Generality & Existence II

Modality & Quantifiers

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ARCHÉ, ST ANDREWS · 2 DECEMBER 2015

To analyse the *quantifiers*

My Aim

To analyse the *quantifiers*
(including their interactions with *modals*)

To analyse the *quantifiers*
(including their interactions with *modals*)
using the tools of *proof theory*

To analyse the *quantifiers*
(including their interactions with *modals*)
using the tools of *proof theory*
in order to better understand
quantification, *existence* and *identity*.

Understanding the interactions between *quantifiers* and *modal operators*.

Sequents & Defining Rules

Hypersequents & Defining Rules

Quantification & the Barcan Formula

Positions & Models

Consequences & Questions

SEQUENTS & DEFINING RULES

$$\Gamma \succ \Delta$$

Don't assert each element of Γ
and deny each element of Δ.

Identity: $A \succ A$

Structural Rules

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Weakening: $\dfrac{\Gamma \succ \Delta}{\Gamma, A \succ \Delta} \quad \dfrac{\Gamma \succ \Delta}{\Gamma \succ A, \Delta}$

Structural Rules

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Weakening: $$\frac{\Gamma \succ \Delta}{\Gamma, A \succ \Delta} \qquad \frac{\Gamma \succ \Delta}{\Gamma \succ A, \Delta}$$

Contraction: $$\frac{\Gamma, A, A \succ \Delta}{\Gamma, A \succ \Delta} \qquad \frac{\Gamma \succ A, A, \Delta}{\Gamma \succ A, \Delta}$$

Structural Rules

$$\textit{Identity:}\quad A \succ A$$

$$\textit{Weakening:}\quad \frac{\Gamma \succ \Delta}{\Gamma, A \succ \Delta} \qquad \frac{\Gamma \succ \Delta}{\Gamma \succ A, \Delta}$$

$$\textit{Contraction:}\quad \frac{\Gamma, A, A \succ \Delta}{\Gamma, A \succ \Delta} \qquad \frac{\Gamma \succ A, A, \Delta}{\Gamma \succ A, \Delta}$$

$$\textit{Cut:}\quad \frac{\Gamma \succ A, \Delta \qquad \Gamma, A \succ \Delta}{\Gamma \succ \Delta}$$

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Cut: $$\frac{\Gamma \succ A, \Delta \quad \Gamma, A \succ \Delta}{\Gamma \succ \Delta}$$

Structural rules govern declarative sentences *as such*.

With Left/Right rules?

$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \land B \succ \Delta}\,[\land L] \qquad \frac{\Gamma \succ A, \Delta \quad \Gamma \succ B, \Delta}{\Gamma \succ A \land B, \Delta}\,[\land R]$$

With Left/Right rules?

$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \wedge B \succ \Delta}\,[\wedge L] \qquad \frac{\Gamma \succ A, \Delta \quad \Gamma \succ B, \Delta}{\Gamma \succ A \wedge B, \Delta}\,[\wedge R]$$

$$\frac{\Gamma, B \succ \Delta}{\Gamma, A \; tonk \; B \succ \Delta}\,[tonkL] \qquad \frac{\Gamma \succ A, \Delta}{\Gamma \succ A \; tonk \; B, \Delta}\,[tonkR]$$

Use $\succ_{\mathcal{L}}$ to *define* $\succ_{\mathcal{L}'}$.

What is involved in going from $\mathcal{L}$ to $\mathcal{L}'$?

Use $\succ_{\mathcal{L}}$ to *define* $\succ_{\mathcal{L}'}$.

Desideratum #1: $\succ_{\mathcal{L}'}$ is *conservative:* $(\succ_{\mathcal{L}'})|_{\mathcal{L}}$ is $\succ_{\mathcal{L}}$.

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Desideratum #1: $\succ_{\mathcal{L}'}$ is *conservative*: $(\succ_{\mathcal{L}'})|_{\mathcal{L}}$ is $\succ_{\mathcal{L}}$.

Desideratum #2: Concepts are defined *uniquely*.

A Defining Rule

$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \land B \succ \Delta}\ [\land Df]$$

(The rule is printed with a double line.)

