

1 WeBridge: Synthesizing Stored Procedures for  
2 Large-Scale Real-World Web Applications

3 (Extra Material)

4 **Abstract**

5 This document contains the extra material of SIGMOD'24 submis-  
6 sion #347. The main content is the formal proof of the correctness of  
7 WeBridge.

# 1 Correctness Proof of WeBridge

In this section, we show the correctness of WeBridge. The correctness condition of WeBridge states that, for any execution possible with WeBridge, there is always a equivalent execution possible in the original application. We first discuss the correctness of sequential requests, which is the Theorem 1.2 in this section.

## 1.1 Sequential Requests

We will introduce some preliminary concepts and notations. The program states of the original application API is denoted as  $\langle D, S \rangle$ , where  $D$  represents the database state,  $S$  represents the application's heap and local variable state. WeBridge extends the program states of the original application with a query result buffer  $B$ . Thus, the program states become  $\langle D, S, B \rangle$ . Additionally, let  $C_{sp}$  be the stored procedures WeBridge generated for the original application. Given the definition of program states, we now describe how the application is evaluated upon each user request. Let  $I$  be the set of all possible request inputs of an API. The input contains the request parameters for the API, and a system environment (which determine the return values of native method calls). For each input  $i \in I$ , suppose the initial program state of the API is  $\langle D, S \rangle$ , the final state of the API is  $\langle D', S' \rangle$ , then evaluating the API with input  $i$  is denoted with:  $\langle D, S \rangle \xrightarrow{i} \langle D', S' \rangle$ . Similarly, evaluating the API transformed by WeBridge is denoted with  $\langle D, S, B \rangle \xrightarrow{i} \langle D', S', B' \rangle$ . During the above evaluation, the program will execute along a *path* (Section 4 in the paper)  $P$ , which is a finite sequence of  $n$  boolean branch decisions denoted as  $P = [b_1, b_2, \dots, b_n]$ . If  $b_i = 1$ , it signifies the  $i$ -th branch decision took the 'then' branch; otherwise the 'else' branch was taken. Let  $\Pi$  be the set of all possible paths and  $\Phi$  be the set of paths optimized by WeBridge, where  $\Phi \subset \Pi$ . A hot path is a path  $p$  where  $p \in \Phi$ , a cold path  $p_c$  is a path where  $p_c \in \Pi \wedge p_c \notin \Phi$ . Additionally, the execution of each path of the program will lead to a sequence of SQL statements issued to the database. Each SQL statements consists of a *template* string and a list of *parameters*. We use the following notation:  $\langle D, S \rangle \xRightarrow{i} Q$  to represent that evaluating the original application API with initial state  $\langle D, S \rangle$  on input  $i$  produces the sequence of SQL statements  $Q$ . We use  $Q[i]$  to denote the  $i$ -th SQL statement in sequence  $Q$ , where  $i \in \{1, \dots, |Q|\}$ . Additionally, we use  $head(Q)$  to denote the first element of sequence  $Q$ , and  $concat(Q_1, Q_2)$  to denote the sequential concatenation of two sequences  $Q_1$  and  $Q_2$ . Similarly, for the application API transformed by WeBridge, we have  $\langle D, S, B \rangle \xRightarrow{i} Q'$ , where  $Q'$  is the sequence of executed SQL statements.

We next define and proof the following lemma based on the above definitions.

**Lemma 1.1.** *Given an API program  $C$  and initial states  $D_0, S_0$  and  $B_0$ , let  $I$  be the set of all possible inputs for  $C$ .  $\forall i \in I$ , If  $\langle D_0, S_0 \rangle \xRightarrow{i} Q$  under the original*

48 application,  $\langle D_0, S_0, B_0 \rangle \xRightarrow{i} Q'$  under the application transformed by WeBridge,  
 49 then  $Q = Q'$ .

