Security

2024/25 Q2

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DAC – UPC

* Part of the material comes from other sources.

Security

- Introduction
- Cryptography
- Public Key Infrastructure (PKI)
- Security in Applications

Security - Introduction

- Threats
- Some considerations
- Protection mechanisms

Security - Introduction

- **Threats**
  - External vs. Internal (w.r.t. “infrastructure”)
  - Access:

    Internal vs. External (w.r.t. equipment)

    Authorized vs. Non
  - Data introduction: virus, hoax, … malware
  - Actions: Read, Copy, Modify, Delete, Add, …
  - Communications:
    Interception, Manipulation, Impersonation (“suplantación”), Repudiation

Security - Introduction

- **Some considerations**
  - Attack to whom? Who is the attacker? Why?
  - On what?
    (HW, SW, data, communications, identity, …)
  - Harm:
    Recoverable?
    Protection/Recovery cost vs. Losses
    Risks

Security - Introduction

- **Protection mechanisms**
  - *Preventive, Detective, Corrective*
  - *Physical, Technical, Organizational*
  - External attacks protection (firewalls, ...)
  - Communication services:
    Confidentiality, Integrity,
    Authentication, No repudiation
  - Basic mechanism: CRYPTOGRAPHY

Security - Cryptography

- **Private key (symmetric)**
- **Public key (asymmetric)**
- *General algorithms for public key*
  - Modular arithmetic:
    Modular multiplicative inverse,
    Extended Euclidean algorithm (“magic box”)
  - Diffie-Hellman
- **Encryption/Decryption** algorithms for public key
  - RSA
  - ElGamal
- **Digital signature**
  - RSA

Security - Cryptography

- **Private key (symmetric)**

  Sender: $c = E_k(m)$; Recipient: $m = D_k(c)$

  E: Encryption algorithm; D: Decryption algorithm;

  k: Key; c: cyphered text; m: clear text (message).

  – Historical: Transposition, Substitution, …

Security - Cryptography

- **Private key (symmetric)**

  Sender: $\mathtt{c = E_k(m)}$; Recipient: $\mathtt{m = D_k(c)}$

  E: Encryption algorithm; D: Decryption algorithm;

  k: Key; c: cyphered text; m: clear text (message).

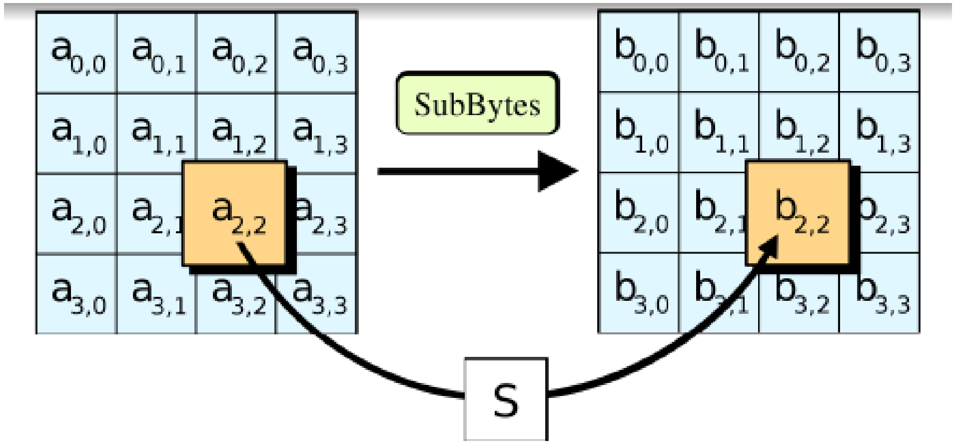
  – Historical: Transposition, Substitution, …

  – Block cipher

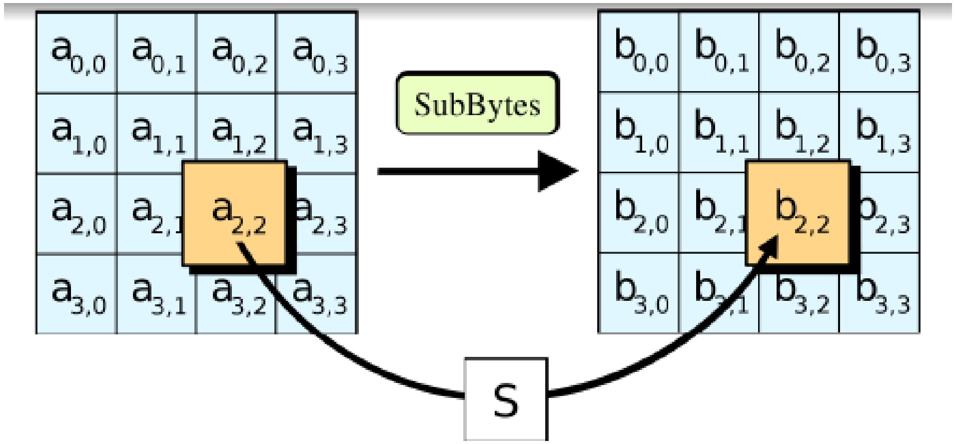
    - Principles: *Confusion* (*key* → *cipher text* independence)
      Diffusion (*clear text* → *cipher text* independence)
    - DES (Data Encryption Standard), 1976 (obsolete 2005)
    - 3DES (Triple DES), 1995 (obsolete/deprecated end 2023!)
    - AES (Advanced Encryption Standard), 2001
      (now in ISO/IEC 18033-3 (block ciphers))

    (permutations-based) (blocks: 128 bits; keys: 128, 192, 256 bits)

