Generality & Existence III

Substitution & Identity

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To analyse the *quantifiers*

To analyse the *quantifiers*
using the tools of *proof theory*

To analyse the *quantifiers*
using the tools of *proof theory*
in order to better understand
existence and *identity*.

Sequents & Defining Rules

Identity & Indistinguishability

Defining Rules & Left/Right Rules

Identity & Predication

Free Logic & Identity

SEQUENTS & DEFINING RULES

$$\Gamma \succ \Delta$$

Don't assert each element of Γ
and deny each element of Δ.

Identity: $A \succ A$

$$Identity: \quad A \succ A$$

$$Weakening: \quad \frac{\Gamma \succ \Delta}{\Gamma, A \succ \Delta} \qquad \frac{\Gamma \succ \Delta}{\Gamma \succ A, \Delta}$$

Structural Rules

Identity: $A \succ A$

Weakening: $$\dfrac{\Gamma \succ \Delta}{\Gamma, A \succ \Delta} \qquad \dfrac{\Gamma \succ \Delta}{\Gamma \succ A, \Delta}$$

Contraction: $$\dfrac{\Gamma, A, A \succ \Delta}{\Gamma, A \succ \Delta} \qquad \dfrac{\Gamma \succ A, A, \Delta}{\Gamma \succ A, \Delta}$$

Structural Rules

$$\textit{Identity:} \quad A \succ A$$

$$\textit{Weakening:} \quad \frac{\Gamma \succ \Delta}{\Gamma, A \succ \Delta} \qquad \frac{\Gamma \succ \Delta}{\Gamma \succ A, \Delta}$$

$$\textit{Contraction:} \quad \frac{\Gamma, A, A \succ \Delta}{\Gamma, A \succ \Delta} \qquad \frac{\Gamma \succ A, A, \Delta}{\Gamma \succ A, \Delta}$$

$$\textit{Cut:} \quad \frac{\Gamma \succ A, \Delta \quad \Gamma, A \succ \Delta}{\Gamma \succ \Delta}$$

Structural Rules

$$Identity: \quad A \succ A$$

$$Weakening: \quad \frac{\Gamma \succ \Delta}{\Gamma, A \succ \Delta} \quad \frac{\Gamma \succ \Delta}{\Gamma \succ A, \Delta}$$

$$Contraction: \quad \frac{\Gamma, A, A \succ \Delta}{\Gamma, A \succ \Delta} \quad \frac{\Gamma \succ A, A, \Delta}{\Gamma \succ A, \Delta}$$

$$Cut: \quad \frac{\Gamma \succ A, \Delta \quad \Gamma, A \succ \Delta}{\Gamma \succ \Delta}$$

Structural rules govern declarative sentences *as such*.

With Left/Right rules?

$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \land B \succ \Delta} \; [\land L] \qquad \frac{\Gamma \succ A, \Delta \quad \Gamma \succ B, \Delta}{\Gamma \succ A \land B, \Delta} \; [\land R]$$

With Left/Right rules?

$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \wedge B \succ \Delta} \; [\wedge L] \qquad \frac{\Gamma \succ A, \Delta \quad \Gamma \succ B, \Delta}{\Gamma \succ A \wedge B, \Delta} \; [\wedge R]$$

$$\frac{\Gamma, B \succ \Delta}{\Gamma, A \; tonk \; B \succ \Delta} \; [tonkL] \qquad \frac{\Gamma \succ A, \Delta}{\Gamma \succ A \; tonk \; B, \Delta} \; [tonkR]$$

Use $\succ_{\mathcal{L}}$ to *define* $\succ_{\mathcal{L}'}$.

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Desideratum #1: $\succ_{\mathcal{L}'}$ is *conservative*: $(\succ_{\mathcal{L}'})|_{\mathcal{L}}$ is $\succ_{\mathcal{L}}$.

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Desideratum #1: $\succ_{\mathcal{L}'}$ is *conservative*: $(\succ_{\mathcal{L}'})|_{\mathcal{L}}$ is $\succ_{\mathcal{L}}$.

Desideratum #2: Concepts are defined *uniquely*.

$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \wedge B \succ \Delta} \ [\wedge Df]$$

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Fully specifies norms governing conjunctions
on the *left* in terms of simpler vocabulary.

