

# Bits, Bytes, and Integers

Introduction to Computer Systems  
2<sup>nd</sup> Lecture, Sep 16, 2021

## Instructors

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# Today: Bits, Bytes, and Integers

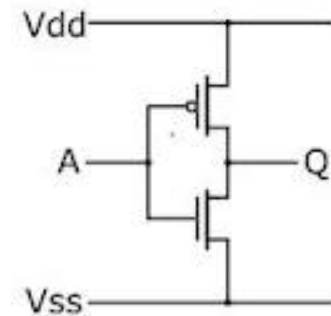
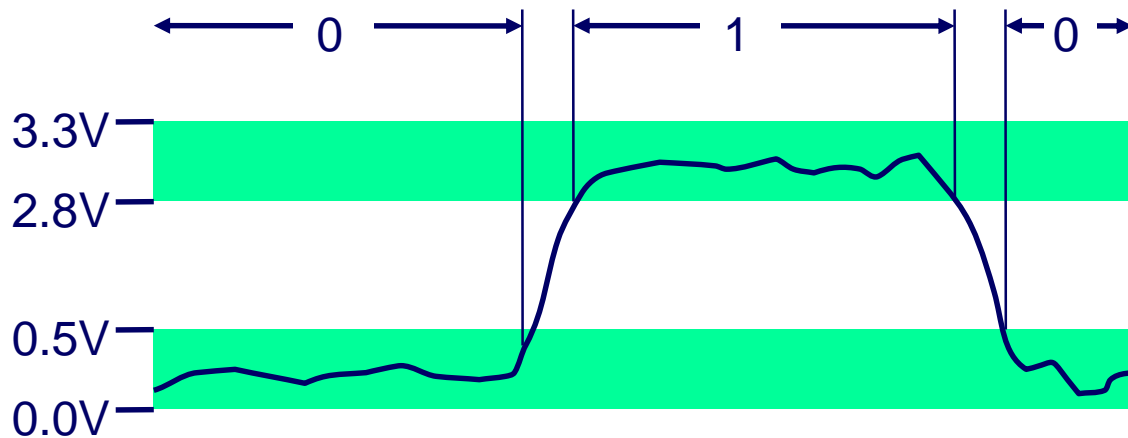
- **Representing information as bits**
- **Bit-level manipulations**
- **Integers**
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- **Representations in memory, pointers, strings**

# Binary Representations

## ■ Base 2 Number Representation

- Represent  $15213_{10}$  as  $11101101101101_2$
- Represent  $1.20_{10}$  as  $1.0011001100110011[0011]..._2$
- Represent  $1.5213 \times 10^4$  as  $1.1101101101101_2 \times 2^{13}$

## ■ Why Computers Use Binary?



**Binary is the most practical system to use!**

# Encoding Byte Values

## ■ Byte = 8 bits

- Binary  $00000000_2$  to  $11111111_2$
- Decimal:  $0_{10}$  to  $255_{10}$
- Hexadecimal  $00_{16}$  to  $FF_{16}$ 
  - Base 16 number representation
  - Use characters '0' to '9' and 'A' to 'F'
  - Write  $FA1D37B_{16}$  in C as
    - `0xFA1D37B`
    - `0xfa1d37b`

**Why 8 bit?**

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

# Data Representations

C Data Type	Typical 32-bit	Intel IA32	x86-64
char	1	1	1
short	2	2	2
int	4	4	4
long	4	4	8
float	4	4	4
double	8	8	8
long double	-	-	10/16
pointer	4	4	8

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# Boolean Algebra

## ■ Developed by George Boole in 19th Century

- Algebraic representation of logic
  - Encode “True” as 1 and “False” as 0

### And

- $A \& B = 1$  when both  $A=1$  and  $B=1$

$\&$	0	1
0	0	0
1	0	1

### Or

- $A | B = 1$  when either  $A=1$  or  $B=1$

$ $	0	1
0	0	1
1	1	1

### Not

- $\sim A = 1$  when  $A=0$

$\sim$	
0	1
1	0

### Exclusive-Or (Xor)

- $A \wedge B = 1$  when either  $A=1$  or  $B=1$ , but not both

$\wedge$	0	1
0	0	1
1	1	0

# General Boolean Algebras

## ■ Operate on Bit Vectors

- Operations applied bitwise

01101001	01101001	01101001	
& 01010101	01010101	^ 01010101	~ 01010101
<u>          </u>	<u>          </u>	<u>          </u>	<u>          </u>
01000001	01111101	00111100	10101010

## ■ All of the Properties of Boolean Algebra Apply



# Example: Representing & Manipulating Sets

## ■ Representation

- Width  $w$  bit vector represents subsets of  $\{0, \dots, w-1\}$
- $a_j = 1$  if  $j \in A$

▪ 01101001       $\{0, 3, 5, 6\}$

▪ 76543210

▪ 01010101       $\{0, 2, 4, 6\}$

▪ 76543210

## ■ Operations

- |                            |          |                        |
|----------------------------|----------|------------------------|
| ▪ &   Intersection         | 01000001 | $\{0, 6\}$             |
| ▪     Union                | 01111101 | $\{0, 2, 3, 4, 5, 6\}$ |
| ▪ ^   Symmetric difference | 00111100 | $\{2, 3, 4, 5\}$       |
| ▪ ~   Complement           | 10101010 | $\{1, 3, 5, 7\}$       |

# Bit-Level Operations in C

## ■ Operations `&`, `|`, `~`, `^` Available in C

- Apply to any “integral” data type
  - long, int, short, char, unsigned
- View arguments as bit vectors
- Arguments applied bit-wise

## ■ Examples (Char data type)

- $\sim 0x41 \rightarrow 0xBE$ 
  - $\sim 01000001_2 \rightarrow 10111110_2$
- $\sim 0x00 \rightarrow 0xFF$ 
  - $\sim 00000000_2 \rightarrow 11111111_2$
- $0x69 \& 0x55 \rightarrow 0x41$ 
  - $01101001_2 \& 01010101_2 \rightarrow 01000001_2$
- $0x69 | 0x55 \rightarrow 0x7D$ 
  - $01101001_2 | 01010101_2 \rightarrow 01111101_2$

# Contrast: Logic Operations in C

## ■ Contrast to Logical Operators

- `&&`, `||`, `!`
  - View 0 as “False”
  - Anything nonzero as “True”
  - Always return 0 or 1
  - **Early termination**

## ■ Examples (char data type)

- `!0x41 → 0x00`
- `!0x00 → 0x01`
- `!!0x41 → 0x01`
  
- `0x69 && 0x55 → 0x01`
- `0x69 || 0x55 → 0x01`
- `p && *p` (avoids null pointer access)

**Watch out for `&&` vs. `&`  
(and `||` vs. `|`)...  
one of the more common  
oopsies in  
C programming**

# Shift Operations

## ■ Left Shift: $x \ll y$

- Shift bit-vector  $x$  left  $y$  positions
  - Throw away extra bits on left
  - Fill with 0's on right

## ■ Right Shift: $x \gg y$

- Shift bit-vector  $x$  right  $y$  positions
  - Throw away extra bits on right
- Logical shift
  - Fill with 0's on left
- Arithmetic shift
  - Replicate most significant bit on left

## ■ Undefined Behavior

- Shift amount  $< 0$  or  $\geq$  word size

Argument $x$	01100010
$\ll 3$	00010000
Log. $\gg 2$	00011000
Arith. $\gg 2$	00011000

Argument $x$	10100010
$\ll 3$	00010000
Log. $\gg 2$	00101000
Arith. $\gg 2$	11101000

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# Encoding Integers

## Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

## Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

```
short int x = 15213;
short int y = -15213;
```

Sign  
Bit



### ■ C short 2 bytes long

	Decimal	Hex	Binary
<b>x</b>	15213	3B 6D	00111011 01101101
<b>y</b>	-15213	C4 93	11000100 10010011

### ■ Sign Bit

- For 2's complement, most significant bit indicates sign
  - 0 for nonnegative
  - 1 for negative

# Two-complement: Simple Example

$$\begin{array}{rcccccc}
 & & -16 & 8 & 4 & 2 & 1 \\
 10 = & 0 & 1 & 0 & 1 & 0 & \\
 & & & & & & 8+2 = 10
 \end{array}$$

$$\begin{array}{rcccccc}
 & & -16 & 8 & 4 & 2 & 1 \\
 -10 = & 1 & 0 & 1 & 1 & 0 & \\
 & & & & & & -16+4+2 = -10
 \end{array}$$

# Encoding Example (Cont.)

$\mathbf{x} =$             15213: 00111011 01101101  
 $\mathbf{y} =$             -15213: 11000100 10010011

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
<b>Sum</b>	<b>15213</b>		<b>-15213</b>	

