MMF 2025H: Risk Management Laboratory

Portfolio and System Documentation

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July 18, 2021

Contents

1 Introduction

”Don’t put all your eggs in one basket.” This tried-and-true aphorism, while valuable in many aspects of life, is even more potent when approaching the topic of your long-term portfolio. The power of diversification has been proven by practitioners and academics alike. Other financial products such as mutual funds or GICs offer unattractive returns and fees that often do not justify these unattractive returns. We cannot predict the future, and the practitioners that try often have high barriers to entry (hedge funds), charge extremely high fees, and are often wrong. Predicting the future is not necessarily the goal of your portfolio–you seek to preserve your wealth and gain exposure to as wide of a range of products as possible.

Look no further–the robo-advisor developed by the best and brightest of the prestigious Master of Mathematical Finance program at the University of Toronto seeks to do exactly that: grow your wealth.

2 Motivation

To leverage the power of diversification, the goal of the portfolio is to invest only in high-quality, low-fee Exchange Traded Funds, otherwise known as ETFs. ETFs themselves are diversified baskets of stocks that generally follow a theme. For example, consider the iShares Global Agriculture Index ETF (COW). According to its own documentation, this ETF seeks exposure to companies involved in the production of agricultural products, fertilizers and agricultural chemicals, farm machinery, and packaged foods and meats. By investing in many companies that are similar, we are able to diversify away any risks involved with one specific company. However, our goal is not simply to invest in agricultural companies. We want as much diversification as possible. Therefore, by creating a basket of ETFs, not only will we hold many stocks from any one specific sector, we will also be holding stocks from many unique sectors.

ETFs are extremely powerful in the sense that they also give investors exposure to products that otherwise may not be available. For example, consider the iShares Global Government Bond Index ETF (CAD-Hedged) (XGGB). This ETF gives investors exposure to global government bonds which otherwise would be extremely difficult for an average retail investor to do. Futhermore, note that ETFs such as XGGB are able to give investors exposure to other geographies as well.

Ultimately, the goal behind this robo-advisor is to take full advantage of the power of diversification by purchasing low-cost baskets of stocks (ETFs) that give investors exposure to:

- Whole sectors/baskets of similar stocks
- Across many geographies
- Investing in a wide range of themes

- Using many unique financial products

The combination of all these ideas ultimately gives investors the maximum possible diversification that they can achieve in a low-cost, hands-off portfolio.

3 Portfolio Construction, Methodology, and Results

3.1 Asset Universe

In Canada, most investment accounts include both a US-Dollar (USD) account and Canadian Dollar (CAD) account allowing investors to purchase both US and Canada listed securities. For this reason, it is mandated that we take advantage of both geographies by splitting our account 5050: half in Canadian ETFs and half on American ETFs. This will allow access to a universe of 134 products hand-picked to be the lowest-cost and most effective diversifiers.

More specifically, the 78 Canadian ETFs are provided by iShares, which is a subsidiary of BlackRock, and Horizons, which is a subsidiary of Mirae Asset. BlackRock is by far the world's largest asset manager with over $9 trillion in assets over management. They offer some of the cheapest (i.e. lowest fee) ETFs in the world which give investors access to many unique financial products. Mirae Asset is an asset manager with $550 billion of assets under management. Their iShares products are extremely unique and offer investors ETFs whose themes cannot be found anywhere else.

The remaining 56 American ETFs are also managed by BlackRock. However, as the ETF market in the US is far more competitive, the fees are much lower. For this reason, these 56 ETFs are classical, i.e. geography and sector focused, rather than unique and unconventional. The rationale behind this decision is that in a 5050 split of assets, we are willing to pay higher fees to get access to more interesting products. For example, there are many ETFs designed to track the S&P 500 index, but the BlackRock ETF is among the absolute cheapest (versus its Canadian counterpart). The ETFs offered by Horizon are unique and have very niche, unconventional themes, thus justifying a slightly more expensive fee structure.

For more information on the universe of ETF products, see Appendix A.

3.2 Methodology

Ultimately, we seek the highest level of diversification. To achieve this, we seek the assets with the lowest levels of correlation with its peers. By breaking our backtesting data into 6-month pieces, we can observe the previous period to gain insight on how we should structure our next 6 months. As previously mentioned, we first calculate the assets with the lowest mean correlation in the previous period. Then, based on the parameters and model type chosen, we construct our next period's portfolio. We will utilize three different optimization algorithms to optimize these portfolios: Mean

Variance Optimization (MVO), Sharpe Ratio Maximization and Risk Parity to achieve the optimal weight for each rebalancing period.

3.2.1 Mean Variance Optimization

Mean-Variance Optimization (MVO) leverages the benefits of diversification to select the portfolio that provides the smallest possible variance (risk) for a given expected return. According to traditional finance literature, the trade-off between a portfolio's variance/risk and return lies on the efficient frontier, which can be constructed using an optimization problem:

$$\text{minimize } \mathbf{x}^T \mathcal{Q}\mathbf{x} \text{ (Variance)}$$

$$\begin{aligned} \text{subject to } & \mu^T \mathbf{x} \geq R \text{ (Target benchmark Return)} \\ & \mathbf{1}^T \mathbf{x} = 1 \text{ (Weights sum to 1)} \\ & (\mathbf{x} \geq \mathbf{0}) \text{ (No short selling)} \end{aligned}$$

where $\mathbf{x}$ is a vector containing asset weights, $\mathcal{Q}$ is a matrix of asset covariances using last period of asset returns, μ is a vector of asset returns, and $\mathbf{1}$ is the 1-vector.

3.2.2 Risk Parity

The problem with traditional mean-variance optimization is that it's not enough to diversify the risk based only on the individual asset variance. When we equalize the risk-contributions, we have to take into account the asset co-variance. It is also an advantage to not have to estimate unreliable returns during the portfolio construction process.

We can define the risk-contribution of an individual asset as:

$$x_i(Q)x_i \qquad \text{where Q is the co-variance matrix}$$

Risk Parity seeks a portfolio where the risk-contribution of every portfolio is equal. This can be formulated as:

$$\text{minimize } \sum_{i=1}^{n} \sum_{j=1}^{n} \mathbf{x_i} \mathcal{Q} \mathbf{x_i} - \mathbf{x_j} \mathcal{Q} \mathbf{x_j}$$

$$\begin{aligned} \text{s.t : } & \mathbf{1}^T \mathbf{x} = 1 \text{ (Weights sum to 1)} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

However, the original objective function of the risk-parity method has a highly non-linear function, and it's difficult to find the gradient of the objective function in an analytical way. Therefore, we introduce the alternative barrier function:

$$f(y) = \frac{1}{2}y^TQy - c\sum_{i=1}^{n} ln(y_i)$$

Clearly $f(y)$ is strictly convex. To solve this equation, by first-order necessary conditions (FONC), $\nabla f(y) = Qy - cy^{-1} = 0$ where $y^{-1} = [\frac{1}{y_1}...\frac{1}{y_n}]$.
So we must have:

$$y_i(Qy)_i = y_jj(Qy)_j \qquad \forall i, j$$

which is exactly risk-parity method as desired. The Alternative Risk-Parity problem is as follows:

$$\text{minimize } \frac{1}{2}y^TQy - c\sum_{i=1}^{n} ln(y_i)$$
$$\text{s.t} \quad y \geq 0$$