AES permutations

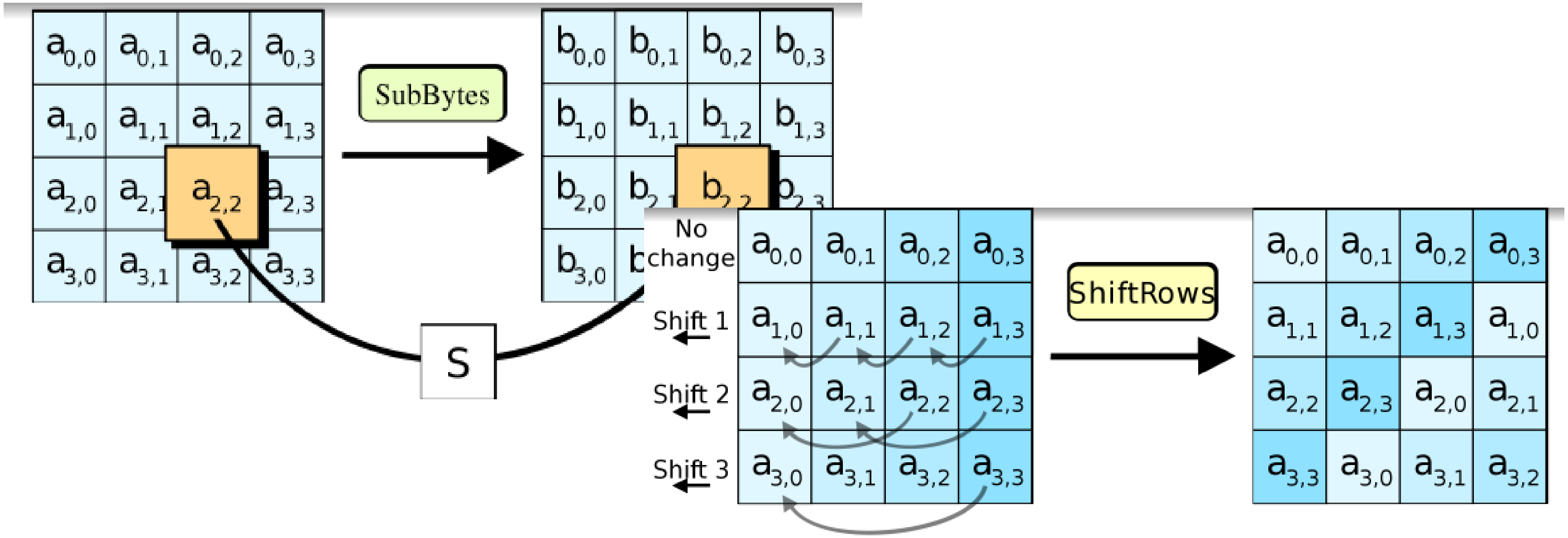


AES permutations

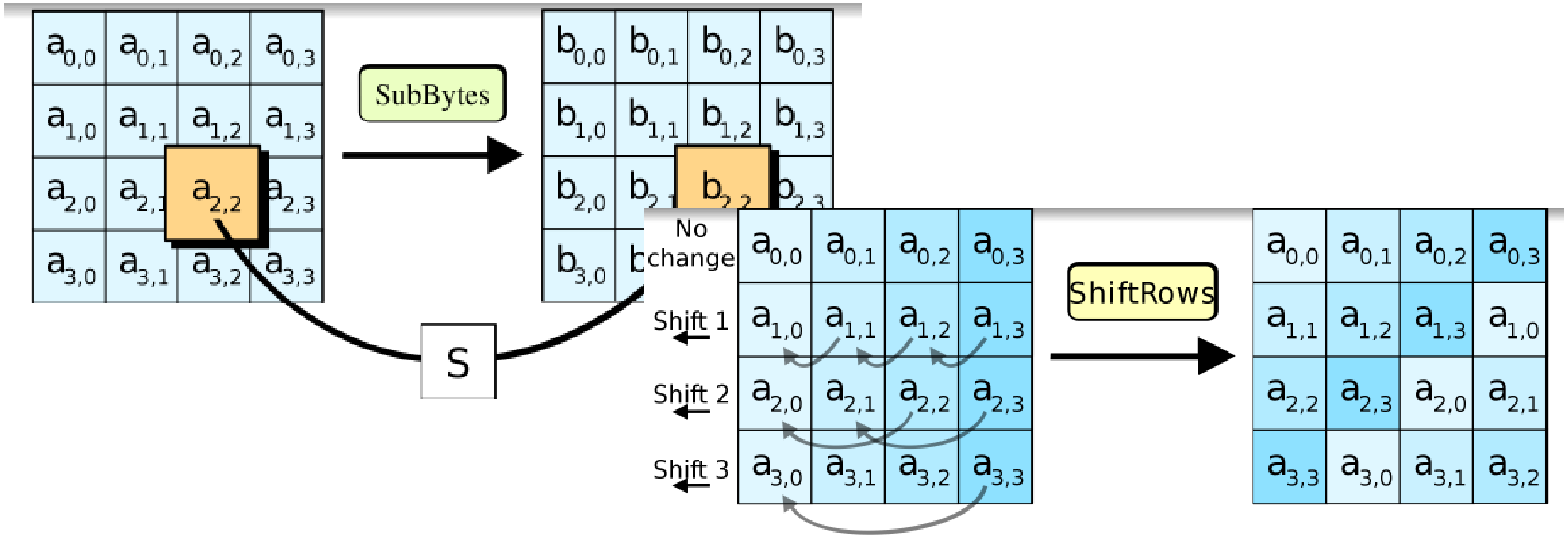


SubBytes: a non-linear substitution step where each byte is replaced with another according to a lookup table

