A metamodelling approach to estimate fair market values

TEAM BIGS

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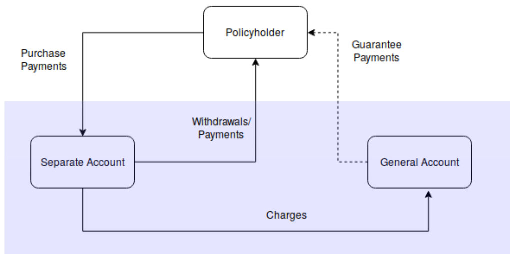
Chapter 1

Introduction

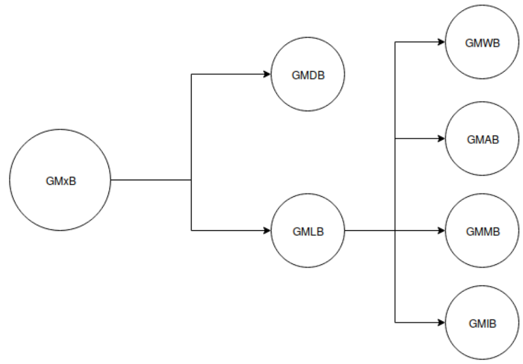
A variable annuity (VA) is a life insurance product created by insurance companies to address concerns that many people have about outliving their assets. Essentially, a VA is a deferred annuity with two phases: the accumulation phase and the payout phase. During the accumulation phase, the policyholder makes purchase payments to the insurance company. During the payout phase, policyholders receive benefit payments from the insurance company. The policyholder has the option of allocating his investments among a set of investment funds.

A major feature of a variable annuity is that it includes guarantees or riders. These can be divided into two broad categories: death benefits and living benefits. A guaranteed minimum death benefit (GMDB) guarantees a specified lump sum to the beneficiary upon the death of the policyholder regardless of the performance of the investment portfolio. There are several types of living benefits, which include the guaranteed minimum withdrawal benefit (GMWB), the guaranteed minimum income benefit (GMIB), the guaranteed minimum maturity benefit (GMMB), and the guaranteed minimum accumulation benefit (GMAB). A GMWB guarantees that policyholders can make systematic annual withdrawals of a specified amount from the benefit base over a period of time, even though the investment portfolio might be depleted. A GMIB guarantees that policyholders can convert the greater of the current account value or the benefit base to an annuity according to a specified rate. A GMMB guarantees that policyholders received a specific amount at the maturity of the contract. A GMAB guarantees that policyholders can renew the contract during a specified window after a specified waiting period (usually 10 years).

Using dynamic hedging to mitigate the financial risks associated with VA guarantees, insurance companies need to estimate the fair market value (FMV) of guarantees for a large portfolio of VA contracts in a timely manner. As the value of



(a) Variable annuity flowchart



(b) Guarantees

Figure 1.1: Variable Annuities Description

the guarantees cannot be determined by closed-form formulas, Monte Carlo (MC) simulations are used to value VA portfolios. This approach can be extremely time-consuming as every contract needs to be projected over many scenarios for a long time horizon. To address this computational problem, metamodeling approaches have been proposed.

Metamodeling approaches can significantly reduce the computational effort in the valuation of a large portfolio of VA contracts for two main reasons: first, only a small group of representative contracts needs to be valued using Monte Carlo simulations; second, metamodels are usually simpler and faster than Monte Carlo simulation models. The basic four steps to build a metamodel are illustrated in Figure 1.2. They consist in: (1) defining a subset of representative VA contracts, (2) computing the FMV for this representative set using MC simulations, (3) fitting a model based on the characteristics and FMV of the contracts, (4) using the estimated model to predict the FMV of the remaining VA contracts.

Table 1.1 shows the main metamodeling approaches applied in the valuation of VA portfolios. Some of the metamodels proposed include: linear regression models with interactions, GB2 regression model, ordinary kriging, universal kriging, rank-order kriging (quantile kriging), and tree-based models.

Gan (2018) investigated the effect of including interactions in linear regression models for the valuation of large VA portfolios. His results show that including interactions in linear regression models can lead to significant improvements in prediction accuracy.

In Gan and Valdez (2017), the GB2 distribution was used to model the fair market values of VA guarantees, because it can capture the skewness found in their empir-

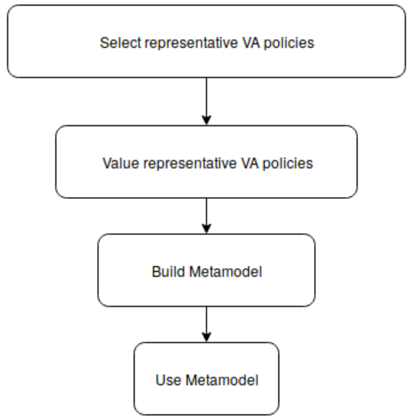


Figure 1.2: Metamodel fluxogram

ical distributions. The numerical results of the approach show that the four-stage optimization, described in the article, performs well and the fitted GB2 regression model performed as expected. A few comparisons made by the authors: (1) the GB2 model captures skewness better than the kriging model; (2) the GB2 model outperforms the kriging model in computational speed; (3) GB2 model produces comparably accurate predictions as the kriging model at portfolio level.

An important step in the metamodeling process is the selection of representative policies. Gan and Valdez (2016) compared five different experimental design methods for the GB2 regression model: random sampling, low-discrepancy sequence, data clustering (hierarchical k-means), Latin hypercube sampling, and conditional Latin hypercube sampling.

Hejazi and Jackson (2016) proposed a machine learning approach. After a small set of representative VA contracts was selected and valued via Monte Carlos simulations, the values of these representatives contracts were then used in a spatial interpolation method that found the value of the contracts in the input portfolio as a linear combination of the values of the representative contracts. The traditional spatial interpolation methods as kriging, IDW and RBF (Hejazi 2015) have a strong dependence on the distance function used in estimations. Therefore, the authors proposed a neural network implementation of the spatial interpolation technique that learns an effective choice of distance function. The results show the superior accuracy of the neural network approach in estimation of the delta value for the input portfolio when compared to the traditional spatial interpolation techniques.