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$



# Numeric Ranges

## ■ Unsigned Values

- $UMin = 0$   
000...0
- $UMax = 2^w - 1$   
111...1

## ■ Two's Complement Values

- $TMin = -2^{w-1}$   
100...0
- $TMax = 2^{w-1} - 1$   
011...1

## ■ Other Values

- Minus 1  
111...1

### Values for $W = 16$

	Decimal	Hex	Binary
<b>UMax</b>	<b>65535</b>	<b>FF FF</b>	<b>11111111 11111111</b>
<b>TMax</b>	<b>32767</b>	<b>7F FF</b>	<b>01111111 11111111</b>
<b>TMin</b>	<b>-32768</b>	<b>80 00</b>	<b>10000000 00000000</b>
<b>-1</b>	<b>-1</b>	<b>FF FF</b>	<b>11111111 11111111</b>
<b>0</b>	<b>0</b>	<b>00 00</b>	<b>00000000 00000000</b>

# Values for Different Word Sizes

	W			
	8	16	32	64
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

## ■ Observations

- $|TMin| = TMax + 1$ 
  - Asymmetric range
- $UMax = 2 * TMax + 1$

## ■ C Programming

- `#include <limits.h>`
- Declares constants, e.g.,
  - `ULONG_MAX`
  - `LONG_MAX`
  - `LONG_MIN`
- Values platform specific

# Unsigned & Signed Numeric Values

$X$	$B2U(X)$	$B2T(X)$
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	-8
1001	9	-7
1010	10	-6
1011	11	-5
1100	12	-4
1101	13	-3
1110	14	-2
1111	15	-1

## ■ Equivalence

- Same encodings for nonnegative values

## ■ Uniqueness

- Every bit pattern represents unique integer value
- Each representable integer has unique bit encoding

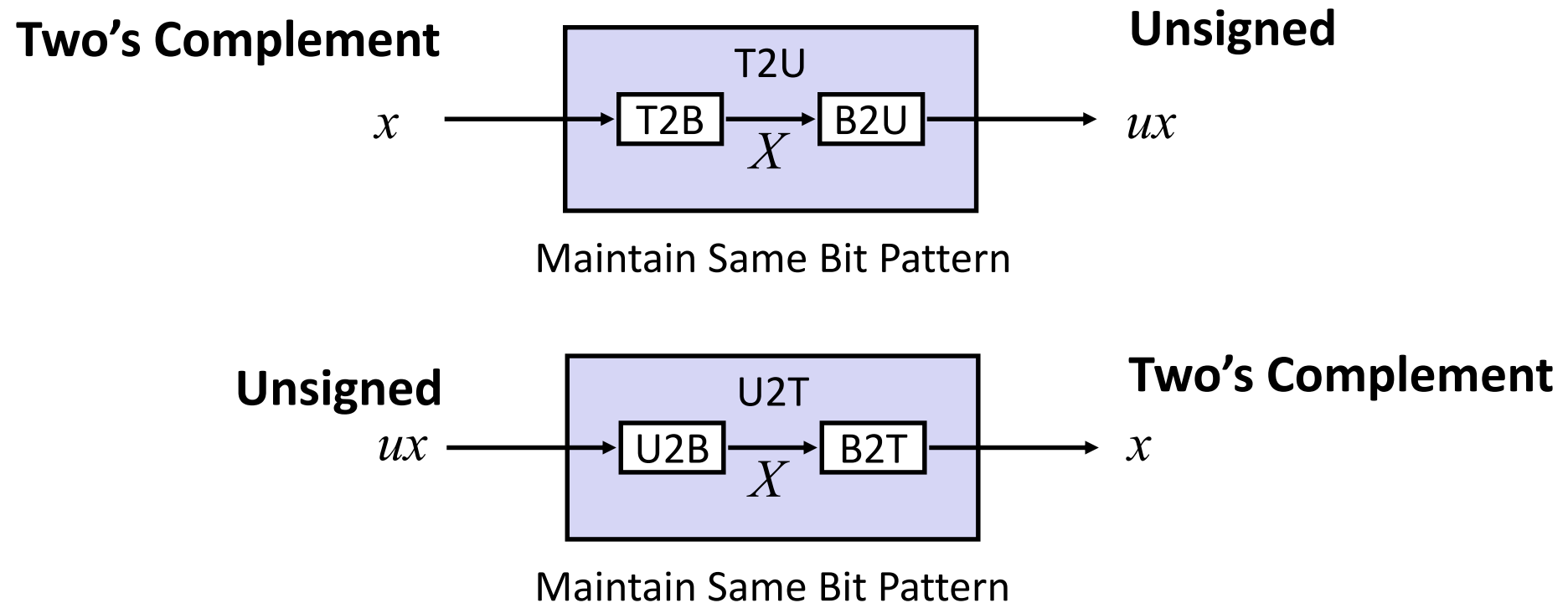
## ■ $\Rightarrow$ Can Invert Mappings

- $U2B(x) = B2U^{-1}(x)$ 
  - Bit pattern for unsigned integer
- $T2B(x) = B2T^{-1}(x)$ 
  - Bit pattern for two's comp integer

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# Mapping Between Signed & Unsigned



- Mappings between unsigned and two's complement numbers:  
**keep bit representations and reinterpret**

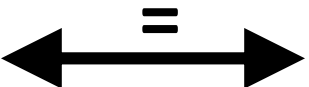
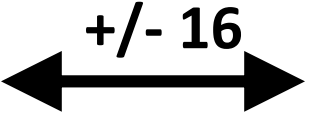
# Mapping Signed $\leftrightarrow$ Unsigned

Bits	Signed		Unsigned
0000	0		0
0001	1		1
0010	2		2
0011	3		3
0100	4		4
0101	5		5
0110	6		6
0111	7		7
1000	-8		8
1001	-7		9
1010	-6		10
1011	-5		11
1100	-4		12
1101	-3		13
1110	-2		14
1111	-1		15

T2U

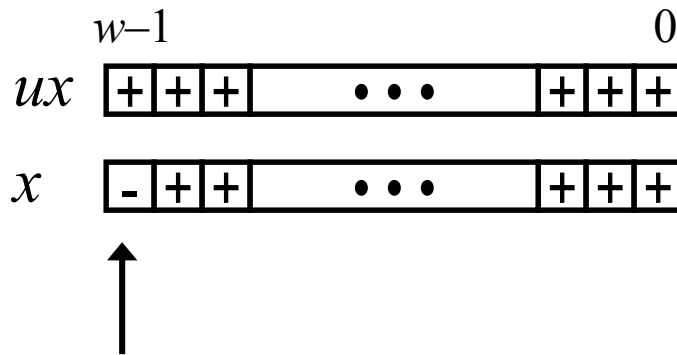
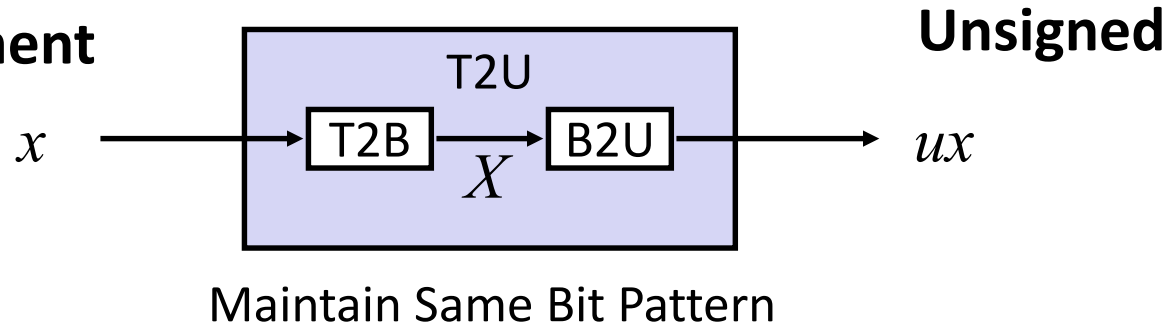
U2T

# Mapping Signed $\leftrightarrow$ Unsigned

Bits	Signed		Unsigned
0000	0		0
0001	1		1
0010	2		2
0011	3		3
0100	4		4
0101	5		5
0110	6		6
0111	7		7
1000	-8		8
1001	-7		9
1010	-6		10
1011	-5		11
1100	-4		12
1101	-3		13
1110	-2		14
1111	-1		15