$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \wedge B \succ \Delta}\ [\wedge Df]$$

Fully specifies norms governing conjunctions on the *left* in terms of simpler vocabulary.

A Defining Rule

$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \land B \succ \Delta}\,[\land Df]$$

Fully specifies norms governing conjunctions on the *left* in terms of simpler vocabulary.

Identity and *Cut* determine the behaviour of conjunctions on the *right*.

$$
\dfrac{\Gamma \succ A, \Delta \qquad \dfrac{\Gamma \succ B, \Delta \qquad \dfrac{\dfrac{}{\color{red}{A \wedge B \succ A \wedge B}}\,[Id]}{A, B \succ A \wedge B}\,[\wedge Df]}{\Gamma, A \succ A \wedge B, \Delta}\,[Cut]}{\Gamma \succ A \wedge B, \Delta}\,[Cut]
$$

From $[\wedge Df]$ to $[\wedge L/R]$

$$
\dfrac{\Gamma \succ A, \Delta \qquad \dfrac{\Gamma \succ B, \Delta \qquad \dfrac{\dfrac{}{A \wedge B \succ A \wedge B}\,[Id]}{A, B \succ A \wedge B}\,[\wedge Df]}{\Gamma, A \succ A \wedge B, \Delta}\,[Cut]}{\Gamma \succ A \wedge B, \Delta}\,[Cut]
$$

From [$\wedge Df$] to [$\wedge L/R$]

$$
\dfrac{\Gamma \succ A, \Delta \qquad \dfrac{\Gamma \succ B, \Delta \qquad \dfrac{\dfrac{}{A \wedge B \succ A \wedge B}\,[Id]}{A, B \succ A \wedge B}\,[\wedge Df]}{\Gamma, A \succ A \wedge B, \Delta}\,[Cut]}{\Gamma \succ A \wedge B, \Delta}\,[Cut]
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$$

From [$\wedge Df$] to [$\wedge L/R$]

$$
\dfrac{\Gamma \succ A, \Delta \qquad \dfrac{\Gamma \succ B, \Delta \qquad \dfrac{\overline{A \wedge B \succ A \wedge B}\,[Id]}{A, B \succ A \wedge B}\,[\wedge Df]}{\Gamma, A \succ A \wedge B, \Delta}\,[Cut]}{\Gamma \succ A \wedge B, \Delta}\,[Cut]
$$

$$
\dfrac{\Gamma \succ A, \Delta \qquad \Gamma \succ B, \Delta}{\Gamma \succ A \wedge B, \Delta}\,[\wedge R]
$$

And Back

$$\dfrac{\dfrac{A \succ A \quad B \succ B}{A, B \succ A \land B}\,[\land R] \qquad \Gamma, A \land B \succ \Delta}{\Gamma, A, B \succ \Delta}\,[Cut]$$

I want to see how this works for modal operators, and examine their interaction with the quantifiers.

Explaining *why* the modal operators have the logical properties they exhibit is an open question.

… possible worlds, in the sense of possible states of affairs are not *really* individuals (just as numbers are not *really* individuals).

To say that a state of affairs obtains is just to say that something is the case; to say that something is a possible state of affairs is just to say that something could be the case; and to say that something is the case 'in' a possible state of affairs is just to say that the thing in question would necessarily be the case if that state of affairs obtained, i.e. if something else were the case …

We understand 'truth in states of affairs' because we understand 'necessarily'; not *vice versa*.
— "Worlds, Times and Selves" (1969)

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- Why is it that modal concepts (which are conceptually prior to worlds) have a structure that fits possible worlds models?
- (Why does possibility distribute of disjunction, necessity over disjunction? Why do the modalities *work* like normal modal logics?)

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- (Why does possibility distribute of disjunction, necessity over disjunction? Why do the modalities *work* like normal modal logics?)
- If modality is *primitive* we have no explanation.
- If modality is governed by the rules introduced here, then we can see *why* possible worlds are useful, and model the behaviour of modal concepts.

HYPERSEQUENTS & DEFINING RULES

Modal Reasoning involves *Shifts*

Suppose it's necessary that p and necessary that q.

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Is it necessary that both p and q?

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Consider any way it could go:

Since it's necessary that p, here we have p.