Xu et al. (2018) propose a moment matching machine learning (MMML) approach to compute dollar deltas, VaRs and CVaRs for large portfolios. There are

Publication	Experimental Design	Metamodel
Gan (2013)	Clustering	Kriging
Gan and Lin (2015)	Clustering	Kriging
Gan (2015)	LHS	Kriging
Hejazi and Jackson (2016)	Uniform sampling	Neural network
Gan and Valdez (2016)	Clustering, LHS	GB2 regression
Gan and Valdez (2017)	Clustering	gamma regression
Gan and Lin (2017)	LHS, conditional LHS	Kriging
Hejazi et al. (2017)	Uniform sampling	Kriging, IDW, RBF
Gan and Huang (2017)	Clustering	Kriging
Xu et al (2018)	Random sampling	Neural Network, regression trees
Gan and Valdez (2018)	Clustering	GB2 regression
Quan, Gan and Valdez (2019)	Clustering	Regression trees

Table 1.1: Metamodeling approaches for the valuation of variable annuities.

two main contributions that could be highlighted in their paper. First, they proposed a moment matching method for single VA contracts. Computations using this method are much faster than nested MC simulations. The second contribution is that they combine the moment matching method with classical machine learning methods to manage the risk of large VA portfolios. Their MMML approach can easily handle large portfolios, being a remarkably efficient alternative to the standard nested simulation methods to hedge and manage the risk of large portfolios arising in the insurance industry.

Chapter 2

The synthetic data set

Researchers usually do not have access to real data sets from insurance companies. As a result, most of the papers on the valuation of variable annuity portfolios rely on synthetic data sets to test the performance of the proposed techniques. To assist in the development and dissemination of research related to the efficient valuation of large variable annuity portfolios, Gan and Valdez (2017) created synthetic data sets that are based on the properties of real portfolios of variable annuities. They have also implemented a simple Monte Carlo valuation engine that is used to calculate the fair market value and the Greeks of the guarantees embedded in the synthetic variable annuity contracts.

Contracts found in real portfolios of variable annuities are typically characterized by having different types of guarantees, different investment fund allocations, and different issue and maturity dates. To account for the different types of guarantees, Gan and Valdez (2017) consider the 19 products shown in Table 2.1. Rider fees are set in the range of 0.25% to 0.75%, and the rider fee of the combined guarantees is equal to the sum of individual ones minus 0.2%. The investment choices of a policyholder are mapped to a combination of tradable and liquid indices such as the S&P500 index. In the synthetic portfolio, account values of the investment funds were generated randomly from a specified range and allocated to investment funds in equal proportions. In practice, variable annuity policies in a portfolio are issued at different dates. To value the policies at the valuation date, the policies are aged from the issue dates to the valuation date. Finally, the parameters used to generate others variables, such as time to maturity and age come from the variables displayed in Table 2.2.

The data sets generated by Gan and Valdez (2017) contain 10,000 synthetic variable annuity policies for each of the guarantees types described in Table 2.1. There-

Product	Description	Rider Fee
DBRP	GMDB with return of premium	0.25%
DBRU	GMDB with annual roll-up	0.35%
DBSU	GMDB with annual ratchet	0.35%
ABRP	GMAB with return of premium	0.50%
ABRU	GMAB with annual roll-up	0.60%
ABSU	GMAB with annual ratchet	0.60%
IBRP	GMIB with return of premium	0.60%
IBRU	GMIB with annual roll-up	0.70%
IBSU	GMIB with annual ratchet	0.70%
MBRP	GMMB with return of premium	0.50%
MBRU	GMMB with annual roll-up	0.60%
MBSU	GMMB with annual ratchet	0.60%
WBRP	GMWB with return of premium	0.65%
WBRU	GMWB with annual roll-up	0.75%
WBSU	GMWB with annual ratchet	0.75%
DBAB	GMDB + GMAB with annual ratchet	0.75%
DBIB	GMDB + GMIB wwith annual ratchet	0.85%
DBMB	GMDB + GMMB with annual ratchet	0.75%
DBWB	GMDB + GMWB with annual ratchet	0.90%

Table 2.1: Variable annuity products in the synthetic database.

Feature	Value
Policyholder birth date	[1/1/1950,1/1/1980]
Issue date	[1/1/2000,1/1/2014]
Valuation date	1/6/2014
Maturity	[15,30]years
Initial account value	[50000,500000]
Female percent	40%
Fund fee	30, 50, 60, 80, 10, 38, 45, 55, 57, 46 bps for Funds 1 to 10
M&E fee	200 bps

Table 2.2: Parameters used in the generation of the synthetic database of variable annuities.

Field	Description
recordID	Unique identifier of the policy
survivorShip	Positive weighting number
gender	Gender of the policyholder
productType	Product type
issueDate	Issue Date
matDate	Maturity date
birthDate	Birth date of the policyholder
currentDate	Current date
baseFee	M&E (Mortality & Expense) fee
riderFee	Rider fee
rollUprate	Roll-up rate
rollUprate	Guaranteed benefit
rollUprate	GMWB balance
wbWithdrawalRate	Guaranteed withdrawal rate
withdrawal	Withdrawal so far
FundValuei	Fund value of the ith investment fund
FundNumi	Fund number of the ith investment fund
FundFeei	Fund management fee of the ith investment fund

Table 2.3: Description of fields in the specification of variable annuities policies.

fore, the synthetic portfolio contains 190,000 policies. There are 45 fields in each policy, including 10 fund values, 10 fund numbers and 10 fund fees. The description of the policy fields in the synthetic data set is shown in Table 2.3.

The main goal of Monte Carlo simulation engine is calculate the fair market values (FMV), partial dollar deltas and partial dollar rhos of the guarantees for the synthetic portfolios. Total fair market values are usually positive, as guarantee benefit payoffs are larger than their associated risk. Since the VA contracts are usually long-terms contracts, guarantees are more sensitive to long-term interest rates than to short-term interest rates.

Chapter 3

The challenge

1 Problem statement

The objective of the project is to improve the consistency in the determination of statistically significant interaction terms in linear models with interactions used for the valuation of portfolios of variable annuity contracts. The approach has been introduced in Gan and Valdez (2018) with excellent prediction results. However, the non-Gaussian residuals of the fitted models compromise any inference based on the hypothesis of gaussianity. Consequently, one cannot identify the significant interaction terms and interpret the predictions provided by the models.

Initially, an investigation of interaction terms in the framework of generalized linear models was considered. However, given the time constraints of the challenge, other alternative solutions were explored. First, as described in Section 2.1, we replicate the results obtained in Gan and Valdez (2018) as a starting point. In the following section, we explore the Box-Cox regression model as an alternative to improve the original results. Finally, in Section 2.3, we explore tree-based models as a metamodeling approach, as in Quan et al. (2018).

