A metamodelling approach to estimate fair market values

TEAM BIGS

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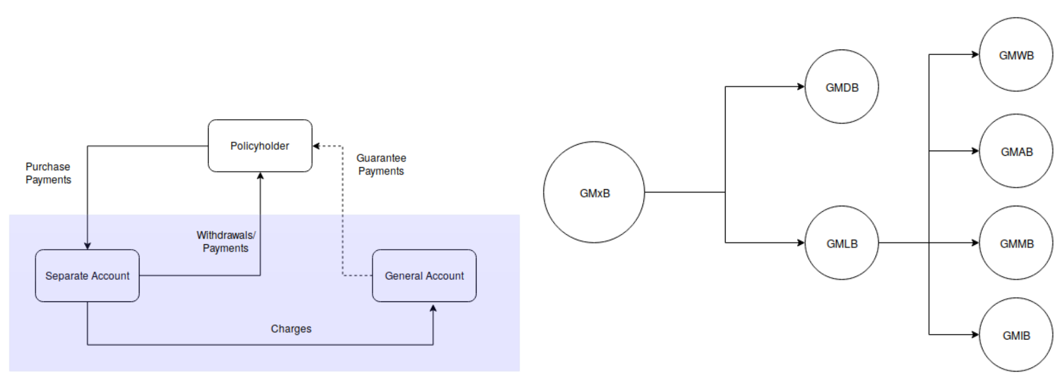
Chapter 1

Introduction

A variable annuity (VA) is a life insurance product created by insurance companies to address concerns that many people have about outliving their assets. Essentially, a VA ias deferred annuity with two phases: the accumulation phase and the payout phase. During the accumulation phase, the policyholder makes purchase payments to the insurance company. During the payout phase, policyholder received benefit payments from the insurance company. The policyholder has the option of allocating the money among this set of investment funds. A major feature of a variable annuity is that it includes guarantees or riders.

These guarantees can be divided into two broad categories: death benefits and living benefits. A guaranteed minimum death benefit (GMDB) guarantees a specified lump sum to the beneficiary upon the death of the policyholder regardless of



(a) Variable annuity fluxogram (b) Guarantees

Figure 1.1: Variable Annuities Description

the performance of the investment portfolio.There are several types of living benefits. Popular living benefits include the guaranteed minimum withdrawal benefit (GMWB), the guaranteed minimum income benefit (GMIB), the guaranteed minimum maturity benefit (GMMB), and the guaranteed minimum accumulation benefit (GMAB). A GMWB guarantees that the policyholder can make systematic annual withdrawals of a specified amount from the benefit base over a period of time, even though the investment portfolio might be depleted. A GMIB guarantees that the policyholder can convert the greater of the actual account value or the benefit base to an annuity according to a specified rate. A GMMB guarantees the policyholder a specific amount at the maturity of the contract. A GMAB guarantees that the policyholder can renew he contract during a specified window after a specified waiting period, which is usually 10 years.

Using dynamic headging to mitigate the financial risks associated with VA guarantees, insurance companies first have to quantify the risks. This usually requires calculating the fair market values (FMV) of the guarantees for large portfolio of VA contracts in a timely manner. Dynamic hedging requires calculating the dollar Deltas of a portfolio of variable annuity policies within a short interval. The value of the guarantees cannot be determined by closed-form formula. Monte Carlo simulation can be used to value the VA protfolio, but it is extremly time-consuming because every contract needs to be projected over many scenarios for a long time horizon. In order to deal with this problem, metamodeling approaches have been proposed to address the aforementioned computational problem.

Using metamodeling approaches can reduce significantly the runtime of valuing a large portfolio of VA contracts for two main reasons: first, building a metamodel only requires using the Monte Carlo simulation model to value a small number of representative VA contracts; second, the metamodel is usually much simpler and faster than the Monte Carlo simulation model. The basics four steps to build a metamodel can be written in the following picture.

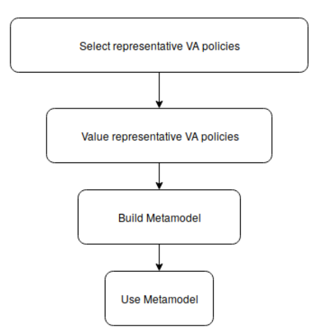


Figure 1.2: Metamodel fluxogram

In other words it consists in: (1)define a subset of representative VA contract, (2)compute the FMV for this represntative set using MC simulation, (3) fit a model based on the characteristics and FMV of the contracts, (4)Use the estimated model to predict the FMV of the remaining VA contracts. The metamodels investigated in literature are sophisticated predictive models, which might cause difficulties in terms of interpretation or calibration.

