Generality & Existence III

Predication & Identity

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My Aim

To analyse the *quantifiers*

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(including their interactions with *modals*)

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To analyse the *quantifiers*
(including their interactions with *modals*)
using the tools of *proof theory*
in order to better understand
quantification, *existence* and *identity*.

My Aim for This Talk

Understanding the behaviour
of the identity predicate.

Today's Plan

Sequents & Defining Rules

Identity & Indistinguishability

Defining Rules & Left/Right Rules

Identity & Predication

Free Logic & Identity

SEQUENTS & DEFINING RULES

Sequents

$$\Gamma \succ \Delta$$

Don’t assert each element of Γ
and deny each element of Δ.

Structural Rules

Identity: $A \succ A$

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Weakening: $\dfrac{\Gamma \succ \Delta}{\Gamma, A \succ \Delta} \quad \dfrac{\Gamma \succ \Delta}{\Gamma \succ A, \Delta}$

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Contraction: $$\frac{\Gamma, A, A \succ \Delta}{\Gamma, A \succ \Delta} \qquad \frac{\Gamma \succ A, A, \Delta}{\Gamma \succ A, \Delta}$$

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Cut: $$\frac{\Gamma \succ A, \Delta \quad \Gamma, A \succ \Delta}{\Gamma \succ \Delta}$$

Structural Rules

Identity: $A \succ A$

Weakening: $\dfrac{\Gamma \succ \Delta}{\Gamma, A \succ \Delta} \quad \dfrac{\Gamma \succ \Delta}{\Gamma \succ A, \Delta}$

Contraction: $\dfrac{\Gamma, A, A \succ \Delta}{\Gamma, A \succ \Delta} \quad \dfrac{\Gamma \succ A, A, \Delta}{\Gamma \succ A, \Delta}$

Cut: $\dfrac{\Gamma \succ A, \Delta \quad \Gamma, A \succ \Delta}{\Gamma \succ \Delta}$

Structural rules govern declarative sentences *as such*.

Extending a Language with Specific Vocabulary

With Left/Right rules?

$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \land B \succ \Delta}\ [\land L] \qquad \frac{\Gamma \succ A, \Delta \quad \Gamma \succ B, \Delta}{\Gamma \succ A \land B, \Delta}\ [\land R]$$

Extending a Language with Specific Vocabulary

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$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \land B \succ \Delta}\,[\land L] \qquad \frac{\Gamma \succ A, \Delta \quad \Gamma \succ B, \Delta}{\Gamma \succ A \land B, \Delta}\,[\land R]$$

$$\frac{\Gamma, B \succ \Delta}{\Gamma, A\ \mathit{tonk}\ B \succ \Delta}\,[\mathit{tonkL}] \qquad \frac{\Gamma \succ A, \Delta}{\Gamma \succ A\ \mathit{tonk}\ B, \Delta}\,[\mathit{tonkR}]$$

What is involved in going from $\mathcal{L}$ to $\mathcal{L}'$?

Use $\succ_{\mathcal{L}}$ to *define* $\succ_{\mathcal{L}'}$.

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Desideratum #1: $\succ_{\mathcal{L}'}$ is *conservative:* $(\succ_{\mathcal{L}'})|_{\mathcal{L}}$ is $\succ_{\mathcal{L}}$.

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Desideratum #1: $\succ_{\mathcal{L}'}$ is *conservative:* $(\succ_{\mathcal{L}'})|_{\mathcal{L}}$ is $\succ_{\mathcal{L}}$.

Desideratum #2: Concepts are defined *uniquely*.

A Defining Rule

$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \land B \succ \Delta}\,[\land Df]$$

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Fully specifies norms governing conjunctions on the *left* in terms of simpler vocabulary.

Identity and *Cut* determine the behaviour of conjunctions on the *right*.

