Optional Midsemester Assignment Submission: EE 338, Spring 2024-25

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Filter Number (M): 104

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1 Aim Of the Assignment

We are required to design an Infinite-Impulse-Response (IIR) Filter, with bands from both Group I and Group II, acting as **equiripple / oscillatory passband**, and all the **stopbands are monotonic / non-oscillatory**. Now, we know that for the case when passband is oscillatory and stopband is monotonic, then the desired Filter type is **Chebyshev**.

2 Specifications Required

The analog signal is bandlimited to 280kHz and it's ideally sampled with a sampling rate of 630kHz. Now 2 x bandwidth ¡ sampling rate. The sampling rate obeys the Nyquist criteria and can be reconstructed without any loss.

1. The tolerance of stopband and passband are $\delta = 0.15$ in magnitude.
2. The Filter number assigned to me is: M = 104
   (a) Q = floor (M / 11) = 9
   (b) R = M mod 11 = 5
3. Range of Group I frequency: (40 + 5D) to (70 + 5D); where D = Q
   (a) **Lower Edge** = 40 + 5D = 40 + 45 = **85kHz**
   (b) **Upper Edge** = 70 + 5D = 70 + 45 = **115kHz**
4. Range of Group II frequency: (170 + 5D) to (200 + 5D); where D = R
   (a) **Lower Edge** = 170 + 5D = 170 + 25 = **195kHz**
   (b) **Upper Edge** = 200 + 5D = 200 + 25 = **225kHz**
5. The transition band on either side of the passband is 5kHz. Group I and II are the passbands.

3 Desired Frequency Response

As per the above specifications, below is the plot of the desired frequency specifications.

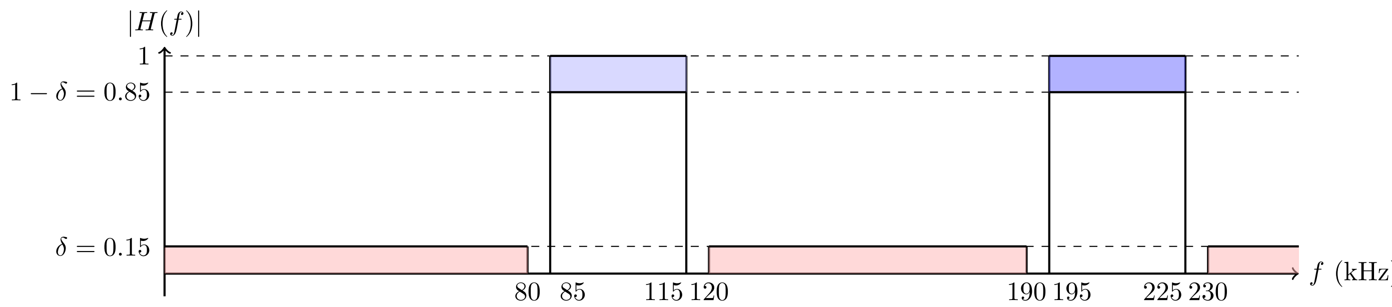

Figure 1: Desired Frequency Response

Now, it should be noted that, the blue regions represents the equiripple / oscillatory passband and the red region represents the monotonic stopband. It should also be noted that, since this is an IIR filter, the upper band is capped to 1.
The light blue region, on the left, corresponds to the Group I of frequency bands, while the slightly darker region on the right, corresponds to the Group II of frequency bands.

3.1 Realization of the Frequency Response

The filter response can be realised by cascading a BandPass Filter with a BandPass Filter *(Series Connection)* OR by adding the result obtained by two BandPass Filters *(Parallel Connection)*. For the purpose of this assignment, we will be using the Cascading option.

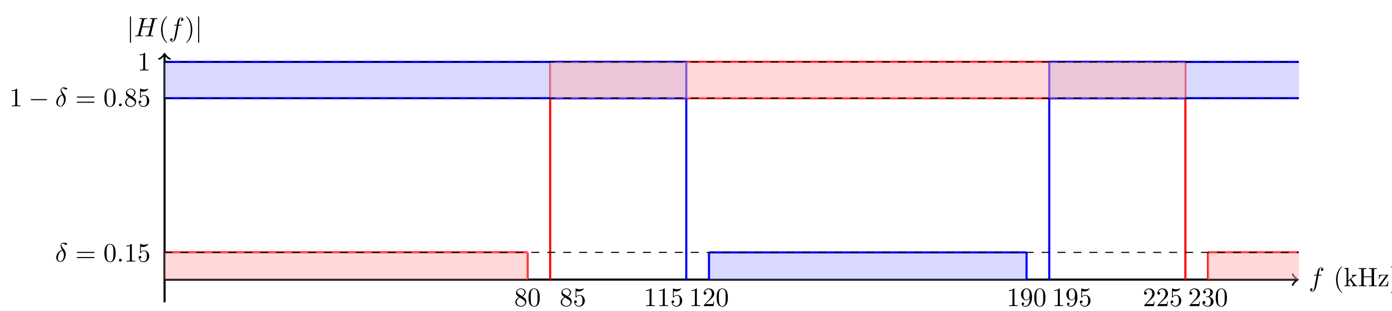

Figure 2: Cascade of BandPass and BandStop Filters

The Blue filter represents the BandStop Filter with stop band from 120kHz to 190kHz, and 5kHz transition band, across the stopband. The red filter represents the BandPass Filter with passband between 85kHz and 225kHz, with a transition band of 5kHz across the passband.