$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \land B \succ \Delta} \ [\land Df]$$

Fully specifies norms governing conjunctions
on the *left* in terms of simpler vocabulary.

Identity and *Cut* determine the behaviour
of conjunctions on the *right*.

$$
\cfrac{
  \cfrac{
    \cfrac{}{A \wedge B \succ A \wedge B}\ {\scriptstyle[Id]}
  }{
    \Gamma \succ B, \Delta \qquad A, B \succ A \wedge B
  }\ {\scriptstyle[\wedge Df]}
}{}
$$

$$
\cfrac{
  \Gamma \succ A, \Delta \qquad
  \cfrac{\Gamma \succ B, \Delta \qquad A, B \succ A \wedge B}{\Gamma, A \succ A \wedge B, \Delta}\ {\scriptstyle[Cut]}
}{
  \Gamma \succ A \wedge B, \Delta
}\ {\scriptstyle[Cut]}
$$

$$\frac{\dfrac{}{A \wedge B \succ A \wedge B} \, [Id]}{\dfrac{\Gamma \succ B, \Delta \quad \dfrac{}{A, B \succ A \wedge B} \, [\wedge Df]}{\dfrac{\Gamma \succ A, \Delta \quad \dfrac{\Gamma, A \succ A \wedge B, \Delta}{} \, [Cut]}{\Gamma \succ A \wedge B, \Delta} \, [Cut]}}$$

$$\frac{\Gamma \succ A, \Delta \quad \dfrac{\Gamma \succ B, \Delta \quad \dfrac{\dfrac{}{A \wedge B \succ A \wedge B}\ [Id]}{A, B \succ A \wedge B}\ [\wedge Df]}{\Gamma, A \succ A \wedge B, \Delta}\ [Cut]}{\Gamma \succ A \wedge B, \Delta}\ [Cut]$$

From [∧*Df*] to [∧*L/R*]

$$
\cfrac{
  \Gamma \succ A, \Delta \qquad
  \cfrac{
    \Gamma \succ B, \Delta \qquad
    \cfrac{
      \cfrac{}{A \wedge B \succ A \wedge B} \ [Id]
    }{A, B \succ A \wedge B} \ [\wedge Df]
  }{\Gamma, A \succ A \wedge B, \Delta} \ [Cut]
}{\Gamma \succ A \wedge B, \Delta} \ [Cut]
$$

$$\dfrac{\dfrac{}{A \wedge B \succ A \wedge B} \, [Id]}{\dfrac{\Gamma \succ B, \Delta \quad \dfrac{}{A, B \succ A \wedge B} \, [\wedge Df]}{\dfrac{\Gamma, A \succ A \wedge B, \Delta}{\Gamma \succ A \wedge B, \Delta} \, [Cut]}} \, [Cut]$$

$$\dfrac{\Gamma \succ A, \Delta \quad \Gamma \succ B, \Delta}{\Gamma \succ A \wedge B, \Delta} \, [\wedge R]$$

$$\cfrac{\cfrac{A \succ A \quad B \succ B}{A, B \succ A \wedge B} \; [\wedge R] \quad \Gamma, A \wedge B \succ \Delta}{\Gamma, A, B \succ \Delta} \; [Cut]$$

$$\frac{\Gamma \succ A(\mathfrak{n}), \Delta}{\Gamma \succ (\forall x)A(x), \Delta} \; [\forall Df] \qquad \frac{\Gamma, A(\mathfrak{n}) \succ \Delta}{\Gamma, (\exists x)A(x) \succ \Delta} \; [\exists Df]$$

Deductive Generality

$$\mathcal{L}[\wedge Df, Cut] = \mathcal{L}[\wedge L/R, Cut]$$

$$\mathcal{L}[\wedge Df, Cut] = \mathcal{L}[\wedge L/R, Cut] = \mathcal{L}[\wedge L/R]$$

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This *generalises*: $\wedge$, $\vee$, $\supset$, $\neg$ work in the same way.

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This *generalises*: $\wedge$, $\vee$, $\supset$, $\neg$ work in the same way.

I want to see how this works for *identity*.

