Merely Verbal Disputes and Coordinating on Logical Constants

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THE UNIVERSITY OF MELBOURNE

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My Plan

BACKGROUND

Merely Verbal Disagreement

I'm interested in *disagreement*…

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…and I'm interested in *words*,
and what they mean.

In particular, I'm interested in the role that *logic* and *logical concepts* might play in *clarifying* and *managing* disagreement.

This topic not only has connections with *logic*, but also *semantics*, *epistemology* and *metaphysics*.

Particular Issues

- *Monism* and *Pluralism* about logic.

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- *Monism* and *Pluralism* about logic.
- Disagreement between rival logicians
- Ontological relativity
- The status of modal vocabulary
- and much more.

A DEFINITION

A man walks rapidly around a tree, while a squirrel moves on the tree trunk. Both face the tree at all times, but the tree trunk stays between them. A group of people are arguing over the question:

A man walks rapidly around a tree, while a squirrel moves on the tree trunk. Both face the tree at all times, but the tree trunk stays between them. A group of people are arguing over the question:

Does the man go round the squirrel or not?

α: The man *goes round* the squirrel.

δ: The man doesn't *go round* the squirrel.

William James, a Tree, a Squirrel and a Man

Which party is right depends on what you practically mean by 'going round' the squirrel. If you mean passing from the north of him to the east, then to the south, then to the west, and then to the north of him again, obviously the man does go round him, for he occupies these successive positions. But if on the contrary you mean being first in front of him, then on the right of him then behind him, then on his left, and finally in front again, it is quite as obvious that the man fails to go round him …

Make the distinction, and there is no occasion for any farther dispute.
— *William James,* Pragmatism (1907)

α: The man *goes round*$_1$ the squirrel.

δ: The man doesn't *go round*$_2$ the squirrel.

Once we *disambiguate* “going round” there is no disagreement any more.

- For James, "going round_1" and "going round_2" are explicated in other terms of α and δ's vocabulary.

Resolution by translation

- For James, "going round$_1$" and "going round$_2$" are explicated in other terms of α and δ's vocabulary.
- Perhaps terms t_1 and t_2 *can't* be explicated in terms of prior vocabulary. No matter.

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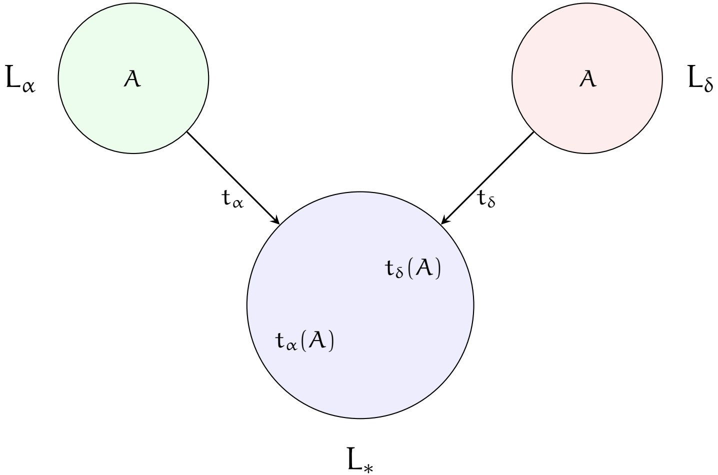
- Perhaps terms t_1 and t_2 *can't* be explicated in terms of prior vocabulary. No matter.
- α could learn t_2 while δ could learn t_1.


L_α
A
A
L_δ


L_α
A
A
L_δ
t_α
t_δ
$t_\delta(A)$
$t_\alpha(A)$
L_*

What is a *Language*?

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+ CUT: If $X \vdash A, Y$ and $X, A \vdash Y$ then $X \vdash Y$.

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$$t : L_1 \rightarrow L_2$$

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$$t : L_1 \longrightarrow L_2$$

- t may be INCOHERENCE PRESERVING: $X \vdash_{L_1} Y \;\Rightarrow\; t(X) \vdash_{L_2} t(Y)$.