AES permutations

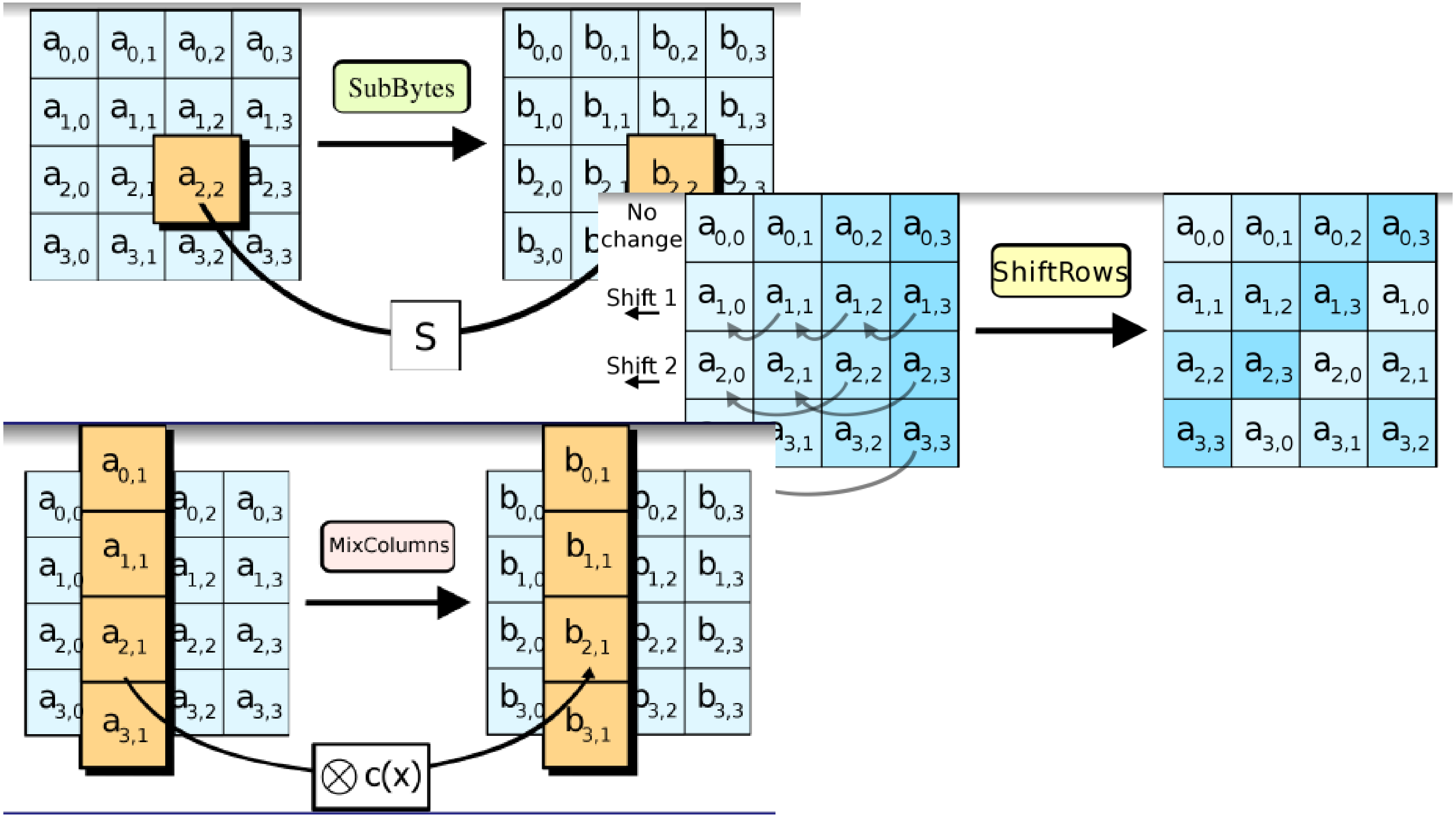


AES permutations

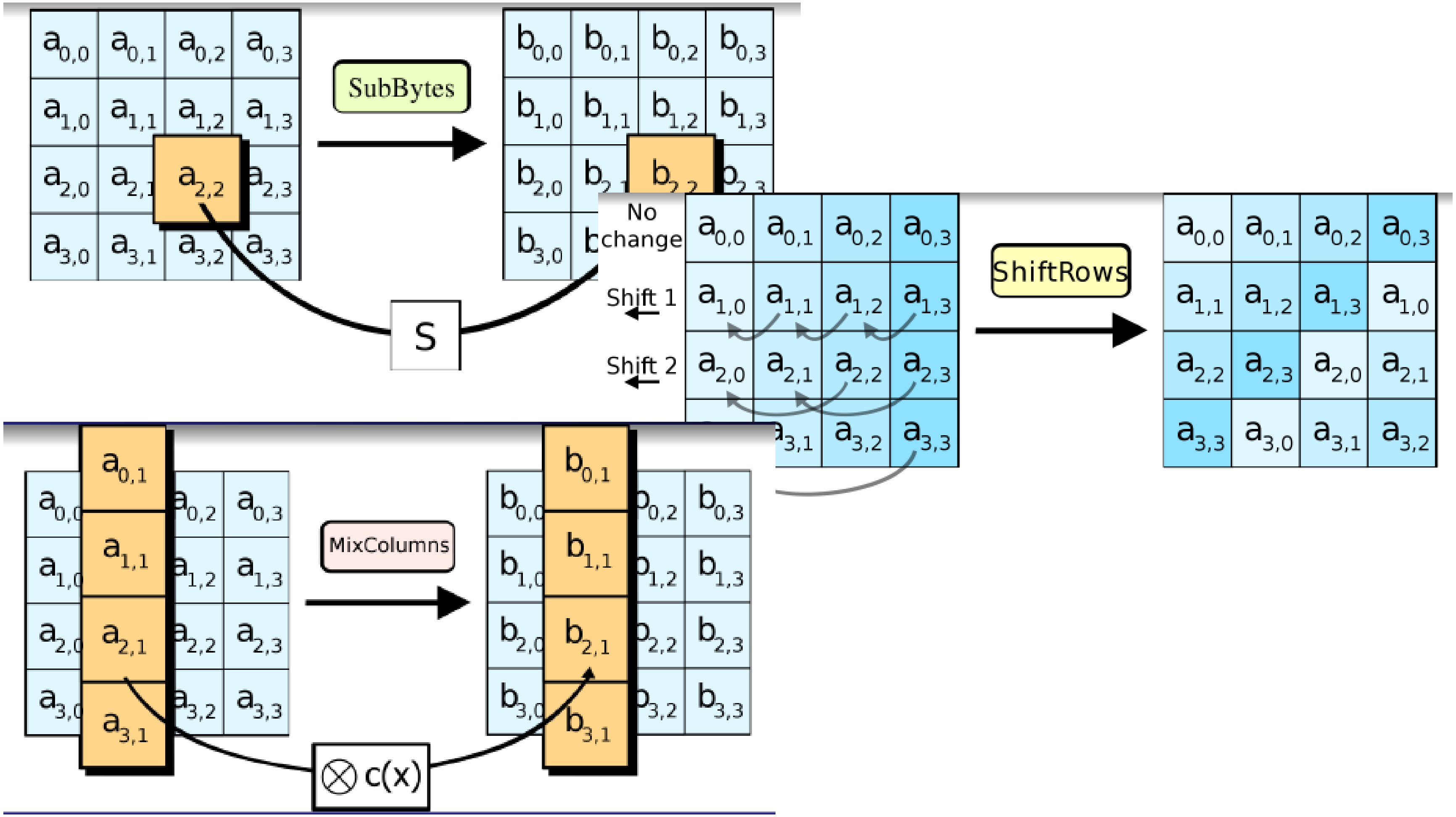


ShiftRows: a transposition step where each row of the state is shifted cyclically a certain number of steps

