

Assignment Fall 20-21
Course: Mathematical Analysis for Computer Science
Course code: CSE 313

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1(a) Question Answer

Part-1

Given,
~~Pb~~ Numbers = 160 , 4A = 40 ($\because$ My ID = 20)

Prime Exponent representation method

$$160 = 2^5 \cdot 3^0 \cdot 5^1 = (5, 0, 1, 0, 0, \ldots)$$

$$40 = 2^3 \cdot 3^0 \cdot 5^1 = (3, 0, 1, 0, 0 \ldots)$$

$$\text{LCM}(160, 40) = 2^{\max(5,3)} \cdot 3^{\max(0,0)} \cdot 5^{\max(1,1)}$$

$$= 2^5 \cdot 3^0 \cdot 5^1$$

$$= 160$$

$$\text{GCD}(160, 40) = 2^{\min(5,3)} \cdot 3^{\min(0,0)} \cdot 5^{\min(1,1)}$$

$$= 2^3 \cdot 3^0 \cdot 5^1$$

$$= 40$$

Part-2

There are three methods for finding the GCD (Greatest common divisor), one of them is prime exponent representation.

Merits

1) Prime exponent representation method can be useful over traditional method when we need to find the highest common Factors or lowest common ~~Factors~~ multiple of

sets of numbers to find GCD.

2) We can always exchange one of the factors for a product which that factor is equal to.

$$40 = 4 \times 10$$

~~4×2~~ $4 = 2\times 2$ $10 = 2\times 5$

$$40 = (2\times 2)\times 10 \qquad 40 = 4\times(2\times 5)$$

Now, if we continue with our example for 40, we will get to $40 = 2\times 2\times 2\times 5$, at which point we cann't break down the factors into smaller numbers because they all are prime.

Demerits

1) If we don't already have prime exponents or prime factorizations, the Euclid's algorithm will be much quicker.

Ex: It is easy to find 36, 360, 180 etc. used using this.
But it's not easy to find 1607, 1232 etc. numbers using prime exponent representation method.

2) Prime factorization method is much harder ~~to~~ to find Gcd then the other methods (espicially for large numbers). So, Euclidean algorithm will be much easier to find GCD for large ~~nu~~ numbers like: 1650, 1320, 3420 etc.

1(b) Question Answer

Soln: Let, $a = p_1^{\alpha_1} \cdots p_k^{a_k}$

$b = p_1^{\beta_1} \cdots p_k^{b_k}$

and $\gcd(a,b) = p_1^{\gamma_1} \cdots p_k^{\gamma_k}$, (where, $\gamma_i = \min(\alpha_i, \beta_i)$)

$\text{lcm}(a,b) = p_1^{\gamma_1} \cdots p_k^{\gamma_k}$, (where, $\gamma_i = \max(\alpha_i, \beta_i)$)

Proof

$$\gcd(a,b) \cdot \text{lcm}(a,b) = p_1^{\min(\alpha_1,\beta_1)} \cdots p_k^{\min(\alpha_k,\beta_k)} \cdot p_1^{\max(\alpha_1,\beta_1)} \cdots p_k^{\max(\alpha_k,\beta_k)}$$

$$= p_1^{\min(\alpha_1,\beta_1)+\max(\alpha_1,\beta_1)} \cdots p_k^{\min(\alpha_k,\beta_k)+\max(\alpha_k,\beta_k)}$$

$$= p_1^{\alpha_1+\beta_1} \cdots p_k^{\alpha_k+\beta_k}.$$

$$= ab$$

Example

let, $a = 20 = 2^2 \cdot 3^0 \cdot 5^1 = (2,0,1,0,0\ldots)$

$b = 4 = 2^2 \cdot 3^0 \cdot 5^0 = (2,0,0\ldots)$

We know,
20×4
$= 80$

$$\gcd(20,4) = 2^{\min(2,2)} \cdot 3^{\min(0,0)} \cdot 5^{\min(1,0)}$$

$$= 2^2 \times 3^0 \times 5^0$$

$$= 4$$

$$\text{lcm}(20,4) = 2^{\max(2,2)} \cdot 3^{\max(0,0)} \cdot 5^{\max(1,0)}$$

$$= 2^2 \times 3^0 \times 5^1$$

$$= 20$$

$\gcd(20,4) \cdot \text{lcm}(20,4)$

$= 4 \times 20$

$= 80$

$= ab$

Ans:

3(a) Question Answer

Given,

poison random variable with parameter, $\lambda = 3$

My ID is = 20 ↳ last digit

so, I have to calculate no watches sold in a

2(a) Question Answer

Part-1

Binomial Random Variable:

A binomial random variable counts how often a particular event occurs in a fixed number of tries or trials. For a variable to be a binomial random variable, all of the following conditions must be met:

1) There are a fixed ~~&~~ number of trails.

2) On each trial, the event of interest either occurs or does not.

3) The probability of occurrence (or not) is the same on each trail.

4) Trails are independent of one ~~an~~ another.

Example: 1) Number of winning lottery tickets when I buy 10 tickets of the same kind.

2) Suppose that n independent trails, each of which results in a "success" with probability p and in a "failure" with probability 1-p, are to be performed. If X represents the number of successes that occure in the n trials, then X is said to be a binomial random variable with parameters (n, p).

The probability mass function of a binomial random variable having parameters (n, p) is given by,

$$p(i) = \binom{n}{i} p^i (1-p)^{n-i}, \quad i = 0, 1, \ldots, n$$

where, $\binom{n}{i} = \frac{n!}{(n-i)!\, i!}$

equals the number of different groups of i objects that can be chosen from a set of n objects.

Part-2

Given,

X = no of total students in our class = 30

Y = last two digit of my ID = 20

Z = First letter of my name = N (Nowshin)

The Binomial Random variable Having Parameters: (Y, N) is given by,

$$N(X) = \binom{Y}{X} N^X (1-N)^{Y-X} \qquad X = 0, 1 \cdots 30$$

Where, N = probability of success.

$1 - N$ = " " failure.

X = trials

Y = A set of objects.

Ans.

Q(b) Question Answer

Part - 1

If X is the number of defective items in the sample, then X is a ~~bit~~ bi-nomial random variable with parameters $(\underset{n}{6}, \underset{p}{0.4})$. Hence, the desired probability is given by,

$$\cancel{P\{X=0\}+P\{X=1\}} = \cancel{\binom{6}{0}(0.4)^0(0.6)^3 + \binom{6}{1}(0.4)^1(0.6)^2}$$

$$\cancel{= 0.0864 + 0.864}$$

$$\cancel{= 0.9504}$$

We know, $p(i) = \binom{n}{i} p^i (1-p)^{n-i} \quad , i = 0, 1 \cdots n$

$$P\{X=0\} + P\{X=1\} = \binom{6}{0}(0.4)^0 (1-0.4)^{6-0} + \binom{6}{1}(0.4)^1(1-0.4)^{6-1}$$

$$= 1 \times 1 \times 0.047 + 6 \times 0.4 \times 0.078$$

$$= 0.2342$$

Part - 2

Independent probability

If two events are independent if the result of the second event is not affected by the result of the first event. If A and B are independent events, the probability of both events is independent.

$P(A \text{ and } B) = P(A) \cdot P(B)$

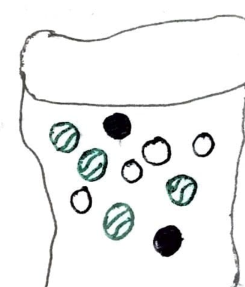
Dependent ~~Event~~ probability

Two events are dependent if the result of the first event affects the outcome of the second event so that the probability is changed. Then this probability is called dependent probability.

$$P(A \text{ and } B) = P(A) \times \underbrace{P(B|A)}_{\text{Probability of B given A}}$$

Example

Using a bag of marbles ~~on the left~~ ~~Lo~~



What is the probability of pulling a black marble two times in a row? P(black, black)