And we could retrieve optimal asset weights through $x_i = \frac{y_i}{\sum_{i=1}^{n} y_i}$

3.2.3 Sharpe Ratio Maximization

Since Sharpe ratio is an important measure to the performance of our optimized portfolio, we introduced the Sharpe Ratio strategy to maximize this measure. Sharpe ratio is a measure of excess return received for the extra volatility endured to hold a riskier asset. The higher the Sharpe ratio, the better the portfolio. Mathematically, the Sharpe ratio has the form:

$$Sharpe\ Ratio = \frac{\bar{r} - r_f}{\sigma}, \text{ where } \sigma \text{ is the standard deviation of the portfolio.}$$

As an alternative to the standard MVO as mentioned, we consider a different portfolio optimization task. We denote risk-free interest rate as r_f and formulated the problem as following:

$$\max_{\boldsymbol{x}} \frac{\boldsymbol{\mu^T x} - r_f}{\sqrt{\boldsymbol{x^T Q x}}}$$
$$\text{s.t.} \sum_j x_j = 1$$
$$\boldsymbol{x} \geq \boldsymbol{0}$$

3.3 Backtest Results

In the period between January 1, 2011 and April 1, 2016, we put our portfolio construction methodology to the test. Almost immediately, we recognized the poor performance of Mean-Variance Optimization versus its peers, and scrapped the inclusion altogether. For this reason, the following results only inlcude the maximum sharpe ratio and risk parity results.

The following table demonstrates the results of including the 10 lowest-correlated assets in the Canada and USA universes:

	SR	max drawdown
SR max	0.8983	-23.34%
Risk parity	-0.1884	-51.15%

As we can see, the backtest maximum drawdown is not very good in both scenarios. This fact is bolstered by the poor Sharpe Ratios of the strategies. As such, we move onto 15 assets per geographical universe and observe those results:

	SR	max drawdown
SR max	1.701	-14.02%
Risk parity	0.0625	-44.80%

Here, we can see an improvement in the Sharpe Ratio, but still relatively unattractive drawdowns for such Sharpes. We move onto the last backtest:

	SR	max drawdown
SR max	2.0458	-14.28%
Risk parity	0.2768	-37.89%

It is quite obvious that the maximum Sharpe Ratio strategy offers the most attractive results. It is for this reason that in our final portfolio, we selected the SR max algorithm with 20 assets from each geographical universe.

3.4 Portfolio strategy

The goal of this portfolio construction is to seek for global macro ETFs with diversification as much as possible. The optimization strategy discussed in section 3.2 will help the investor search for optimized portfolio weights using uncorrelated pools of ETFs, which helps investor achieve diversification. This is the reason why we decided to use the 20-asset methodology. Going forwards, we wanted to give the investor access to different levels of risks to fit their risk appetite. Therefore, we introduce an options strategy which utilizes the leverage allowed in our mandate. By varying the leverage used in the options strategy, we were able to construct portfolios of conservative, balanced, and aggressive risk profiles. There are a few intricacies to this options methodology which is why we reserved the next section to elaborate.

3.5 Portfolio strategy: Foreign Exchange Options Strategy

The portfolio strategy is to be diversified around the world. We have a side strategy which uses the approved leverage of 20% (when the risk profile goes accordingly), in option premiums. The main idea is to use an optimization method to decide which position to take and in which currencies. The position will be taken by buying short term options, from one month up to three months. We will buy either a call or a put, in one or many of the available FX markets. We have access to the following option markets: from America, the Canadian dollar (CAD), the Mexican Peso (MXN), and the Brazilian Real (BRL); from Europe, the Euro (EUR), and the Russian Ruble (RUB); from Asia, the Chinese Yuan (CNY), and Japanese Yen (JPY); and from Africa, the South African rand (ZAR). We decided to use short term options given that the liquidity in the FX markets is concentrated in these tenors and also because FX markets tend to change trend very quickly. After running the optimization method, which will allow us to take short positions, we will choose the higher weights in absolute value and take those positions by buying plain vanilla options. If the weight is positive we will buy a call option, if it is negative we will buy a put options. To decide in how many currencies from the 8 available we will take a position, we will do back-testing from April 2011 to April 2016 (current date). In this back-testing we will also decide the out-of-sample period window to use during the performance of the portfolio. For instance, to buy options at inception date in April 2016, we need to decide how much information we are going to use in the optimization method. Given that we want to take advantage of the current trend of the FX market, the calibration period will be short as well, from one month up to six months.

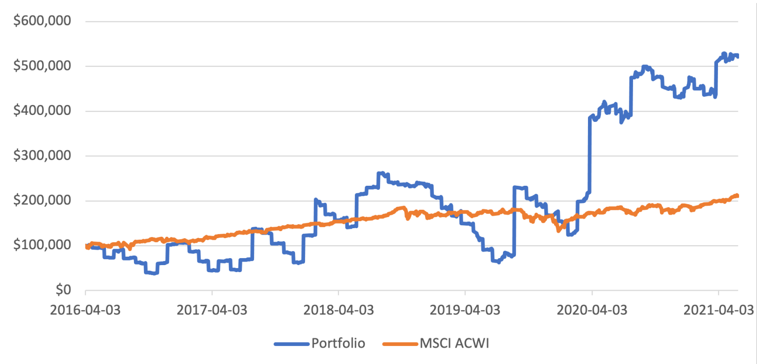
Depending on the risk profile of the client, we will choose to use 0%, 10% or 20% of leverage. Conservative clients will not use the leverage to invest in FX options; balanced clients will use 10% of their portfolio in leverage to buy FX options; and aggressive clients will uso the 20% available of leverage to buy FX options. The strategy will be the same in all risk profiles, the only thing that will change is the amount of options they will buy.

To buy the options we will assume we can borrow money at the risk free rate plus 100 basis points. When the option matures we will use the payout to pay the loan and the remaining cash we will send it to the portfolio to buy more ETFs. In case we do not earn any money from the option, we will sell the equivalent portion of ETFs to pay back the loan. After this process we will calculate again the available cash to buy options according to the risk profile. We will repeat all the life of the investment.

4 Final Results and Benchmark Comparison

4.1 Final Results

To give our investor the best possible set up to maximize return, we gave them the aggressive 20% options strategy to supplement the normally tame sharpe ratio maximization portfolio algorithm. By calculating the ETF portfolio and the aggressive option portfolio, we were able to construct a rather aggressive PnL profile:



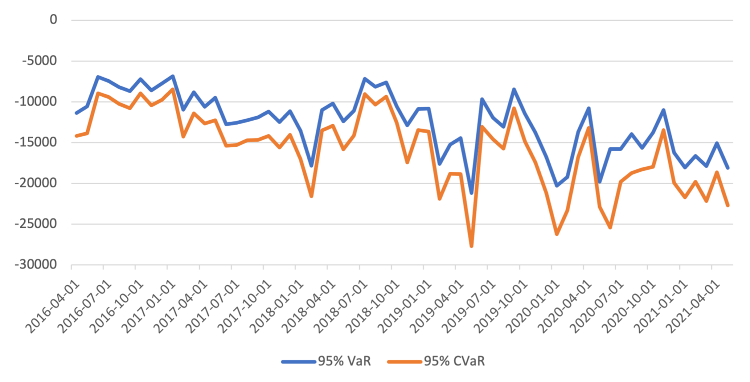
We can see that the value of the portfolio swings rapidly, especially during 2019. Suprisingly, the COVID pandemic had less of an effect on our portfolio than the previous downward period. As demonstrated in the chart, for the benchmark, we utilized the MSCI All Country World Index (ACWI). According to the MSCI website, the ACWI is designed to represent performance of the full opportunity set of large- and mid-cap stocks across 23 developed and 27 emerging markets. As of June 2021, the ACWI covers more than 2,900 constituents across 11 sectors and approximately 85% of the free float-adjusted market capitalization in each market. Despite being composed solely of equities, whereas our portfolio is constructed with many financial instruments in mind, the ACWI gives us similar global exposures. Therefore, it acts as a fitting benchmark.

