CSE 351: Artificial Intelligence

Propositional Logic

Course Teacher: Dr. M. Firoz Mridha

Logic and AI

- Would like our AI to have knowledge about the world, and logically draw conclusions from it
- Search algorithms generate successors and evaluate them, but do not “understand” much about the setting
- Example question: is it possible for a chess player to have 8 pawns and 2 queens?
  - Search algorithm could search through tons of states to see if this ever happens, but…

A story

- You roommate comes home; he/she is completely wet
- You know the following things:
  - Your roommate is wet
  - If your roommate is wet, it is because of rain, sprinklers, or both
  - If your roommate is wet because of sprinklers, the sprinklers must be on
  - If your roommate is wet because of rain, your roommate must not be carrying the umbrella
  - The umbrella is not in the umbrella holder
  - If the umbrella is not in the umbrella holder, either you must be carrying the umbrella, or your roommate must be carrying the umbrella
  - You are not carrying the umbrella
- Can you conclude that the sprinklers are on?
- Can AI conclude that the sprinklers are on?

Knowledge base for the story

- RoommateWet
- RoommateWet => (RoommateWetBecauseOfRain OR RoommateWetBecauseOfSprinklers)
- RoommateWetBecauseOfSprinklers => SprinklersOn
- RoommateWetBecauseOfRain => NOT(RoommateCarryingUmbrella)
- UmbrellaGone
- UmbrellaGone => (YouCarryingUmbrella OR RoommateCarryingUmbrella)
- NOT(YouCarryingUmbrella)

Syntax

- What do well-formed sentences in the knowledge base look like?
- A BNF grammar:
- *Symbol* → P, Q, R, …, RoommateWet, …
- *Sentence* → True | False | *Symbol* | NOT(*Sentence*) | (*Sentence* AND *Sentence*) | (*Sentence* OR *Sentence*) | (*Sentence* => *Sentence*)
- We will drop parentheses sometimes, but formally they really should always be there

Semantics

- A model specifies which of the proposition symbols are true and which are false
- Given a model, I should be able to tell you whether a sentence is true or false
- Truth table defines semantics of operators:

a	b	NOT(a)	a AND b	a OR b	a => b
false	false	true	false	false	true
false	true	true	false	true	true
true	false	false	false	true	false
true	true	false	true	true	true

- Given a model, can compute truth of sentence recursively with these

Caveats

- TwoIsAnEvenNumber OR ThreeIsAnOddNumber

  is true (not exclusive OR)
- TwoIsAnOddNumber => ThreeIsAnEvenNumber

  is true (if the left side is false it’s always true)

All of this is assuming those symbols are assigned their natural values…

Tautologies

- A sentence is a tautology if it is true for any setting of its propositional symbols

P	Q	P OR Q	NOT(P) AND NOT(Q)	(P OR Q) OR (NOT(P) AND NOT(Q))
false	false	false	true	true
false	true	true	false	true
true	false	true	false	true
true	true	true	false	true

- (P OR Q) OR (NOT(P) AND NOT(Q)) is a tautology

Is this a tautology?

- $(P \Rightarrow Q)$ OR $(Q \Rightarrow P)$

Logical equivalences

- Two sentences are logically equivalent if they have the same truth value for every setting of their propositional variables

P	Q	P OR Q	NOT(NOT(P) AND NOT(Q))
false	false	false	false
false	true	true	true
true	false	true	true
true	true	true	true

- P OR Q and NOT(NOT(P) AND NOT(Q)) are logically equivalent
- Tautology = logically equivalent to True

Famous logical equivalences

- (a OR b) ≡ (b OR a) *commutatitvity*
- (a AND b) ≡ (b AND a) *commutatitvity*
- ((a AND b) AND c) ≡ (a AND (b AND c)) *associativity*
- ((a OR b) OR c) ≡ (a OR (b OR c)) *associativity*
- NOT(NOT(a)) ≡ a *double-negation elimination*
- (a => b) ≡ (NOT(b) => NOT(a)) *contraposition*
- (a => b) ≡ (NOT(a) OR b) *implication elimination*
- NOT(a AND b) ≡ (NOT(a) OR NOT(b)) *De Morgan*
- NOT(a OR b) ≡ (NOT(a) AND NOT(b)) *De Morgan*
- (a AND (b OR c)) ≡ ((a AND b) OR (a AND c)) *distributitivity*
- (a OR (b AND c)) ≡ ((a OR b) AND (a OR c)) *distributitivity*

Inference

- We have a knowledge base of things that we know are true
  – RoommateWetBecauseOfSprinklers
  – RoommateWetBecauseOfSprinklers => SprinklersOn
- Can we conclude that SprinklersOn?
- We say SprinklersOn is entailed by the knowledge base if, for every setting of the propositional variables for which the knowledge base is true, SprinklersOn is also true

RWBOS	SprinklersOn	Knowledge base
false	false	false
false	true	false
true	false	false
true	true	true

- SprinklersOn is entailed!