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- t may be COHERENCE PRESERVING: $X \nvdash_{L_1} Y \;\Rightarrow\; t(X) \nvdash_{L_2} t(Y)$.
- t may be *compositional* (e.g., $t(A \land B) = \neg(\neg t(A) \lor \neg t(A))$, so $t(\lambda p.\lambda q.(p \land q)) = \lambda p.\lambda q.(\neg(\neg p \lor \neg q))$.)

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  This is *compositional* and *coherence preserving*, but *not incoherence preserving* for FOL derivability. $(\forall x)(\exists y)(y = x + 1)$ is true in *all* models (whether the axioms of PA hold or not). Its translation $(\forall x \in \omega)(\exists y \in \omega)(\forall z)(z \in y \equiv (z \in x \vee z = x))$ is a ZF *theorem* but not true in all models.

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  $\vdash (\forall x)(\exists y)(y = x + 1)$ while $\not\vdash t[(\forall x)(\exists y)(y = x + 1)]$.

A *dispute*

A *dispute* between a speaker α of language L_α, A *dispute* between a speaker α of language L_{α}, and δ of language L_{δ}, A *dispute* between a speaker α of language L_α, and δ of language L_δ, over C

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A General Scheme...

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- For some language L_*, $t_\alpha : L_\alpha \to L_*$, and $t_\delta : L_\delta \to L_*$,
- and $t_\alpha(C) \not\vdash_{L_*} t_\delta(C)$.

Given a resolution by translation, there is no disagreement over C in the shared language L_*.

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The position $[t_\alpha(C) : t_\delta(C)]$ is coherent.

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Taking Disputes to be Resolved by Translation

To *take* a dispute to be resolved by translation
is to take there to be a pair of translations
that resolves the dispute.

(You may not even *have* the translations in hand.)

A METHOD …

… to resolve *any* dispute by translation.

Resolution by *Disjoint Union*

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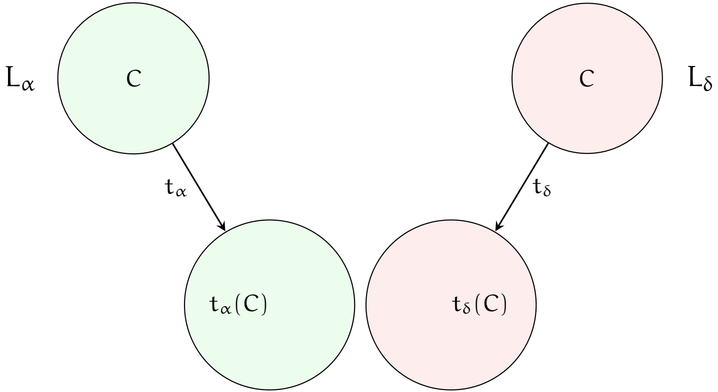
Or, what I like to call "the way of the undergraduate relativist."

Resolution by *Disjoint Union*



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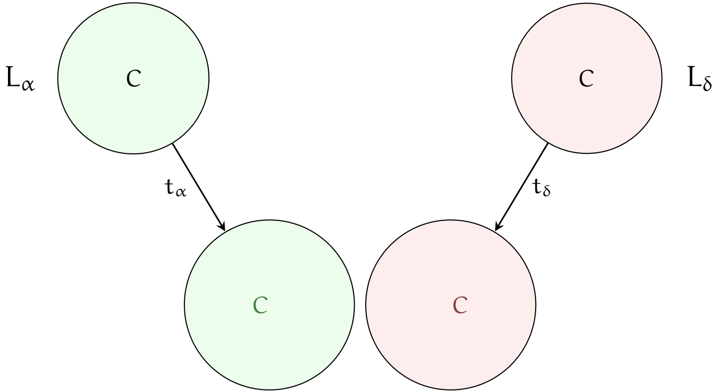

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$(X_\alpha, X_\delta \vdash Y_\alpha, Y_\delta)$ iff $(X_\alpha \vdash Y_\alpha)$ or $(X_\delta \vdash Y_\delta)$.

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This 'translation' is structure preserving,
and coherence and incoherence preserving too.

This 'resolves' the dispute over C

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then $C \nvdash_{L_{\alpha|\delta}} C$

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then $C \nvdash_{L_{\alpha|\delta}} C$

(Asserting C-from-L_α and denying C-from-L_δ is coherent.)