2 Results

2.1 Linear models with interactions

In this section, following Gan (2018), a linear regression model with interaction effects is used in the valuation of VA contracts. The main advantages of the approach discussed here is the simplicity in the metamodeling process, and the possibility of exploring interactions terms as features to predict the FMV of VA contracts.

Interaction model

By definition, interactions exist in a regression model when the response variable cannot be explained by additive functions of the independent explanatory variables. Let Y be a continuous response variable, and let $X_1, X_2, X_3, ..., X_n$ be the explanatory variables, including categorical and continuous, that will be used to model Y. The first-order interaction model can be write as:

$$Y = \mu + \sum_{i=1}^{p} X_i \beta_i + \sum_{i<j} X_{i:j} \beta_{i:j} + \varepsilon, \tag{3.1}$$

where the term X_i denotes the individual effect of X_i on Y; and $X_{i:j}$ denotes the effect of interaction between X_i and X_j on Y. In general lines, the equation (5.1) corresponds to an extension of the multiple linear regression model that is obtained with the inclusion of interaction terms.

The inclusion of interaction terms as explanatory variables can be viewed as the main virtue of the presented approach, since the added variables help to increase the predictive power of the model; however if the number of interaction terms to be included in the regression model is large, the estimated model can overfit the data. To avoid overfitting problem, two shrinkage methods are used to select the relevant interactions and estimate the parameters: group-LASSO and overlapped group-LASSO, to be presented in the next subsection.

Estimation of parameters

Group-LASSO (GLASSO) and overlapped Group-LASSO (OGLASSO) can be considered general versions of the LASSO, Tibshirani (1996). While the latter is designed for selecting individual input variables, GLASSO and OGLASSO are able to select important factors, or group of relevant variables.

Suppose that there are p groups of variables. For $j = 1, 2, 3, ..., p$, let $\mathbf{X}_j$ denotes the feature matrix for group j. The GLASSO can be formulated as:

$$\hat{\beta}^{GLASSO} = \arg\min_{\beta} \left(\frac{1}{2} \left\| \mathbf{y} - \beta_0 \mathbf{1} - \sum_{j=1}^{p} \mathbf{X}_j \boldsymbol{\beta}_j \right\|_2^2 + \lambda \sum_{j=1}^{p} \gamma_j \left\| \boldsymbol{\beta}_j \right\|_1 \right), \tag{3.2}$$

where $\mathbf{1}$ is a vector of ones, $\|\bullet\|_2$ denotes the ℓ^2-norm, and $\lambda, \gamma_1, ..., \gamma_p$ are tuning parameters. The parameter λ controls the overall amount of regularization while the parameters $\gamma_1, ..., \gamma_p$ allow each group to be penalized to different extents. An attractive property of the GLASSO is that if $\hat{\boldsymbol{\beta}}_{\boldsymbol{j}}$ is nonzero, then all of its components are typically nonzero.

In an intuitive way, GLASSO gives us the $\hat{\boldsymbol{\beta}}_{\boldsymbol{j}}$ that minimize the mean squared error given a penalization term that shrinks parameters to zero. The selected group of variables are those related to the parameters that are nonzero. Optimal penalization terms can be chosen by cross-validation.

The OGLASSO extends the GLASSO by adding an overlapped Group-LASSO penalty to the loss function in order to obtain hierarchical interaction models that can be formulated as the following unconstrained optimization problem:

$$\begin{aligned} \hat{\beta}^{OGLASSO} = \arg\min_{\beta} \Bigg(\frac{1}{2} \left\| \mathbf{y} - \beta_0 \mathbf{1} - \Sigma_{j=1}^{p} \mathbf{X}_j \beta_j - \sum_{s<t} \boldsymbol{X}_{s:t} \boldsymbol{\beta}_{s:t} \right\|_2^2 \\ + \lambda \left(\sum_{j=1}^{p} \left\| \boldsymbol{\beta}_j \right\|_1 + \sum_{s<t} \|\boldsymbol{\beta}_{s:t}\|_1 \right) \Bigg) \end{aligned}$$

Unlike the GLASSO, the OGLASSO is capable of selecting interactions between groups as relevant explanatory variables.

Empirical results

The following results were computed for two independent representative portfolios of VA contracts, extracted from the synthetic data set described in Chapter 2. For each of them, a linear model with interactions was estimated based on the

OGLASSO. The plots in Figure 3.1 show histograms of the FMVs as a preliminary examination of the target variable. As illustrated, the distribution is skewed to the right and there are many contracts with FMV close to zero. The QQ-plots in Figure 3.1 compare the theoretical distribution of the FMV (Normal) with its empirical distribution. Based on the graphical analysis it is possible to assert that the FMVs do not follow a Normal distribution, as many points on the plot are located far away from the straight line.