Simple algorithm for inference

- Want to find out if sentence a is entailed by knowledge base…
- *For every possible setting of the propositional variables,*
  - *If knowledge base is true and a is false, return false*
- *Return true*
- Not very efficient: $2^{\#\text{propositional variables}}$ settings

Inconsistent knowledge bases

- Suppose we were careless in how we specified our knowledge base:
- PetOfRoommateIsABird => PetOfRoommateCanFly
- PetOfRoommateIsAPenguin => PetOfRoommateIsABird
- PetOfRoommateIsAPenguin => NOT(PetOfRoommateCanFly)
- PetOfRoommateIsAPenguin
- No setting of the propositional variables makes all of these true
- Therefore, technically, this knowledge base implies anything
- TheMoonIsMadeOfCheese

Reasoning patterns

- Obtain new sentences directly from some other sentences in knowledge base according to reasoning patterns
- If we have sentences a and a => b , we can correctly conclude the new sentence b
  - This is called modus ponens
- If we have a AND b , we can correctly conclude a
- All of the logical equivalences from before also give reasoning patterns

Formal proof that the sprinklers are on

1) RoommateWet
2) RoommateWet => (RoommateWetBecauseOfRain OR RoommateWetBecauseOfSprinklers)
3) RoommateWetBecauseOfSprinklers => SprinklersOn
4) RoommateWetBecauseOfRain => NOT(RoommateCarryingUmbrella)
5) UmbrellaGone
6) UmbrellaGone => (YouCarryingUmbrella OR RoommateCarryingUmbrella)
7) NOT(YouCarryingUmbrella)
8) YouCarryingUmbrella OR RoommateCarryingUmbrella *(modus ponens on 5 and 6)*
9) NOT(YouCarryingUmbrella) => RoommateCarryingUmbrella *(equivalent to 8)*
10) RoommateCarryingUmbrella *(modus ponens on 7 and 9)*
11) NOT(NOT(RoommateCarryingUmbrella) *(equivalent to 10)*
12) NOT(NOT(RoommateCarryingUmbrella)) => NOT(RoommateWetBecauseOfRain) *(equivalent to 4 by contraposition)*
13) NOT(RoommateWetBecauseOfRain) *(modus ponens on 11 and 12)*
14) RoommateWetBecauseOfRain OR RoommateWetBecauseOfSprinklers *(modus ponens on 1 and 2)*
15) NOT(RoommateWetBecauseOfRain) => RoommateWetBecauseOfSprinklers *(equivalent to 14)*
16) RoommateWetBecauseOfSprinklers *(modus ponens on 13 and 15)*
17) SprinklersOn *(modus ponens on 16 and 3)*

Reasoning about penguins

1) PetOfRoommateIsABird => PetOfRoommateCanFly
2) PetOfRoommateIsAPenguin => PetOfRoommateIsABird
3) PetOfRoommateIsAPenguin => NOT(PetOfRoommateCanFly)
4) PetOfRoommateIsAPenguin
5) PetOfRoommateIsABird *(modus ponens on 4 and 2)*
6) PetOfRoommateCanFly *(modus ponens on 5 and 1)*
7) NOT(PetOfRoommateCanFly) *(modus ponens on 4 and 3)*
8) NOT(PetOfRoommateCanFly) => FALSE *(equivalent to 6)*
9) FALSE *(modus ponens on 7 and 8)*
10) FALSE => TheMoonIsMadeOfCheese *(tautology)*
11) TheMoonIsMadeOfCheese *(modus ponens on 9 and 10)*

Getting more systematic

- Any knowledge base can be written as a single formula in conjunctive normal form (CNF)
  - CNF formula: (… OR … OR …) AND (… OR …) AND …
  - … can be a symbol x, or NOT(x) (these are called literals)
  - Multiple facts in knowledge base are effectively ANDed together