From $[\land \mathit{Df}]$ to $[\land L/R]$

$$\dfrac{\Gamma \succ A, \Delta \qquad \dfrac{\Gamma \succ B, \Delta \qquad \dfrac{\dfrac{}{A \land B \succ A \land B}\,[\mathit{Id}]}{A, B \succ A \land B}\,[\land \mathit{Df}]}{\Gamma, A \succ A \land B, \Delta}\,[\mathit{Cut}]}{\Gamma \succ A \land B, \Delta}\,[\mathit{Cut}]$$

From [$\wedge Df$] to [$\wedge L/R$]

$$\dfrac{\Gamma \succ A, \Delta \qquad \dfrac{\Gamma \succ B, \Delta \qquad \dfrac{\dfrac{}{A \wedge B \succ A \wedge B}\,[\mathit{Id}]}{A, B \succ A \wedge B}\,[\wedge \mathit{Df}]}{\Gamma, A \succ A \wedge B, \Delta}\,[\mathit{Cut}]}{\Gamma \succ A \wedge B, \Delta}\,[\mathit{Cut}]$$

From $[\wedge \mathit{Df}]$ to $[\wedge L/R]$

$$\dfrac{\Gamma \succ A, \Delta \qquad \dfrac{\Gamma \succ B, \Delta \qquad \dfrac{\dfrac{}{A \wedge B \succ A \wedge B}\,[\mathit{Id}]}{A, B \succ A \wedge B}\,[\wedge \mathit{Df}]}{\Gamma, A \succ A \wedge B, \Delta}\,[\mathit{Cut}]}{\Gamma \succ A \wedge B, \Delta}\,[\mathit{Cut}]$$

From [$\bigwedge Df$] to [$\bigwedge L/R$]

$$\dfrac{\Gamma \succ A, \Delta \qquad \dfrac{\Gamma \succ B, \Delta \qquad \dfrac{\dfrac{}{A \land B \succ A \land B}\,[\mathit{Id}]}{A, B \succ A \land B}\,[\land \mathit{Df}]}{\Gamma, A \succ A \land B, \Delta}\,[\mathit{Cut}]}{\Gamma \succ A \land B, \Delta}\,[\mathit{Cut}]$$

From $[\land Df]$ to $[\land L/R]$

$$\dfrac{\Gamma \succ A, \Delta \qquad \dfrac{\Gamma \succ B, \Delta \qquad \dfrac{\dfrac{}{A \land B \succ A \land B}\,[\mathit{Id}]}{A, B \succ A \land B}\,[\land \mathit{Df}]}{\Gamma, A \succ A \land B, \Delta}\,[\mathit{Cut}]}{\Gamma \succ A \land B, \Delta}\,[\mathit{Cut}]$$

$$\dfrac{\Gamma \succ A, \Delta \qquad \Gamma \succ B, \Delta}{\Gamma \succ A \land B, \Delta}\,[\land R]$$

And Back

$$\dfrac{\dfrac{A \succ A \qquad B \succ B}{A, B \succ A \wedge B}\,[\wedge R] \qquad \Gamma, A \wedge B \succ \Delta}{\Gamma, A, B \succ \Delta}\,[\mathit{Cut}]$$

Quantifier Rules

$$\begin{array}{c} \Gamma \succ A(n), \Delta \\ \hline\hline \Gamma \succ (\forall x)A(x), \Delta \end{array}\ [\forall Df] \qquad \begin{array}{c} \Gamma, A(n) \succ \Delta \\ \hline\hline \Gamma, (\exists x)A(x) \succ \Delta \end{array}\ [\exists Df]$$

Deductive Generality

Equivalence

$$\mathcal{L}[\bigwedge Df, Cut] = \mathcal{L}[\bigwedge L/R, Cut]$$

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This *generalises*: $\wedge$, $\vee$, $\supset$, $\neg$ work in the same way.

Equivalence

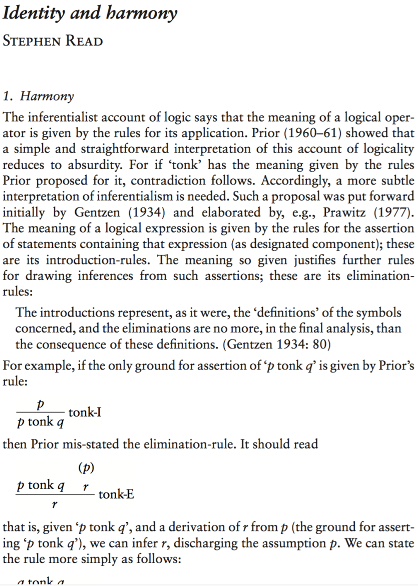
$$\mathcal{L}[\wedge Df, Cut] = \mathcal{L}[\wedge L/R, Cut] = \mathcal{L}[\wedge L/R]$$

This *generalises*: $\wedge$, $\vee$, $\supset$, $\neg$ work in the same way.

I want to see how this works for *identity*.