*Proof.* The premise contains

$$\langle D_0, S_0 \rangle \xRightarrow{i} Q \quad (1)$$

and

$$\langle D_0, S_0, B_0 \rangle \xRightarrow{i} Q' \quad (2)$$

The conclusion is

$$Q = Q' \quad (3)$$

50 The conclusion is proved by classifying the type of the path taken by  $C$  into  
 51 two cases.

- A Hot path  $p$ . By definition of the hot path, we have

$$p \in \Phi \quad (4)$$

indicating that  $C$  is taking a path that have already been optimized by WeBridge. If  $p$  is optimized by WeBridge, then all the database accesses along the path are replace with calls to  $C_{sp}$ , which is the stored procedures generated by WeBridge for  $p$ . Let  $Q_{sp}$  be the sequence of SQL statements of  $C_{sp}$ , in which the SQL statement template string and parameters are extracted by concolic execution in in Algorithm 1. Since the concolic execution in Algorithm 1 is done by a deterministic reply of the hot path  $p$ , which implies that the collected concrete SQL templates in  $Q_{sp}$  must be the same with  $Q$ . Along with (1), we have:

$$|Q| = |Q_{sp}| \quad (5)$$

and

$$\forall k \in \{1, \dots, |Q|\}. Q_{sp}[k].template = Q[k].template \quad (6)$$

For the query parameters, since WeBridge assumes the absence of global application states, the value of parameters are only determined by the external input states (Section 4 in the paper). All these states have been symbolized and their related computations are tracked in symbolic form via Algorithm 1. These symbolic computations are then transformed into equivalent stored procedure code in  $C_{sp}$  by Algorithm 4, by transformation rules that WeBridge assume to be semantic preserving. Consequently, any computations that might change the value of a parameter have been transformed into equivalent symbolic computations in the stored procedure. These computations are the dynamic backward slices [1, 2] of the

query parameters. Therefore, given the same set of concrete input states to the original application and stored procedure  $C_{sp}$ , the parameter values computed from these input states must be the same:

$$\forall k \in \{1, \dots, |Q|\}. Q_{sp}[k].parameters = Q[k].parameters \quad (7)$$

By the definition of a SQL statement, (5), (6) and (7), we have:

$$Q_{sp} = Q \quad (8)$$

Since  $p$  is a hot path, the path conditions for all SQL statements in  $Q_{sp}$  should evaluate to **true**, which means that all SQL statements in  $Q_{sp}$  will execute. Along with (2), we have:

$$Q_{sp} = Q' \quad (9)$$

By (8) and (9), the conclusion (3) is proved in this case.

• A Cold path  $p_c$ . By definition of cold path, we can further divide the type of cold path into two cases.

- $\forall p' \in \Phi. head(p') \neq head(p_c)$ . In this case, the cold path  $p_c$  diverges on the first branch decision of hot paths. Let  $b = head(p_c)$  for the cold path, then the first branch decision for all hot paths must be  $\neg b$ :

$$\forall p' \in \Phi. head(p_c) = \neg head(p') \quad (10)$$

By (10), we divide  $Q_{sp}$  into two sub-sequences of SQL statements  $Q_{sp_1}$  and  $Q_{sp_2}$ , where:

$$Q_{sp} = concat(Q_{sp_1}, Q_{sp_2}) \quad (11)$$

And the path conditions of all SQL statements in  $Q_{sp_1}$  evaluates to **true**, which represent the queries that issued before the first branch condition is made; the path conditions of all SQL statements in  $Q_{sp_2}$  evaluates to **false**. According to the number of SQL statements in  $Q_{sp_1}$ , we can divide the proof into two cases:

- \*  $|Q_{sp_1}| = 0$ . In this case, no SQL statement will be issued by  $C_{sp}$ , and the application trivially fallbacks to normal execution to issue all SQL statements in interactive mode. This indicates that  $Q = Q'$ , and the conclusion (3) is proved in this case.
- \*  $|Q_{sp_1}| > 0$ . In this case, the path conditions for  $Q_{sp_1}$  in  $C_{sp}$  will all be **true**, indicating that these SQL statements will execute unconditionally for both hot paths and cold paths. Thus, with (8) and (9) we have:

$$\forall i \in \{1, \dots, |Q_{sp_1}|\}. Q_{sp_1}[i] = Q[i] \quad (12)$$

additionally, since SQL statements in  $Q_{sp_1}$  are all executed by stored procedure  $C_{sp}$ , and by (2), we have:

$$\forall i \in \{1, \dots, |Q_{sp_1}|\}. Q_{sp_1}[i] = Q'[i] \quad (13)$$

For SQL statements in  $Q_{sp_2}$ , by (10) their path conditions will evaluate to **false**. This indicates that no SQL statement in  $C_{sp_2}$  will execute. Thus, the application fallbacks to normal execution to issue the following SQL statements in interactive mode. Then, we have:

$$\forall i \in \{|Q_{sp_1}| + 1, |Q|\}. Q[i] = Q'[i] \quad (14)$$

By (12), (13) and (14), the conclusion (3) is proved in this case.  
 64 –  $\forall p' \in \Phi. head(p') = head(p_c)$ . In this case, the cold path  $p_c$  and hot  
 65 path  $p'$  “share” a common prefix of branch decisions. The hot path  
 66  $p'$  that has the longest prefix of branch decisions with  $p_c$  is:  
 67

$$\begin{aligned} \exists p' \in \Phi, n \in \{2, \dots, |p'|\}, \forall j \in \{1, \dots, n-1\}. \\ p'[j] = p_c[j] \wedge p'[n] \neq p_c[n] \wedge \\ (\nexists p'' \in \Phi. p''[n] = p_c[n]) \end{aligned} \quad (15)$$

Where  $n-1$  in (15) is the length of the longest prefix branch decisions for  $p_c$  and  $p'$ . Additionally, any SQL statement in  $Q_{sp}$  with the  $n$ -th branch decision encoded in path conditions will not get executed, as  $p'[n] \neq p_c[n]$ . Let  $Q_{bf}$  be the sequence of SQL statements that do not encode the  $n$ -th branch decision in their path conditions, and let  $Q_{af}$  be the sequence of SQL statements that encode the  $n$ -th branch decision in their path conditions, we have:

$$Q_{sp} = concat(Q_{bf}, Q_{af}) \quad (16)$$

Then, by (15) and (16), we know that only the SQL statements in  $Q_{sp_1}$  will execute along path  $p_c$ . Then we know that:

$$\forall i \in \{1, \dots, |Q_{bf}|\}. Q_{bf}[i] = Q[i] \quad (17)$$

and since SQL statements in  $Q_{bf}$  are executed by  $C_{sp}$ , and by (2), we have:

$$\forall i \in \{1, \dots, |Q_{bf}|\}. Q_{bf}[i] = Q'[i] \quad (18)$$

For SQL statements in  $Q_{af}$ , their path conditions must include the  $n$ -th branch decision, which evaluates to **false** in the cold path  $p_c$ . This indicates that no SQL statement in  $C_{sp}$  will execute after the  $n$ -th branch decision is made. Thus, the application fallbacks to normal execution to issue the following SQL statements in interactive mode. Then, we have:

$$\forall i \in \{|Q_{bf}| + 1, |Q|\}. Q[i] = Q'[i] \quad (19)$$

By (17), (18) and (19), the conclusion (3) is proved in this case.  
 68

69 Then we have proved the conclusion (3), and Lemma 1.1 is proved. □

70  
71 Next, we prove the Theorem 1.2.

72 **Theorem 1.2.** *Given an API program  $C$  and initial states  $D_0, S_0$  and  $B_0$ , let*  
73  *$I$  be the set of all possible inputs for  $C$ .  $\forall i \in I$ , if  $\langle D_0, S_0 \rangle \xrightarrow{i} \langle D_c, S_c \rangle$  under*  
74 *the original application,  $\langle D_0, S_0, B_0 \rangle \xrightarrow{i} \langle D_{c'}, S_{c'}, B_{c'} \rangle$  under the application*  
75 *transformed by WeBridge, then  $D_c = D_{c'} \wedge S_c = S_{c'}$ .*

*Proof.* The premise contains

$$\langle D_0, S_0 \rangle \xrightarrow{i} \langle D_c, S_c \rangle \quad (20)$$

under the original application, and

$$\langle D_0, S_0, B_0 \rangle \xrightarrow{i} \langle D_{c'}, S_{c'}, B_{c'} \rangle \quad (21)$$

76 under the application transformed by WeBridge.

The conclusion includes

$$D_c = D_{c'} \quad (22)$$

and

$$S_c = S_{c'} \quad (23)$$

77 We first proof (22). By Lemma 1.1, (20) and (21), we know that the SQL state-  
78 ments issued by the original application and the application transformed by  
79 WeBridge are the same. Since the initial database state  $D_0$  could only updated  
80 by the SQL statements, the database state after executing the same sequence of  
81 SQL statements should be the same. Thus, we have  $D_c = D_{c'}$ , and conclusion  
82 (22) is proved.

83  
84 We next proof (23). Because WeBridge assumes the absence of global states  
85 in the application, the application state could only be affected by external in-  
86 put states. Thus, given the same input  $i$  for the original application and the  
87 application transformed by WeBridge, it is sufficient to proof (23) by showing  
88 the query execution results of each SQL statement in the original application  
89 and the application transformed by WeBridge are the same. We proof (23) by  
90 classifying the type of the path taken by  $C$  into two cases.

