

# Edge and Corner Detection

# Basics

- **Edge pixels** are those at which image intensity changes abruptly.
- **Edge segments** or edges are sets of connected edge pixels.

# Digital derivatives: first versus second

- Along a ramp (i.e. intensity change with constant slope), first derivative is a non-zero constant.
- Second derivative is zero along the ramp, except at the start and end!
- Second derivative is preferable for image sharpening! Many edges in images are ramp-like, in which case **first** derivative will give **thick** edges (undesirable), whereas **second** derivative gives **thin** edges two pixels wide (desirable).
- Second derivative changes sign midway at a step edge (**zero crossing property**).

# Digital derivatives: first versus second

- At **roof edges**, second derivative gives stronger response than first derivative (i.e. has larger magnitude) – except if both parts of the roof were at 45 degrees, where both will have equal response.
- At **isolated points**, second derivative gives stronger response than first derivative
- A rotationally invariant second-derivative operator is used for edge detection: the Laplacian, in fact zero-crossings of a Laplacian.

# Laplacian of an image

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \longrightarrow \text{Rotationally symmetric operator (in the continuous domain)}$$

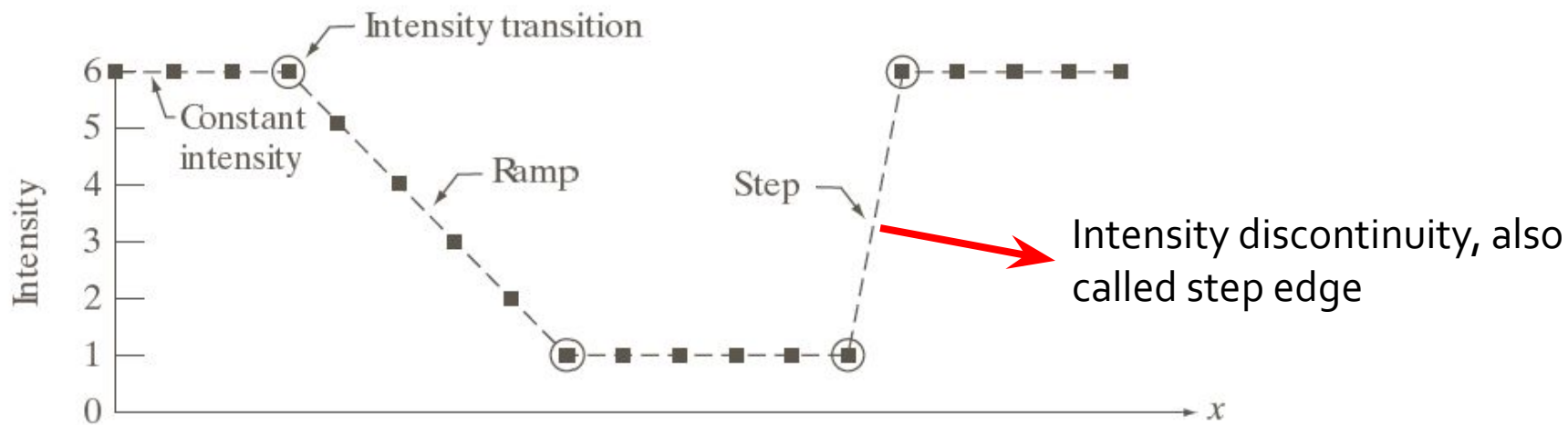
$$= f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$+ f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$= f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Laplacian operators: second operator is obtained by adding second derivatives along both the diagonals, to the first operator

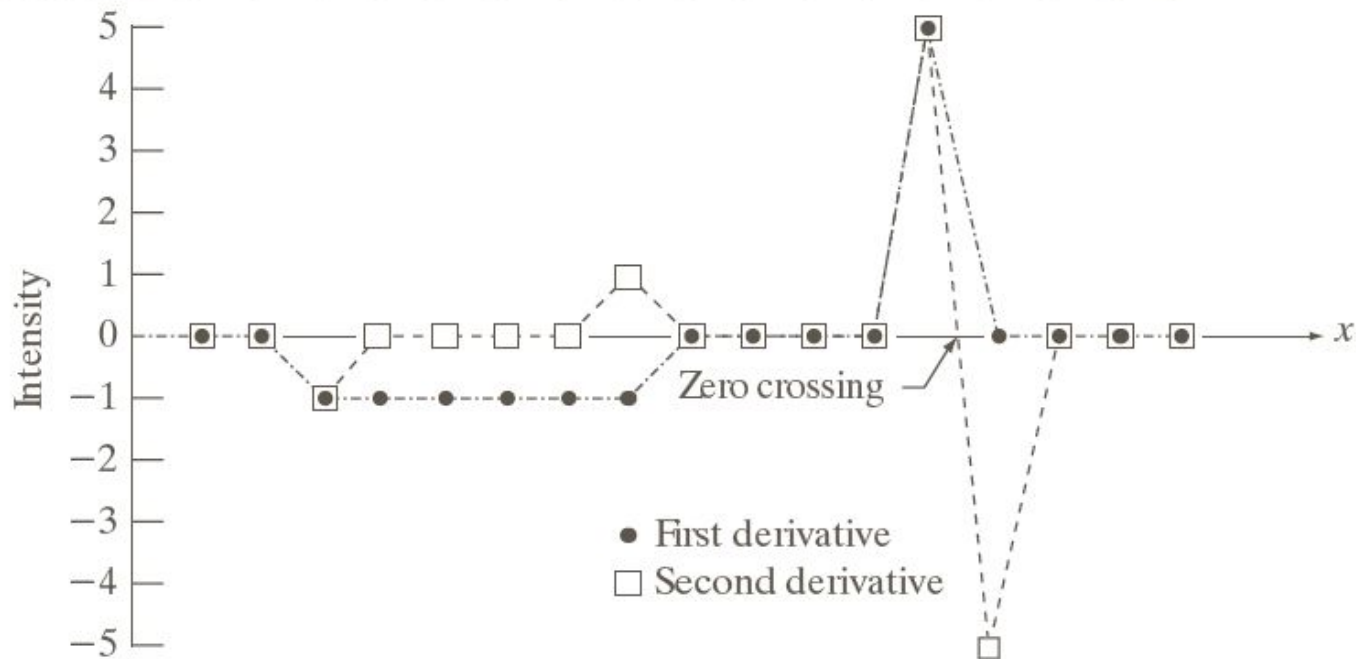


Scan line

	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6
	$x$																		

1st derivative 0 0 -1 -1 -1 -1 -1 0 0 0 0 0 0 5 0 0 0 0

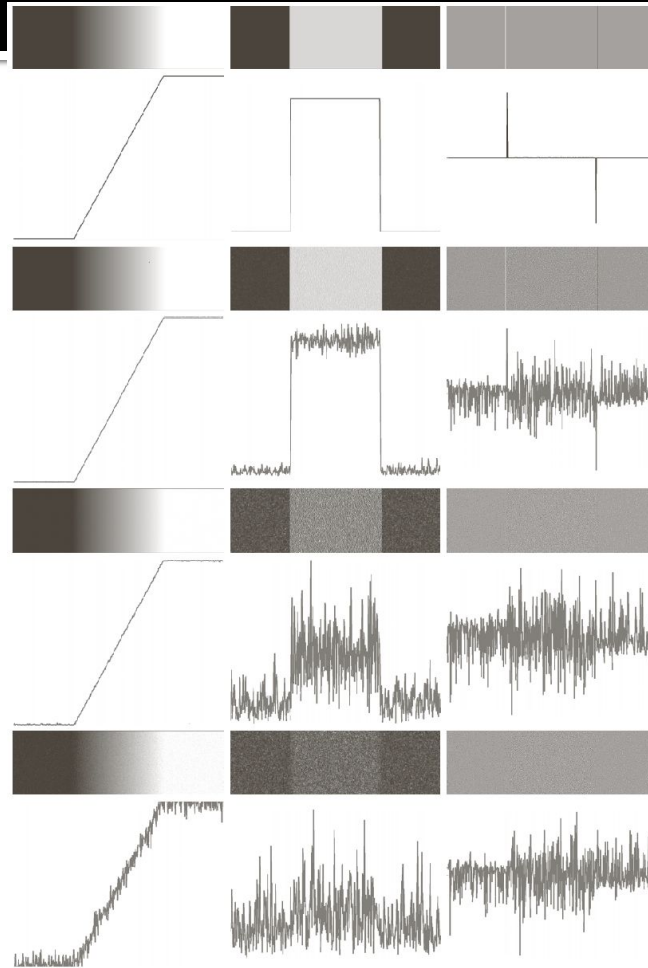
2nd derivative 0 0 -1 0 0 0 0 1 0 0 0 0 0 5 -5 0 0 0



From book  
by Gonzalez  
and Woods

# Edge detection under noise

First and second derivatives are very sensitive to noise!  
**Left to right in each row:** original image, first derivative, second derivative  
**In any column:** increase in noise level



**FIGURE 10.11** First column: Images and intensity profiles of a ramp edge corrupted by random Gaussian noise of zero mean and standard deviations of 0.0, 0.1, 1.0, and 10.0 intensity levels, respectively. Second column: First-derivative images and intensity profiles. Third column: Second-derivative images and intensity profiles.

# Steps in robust edge detection

- Image smoothing to reduce noise: because first and second derivatives are noisy
- Actual detection of edge points (may produce thick or disconnected edges)
- Pruning or linking of the edge points



# Edge detection: image gradient

- An image gradient is defined as follows:

$$\nabla f = \begin{pmatrix} \partial f / \partial x \\ \partial f / \partial y \end{pmatrix} = \begin{pmatrix} f_x \\ f_y \end{pmatrix}$$

$$\text{Gradient magnitude} = \|\nabla f\|_2 = \sqrt{f_x^2 + f_y^2}$$

$$\text{Gradient direction} = \theta = \tan^{-1}(f_y / f_x)$$

Gradient operator :

$$f_x(x, y) = f(x+1, y) - f(x, y)$$

$$f_y(x, y) = f(x, y+1) - f(x, y)$$

# Robust edge operators

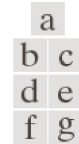
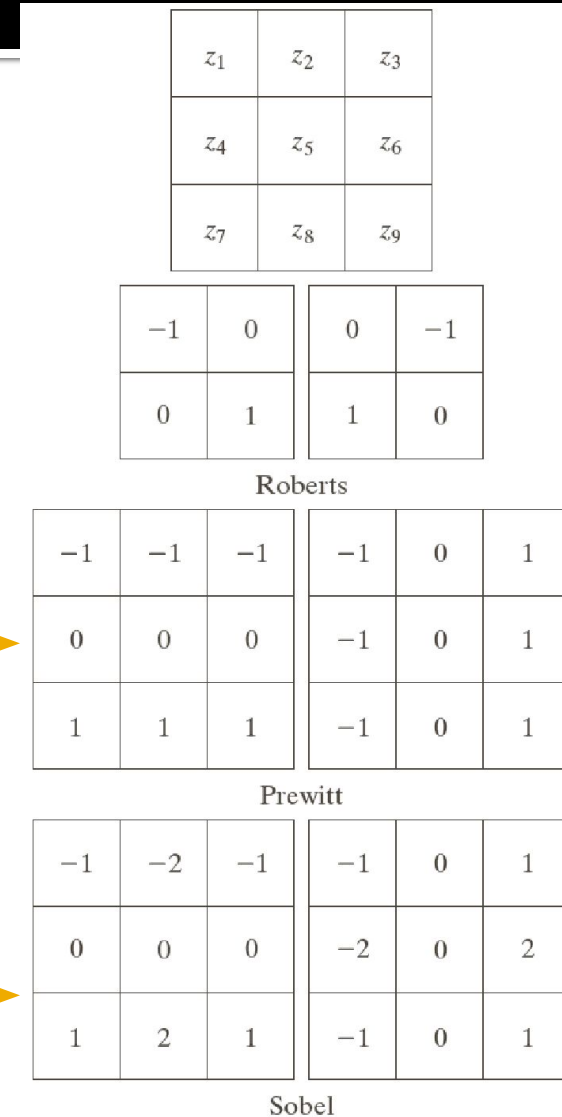
These operators: Prewitt and Sobel perform some smoothing prior to difference computation, and hence are more robust than the simple finite differences from the previous slide.

$$f_y = \frac{\partial f}{\partial y} = (z_7 + z_8 + z_9) - (z_1 + z_2 + z_3)$$

$$f_x = \frac{\partial f}{\partial x} = (z_3 + z_6 + z_9) - (z_1 + z_4 + z_7)$$

$$f_y = \frac{\partial f}{\partial y} = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

$$f_x = \frac{\partial f}{\partial x} = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$



**FIGURE 10.14**  
A  $3 \times 3$  region of an image (the  $z$ 's are intensity values) and various masks used to compute the gradient at the point labeled  $z_5$ .

