Fakultas Informatika
School of Computing
Telkom University

CAK2HAB3 - Dasar Kecerdasan Artifisial

Reasoning: Fuzzy Logic

Lecturer Team

Five types of Logic

Type of logic	Reference to real world	Type of knowledge known by Agent
Propositional Logic	Facts	True / False / Unknown
First-order Logic	Facts, Object, Relation	True / False / Unknown
Temporal Logic	Facts, Object, Relation, Time	True / False / Unknown
Probability Theory	Facts	Degree of belief [0, 1] (certainty factor)
Fuzzy Logic	Truth values (degree of truth)	Confidence level [0, 1] (certainty factor)

What is Fuzzy Logic?

Fuzzy Logic

- Experts rely on common sense when they solve problems.

- “People do not require precise, numerical information input, and yet they are capable of highly adaptive control.”
  - Professor Lotfi Zadeh, UC Berkeley, 1965

Fuzzy Logic

- How can we represent expert knowledge that uses vague and ambiguous terms in a computer?
- Accepts noisy, imprecise input!
- Decisions based on “degree of truth”
  - Graded truth.
  - Truth values between True and False.
  - Not everything is either/or, true/false, black/white, on/off etc.

Fuzzy Logic

- Fuzzy logic is based on the idea that all things admit of degrees.
  - Temperature, height, speed, distance, beauty
    - all come on a sliding scale.
  - Model uncertainty in natural language.
    - The motor is running really hot.
    - Tom is a very tall guy.

Fuzzy Logic

- Reasoning using linguistic terms. Natural to express expert knowledge.
  - If the weather is cold, then wear warm clothing

Fuzzy Logic

- Fuzzy logic is not logic that is fuzzy, but logic that is used to describe fuzziness.
- Fuzzy logic is the theory of fuzzy sets, sets that calibrate vagueness.
- It's not a method for reasoning under uncertainty
  - that's probability

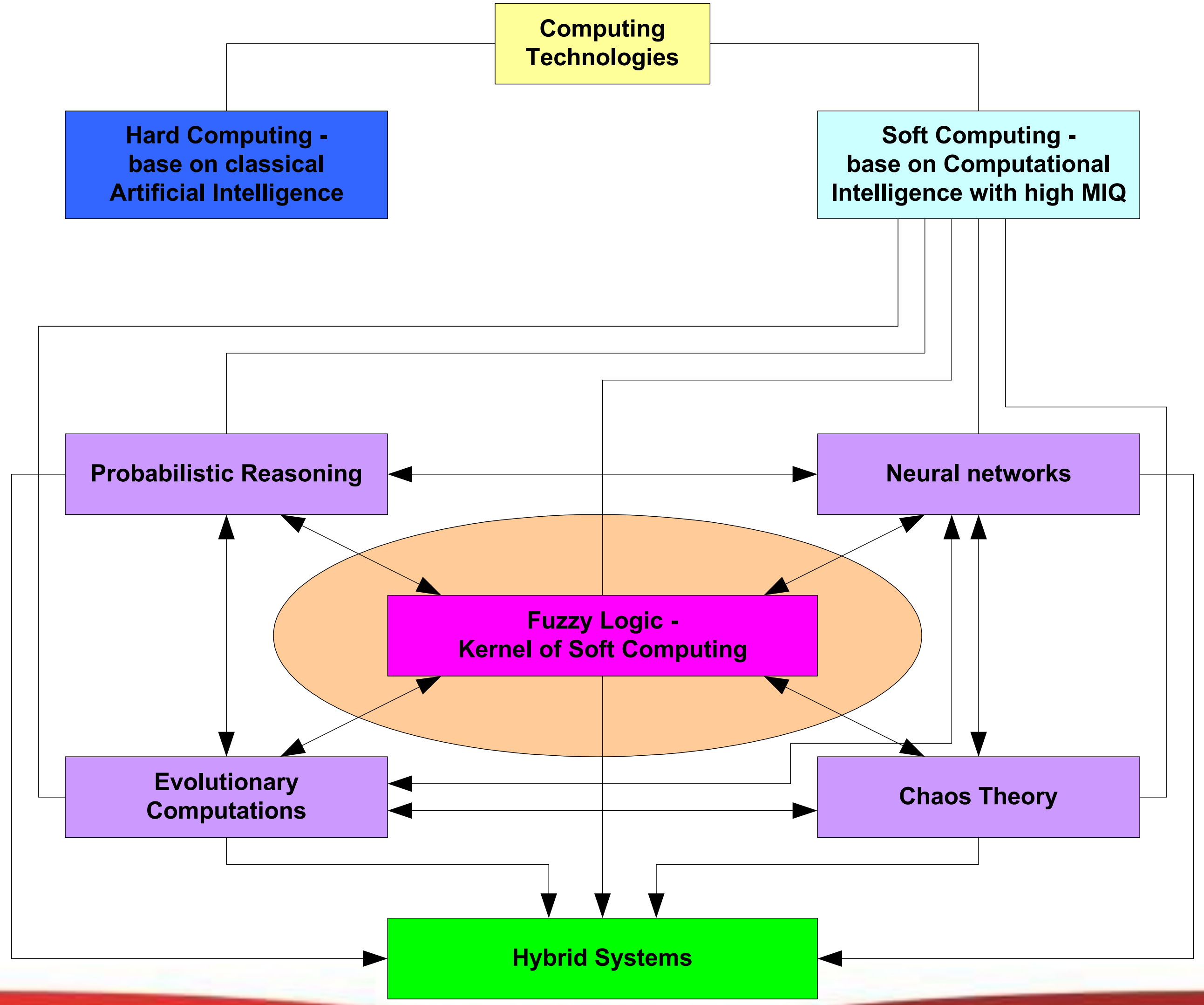
Fuzzy Logic

- Fuzzy set theory resembles human reasoning in its use of approximate information and uncertainty to generate decisions.
- It was specifically designed to mathematically represent uncertainty and vagueness and provide formalized tools for dealing with the imprecision intrinsic to many problems.

Soft Computing

Evolving collection of methodologies, which aims to exploit tolerance for imprecision, uncertainty, and partial truth to achieve robustness, tractability and low cost

[Lotfi A. Zadeh, 2006]


Computing Technologies
Hard Computing - base on classical Artificial Intelligence
Soft Computing - base on Computational Intelligence with high MIQ
Probabilistic Reasoning
Neural networks
Fuzzy Logic - Kernel of Soft Computing
Evolutionary Computations
Chaos Theory
Hybrid Systems

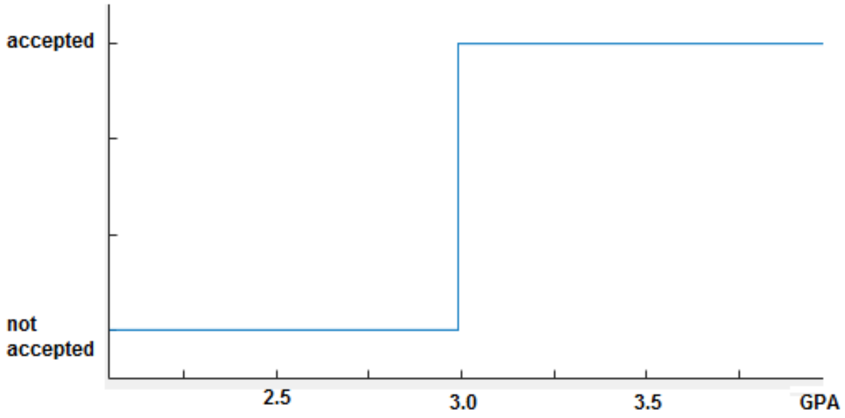
Case Example
that can be solved using Fuzzy

Scholarship Selection

- A university wants to provide some scholarships to their outstanding students

- Define classical rule
  - Select students based on their current GPA

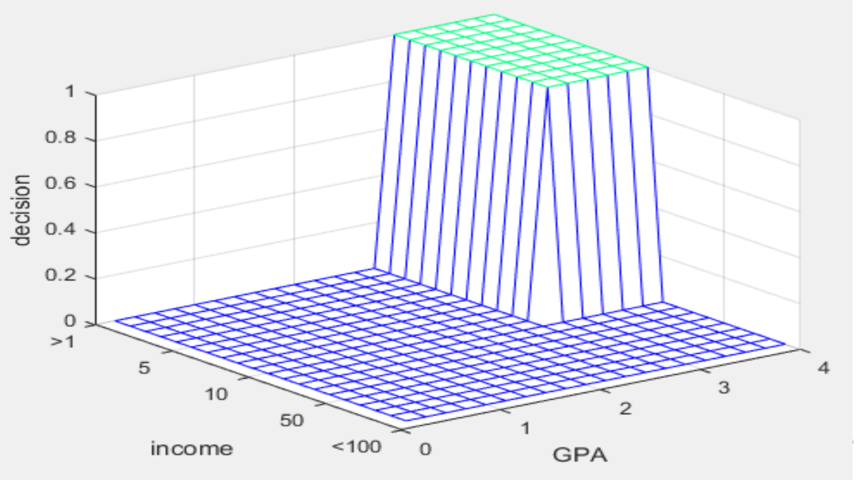
```
if GPA >= 3.0 then
    accept
```



Scholarship Selection

- The selection criteria is too broad, add more parameter
- Add more classic rule
  - Select students based on their parent’s income

```
if GPA >= 3.0 and income<10 then
      accept
```

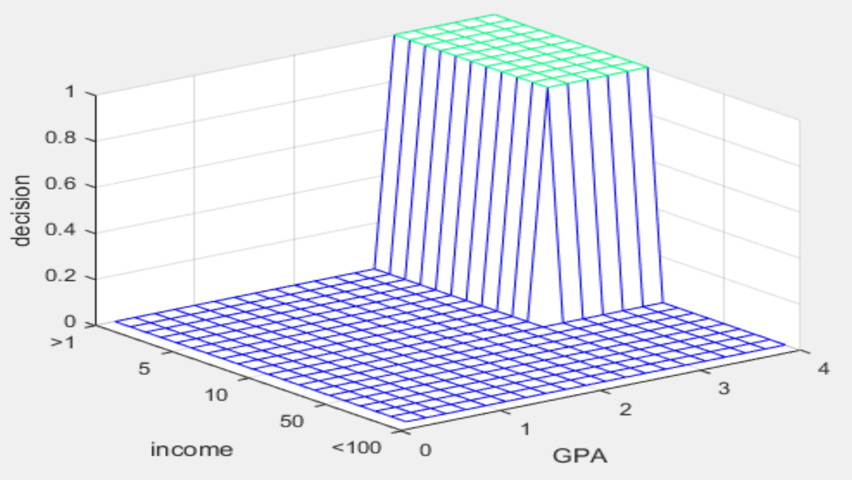


Scholarship Selection

- Then come a specific case:

Candidate	GPA	Parents' Income IDR/month
A	3.00	9,500,000
B	2.98	1,000,000

```
if GPA >= 3.0 and income<10 then
      accept
```



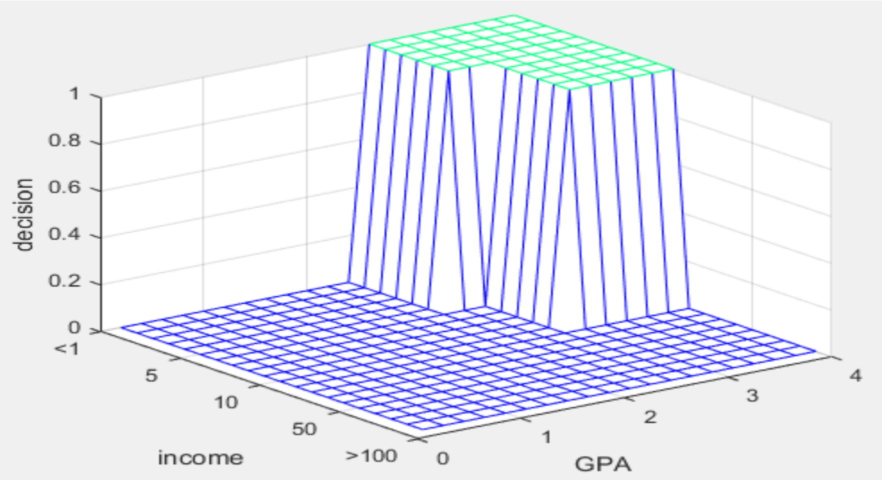
Scholarship Selection

- Then come a specific case:

Candidate	GPA	Parents’ Income IDR/month
A	3.00	9,500,000
B	2.98	1,000,000

  - add a new rule

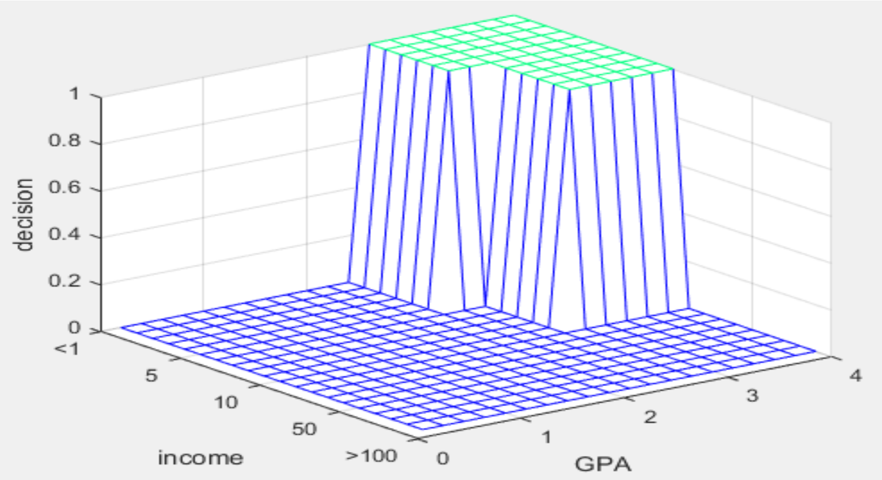
```
if (GPA >= 3.0 and income<10) or
      (GPA>=2.5 and income<5)
then
      accept
```



Scholarship Selection

- What if the University can only provide 5 scholarships, and there are 6 qualified applicants?

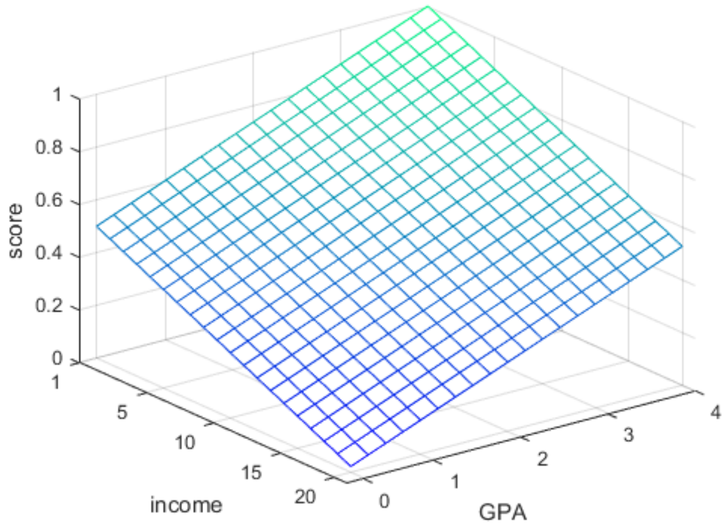
```
if (GPA >= 3.0 and income<10) or
      (GPA>=2.5 and income<5)
then
      accept
```



Scholarship Selection

- What if the University can only provide 5 scholarships, and there are 6 qualified applicants?

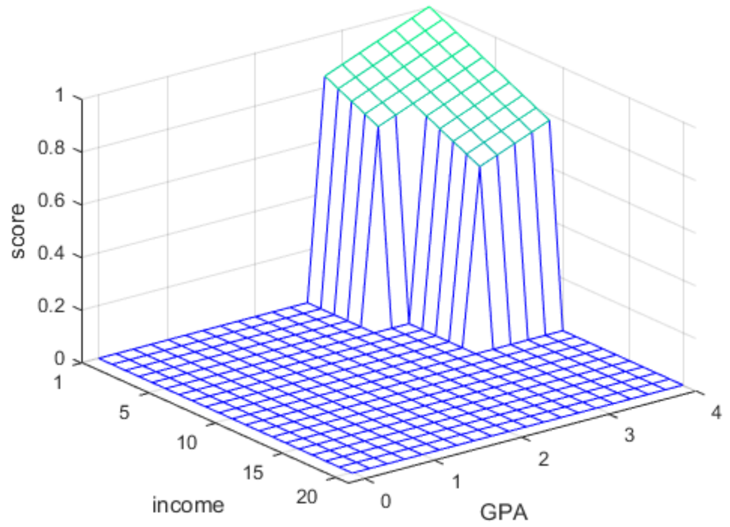
- Change the binary decision to a scoring system [0,1]
  - Score = (GPA/4 + (20-income)/19)/2



Scholarship Selection

- Combine and Adjust to original rule:

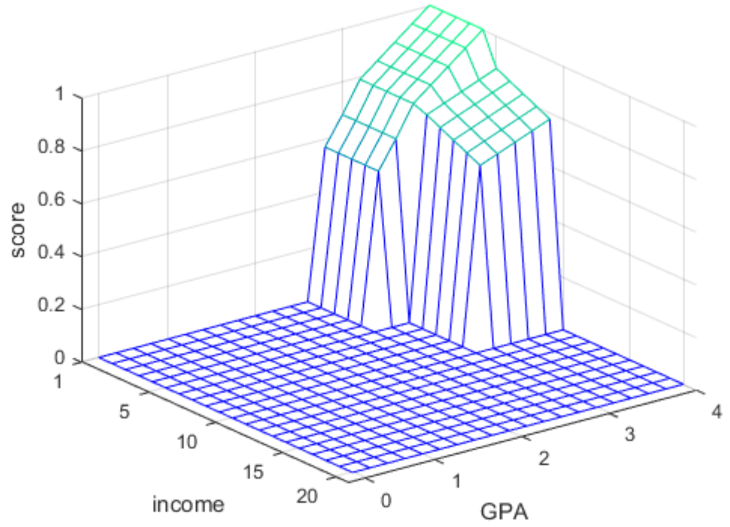
```
if (GPA >= 3.0 and income<10) or
      (GPA>=2.5 and income<5)
then
      score = (GPA/4+(20-income)/19)/2
else
      score = 0
```



Scholarship Selection

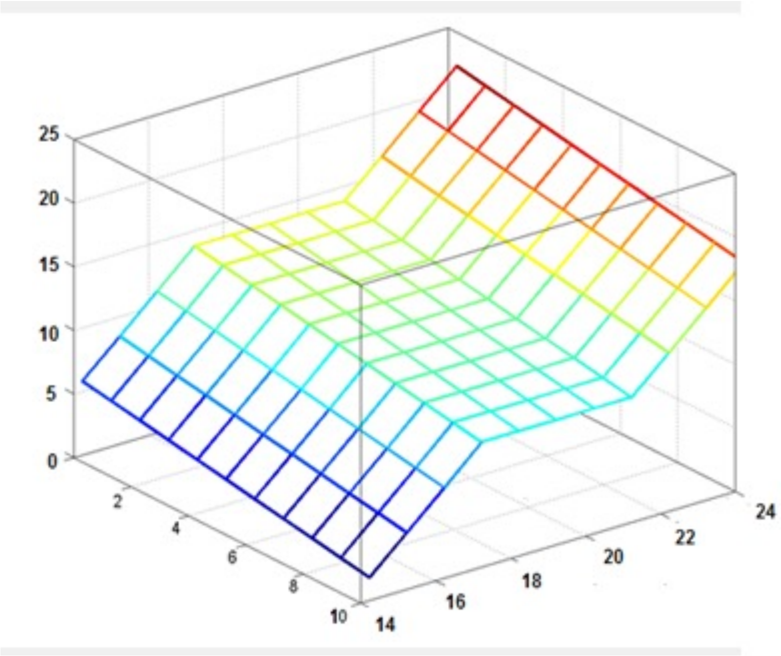
- More specific cases?
- Further modifying the rule to adapt the situation

```
if(GPA>=2.5 and income<6) then
      score = 2*(20-income)/19-0.8;
if(GPA>=3 and income<6) then
      score = (20-income)/19
if(GPA<3 and 6<income<10) then
      score = (GPA/4+(20-income)/19)/2;
```



Scholarship Selection

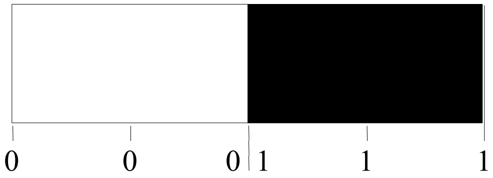
- Extended Rule
  - Complicated function
  - Not easy to modify
  - Not intuitive
  - Many hard-coded parameters
  - Not easy to understand
- Fuzzy can help this



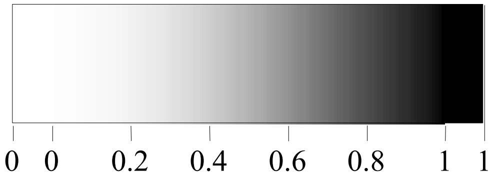
Fuzzy Value and Fuzzy Set

Fuzzy, Crisp, and Probability

- Crisp Facts
  - clear and distinct boundaries
  - “either-or”
- Fuzzy Facts
  - imprecise boundaries
  - “more-or-less”
- Probability
  - incomplete facts
  - “might-be”, “chance”



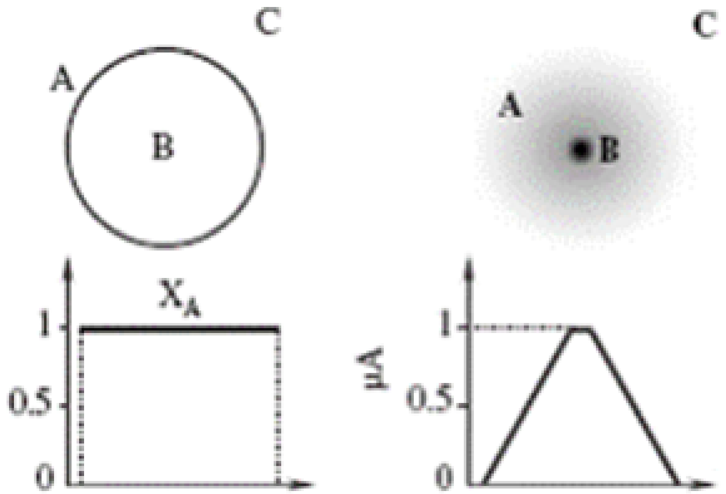
(a) Boolean Logic.

