Merely Verbal Disputes and Coordinating on Logical Constants

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My Plan

Background

A Definition

A Method …

… and its Cost

Preservation

Examples

The Upshot

BACKGROUND

I'm interested in *disagreement*…

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…and I'm interested in *words*,
and what they mean.

In particular, I'm interested in the role that *logic* and *logical concepts* might play in *clarifying* and *managing* disagreement.

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- *Monism* and *Pluralism* about logic
- *Ontological* relativity ($\exists$)
- The status of modal vocabulary ($\Diamond$)

A DEFINITION

A man walks rapidly around a tree, while a squirrel moves on the tree trunk. Both face the tree at all times, but the tree trunk stays between them. A group of people are arguing over the question:

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> *A man walks rapidly around a tree, while a squirrel moves on the tree trunk. Both face the tree at all times, but the tree trunk stays between them. A group of people are arguing over the question:*
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> *Does the man go round the squirrel or not?*

α: The man *goes round* the squirrel.

δ: The man doesn't *go round* the squirrel.

William James, a Tree, a Squirrel and a Man

Which party is right depends on what you practically mean by 'going round' the squirrel. If you mean passing from the north of him to the east, then to the south, then to the west, and then to the north of him again, obviously the man does go round him, for he occupies these successive positions. But if on the contrary you mean being first in front of him, then on the right of him then behind him, then on his left, and finally in front again, it is quite as obvious that the man fails to go round him …

Make the distinction, and there is no occasion for any farther dispute.

— *William James,* Pragmatism (1907)

Resolving a dispute by clarifying meanings

α: The man *goes round*$_1$ the squirrel.

δ: The man doesn't *go round*$_2$ the squirrel.

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α: The man *goes round*$_1$ the squirrel.

δ: The man doesn't *go round*$_2$ the squirrel.

Once we *disambiguate* "going round"
no disagreement remains.

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- Perhaps terms t_1 and t_2 *can’t* be explicated in terms of prior vocabulary. No matter.
- α could learn t_2 while δ could learn t_1.

Introducing General Scheme

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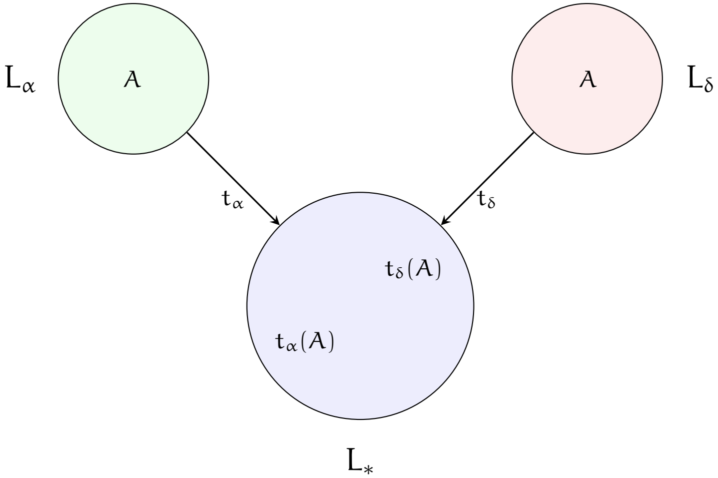

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- t may be INCOHERENCE PRESERVING: $X \vdash_{L_1} Y \;\Rightarrow\; t(X) \vdash_{L_2} t(Y)$.
- t may be COHERENCE PRESERVING: $X \not\vdash_{L_1} Y \;\Rightarrow\; t(X) \not\vdash_{L_2} t(Y)$.
- t may be COMPOSITIONAL (e.g., $t(A \wedge B) = \neg(\neg t(A) \vee \neg t(A))$, so $t(\lambda p.\lambda q.(p \wedge q)) = \lambda p.\lambda q.(\neg(\neg p \vee \neg q))$.)

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  $\vdash (\forall x)(\exists y)(y = x + 1)$ while $\not\vdash t[(\forall x)(\exists y)(y = x + 1)]$.

