Assignment 2: CS 663, Fall 2024

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1. Prove or disprove: (a) The Laplacian mask with a -8 in the center (see class slides) is a separable filter. (b) The Laplacian mask with a -4 in the center (see class slides) can be implemented entirely using 1D convolutions. **[5+5=10 points]**

Soln:

(a)Laplacian mask with a -8 in the center. For the mask to be separable, it should be possible to express it as the outer product of two 1D vectors, say v_1 and v_2.

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \times \begin{bmatrix} d & e & f \end{bmatrix} = \begin{bmatrix} ad & ae & af \\ bd & be & bf \\ cd & ce & cf \end{bmatrix}$$

This would mean, ad = ae = af = -1. And if a != 0 (which will always hold, because if a was zero, ad = ae = af = 0), then d = e = f. This would mean, bd = be = bf = -1. But be = 8. Hence this mask, can not be split into two vectors, and thereby, **the Laplacian mask with -8 in the center is not separable**.

Conclusion for (a):
Disproved. The Laplacian mask with -8 in the center is not a separable filter.

(b) Laplacian mask with a -4 in the center.

We can split our original Laplacian operator into a sum of two matrices:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Further, splitting each of these masks into 1D vectors, we obtain:

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 1 & -2 & 1 \end{bmatrix} \Rightarrow [f(x+1,y) - 2f(x,y) + f(x-1,y)]$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \Rightarrow [f(x,y+1) - 2f(x,y) + f(x,y-1)]$$

$$\nabla f = [f(x+1,y) - 2f(x,y) + f(x-1,y)] + [f(x,y+1) - 2f(x,y) + f(x,y-1)]$$

Thus, **the Laplacian mask with -4 in the center can be implemented entirely using 1D convolutions**.