$$Au_t + Bu_y = 0 \qquad A = \begin{bmatrix} 2 & -6 \\ -1 & 0 \end{bmatrix} \cdot 2 \qquad B = \begin{bmatrix} -3 & 8 \\ 3 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -6 \\ -1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 3 & 0 \\ 3 & -4 \end{bmatrix}$$

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \rightsquigarrow \tilde{u} = \begin{bmatrix} u_2 \\ u_1 \end{bmatrix}$$

$$6 \cdot \begin{bmatrix} -1 & 0 \\ 0 & -6 \end{bmatrix} \qquad \begin{bmatrix} 3 & -4 \\ 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -6 & 0 \\ 0 & -6 \end{bmatrix} := D \qquad \begin{bmatrix} 18 & -24 \\ 3 & 0 \end{bmatrix} := A$$

$$D\tilde{u}_t + A\tilde{u}_x = 0$$

$$A = CJC^{-1}$$

$$CC^{-1}D\tilde{u}_t + CJC^{-1}\tilde{u}_x = 0$$

$$CDC^{-1}\tilde{u}_t + CJC^{-1}\tilde{u}_x = 0 \quad \Big|\, C^{-1}\cdot$$

$$DC^{-1}\tilde{u}_t + JC^{-1}\tilde{u}_x = 0$$

$$Dv_t + Jv_x = 0 \quad, \text{ где } v = C^{-1}\tilde{u}$$

$$\begin{bmatrix} -6 & 0 \\ 0 & -6 \end{bmatrix}\begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix} + \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}\begin{bmatrix} v_{1x} \\ v_{2x} \end{bmatrix} = 0$$

$$\begin{cases} -6v_{1t} + \lambda_1 v_{1x} = 0 \\ -6v_{2t} + \lambda_2 v_{2x} = 0 \end{cases} \qquad \begin{matrix} v_1 = F_1(x,t) \\ v_2 = F_2(x,t) \end{matrix} \text{ - реш-е}$$

Найдем $Sp\,A$: $\begin{bmatrix} 18 & -24 \\ 3 & 0 \end{bmatrix} = 3\begin{bmatrix} 6 & -8 \\ 1 & 0 \end{bmatrix}$

$$\begin{matrix} \lambda_1 + \lambda_2 = 6 \\ \lambda_1\lambda_2 = 8 \end{matrix} \qquad Sp = \{2, 4\}$$

$$v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} 6 & 0 \\ 0 & 12 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$$

$$v = C^{-1}\tilde{u}$$

① $V_{1t} - V_{1x} = 0$

$\frac{dx}{dt} = -1$ $\qquad x = -t + c_1$

$\qquad\qquad\quad C_1 = t + x \quad \Rightarrow \quad V_1 = F_1(t+x)$

② $V_{2t} - 2V_{2,x} = 0$ $\qquad\qquad$ Характеристики: $x+t,\ x+2t$

$\frac{dx}{dt} = -2$ $\qquad V_2 = F_2(x+2t)$

$V = \begin{pmatrix} F_1(x+t) \\ F_2(x+2t) \end{pmatrix}$ $\qquad \tilde{u} = CV$ $\qquad \begin{bmatrix} u_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 2F_1(x+t) + 4F_2(x+2t) \\ F_1(x+t) + F_2(x+2t) \end{bmatrix}$

Определение характеристик:

$Au_t + Bu_x = 0$

$S = \{(t,x) \mid \varphi(t,x) = 0\}$ $\qquad \nabla\varphi|_S = 0$

$\det\left[\varphi_t A + \varphi_x B\right]\Big|_S = 0 \quad \Rightarrow \quad S$ - хар. пов-ть

скаляры

$\det A \cdot \det\left(\varphi_t + \varphi_x A^{-1}B\right)\Big|_S = 0$

$A^{-1} = \frac{1}{-6}\begin{bmatrix} 0 & 6 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -3 & 8 \\ 3 & 4 \end{bmatrix} = \frac{1}{-6}\begin{bmatrix} 18 & -24 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} -3 & -4 \\ -1/2 & 0 \end{bmatrix}$