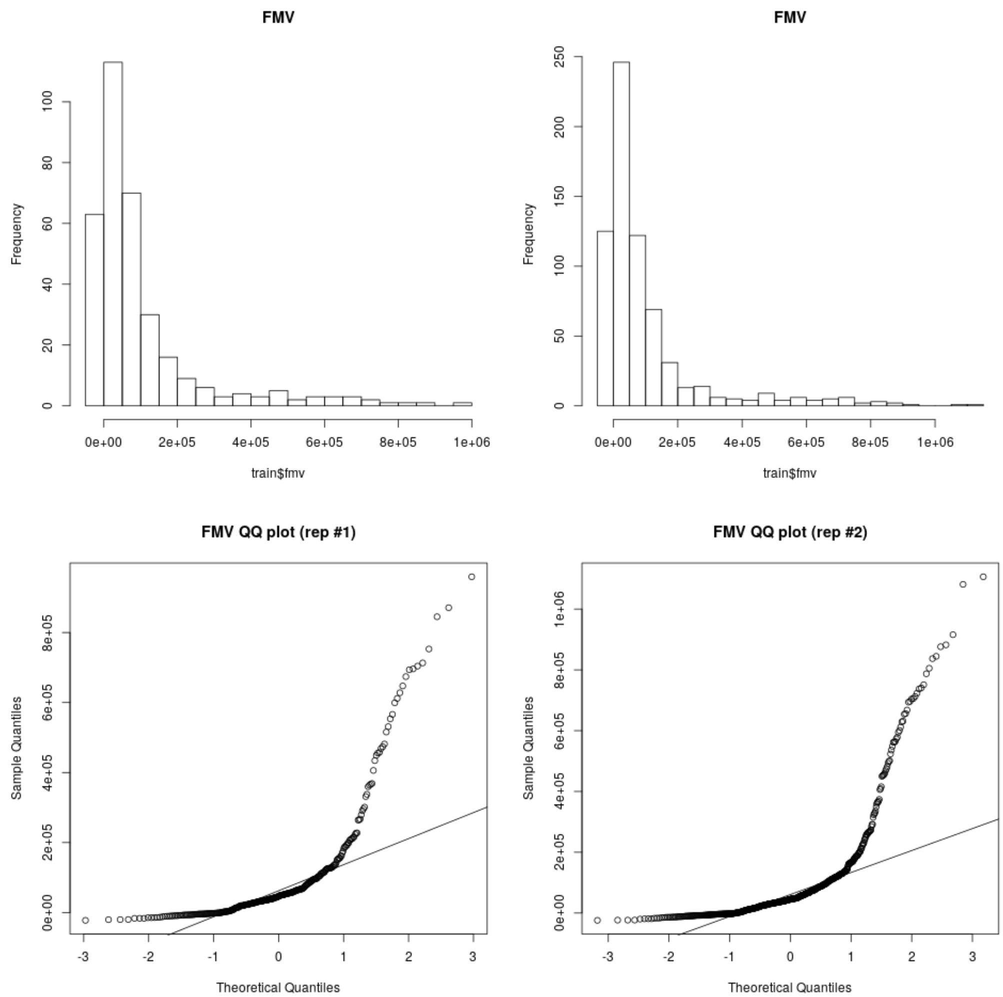


Figure 3.1: Histograms and QQ-plots for the fair market values of the contracts in the representative portfolios.

After estimating the model parameters for both representative portfolios, we analyse the residuals of the OGLASSO models. Histograms and QQ-plots are shown in Figure 3.2. As expected, the residuals illustrated in the histograms are

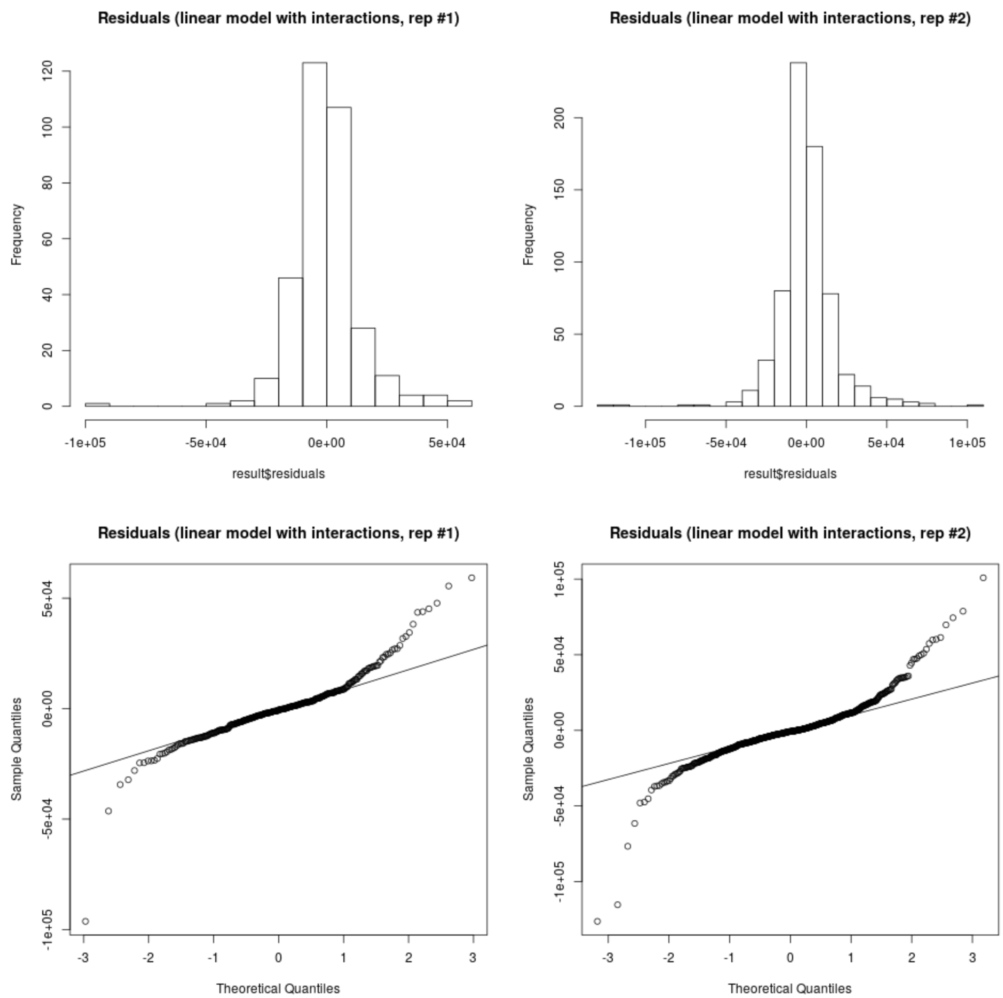


Figure 3.2: Histograms and QQ-plots for the residuals of the OGLASSO models fitted for each representative portfolio.

centered around zero. In addition, as the QQ-plots indicate, the residuals present heavy tails, a feature that is not consistent with the hypothesis of a Normal distribution. Normally distributed residuals are a desirable feature to validate any inference built on top of the estimated parameters. As illustrated in Figure 3.3, the statistically significant interaction terms identified for each representative portfolio are quite different.

Despite the problems identified in the residuals of the fitted models, their predictions are quite satisfactory. These are illustrated using the QQ-plots in Figure 3.4, where predicted FMVs are compared against their correct values. The in-sample

