

$$1a) \boxed{q=69 \ r=19}$$

$$a=2020 \ b=29 \ 0 \leq r < 29$$

$$2020 = q \cdot 29 + r$$

$$b) \boxed{q=69 \ r=19}$$

$$-87 \leq r < 58$$

$$c) \boxed{q=70 \ r=-10}$$

$$0 \leq |r| \leq 14$$

$$d) \boxed{q=87 \ r=19}$$

$$b=23 \ 0 \leq r < 23$$

$$e) \boxed{q=88 \ r=-4}$$

$$-11.5 < r \leq 11.5$$

$$2) \ a, b \in \mathbb{N} \ \&\& \ a > b$$

$$a = \tilde{q} \cdot b + \tilde{r} \quad -\frac{b}{2} < \tilde{r} \leq \frac{b}{2}$$

$$a = \lceil \tilde{q} \cdot b \rceil + \tilde{r} \quad \lceil \tilde{q} \cdot b \rceil - \lfloor \tilde{q} \cdot b \rfloor \leq 1$$

Since the difference between floor & ceiling of $\tilde{q} \cdot b$ is no greater than 1 we should be able to use the ceiling combined with $-\frac{b}{2} < \tilde{r} \leq \frac{b}{2}$

For example: $2020 = \lceil 88 \cdot 23 \rceil = 2024$
 $2020 = \lceil 88 \cdot 23 \rceil - 4$

$$\text{also } \frac{b}{2} - \frac{-b}{2} = b \equiv b - 0$$

$$3a) \gcd(2020, 1234) = 2$$

$$2020 = 1234 + 786$$

$$1234 = 786 + 448$$

$$786 = 448 + 338$$

$$448 = 338 + 110$$

$$338 = 3 \cdot 110 + 8$$

$$110 = 13 \cdot 8 + 6$$

$$8 = 6 + \boxed{2}$$

$$6 = 3 \cdot 2 + 0$$

$$b) 2020x + 1234y = \gcd(2020, 1234)$$

$$2020x + 1234y = 2$$

$$x = 239 \quad y = -257$$

$$4a) \gcd(20202020, 174675) = 685$$

$$20202020 = 115 \cdot 174675 + 114395$$

$$174675 = 114395 + 60280$$

$$114395 = 60280 + 54115$$

$$60280 = 54115 + 6165$$

$$54115 = 8 \cdot 6165 + 4795$$

$$6165 = 4795 + 1370$$

$$4795 = 3 \cdot 1370 + \boxed{685}$$

$$1370 = 2 \cdot 685 + 0$$

$$b) 20202020x + 174675y = 685$$

$$x = 3354 \quad y = -13069$$

5a) divisors: $1, 2, 4, 5, 10, 20, 101, 202, 404, 505, 1010, 2020$

b) 5050: $1, 2, 5, 10, 25, 50, 101, 202, 505, 1010, 5225, 5050$

c) Common: $1, 2, 5, 10, 101, 202, 505, 1010$

d) gcd: 1010

6a) 2020 has 12 positive divisors

b) 2021 has 4 positive divisors

c) 2022 has 8 positive divisors

d) Other than counting divisors we could use the Tau function where $a = p^b \cdot q^d \dots$ and number of divisors $(n) = (b+1)(d+1)$
For example 2020 factors into $2^2 \cdot 5^1 \cdot 101^1 \therefore$
 $n = (2+1)(1+1)(1+1) = 12 = \text{divisor count}$

7) $\text{gcd}(1548482876424, 3367586192850) = 2 \cdot 3^2 \cdot 11^3 \cdot 37^3$
 $\text{lcm}(1548482876424, 3367586192850) = 2^3 \cdot 3^3 \cdot 5^2 \cdot 11^4 \cdot 29 \cdot 37^4$