- 91 • A Hot path. In this case, all SQL statements are issued by the stored pro-  
92 cedure, and the query execution results are first stored in the stored pro-  
93 cedure buffer, which is initially  $B_0$ , then returned to the application upon  
94 following query invocations in sequential order. As the initial database  
95 states for the original application and transformed application are both

96  $D_0$ , by Lemma 1.1, we can conclude that the query execution result of  
 97 each SQL statement in the original application and the execution results  
 98 stored in the buffer of the transformed application should be all the same.  
 99 Consequently, the final execution result retrieved by the transformed ap-  
 100 plication should be the same with the originally application. Thus, (23)  
 101 is proved in this case.

102 • A Cold path. We can further divide the cold path into two cases by the  
 103 number of SQL statements that executed by stored procedure.

104 – No SQL statement is executed in the stored procedure. In this case,  
 105 there will be no query execution result stored in the buffer. At the  
 106 same time, all possible SQL statements will be issued interactively,  
 107 which is exactly the same with the original application without a  
 108 buffer. Thus, both applications should observe the same query exe-  
 109 cution results, and (23) is proved in this case.

– At lease one SQL statement is executed in the stored procedure. In  
 this case, we can divide the sequence of executed SQL statements  $Q_{sp}$   
 into two sub-sequences of SQL statements  $Q_{sp_1}$  and  $Q_{sp_2}$ , where:

$$Q_{sp} = \text{concat}(Q_{sp_1}, Q_{sp_2}) \quad (24)$$

110 In (24),  $Q_{sp_1}$  is the sequence of SQL statements executed in the stored  
 111 procedure, and  $Q_{sp_2}$  is the sequence of SQL statements executed  
 112 interactively. Query execution results of  $Q_{sp_1}$  are processed just the  
 113 same as how the results are processed in a hot path, except for the  
 114 hot path will “terminate” after the last SQL statement of  $Q_{sp_1}$  is  
 115 executed. The equivalence proof of query results for SQL statements  
 116 in  $Q_{sp_1}$  is then the same with the hot path we proved before. For  
 117  $Q_{sp_2}$ , no SQL statement will be executed in the stored procedure,  
 118 and the equivalence proof of query results follows the same proof of  
 119 the previous case. Consequently, (23) is proved in this case.

120 As a result, Theorem 1.2 is proved.

121 Additionally, the correctness of the speculative execution optimization in  
 122 Section 7 of the paper can be shown as follows. First, the validation and recovery  
 123 algorithm in Section 7 ensures that the SQL statements executed by WeBridge  
 124 is exactly the same with the original application, which implies (3). Next, by  
 125 Lemma 1.2 and (3), the (22) and (23) can be proved just as as shown before,  
 126 indicating Theorem 1.2 holds.  $\square$

## 127 1.2 Concurrent Requests

128 Before delving into the scenario of concurrent requests, we will establish some  
 129 notations and definitions. We represent the executions of concurrent APIs as  
 130 an *execution of events*. We identify six types of events:

- 131 • A SQL statement's invocation to the database in the original application,  
132 denoted as  $e_{db}$ .
- 133 • An invocation of an external (native) call, denoted as  $e_{ext}$ .
- 134 • Local computation(s) performed by the application, denoted as  $e_{loc}$ .
- 135 • An invocation of a SQL statement by the stored procedure, denoted as  
136  $e_{sp}$ .
- 137 • An invocation of a SQL statement in the application by WeBridge, an-  
138 swered with the results from the stored procedure buffer, denoted as  $e_{buf}$ .
- 139 • An empty event, denoted as  $\emptyset$ .

140 Each event records the values of the arguments provided to the event during  
141 execution, along with the return values of the event. We define an *execution* to  
142 be an finite sequence of events. We consider two events  $e_1$  and  $e_2$  *conflict*, if  
143 given the same input parameters to  $e_1$  and  $e_2$ , executing  $e_1$  before  $e_2$  produces  
144 return results different from  $e_2$  executes before  $e_1$ . We then formally define the  
145 equivalence of executions:

146 **Definition 1** (Equivalence of executions). *Given two executions  $E_1$  and  $E_2$ , if:*