… AND ITS COST

Nothing α says has any bearing on δ, or *vice versa*.

What is $A \wedge B$?

What is $A \wedge B$?

There's *no such sentence* in $L_{\alpha|\delta}$!

The Case of the Venusians

Suppose aliens land on earth speaking our languages and familiar with our cultures and tell us that for more complete communication it will be necessary that we increase our vocabulary by the addition of a 1-ary sentence connective $\mathbb{V}$ … concerning which we should note immediately that certain restrictions to our familiar inferential practices will need to be imposed. As these Venusian logicians explain, ($\wedge$E) will have to be curtailed. Although for purely terrestrial sentences A and B, each of A and B follows from their conjunction $A \wedge B$, it will not in general be the case that $\mathbb{V}A$ follows from $\mathbb{V}A \wedge B$, or that $\mathbb{V}B$ follows from $A \wedge \mathbb{V}B$…

— Lloyd Humberstone, *The Connectives* §4.34

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So, there's *no* conjunction.

We might have had conjunction in L_α
and conjunction in L_δ, too
but we *lost it* from $L_{\alpha|\delta}$.

PRESERVATION

Have we got conjunction in L?

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for *all* X, Y, A and B in L.

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for *all* X, Y, A and B in L.

(There is no conjunction in $L_{\alpha|\delta}$. There is no sentence “A and B”.)

A translation $t : L_1 \to L_2$ is CONJUNCTION PRESERVING if a conjunction in L_1 is translated by a conjunction in L_2.

Preservation seems like a good idea

Translations should keep *some things* preserved.

Let's see what we can do with this.

EXAMPLES

Obviously, there some disagreements can resolved
by a disambiguation of different senses of the word 'and.'

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$$\text{`and}_\alpha\text{'} \xrightarrow{t_\alpha} \text{`}\wedge\text{'} \qquad \text{`and}_\delta\text{'} \xrightarrow{t_\delta} \text{`and } \textit{then}\text{'}$$

No Verbal Disagreement Between Two *Conjunctions*

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That is, in L_*, $A \wedge B \vdash A \,\&\, B$ and $A \,\&\, B \vdash A \wedge B$.

Why?

Here's why

$$\cfrac{\cfrac{A \,\&\, B \vdash A \,\&\, B}{A, B \vdash A \,\&\, B}\,[\&\uparrow]}{A \land B \vdash A \,\&\, B}\,[\land\downarrow] \qquad \cfrac{\cfrac{A \land B \vdash A \land B}{A, B \vdash A \land B}\,[\land\uparrow]}{A \,\&\, B \vdash A \land B}\,[\&\downarrow]$$

(Since $\land$ and $\&$ are both conjunctions in L_*.)

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$$\dfrac{A \vdash A' \qquad \dfrac{B \vdash B' \qquad \dfrac{A' \mathbin{\&} B' \vdash A' \mathbin{\&} B'}{A', B' \vdash A' \mathbin{\&} B'}[\&\uparrow]}{A', B \vdash A' \mathbin{\&} B'}[Cut]}{\dfrac{A, B \vdash A' \mathbin{\&} B'}{A \wedge B \vdash A' \mathbin{\&} B'}[\wedge\downarrow]}[Cut]$$

If A/A' and $B/B'$are equivalent, so are $A \wedge B$ and $A' \mathbin{\&} B'$.

This is not surprising...

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… since the rules for conjunction are *very strong*.

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Sort of.

Negation

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CLASSICAL LOGIC:

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INTUITIONIST LOGIC:

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Let's call something a NEGATION in L
if it satisfies at least the intuitionist negation rules.

And let's say that t PRESERVES NEGATION
if it translates a negation by a negation.

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2. $t_1 : L_1 \to L_*$, and $t_2 : L_2 \to L_*$ are both *negation preserving*.

No Verbal Disagreement Between Two *Negations*

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Why?

Collapse?

$$\dfrac{\dfrac{-A \vdash -A}{-A, A \vdash}\,[-\uparrow]}{-A \vdash \neg A}\,[\neg\downarrow] \qquad \dfrac{\dfrac{\neg A \vdash \neg A}{\neg A, A \vdash}\,[\neg\uparrow]}{\neg A \vdash -A}\,[-\downarrow]$$

Any disagreement, where one asserts $\neg A$ and the other denies $-A$ (or vice versa) must resolve into a disagreement over A.