$$8a) \quad a^2 \mid b^3 \Rightarrow a \mid b$$

$$a^2 \geq b^3$$

$$\frac{b^3}{a^2} = 2p + 1q \dots$$

$$\frac{b^3}{a^2} = c \frac{b}{a}, a \mid b$$

True

$$b) \quad a^3 \mid b^2 \Rightarrow a \mid b$$

$$a^3 \geq b^2$$

$$\frac{b^2}{a^3} = 2p + 1q \dots$$

$$\frac{b^2}{a^3} = c \frac{b}{a}, a \mid b$$

True

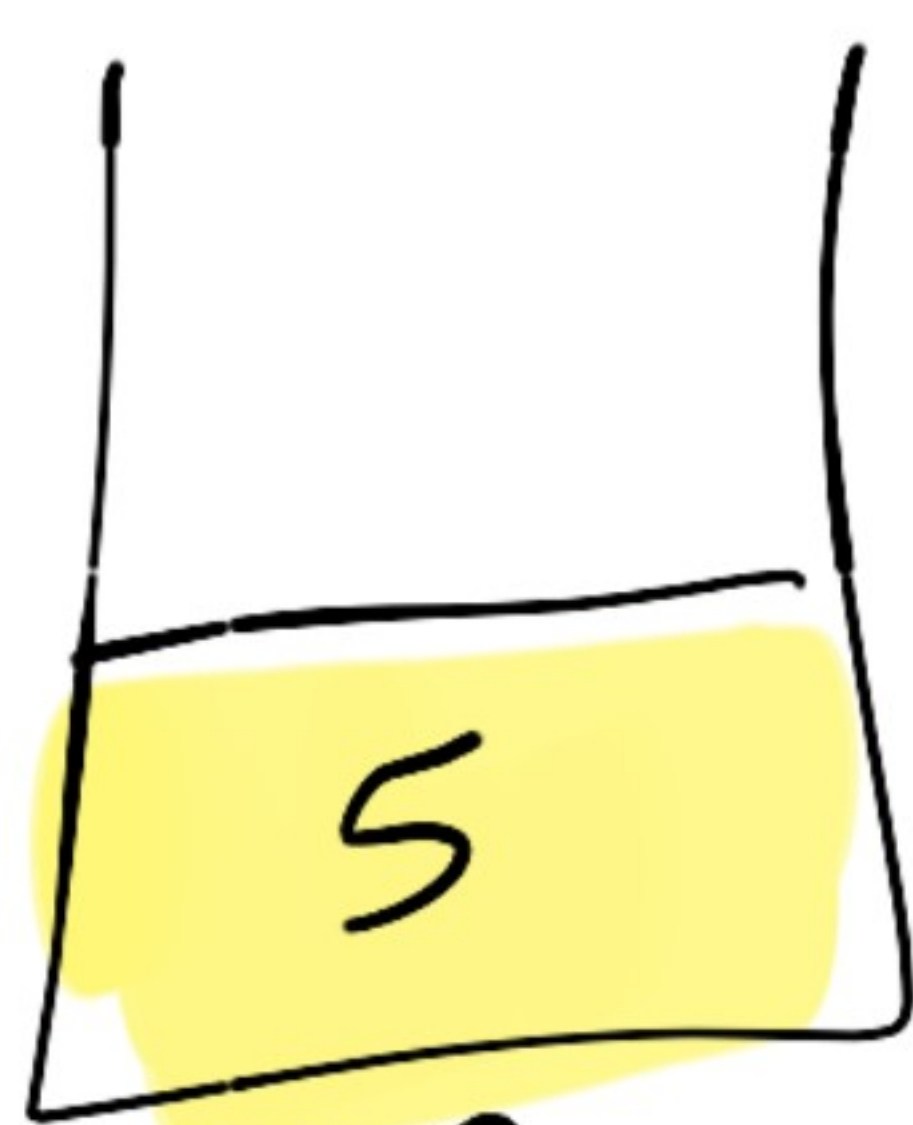
9a) Containers: 8 pint, 11 pint, 40 pint