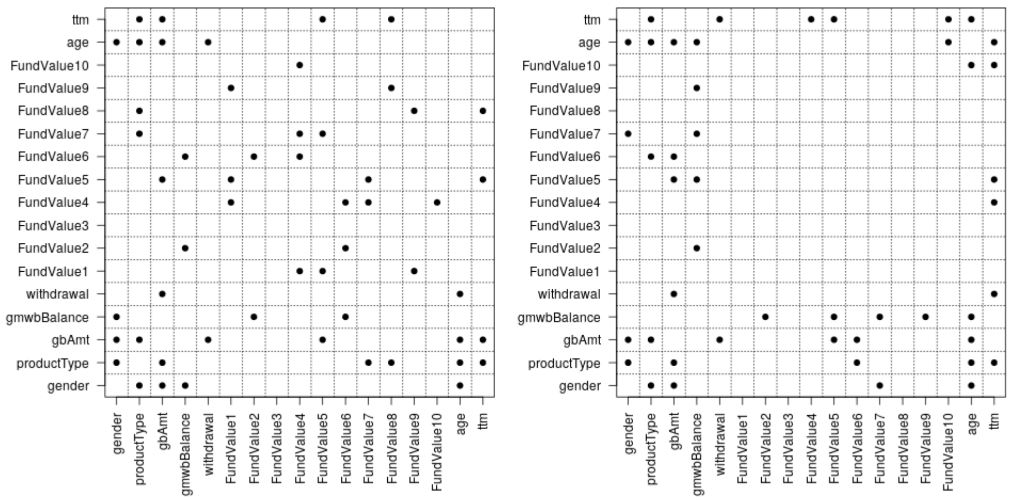


Figure 3.3: Interaction matrices for the OGLASSO models, showing the statistically significant interaction terms in each model.

results, computed exclusively with the contracts in the representative portfolios, give us R^2 values equal to 0.9928 and 0.9891 for each representative portfolio. For the full data set of VA contracts, R^2 values are equal to 0.9526 and 0.9618.

In summary, as expected, the numeric results presented above confirm that linear models with interaction terms provide excellent results for the valuation of portfolios of VA contracts. However, as an important assumption is violated, inference based on the estimated parameters is inconsistent, as illustrated in Figure 3.3. In the next section, we discuss a technique that improves the consistency of the results.

2.2 Box-Cox regression model

The Box-Cox transform, introduced by Box and Cox (1964), is commonly used in statistics to correct the skewness and non-gaussianity of regression residuals. It is, therefore, a natural choice to improve the gaussianity of the residuals in the OGLASSO model and, consequently, improve the consistency in the significance of the interaction terms of the regression models used in the valuation of the VA portfolios.

The Box-Cox regression model is given by

$$Y(\lambda) = \beta_0 + \beta_1 X_1 + \cdots + \beta_k X_k + \varepsilon, \tag{3.3}$$

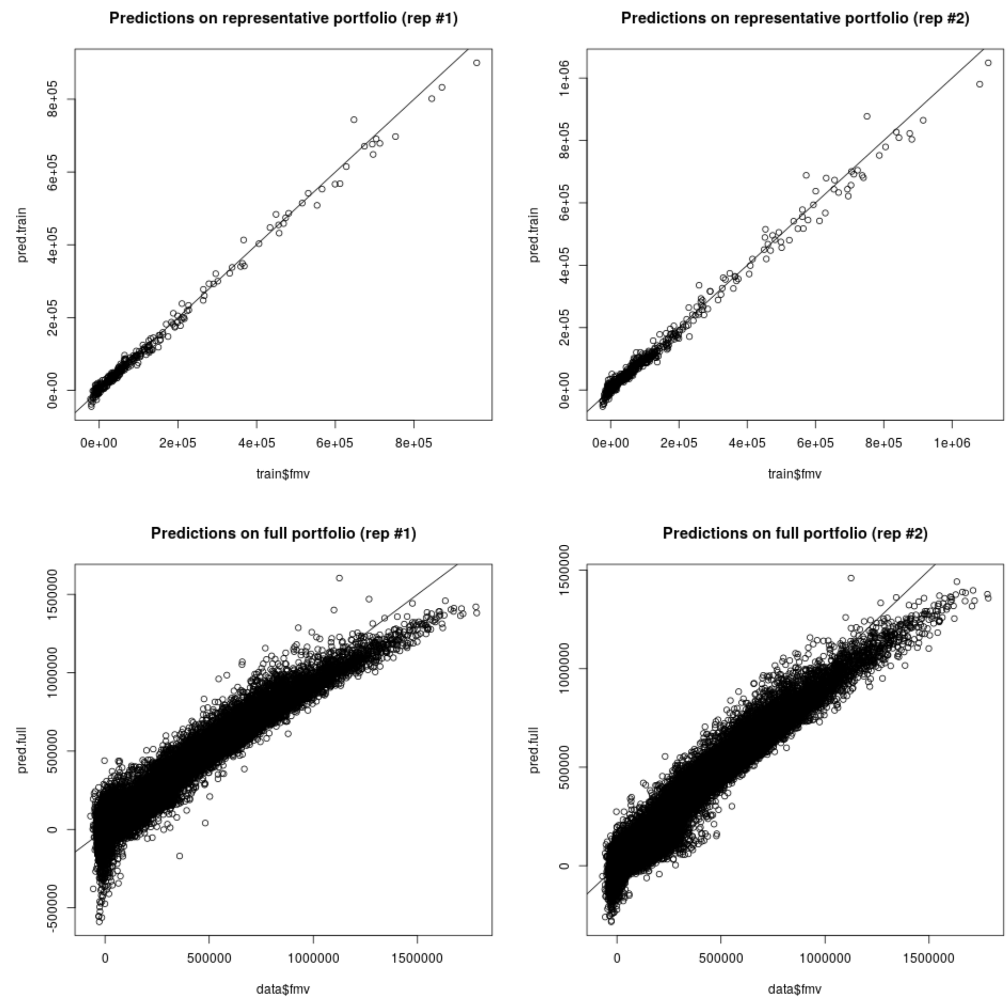


Figure 3.4: In-sample (representative portfolio) and out-of-sample (full portfolio) analysis of the predicted fair market values for VA contracts

where $Y(\lambda)$ is the transformed response variable, $X_1 \dots X_n$ the explanatory variables and $\varepsilon \sim N(0, \sigma^2)$. The transform, with parameters $\lambda = (\lambda_1, \lambda_2)$, is defined as

$$Y(\lambda) = \begin{cases} \frac{(Y+\lambda_2)^{\lambda_1 - 1}}{\lambda_1}, & \text{if } \lambda_1 \neq 0 \\ \log(Y + \lambda_2), & \text{if } \lambda_1 = 0 \end{cases} \tag{3.4}$$

As described in Box and Cox (1964), the parameters λ can be determined by maximizing the likelihood of the regression residuals. However, given the lack of appropriate software packages for the OGLASSO with the Box-Cox transform, we adopt a two-step procedure. First, we choose parameters to maximize the gaussianity of the FMVs, and then we fit the OGLASSO. Finally, we test the adequacy of the model by computing the FMVs for the entire portfolio of VA contracts, as in Section 2.1.