RoommateWet => (RoommateWetBecauseOfRain OR RoommateWetBecauseOfSprinklers)

becomes

(NOT(RoommateWet) OR RoommateWetBecauseOfRain OR RoommateWetBecauseOfSprinklers)

Converting story problem to conjunctive normal form

- RoommateWet
  - RoommateWet
- RoommateWet => (RoommateWetBecauseOfRain OR RoommateWetBecauseOfSprinklers)
  - NOT(RoommateWet) OR RoommateWetBecauseOfRain OR RoommateWetBecauseOfSprinklers
- RoommateWetBecauseOfSprinklers => SprinklersOn
  - NOT(RoommateWetBecauseOfSprinklers) OR SprinklersOn
- RoommateWetBecauseOfRain => NOT(RoommateCarryingUmbrella)
  - NOT(RoommateWetBecauseOfRain) OR NOT(RoommateCarryingUmbrella)
- UmbrellaGone
  - UmbrellaGone
- UmbrellaGone => (YouCarryingUmbrella OR RoommateCarryingUmbrella)
  - NOT(UmbrellaGone) OR YouCarryingUmbrella OR RoommateCarryingUmbrella
- NOT(YouCarryingUmbrella)
  - NOT(YouCarryingUmbrella)

Unit resolution

- Unit resolution: if we have
- l_1 OR l_2 OR … OR l_k

  and
- NOT(l_i)

  we can conclude
- l_1 OR l_2 OR … l_{i-1} OR l_{i+1} OR … OR l_k
- Basically modus ponens

Applying resolution to story problem

1) RoommateWet
2) NOT(RoommateWet) OR RoommateWetBecauseOfRain OR RoommateWetBecauseOfSprinklers
3) NOT(RoommateWetBecauseOfSprinklers) OR SprinklersOn
4) NOT(RoommateWetBecauseOfRain) OR NOT(RoommateCarryingUmbrella)
5) UmbrellaGone
6) NOT(UmbrellaGone) OR YouCarryingUmbrella OR RoommateCarryingUmbrella
7) NOT(YouCarryingUmbrella)
8) NOT(UmbrellaGone) OR RoommateCarryingUmbrella (6,7)
9) RoommateCarryingUmbrella (5,8)
10) NOT(RoommateWetBecauseOfRain) (4,9)
11) NOT(RoommateWet) OR RoommateWetBecauseOfSprinklers (2,10)
12) RoommateWetBecauseOfSprinklers (1,11)
13) SprinklersOn (3,12)

Limitations of unit resolution

- P OR Q
- NOT(P) OR Q
- Can we conclude Q?

(General) resolution

- General resolution: if we have
- l_1 OR l_2 OR … OR l_k

  and
- m_1 OR m_2 OR … OR m_n

  where for some i,j, l_i = NOT(m_j)

  we can conclude
- l_1 OR l_2 OR … l_{i-1} OR l_{i+1} OR … OR l_k OR m_1 OR m_2 OR … OR m_{j-1} OR m_{j+1} OR … OR m_n
- Same literal may appear multiple times; remove those

Applying resolution to story problem (more clumsily)

1) RoommateWet
2) NOT(RoommateWet) OR RoommateWetBecauseOfRain OR RoommateWetBecauseOfSprinklers
3) NOT(RoommateWetBecauseOfSprinklers) OR SprinklersOn
4) NOT(RoommateWetBecauseOfRain) OR NOT(RoommateCarryingUmbrella)
5) UmbrellaGone
6) NOT(UmbrellaGone) OR YouCarryingUmbrella OR RoommateCarryingUmbrella
7) NOT(YouCarryingUmbrella)
8) NOT(RoommateWet) OR RoommateWetBecauseOfRain OR SprinklersOn (2,3)
9) NOT(RoommateCarryingUmbrella) OR NOT(RoommateWet) OR SprinklersOn (4,8)
10) NOT(UmbrellaGone) OR YouCarryingUmbrella OR NOT(RoommateWet) OR SprinklersOn (6,9)
11) YouCarryingUmbrella OR NOT(RoommateWet) OR SprinklersOn (5,10)
12) NOT(RoommateWet) OR SprinklersOn (7,11)
13) SprinklersOn (1,12)

Systematic inference?