What options are there for disagreement?

- Disagreement over the consequence relation '$\vdash$' (*pluralism*).
- The classical logician thinks the intuitionist is mistaken to take '$\neg$' to be so weak, or the intuitionist thinks that the classical logician is mistaken to take '$-$' to be so strong.

Can we have merely verbal disagreement about ‘exists’?

~~Can we have merely verbal disagreement about ‘exists’?~~

Can we have merely verbal disagreement about ‘$(\exists x)$’?

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Can we have merely verbal disagreement about '$(\exists x)$'?

Surely!

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Can we have merely verbal disagreement about '$(\exists x)$'?

Surely! Take *multi-sorted* first order logic. α says that there are numbers ($(\exists x)Nx$). δ denies it ($\neg(\exists x)Nx$). Can we make this difference *merely verbal*? While respecting some of the semantics of $(\exists x)$?

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Translate into a vocabulary with two quantifiers and two *two* domains: D_1 and D_2 with two quantifiers $(\exists_1 x)$ and $(\exists_2 x)$ ranging over each. Let N have a non-empty extension on D_1 but an empty one on D_2. Both α and δ can happily endorse $(\exists_1 x)Nx$ and deny $(\exists_2 x)Nx$ and be done with it.

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Isn't *this* a merely verbal disagreement over what exists?

Not so fast...

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$$\frac{X, A(v) \vdash Y}{X, (\exists x)A(x) \vdash Y}\ [\exists\updownarrow]$$

(Where v is not free in X and Y.)

This is what it takes to be an *existential quantifier* in L.

Existential Quantifier Collapse

$$\dfrac{\dfrac{(\exists_2 x)A(x) \vdash (\exists_2 x)A(x)}{A(v) \vdash (\exists_2 x)A(x)}\,[\exists_2\uparrow]}{(\exists_1 x)A(x) \vdash (\exists_2 x)A(x)}\,[\exists_1\downarrow] \qquad \dfrac{\dfrac{(\exists_1 x)A(x) \vdash (\exists_1 x)A(x)}{A(v) \vdash (\exists_1 x)A(x)}\,[\exists_1\uparrow]}{(\exists_2 x)A(x) \vdash (\exists_1 x)A(x)}\,[\exists_2\downarrow]$$

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If the term v appropriate to $[\exists_1\updownarrow]$ also applies in $[\exists_2\updownarrow]$, and *vice versa*, then indeed, the two quantifiers *collapse*.

Coordination on *terms* brings coordination on $(\exists x)$

If the following *three* conditions hold:

1. '$(\exists_1 x)$' is an existential quantifier in L_1 and '$(\exists_2 x)$' is an existential quantifier in L_2, and
2. $t_1 : L_1 \longrightarrow L_*$, and $t_2 : L_2 \longrightarrow L_*$, are both *existential quantifier preserving*, and
3. In L_*, the term v is *appropriate* for $(\exists_1 x)$ iff it is appropriate for $(\exists_2 x)$

then $(\exists_1 x)$ and $(\exists_2 x)$ are *equivalent* in L_*, in that in L_* we have $(\exists_1 x)A \vdash (\exists_2 x)A$ and $(\exists_2 x)A \vdash (\exists_1 x)A$.

It's important to recognise what this is *not*

The appropriateness condition for eigenvariables (demonstratives, terms) is *grammatical*. It doesn't force agreement on *what exists*.

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The appropriateness condition for eigenvariables (demonstratives, terms) is *grammatical*. It doesn't force agreement on *what exists*.

You could coherently be a *monist* and argue with someone with a more conventional ontology—with the *same* quantifiers, provided that you both took the same terms (demonstratives, eigenvariables, whatever) to be in order.

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You don't need to take these terms to *refer* to the same things.

A Dispute between a Monist and a Pluralist

MONIST:

- $(\forall x)(\forall y)x = y$

PLURALIST:

A Dispute between a Monist and a Pluralist

MONIST:

- $(\forall x)(\forall y)x = y$

PLURALIST:

- $(\exists x)(\exists y)x \neq y$

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- $a \neq b$

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Can we have merely verbal disagreement about ‘possibility’?