# Prewitt and Sobel

-1	-1	-1	-1	0	1
0	0	0	-1	0	1
1	1	1	-1	0	1

Prewitt

-1	-2	-1	-1	0	1
0	0	0	-2	0	2
1	2	1	-1	0	1

Sobel

- Apply the differencing mask  $[-1 \ 0 \ 1]^T$  and then apply the smoothing mask  $[1 \ 1 \ 1]$  on the result. This is equivalent to applying the first Prewitt mask (i.e. for  $f_y$ ).
- For the Prewitt operator for  $f_x$ , apply  $[-1 \ 0 \ 1]$  and then  $[1 \ 1 \ 1]^T$ .
- Apply the differencing mask  $[-1 \ 0 \ 1]^T$  and then apply the smoothing mask  $[1 \ 2 \ 1]$  on the result. This is equivalent to applying the first Sobel mask (i.e. for  $f_y$ ).
- For the Sobel operator for  $f_x$ , apply  $[-1 \ 0 \ 1]$  and then  $[1 \ 2 \ 1]^T$ .

## Edges with Sobel Mask

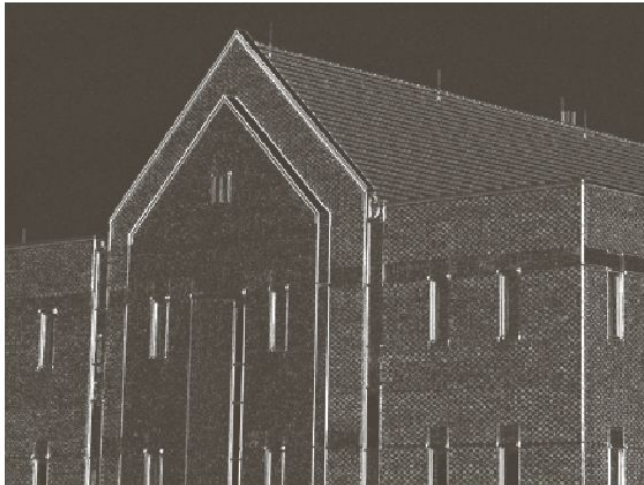
Original  
image



Derivative  
s in y  
direction:  
 $f_y$



Derivative  
s in x  
direction:  
 $f_x$



$|f_x| + |f_y|$

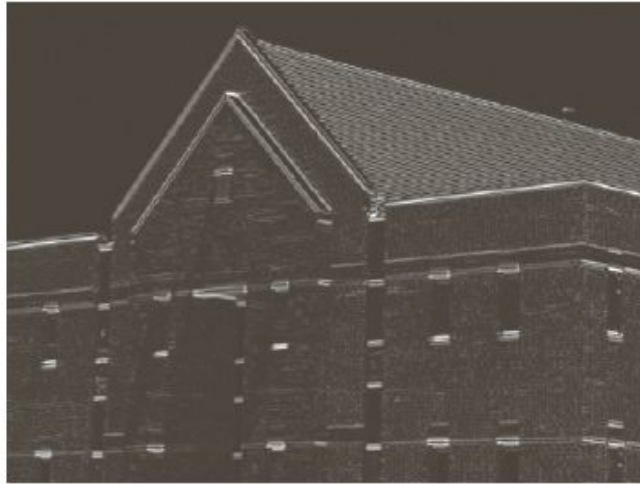


## Edges with 5 x 5 averaging followed by Sobel Mask

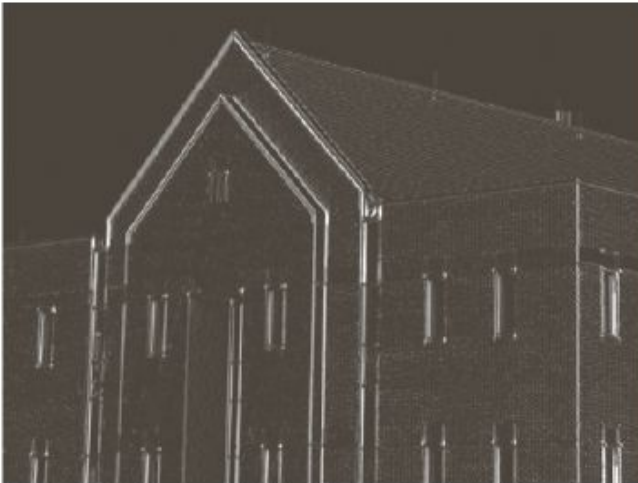
Original  
image



Derivative  
s in y  
direction:  
 $f_y$



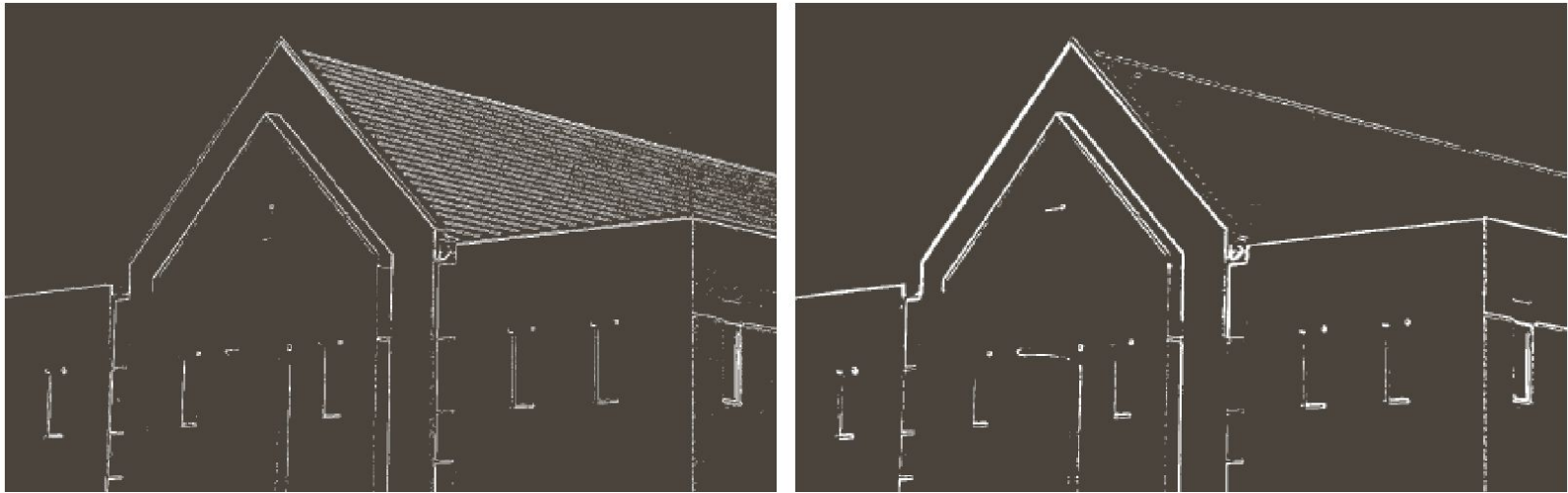
Derivative  
s in x  
direction:  
 $f_x$



$|f_x| + |f_y|$



Notice: finer details removed



a b

**FIGURE 10.20** (a) Thresholded version of the image in Fig. 10.16(d), with the threshold selected as 33% of the highest value in the image; this threshold was just high enough to eliminate most of the brick edges in the gradient image. (b) Thresholded version of the image in Fig. 10.18(d), obtained using a threshold equal to 33% of the highest value in that image.

Edge map after Thresholding the earlier map.



# Marr-Hildreth edge detector

- Principled approach to combined edge detection and image smoothing
- An edge operator should satisfy two criteria:
  - (1) Notion of **scale** in edge detection (operators of different sizes)
  - (2) Differential operator to compute first or second derivatives of the intensity
- Uses the **Laplacian of Gaussian** operator on an image satisfies both criteria

# Laplacian of Gaussian (LoG or Mexican hat)

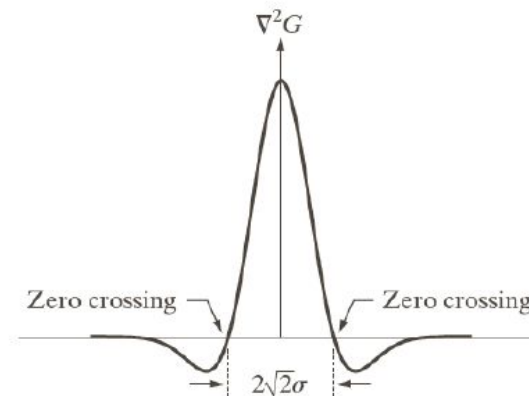
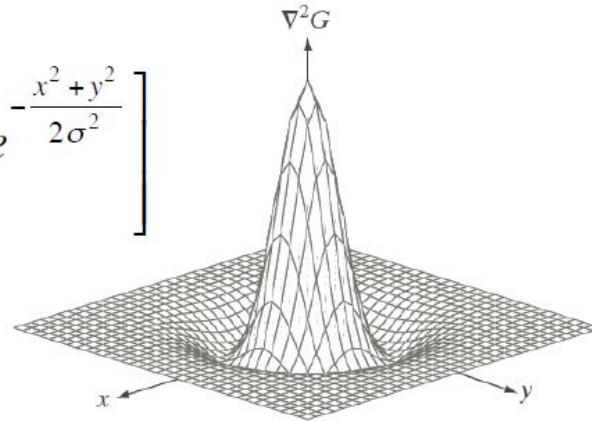
$$\nabla^2 G(x,y) = \frac{\partial^2 G(x,y)}{\partial x^2} + \frac{\partial^2 G(x,y)}{\partial y^2}$$

$$\frac{\partial^2 G(x,y)}{\partial x^2} = \frac{\partial^2}{\partial x^2} \left[ e^{-\frac{x^2+y^2}{2\sigma^2}} \right] = \frac{\partial}{\partial x} \left[ -\frac{x}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \right]$$

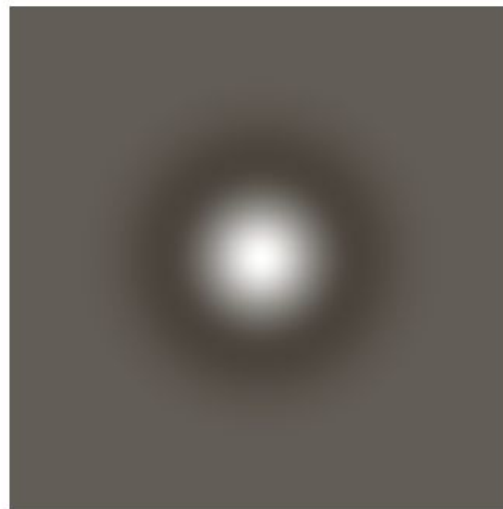
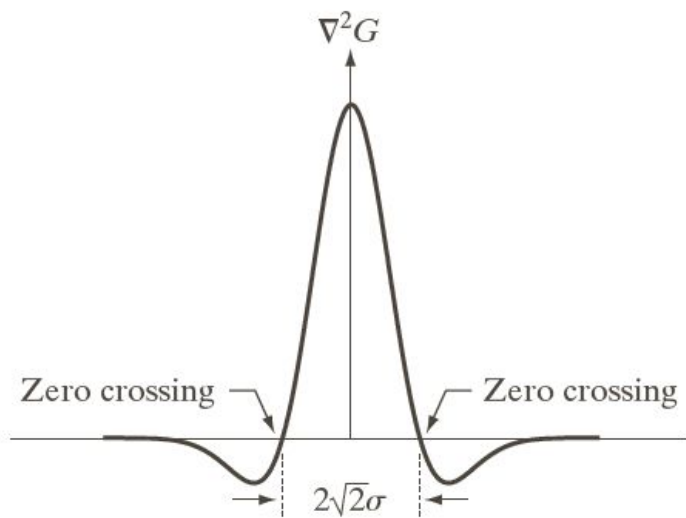
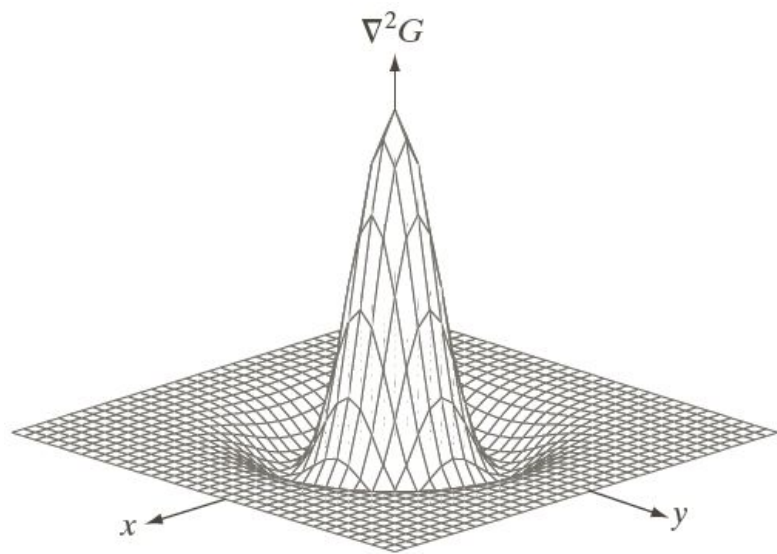
$$= \left( \frac{x^2}{\sigma^4} - \frac{1}{\sigma^2} \right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial^2 G(x,y)}{\partial y^2} = \left( \frac{y^2}{\sigma^4} - \frac{1}{\sigma^2} \right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\nabla^2 G(x,y) = \left( \frac{x^2}{\sigma^4} + \frac{y^2}{\sigma^4} - \frac{2}{\sigma^2} \right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$







0	0	-1	0	0
0	-1	-2	-1	0
-1	-2	16	-2	-1
0	-1	-2	-1	0
0	0	-1	0	0

a	b
c	d

**FIGURE 10.21**

(a) Three-dimensional plot of the *negative* of the LoG. (b) Negative of the LoG displayed as an image. (c) Cross section of (a) showing zero crossings. (d)  $5 \times 5$  mask approximation to the shape in (a). The negative of this mask would be used in practice.

# Why LoG for edge detection?

- Remember: we are interested in zero-crossings of the second derivative!
- We want the second derivative operator to be rotationally invariant. The Laplacian is the simplest (though not the only) such operator.
- The Gaussian is a low-pass filter, which blurs structures with intensity difference smaller than  $\sigma$ .
- Gaussian is smooth in spatial and frequency domains.

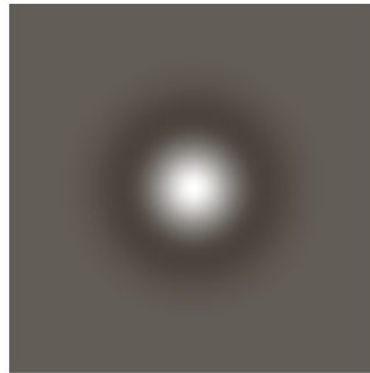
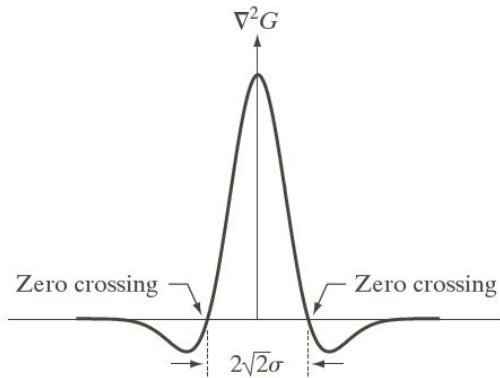
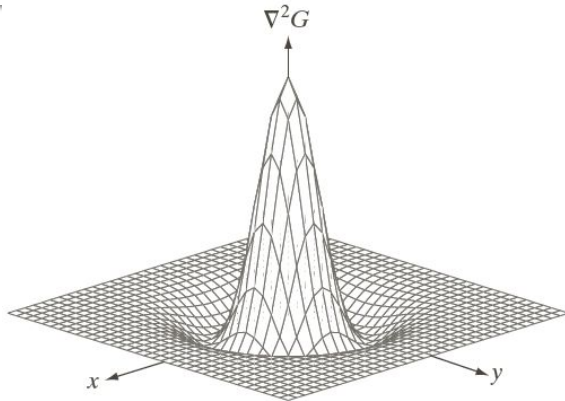
# Using LoG for edge detection

- (1) Convolve the image with the LoG operator, i.e.:

$$g(x, y) = [\nabla^2 G(x, y)] * f(x, y) = \nabla^2 [G(x, y) * f(x, y)]$$

- (2) Find the zero-crossings in the resultant image.

# Implementing the convolution



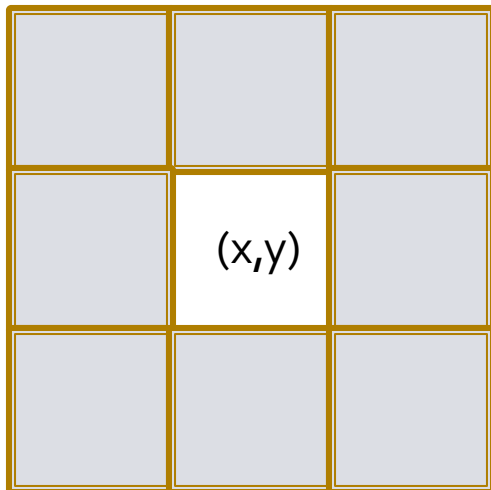
0	0	-1	0	0
0	-1	-2	-1	0
-1	-2	16	-2	-1
0	-1	-2	-1	0
0	0	-1	0	0

Use an  $n \times n$  mask, created by sampling the formula for LoG on a discrete Cartesian grid of size  $n \times n$ .

How to choose  $n$ ? Depends on  $\sigma$ :  $n$  should be  $\geq 6\sigma$ , because more than 99% of the volume underneath a 2D Gaussian lies between  $\mu - 3\sigma$  and  $\mu + 3\sigma$ .

# Finding zero-crossings

- Look at a  $3 \times 3$  window around a pixel.
- There exists a zero-crossing at that pixel if the signs of at least one pair of opposing neighbors (top & bottom, left & right, top-left & right-bottom, top-right & left-bottom) differ and the difference in their values must exceed a threshold  $p$ .



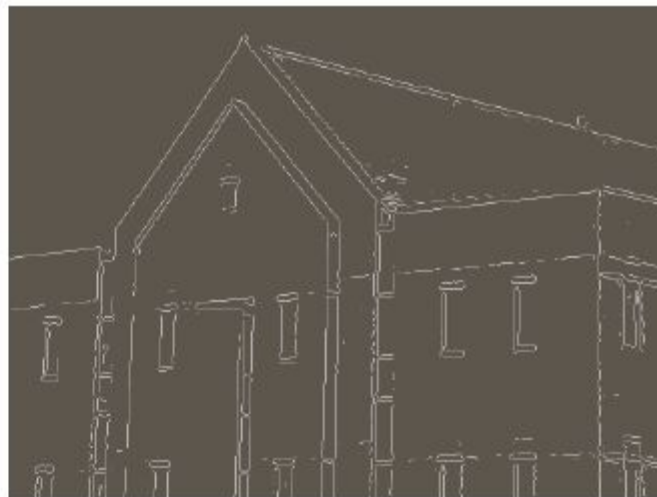
$\sigma = 4,$   
 $n = 25$



Convolution  
with LoG



Zero-crossings



Zero-crossings  
+ thresholding

# Some properties of the Marr-Hildreth Edge Detector

- Detected edges are 1-pixel in thickness
- Can be used for different sigma values, keeping only those edge points that are common to multiple scales

# Canny Edge Detector: using 1<sup>st</sup> derivatives

- Three aims/objectives:
  - (a) Low error (no spurious responses due to noise)
  - (b) Well-localized edge points
  - (c) Single pixel wide edges

Canny's detector was formulated by analyzing step edges in 1D under zero-mean i.i.d. Gaussian noise, and expressing the preceding three criteria mathematically and deriving an (approximate) operator that obeys all criteria.



# Canny's Edge Detector

- In 1D, the approximate operator was the first derivative of the Gaussian:  $\frac{d}{dx} e^{-\frac{x^2}{2\sigma^2}} = \frac{-x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}$
- In 2D, the approximate operator was the first derivative of the Gaussian in the direction of the local image gradient (i.e. perpendicular to the edge).
- But the true gradient is not known, so we first smooth the image with a 2D Gaussian and then use its gradient magnitude and direction.

# Canny's Edge Detector

- Gradient magnitude and direction:

$$g(x, y) = G(x, y) * f(x, y)$$

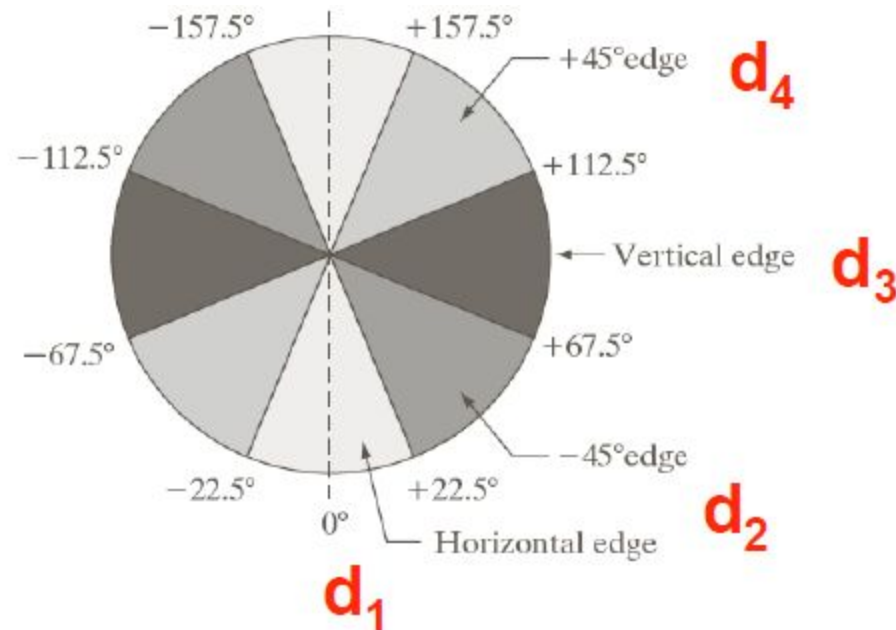
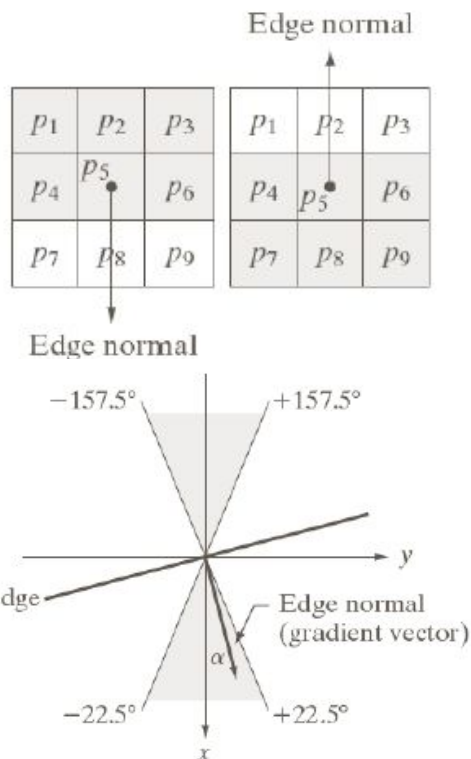
$$M(x, y) = |\nabla g(x, y)| = \sqrt{g_x^2(x, y) + g_y^2(x, y)}$$

$$\theta(x, y) = \tan^{-1} \left( \frac{g_y(x, y)}{g_x(x, y)} \right)$$

- $M(x, y)$  contains wide ridges along the true edges of the image.
- Though the maximum values are along the edge, those values don't immediately fall off to 0 on either side of the edge.
- Recall ramp edges.

# Canny edge detector

- This requires us to perform a **thinning** operation – called as **non-maximal suppression** (i.e. we set to zero those values of  $M(x,y)$  which are likely to NOT be local maxima).

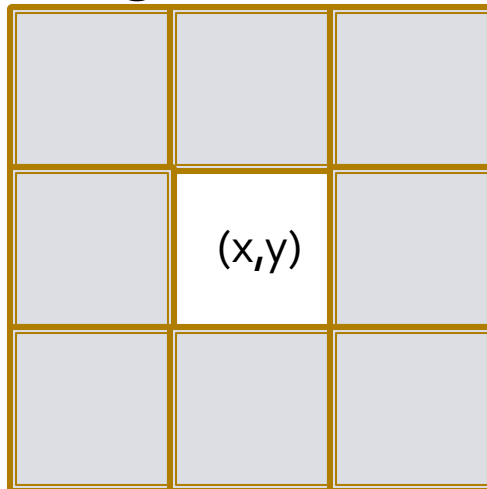


# Canny edge detector

- Consider the four quantized directions shown on the previous slide. Pick the direction  $d_{k'}$  which is closest to  $\Theta(x,y)$ .
- If  $M(x,y)$  is less than either of its two neighbors along the direction  $d_{k'}$ , then set it to 0 (i.e. suppress the non-maximum), otherwise we leave it as is.
- After this processing,  $M(x,y)$  is the non-maximal suppressed image.

# Canny Edge Detector

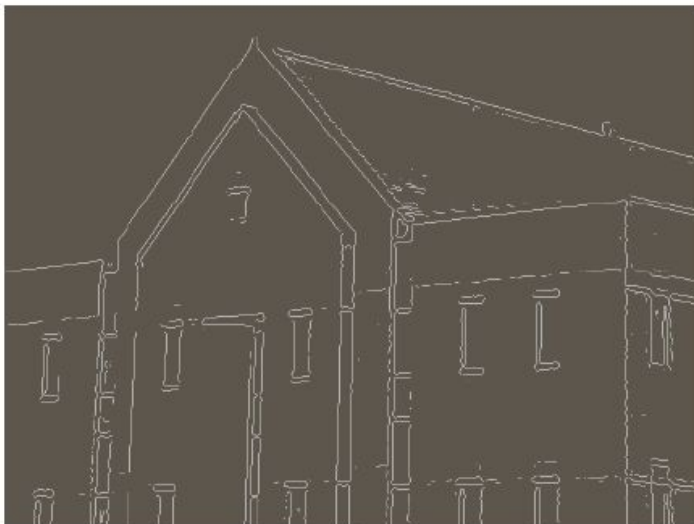
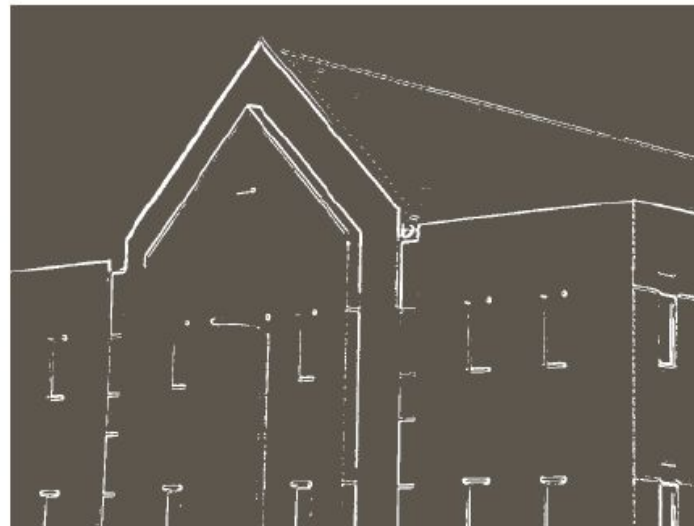
- The (thinned) edge map so far may still contain some false positives. This is taken care of (to some extent) by **hysteresis thresholding**.
- This uses a low-threshold TL and a high threshold TH.
- We keep only those pixels for which
  - $M(x,y) > TH$ , OR
  - $M(x,y) > TL$  AND  $(x,y)$  is a neighbor of at least one pixel with  $M(x,y) > TH$ .



The 8 neighbors  
of pixel  $(x,y)$

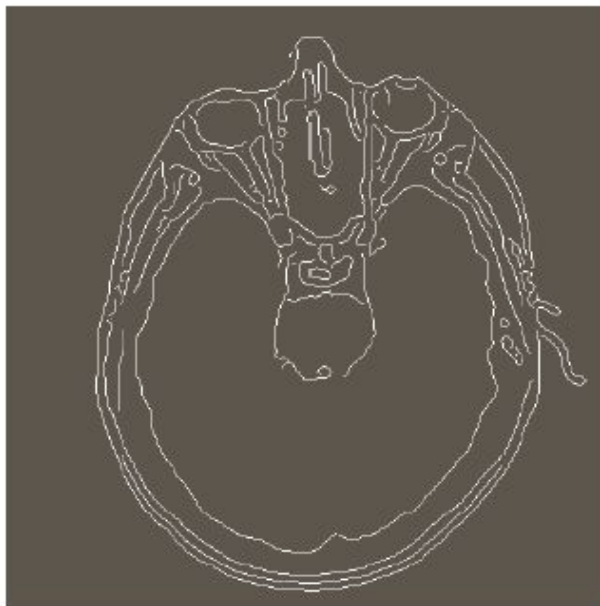
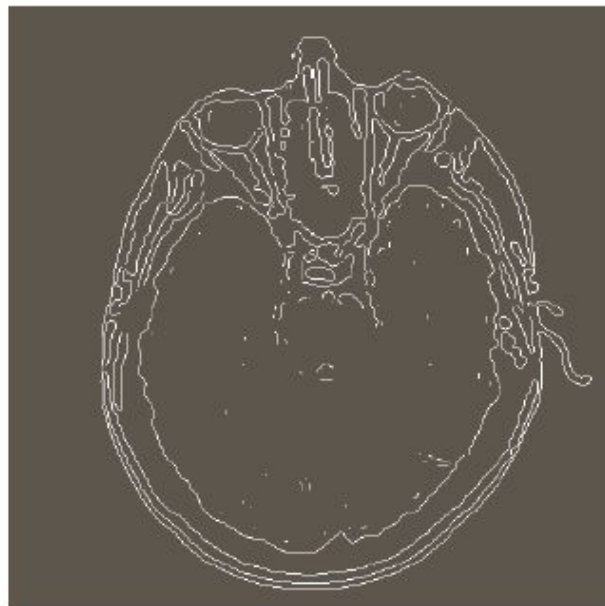
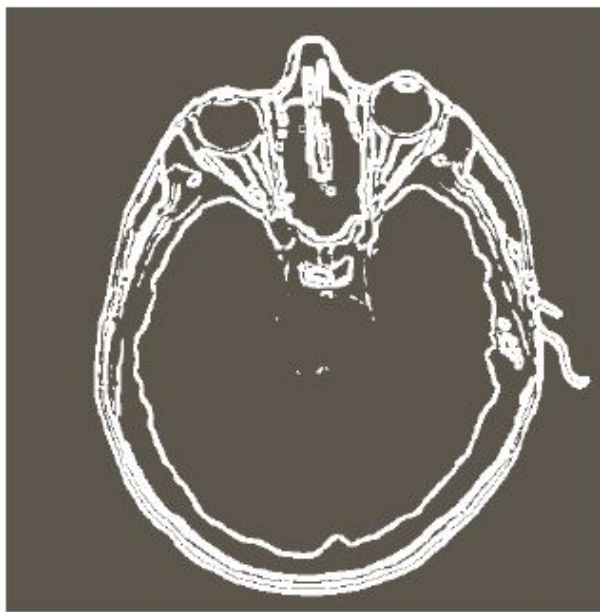
# Summary: Canny Edge detector

- (1) Convolve the image with a Gaussian Filter. Compute the gradient magnitudes and orientations.
- (2) Perform non-maximal suppression.
- (3) Perform hysteresis thresholding.



a	b
c	d

**FIGURE 10.25**  
 (a) Original image of size  $834 \times 1114$  pixels, with intensity values scaled to the range  $[0, 1]$ .  
 (b) Thresholded gradient of smoothed image.  
 (c) Image obtained using the Marr-Hildreth algorithm.  
 (d) Image obtained using the Canny algorithm. Note the significant improvement of the Canny image compared to the other two.



a	b
c	d

**FIGURE 10.26**

(a) Original head CT image of size  $512 \times 512$  pixels, with intensity values scaled to the range  $[0, 1]$ .

(b) Thresholded gradient of smoothed image.

(c) Image obtained using the Marr-Hildreth algorithm.

(d) Image obtained using the Canny algorithm.

(Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

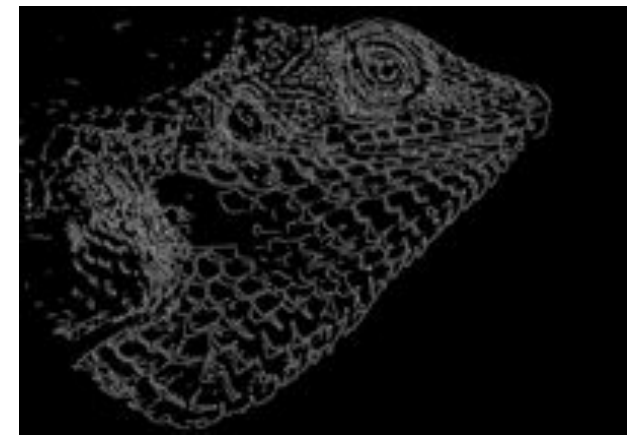
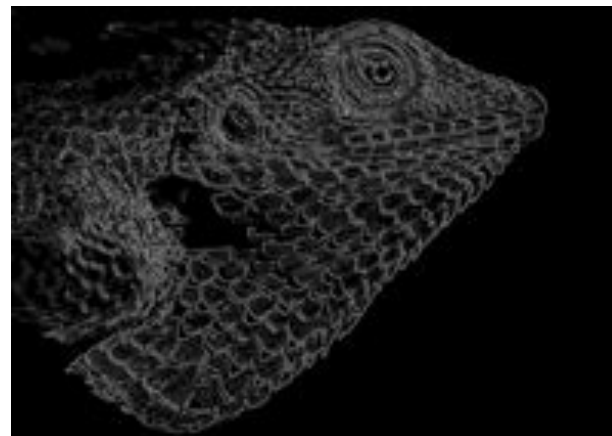
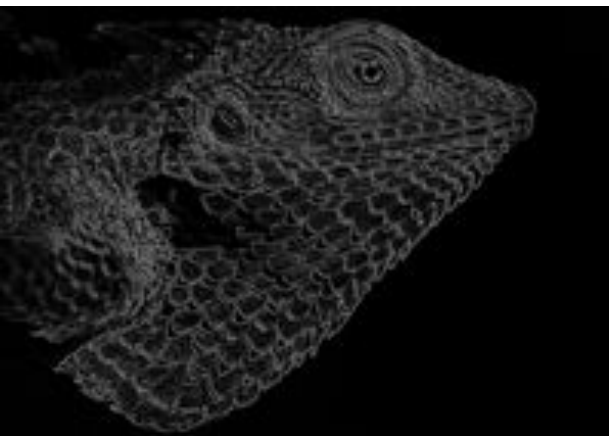




Gaussian filtered  
(sigma = 1.4, 5 × 5  
mask)

Intensity gradients  
superimposed

[https://en.wikipedia.org/wiki/Canny\\_edge\\_detector](https://en.wikipedia.org/wiki/Canny_edge_detector)



Non-maximum  
suppression

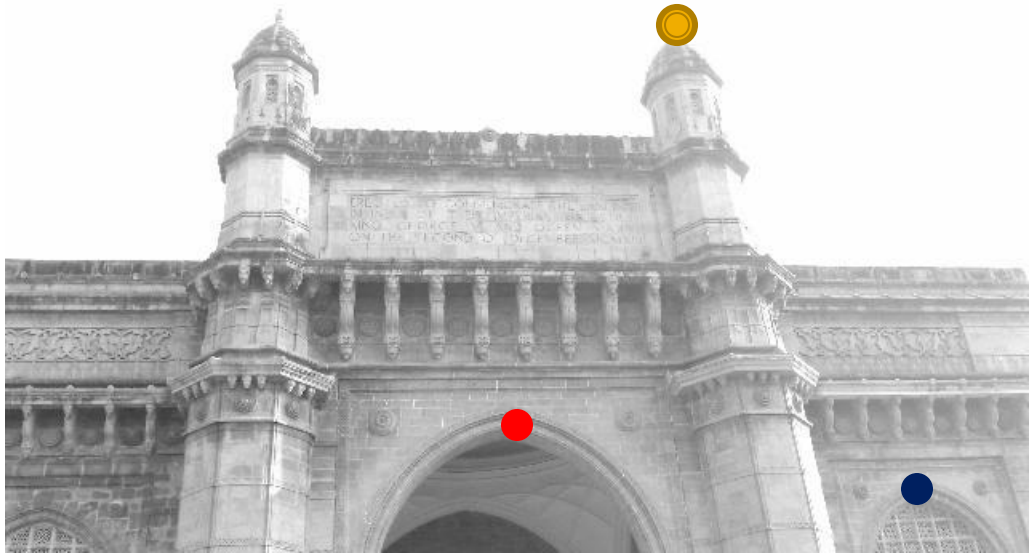
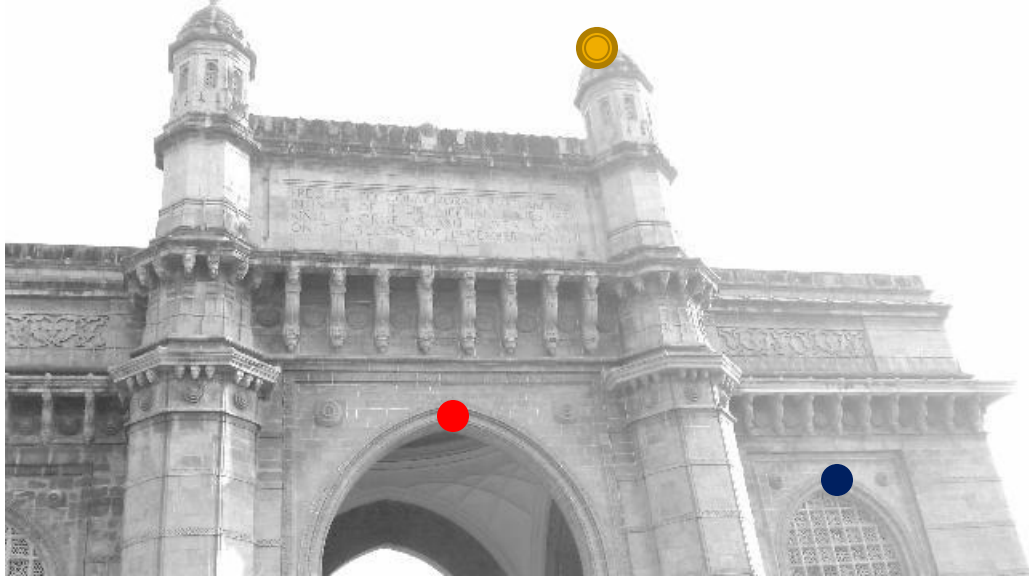
Double thresholding  
(TL = 0.1, TH = 0.3)

After edge-linking  
(hysteresis)

# Corner Detection

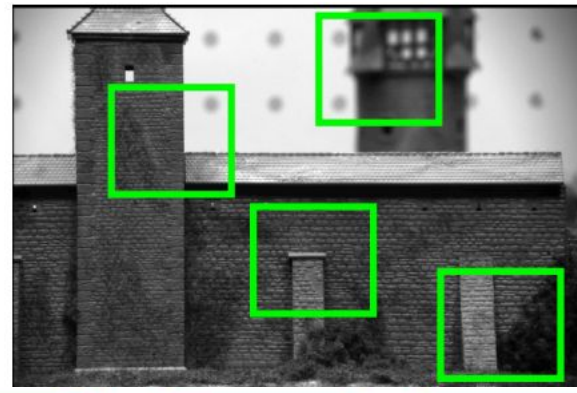
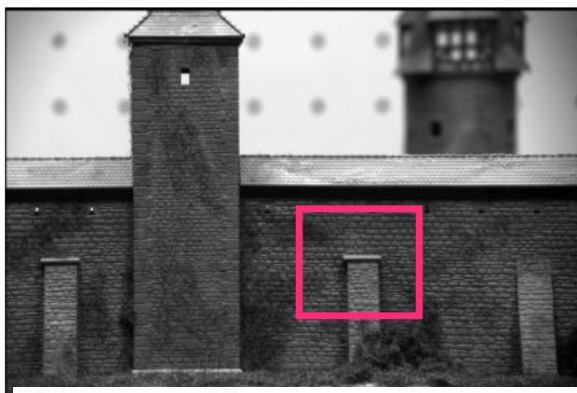
# Importance of corners

- Corners are considered salient feature points.
- Useful in many tasks in computer vision and image processing
- Example: corresponding control points for image alignment



# Importance of corners

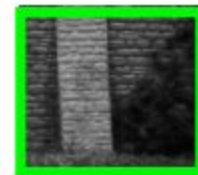
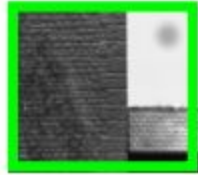
- In some applications, we need to find matching patches.
- But some patch matches are easier to find than others.
  - Each patch is assumed to be centered at a certain point (say  $(x,y)$ )
  - The patch encodes information about the local geometry of the image intensities about that point.
- See next slide for examples.



Good patch  
for matching



?  
=



Bad patch for  
matching



?  
=



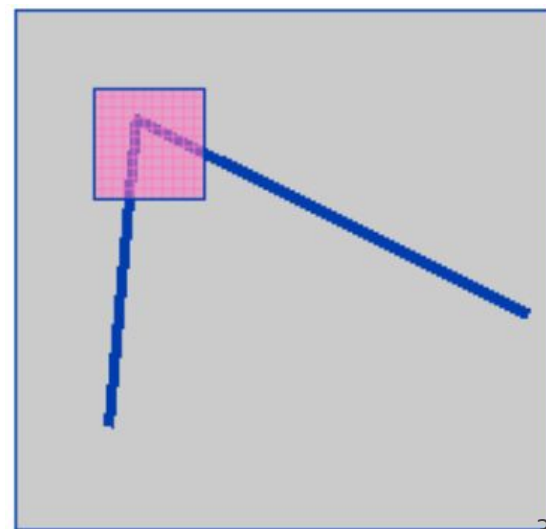
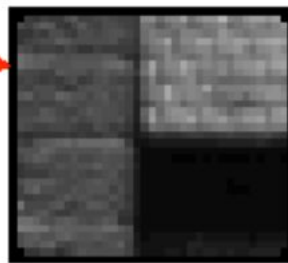
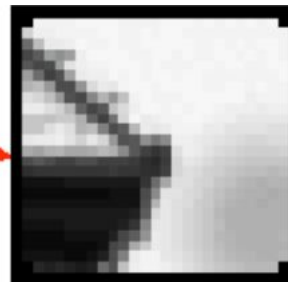
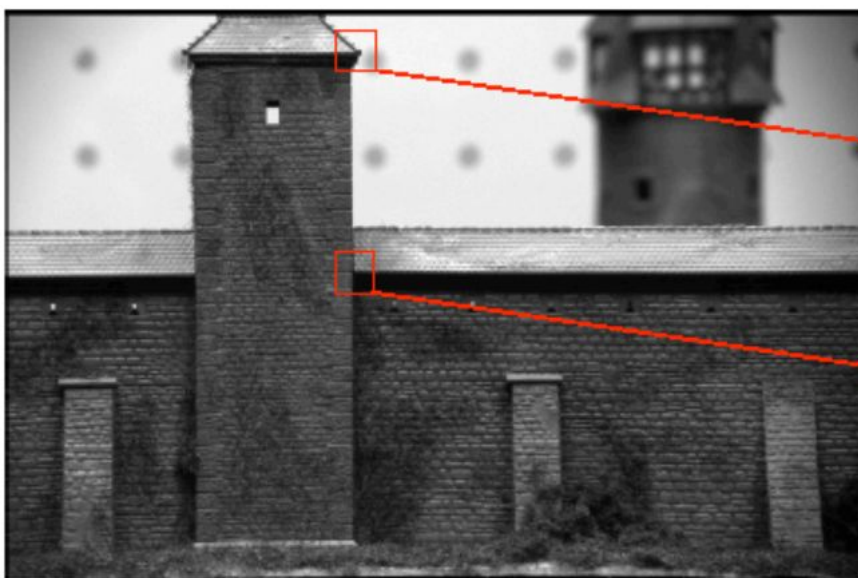
<http://www.cse.psu.edu/~rtc12/CSE486/lecture06.pdf>

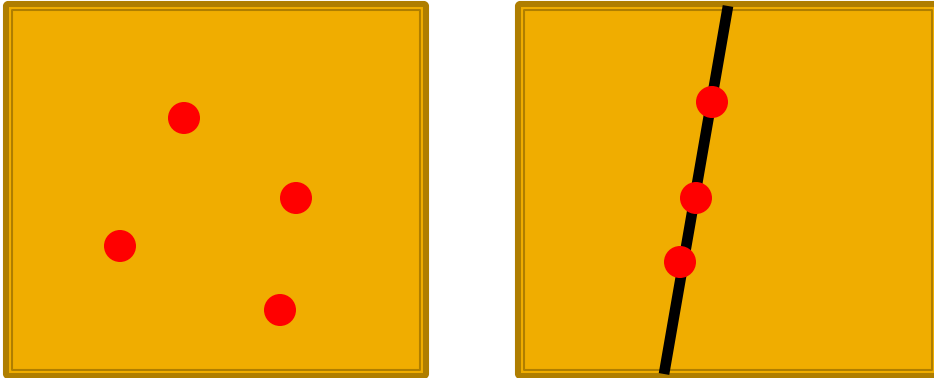


# Importance of corners

- Good patches should contain corners – large intensity variations in all directions.
- Shifting the window in any direction yields large appearance change.

<http://www.cse.psu.edu/~rtc12/CSE486/lecture06.pdf>





Ambiguity in point matching in flat intensity regions or along edges



# Principle behind Harris corner detector

- Consider two patches – one each centered at  $(x,y)$  and  $(x+u,y+v)$  in two images (**patch** domain:  $\Omega$ , typically rectangular, like say  $7 \times 7$  or  $5 \times 5$ ) with  $u,v$  being small.
- The image have same intensity at physically corresponding locations.
- The sum of squared difference (SSD) between their intensities is given as:

$$SSD = \sum_{(x,y) \in \Omega} (I(x,y) - I(x+u, y+v))^2$$

By first order Taylor series,

$$I(x+u, y+v) \approx I(x,y) + uI_x(x,y) + vI_y(x,y)$$

$$SSD = \sum_{(x,y) \in \Omega} (I(x,y) - I(x+u, y+v))^2$$

By first order Taylor series,

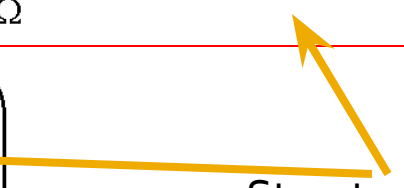
$$I(x+u, y+v) \approx I(x,y) + uI_x(x,y) + vI_y(x,y)$$

$$\therefore SSD = \sum_{(x,y) \in \Omega} (uI_x(x,y) + vI_y(x,y))^2$$

$$= \begin{pmatrix} u & v \end{pmatrix} \begin{pmatrix} \sum_{(x,y) \in \Omega} I_x^2(x,y) & \sum_{(x,y) \in \Omega} I_x(x,y)I_y(x,y) \\ \sum_{(x,y) \in \Omega} I_x(x,y)I_y(x,y) & \sum_{(x,y) \in \Omega} I_y^2(x,y) \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$$= \begin{pmatrix} u & v \end{pmatrix} A \begin{pmatrix} u \\ v \end{pmatrix}$$

Structure tensor matrix (size 2 x 2)



# Principle behind Harris corner detector

- The local **A** matrix on the previous slide is called the structure tensor.
- **A** carries information about local image geometry.

$$A = \lambda_1 u_1 u_1^t + \lambda_2 u_2 u_2^t; \text{eigenvals} : \lambda_1, \lambda_2; \text{eigenvecs} : u_1, u_2$$

- It is always positive semi-definite, i.e. its two eigenvalues are always non-negative.
- In locally flat regions, **A** will be close to a zero matrix and hence both its eigenvalues will be close to 0.

# Proof that the structure tensor matrix is positive semi-definite: Method 1

$$\begin{aligned}
 A &= \begin{pmatrix} \sum_{(x,y) \in \Omega} I_x^2(x,y) & \sum_{(x,y) \in \Omega} I_x(x,y)I_y(x,y) \\ \sum_{(x,y) \in \Omega} I_x(x,y)I_y(x,y) & \sum_{(x,y) \in \Omega} I_y^2(x,y) \end{pmatrix} \\
 &= \begin{pmatrix} I_x(x_1, y_1) & I_x(x_2, y_2) & \cdot & \cdot & I_x(x_N, y_N) \\ I_y(x_1, y_1) & I_y(x_2, y_2) & \cdot & \cdot & I_y(x_N, y_N) \end{pmatrix} \begin{pmatrix} I_x(x_1, y_1) & I_y(x_1, y_1) \\ I_x(x_2, y_2) & I_y(x_2, y_2) \\ \cdot & \cdot \\ \cdot & \cdot \\ I_x(x_N, y_N) & I_y(x_N, y_N) \end{pmatrix} \\
 &= ZZ^T
 \end{aligned}$$

Any matrix that can be written in the form  $A = ZZ^T$  is always positive semi-definite (quoting a result from linear algebra)

# Proof that the structure tensor matrix is positive semi-definite: Method 2

$$A = \begin{pmatrix} \sum_{(x,y) \in \Omega} I_x^2(x,y) & \sum_{(x,y) \in \Omega} I_x(x,y)I_y(x,y) \\ \sum_{(x,y) \in \Omega} I_x(x,y)I_y(x,y) & \sum_{(x,y) \in \Omega} I_y^2(x,y) \end{pmatrix}$$

$$\text{trace}(A) = \lambda_1 + \lambda_2 = \sum_{(x,y) \in \Omega} I_x^2(x,y) + \sum_{(x,y) \in \Omega} I_y^2(x,y)$$

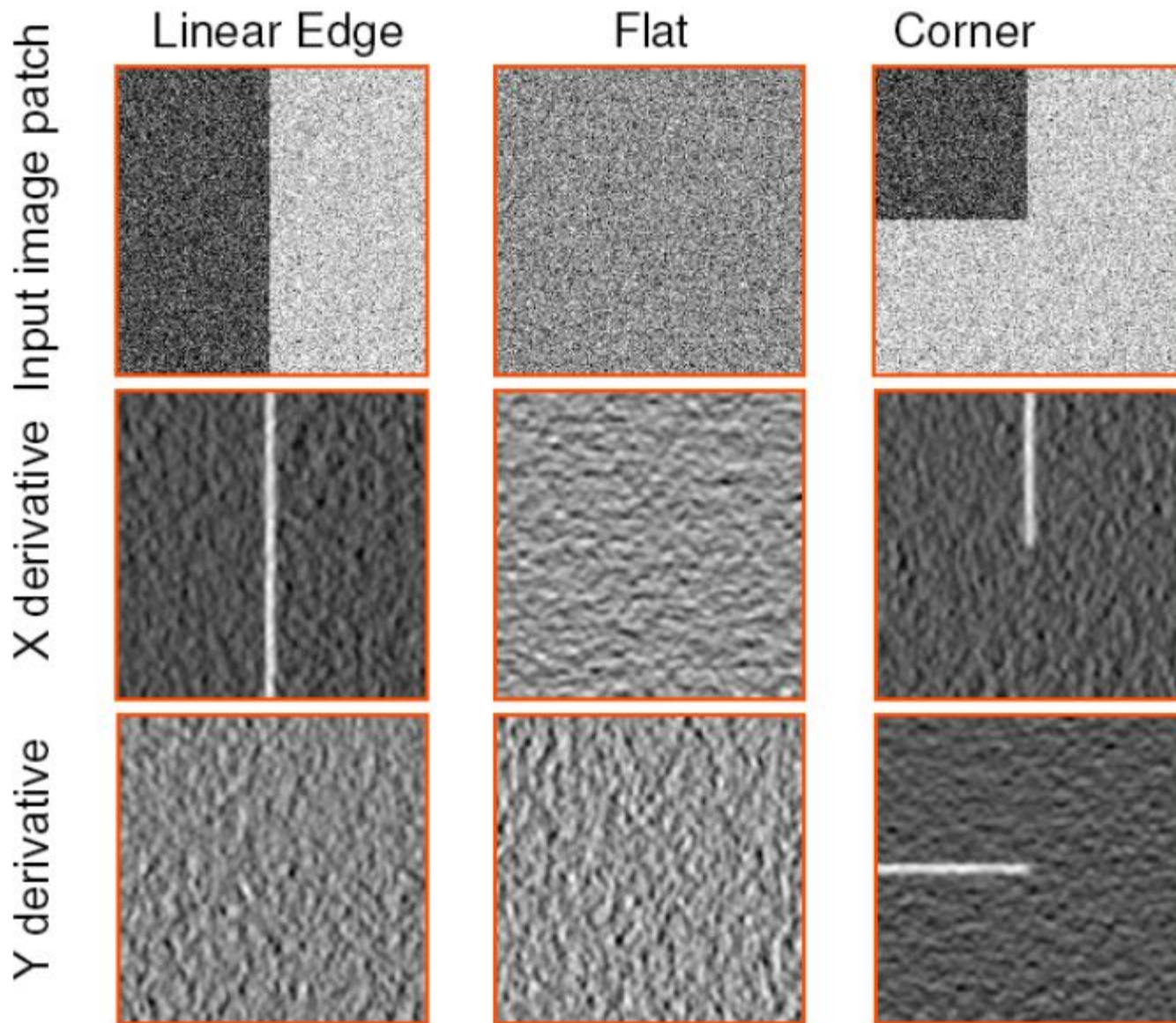
$\text{trace}(A)$  is clearly non-negative as  $I_x^2$  and  $I_y^2$  are both non-negative

$$|A| = \sum_{(x,y) \in \Omega} I_x^2(x,y) \sum_{(x,y) \in \Omega} I_y^2(x,y) - \left( \sum_{(x,y) \in \Omega} I_x(x,y)I_y(x,y) \right)^2$$

$|A| \geq 0$  by Cauchy-Schwarz inequality (the first term is  $\|a\|^2 \|b\|^2$ , and the second term is  $(a \bullet b)^2$  for  $N$  element vectors  $a$  and  $b$ , respectively containing the  $I_x$  and  $I_y$  values)

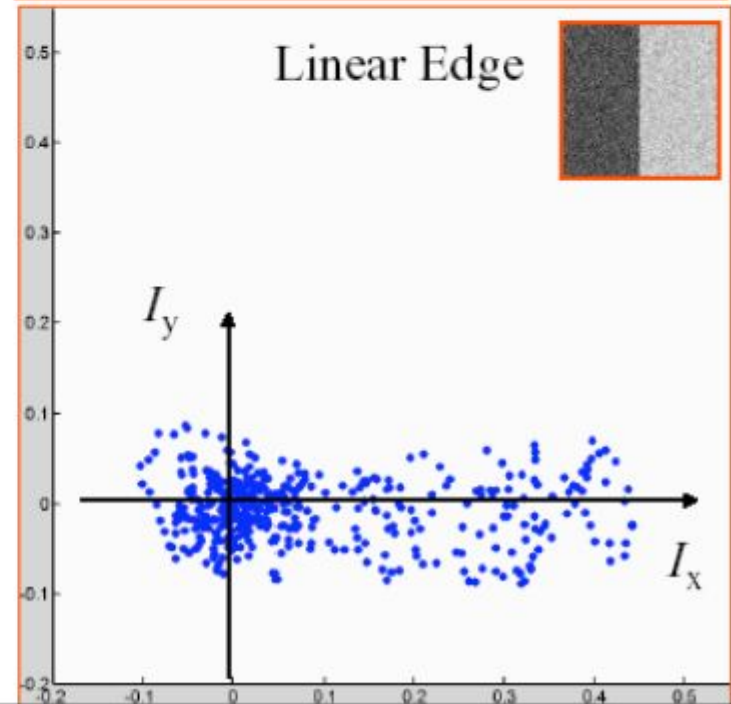
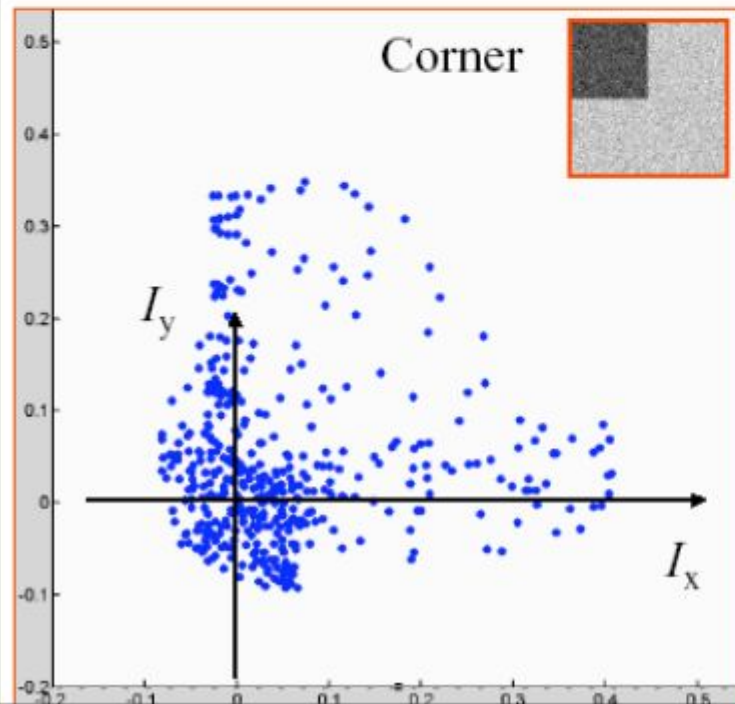
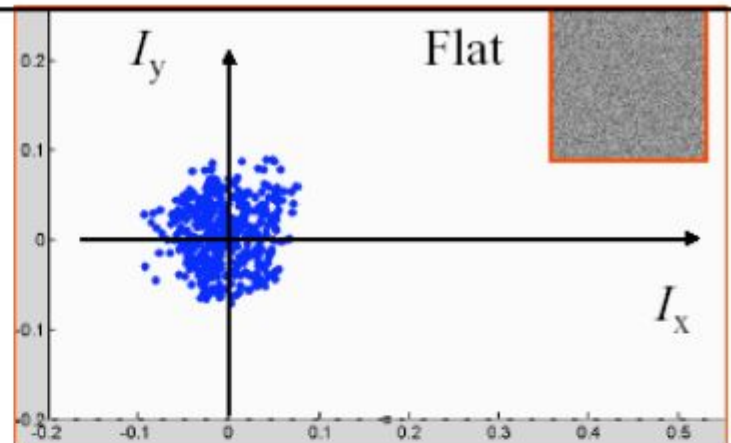
# Principle behind Harris corner detector

- At a point lying on an edge, only one of the eigenvalues is large (corresponding to the eigenvector that points across the edge) and the other is close to 0 (corresponding to the eigenvector that points along the edge)
- At a corner point, both eigenvalues will be large.
- We would like the SSD to be large for **all** non-zero shifts  $(u,v)$  so that we can allow for maximum discriminability and easier point matching.