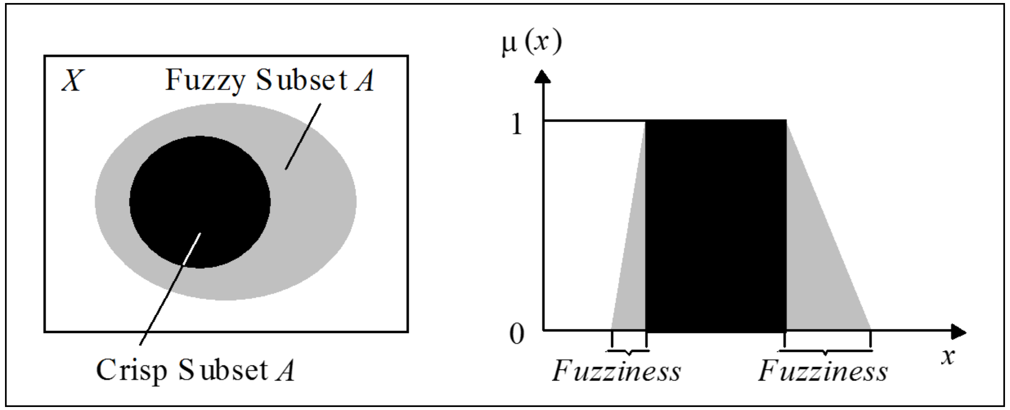


(b) Multi-valued Logic.

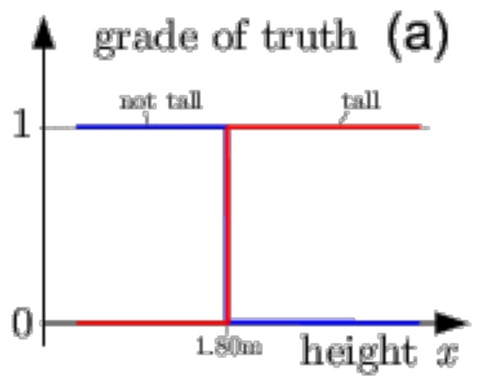
Fuzzy vs Crisp value

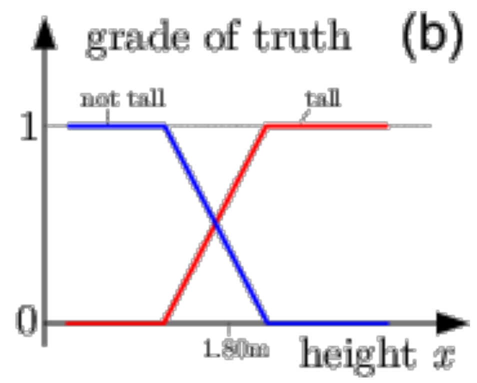
- Crisp value has distinct boundary between 1 and 0
- Fuzzy vale consist of a crisp value, and a gray area to bridge the values

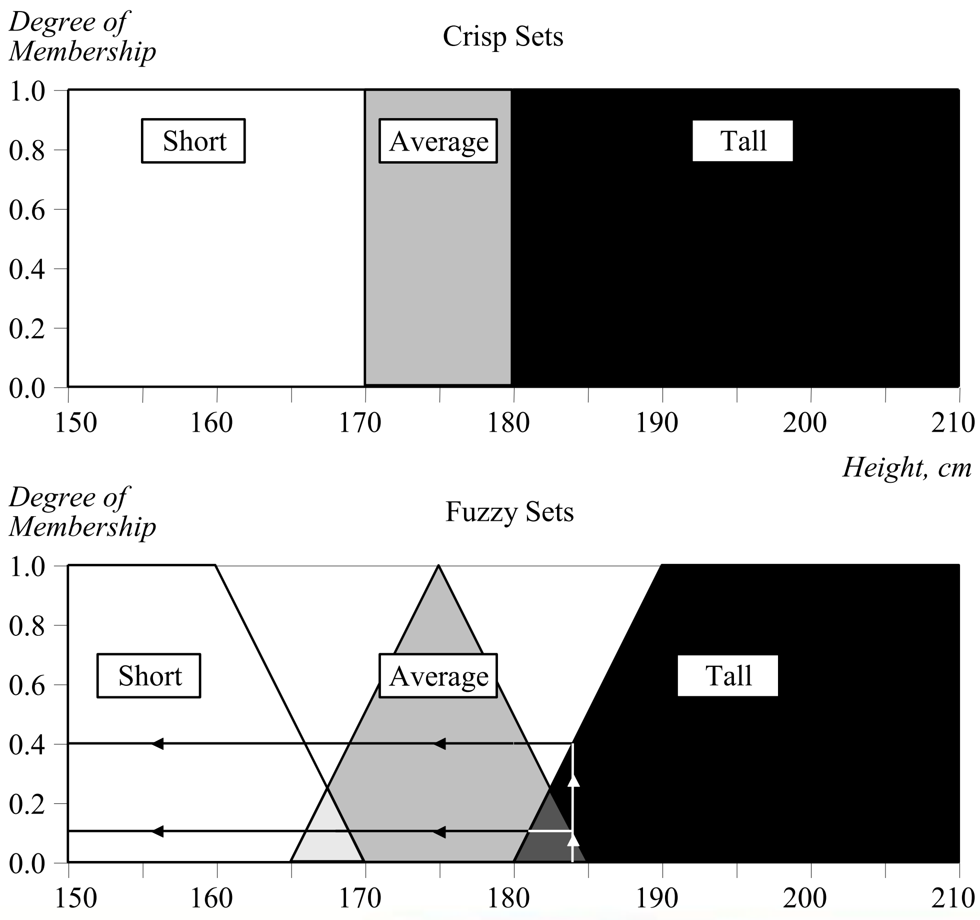




Fuzzy vs Crisp value







Fuzzy vs Probability

- Probability
  - likelihood that a future event will occur
  - Probability event is in a set
- Fuzzy Logic
  - Measures ambiguity of event that has already occurred
  - Degree of membership in a set

Linguistic Variables

- A linguistic variable is a fuzzy variable
- A variable whose values are words or sentences in a natural or artificial language
- Age is a linguistic variable if its values are linguistic rather than numerical,
  - i.e., *young*, *not young*, *very young*, *quite young*, *old*, *not very old* and *not very young*, etc.,
  - rather than 20, 21,22, 23

Linguistic Variables

- Treating **Truth** as a linguistic variable with values such as true, very true, completely *true*, *not very true*, *untrue*, *etc*., leads to what is called fuzzy logic.
- By providing a basis for approximate reasoning, that is, a mode of reasoning which is not exact nor very inexact, such logic may offer a more realistic framework for human reasoning than the traditional two-valued logic.

Membership Function

Membership Function

- Formula used for more *humane* reasoning.
  - Let U be the universe of discourse and x is a member of U.
  - A fuzzy set A in U is defined as a membership function, which maps every object in U into a real value in the interval $[0, 1]$.
  - Values denote the degree of membership x in A.

Degree of Membership Example

- U is universe of discourse of temperature $[-\infty, \infty]$
- A is a Fuzzy set Temperature with 3 Linguistics:
  Cold, Warm, Hot
- Fuzzy set maps U into A as degree of membership in range of [0,1]

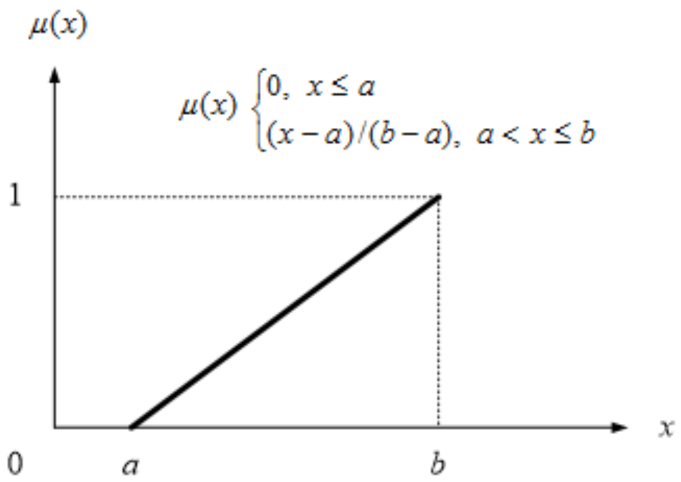
Crisp Value (x)	Fuzzy Value		
Temperature (°C)	**Cold**	**Warm**	**Hot**
5	1	0.1	0
15	0.9	0.8	0
25	0.5	1	0.6
35	0.1	0.6	0.9
45	0	0.2	1

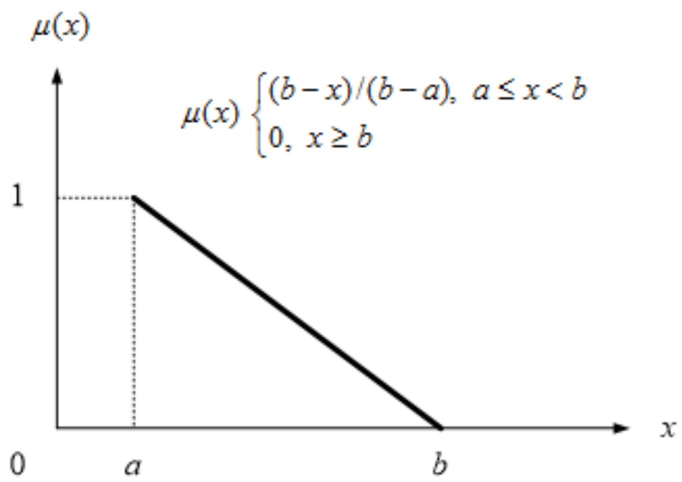
Membership Functions

- Membership function usually described as a mathematical graph function
- Linear Function:
  - Triangles, Trapezoidal
- Sigmoid Function and Bell Function:
  - Phi, Beta, Gauss