4 Designing Filter

We now require to design the BandPass and BandStop Filter ony by one, to realise the final filter.

1. BandPass Filter
   (a) Passband Range Required: 85kHz - 225kHz
   (b) PassBand Tolerance (arbitrary): δ_1
   (c) StopBand Tolerance (arbitrary): δ_2
   (d) passband Tolerance: 5kHz
2. BandStop Filter
   (a) Passband Range Required: 120kHz - 190kHz
   (b) PassBand Tolerance (arbitrary): δ_3
   (c) StopBand Tolerance (arbitrary): δ_4
   (d) stopband Tolerance: 5kHz

After cascading the two filters; following will be the value of tolerances and the respective frequency range:

$$Tolerances = \begin{cases} \delta_2(1-\delta_3), & 0^+ \leq f < 80kHz \\ (1-\delta_1)(1-\delta_3), & 85kHz \leq f < 115kHz \\ \delta_4(1-\delta_3), & 120kHz \leq f < 190kHz \\ (1-\delta_1)(1-\delta_3), & 195kHz \leq f < 225kHz \\ \delta_2(1-\delta_3), & 230kHz \leq f < 315kHz \end{cases}$$

Comparing the Tolerances from the above table to the Tolerance level values from Figure - 1. Equating for 85kHz $\leq$ f $\leq$ 115kHz and 195kHz $\leq$ f $\leq$ 225kHz:

$$(1-\delta_1)(1-\delta_3) = 1 - \delta = 0.85$$

$$1 - \delta_1 - \delta_3 - \delta_1\delta_3 = 0.85$$

$$\delta_1 + \delta_3 - \delta_1\delta_3 = 0.15$$

Equating for 0kHz $\leq$ f $\leq$ 80kHz and 230kHz $\leq$ f $\leq$ 315kHz:

$$\delta_2(1-\delta_3) = \delta = 0.15$$

But equating for 120kHz $\leq$ f $\leq$ 190kHz gives us:

$$\delta_4(1-\delta_1) = \delta = 0.15$$

For the ease of solving, we can assume: $\delta_1 = \delta_3$ and $\delta_2 = \delta_4$. Thus the above equations become:

$$\delta_1 + \delta_3 - \delta_1\delta_3 = 2\delta_1 - \delta_1^2 = 0.15$$

Solving the quadratic equation,

$$\delta_1 = \frac{+2 \pm \sqrt{(-2)^2 - 4(1)(0.15)}}{2*1}$$

Thus, δ_1 can either be 1.92 or 0.08, but we know that δ_1 can't take a value more than 1, thus, $\delta_1 = 0.08 = \delta_3$ Similarly,

$$\delta_2(1-\delta_3) = 0.15$$

$$\delta_2 = \frac{0.15}{0.92} = 0.16 = \delta_4$$

4.1 Normalizing the Frequency

To normalize the frequency, so that the range of frequencies lie between -π and +π. We can use the below formula:

$$\omega = \frac{\Omega \cdot 2\pi}{\Omega_{sampling}} = \frac{\Omega \cdot \pi}{315 \cdot 10^3}$$

4.2 Converting the values to analog

We can convert these values by using the below formula:

$$\Omega = tan(\frac{\omega}{2})$$

4.3 Tabulated Values

Below is the table to convert between the formulas:

f(kHz)	80	85	115	120	190	195	225	230
Normalized Frequency (ω)	0.25π	0.27π	0.365π	0.38π	0.60π	0.62π	0.71π	0.73π
Analog Equivalent (Ω)	0.41	0.45	0.645	0.68	1.38	1.47	2.04	2.21

Table 1: Frequency values and their corresponding normalized and analog equivalents

4.4 Calculating BandPass Filter Analog Characteristics

Below is the expected Filter value for the BandPass Filter.

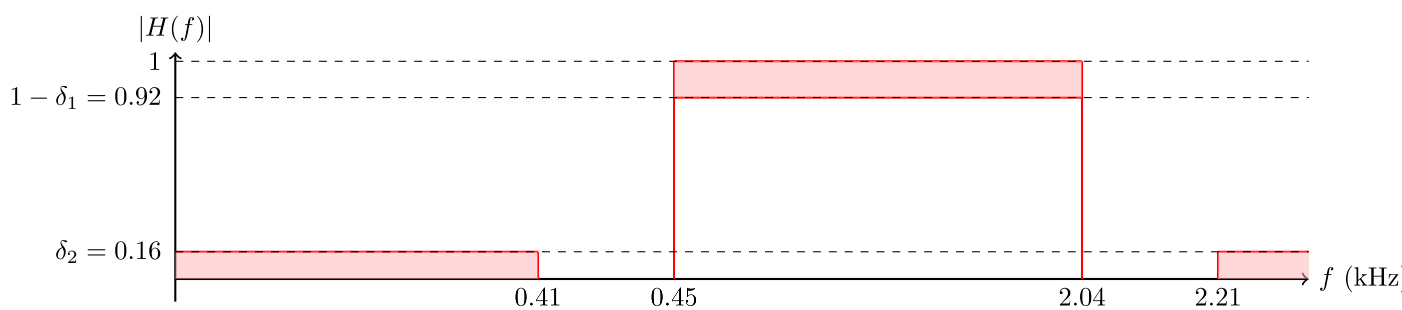

Figure 3: Desired BandPass Filters (Not drawn to Scale)

Using the result for converting a Bandpass filter to Chebyshev LPF; as taught in class on "10th February 2025 (Monday) (L-16)".