The scope of this work is to investigate interaction terms in Generalized Linear Model framework as a metamodel for valuing the complex financial guarantees associated with VA contracts. We choose statiscally significant interaction terms to the model, evaluate performance as predictive model, and interpret the resulting effect of the addition of interaction terms. We propose others metamodels: Box Cox regression and Neural Network regression. After describe these methods we will compare both e suggest the best of them to evaluate a VA portfolio.

Chapter 2

Literature

There are many publications on metamodeling approaches and recently these approaches have been proposed to speed up the valuation of large VA portfolios and produce accurate results. Some of the metamodels proposed and examined can be listed below:

- Ordinary Kriging
- Universal Kriging
- GB2 regression model
- Rank-order Kriging (quantile Kriging)
- Linear models with interactions
- Tree-based models

In Gan (2018) author investigated the effect of including interactions in linear regression models for the valuation of those large VA portfolios. Since there were many features of VA contract, there were a large number of possible interactions between the features. So he selected the important interactions using overlaped group-lasso that could produce hierarchical interaction models. The numeric results obtained in Gan (2018) show that including interactions in linear regression models can lead to significant improvements in prediction accuracy.

In Gan and A.Valdez (2017), GB2 distribution was used to model the fair market values of VA guarantees, because it can captures the skewness shown in the empirical distribution of them. The GB2 distribution is a flexible statistical distribution that contains three shape parameters and one scale parameter. However, finding the optimum parameters for the GB2 regression model was not strightfoward and therefore some difficult challenges were present.

The numerical results of that approach show that the four-stage optimization, described in that article, worked well and the result fitted in GB2 regression model performed as expected. Some comparison was made by the authors: (1) GB2 model captures skewness better than Kriging model, (2) GB2 model outperforms the Krigingmodel in computational speed, (3)GB2 model produces comparably accurate predictions as teh Kriging model at portfolio level.

An important step in the metamodeling process is the selection of representative policies. Gan and Valdez (2016) compared five different experimental design methods for the GB2 regression model:

- Random Sampling
- Low-discrepancy sequence
- Data clustering (Hierarchical k-means)
- Latin Hypercube sampling
- Conditional Latin Hypercube sampling

Hejazi and Jackson (2016), proposed a machine learning approach inside the metamodel. After a small set of representative VA contracts was selected and valued via Monte Carlos simulations, the values of these representatives contracts were then used in a spatial interpolation method that found the value of the contarcts in the input portfolio as a linear combination of the values of the representative contracts.

The traditional spatial interpolation methods as Kriging, IDW and RBF (Hejazi 2015) have a strong dependence with the distance function used in estimations. So authors proposed a neural network implementation of the spatial interpolation techinique that learns an effective choice of that distance function. The results obtained by the authors show the superior accuracy of the neural network approach in estimation of the delta value for the input portfolio compared to the traditional spatial interpolation techniques.

Xu et al.(2018) propose a moment matching machine learning (MMML) approach to compute dollar deltas, VaRs and CVaRs for large portfolios. There are two main contributions that coulb be highlighted from this paper. First, they proposed a moment matching method to compute annual dollar deltas, VaRs, and CVaRs for a single VA contract. Due to these selected scenarios, the moment matching method can compute the annual dollar deltas, VaRs and CVaRs as accurately as the nested simulations, but only takes far less computational time as nested simulations requires.

The second contribution is that they combine the moment matching method with some classical machine learning methods to manage the risk of a large VA portfolio. The machine is trained with a standard machine learning method, such as neutral network or tree regression. Their MMML approach can easily handle huge portfolios (which cannot be handled via the nested simulation method due to cost). Their approach appears to be a remarkably efficient alternative to the standard nested simulation methodology to hedge and manage the risk of large portfolios arising in the insurance industry.

Below we can see a table that show the main core of metamodeling approachs applied to evaluate VA portfolios.

Publication	Experimental Design	Metamodel
Gan (2013)	Clustering	Kriging
Gan and Lin (2015)	Clustering	Kriging
Gan (2015)	LHS	Kriging
Hejazi and Jackson (2016)	Uniform sampling	Neural network
Gan and Valdez (2016)	Clustering, LHS	GB2 regression
Gan and Valdez (2017)	Clustering	gamma regression
Gan and Lin (2017)	LHS, conditional LHS	Kriging
Hejazi et al. (2017)	Uniform sampling	Kriging, IDW, RBF
Gan and Huang (2017)	Clustering	Kriging
Xu et al (2018)	Random sampling	Neural Network, regression trees
Gan and Valdez (2018)	Clustering	GB2 regression
Quan, Gan and Valdez (2019)	Clustering	Regression trees

Chapter 3

Synthetic Data

It is dificcult for researchers to obtain real datasets from insurance companies to assess the performance of those metamodeling techiniques.As a result, most of the papers on variable annuity portfolio valuation use synthetic datasets to test the performance of the proposed metamodeling techiniques. Gan and A.Valdez (2017) creates synthetic datasets to facilitate the development and dissemination of reaserch related to the efficient valuation of large variable annuity portfolios. This synthetic dataset was based on the properties of real portfolios of variable annuities and implement a simple Monte Carlo valuation engine that is used to calculate the fair market values and the Greeks of the guarantees embedded in those synthetic variable annuity contracts.

