



北京大学高能计算与应用中心
Center for Energy-efficient Computing and Applications

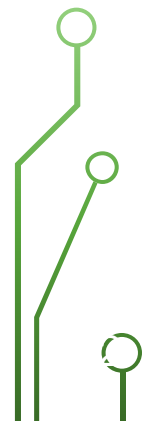
FPGA Routing 并行加速

2024年 并行与分布式导论

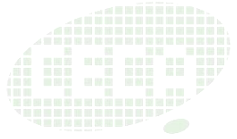
2024/05

OUTLINE

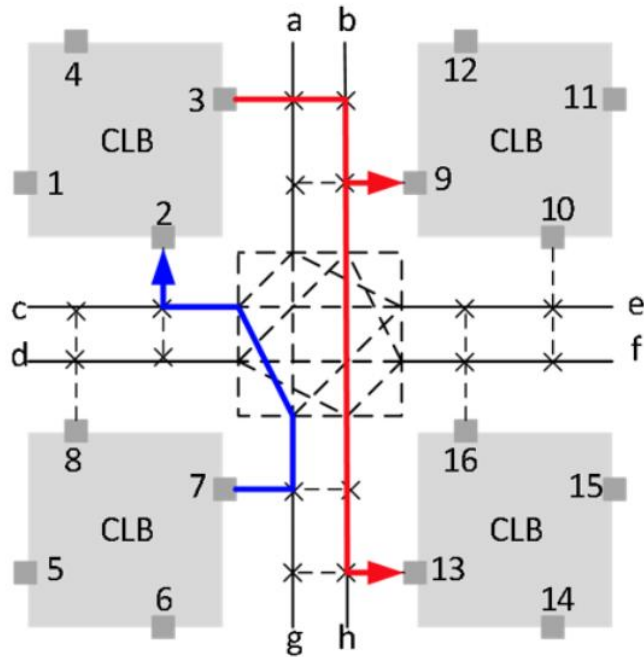
- 基本概念与算法
- Intra-Connection Strategy
- Inter-Connection Strategy
- 评分标准



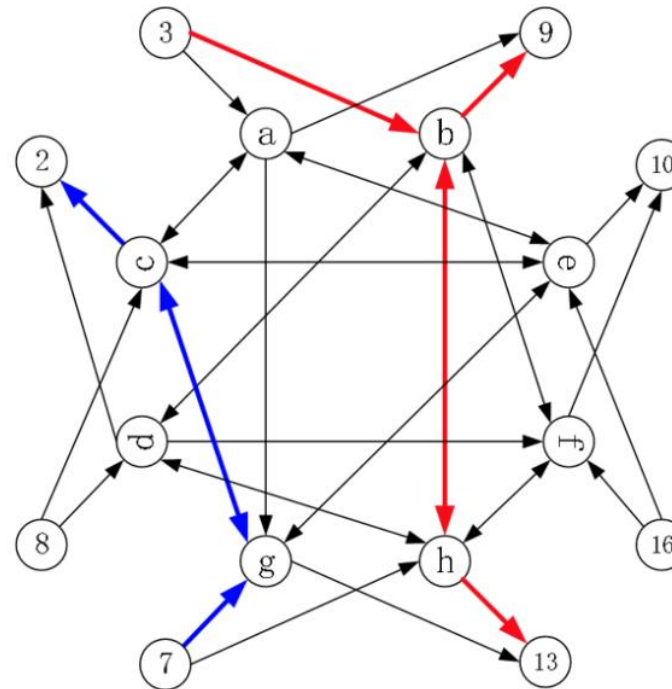
FPGA Routing



(Optimization version) Given a directed graph $G = \langle V, E \rangle$, edge weights $c : E \rightarrow R^+$, and a collection $M \subseteq V \times 2^V$ of source-destination pairs. Find the node-disjoint subgraphs $G_m = \langle V_m, E_m \rangle \subseteq G$ for all $m \in M$ and minimize $\sum_{m \in M} \sum_{e \in E_m} c(e)$.



FPGA Architecture and
Signals to be Routed



Routing Graph G and

$$M = \{ (7, \{2\}) , (3, \{9, 13\}) \}$$

基本概念



➤ Connection

- 单个 source 到 单个 sink 的连接路径

➤ Net

- 由多条 connection 组成
- 每条 connection 的 source 相同，sink 不同
- 同一 Net 中的不同 connection 可以共享节点
- 不同 Net 中的 connection 不可以共享节点

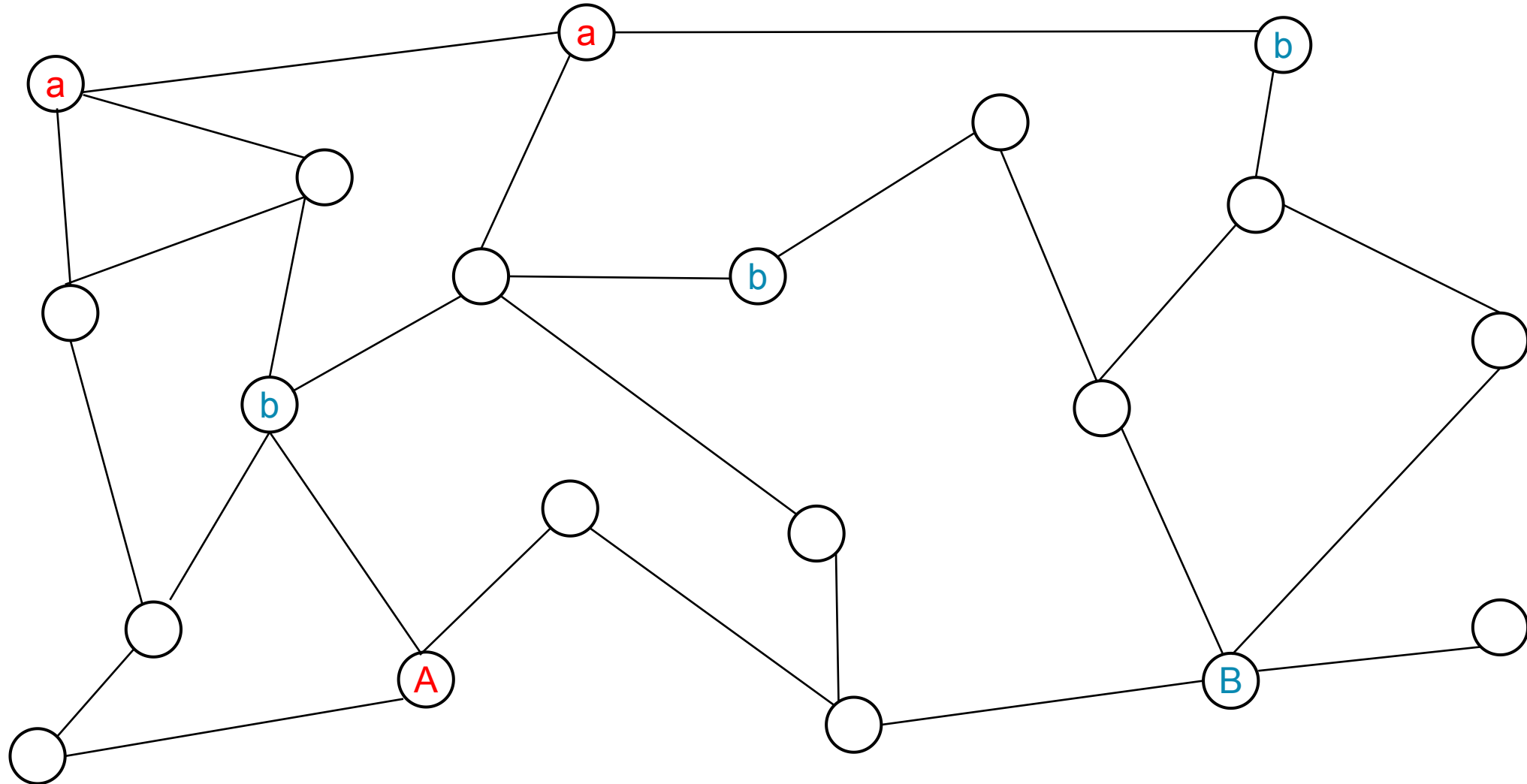
➤ Node

- 一条物理的线，可以抽象成图中的一个节点

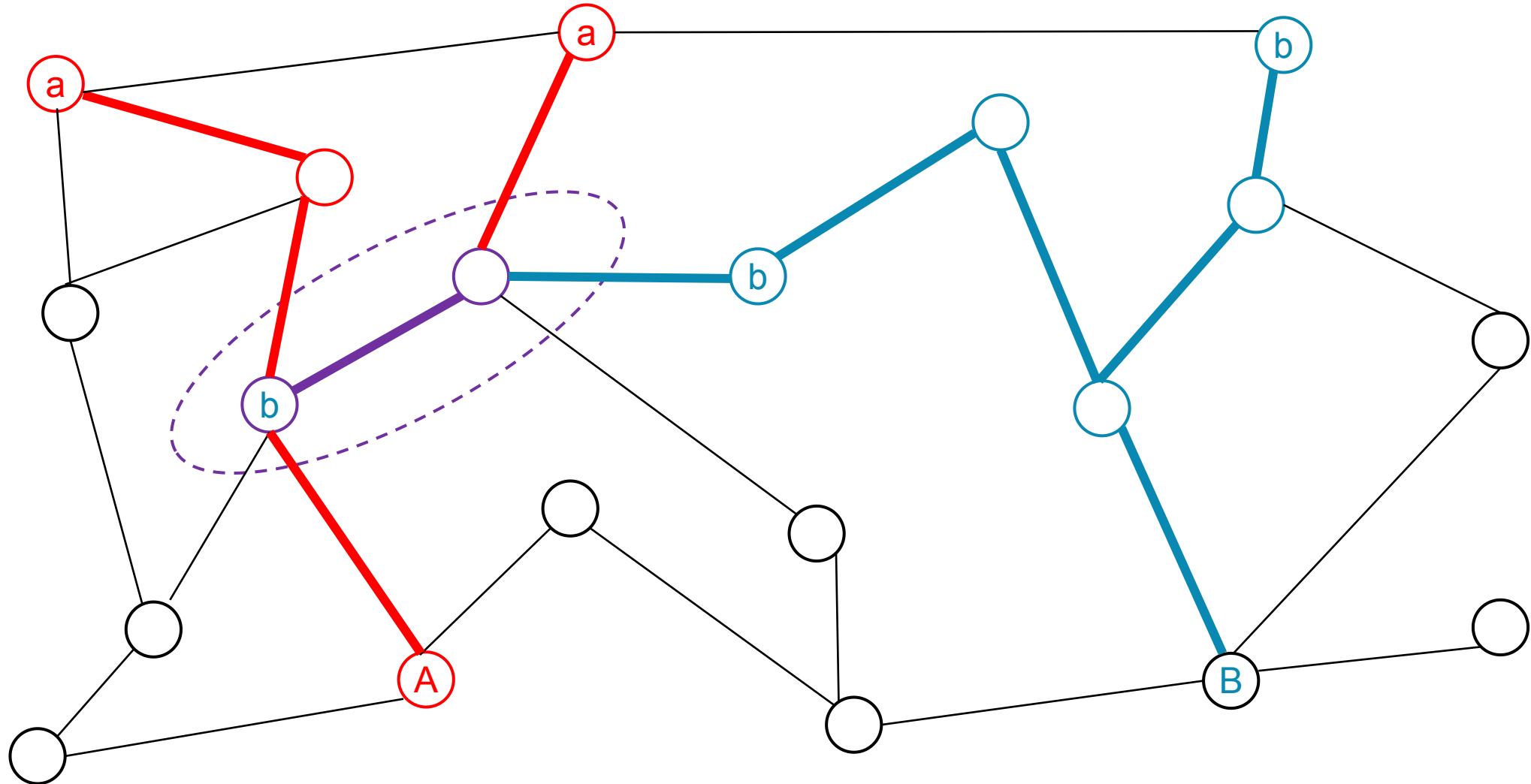
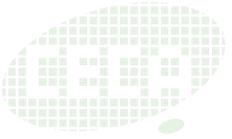
➤ Routing 目标

- 寻找到一组 connection 方案，使所有 connection cost 的最大值尽可能小

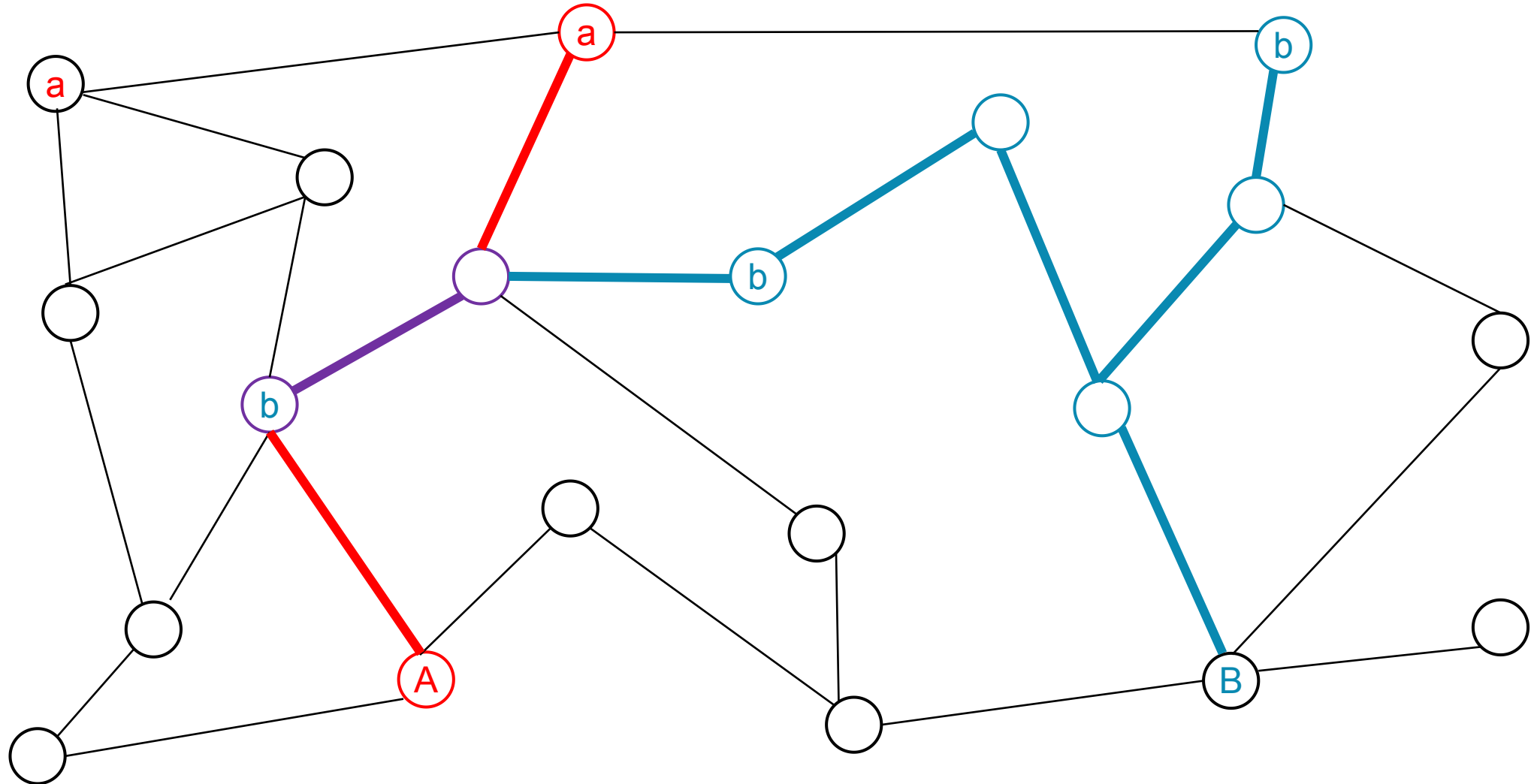
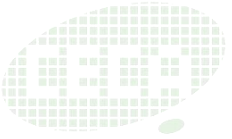
Problem Statement