The first parameter determined in our experiment is the shift λ_2, chosen to guarantee non-negative values for all transformed FMVs in the synthetic database. In the sequence, values for λ_1 were chosen to maximize the gaussianity of the transformed FMVs in each representative portfolio. Using the estimated parameters, we found improvements in the symmetry of the transformed FMVs and also smaller deviations from the Gaussian distribution, as illustrated by the histograms and the QQ-plots in Figure 3.5. More importantly, after fitting the OGLASSO model for each representative portfolio, we obtain residuals that are significantly more Gaussian, as illustrated by the QQ-plots in Figure 3.6 and their counterparts in Figure 3.2, where no transformations are used.

Analysing the interaction matrices in figures 3.3 and 3.7, we conclude that the detection of statistically significant interactions has also improved. First, we could observe an increase in the number of interactions for both representative portfolios. For the first, we notice an increased from 54 to 64 interactions, and from 46 to 76 for the second. The number of matching interactions (identified in both portfolios) has also increased, from 22 to 48. In addition, if we consider these matching interactions as correct, we also find an increase in accuracy: from 40% to 75% in the case of the first portfolio, and from 47% to 63% for the second.

The models estimated for both representative portfolios fit the transformed FMVs quite well, as their R^2 values are, respectively, 99.2% and 98.6%. In-sample predictions for the transformed FMVs and regular FMVs are illustrated in Figure 3.8, where they are compared against their correct values using QQ-plots. It is interesting to notice that, due to the Box-Cox transform, the transformed FMVs do not seem as concentrated as the non-transformed ones.

We test the out-of-sample performance of the models by computing predictions for the entire data set of variable annuities. Results for both models can be found in Figure 3.9. Unfortunately, the improvements found in the in-sample results are not repeated in this test. Prediction errors are naturally larger in out-sample tests, however these seem to be inflated by the inverse Box-Cox transform, unexpectedly resulting in a poor out-of-sample performance.

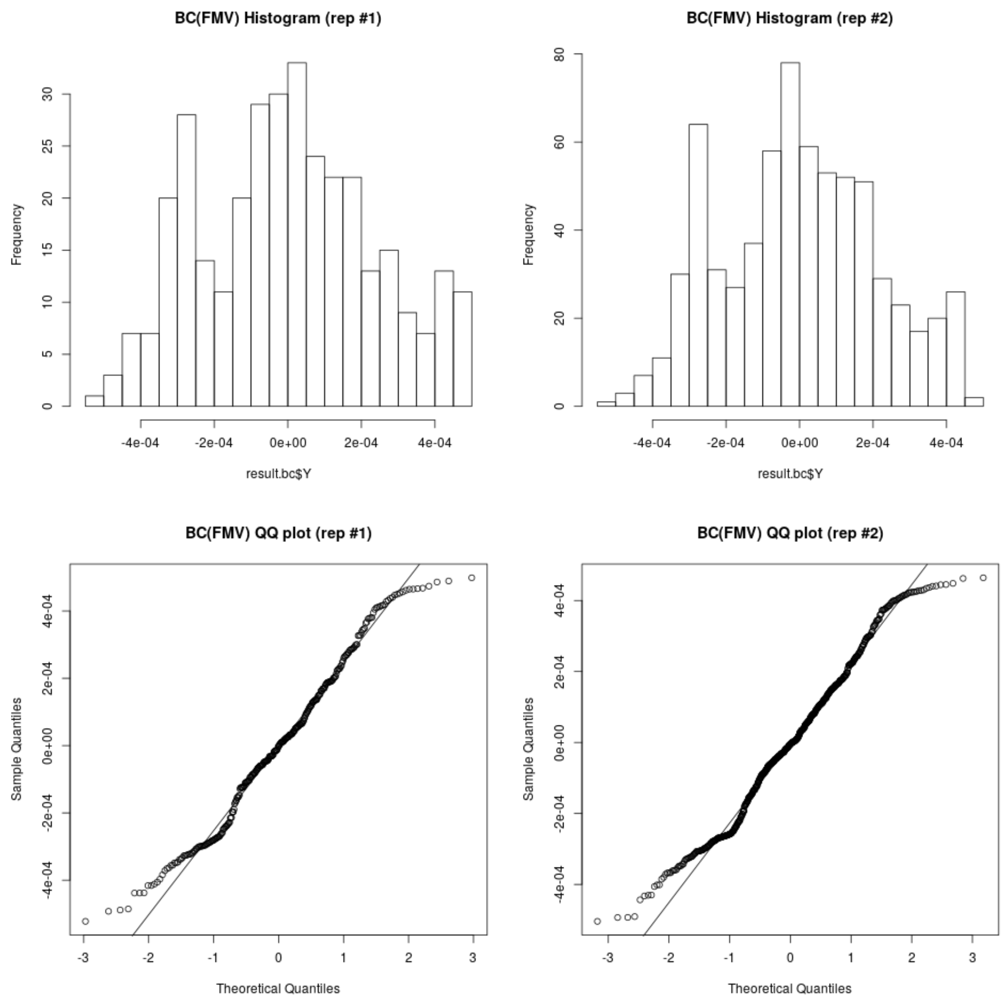


Figure 3.5: Histograms and QQ-plots of transformed fair market values of contracts in the representative portfolios.

2.3 Tree-based models

We also considered tree-based models as a meta-modelling approach. One advantage of these models is that they can handle categorical variables naturally, such as gender and product type. Moreover, they can capture non linear effects and interactions between variables. Other great advantages of tree-based models are their ability to perform feature selection and the fact that they are also easy to interpret by checking the tree structure.

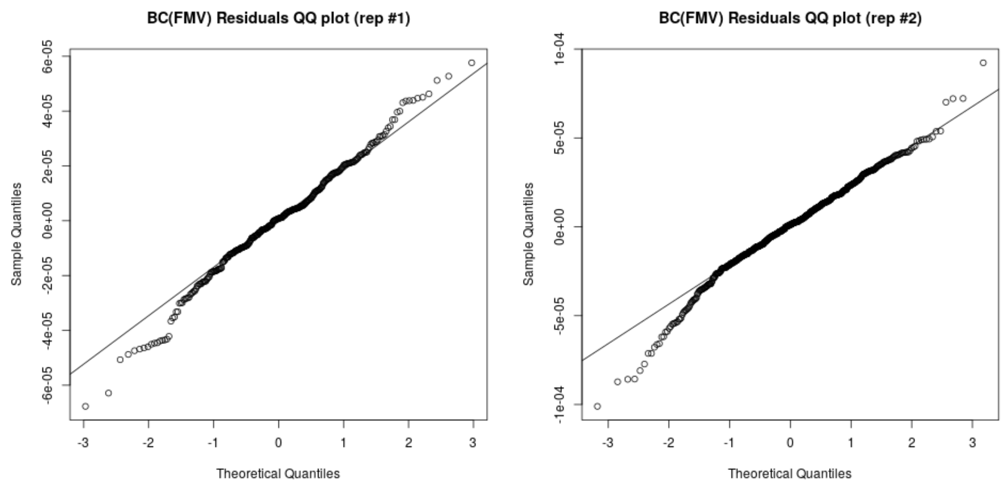


Figure 3.6: QQ-plots of residuals for the models fitted on transformed fair market values.