- General strategy: if we want to see if sentence a is entailed, add NOT(a) to the knowledge base and see if it becomes inconsistent (we can derive a contradiction)
- CNF formula for modified knowledge base is satisfiable if and only if sentence a is not entailed
  - Satisfiable = there exists a model that makes the modified knowledge base true = modified knowledge base is consistent

Resolution algorithm

- Given formula in conjunctive normal form, repeat:
- Find two clauses with complementary literals,
- Apply resolution,
- Add resulting clause (if not already there)
- If the empty clause results, formula is not satisfiable
  - Must have been obtained from P and NOT(P)
- Otherwise, if we get stuck (and we will **eventually**), the formula is guaranteed to be satisfiable (proof in a couple of slides)

Example

- Our knowledge base:
  - 1) RoommateWetBecauseOfSprinklers
  - 2) NOT(RoommateWetBecauseOfSprinklers) OR SprinklersOn
- Can we infer SprinklersOn? We add:
  - 3) NOT(SprinklersOn)
- From 2) and 3), get
  - 4) NOT(RoommateWetBecauseOfSprinklers)
- From 4) and 1), get empty clause

If we get stuck, why is the formula satisfiable?

- Consider the final set of clauses C
- Construct satisfying assignment as follows:
- Assign truth values to variables in order x_1, x_2, …, x_n
- If x_j is the last chance to satisfy a clause (i.e., all the other variables in the clause came earlier and were set the wrong way), then set x_j to satisfy it
  - Otherwise, doesn't matter how it's set
- Suppose this fails (for the first time) at some point, i.e., x_j must be set to true for one last-chance clause and false for another
- These two clauses would have resolved to something involving only up to x_{j-1} (not to the empty clause, of course), which must be satisfied
- But then one of the two clauses must also be satisfied - contradiction

Special case: Horn clauses

- Horn clauses are implications with only positive literals
- x_1 AND x_2 AND x_4 => x_3 AND x_6
- TRUE => x_1
- Try to figure out whether some x_j is entailed
- Simply follow the implications (modus ponens) as far as you can, see if you can reach x_j
- x_j is entailed if and only if it can be reached (can set everything that is not reached to false)
- Can implement this more efficiently by maintaining, for each implication, a count of how many of the left-hand side variables have been reached

First-Order Logic

Logic roadmap overview

- Propositional logic (review)
- Problems with propositional logic
- First-order logic (review)
  - Properties, relations, functions, quantifiers, …
  - Terms, sentences, wffs, axioms, theories, proofs, …
- Extensions to first-order logic
- Logical agents
  - Reflex agents
  - Representing change: situation calculus, frame

Disclaimer

"Logic, like whiskey, loses its beneficial effect when taken in too large quantities."

- *Lord Dunsany*

Propositional Logic: Review

Big Ideas

- Logic is a great knowledge representation language for many AI problems
- **Propositional logic** is the simple foundation and fine for some AI problems
- **First order logic** (FOL) is much more expressive as a KR language and more commonly used in AI

Propositional logic

- **Logical constants**: true, false
- **Propositional symbols**: P, Q,... (**atomic sentences**)
- Wrapping **parentheses**: (…)
- Sentences are combined by **connectives**:

  $\wedge$ and [conjunction]

  $\vee$ or [disjunction]

  $\Rightarrow$ implies [implication / conditional]

  $\Leftrightarrow$ is equivalent [biconditional]

  $\neg$ not [negation]
- **Literal**: atomic sentence or negated atomic sentence

  P, $\neg$ P

Examples of PL sentences

- $(P \wedge Q) \rightarrow R$

  "If it is hot and humid, then it is raining"

- $Q \rightarrow P$

  "If it is humid, then it is hot"

- Q

  "It is humid."

- We're free to choose better symbols, btw:

  Ho = "It is hot"

  Hu = "It is humid"

  R = "It is raining"

Propositional logic (PL)

- Simple language for showing key ideas and definitions
- User defines set of propositional symbols, like P and Q
- User defines **semantics** of each propositional symbol:
  - P means “It is hot”, Q means “It is humid”, etc.
- A sentence (well formed formula) is defined as follows:
  - A symbol is a sentence
  - If S is a sentence, then $\neg$S is a sentence

Some terms

- The meaning or **semantics** of a sentence determines its **interpretation**
- Given the truth values of all symbols in a sentence, it can be "evaluated" to determine its **truth value** (True or False)
- A **model** for a KB is a *possible world* – an assignment of truth values to propositional symbols that makes each