IDENTITY & INDISTINGUISHABILITY

Identity and harmony

STEPHEN READ

1. *Harmony*

The inferentialist account of logic says that the meaning of a logical operator is given by the rules for its application. Prior (1960–61) showed that a simple and straightforward interpretation of this account of logicality reduces to absurdity. For if 'tonk' has the meaning given by the rules Prior proposed for it, contradiction follows. Accordingly, a more subtle interpretation of inferentialism is needed. Such a proposal was put forward initially by Gentzen (1934) and elaborated by, e.g., Prawitz (1977). The meaning of a logical expression is given by the rules for the assertion of statements containing that expression (as designated component); these are its introduction-rules. The meaning so given justifies further rules for drawing inferences from such assertions; these are its elimination-rules:

> The introductions represent, as it were, the 'definitions' of the symbols concerned, and the eliminations are no more, in the final analysis, than the consequence of these definitions. (Gentzen 1934: 80)

For example, if the only ground for assertion of '*p* tonk *q*' is given by Prior's rule:

$$\frac{p}{p \text{ tonk } q} \text{ tonk-I}$$

then Prior mis-stated the elimination-rule. It should read

$$\frac{p \text{ tonk } q \qquad \overset{(p)}{r}}{r} \text{ tonk-E}$$

that is, given '*p* tonk *q*', and a derivation of *r* from *p* (the ground for asserting '*p* tonk *q*'), we can infer *r*, discharging the assumption *p*. We can state the rule more simply as follows:

$$q \text{ tonk } q$$

$$t = t$$

$$t = t$$

$$s = t \supset t = s \qquad s = t \supset (t = u \supset s = u)$$

Identity Axioms

$$t = t$$

$$s = t \supset t = s \qquad s = t \supset (t = u \supset s = u)$$

$$s = t \supset (A(s) \equiv A(t))$$

$$\frac{\begin{array}{c} [Fs] \\ \vdots \\ Ft \end{array}}{s = t} \; [=I]$$

$$\begin{array}{c} [Fs] \\ \vdots \\ Ft \\ \hline s = t \end{array} \; [=I]$$

$$\frac{s = t \quad A(s)}{A(t)} \; [=E]$$

$$\frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s = t, \Delta} \ [{=}Df]$$

$$\frac{\Gamma \succ \Delta}{\Gamma|^{Fx}_{A(x)} \succ \Delta|^{Fx}_{A(x)}} \; [Spec^{Fx}_{A(x)}]$$

$$\frac{\Gamma \succ \Delta}{\Gamma|_{A(x)}^{Fx} \succ \Delta|_{A(x)}^{Fx}} \; [Spec_{A(x)}^{Fx}]$$

No norm holds of Fx that doesn't also hold of the sentence context $A(x)$.

DEFINING RULES & LEFT/RIGHT RULES

$$\dfrac{\dfrac{\dfrac{s = t \succ s = t}{s = t, Fs \succ Ft} \; [=Df]}{s = t, A(s) \succ A(t)} \; [Spec^{Fx}_{A(x)}]}{\dfrac{\dfrac{\Gamma \succ A(s), \Delta \qquad s = t, A(s) \succ A(t)}{s = t, \Gamma \succ A(t), \Delta} \; [Cut] \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}} \; [Cut]$$

From [=*Df*] to [=*L*]

$$\dfrac{\dfrac{\dfrac{s=t \succ s=t}{s=t, Fs \succ Ft}\ [=Df]}{\Gamma \succ A(s), \Delta \qquad s=t, A(s) \succ A(t)}\ [Spec^{Fx}_{A(x)}]}{\dfrac{s=t, \Gamma \succ A(t), \Delta}{s=t, \Gamma \succ \Delta}\ [Cut] \qquad \Gamma, A(t) \succ \Delta}\ [Cut]$$

$$\dfrac{\dfrac{\dfrac{s = t \succ s = t}{s = t, Fs \succ Ft}\ [=Df]}{\Gamma \succ A(s), \Delta \quad \dfrac{s = t, Fs \succ Ft}{s = t, A(s) \succ A(t)}\ [Spec^{Fx}_{A(x)}]}{\dfrac{s = t, \Gamma \succ A(t), \Delta}{s = t, \Gamma \succ \Delta}\ [Cut] \qquad \Gamma, A(t) \succ \Delta}\ [Cut]}$$