Linear Membership Function

- Illustrates linear progression in fuzzy value that bridge one value to another

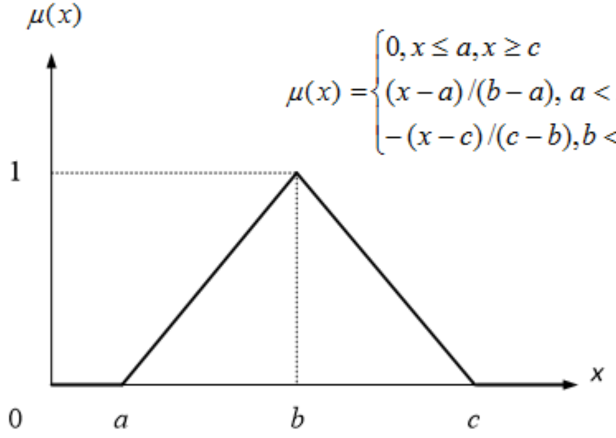




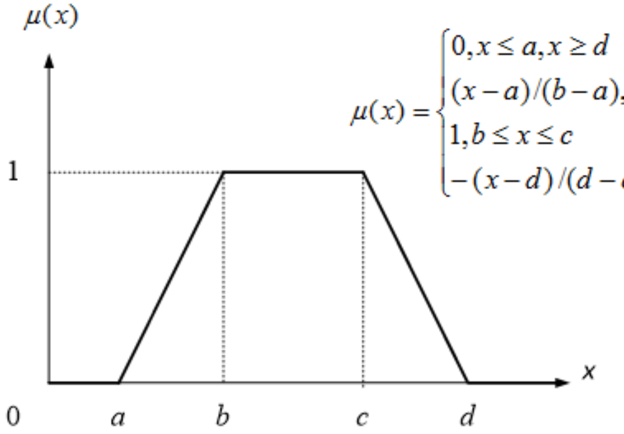
Linear Membership Function

- Combination of two linear membership function make the function looks triangular or trapezoidal

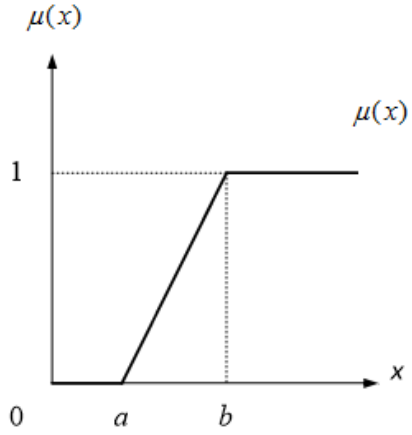
$$\mu(x) = \begin{cases} 0, x \le a, x \ge c \\ (x-a)/(b-a),\ a < x \le b \\ -(x-c)/(c-b), b < x \le c \end{cases}$$



$$\mu(x) = \begin{cases} 0, x \le a, x \ge d \\ (x-a)/(b-a), a < x < b \\ 1, b \le x \le c \\ -(x-d)/(d-c), c < x \le d \end{cases}$$

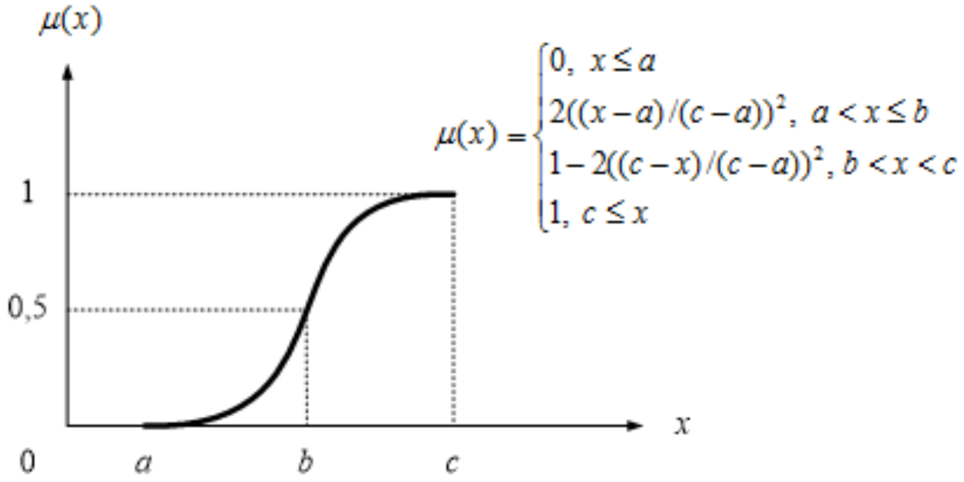


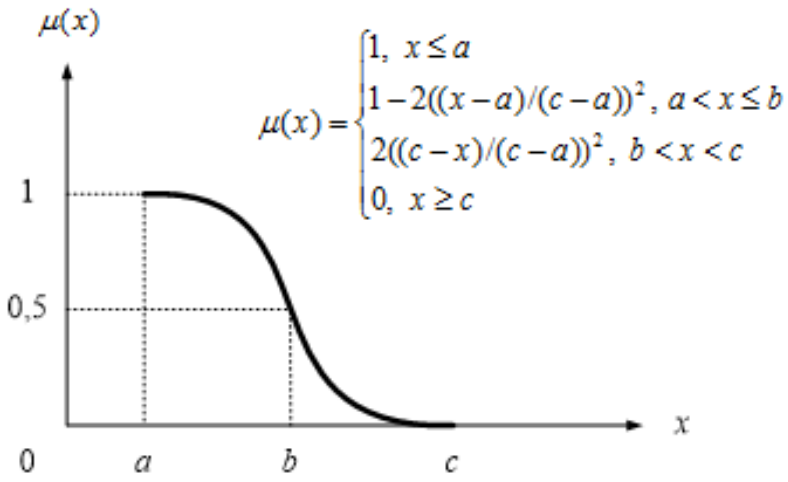
$$\mu(x) = \begin{cases} 0, x \le a \\ (x-a)/(b-a), a < x < b \\ 1,\ x \ge b \end{cases}$$



Sigmoid Membership Function

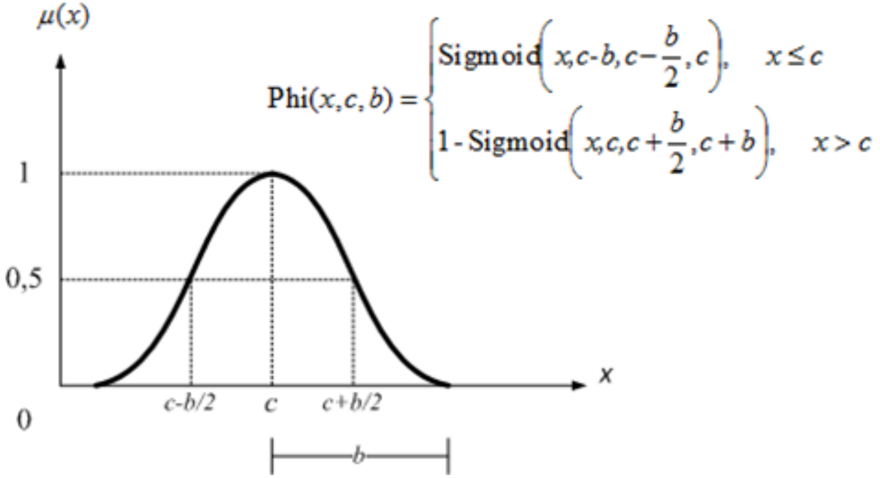
- Illustrates progression from small beginnings that accelerates and approaches a climax over time
  - When a specific mathematical model is lacking, a sigmoid function is often used



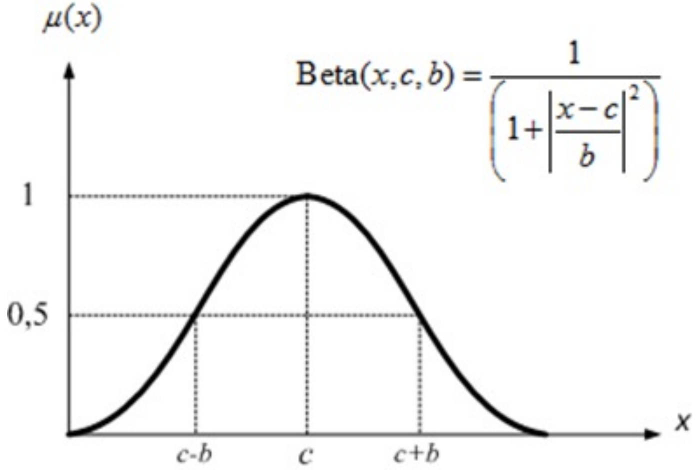


Sigmoid Membership Function

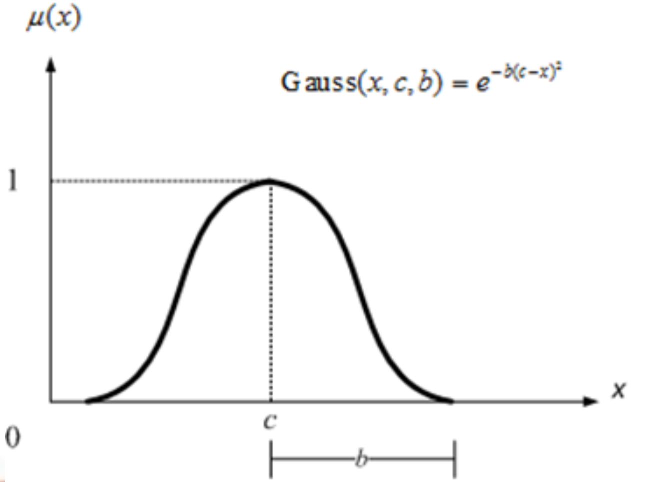
- Various functions to represent sigmoid such ass bell and gaussian



$$\mathrm{Phi}(x,c,b)=\begin{cases}\mathrm{Sigmoid}\left(x,c\text{-}b,c-\frac{b}{2},c\right), & x\le c\\ 1\text{-}\mathrm{Sigmoid}\left(x,c,c+\frac{b}{2},c+b\right), & x>c\end{cases}$$