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Since it's necessary that p, here we have p.

Since it's necessary that q, here we have q.

So, we have both p and q.

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Could we *avoid* p and q?

Consider any way it could go:

Since it's necessary that p, here we have p.

Since it's necessary that q, here we have q.

So, we have both p and q.

So, no matter how things go, we have p and q.

Modal Reasoning involves *Shifts*

Suppose it's necessary that p and necessary that q.
Is it necessary that both p and q?

Could we *avoid* p and q?

Consider any way it could go:

Since it's necessary that p, here we have p.

Since it's necessary that q, here we have q.

So, we have both p and q.

So, no matter how things go, we have p and q.

So the conjunction p and q is necessary.

Exposing the Structure of that Deduction

$$\dfrac{\dfrac{\dfrac{\Box p, \Box q \succ \Box p}{\Box p, \Box q \succ \ \mid\ \succ p}[\Box Df] \quad \dfrac{\Box p, \Box q \succ \Box q}{\Box p, \Box q \succ \ \mid\ \succ q}[\Box Df]}{\dfrac{\Box p, \Box q \succ \ \mid\ \succ p \land q}{\Box p, \Box q \succ \Box(p \land q)}[\Box Df]}[\land R]}{\Box p \land \Box q \succ \Box(p \land q)}[\land Df]$$

Exposing the Structure of that Deduction

$$\dfrac{\dfrac{\dfrac{\Box p, \Box q \succ \Box p}{\Box p, \Box q \succ \;\mid\; \succ p}[\Box Df] \quad \dfrac{\Box p, \Box q \succ \Box q}{\Box p, \Box q \succ \;\mid\; \succ q}[\Box Df]}{\dfrac{\Box p, \Box q \succ \;\mid\; \succ p \land q}{\Box p, \Box q \succ \Box(p \land q)}[\Box Df]}[\land R]}{\color{red}{\Box p \land \Box q} \succ \Box(p \land q)}[\land Df]$$

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Exposing the Structure of that Deduction

$$
\dfrac{
\dfrac{\Box p, \Box q \succ \Box p}{\color{red}{\Box p, \Box q \succ \; \mid \; \succ p}}\,[\Box Df]
\qquad
\dfrac{\Box p, \Box q \succ \Box q}{\Box p, \Box q \succ \; \mid \; \succ q}\,[\Box Df]
}{
\dfrac{
\dfrac{\Box p, \Box q \succ \; \mid \; \succ p \land q}{\Box p, \Box q \succ \Box(p \land q)}\,[\Box Df]
}{\Box p \land \Box q \succ \Box(p \land q)}\,[\land Df]
}\,[\land R]
$$

Exposing the Structure of that Deduction

$$\dfrac{\dfrac{\Box p, \Box q \succ \Box p}{\Box p, \Box q \succ \ \mid\ \succ p}[\Box Df] \qquad \dfrac{\Box p, \Box q \succ \Box q}{\color{red}{\Box p, \Box q \succ \ \mid\ \succ q}}[\Box Df]}{\dfrac{\dfrac{\Box p, \Box q \succ \ \mid\ \succ p \land q}{\Box p, \Box q \succ \Box(p \land q)}[\Box Df]}{\Box p \land \Box q \succ \Box(p \land q)}[\land Df]}[\land R]$$

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$$\Box p, \Box q \succ \quad \mid \quad \succ p \wedge q$$

Don't assert $\Box p$ and $\Box q$ in one 'zone'
and deny $p \wedge q$ in another.

$$\Gamma \succ \Delta \mid \Gamma' \succ \Delta'$$

Don't assert each member of Γ
and deny each member of Δ in one 'zone'
and assert each member of Γ'
and deny each member of Δ' in another.

Two Kinds of Zone Shift

INDICATIVE: suppose I'm wrong and that. . .

SUBJUNCTIVE: suppose things go differently.
or *had gone* differently.

SPECIAL COMMEMORATIVE SECTION

The New York Times.