A *dispute*

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- For some language L_*, $t_\alpha : L_\alpha \longrightarrow L_*$, and $t_\delta : L_\delta \longrightarrow L_*$,
- and $t_\alpha(C) \not\vdash_{L_*} t_\delta(C)$.

Given a resolution by translation,
there is no disagreement over C
in the shared language L_*.

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The position $[t_\alpha(C) : t_\delta(C)]$ (in L_*) is coherent.

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Taking Disputes to be Resolved by Translation

To *take* a dispute to be resolved by translation is to take there to be a pair of translations that resolves the dispute.

(You may not even *have* the translations in hand.)

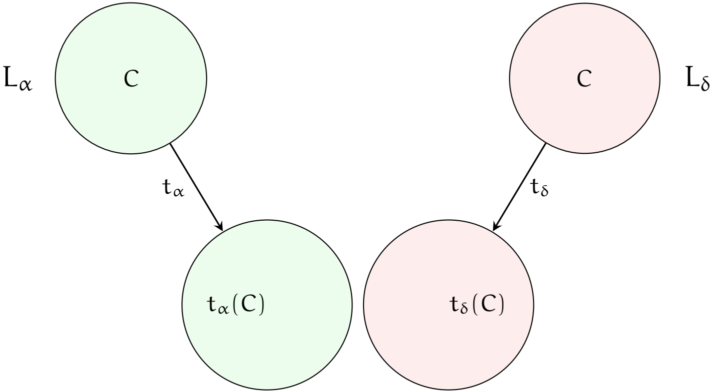
A METHOD …

… to resolve *any* dispute by translation.

Resolution by *Disjoint Union*

Or, what I like to call "the way of the undergraduate relativist."

Resolution by *Disjoint Union*



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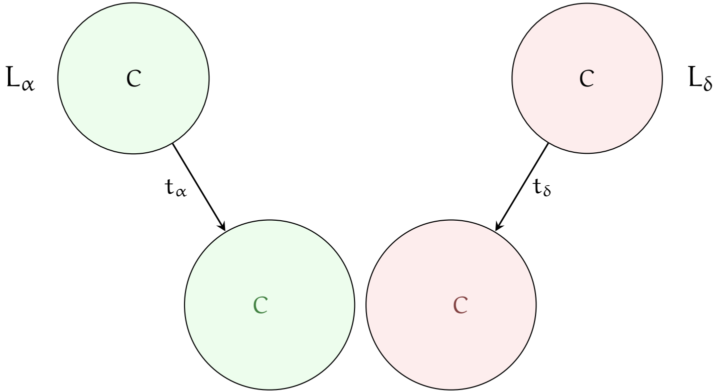

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This ‘translation’ is structure preserving,
and coherence and incoherence preserving too.

This ‘resolves’ the dispute over C

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$$\text{If } C \not\vdash_{L_\alpha}$$

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$$\text{then } C \not\vdash_{L_{\alpha|\delta}} C$$

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If $C \nvdash_{L_\alpha}$

(α's assertion of C is coherent)

and $\nvdash_{L_\delta} C$

(δ's denial of C is coherent)

then $C \nvdash_{L_{\alpha|\delta}} C$

(Asserting C-from-L_α and denying C-from-L_δ is coherent.)

… AND ITS COST

Nothing α says has any bearing on δ, or *vice versa*.

What is $A \wedge B$?

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There's *no such sentence* in $L_{\alpha|\delta}$!