$$\mathrm{Beta}(x,c,b)=\frac{1}{\left(1+\left|\frac{x-c}{b}\right|^2\right)}$$

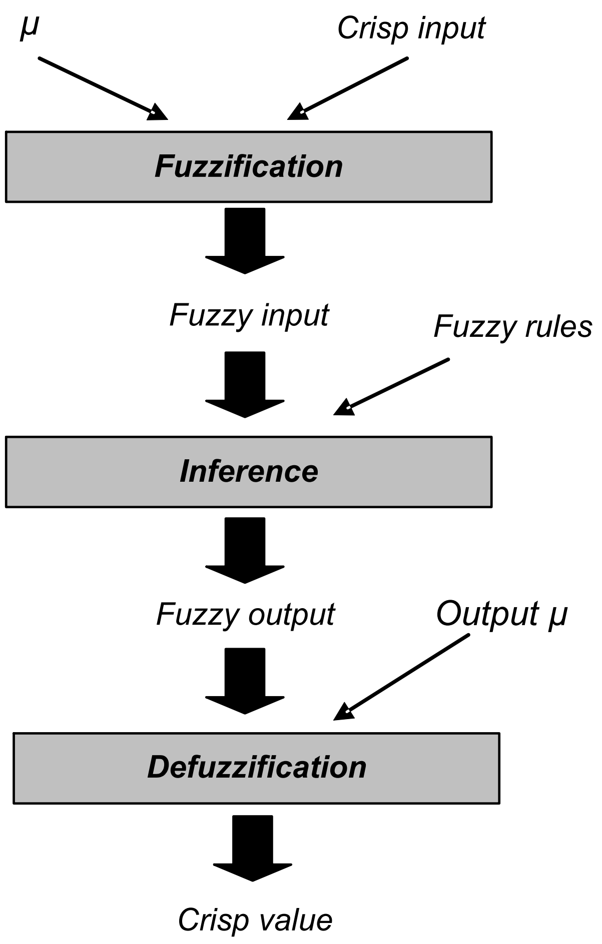


$$\mathrm{Gauss}(x,c,b)=e^{-b(c-x)^2}$$

Building Fuzzy System

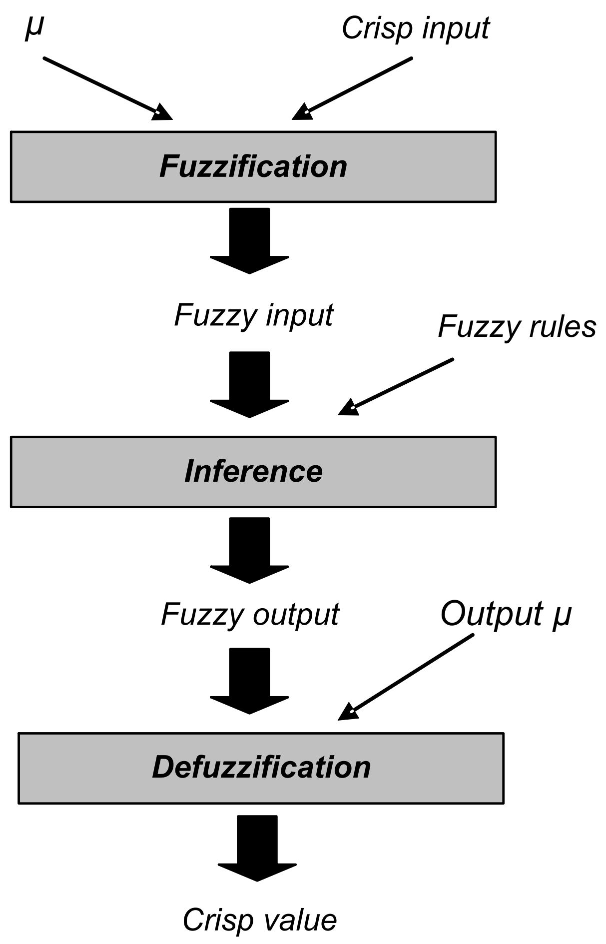
Fuzzy System

- Consist of 3 main process
- Fuzzification
  - Transform crisp input into fuzzy values based-on its corresponding Membership Function
- Inference
  - Determine the fuzzy output using predefined rule
- Defuzzification
  - Transform back the output fuzzy value into crisp value



Fuzzy System

- To build a fuzzy system,
  we need to design and specify:
  - Number of input and output Linguistic
  - Input Membership Functions
  - Fuzzy Rules
  - Defuzzification Method
  - Output Membership Function
- Ideally these rules and functions
  are provided by “the expert in the appropriate field”



Designing
Fuzzy System for Scholarship Selection

For this example, let’s pretend that we are
“the expert in this field”

Step 1: Determine the input and output

- Just as the example has been given previously, the inputs for the selection are GPA and Parent's income, while the output is the Acceptance score

- Design the Linguistic Variable for each Input, let's specify each into three (3) linguistics such as:
  - GPA: **High**, **Average**, **Low**
  - Income: **Upper**, **Middle**, **Bottom**

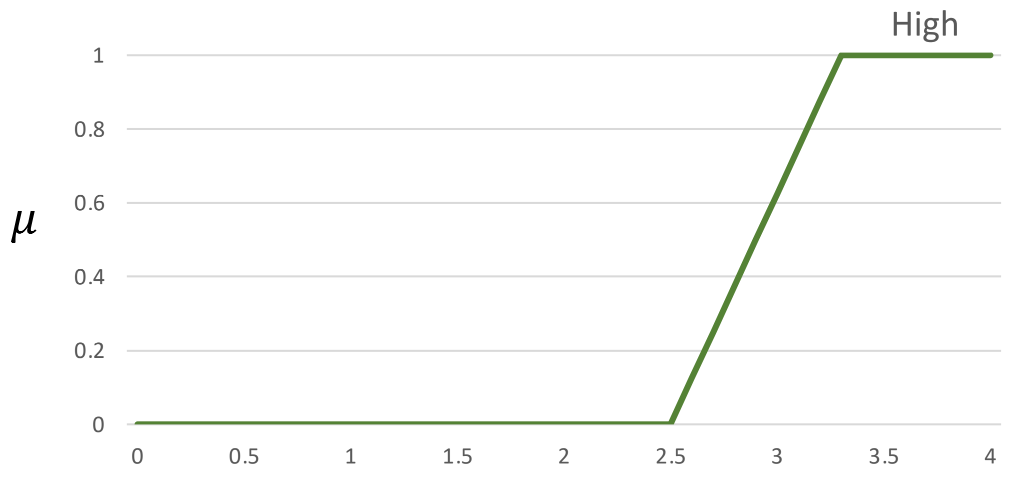
- For the output, we design also three (3) Linguistic
  - Score: **Accepted**, **Considered**, **Rejected**

Step 2: Design the Membership Functions

- Design the membership function for each Linguistic
- Membership Function for GPA(**High**)
  - The range of GPA is [0,4]
  - From there, we determine that
  - GPA>3.25 is definitely **High**, and GPA≤2.5 is definitely **Not High**
  - Therefore the range (2.5, 3.25] is the fuzzy area between High and Not High
  - Let's say that the progression between GPA High and Not High is linear,
  - thus we have

Step 2: Design the Membership Functions

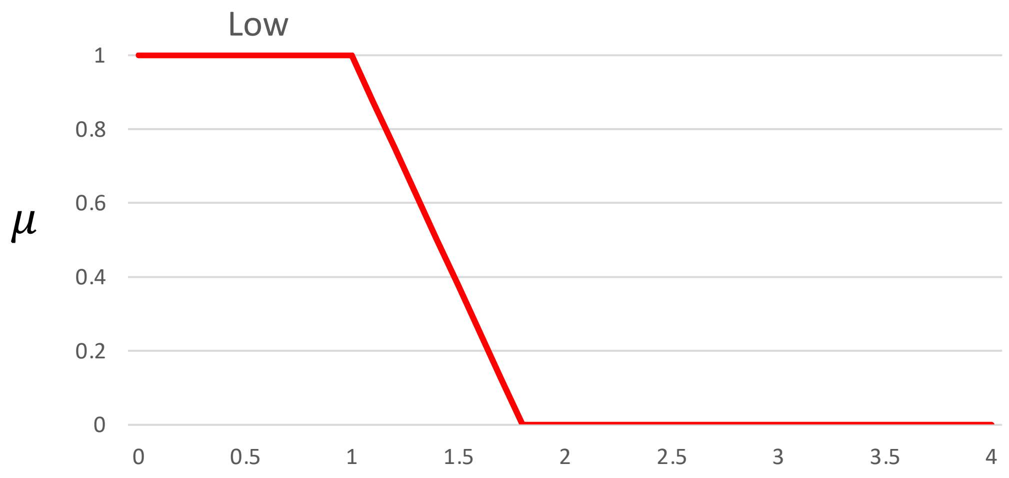
- Design the membership function for each Linguistic
- Membership Function for GPA(**High**)



$$\mu(x) = \begin{cases} 0, & x \leq 2.5 \\ 1, & x > 3.25 \\ \dfrac{x - 2.5}{3.25 - 2.5}, & 2.5 < x \leq 3.25 \end{cases}$$

Step 2: Design the Membership Functions

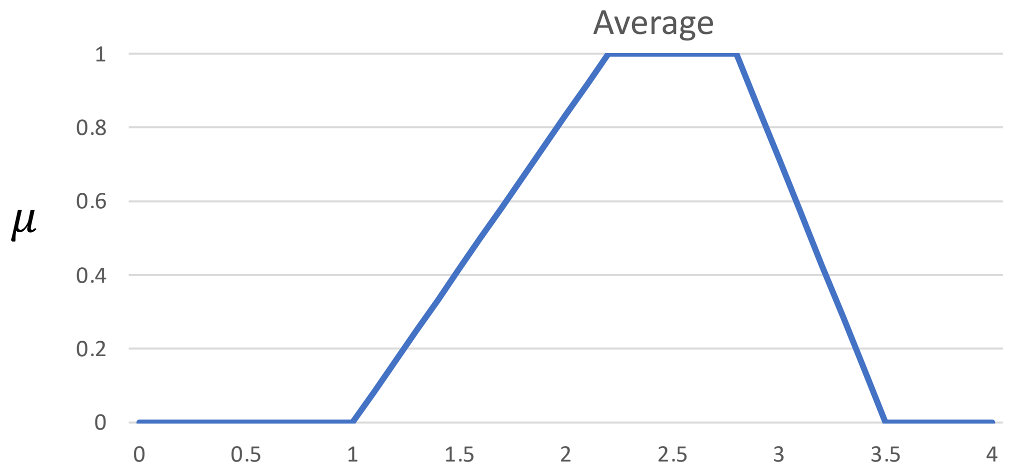
- Membership Function for GPA(**Low**)
  - Here, we determine that GPA≤1 is **Low**, and GPA>1.75 is **Not Low**
  - Using the same linear function, we have



$$\mu(x) = \begin{cases} 1, & x \leq 1 \\ 0, & x > 1.75 \\ \dfrac{1.75 - x}{1.75 - 1}, & 1 < x \leq 1.75 \end{cases}$$