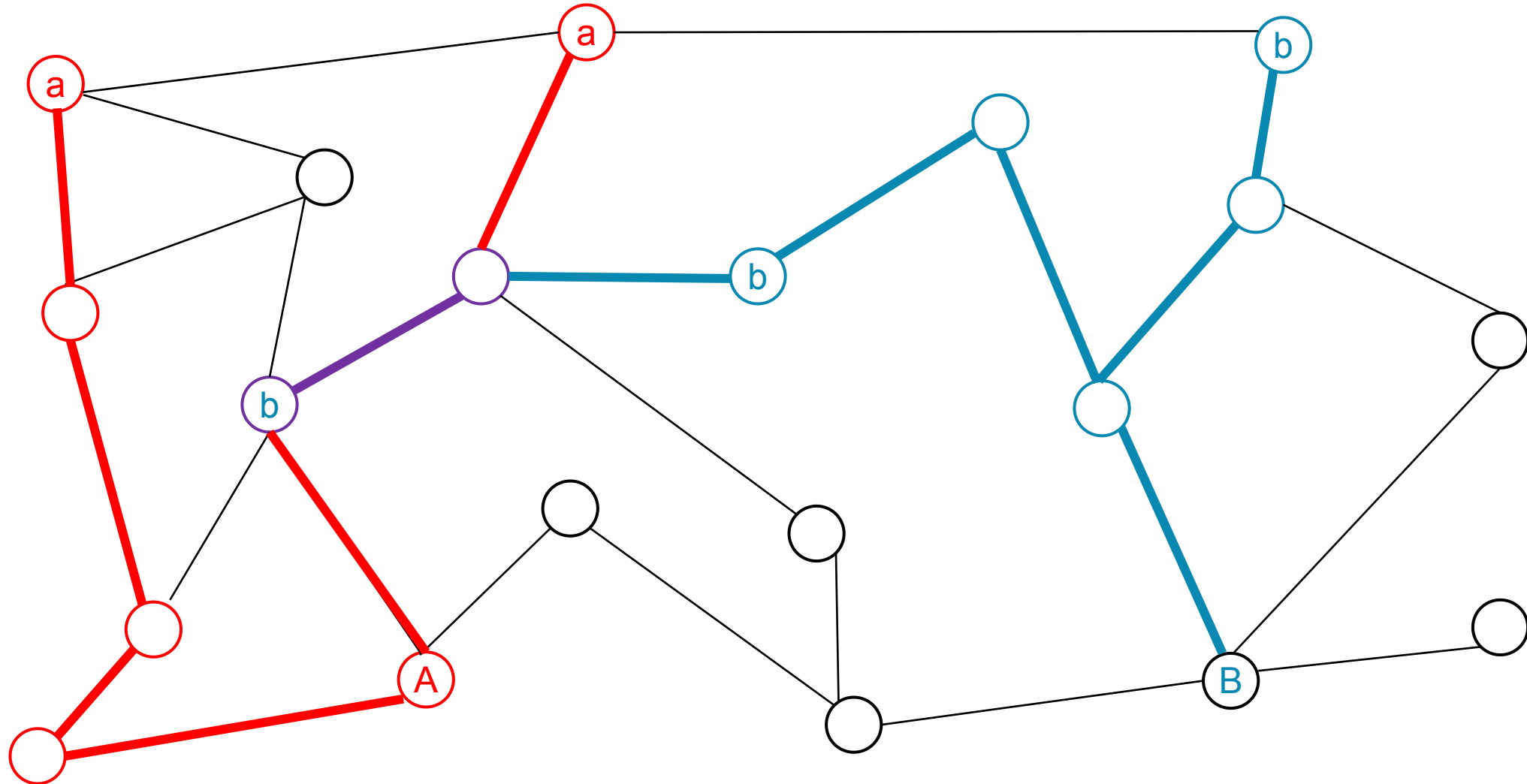
Negotiation Algorithm



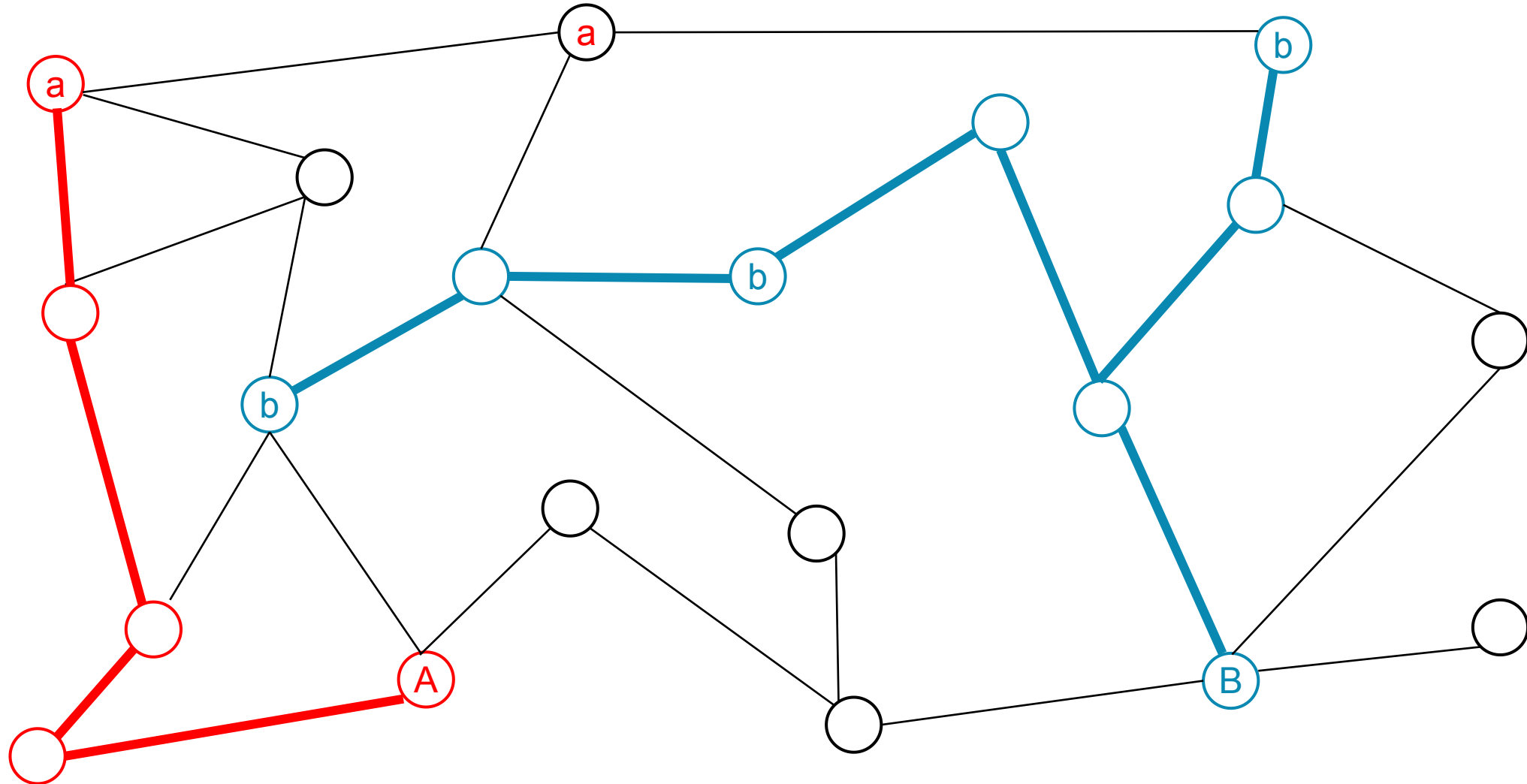
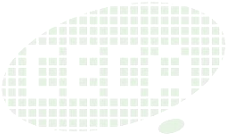
Negotiation Algorithm

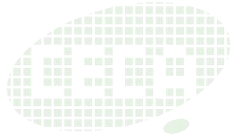


Negotiation Algorithm

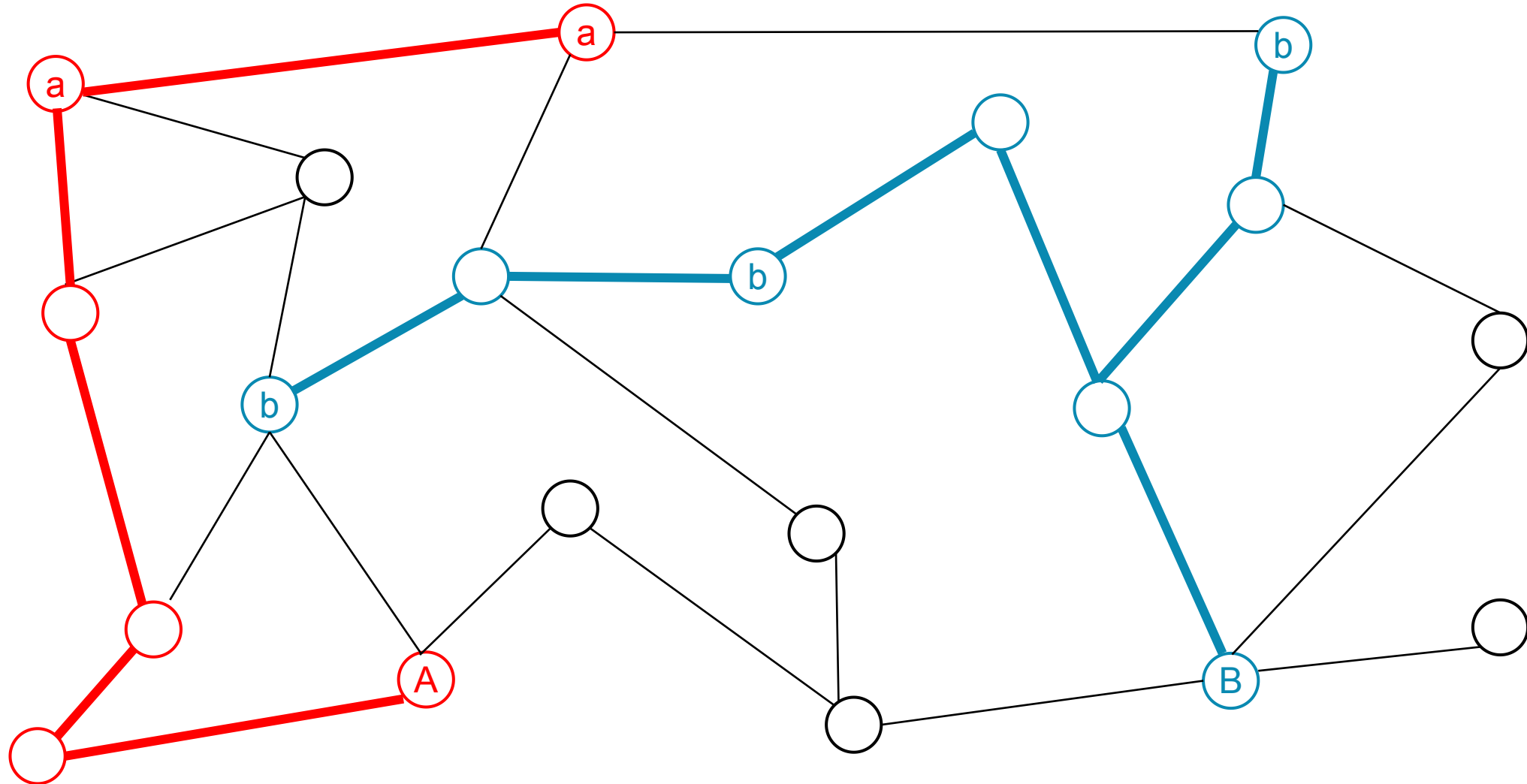


Negotiation Algorithm

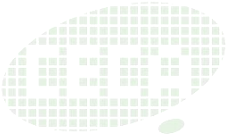




10 Negotiation Algorithm



A* Basics



➤ Best-first search

- Closed/Open set: nodes that has/has not been expanded.

