16/4/21

$$\begin{cases} u_{tt} - \Delta u = f(x) \\ u|_{t=0} = \varphi_1(x) \\ u_t|_{t=0} = \varphi_2(x) \end{cases}$$

$$\Delta^4 \varphi_1 = \Delta^4 \varphi_2 = \Delta^4 f = 0$$

Применим к задаче Δ^4

$$\begin{cases} \Delta^4 u_{tt} - \Delta^5 u = 0 \\ \Delta^4 u|_{t=0} = 0 \\ \Delta^4 u_t|_{t=0} = 0 \end{cases} \Rightarrow \Delta^4 u = 0 \quad (\text{в силу единств. реш})$$

А теперь Δ^3:

$$\begin{cases} \Delta^3 u_{tt} = \Delta^3 f(x) \\ \Delta^3 u|_{t=0} = \Delta^3 \varphi_1 \\ \Delta^3 u_t|_{t=0} = \Delta^3 \varphi_2 \end{cases}$$

$$\Delta^3 u = \Delta^3 \varphi_1 + \Delta^3 \varphi_2 t + \frac{t^2}{2} \Delta^3 f(x)$$

Теперь Δ^2

$$\begin{cases} \Delta^2 u_{tt} - \Delta^3 u = \Delta^2 u_{tt} \underbrace{- \Delta^3 \varphi_1 - \Delta^3 \varphi_2 t - \frac{t^2}{2} \Delta^3 f}_{f_1(t,x)} = \Delta^2 f \\ \Delta^2 u|_{t=0} = \Delta^2 \varphi_1 \\ \Delta^2 u_t|_{t=0} = \Delta^2 \varphi_2 \end{cases}$$

$$\Delta^2 u = \int_0^t (t-s) f_1(s,x)\, ds + \frac{t^2}{2} \Delta^2 f + \Delta^2 \varphi_1 + t \Delta^2 \varphi_2$$

<u>Сб 12.37(б)</u>

$$n=2 \qquad \begin{cases} u_{tt} - 3\Delta u = x^3 + y^3 \\ u|_{t=0} = x^2 \\ u_t|_{t=0} = y^2 \end{cases}$$

Переставлять Δ и $\frac{\partial}{\partial t}$ законно, так как ф-ии хорошие, т.к. f, φ_1 и φ_2 хорошие (т.е. реш-е обладает большой гладкостью)

Сначала Δ^2:

$$\begin{cases} \Delta^2 u_{tt} - 3\Delta^3 u = 0 \\ \Delta^2 u|_{t=0} = 0 \\ \Delta^2 u_t|_{t=0} = 0 \end{cases} \Rightarrow \Delta^2 u = 0$$

$$\begin{cases} \Delta u_{tt} - 3\Delta^2 u = \Delta u_{tt} = \Delta(x^3 + y^3) = 6x + 6y \\ \Delta u|_{t=0} = \Delta x^2 = 2 \\ \Delta u_t|_{t=0} = \Delta y^2 = 2 \end{cases}$$

$$\Delta u = 2 + 2t + \frac{t^2}{2}(6x + 6y) = 2 + 2t + 3t^2(x+y)$$

Т.о. $\begin{cases} u_{tt} = 6+6t+9t^2(x+y)+x^3+y^3 \\ u|_{t=0} = x^2 \\ u_t|_{t=0} = y^2 \end{cases}$

$$u = x^2 + y^2 t + \int_0^t (t-s)(6+6s+9s^2(x+y)+x^3+y^3)\,ds$$

ф-ла для реш-я

Д/З $\quad \Pi = \{ t+\alpha x+\beta y = 0 \}$

$$\begin{cases} u_{tt} - u_{xx} - u_{yy} \\ u|_\Pi = \varphi_1(t,x,y) \\ \frac{\partial u}{\partial \nu}\big|_\Pi = \varphi_2(t,x,y) \end{cases}$$

Пытаемся строить пример Адамара

$e^{n(t+\alpha x+\beta y+ipx+iqy)-\sqrt{n}}$

$n^2 e^{t+\alpha x+\beta y+ipx+iqy}(1-\alpha^2-\beta^2+p^2+q^2)e^{-\sqrt{n}}$

$p^2+q^2 = \alpha^2+\beta^2+1$

<u>Напом</u> $\quad S = \{ \Phi(t,x,y) = 0 \}$

$$(\Phi_t)^2 - (\Phi_x)^2 - (\Phi_y)^2 \Big|_S = 0$$

тут ещё ге-та было

$t+\alpha x+\beta y = 0 \iff \hat{t} = 0$

$$\begin{cases} \hat{t} = t+\alpha x+\beta y \\ \hat{x} = x \\ \hat{y} = y \end{cases} \qquad \begin{pmatrix} 1 & 0 & 0 \\ \alpha & 1 & 0 \\ \beta & 0 & 1 \end{pmatrix} \text{ – замена невырожд.}$$

