Assignment 1: CS 754, Spring 2024-25

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Declaration: The work submitted is our own, and we have adhered to the principles of academic honesty while completing and submitting this work. We have not referred to any unauthorized sources, and we have not used generative AI tools for the work submitted here.

1. Construct a synthetic image $\boldsymbol{f}$ of size 32×32 in the form of a sparse linear combination of k randomly chosen 2D DCT basis vectors. Simulate m compressive measurements of this image in the form $\boldsymbol{y} = \boldsymbol{\Phi}\text{vec}(\boldsymbol{f})$ where $\text{vec}(\boldsymbol{f})$ stands for a vectorized form of $\boldsymbol{f}$, $\boldsymbol{y}$ contains m elements and $\boldsymbol{\Phi}$ has size $m \times 1024$. The elements of $\boldsymbol{\Phi}$ should be independently drawn from a Rademacher matrix (i.e. the values of the entries should independently be -1 and $+1$ with probability 0.5). Your job is to implement the OMP algorithm to recover $\boldsymbol{f}$ from $\boldsymbol{y}, \boldsymbol{\Phi}$ for $k \in \{5, 10, 20, 30, 50, 100, 150, 200\}$ and $m \in \{100, 200, ..., 1000\}$. In the OMP iterations, you may assume knowledge of the true value of k. Each time, you should record the value of the RMSE given by $\|\text{vec}(\boldsymbol{f}) - \text{vec}(\hat{\boldsymbol{f}})\|_2 / \|\text{vec}(\boldsymbol{f})\|_2$. For $k \in \{5, 50, 200\}$, you should plot a graph of RMSE versus m and plot the reconstructed images with appropriate captions declaring the value of k, m. Also plot the ground truth image. For $m \in \{500, 700\}$, you should plot a graph of RMSE versus k and plot the reconstructed images with appropriate captions declaring the value of k, m. Also plot the ground truth image. Comment on the behaviour of these plots. Repeat all these tasks with the CoSAMP, another greedy algorithm from equation (10) of the paper 'CoSaMP: iterative signal recovery from incomplete and inaccurate samples' which you can find at `https://dl.acm.org/doi/10.1145/1859204.1859229`. For implementing this algorithm, you should again assume knowledge of the true k. A local copy of this paper is also uploaded onto the homework folder. [15 + 15 = 30 points]

Soln:

