

Optional Midsemester Assignment Submission: EE 338, Spring 2024-25

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Filter Number (M): 104

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1 Aim Of the Assignment

We are required to design an Infinite-Impulse-Response (IIR) Filter, with bands from both Group I and Group II, acting as **equiripple / oscillatory passband**, and all the **stopbands are monotonic / non-oscillatory**. Now, we know that for the case when passband is oscillatory and stopband is monotonic, then the desired Filter type is **Chebyshev**.

2 Specifications Required

The analog signal is bandlimited to 280kHz and it's ideally sampled with a sampling rate of 630kHz. Now 2 x bandwidth ; sampling rate. The sampling rate obeys the Nyquist criteria and can be reconstructed without any loss.

1. The tolerance of stopband and passband are $\delta = 0.15$ in magnitude.
2. The Filter number assigned to me is: $M = 104$
 - (a) $Q = \text{floor}(M / 11) = 9$
 - (b) $R = M \bmod 11 = 5$
3. Range of Group I frequency: $(40 + 5D)$ to $(70 + 5D)$; where $D = Q$
 - (a) **Lower Edge** $= 40 + 5D = 40 + 45 = \mathbf{85kHz}$
 - (b) **Upper Edge** $= 70 + 5D = 70 + 45 = \mathbf{115kHz}$
4. Range of Group II frequency: $(170 + 5D)$ to $(200 + 5D)$; where $D = R$
 - (a) **Lower Edge** $= 170 + 5D = 170 + 25 = \mathbf{195kHz}$
 - (b) **Upper Edge** $= 200 + 5D = 200 + 25 = \mathbf{225kHz}$
5. The transition band on either side of the passband is 5kHz. Group I and II are the passbands.

3 Desired Frequency Response

As per the above specifications, below is the plot of the desired frequency specifications.

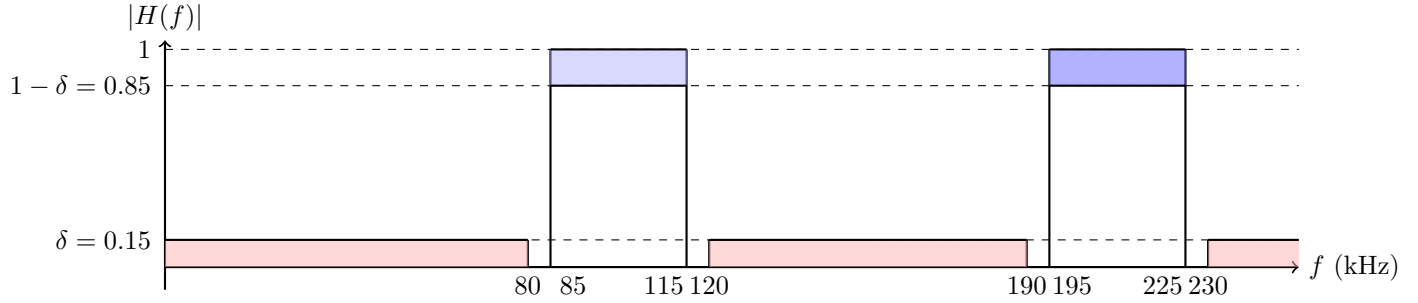


Figure 1: Desired Frequency Response

Now, it should be noted that, the blue regions represents the equiripple / oscillatory passband and the red region represents the monotonic stopband. It should also be noted that, since this is an IIR filter, the upper band is capped to 1.

The light blue region, on the left, corresponds to the Group I of frequency bands, while the slightly darker region on the right, corresponds to the Group II of frequency bands.

3.1 Realization of the Frequency Response

The filter response can be realised by cascading a BandPass Filter with a BandPass Filter (*Series Connection*) OR by adding the result obtained by two BandPass Filters (*Parallel Connection*). For the purpose of this assignment, we will be using the Cascading option.

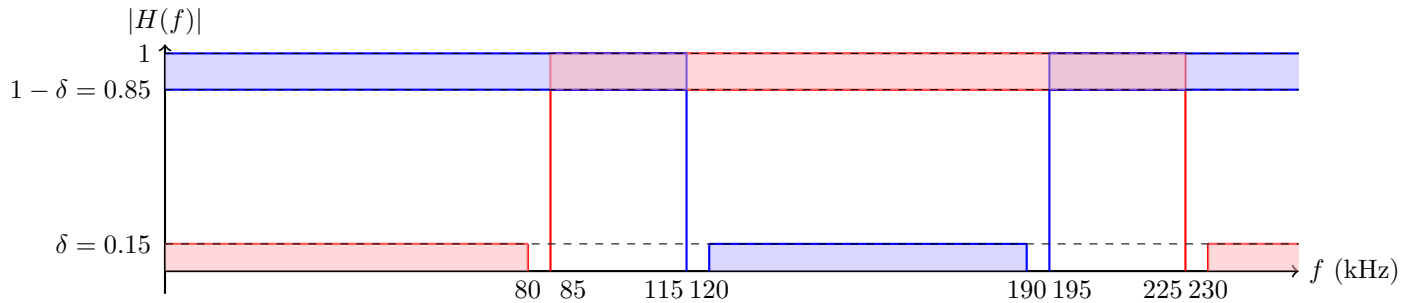


Figure 2: Cascade of BandPass and BandStop Filters

The Blue filter represents the **BandStop Filter** with **stop band from 120kHz to 190kHz**, and 5kHz transition band, across the stopband. The red filter represents the **BandPass Filter** with **passband between 85kHz and 225kHz**, with a transition band of 5kHz across the passband.

4 Designing Filter

We now require to design the BandPass and BandStop Filter one by one, to realise the final filter.