The Case of the Venusians

Suppose aliens land on earth speaking our languages and familiar with our cultures and tell us that for more complete communication it will be necessary that we increase our vocabulary by the addition of a 1-ary sentence connective $\mathbb{V}$ … concerning which we should note immediately that certain restrictions to our familiar inferential practices will need to be imposed. As these Venusian logicians explain, ($\wedge$E) will have to be curtailed. Although for purely terrestrial sentences A and B, each of A and B follows from their conjunction $A \wedge$ B, it will not in general be the case that $\mathbb{V}A$ follows from $\mathbb{V}A \wedge$ B, or that $\mathbb{V}$B follows from $A \wedge \mathbb{V}$B…

— Lloyd Humberstone, *The Connectives* §4.34

If some statements A (from L_α) and B (from L_δ) are both *deniable* (so $\nvdash A$, and $\nvdash B$) then no sentence in $L_{\alpha|\delta}$ entails both A and B.

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So, there's *no* conjunction in $L_{\alpha|\delta}$.

PRESERVATION

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$$\frac{X, A, B \vdash Y}{\overline{X, A \text{ and } B \vdash Y}}\ [\text{and}\updownarrow]$$

for *all* X, Y, A and B in L.

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for *all* X, Y, A and B in L.

(There is no conjunction in $L_{\alpha|\delta}$. There is no sentence “A and B”.)

A translation $t : L_1 \rightarrow L_2$ is CONJUNCTION PRESERVING if a conjunction in L_1 is translated by a conjunction in L_2.

Preservation seems like a good idea

Translations should keep *some things* preserved.

Let's see what we can do with this.

EXAMPLES

Obviously, there some disagreements can resolved
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$$\text{`and}_\alpha\text{'} \xrightarrow{t_\alpha} \text{`}\wedge\text{'} \qquad \text{`and}_\delta\text{'} \xrightarrow{t_\delta} \text{`and } \textit{then}\text{'}$$

No Verbal Disagreement Between Two *Conjunctions*

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That is, in L_*, $A \wedge B \vdash A \,\&\, B$ and $A \,\&\, B \vdash A \wedge B$.

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Why?

$$\dfrac{\dfrac{A \mathbin{\&} B \vdash A \mathbin{\&} B}{A, B \vdash A \mathbin{\&} B}\,[\&\uparrow]}{A \wedge B \vdash A \mathbin{\&} B}\,[\wedge\downarrow] \qquad \dfrac{\dfrac{A \wedge B \vdash A \wedge B}{A, B \vdash A \wedge B}\,[\wedge\uparrow]}{A \mathbin{\&} B \vdash A \wedge B}\,[\&\downarrow]$$

(Since $\wedge$ and & are both conjunctions in L_*.)

Equivalence and Verbal Disagreements

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$$\dfrac{A \vdash A' \qquad \dfrac{B \vdash B' \qquad \dfrac{A' \,\&\, B' \vdash A' \,\&\, B'}{A', B' \vdash A' \,\&\, B'}\,[\&\uparrow]}{A', B \vdash A' \,\&\, B'}\,[Cut]}{\dfrac{A, B \vdash A' \,\&\, B'}{A \wedge B \vdash A' \,\&\, B'}\,[\wedge\downarrow]}\,[Cut]$$

If A/A' and $B/B'$are equivalent, so are $A \wedge B$ and $A' \,\&\, B'$.

This is not surprising…

… since the rules for conjunction are *very strong*.

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Sort of.

When is something a *negation*?

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CLASSICAL LOGIC:

$$\frac{X \vdash A, Y}{\overline{X, -A \vdash Y}} \, [-\updownarrow]$$

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INTUITIONIST LOGIC:

$$\frac{X, A \vdash}{X \vdash \neg A}\ [\neg\updownarrow]$$

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Let's call something a NEGATION in L
if it satisfies at least the intuitionist negation rules.

And let's say that $t : L_1 \to L_2$ PRESERVES NEGATION
if it translates a negation in L_1 by a negation in L_2.

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No Verbal Disagreement Between Two *Negations*

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1. '$\neg$' is a negation in L_1 and '$-$' is a negation in L_2, and
2. $t_1 : L_1 \to L_*$, and $t_2 : L_2 \to L_*$ are both *negation preserving*.

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Why?

