Question 2, Assignment 5: CS 754, Spring 2024-25

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1. Read the wiki article on L1-norm PCA: `https://en.wikipedia.org/wiki/L1-norm_principal_component_analysis`. List any three fundamental ways in which robust PCA that we did in class differs from L1-norm PCA. [15 points]

Soln:

1. Problem Formulation:

- *RPCA* decomposes a matrix into low-rank + sparse components. RPCA tries solves the following optimization problem

$$min_{L,S}||L||_* + \frac{1}{\sqrt{max(n_1, n_2)}}||S||_1 \text{ where M = S + L and M } \in \mathbb{R}^{n_1 \times n_2}$$

- *L1-norm PCA* finds principal components by maximizing L1 dispersion of projected data

$$max_Q||X^T Q||_1 \text{ where } Q = [q_1, q_2, ..., q_K] \in \mathbb{R}^{D \times K} \text{ such that } Q^T Q = I_k$$

2. Outlier Handling Mechanism:

- *RPCA* ensures that the outliers are isolated in S. Under incoherence conditions, RPCA provably recovers L and S even if outliers are large but sparse.
- *L1-norm PCA* has a median-like behavior that reduces sensitivity to outliers without explicitly modeling them. Outliers influence the principal components less than in L2-PCA, but they are not identified explicitly.

3. Computational and Theoretical Properties:

- *RPCA* is convex, ensuring a global optimum
- *L1-norm PCA* is non-convex, making it prone to getting stuck in local optima

Output

- *RPCA* outputs seperable components (L/S)
- *L1-norm PCA* outputs robust eigenvectors

Output

- *RPCA* is used for data with sparse and large corruptions (eg incase of sensor failures)
- *L1-norm PCA* is used on data with diffuse outliers (eg heavy tailed noise)

References

1. `https://en.wikipedia.org/wiki/L1-norm_principal_component_analysis`
2. `https://candes.su.domains/teaching/math301/Lectures/rpca.pdf`