AES permutations

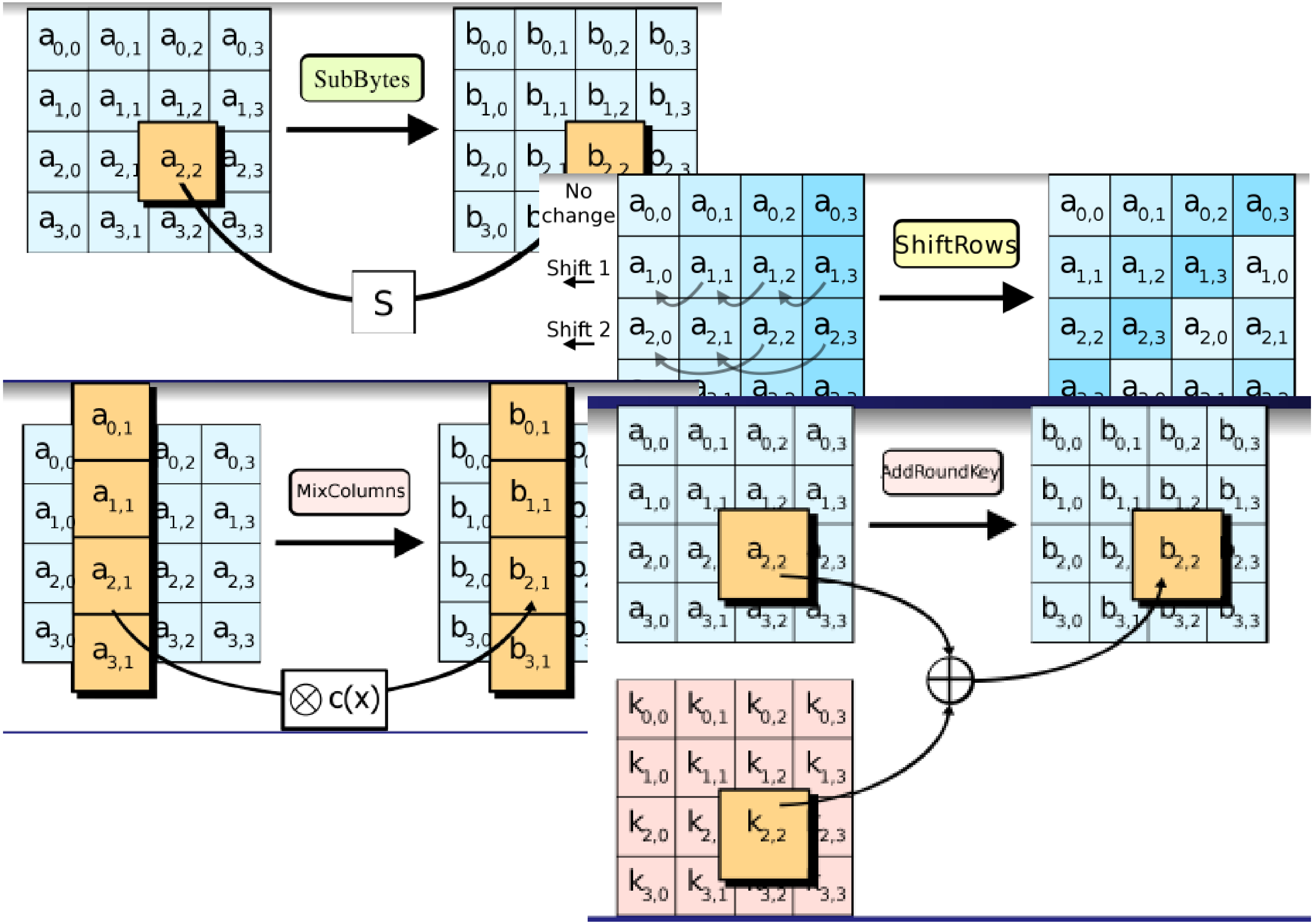


AES permutations

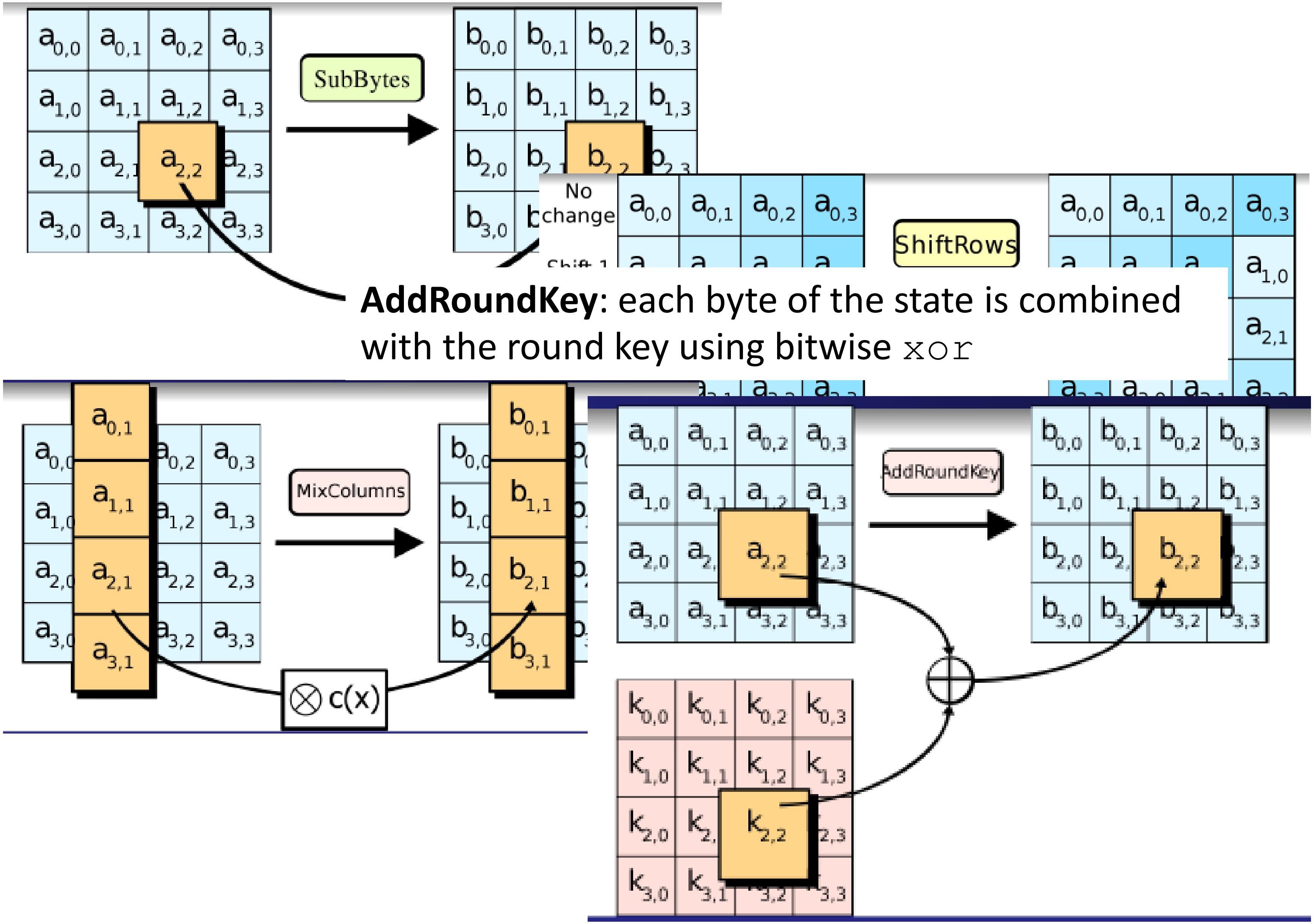


MixColumns: a mixing operation which operates on
the columns of the
state, combining the four bytes in each column

AES permutations



AES permutations



AddRoundKey: each byte of the state is combined with the round key using bitwise `xor`

Security - Cryptography

- Private key (symmetric)
- **Public key (asymmetric)**
  - Secret + Public parts of the key (Ks, Kp)
  - No need for key distribution
  - *Encryption*

    Sender: $c = E_{Kp}(m)$; Recipient: $m = D_{Ks}(c)$

    (Ks,Kp) from the Recipient.
  - *Signature*

    Sender: $s = E_{Ks}(m)$; Recipient: $m = D_{Kp}(s)$

    *"Sender" is the **Signer**. (Ks,Kp) from the Signer.*

Security - Cryptography

- Private key (symmetric)
- Public key (asymmetric)
- **General algorithms for public key**
  - Modular arithmetic:
    Modular multiplicative inverse,
    Extended Euclidean algorithm ("magic box")
  - Diffie-Hellman

Modular arithmetics

- (Exponentiation by squaring)
- Modular multiplicative inverse
- Extended Euclidean algorithm (“magic box”)

Exponentiation by squaring

Modular arithmetic allows us to compute exponentiations without managing very big numbers!

Exponentiation by squaring (a,z,n) $x = a^z \bmod n$

---

begin
- $x = 1$;
- $z^1 =$ binary representation of z;
- `// starting by the most significant bit`
- **foreach** *bit* $z^1_i \in z^1$ **do**
  - $x = x^2 \bmod n$;
  - `// multiply` x `by` a `if` z_1 `is equal to one`
  - **if** $z^1_i == 1$ **then**
    - $x = x \cdot a \bmod n$
- **return** x

Exponentiation by squaring - Example

Example

Compute $5^{27} \bmod 217$

27 is 11011 in binary

