Mission Planning Document

Verification of Dark Matter
Mission Planning Document
- PHYSICS -
NovaScience

Background

You are a group of scientists specialized in the detection of dark matter in space.

Your mission: **To send your pilot to the space station** to get **information about several stars**. In exchange, NASA will give you information on five stars from the galaxy UGC 11748, a galaxy we have strong reason to believe contains dark matter.

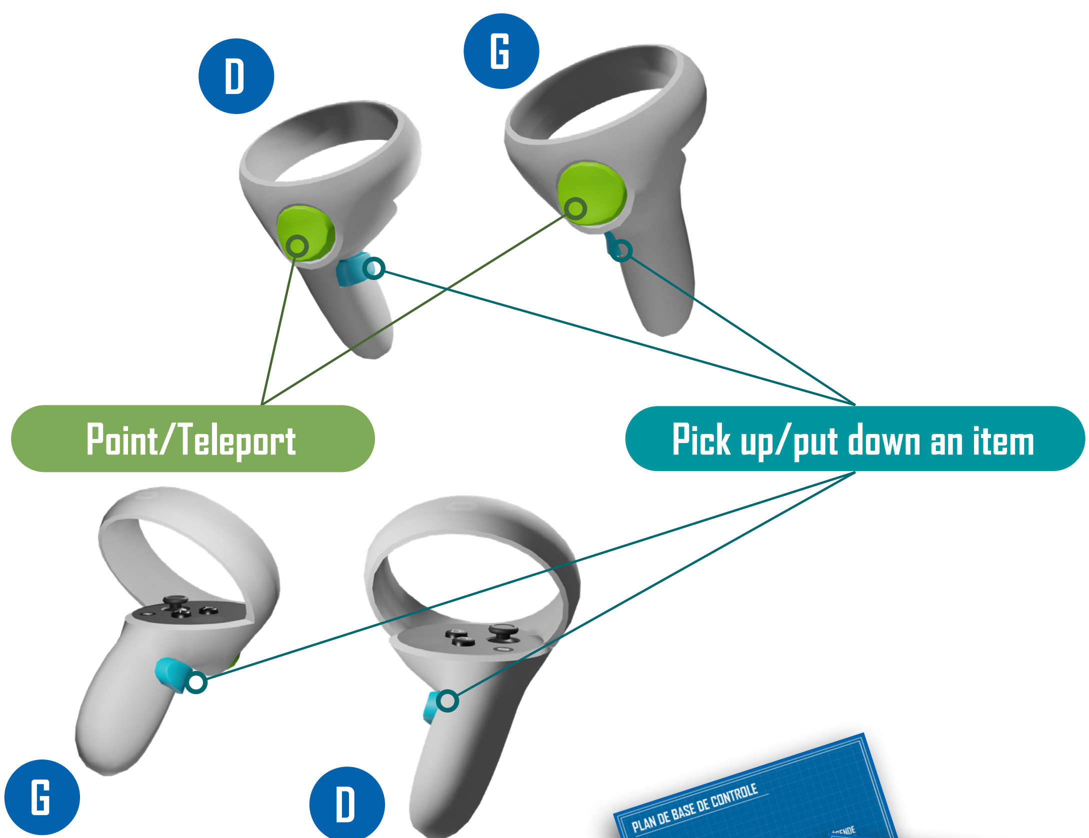
The team will have to first prove their competencies by carrying out several simulations **before blasting off to the space station.**

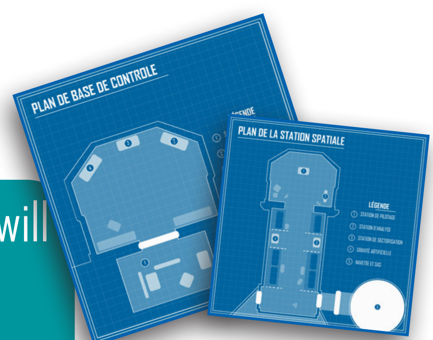
To accomplish the mission, the pilot, mission chief and engineer will have to work together and pool the complementary information available to them.