$$\dfrac{\dfrac{\dfrac{\mathsf{s=t \succ s=t}}{\mathsf{s=t, \overline{F}s \succ \overline{F}t}} \ [=Df]}{\Gamma \succ A(s), \Delta \qquad \mathsf{s=t, A(s) \succ A(t)}} \ [Spec^{Fx}_{A(x)}]}{\dfrac{\mathsf{s=t, \Gamma \succ A(t), \Delta}}{\mathsf{s=t, \Gamma \succ \Delta}} \ [Cut] \qquad \Gamma, A(t) \succ \Delta} \ [Cut]$$

$$\dfrac{\dfrac{\dfrac{\dfrac{s = t \succ s = t}{s = t, \overline{F}s \succ \overline{F}t} \; [=Df]}{s = t, A(s) \succ A(t)} \; [Spec^{Fx}_{A(x)}]}{s = t, \Gamma \succ A(t), \Delta} \quad \Gamma \succ A(s), \Delta}{\quad} [Cut]$$

$$\dfrac{\dfrac{\dfrac{\dfrac{s = t \succ s = t}{s = t, \overline{F}s \succ \overline{F}t} \, [=Df]}{s = t, A(s) \succ A(t)} \, [Spec^{\overline{F}x}_{A(x)}]}{\Gamma \succ A(s), \Delta \qquad s = t, A(s) \succ A(t)}{s = t, \Gamma \succ A(t), \Delta} \, [Cut] \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta} \, [Cut]$$

$$\dfrac{\dfrac{\dfrac{\dfrac{s = t \succ s = t}{s = t, Fs \succ Ft}\ [=Df]}{s = t, A(s) \succ A(t)}\ [Spec^{Fx}_{A(x)}]}{s = t, \Gamma \succ A(t), \Delta}\ [Cut] \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\ [Cut]$$

$$\dfrac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\ [=L]$$

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta} \; [=L]$$

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This is *valid,* but *ugly.*

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This is *valid,* but *ugly*.

Proof search?

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta} \ [=L]$$

This is *valid,* but *ugly.*

Proof search?
Subformula property?

$$\dfrac{\Gamma \succ A(s), \Delta \quad \dfrac{\dfrac{\dfrac{s = t \succ s = t}{s = t, Fs \succ Ft} \; [=Df]}{s = t, A(s) \succ A(t)} \; [Spec_{A(x)}^{Fx}]}{s = t, \Gamma \succ A(t), \Delta} \; [Cut] \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta} \; [Cut]$$

$$\dfrac{\begin{array}{c} \dfrac{\dfrac{s = t \succ s = t}{s = t, \mathsf{F}s \succ \mathsf{F}t}\,{}_{[=Df]}}{s = t, A(s) \succ A(t)}\,{}_{[Spec^{\mathsf{F}x}_{A(x)}]} \\ \dfrac{\Gamma \succ A(s), \Delta \qquad s = t, A(s) \succ A(t)}{s = t, \Gamma \succ A(t), \Delta}\,{}_{[Cut]} \qquad \Gamma, A(t) \succ \Delta \end{array}}{s = t, \Gamma \succ \Delta}\,{}_{[Cut]}$$

$$\dfrac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta}\,{}_{[=L']}$$

$$\dfrac{\dfrac{\Gamma \succ s = t, \Delta}{\Gamma, Fs \succ s = t, Ft, \Delta} \; [K] \qquad \dfrac{\dfrac{\overline{Ft \succ Ft} \; [Id]}{s = t, Fs \succ Ft} \; [=L']}{\Gamma, s = t, Fs \succ Ft, \Delta} \; [K]}{\Gamma, Fs \succ Ft, \Delta} \; [Cut]$$

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta} \; [=L']$$

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta} \; [=L']$$

This is *better*...

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta} \ [=L']$$

This is *better*...

But it is still *strange*.

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta} \; [=L']$$

This is *better*...

But it is still *strange*.

It operates at *two places* in the concluding sequent.
This puts *compositionality* in question.

IDENTITY & PREDICATION

$$\frac{\dfrac{\dfrac{\Gamma \succ A(s) \wedge B(s), \Delta}{\Gamma \succ A(s), \Delta} \, [\wedge E]}{s = t, \Gamma \succ A(t), \Delta} \, [=L'] \qquad \dfrac{\dfrac{\Gamma \succ A(s) \wedge B(s), \Delta}{\Gamma \succ B(s), \Delta} \, [\wedge E]}{s = t, \Gamma \succ B(t), \Delta} \, [=L']}{s = t, \Gamma \succ A(t) \wedge B(t), \Delta} \, [\wedge R]$$

(Where the [$\wedge E$] is given by a *Cut* on $A(t) \wedge B(t) \succ A(t)$, or $A(t) \wedge B(t) \succ B(t)$.)

$$\dfrac{\dfrac{\dfrac{\Gamma \succ A(s) \wedge B(s), \Delta}{\Gamma \succ A(s), \Delta} \ [\wedge E]}{s = t, \Gamma \succ A(t), \Delta} \ [=L'] \qquad \dfrac{\dfrac{\Gamma \succ A(s) \wedge B(s), \Delta}{\Gamma \succ B(s), \Delta} \ [\wedge E]}{s = t, \Gamma \succ B(t), \Delta} \ [=L']}{s = t, \Gamma \succ A(t) \wedge B(t), \Delta} \ [\wedge R]$$

(Where the [$\wedge E$] is given by a *Cut* on $A(t) \wedge B(t) \succ A(t)$, or $A(t) \wedge B(t) \succ B(t)$.)

[=L'] on *conjunctions* is given by [=L'] on its *conjuncts*.

$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{\Gamma \succ A(s) \lor B(s), \Delta}{\Gamma \succ A(s), B(s), \Delta} \; [\lor Df]}{s = t, \Gamma \succ A(t), B(s), \Delta} \; [=L']}{s = t, s = t, \Gamma \succ A(t), B(t), \Delta} \; [=L']}{s = t, \Gamma \succ A(t), B(t), \Delta} \; [W]}{s = t, \Gamma \succ A(t) \lor B(t), \Delta} \; [\lor Df]$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Gamma \succ A(s) \vee B(s), \Delta}{\Gamma \succ A(s), B(s), \Delta} \, [\vee Df]}{s = t, \Gamma \succ A(t), B(s), \Delta} \, [=L']}{s = t, s = t, \Gamma \succ A(t), B(t), \Delta} \, [=L']}{s = t, \Gamma \succ A(t), B(t), \Delta} \, [W]}{s = t, \Gamma \succ A(t) \vee B(t), \Delta} \, [\vee Df]$$

[=L'] on *disjunctions* is given by [=L'] on its *disjuncts*.

$$\frac{\dfrac{\dfrac{\Gamma \succ (\forall x)A(x,s), \Delta}{\Gamma \succ A(n,s), \Delta} \; [\forall Df]}{s = t, \Gamma \succ A(n,t), \Delta} \; [=L']}{s = t, \Gamma \succ (\forall x)A(x,t), \Delta} \; [\forall Df]$$

[$=L'$] on *a universally quantified statement* is given by [$=L'$] on an *instance*.

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{A(n,s) \succ A(n,s)}\ ^{[Id]}}{s = t, A(n,s) \succ A(n,t)}\ ^{[=L']}}{s = t, A(n,s) \succ (\exists x)A(x,t)}\ ^{[\exists R]}}{s = t, (\exists x)A(x,s) \succ (\exists x)A(x,t)}\ ^{[\exists Df]} \qquad \Gamma \succ (\exists x)A(x,s), \Delta}{s = t, \Gamma \succ (\exists x)A(x,t), \Delta}\ ^{[Cut]}$$

[=L'] on *an existentially quantified statement* is given by [=L'] on an *instance*.

$$\frac{\dfrac{\Gamma \succ \neg A(s), \Delta}{\Gamma, A(s) \succ \Delta} \; [\neg Df]}{\dfrac{s = t, A(t), \Gamma \succ \Delta}{s = t, \Gamma \succ \neg A(t), \Delta} \; [\neg Df]} \; [=L' \text{ on the wrong side!}]$$

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta} \; [=L]$$

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta} \; [=L]$$

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta} \; [=L_r^f]$$

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta} \, [=L]$$

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta} \, [=L_r^f] \qquad \frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta} \, [=R]$$

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta} \; [=L]$$

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta} \; [=L_r^f] \qquad \frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta} \; [=R]$$

$$\frac{\Gamma \succ Fs, \Delta}{s = t, \Gamma \succ Ft, \Delta} \; [=L_r^p]$$

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta} \; [=L]$$

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta} \; [=L_r^f] \qquad \frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta} \; [=R]$$

$$\frac{\Gamma \succ Fs, \Delta}{s = t, \Gamma \succ Ft, \Delta} \; [=L_r^p] \qquad \frac{Fs, \Gamma \succ \Delta}{s = t, Ft, \Gamma \succ \Delta} \; [=L_l^p]$$

$$\dfrac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta} \; [=L]$$

$$\dfrac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta} \; [=L_r^f] \qquad \dfrac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta} \; [=R]$$

$$\dfrac{\Gamma \succ Fs, \Delta}{s = t, \Gamma \succ Ft, \Delta} \; [=L_r^p] \qquad \dfrac{Fs, \Gamma \succ \Delta}{s = t, Ft, \Gamma \succ \Delta} \; [=L_l^p]$$

$$\dfrac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta} \; [=Df]$$

Different Identity Rules

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta} \ [=L]$$

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta} \ [=L_r^f] \qquad \frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta} \ [=R]$$

$$\frac{\Gamma \succ Fs, \Delta}{s = t, \Gamma \succ Ft, \Delta} \ [=L_r^p] \qquad \frac{Fs, \Gamma \succ \Delta}{s = t, Ft, \Gamma \succ \Delta} \ [=L_l^p]$$

$$\frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta} \ [=Df] \qquad \frac{\Gamma \succ \Delta}{\Gamma|_{A(x)}^{Fx} \succ \Delta|_{A(x)}^{Fx}} \ [Spec_{A(x)}^{Fx}]$$

$$\frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta} \; [=Df] \quad \frac{\Gamma \succ \Delta}{\Gamma|^{Fx}_{A(x)} \succ \Delta|^{Fx}_{A(x)}} \; [Spec^{Fx}_{A(x)}]$$

$$\mathcal{L}[=Df, Spec, Cut]$$

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta} \; [=L]$$

$$\mathcal{L}[=Df, Spec, Cut] \;\; = \;\; \mathcal{L}[=L/R, Cut]$$

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta} \, [{=}L_r^f]$$

$$
\begin{aligned}
\mathcal{L}[{=}Df, Spec, Cut] &= \mathcal{L}[{=}L/R, Cut] \\
&= \mathcal{L}[{=}L_r^f/R, Cut]
\end{aligned}
$$

$$\frac{\Gamma \succ Fs, \Delta}{s = t, \Gamma \succ Ft, \Delta} \; [=L_r^p] \quad \frac{Fs, \Gamma \succ \Delta}{s = t, Ft, \Gamma \succ \Delta} \; [=L_l^p] \quad \frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta} \; [=R]$$

$$\begin{aligned}
\mathcal{L}[=Df, Spec, Cut] &= \mathcal{L}[=L/R, Cut] \\
&= \mathcal{L}[=L_r^f/R, Cut] \\
&= \mathcal{L}[=L_r^p/L_l^p/R, Cut]
\end{aligned}$$

$$\frac{\Gamma \succ Fs, \Delta}{s = t, \Gamma \succ Ft, \Delta} \ [=L_r^p] \quad \frac{Fs, \Gamma \succ \Delta}{s = t, Ft, \Gamma \succ \Delta} \ [=L_l^p] \quad \frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta} \ [=R]$$

$$\begin{aligned}
\mathcal{L}[=Df, Spec, Cut] &= \mathcal{L}[=L/R, Cut] \\
&= \mathcal{L}[=L_r^f/R, Cut] \\
&= \mathcal{L}[=L_r^p/L_l^p/R, Cut] \\
&= \mathcal{L}[=L_r^p/L_l^p/R]
\end{aligned}$$