Model for a KB

- Let the KB be [$P \wedge Q \rightarrow R$, $Q \rightarrow P$]
- What are the possible models? Consider all possible assignments of T|F to P, Q and R and check truth tables
  - **FFF: OK**
  - **FFT: OK**
  - FTF: NO
  - FTT: NO
  - **TFF: OK**
  - **TFT: OK**
  - TTF: NO
  - **TTT: OK**

P: it's hot
Q: it's humid
R: it's raining

- If KB is [$P \wedge Q \rightarrow R$, $Q \rightarrow P$, Q], then the only model is TTT

More terms

- A **valid sentence** or **tautology** is a sentence that is True under all interpretations, no matter what the world is actually like or what the semantics is. Example: “It’s raining or it’s not raining”
- An **inconsistent sentence** or **contradiction** is a sentence that is False under all interpretations. The world is never like what it describes, as in “It’s raining and it’s not raining.”

Truth tables

- Truth tables are used to define logical connectives
- and to determine when a complex sentence is true given the values of the symbols in it

Truth tables for the five logical connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
False	*False*	*True*	*False*	*False*	*True*	*True*
False	*True*	*True*	*False*	*True*	*True*	*False*
True	*False*	*False*	*False*	*True*	*False*	*False*
True	*True*	*False*	*True*	*True*	*True*	*True*

Example of a truth table used for a complex sentence

P	H	$P \vee H$	$(P \vee H) \wedge \neg H$	$((P \vee H) \wedge \neg H) \Rightarrow P$
False	*False*	*False*	*False*	*True*
False	*True*	*True*	*False*	*True*
True	*False*	*True*	*True*	*True*
True	*True*	*True*	*False*	*True*

On the implies connective: P → Q

- Note that → is a logical connective
- So *P→Q* is a logical sentence and has a truth value, i.e., is either true or false
- If we add this sentence to the KB, it can be used by an inference rule, *Modes Ponens*, to derive/infer/prove Q if P is also in the KB
- Given a KB where P=True and Q=True, we can also derive/infer/prove

$P \rightarrow Q$

- When is $P \rightarrow Q$ true? Check all that apply
  - ❑ P=Q=true
  - ❑ P=Q=false
  - ❑ P=true, Q=false
  - ❑ P=false, Q=true

$P \rightarrow Q$

- When is $P \rightarrow Q$ true? Check all that apply
  - ❑ P=Q=true
  - ❑ P=Q=false
  - ❑ P=true, Q=false
  - ❑ P=false, Q=true
- We can get this from the truth table for $\rightarrow$

Inference rules

- **Logical inference** creates new sentences that logically follow from a set of sentences (KB)
- An inference rule is **sound** if every sentence X it produces when operating on a KB logically follows from the KB
  - i.e., inference rule creates no contradictions
- An inference rule is **complete** if it can

Sound rules of inference

- Here are some examples of sound rules of inference
- Each can be shown to be sound using a truth table

RULE	PREMISE	CONCLUSION
Modus Ponens	$A, A \rightarrow B$	B
And Introduction	A, B	$A \wedge B$
And Elimination	$A \wedge B$	A
Double Negation	$\neg\neg A$	A
Unit Resolution	$A \vee B, \neg B$	A

Soundness of modus ponens

A	**B**	**$A \rightarrow B$**	**OK?**
True	True	True	√
True	False	False	√
False	True	True	√
False	False	True	√

Resolution

- **Resolution** is a valid inference rule producing a new clause implied by two clauses containing *complementary literals*
  – A literal is an atomic symbol or its negation, i.e., P, ~P
- Amazingly, this is the only interference rule you need to build a sound and complete theorem prover
  – Based on proof by contradiction and usually called resolution refutation

Resolution

- A KB is actually a set of sentences all of which are true, i.e., a conjunction of sentences.
- To use resolution, put KB into *conjunctive normal form* (CNF), where each sentence written as a disjunc- tion of (one or mo

Tautologies

$(A \rightarrow B) \leftrightarrow (\sim A \vee B)$

$(A \vee (B \wedge C))$

$\leftrightarrow (A \vee B) \wedge (A \vee C)$

Example

- KB: $[P \rightarrow Q \;,\; Q \rightarrow R \wedge S]$
- KB in CNF: $[\sim P \vee Q \;,\; \sim Q \vee R \;,\; \sim Q \vee S]$
- Resolve KB(1) and KB(2) producing: $\sim P \vee R$ *(i.e., $P \rightarrow R$)*
- Resolve KB(1) and KB(3) producing: $\sim P \vee S$ *(i.e.*