LATE CITY EDITION

KENNEDY IS KILLED BY SNIPER AS HE RIDES IN CAR IN DALLAS; JOHNSON SWORN IN ON PLANE

TEXAN ASKS UNITY

LEFTIST ACCUSED

Gov. Connally Shot; Mrs. Kennedy Safe

Why America Weeps

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Why America Weeps

- Suppose Oswald *didn't* shoot JFK.

- Suppose Oswald *didn't* shoot JFK.
- Suppose Oswald *hadn't* shot JFK.

Philosophers' Imprint

volume 7, no. 4
may 2007

STEREOSCOPIC VISION:

Persons, Freedom, and Two Spaces of Material Inference

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Freedom, oh freedom, well that's just some people talkin'.
— The Eagles

What is a person, as opposed to a non-person? One might begin to address the question by appealing to a second distinction: between *agents*, characterized by the ability to act freely and intentionally, and mere patients, caught up in events but in no sense authors of the happenings involving them. An alternative way to address the question appeals to a third distinction: between *subjects* — bearers of rights and responsibilities, commitments and entitlements, makers of claims, thinkers of thoughts, issuers of orders, and posers of questions — and mere objects, graspable or evaluable by subjects but not themselves graspers or evaluators.

We take it as a methodological point of departure that these three distinctions are largely coextensive, indeed coextensive in conceptually central cases. Granted, these distinctions can come apart. One might think that 'person' applies to anything that is worthy of a distinctive sort of moral respect and think this applicable to some fetuses or the deeply infirm elderly. Even if the particular respect due such beings is importantly different from "what we owe each other", such respect could still be thought to be of the kind distinctively due people, and think this even while holding that such people lack agentive or subjective capacity. Similarly, one might think dogs or various severely impaired humans to be attenuated subjects but not agents.

Without taking any particular stand on such examples, our methodological hypothesis is that such cases, if they exist, are understood as persons (agents, subjects) essentially by reference to paradigm cases and, indeed, to a single paradigm within which person/non-person, subject/object, and agent/patient are conceptually connected.[1] Stated

1. For one detailed development of this sort of paradigm-riff structure, and a defense of the possibility of concepts essentially governed by such a structure, see Lance and Little (2004). Discussions with Hilda Lindeman have helped

- DISAGREEMENT: We *disagree*. We have reason to come to shared positions.

We are *social* creatures, who *act* on the basis of *views*

- DISAGREEMENT: We *disagree*. We have reason to come to shared positions.

- PLANNING: We *plan*. We have reason to consider options (prospectively) or replay scenarios (retrospectively).

We are *social* creatures, who *act* on the basis of *views*

- DISAGREEMENT: We *disagree*. We have reason to come to shared positions.
- PLANNING: We *plan*. We have reason to consider options (prospectively) or replay scenarios (retrospectively).
- We do *many different and strange things* in our messy zone-shifting practices, but we can isolate a particular convention or practice, idealise it, to see what we could do following *those* rules.

We are *social* creatures, who *act* on the basis of *views*

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- PLANNING: We *plan*. We have reason to consider options (prospectively) or replay scenarios (retrospectively).
- We do *many different and strange things* in our messy zone-shifting practices, but we can isolate a particular convention or practice, idealise it, to see what we could do following *those* rules.
  - (*Analogies*: $\forall x$ from first order logic and natural language's 'all.' Frictionless planes. etc.)

Example Subjunctive Shifts

Oswald *did* shoot JFK, but suppose he *hadn't*? How would history have gone differently then?

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$$[\mathrm{oSk} \ : \quad]_{@} \ \mid \ [@\mathrm{oSk} \ : \ \mathrm{oSk}]$$

Oswald *did* shoot JFK, but suppose he *hadn't*? How would history have gone differently then?

$$[\mathrm{oSk} \; : \quad]_{@} \mid [@\mathrm{oSk} \; : \; \mathrm{oSk}]$$

We open up a zone for consideration, in which we deny oSk, while keeping track of the initial zone where we assert it.

(And if we like, we can assert @oSk in the zone under the counterfactual supposition.)

Disagreement and Indicative Shifting

I think that Oswald shot JFK, but you don't.

I consider what it would mean for you to be right.

If you're right, Oswald *actually didn't* shoot JFK.