1. BandPass Filter

- (a) Passband Range Required: 85kHz - 225kHz
- (b) PassBand Tolerance (arbitrary): δ_1
- (c) StopBand Tolerance (arbitrary): δ_2
- (d) passband Tolerance: 5kHz

2. BandStop Filter

- (a) Passband Range Required: 120kHz - 190kHz
- (b) PassBand Tolerance (arbitrary): δ_3
- (c) StopBand Tolerance (arbitrary): δ_4
- (d) stopband Tolerance: 5kHz

After cascading the two filters; following will be the value of tolerances and the respective frequency range:

$$Tolerances = \begin{cases} \delta_2(1 - \delta_3), & 0^+ \leq f < 80kHz \\ (1 - \delta_1)(1 - \delta_3), & 85kHz \leq f < 115kHz \\ \delta_4(1 - \delta_3), & 120kHz \leq f < 190kHz \\ (1 - \delta_1)(1 - \delta_3), & 195kHz \leq f < 225kHz \\ \delta_2(1 - \delta_3), & 230kHz \leq f < 315kHz \end{cases}$$

Comparing the Tolerances from the above table to the Tolerance level values from Figure - 1. Equating for $85kHz \leq f \leq 115kHz$ and $195kHz \leq f \leq 225kHz$:

$$(1 - \delta_1)(1 - \delta_3) = 1 - \delta = 0.85$$

$$1 - \delta_1 - \delta_3 - \delta_1\delta_3 = 0.85$$

$$\delta_1 + \delta_3 - \delta_1\delta_3 = 0.15$$

Equating for $0kHz \leq f \leq 80kHz$ and $230kHz \leq f \leq 315kHz$:

$$\delta_2(1 - \delta_3) = \delta = 0.15$$

But equating for $120kHz \leq f \leq 190kHz$ gives us:

$$\delta_4(1 - \delta_1) = \delta = 0.15$$

For the ease of solving, we can assume: $\delta_1 = \delta_3$ and $\delta_2 = \delta_4$. Thus the above equations become:

$$\delta_1 + \delta_3 - \delta_1\delta_3 = 2\delta_1 - \delta_1^2 = 0.15$$

Solving the quadratic equation,

$$\delta_1 = \frac{+2 \pm \sqrt{(-2)^2 - 4(1)(0.15)}}{2 * 1}$$

Thus, δ_1 can either be 1.92 or 0.08, but we know that δ_1 can't take a value more than 1, thus, $\delta_1 = 0.08 = \delta_3$. Similarly,

$$\delta_2(1 - \delta_3) = 0.15$$

$$\delta_2 = \frac{0.15}{0.92} = 0.16 = \delta_4$$

4.1 Normalizing the Frequency

To normalize the frequency, so that the range of frequencies lie between $-\pi$ and $+\pi$. We can use the below formula:

$$\omega = \frac{\Omega \cdot 2\pi}{\Omega_{sampling}} = \frac{\Omega \cdot \pi}{315 \cdot 10^3}$$

4.2 Converting the values to analog

We can convert these values by using the below formula:

$$\Omega = \tan\left(\frac{\omega}{2}\right)$$

4.3 Tabulated Values

Below is the table to convert between the formulas:

f(kHz)	80	85	115	120	190	195	225	230
Normalized Frequency (ω)	0.25π	0.27π	0.365π	0.38π	0.60π	0.62π	0.71π	0.73π
Analog Equivalent (Ω)	0.41	0.45	0.645	0.68	1.38	1.47	2.04	2.21

Table 1: Frequency values and their corresponding normalized and analog equivalents

4.4 Calculating BandPass Filter Analog Characteristics

Below is the expected Filter value for the BandPass Filter.

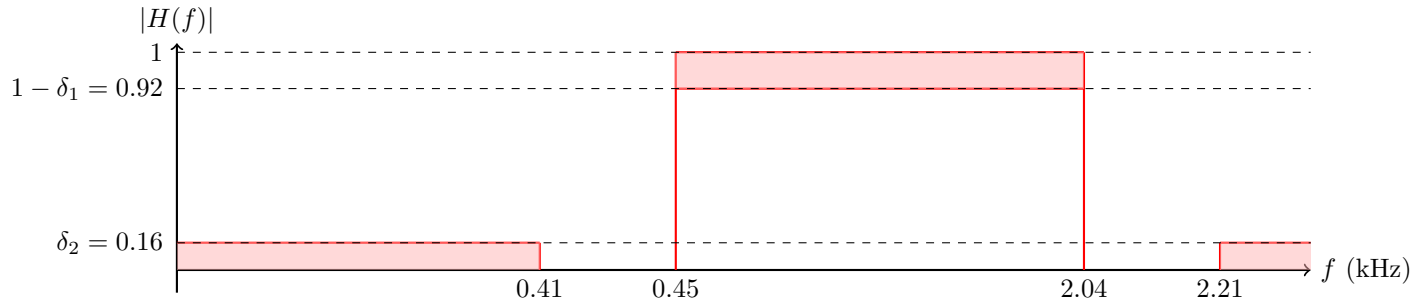


Figure 3: Desired BandPass Filters (Not drawn to Scale)

Using the result for converting a Bandpass filter to Chebyshev LPF; as taught in class on "10th February 2025 (Monday) (L-16)".

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

$$\Omega_0^2 = \Omega_{p1}\Omega_{p2}$$

$$B = \Omega_{p2} - \Omega_{p1}$$

The below is the filter response of a typical bandpass filter:

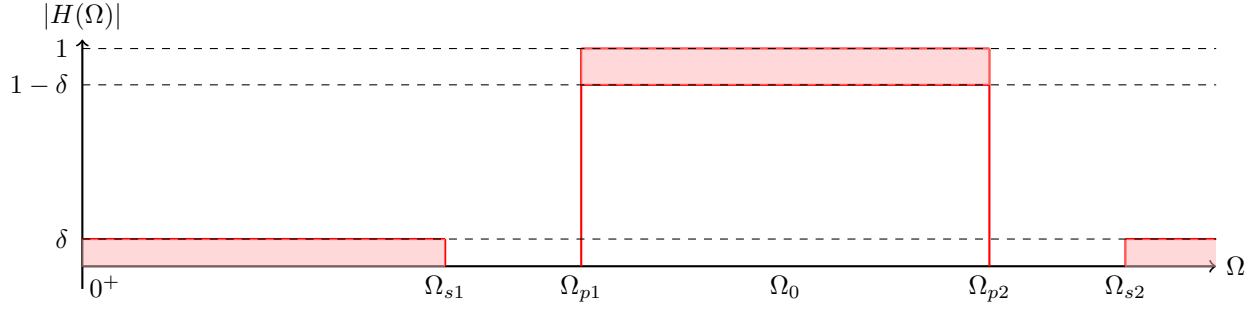


Figure 4: Frequency Response of a typical BandPass Filters

After the above operation on the filter response, the below LPF is obtained:

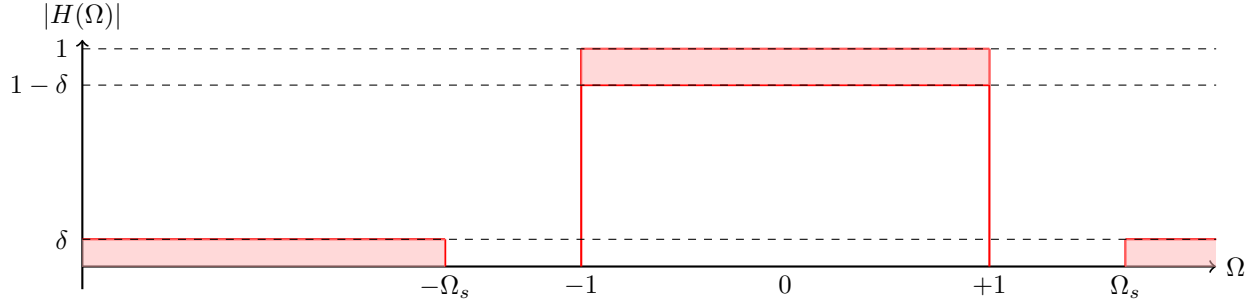


Figure 5: Frequency Response of a typical BandPass Filters

Comparing the graphs of Figure 5 and Figure 4 and equating the respective values, we obtain:

$$\Omega_{s1} = 0.41; \Omega_{s2} = 2.21$$

$$\Omega_{p1} = 0.45; \Omega_{p2} = 2.04$$

Substituting the above values of Ω_{p1} and Ω_{p2} into the formulae of Ω_L :

$$\Omega_L = \frac{\Omega^2 - (0.45)(2.04)}{\Omega \cdot (2.04 - 0.45)} = \frac{\Omega^2 - 0.918}{1.59\Omega}$$

Also, Ω_{p2} gets mapped to +1, Ω_{p1} gets mapped to -1. Ω_{s1} gets mapped to Ω_{Ls1} while Ω_{s2} gets mapped to Ω_{Ls2} . We can calculate the values of Ω_{Ls1} and Ω_{Ls2} :

$$\Omega_{Ls1}(\Omega) = \Omega_{Ls1}(0.41) = \frac{(0.41)^2 - 0.918}{0.41 \cdot 1.59} = -1.15$$

$$\Omega_{Ls2}(\Omega) = \Omega_{Ls2}(2.21) = \frac{(2.21)^2 - 0.918}{2.21 \cdot 1.59} = 1.128 \approx 1.13$$

4.4.1 Specifications of a Low Pass Filter

Ω_p (passband edge) = +1

Ω_s (stopband edge) = $\min(\Omega_{Ls1}, \Omega_{Ls2}) = \min(1.13, 1.15) = 1.13$

$$D_1 = \frac{1}{(1-\delta_1)^2} - 1 = \frac{1}{0.92^2} - 1 = 0.18$$

$$D_2 = \frac{1}{\delta_2^2} - 1 = \frac{1}{0.16^2} - 1 = 38$$

Using the above obtained results to calculate the order of a Chebyshev filter; using information from the

NPTEL 22B Lecture on Youtube as well as NPTEL website

$$N = \left\lceil \frac{\cosh^{-1}(\sqrt{\frac{D_2}{D_1}})}{\cosh^{-1}(\frac{\Omega_s}{\Omega_p})} \right\rceil$$

$$N \geq \left\lceil \frac{\cosh^{-1}(\sqrt{\frac{38}{0.18}})}{\cosh^{-1}(\frac{1.13}{1})} \right\rceil = \left\lceil \frac{3.368}{0.504} \right\rceil = \left\lceil 6.675 \right\rceil = 7$$

Since, we choose the minimum possible value of N to save resources, the **Order of the Chebyshev Filter is 7**.

4.5 Calculating BandStop Filter Analog Characteristics

Below is the expected Filter Response required for the Design.

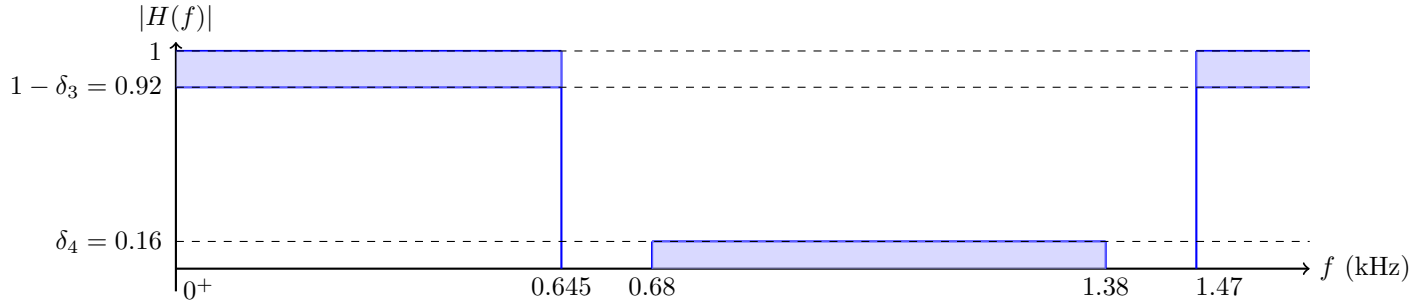


Figure 6: Desired BandStop Filters (Not drawn to Scale)

Using the result for converting a Bandstop filter to Chebyshev LPF; as taught in class on "10th February 2025 (Monday) (L-16)".

$$\Omega_L = \frac{B\Omega}{\Omega_0^2 - \Omega^2}$$

$$B = \Omega_{p2} - \Omega_{p1}$$

$$\Omega_0^2 = \Omega_{p1}\Omega_{p2}$$

The below is the filter response of a typical bandpass filter:

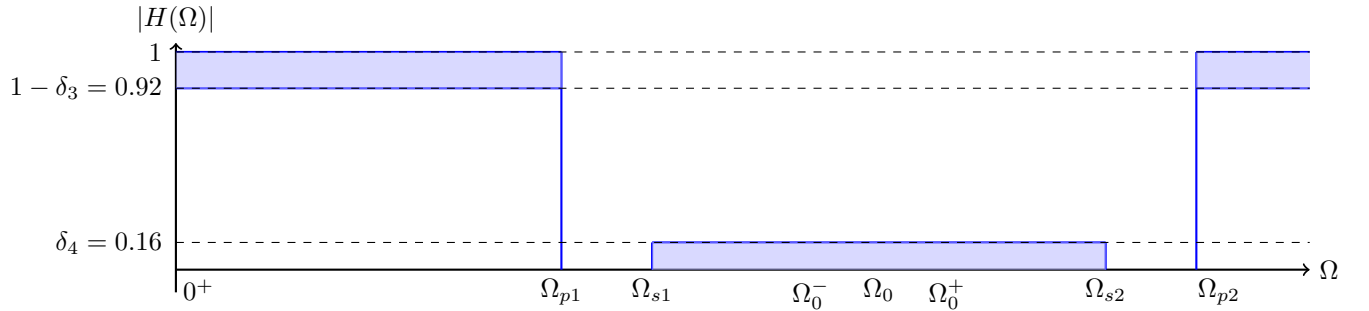


Figure 7: Frequency of a typical Bandstop filter

After the above operation on the filter response, the below LPF is obtained:

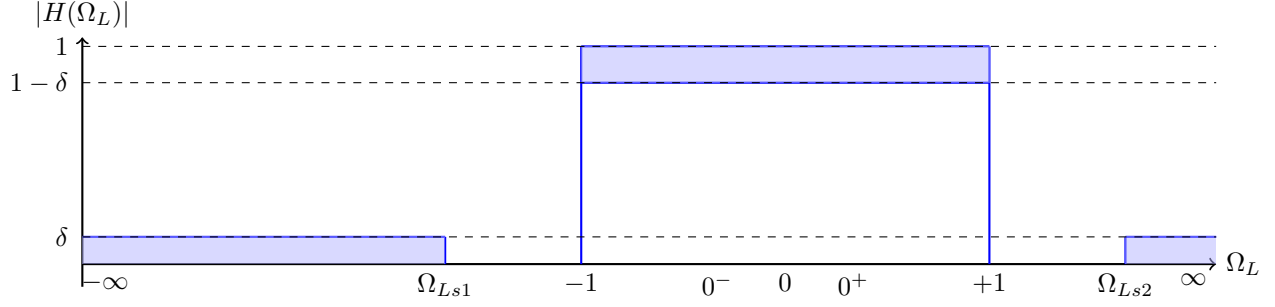


Figure 8: Frequency Response of a typical Lowpass Filters

Unlike the BandPass filter, this one typically doesn't follow a one-to-one mapping, instead it follows the below mapping: Comparing the values from Figure 7 and Figure 8:

Ω	Ω_0^+	Ω_{s2}	Ω_{p2}	$+\infty$	Ω_0	0^+	Ω_{p1}	Ω_{s1}	Ω_0^-
Ω_L	$-\infty$	Ω_{Ls2}	-1	0^-	0	0^+	1	Ω_{Ls1}	$+\infty$

Table 2: Conversion from BandStop to LPF

$$\Omega_{s1} = 0.68; \Omega_{s2} = 1.38$$

$$\Omega_{p1} = 0.645; \Omega_{p2} = 1.47$$

$$\Omega_0^2 = \Omega_{p1} \cdot \Omega_{p2} = 0.645 \cdot 1.47 = 0.94815$$

$$B = \Omega_{p2} - \Omega_{p1} = 1.47 - 0.645 = 0.825$$

$$\Omega_{Ls1}(\Omega) = \Omega_{Ls1}(0.68) = \frac{0.825 \cdot 0.68}{0.94815 - (0.68)^2} = 1.154 \approx 1.15$$

$$\Omega_{Ls2}(\Omega) = \Omega_{Ls2}(1.38) = \frac{0.825 \cdot 1.38}{0.94815 - (1.38)^2} = -1.19$$

4.5.1 Specifications of a Low Pass Filter

$$\Omega_p \text{ (passband edge)} = +1$$

$$\Omega_s \text{ (stopband edge)} = \min(\Omega_{Ls1}, \Omega_{Ls2}) = \min(1.15, 1.19) = 1.15$$

$$D_1 = \frac{1}{(1-\delta_1)^2} - 1 = \frac{1}{0.92^2} - 1 = 0.18$$

$$D_2 = \frac{1}{\delta_2^2} - 1 = \frac{1}{0.16^2} - 1 = 38$$

Using the above obtained results to calculate the order of a Chebyshev filter; using information from the NPTEL 22B Lecture on Youtube as well as NPTEL website

$$N = \left\lceil \frac{\cosh^{-1}\left(+\sqrt{\frac{D_2}{D_1}}\right)}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)} \right\rceil$$

$$N \geq \left\lceil \frac{\cosh^{-1}\left(+\sqrt{\frac{38}{0.18}}\right)}{\cosh^{-1}\left(\frac{1.15}{1}\right)} \right\rceil = \left\lceil \frac{3.368}{0.541} \right\rceil = \left\lceil 6.224 \right\rceil = 7$$

Since, we choose the minimum possible value of N to save resources, the **Order of the Chebyshev Filter is 7**.

5 Manual Calculation for Poles

We know that the roots of a chebysehv filter are given by:

$$s_k = \sigma_k + j\Omega_k$$

$$\sigma_k = -a \cdot \sin \frac{(2k+1)\pi}{2N}$$

$$\Omega_k = b \cdot \cos \frac{(2k+1)\pi}{2N}$$

$$s_k = -\sinh\left(\frac{1}{N} \cdot \sinh^{-1} \frac{1}{\epsilon}\right) \cdot \sin \frac{(2k+1)\pi}{2N} + j \cdot \cosh\left(\frac{1}{N} \cdot \sinh^{-1} \frac{1}{\epsilon}\right) \cdot \cos \frac{(2k+1)\pi}{2N}$$

5.1 BandPass Filter

Ω_s	1.13
Ω_p	1
$\epsilon (= \sqrt{D_1})$	$0.18^{0.5} = 0.424$
D_2	38
N (Number of Poles)	7
a (= $\sinh(\frac{1}{N} \cdot \sinh^{-1} \frac{1}{\epsilon})$)	0.229
b (= $\cosh(\frac{1}{N} \cdot \sinh^{-1} \frac{1}{\epsilon})$)	1.026