Step 2: Design the Membership Functions

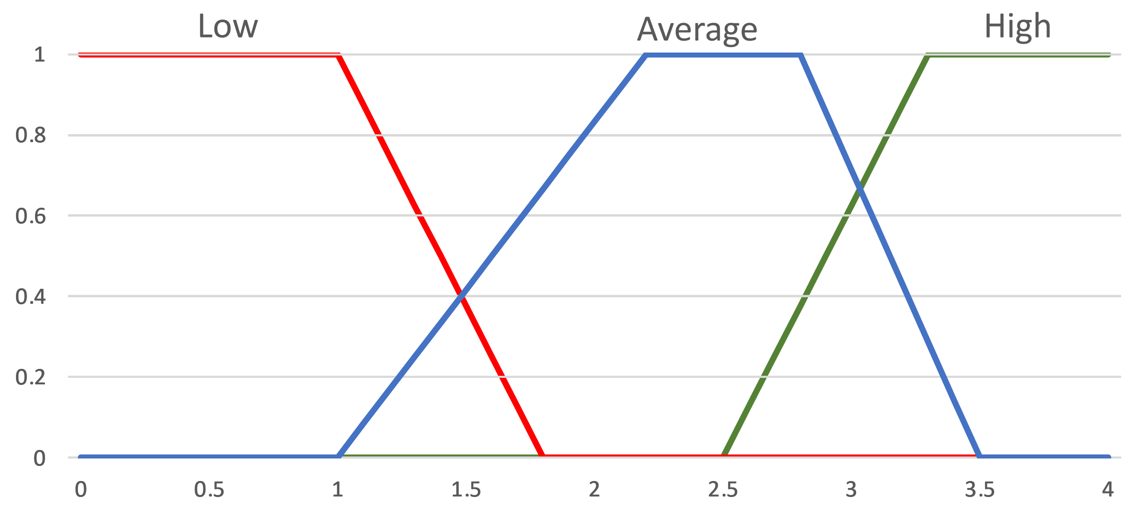
- Membership Function for GPA(**Average**)
  - The **Average** GPA is considered between 2.2 to 2.8
  - While GPA≤1 or GPA>3.5 is considered **Not Average**
  - Using the same linear function, we have



$$\mu(x) = \begin{cases} 0, & x \leq 1, x > 3.5 \\ \dfrac{x-1}{2.2-1}, & 1 < x \leq 2.2 \\ 1, & 2.2 < x \leq 2.8 \\ \dfrac{3.5-x}{3.5-2.8}, & 2.8 < x \leq 3.5 \end{cases}$$

Step 2: Design the Membership Functions

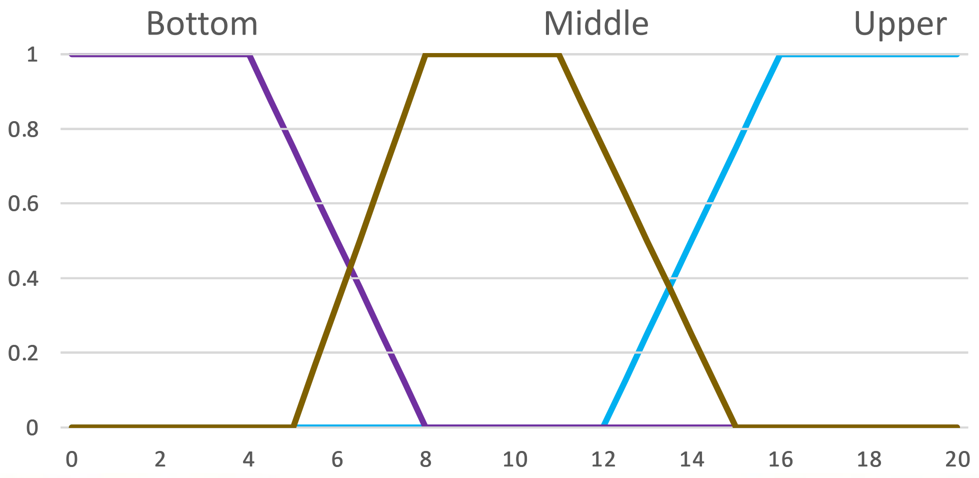
- If we combine the Membership Function for GPA
  it’ll look like:



- Do the same for Income

Step 2: Design the Membership Functions

- Membership Function for Parent's Income
  - We determine that Income>16 million is considered as **Upper** with fuzziness starts from 12 million, and
  - Income≤4 million is considered as **Bottom**, and by 8 million it's no longer considered Bottom
  - Income between 8 to 11 million is considered as **Middle** with fuzziness starts from 5 to 15 million



Step 3: Design the Fuzzy Rules

- Define rules that will determine Accepted or Rejected score based on GPA and Income inputs
- With 2 inputs and 3 linguistics each, there will be 3x3=9 rules (combinatorial)
- Basic rule form:

```
if GPA='High' and Income='Bottom' then Score='Accepted'
...
if GPA='Average' and Income='Middle' then Score='Considered'
...
if GPA='Low' and Income='Upper' then Score='Rejected'
...
```

Step 3: Design the Fuzzy Rules

- Or define it as a table

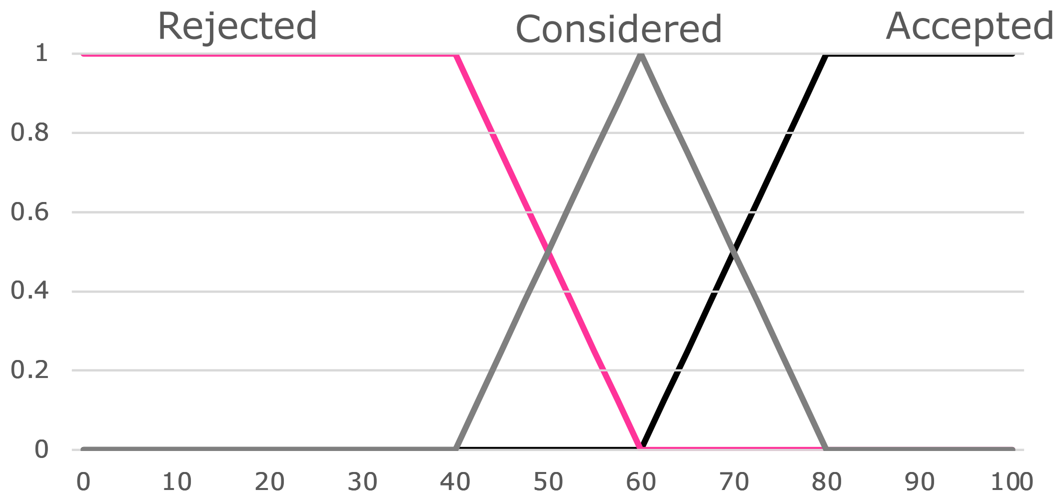
GPA	Income	Score
High	Upper	Considered
High	Middle	Accepted
High	Bottom	Accepted
Average	Upper	Considered
Average	Middle	Considered
Average	Bottom	Accepted
Low	Upper	Rejected
Low	Middle	Rejected
Low	Bottom	Considered

Step 4: Choose Defuzzification Method

- Process that maps a fuzzy set back to a crisp set
- Common methods:
  - Center of Gravity (Mamdani-style)
  - Constant Defuzzification (Takagi-Sugeno-style)
- Other common methods:
  - Mean of Maxima
  - Weighted Average
  - Middle of Maxima
  - Singleton, Etc.

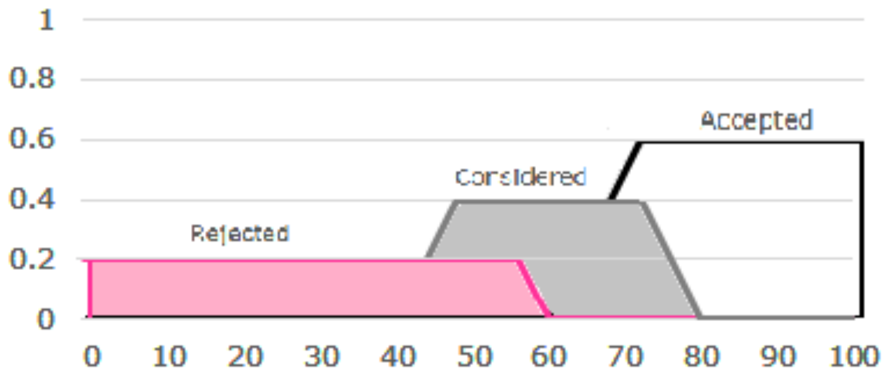
Step 4: Choose Defuzzification Method

- Center of Gravity (Mamdani-style)
  - Has the same Membership form as input
  - For example, let the output membership function be:



Step 4: Choose Defuzzification Method

- Center of Gravity (Mamdani-style)
  - Clipping using MAX-MIN method to “chop off” parts of Membership based on the fuzzy output obtained from rule inferencing, then Aggregate the clipped membership
  - For example, we obtained fuzzy output
    - Rejected(0.2)
    - Considered(0.4)
    - Accepted(0.6)



Step 4: Choose Defuzzification Method

- Center of Gravity (Mamdani-style)
  - Calculate center of gravity using

$$\boxed{z^* = \frac{\int \mu B(z_r).\, z_r \, dz}{\int \mu B(z_r)\, dz}}$$

  - If the crisp values (z_r) are discrete, we can replace integration with summation

$$\boxed{z^* = \frac{\sum_{i=1}^{n} \mu B(z_i).\, z_i}{\sum_{i=1}^{n} \mu B(z_i)}}$$

$$z_i = i^{th}\ random\ number$$
$$\mu B(z_i) = membership\ for\ i^{th}\ number$$

Step 4: Choose Defuzzification Method

- Constant Defuzzification (Takagi-Sugeno-style)
  - Choose a constant value to represent each output linguistic
  - For example, set the constant value to 50, 70, and 100

Step 4: Choose Defuzzification Method

- Constant Defuzzification (Takagi-Sugeno-style)
  - Using fuzzy output obtained from rule inferencing
  - Calculate crisp output using

$$\boxed{z^* = \frac{\sum_{i=1}^{l} \mu B_i . c_i}{\sum_{i=1}^{l} \mu B_i}}$$

$c_i = constant\ for\ i^{th}\ linguistic$
$\mu B_i = membership\ for\ i^{th}\ linguistic$