➤ Cost

$$f(n) = g(n) + h(n) \text{ for node } n$$

Cost of path from
source to n

Heuristic function of estimated
cheapest-cost path from n to the goal

- **Admissible:** $h(n)$ never overestimates the actual cost
- **Monotonic (Consistent):**
 $h(n) \leq c(n, n_0) + h(n_0)$ for each n_0 being a successor of n

A* Basics



Algorithm 1: A*

```
1 Initialize OPEN to  $\{s_0\}$ ;  
2 while  $OPEN \neq \emptyset$  do  
3   Get and remove from OPEN a node  $n$  with a smallest  $f(n)$ ;  
4   Add  $n$  to CLOSED;  
5   if  $n$  is a goal node then  
6     | Return solution path from  $s_0$  to  $n$ ;  
7   for every successor  $n'$  of  $n$  do  
8     |  $g_1 = g(n) + c(n, n')$ ;  
9     | if  $n' \in CLOSED$  then  
10      | if  $g_1 < g(n')$  then  
11        | Remove  $n'$  from CLOSED and add it to OPEN;  
12      | else  
13        | Continue;  
14      | else  
15        | if  $n' \notin OPEN$  then  
16          | Add  $n'$  to OPEN;  
17        | else if  $g_1 \geq g(n')$  then  
18          | Continue;  
19      | Set  $g(n') = g_1$ ;  
20      | Set  $f(n') = g(n') + h(n')$ ;  
21      | Set  $\text{parent}(n') = n$ ;  
22 Return failure (no path exists);
```

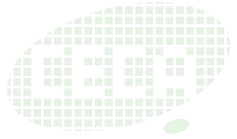


PathFinder

- 我们已实现基于 A* 搜索的串行 PathFinder
 - *fpga-route/src/ace-route.cc/path_finder()*

Algorithm 5: Pseudo-code of the Pathfinder routing algorithm [31]

```
Let:  $RT_i$  be the set of nodes in the current routing of net  $i$ 
while shared resources exist do
  /*Illegal routing*/
  foreach net, i do
    rip-up routing tree  $RT_i$ ;
     $RT(i) = s_i$  foreach sink  $t_{ij}$  do
      Initialize priority queue PQ to  $RT_i$  at cost 0;
      while sink  $t_{ij}$  not found do
        Remove lowest cost node  $m$  from PQ;
        foreach fanout node  $n$  of node  $m$  do
          | Add  $n$  to PQ at  $\text{PathCost}(n) = c_n + \text{PathCost}(m)$ ;
        end
      end
      foreach node  $n$  in path  $t_{ij}$  to  $s_i$  do
        /*backtrace*/
        Update  $c_n$ ;
        Add  $n$  to  $RT_i$ ;
      end
    end
  end
  update  $h_n$  for all  $n$ ;
end
```



Strategy 1 : Intra-Connection Parallelization

- 在单个 connection 寻路过程中并行搜索
- *Parallel A**
- *Hash Distributed A**
- *Parallel Bidirectional A**

Parallel A* Paradigms

► Centralized: Simple Parallel A* (SI)

- Share an open list among threads
- Straightforward, Easy to implement.

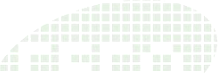
What's the problem?

Algorithm 2: Simple Parallel A* (SPA*)

```

1 Initialize  $OPEN_{shared}$  to  $\{s_0\}$ ;
2 Initialize Lock  $l_o, l_i$ ;
3 Initialize  $incumbent.cost = \infty$ ;
4 In parallel, on each thread, execute 5-32;
5 while  $TerminateDetection()$  do
6   if  $OPEN_{shared} = \emptyset$  or Smallest  $f(n)$  value of  $n \in OPEN_{shared} \geq incumbent.cost$  then
7     Continue;
8   AcquireLock( $l_o$ );
9   Get and remove from  $OPEN_{shared}$  a node  $n$  with a smallest  $f(n)$ ;
10  ReleaseLock( $l_o$ );
11  Add  $n$  to  $CLOSED_{shared}$ ;
12  if  $n$  is a goal node then
13    AcquireLock( $l_i$ );
14    if path cost from  $s_0$  to  $n < incumbent.cost$  then
15       $incumbent =$  path from  $s_0$  to  $n$ ;
16       $incumbent.cost =$  path cost from  $s_0$  to  $n$ ;
17    ReleaseLock( $l_i$ );
18  for every successor  $n'$  of  $n$  do
19     $g_1 = g(n) + c(n, n')$ ;
20    if  $n' \in CLOSED_{shared}$  then
21      if  $g_1 < g(n')$  then
22        Remove  $n'$  from  $CLOSED_{shared}$  and add it to  $OPEN_{shared}$ ;
23      else
24        Continue;
25    else
26      if  $n' \notin OPEN_{shared}$  then
27        Add  $n'$  to  $OPEN_{shared}$ ;
28      else if  $g_1 \geq g(n')$  then
29        Continue;
30    Set  $g(n') = g_1$ ;
31    Set  $f(n') = g(n') + h(n')$ ;
32    Set parent( $n') = n$ ;
33 if  $incumbent.cost = \infty$  then
34   Return failure (no path exists);
35 else
36   Return solution path from  $s_0$  to  $n$ ;

```



Parallel A* Paradigms

Decentralized:

- Each thread has its own open list

What's the problem?

- Load balancing
- Termination detection
- Duplication elimination

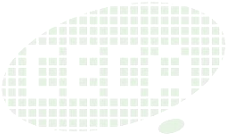
Algorithm 3: Decentralized A* with Local OPEN/CLOSED lists

```

1 Initialize  $OPEN_p$  for each thread  $p$ ;
2 Initialize  $incumbent.cost = \infty$ ;
3 Add  $s_0$  to  $OPEN_{ComputeRecipient(s_0)}$ ;
4 In parallel, on each thread  $p$ , execute 5-31;
5 while  $TerminateDetection()$  do
6   while  $BUFFER_p \neq \emptyset$  do
7     Get and remove from  $BUFFER_p$  a triplet  $(n', g_1, n)$ ;
8     if  $n' \in CLOSED_p$  then
9       if  $g_1 < g(n')$  then
10        | Remove  $n'$  from  $CLOSED_p$  and add it to  $OPEN_p$ ;
11      else
12        | Continue;
13    else
14      if  $n' \notin OPEN_p$  then
15        | Add  $n'$  to  $OPEN_p$ ;
16      else if  $g_1 \geq g(n')$  then
17        | Continue;
18      Set  $g(n') = g_1$ ;
19      Set  $f(n') = g(n') + h(n')$ ;
20      Set  $parent(n') = n$ ;
21 if  $OPEN_p = \emptyset$  or  $Smallest\ f(n)\ value\ of\ n \in OPEN_p \geq incumbent.cost$  then
22   | Continue;
23 Get and remove from  $OPEN_p$  a node  $n$  with a smallest  $f(n)$ ;
24 Add  $n$  to  $CLOSED_p$ ;
25 if  $n$  is a goal node then
26   if path cost from  $s_0$  to  $n < incumbent.cost$  then
27     |  $incumbent =$  path from  $s_0$  to  $n$ ;
28     |  $incumbent.cost =$  path cost from  $s_0$  to  $n$ ;
29 for every successor  $n'$  of  $n$  do
30   | Set  $g_1 = g(n) + c(n, n')$ ;
31   | Add  $(n', g_1, n)$  to  $BUFFER_{ComputeRecipient(n)}$ ;
32 if  $incumbent.cost = \infty$  then
33   | Return failure (no path exists);
34 else
35   | Return solution path from  $s_0$  to  $n$ ;

```


Hash Distributed A^*



HDA* starts by expanding the initial state at the root processor. Then, each processor P executes the following loop until an optimal solution is found:

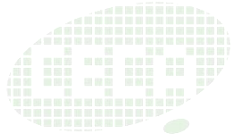
1. First, P checks whether one or more new states have been received in its message queue. If so, P checks for each new state s in $Closed_P$, in order to determine whether s is a duplicate, or whether it should be inserted in $Open_P$.²
2. If the message queue is empty, then P selects a highest priority state from $Open_P$ and expands it, resulting in newly generated states. For each newly generated state s , a hash key $K(s)$ is computed based on the state representation, and the $reK(s)$ and s is sent to the processor that owns $K(s)$. This send is asynchronous and non-blocking. P continues its computation without waiting for a reply from the destination.



Parallel Bidirectional A*

Algorithm 1 PNBA*, Parallel New Bidirectional A*

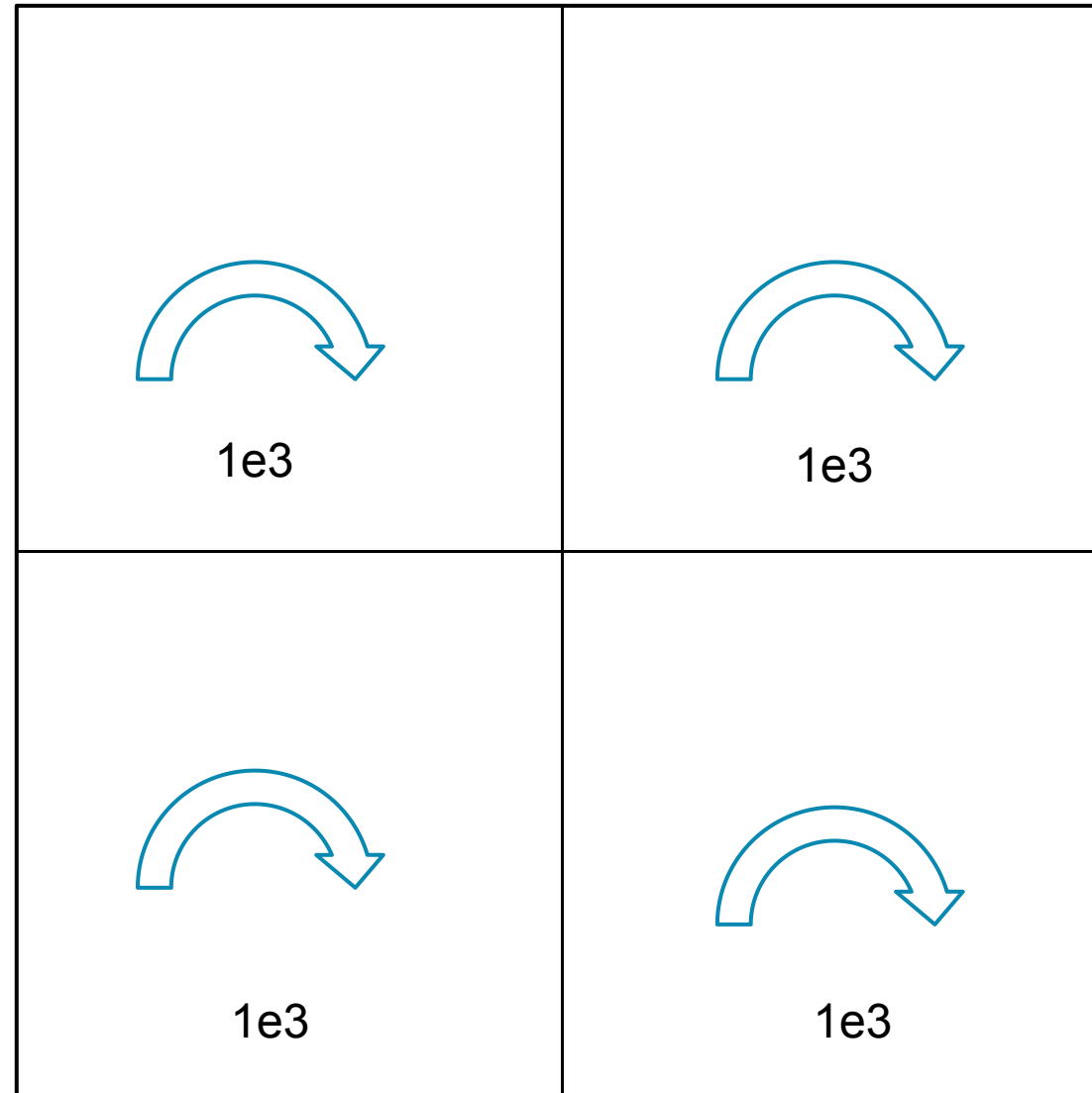
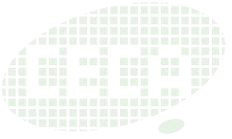
```
1: while  $\neg$ finished do
2:    $x \leftarrow \text{open}_1.\text{pop}()$ 
3:   if  $x \in \mathcal{M}$  then
4:     if  $(f_1(x) < \mathcal{L}) \wedge (g_1(x) + F_2 - h_2(x) < \mathcal{L})$  then
5:       for all edges  $(x, y)$  of the graph being explored do
6:         if  $(y \in \mathcal{M}) \wedge (g_1(y) > g_1(x) + d_1(x, y))$  then
7:            $g_1(y) \leftarrow g_1(x) + d_1(x, y)$ 
8:            $f_1(y) \leftarrow g_1(y) + h_1(y)$ 
9:           if  $y \in \text{open}_1$  then
10:             $\text{open}_1.\text{remove}(y)$ 
11:             $\text{open}_1.\text{insert}(y)$ 
12:            if  $g_1(y) + g_2(y) < \mathcal{L}$  then
13:              lock
14:              if  $g_1(y) + g_2(y) < \mathcal{L}$  then
15:                 $\mathcal{L} \leftarrow g_1(y) + g_2(y)$ 
16:              unlock
17:            $\mathcal{M} \leftarrow \mathcal{M} - \{x\}$ 
18:   if  $\text{open}_1.\text{size}() > 0$  then
19:      $F_1 \leftarrow f_1(\text{open}_1.\text{peek}())$ 
20:   else
21:     finished  $\leftarrow$  true
```



Strategy 2 : Inter-Connection Parallelization

- 不同 connection 并行进行寻路过程
- 按区域划分 connection
- *Intuition Parallel*
- *Dense Parallel*