# Relation between Signed & Unsigned

Two's Complement



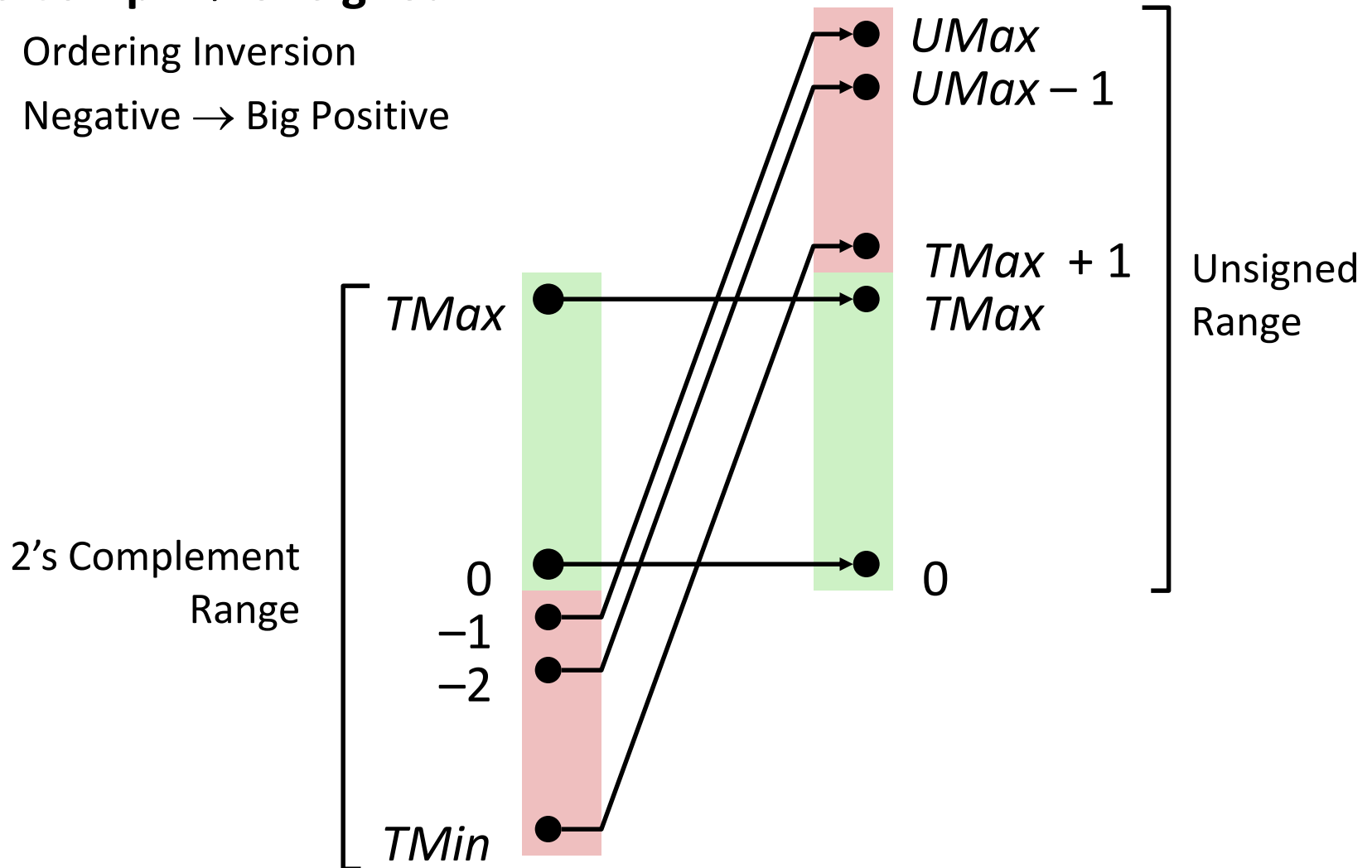
Large negative weight  
*becomes*  
 Large positive weight



# Conversion Visualized

## ■ 2's Comp. → Unsigned

- Ordering Inversion
- Negative → Big Positive



# Signed vs. Unsigned in C

## ■ Constants

- By default are considered to be signed integers
- Unsigned if have “U” as suffix

`0U, 4294967259U`

## ■ Casting

- Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;  
unsigned ux, uy;  
tx = (int) ux;  
uy = (unsigned) ty;
```

- Implicit casting also occurs via assignments and procedure calls

```
tx = ux;  
uy = ty;
```

# Casting Surprises

## ■ Expression Evaluation

- If there is a mix of unsigned and signed in single expression,  
*signed values implicitly cast to unsigned*
- Including comparison operations  $<$ ,  $>$ ,  $==$ ,  $<=$ ,  $>=$
- Examples for  $W = 32$ : **TMIN = -2,147,483,648**, **TMAX = 2,147,483,647**

■ Constant <sub>1</sub>	Constant <sub>2</sub>	Relation	Evaluation
0	0U	$==$	unsigned
-1	0	$<$	signed
-1	0U	$>$	unsigned
2147483647	-2147483647-1	$>$	signed
2147483647U	-2147483647-1	$<$	unsigned
-1	-2	$>$	signed
(unsigned)-1	-2	$>$	unsigned
2147483647	2147483648U	$<$	unsigned
2147483647	(int) 2147483648U	$>$	signed

# Summary

## Casting Signed $\leftrightarrow$ Unsigned: Basic Rules

- Bit pattern is maintained
- But reinterpreted
- Can have unexpected effects: adding or subtracting  $2^w$
- Expression containing signed and unsigned int
  - `int` is cast to `unsigned`!!

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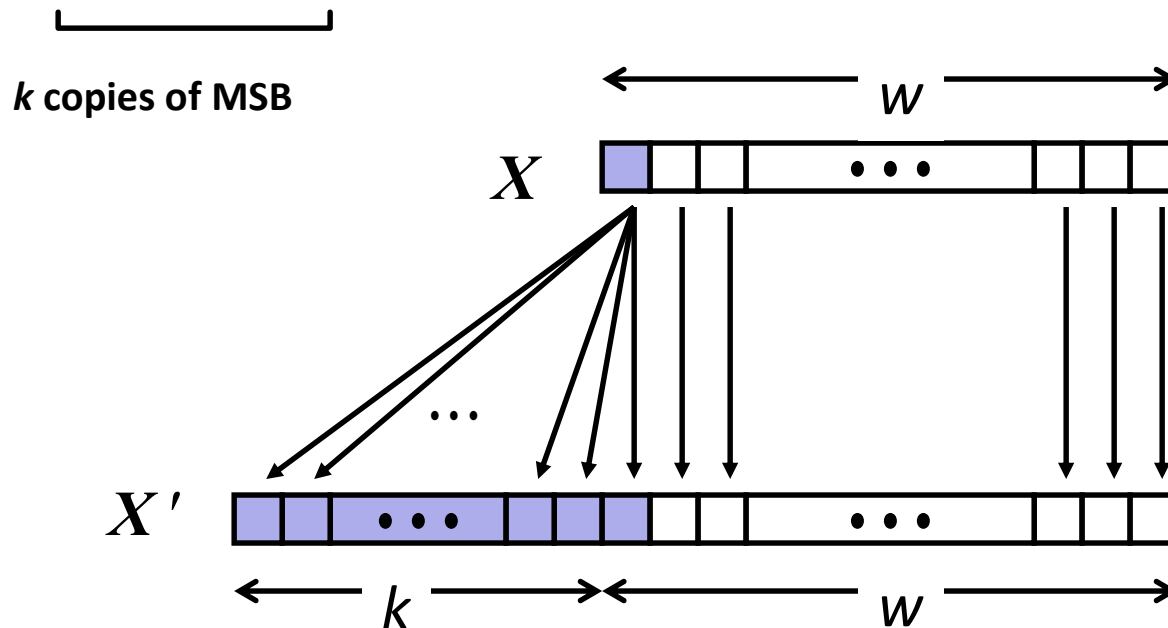
# Sign Extension

## ■ Task:

- Given  $w$ -bit signed integer  $x$
- Convert it to  $w+k$ -bit integer with same value

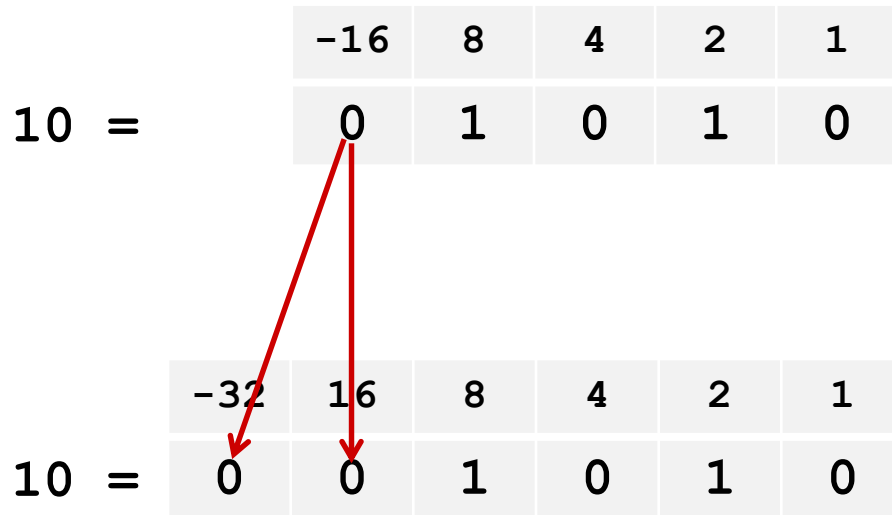
## ■ Rule:

- Make  $k$  copies of sign bit:
- $X' = \underbrace{x_{w-1}, \dots, x_{w-1}}_{k \text{ copies of MSB}}, x_{w-1}, x_{w-2}, \dots, x_0$

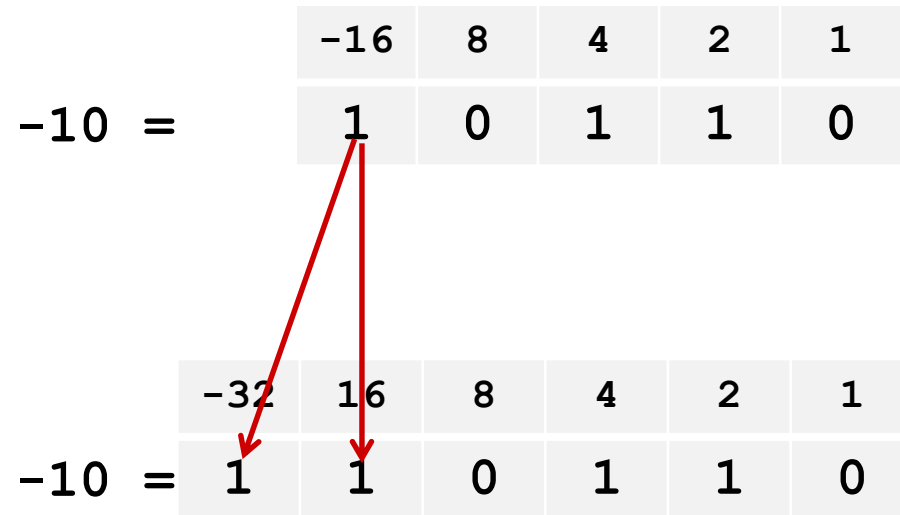


# Sign Extension: Simple Example

## Positive number



## Negative number



# Sign Extension Example