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

$$\Omega_0^2 = \Omega_{p1}\Omega_{p2}$$

$$B = \Omega_{p2} - \Omega_{p1}$$

The below is the filter response of a typical bandpass filter:

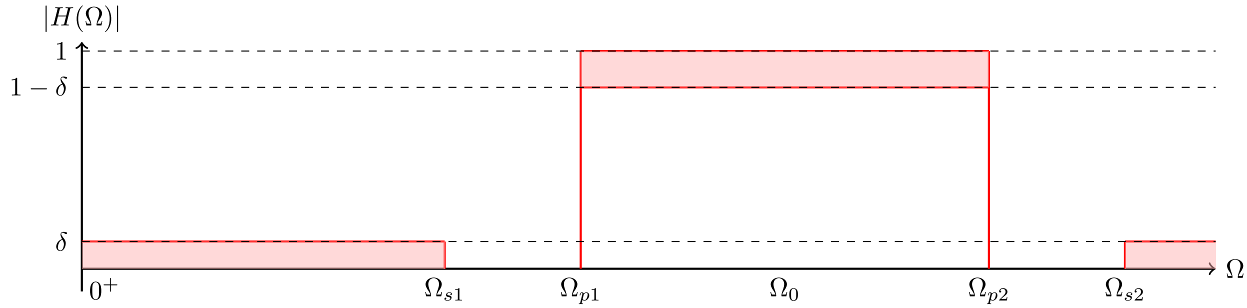

Figure 4: Frequency Response of a typical BandPass Filters

After the above operation on the filter response, the below LPF is obtained:

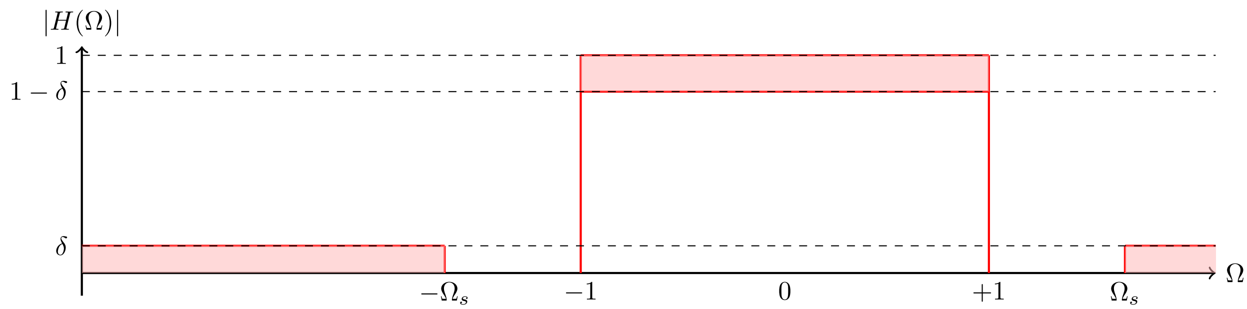

Figure 5: Frequency Response of a typical BandPass Filters

Comparing the graphs of Figure 5 and Figure 4 and equating the respective values, we obtain:

$$\Omega_{s1} = 0.41; \Omega_{s2} = 2.21$$

$$\Omega_{p1} = 0.45; \Omega_{p2} = 2.04$$

Substituting the above values of Ω_{p1} and Ω_{p2} into the formulae of Ω_L:

$$\Omega_L = \frac{\Omega^2 - (0.45)(2.04)}{\Omega \cdot (2.04 - 0.45)} = \frac{\Omega^2 - 0.918}{1.59\Omega}$$

Also, Ω_{p2} gets mapped to +1, Ω_{p1} gets mapped to -1. Ω_{s1} gets mapped to Ω_{Ls1} while Ω_{s2} gets mapped to Ω_{Ls2}. We can calculate the values of Ω_{Ls1} and Ω_{Ls2}:

$$\Omega_{Ls1}(\Omega) = \Omega_{Ls1}(0.41) = \frac{(0.41)^2 - 0.918}{0.41 \cdot 1.59} = -1.15$$

$$\Omega_{Ls2}(\Omega) = \Omega_{Ls2}(2.21) = \frac{(2.21)^2 - 0.918}{2.21 \cdot 1.59} = 1.128 \approx 1.13$$

4.4.1 Specifications of a Low Pass Filter

Ω_p (passband edge) = +1
Ω_s (stopband edge) = $\min(\Omega_{Ls1}, \Omega_{Ls2}) = \min(1.13, 1.15) = 1.13$
$D_1 = \frac{1}{(1-\delta_1)^2} - 1 = \frac{1}{0.92^2} - 1 = 0.18$
$D_2 = \frac{1}{\delta_2^2} - 1 = \frac{1}{0.16^2} - 1 = 38$
Using the above obtained results to calculate the order of a Chebyshev filter; using information from the

NPTEL 22B Lecture on Youtube as well as NPTEL website

$$N = \left\lceil \frac{cosh^{-1}(+\sqrt{\frac{D_2}{D_1}})}{cosh^{-1}(\frac{\Omega_s}{\Omega_p})} \right\rceil$$

$$N \geq \left\lceil \frac{cosh^{-1}(+\sqrt{\frac{38}{0.18}})}{cosh^{-1}(\frac{1.13}{1})} \right\rceil = \left\lceil \frac{3.368}{0.504} \right\rceil = \left\lceil 6.675 \right\rceil = 7$$

Since, we choose the minimum possible value of N to save resources, the **Order of the Chebyshev Filter is 7**.