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Can we have merely verbal disagreement about ‘$\Diamond$’?

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Can we have merely verbal disagreement about ‘$\Diamond$’?

Surely!

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Can we have merely verbal disagreement about ‘$\Diamond$’?

Surely! Take *multi-modal* logic. $\Diamond_1$ ranges over *possible worlds*; $\Diamond_2$ ranges over *times*.

Isn’t *this* a merely verbal disagreement over what *possible*?

Surely it's impossible to force coordination something as flexible as $\Diamond$!

Not so fast...

Let's consider more closely what might be involved in *possibility preservation*.

$$\frac{A \vdash \ \mid X \vdash Y \mid \Delta}{X, \Diamond A \vdash Y \mid \Delta}\ [\Diamond\updownarrow]$$

The separated sequents indicate positions in which assertions and denials are made in different *zones* of a discourse.

$$\dfrac{\dfrac{\Diamond_2 A \vdash \Diamond_2 A}{A \vdash \;\mid\; \vdash \Diamond_2 A}\,[\Diamond_2\Uparrow]}{\Diamond_1 A \vdash \Diamond_2 A}\,[\Diamond_1\Downarrow] \qquad \dfrac{\dfrac{\Diamond_1 A \vdash \Diamond_1 A}{A \vdash \;\mid\; \vdash \Diamond_1 A}\,[\Diamond_1\Uparrow]}{\Diamond_2 A \vdash \Diamond_1 A}\,[\Diamond_2\Downarrow]$$

If the *zone* appropriate to $[\Diamond_1\Updownarrow]$ also applies in $[\Diamond_2\Updownarrow]$, and *vice versa* then indeed, the two operators *collapse*.

Coordination on *zones* brings coordination on $\Diamond$

If the following *three* conditions hold:

1. '$\Diamond_1$' is an possibility in L_1 and '$\Diamond_2$' is an possibility in L_2, and
2. $t_1 : L_1 \to L_*$, and $t_2 : L_2 \to L_*$, are both *possibility preserving*, and
3. In L_*, a zone *appropriate* for $\Diamond_1$ iff it is appropriate for $\Diamond_2$

then $\Diamond_1$ and $\Diamond_2$ are *equivalent* in L_*, in that in L_* we have $\Diamond_1 A \vdash \Diamond_2 A$ and $\Diamond_2 A \vdash \Diamond_1 A$.

It's important to recognise what *this* is not

The appropriateness condition for zones is *dialogical*. It doesn't force agreement on *what is possible*.

It's important to recognise what *this* is not

The appropriateness condition for zones is *dialogical*. It doesn't force agreement on *what is possible*.

You could coherently be a *modal fatalist* and argue with someone with a more conventional modal views—with the *same* modal operators, provided that you both took the same zones to be in order.

It's important to recognise what *this* is not

The appropriateness condition for zones is *dialogical*. It doesn't force agreement on *what is possible*.

You could coherently be a *modal fatalist* and argue with someone with a more conventional modal views—with the *same* modal operators, provided that you both took the same zones to be in order.

(You don't need to take the same things to *hold* in each zone.)

THE UPSHOT

… and the fewer ways there are to settle verbal disputes.

It's one thing to think of a logical concept…

… as something satisfying a set of *axioms*.

... as something satisfying a set of *axioms*.

But that is *cheap*. Defining rules are *more powerful*.

And defining rules are natural, given the conception of logical constants as topic neutral, and definable in general terms.

Generality comes in degrees

1. Propositional connectives: *sequents alone*.
2. Modals: *hypersequents*.
3. Quantifiers: *predicate structure*.

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1. Propositional connectives: *sequents alone*.
2. Modals: *hypersequents*.
3. Quantifiers: *predicate structure*.

Using this structure to define the behaviour of a logical concepts allows for them to be preserved in translation and used as a fixed point in the midst of disagreement.

The defining rule is the fulcrum...

...which stays fixed while other things change arond it.

THANK YOU!

http://consequently.org/presentation/2015/mvd-logicmelb/

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