5.2 BandStop Filter

Ω_s	1.15
Ω_p	1
$\epsilon (= \sqrt{D_1})$	$0.18^{0.5} = 0.424$
D_2	38
N (Number of Poles)	7
a (= $\sinh(\frac{1}{N} \cdot \sinh^{-1} \frac{1}{\epsilon})$)	0.229
b (= $\cosh(\frac{1}{N} \cdot \sinh^{-1} \frac{1}{\epsilon})$)	1.026

5.3 Poles Calculation

Since the value of N and $\Omega_p = 1$ is same, the poles will remain same. Thus, poles are given by:
For N = 7, k will vary from k = 0 to k = 6.

$$s_k = -0.229 \cdot \sin \left(\frac{(2k+1)\pi}{14} \right) + j \cdot 1.026 \cdot \cos \left(\frac{(2k+1)\pi}{14} \right)$$

$$s_0 = -0.229 \cdot \sin \left(\frac{\pi}{14} \right) + j \cdot 1.026 \cdot \cos \left(\frac{\pi}{14} \right)$$

$$s_1 = -0.229 \cdot \sin \left(\frac{3\pi}{14} \right) + j \cdot 1.026 \cdot \cos \left(\frac{3\pi}{14} \right)$$

$$s_2 = -0.229 \cdot \sin\left(\frac{5\pi}{14}\right) + j \cdot 1.026 \cdot \cos\left(\frac{5\pi}{14}\right)$$

$$s_3 = -0.229 \cdot \sin\left(\frac{7\pi}{14}\right) + j \cdot 1.026 \cdot \cos\left(\frac{7\pi}{14}\right)$$

$$s_4 = -0.229 \cdot \sin\left(\frac{9\pi}{14}\right) + j \cdot 1.026 \cdot \cos\left(\frac{9\pi}{14}\right)$$

$$s_5 = -0.229 \cdot \sin\left(\frac{11\pi}{14}\right) + j \cdot 1.026 \cdot \cos\left(\frac{11\pi}{14}\right)$$

$$s_6 = -0.229 \cdot \sin\left(\frac{13\pi}{14}\right) + j \cdot 1.026 \cdot \cos\left(\frac{13\pi}{14}\right)$$

The system function is given by:

$$H(s) = \frac{K}{\prod_k (s - s_k)} = \frac{K}{D_N(s)}$$

To find K for odd value of N , we use the given formula:

$$K = D_N(s) \Big|_{s=0}$$

where $D_N(s)$ is the denominator of the system function:

$$D_N(s) = \prod_{k=0}^{N-1} (s - s_k)$$

For $N = 7$, we substitute $s = 0$:

$$K = \prod_{k=0}^6 (0 - s_k) = (-1)^7 \prod_{k=0}^6 s_k$$

Since $(-1)^7 = -1$, we get:

$$K = - \prod_{k=0}^6 s_k$$

Each s_k is given by:

$$s_k = -0.229 \cdot \sin\left(\frac{(2k+1)\pi}{14}\right) + j \cdot 1.026 \cdot \cos\left(\frac{(2k+1)\pi}{14}\right)$$

Thus, the computed value of K is:

$$K \approx 0.0367$$

$$H(s) = \frac{0.0367}{\prod_k (s - s_k)}$$

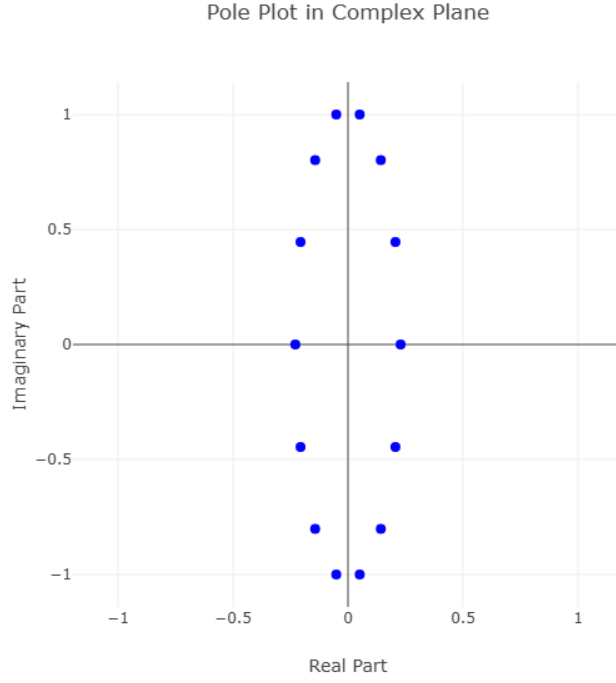


Figure 9: Plot of Chebyshev Filter for BandPass / BandStop Filter (They have the same poles in this case)

5.4 Filter Response for BandPass Filter

$$f(x) = \frac{1 - e^{-ix}}{1 + e^{-ix}}$$

$$\frac{\prod_{k=0}^6 \left(-i \cos \left(\frac{(2k+1)\pi}{14} \right) \cosh(0.227540122101) + \sin \left(\frac{(2k+1)\pi}{14} \right) \sinh(0.227540122101) \right)}{\prod_{k=0}^6 \left(\frac{(f(x))^2 + 0.918}{1.59 \cdot f(x)} - i \cos \left(\frac{(2k+1)\pi}{14} \right) \cosh(0.227540122101) + \sin \left(\frac{(2k+1)\pi}{14} \right) \sinh(0.227540122101) \right)}$$

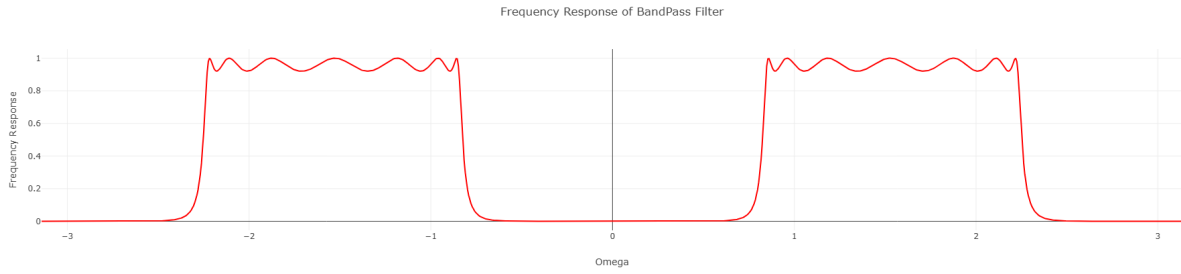


Figure 10: Plot of Filter Response for BandPass Filter

5.5 Filter Response for BandStop Filter

$$f(x) = \frac{1 - e^{-ix}}{1 + e^{-ix}}$$

$$\frac{\prod_{k=0}^6 \left(-i \cos \left(\frac{(2k+1)\pi}{14} \right) \cosh(0.227540122101) + \sin \left(\frac{(2k+1)\pi}{14} \right) \sinh(0.227540122101) \right)}{\prod_{k=0}^6 \left(\frac{0.825 \cdot f(x)}{(f(x))^2 + 0.94815} - i \cos \left(\frac{(2k+1)\pi}{14} \right) \cosh(0.227540122101) + \sin \left(\frac{(2k+1)\pi}{14} \right) \sinh(0.227540122101) \right)}$$