IDENTITY & INDISTINGUISHABILITY

Identity and Harmony



Identity and harmony

STEPHEN READ

1. *Harmony*

The inferentialist account of logic says that the meaning of a logical operator is given by the rules for its application. Prior (1960–61) showed that a simple and straightforward interpretation of this account of logicality reduces to absurdity. For if 'tonk' has the meaning given by the rules Prior proposed for it, contradiction follows. Accordingly, a more subtle interpretation of inferentialism is needed. Such a proposal was put forward initially by Gentzen (1934) and elaborated by, e.g., Prawitz (1977). The meaning of a logical expression is given by the rules for the assertion of statements containing that expression (as designated component); these are its introduction-rules. The meaning so given justifies further rules for drawing inferences from such assertions; these are its elimination-rules:

> The introductions represent, as it were, the 'definitions' of the symbols concerned, and the eliminations are no more, in the final analysis, than the consequence of these definitions. (Gentzen 1934: 80)

For example, if the only ground for assertion of '*p* tonk *q*' is given by Prior's rule:

$$\frac{p}{p \text{ tonk } q}\ \text{tonk-I}$$

then Prior mis-stated the elimination-rule. It should read

$$\frac{p \text{ tonk } q \qquad \begin{matrix}(p)\\ r\end{matrix}}{r}\ \text{tonk-E}$$

that is, given '*p* tonk *q*', and a derivation of *r* from *p* (the ground for asserting '*p* tonk *q*'), we can infer *r*, discharging the assumption *p*. We can state the rule more simply as follows:

Identity Axioms

$$t = t$$

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$$s = t \supset t = s \qquad s = t \supset (t = u \supset s = u)$$

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$$s = t \supset t = s \qquad s = t \supset (t = u \supset s = u)$$

$$s = t \supset (A(s) \equiv A(t))$$

Identity Rules in Natural Deduction

$$\dfrac{\begin{matrix}[Fs] \\ \vdots \\ Ft\end{matrix}}{s = t}\,[{=}I]$$

Identity Rules in Natural Deduction

$$\dfrac{\begin{matrix}[Fs] \\ \vdots \\ Ft\end{matrix}}{s = t}\,[{=}I] \qquad\qquad \dfrac{s = t \quad A(s)}{A(t)}\,[{=}E]$$

Defining Rule for Identity

$$\frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s = t, \Delta}\,[{=}Df]$$

Generality in Predicate Position

$$\frac{\Gamma \succ \Delta}{\Gamma|^{Fx}_{A(x)} \succ \Delta|^{Fx}_{A(x)}}\;[\mathit{Spec}^{Fx}_{A(x)}]$$

Generality in Predicate Position

$$\frac{\Gamma \succ \Delta}{\Gamma|^{Fx}_{A(x)} \succ \Delta|^{Fx}_{A(x)}}\ [\mathit{Spec}^{Fx}_{A(x)}]$$

No norm holds of Fx that doesn't also hold of the sentence context $A(x)$.

DEFINING RULES & LEFT/RIGHT RULES

From [$=$*Df*] to [$=$*L*]

$$\dfrac{\dfrac{\Gamma \succ A(s), \Delta \qquad \dfrac{\dfrac{s = t \succ s = t}{s = t, Fs \succ Ft}\,[{=}Df]}{s = t, A(s) \succ A(t)}\,[Spec^{Fx}_{A(x)}]}{s = t, \Gamma \succ A(t), \Delta}\,[Cut] \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[Cut]$$

From $[{=}Df]$ to $[{=}L]$

$$\dfrac{\dfrac{\Gamma \succ A(s), \Delta \qquad \dfrac{\dfrac{s = t \succ s = t}{s = t, Fs \succ Ft}\,[{=}Df]}{s = t, A(s) \succ A(t)}\,[Spec^{Fx}_{A(x)}]}{s = t, \Gamma \succ A(t), \Delta}\,[Cut] \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[Cut]$$

From [=*Df*] to [=*L*]

$$\dfrac{\dfrac{\Gamma \succ A(s), \Delta \qquad \dfrac{\dfrac{s = t \succ s = t}{s = t, Fs \succ Ft}\,[{=}Df]}{s = t, A(s) \succ A(t)}\,[Spec^{Fx}_{A(x)}]}{s = t, \Gamma \succ A(t), \Delta}\,[Cut] \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[Cut]$$