$$\frac{\Gamma \succ Fs, \Delta}{s = t, \Gamma \succ Ft, \Delta} \; [=L_r^p] \quad \frac{Fs, \Gamma \succ \Delta}{s = t, Ft, \Gamma \succ \Delta} \; [=L_l^p] \quad \frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta} \; [=R]$$

$$\begin{aligned}
\mathcal{L}[=Df, Spec, Cut] &= \mathcal{L}[=L/R, Cut] \\
&= \mathcal{L}[=L_r^f/R, Cut] \\
&= \mathcal{L}[=L_r^p/L_l^p/R, Cut] \\
&= \mathcal{L}[=L_r^p/L_l^p/R]
\end{aligned}$$

Each system gives you classical first-order predicate logic with identity.

Non-Symmetric 'Identity'

$$\frac{\Gamma \succ Fs, \Delta}{t \ is \ s, \Gamma \succ Ft, \Delta} \ [isL_r^p] \qquad \frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ t \ is \ s, \Delta} \ [isR]$$

$$\frac{\Gamma \succ Fs, \Delta}{t \ is \ s, \Gamma \succ Ft, \Delta} \ [isL_r^p] \qquad \frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ t \ is \ s, \Delta} \ [isR]$$

There are models of this system in which s *is* t $\not\succ$ t *is* s.

Non-Symmetric 'Identity'

$$\frac{\Gamma \succ Fs, \Delta}{t \; is \; s, \Gamma \succ Ft, \Delta} \; [isL_r^p] \qquad \frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ t \; is \; s, \Delta} \; [isR]$$

There are models of this system in which s *is* t $\not\succ$ t *is* s.

DOMAIN: *Animal* < *Mammal* < *Human*.

ATOMIC PREDICATES: closed upward under <.

Spec: holds for atomic predicates, closed under $\wedge, \vee, \forall, \exists$ but not $\neg$ or $\supset$.

FREE LOGIC & IDENTITY

$$\frac{\Gamma, n \succ A(n), \Delta}{\Gamma \succ (\forall x)A(x), \Delta} \; [\forall Df] \qquad \frac{\Gamma, n, A(n) \succ \Delta}{\Gamma, (\exists x)A(x) \succ \Delta} \; [\exists Df]$$

Free Quantification

$$\frac{\Gamma, n \succ A(n), \Delta}{\Gamma \succ (\forall x)A(x), \Delta} \; [\forall Df] \qquad \frac{\Gamma, n, A(n) \succ \Delta}{\Gamma, (\exists x)A(x) \succ \Delta} \; [\exists Df]$$

$$\frac{\Gamma, t \succ \Delta}{\Gamma, t\downarrow \succ \Delta} \; [\downarrow Df]$$

$$\frac{\Gamma, n \succ A(n), \Delta}{\Gamma \succ (\forall x)A(x), \Delta} \; [\forall Df] \qquad \frac{\Gamma, n, A(n) \succ \Delta}{\Gamma, (\exists x)A(x) \succ \Delta} \; [\exists Df]$$

$$\frac{\Gamma, t \succ \Delta}{\Gamma, t\downarrow \succ \Delta} \; [\downarrow Df]$$

$(\forall x)Fx \nsucc Ft$

$$\frac{\Gamma, n \succ A(n), \Delta}{\Gamma \succ (\forall x)A(x), \Delta} \ [\forall Df] \qquad \frac{\Gamma, n, A(n) \succ \Delta}{\Gamma, (\exists x)A(x) \succ \Delta} \ [\exists Df]$$

$$\frac{\Gamma, t \succ \Delta}{\Gamma, t\downarrow \succ \Delta} \ [\downarrow Df]$$

$(\forall x)Fx \nsucc Ft \quad A(t) \nsucc (\exists x)A(x)$

Free Quantification

$$\frac{\Gamma, n \succ A(n), \Delta}{\Gamma \succ (\forall x)A(x), \Delta} \; [\forall Df] \qquad \frac{\Gamma, n, A(n) \succ \Delta}{\Gamma, (\exists x)A(x) \succ \Delta} \; [\exists Df]$$

$$\frac{\Gamma, t \succ \Delta}{\Gamma, t\downarrow \succ \Delta} \; [\downarrow Df]$$