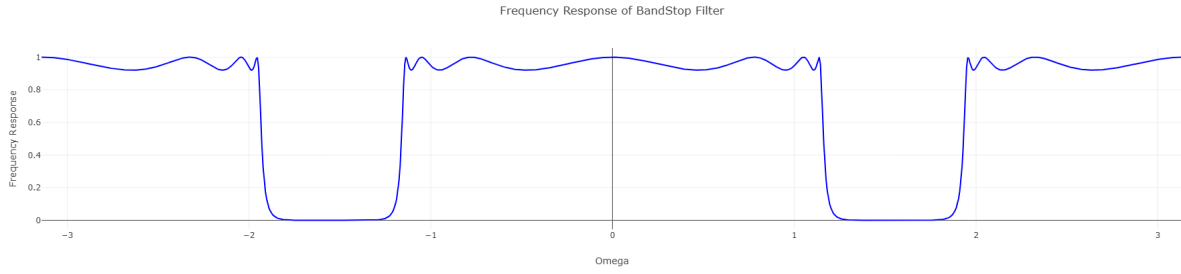


Figure 11: Plot of Filter Response for BandStop Filter

5.6 Filter Response for Cascaded Filter

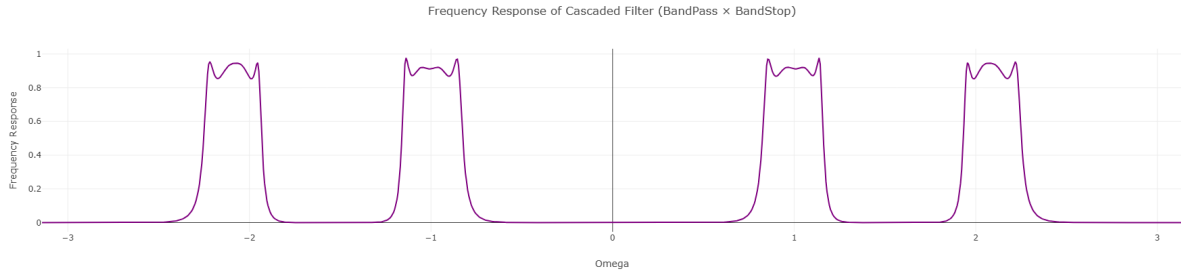


Figure 12: Plot of Filter Response for Cascaded Filter

6 Code for Poles Plot and Filter Response Plot

The following section provides the code used to generate the plots in Python.

6.1 JavaScript Code for Poles Plot

```
1 function plotPoles() {
2     const A_values = [-0.229, 0.229];
3     const B = 1.026;
4     let theta = Array.from({length: 7}, (_, k) => (Math.PI * (2 * k + 1)) / 14);
5     let poles = [];
6
7     A_values.forEach(A => {
8         theta.forEach(t => {
9             let realPart = A * Math.sin(t);
10            let imagPart = B * Math.cos(t);
11            poles.push({x: realPart, y: imagPart});
12        });
13    });
14
15    let trace = {
16        x: poles.map(p => p.x),
17        y: poles.map(p => p.y),
18        mode: 'markers',
19        marker: { color: 'blue', size: 8 },
20        type: 'scatter'
21    };
22
23    let layout = {
24        title: 'Pole Plot in Complex Plane',
25        xaxis: { title: 'Real Part', zeroline: true, scaleanchor: "y" },
26        yaxis: { title: 'Imaginary Part', zeroline: true },
27        showlegend: false
28    };
29
30    Plotly.newPlot('pole-plot', [trace], layout);
31 }
32
33 plotPoles();
```

Listing 1: Filter Type Definition

6.2 JavaScript Code for BandPass Filter Response

```
1 window.onload = function () {
2     console.log("Math.js Loaded:", typeof math !== "undefined");
3     console.log("Plotly Loaded:", typeof Plotly !== "undefined");
4
5     if (typeof math === "undefined" || typeof Plotly === "undefined") {
6         alert("Error: Math.js or Plotly.js failed to load. Check your internet connection.");
7     }
8     return;
9 }
10
11 function BandPassFilter(x) {
12     let Numerator = math.complex(1, 0);
13     let Denominator = math.complex(1, 0);
14
15     for (let k = 0; k < 7; k++) {
16         let angle = (2 * k + 1) * Math.PI / 14;
17         let term1 = math.multiply(
18             math.complex(0, -1),
19             math.multiply(Math.cos(angle),
20                 Math.cosh(0.227540122101)));
21     }
```

```

20     let term2 = math.multiply(Math.sin(angle), Math.sinh(0.227540122101));
21
22     let exponent = math.exp(math.multiply(math.complex(0, -1), x));
23     let fraction = math.divide(math.subtract(1, exponent), math.add(1, exponent));
24
25     let term3 = math.divide(math.add(math.pow(fraction, 2), 0.918), math.multiply
(1.59, fraction));
26
27     Numerator = math.multiply(Numerator, math.add(term1, term2));
28     Denominator = math.multiply(Denominator, math.subtract(math.subtract(term3,
term1), term2));
29     }
30
31     return math.divide(Numerator, Denominator);
32 }
33
34 // Values on the x-axis
35 let x_values = math.range(-Math.PI, Math.PI, 2 * Math.PI / 1000).toArray();
36
37 // BandPass Filter Response
38 let y_bandpass = x_values.map(x => BandPassFilter(x));
39 let mag_bandpass = y_bandpass.map(y => math.abs(y));
40
41 let trace1 = {
42     x: x_values,
43     y: mag_bandpass,
44     mode: 'lines',
45     line: { color: 'red' },
46     name: 'BandPass Magnitude'
47 };
48
49 let layout1 = {
50     title: "Frequency Response of BandPass Filter",
51     xaxis: { title: "Omega" },
52     yaxis: { title: "Frequency Response" },
53     grid: { visible: true }
54 };
55
56 Plotly.newPlot("bandpass-plot", [trace1], layout1);
57 };