There are a few key items to watch out for: we will be exposed to products and regions that vary between being more or less risky. For example, there are some frontier ETFs included in our portfolio, and frontier markets are generally more volatile and risky versus even emerging markets. Furthermore, we also have ETFs which hold government debt, widely regarded to be one of the safest financial instruments.

4.1.1 ETF VaR Analysis

As the ETF part of the portfolio forms the backbone of our strategy, it is valuable to take a peek at the VaR and CVaR of our results:

As we can see, our VaR and CVaR drop sharply in a few occasions. Naturally the COVID-19 pandemic is one of the lowest points of the chart, with an interesting global drop happening some time in 2019. Further analysis would be necessary to diagnose these drops.



5 Portfolio Activity

The rebalancing period of the portfolio is half year.For every period, our momentum algorithm will auto adjust the ETF exposure base on their performances last period. In other words, To best protect your money and grow wealth steady, We always pick the best-performed ETF each period with certain cutoff numbers.

After selecting the best-performed ETF portfolio, we try several asset allocation optimization algorithms to construct an optimal static ETF weights for each period. The idea is to gain maximum momentum factor profit from last period, and reduce the portfolio exposure if the risk grows over the limit.

6 Models

6.1 Call option valuation and sensitivities

We know the broadly accepted model to obtain the price of a call option is Black-scholes and is calculated as follows:

$$
\begin{aligned}
V_c(S,t) &= Se^{-q\tau}\Phi(d_1) - Ke^{-r\tau}\Phi(d_2) \\
d_1 &= \frac{ln(\frac{S}{K}) + (r - q + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \\
d_2 &= \frac{ln(\frac{S}{K}) + (r - q - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \\
&= d_1 - \sigma\sqrt{\tau}
\end{aligned}
$$

The Delta of an option is how much the value of the option changes with respect to the change of the price of the underling asset. The formula for the Delta of a Call Option is derived as follows:

$$
\begin{aligned}
\Delta_c &= \frac{\partial V_c}{\partial S} \\
&= e^{-q\tau}\Phi(d_1) + Se^{-q\tau}\Phi'(d_1)\frac{\partial d_1}{\partial S} - Ke^{-r\tau}\Phi'(d2)\frac{\partial d_2}{\partial S} \\
&= e^{-q\tau}\Phi(d_1) + Se^{-q\tau}\frac{1}{\sqrt{2\pi}}e^{\frac{-d_1^2}{2}}\frac{1}{S\sigma\sqrt{\tau}} - Ke^{-r\tau}\frac{1}{\sqrt{2\pi}}e^{\frac{-(d_1-\sigma\sqrt{\tau})^2}{2}}\frac{1}{S\sigma\sqrt{\tau}} \\
&= e^{-q\tau}\Phi(d_1) + \frac{e^{\frac{-d_1^2}{2}}}{\sigma\sqrt{2\pi\tau}}\left[e^{-q\tau} - \frac{Ke^{-r\tau}}{S}e^{d_1\sigma\sqrt{\tau}-\frac{\sigma^2\tau}{2}}\right] \\
&= e^{-q\tau}\Phi(d_1) + \frac{e^{\frac{-d_1^2}{2}}}{\sigma\sqrt{2\pi\tau}}\left[e^{-q\tau} - \frac{Ke^{-r\tau}}{S}e^{ln(\frac{S}{K})+(r-q+\frac{\sigma^2}{2})\tau-\frac{\sigma^2\tau}{2}}\right] \\
&= e^{-q\tau}\Phi(d_1) + \frac{e^{\frac{-d_1^2}{2}}}{\sigma\sqrt{2\pi\tau}}\left[e^{-q\tau} - \frac{Ke^{-r\tau}}{S}e^{ln(\frac{S}{K})+(r-q)\tau}\right] \\
&= e^{-q\tau}\Phi(d_1) + \frac{e^{\frac{-d_1^2}{2}}}{\sigma\sqrt{2\pi\tau}}\left[e^{-q\tau} - e^{-q\tau}\right] \\
&= e^{-q\tau}\Phi(d_1)
\end{aligned}
$$

The Gammma of an option is the measure of how much does the Delta changes with respect to the change of the price of the underlying asset. The formula for the Gamma sensitivity of a Call Options is as follows:

$$
\begin{aligned}
\Gamma_c &= \frac{\partial \Delta_c}{\partial S} \\
&= e^{-q\tau}\phi(d_1)\frac{1}{S\sigma\sqrt{\tau}}
\end{aligned}
$$

The Vega of an option is how much the value of the option changes with respect to the change of the volatility. The formula for the Vega of a Call Option is as follows:

$$
\begin{aligned}
\Lambda_c &= \frac{\partial V_c}{\partial \sigma} \\
&= Se^{-q\tau}\phi(d_1)\frac{\partial d_1}{\partial \sigma} - Ke^{-r\tau}\phi(d_2)\frac{\partial d_2}{\partial \sigma} \\
&= Se^{-q\tau}\phi(d_1)(\frac{\partial d_2}{\partial \sigma} + \sqrt{\tau}) - Ke^{-r\tau}\phi(d_2)\frac{\partial d_2}{\partial \sigma} \\
&= Se^{-q\tau}\phi(d_1)\sqrt{\tau} + \frac{\partial d_2}{\partial \sigma}\underbrace{\left[Se^{-q\tau}\phi(d_1) - Ke^{-r\tau}\phi(d_2)\right]}_{=0}
\end{aligned}
$$

We know that term is zero as in the Delta derivation. So the formula for Vega is:

$$\Lambda_c = Se^{-q\tau}\phi(d_1)\sqrt{\tau}$$

To obtain Theta, which is the sensitivity of the option price to time, we derive the price formula with respect to τ. The formula of Theta of call options is the following:

$$\begin{aligned}
\Theta_c &= \frac{\partial V_c}{\partial \tau} \\
&= \left(-\frac{Se^{-q\tau}\phi(d_1)\sigma}{2\sqrt{\tau}} - rKe^{-r\tau}\Phi(d_2) + qSe^{-q\tau}\Phi(d_1)\right)/365
\end{aligned}$$

6.2 Put option valuation and sensitivities

We know the price of a Put Option is calculated as follows:

$$\begin{aligned}
V_p(S,t) &= Ke^{-r\tau}\Phi(-d_2) - Se^{-q\tau}\Phi(-d_1) \\
d_1 &= \frac{ln(\frac{S}{K}) + (r - q + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \\
d_2 &= \frac{ln(\frac{S}{K}) + (r - q - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}
\end{aligned}$$