4.5 Calculating BandStop Filter Analog Characteristics

Below is the expected Filter Response required for the Design.

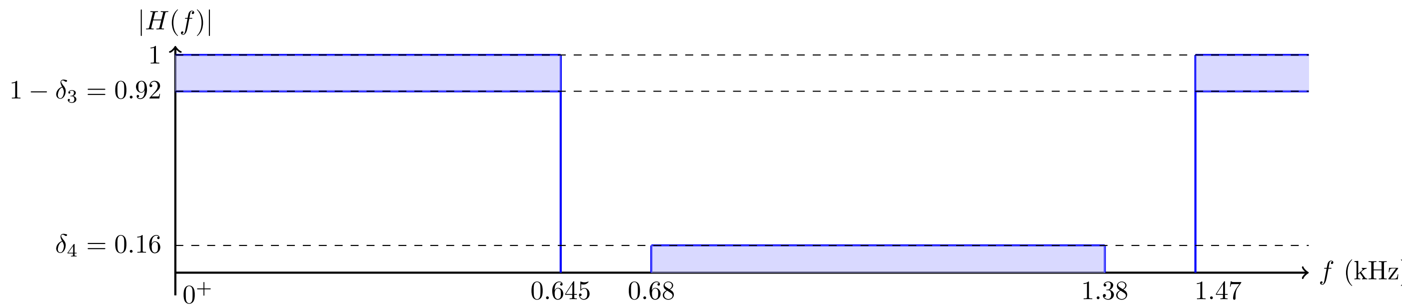

Figure 6: Desired BandStop Filters (Not drawn to Scale)

Using the result for converting a Bandstop filter to Chebyshev LPF; as taught in class on "10th February 2025 (Monday) (L-16)".

$$\Omega_L = \frac{B\Omega}{\Omega_0^2 - \Omega^2}$$

$$B = \Omega_{p2} - \Omega_{p1}$$

$$\Omega_0^2 = \Omega_{p1}\Omega_{p2}$$

The below is the filter response of a typical bandpass filter:

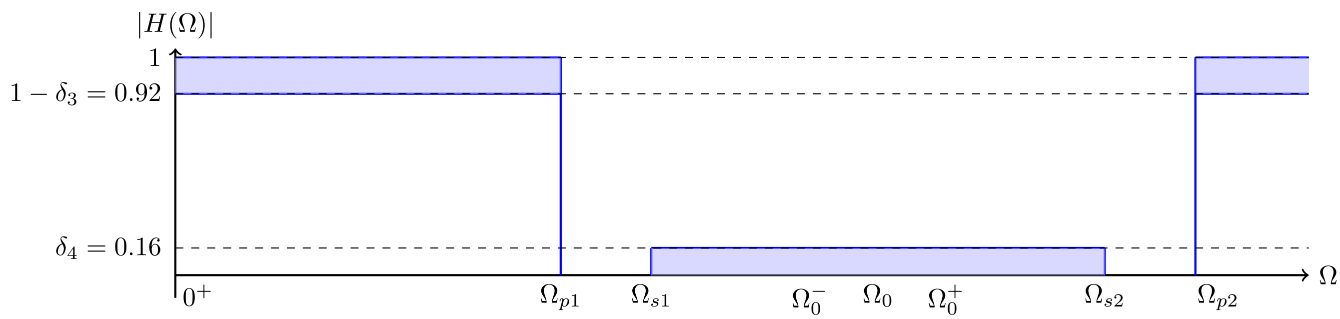

Figure 7: Frequency of a typical Bandstop filter

After the above operation on the filter response, the below LPF is obtained:

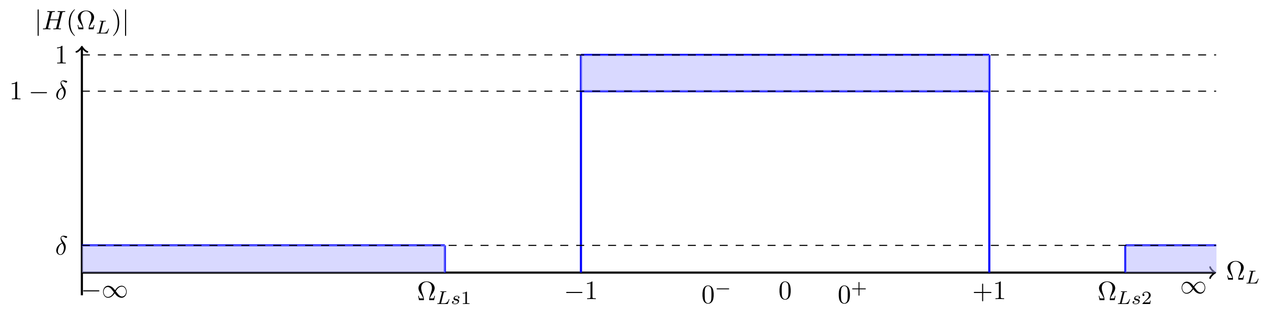

Figure 8: Frequency Response of a typical Lowpass Filters

Unlike the BandPass filter, this one typically doesn't follow a one-to-one mapping, instead it follows the below mapping: Comparing the values from Figure 7 and Figure 8:

Ω	Ω_0^+	Ω_{s2}	Ω_{p2}	$+\infty$	Ω_0	0^+	Ω_{p1}	Ω_{s1}	Ω_0^-
Ω_L	$-\infty$	Ω_{Ls2}	-1	0^-	0	0^+	1	Ω_{Ls1}	$+\infty$

Table 2: Conversion from BandStop to LPF

$$\Omega_{s1} = 0.68; \Omega_{s2} = 1.38$$

$$\Omega_{p1} = 0.645; \Omega_{p2} = 1.47$$

$$\Omega_0^2 = \Omega_{p1} \cdot \Omega_{p2} = 0.645 \cdot 1.47 = 0.94815$$

$$B = \Omega_{p2} - \Omega_{p1} = 1.47 - 0.645 = 0.825$$

$$\Omega_{Ls1}(\Omega) = \Omega_{Ls1}(0.68) = \frac{0.825 \cdot 0.68}{0.94815 - (0.68)^2} = 1.154 \approx 1.15$$

$$\Omega_{Ls2}(\Omega) = \Omega_{Ls2}(1.38) = \frac{0.825 \cdot 1.38}{0.94815 - (1.38)^2} = -1.19$$

4.5.1 Specifications of a Low Pass Filter

Ω_p (passband edge) = +1
Ω_s (stopband edge) = $\min(\Omega_{Ls1}, \Omega_{Ls2}) = \min(1.15, 1.19) = 1.15$
$D_1 = \frac{1}{(1-\delta_1)^2} - 1 = \frac{1}{0.92^2} - 1 = 0.18$
$D_2 = \frac{1}{\delta_2^2} - 1 = \frac{1}{0.16^2} - 1 = 38$
Using the above obtained results to calculate the order of a Chebyshev filter; using information from the NPTEL 22B Lecture on Youtube as well as NPTEL website

$$N = \left\lceil \frac{cosh^{-1}(+\sqrt{\frac{D_2}{D_1}})}{cosh^{-1}(\frac{\Omega_s}{\Omega_p})} \right\rceil$$

$$N \geq \left\lceil \frac{cosh^{-1}(+\sqrt{\frac{38}{0.18}})}{cosh^{-1}(\frac{1.15}{1})} \right\rceil = \left\lceil \frac{3.368}{0.541} \right\rceil = \left\lceil 6.224 \right\rceil = 7$$

Since, we choose the minimum possible value of N to save resources, the **Order of the Chebyshev Filter is 7**.

5 Manual Calculation for Poles

We know that the roots of a chebysehv filter are given by:

$$s_k = \sigma_k + j\Omega_k$$

$$\sigma_k = -a \cdot sin\frac{(2k+1)\pi}{2N}$$

$$\Omega_k = b \cdot cos\frac{(2k+1)\pi}{2N}$$

$$s_k = -sinh(\frac{1}{N} \cdot sinh^{-1}\frac{1}{\epsilon}) \cdot sin\frac{(2k+1)\pi}{2N} + j \cdot cosh(\frac{1}{N} \cdot sinh^{-1}\frac{1}{\epsilon}) \cdot cos\frac{(2k+1)\pi}{2N}$$

5.1 BandPass Filter

Ω_s	1.13
Ω_p	1
$\epsilon(=\sqrt{D_1})$	$0.18^{0.5} = 0.424$
D_2	38
N (Number of Poles)	7
a ($= sinh(\frac{1}{N} \cdot sinh^{-1}\frac{1}{\epsilon})$	0.229
b ($= cosh(\frac{1}{N} \cdot sinh^{-1}\frac{1}{\epsilon})$	1.026

5.2 BandStop Filter

Ω_s	1.15
Ω_p	1
$\epsilon(=\sqrt{D_1})$	$0.18^{0.5} = 0.424$
D_2	38
N (Number of Poles)	7
a ($= sinh(\frac{1}{N} \cdot sinh^{-1}\frac{1}{\epsilon})$	0.229
b ($= cosh(\frac{1}{N} \cdot sinh^{-1}\frac{1}{\epsilon})$	1.026

5.3 Poles Calculation

Since the value of N and $\Omega_p = 1$ is same, the poles will remain same. Thus, poles are given by:
For N = 7, k will vary from k = 0 to k = 6.