<http://www.cse.psu.edu/~rtc12/CSE486/lecture06.pdf>

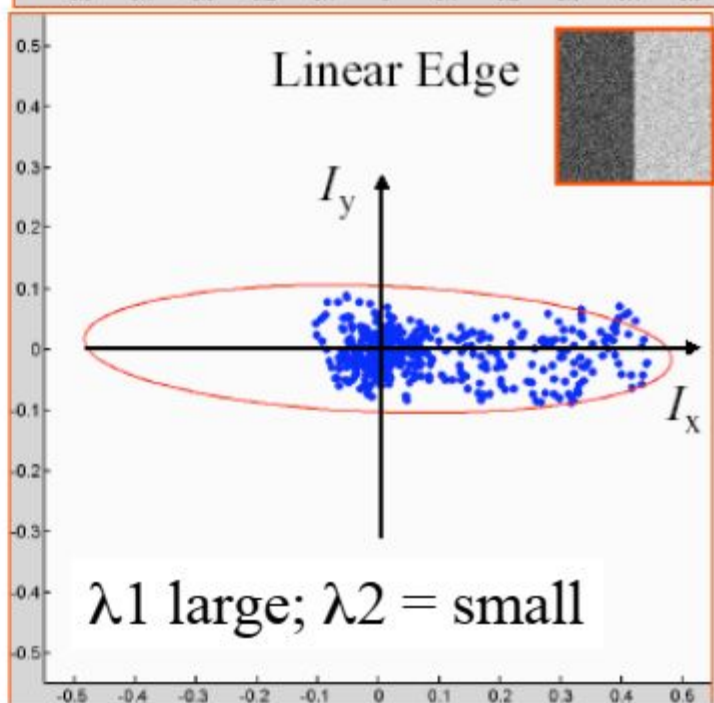
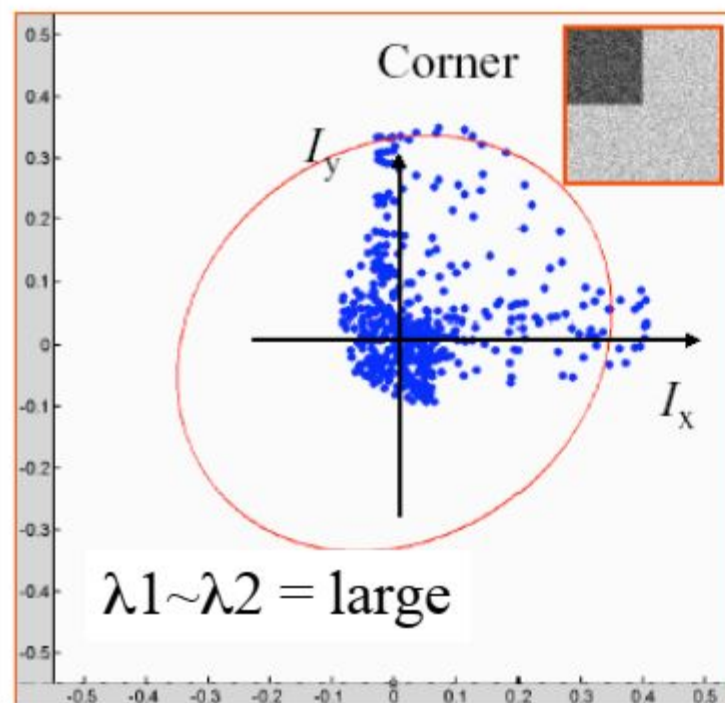
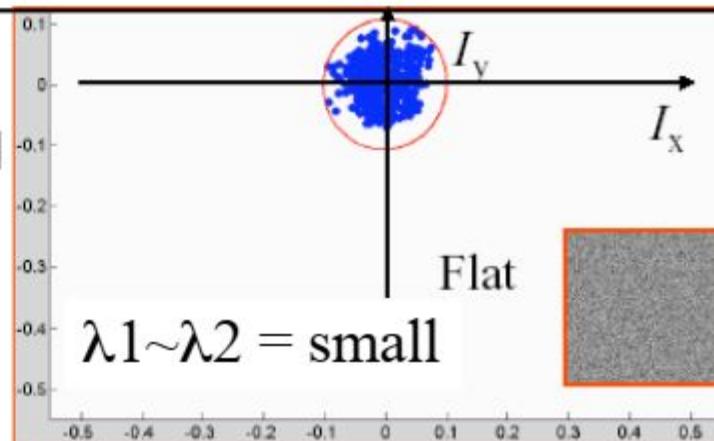
The distribution of the  $x$  and  $y$  derivatives is very different for all three types of patches



<http://www.cse.psu.edu/~rtc12/CSE486/lecture06.pdf>



The distribution of  $x$  and  $y$  derivatives can be characterized by the shape and size of the principal component ellipse



# “Corner”-ness measure

- We would like both eigenvalues of this matrix to be large.
- Corner response or “corner”ness measure:  
 $R_H = \det(A) - k (\text{trace}(A))^2$ ,  $k$  between 0.04 to 0.06
- Does not require explicit eigenvalue/eigenvector computation:

$$|A| = \lambda_1 \lambda_2 = A_{11}A_{22} - A_{12}A_{21};$$

$$\text{trace}(A) = \lambda_1 + \lambda_2 = A_{11} + A_{22}$$

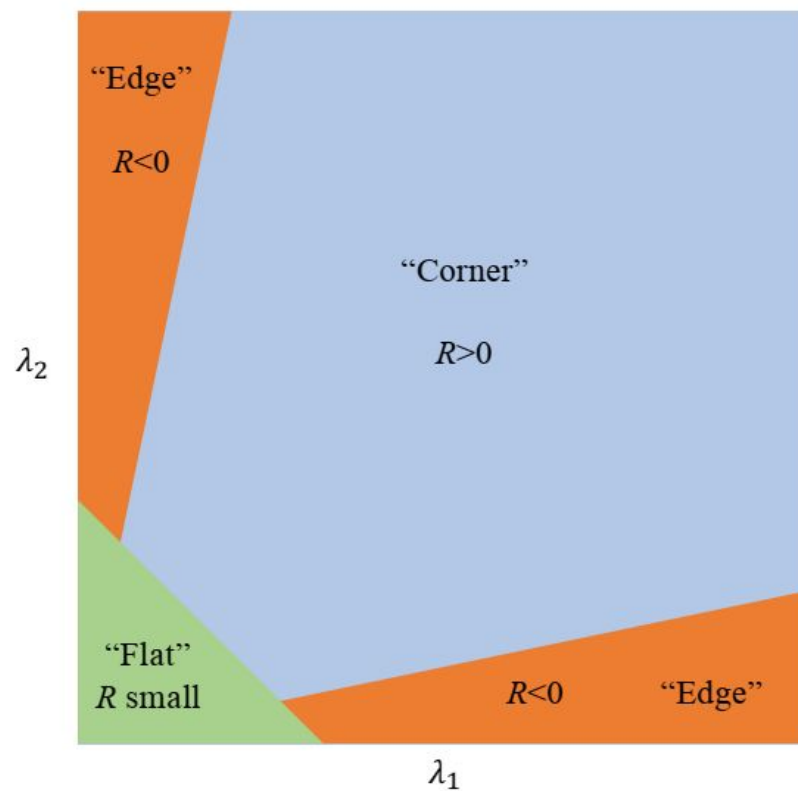
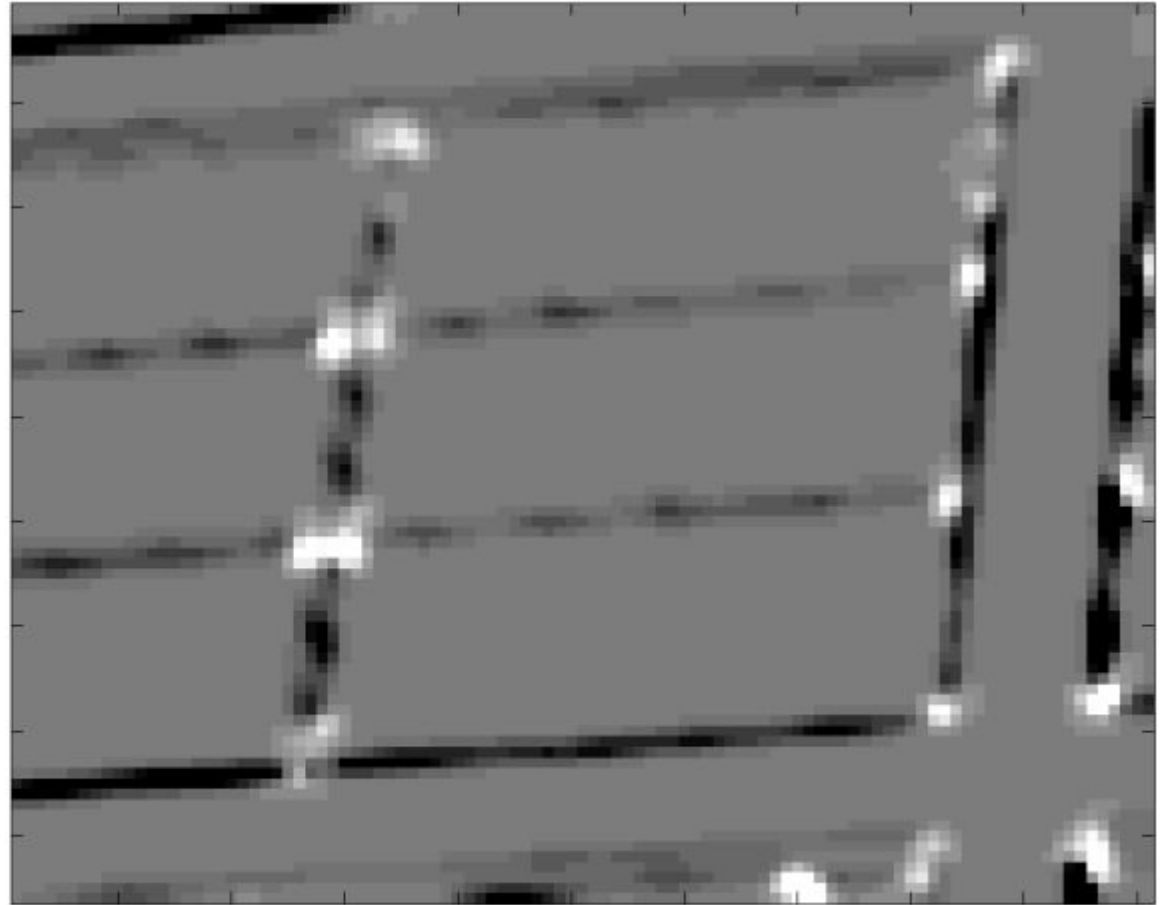
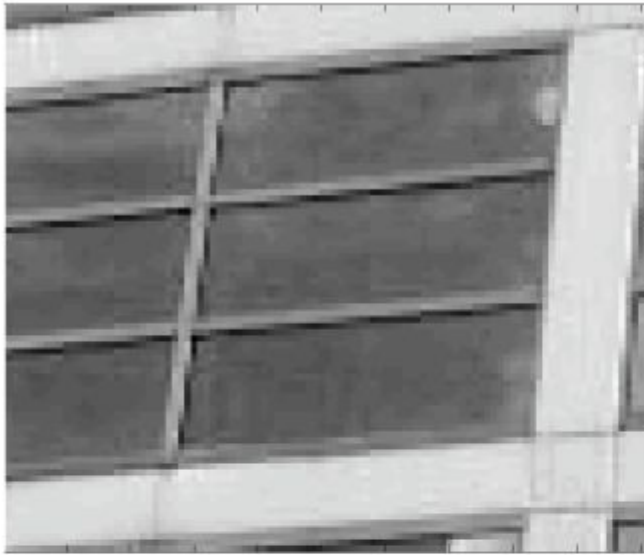


Figure 5: Harris measure. If both eigenvalues are large, then  $R$  is large and positive, providing a clue to detect a corner. If both eigenvalues are small, then  $R$  is also small and positive, which means that the point is probably part of a homogeneous region. Finally, if one of the eigenvalues is much larger than the other,  $R$  becomes negative, and the point belongs to an edge.

[http://www.ipol.im/pub/art/2018/229/article\\_lr.pdf](http://www.ipol.im/pub/art/2018/229/article_lr.pdf)



Harris R score.