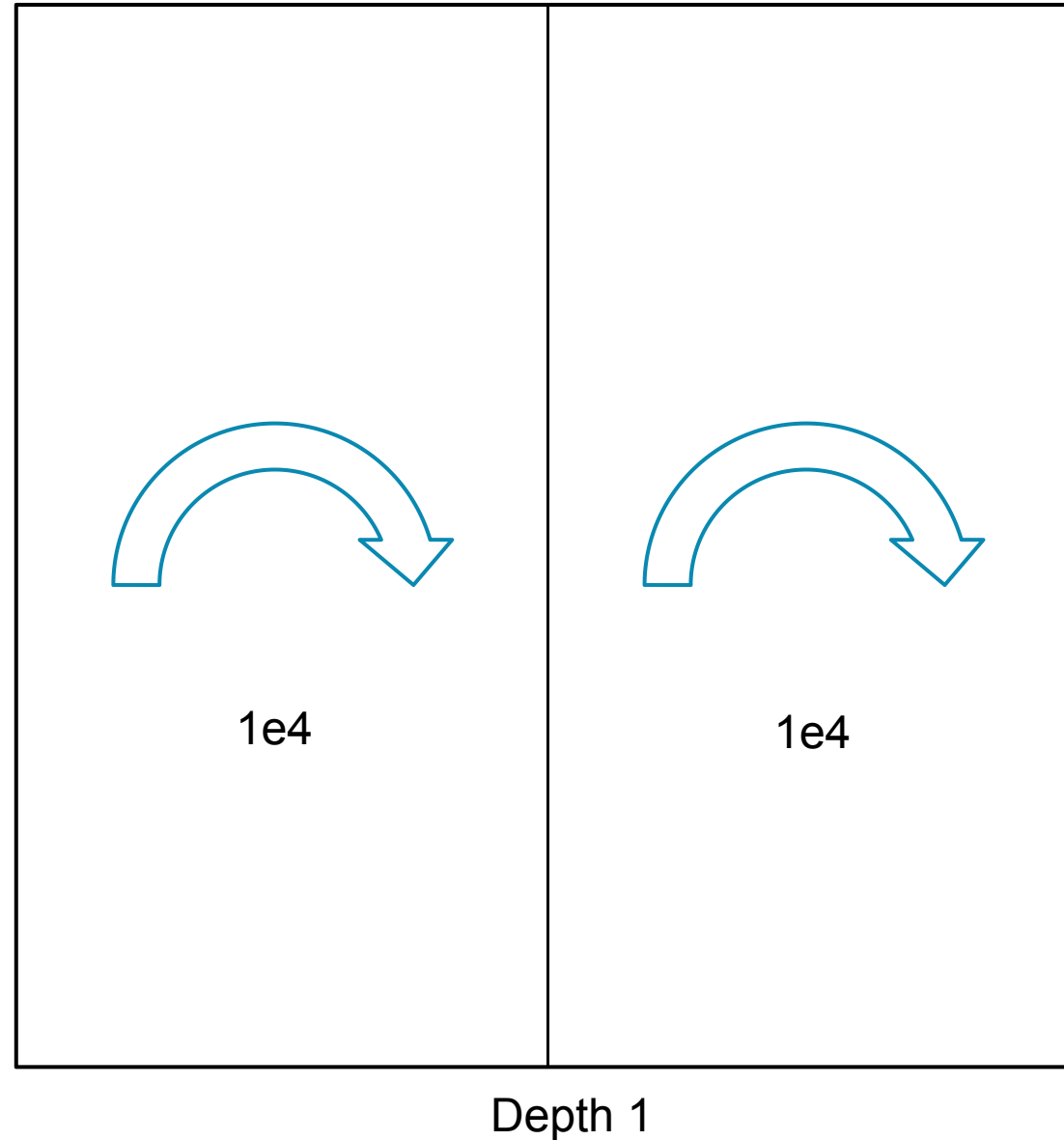
Intuition Parallel Method



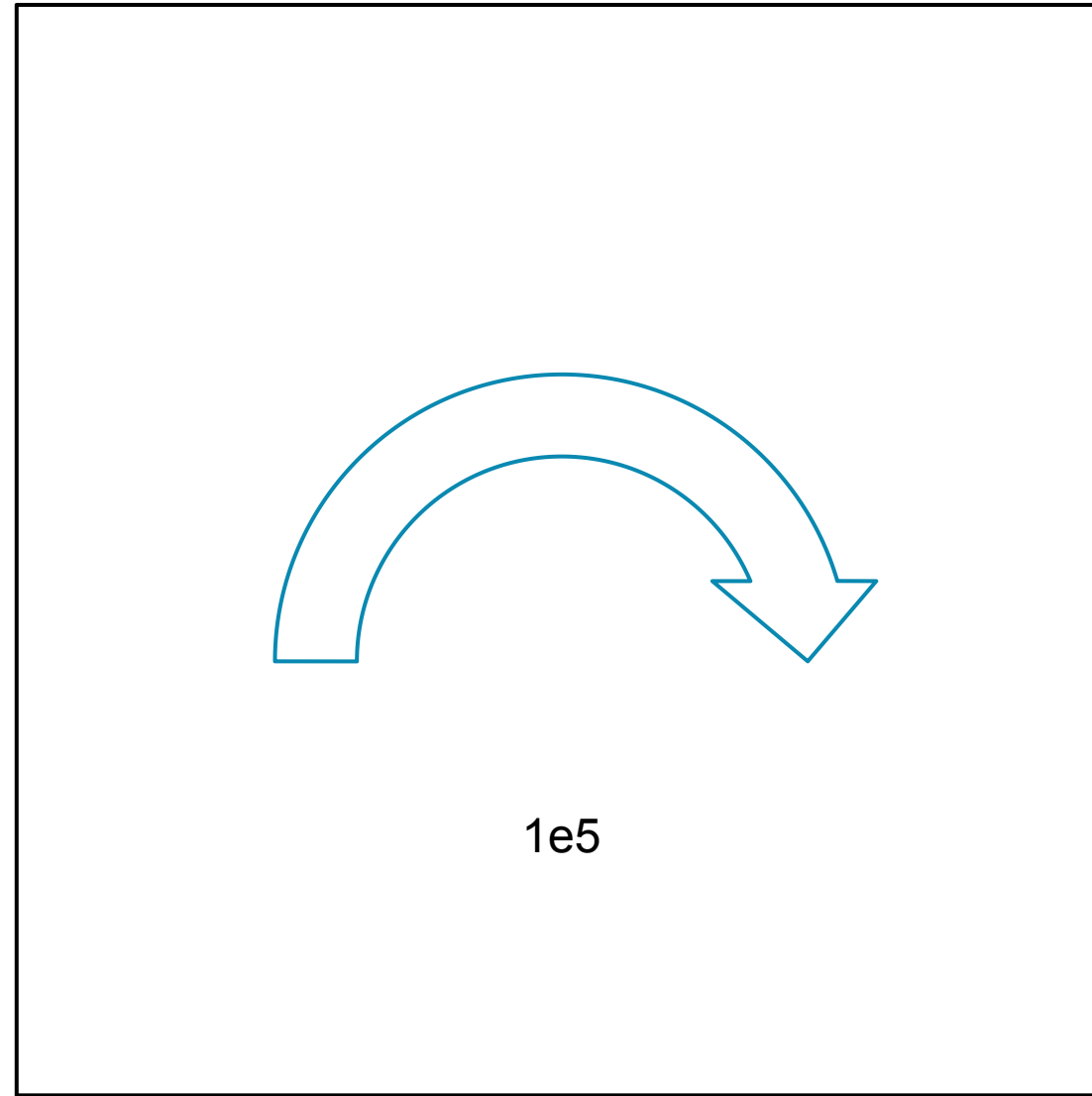
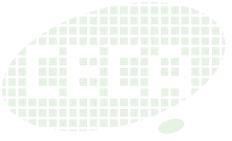
Parallelizable division and conquer based on spatial partitioning
Depth 2



21 Intuition Parallel Method



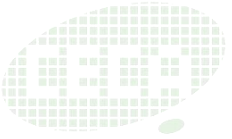
Intuition Parallel Method



1e5

Depth 0

Intuition Parallel Method



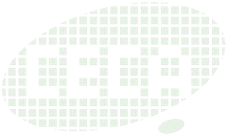
➤ Parallelism is restricted by overlapped nets.

➤ E.g.
$$\frac{1000 + 10000 + 100000}{1000 * 4 + 10000 * 2 + 100000} \approx 89.5\%$$

Dense Parallel



- Idea: Allow Overlapped Nets to be paralleled
- I. Modify underlying data structure and design synchronization mechanism to allow one node to be explored in multiple threads simultaneously
- II. Optimize synchronization mechanism to limit overhead



Dependencies in this A* search:

READ

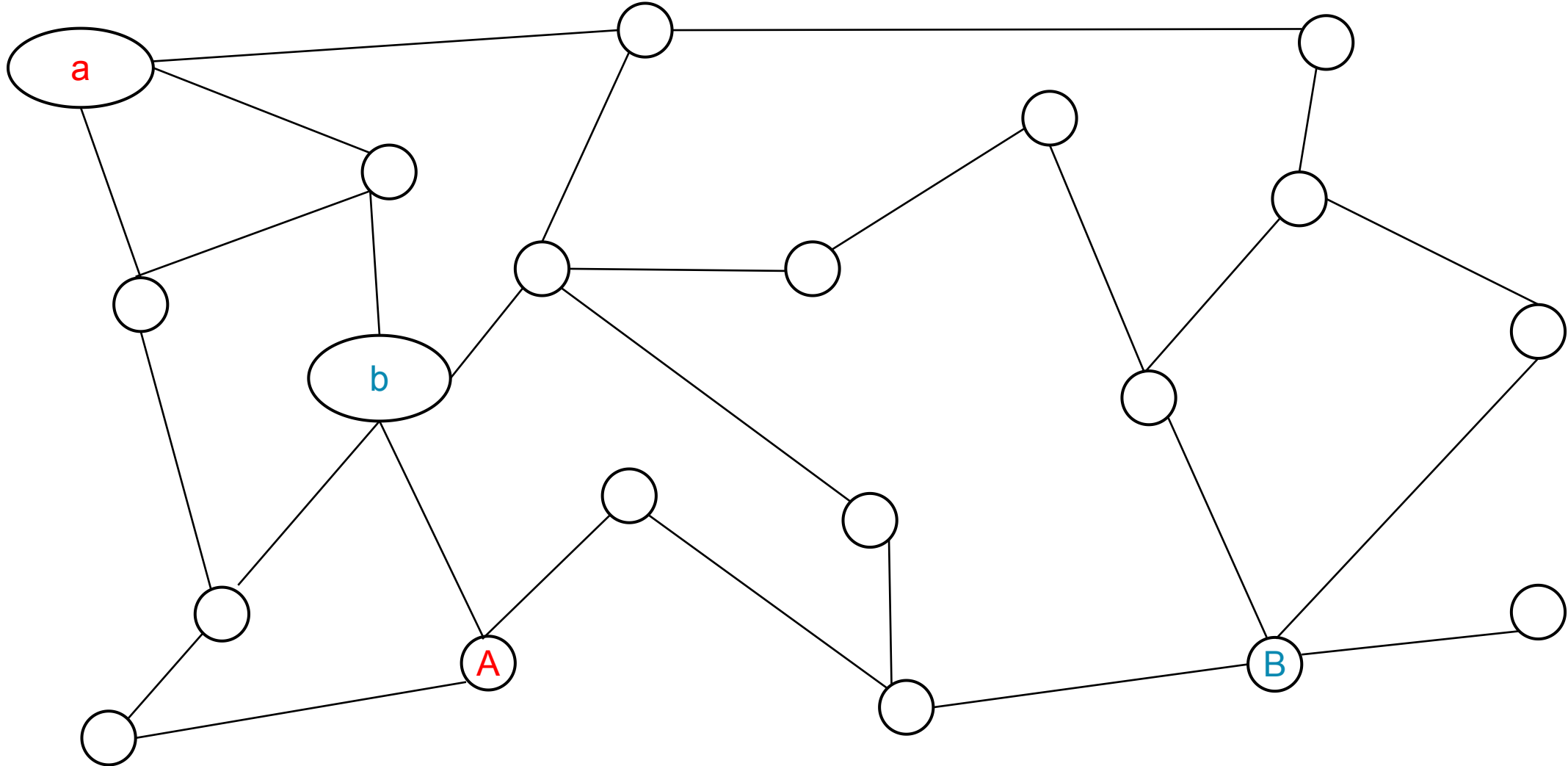
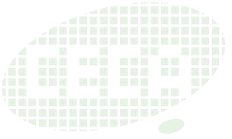
- Underlying graph
- Historical congestion cost