$5^{27} \bmod 217 \Rightarrow 1 \rightarrow S \rightarrow 1 \rightarrow M \rightarrow 5 \rightarrow S \rightarrow 25 \rightarrow M \rightarrow 125 \rightarrow S \rightarrow 15625 \equiv 1 \rightarrow S \rightarrow 1 \rightarrow M \rightarrow 5 \rightarrow S \rightarrow 25 \rightarrow M \rightarrow 125$

S: squaring
M: multiply

Extended Euclidean algorithm

- Extended Euclidean algorithm: computes the *greatest common divisor (gcd)* of two integers *a* and *n*
- When *a* and *n* are *coprime* (i.e. *gcd (a,n) = 1*), its output is the **modular multiplicative inverse** of *a* mod *n*

'The Magic Box' method

a	b	d	k
1	0	n	
0	1	a	

$$d_i = d_{i-2} \bmod d_{i-1}$$
k_i is the quotient of d_{i-1}/d_i
$$a_i = a_{i-2} - k_{i-1} \cdot a_{i-1}$$
$$b_i = b_{i-2} - k_{i-1} \cdot b_{i-1}$$

The procedure finishes when $d_i = 1$ (if a,n are coprime)

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'The Magic Box' method

a	b	d	k
1	0	n	
0	1	a	

Starting point

$d_i = d_{i-2} \bmod d_{i-1}$

k_i is the quotient of d_{i-1}/d_i

$= a_{i-2} - k_{i-1} \cdot a_{i-1}$

$b_i = b_{i-2} - k_{i-1} \cdot b_{i-1}$

The procedure finishes when d_i =1 (if a,n are coprime)

"Magic Box" example

Compute the GCD of 120 and 23

a	b	d	k
1	0	120	
0	1	23	5
1	-5	5	4
-4	21	3	1
5	-26	2	1
-9	47	1	2

$d_i = d_{i-2} \bmod d_{i-1}$

k_i is the quotient of d_{i-1}/d_i

$a_i = a_{i-2} - k_{i-1} \cdot a_{i-1}$

$b_i = b_{i-2} - k_{i-1} \cdot b_{i-1}$

$d_3 = 120 \bmod 23 = 5$ k_2 is the quotient of $120/23$

$a_3 = 1 - 5 \cdot 0 = 1$

$b_3 = 0 - 5 \cdot 1 = -5$

- 47 is the **modular multiplicative inverse** of 23 mod 120.
- Bézout identity in the example: 1 = 23*47 + 120*(-9)

Diffie-Hellman

- Modular arithmetic algorithm with several uses:
  - Part of asymmetric key mechanisms
  - **Private key generation** in symmetric key mechanisms

- A: takes random element $a \in G$; computes $\alpha^a \in G$;
  B: id. with $b \in G$; $\alpha^b \in G$

  (A: Sender; B: Recipient; *G* and *α* known by both;
  G: *multiplicative finite group with generator* $\alpha \in G$)

- A & B interchange α^a and α^b
- A computes $(\alpha^b)^a$; B computes $(\alpha^a)^b$

Diffie-Hellman

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  G: *multiplicative finite group with generator* $\alpha \in G$)
- A & B interchange α^a and α^b
- A computes $(\alpha^b)^a$; B computes $(\alpha^a)^b$
- $(\alpha^b)^a = (\alpha^a)^b$ is the private key !!! Only A & B know

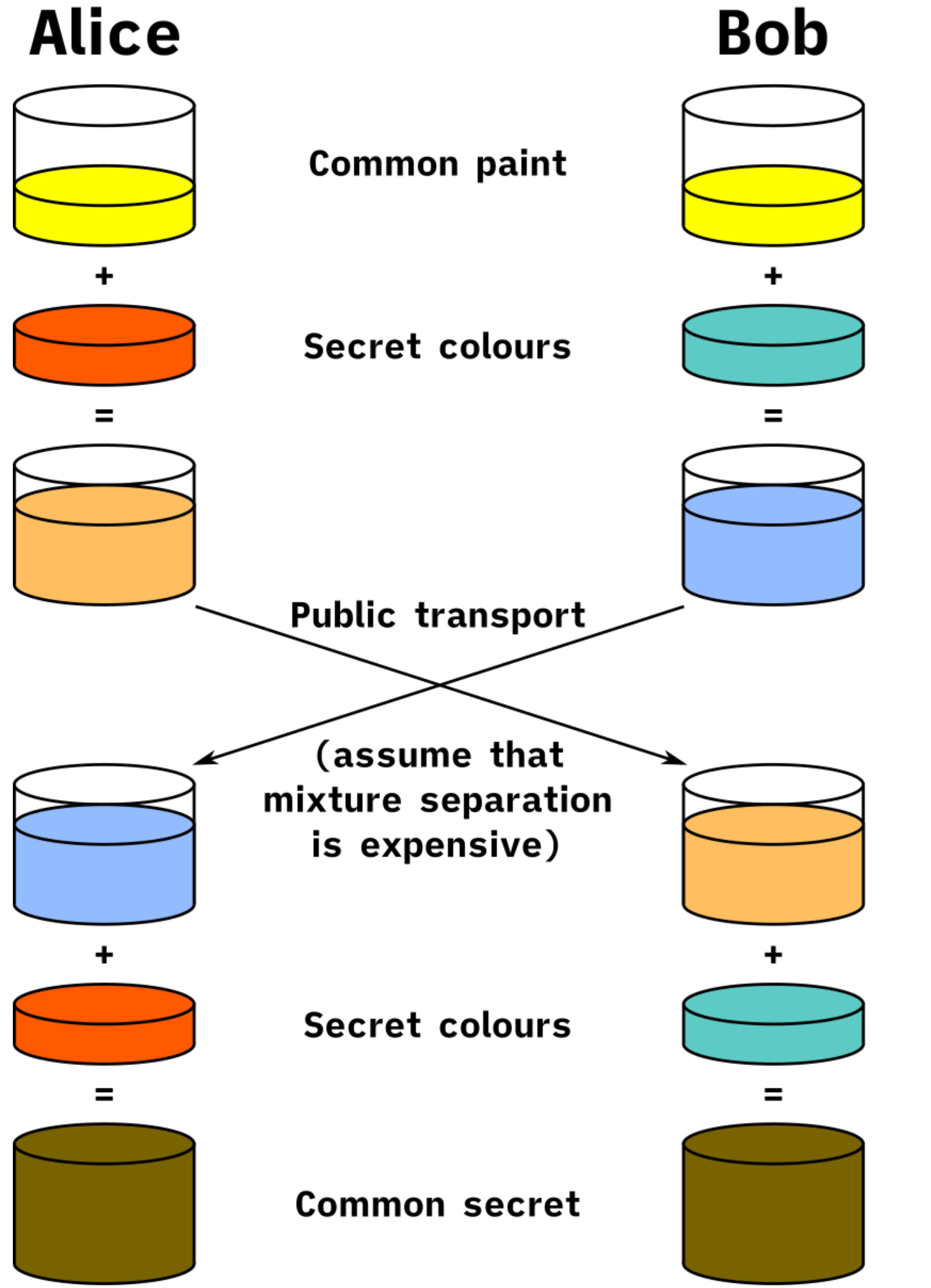
Diffie-Hellman example

Example

1. A and B choose publicly $G = \mathbb{Z}_{53}^*$ and the generator $\alpha = 2$
2. A chooses $a = 29$, computes $\alpha^a = 2^{29} \bmod 53 = 45$ and sends 45 to B
3. B chooses $b = 19$, computes $\alpha^b = 2^{19} \bmod 53 = 12$ and sends 12 to A
4. A receives 12 and computes $12^{29} \bmod 53 = 21$
5. B receives 45 and computes $45^{19} \bmod 53 = 21$

The **private key** is 21

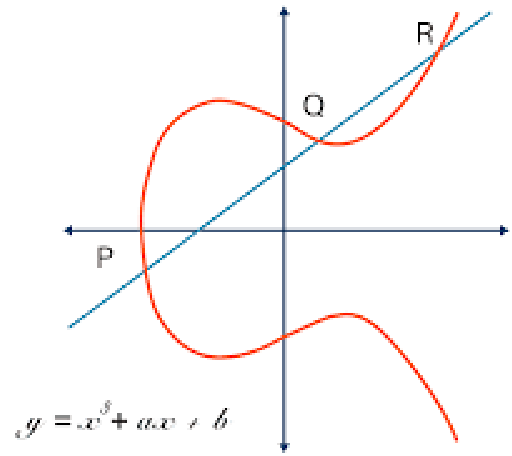
Diffie-Hellman key exchange



Source: https://www.encryptionconsulting.com/diffie-hellman-key-exchange-vs-rsa/ and others

Elliptic-curve Diffie-Hellman (ECDH)

- Use of “elliptic curves” to generate keys **(“discrete logarithm” problem)** instead of large prime numbers
- Parameters:
  - agreed specific elliptic curve E
  - a point G in E (base point)



- Steps:

1. UserA:

– takes random element *a (integer)* → private key

– computes public key *A = a·G* (*G* multiplied by itself *a* times)

2. UserB: id. with *b, B = b·G* (id. for *b*)

3. UserA & UserB interchange public keys (*A & B*)

4. UserA & UserB compute **K** (”Private Key”·“received Public Key”): UserA: K = aB (=ab·G); UserB: K = bA (=ba·G) → **Same number!**

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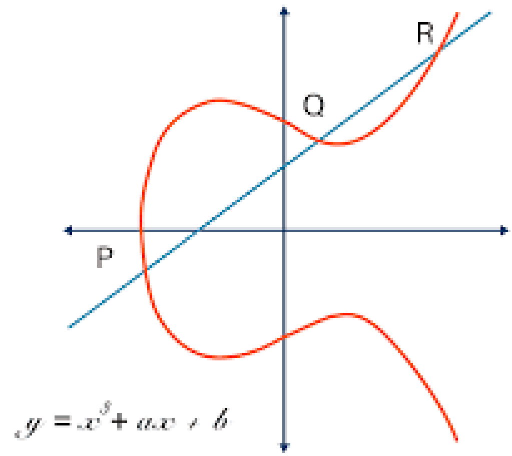

- Steps:
  1. UserA:
  - takes random element *a (integer)* → private key
  - computes public key *A*
  2. UserB: id. with *b, B = b*
  3. UserA & UserB interch
  4. UserA & UserB compu :

  UserA: K = aB (=ab·G); UserB: K = bA (=ba·G) → **Same number!**

Detailed explanations f.e. in:

https://www.allaboutcircuits.com/technical-articles/elliptic-curve-cryptography-in-embedded-systems/

Security - Cryptography

- Private key (symmetric)
- Public key (asymmetric)
- General algorithms for public key
- **Encryption/Decryption algorithms for public key**
  - RSA (Rivest, Shamir, Adleman)
  - ElGamal
- **Digital signature**
  - RSA

RSA

KEYS GENERATION

- Choose 2 distinct **very big** prime numbers p and q.
- Compute $n=p*q$; n defines the multiplicative group $\mathbb{Z}n$
- Compute Euler's totient function: $\Phi(n)=(p-1)*(q-1)$
- Choose an integer e such that $1<e<\Phi(n)$, and $GCD(e,\Phi(n))=1$ (i.e., e and $\Phi(n)$ are coprime)
- Determine $d=e^{-1} mod \Phi(n)$ using Modular multiplicative inverse (extended Euclidean algorithm)
- The **public key** is $\mathbf{(n,e)}$
- The **secret key** is $\mathbf{d}$ or $\mathbf{(n,d)}$

p, q and $\Phi(n)$ are also secret

RSA

ENCRYPTION

- B has keys:

  PUBLIC (n,e); SECRET (d)

- A wants to send m to B:

$$c = m^e \bmod n$$

DECRYPTION

- B receives c and computes:

$$m = c^d \bmod n$$

RSA

EXAMPLE

Keys generation:

- *p=61, q=53*; n=61*53=3223 $\boxed{n=p*q}$
- Φ(3233)=60*52=3120 $\boxed{\Phi(n)=(p-1)*(q-1)}$
- *e=17* (1<17<3120)
  e coprime to 3120 (i.e. *e* not a divisor of 3120)
- d = 17^{-1} mod 3120 = 2753 $\boxed{d=e^{-1}\bmod\Phi(n)}$
  ext. Eucl. alg.

Public key: (n=3233, e=17)

Secret key: (n=3233, **d=2753)**

RSA

EXAMPLE

Keys generation (d calculation):

- $d = 17^{-1} \bmod 3120 = 2753$

$d = e^{-1} \bmod \Phi(n)$

- Extended Euclidean algorithm:

b	d	k
0	**3120**	–
1	**17**	

RSA

EXAMPLE

Keys generation (d calculation):

- $d = 17^{-1} \bmod 3120 = 2753$

$d = e^{-1} \bmod \Phi(n)$

- Extended Euclidean algorithm:

b	d	k
0	**3120**	-
1	**17**	183
	9	

RSA

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	8	

RSA

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Keys generation (`d` calculation):

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$d = e^{-1} \bmod \Phi(n)$

- Extended Euclidean algorithm:

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1	**17**	183
	9	1
	8	1
	1	

RSA

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Keys generation (d calculation):

- $d = 17^{-1} \bmod 3120 = 2753$

$d=e^{-1} \bmod \Phi(n)$

- Extended Euclidean algorithm:

b	d	k
0	**3120**	-
1	**17**	183
	9	1
	8	1
	1	

$bi=b_{i-2}-(k_{i-1}*b_{i-1})$

RSA

EXAMPLE

Keys generation (d calculation):

- $d = 17^{-1} \bmod 3120 = 2753$

$d = e^{-1} \bmod \Phi(n)$

- Extended Euclidean algorithm:

b	d	k
0	**3120**	-
1	**17**	183
-183	9	1
184	8	1
-367	1	

$b_i = b_{i-2} - (k_{i-1} * b_{i-1})$

RSA

EXAMPLE

Keys generation (d calculation):

- $d = 17^{-1} \bmod 3120 = 2753$

$d=e^{-1} \bmod \Phi(n)$

- Extended Euclidean algorithm:

b	d	k
0	**3120**	-
1	**17**	183
-183	9	1
184	8	1
-367	1	

$bi=b_{i-2}-(k_{i-1}*b_{i-1})$

3120-367=**2753**

RSA

EXAMPLE

Encryption / Decryption:

- *m=65*;

$$c = m^e \bmod n$$

- $c = 65^{17} \bmod 3233 = 2790$

$$m = c^d \bmod n$$

- $m = 2790^{2753} \bmod 3233 = 65$

ElGamal

KEYS GENERATION

- Choose a cyclic (generated by a single element) multiplicative finite group G and one element $\alpha \in G$
- Users choose a random number a → Secret key *Ks*
- Also compute $\alpha^a \in G$ → Public key *Kp*

  (Although α, G and α^a (i.e. *Kp*) are publicly known,
  a (i.e. *Ks*) is not known)

ElGamal

ENCRYPTION

- A has keys:

  PUBLIC ($Kp=\alpha^a$); SECRET ($Ks=a$)