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Epistemic alternatives interact differently with actuality.

$$[\mathrm{oSk} : \ ]_{@} \Longrightarrow [\ : \mathrm{oSk}]_{@}$$

I think that Hesperus is Phosphorous, but I recognise that *you don't*. I don't take *you* to be *inconsistent* or misusing names.

Indicative Shifting

I think that Hesperus is Phosphorous, but I recognise that *you don't*. I don't take *you* to be *inconsistent* or misusing names.

We *don't* have this:

$$a = b \succ \quad \Longrightarrow \quad Fa \succ Fb$$

It's coherent for you to assert Fa and deny Fb even if I take it that $a = b$, and it's coherent for me to consider an alternative in which $a \neq b$ even if I don't agree.

- Let's take any zone found by indicatively shifting from here (or from anywhere from here, etc.) to be an alternative indicative context from any *other* context indicatively shifted from here.

Idealised Indicative Shifts

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- This is as *liberal* as possible about what counts as an *alternative* from any alternative zone.

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- (And each are *actual* zones.)
- This is as *liberal* as possible about what counts as an *alternative* from any alternative zone.
- This gives us a motivation for a richer family of hypersequents.

Two Dimensional Hypersequents

$$
\begin{array}{ccccccccc}
X_1^1 \succ_@ Y_1^1 & \mid & X_2^1 \succ Y_2^1 & \mid & \cdots & \mid & X_{m_1}^1 \succ Y_{m_1}^1 & \| \\
X_1^2 \succ_@ Y_1^2 & \mid & X_2^2 \succ Y_2^2 & \mid & \cdots & \mid & X_{m_2}^2 \succ Y_{m_2}^2 & \| \\
\vdots & & \vdots & & & & \vdots & \\
X_1^n \succ_@ Y_1^n & \mid & X_2^n \succ Y_2^n & \mid & \cdots & \mid & X_{m_n}^n \succ Y_{m_n}^n &
\end{array}
$$

Two Dimensional Hypersequents

$$
\begin{array}{ccccccccc}
X_1^1 \succ_@ Y_1^1 & | & X_2^1 \succ Y_2^1 & | & \cdots & | & X_{m_1}^1 \succ Y_{m_1}^1 & \| \\
X_1^2 \succ_@ Y_1^2 & | & X_2^2 \succ Y_2^2 & | & \cdots & | & X_{m_2}^2 \succ Y_{m_2}^2 & \| \\
\vdots & & \vdots & & & & \vdots & \\
X_1^n \succ_@ Y_1^n & | & X_2^n \succ Y_2^n & | & \cdots & | & X_{m_n}^n \succ Y_{m_n}^n &
\end{array}
$$

Think of these as *scorecards*, keeping track of assertions and denials.

Notation

$$\mathcal{H}[\Gamma \succ \Delta]$$

Notation

$$\mathcal{H}[\Gamma' \succ \Delta']$$

Notation

$$\mathcal{H}[\Gamma' \succ \Delta']$$

$$\mathcal{H}[\Gamma \succ \Delta \mid \Gamma' \succ \Delta']$$

Notation

$$\mathcal{H}[\Gamma' \succ \Delta']$$

$$\mathcal{H}[\Gamma \succ \Delta \mid \Gamma' \succ \Delta']$$

$$\mathcal{H}[\Gamma \succ \Delta \parallel \Gamma' \succ \Delta']$$

Defining Rule for $\Box$

$$\frac{\mathcal{H}[\Gamma \succ \Delta \;\mid\; \succ A]}{\mathcal{H}[\Gamma \succ \Box A, \Delta]}\,[\Box Df]$$

Defining Rule for @

$$\frac{\mathcal{H}[\Gamma, A \succ_@ \Delta \mid \Gamma' \succ \Delta']}{\mathcal{H}[\Gamma \succ_@ \Delta \mid \Gamma', @A \succ \Delta']}\ [@Df]$$

Defining Rule for [e]

$$\frac{\mathcal{H}[\Gamma \succ \Delta \;\; \| \;\; \succ_{@} A]}{\mathcal{H}[\Gamma \succ [\mathsf{e}]A, \Delta]}\;[[\mathsf{e}]Df]$$