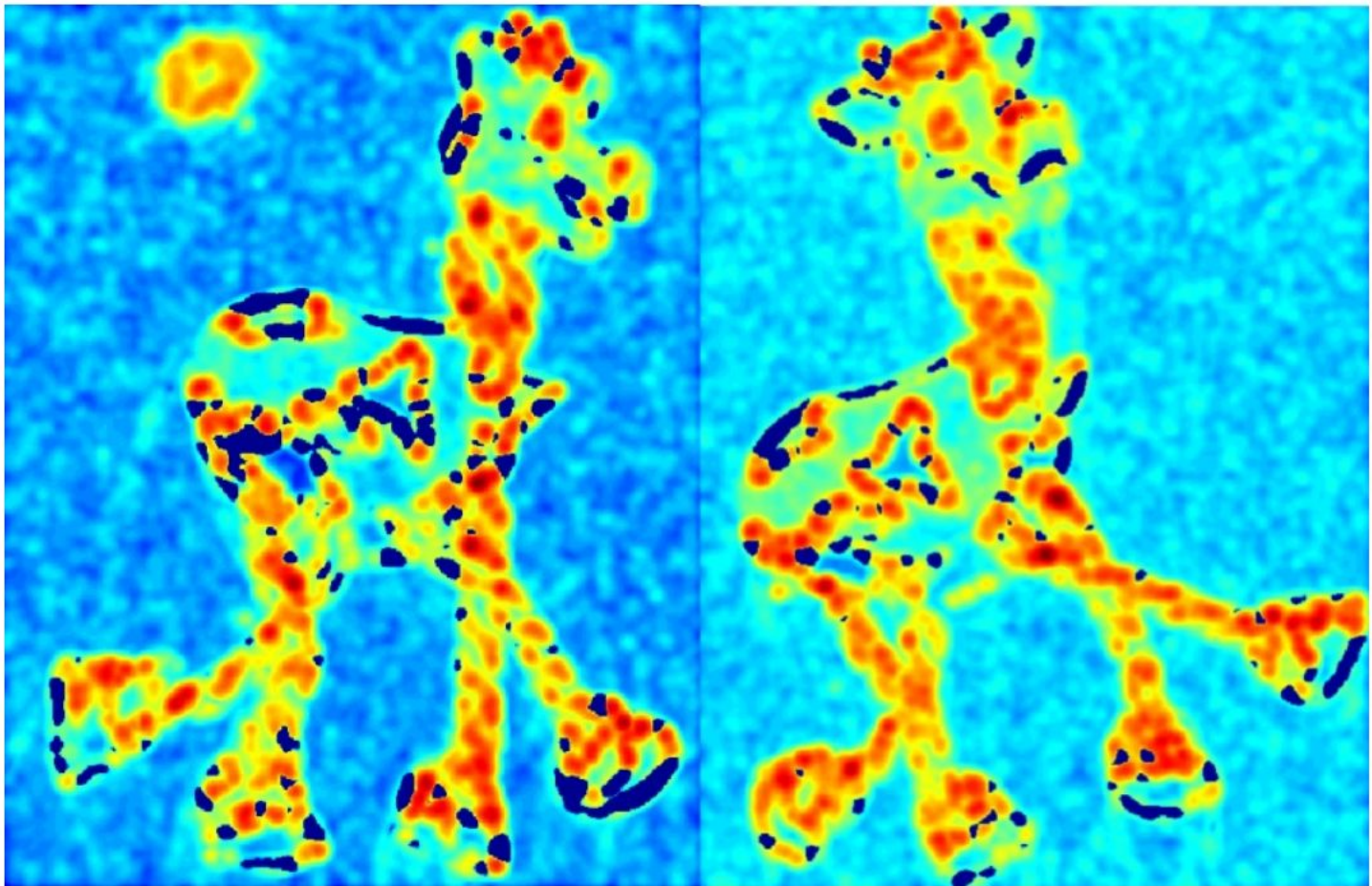
$I_x$ ,  $I_y$  computed using Sobel operator  
Windowing function  $w$  = Gaussian,  $\sigma=1$

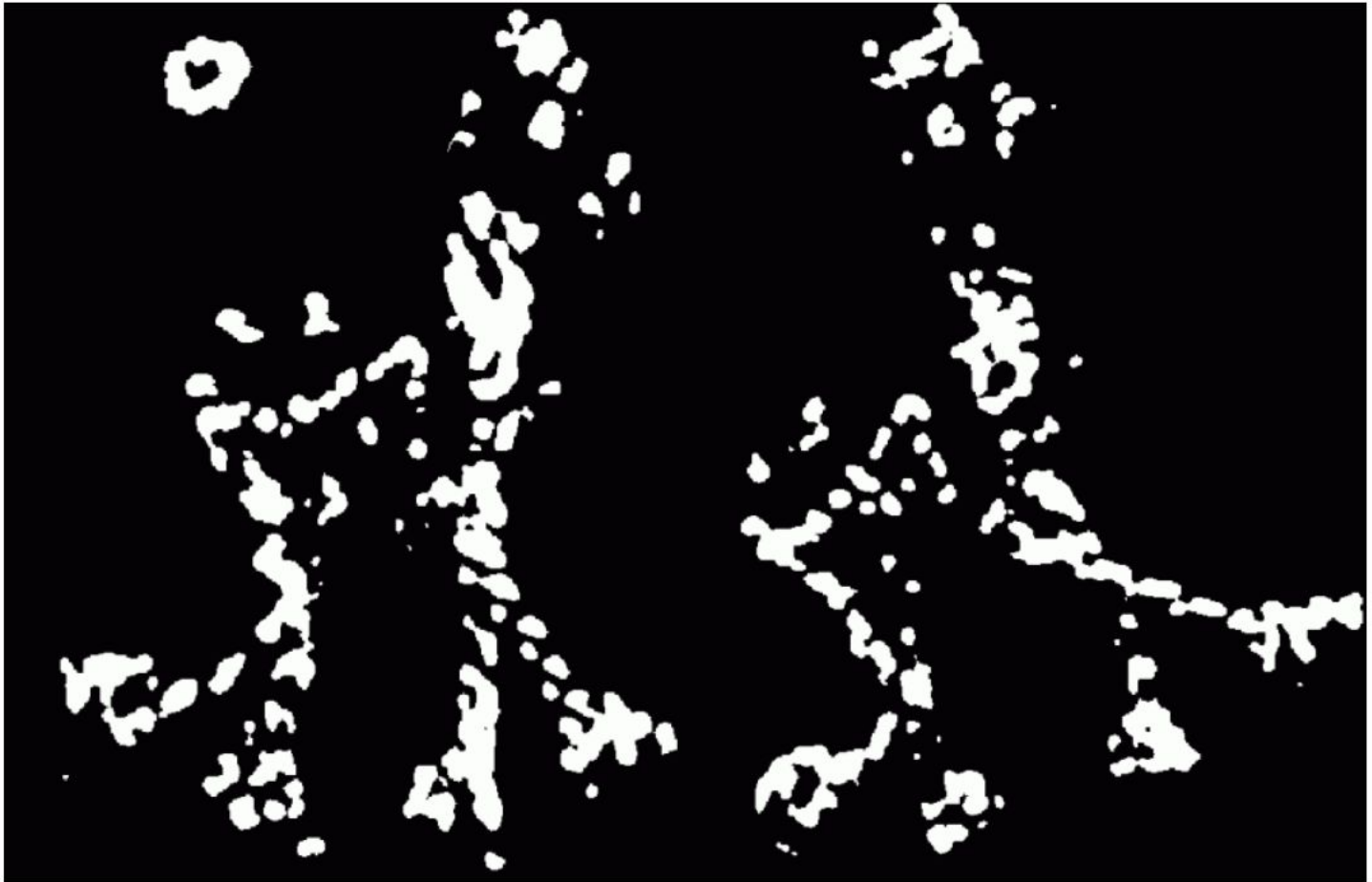
# Computing corners

- Smooth image  $I$  to compute various derivatives
- Choose a window size (say  $7 \times 7$ ) and compute the structure tensor at each pixel, given this window size
- Compute the corner-ness measure for each pixel
- Perform thresholding or non-maximal suppression to create a corners map.

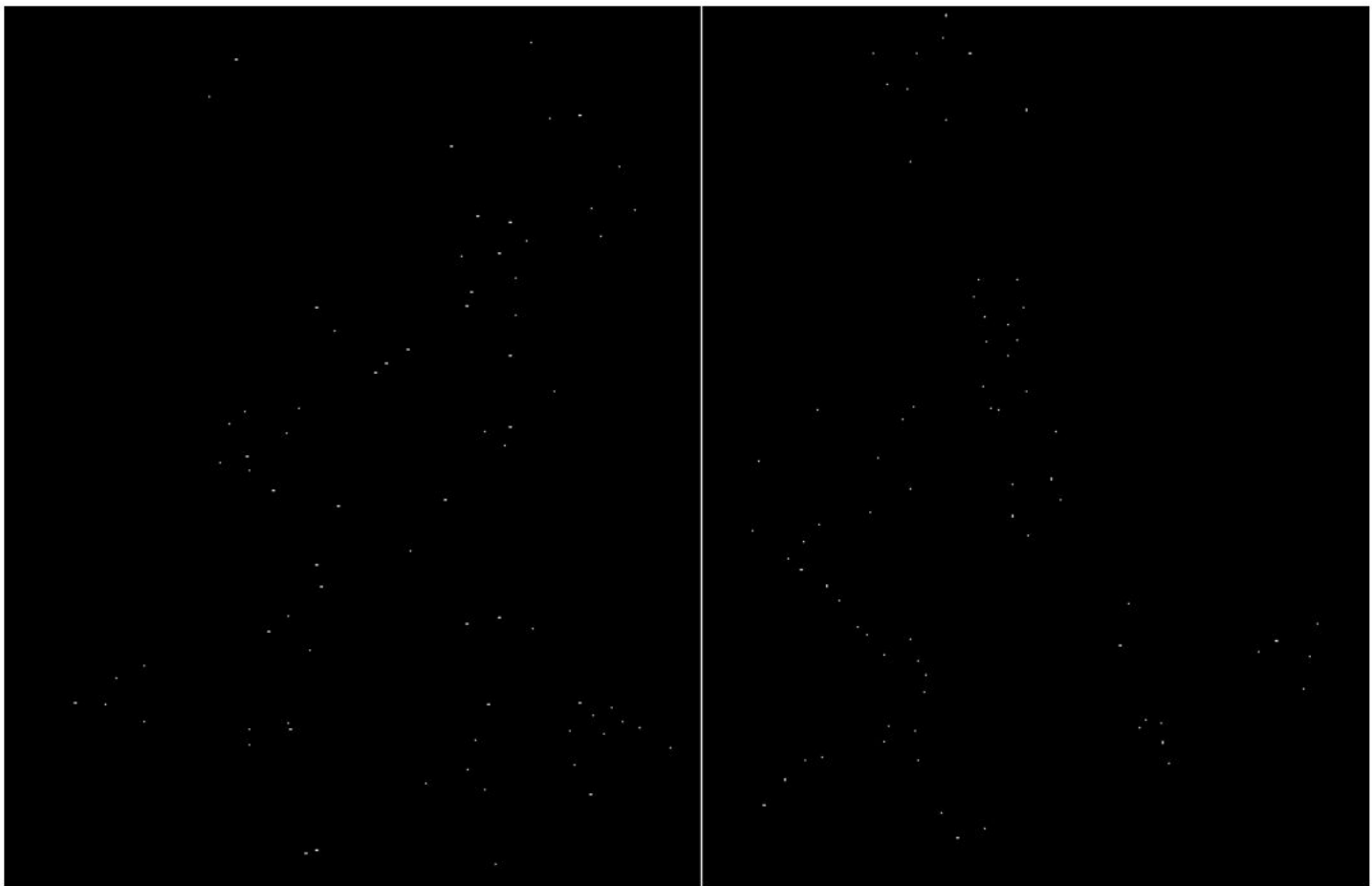














# Invariance

- The detected corners are invariant under affine intensity change, i.e. image  $J$  replaced by  $aJ + b$  for scalars  $a, b$ .
- The corners are also invariant to rotation and translation.

# Other measures of corner-ness

- Based on just the minimum eigenvalue – declared a corner if the minimum eigenvalue exceeds a threshold
- Harmonic mean of eigenvalues

$$R_{HM} = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} = \frac{\det(M)}{\text{trace}(M)}$$