We can prove the Call-Put parity with the follwing derivation:

$$\begin{aligned}
V_p(S,t) &= Ke^{-r\tau}\Phi(-d_2) - Se^{-q\tau}\Phi(-d_1) \\
&= Ke^{-r\tau}(1 - \Phi(d_2)) - Se^{-q\tau}(1 - \Phi(d_1)) \\
&= Se^{-q\tau}\Phi(d_1) - Ke^{-r\tau}\Phi(d_2) + Ke^{-r\tau} - Se^{-q\tau} \\
&= V_c(S,t) + Ke^{-r\tau} - Se^{-q\tau}
\end{aligned}$$

The Delta of a Put Options is as follows:

$$\begin{aligned}
\Delta_p &= \frac{\partial V_p}{\partial S} \\
&= \Delta_c - e^{-q\tau} \\
&= e^{-q\tau}\Phi(d_1) - e^{-q\tau} \\
&= -e^{-q\tau}(1 - \Phi(d_1)) \\
&= -e^{-q\tau}\Phi(-d_1)
\end{aligned}$$

The Gamma of a Put Option is the same as in a Call options. The formula is the following:

$$\begin{aligned}
\Gamma_p &= \frac{\partial \Delta_p}{\partial S} \\
&= e^{-q\tau}\phi(d_1)\frac{1}{S\sigma\sqrt{\tau}}
\end{aligned}$$

The Vega of a Put Option is the following:

$$\begin{aligned}\Lambda_p &= \frac{\partial V_p}{\partial \sigma} \\ &= \frac{\partial V_c}{\partial \sigma} \\ &= Se^{-q\tau}\phi(d_1)\sqrt{\tau}\end{aligned}$$

The formula of Theta of a put options is the following:

$$\begin{aligned}\Theta_p &= \frac{\partial V_p}{\partial \tau} \\ &= \left(-\frac{Se^{-q\tau}\phi(d_1)\sigma}{2\sqrt{\tau}} - rKe^{-r\tau}(1-\Phi(d_2)) + qSe^{-q\tau}(1-\Phi(d_1))\right)/365\end{aligned}$$

6.3 Volatility Surface construction

The first step is to define the model for the volatility smile at each tenor. In the Foreign Exchange Options markets, we have three different types of quotes for each tenor, the At-the-money, 25-delta Risk Reversal and 25-delta Butterfly. These quotes give us information about the volatility smile in 3 different points: the 25-delta call, 25-delta put and the at-the-money volatility. To translate this quotes into the volatility figures we need to do the following. First we use the At-the-money quote as the At-the-money volatility, given that the quote price is already the volatility at the At-the-money strike. To we calculate the strike we use the following equation that corresponds to the dynamics of the asset price:

$$K_{ATM} = Se^{(r-q+\frac{1}{2}\sigma_{ATM}^2)\tau}$$

Then we will use the Risk Reversals and the butterfly quotes to solve the volatility at the 25-delta call and 25-delta put. The quotes represent the following:

$$\sigma_{RR} = \sigma_{25\Delta c} - \sigma_{25\Delta p}$$

This means that when the 25-delta Risk Reversal σ_{RR} is positive, the market is giving a higher price to the calls than the puts, which represent that there is higher probability that the underlying will rise than the underlying falling. The 25-delta butterfly is the following:

$$\sigma_{BF} = \frac{\sigma_{25\Delta c} + \sigma_{25\Delta p}}{2} - \sigma_{ATM}$$

A low 25-delta Butterfly quote means that the volatility smile is flatter that when the quote is high. Solving the system of two equations we get the following volatility points:

$$\sigma_{25\Delta c} = \sigma_{ATM} + \sigma_{BF} + \frac{1}{2}\sigma_{RR}$$
$$\sigma_{25\Delta p} = \sigma_{ATM} + \sigma_{BF} - \frac{1}{2}\sigma_{RR}$$

Now we can use the delta formula to calculate the strike price that corresponds to that level of deltas. For instance, for a 25Δ put we have:

$$-e^{-q\tau}\Phi\left(-\frac{\ln\frac{S}{K_{25\Delta p}} + (r - q + \frac{1}{2}\sigma^2_{25\Delta p})\tau}{\sigma_{25\Delta p}\sqrt{\tau}}\right)$$

After straight forward algebra this leads to the following strike price:

$$K_{25\Delta p} = Se^{\Phi^{-1}(\frac{1}{4}e^{q\tau})\sigma_{25\Delta p}\sqrt{\tau}+(r-q+\frac{1}{2}\sigma^2_{25\Delta p})\tau}$$

And for a 25-delta call:

$$K_{25\Delta c} = Se^{-\Phi^{-1}(\frac{1}{4}e^{q\tau})\sigma_{25\Delta c}\sqrt{\tau}+(r-q+\frac{1}{2}\sigma^2_{25\Delta c})\tau}$$

In Malz (1997) a paper called "Option-Implied Probability Distributions and Currency Excess Returns", a polynomial of degree 2 is proposed to interpolate the volatility smile. The polynomial matches the 3 volatility points we just derived. The parabolic interpolation formula is the following:

$$\sigma(\Delta_f) = \sigma_{ATM} - 2\sigma_{RR}(\Delta_f - 0.5) + 16\sigma_{BF}(\Delta_f - 0.5)^2$$

where Δ_f is the delta level from which we want to obtain the volatility. Now with having the 3 quotes at each we can construct a Moneyness volatility surface. Here one arbitrary example of how should it looks:

Moneyness Volatility Surface USDMXN									
	Delta put				ATM	Delta call			
Tenor	10	20	30	40	50	40	30	20	10
0.08	0.1654	0.1649	0.162	0.159	0.1575	0.171	0.172	0.1786	0.1823
0.25	0.1765	0.1693	0.1671	0.164	0.1589	0.162	0.1758	0.1767	0.1883
0.50	0.1696	0.1636	0.1603	0.1583	0.1635	0.1686	0.1693	0.1788	0.1908
1.00	0.1766	0.17	0.1642	0.1639	0.1649	0.1729	0.1758	0.1813	0.1818

Table 1: Example of a moneyness volatility surface

Now we want to change the surface structure to have strike values in the x-axis instead of delta values. This change wants to be done in order to facilitate the valuation process of options when we already have a fixed strike and we want to interpolate the surface using the strike and the tenor to obtain the volatility that will be used for calculating the mark-to-market. To make this change first we are going to get the strike value of each one of the volatility points of the surface. Next we will define the new strike grid we want to have in the surface. We chose the first strike to be the 10-delta put of the longest tenor we have, which in our case is 3 months, and for the last strike of the grid we chose the 10-delta call strike, then we interpolate linearly in the remaining spaces to have the complete grid. Then we will interpolate each of the grid strikes and tenors with the moneyness surface and the strikes matrix; this process will give us a volatility surface that we are going to call the pre-calibration volatility surface. This surface would be enough to calculate approximate volatilites, but we are going to calibrate this to the volatility smile we got with the market quotes. The calibration process consists in finding the volatility that when used with that strike gives us certain delta, and that delta when interpolated using that parabolic interpolation in the volatility smiles, give us exactly the same volatility used to get that delta.

$$K \mapsto K(\sigma) \mapsto \Delta(K, \sigma) \mapsto \sigma(\Delta)$$

After calibrating the volatility surface we get a precise market volatility surface with a structure strike-tenor from which we can interpolate to get vol values.