Example Derivation

$$
\begin{array}{c}
\succ \quad \mid \; [\mathrm{e}]A \succ [\mathrm{e}]A \\
\hline
\succ \quad \mid \; [\mathrm{e}]A \succ \quad \| \quad \succ_{@} A \\
\hline
\succ \quad \mid \; [\mathrm{e}]A \succ \quad \| \quad \succ_{@} @A \\
\hline
\succ \quad \mid \; [\mathrm{e}]A \succ [\mathrm{e}]@A \\
\hline
\succ \quad \mid \quad \succ [\mathrm{e}]A \supset [\mathrm{e}]@A \\
\hline
\succ \Box([\mathrm{e}]A \supset [\mathrm{e}]@A)
\end{array}
\qquad
\begin{array}{l}
\\
[[\mathrm{e}]Df] \\
[@Df] \\
[[\mathrm{e}]Df] \\
[\supset Df] \\
[\Box Df]
\end{array}
$$

QUANTIFICATION & THE BARCAN FORMULA

The Standard Quantifier Rules

$$\frac{\Gamma \succ A(n), \Delta}{\Gamma \succ (\forall x)A(x), \Delta}\,[\forall Df] \qquad \frac{\Gamma, A(n) \succ \Delta}{\Gamma, (\exists x)A(x) \succ \Delta}\,[\exists Df]$$

Deriving the Barcan Formula

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(\forall x)\Box Fx \succ (\forall x)\Box Fx}{(\forall x)\Box Fx \succ \Box Fn}\,[\forall Df]}{(\forall x)\Box Fx \succ \quad | \quad \succ Fn}\,[\Box Df]}{(\forall x)\Box Fx \succ \quad | \quad \succ (\forall x)Fx}\,[\forall Df]}{(\forall x)\Box Fx \succ \Box(\forall x)Fx}\,[\Box Df]}{\succ (\forall x)\Box Fx \supset \Box(\forall x)Fx}\,[\supset Df]}{}$$

Where the derivation breaks down

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\color{red}{(\forall x)\Box Fx \succ (\forall x)\Box Fx}}{(\forall x)\Box Fx \succ \Box Fn}\,[\forall \mathit{Df}]}{(\forall x)\Box Fx \succ \quad \mid \quad \succ Fn}\,[\Box \mathit{Df}]}{(\forall x)\Box Fx \succ \quad \mid \quad \succ (\forall x)Fx}\,[\forall \mathit{Df}]}{(\forall x)\Box Fx \succ \Box(\forall x)Fx}\,[\Box \mathit{Df}]}{\succ (\forall x)\Box Fx \supset \Box(\forall x)Fx}\,[\supset \mathit{Df}]
$$

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$$
\begin{array}{cl}
(\forall x)\Box Fx \succ (\forall x)\Box Fx & \\
\hline
(\forall x)\Box Fx \succ \Box Fn & [\forall Df] \\
\hline
\color{red}{(\forall x)\Box Fx \succ \quad \mid \quad \succ Fn} & [\Box Df] \\
\hline
(\forall x)\Box Fx \succ \quad \mid \quad \succ (\forall x)Fx & [\forall Df] \\
\hline
(\forall x)\Box Fx \succ \Box(\forall x)Fx & [\Box Df] \\
\hline
\succ (\forall x)\Box Fx \supset \Box(\forall x)Fx & [\supset Df]
\end{array}
$$

Pro and *Con* attitudes to Terms

To rule a term *in* is to take it as suitable
to substitute into a quantifier,
i.e., to take the term to *denote*.

To rule a term *out* is to take it as unsuitable
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We add terms to the LHS and RHS of sequents $\Gamma \succ \Delta$.