Soundness of the resolution inference rule

α	β	γ	$\alpha \vee \beta$	$\neg\beta \vee \gamma$	$\alpha \vee \gamma$
False	*False*	*False*	*False*	*True*	*False*
False	*False*	*True*	*False*	*True*	*True*
False	*True*	*False*	*True*	*False*	*False*
False	*True*	*True*	*True*	*True*	*True*
True	*False*	*False*	*True*	*True*	*True*
True	*False*	*True*	*True*	*True*	*True*
True	*True*	*False*	*True*	*False*	*True*
True	*True*	*True*	*True*	*True*	*True*

From the rightmost three columns of this truth table, we can see that

$$(\alpha \vee \beta) \wedge (\beta \vee \gamma) \leftrightarrow (\alpha \vee \gamma)$$

is valid (i.e., always true regardless of the truth values assigned to α, β and γ

Proving things

- A **proof** is a sequence of sentences, where each is a premise or is derived from earlier sentences in the proof by an inference rule
- The last sentence is the **theorem** (also called goal or query) that we want to prove
- Example for the “weather problem”

1 Hu	premise	“It’s humid”
2 $Hu \rightarrow Ho$	premise	“If it’s humid, it’s hot”
3 Ho	modus ponens(1,2)	“It’s hot”
4 $(Ho \wedge Hu) \rightarrow R$	premise	“If it’s hot & humid, it’s raining”
5 $Ho \wedge Hu$	and introduction(1,3)	“It’s hot and humid”

Horn sentences

- A **Horn sentence** or **Horn clause** has the form:

  $P1 \wedge P2 \wedge P3 \ldots \wedge Pn \rightarrow Qm$ *where n>=0, m in{0,1}*

- Note: a conjunction of 0 or more symbols to left of $\rightarrow$ and 0-1 symbols to right
- Special cases:
  - n=0, m=1: P (assert P is true)
  - n>0, m=0: $P \wedge Q \rightarrow$ (constraint: both P and Q can't be true)

$\boxed{(P \rightarrow Q) = (\neg P \vee Q)}$

Significance of Horn logic

- We can also have horn sentences in FOL
- Reasoning with horn clauses is much simpler
  - Satisfiability of a propositional KB (i.e., finding values for a symbols that will make it true) is NP complete
  - Restricting KB to horn sentences, satisfiability is in P
- For this reason, FOL Horn sentences are the basis for Prolog and Datalog
- What Horn sentences give up are handling, in

Entailment and derivation

- **Entailment: KB |= Q**
  - Q is entailed by KB (set sentences) iff there is no logically possible world where Q is false while all the sentences in KB are true
  - Or, stated positively, Q is entailed by KB iff the conclusion is true in every logically possible world in which all the premises in KB are true
- **Derivation: KB |- Q**
  - We can derive Q from KB if there's a proof

Two important properties for inference

Soundness: If KB |- Q then KB |= Q

- If Q is derived from KB using a given set of rules of inference, then Q is entailed by KB
- Hence, inference produces only real entailments, or any sentence that follows deductively from the premises is valid

Completeness: If KB |= Q then KB |- Q

- If Q is entailed by KB, then Q can be derived from KB using the rules of inference

Problems with Propositional Logic

Propositional logic: pro and con

- Advantages
  - Simple KR language sufficient for some problems
  - Lays the foundation for higher logics (e.g., FOL)
  - Reasoning is decidable, though NP complete, and efficient techniques exist for many problems
- Disadvantages

PL is a weak KR language

- Hard to identify "individuals" (e.g., Mary, 3)
- Can't directly talk about properties of individuals or relations between individuals (e.g., "Bill is tall")
- Generalizations, patterns, regularities can't easily be represented (e.g., "all triangles have 3 sides")
- First-Order Logic (FOL) is expressive enough to represent this kind of information using relations, variables and quantifiers, e.g.,

PL Example

- Consider the problem of representing the following information:
  - Every person is mortal.
  - Confucius is a person.
  - Confucius is mortal.
- How can these sentences be represented so that we can infer the third sentence from the first two?

