Assignment 1: CS 754, Spring 2024-25

Amitesh Shekhar
IIT Bombay
22b0014@iitb.ac.in

Anupam Rawat
IIT Bombay
22b3982@iitb.ac.in

Toshan Achintya Golla
IIT Bombay
22b2234@iitb.ac.in

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Declaration: The work submitted is our own, and we have adhered to the principles of academic honesty while completing and submitting this work. We have not referred to any unauthorized sources, and we have not used generative AI tools for the work submitted here.

1. For a unique solution to the P1 problem, we require that $\delta_{2s} < 0.41$ as given in class. What is the corresponding upper bound for δ_{2s} in order for the P0 problem to give a unique solution? (Hint: Look at the proof of the uniqueness of the solutions to the P0 problem, and see the definition of RIC) [15 points]

Soln:

The P0 problem requires us to minimize $\|\theta\|_0$ for $y = A\theta$.
To find the upper bound of δ_{2s} in order for the P0 problem to give a unique solution, we begin by assuming that there exist 2 solutions θ_1 and θ_2; such that $\theta_1 \neq \theta_2$. Thus,

$$\|\theta_1\|_0 = \|\theta_2\|_0 = S$$

$$y = A\theta_1 = A\theta_2$$

$$A(\theta_1 - \theta_2) = 0$$

Thus, either the below is true

$$\theta_1 = \theta_2$$

or the below is true

$$\theta_1 - \theta_2 \in nullspace(A)$$

As per our beginning statement, $\theta_1 \neq \theta_2$, thus

$$\|\theta_1 - \theta_2\|_0 \leq 2S$$

Now,

$$A(\theta_1 - \theta_2) \neq 0$$

as A has Restrictive Isometric Property of 2S-order.

$$(1 - \delta_S)\|\theta_{2S}\|^2 \leq \|A\theta_{2S}\|^2 \leq (1 + \delta_{2S})\|\theta_{2S}\|^2$$

Replacing θ_{2S} with $\theta_1 - \theta_2$,

$$(1 - \delta_S)\|\theta_1 - \theta_2\|^2 \leq \|A(\theta_1 - \theta_2)\|^2 \leq (1 + \delta_{2S})\|\theta_1 - \theta_2\|^2$$

For, this to not to be equal to zero,

$$(1 - \delta_S)\|\theta_1 - \theta_2\|^2 > 0$$

Since, our definition $\theta_1 \neq \theta_2$

$$(1 - \delta_{2S}) > 0$$

$$\delta_{2S} < 1$$

Thus, the upper bound for δ_{2s} in order for the P0 problem to give a unique solution