Structural Rules remain as before

Identity: $\mathsf{X} \succ \mathsf{X}$

Weakening: $$\frac{\mathcal{H}[\Gamma \succ \Delta]}{\mathcal{H}[\Gamma, \mathsf{X} \succ \Delta]} \qquad \frac{\mathcal{H}[\Gamma \succ \Delta]}{\mathcal{H}[\Gamma \succ \mathsf{X}, \Delta]}$$

Contraction: $$\frac{\mathcal{H}[\Gamma, \mathsf{X}, \mathsf{X} \succ \Delta]}{\mathcal{H}[\Gamma, \mathsf{X} \succ \Delta]} \qquad \frac{\mathcal{H}[\Gamma \succ \mathsf{X}, \mathsf{X}, \Delta]}{\mathcal{H}[\Gamma \succ \mathsf{X}, \Delta]}$$

Cut: $$\frac{\mathcal{H}[\Gamma \succ \mathsf{X}, \Delta] \quad \mathcal{H}[\Gamma, \mathsf{X} \succ \Delta]}{\mathcal{H}[\Gamma \succ \Delta]}$$

Here X is either a sentence or a term.

...and there are some more

Ext. Weak.: $$\frac{\mathcal{H}[\Gamma \succ \Delta]}{\mathcal{H}[\Gamma \succ \Delta \mid \Gamma' \succ \Delta']} \quad \frac{\mathcal{H}[\Gamma \succ \Delta]}{\mathcal{H}[\Gamma \succ \Delta \parallel \Gamma' \succ \Delta']}$$

Ext. Contr.: $$\frac{\mathcal{H}[\Gamma \succ \Delta \mid \Gamma \succ \Delta]}{\mathcal{H}[\Gamma \succ \Delta]} \quad \frac{\mathcal{H}[\mathcal{S} \parallel \mathcal{S}]}{\mathcal{H}[\mathcal{S}]}$$

Quantifier Rules, allowing for non-denoting terms

$$\frac{\mathcal{H}[\Gamma, n \succ A(n), \Delta]}{\mathcal{H}[\Gamma \succ (\forall x)A(x), \Delta]} \; [\forall Df] \qquad \frac{\mathcal{H}[\Gamma, n, A(n) \succ \Delta]}{\mathcal{H}[\Gamma, (\exists x)A(x) \succ \Delta]} \; [\exists Df]$$

Now you *can't* derive the Barcan Formula

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$$(\forall x)\Box Fx \succ \Box(\forall x)Fx \quad | \quad \succ \color{red}{(\forall x)Fx}$$

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$$(\forall x)\Box Fx \succ \Box(\forall x)Fx \;\mid\; b \succ Fb, (\forall x)Fx$$

Now you *can't* derive the Barcan Formula

$$(\forall x)\Box Fx \succ b, Fb, \Box(\forall x)Fx \mid b \succ Fb, (\forall x)Fx$$

Now you *can't* derive the Barcan Formula

$$a, (\forall x)\Box Fx \succ b, Fb, \Box(\forall x)Fx \;\mid\; a, b \succ Fb, (\forall x)Fx$$

Now you *can't* derive the Barcan Formula

$$a, \Box Fa, (\forall x)\Box Fx \succ b, Fb, \Box(\forall x)Fx \;\mid\; a, b \succ Fb, (\forall x)Fx$$

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This hypersequent is underivable…

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This hypersequent is underivable...

...and it's *fully refined*.

Epistemic Barcan Formula

$$(\forall x)[\mathsf{e}]Fx \succ [\mathsf{e}](\forall x)Fx$$

Epistemic Barcan Formula

$$(\forall x)[e]Fx \succ [e](\forall x)Fx$$

$$\langle e\rangle(\exists x)Fx \succ (\exists x)\langle e\rangle Fx$$

Suppose $\langle e \rangle (\exists x)(\exists y)(Mx \mathbin{\&} Ey \mathbin{\&} x \neq y)$.

Morning Star and Evening Star

Suppose $\langle e\rangle(\exists x)(\exists y)(Mx \mathbin{\&} Ey \mathbin{\&} x \neq y)$.

Do we have $(\exists x)\langle e\rangle((\exists y)(Mx \mathbin{\&} Ey \mathbin{\&} x \neq y)$?

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Do we have $(\exists x)\langle e \rangle ((\exists y)(Mx \mathbin{\&} Ey \mathbin{\&} x \neq y)$?

And $(\exists x)(\exists y)\langle e \rangle (Mx \mathbin{\&} Ey \mathbin{\&} x \neq y)$?

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Suppose $\langle e \rangle (\exists x)(\exists y)(Mx \;\&\; Ey \;\&\; x \neq y)$.

