Question 4, Assignment 3: CS 754, Spring 2024-25

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Declaration: The work submitted is our own, and we have adhered to the principles of academic honesty while completing and submitting this work. We have not referred to any unauthorized sources, and we have not used generative AI tools for the work submitted here.

1. Let $R_\theta f(\rho)$ be the Radon transform of the image $f(x,y)$ in the direction given by θ for bin index ρ. Let g be a version of f shifted by (x_0,y_0). Then, prove that $R_\theta g(\rho) = R_\theta f(\rho - (x_0,y_0)\cdot(\cos\theta,\sin\theta))$. [8 points]

Soln:

We know that Radon transform of a 2-dimensional image $f(x,y)$, can be obtained by:

$$R_\theta f(\rho) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\cdot\delta(\rho-(xcos\theta+ysin\theta))\cdot dx\cdot dy \tag{1}$$

Since, $g(x,y)$ is a shifted version of $f(x,y)$ by (x_0,y_0), it can be written as:

$$g(x,y) = f(x-x_0, y-y_0) \tag{2}$$

Similarly Radon Transform of $g(x,y)$ defined at angle θ can be written as

$$R_\theta g(\rho) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y)\cdot\delta(\rho-(xcos\theta+ysin\theta))\cdot dx\cdot dy \tag{3}$$

Substituting the value of $g(x,y)$ from equation (2), we get:

$$R_\theta g(\rho) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x-x_0,y-y_0)\cdot\delta(\rho-(xcos\theta+ysin\theta))\cdot dx\cdot dy \tag{4}$$

Performing the variable substitution: $x' = x - x_0$ and $y' = y - y_0$ in the above equation, we get

$$R_\theta g(\rho) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x',y')\cdot\delta(\rho-((x'+x_0)cos\theta+(y'+y_0)sin\theta))\cdot dx'\cdot dy' \tag{5}$$

$$R_\theta g(\rho) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x',y')\cdot\delta((\rho-x_0cos\cdot\theta-y_0\cdot sin\theta)-x'\cdot cos\theta+y'\cdot sin\theta)\cdot dx'\cdot dy' \tag{6}$$

Comparing the equation (6) with equation (1):

$$R_\theta g(\rho) = R_\theta f(\rho - x_0\cdot cos\theta - y_0\cdot sin\theta)$$

$$R_\theta g(\rho) = R_\theta f(\rho - \langle(x_0,y_0)\|(cos\theta,sin\theta)\rangle)$$

Hence Proved!