OMP

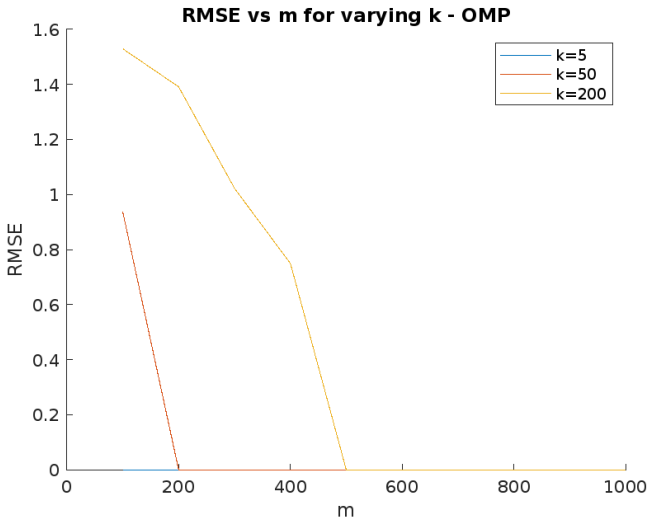

Figure 1: Plot for RMSE vs m for OMP

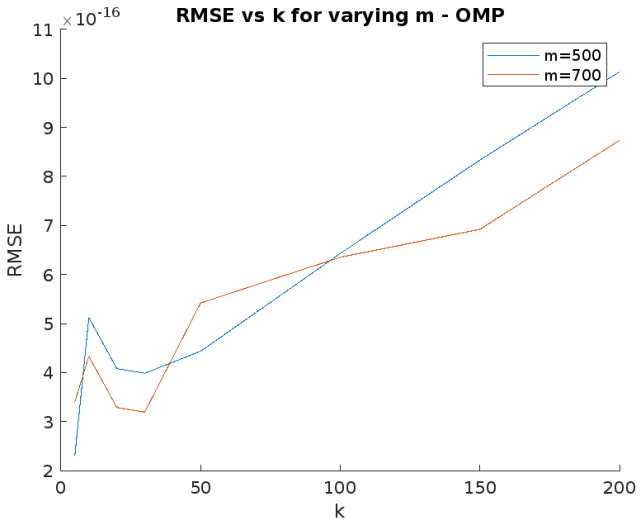

Figure 2: Plot for RMSE vs k for OMP

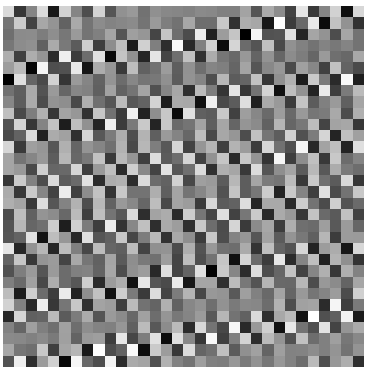

Figure 3: Original Image for k=5

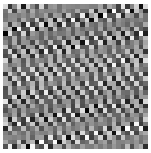

Figure 4: Reconstructed for m=100

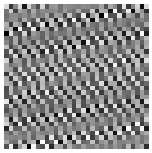

Figure 5: Reconstructed for m=200

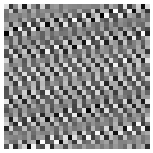

Figure 6: Reconstructed for m=300

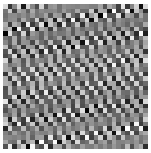

Figure 7: Reconstructed for m=400

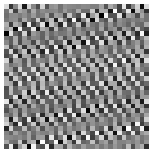

Figure 8: Reconstructed for m=500

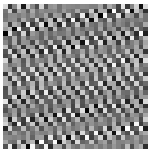

Figure 9: Reconstructed for m=600

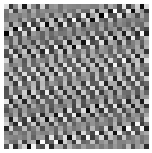

Figure 10: Reconstructed for m=700

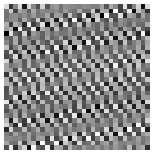

Figure 11: Reconstructed for m=800

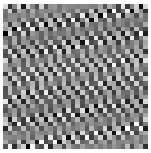

Figure 12: Reconstructed for m=900

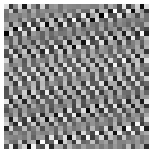

Figure 13: Reconstructed for m=1000

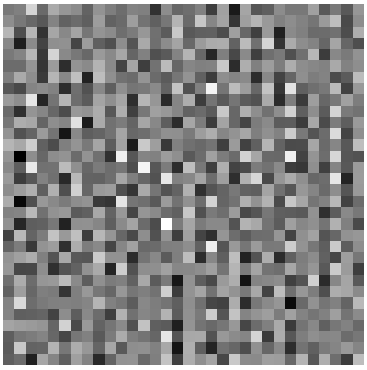

Figure 14: Original Image for k=50

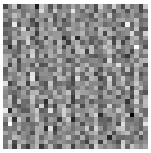

Figure 15: Reconstructed for m=100

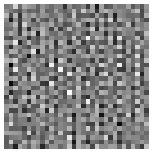

Figure 16: Reconstructed for m=200

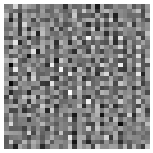

Figure 17: Reconstructed for m=300

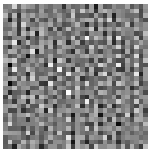

Figure 18: Reconstructed for m=400

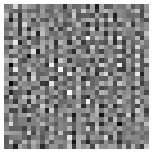

Figure 19: Reconstructed for m=500

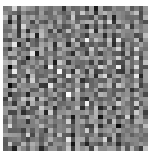

Figure 20: Reconstructed for m=600

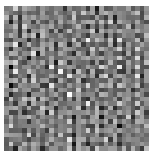

Figure 21: Reconstructed for m=700

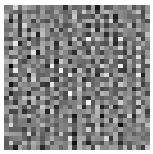

Figure 22: Reconstructed for m=800

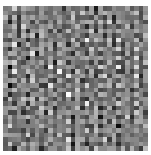

Figure 23: Reconstructed for m=900

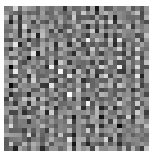

Figure 24: Reconstructed for m=1000

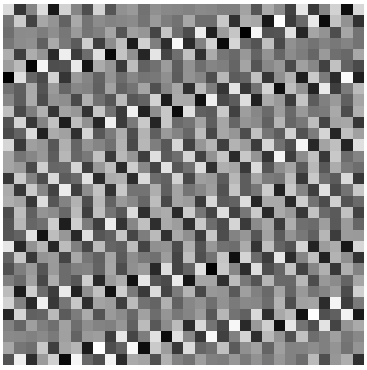

Figure 25: Original Image for k=200

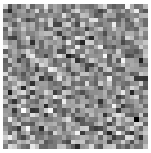

Figure 26: Reconstructed for m=100

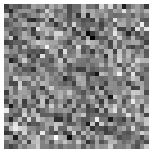

Figure 27: Reconstructed for m=200

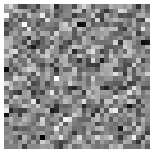

Figure 28: Reconstructed for m=300

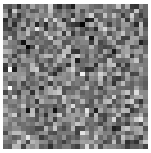

Figure 29: Reconstructed for m=400

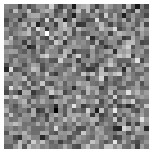

Figure 30: Reconstructed for m=500