$u(t,x,y) = u(t(\hat{t},x,y),x,y) = \tilde{u}(\hat{t},x,y)$

$u_x = \tilde{u}_x + \tilde{u}_{\hat{t}}$

$u_{xx} = \tilde{u}_{xx} + 2\alpha\tilde{u}_{\hat{t}x} + \alpha^2\tilde{u}_{\hat{t}\hat{t}}$

$u_{yy} = \tilde{u}_{yy} + 2\beta\tilde{u}_{\hat{t}y} + \beta^2\tilde{u}_{\hat{t}\hat{t}}$

$$(1-\alpha^2-\beta^2)\hat u_{\hat t\hat t} - \Delta \tilde u - 2(\alpha \hat u_{\hat t x} + \beta \tilde u_{\hat t y}) = 0$$

$$\tilde u|_{\hat t=0} = \tilde\varphi_1(\hat t, x, y) = \varphi_1(t+\alpha x+\beta y, x, y)$$

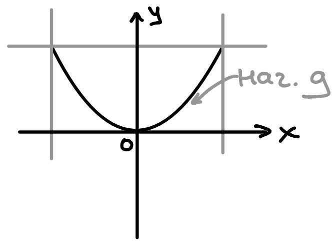
$$\hat u_{\hat t}|_{\hat t=0} = \tilde\varphi_3$$

$$u_t \frac{1}{\sqrt{1+\alpha^2+\beta^2}} + u_x\frac{\alpha}{\sqrt{\dots}} + u_y\frac{\beta}{\sqrt{\dots}} = \left(\frac{1+\alpha^2+\beta^2}{\sqrt{1+\alpha^2+\beta^2}}\right)\tilde u_{\hat t} + \frac{\alpha}{\sqrt{\dots}}\tilde u_{\hat x} + \frac{\beta}{\sqrt{\dots}}\hat u_{\hat y} = \varphi_2$$

$$\tilde u_{\hat t} = \frac{1}{\sqrt{1+\alpha^2+\beta^2}}\left(\varphi_2 - \frac{\alpha}{\sqrt{\dots}}\tilde u_{\hat x} - \frac{\beta}{\sqrt{\dots}}\hat u_{\hat y}\right)$$

<u>Б012.4</u>

$$\begin{cases} u_{xy} = 0, \quad |x|\le 1, \quad 0\le y\le 1 \\ u|_{y=x^2} = 0 \\ u_y|_{y=x^2} = u_1(x) \end{cases}$$



$\int dx$: $u_y = A(y)$ Рассм. $x>0$ $A(x^2)=u_1(x) \Rightarrow A(t)=u_1(\sqrt t)$

$$u = \int_{y_0\in[0,1]}^{y} A(t)\,dt + B(x) \overset{u_y|_{y=x^2}=u_1}{=} \int_{y_0}^{y} u_1(\sqrt t)\,dt + B(x)$$

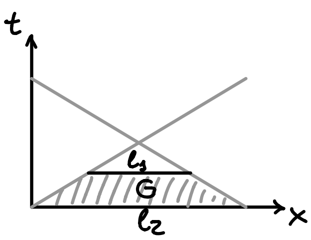
$$u|_{y=x^2}=0 \Rightarrow \int_{y_0}^{y} u_1(\sqrt t)\,dt - \int_{y_0}^{x^2} u_1(\sqrt t)\,dt = \int_{x^2}^{y} u_1(\sqrt t)\,dt =$$

$$= \Big/ \begin{matrix}\sqrt t = s\\ t = 2s\,ds\end{matrix} \Big/ = 2\int_x^{\sqrt y} u_1(s)\, s\, ds$$

А если $x<0$, то $2\int_x^{\sqrt y} u_1(-s)\,ds$

Ф-ия должна быть непр $\Rightarrow \int_0^{\sqrt y} s u_1(s)\,ds = \int_0^{\sqrt y} s u_1(-s)\,ds$ $\forall y\in[0,1] \Rightarrow$

$$u_1(t) \equiv u_1(-t)$$



$$\int_{l_1}\left((u_x)^2+(u_t)^2\right)dt \le \int_{l_2}\left((u_x)^2+(u_t)^2\right.dx$$

$$\int_G (u_t u_{tt} - u_t u_{xx})\,dx\,dt = 0$$

$$\int_G\left(\frac12(u_t)^2_t - (u_t u_x)_x + \frac12(u_x)^2_t\right) dx\,dt = 0$$

$$\int_{l_1}\left(\frac12 u_t^2 + \frac12 u_x\right)dz - \int_{l_2}\left(\frac12 u_t^2+\frac12 u_x^2\right)dz +$$

$$+\int_{S_{\text{БОК}}} \left(\frac{1}{2}(u_t)^2 + \frac{1}{2}(u_x)^2 \right) \cos(\nu, t) - u_t u_x \cos(\nu, x) \, dz$$