From [=*Df*] to [=*L*]

$$\dfrac{\dfrac{\Gamma \succ A(s), \Delta \qquad \dfrac{\dfrac{s = t \succ s = t}{s = t, Fs \succ Ft}\,[{=}Df]}{s = t, A(s) \succ A(t)}\,[Spec^{Fx}_{A(x)}]}{s = t, \Gamma \succ A(t), \Delta}\,[Cut] \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[Cut]$$

From $[{=}\mathit{Df}]$ to $[{=}L]$

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From [$=\mathit{Df}$] to [$=L$]

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From [=*Df*] to [=*L*]

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$$\dfrac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[{=}L]$$

$[=L]$

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[=L]$$

[=*L*]

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\ [{=}L]$$

This is *valid*, but *ugly*.

[=*L*]

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[{=}L]$$

This is *valid*, but *ugly*.

Proof search?

[=*L*]

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[{=}L]$$

This is *valid*, but *ugly*.

Proof search?

Subformula property?

Backtracking a little

$$\dfrac{\dfrac{\Gamma \succ A(s), \Delta \qquad \dfrac{\dfrac{s = t \succ s = t}{s = t, Fs \succ Ft}\,[{=}\mathit{Df}]}{s = t, A(s) \succ A(t)}\,[\mathit{Spec}^{Fx}_{A(x)}]}{s = t, \Gamma \succ A(t), \Delta}\,[\mathit{Cut}] \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[\mathit{Cut}]$$

Backtracking a little

$$\dfrac{\dfrac{\Gamma \succ A(s), \Delta \qquad \dfrac{\dfrac{s = t \succ s = t}{s = t, Fs \succ Ft}\,[{=}\mathit{Df}]}{s = t, A(s) \succ A(t)}\,[\mathit{Spec}^{Fx}_{A(x)}]}{s = t, \Gamma \succ A(t), \Delta}\,[\mathit{Cut}] \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[\mathit{Cut}]$$

$$\dfrac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta}\,[{=}L']$$

$[{=}L']$ is Enough to recover $[{=}Df]$

$$\dfrac{\dfrac{\Gamma \succ s = t, \Delta}{\Gamma, Fs \succ s = t, Ft, \Delta}\,[K] \qquad \dfrac{\dfrac{\dfrac{}{Ft \succ Ft}\,[Id]}{s = t, Fs \succ Ft}\,[{=}L']}{\Gamma, s = t, Fs \succ Ft, \Delta}\,[K]}{\Gamma, Fs \succ Ft, \Delta}\,[Cut]$$

$[{=}L']$

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta}\,[{=}L']$$

$[{=}L']$

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta}\,[{=}L']$$

This is *better*…

$[{=}L']$

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta}\,[{=}L']$$

This is *better*…

But it is still *strange*.

$[{=}L']$

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta}\ [{=}L']$$

This is *better*…

But it is still *strange*.

It operates at *two places* in the concluding sequent.
This puts *compositionality* in question.

IDENTITY & PREDICATION

Decomposing $[{=}L']$: conjunctions

$$\dfrac{\dfrac{\dfrac{\Gamma \succ A(s) \land B(s), \Delta}{\Gamma \succ A(s), \Delta}\,[\land E]}{s = t, \Gamma \succ A(t), \Delta}\,[{=}L'] \qquad \dfrac{\dfrac{\Gamma \succ A(s) \land B(s), \Delta}{\Gamma \succ B(s), \Delta}\,[\land E]}{s = t, \Gamma \succ B(t), \Delta}\,[{=}L']}{s = t, \Gamma \succ A(t) \land B(t), \Delta}\,[\land R]$$

(Where the $[\land E]$ is given by a *Cut* on $A(t) \land B(t) \succ A(t)$, or $A(t) \land B(t) \succ B(t)$.)

Decomposing [$=L'$]: conjunctions

$$\dfrac{\dfrac{\dfrac{\Gamma \succ A(s) \land B(s), \Delta}{\Gamma \succ A(s), \Delta}\,[\land E]}{s = t, \Gamma \succ A(t), \Delta}\,[=L'] \qquad \dfrac{\dfrac{\Gamma \succ A(s) \land B(s), \Delta}{\Gamma \succ B(s), \Delta}\,[\land E]}{s = t, \Gamma \succ B(t), \Delta}\,[=L']}{s = t, \Gamma \succ A(t) \land B(t), \Delta}\,[\land R]$$

(Where the [$\land E$] is given by a *Cut* on $A(t) \land B(t) \succ A(t)$, or $A(t) \land B(t) \succ B(t)$.)