```

Listing 2: Filter Type Definition

6.3 JavaScript Code for BandStop Filter

```

1 window.onload = function () {
2     console.log("Math.js Loaded:", typeof math !== "undefined");
3     console.log("Plotly Loaded:", typeof Plotly !== "undefined");
4
5     if (typeof math === "undefined" || typeof Plotly === "undefined") {
6         alert("Error: Math.js or Plotly.js failed to load. Check your internet connection.");
7     }
8     return;
9 }
10
11 function BandStopFilter(x) {
12     let Numerator = math.complex(1, 0);
13     let Denominator = math.complex(1, 0);
14
15     for (let k = 0; k < 7; k++) {
16         let angle = (2 * k + 1) * Math.PI / 14;
17         let term1 = math.multiply(math.complex(0, -1), math.multiply(Math.cos(angle),
Math.cosh(0.227540122101)));
18         let term2 = math.multiply(Math.sin(angle), Math.sinh(0.227540122101));
19         let exponent = math.exp(math.multiply(math.complex(0, -1), x));

```

```

20         let fraction = math.divide(math.subtract(1, exponent), math.add(1, exponent));
21
22         let term3 = math.divide(math.multiply(0.825, fraction), math.add(math.pow(
fraction, 2), 0.94815));
23
24         Numerator = math.multiply(Numerator, math.add(term1, term2));
25         Denominator = math.multiply(Denominator, math.subtract(math.add(term3, term2),
term1));
26     }
27
28     return math.divide(Numerator, Denominator);
29 }
30
31 // Values on the x-axis
32 let x_values = math.range(-Math.PI, Math.PI, 2 * Math.PI / 1000).toArray();
33
34 // BandStop Filter Response
35 let y_bandstop = x_values.map(x => BandStopFilter(x));
36 let mag_bandstop = y_bandstop.map(y => math.abs(y));
37
38 let trace2 = {
39     x: x_values,
40     y: mag_bandstop,
41     mode: 'lines',
42     line: { color: 'blue' },
43     name: 'BandStop Magnitude'
44 };
45
46 let layout2 = {
47     title: "Frequency Response of BandStop Filter",
48     xaxis: { title: "Omega" },
49     yaxis: { title: "Frequency Response" },
50     grid: { visible: true }
51 };
52
53 Plotly.newPlot("bandstop-plot", [trace2], layout2);
54 };

```

Listing 3: Filter Type Definition

6.4 JavaScript Code for Cascaded Filter Response

```

1 window.onload = function () {
2     console.log("Math.js Loaded:", typeof math !== "undefined");
3     console.log("Plotly Loaded:", typeof Plotly !== "undefined");
4
5     if (typeof math === "undefined" || typeof Plotly === "undefined") {
6         alert("Error: Math.js or Plotly.js failed to load. Check your internet connection.");
7     }
8     return;
9 }
10
11 // Values on the x-axis
12 let x_values = math.range(-Math.PI, Math.PI, 2 * Math.PI / 1000).toArray();
13 // Cascaded Filter Response
14 let y_cascaded = y_bandpass.map((bp, i) => math.multiply(bp, y_bandstop[i]));
15 let mag_cascaded = y_cascaded.map(y => math.abs(y));
16
17 let trace3 = {
18     x: x_values,
19     y: mag_cascaded,
20     mode: 'lines',
21     line: { color: 'purple' },
22     name: 'Cascaded Filter Magnitude'
23 };
24
25

```

```

24     let layout3 = {
25         title: "Frequency Response of Cascaded Filter (BandPass      BandStop)",
26         xaxis: { title: "Omega" },
27         yaxis: { title: "Frequency Response" },
28         grid: { visible: true }
29     };
30
31     Plotly.newPlot("cascaded-plot", [trace3], layout3);
32 };

```

Listing 4: Filter Type Definition

6.5 Combined JavaScript Code for Filter Response

```

1  <!DOCTYPE html>
2  <html lang="en">
3  <head>
4      <meta charset="UTF-8">
5      <meta name="viewport" content="width=device-width, initial-scale=1.0">
6      <title>Filter Frequency Response</title>
7      <script src="https://cdnjs.cloudflare.com/ajax/libs/mathjs/11.3.2/math.js" defer></script>
8      <script src="https://cdn.plot.ly/plotly-2.20.0.min.js" defer></script>
9  </head>
10 <body>
11
12 <h2>Frequency Response of BandPass Filter</h2>
13 <div id="bandpass-plot"></div>
14
15 <h2>Frequency Response of BandStop Filter</h2>
16 <div id="bandstop-plot"></div>
17
18 <h2>Frequency Response of Cascaded Filter (BandPass      BandStop)</h2>
19 <div id="cascaded-plot"></div>
20
21 <script>
22     window.onload = function () {
23         console.log("Math.js Loaded:", typeof math !== "undefined");
24         console.log("Plotly Loaded:", typeof Plotly !== "undefined");
25
26         if (typeof math === "undefined" || typeof Plotly === "undefined") {
27             alert("Error: Math.js or Plotly.js failed to load. Check your internet
28 connection.");
29             return;
30         }
31
32         function BandPassFilter(x) {
33             let Numerator = math.complex(1, 0);
34             let Denominator = math.complex(1, 0);
35
36             for (let k = 0; k < 7; k++) {
37                 let angle = (2 * k + 1) * Math.PI / 14;
38                 let term1 = math.multiply(math.complex(0, -1), math.multiply(Math.cos(angle)
39 , Math.cosh(0.227540122101)));
40                 let term2 = math.multiply(Math.sin(angle), Math.sinh(0.227540122101));
41
42                 let exponent = math.exp(math.multiply(math.complex(0, -1), x));
43                 let fraction = math.divide(math.subtract(1, exponent), math.add(1, exponent)
44 );
45
46                 let term3 = math.divide(math.add(math.pow(fraction, 2), 0.918), math.
47 multiply(1.59, fraction));
48
49                 Numerator = math.multiply(Numerator, math.add(term1, term2));
50                 Denominator = math.multiply(Denominator, math.subtract(math.subtract(term3,
51 term1), term2));