```
short int x = 15213;
int      ix = (int) x;
short int y = -15213;
int      iy = (int) y;
```

	Decimal	Hex	Binary
<b>x</b>	15213	3B 6D	00111011 01101101
<b>ix</b>	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
<b>y</b>	-15213	C4 93	11000100 10010011
<b>iy</b>	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

- Converting from smaller to larger integer data type
- C automatically performs sign extension



# Truncation: Simple Example

## No sign change

	-16	8	4	2	1
2 =	0	0	0	1	0

	-8	4	2	1
2 =	0	0	1	0

$$2 \bmod 16 = 2$$

	-16	8	4	2	1
-6 =	1	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

$$-6 \bmod 16 = 26 \text{U} \bmod 16 = 10 \text{U} = -6$$

## Sign change

	-16	8	4	2	1
10 =	0	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

$$10 \bmod 16 = 10 \text{U} \bmod 16 = 10 \text{U} = -6$$

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-8	4	2	1
6 =	0	1	1	0

$$-10 \bmod 16 = 22 \text{U} \bmod 16 = 6 \text{U} = 6$$

# Summary:

## Expanding, Truncating: Basic Rules

- **Expanding (e.g., short int to int)**
  - Unsigned: zeros added
  - Signed: sign extension
  - Both yield expected result
  
- **Truncating (e.g., unsigned to unsigned short)**
  - Unsigned/signed: bits are truncated
  - Result reinterpreted
  - Unsigned: mod operation
  - Signed: similar to mod

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# Unsigned Addition

Operands:  $w$  bits

$u$

$+ v$

True Sum:  $w+1$  bits

$u + v$

Discard Carry:  $w$  bits

$\text{UAdd}_w(u, v)$

## ■ Standard Addition Function

- Ignores carry output

## ■ Implements Modular Arithmetic

$$s = \text{UAdd}_w(u, v) = u + v \bmod 2^w$$

unsigned char	1110 1001	E9	233
	+ 1101 0101	+ D5	+ 213
	<hr/>	<hr/>	<hr/>
	<hr/>	<hr/>	<hr/>

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111


# Unsigned Addition

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$u + v$  

Discard Carry:  $w$  bits

$\text{UAdd}_w(u, v)$  

## ■ Standard Addition Function

- Ignores carry output

## ■ Implements Modular Arithmetic

$$s = \text{UAdd}_w(u, v) = u + v \bmod 2^w$$

unsigned char

```

      1110 1001
    + 1101 0101
    -----
  1 1011 1110
    -----
      1011 1110
  
```

```

      E9
    + D5
    -----
    1BE
    -----
      BE
  
```

```

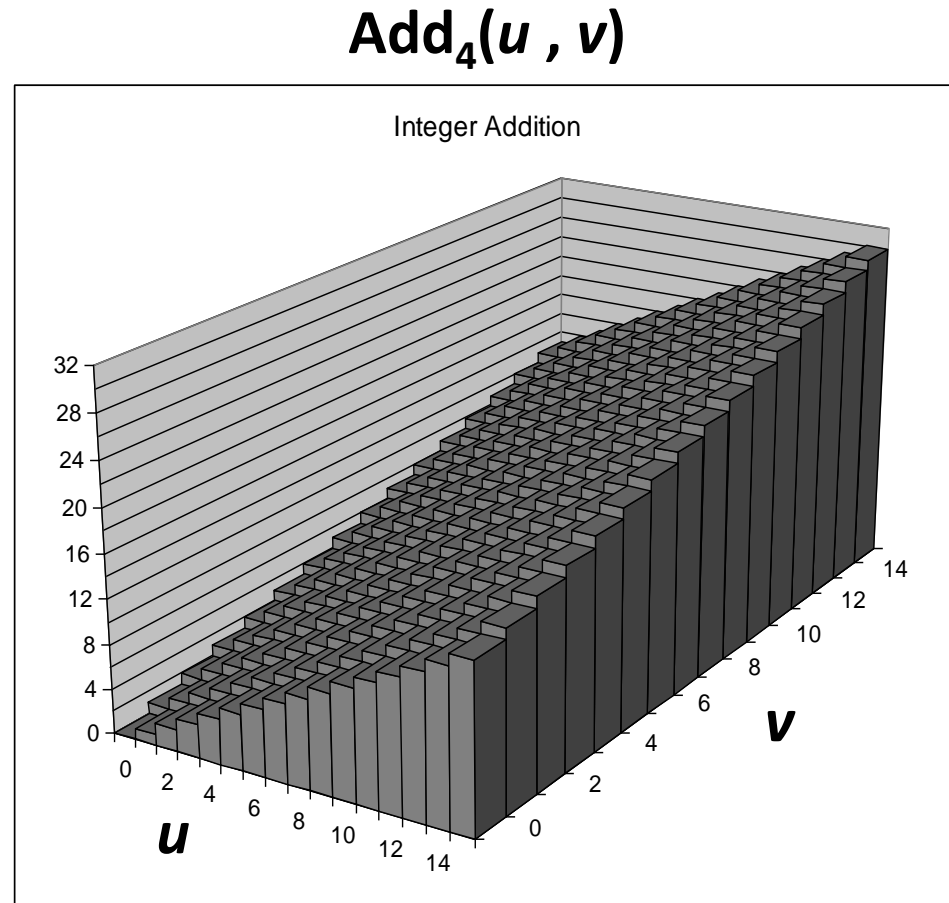
      233
    + 213
    -----
    446
    -----
    190
  
```

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

# Visualizing (Mathematical) Integer Addition

## ■ Integer Addition

- 4-bit integers  $u, v$
- Compute true sum  $\text{Add}_4(u, v)$
- Values increase linearly with  $u$  and  $v$
- Forms planar surface

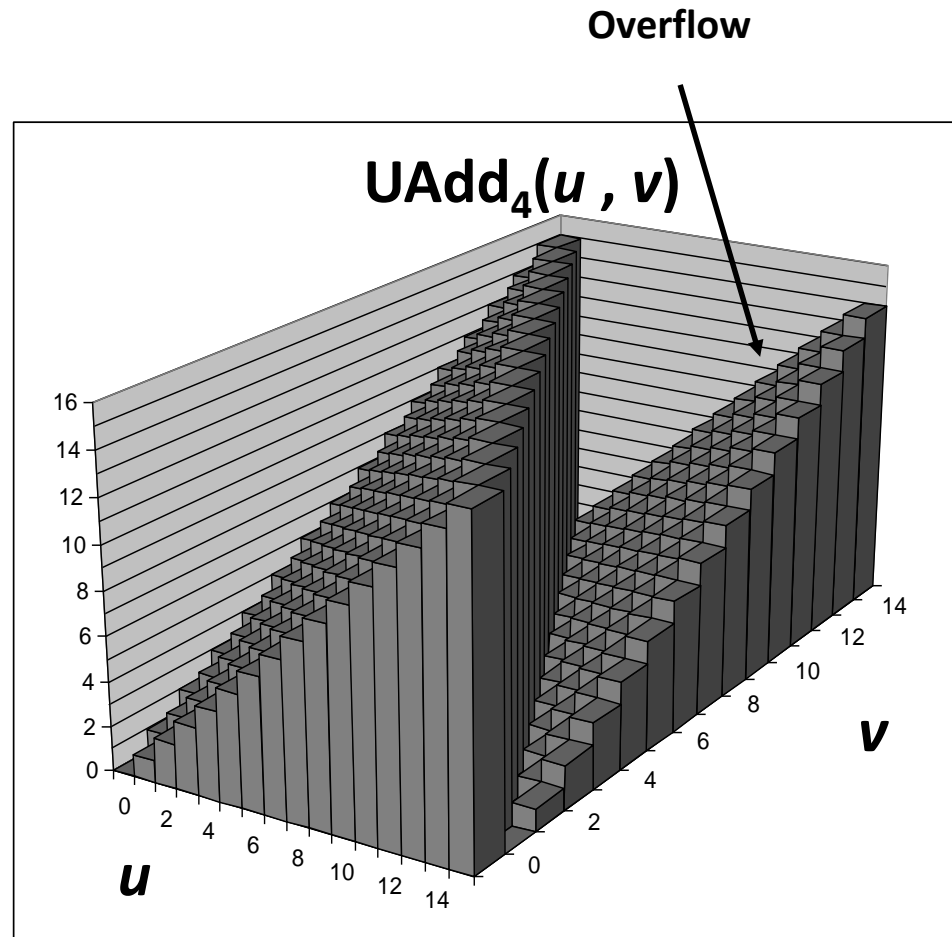
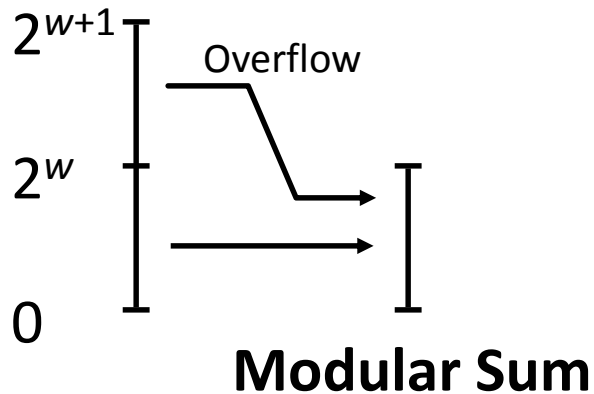


# Visualizing Unsigned Addition

## ■ Wraps Around

- If true sum  $\geq 2^w$
- At most once

True Sum



# Two's Complement Addition

Operands:  $w$  bits

$u$

$+$   $v$

True Sum:  $w+1$  bits

$u + v$

Discard Carry:  $w$  bits

$\text{TAdd}_w(u, v)$

## ■ TAdd and UAdd have Identical Bit-Level Behavior

- Signed vs. unsigned addition in C:

```
int s, t, u, v;
```

```
s = (int) ((unsigned) u + (unsigned) v);
```

```
t = u + v
```

- Will give `s == t`

```

      1110 1001
+     1101 0101
-----
  1 1011 1110
-----
    1011 1110

```

```

      E9
+     D5
-----
    1BE
-----
     BE

```

```

      -23
+    -43
-----
    -66
-----
    -66

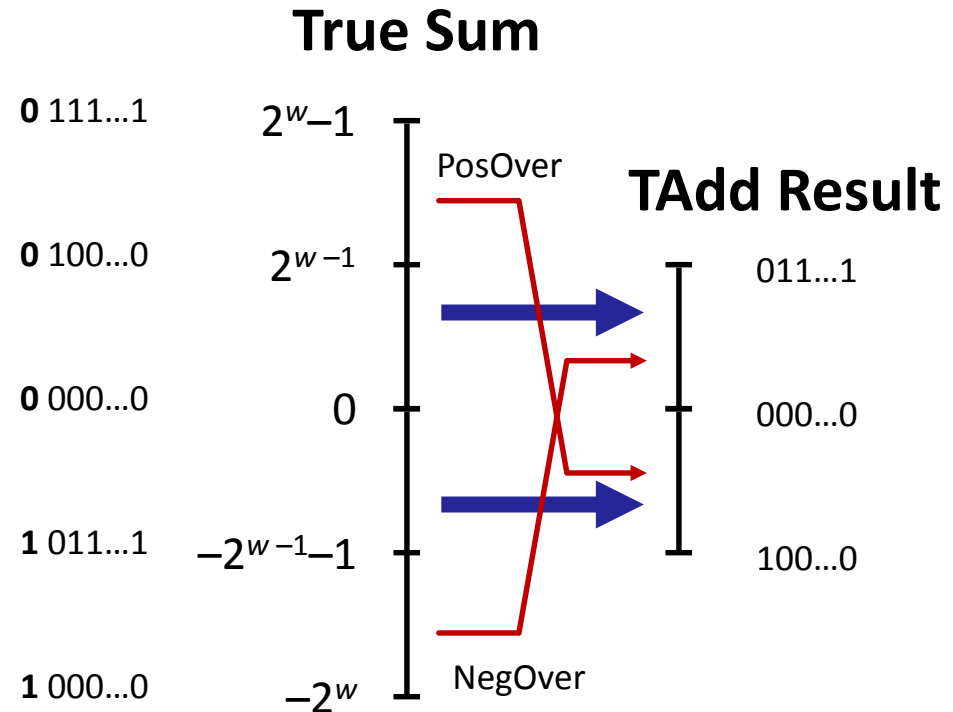
```



# TAdd Overflow

## ■ Functionality

- True sum requires  $w+1$  bits
- Drop off MSB
- Treat remaining bits as 2's comp. integer



# Visualizing 2's Complement Addition

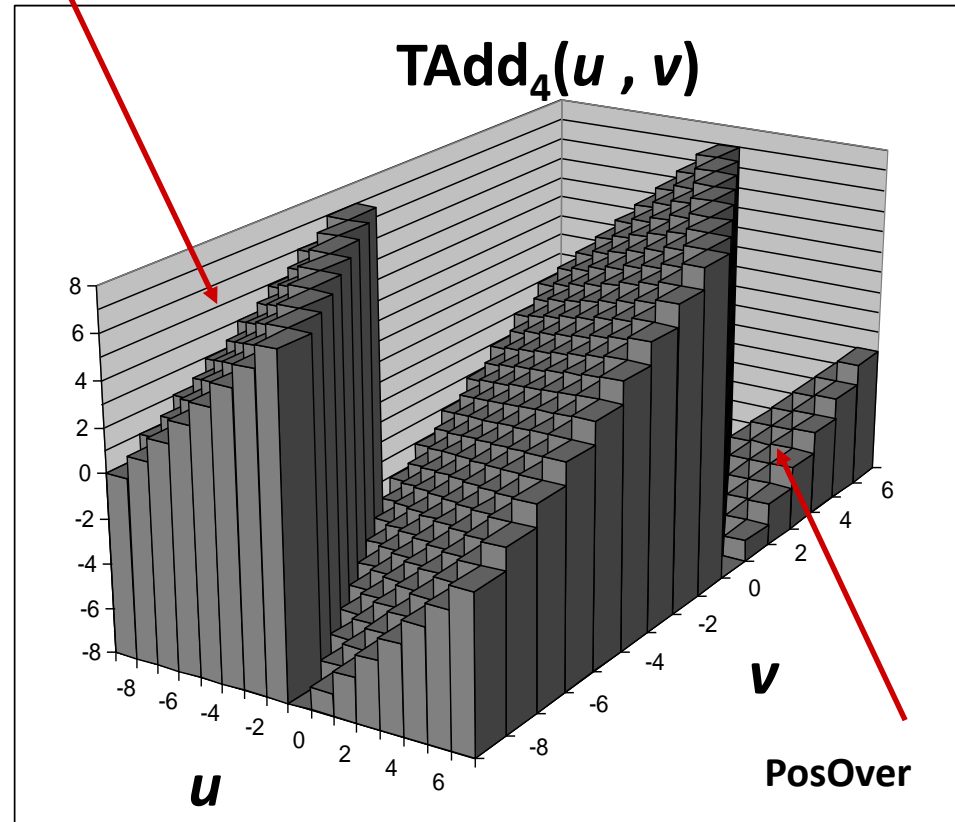
## ■ Values

- 4-bit two's comp.
- Range from -8 to +7

## ■ Wraps Around

- If  $\text{sum} \geq 2^{w-1}$ 
  - Becomes negative
  - At most once
- If  $\text{sum} < -2^{w-1}$ 
  - Becomes positive
  - At most once

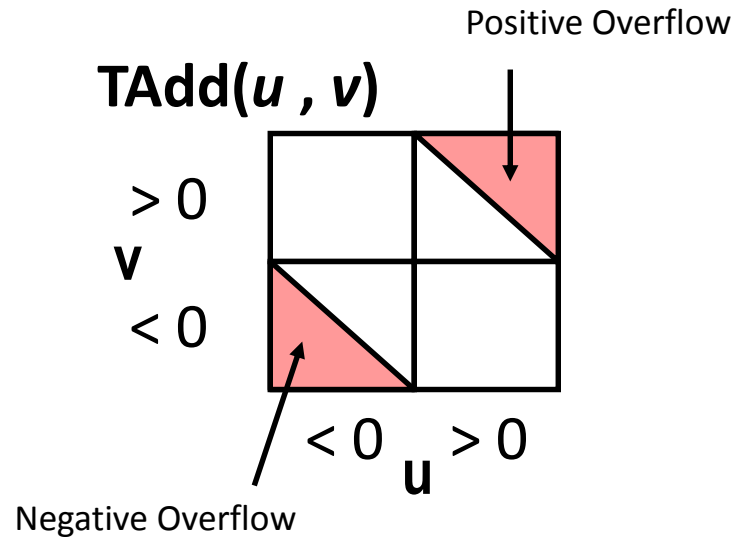
NegOver



# Characterizing TAdd

## ■ Functionality

- True sum requires  $w+1$  bits
- Drop off MSB
- Treat remaining bits as 2's comp. integer

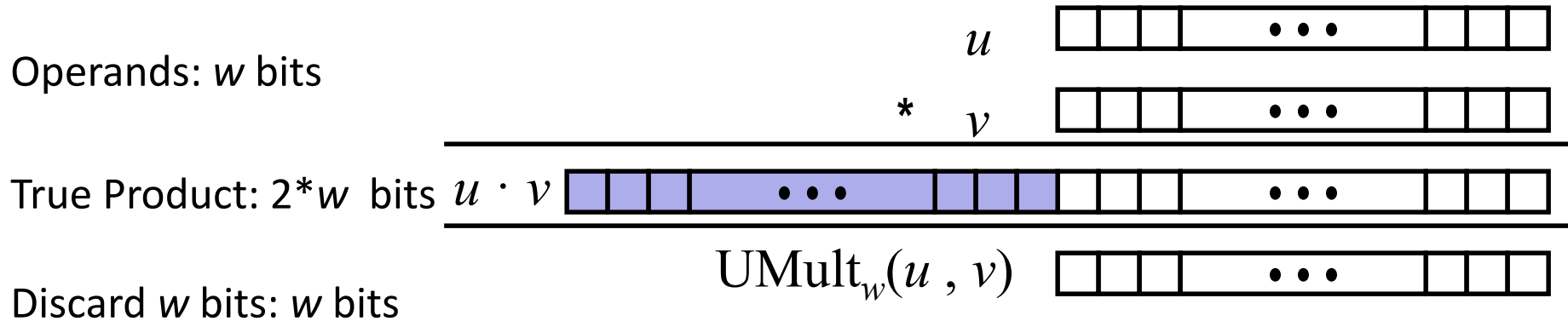


$$TAdd_w(u, v) = \begin{cases} u + v + 2^w & u + v < TMin_w \text{ (NegOver)} \\ u + v & TMin_w \leq u + v \leq TMax_w \\ u + v - 2^w & TMax_w < u + v \text{ (PosOver)} \end{cases}$$

# Multiplication

- **Goal: Computing Product of  $w$ -bit numbers  $x, y$** 
  - Either signed or unsigned
- **But, exact results can be bigger than  $w$  bits**
  - Unsigned: up to  $2w$  bits
    - Result range:  $0 \leq x * y \leq (2^w - 1)^2 = 2^{2w} - 2^{w+1} + 1$
  - Two's complement min (negative): Up to  $2w-1$  bits
    - Result range:  $x * y \geq (-2^{w-1}) * (2^{w-1} - 1) = -2^{2w-2} + 2^{w-1}$
  - Two's complement max (positive): Up to  $2w$  bits, but only for  $(TMin_w)^2$ 
    - Result range:  $x * y \leq (-2^{w-1})^2 = 2^{2w-2}$
- **So, maintaining exact results...**
  - would need to keep expanding word size with each product computed
  - is done in software, if needed
    - e.g., by “arbitrary precision” arithmetic packages

# Unsigned Multiplication in C



## ■ Standard Multiplication Function

- Ignores high order  $w$  bits

## ■ Implements Modular Arithmetic

$$\text{UMult}_w(u, v) = u \cdot v \bmod 2^w$$

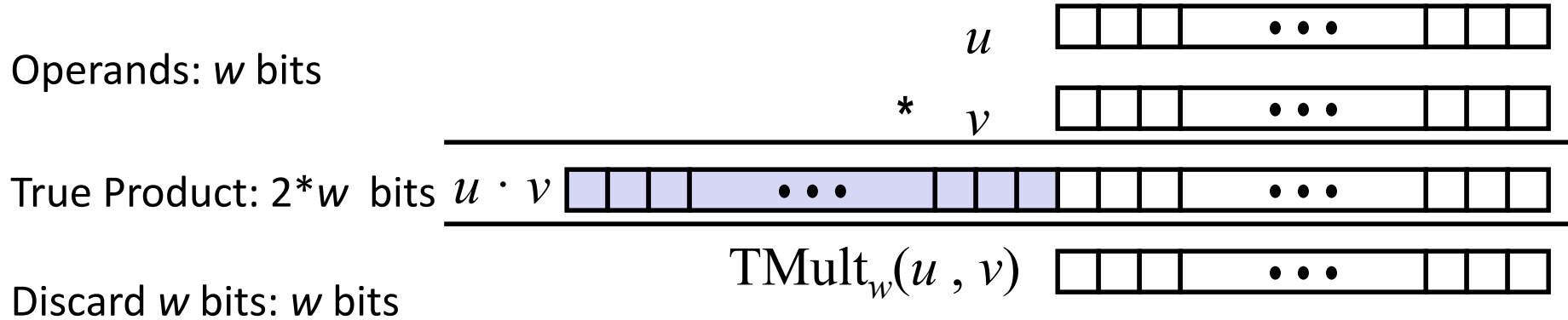
$$\begin{array}{r}
 \phantom{1110} 1110 \ 1001 \\
 * \phantom{1101} 1101 \ 0101 \\
 \hline
 1100 \ 0001 \ 1101 \ 1101 \\
 \phantom{1100} 1101 \ 1101 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \phantom{E9} E9 \\
