# WeBridge: Synthesizing Stored Procedures for Large-Scale Real-World Web Applications

(Extra Material)

#### Abstract

This document contains the extra material of SIGMOD'24 submission #347. The main content is the formal proof of the correctness of WeBridge.

### 1 Correctness Proof of WeBridge

In this section, we show that the transformation performed by WeBridge preserves the semantic of the original web application, which is the Theorem 1.2 in this section.

We first introduce some preliminary concepts. The program states of the original application API is denoted as  $\langle D, S \rangle$ , where D represents the database state, S represents the application's heap and local variable state. WeBridge extends the program states of the original application with a query result buffer B. Thus, the program states become  $\langle D, S, B \rangle$ . Additionally, let  $C_{sp}$  be the stored procedures WeBridge generated for the original application. Given the definition of program states, we now describe how the application is evaluated upon each user request. Let I be the set of all possible request inputs of an API. The input contains the request parameters for the API, and a system environment (which determine the return values of native method calls). For each input  $i \in I$ , suppose the initial program state of the API is  $\langle D, S \rangle$ , the final state of the API is  $\langle D', S' \rangle$ , then evaluating the API with input i is denoted with:  $\langle D, S \rangle \xrightarrow{i} \langle D', S' \rangle$ . Similarly, evaluating the API transformed by WeBridge is denoted with  $\langle D, S, B \rangle \xrightarrow{i} \langle D', S', B' \rangle$ . During the above evaluation, the program will execute along a path (Section 4 in the paper) P, which is a finite sequence of n boolean branch decisions denoted as  $P = [b_1, b_2, ..., b_n]$ . If  $b_i = 1$ , it signifies the i-th branch decision took the 'then' branch; otherwise the 'else' branch was taken. Let  $\Pi$  be the set of all possible paths and  $\Phi$  be the set of paths optimized by WeBridge, where  $\Phi \subset \Pi$ . A hot path is a path p where  $p \in \Phi$ , a cold path  $p_c$  is a path where  $p_c \in \Pi \land p_c \notin \Phi$ . Additionally, the execution of each path of the program will lead to a sequence of SQL statements issued to the database. Each SQL statements consists of a template string and a list of parameters. We use the following notation:  $\langle D, S \rangle \stackrel{\imath}{\Rightarrow} Q$  to represent that evaluating the original application API with initial state  $\langle D, S \rangle$  on input i produces the sequence of SQL statements Q. We use Q[i] to denote the ith SQL statement in sequence Q, where  $i \in \{1, \ldots, |Q|\}$ . Additionally, we use head(Q) to denote the first element of sequence Q, and  $concat(Q_1, Q_2)$  to denote the sequential concatenation of two sequences  $Q_1$  and  $Q_2$ . Similarly, for the application API transformed by WeBridge, we have  $\langle D, S, B \rangle \stackrel{i}{\Rightarrow} Q'$ , where Q' is the sequence of executed SQL statements.

We next define and proof the following lemma based on the above definitions.

**Lemma 1.1.** Given an API program C and initial states  $D_0, S_0$  and  $B_0$ , let I be the set of all possible inputs for C.  $\forall i \in I$ , If  $\langle D_0, S_0 \rangle \stackrel{i}{\Rightarrow} Q$  under the original application,  $\langle D_0, S_0, B_0 \rangle \stackrel{i}{\Rightarrow} Q'$  under the application transformed by WeBridge, then Q = Q'.

#### Proof

The premise contains

$$\langle D_0, S_0 \rangle \stackrel{i}{\Rightarrow} Q$$
 (1)

and

$$\langle D_0, S_0, B_0 \rangle \stackrel{i}{\Rightarrow} Q'$$
 (2)

The conclusion is

$$Q = Q' \tag{3}$$

The conclusion is proved by classifying the type of the path taken by C into two cases.

• A Hot path p. By definition of the hot path, we have

$$p \in \Phi \tag{4}$$

indicating that C is taking a path that have already been optimized by WeBridge. If p is optimized by WeBridge, then all the database accesses along the path are replace with calls to  $C_{sp}$ , which is the stored procedures generated by WeBridge for p. Let  $Q_{sp}$  be the sequence of SQL statements of  $C_{sp}$ , in which the SQL statement template string and parameters are extracted by concolic execution in Algorithm 1. Since the concolic execution in Algorithm 1 is done by a deterministic reply of the hot path p, which implies that the collected concrete SQL templates in  $Q_{sp}$  must be the same with Q. Along with (1), we have:

$$|Q| = |Q_{sp}| \tag{5}$$

and

$$\forall k \in \{1, \dots, |Q|\}. \ Q_{sp}[k].template = Q[k].template \tag{6}$$