$$3(11) + 3(2) + 1 = 40$$

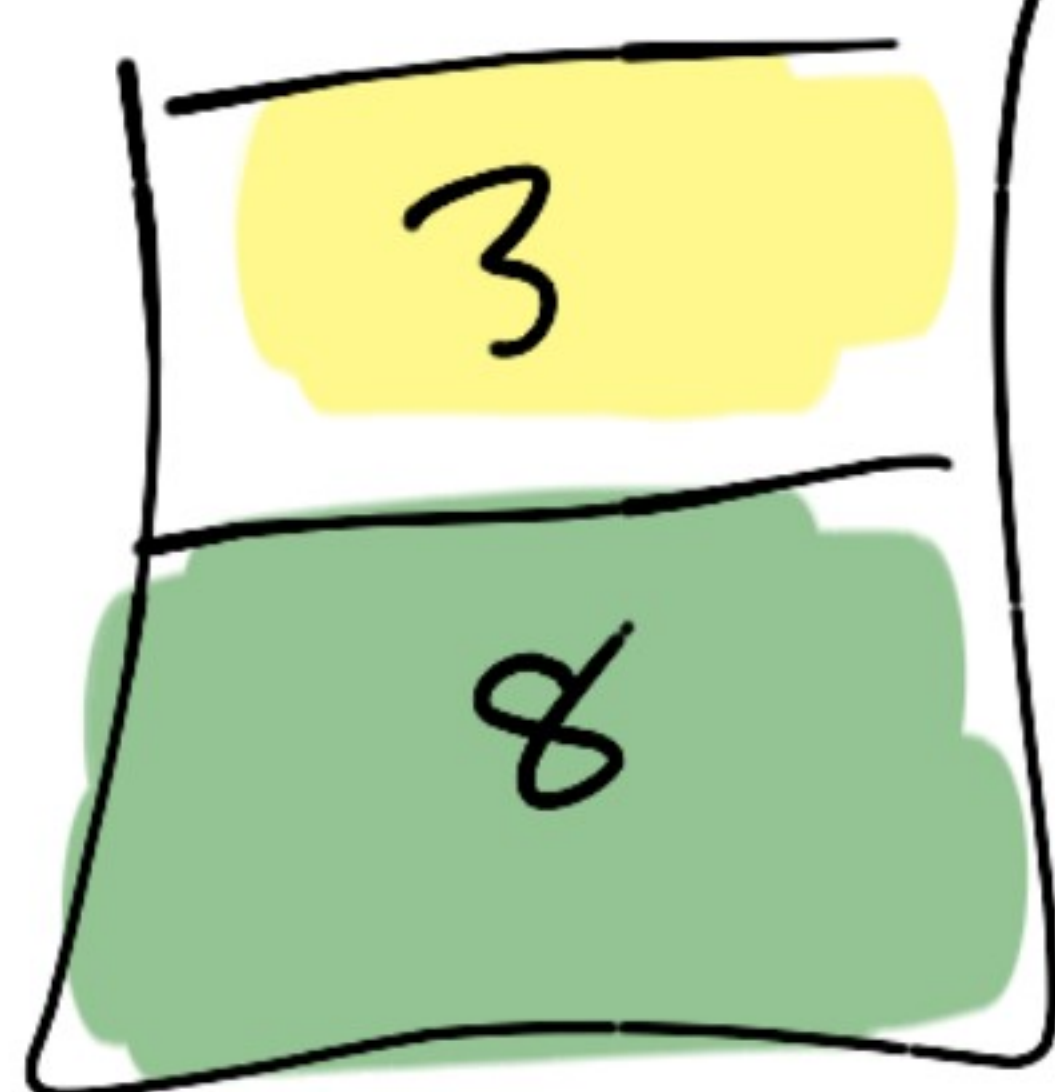
- Pour 3×11 pints into the 5gal Bucket
- fill 11 pint container & empty 8 pints into 8 pint container
 - Pour remaining 2 pints into 5gal Bucket three times
- fill 11 pint container & empty 8 pints into 8 pint container
 - Pour 1 pint into 5gal Bucket - leaving 1 pint in the 8 pint container