PL Example

- In PL we have to create propositional symbols to stand for all or part of each sentence, e.g.:

  P = “person”; Q = “mortal”; R = “Confucius”

- The above 3 sentences are represented as:

  $P \rightarrow Q$; $R \rightarrow P$; $R \rightarrow Q$

- The 3rd sentence is entailed by the first two, but we need an explicit symbol, R, to represent an individual, Confucius, who is a member of the classes *person* and *mortal*

- Representing other individuals requires introducing separate symbols for each, with some way to

Hunt the Wumpus domain

- Some atomic propositions:

  S12 = There is a stench in cell (1,2)

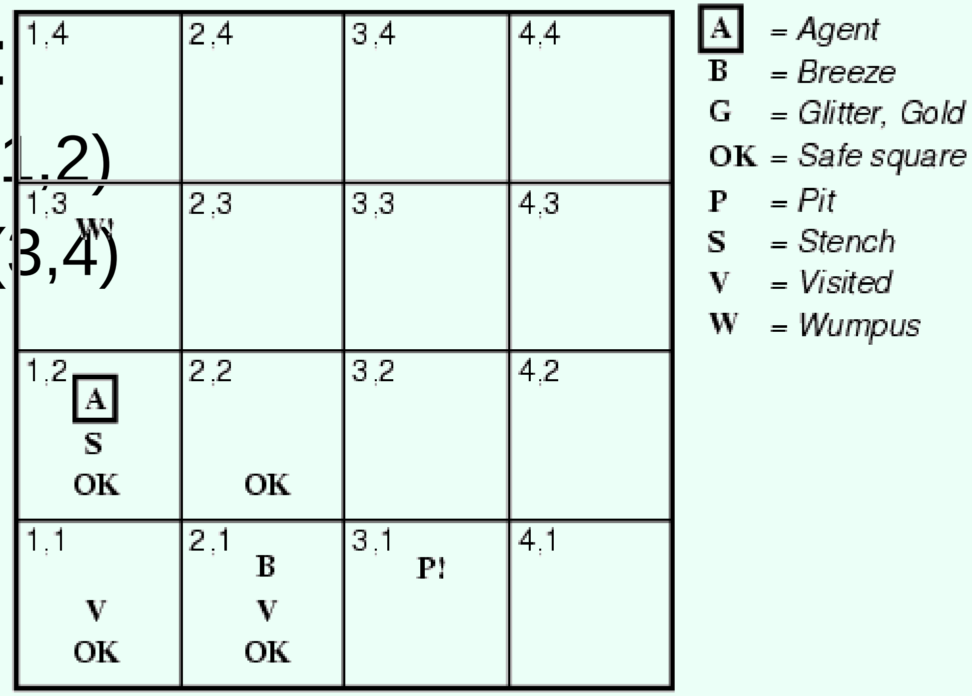
  B34 = There is a breeze in cell (3,4)

  W22 = Wumpus is in cell (2,2)

  V11 = We've visited cell (1,1)

  OK11 = Cell (1,1) is safe.

  …



- Some rules:

  (R1) $\neg S11 \rightarrow \neg W11 \wedge \neg W12 \wedge \neg W21$

  (R2) $\neg S21 \rightarrow \neg W11 \wedge \neg W21 \wedge \neg W22 \wedge \neg W31$

  (R3) $\neg S12 \rightarrow \neg W11 \wedge \neg W12 \wedge \neg W22 \wedge \neg W13$