$$s_k = -0.229 \cdot sin\left(\frac{(2k+1)\pi}{14}\right) + j \cdot 1.026 \cdot cos\left(\frac{(2k+1)\pi}{14}\right)$$

$$s_0 = -0.229 \cdot \sin\left(\frac{\pi}{14}\right) + j \cdot 1.026 \cdot \cos\left(\frac{\pi}{14}\right)$$

$$s_1 = -0.229 \cdot \sin\left(\frac{3\pi}{14}\right) + j \cdot 1.026 \cdot \cos\left(\frac{3\pi}{14}\right)$$

$$s_2 = -0.229 \cdot \sin\left(\frac{5\pi}{14}\right) + j \cdot 1.026 \cdot \cos\left(\frac{5\pi}{14}\right)$$

$$s_3 = -0.229 \cdot \sin\left(\frac{7\pi}{14}\right) + j \cdot 1.026 \cdot \cos\left(\frac{7\pi}{14}\right)$$

$$s_4 = -0.229 \cdot \sin\left(\frac{9\pi}{14}\right) + j \cdot 1.026 \cdot \cos\left(\frac{9\pi}{14}\right)$$

$$s_5 = -0.229 \cdot \sin\left(\frac{11\pi}{14}\right) + j \cdot 1.026 \cdot \cos\left(\frac{11\pi}{14}\right)$$

$$s_6 = -0.229 \cdot \sin\left(\frac{13\pi}{14}\right) + j \cdot 1.026 \cdot \cos\left(\frac{13\pi}{14}\right)$$

The system function is given by:

$$H(s) = \frac{K}{\Pi_k(s - s_k)} = \frac{K}{D_N(s)}$$

To find K for odd value of N, we use the given formula:

$$K = D_N(s)\Big|_{s=0}$$

where $D_N(s)$ is the denominator of the system function:

$$D_N(s) = \prod_{k=0}^{N-1}(s - s_k)$$

For $N = 7$, we substitute $s = 0$:

$$K = \prod_{k=0}^{6}(0 - s_k) = (-1)^7 \prod_{k=0}^{6} s_k$$

Since $(-1)^7 = -1$, we get:

$$K = -\prod_{k=0}^{6} s_k$$

Each s_k is given by:

$$s_k = -0.229 \cdot \sin\left(\frac{(2k+1)\pi}{14}\right) + j \cdot 1.026 \cdot \cos\left(\frac{(2k+1)\pi}{14}\right)$$

Thus, the computed value of K is:

$$K \approx 0.0367$$

$$H(s) = \frac{0.0367}{\Pi_k(s - s_k)}$$

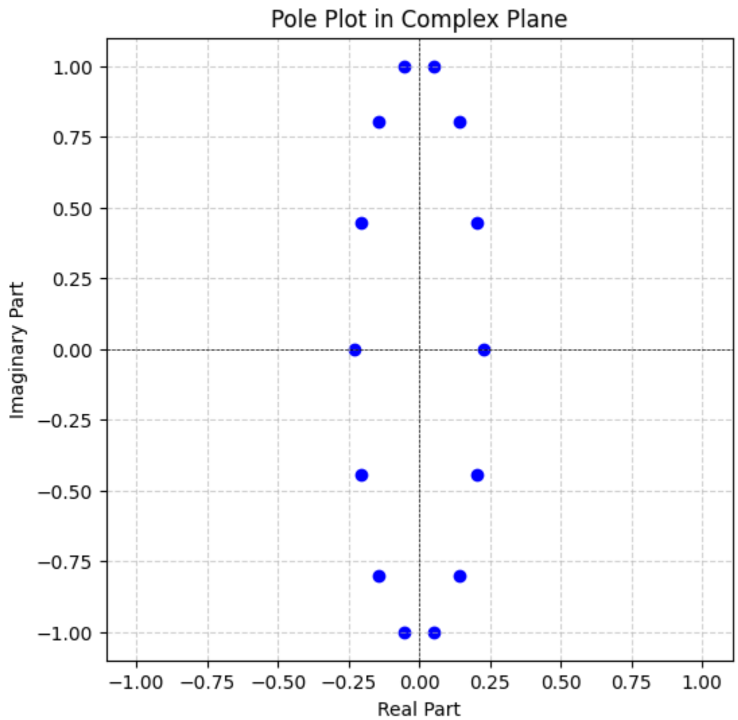

Figure 9: Plot of Chebyshev Filter for BandPass / BandStop Filter (They have the same poles in this case)

5.4 Filter Response for BandPass Filter

$$f(x) = \frac{1 - e^{-ix}}{1 + e^{-ix}}$$

$$\frac{\prod_{k=0}^{6} \left(-i \cos\left(\frac{(2k+1)\pi}{14}\right) \cosh(0.227540122101) + \sin\left(\frac{(2k+1)\pi}{14}\right) \sinh(0.227540122101) \right)}{\prod_{k=0}^{6} \left(\frac{(f(x))^2 + 0.918}{1.59 \cdot f(x)} - i \cos\left(\frac{(2k+1)\pi}{14}\right) \cosh(0.227540122101) + \sin\left(\frac{(2k+1)\pi}{14}\right) \sinh(0.227540122101) \right)}$$