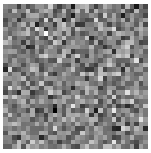

Figure 31: Reconstructed for m=600

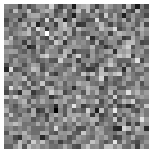

Figure 32: Reconstructed for m=700

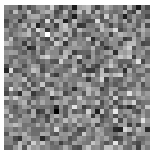

Figure 33: Reconstructed for m=800

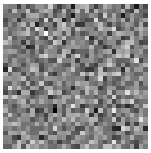

Figure 34: Reconstructed for m=900

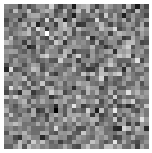

Figure 35: Reconstructed for m=1000

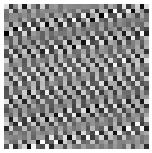

Figure 36: Qriginal k=5

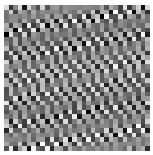

Figure 37: Reconstructed for m=500

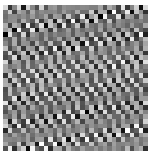

Figure 38: Reconstructed for m=700

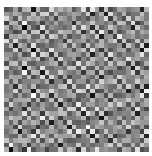

Figure 39: Original k=10

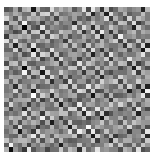

Figure 40: Reconstructed for m=500

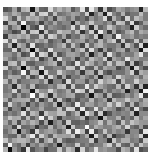

Figure 41: Reconstructed for m=700

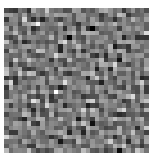

Figure 42: Original k=20

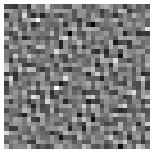

Figure 43: Reconstructed for m=500

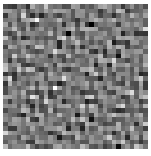

Figure 44: Reconstructed for m=700

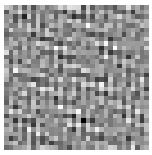

Figure 45: original k=30

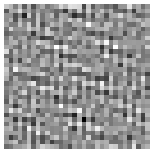

Figure 46: Reconstructed for m=500

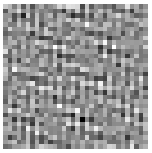

Figure 47: Reconstructed for m=700

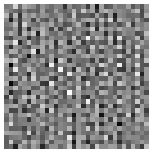

Figure 48: Original k=50

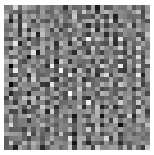

Figure 49: Reconstructed for m=500

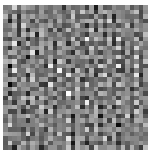

Figure 50: Reconstructed for m=700

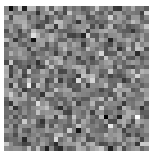

Figure 51: Qriginal k=100

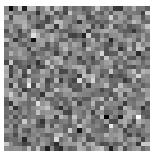

Figure 52: Reconstructed for m=500

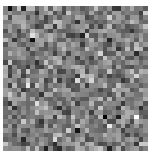

Figure 53: Reconstructed for m=700

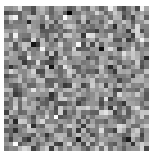

Figure 54: Qriginal k=150

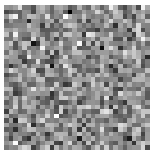

Figure 55: Reconstructed for m=500

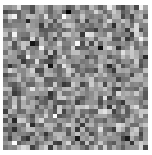

Figure 56: Reconstructed for m=700

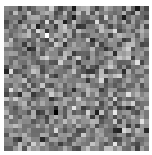

Figure 57: Qriginal k=200

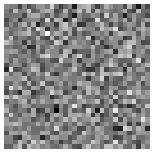

Figure 58: Reconstructed for m=500

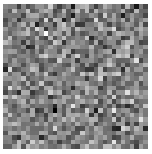

Figure 59: Reconstructed for m=700

Comments on OMP

We observe that RMSE increases for increasing k at a fixed m. Also, RMSE decreases for increasing m with a fixed k.