[$=L'$] on *conjunctions* is given by [$=L'$] on its *conjuncts*.

Decomposing $[{=}L']$: disjunctions

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Gamma \succ A(s) \lor B(s), \Delta}{\Gamma \succ A(s), B(s), \Delta}\,[\lor Df]}{s = t, \Gamma \succ A(t), B(s), \Delta}\,[{=}L']}{s = t, s = t, \Gamma \succ A(t), B(t), \Delta}\,[{=}L']}{s = t, \Gamma \succ A(t) \lor B(t), \Delta}\,[W]}{s = t, \Gamma \succ A(t) \lor B(t), \Delta}\,[\lor Df]$$

Decomposing $[{=}L']$: disjunctions

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Gamma \succ A(s) \lor B(s), \Delta}{\Gamma \succ A(s), B(s), \Delta}\,[\lor Df]}{s = t, \Gamma \succ A(t), B(s), \Delta}\,[{=}L']}{s = t, s = t, \Gamma \succ A(t), B(t), \Delta}\,[{=}L']}{s = t, \Gamma \succ A(t) \lor B(t), \Delta}\,[W]}{s = t, \Gamma \succ A(t) \lor B(t), \Delta}\,[\lor Df]$$

$[{=}L']$ on *disjunctions* is given by $[{=}L']$ on its *disjuncts*.

Decomposing [$=L'$]: universal quantifiers

$$
\dfrac{\dfrac{\dfrac{\Gamma \succ (\forall x)A(x,s), \Delta}{\Gamma \succ A(n,s), \Delta}[\forall Df]}{s = t, \Gamma \succ A(n,t), \Delta}[=L']}{s = t, \Gamma \succ (\forall x)A(x,t), \Delta}[\forall Df]
$$

[$=L'$] on *a universally quantified statement* is given by [$=L'$] on an *instance*.

Decomposing [$=L'$]: existential quantifiers

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A(n,s) \succ A(n,s)}\,[\mathit{Id}]}{s = t, A(n,s) \succ A(n,t)}\,[=L']}{s = t, A(n,s) \succ (\exists x)A(x,t)}\,[\exists R]}{s = t, (\exists x)A(x,s) \succ (\exists x)A(x,t)}\,[\exists \mathit{Df}] \qquad \Gamma \succ (\exists x)A(x,s), \Delta}{s = t, \Gamma \succ (\exists x)A(x,t), \Delta}\,[\mathit{Cut}]$$

[$=L'$] on *an existentially quantified statement* is given by [$=L'$] on an *instance*.

But for *negation*…

$$\dfrac{\dfrac{\dfrac{\Gamma \succ \neg A(s), \Delta}{\Gamma, A(s) \succ \Delta}[\neg Df]}{s = t, A(t), \Gamma \succ \Delta}[{=}L' \text{ on the wrong side!}]}{s = t, \Gamma \succ \neg A(t), \Delta}[\neg Df]$$

Different Identity Rules

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[{=}L]$$

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$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta}\,[{=}L^{f}_{r}]$$

Different Identity Rules

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[{=}L]$$

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta}\,[{=}L^{f}_{r}] \qquad \frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta}\,[{=}R]$$

Different Identity Rules

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[{=}L]$$

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta}\,[{=}L_r^f] \qquad \frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta}\,[{=}R]$$

$$\frac{\Gamma \succ Fs, \Delta}{s = t, \Gamma \succ Ft, \Delta}\,[{=}L_r^p]$$

Different Identity Rules

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[{=}L]$$

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta}\,[{=}L^{\mathrm{f}}_{\mathrm{r}}] \qquad \frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta}\,[{=}R]$$

$$\frac{\Gamma \succ Fs, \Delta}{s = t, \Gamma \succ Ft, \Delta}\,[{=}L^{\mathrm{p}}_{\mathrm{r}}] \qquad \frac{Fs, \Gamma \succ \Delta}{s = t, Ft, \Gamma \succ \Delta}\,[{=}L^{\mathrm{p}}_{\mathrm{l}}]$$