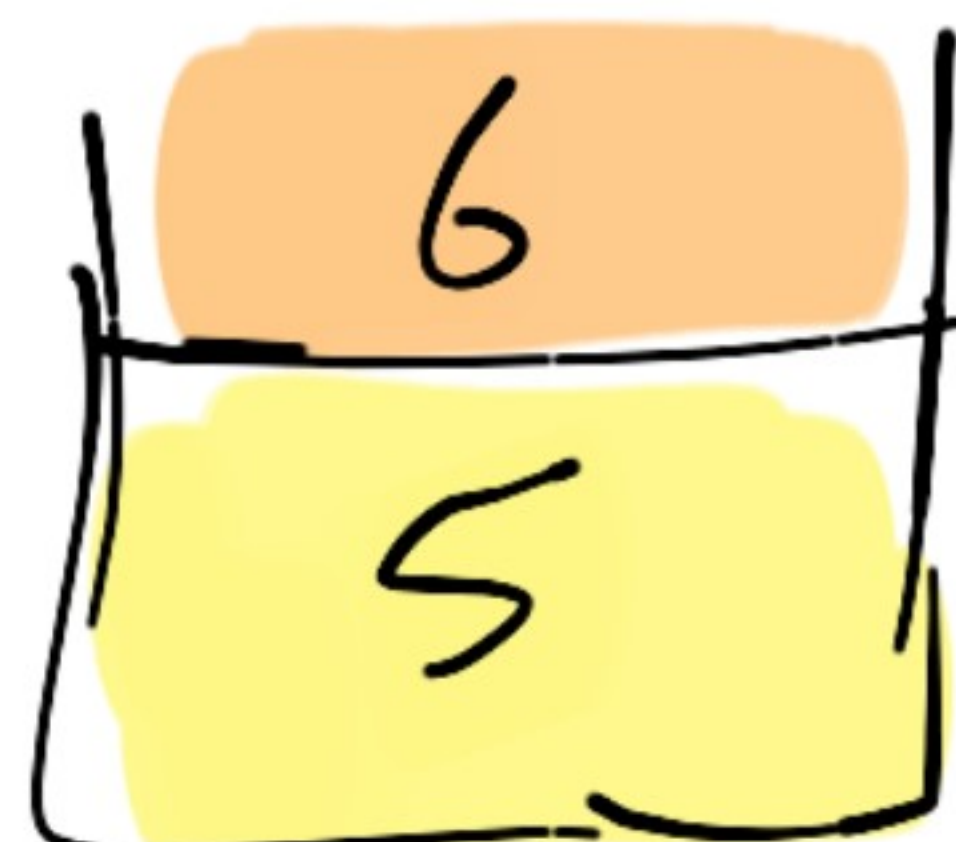
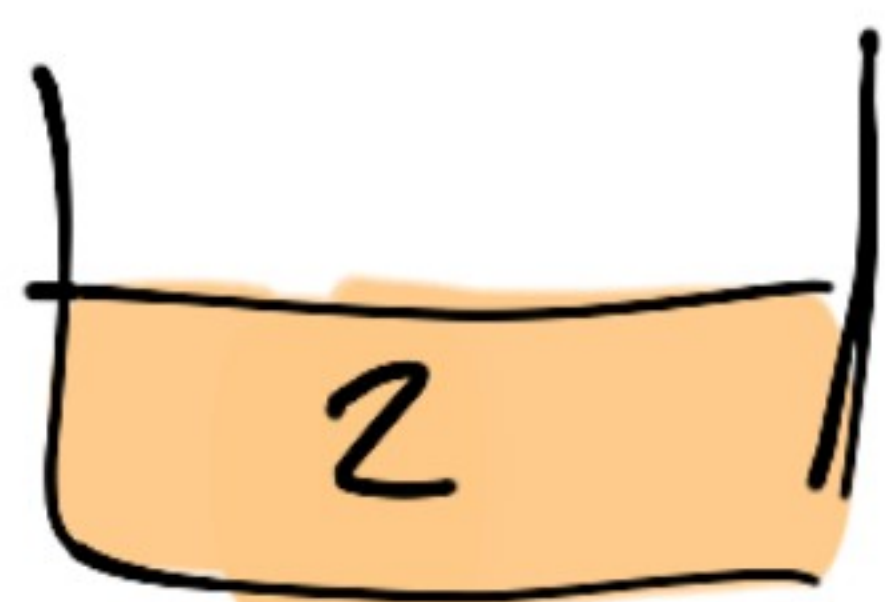
9b)