Strike-tenor Volatility Surface USDMXN									
	Strikes								
Tenor	18.1164	18.6262	19.1359	19.6457	20.1554	20.6419	21.1285	21.615	22.1015
0.08	0.1654	0.1649	0.162	0.159	0.1575	0.171	0.172	0.1786	0.1823
0.25	0.1765	0.1693	0.1671	0.164	0.1589	0.162	0.1758	0.1767	0.1883
0.50	0.1696	0.1636	0.1603	0.1583	0.1635	0.1686	0.1693	0.1788	0.1908
1.00	0.1766	0.17	0.1642	0.1639	0.1649	0.1729	0.1758	0.1813	0.1818

Table 2: Example of a strike-tenor volatility surface

It is import to mention that between longer is the strike grid but with as much granularity, preciser is the volatility we will interpolate. In our case that we do not want to do any market making, like selling volatility before expiry, we do not so much granularity in the volatility surface strike grid. In addition, the interpolation method used is bilinear interpolation, which may not be the smoothest, it is enough to what we need to get a fair mark-to-market for the options.

6.4 Risk model definition, calibration and market data, validation (back-testing) of risk model

6.5 Risk model definition, calibration and market data, validation (back-testing) of the options model

We will have access to 8 different FX volatility markets. From America, the Canadian dollar (CAD), the Mexican Peso (MXN), and the Brazilian real (BRL); from Europe, the Euro (EUR), and the Russian Ruble (RUB); from Asia, the Chinese Yuan (CNY), and Japanese Yen (JPY); and from Africa, the South African rand (ZAR). In each calibration period we will buy plain vanilla calls or puts from some of these currencies. To choose the length of the calibration period and the maturity of the options we will perform a 5-year back testing, specifically we will use data from April 2011 to April 2016 that is when the strategy starts.

We will compare the performance of 1 month options and 3 months options. We are only going to analyze these two maturities because this are tenors where market quotes exist and also they are cheaper than any other because here the liquidity is concentrated. Adding to that, it is easy to build the volatility smile where quotes are available and avoid any type of arbitrage against you in the execution. We will analyze also to use a historic window of 1 month up to 6 months to calibrate the optimization method. As we mentioned earlier, we did not analyze longer time periods because the FX market tend to change trends very quickly and we are looking for short term strategies. One more thing we will analyze in the back testing, is in how many currencies to take a position. We will analyze in the back-testing the usage of 1 up to 4 currencies. The reason we are limiting the result into 4 currencies, is because we prefer to choose only the currencies that show the higher weights, in other words, the positions were you will allocate most of your money in a traditional strategy.The results of the back-testing are the following:

Profit and loss of 1-month options according to strategy					
	Number of currency positions				
Calibration period	1	2	3	4	Average
1 month	18683	14640	15497	10007	14706
2 months	22841	16582	14166	17372	17740
3 months	26784	11964	8936	7876	13890
4 months	630	3814	7329	6231	4501
5 months	13171	17058	6601	6513	10835
6 months	-6997	7990	10565	12149	5926
Average	12518	12008	10515	10024	11266

Table 3: Back-testing of 1-month tenor options

The results of the back-testing using 3-month options are the following:

Profit and loss of 3-month options according to strategy					
	Number of currency positions				
Calibration period	1	2	3	4	Average
1 month	292	6589	8931	5849	5415.25
2 months	5104	6700	7690	6639	6533.25
3 months	-1848	3359	5707	3930	2787
4 months	-3982	350	2525	5169	1015.5
5 months	-3757	1507	3374	6261	1846.25
6 months	-1133	6187	5140	5519	3928.25
Average	-887	4115	5561	5561	3587

Table 4: Back-testing of 3-month tenor options

From the tables above we can see that is more profitable to use 1-month options rather than 3 month options. The average pL with 1-month options is 11,267 USD whereas with 3-month options is 3,588. For this reason we will use 1-month options in our portfolio. Then looking at the results of 1-month options, we can see that the best results are obtained when only taking positions in one currency. So in our portfolio we will only choose the highest weight in the optimization method and depending on the sign of the weight we will choose call or put. And finally we see that the maximum pl is reached when choosing a 3-month calibration period.

6.6 Scenarios – detailed description and construction

7 Portfolio analytic (light-cone analytic) methodologies

7.1 Exposures metrics

In the case of the options we will use the traditional metrics to asses the risk taken in the position. The following chart shows the level of delta taken in the portfolio during the performance period.

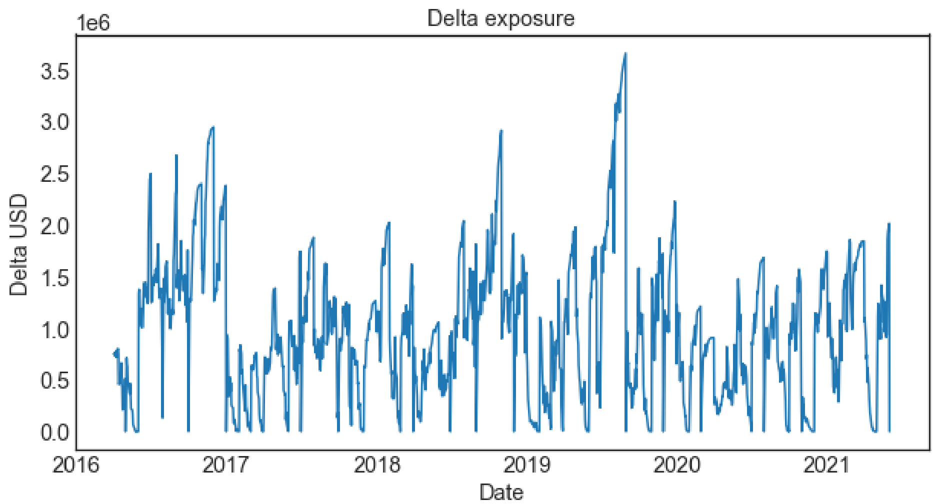


Figure 1: Delta exposure in the performance period

We can see that in average the delta exposure is around 1 million USD. This is almost 10 times the exposure in the ETFs. So the amount of leverage taken is huge. The next chart show the level of vega exposure taken in the performance period:

We see that the average Vega exposure is around 1 thousand US dollars. This may not look very aggressive but if we consider that the initial investment was 100 thousand dollars, the Vega exposure is around 1% of the initial value of the portfolio. Once again is very aggressive. The next chart is the gamma exposure:

The gamma exposure is always tricky, becuase it shows how fast the delta changes but many times the gamma could be higher than the maximum delta that an option could have. In our case is similar, our maximum gamma is around 8 million US dollars, but our delta exposure could never be that. However, this metric only show that our positions could have drastic changes in the delta position. The last chart of the options risk exposure is the theta:

We can see that the average value of theta is around one thousand US dollars.

The next graphs shows the risk metrics VaR and cVaR using the alphas of 95% and 99%.