Independent Probability

When we put 1^{st} marble back in:

$$\frac{2}{10} \times \frac{2}{10}$$

$$= \frac{1}{5} \times \frac{1}{5}$$

$$= \frac{1}{25}$$

Dependent Probability

When we keep 1^{st} marble:

$$\frac{2}{10} \times \frac{1}{9}$$

$$= \frac{1}{5} \times \frac{1}{9}$$

$$= \frac{1}{45}$$

Ans:

3(a) Question Answer

Part-1

The ~~poison~~ poisson Random Variable:

A random variable X, taking on one of the values 0, 1, 2..., is said to be a poisson random variable with parameter λ, if for some $\lambda > 0$,

$$p(i) = P\{X = i\} = e^{-\lambda} \frac{\lambda^i}{i!}, \quad i = 0, 1, \ldots \longrightarrow (1)$$

Equation (1) Defines a probability mass function since

$$\sum_{i=0}^{\infty} p(i) = e^{-\lambda} \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = e^{-\lambda} e^{\lambda} = 1$$

The poisson random variable has a wide range of applications in a diverse number of areas. A important property of the poisson random variable is that it may be used to approximate a binomial random variable when the binomial parameter n is large and p is small. ~~To~~

Suppose that X is a binomial random variable with parameters (n, p) and let $\lambda = np$. Then

$$P\{X = i\} = \frac{n!}{(n-i)!\, i!} p^i (1-p)^{n-i}$$

$$= \frac{n!}{(n-i)!\, i!} \left(\frac{\lambda}{n}\right)^i \left(1 - \frac{\lambda}{n}\right)^{n-i}$$

$$= \frac{n(n-1)\cdots(n-i+1)}{n^i} \frac{\lambda^i}{i!} \frac{(1-\lambda/n)^n}{(1-\lambda/n)^i}$$

Now, for n large and p small

$$\left(1-\frac{\lambda}{n}\right)^n \approx e^{-\lambda}, \quad \frac{n(n-1)\cdots(n-i+1)}{n^i} \approx 1, \quad \left(1-\frac{\lambda}{n}\right)^i \approx 1$$

Hence, for n large and p small,

$$P\{X=i\} \approx e^{-\lambda} \frac{\lambda^i}{i!}$$

Math Solution

Part-2

Given, poison random variable. Parameter, $\lambda = 3$

My ID = 20 → last digit.

So, I have to find the probability that no watches has ~~h~~. been sold in each day.

We know, $P\{X=i\} \approx e^{-\lambda} \frac{\lambda^i}{i!}$

Here, $i=0$

$$\therefore P\{X=0\} = e^{-3}$$

$$= 0.04978$$

$$\approx 0.05$$

Ans:

3(b) Question Answer

Part - 1

Given, ~~Cheerfull = C~~

Happy = H

Sad = S

Angry = A

Letting X_n denote ~~&~~ Sana's mood on the nth day, then $\{X_n, n \geq 0)$ is a three State Markov chain.

State 0 = Happy

State 1 = Sad

State 2 = Angry

Now, transition probability matrix :

$$P = \begin{array}{ccc} (H) & (S) & (A) \end{array}$$
$$P = \begin{Vmatrix} 0.6 & 0.3 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.6 & 0.3 \end{Vmatrix}$$

Part-2

Four-step transition probability matrix P^4:

$$P^{(2)} = P^2 = \begin{Vmatrix} 0.6 & 0.3 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.6 & 0.3 \end{Vmatrix} \cdot \begin{Vmatrix} 0.6 & 0.3 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.6 & 0.3 \end{Vmatrix}$$

$$= \begin{Vmatrix} (0.6\times0.6)+(0.3\times0.3)+(0.1\times0.1) & (0.6\times0.3)+(0.3\times0.3)+(0.1\times0.6) & (0.6\times0.1)+(0.3\times0.4)+(0.1\times0.3) \\ (0.3\times0.6)+(0.3\times0.3)+(0.4\times0.1) & (0.3\times0.3)+(0.33\times0.3)+(0.4\times0.6) & (0.3\times0.1)+(0.3\times0.4)+(0.4\times0.3) \\ (0.1\times0.6)+(0.6\times0.3)+(0.3\times0.1) & (0.1\times0.3)+(0.6\times0.3)+(0.3\times0.6) & (0.1\times0.1)+(0.4\times0.6)+(0.3\times0.3) \end{Vmatrix}$$