$(\forall x)Fx \not\succ Ft \quad A(t) \not\succ (\exists x)A(x) \quad (\forall x)Fx, t \succ Ft$

$$\frac{\Gamma, n \succ A(n), \Delta}{\Gamma \succ (\forall x)A(x), \Delta} \ [\forall Df] \qquad \frac{\Gamma, n, A(n) \succ \Delta}{\Gamma, (\exists x)A(x) \succ \Delta} \ [\exists Df]$$

$$\frac{\Gamma, t \succ \Delta}{\Gamma, t\!\downarrow \ \succ \Delta} \ [\downarrow Df]$$

$(\forall x)Fx \not\succ Ft \quad A(t) \not\succ (\exists x)A(x) \quad (\forall x)Fx, t \succ Ft \quad A(t), t\!\downarrow \ \succ (\exists x)A(t)$

$$\frac{t_i, \Gamma \succ \Delta}{Ft_1 \cdots t_n, \Gamma \succ \Delta} \; [\text{F}L]$$

NON-COMMITAL:
$$\frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s =_n t, \Delta} \ [=_n Df]$$

Which Identity Rule?

NON-COMMITAL:

$$\frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s =_n t, \Delta} \; [=_n Df]$$

COMMITTAL:

$$\frac{\Gamma \succ s, \Delta \quad \Gamma \succ t, \Delta \quad \Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s =_c t, \Delta} \; [=_c Df]$$

Which Identity Rule?

NON-COMMITAL:
$$\frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s =_n t, \Delta} \; [=_n Df]$$

COMMITTAL:
$$\frac{\Gamma \succ s, \Delta \quad \Gamma \succ t, \Delta \quad \Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s =_c t, \Delta} \; [=_c Df]$$

$$\succ t =_n t$$

$$\text{NON-COMMITAL:} \quad \frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s =_n t, \Delta} \; [=_n Df]$$

$$\text{COMMITTAL:} \quad \frac{\Gamma \succ s, \Delta \quad \Gamma \succ t, \Delta \quad \Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s =_c t, \Delta} \; [=_c Df]$$

$$\succ t =_n t \qquad \not\succ t =_c t$$

$$\text{NON-COMMITAL:} \quad \frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s =_n t, \Delta} \; [=_n Df]$$

$$\text{COMMITTAL:} \quad \frac{\Gamma \succ s, \Delta \quad \Gamma \succ t, \Delta \quad \Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s =_c t, \Delta} \; [=_c Df]$$

$$\succ t =_n t \qquad \not\succ t =_c t \qquad s =_c t \succ t\!\downarrow \qquad s =_c t \succ s\!\downarrow$$

$$\cfrac{\cfrac{\cfrac{\cfrac{s \succ s, \mathsf{F}t}{\mathsf{F}s \succ s, \mathsf{F}t} \; [\mathsf{F}L]}{\mathsf{F}s \succ s{\downarrow}, \mathsf{F}t} \; [{\downarrow}Df]}{\neg s{\downarrow}, \mathsf{F}s \succ \mathsf{F}t} \; [\neg Df]}{\neg s{\downarrow} \succ s =_n t} \; [=_n Df]$$

$$\frac{\dfrac{s \succ s, Ft}{Fs \succ s, Ft} \; [FL]}{\dfrac{Fs \succ s\downarrow, Ft}{\dfrac{\neg s\downarrow, Fs \succ Ft}{\neg s\downarrow \succ s =_n t} \; [=_n Df]} \; [\neg Df]} \; [\downarrow Df]$$

MORAL: For non-committal identity, allow F to be *negative* as well as *positive* (e.g., *nonexistence*) so FL might fail for this predicate.

$$\frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s = t, \Delta} \ [=Df]$$

What can be substituted for the F here makes a *real* difference.

$$\frac{\Gamma, Fs \succ Ft, \Delta}{\Gamma \succ s = t, \Delta} \ [=Df]$$

What can be substituted for the F here makes a *real* difference.

I'll consider this more on this in the next talk,
when I consider the interaction with modality.

THANK YOU!

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generality-and-existence-3-melbourne-logic-workshop

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