 * \phantom{D5} D5 \\
 \hline
 C1DD \\
 \phantom{DD} DD \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \phantom{233} 233 \\
 * \phantom{213} 213 \\
 \hline
 49629 \\
 \phantom{49629} 221 \\
 \hline
 \end{array}$$

# Signed Multiplication in C

Operands:  $w$  bits



## ■ Standard Multiplication Function

- Ignores high order  $w$  bits
- Some of which are different for signed vs. unsigned multiplication
- Lower bits are the same

	1110 1001	E9	-23
*	1101 0101	* D5	* -43
	0000 0011 1101 1101	03DD	989
	1101 1101	DD	-35

# Power-of-2 Multiply with Shift

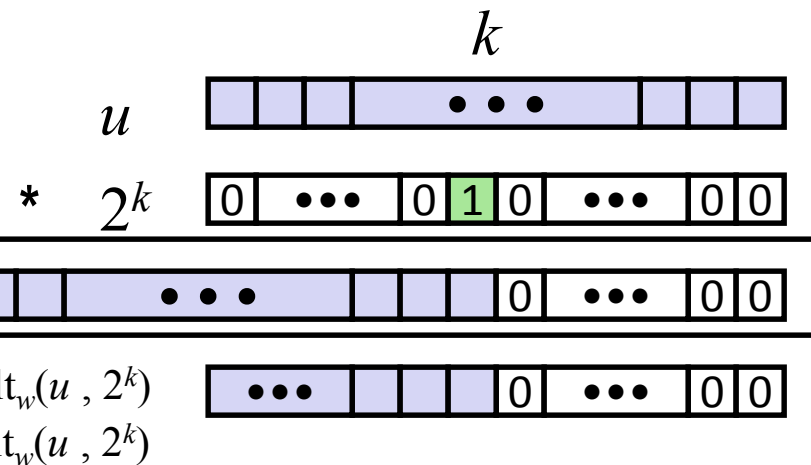
## ■ Operation

- $u \ll k$  gives  $u * 2^k$
- Both signed and unsigned

Operands:  $w$  bits

True Product:  $w+k$  bits

Discard  $k$  bits:  $w$  bits



## ■ Examples

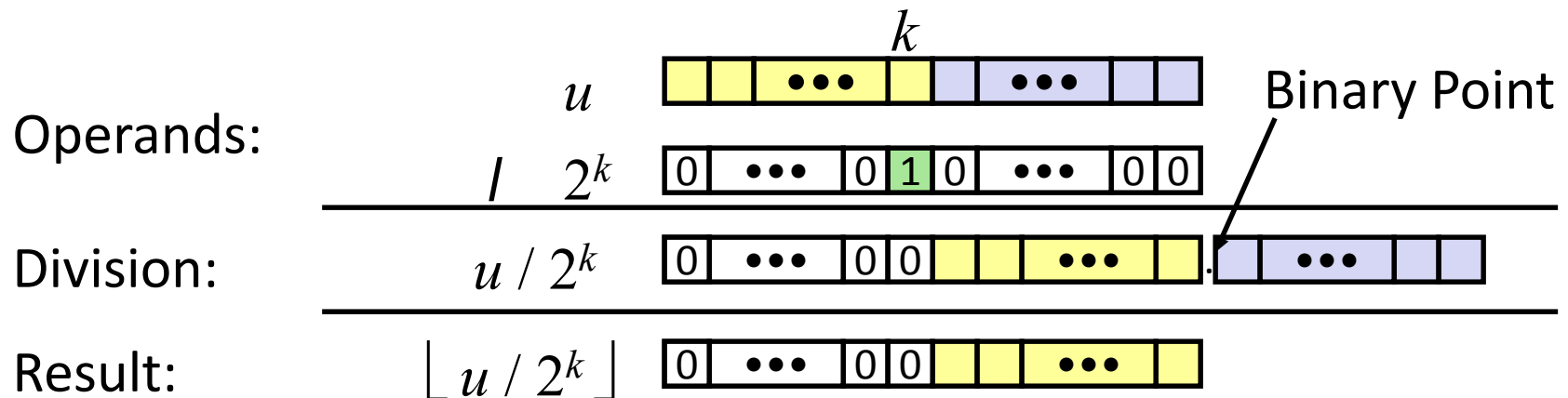
- $u \ll 3 \quad == \quad u * 8$
- $(u \ll 5) - (u \ll 3) == u * 24$
- Most machines shift and add faster than multiply
  - Compiler generates this code automatically

**Important Lesson:**  
Trust Your Compiler!

# Unsigned Power-of-2 Divide with Shift

## ■ Quotient of Unsigned by Power of 2

- $u \gg k$  gives  $\lfloor u / 2^k \rfloor$
- Uses logical shift



	Division	Computed	Hex	Binary
<b>x</b>	<b>15213</b>	<b>15213</b>	3B 6D	00111011 01101101
<b>x &gt;&gt; 1</b>	<b>7606.5</b>	<b>7606</b>	1D B6	00011101 10110110
<b>x &gt;&gt; 4</b>	<b>950.8125</b>	<b>950</b>	03 B6	00000011 10110110
<b>x &gt;&gt; 8</b>	<b>59.4257813</b>	<b>59</b>	00 3B	00000000 00111011



# Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- **Integers**
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - **Summary**
- Representations in memory, pointers, strings

# Arithmetic: Basic Rules

## ■ Addition:

- Unsigned/signed: Normal addition followed by truncate, same operation on bit level
- Unsigned: addition mod  $2^w$ 
  - Mathematical addition + possible subtraction of  $2^w$
- Signed: modified addition mod  $2^w$  (result in proper range)
  - Mathematical addition + possible addition or subtraction of  $2^w$

## ■ Multiplication:

- Unsigned/signed: Normal multiplication followed by truncate, same operation on bit level
- Unsigned: multiplication mod  $2^w$
- Signed: modified multiplication mod  $2^w$  (result in proper range)

# Why Should I Use Unsigned?

## ■ *Don't* use without understanding implications

- Easy to make mistakes

