Y}_\nu\,, \tag{41}$$

for light ν_l neutrino masses, whereas the heavy ν_h ones are given by

$$\mathbf{m}_{\nu_h}^{Type-I} \approx \frac{1}{\sqrt{2}} \mathbf{Y}_\chi x\,, \tag{42}$$

where we have assumed a flavour diagonal basis. Note that the smallness of light neutrino masses imply that either the x VEV is very large or (if we fix it to be at the $\mathcal{O}\,(TeV)$ scale and $\mathbf{Y}_\chi \sim \mathcal{O}\,(1)$) the corresponding Yukawa coupling should be tiny, $\mathbf{Y}_\nu < 10^{-6}$. It is clear that the low scale character of the type-I seesaw mechanism in the minimal B-L-SM is *faked* by small Yukawa couplings to the Higgs boson. A more elegant description was proposed in Ref. [**?**] where small SM neutrino masses naturally result from an inverse seesaw mechanism. In this work, however, we will not study the neutrino sector and thus, for an improved efficiency of our numerical analysis of Z' observables, it will be sufficient to fix the Yukawa couplings to $\mathbf{Y}_\chi = 10^{-1}$ and $\mathbf{Y}_\nu = 10^{-7}$ values such that the three lightest neutrinos lie in the sub-eV domain.

3.3 Phenomenological analysis

The study of this model

4 3HDM

5 Conclusions and Future Work

6 Appendix

6.1 Gamma Matrices

The γ matrices are defined as,

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} I \tag{43}$$

where,

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{44}$$

and if $\gamma_\mu = (\gamma^0, \gamma)$ then it is usual to require for the hermitian conjugate matrices,

$$\gamma^{0\dagger} = \gamma^0 \quad \text{and} \quad \gamma^\dagger = -\gamma \tag{45}$$

6.2 Lagrangian Dynamics

In Lagrangian dynamics we define the action S has,

$$\mathcal{S} = \int L\, dt = \int \mathcal{L}(\phi, \partial\phi)\, d^4x \tag{46}$$

where L is the Lagrangian, and the $\mathcal{L}$ is designated as the *Lagrangian density*, note these terms are usually used interchangeable. Here $\mathcal{L}$ is a function of the field ϕ and it's spatial derivatives.

The action S is constrained by the principle of least action, this requires the "path" taken by a field between an initial and final set of coordinates to leave the action invariant, this can be expressed by,

$$\partial\mathcal{S} = 0 \tag{47}$$

from here one can deduce the *Euler-Lagrange* equations,

$$\partial_\mu \left(\frac{\partial \mathcal{L}(\phi, \partial\phi)}{\partial(\partial_\mu)} \right) - \frac{\partial \mathcal{L}(\phi, \partial\phi)}{\partial\phi} = 0 \tag{48}$$