WRITE

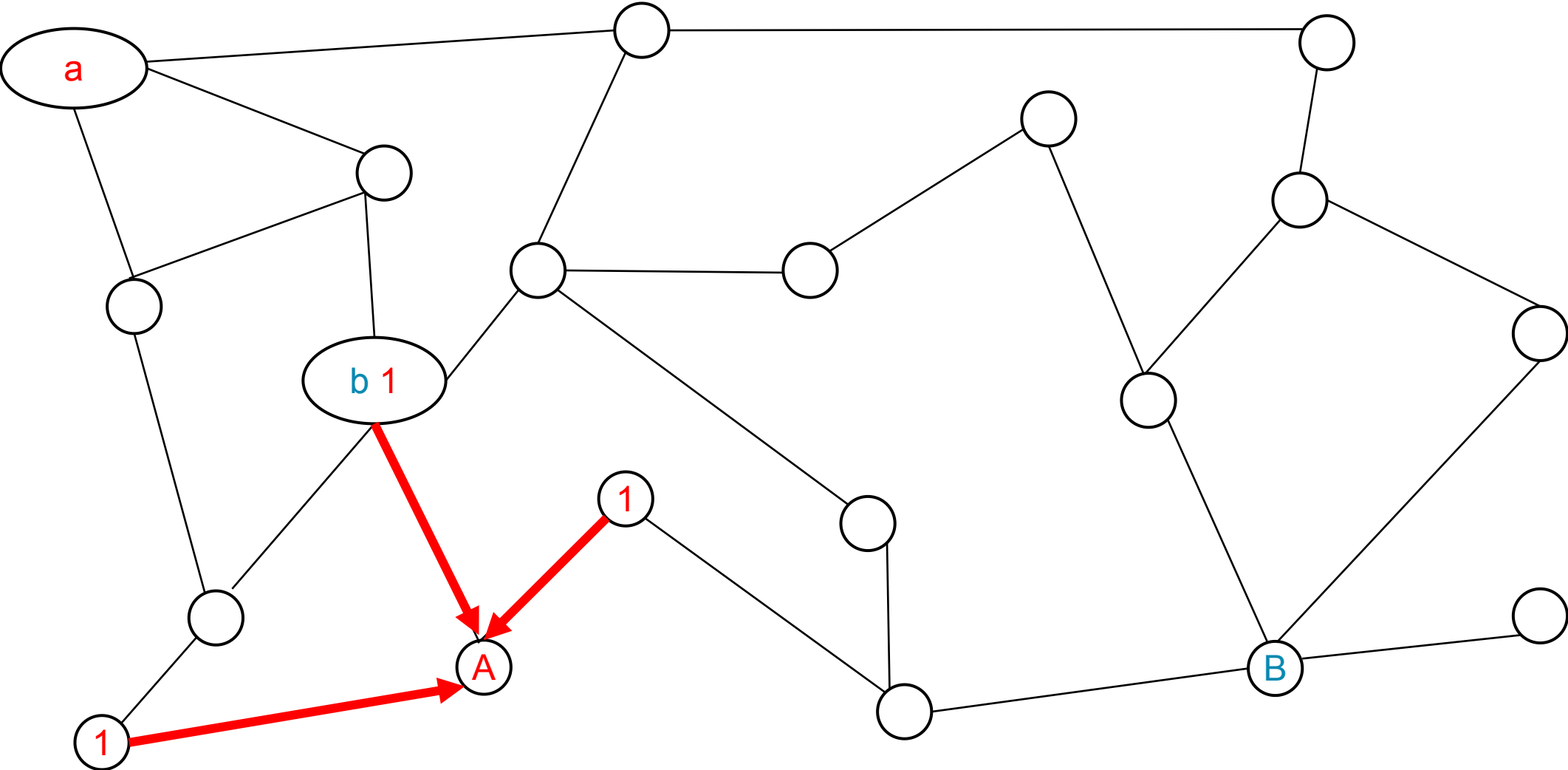
- Nodes visited
- Node cost in path
- Node's previous node path
- Node occupancy status

Part I

Parallelism Construction

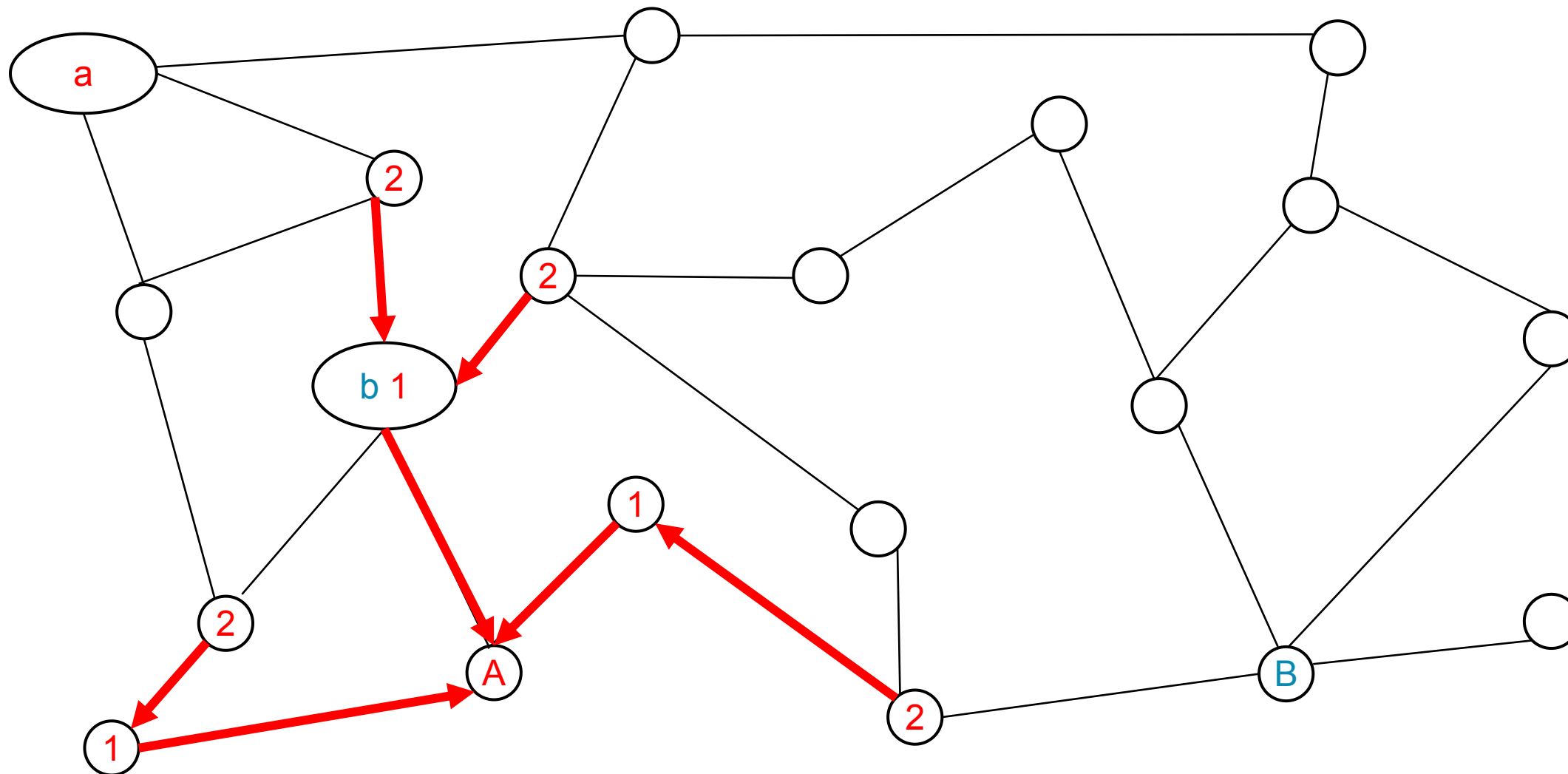


Part I Parallelism Construction



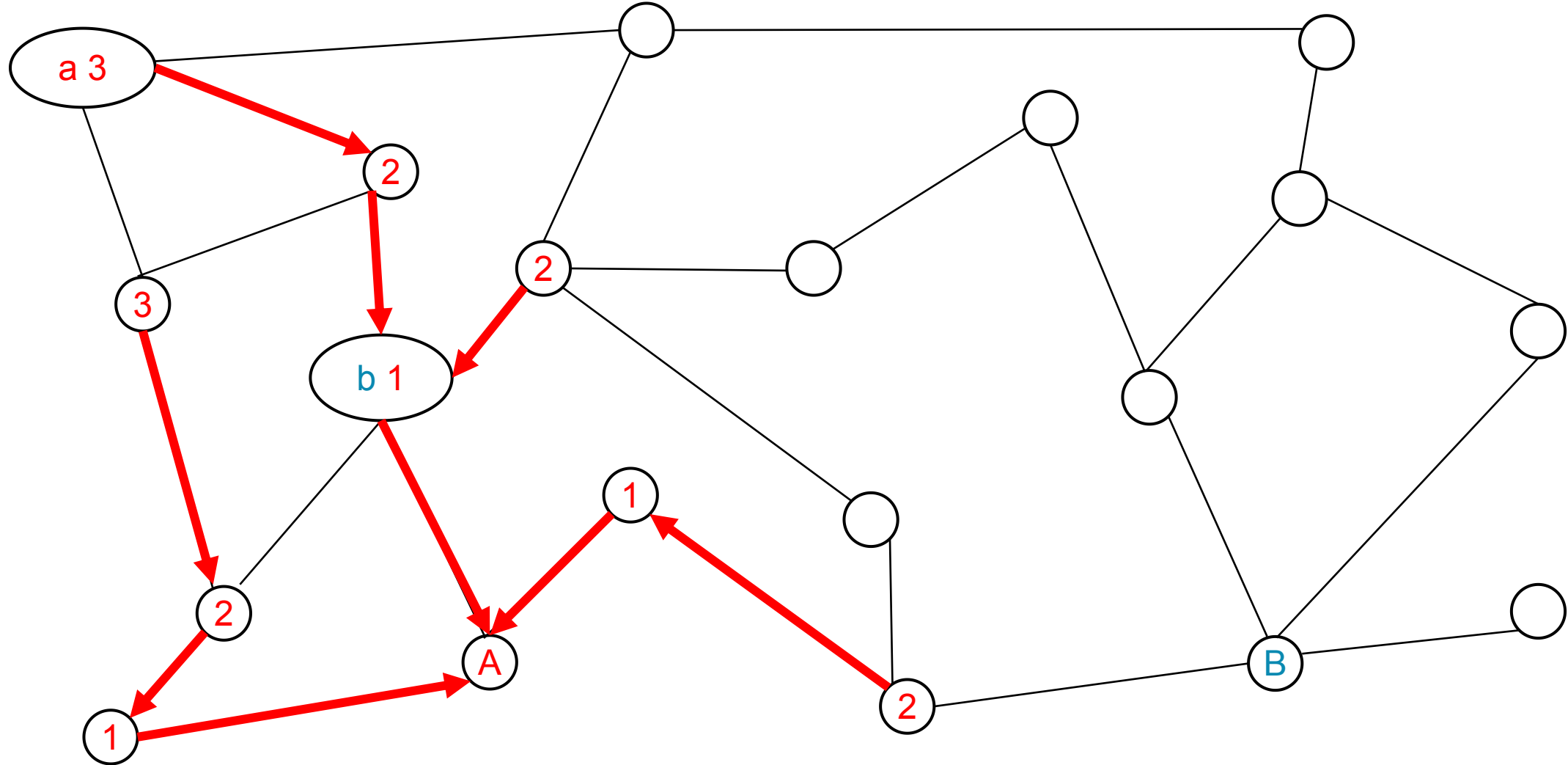
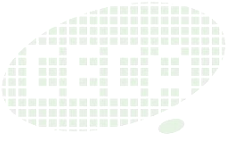
Part I

Parallelism Construction



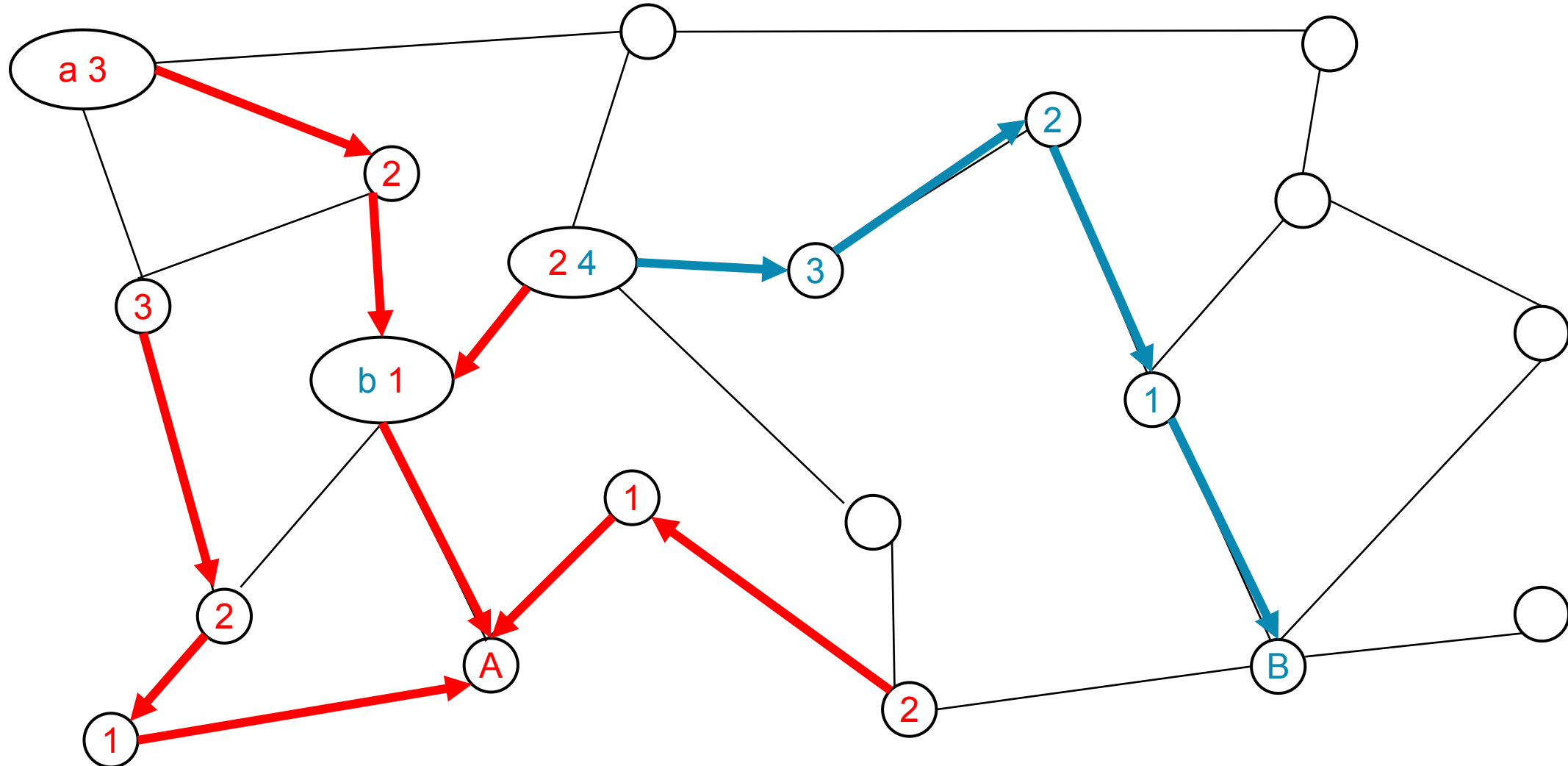
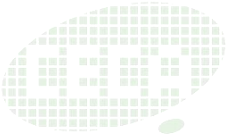
Part I

Parallelism Construction

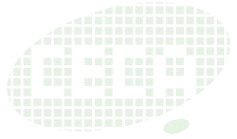


Part I

Parallelism Construction



Thread Private: Visited; Cost in Path; Path(Previous Node)



Dependencies in this A* search:

READ

Shared

- Underlying graph
- Historical congestion cost

WRITE

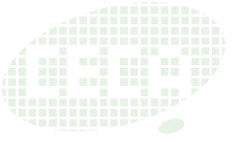
Thread Private

- Nodes visited
- Node cost in path
- Node's previous node path

Shared With Lock

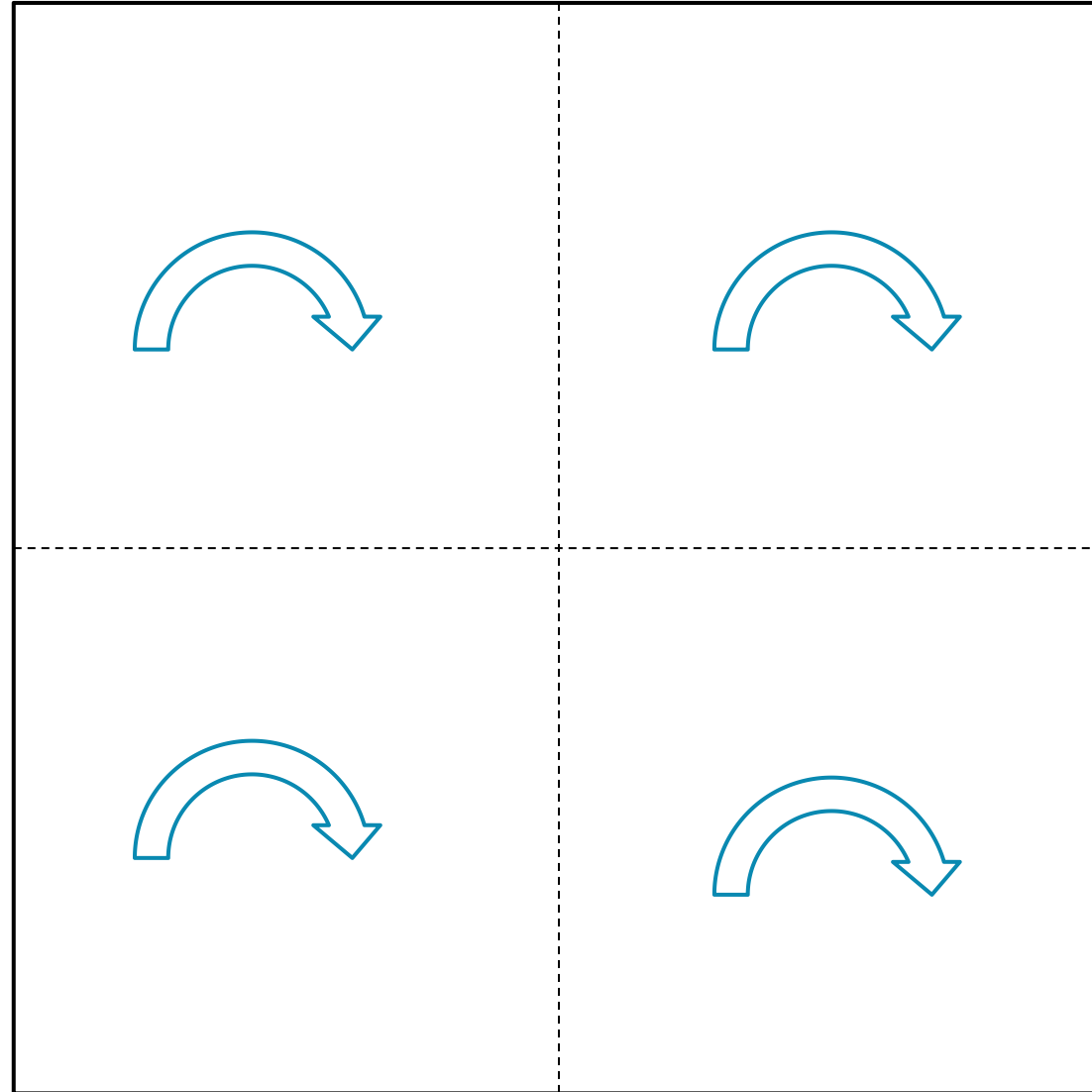
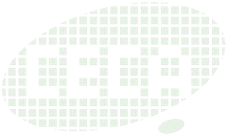
- Node occupancy status

Part II Synchronization Optimization



- 1. Sort Strategy
- 2. Atomic Built flag
- 3. Dynamic bunch node allocator
- 4. Lock-Free No-Delete hash map

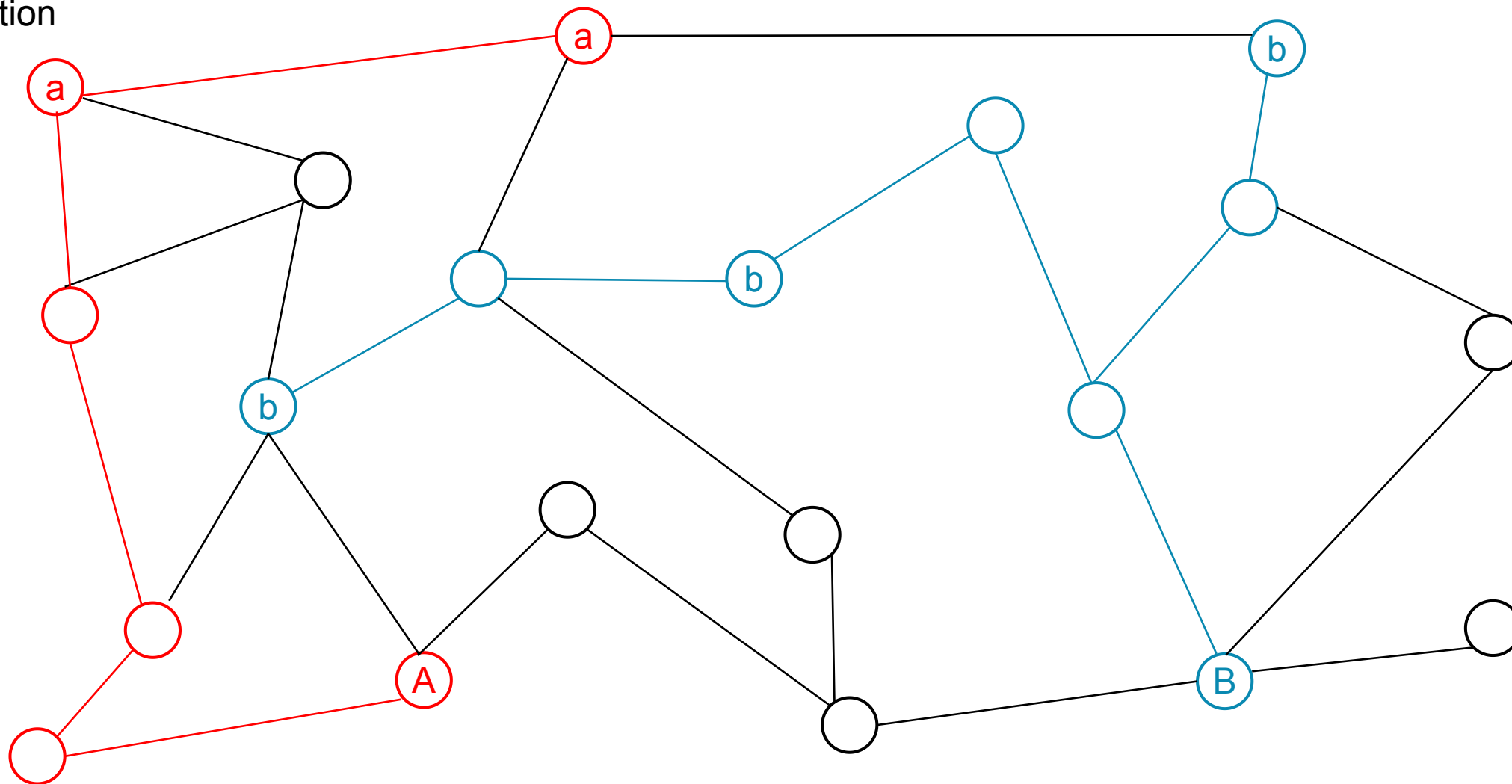
1. Sort Strategy

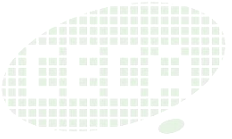


2. Atomic Built flag

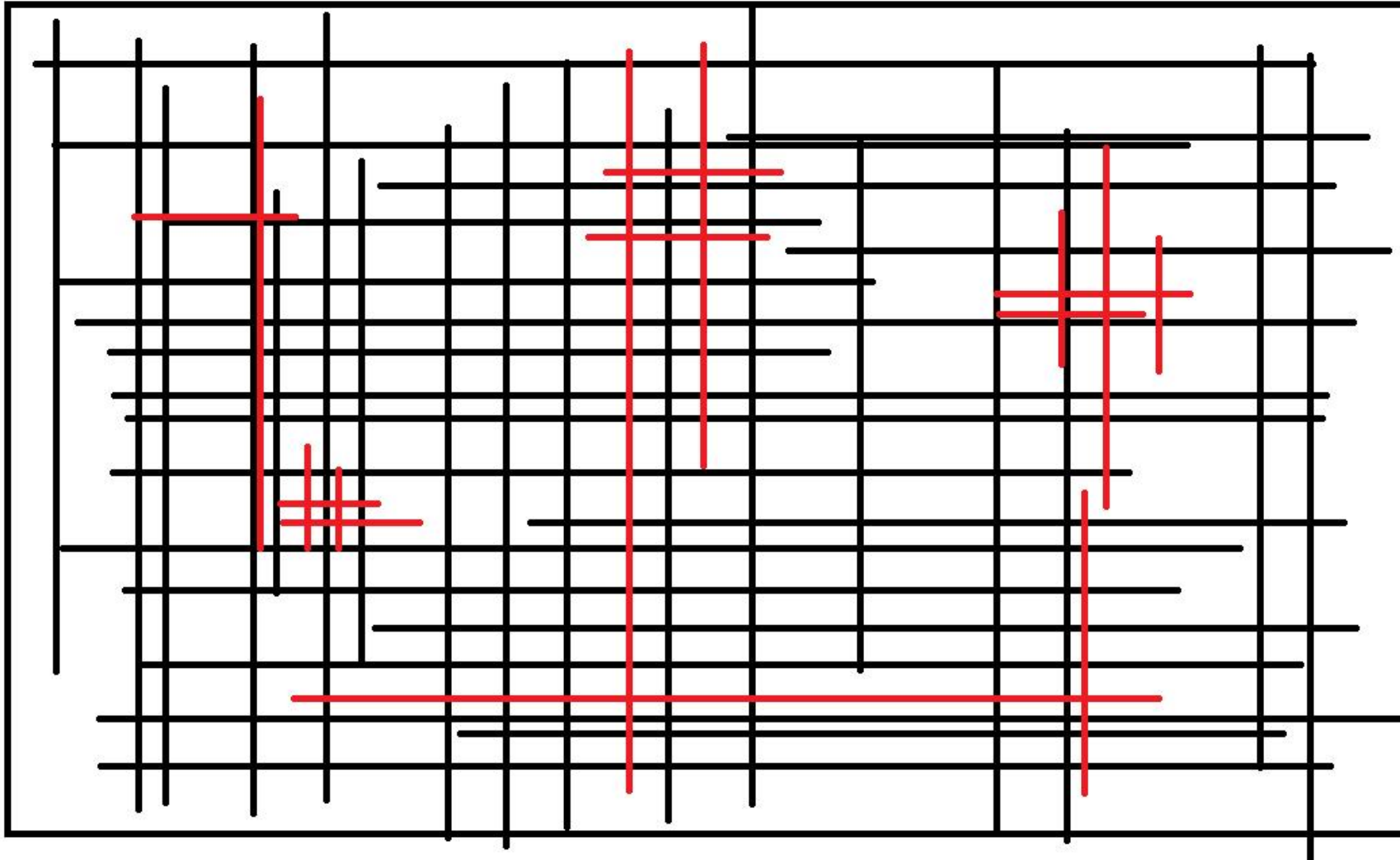


In Imagination



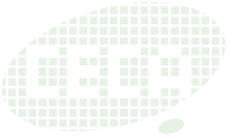


In Fact



Build real nodes and edges dynamically to cut branches and save construction time

2. Atomic Built flag



Dependencies in this A* search:

READ

Shared

- Historical congestion cost
- Underlying graph

WRITE

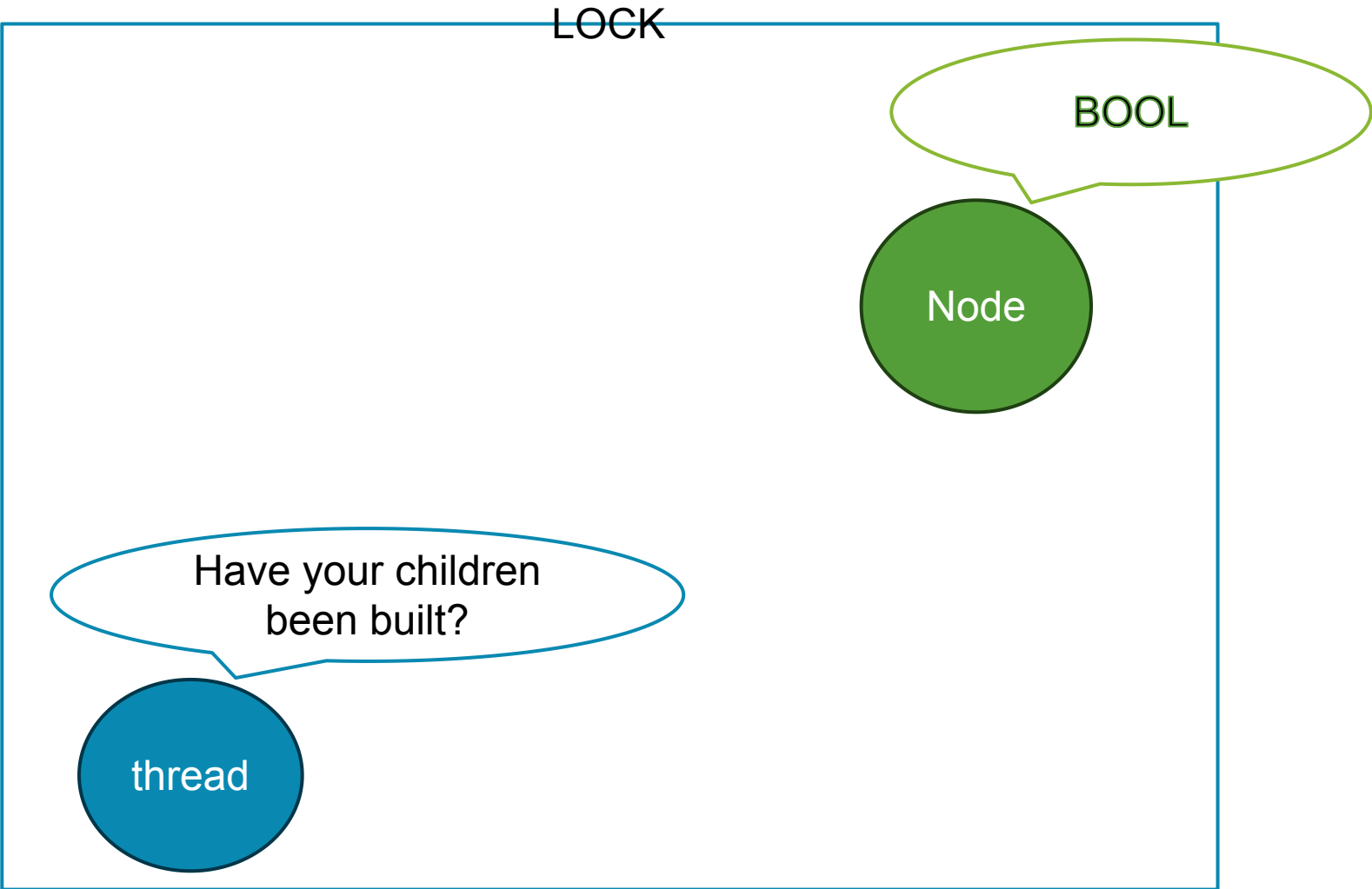
Thread Private

- Nodes visited
- Node cost in path
- Node's previous node path

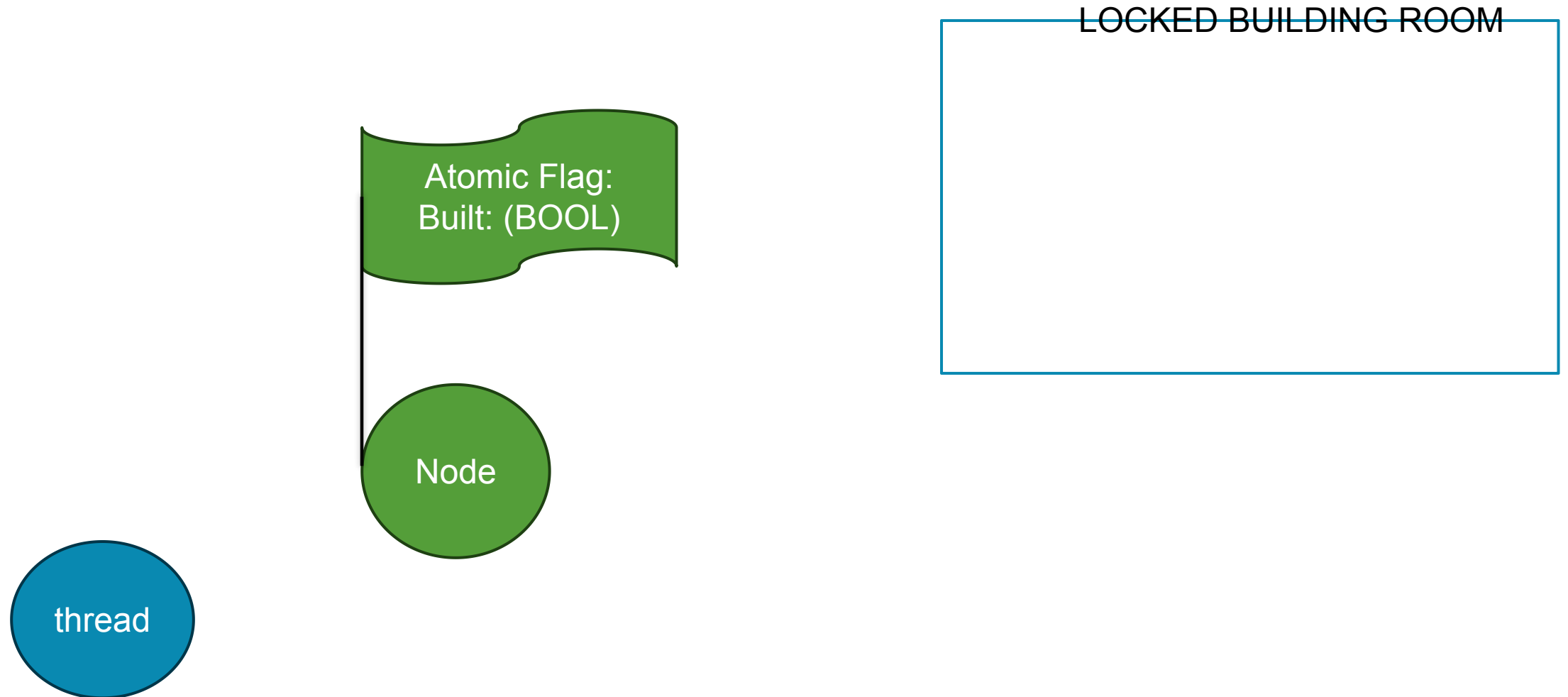
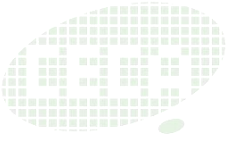
Shared With Lock

