Mid-sem: CS 754, Computer Vision, 20th February

Instructions: There are 120 minutes for this exam. This exam is worth 10% of your final grade. Attempt all questions. Avoid writing lengthy answers. Each question carries 10 points.

1. Why is the Gaussian considered unfit for modelling the distribution of the DCT coefficients of natural images, as opposed to (say) the Laplacian?

2. In compressed sensing, the following optimization problem, known as Basis Pursuit (BP), is often considered: $\min_{\boldsymbol{\theta}} \|\boldsymbol{\theta}\|_1$ such that $\|\boldsymbol{y} - \boldsymbol{\Phi\Psi\theta}\|_2 \leq \epsilon$ where $\boldsymbol{\theta} \in \mathbb{R}^n$ is a sparse vector of signal coefficients in some $n \times n$ orthonormal basis $\boldsymbol{\Psi}$ yielding signal $\boldsymbol{x} = \boldsymbol{\Psi\theta}$, $\boldsymbol{\Phi}$ is a $m \times n$ measurement matrix with $m \ll n$, $\boldsymbol{y} \in \mathbb{R}^m$ is a measurement vector, and ϵ is an upper bound on the noise in $\boldsymbol{y}$. The LASSO problem defined as $\min_{\boldsymbol{\theta}} \|\boldsymbol{y} - \boldsymbol{\Phi\Psi\theta}\|^2 + \lambda \|\boldsymbol{\theta}\|_1$ is provably equivalent to BP for specific values of λ and ϵ. Is the choice of L_1 norm on $\boldsymbol{\theta}$ justifiable (in terms of accuracy of recovery) even if the values in $\boldsymbol{\theta}$ were not Laplacian distributed, but distributed as per a Generalized Gaussian Distribution with shape parameter less than 1? Assume a sufficient number of measurements m are available.

3. Consider a noisy and blurred image $y = h * x + \eta$ where h is a known blur kernel, x is the unknown noise-free and blur-free image and η is zero-mean Gaussian noise with known standard deviation σ. State the objective function to be minimized when you apply the MAP method in order to obtain x. Assume x was a sparse image (in the spatial domain itself). Write down the equations for the prior and the likelihood.

4. State one advantage of mutual coherence over restricted isometry property (RIP) in compressed sensing, and vice versa.

5. What is the key difference between ISTA and OMP algorithms for compressed sensing recovery?

6. Prove that to allow for successful reconstruction of a k-sparse signal $\boldsymbol{\theta}$ from compressive measurements $\boldsymbol{y} = \boldsymbol{A\theta}$, it is necessary that the matrix $\boldsymbol{A}$ have the property that any subset of $2k$ columns from $\boldsymbol{A}$ be linearly independent. How would you reconstruct $\boldsymbol{\theta}$ using L_0 minimization, and what is the smallest number of measurements required in terms of k and/or n? If you switched over to L_1 minimization, what is the smallest number of measurements required in terms of k and/or n?

7. Apart from sparsity and power law, briefly mention any two statistical properties of natural images.