Collapse?

$$\dfrac{\dfrac{-A \vdash -A}{-A, A \vdash}\,[-\uparrow]}{-A \vdash \neg A}\,[\neg\downarrow] \qquad \dfrac{\dfrac{\neg A \vdash \neg A}{\neg A, A \vdash}\,[\neg\uparrow]}{\neg A \vdash -A}\,[-\downarrow]$$

It follows that any disagreement, where one asserts $\neg A$ and the other denies $-A$ (or *vice versa*) must resolve into a disagreement over A.

What options are there for disagreement?

- Disagreement over the consequence relation '$\vdash$' (*pluralism*).
- The classical logician thinks the intuitionist is mistaken to take '$\neg$' to be so weak, or the intuitionist thinks that the classical logician is mistaken to take '$-$' to be so strong.

Ontological Relativity

Can we have merely verbal disagreement about ‘exists’?

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Can we have merely verbal disagreement about '$(\exists x)$'?

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Surely!

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Surely! Take *multi-sorted* first order logic. α says that there are numbers ($(\exists x)Nx$). δ denies it ($\neg(\exists x)Nx$). Can we make this difference *merely verbal*? While respecting some of the semantics of $(\exists x)$?

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Translate into a vocabulary with two quantifiers and two *two* domains: D_1 and D_2 with two quantifiers $(\exists_1 x)$ and $(\exists_2 x)$ ranging over each. Let N have a non-empty extension on D_1 but an empty one on D_2. Both α and δ can happily endorse $(\exists_1 x)Nx$ and deny $(\exists_2 x)Nx$ and be done with it.

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Isn't *this* a merely verbal disagreement over what exists?

Not so fast...

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$$\frac{X, A(v) \vdash Y}{X, (\exists x)A(x) \vdash Y}\ [\exists\updownarrow]$$

(Where v is not free in X and Y.)

This is what it takes to be an *existential quantifier* in L.

Existential Quantifier Collapse

$$\dfrac{\dfrac{(\exists_2 x)A(x) \vdash (\exists_2 x)A(x)}{A(v) \vdash (\exists_2 x)A(x)}\,[\exists_2\uparrow]}{(\exists_1 x)A(x) \vdash (\exists_2 x)A(x)}\,[\exists_1\downarrow] \qquad \dfrac{\dfrac{(\exists_1 x)A(x) \vdash (\exists_1 x)A(x)}{A(v) \vdash (\exists_1 x)A(x)}\,[\exists_1\uparrow]}{(\exists_2 x)A(x) \vdash (\exists_1 x)A(x)}\,[\exists_2\downarrow]$$

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If the term v appropriate to $[\exists_1\updownarrow]$ also applies in $[\exists_2\updownarrow]$, and *vice versa*, then indeed, the two quantifiers *collapse*.

Coordination on *terms* brings coordination on $(\exists x)$

If the following *three* conditions hold:

1. '$(\exists_1 x)$' is an existential quantifier in L_1 and '$(\exists_2 x)$' is an existential quantifier in L_2, and
2. $t_1 : L_1 \longrightarrow L_*$, and $t_2 : L_2 \longrightarrow L_*$, are both *existential quantifier preserving*, and
3. In L_*, some fresh term v is *appropriate for both* $(\exists_1 x)$ *and* $(\exists_2 x)$

then $(\exists_1 x)$ and $(\exists_2 x)$ are *equivalent* in L_*, in that in L_* we have $(\exists_1 x)A \vdash (\exists_2 x)A$ and $(\exists_2 x)A \vdash (\exists_1 x)A$.

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It's important to recognise what this is *not*

The appropriateness condition for eigenvariables (demonstratives, terms) is *grammatical*. It doesn't force agreement on *what exists*.

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You could coherently be a *monist* and argue with someone with a more conventional ontology—with the *same* quantifiers—provided that you both took the same terms (demonstratives, eigenvariables, whatever) to be in order for that quantifier.

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You *don't* need to take these terms to *refer* to (or range over) the same things in any substantial sense.