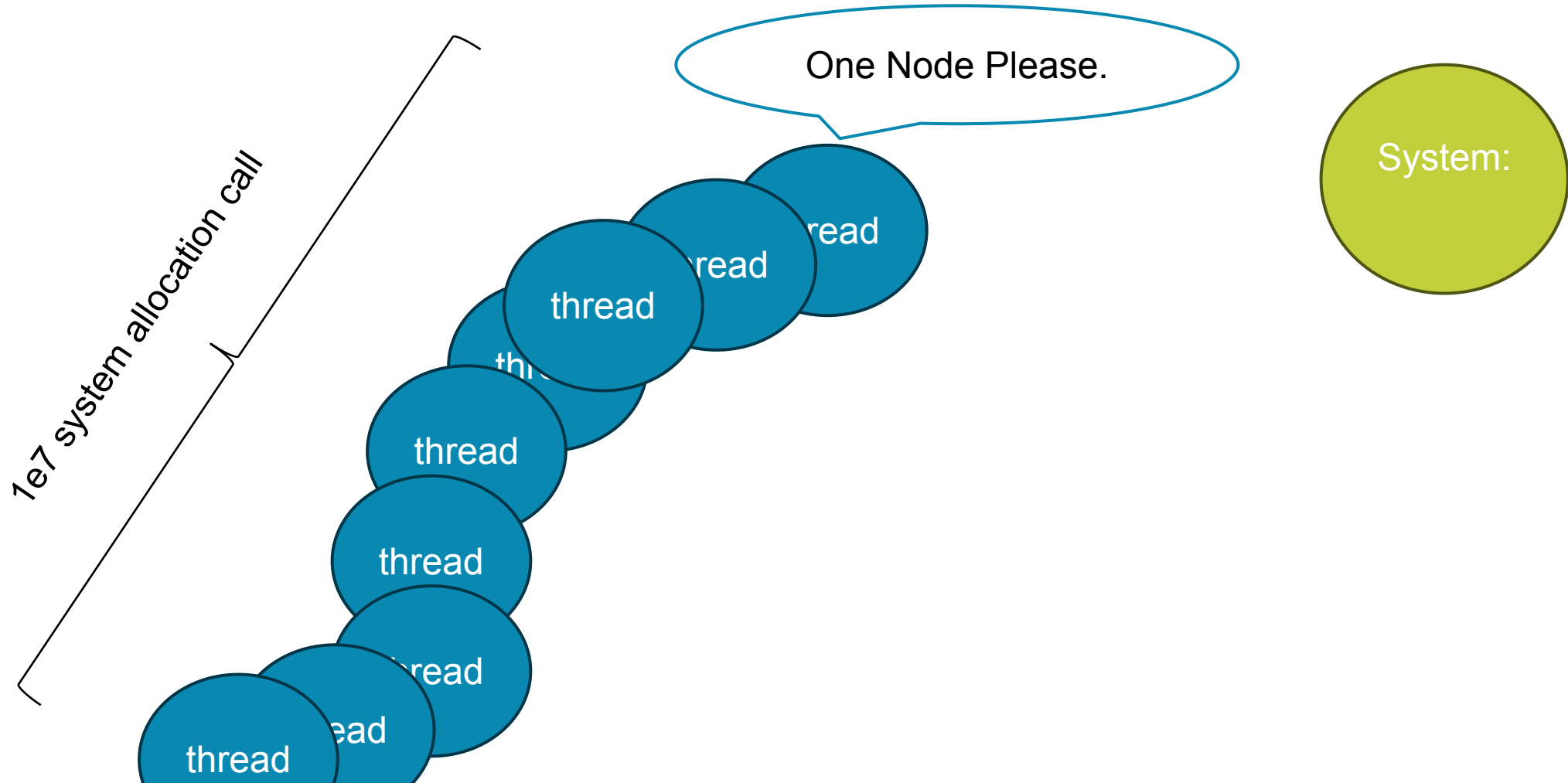
- Node occupancy status

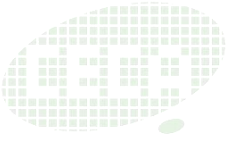
2. Atomic Built flag



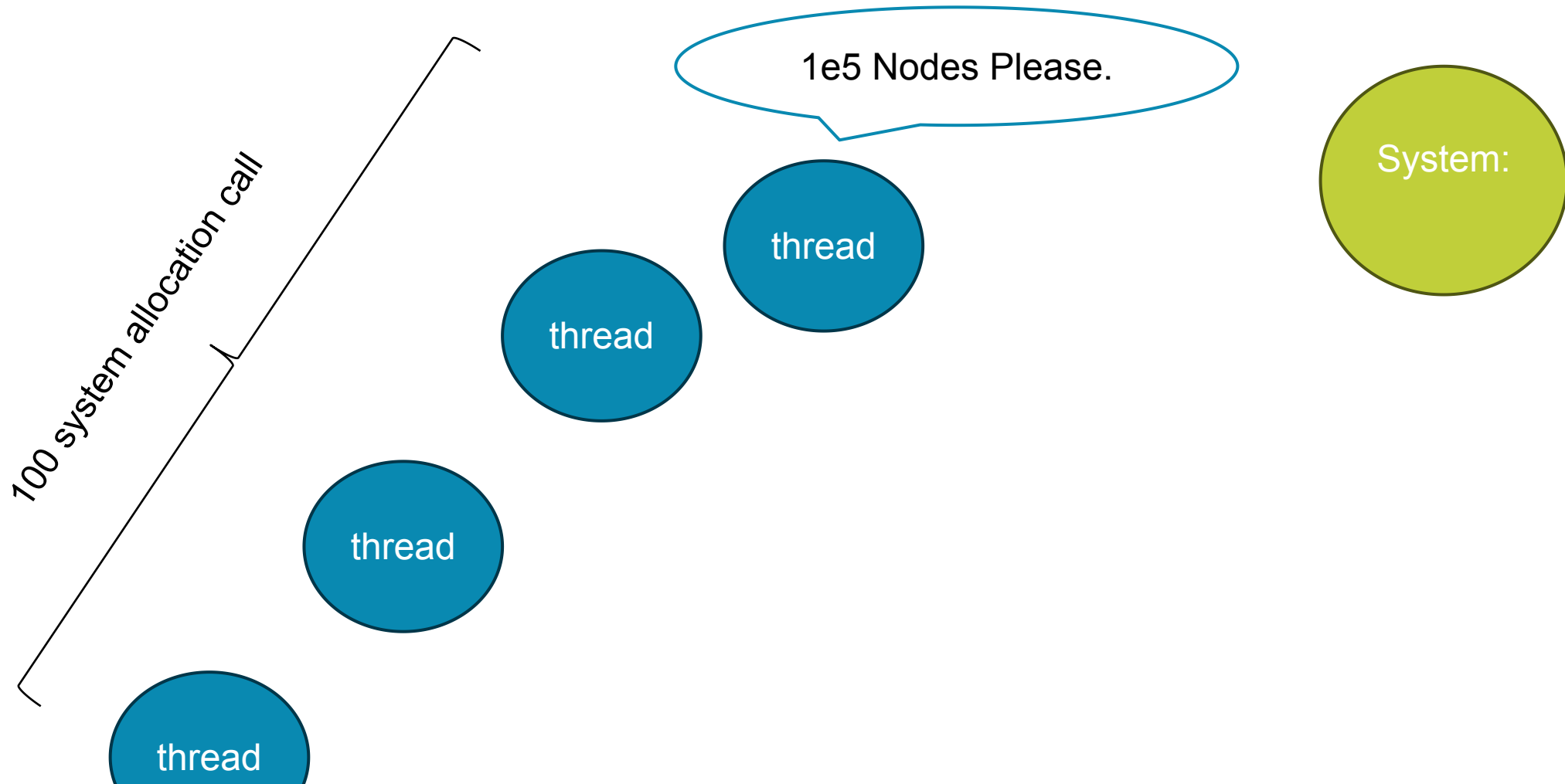
2. Atomic Built flag





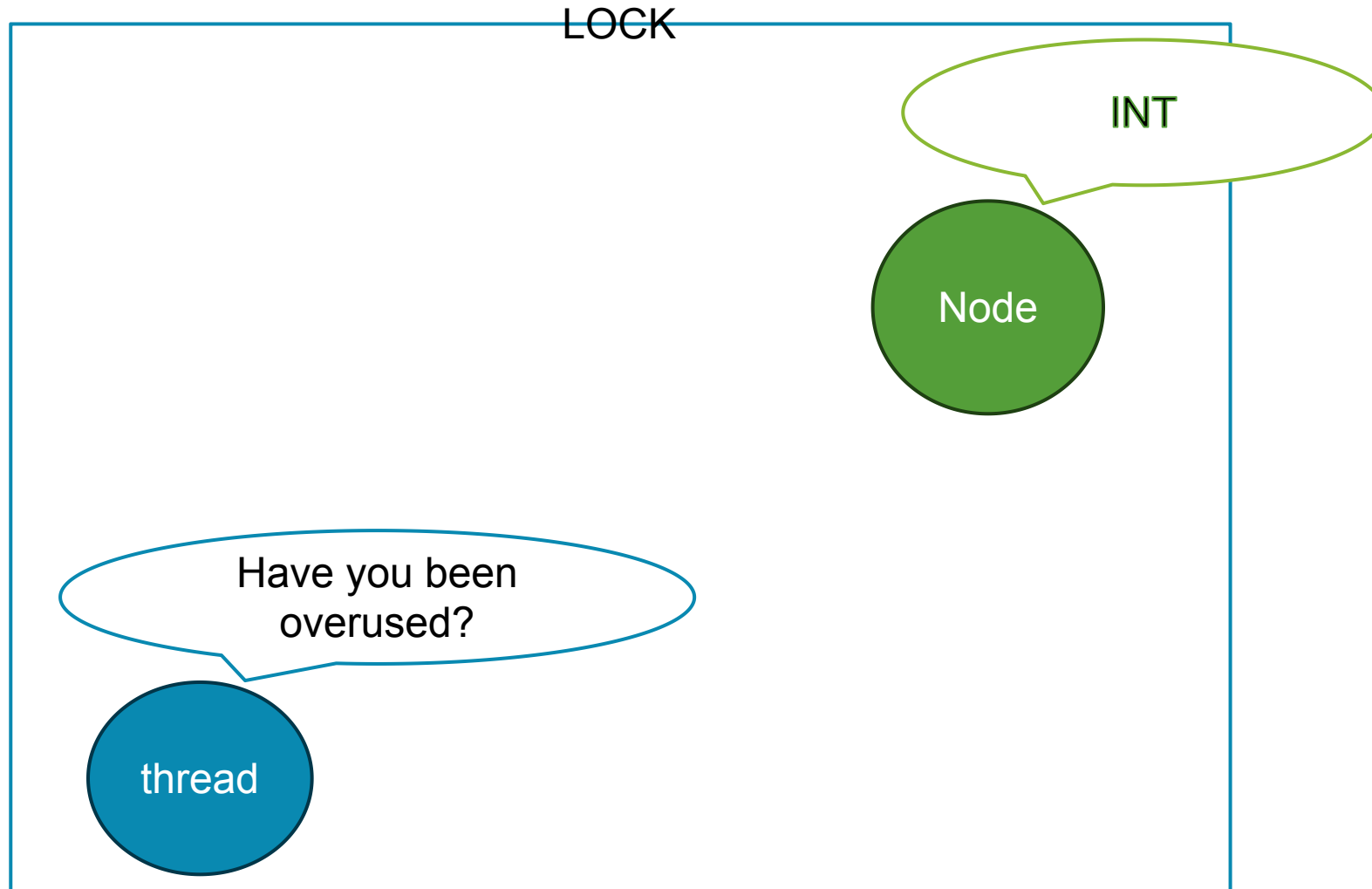


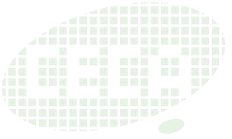
After Implementing a thread private node allocator



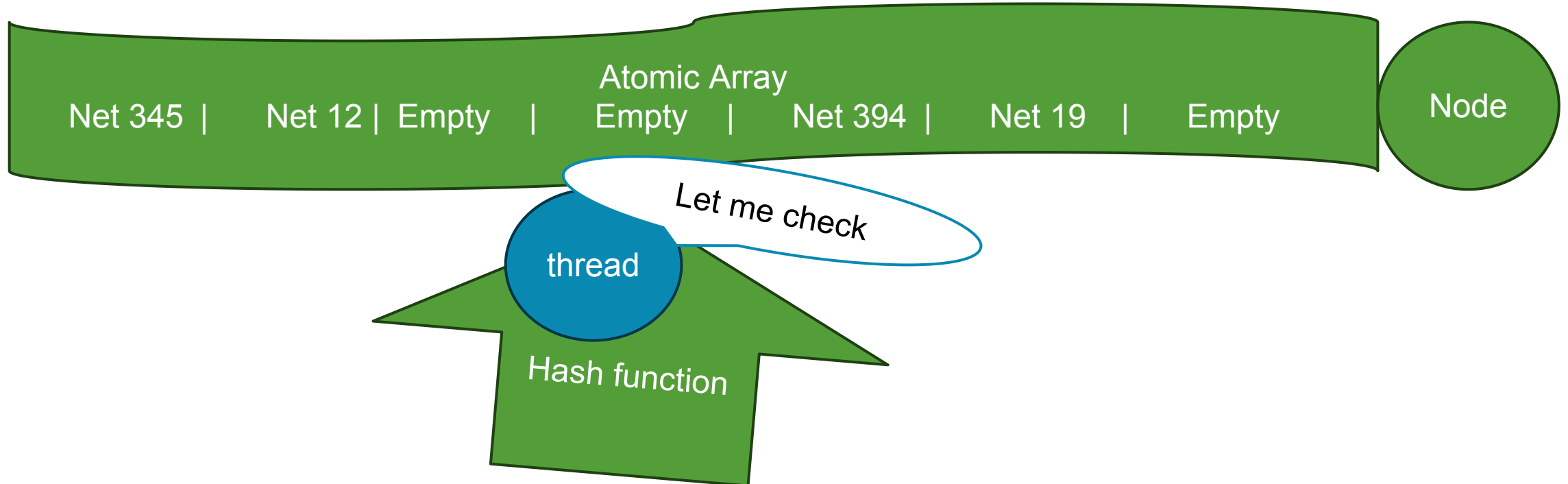


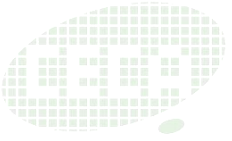
4.Lock-Free No-Delete hash map





LOCK





Under 32 Threads:

➤ 1.Sort Strategy

➤ 80%->60%

➤ 2. Atomic Built flag

➤ 60%->40%

➤ 3. Dynamic bunch node allocator

➤ 40%->10%

➤ 4. Lock-Free No-Delete hash map

➤ 10%->5%

Result Example (Bidirectional Parallel A*)



Table 2: Critical-path wirelength normalized with RWRRoute across all designs (min/max/mean) for different routers.

		Min	Max	Mean
Base Routers	RWRRoute	1.00	1.00	1.00
	Vivado	0.83	1.51	1.12
Intra-Connection Opt. Only	Basic	0.91	1.23	1.03
	BE (Serial)	0.82	1.26	1.02
	BE (Parallel)	0.89	1.62	1.08
	Adaptive	0.92	1.25	1.04
Inter-Connection Par. × Intra-Connection Opt.	Basic	0.83	1.64	1.03
	BE (Parallel)	0.90	1.26	1.04
	Adaptive	0.82	1.27	1.01

Table 3: Accelerated inter-connection parallelism by intra-connection optimizations, with speedup over RWRRoute.

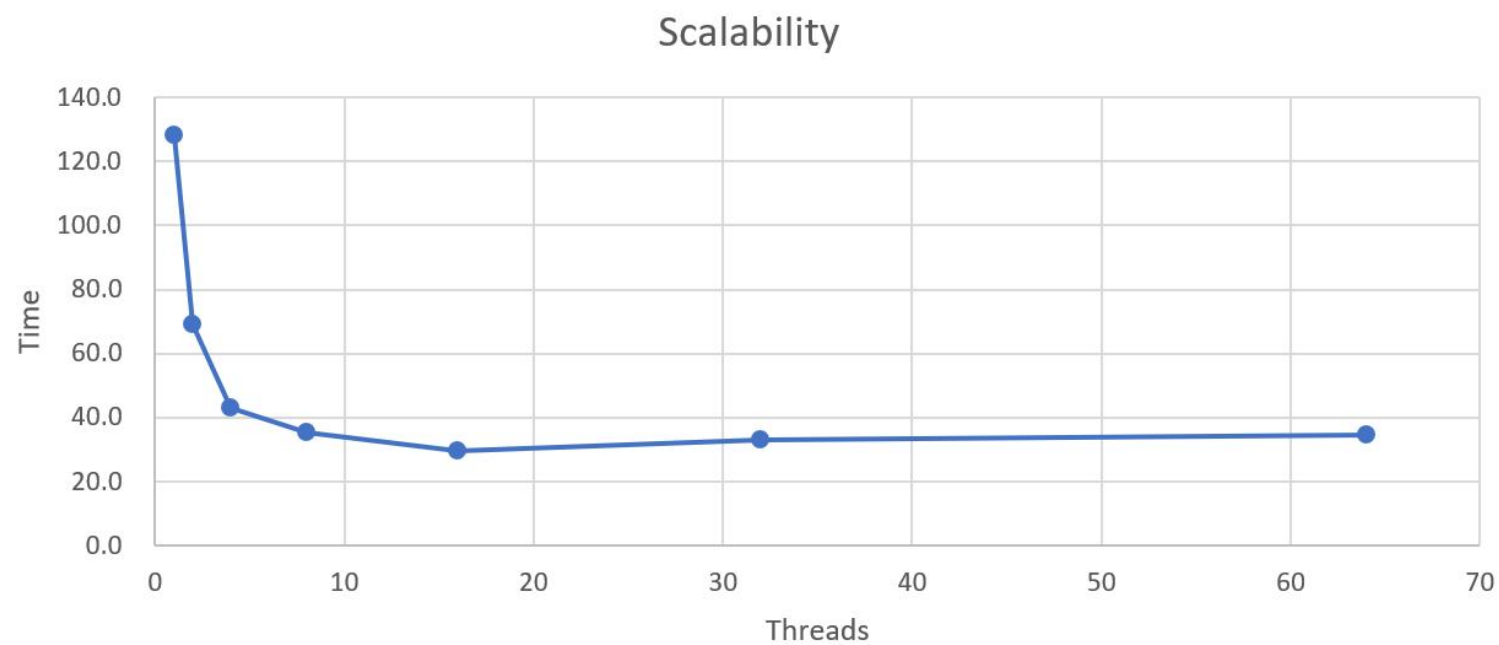
Bi-partition Par. ×	Basic Speedup	BE (2-thread)		Adapt Speedup
		Runtime Var.	Speedup	
boom_med_pb	1.41×	(-5.8%, +9.1%)	3.48×	2.87×
boom_soc	1.61×	(-11.7%, +10.8%)	3.93×	3.74×
boom_soc_v2	1.94×	(-5.8%, +4.7%)	6.90×	6.19×
corescore_500	2.86×	(-4.3%, +3.8%)	3.65×	2.93×
corescore_500_pb	2.57×	(-12.0%, +8.3%)	4.11×	3.47×
corescore_1700	3.26×	(-3.5%, +3.9%)	8.02×	6.64×
corundum_25g	1.90×	(-1.3%, +3.8%)	2.75×	2.21×
ispd16_example2	3.01×	(-6.4%, +12.7%)	2.61×	2.63×
koios_dla_large	3.57×	(-1.1%, +1.4%)	4.25×	3.31×
mlcad_d181	1.23×	(-8.0%, +11.8%)	6.13×	6.09×
mlcad_d181_left...	1.59×	(-6.1%, +5.5%)	9.80×	10.23×
rosetta_fd	1.46×	(-3.6%, +4.1%)	3.61×	2.77×
vtr_lu64peeng	2.45×	(-8.8%, +7.7%)	3.53×	2.74×
vtr_mcml	2.25×	(-4.0%, +5.8%)	7.17×	5.23×
Average	2.22×	(-5.9%, +6.7%)	5.01×	4.36×

*Like Tab. 1, BE (2-thread) integrated router has 8 runs, and others 3 runs.

Result Example (Dense Parallel)



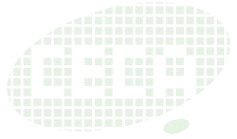
threads	time/s	ratio	This is running on corescore_500_unrouted Intel(R) Xeon(R) Gold 6248R CPU @ 3.00GHz
1	128.3		
2	69.3	1.851371	
4	43.0	1.611628	
8	35.2	1.220204	
16	29.7	1.184936	
32	33.0	0.901212	
64	34.7	0.952381	



评分标准



- Final Project
 - 20% of the total grade
- You will get at most 80% of the final project score if you only implement
 - Basic Parallel A* or Intuition Parallel
- You will get 90% of the final project score if you implement
 - Hash Distributed A* or Parallel Bidirectional A* or Dense Parallel
- You will get 100% of the final project score if you
 - create some novel methods and get better performance compared with our experiment results
- You should submit your **source code** and **experiment report**
 - Your report should include :
 - Brief introduction of your implementation
 - Experiment results (time acceleration and critical path cost)



附录：问题简化

➤ 1. Alt Source 化简

- 每个 net 实际可能有 source 和 alternative_source，其中一个 source 建立连接即可完成 routing
- 我们删掉了需要使用 alt_source 的 connection，每个 connection 只有一个 source 到一个 sink

➤ 2. Connection 化简

- Indirect Connection: source -> source_int -> sink_int -> sink
- Direct Connection: source -> sink
- 本次作业只需要考虑 indirect connection 中 source_int 到 sink_int 的计算

附录：START UP



➤ 从北大网盘下载压缩包

- <https://disk.pku.edu.cn/link/AAD923CB8CC59A48BFA9886F11A026AB9E>

➤ 解压

- `tar -zxvf fpga-route.tar.gz`

➤ 环境配置

- 根据 README 配置环境
- 你可以选择在自己的机器 或 我们提供的AMD服务器配置环境并编程

➤ Benchmark

- 你可以在较小的benchmark上验证算法的初步效果（如 boom_med_pb，约1分钟内布线完成）
- 之后在较大的benchmark进行验证（有些用例可能花费 1000-2000 s）