For the query parameters, since WeBridge assumes the absence of global application states, the value of parameters are only determined by the external input states (Section 4 in the paper). All these states have been symbolized and their related computations are tracked in symbolic form via Algorithm 1. These symbolic computations are then transformed into equivalent stored procedure code in  $C_{sp}$  by Algorithm 4, by transformation rules that WeBridge assume to be semantic preserving. Consequently, any computations that might change the value of a parameter have been transformed into equivalent symbolic computations in the stored procedure. These computations are the dynamic backward slices [1, 2] of the query parameters. Therefore, given the same set of concrete input states to the original application and stored procedure  $C_{sp}$ , the parameter values computed from these input states must be the same:

$$\forall k \in \{1, \dots, |Q|\}. \ Q_{sp}[k].parameters = Q[k].parameters$$
 (7)

By the definition of a SQL statement, (5), (6) and (7), we have:

$$Q_{sp} = Q \tag{8}$$

Since p is a hot path, the path conditions for all SQL statements in  $Q_{sp}$  should evaluate to true, which means that all SQL statements in  $Q_{sp}$  will execute. Along with (2), we have:

$$Q_{sp} = Q' \tag{9}$$

By (8) and (9), the conclusion (3) is proved in this case.

- A Cold path  $p_c$ . By definition of cold path, we can further divide the type of cold path into two cases.
  - $-\forall p' \in \Phi$ .  $head(p') \neq head(p_c)$ . In this case, the cold path  $p_c$  diverges on the first branch decision of hot paths. Let  $b = head(p_c)$  for the cold path, then the first branch decision for all hot paths must be  $\neg b$ :

$$\forall p' \in \Phi.head(p_c) = \neg head(p') \tag{10}$$

By (10), we divide  $Q_{sp}$  into two sub-sequences of SQL statements  $Q_{sp_1}$  and  $Q_{sp_2}$ , where:

$$Q_{sp} = concat(Q_{sp_1}, Q_{sp_2}) \tag{11}$$

And the path conditions of all SQL statements in  $Q_{sp_1}$  evaluates to true, which represent the queries that issued before the first branch condition is made; the path conditions of all SQL statements in  $Q_{sp_2}$  evaluates to false. According to the number of SQL statements in  $Q_{sp_1}$ , we can divide the proof into two cases:

- \*  $|Q_{sp_1}| = 0$ . In this case, no SQL statement will be issued by  $C_{sp}$ , and the application trivially fallbacks to normal execution to issue all SQL statements in interactive mode. This indicates that Q = Q', and the conclusion (3) is proved in this case.
- \*  $|Q_{sp_1}| > 0$ . In this case, the path conditions for  $Q_{sp_1}$  in  $C_{sp}$  will all be true, indicating that these SQL statements will execute unconditionally for both hot paths and cold paths. Thus, with (8) and (9) we have:

$$\forall i \in \{1, ..., |Q_{sp_1}|\}. \ Q_{sp_1}[i] = Q[i] \tag{12}$$

additionally, since SQL statements in  $Q_{sp_1}$  are all executed by stored procedure  $C_{sp}$ , and by (2), we have:

$$\forall i \in \{1, ..., |Q_{sp_1}|\}. \ Q_{sp_1}[i] = Q'[i]$$
(13)

For SQL statements in  $Q_{sp_2}$ , by (10) their path conditions will evaluate to false. This indicates that no SQL statement in  $C_{sp_2}$ 

will execute. Thus, the application fallbacks to normal execution to issue the following SQL statements in interactive mode. Then, we have:

$$\forall i \in \{|Q_{sp_1}| + 1, |Q|\}. \ Q[i] = Q'[i] \tag{14}$$

By (12), (13) and (14), the conclusion (3) is proved in this case.  $-\forall p' \in \Phi$ .  $head(p') = head(p_c)$ . In this case, the cold path  $p_c$  and hot path p' "share" a common prefix of branch decisions. The hot path p' that has the longest prefix of branch decisions with  $p_c$  is:

$$\exists p' \in \Phi, n \in \{2, ..., |p'|\}, \forall j \in \{1, ..., n-1\}.$$

$$p'[j] = p_c[j] \land p'[n] \neq p_c[n] \land$$

$$(\nexists p'' \in \Phi. p''[n] = p_c[n])$$
(15)

Where n-1 in (15) is the length of the longest prefix branch decisions for  $p_c$  and p'. Additionally, any SQL statement in  $Q_{sp}$  with the n-th branch decision encoded in path conditions will not get executed, as  $p'[n] \neq p_c[n]$ . Let  $Q_{bf}$  be the sequence of SQL statements that do not encode the n-th branch decision in their path conditions, and let  $Q_{af}$  be the sequence of SQL statements that encode the n-th branch decision in their path conditions, we have:

$$Q_{sp} = concat(Q_{bf}, Q_{af}) \tag{16}$$

Then, by (15) and (16), we know that only the SQL statements in  $Q_{sp_1}$  will execute along path  $p_c$ . Then we know that:

$$\forall i \in \{1, \dots, |Q_{bf}|\}. \ Q_{bf}[i] = Q[i] \tag{17}$$

and since SQL statements in  $Q_{bf}$  are executed by  $C_{sp}$ , and by (2), we have:

$$\forall i \in \{1, ..., |Q_{bf}|\}. \ Q_{bf}[i] = Q'[i] \tag{18}$$

For SQL statements in  $Q_{af}$ , their path conditions must include the n-th branch decision, which evaluates to false in the cold path  $p_c$ . This indicates that no SQL statement in  $C_{sp}$  will execute after the n-th branch decision is made. Thus, the application fallbacks to normal execution to issue the following SQL statements in interactive mode. Then, we have:

$$\forall i \in \{|Q_{bf}| + 1, |Q|\}. \ Q[i] = Q'[i] \tag{19}$$

By (17), (18) and (19), the conclusion (3) is proved in this case.

Then we have proved the conclusion (3).

Next, we prove the Theorem 1.2.

**Theorem 1.2.** Given an API program C and initial states  $D_0, S_0$  and  $B_0$ , let I be the set of all possible inputs for C.  $\forall i \in I$ , if  $\langle D_0, S_0 \rangle \xrightarrow{i} \langle D_c, S_c \rangle$  under the original application,  $\langle D_0, S_0, B_0 \rangle \xrightarrow{i} \langle D_{c'}, S_{c'}, B_{c'} \rangle$  under the application transformed by WeBridge, then  $D_c = D_{c'} \wedge S_c = S_{c'}$ .

#### Proof

The premise contains

$$\langle D_0, S_0 \rangle \xrightarrow{i} \langle D_c, S_c \rangle$$
 (20)

under the original application, and

$$\langle D_0, S_0, B_0 \rangle \xrightarrow{i} \langle D_{c'}, S_{c'}, B_{c'} \rangle$$
 (21)

under the application transformed by WeBridge.

The conclusion includes

$$D_c = D_{c'} \tag{22}$$

and

$$S_c = S_{c'} \tag{23}$$

We first proof (22). By Lemma 1.1, (20) and (21), we know that the SQL statements issued by the original application and the application transformed by WeBridge are the same. Since the initial database state  $D_0$  could only updated by the SQL statements, the database state after executing the same sequence of SQL statements should be the same. Thus, we have  $D_c = D_{c'}$ , and conclusion (22) is proved.

We next proof (23). Because WeBridge assumes the absence of global states in the application, the application state could only be modified by external input states. Thus, given the same input i for the original application and the application transformed by WeBridge, it is sufficient to proof (23) by showing the query execution results of each SQL statement in the original application and the application transformed by WeBridge are the same. We proof (23) by classifying the type of the path taken by C into two cases.

• A Hot path. In this case, all SQL statements are issued by the stored procedure, and the query execution results are first stored in the stored procedure buffer, which is initially  $B_0$ , then returned to the application upon following query invocations in sequential order. As the initial database states for the original application and transformed application are both  $D_0$ , by Lemma 1.1, we can conclude that the query execution result of each SQL statement in the original application and the execution results stored in the buffer of the transformed application should be all the same. Consequently, the final execution result retrieved by the transformed application should be the same with the originally application. Thus, (23) is proved in this case.

- A Cold path. We can further divide the cold path into two cases by the number of SQL statements that executed by stored procedure.
  - No SQL statement is executed in the stored procedure. In this case, there will be no query execution result stored in the buffer. At the same time, all possible SQL statements will be issued interactively, which is exactly the same with the original application without a buffer. Thus, both applications should observe the same query execution results, and (23) is proved in this case.
  - At lease one SQL statement is executed in the stored procedure. In this case, we can divide the sequence of executed SQL statements  $Q_{sp}$  into two sub-sequences of SQL statements  $Q_{sp_1}$  and  $Q_{sp_2}$ , where:

$$Q_{sp} = concat(Q_{sp_1}, Q_{sp_2}) \tag{24}$$

In (24),  $Q_{sp_1}$  is the sequence of SQL statements executed in the stored procedure, and  $Q_{sp_2}$  is the sequence of SQL statements executed interactively. Query execution results of  $Q_{sp_1}$  are processed just the same as how the results are processed in a hot path, except for the hot path will "terminate" after the last SQL statement of  $Q_{sp_1}$  is executed. The equivalence proof of query results for SQL statements in  $Q_{sp_1}$  is then the same with the hot path we proved before. For  $Q_{sp_2}$ , no SQL statement will be executed in the stored procedure, and the equivalence proof of query results follows the same proof of the previous case. Consequently, (23) is proved in this case.

As a result, Theorem 1.2 is proved.

## References

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- [2] X. Zhang, R. Gupta, and Y. Zhang. Precise dynamic slicing algorithms. In 25th International Conference on Software Engineering, 2003. Proceedings., pages 319–329, 2003.