Q2) We are given a 1 D convolution mask $(w_0, w_1, \ldots\ldots w_6)$ and a 1 D image of size n. We shall map our individual convolution result to that pixel in the image which aligns with the centre of the mask.

Assuming zero padding of size $7-1=6$ on either sides of the image, we get the following relation:

$$Mf = g$$

$$\underbrace{\begin{bmatrix}
w_3 & w_2 & w_1 & w_0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
w_4 & w_3 & w_2 & w_1 & w_0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
w_5 & w_4 & w_3 & w_2 & w_1 & w_0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
w_6 & w_5 & w_4 & w_3 & w_2 & w_1 & w_0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & w_6 & w_5 & w_4 & w_3 & w_2 & w_1 & w_0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & w_6 & w_5 & w_4 & w_3 & w_2 & w_1 & w_0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & w_6 & w_5 & w_4 & w_3
\end{bmatrix}}_{n \times n}
\underbrace{\begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ \vdots \\ f_{n-1} \end{bmatrix}}_{n \times 1}
=
\underbrace{\begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \\ \vdots \\ g_{n-1} \end{bmatrix}}_{n \times 1}$$

Here, M is the required $n \times n$ matrix.

We can see that the first row is $w_3\ w_2\ w_1\ w_0$ followed by all zeroes. As the row index increases, the values shift accordingly until we reach $n-4$ zeroes followed by $w_6\ w_5\ w_4\ w_3$.

Some properties of matrix M:

(i) The matrix M has constant values along each diagnol. Such a matrix is said to be a Toeplitz matrix.

(ii) For a given row of the matrix, there are atleast $4\ (= \frac{7+1}{2})$ and atmost 7 non-zero values. For a m sized mask, there would have been atleast $\frac{m+1}{2}$ and atmost m non-zero entries.

(iii) If the mask is symmetric, that is, $w_{3-i} = w_{3+i}\ \forall i \in \{0,1,2,3\}$, then the matrix M is also symmetric.

Such a matrix-based construction allows convolution to be performed more efficiently because implementing $Mf = g$ is relatively easier compared to $(w * f)(x) = \sum_{s=-a}^{a} w(s) f(x-s)$.

Also, using principles of linear Algebra, we can analyze operations like Deconvolution (undoing the effects of convolution) as $f = M^{-1} g$.