From this graph we can see that the short fall with alpha 99% of the options strategy started from levels in around sixteen thousand dollars, and they reach a maximum level in 2020 it reach almost 25

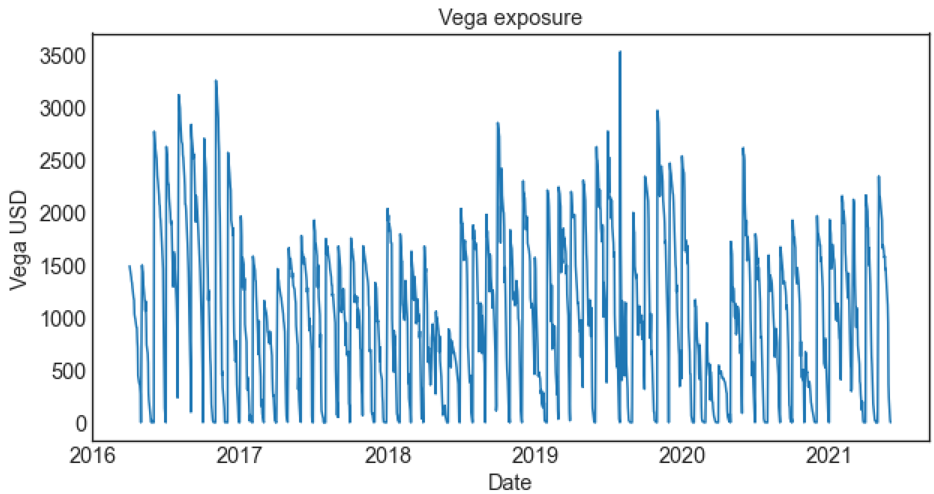


Figure 2: Vega exposure in the performance period

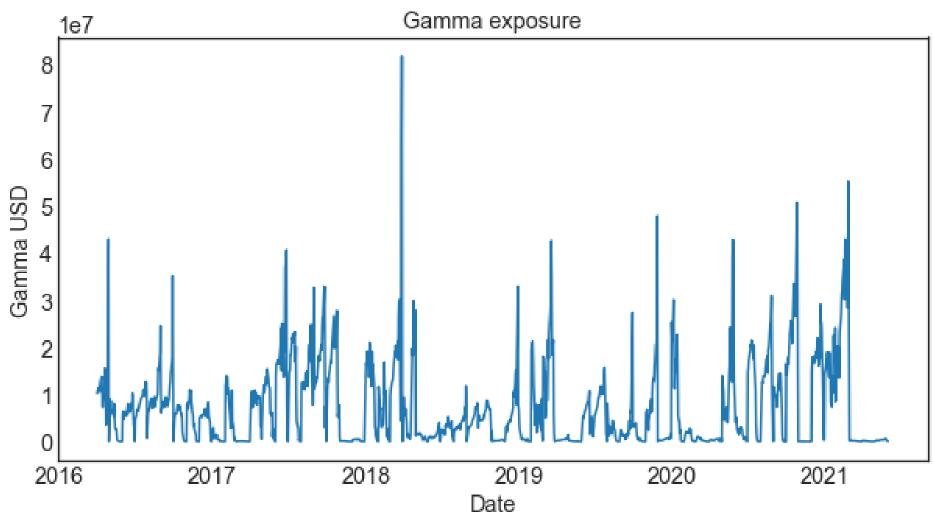


Figure 3: Gamma exposure in the performance period

thousand US dollars.

8 Model Risk assessment

8.1 Risk and scenario metrics

Three scenarios are applied to the portfolio returns:

- **Random ETF crash**: each ETF have equal probability to crash randomly over the period.
- **Random market crash**: the market have some probability to crash together over the period.

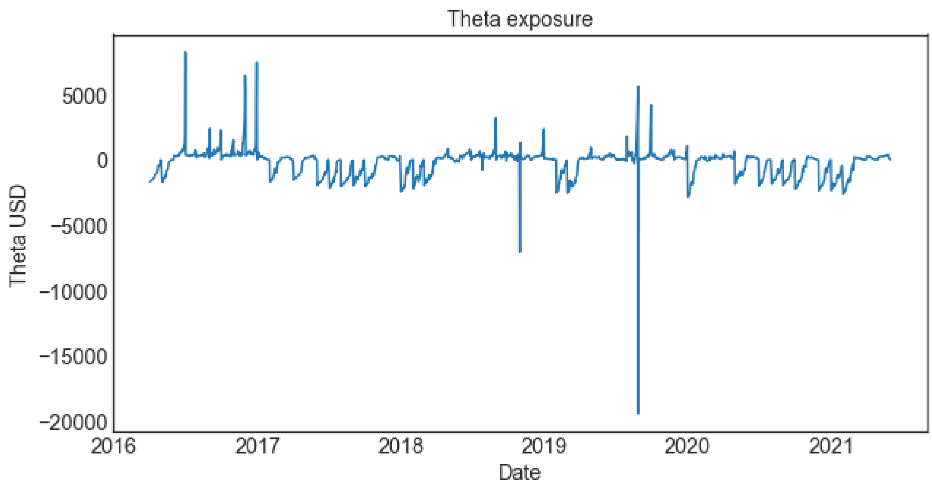


Figure 4: Theta exposure in the performance period

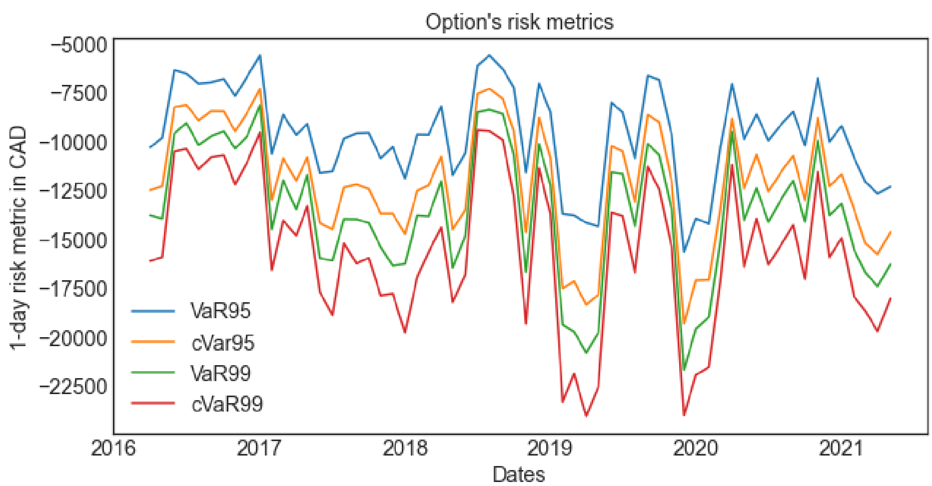


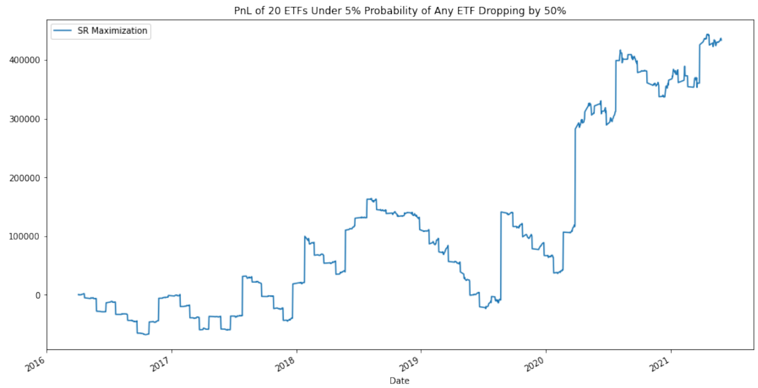
Figure 5: Evolution of FX Options risk metrics

- **Moving average crash**: the market have some probability to crash. And if yesterday crash, today have more probability to crash again.