Fuzzy System for Scholarship Selection

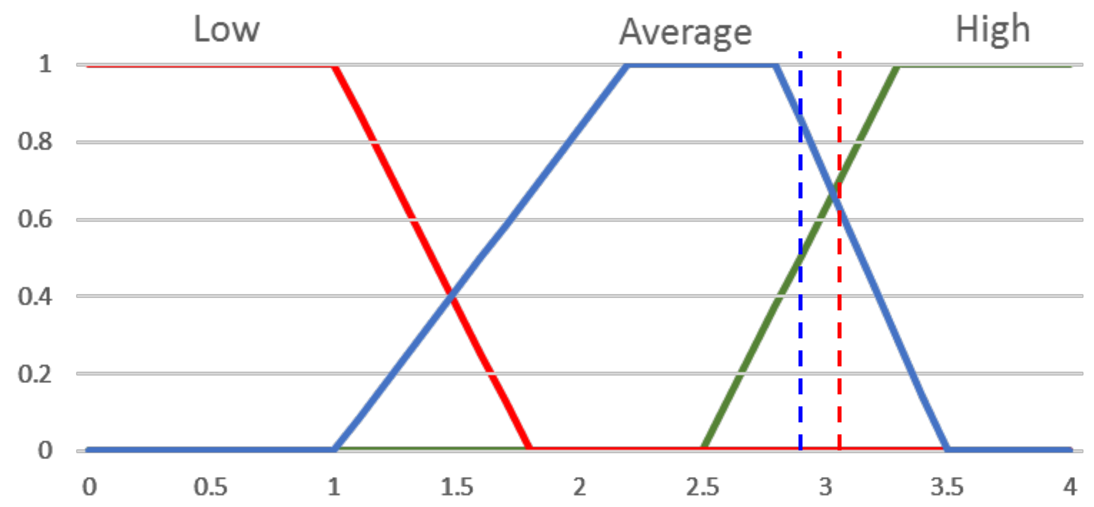
Test to an example

Case Example

- Student A has GPA of 3.01 with Parental Income of 14M
- Student B has GPA of 2.90 with Parental Income of 5.5M

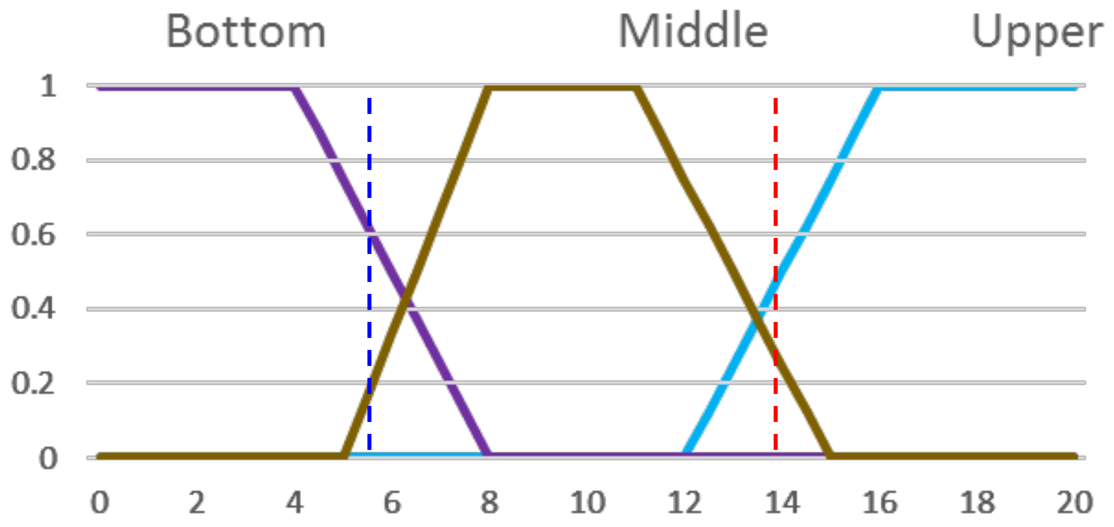
Fuzzification - GPA

- Student A has GPA of 3.01
  - High $= \frac{3.01-2.5}{3.25-2.5} = \frac{0.51}{0.75} = 0.68$
  - Average $= \frac{3.5-3.01}{3.5-2.8} = \frac{0.49}{0.7} = 0.70$
  - Low $= 0$
- Student B has GPA of 2.90
  - High $= \frac{2.9-2.5}{3.25-2.5} = \frac{0.4}{0.75} = 0.53$
  - Average $= \frac{3.5-2.9}{3.5-2.8} = \frac{0.6}{0.7} = 0.85$
  - Low $= 0$



Fuzzification - Income

- Student A has Parental Income of 14M
  - Upper $= \frac{14-12}{16-12} = \frac{2}{4} = 0.5$
  - Middle $= \frac{15-14}{15-11} = \frac{1}{4} = 0.25$
  - Bottom $= 0$
- Student B has Parental Income of 5.5M
  - Upper $= 0$
  - Middle $= \frac{5.5-5}{8-5} = \frac{0.5}{3} = 0.167$
  - Bottom $= \frac{8-5.5}{8-4} = \frac{2.5}{4} = 0.625$



Inference

- Apply the inference rule to the fuzzy inputs to obtain the fuzzy output
- Using Clipping technique, the conjunction rule will get the minimum value of fuzzy input as the fuzzy output
- For example, for Student A, we will get

```
if GPA='High(0.68)' and Income='Bottom(0)' then Score='Accepted(0)'
...
if GPA='Average(0.70)' and Income='Middle(0.25)' then Score='Considered(0.25)'
...
if GPA='Low(0)' and Income='Upper(0.5)' then Score='Rejected(0)'
...
```

Inference

- Thus we have

Student A					
GPA		**Income**		**Score**	
High	0.68	Upper	0.5	Considered	0.5
High	0.68	Middle	0.25	Accepted	0.25
High	0.68	Bottom	0	Accepted	0
Average	0.70	Upper	0.5	Considered	0.5
Average	0.70	Middle	0.25	Considered	0.25
Average	0.70	Bottom	0	Accepted	0
Low	0	Upper	0.5	Rejected	0
Low	0	Middle	0.25	Rejected	0
Low	0	Bottom	0	Considered	0

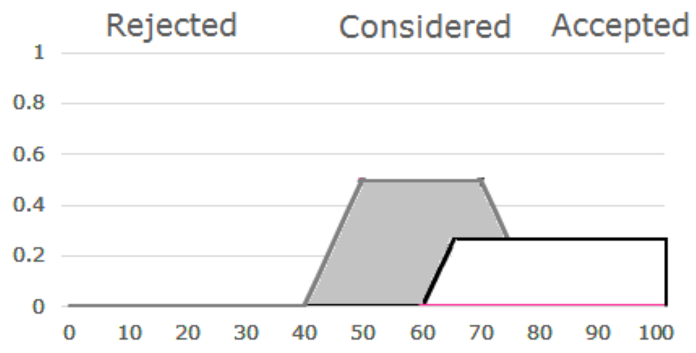
Student B					
GPA		**Income**		**Score**	
High	0.53	Upper	0	Considered	0
High	0.53	Middle	0.167	Accepted	0.167
High	0.53	Bottom	0.625	Accepted	0.53
Average	0.85	Upper	0	Considered	0
Average	0.85	Middle	0.167	Considered	0.167
Average	0.85	Bottom	0.625	Accepted	0.625
Low	0	Upper	0	Rejected	0
Low	0	Middle	0.167	Rejected	0
Low	0	Bottom	0.625	Considered	0

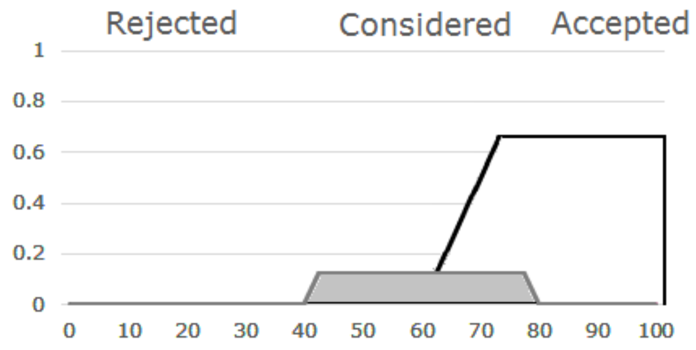
Inference

- Using Disjunction rule, get the maximum value for each fuzzy output
- Student A
  - Accepted $= 0.25 \vee 0 \vee 0 = 0.25$
  - Considered $= 0.5 \vee 0.5 \vee 0.25 \vee 0 = 0.5$
  - Rejected $= 0 \vee 0 = 0$
- Student B
  - Accepted $= 0.285 \vee 0.53 \vee 0.625 = 0.625$
  - Considered $= 0 \vee 0 \vee 0.167 \vee 0 = 0.167$
  - Rejected $= 0 \vee 0 = 0$

Defuzzification – Mamdani

- Using Clipping technique, we will get
- Student A
  - Accepted $= 0.25$
  - Considered $= 0.5$
  - Rejected $= 0$
- Student B
  - Accepted $= 0.625$
  - Considered $= 0.167$
  - Rejected $= 0$





Defuzzification – Mamdani

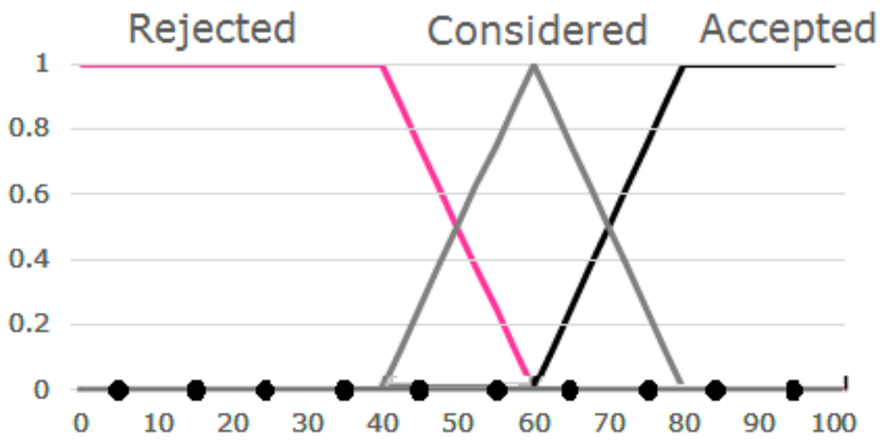
- To defuzzify it, we can use either equation

$$\boxed{z^* = \frac{\int \mu B(z_r).z_r \, dz}{\int \mu B(z_r) \, dz} \quad \text{or } z^* = \frac{\sum_{i=1}^{n} \mu B(z_i).z_i}{\sum_{i=1}^{n} \mu B(z_i)}}$$

- To simplify the calculation, we use the discrete equation
- To calculate the crisp output, first generate n random number
  - For example = 5, 15, 25, 35, 45, 55, 65, 75, 85, 95
- Then for each number, calculate the membership using the clipped membership function

