$\frac{dx}{1} = \frac{dy}{-x}$ ← тут ещё страница

$y' = -x$

$y = -\frac{x^2}{2} + c$

$y + \frac{x^2}{2} = \varphi(x,y)$

$\tilde{x} = \varphi(x,y) = y + \frac{x^2}{2}$

$\tilde{y} = \psi(x,y) = \alpha x + \beta y$

$\tilde{a}(x,y) = 0 \Rightarrow \tilde{b} = 0$

$\tilde{c}\tilde{u}_{\tilde{y}\tilde{y}} + \ldots = 0$

$u_{xx} = ?\,\tilde{u}_{\tilde{x}} + ?\,\tilde{u}_{\tilde{y}} + \ldots$

$u_{xy} \;//\; u_{yy}$

$\tilde{c} = (\psi_x + x\psi_y)^2 =$
$= (\alpha - \beta x)^2$

$u(x,y) = u(x(\tilde{x},\tilde{y}), y(\tilde{x},\tilde{y})) = u(\tilde{x},\tilde{y})$

$u(x,y) = \tilde{u}\left(y + \frac{x^2}{2}, \alpha x + \beta y\right)$

$(\alpha - x\beta)^2 \tilde{u}_{\tilde{y}\tilde{y}} + \ldots = 0$

это ур-е исходное, на которое кое-кто опозорал

$u_{xx} - 2xu_{xy} + x^2 u_{yy} - 2u_y = 0$

$u_x = \tilde{u}_{\tilde{x}} \cdot x + \tilde{u}_{\tilde{y}} \cdot \alpha$ $\qquad u_y = \tilde{u}_{\tilde{x}} + \tilde{u}_{\tilde{y}} \cdot \beta$

$u_{xx} = \tilde{u}_{\tilde{x}} + \ldots$ на u_{xy} и u_{yy} всё равно

$\tilde{u}_{\tilde{x}} + (\alpha - x\beta)^2 \tilde{u}_{\tilde{y}\tilde{y}} - 2\tilde{u}_{\tilde{x}} - 2\beta\tilde{u}_{\tilde{y}} = 0$

$$(\alpha - x\beta)^2 \hat{u}_{\hat{y}\hat{y}} - \hat{u}_{\hat{x}} - 2\beta \hat{u}_{\hat{y}} = 0$$

Попробуем взять $\alpha = 1$ $\beta = 0$:

$$\hat{u}_{\hat{y}\hat{y}} - \hat{u}_{\hat{x}} = 0$$

ур-е теплопров-ти

$$u_t - u_{xx} = 0$$

(параболич.)

это получилось с заменой: $\begin{cases} \hat{x} = y + \frac{x^2}{2} \\ \hat{y} = x \end{cases}$

считаем опр-ль

$$\begin{vmatrix} x & 1 \\ 1 & 0 \end{vmatrix} = -1 \neq 0 \Rightarrow \text{невырожд.}, \text{ ура!}$$

а если взять замену

$$\begin{cases} \hat{x} = y + \frac{x^2}{2} \\ \hat{y} = y \end{cases} \quad \begin{vmatrix} x & 1 \\ 0 & 1 \end{vmatrix} = x \begin{matrix} >0 \\ <0 \end{matrix}$$

$$x^2 \hat{u}_{\hat{y}\hat{y}} + \hat{u}_{\hat{x}} - 2\hat{u}_{\hat{y}} = 0$$

"какая-то абракадабра"

2.11.5) $\underline{\hat{a} = 0} \Rightarrow \hat{x} = \Phi(x,y) \Rightarrow$ <u>если $d = 0$</u>, то $\hat{b} = 0$

$$\hat{b} = a(x,y)\Phi_x\Psi_x + b(x,y)(\Phi_x\Psi_y + \Phi_y\Psi_x) + c(x,y)\Phi_y\Psi_y =$$

неважно при этом, какая Ψ $(\hat{y} = \Psi(x,y))$

$$\hat{a} = 0 \Rightarrow a(\Phi_x)^2 + 2b\Phi_x\Phi_y + c(\Phi_y)^2 = 0$$

$$d = b^2 - ac = 0$$

$$a(\Phi_x - \lambda\Phi_y)^2 = 0 \qquad \lambda = -\frac{b}{a}$$

$$\Phi_x + \frac{b}{a}\Phi_y = 0 \qquad \Phi_x = -\frac{b}{a}\Phi_y$$

$$= \cancel{-b\Phi_y\Psi_x} - \frac{b^2}{a}\Phi_y\Psi_y + \cancel{b\Phi_y\Psi_x} + c\Phi_y\Psi_y = \Big(\underbrace{c - \frac{b^2}{a}}_{0 \text{ (т.к. } d=0)}\Big)\Phi_y\Psi_y = 0$$

ч.т.д.