```

```

47     }
48
49     return math.divide(Numerator, Denominator);
50 }
51
52 function BandStopFilter(x) {
53     let Numerator = math.complex(1, 0);
54     let Denominator = math.complex(1, 0);
55
56     for (let k = 0; k < 7; k++) {
57         let angle = (2 * k + 1) * Math.PI / 14;
58         let term1 = math.multiply(math.complex(0, -1), math.multiply(Math.cos(angle)
, Math.cosh(0.227540122101)));
59         let term2 = math.multiply(Math.sin(angle), Math.sinh(0.227540122101));
60
61         let exponent = math.exp(math.multiply(math.complex(0, -1), x));
62         let fraction = math.divide(math.subtract(1, exponent), math.add(1, exponent)
);
63
64         let term3 = math.divide(math.multiply(0.825, fraction), math.add(math.pow(
fraction, 2), 0.94815));
65
66         Numerator = math.multiply(Numerator, math.add(term1, term2));
67         Denominator = math.multiply(Denominator, math.subtract(math.add(term3, term2
), term1));
68     }
69
70     return math.divide(Numerator, Denominator);
71 }
72
73 // Values on the x-axis
74 let x_values = math.range(-Math.PI, Math.PI, 2 * Math.PI / 1000).toArray();
75
76 // BandPass Filter Response
77 let y_bandpass = x_values.map(x => BandPassFilter(x));
78 let mag_bandpass = y_bandpass.map(y => math.abs(y));
79
80 // BandStop Filter Response
81 let y_bandstop = x_values.map(x => BandStopFilter(x));
82 let mag_bandstop = y_bandstop.map(y => math.abs(y));
83
84 // Cascaded Filter Response
85 let y_cascaded = y_bandpass.map((bp, i) => math.multiply(bp, y_bandstop[i]));
86 let mag_cascaded = y_cascaded.map(y => math.abs(y));
87
88
89
90 let trace1 = {
91     x: x_values,
92     y: mag_bandpass,
93     mode: 'lines',
94     line: { color: 'red' },
95     name: 'BandPass Magnitude'
96 };
97
98 let layout1 = {
99     title: "Frequency Response of BandPass Filter",
100     xaxis: { title: "Omega" },
101     yaxis: { title: "Frequency Response" },
102     grid: { visible: true }
103 };
104
105 Plotly.newPlot("bandpass-plot", [trace1], layout1);
106
107
108
109 let trace2 = {
110     x: x_values,

```



```

111         y: mag_bandstop,
112         mode: 'lines',
113         line: { color: 'blue' },
114         name: 'BandStop Magnitude'
115     };
116
117     let layout2 = {
118         title: "Frequency Response of BandStop Filter",
119         xaxis: { title: "Omega" },
120         yaxis: { title: "Frequency Response" },
121         grid: { visible: true }
122     };
123
124     Plotly.newPlot("bandstop-plot", [trace2], layout2);
125
126
127
128     let trace3 = {
129         x: x_values,
130         y: mag_cascaded,
131         mode: 'lines',
132         line: { color: 'purple' },
133         name: 'Cascaded Filter Magnitude'
134     };
135
136     let layout3 = {
137         title: "Frequency Response of Cascaded Filter (BandPass      BandStop)",
138         xaxis: { title: "Omega" },
139         yaxis: { title: "Frequency Response" },
140         grid: { visible: true }
141     };
142
143     Plotly.newPlot("cascaded-plot", [trace3], layout3);
144     };
145 </script>
146
147 </body>
148 </html>

```

Listing 5: Filter Type Definition

7 Comparison

This document compares the design characteristics of two types of IIR filters: **Butterworth** and **Chebyshev**. The comparison considers parameters such as filter order, passband behavior, stopband characteristics, and computational complexity.

Two filter types are considered:

- **Filter Type 1:** Butterworth Bandpass (Order 22) and Bandstop (Order 19).
- **Filter Type 2:** Chebyshev Bandpass (Order 7) and Bandstop (Order 7).

8 Conclusion

Both filters have their own advantages. The **Butterworth filter** provides a maximally flat passband with a smooth roll-off but requires a higher order for sharp transitions. The **Chebyshev filter**, on the other hand, achieves steeper roll-off with a lower order but introduces ripples in the passband and stopband. The choice depends on the application requirements, balancing flatness, roll-off, and computational efficiency.

Aspect	Butterworth Filter	Chebyshev Filter
Design Approach	Bandpass (N = 22), Bandstop (N = 19)	Bandpass (N = 7), Bandstop (N = 7)
Filter Order	Higher (22/19) – More complexity	Lower (7/7) – Less complexity
Passband Characteristics	Maximally flat, no ripples	Equiripple passband (Type I Chebyshev)
Stopband Characteristics	Smooth attenuation (monotonic)	Sharper roll-off with ripples
Transition Band	Gradual roll-off, requiring high order	Steeper roll-off, lower order suffices
Computational Complexity	Higher due to more coefficients	Lower due to fewer coefficients
Group Delay	Higher due to high order	Lower but with variations from ripple
Implementation Feasibility	More challenging	Easier due to lower order

Table 3: Comparison of Butterworth and Chebyshev Filters

9 Peer Review

Name of student: Anupam Rawat

Name and Roll Number of the reviewer : Jatin Kumar 22B3922

Group Number : 34

Review Comments :

I have reviewed the filter design assignment of Anupam Rawat, Roll Number 22B3982. The filter number assigned to him is 104. Following are my comments on his assignment:

He has correctly implemented the filter, and the response are in accordance to the expected frequency response. He has also correctly used the chebyshev approximations to implement the IIR designs for each filter. He has included all the code, results and their plots in the report.

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I would like to express my sincere gratitude to **Professor V. M. Gadre** for introducing and teaching me the fundamental and profoundly important course on *Digital Signal Processing*. The structured tasks and exercises in this course have greatly enhanced my understanding of various concepts in this field.

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I truly appreciate the guidance and teamwork that is making this journey both enriching and intellectually stimulating.