The major properties typically observed on real portfolios of variables annuities contracts:

- Different contracts may contain different types of guarantees.
- The contract holder has the option to allocate the money among multiple investment funds.
- Real variable annuity contracts are issued at different dates and have different times to maturity.

There are several types of guarantees, but to create a synthetic portfolio of varriabel annuity contracts, Gan and A.Valdez (2017) considers 19 products shown in table. For the synthetic variable annuity policies, they set the rider fees of individual riders in the range of 0.25% to 0.75%. The rider fee of the combined guarantees is set equal to the sum of the fees of the individual gurantees minus 0.20%.

Product	Description	Rider Fee
DBRP	GMDB with return of premium	0.25%
DBRU	GMDB with annual roll-up	0.35%
DBSU	GMDB with annual ratchet	0.35%
ABRP	GMAB with return of premium	0.50%
ABRU	GMAB with annual roll-up	0.60%
ABSU	GMAB with annual ratchet	0.60%
IBRP	GMIB with return of premium	0.60%
IBRU	GMIB with annual roll-up	0.70%
IBSU	GMIB with annual ratchet	0.70%
MBRP	GMMB with return of premium	0.50%
MBRU	GMMB with annual roll-up	0.60%
MBSU	GMMB with annual ratchet	0.60%
WBRP	GMWB with return of premium	0.65%
WBRU	GMWB with annual roll-up	0.75%
WBSU	GMWB with annual ratchet	0.75%
DBAB	GMDB + GMAB with annual ratchet	0.75%
DBIB	GMDB + GMIB wwith annual ratchet	0.85%
DBMB	GMDB + GMMB with annual ratchet	0.75%
DBWB	GMDB + GMWB with annual ratchet	0.90%

The policyholder is allowed to select the investment funds. In dynamic hedging, a fund mapping is used to map an investment fund to a combination of tradable and liquid indices suach as the S&P500 index. In the synthetic portfolio, account values of the investment funds of a policy were generated randomly from a specified range. The total account values are allocated to the investments funds equally.

In practice, variable annuity policies in a portfolio are issued at different dates. To value the policies at the valuation date, the policies are aged from the issues dates to the valuation date. The parameters used to generate others variables, as time to maturity and age come from the raw variables displayed in table below.

Feature	Value
Policyholder birth date	[1/1/1950,1/1/1980]
Issue date	[1/1/2000,1/1/2014]
Valuation date	1/6/2014
Maturity	[15,30]years
Initial account value	[50000,500000]
Female percent	40%
Fund fee	30, 50, 60, 80, 10, 38, 45, 55, 57, 46 bps for Funds 1 to 10
M&E fee	200 bps

There are two types of scenarios: risk-neutral and real-word. Each of these two scenarios are generated by each measures respectively. Risk-neutral scenarios are used to calculate the fair market values of financial derivatives such as the guarantees embedded in variable annuities. Real-world scenarios are used to calculate solvency capitals or evaluate hedging strategies.

The description of the policy fields at the synthetic dataset is shown below. Gan and A.Valdez (2017), generated 10,000 synthetic variable annuity policies for each of the guarantees types described before. This synthetic portfolio contains 190,000 policies. There are 45 fields in total, including 10 fund values, 10 fund numbers and 10 fund fees.

Field	Description
recordID	Unique identifier of the policy
survivorShip	Positive weighting number
gender	Gender of the policyholder
productType	Product type
issueDate	Issue Date
matDate	Maturity date
birthDate	Birth date of the policyholder
currentDate	Current date
baseFee	M&E (Mortality & Expense) fee
riderFee	Rider fee
rollUprate	Roll-up rate
rollUprate	Guaranteed benefit
rollUprate	GMWB balance
wbWithdrawalRate	Guaranteed withdrawal rate
withdrawal	Withdrawal so far
FundValuei	Fund value of the ith investment fund
FundNumi	Fund number of the ith investment fund
FundFeei	Fund management fee of the ith investment fund

Chapter 4

Metamodeling Approach

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Chapter 5

Conclusion

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