Different Identity Rules

$$\dfrac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[{=}L]$$

$$\dfrac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta}\,[{=}L_r^f] \qquad \dfrac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta}\,[{=}R]$$

$$\dfrac{\Gamma \succ Fs, \Delta}{s = t, \Gamma \succ Ft, \Delta}\,[{=}L_r^p] \qquad \dfrac{Fs, \Gamma \succ \Delta}{s = t, Ft, \Gamma \succ \Delta}\,[{=}L_l^p]$$

$$\begin{array}{c} \Gamma, Fa \succ Fb, \Delta \\ \hline\hline \Gamma \succ a = b, \Delta \end{array}\,[{=}Df]$$

Different Identity Rules

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[{=}L]$$

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta}\,[{=}L^f_r] \qquad \frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta}\,[{=}R]$$

$$\frac{\Gamma \succ Fs, \Delta}{s = t, \Gamma \succ Ft, \Delta}\,[{=}L^p_r] \qquad \frac{Fs, \Gamma \succ \Delta}{s = t, Ft, \Gamma \succ \Delta}\,[{=}L^p_l]$$

$$\begin{array}{c} \Gamma, Fa \succ Fb, \Delta \\ \hline\hline \Gamma \succ a = b, \Delta \end{array}\,[{=}Df] \qquad \frac{\Gamma \succ \Delta}{\Gamma|^{Fx}_{A(x)} \succ \Delta|^{Fx}_{A(x)}}\,[Spec^{Fx}_{A(x)}]$$

Equivalences

$$\frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta}\,[{=}Df] \qquad \frac{\Gamma \succ \Delta}{\Gamma|^{Fx}_{A(x)} \succ \Delta|^{Fx}_{A(x)}}\,[Spec^{Fx}_{A(x)}]$$

$\mathcal{L}[{=}Df, Spec, Cut]$

Equivalences

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[{=}L]$$

$$\mathcal{L}[{=}\mathit{Df}, \mathit{Spec}, \mathit{Cut}] \quad = \quad \mathcal{L}[{=}L/R, \mathit{Cut}]$$

Equivalences

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta} \; [{=}L_r^f]$$

$$\begin{aligned} \mathcal{L}[{=}Df, Spec, Cut] &= \mathcal{L}[{=}L/R, Cut] \\ &= \mathcal{L}[{=}L_r^f/R, Cut] \end{aligned}$$

Equivalences

$$\frac{\Gamma \succ Fs, \Delta}{s = t, \Gamma \succ Ft, \Delta}\,[{=}L^p_r] \quad \frac{Fs, \Gamma \succ \Delta}{s = t, Ft, \Gamma \succ \Delta}\,[{=}L^p_l] \quad \frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta}\,[{=}R]$$

$$\begin{aligned} \mathcal{L}[{=}Df, Spec, Cut] &= \mathcal{L}[{=}L/R, Cut] \\ &= \mathcal{L}[{=}L^f_r/R, Cut] \\ &= \mathcal{L}[{=}L^p_r/L^p_l/R, Cut] \end{aligned}$$

Equivalences

$$\frac{\Gamma \succ Fs, \Delta}{s = t, \Gamma \succ Ft, \Delta}\,[{=}L_r^p] \quad \frac{Fs, \Gamma \succ \Delta}{s = t, Ft, \Gamma \succ \Delta}\,[{=}L_l^p] \quad \frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta}\,[{=}R]$$

$$\begin{aligned}
\mathcal{L}[{=}Df, Spec, Cut] &= \mathcal{L}[{=}L/R, Cut] \\
&= \mathcal{L}[{=}L_r^f/R, Cut] \\
&= \mathcal{L}[{=}L_r^p/L_l^p/R, Cut] \\
&= \mathcal{L}[{=}L_r^p/L_l^p/R]
\end{aligned}$$

Equivalences

$$\frac{\Gamma \succ Fs, \Delta}{s = t, \Gamma \succ Ft, \Delta}\,[{=}L^{p}_{r}] \quad \frac{Fs, \Gamma \succ \Delta}{s = t, Ft, \Gamma \succ \Delta}\,[{=}L^{p}_{l}] \quad \frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta}\,[{=}R]$$

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\mathcal{L}[{=}Df, Spec, Cut] &= \mathcal{L}[{=}L/R, Cut] \\
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&= \mathcal{L}[{=}L^{p}_{r}/L^{p}_{l}/R, Cut] \\
&= \mathcal{L}[{=}L^{p}_{r}/L^{p}_{l}/R]
\end{aligned}$$

Each system gives you classical first-order predicate logic with identity.