A *Monist* arguing with a *Pluralist* (agreeing on terms)

MONIST:

- $(\forall x)(\forall y)x = y$

PLURALIST:

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If the pluralist had argued instead:

- $(\exists x)(\exists y)x \neq y$, because

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But to *not* take these to be predications of the form Fa and $\neg Fb$, and so, to not be appropriate to substitute into the quantifier.

Can we have merely verbal disagreement about ‘possibility’?

Modal Relativity

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Isn't *this* a merely verbal disagreement over what *possible*?

Not so fast...

Let's consider more closely what might be involved in *possibility preservation*.

$$\dfrac{A \vdash \ \mid X \vdash Y \mid \Delta}{X, \Diamond A \vdash Y \mid \Delta}\ [\Diamond\Updownarrow]$$

The separated sequents indicate positions in which assertions and denials are made in different *zones* of a discourse.

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For details, see

- Greg Restall "Proofnets for S5" pages 151–172 in *Logic Colloquium 2005*, C. Dimitracopoulos, L. Newelski, and D. Normann (eds.),in *Lecture Notes in Logic* #28, Cambridge University Press, 2007 «http://consequently.org/writing/s5nets/»
- Greg Restall "A Cut-Free Sequent System for Two-Dimensional Modal Logic—and why it matters," *Annals of Pure and Applied Logic* 2012 (163) 1611–1623. «http://consequently.org/writing/cfss2dml/»

$$\dfrac{\dfrac{\Diamond_2 A \vdash \Diamond_2 A}{A \vdash \ \mid\ \vdash \Diamond_2 A}[\Diamond_2\uparrow]}{\Diamond_1 A \vdash \Diamond_2 A}[\Diamond_1\downarrow] \qquad \dfrac{\dfrac{\Diamond_1 A \vdash \Diamond_1 A}{A \vdash \ \mid\ \vdash \Diamond_1 A}[\Diamond_1\uparrow]}{\Diamond_2 A \vdash \Diamond_1 A}[\Diamond_2\downarrow]$$

If the *zone* appropriate to $[\Diamond_1\updownarrow]$ also applies in $[\Diamond_2\updownarrow]$, and *vice versa* then indeed, the two operators *collapse*.

Coordination on *zones* brings coordination on $\Diamond$

If the following *three* conditions hold:

1. '$\Diamond_1$' is an possibility in L_1 and '$\Diamond_2$' is an possibility in L_2, and
2. $t_1 : L_1 \to L_*$, and $t_2 : L_2 \to L_*$, are both *possibility preserving*, and
3. In L_*, a zone is *appropriate* for $\Diamond_1$ iff it is appropriate for $\Diamond_2$

then $\Diamond_1$ and $\Diamond_2$ are *equivalent* in L_*, in that in L_* we have $\Diamond_1 A \vdash \Diamond_2 A$ and $\Diamond_2 A \vdash \Diamond_1 A$.

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You could coherently be a *modal fatalist* and argue with someone with a more conventional modal views—with the *same* modal operators, provided that you both took the same zones to be in order.

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(You don't need to take the same things to *hold* in each zone.)

THE UPSHOT

Upshot #1: The Power of Keeping Some Things Fixed

The more you want from a translation,
the fewer translations you have,
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to settle disputes as merely verbal.

And the more chance you have to *locate* that dispute
in some particular part of your vocabulary.

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But that is *cheap*. Defining rules are *more powerful*.

And defining rules are natural, given the conception of logical constants as topic neutral, and definable in general terms.

Upshot #3: Generality Comes in Degrees

1. Propositional connectives: *sequents alone*.
2. Modals: *hypersequents*.
3. Quantifiers: *predicate structure*.

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1. Propositional connectives: *sequents alone*.
2. Modals: *hypersequents*.
3. Quantifiers: *predicate structure*.

Using this structure to define the behaviour of a logical concepts allows for them to be preserved in translation and used as a fixed point in the midst of disagreement.

THANK YOU!

http://consequently.org/presentation/2015/verbal-disputes-oxford/

@consequently on Twitter