Defuzzification – Mamdani

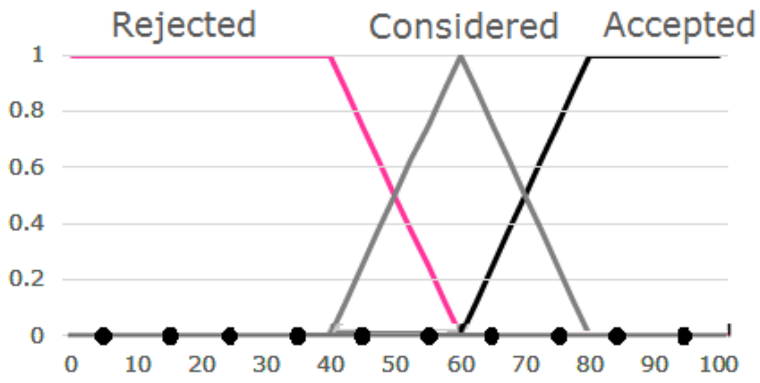
- First calculate the membership of each random generated number using the designed output Membership Function

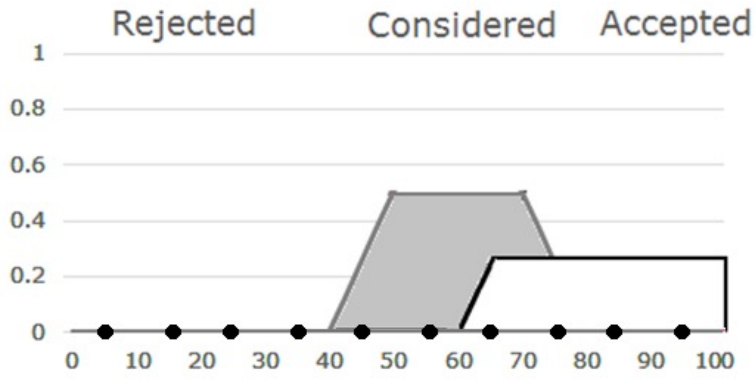


z_i	R	C	A
5	1	0	0
15	1	0	0
25	1	0	0
35	1	0	0
45	0.75	0.25	0
55	0.25	0.75	0
65	0	0.75	0.25
75	0	0.25	0.75
85	0	0	1
95	0	0	1

Defuzzification – Mamdani

- Maximized (threshold) the value based on the clipped membership
- Student A
  - Accepted $= 0.25$, Considered $= 0.5$, Rejected $= 0$

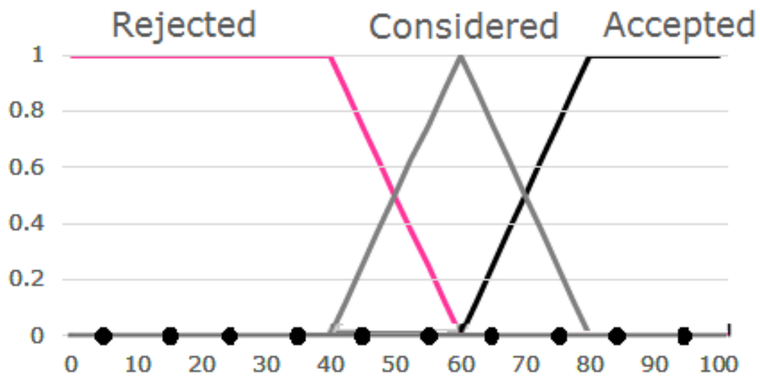


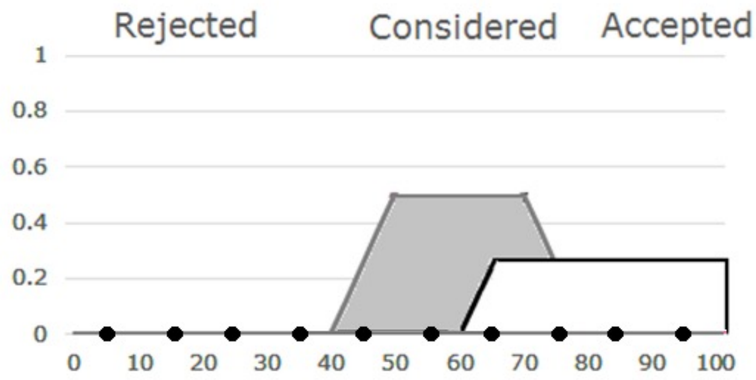


z_i	R	C	A
5	0	0	0
15	0	0	0
25	0	0	0
35	0	0	0
45	0	0.25	0
55	0	0.5	0
65	0	0.5	0.25
75	0	0.25	0.25
85	0	0	0.25
95	0	0	0.25

Defuzzification – Mamdani

- Output Membership is the maximum value from each linguistic
- Student A
  - Accepted $= 0.25$, Considered $= 0.5$, Rejected $= 0$

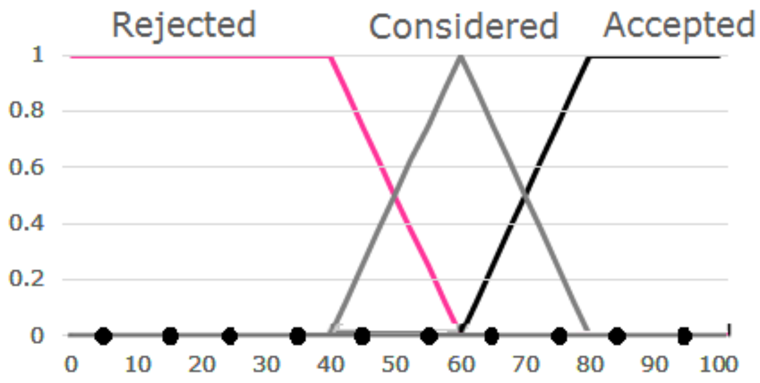


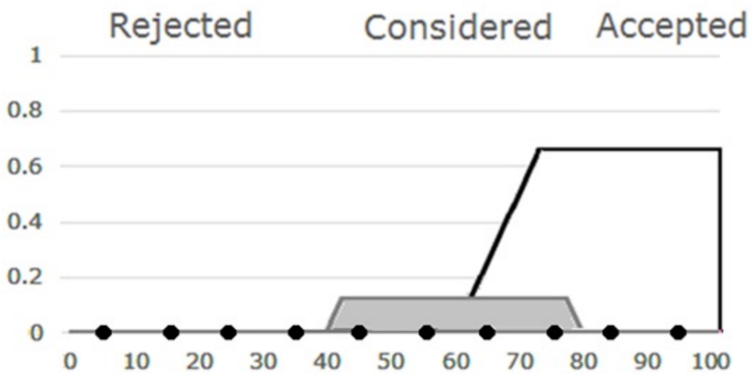


z_i	R	C	A	μB_i
5	0	0	0	0
15	0	0	0	0
25	0	0	0	0
35	0	0	0	0
45	0	0.25	0	0.25
55	0	0.5	0	0.5
65	0	0.5	0.25	0.5
75	0	0.25	0.25	0.25
85	0	0	0.25	0.25
95	0	0	0.25	0.25

Defuzzification – Mamdani

- Do the same for Student B
- Student B
  - Accepted $= 0.625$, Considered $= 0.167$, Rejected $= 0$





z_i	R	C	A	μB_i
5	0	0	0	0
15	0	0	0	0
25	0	0	0	0
35	0	0	0	0
45	0	0.167	0	0.167
55	0	0.167	0	0.167
65	0	0.167	0.25	0.25
75	0	0.167	0.625	0.625
85	0	0	0.625	0.625
95	0	0	0.625	0.625

Defuzzification – Mamdani

- Lastly, calculate the crisp output using equation
- Student A

$$z^* = \frac{\sum_{i=1}^{n} \mu B(z_i).z_i}{\sum_{i=1}^{n} \mu B(z_i)}$$

$$z^* = \frac{(5*0)+(15*0)+(25*0)+(35*0)+(45*0.25)+(55*0.5)+(65*0.5)+(75*0.25)+(85*0.25)+(95*0.25)}{0+0+0+0+0.25+0.5+0.5+0.25+0.25+0.25}$$

$$z^* = \frac{11.25+27.5+32.5+21.25+23.75}{2} = \frac{135}{2}$$

$$z^* = 67.5$$

z_i	μB_i	$z_i * \mu B_i$
5	0	0
15	0	0
25	0	0
35	0	0
45	0.25	11.25
55	0.5	27.5
65	0.5	32.5
75	0.25	18.75
85	0.25	21.25
95	0.25	23.75
Σ	2	135

Defuzzification – Mamdani

- Lastly, calculate the crisp output using equation
- Student B

$$z^* = \frac{\sum_{i=1}^{n} \mu B(z_i).z_i}{\sum_{i=1}^{n} \mu B(z_i)}$$

$$z^* = \frac{(5*0)+(15*0)+(25*0)+(35*0)+(45*0.167)+(55*0.167) +(65*0.25)+(75*0.625)+(85*0.625)+(95*0.625)}{0+0+0+0+0.167+0.167+0.25+0.625+0.625+0.625}$$

$$z^* = \frac{7.51+9.18+16.25+46.87+53.12+59.37}{2.459} = \frac{192.325}{2.459}$$

$$z^* = 78.21$$

z_i	μB_i	$z_i * \mu B_i$
5	0	0
15	0	0
25	0	0
35	0	0
45	0.25	11.25
55	0.5	27.5
65	0.5	32.5
75	0.25	18.75
85	0.25	21.25
95	0.25	23.75
Σ	2	135

Defuzzification – Mamdani

- Apply the fuzzy output to the equation
- Student A

$$z^* = \frac{11.25+27.5+32.5+21.25+23.75}{2}$$
$$z^* = \frac{135}{2}$$

$$z^* = \mathbf{67.5}$$

- Student B

$$z^* = \frac{7.51+9.18+16.25+46.87+53.12+59.37}{2.459}$$
$$z^* = \frac{192.325}{2.459}$$

$$z^* = \mathbf{78.21}$$

Defuzzification – Sugeno

- Apply the fuzzy output to the equation

$$z^* = \frac{\sum_{i=1}^{l} \mu B_i . c_i}{\sum_{i=1}^{l} \mu B_i}$$

- Student A
  - Accepted $= 0.25$
  - Considered $= 0.5$
  - Rejected $= 0$
- Student B
  - Accepted $= 0.625$
  - Considered $= 0.167$
  - Rejected $= 0$

Defuzzification – Sugeno

- Apply the fuzzy output to the equation
- Student A

$$z^* = \frac{(0.25*100)+(0.5*70)+(0*50)}{0.25+0.5+0}$$
$$z^* = \frac{25+35+0}{0.75}$$

$$z^* = \mathbf{80}$$

- Student B

$$z^* = \frac{(0.625*100)+(0.167*70)+(0*50)}{0.625+0.167+0}$$
$$z^* = \frac{62.5+11.69+0}{0.792}$$

$$z^* = \mathbf{93.67}$$

Question?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THANK YOU