```
unsigned i;  
for (i = cnt-2; i >= 0; i--)  
    a[i] += a[i+1];
```

- Can be very subtle

```
#define DELTA sizeof(int)  
int i;  
for (i = CNT; i-DELTA >= 0; i-= DELTA)  
    . . .
```

# Counting Down with Unsigned

## ■ Proper way to use unsigned as loop index

```
unsigned i;  
for (i = cnt-2; i < cnt; i--)  
    a[i] += a[i+1];
```

## ■ See Robert Seacord, *Secure Coding in C and C++*

- C Standard guarantees that unsigned addition will behave like modular arithmetic
  - $0 - 1 \rightarrow UMax$

## ■ Even better

```
size_t i;  
for (i = cnt-2; i < cnt; i--)  
    a[i] += a[i+1];
```

- Data type `size_t` defined as unsigned value with length = word size
- Code will work even if `cnt = UMax`
- What if `cnt` is signed and `< 0`?

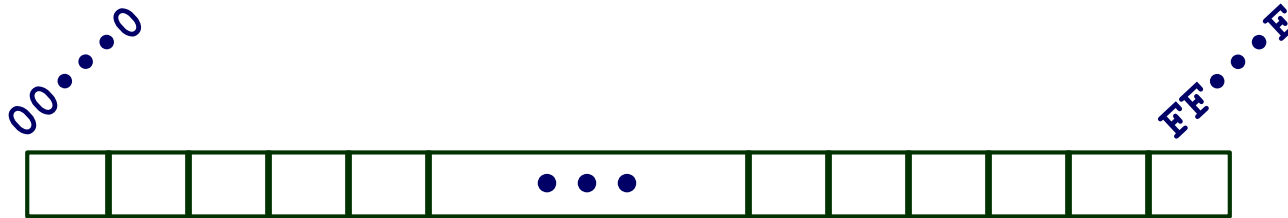
# Why Should I Use Unsigned? (cont.)

- **Do Use When Performing Modular Arithmetic**
  - Multiprecision arithmetic
- **Do Use When Using Bits to Represent Sets**
  - Logical right shift, no sign extension

# Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- **Representations in memory, pointers, strings**

# Byte-Oriented Memory Organization



## ■ Programs refer to data by address

- Conceptually, envision it as a very large array of bytes
  - In reality, it's not, but can think of it that way
- An address is like an index into that array
  - and, a pointer variable stores an address

## ■ Note: system provides private address spaces to each “process”

- Think of a process as a program being executed
- So, a program can clobber its own data, but not that of others

# Machine Words

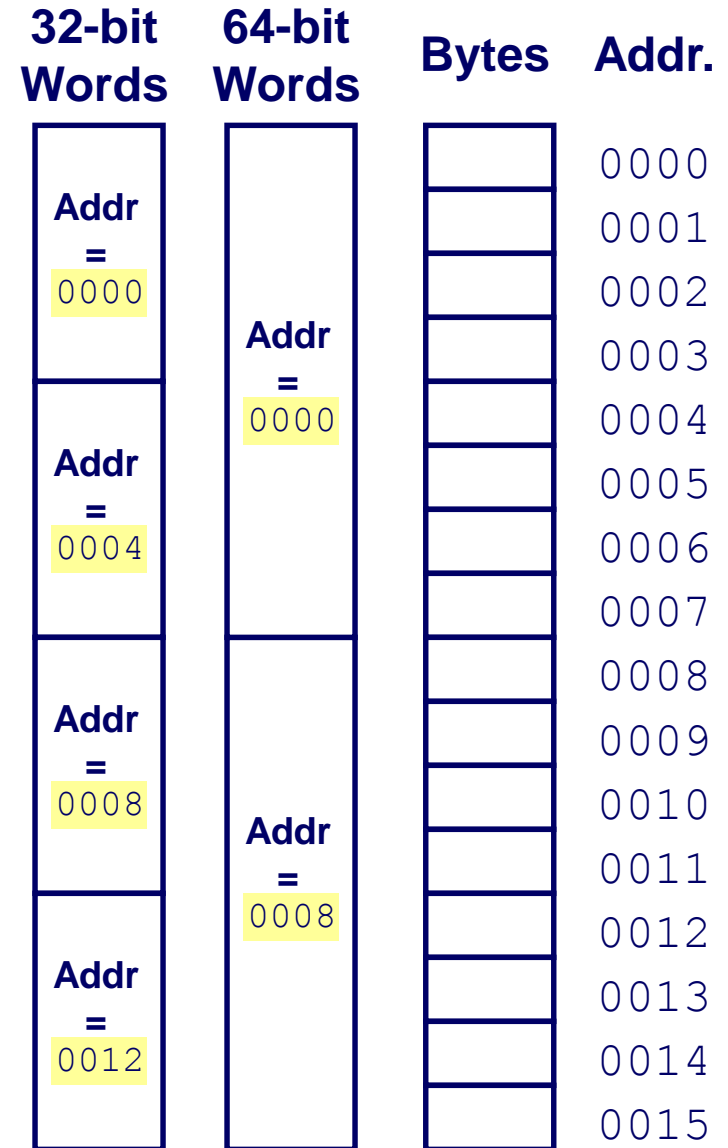
- **Any given computer has a “Word Size”**
  - Nominal size of integer-valued data
    - and of addresses
  - Until recently, most machines used 32 bits (4 bytes) as word size
    - Limits addresses to 4GB ( $2^{32}$  bytes)
  - Increasingly, machines have 64-bit word size
    - Potentially, could have 18 EB (exabytes) of addressable memory
    - That's  $18.4 \times 10^{18}$
  - Machines still support multiple data formats
    - Fractions or multiples of word size
    - Always integral number of bytes



# Word-Oriented Memory Organization

## ■ Addresses Specify Byte Locations

- Address of first byte in word
- Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



# Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
<code>char</code>	1	1	1
<code>short</code>	2	2	2
<code>int</code>	4	4	4
<code>long</code>	4	8	8
<code>float</code>	4	4	4
<code>double</code>	8	8	8
<code>long double</code>	–	–	10/16
<code>pointer</code>	4	8	8

# Byte Ordering

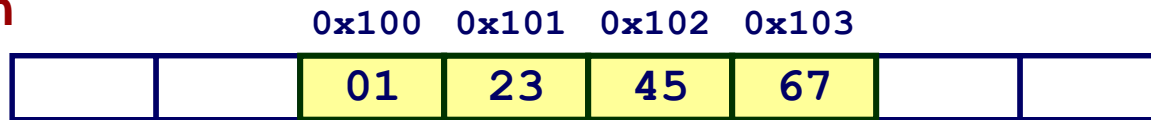
- So, how are the bytes within a multi-byte word ordered in memory?
- Conventions
  - Big Endian: Sun, PPC Mac, Internet
    - Least significant byte has highest address
  - Little Endian: x86, ARM processors running Android, iOS, and Windows
    - Least significant byte has lowest address

# Byte Ordering Example

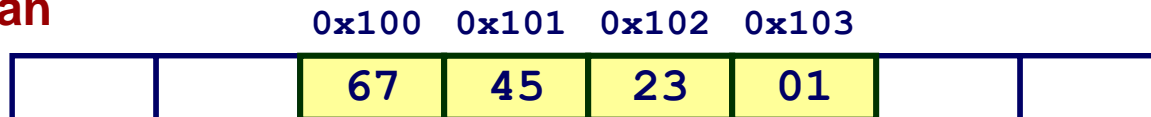
## ■ Example

- Variable x has 4-byte value of 0x01234567
- Address given by &x is 0x100

### Big Endian



### Little Endian



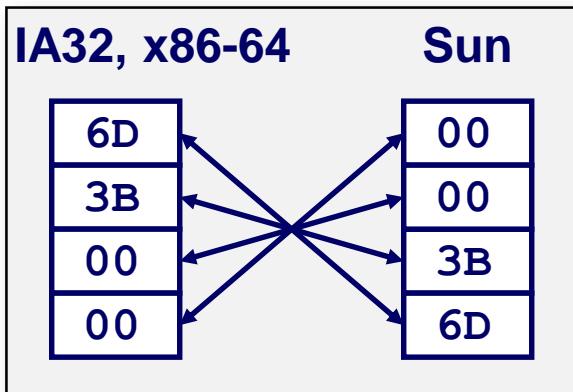
# Representing Integers

**Decimal:** 15213

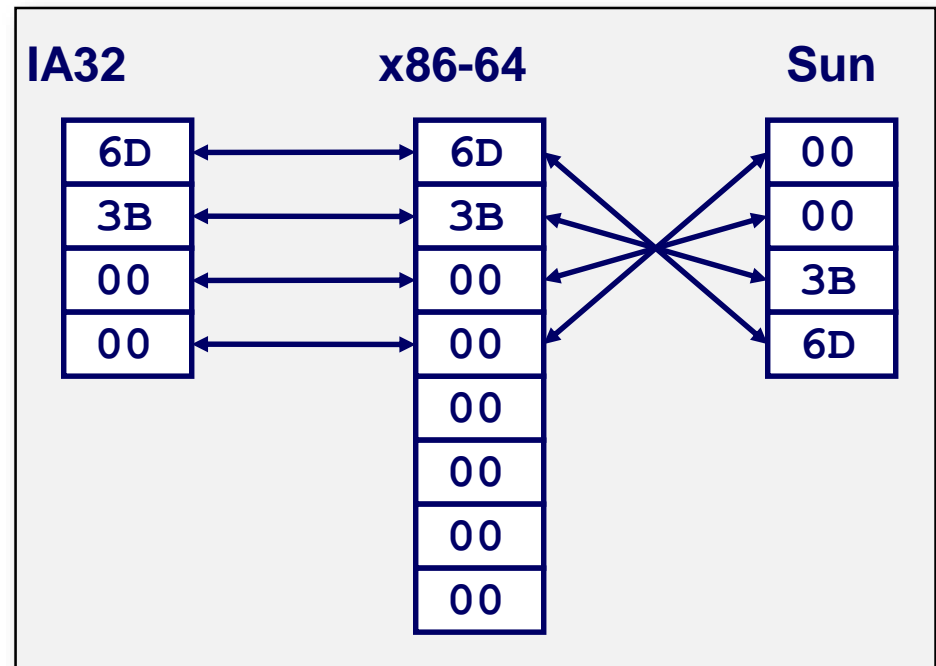
**Binary:** 0011 1011 0110 1101

**Hex:** 3 B 6 D

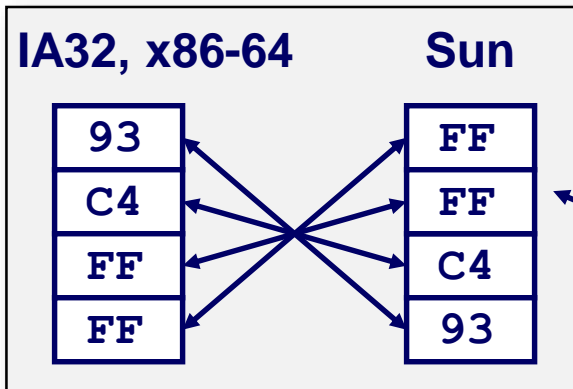
int A = 15213;



long int C = 15213;



int B = -15213;



Two's complement representation

# Examining Data Representations

## ■ Code to Print Byte Representation of Data

- Casting pointer to unsigned char \* allows treatment as a byte array