$$= \begin{Vmatrix} 0.46 & 0.33 & 0.33 \\ 0.31 & 0.42 & 0.27 \\ 0.27 & 0.39 & 0.34 \end{Vmatrix}$$

$$P^{(4)} = (P^2)^2 = \begin{Vmatrix} 0.46 & 0.33 & 0.33 \\ 0.31 & 0.42 & 0.27 \\ 0.27 & 0.39 & 0.34 \end{Vmatrix} \cdot \begin{Vmatrix} 0.46 & 0.33 & 0.33 \\ 0.31 & 0.42 & 0.27 \\ 0.27 & 0.39 & 0.34 \end{Vmatrix}$$

$$= \begin{Vmatrix} 0.403 & 0.419 & 0.353 \\ 0.346 & 0.384 & 0.306 \\ 0.337 & 0.386 & 0.31 \end{Vmatrix} \text{(calculated it)}$$

Ans:

4(a) question Answer

Soln: If we let the state at time n depend only on whether or not it is sunny at time n, then the proceding model is not a "markov chain". However, we can transform this model into a Markov chain by saying that the state at any time is determined by the weather conditions during both day and the previous day. In other words, we can say that the process is in.

State 0 If it is sunny both today and yesterday.

State 1 If it is sunny today but not yesterday.

State 2 If it was sunny yesterday but not today.

State 3 If it ~~di~~ was n't sunny either yesterday or today.

This ~~procedd~~. proceding would then represent a four state Markov chain having transition probability matrix.

$$P = \begin{Vmatrix} 0.6 & 0 & 0.4 & 0 \\ 0.2 & 0 & 0.8 & 0 \\ 0 & 0.3 & 0 & 0.7 \\ 0 & 0.5 & 0 & 0.5 \end{Vmatrix}$$

4(b) question Answer

Part-1

Congruences: Integer a is congruent to integer b modulo $m>0$, if a and b give the same remainder when divided by m Notation $a \equiv b \pmod{m}$.

Congruence is related to equivalence relation:-

Reflectivity: $a \equiv a \pmod{m}$

Ex: $3 \equiv 3 \pmod{5}$

They are reflexive: a is related to a.

Symmetry: $a \equiv b \pmod{m} \Rightarrow b \equiv a \pmod{m}$

Ex: if $3 \equiv 8 \pmod{5}$ then $8 \equiv 3 \pmod{5}$

They are symmetric: if a is related to b and b is related to a.

Transitivity: $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m} \Rightarrow a \equiv c \pmod{m}$

Ex: if $3 \equiv 8 \pmod{5}$ and if $8 \equiv 18 \pmod{5}$

then $3 \equiv 18 \pmod{5}$

They are transitive: if a is related to b and b is related to c then a is related to c.

Part-2

Given, Two fair dice are rolled at the same time. So, the sample space consists of 36 points.

$$S = \left\{ \begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{array} \right\}$$

Where the outcome (i,j) is said to occur if i appears on the first die and j on the second die.

(i)

The value of the second dice minus the value of the first dice is two: Event $= \{(1,3),(3,5),(4,6)\}$,

$$\text{Probability} = \frac{3}{36} = \frac{1}{12}$$

(ii)

The value of the first dice is odd and the value of the second dice is even: Event $= \{(1,2),(3,2),(5,2),$
$(1,4),(3,4),(5,4),$
$(1,6),(3,6),(5,6)\}$

$$\text{Probability} = \frac{9}{36} = \frac{1}{4}$$

(iii)

Difference of two dice is one: Event $= \{(1,2),(3,2),(2,3),(4,3),$
$(3,4),(5,4),(4,5),(6,5),$
$(5,6)\},(2,1)\}$

$$\text{Probability} = \frac{10}{36} = \frac{5}{18}$$

Ans: