Bayesian Networks: Preliminaries

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Introduction

Vint Cerf, chief Internet evangelist at Google and Turing Award

We are huge consumers of Bayesian methods

- Speech-recognition software
- Spam filters
- Weather forecasting
- Evaluation of potential oil wells
- Skill ranking at Microsoft Xbox
- Microsoft wizards
- Cell phone decoding

Two Variables

Example

Consider a COVID antigen test with 65.5% sensitivity and specificity of 99.9% and a region with a prevalence of 5%

Variables

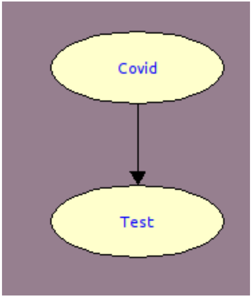
Covid C and Test result T

Data

$P(c) = 0.05$, $P(t|c) = 0.655$, $P(\neg t|\neg c) = 0.999$

Chain Rule

$P(T, C) = P(T|C)P(C)$



- $P(c|t) = \frac{0.655 \times 0.05}{0.655 \times 0.05 + 0.001 \times 0.95} = 0.972$
- $P(c|\neg t) = 0.982$

Junction

Example

Consider a smoke detector that detects smoke as a proxy for fire

Variables

Fire F produces Smoke S, which triggers the Alarm A

Junction

Example

Consider a smoke detector that detects smoke as a proxy for fire

Variables

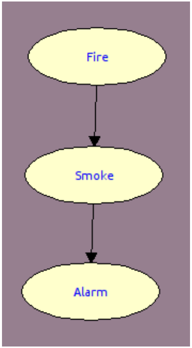
Fire F produces Smoke S, which triggers the Alarm A

Assumptions

Fire and Alarm are independent, given Smoke ($F \perp A | S$)

Chain Rule (exploiting independencies)

$P(A, S, F) =$
$P(A|S, F)P(S|F)P(F) =$
$P(A|S)P(S|F)P(F)$



Junction cont.

$P(f) =$
$P(f|a) =$
$P(a) =$
$P(a|f) =$
$P(f|s) =$
$P(f|a,s) =$
Note that F and A are NOT independent

Fork

Example

Consider the shoe size and the reading ability of kids as they age

Variables

Age $A \in 10..15$, Shoe size $S \in 37..42$, Reading ability $R \in 5..10$

Fork

Example

Consider the shoe size and the reading ability of kids as they age

Variables

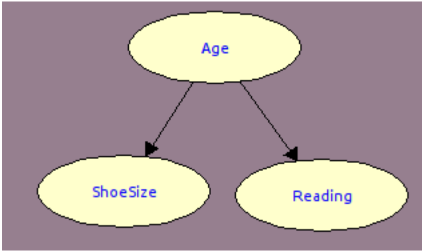
Age $A \in 10..15$, Shoe size $S \in 37..42$, Reading ability $R \in 5..10$

Assumptions

Shoe size and reading ability are independent, given age ($S \perp R | A$)

Chain Rule (exploiting independencies)

$P(S, R, A) =$
$P(S|R, A)P(R|A)P(A) =$
$P(S|A)P(R|A)P(A)$



Fork cont.

$P(S = 39) =$
$P(S = 39|R = good) =$
$P(S = 39|A = 11) =$
$P(S = 39|A = 11, R = good) =$
Note that S and R are NOT independent

Collider

Example

Consider the probability of success of actors taking into account their talent and their beauty.

Variables

Success $S \in 1..3$, Talent $T \in 1..3$, Beauty $B \in 1..3$

Collider

Example

Consider the probability of success of actors taking into account their talent and their beauty.

Variables

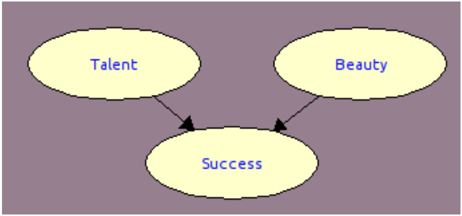
Success $S \in 1..3$, Talent $T \in 1..3$, Beauty $B \in 1..3$

Assumptions

Talent and Beauty are independent ($T \perp B$)

Chain Rule (exploiting independencies)

$P(S, T, B) =$
$P(S|T, B)P(T|B)P(B) =$
$P(S|T, B)P(T)P(B)$



Collider cont.

$P(T = 1) =$
$P(T = 1|B = 1) =$
$P(T = 1|S = 3) =$
$P(T = 1|B = 1, S = 3) =$
Note that T and B are NOT independent, given S

Clique

Example

COVID test with 65.5% and 45% sensitivity for sympt. and asympt. patients and specificity of 99.9% and a prevalence of 5%

Variables

Covid C, Test result T, Symptoms S.

Clique

Example

COVID test with 65.5% and 45% sensitivity for sympt. and asympt. patients and specificity of 99.9% and a prevalence of 5%

Variables

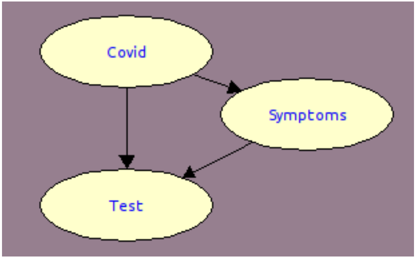
Covid C, Test result T, Symptoms S.

Assumptions

There is no (conditional) independence (note that it is neither a **junction**, a **fork**, nor a **collider**)

Chain Rule

$P(T, S, C) =$
$P(T|S, C)P(S|C)P(C)$



Another Example: **Junction**

Variables

Smoking $S \in \{s, \neg s\}$, Lung Cancer $C \in \{c, \neg c\}$, Test $S \in \{t, \neg t\}$

Assumptions

Smoking and Test are independent, given Cancer $(S \perp T | C)$

Chain Rule (exploiting independencies)

$P(T, C, S) = P(T|C, S)P(C|S)P(S) = P(T|C)P(C|S)P(S)$

Another Example: **Fork**

Variables

Smoking $S \in \{s, \neg s\}$, Lung Cancer $C \in \{c, \neg c\}$, Heart Condition $H \in \{h, \neg h\}$

Assumptions

C and H are conditionally independent ($C \perp H | S$)

Chain Rule (exploiting independence)

$$P(C, H, S) = P(C|H, S)P(H|S)P(S) = P(C|S)P(H|S)P(S)$$

Another Example: **Collider**

Variables

Smoking $S \in \{s, \neg s\}$, Lung Cancer $C \in \{c, \neg c\}$, Exposure to Pollution $P \in \{p, \neg p\}$

Assumptions

S and P are independent ($S \perp P$)

Chain Rule (exploiting independence)

$P(C, P, S) = P(C|P, S)P(P|S)P(S) = P(C|P, S)P(P)P(S)$