```
typedef unsigned char *pointer;  
  
void show_bytes(pointer start, int len){  
    int i;  
    for (i = 0; i < len; i++)  
        printf("%p\t0x%.2x\n", start+i, start[i]);  
    printf("\n");  
}
```

### Printf directives:

%p: Print pointer

%x: Print Hexadecimal

# show\_bytes Execution Example

```
int a = 15213;  
printf("int a = 15213;\n");  
show_bytes((pointer) &a, sizeof(int));
```

## Result (Linux x86-64):

```
int a = 15213;  
0x7fffb7f71dbc    6d  
0x7fffb7f71dbd    3b  
0x7fffb7f71dbe    00  
0x7fffb7f71dbf    00
```

# Representing Pointers

```
int B = -15213;  
int *P = &B;
```

Sun	IA32	x86-64
EF	AC	3C
FF	28	1B
FB	F5	FE
2C	FF	82
		FD
		7F
		00
		00

Different compilers & machines assign different locations to objects

Even get different results each time run program



# Representing Strings

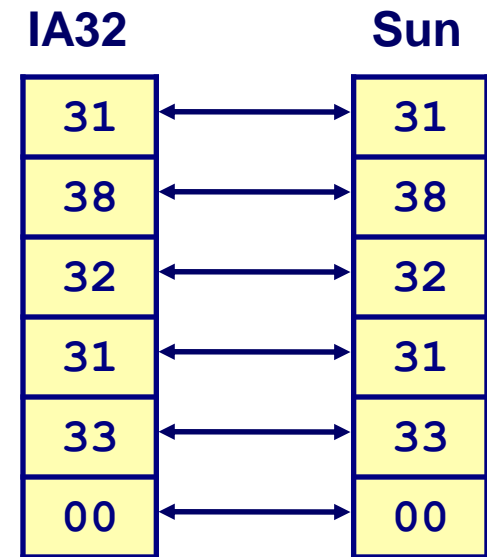
```
char S[6] = "18213";
```

## ■ Strings in C

- Represented by array of characters
- Each character encoded in ASCII format
  - Standard 7-bit encoding of character set
  - Character “0” has code 0x30
    - Digit  $i$  has code  $0x30+i$
  - *man ascii for code table*
- String should be null-terminated
  - Final character = 0

## ■ Compatibility

- Byte ordering not an issue



# Reading Byte-Reversed Listings

## ■ Disassembly

- Text representation of binary machine code
- Generated by program that reads the machine code

## ■ Example Fragment

Address	Instruction Code	Assembly Rendition
8048365:	5b	pop %ebx
8048366:	81 c3 ab 12 00 00	add \$0x12ab,%ebx
804836c:	83 bb 28 00 00 00 00	cmpl \$0x0,0x28(%ebx)

## ■ Deciphering Numbers

- Value: 0x12ab
- Pad to 32 bits: 0x000012ab
- Split into bytes: 00 00 12 ab
- Reverse: ab 12 00 00

# Summary

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
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- Representations in memory, pointers, strings
- Summary

# Integer C Puzzles

## Initialization

```
int x = foo();
int y = bar();
unsigned ux = x;
unsigned uy = y;
```

<code>x &lt; 0</code>	$\Rightarrow (x*2) < 0$	✗
<code>ux &gt;= 0</code>		✓
<code>x &amp; 7 == 7</code>	$\Rightarrow (x \ll 30) < 0$	✓
<code>ux &gt; -1</code>		✗
<code>x &gt; y</code>	$\Rightarrow -x < -y$	✗
<code>x * x &gt;= 0</code>		✗
<code>x &gt; 0 &amp;&amp; y &gt; 0</code>	$\Rightarrow x + y > 0$	✗
<code>x &gt;= 0</code>	$\Rightarrow -x \leq 0$	✓
<code>x &lt;= 0</code>	$\Rightarrow -x \geq 0$	✗
<code>(x -x)&gt;&gt;31 == -1</code>		✗
<code>ux &gt;&gt; 3 == ux/8</code>		✓
<code>x &gt;&gt; 3 == x/8</code>		✗
<code>x &amp; (x-1) != 0</code>		✗