For each scenario , the portfolio gets some random market events, and we do stress testing base on that.

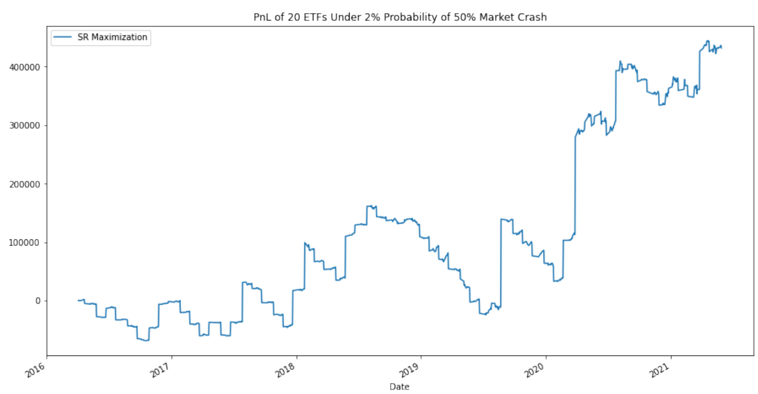
8.1.1 Random ETF Crash

As previously mentioned, each ETF will have a 2% probability to randomly crash 50% over the period of the portfolio:



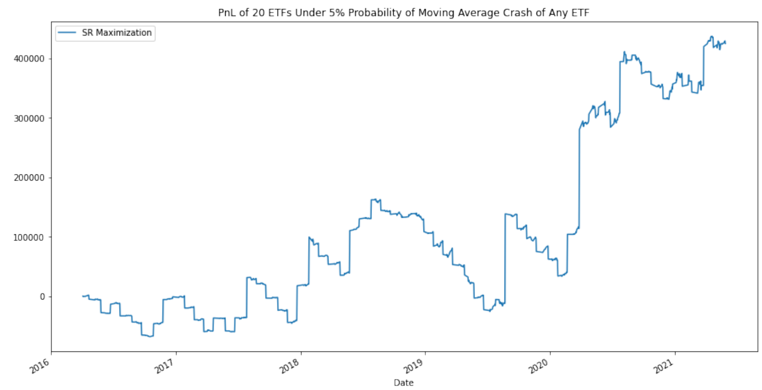
8.1.2 Random Market Crash

Similarly this time, the whole market will have a 2% probability to randomly crash 50% over the period of the portfolio:



8.1.3 Random ETF Moving Average Crash

Finally, the whole market will have a 2% probability to randomly crash an amount relative to its moving average over the period of the portfolio:



9 System's documentation: conceptual design and components, code, data, validation

Figure 6: User Interface Sample

9.1 User interface User Guide

- Section 1: Entering your name.
- Section 2: Loading data section. User will need to select the directory from your computer to load the asset returns, risk free rate, and the benchmark returns. Once a user finish entering the directory, he/she can hit the 'Load data' button.
- Section 3: Loading initial capitals. User will need to enter a CAD dollar amount of initial number of capitals that he/she would like to invest.
- Section 4: Loading cutoffs. User will need to select the number of ETFs that he/she wants to include in United State and Canada region.

- Section 5: Loading the optimization algorithm. User will need to specify the preferred optimization method for the portfolio construction.

- Section 6: Loading rebalance period. User will need to select their preferred rebalancing period for the portfolio construction.

- Section 7: Loading start date of the portfolio. User will need to enter the date he/she wants to start the portfolio. The date should be in MM/DD/YYYY format.

- Section 8: Selecting the risk appetite. User will choose their risk appetite between: conservative, balanced and aggressive.

- Section 9: Backtesting Optimization. As soon as a user is filling all informaton from section 1 to 7, he/she can click two buttons in this section to optimize and backtest the portfolio. The backtesting period will use the last five years before the starting date, and the portfolio will start to run from the start date till now.

- Section 10: Backtesting result. User can click 'Show bact-test metrics' to have all backtesting result shown on right text. User can click 'Plot back-testing PnL v.s. Benchmark' and 'Plot Pie chart' to see the backtesting PnL plot as well as the final weight of each ETFs in pie chart. The plot will be saved in same directionary of this UI.

- Section 11: Portfolio performance. User can click 'Show your optimized portfolio metrics' to have all portfolio performance shown on right text. User can click 'Plot your portfolio PnL v.s. Benchmark' and 'Plot Pie chart' to see the portfolio PnL plot as well as the final weight of each ETFs in pie chart. The plot will be saved in same directionary of this UI.

9.2 Coding components

9.2.1 portfolio_optimizer.py

This module is the main class that is used to simulate the global macro portfolio based on the defined strategy. It takes a dictionary of semiannual returns dataframes for each ETFs in each rebalancing periods as attribute.
The class has a function called 'portfolio_simulator'. It has 9 inputs:

- initial_capital: the assumed initial capital amount for our portfolio.

- riskfree: a dataframe of 3-m US treasury yield in each time.

- top_cor: top cutoff number of ETFs with lowest correlations.

- cutoff: the number of ETFs we want to include in the portfolio.

- VaRcutoff: the VaR threshold that is used to determine the scaling factor in each small period.
- optimization_type: four different types available: 'MVO', 'Risk parity', 'Sharpe ratio maximization', 'Equally weighted'.
- leverage: the leverage ratio for option.
- benchmark: the benchmark excess returns used for MVO. (default=None)
- scenario: which scenario is used to analyze. (default=None)

9.2.2 Risk_analytics.py

This module contains two classes. The risk class is used to calculate different risk metrics for the constructed portfolio. Metrics include: Value at Risk (VaR), conditional Value at Risk (CVaR), max drawdowns and Sharpe Ratio.
The scenario_analysis class is used to perform scenario analysis dicussed in section 8.3. With function all_crush, random_crush, mv_crush, a dataframe of returns with these crushes happened is outputed.

9.2.3 FX_Volatility.py

This module is used to perform option valuation and sensitivities analysis.

9.2.4 Robo_advisor.py

This module is utilized python GUI package 'tkinter' to realize the functionality of User Interface discussed in section 10.1.

9.3 Data components

- returns.csv: the return dataset for all of our ETFs.
- 3mTbill.csv: the 3-month US treasury bill, which is used as risk free rate.
- benchmark.csv: the MSCI All Country World Index (ACWI), which is used as our benchmark.
- FXspot_ATMvols_edates.csv: the spot rate for each Foreign exchange, which is used in option strategy.
- vol_data_edates.csv: the volatility data of FX spot rate, which is used in option strategy.

9.4 Validation components

- portfolio_construction.ipynb: the jupyter notebook that is used to perform backtesting, portfolio performance analysis, model validation and scenario analysis using different parameters.
- Options_PnL.jpynb: the jupyter notebook that is used to calculate Profit and loss as well as risk metrics of option strategy for different leverage levels.