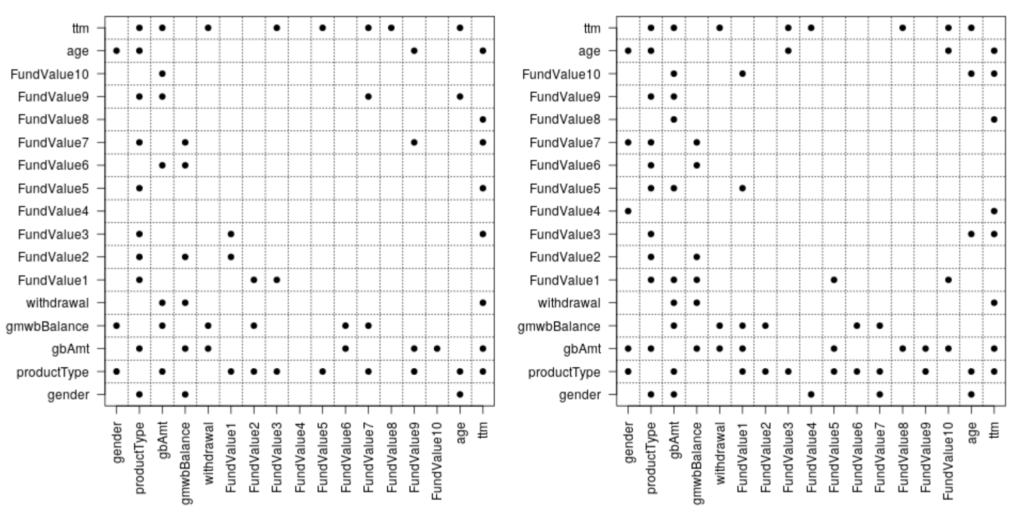


Figure 3.7: Interaction matrices for the models fitted on transformed fair market values.

To study such models we used primarily Quan et al. (2018), where the authors compare the performance of several different tree models in predicting the fair market value of VAs. As shown in Table 3.1, we can see that generally, the Gradient boosting performed better. With that in mind, we focus on this method.

A gradient boosting regression tree is an ensemble model which grows trees by

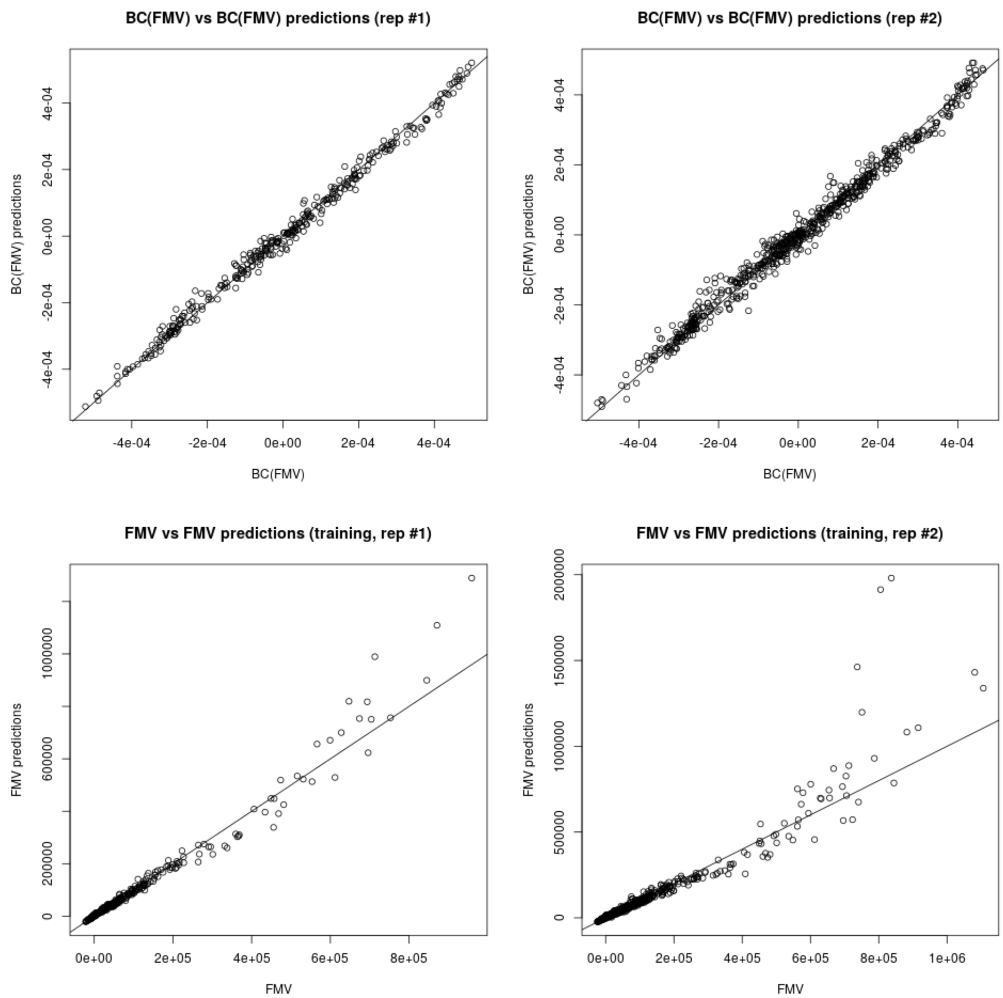


Figure 3.8: In-sample analysis of predictions for transformed fair market values (on top) and fair market values (below). The illustrated QQ-plots compare predicted values against the correct ones, available in the data set of variable annuity contracts.

sequentially putting more weights on residuals of previous trees. The final tree is given by:

$$f_B(\mathbf{X}) = \sum_{b=1}^{B} T_b\left(\mathbf{X}; \Theta_b\right)$$

where $T_b\left(\mathbf{X}; \Theta_b\right)$ are the regression trees, and B is the number of iterations or the number of additive trees. In each step b, for $b = 1, \ldots, B$, we need to find the

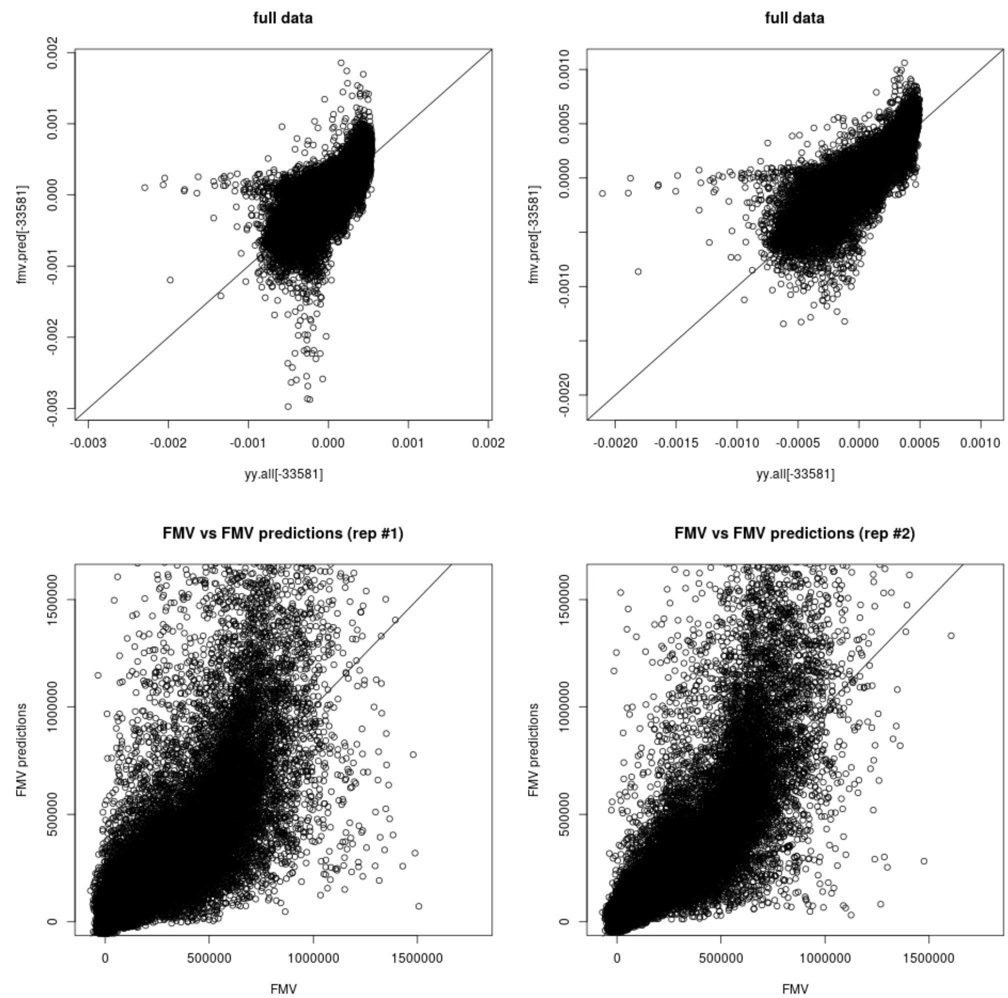


Figure 3.9: Out-of-sample analysis of predictions for transformed fair market values (on top) and fair market values (below). The illustrated QQ-plots compare predicted values against the correct ones, available in the data set of variable annuity contracts.

regression Θ_b based on the following optimization problem:

$$\hat{\Theta}_b = \arg\min_{\Theta_b} \sum_{i=1}^{N} L\left(y_i, f_{b-1}\left(\mathbf{X}_i\right) + T_b\left(\mathbf{X}_i; \Theta_b\right)\right)$$

Being that the gradient boosting regression tree was the best model for our case, we wanted to compare its performance against the overlapped group lasso model, which had very good results, as shown in Section 2.1. Furthermore, we wished to

Model	Gini	R^2	CCC	ME	PE	MSE	MAE
Regression tree (CART)	0.786	0.845	0.917	1.678	-0.025	3278.578	31.421
Bagged trees	0.842	0.918	0.954	2.213	-0.033	1720.725	20.334
Gradient boosting	0.836	0.942	0.969	1.311	-0.019	1214.899	19.341
Conditional inference trees	0.824	0.869	0.930	0.905	-0.013	2754.853	26.536
Conditional random forests	0.836	0.892	0.940	1.596	-0.024	2273.385	23.219
Ordinary Kriging	0.815	0.857	0.912	-0.812	0.012	3006.192	27.429
GB2	0.827	0.879	0.930	0.106	-0.002	2554.246	27.772

Table 3.1: Prediction accuracy of different models

test the robustness of each model to the set of representative contracts. In order to do so, we trained each model with 100 random sets of representative VAs, each containing 340 contracts. We used 100 trees and maximum depth of 3 for the GBM. As the tables 3.2 and 3.3 show, the error measures are smaller for the overlapped grouped lasso model. Additionally, the error measures vary a lot more for the GBM tree model then for the overlapped grouped lasso model, suggesting that the former is more robust to the selection of the representative contracts.

	|PE|	R2	|ME|	MAE	MSE
Min.	0.00	0.87	0.00	28.46	2180.66
1st Qu.	0.01	0.89	1.41	30.72	2517.63
Median	0.03	0.90	2.66	31.63	2776.60
Mean	0.03	0.90	2.80	31.91	2822.27
3rd Qu.	0.04	0.91	3.82	33.00	3053.23
Max.	0.10	0.92	9.37	36.60	3858.51

Table 3.2: GBM Tree: Error measures for multiple sets of representatives contracts

	|PE|	R2	|ME|	MAE	MSE
Min.	0.00	0.93	0.08	18.59	810.84
1st Qu.	0.00	0.96	0.40	20.79	1074.65
Median	0.01	0.96	1.09	21.28	1163.91
Mean	0.01	0.96	1.31	21.33	1182.15
3rd Qu.	0.02	0.96	1.82	21.88	1248.26
Max.	0.05	0.97	4.79	23.70	1911.56

Table 3.3: Overlapped Group Lasso: Error measures for multiple sets of representatives contracts

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