Good luck!

Partner assistance

Your pilot will have to handle **specific equipment**. In case the pilot needs assistance with the manoeuvres, here is the instruction manual.





* In the Mission Chief's Document, you will find the plans required for the mission.

* Information will also be sent to your pilot's arm screen. Remind the pilot to check it!

Rocket launch

Your first task is to determine the energy required to launch a rocket so that it can reach the space station orbiting around the Earth.

Since the rocket will travel a significant distance from the Earth's surface, it will be subjected to different levels of gravitational force. Therefore, you have to use the general formula for calculating the gravitational potential energy, shown at the bottom of this page. In this formula, $\mathbf{U_g}$ is deemed to be zero when the two objects are separated by an infinite distance. The variables shown in this formula are described on the next page.

Equation

$$U_g = -\frac{G m_E m_R}{r}$$

Rocket launch

Consider the initial situation where the rocket is on the launch pad and is moving at a speed equal to the speed of the rotation of the Earth and the final situation where the rocket is docked at the space station and therefore is moving at the same speed as the station.

On the next page, first calculate the initial mechanical energy of the rocket (E_i) and its final mechanical energy (E_f). Using the formula for the conservation of energy ($E_i + W_{nc} = E_f$), calculate the energy that must be provided to the rocket to get it to the space station, which corresponds to the term W_{nc}.

Mass of the Earth : m_E (Celestial analysis station)

Mass of the rocket : m_R *(Piloting simulation station)*

Distance from the centre of the Earth to the launch pad : r_E
(Piloting simulation station)

Distance from the centre of the Earth to the space station: r_{ISS}
(Celestial analysis station)

Orbital speed of the space station : v_{ISS}
(Piloting simulation station)

Rotational speed of the Earth : v_E (Celestial analysis station)

Gravitational constant : G (Engineer's Document)

During the mission, you will find the numerical value of each of the variables at the place mentioned between the brackets.

Rocket launch

Initial situation: Rocket launch

Gravitational potential energy:

Kinetic energy:

Final situation: Arrival at the space station

Gravitational potential energy:

Kinetic energy:

Rocket Launch

With the help of the formulas above, use the formula for the conservation of mechanical energy to calculate the energy to provide to the rocket (W_{nc}).

Write the final expression here. You will need it during the mission, **on page 3 of the Engineer's Document.**

Preparation for liftoff

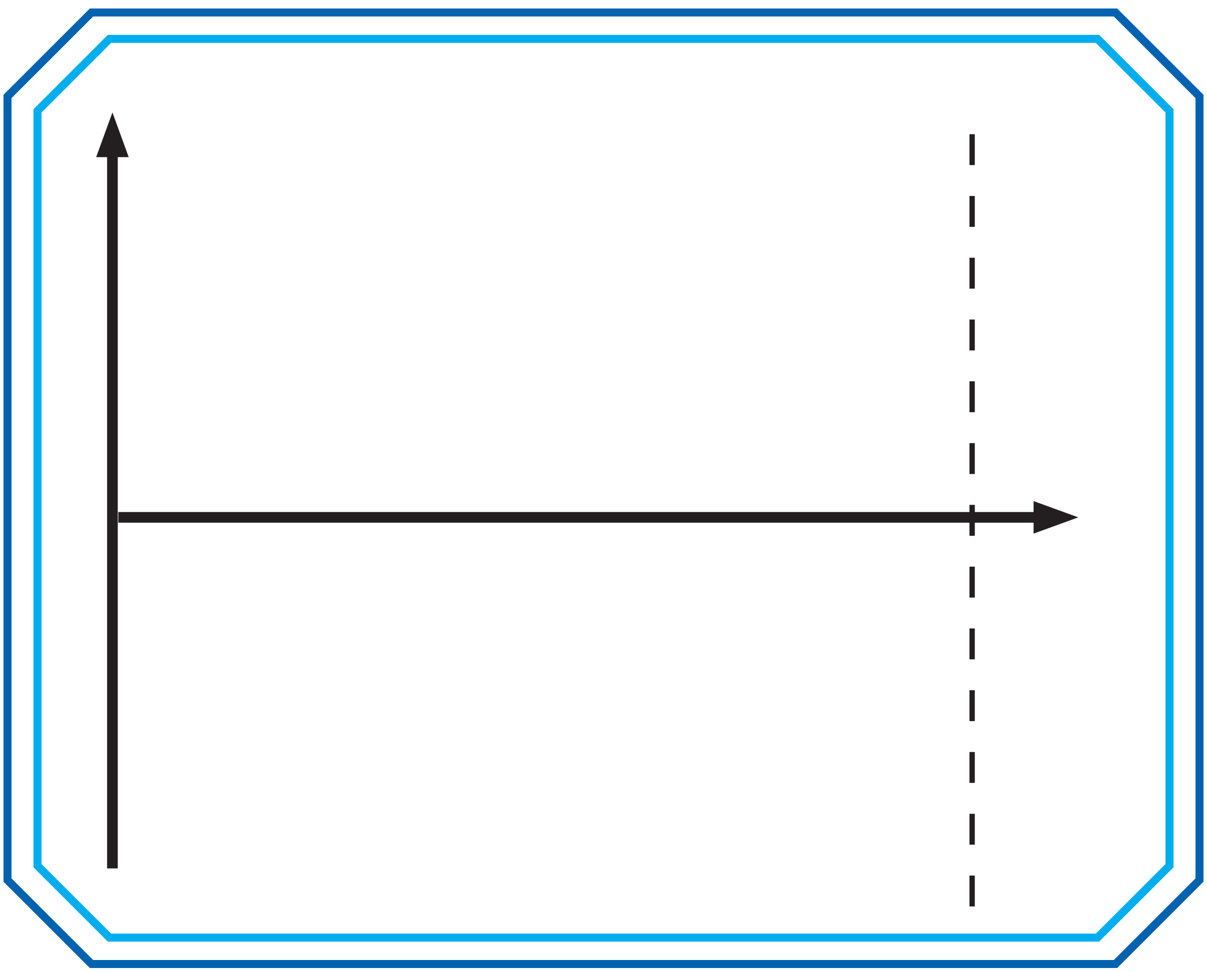
Having analyzed the energies in play, now **try to predict the shape of the graphs** depicting energy as a function of time for each of the three following energies between lift off and docking at the space station.

- **Kinetic energy**

  The rocket begins at a low speed, corresponding to the rotational speed of the Earth. It ultimately has to dock at the space station orbiting the Earth.

- **Potential gravitational energy**

- **The potential energy released by the rocket fuel.**



Calculation of speeds

Once the team has arrived at the space station, they can focus on their exploration of dark matter. To do this, they will have to calculate the expected speed of a star based on its distance from the centre of the galaxy. By measuring the luminosity of a galaxy, it is possible to establish an indirect measurement of its mass and distribution. These measurements reveal that a clear majority of the stars in galaxy UGC 11748, which should represent over **95% of its mass,** are within a radius of $1{,}64\times10^{20}$ m and that the mass contained within that radius should be $1{,}54\times10^{41}$ **kg.** This mass only takes into consideration the matter we can see.

These measurements can be verified by measuring the orbital speed of the stars located around the perimeter of the galaxy. Consider a star located at a distance r from the centre of the galaxy. On the next page, write out Newton's second law algebraically for this star. In your formula, use the variables r for the distance between the star in question and the centre of the galaxy, m_s for the mass of the star, m_g for the total mass of the stars within the radius mentioned earlier (we can establish approximately that this mass is concentrated at a point in the centre of the galaxy) and v for the speed of the star.

Then, isolate the speed in this equation. You will use this equation in the simulation to calculate the expected speed of several stars based on their distance from the centre of the galaxy. These speeds will be calculated speeds.

Calculation of speeds

Using Newton's second law, fin the equation for the calculad speed

Equation of the final calculated speed :

Trace a sketch of the graph of the speed against distance