CoSAMP

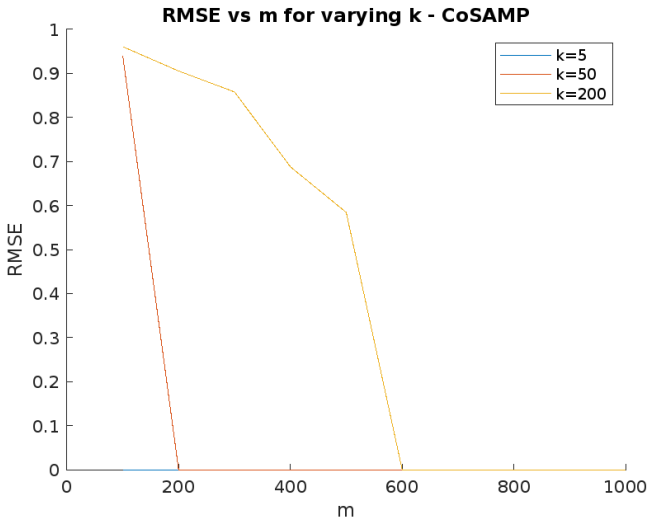

Figure 60: Plot for RMSE vs m for CoSAMP

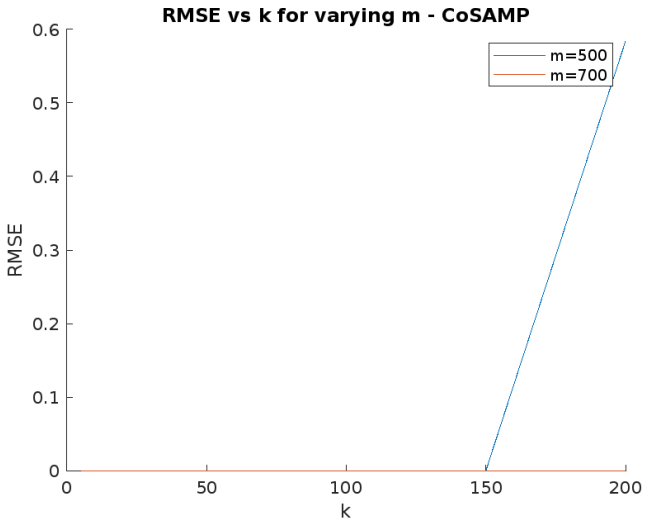

Figure 61: Plot for RMSE vs k for CoSAMP

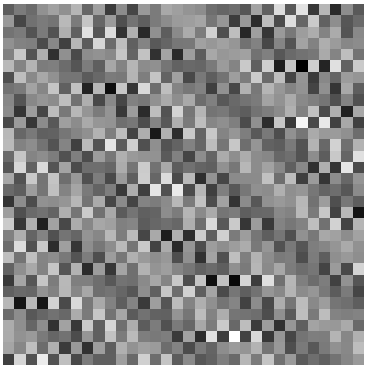

Figure 62: Original Image for k=5

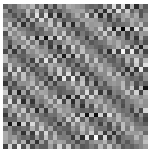

Figure 63: Reconstructed for m=100

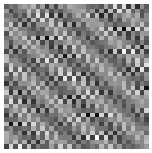

Figure 64: Reconstructed for m=200

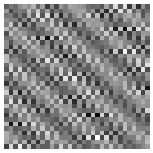

Figure 65: Reconstructed for m=300

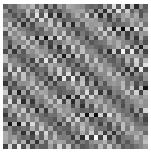

Figure 66: Reconstructed for m=400

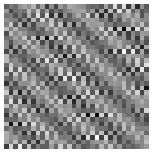

Figure 67: Reconstructed for m=500

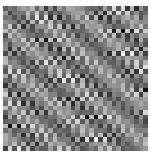

Figure 68: Reconstructed for m=600

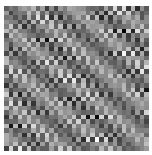

Figure 69: Reconstructed for m=700

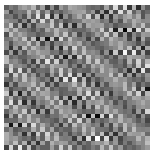

Figure 70: Reconstructed for m=800

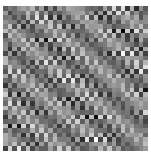

Figure 71: Reconstructed for m=900

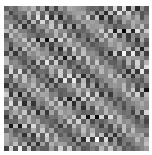

Figure 72: Reconstructed for m=1000

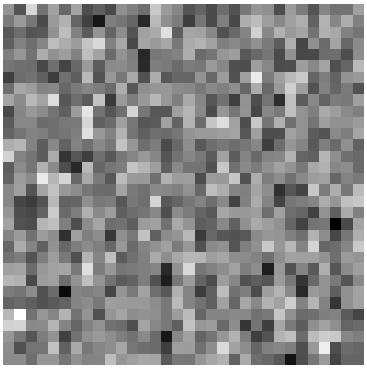

Figure 73: Original Image for k=50

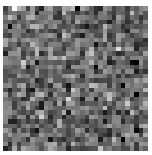

Figure 74: Reconstructed for m=100

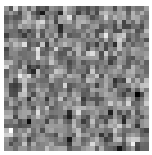

Figure 75: Reconstructed for m=200

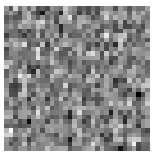

Figure 76: Reconstructed for m=300

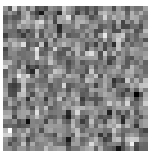

Figure 77: Reconstructed for m=400

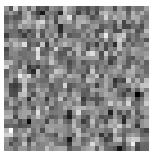

Figure 78: Reconstructed for m=500

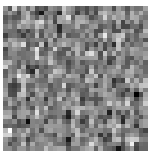

Figure 79: Reconstructed for m=600

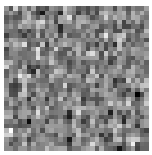

Figure 80: Reconstructed for m=700