10 Appendix

10.1 A. Canada ETFs

Ticker	Name
CGL	iShares Gold Bullion ETF (CAD-Hedged)
SVR	iShares Silver Bullion ETF (CAD-Hedged)
XUU	iShares Core S&P U.S. Total Market Index ETF
XGD	iShares S&P/TSX Global Gold Index ETF
COW	iShares Global Agriculture Index ETF
CWW	iShares Global Water Index ETF
CGR	iShares Global Real Estate Index ETF
XPF	iShares S&P/TSX North American Preferred Stock Index ETF (CAD-Hedged)
XDG	iShares Core MSCI Global Quality Dividend Index ETF
XBM	iShares S&P/TSX Global Base Metals Index ETF
XDGH	iShares Core MSCI Global Quality Dividend Index ETF (CAD-Hedged)
CBO	iShares 1-5 Year Laddered Corporate Bond Index ETF
CLF	iShares 1-5 Year Laddered Government Bond Index ETF
XGGB	iShares Global Government Bond Index ETF (CAD-Hedged)
CLG	iShares 1-10 Year Laddered Government Bond Index ETF
XHY	iShares U.S. High Yield Bond Index ETF (CAD-Hedged)
CBH	iShares 1-10 Year Laddered Corporate Bond Index ETF
XSE	iShares Conservative Strategic Fixed Income ETF
XFR	iShares Floating Rate Index ETF
CMR	iShares Premium Money Market ETF
XIG	iShares U.S. IG Corporate Bond Index ETF (CAD-Hedged)
XSC	iShares Conservative Short Term Strategic Fixed Income ETF
CHB	iShares U.S. High Yield Fixed Income Index ETF (CAD-Hedged)
CVD	iShares Convertible Bond Index ETF
CSD	iShares Short Duration High Income ETF (CAD-Hedged)
HBGD	Horizons Big Data & Hardware Index ETFÊ
HBGD.U	Horizons Big Data & Hardware Index ETFÊ
HSAV	Horizons Cash Maximizer ETF

10.2 A. USA ETFs

Ticker	Name HXH
Horizons Cdn High Dividend Index ETF	
HBB	Horizons CDN Select Universe Bond ETF
HUC	Horizons Crude Oil ETF
HXEM	Horizons Emerging Markets Equity Index ETF
HEWB	Horizons Equal Weight Canada Banks Index ETF
HCRE	Horizons Equal Weight Canada REIT Index ETF
HXX	Horizons Europe 50 Index ETF
BBIG	Horizons Global BBIG Technology ETF
BBIG.U	Horizons Global BBIG Technology ETF
HYDR	Horizons Global Hydrogen Index ETF
HLIT	Horizons Global Lithium Producers Index ETF
CHPS	Horizons Global Semiconductor Index ETF
CHPS.U	Horizons Global Semiconductor Index ETF
ETHI	Horizons Global Sustainability Leaders Index ETF
HURA	Horizons Global Uranium Index ETF
HUG	Horizons Gold ETF
FOUR	Horizons Industry 4.0 Index ETF
INOC	Horizons Inovestor Canadian Equity Index ETF
HXDM	Horizons Intl Developed Markets Equity Index ETF
HXDM.U	Horizons Intl Developed Markets Equity Index ETF
HLPR	Horizons Laddered Canadian Preferred Share Index ETF
HMMJ	Horizons Marijuana Life Sciences Index ETF
HMMJ.U	Horizons Marijuana Life Sciences Index ETF
HHF	Horizons Morningstar Hedge Fund Index ETF
HXQ	Horizons NASDAQ-100¨ÊIndex ETF
HXQ.U	Horizons NASDAQ-100¨ÊIndex ETF
HUN	Horizons Natural Gas ETF
HOG	Horizons Pipelines & Energy Services Index ETFÊ
PSYK	Horizons Psychedelic Stock Index ETF
RBOT	Horizons Robotics and Automation Index ETF
RBOT.U	Horizons Robotics and Automation Index ETF
HSH	Horizons S&P 500 CAD Hedged Index ETF
HXS	Horizons S&P 500¨ÊIndex ETF
HXS.U	Horizons S&P 500¨ÊIndex ETF
HGGB	Horizons S&P Green Bond Index ETF
HXT	Horizons S&P/TSX 60ª Index ETF
HXT.U	Horizons S&P/TSX 60ª Index ETF
HXCN	Horizons S&P/TSX Capped Composite Index ETF
HXE	Horizons S&P/TSX Capped Energy Index ETF
HXF	Horizons S&P/TSX Capped Financials Index ETF
HUZ	Horizons Silver ETF
HTB	Horizons US 7-10 Year Treasury Bond ETF
HTB.U	Horizons US 7-10 Year Treasury Bond ETF
DLR	Horizons US Dollar Currency ETF
DLR.U	Horizons US Dollar Currency ETF
HULC	Horizons US Large Cap Index ETF
HULC.U	Horizons US Large Cap Index ETF
HMUS	Horizons US Marijuana Index ETF
HMUS.U	Horizons US Marijuana Index ETF
HSUV.U	Horizons USD Cash Maximizer ETF

Ticker	Name
EWJ	iShares MSCI Japan ETF
EWT	iShares MSCI Taiwan ETF
MCHI	iShares MSCI China ETF
EWY	iShares MSCI South Korea ETF
EWZ	iShares MSCI Brazil ETF
INDA	iShares MSCI India ETF
ACWV	iShares MSCI Global Min Vol Factor ETF
IDV	iShares International Select Dividend ETF
EWC	iShares MSCI Canada ETF
EWU	iShares MSCI United Kingdom ETF
IXJ	iShares Global Healthcare ETF
IXG	iShares Global Financials ETF
IGF	iShares Global Infrastructure ETF
EWG	iShares MSCI Germany ETF
EWL	iShares MSCI Switzerland ETF
EWA	iShares MSCI Australia ETF
IXC	iShares Global Energy ETF
EWW	iShares MSCI Mexico ETF
EWH	iShares MSCI Hong Kong ETF
MXI	iShares Global Materials ETF
KSA	iShares MSCI Saudi Arabia ETF
EWQ	iShares MSCI France ETF
EWS	iShares MSCI Singapore ETF
EWI	iShares MSCI Italy ETF
ERUS	iShares MSCI Russia ETF
IHAK	iShares Cybersecurity and Tech ETF
EWP	iShares MSCI Spain ETF
KXI	iShares Global Consumer Staples ETF
EWD	iShares MSCI Sweden ETF
ECH	iShares MSCI Chile ETF
FM	iShares MSCI Frontier and Select EM ETF
RXI	iShares Global Consumer Discretionary ETF
EXI	iShares Global Industrials ETF
THD	iShares MSCI Thailand ETF
EIDO	iShares MSCI Indonesia ETF
EZA	iShares MSCI South Africa ETF
IXP	iShares Global Comm Services ETF
EWN	iShares MSCI Netherlands ETF
EPOL	iShares MSCI Poland ETF
TUR	iShares MSCI Turkey ETF
EWM	iShares MSCI Malaysia ETF
ENZL	iShares MSCI New Zealand ETF
EDEN	iShares MSCI Denmark ETF
EIS	iShares MSCI Israel ETF
EPU	iShares MSCI Peru ETF
JXI	iShares Global Utilities ETF
EPHE	iShares MSCI Philippines ETF
QAT	iShares MSCI Qatar ETF
EWO	iShares MSCI Austria ETF
EIRL	iShares MSCI Ireland ETF
ENOR	iShares MSCI Norway ETF
EWK	iShares MSCI Belgium ETF
ICOL	iShares MSCI Colombia ETF
EFNL	iShares MSCI Finland ETF
UAE	iShares MSCI UAE ETF
KWT	iShares MSCI Kuwait ETF