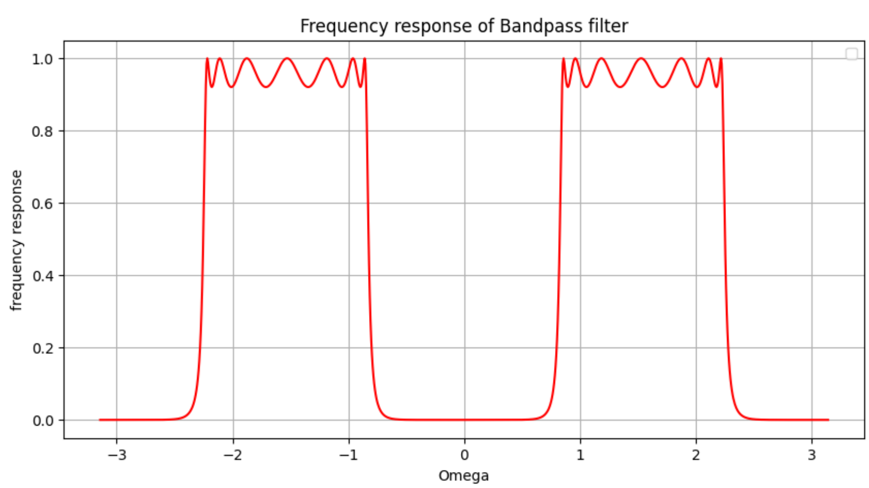

Figure 10: Plot of Filter Response for BandPass Filter

5.5 Filter Response for BandStop Filter

$$f(x) = \frac{1 - e^{-ix}}{1 + e^{-ix}}$$

$$\frac{\prod_{k=0}^{6}\left(-i\cos\left(\frac{(2k+1)\pi}{14}\right)\cosh(0.227540122101) + \sin\left(\frac{(2k+1)\pi}{14}\right)\sinh(0.227540122101)\right)}{\prod_{k=0}^{6}\left(\frac{0.825\cdot f(x)}{(f(x))^2+0.94815} - i\cos\left(\frac{(2k+1)\pi}{14}\right)\cosh(0.227540122101) + \sin\left(\frac{(2k+1)\pi}{14}\right)\sinh(0.227540122101)\right)}$$

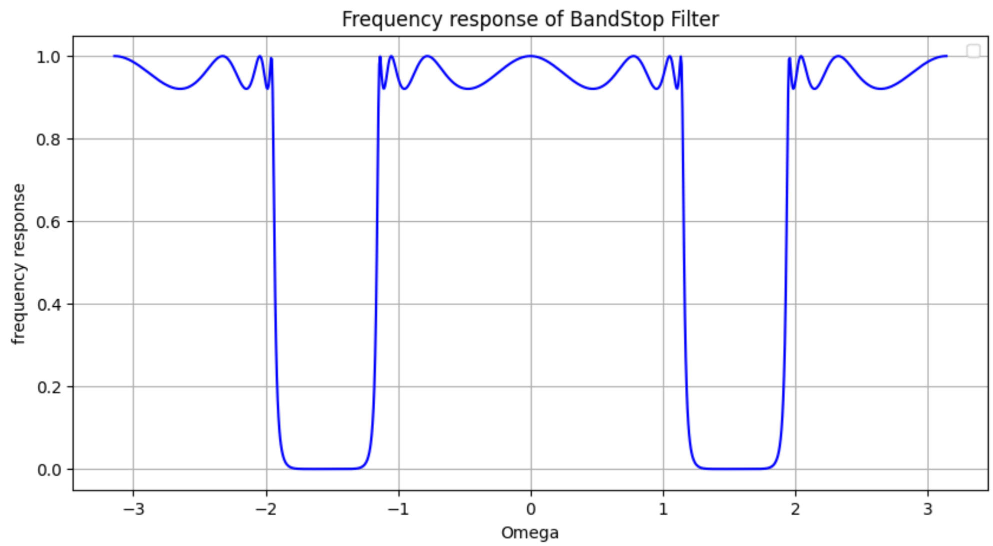

Figure 11: Plot of Filter Response for BandStop Filter

5.6 Filter Response for Cascaded Filter

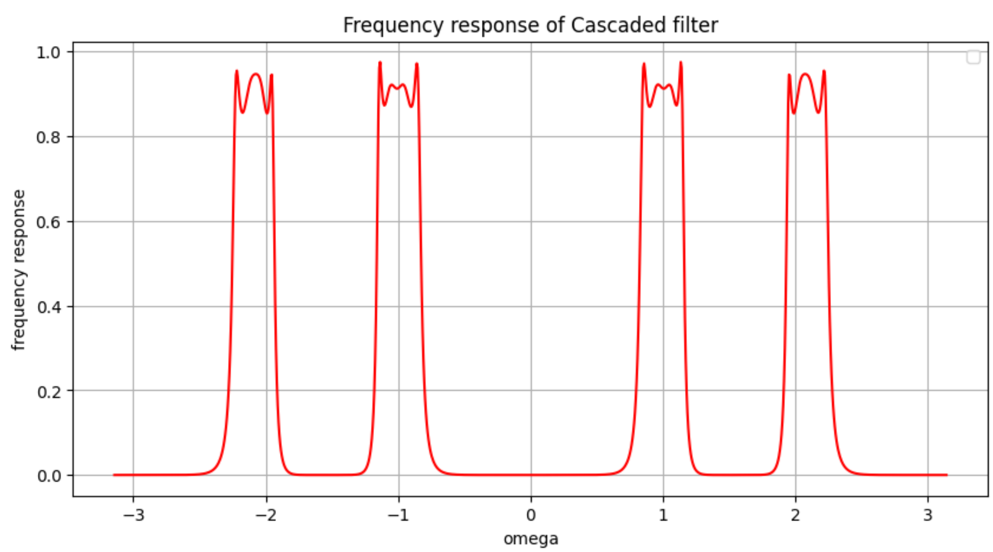

Figure 12: Plot of Filter Response for Cascaded Filter

6 Code for Plot Generation

The following section provides the code used to generate the plots in Python.

