(Q4.) $I(x,y) \leftarrow$ Original Image

Apply the transformation

$$I(x,y) \leftarrow I(x,y) + \alpha \nabla^2 I(x,y)$$

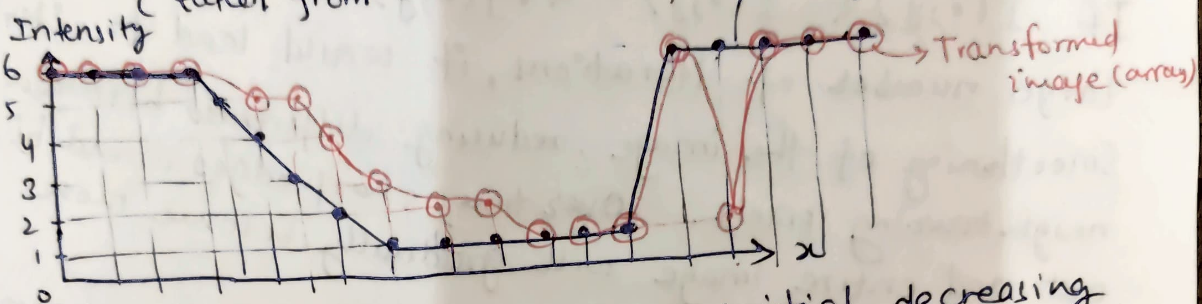
where $\alpha > 0$ and $\alpha \approx 0$ (very small constant)

For 1-D case :-

Consider the 1-D Array :

$a = [6, 6, 6, 6, 5, 4, 3, 2, 1, 1, 1, 1, 1, 1, 6, 6, 6, 6, 6]$

(taken from slides)



The image (1-D) consists of an initial decreasing ramp (6-1) and then a step-edge (1-6).

2^{nd}-derivative of a is as follows:-

$$\frac{d^2a}{dx^2} = [0, 0, -1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 5, -5, 0, 0, 0]$$

At the point the graph (image) starts to decrease, there's a negative $\nabla^2\{a(x)\}$ and at the point the ramp ends (stops decreasing), there's a positive $\nabla^2\{a(x)\}$.

Adding $\nabla^2\{a(x)\}$ (or a fraction of it) will reduce the high peaks while ~~reduce the~~ increase the low-peaks, thus decreasing sharpness and increasing blurring (smoother transitions).

The same holds true for step edge where there's a large positive $\nabla^2\{a(x)\}$ at the lower end and a large negative $\nabla^2\{a(x)\}$ at the ~~x~~ higher end. Adding $\alpha \cdot \nabla^2\{a(x)\}$ to a(x) would decrease this height of the step leading to smoother edges (increasing blurring).

For $\alpha = 1$ (since for very small α, effects would only be visible after many iterations), the enhanced image would be :-

$$\tilde{a}(x) = a(x) + \nabla^2 \{a(x)\}$$

$$= [6, 6, 6, 5, 5, 4, 3, 2, 2, 1, 1, 1, 1, 6, 1, 6, 6, 6, 6]$$

If $I(x,y) \leftarrow I(x,y) + \alpha \nabla^2 I(x,y)$ were to run for a large number of iterations, it would lead to iterative smoothning of the image, reducing differences between neighbouring pixels. Over time, sharp edges would blur out and entire image will gradually become more homogeneous.

In the extreme / limiting case when # iterations $\to \infty$, $I(x,y) \Rightarrow$ constant and image would flatten out completely.

If $I(x,y) \leftarrow I(x,y) - \alpha \nabla^2 I(x,y)$ is run for a large number of iterations, the following would happen:

(i) Since subtracting the laplacian amplifies the differences between neighbouring pixels, sharp edges become sharper and noise gets amplified.

(ii) In the limiting condition, the image would become highly unstable with extreme values. Since this transformation causes a zero-crossing at the step edge (mentioned in slides), it would lead to sharp black and white contrasting edges at such places in the image.

[The explanation as to why this transformation increases sharpness is : For an edge with decreasing intensity, it is negative ($\nabla^2 I(x,y)$) at the beginning and positive at the end. Subtracting it would thus sharpen the edge.]

(explanation taken from SLIDES)