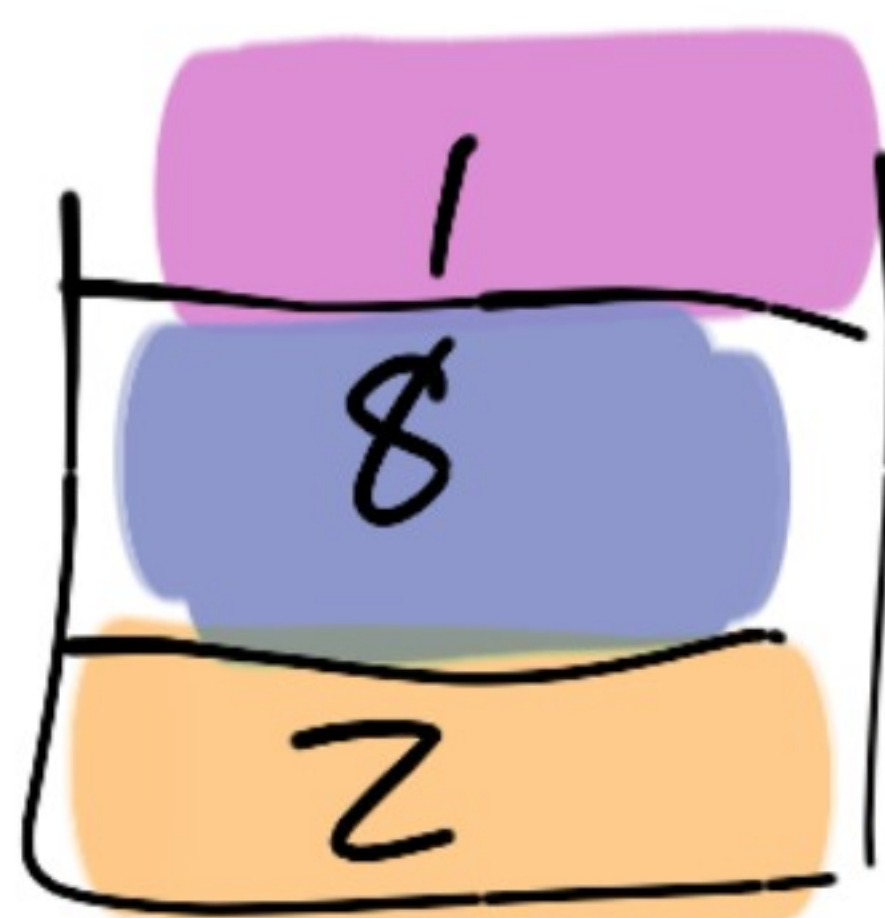
8 pint Container



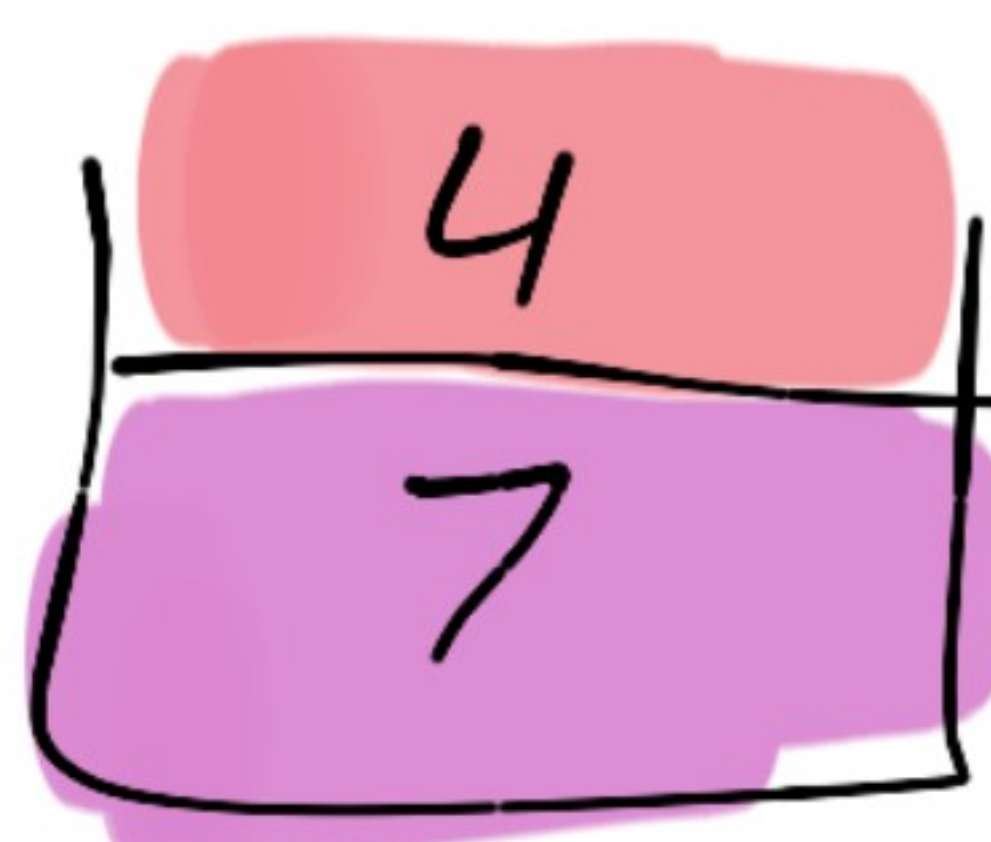
11 pint Container



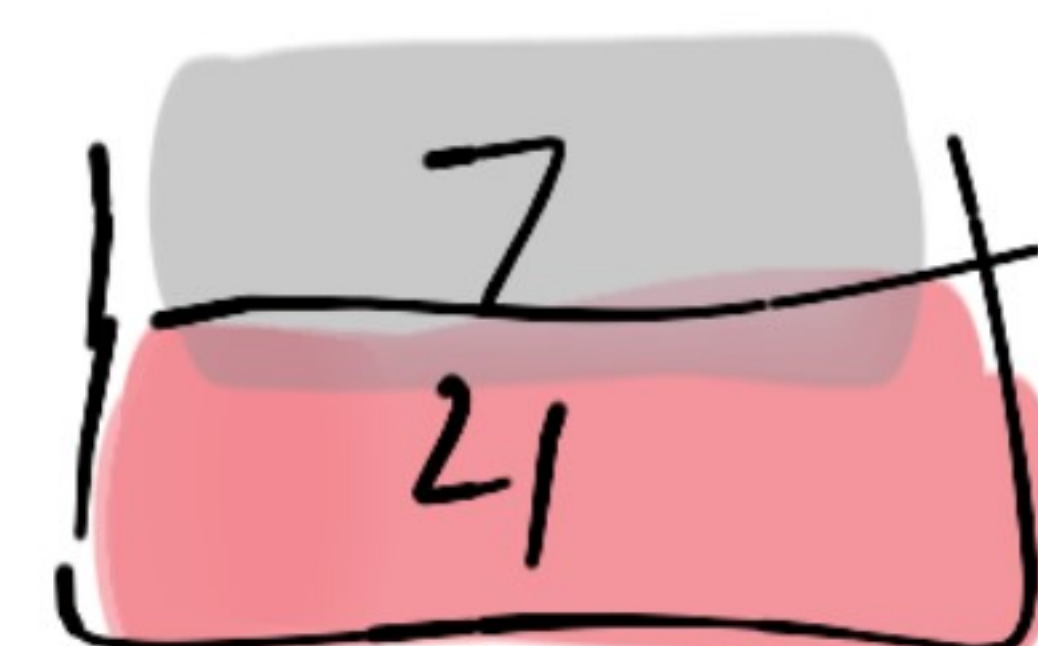
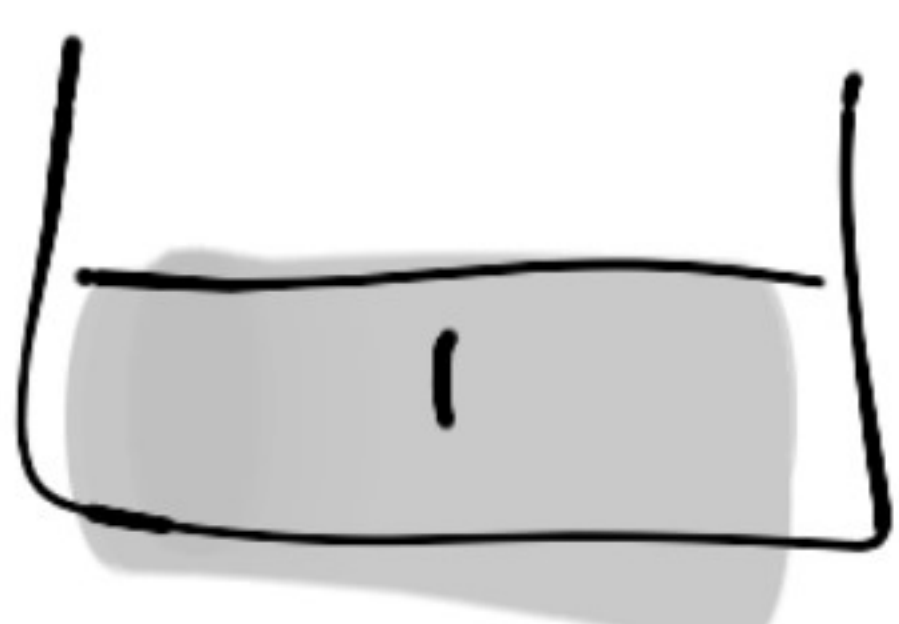
(empty then move 8 pint \rightarrow 11 pint)



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"



"

10)

euclid(a,b) =

func

local g,r,aa,bb,xy,t

a \rightarrow aa

b \rightarrow bb

1 \rightarrow r

[0 1] \rightarrow t

[1 0] \rightarrow xy

while r \neq 0

intD.v(a,b) \rightarrow b

a - qb \rightarrow r

-q \rightarrow t[2,2]

t.xy \rightarrow xy

b \rightarrow a

r \rightarrow b

End while