Non-Symmetric 'Identity'

$$\frac{\Gamma \succ Fs, \Delta}{s \; \mathit{is} \; t, \Gamma \succ Ft, \Delta}\,[\mathit{isL}^{p}_{r}] \qquad \frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s \; \mathit{is} \; t, \Delta}\,[\mathit{isR}]$$

Non-Symmetric 'Identity'

$$\frac{\Gamma \succ Fs, \Delta}{s\ \mathit{is}\ t, \Gamma \succ Ft, \Delta}\ [\mathit{isL}^{p}_{r}] \qquad \frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s\ \mathit{is}\ t, \Delta}\ [\mathit{isR}]$$

There are models of this system in which $s\ \mathit{is}\ t \not\succ t\ \mathit{is}\ s$.

Non-Symmetric ‘Identity’

$$\frac{\Gamma \succ Fs, \Delta}{s\ \mathit{is}\ t, \Gamma \succ Ft, \Delta}\,[\mathit{isL}_{r}^{p}] \qquad \frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s\ \mathit{is}\ t, \Delta}\,[\mathit{isR}]$$

There are models of this system in which $s\ \mathit{is}\ t \not\succ t\ \mathit{is}\ s$.

DOMAIN: *Animal* $<$ *Mammal* $<$ *Human*.

ATOMIC PREDICATES: closed upward under $<$.

Spec: holds for atomic predicates, closed under $\wedge, \vee, \forall, \exists$ but not $\neg$ or $\supset$.

FREE LOGIC & IDENTITY

Free Quantification

$$\frac{\Gamma, n \succ A(n), \Delta}{\Gamma \succ (\forall x)A(x), \Delta}\,[\forall Df] \qquad \frac{\Gamma, n, A(n) \succ \Delta}{\Gamma, (\exists x)A(x) \succ \Delta}\,[\exists Df]$$

Free Quantification

$$\frac{\Gamma, n \succ A(n), \Delta}{\Gamma \succ (\forall x)A(x), \Delta}\,[\forall Df] \qquad \frac{\Gamma, n, A(n) \succ \Delta}{\Gamma, (\exists x)A(x) \succ \Delta}\,[\exists Df]$$

$$\frac{\Gamma, t \succ \Delta}{\Gamma, t{\downarrow} \succ \Delta}\,[{\downarrow}Df]$$

Free Quantification

$$\frac{\Gamma, n \succ A(n), \Delta}{\Gamma \succ (\forall x)A(x), \Delta}\ [\forall Df] \qquad \frac{\Gamma, n, A(n) \succ \Delta}{\Gamma, (\exists x)A(x) \succ \Delta}\ [\exists Df]$$

$$\frac{\Gamma, t \succ \Delta}{\Gamma, t{\downarrow} \succ \Delta}\ [{\downarrow}Df]$$

$(\forall x)Fx \not\succ Ft$

Free Quantification

$$\begin{array}{c} \Gamma, n \succ A(n), \Delta \\ \hline\hline \Gamma \succ (\forall x)A(x), \Delta \end{array}\ [\forall Df] \qquad \begin{array}{c} \Gamma, n, A(n) \succ \Delta \\ \hline\hline \Gamma, (\exists x)A(x) \succ \Delta \end{array}\ [\exists Df]$$

$$\begin{array}{c} \Gamma, t \succ \Delta \\ \hline\hline \Gamma, t{\downarrow} \succ \Delta \end{array}\ [{\downarrow} Df]$$

$(\forall x)Fx \not\succ Ft \quad A(t) \not\succ (\exists x)A(x)$

Free Quantification

$$\frac{\Gamma, n \succ A(n), \Delta}{\Gamma \succ (\forall x)A(x), \Delta}\,[\forall Df] \qquad \frac{\Gamma, n, A(n) \succ \Delta}{\Gamma, (\exists x)A(x) \succ \Delta}\,[\exists Df]$$

$$\frac{\Gamma, t \succ \Delta}{\Gamma, t{\downarrow} \succ \Delta}\,[{\downarrow}Df]$$

$(\forall x)Fx \not\succ Ft \quad A(t) \not\succ (\exists x)A(x) \quad (\forall x)Fx, t \succ Ft$