$\begin{vmatrix} \varphi_t - 3\varphi_x & 4\varphi_x \\ -\frac{1}{2}\varphi_x & \varphi_t \end{vmatrix}$ $\qquad \varphi_t^2 - 3\varphi_x\varphi_t + 2\varphi_x^2 =$

$= \varphi_t^2 - \varphi_x\varphi_t + 2(\varphi_x^2 - \varphi_x\varphi_t) =$

$= \varphi_t(\varphi_t - \varphi_x) - 2\varphi_x(\varphi_t - \varphi_x) = (\varphi_t - \varphi_x)(\varphi_t - 2\varphi_x)$

① $\varphi_t \neq 0 \Rightarrow t = \varphi(x) \Rightarrow (1 + \varphi_x)(1 + 2\varphi_x) = 0$

$\nabla\varphi = \begin{bmatrix} 1 \\ -\varphi_x \end{bmatrix}$ $\qquad t = -x + c$

$\qquad\qquad\qquad t = -x/2 + c$

② $\varphi_x \neq 0 \Rightarrow x = \alpha(t)$ $\qquad \nabla F = \begin{bmatrix} -\alpha' \\ 1 \end{bmatrix} \Rightarrow \dots$ $\qquad x = -t + c$

$\qquad\qquad\qquad x = -2t + c$

$$\begin{bmatrix} 1 & -5 \\ 3 & 1 \end{bmatrix} u_t + \begin{bmatrix} 2 & 2 \\ 0 & -4 \end{bmatrix} u_x = 0$$

$$A^{-1} = \frac{1}{16}\begin{bmatrix} 1 & 5 \\ -3 & 1 \end{bmatrix} \qquad A^{-1}B = \underbrace{\frac{1}{16}\begin{bmatrix} 2 & -18 \\ -6 & -10 \end{bmatrix}}_{:= CJC^{-1}} \qquad \lambda^2 + 8\lambda - 128 = 0$$

$$\lambda_1 = -16$$
$$\lambda_2 = 8$$

$$\underbrace{\begin{bmatrix} 18 & -18 \\ -6 & 6 \end{bmatrix}}_{A-\lambda E}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad u_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad u_2 = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix} \qquad Cv_t + \frac{1}{16}CJv_x = 0 \qquad J = \begin{bmatrix} -16 & 0 \\ 0 & 8 \end{bmatrix}$$

$$v_t + \frac{1}{16}Jv_x = 0$$

$$\begin{cases} v_{1t} - v_{1x} = 0 \\ v_{2t} + \frac{1}{2}v_{2x} = 0 \end{cases}$$

Ищем хар-ки

$$\det\left(A\varphi_t + B\varphi_x\right)\Big|_S = 0$$

$$\det A \cdot \det\left(I\varphi_t + A^{-1}B\varphi_x\right)\Big|_S = 0$$

$$\det\left(I\varphi_t + \frac{1}{16}CJC^{-1}\varphi_x\right)\Big|_S = 0$$

$$\det\left(CIC^{-1}\varphi_t + \frac{1}{16}CJC^{-1}\varphi_x\right)\Big|_S = 0$$

$$\det C \cdot \det\left(I\varphi_t + \frac{1}{16}J\varphi_x\right)\Big|_S \cdot \det C^{-1} = 0$$

$$+\left(I\varphi_t + \frac{1}{16}J\varphi_x\right)\Big|_S = 0$$

$$\left.\begin{vmatrix} \varphi_t - \varphi_x & 0 \\ 0 & \varphi_t + \frac{1}{2}\varphi_x \end{vmatrix}\right|_S = 0$$

Корректность

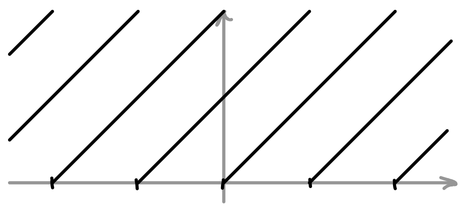
$u_t + u_x = 0 \qquad u_t - u_x = 0$

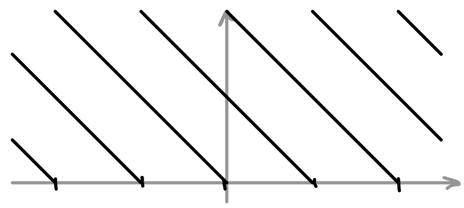
ⓐ $t>0, \ x \in \mathbb{R}$

ⓑ $t>0, \ x>0$

ⓒ $t>0, \ 0<x<\ell$

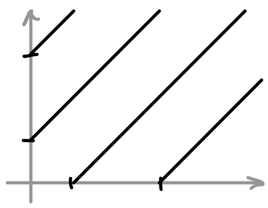
ⓐ Если знаем хар-ки и значения на $t=0$, то знаем везде

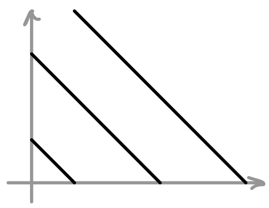




$\begin{cases} u_t + u_x = 0 \\ u|_{t=0} = \varphi(x) \end{cases} \qquad \begin{cases} u_t - u_x = 0 \\ u|_{t=0} = \varphi(x) \end{cases}$

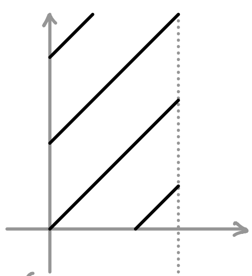
ⓑ

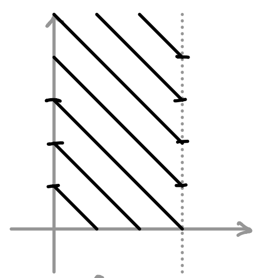




$\begin{cases} u_t + u_x = 0 \\ u|_{t=0} = \varphi(x), \ x>0 \\ u|_{x=0} = \mu(t) \end{cases} \qquad \varphi(0) = \mu(0) \qquad \begin{cases} u_t - u_x = 0 \\ u|_{t=0} = \varphi(x) \end{cases}$

ⓒ





$\begin{cases} u_t + u_x = 0 \\ u|_{t=0} = \varphi(x), \ 0<x<\ell \\ u|_{x=0} = \mu(t) \end{cases} \qquad \varphi(0) = \mu(0) \qquad \begin{cases} u_t - u_x = 0 \\ u|_{t=0} = \varphi(x), \ 0<x<\ell \\ u|_{x=\ell} = \mu(t) \end{cases}$

$\begin{cases} u_{tt} + v_x = 0 \\ v_{tt} - u_x = 0 \end{cases} \qquad \begin{bmatrix} u \\ v \end{bmatrix} = e^{i\alpha x + \beta t} \begin{bmatrix} u_0 \\ v_0 \end{bmatrix}$

$$\begin{cases} \beta^2 e^{i\alpha x+\beta t} u_0 + i\alpha e^{i\alpha x+\beta t} v_0 = 0 \\ \beta^2 e^{i\alpha x+\beta t} v_0 - i\alpha e^{i\alpha x+\beta t} u_0 = 0 \end{cases}$$

$$\begin{pmatrix} \beta^2 & i\alpha \\ -i\alpha & \beta^2 \end{pmatrix} \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} = 0 \qquad \begin{aligned} &\beta^4 - \alpha^2 = 0 \\ &\beta^2 = \pm\alpha \end{aligned} \qquad \beta = \begin{cases} \sqrt{\alpha} \\ \cancel{\sqrt{\alpha}} \end{cases} \qquad \begin{bmatrix} i \\ -1 \end{bmatrix}$$

$$ce^{inx} + \sqrt{n}\,t \begin{bmatrix} i \\ -1 \end{bmatrix} = v_n$$

сил моих больше нет...