2.2. (11) $(y-1)^2 u_{xx} + 2(y-1) u_{xy} + u_{yy} + u_x - u_y = 0$

1/3

$a u_{xx} + 2b u_{xy} + c u_{yy} + \ldots = 0$ ур-е эллиптическое

хотим найти Φ, Ψ : $\hat{x} = \Phi$, $\hat{y} = \Psi$

$\hat{a}(x,y)\hat{u}_{\hat{x}\hat{x}} + 2\hat{b}\hat{u}_{\hat{x}\hat{y}} + \hat{c}(x,y)\hat{u}_{\hat{y}\hat{y}} + \ldots = 0$

$\underbrace{\hat{u}_{\hat{x}\hat{x}} + \hat{u}_{\hat{y}\hat{y}}}_{\Delta u} + \ldots = 0$

1) Пробуем заменить $\hat{a}(x,y) = 0$

$\mathcal{L}_2^0(x, y, \Phi_x, \Phi_y) = a(\Phi_x)^2 + 2b\,\Phi_x\Phi_y + c(\Phi_y)^2 = 0$

$a(\Phi_x^1 - \lambda_1\Phi_y^1)(\Phi_x^2 - \lambda_2\Phi_y^2) = 0$

$d(x,y) = b^2 - ac < 0$ (т.к. эллипт.)

$\lambda_1 = \dfrac{-b + i\sqrt{-d}}{a}$ $\lambda_2 = \dfrac{-b - i\sqrt{-d}}{a}$ т.е. $\lambda_1 = \overline{\lambda_2}$

$\Phi^1 = A(x,y) + iB(x,y)$ $\Phi^2 = \overline{\Phi^1} = A - iB$

Замена: $\begin{cases} \hat{x} = A \\ \hat{y} = B \end{cases}$ ← надо дома подумать, как до этой хуйобы догадаться

Тогда в новых переменных получим:

$\hat{u}_{\hat{x}\hat{x}} + \hat{u}_{\hat{y}\hat{y}} + \ldots = 0$

тут мог быть какой-то один. коэф-т

Теперь пример рассмотрим:

2.6. $xu_{xx} - yu_{yy} + u_x - u = 0$

$d = 0 + xy = xy < 0$

$\hat{a} = 0 \qquad x(\Phi_x - \lambda_1\Phi_y)(\Phi_x - \lambda_2\Phi_y) = 0$

$$\lambda_{12} = \frac{\pm i\sqrt{-xy}}{x} = \pm i\sqrt{\frac{-y}{x}}$$

$$\frac{dy}{dx} = -\lambda_1 = -i\sqrt{\frac{-y}{x}} \qquad y' = \frac{-i\sqrt{|y|}}{\sqrt{x}}$$

рассм-ем во 2й четверти

$$\frac{d\Phi_x}{d\Phi_y} \qquad \frac{y'}{\sqrt{|y|}} = \frac{-i}{\sqrt{|x|}} \qquad \frac{y'}{\sqrt{|y|}} = \frac{-i}{\sqrt{-x}}$$

$$\Rightarrow \sqrt{y} = i\sqrt{-x} + c$$

$\Phi = \sqrt{y} - i\sqrt{-x}$

Замена: $\begin{cases} \hat{x} = \sqrt{y} \\ \hat{y} = -\sqrt{-x} \end{cases}$ $\qquad u_x = \hat{u}_{\hat{x}} \cdot 0 + \hat{u}_{\hat{y}} \cdot \frac{1}{2\sqrt{-x}}$

$$u_y = \hat{u}_{\hat{x}} \cdot \frac{1}{2\sqrt{y}}$$

$$u_{xx} = \hat{u}_{\hat{y}\hat{y}} \cdot \left(\frac{1}{2\sqrt{-x}}\right)^2 + \hat{u}_{\hat{y}} \cdot \frac{1}{4\sqrt{-x^3}} = -\hat{u}_{\hat{y}\hat{y}}\frac{1}{4x} + \hat{u}_{\hat{y}}\frac{1}{4\sqrt{-x^3}}$$

$$u_{yy} = \hat{u}_{\hat{x}\hat{x}}\left(\frac{1}{2\sqrt{y}}\right)^2 - \hat{u}_{\hat{x}} \cdot \frac{1}{4\sqrt{y^3}}$$

$$-\frac{1}{4}\hat{u}_{\hat{y}\hat{y}} + \frac{x}{4x\sqrt{-x}}\hat{u}_{\hat{y}} - \frac{y}{4y}\hat{u}_{\hat{x}\hat{x}} + \frac{1}{4\sqrt{y}}\hat{u}_{\hat{x}} - \frac{\hat{u}_{\hat{y}}}{2\sqrt{-x}} - \hat{u} = 0$$

$$-\frac{1}{4}(\hat{u}_{\hat{x}\hat{x}} + \hat{u}_{\hat{y}\hat{y}}) + \frac{1}{4\hat{y}}\hat{u}_{\hat{y}} + \frac{1}{\hat{x}}\hat{u}_{\hat{x}} + \frac{1}{2\hat{y}}\hat{u}_{\hat{y}} - \hat{u} = 0$$

Д/з:

4) 2.11(5)   5) 2.2(8, 9) – в обл. эллипт.

подумать