Do we have $(\exists x)\langle e \rangle ((\exists y)(Mx \;\&\; Ey \;\&\; x \neq y)$?

And $(\exists x)(\exists y)\langle e \rangle (Mx \;\&\; Ey \;\&\; x \neq y)$?

What could such x and y be?

POSITIONS & MODELS

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- An arbitrary position is a set (*indicative* alternatives) of sets (*subjunctive* alternatives) of pairs of sets of formulas or terms (*components*), where one component in each indicative alternative is marked with an @.

Fully Refined Positions

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  - If $\Box A$ is in the LHS of a component, A is in the LHS of every subjunctive alternative of that component.

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  - If $(\forall x)A(x)$ is in the RHS of a component, so is $A(t)$ for some term t in the LHS of that component.
  - If $\Box A$ is in the LHS of a component, A is in the LHS of every subjunctive alternative of that component.
  - If $\Box A$ is in the RHS of a component, A is in the RHS of some subjunctive alternative of that component.

Models

Fully refinied positions are examples of *models*,
with variable domains and the expected truth conditions for the connectives, quantifiers and modal operators.

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- Any hypersequent that cannot be derived (without *Cut*) can be extended into a fully refined position.
- That fully refined position determines a model in which the hypersequent does not hold.
- So the models are adequate for the logic.
- And in the logic, the cut rule is admissible in the cut-free system.

CONSEQUENCES & QUESTIONS

The structure of modal concepts is explained in terms of the rules for their use.

Principled, Motivated Ersatzism

The structure of modal concepts is explained
in terms of the rules for their use.

Worlds (and their domains) are idealised positions.

Coherent, Well Behaved Contingentism

Inner and Outer Quantification

'Outer' quantification is an issue for contingentism.
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On most approaches to contingentism, it can be *defined*.

This proof theoretical semantics is no different in that regard….

We have *Outer* Quantification

$$\dfrac{\mathcal{H}(n \succ \;|\; \Gamma \succ A(n), \Delta)}{\mathcal{H}(\Gamma \succ (\forall^{\Diamond} x)A(x), \Delta)}\,[\forall^{\Diamond} Df] \qquad \dfrac{\mathcal{H}(n \succ \;|\; \Gamma, A(n) \succ \Delta)}{\mathcal{H}(\Gamma, (\exists^{\Diamond} x)A(x) \succ \Delta)}\,[\exists^{\Diamond} Df]$$

for which the substituted term need be defined in *some* zone.

The Barcan Formula is Derivable

$$\cfrac{\cfrac{\cfrac{\cfrac{(\forall^{\Diamond}x)\Box A(x) \succ (\forall^{\Diamond}x)\Box A(x)}{n \succ \mid (\forall^{\Diamond}x)\Box A(x) \succ \Box A(n)}\,[\forall^{\Diamond}Df]}{n \succ \mid (\forall^{\Diamond}x)\Box A(x) \succ \mid \succ A(n)}\,[\Box Df]}{(\forall^{\Diamond}x)\Box A(x) \succ \mid \succ (\forall^{\Diamond}x)A(x)}\,[\forall^{\Diamond}Df]}{(\forall^{\Diamond}x)\Box A(x) \succ \Box(\forall^{\Diamond}x)A(x)}\,[\Box Df]$$

But we also have *Way Out* Quantification

$$\frac{\mathcal{H}(\Gamma \succ A(n), \Delta)}{\overline{\mathcal{H}(\Gamma \succ (\Pi x)A(x), \Delta)}}\;[\Pi Df] \qquad \frac{\mathcal{H}(\Gamma, A(n) \succ \Delta)}{\overline{\mathcal{H}(\Gamma, (\Sigma x)A(x) \succ \Delta)}}\;[\Sigma Df]$$

for which the term need not be defined *anywhere*.

Higher Order Contingentism?

$$\forall X \Box \phi(X) \succ \Box \forall X \phi(X)$$

$$\forall X \Box \phi(X) \succ \Box \forall X \phi(X)$$

What could it mean to rule a *predicate* in or out?

Identity!

THANK YOU!

http://consequently.org/presentation/2015/generality-and-existence-2-arche

@consequently on Twitter