6.1 Python Code for BandPass Filter

```
def BandPassFilter(x):
    Numerator = 1
    Denominator = 1

    for k in range(7):
        term1 = (-1j) * np.cos((2 * k + 1) * np.pi / 14) * np.cosh(0.227540122101)
        term2 = np.sin((2 * k + 1) * np.pi / 14) * np.sinh(0.227540122101)

        fraction = (1 - np.exp(-1j * x)) / (1 + np.exp(-1j * x))
        term3 = (fraction**2 + 0.918) / (1.59 * fraction)

        Numerator *= term1 + term2
        Denominator *= term3 - term1 + term2

    return Numerator / Denominator

x_values = np.linspace(-np.pi, np.pi, 1000)
y_values = np.array([BandPassFilter(x) for x in x_values])
y_values_real = np.nan_to_num(y_values.real, nan=0.0)
y_values_imag = np.nan_to_num(y_values.imag, nan=0.0)

plt.figure(figsize=(10, 5))
plt.plot(x_values, (y_values_real**2 + y_values_imag**2) ** 0.5, color="r",)
plt.xlabel("Omega")
plt.ylabel("frequency response")
plt.legend()
plt.title("Frequency response of Bandpass filter")
plt.grid()
plt.show()
```

Listing 1: Filter Type Definition

6.2 Python Code for BandStop Filter

```
def BandStopFilter(x):
    Numerator = 1
    Denominator = 1

    for k in range(7):
        term1 = (-1j) * np.cos((2 * k + 1) * np.pi / 14) * np.cosh(0.227540122101)
        term2 = np.sin((2 * k + 1) * np.pi / 14) * np.sinh(0.227540122101)

        fraction = (1 - np.exp(-1j * x)) / (1 + np.exp(-1j * x))
        term3 = (0.825 * fraction) / (fraction**2 + 0.94815)

        Numerator *= term1 + term2
        Denominator *= term3 - term1 + term2

    return Numerator / Denominator


x_values = np.linspace(-np.pi, np.pi, 1000)
y_values = np.array([BandStopFilter(x) for x in x_values])
y_values_real = np.nan_to_num(y_values.real, nan=0.0)
y_values_imag = np.nan_to_num(y_values.imag, nan=0.0)

plt.figure(figsize=(10, 5))
plt.plot(x_values, (y_values_real**2 + y_values_imag**2) ** 0.5, color="b")
```

```python
plt.xlabel("Omega")
plt.ylabel("frequency response")
plt.legend()
plt.title("Frequency response of BandStop Filter")
plt.grid()
plt.show()
```

Listing 2: Filter Type Definition

6.3 Python Code for Cascaded Filter

```python
x_values = np.linspace(-np.pi, np.pi, 1000)
y_values = np.array([BandPassFilter(x)*BandStopFilter(x) for x in x_values])
y_values_real = np.nan_to_num(y_values.real, nan=0.0)
y_values_imag = np.nan_to_num(y_values.imag, nan=0.0)

plt.figure(figsize=(10, 5))
plt.plot(x_values, (y_values_real**2 + y_values_imag**2) ** 0.5, color="r")
plt.xlabel("omega")
plt.ylabel("frequency response")
plt.legend()
plt.title("Frequency response of Cascaded filter")
plt.grid()
plt.show()
```

Listing 3: Filter Type Definition

7 Comparison

This document compares the design characteristics of two types of IIR filters: **Butterworth** and **Chebyshev**. The comparison considers parameters such as filter order, passband behavior, stopband characteristics, and computational complexity.

Two filter types are considered:

- **Filter Type 1**: Butterworth Bandpass (Order 22) and Bandstop (Order 19).
- **Filter Type 2**: Chebyshev Bandpass (Order 7) and Bandstop (Order 7).

Aspect	**Butterworth Filter**	**Chebyshev Filter**
Design Approach	Bandpass (N = 22), Bandstop (N = 19)	Bandpass (N = 7), Bandstop (N = 7)
Filter Order	Higher (22/19) – More complexity	Lower (7/7) – Less complexity
Passband Characteristics	Maximally flat, no ripples	Equiripple passband (Type I Chebyshev)
Stopband Characteristics	Smooth attenuation (monotonic)	Sharper roll-off with ripples
Transition Band	Gradual roll-off, requiring high order	Steeper roll-off, lower order suffices
Computational Complexity	Higher due to more coefficients	Lower due to fewer coefficients
Group Delay	Higher due to high order	Lower but with variations from ripple
Implementation Feasibility	More challenging	Easier due to lower order

Table 3: Comparison of Butterworth and Chebyshev Filters

8 Conclusion

Both filters have their own advantages. The **Butterworth filter** provides a maximally flat passband with a smooth roll-off but requires a higher order for sharp transitions. The **Chebyshev filter**, on the other hand, achieves steeper roll-off with a lower order but introduces ripples in the passband and stopband. The choice depends on the application requirements, balancing flatness, roll-off, and computational efficiency.

9 Peer Review

Name of student: Anupam Rawat
Name and Roll Number of the reviewer : Jatin Kumar 22B3922
Group Number : 34
Review Comments :
I have reviewed the filter design assignment of Anupam Rawat, Roll Number 22B3982. The filter number assigned to him is 104. Following are my comments on his assignment:
He has correctly implemented the filter, and the response are in accordance to the expected frequency response. He has also correctly used the butterworth approximations to implement the IIR designs for each filter. He has included all the code, results and their plots in the report.

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