- 147 • *The sequences of SQL statements executed by the database from events of*  
148  *$e_{db}$  and  $e_{sp}$  are the same in  $E_1$  and  $E_2$ .*
- 149 • *The sequences of external calls from events of  $e_{ext}$  are the same in  $E_1$  and*  
150  *$E_2$ .*
- 151 • *The sequences of local computations performed by the application from*  
152 *events of  $e_{loc}$  are the same in  $E_1$  and  $E_2$ .*
- 153 • *If  $e_1$  and  $e_2$  are two conflicting events in  $E_1$ , and  $e_1$  proceeds  $e_2$ , then  $e_1$*   
154 *and  $e_2$  must also exist in  $E_2$ , and  $e_1$  proceeds  $e_2$ .*

155 *then  $E_1$  and  $E_2$  are equivalent.*

156 *Proof.* The correctness condition of WeBridge is that, for any execution possible  
157 with WeBridge, there is always an equivalent execution possible in the original  
158 application. For sequential API requests, the correctness condition must hold  
159 due to Theorem 1.2. When there are concurrent requests, we need to addi-  
160 tionally consider how one API's SQL statement sequence issued inside a stored  
161 procedure might be interfered by another API's conflicting events, including 1)  
162 SQL statements that have conflicting writes, and 2) external calls that have  
163 conflicting writes. We need to prove that, when such writes interleave with a  
164 stored procedure, the execution is equivalent to one under the original appli-  
165 cation with those writes interleaving corresponding source SQL statements in  
166 the same way. We proof by showing that for any execution produced by the  
167 application with WeBridge, we can construct an equivalent execution, which is  
168 a valid execution of the original application.



169 We first introduce how we construction such an execution and why the con-  
 170 structed execution is equivalent to the execution of WeBridge. The construction  
 171 follows a rule-based manner. Specifically, for each event in an execution of We-  
 172 Bridge:

- 173 • For an invocation of a SQL statement by the stored procedure, construct  
 174 an empty event  $\emptyset$ .
- 175 • For an invocation of a SQL statement in the application by WeBridge,  
 176 answered with the results from the stored procedure buffer, construct an  
 177 event of an invocation of a SQL statement to the database in the original  
 178 application with the same SQL statement.
- 179 • For an event of local computation(s) performed by the application, An  
 180 invocation of an external (native) call, and an empty event, construct the  
 181 same event.

182 Given an execution  $E_1$  from WeBridge, we construct  $E_2$  by the above construc-  
 183 tion rules. For any pair of events  $e_1$  and  $e_2$  that are conflicting in  $E_1$ , we  
 184 highlight the arguments of some of the event types, others are proved similarly.  
 185 If  $e_1$  is of type  $e_{sp}$ , and  $e_2$  of type:

- 186 •  $e_{sp}$ : this means two SQL statements in the stored procedure have a con-  
 187 flict. Since WeBridge collects SQL statements in the execution order of  
 188 SQL statement,  $e_1$  must proceeds  $e_2$ .
- 189 •  $e_{ext}$ : in this case, an external call is conflict with a SQL statement exe-  
 190 cuted in the stored procedure. According to the Algorithm.1 in the paper,  
 191 WeBridge "splits" the stored procedure generation upon each external call,  
 192 so there is no way  $e_{ext}$  executes before  $e_1$  in  $E_2$ .
- 193 •  $e_{db}$ : in this case, a SQL statement outside of stored procedure is executed  
 194 after a SQL statement executed inside stored procedure. This indicates  
 195 that the application must fall backed to execute queries normally, due to  
 196 a failed validation in the speculative execution. As the failure recovery  
 197 must happens after stored procedure invocations,  $e_1$  proceeds  $e_2$ .

198 Consequently, conflicting events are the same in  $E_1$  and  $E_2$ . Additionally, it is  
 199 apparent to see that the sequences of SQL statements executed by the database  
 200 from events of  $e_{db}$  and  $e_{sp}$  are the same in  $E_1$  and  $E_2$ , the sequences of external  
 201 calls from events of  $e_{ext}$  are the same in  $E_1$  and  $E_2$ , and the sequences of local  
 202 computations performed by the application from events of  $e_{loc}$  are the same  
 203 in  $E_1$  and  $E_2$ , from the construction rules. As a result,  $E_1$  and  $E_2$  must be  
 204 equivalent. We next show that the constructed execution must also be a valid  
 205 execution of the original application. Because we have shown that the conflicting  
 206 events are the same in  $E_1$  and  $E_2$ , then proving concurrent requests is reduced  
 207 to proving sequential requests, which follows the previous proof.  $\square$

## 208 **References**

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