Free Quantification

$$\begin{array}{c} \Gamma, n \succ A(n), \Delta \\ \hline\hline \Gamma \succ (\forall x)A(x), \Delta \end{array}\ [\forall Df] \qquad \begin{array}{c} \Gamma, n, A(n) \succ \Delta \\ \hline\hline \Gamma, (\exists x)A(x) \succ \Delta \end{array}\ [\exists Df]$$

$$\begin{array}{c} \Gamma, t \succ \Delta \\ \hline\hline \Gamma, t{\downarrow} \succ \Delta \end{array}\ [{\downarrow}Df]$$

$$(\forall x)Fx \not\succ Ft \quad A(t) \not\succ (\exists x)A(x) \quad (\forall x)Fx, t \succ Ft \quad A(t), t{\downarrow} \succ (\exists x)A(t)$$

Is Predication Existentially Committing?

$$\frac{t_i, \Gamma \succ \Delta}{Ft_1 \cdots t_n, \Gamma \succ \Delta}\,[F\mathit{L}]$$

Which Identity Rule?

NON-COMMITAL: $$\dfrac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s =_n t, \Delta}\ [=_n Df]$$

Which Identity Rule?

NON-COMMITAL: $$\begin{array}{c} \Gamma, Fs \succ Ft, \Delta \\ \hline\hline \Gamma \succ s =_n t, \Delta \end{array}\ [=_n \mathit{Df}]$$

COMMITTAL: $$\begin{array}{c} \Gamma \succ s, \Delta \quad \Gamma \succ t, \Delta \quad \Gamma, Fs \succ Ft, \Delta \\ \hline\hline \Gamma \succ s =_c t, \Delta \end{array}\ [=_c \mathit{Df}]$$

Which Identity Rule?

NON-COMMITAL: $$\frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s =_n t, \Delta}\ [=_n Df]$$

COMMITTAL: $$\frac{\Gamma \succ s, \Delta \quad \Gamma \succ t, \Delta \quad \Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s =_c t, \Delta}\ [=_c Df]$$

$\succ t =_n t$

Which Identity Rule?

NON-COMMITAL: $$\frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s =_n t, \Delta}\ [=_n Df]$$

COMMITTAL: $$\frac{\Gamma \succ s, \Delta \quad \Gamma \succ t, \Delta \quad \Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s =_c t, \Delta}\ [=_c Df]$$

$\succ t =_n t \qquad \not\succ t =_c t$

Which Identity Rule?

$$\text{NON-COMMITAL:}\quad \frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s =_n t, \Delta}\,[{=_n}Df]$$

$$\text{COMMITTAL:}\quad \frac{\Gamma \succ s, \Delta \quad \Gamma \succ t, \Delta \quad \Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s =_c t, \Delta}\,[{=_c}Df]$$

$$\succ t =_n t \qquad \not\succ t =_c t \qquad s =_c t \succ t{\downarrow} \qquad s =_c t \succ s{\downarrow}$$

Non-committal identity clashes with committing predication

$$\dfrac{\dfrac{\dfrac{\dfrac{s \succ s, Ft}{Fs \succ s, Ft}\,[\mathsf{F}L]}{Fs \succ s{\downarrow}, Ft}\,[{\downarrow}Df]}{\neg s{\downarrow}, Fs \succ Ft}\,[\neg Df]}{\neg s{\downarrow} \succ s =_n t}\,[=_n Df]$$

Non-committal identity clashes with committing predication

$$\cfrac{\cfrac{\cfrac{\cfrac{s \succ s, Ft}{Fs \succ s, Ft}\,[\mathrm{F}L]}{Fs \succ s{\downarrow}, Ft}\,[{\downarrow}Df]}{\neg s{\downarrow}, Fs \succ Ft}\,[\neg Df]}{\neg s{\downarrow} \succ s =_n t}\,[=_n Df]$$

MORAL: For non-committal identity, allow F to be *negative* as well as *positive* (e.g., *nonexistence*) so F*L* might fail for this predicate.

The Generality of Predication Matters

$$\frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s = t, \Delta}\ [{=}Df]$$

What can be substituted for the F here makes a *real* difference.

The Generality of Predication Matters

$$\frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s = t, \Delta}\,[{=}Df]$$

What can be substituted for the F here makes a *real* difference.

I'll consider this more on this in the next talk,
when I consider the interaction with modality.

THANK YOU!

`http://consequently.org/presentation/2015/generality-and-existence-3-arche`

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