- B wants to send $m \in G$ to A
- B choses a random number v and computes $\alpha^v \in G$
- B computes

  $c = m * (Kp)^v \in G$; i.e.:

  $c = m * (\alpha^a)^v \bmod G$

- B sends to A:

  (α^v, c)

ElGamal

ENCRYPTION

- A has keys:

  PUBLIC ($Kp=\alpha^a$); SECRET ($Ks=a$)

- B wants to send $m \in G$ to A
- B choses a random number v and computes $\boxed{\alpha^v} \in G$
- B computes

  $c = m * (Kp)^v \in G$; i.e.:

  $\boxed{c} = m * (\alpha^a)^v \bmod G$

- B sends to A:

  $\boxed{(\alpha^v, c)}$

ElGamal

DECRYPTION

- A receives (α^{v}, c)
- A computes

$$(\alpha^{v})^{Ks} \in G\text{; i.e:}$$

$$(\alpha^{v})^{a} \in G = (\alpha^{va}) \bmod G$$

- A computes

$$\mathbf{m = c * (\alpha^{va})^{-1} \in G}\text{; i.e.:}$$

$$\mathbf{m = c * (\alpha^{va})^{-1} \bmod G}$$

Ks = a

$(\alpha^{va})^{-1}$ is the modular multiplicative inverse of (α^{va}) mod G

ElGamal

EXAMPLE

Keys generation:

- $G=13$; $\alpha=2$; **$a=9$**
- Kp = $\alpha^a = 2^9$ mod 13 = 5 $\boxed{\alpha^a \in G}$

Secret key: Ks=9

Public key: Kp=5

ElGamal

EXAMPLE

Encryption:

- $m=11$; $v=\mathbf{10}$; ($G=13$; $\alpha=2$); (Ks=9, Kp=**5**)

ElGamal

EXAMPLE

Encryption:

- *m=11*; *v=**10***; (*G=13*; *α=2*); (Ks=9, Kp=**5**)

- $\alpha^v = 2^{10}$ mod 13 = 10

$\boxed{\alpha^v \in G}$

$\boxed{c = m*(Kp)^v \bmod G}$

ElGamal

EXAMPLE

Encryption:

- $m=11$; $v=\mathbf{10}$; ($G=13$; $\alpha=2$); (Ks=9, Kp=**5**)

- $\alpha^{v} = 2^{10} \bmod 13 = 10$

$\alpha^{v} \in G$

$c = m*(Kp)^{v} \bmod G$

- $c = 11 * \mathbf{5}^{10} \bmod 13 = 11 * 12 \bmod 13 = 2$

$Kp=\alpha^{a}$

ElGamal

EXAMPLE

Encryption:

- *m=11*; *v=**10***; (*G=13*; *α=2*); (Ks=9, Kp=**5**)
- $\alpha^{v} = 2^{10} \bmod 13 = 10$

$\alpha^{v} \in G$

$c = m*(Kp)^{v} \bmod G$

- $c = 11 * \mathbf{5}^{10} \bmod 13 = 11 * 12 \bmod 13 = 2$
- Sends **(10, 2)**

ElGamal

EXAMPLE

Decryption:

- Receives *(α^v,c)= (10,2)*. Ks=**9**; (*G=13*; *α=2*)

- m = 2 * $(10^9)^{-1}$ mod 13

$\boxed{m = c * (\alpha^{va})^{-1} \bmod G}$ $\boxed{Ks=a}$

- m = (2 * ((10^9 mod 13)$^{-1}$ mod 13)) mod 13

$\boxed{(\alpha^v)^a = 10^9 \bmod 13 = 12}$

- m = (2 * (12^{-1} mod 13)) mod 13

$\boxed{\text{“magic box”}}$

ElGamal

EXAMPLE

Decryption ("magic box" calculation):

- Ks=9; (*G=13*; $\alpha=2$). Receives $(\alpha^v, c) = (10, 2)$

- $(\alpha^{va})^{-1} \bmod G = 12^{-1} \bmod 13 = 12$ ("magic box")

b	d	k
0	13	–
1	12	

ElGamal

EXAMPLE

Decryption ("magic box" calculation):

- Ks=9; (*G=13*; *α=2*). Receives $(\alpha^v, c) = (10, 2)$

- $(\alpha^{va})^{-1}$ mod G = 12^{-1} mod 13 = 12 ("magic box")

b	d	k
0	**13**	-
1	**12**	1
-1	1	

$bi=b_{i-2}-(k_{i-1}*b_{i-1})$

13-1=**12**

ElGamal

EXAMPLE

Decryption:

- Receives $(\alpha^v,c)= (10,2)$. Ks=9; ($G=13$; $\alpha=2$)
- $m = 2 * (10^9)^{-1} \bmod 13$

$m = c * (\alpha^{va})^{-1} \bmod G$

Ks=a

- $m = (2 * ((10^9 \bmod 13)^{-1} \bmod 13)) \bmod 13$

$10^9 \bmod 13 = 12$

- $m = (2 * (12^{-1} \bmod 13)) \bmod 13$

12 ("magic box")

m = (2 * 12) mod 13 = 11

Security - Cryptography

- Private key (symmetric)
- Public key (asymmetric)
- General algorithms for public key
- Encryption/Decryption algorithms for public key
  - RSA
  - ElGamal
- **Digital signature**
  - RSA

(Public key) Digital signature

- *Encryption*

  Sender: $\mathbf{c = E_{Kp}(m)}$; Recipient: $\mathbf{m = D_{Ks}(c)}$

  (Ks,Kp) from the Recipient.

- *Signature*

  Sender: $\mathbf{s = E_{Ks}(m)}$; Recipient: $\mathbf{m = D_{Kp}(s)}$

  "Sender" is the ***Signer****. (Ks,Kp) from the Signer.*

- To reduce computational cost,

  sign `Hash(m)` instead of `m`

  (Hash: unidirectional; large variable-size → small fixed-size)

- Signature distributed with message (encrypted or not)
- *RSA algorithm. There are many more*

RSA signature

SIGNATURE

- A has keys:
  PUBLIC (n,e); SECRET (d)
- A signs a message (or its Hash) m:

$$s = m^d \bmod n$$

VERIFICATION

- B receives s and m (encrypted or not)
- Then calculates $s^e \bmod n$
- that should be equal to m