  (R4) $S12 \rightarrow W13 \vee W12 \vee W22 \vee W11$

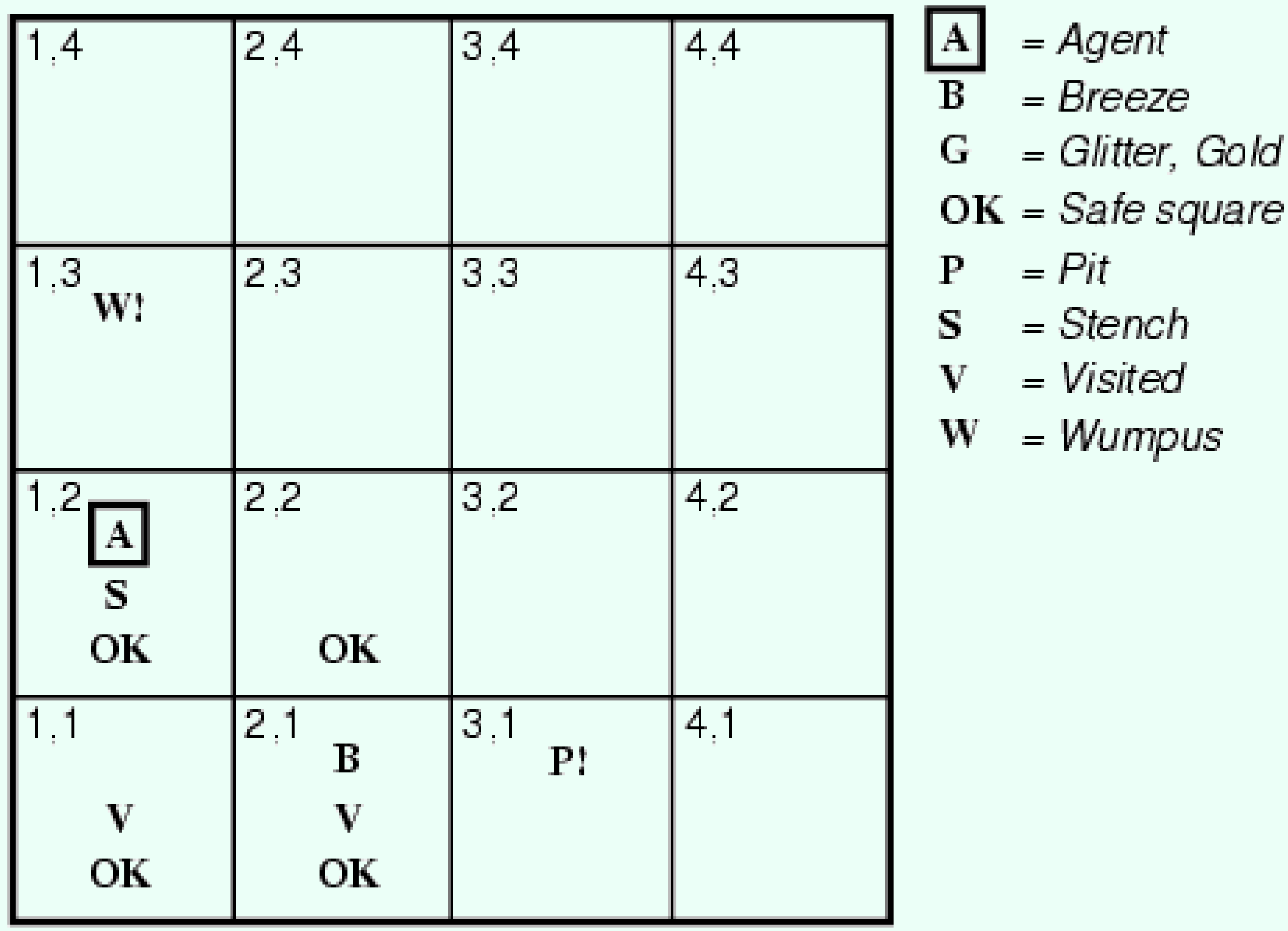
  …

- The lack of variables requires us to give similar rules for each cell!

After the third move

We can prove that the Wumpus is in (1,3) using the four rules given.

See R&N section 7.5



Proving W13

Apply MP with $\neg$S11 and R1:

$$\neg W11 \wedge \neg W12 \wedge \neg W21$$

Apply And-Elimination to this, yielding 3 sentences:

$$\neg W11, \neg W12, \neg W21$$

Apply MP to ~S21 and R2, then apply And-elimination:

$$\neg W22, \neg W21, \neg W31$$

Apply MP to S12 and R4 to obtain:

$$W13 \vee W12 \vee W22 \vee W11$$

Apply Unit resolution on ($W13 \vee W12 \vee W22 \vee W11$) and $\neg$W11:

$$W13 \vee W12 \vee W22$$

Apply Unit Resolution with ($W13 \vee W12 \vee W22$) and $\neg$W22:

Propositional Wumpus hunter problems

- Lack of variables prevents stating more general rules
  - We need a set of similar rules for each cell
- Change of the KB over time is difficult to represent
  - Standard technique is to index facts with the time when they're true
  - This means we have a separate KB for every time point

Propositional logic summary

- Inference is the process of deriving new sentences from old
  - **Sound** inference derives true conclusions given true premises
  - **Complete** inference derives all true conclusions from a set of premises
- A **valid sentence** is true in all worlds under all interpretations
- If an implication sentence can be shown