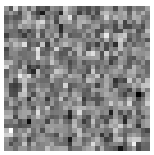

Figure 81: Reconstructed for m=800

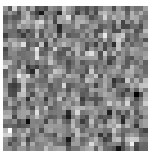

Figure 82: Reconstructed for m=900

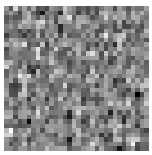

Figure 83: Reconstructed for m=1000

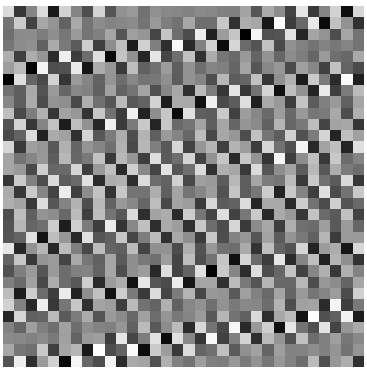

Figure 84: Original Image for k=200

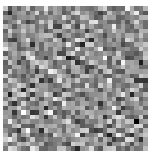

Figure 85: Reconstructed for m=100

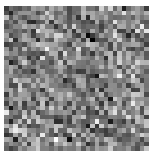

Figure 86: Reconstructed for m=200

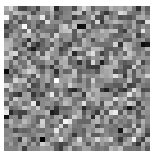

Figure 87: Reconstructed for m=300

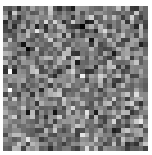

Figure 88: Reconstructed for m=400

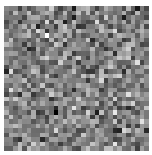

Figure 89: Reconstructed for m=500

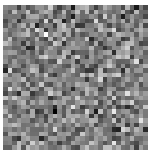

Figure 90: Reconstructed for m=600

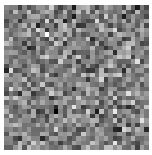

Figure 91: Reconstructed for m=700

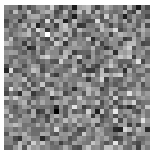

Figure 92: Reconstructed for m=800

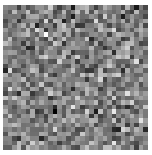

Figure 93: Reconstructed for m=900

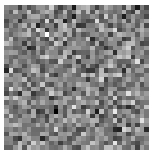

Figure 94: Reconstructed for m=1000

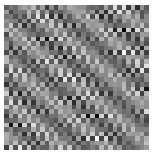

Figure 95: Qriginal k=5

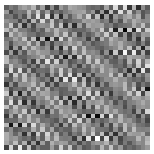

Figure 96: Reconstructed for m=500

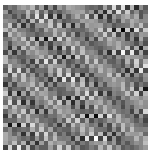

Figure 97: Reconstructed for m=700

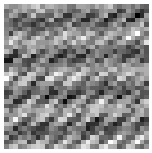

Figure 98: Original k=10

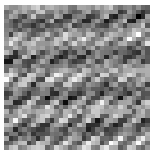

Figure 99: Reconstructed for m=500

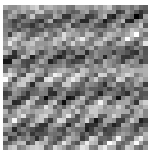

Figure 100: Reconstructed for m=700

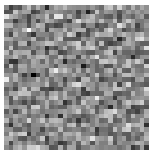

Figure 101: Original k=20

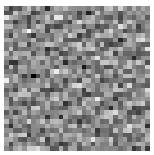

Figure 102: Reconstructed for m=500

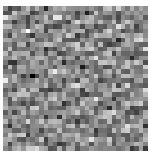

Figure 103: Reconstructed for m=700

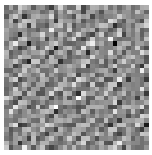

Figure 104: original k=30

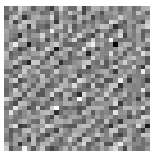

Figure 105: Reconstructed for m=500

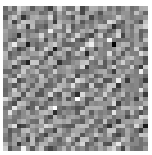

Figure 106: Reconstructed for m=700

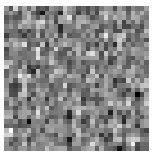

Figure 107: Original k=50

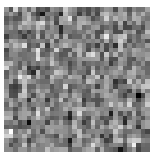

Figure 108: Reconstructed for m=500

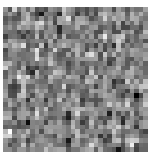

Figure 109: Reconstructed for m=700

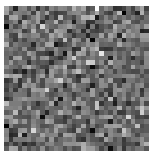

Figure 110: Qriginal k=100

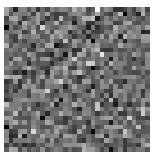

Figure 111: Reconstructed for m=500

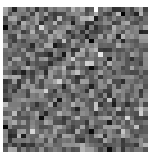

Figure 112: Reconstructed for m=700

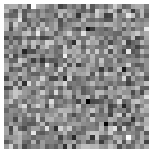

Figure 113: Qriginal k=150

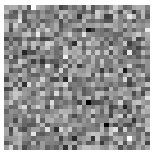

Figure 114: Reconstructed for m=500

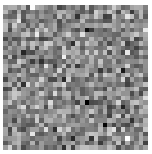

Figure 115: Reconstructed for m=700

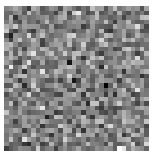

Figure 116: Qriginal k=200

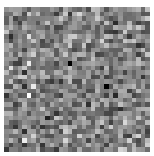

Figure 117: Reconstructed for m=500

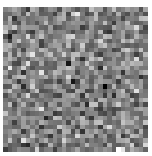

Figure 118: Reconstructed for m=700

Comments on CoSAMP

Similar to OMP, we observe that RMSE increases for increasing k at a fixed m. Also, RMSE decreases for increasing m with a fixed k. However, there is a sudden steep increase in RMSE for m=500,k=200 in case of CoSAMP. This maybe due to the fact that CoSAMP requires more measurements(that is, more m) than OMP. In all other cases, the RMSE is of the order of 10^{-16}.