

Parsing

Part V

Bottom-Up Parsing

- A **bottom-up parser** creates the parse tree of the given input starting from leaves towards the root
- A bottom-up parser tries to find the **right-most derivation** of the given input in the reverse order.

$S \Rightarrow \dots \Rightarrow \omega$ (the right-most derivation of ω)

\leftarrow (the bottom-up parser finds the right-most derivation in the reverse order)

Rightmost Derivation

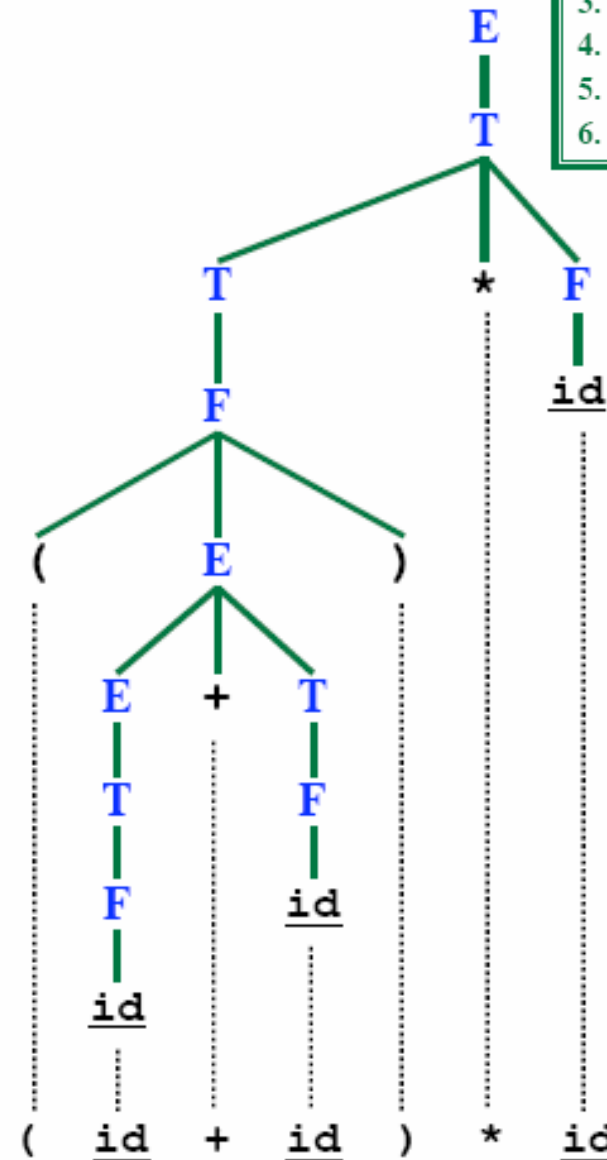
Rules Used:

$E \rightarrow T$
 $T \rightarrow T * F$
 $F \rightarrow \underline{id}$
 $T \rightarrow F$
 $F \rightarrow (E)$
 $E \rightarrow E + T$
 $T \rightarrow F$
 $F \rightarrow \underline{id}$
 $E \rightarrow T$
 $T \rightarrow F$
 $F \rightarrow \underline{id}$

Right-Sentential Forms:

E
 T
 $T * F$
 $T * \underline{id}$
 $F * \underline{id}$
 $(E) * \underline{id}$
 $(E + T) * \underline{id}$
 $(E + F) * \underline{id}$
 $(E + \underline{id}) * \underline{id}$
 $(T + \underline{id}) * \underline{id}$
 $(F + \underline{id}) * \underline{id}$
 $(\underline{id} + \underline{id}) * \underline{id}$

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow \underline{id}$



Rightmost Derivation In reverse

Rules Used:

$F \rightarrow \underline{id}$

$T \rightarrow F$

$E \rightarrow T$

$F \rightarrow \underline{id}$

$T \rightarrow F$

$E \rightarrow E + T$

Right-Sentential Forms:

$(\underline{id} + \underline{id}) * \underline{id}$

$(F + \underline{id}) * \underline{id}$

$(T + \underline{id}) * \underline{id}$

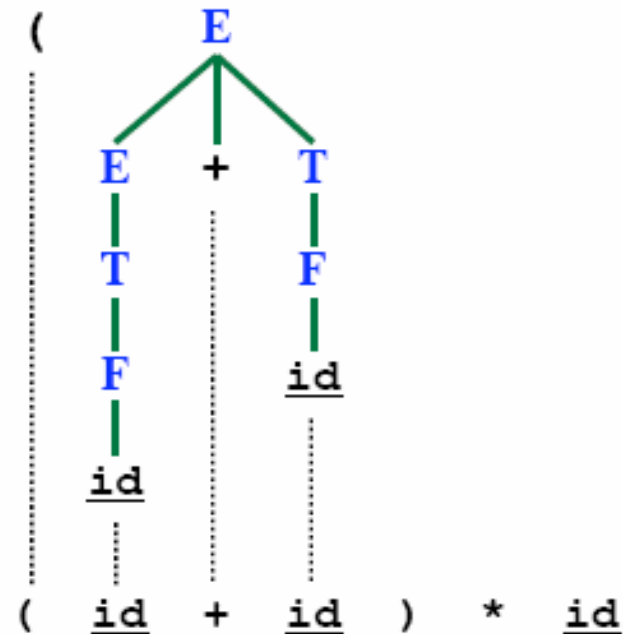
$(E + \underline{id}) * \underline{id}$

$(E + F) * \underline{id}$

$(E + T) * \underline{id}$

$(E) * \underline{id}$

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow \underline{id}$



Rightmost Derivation In reverse

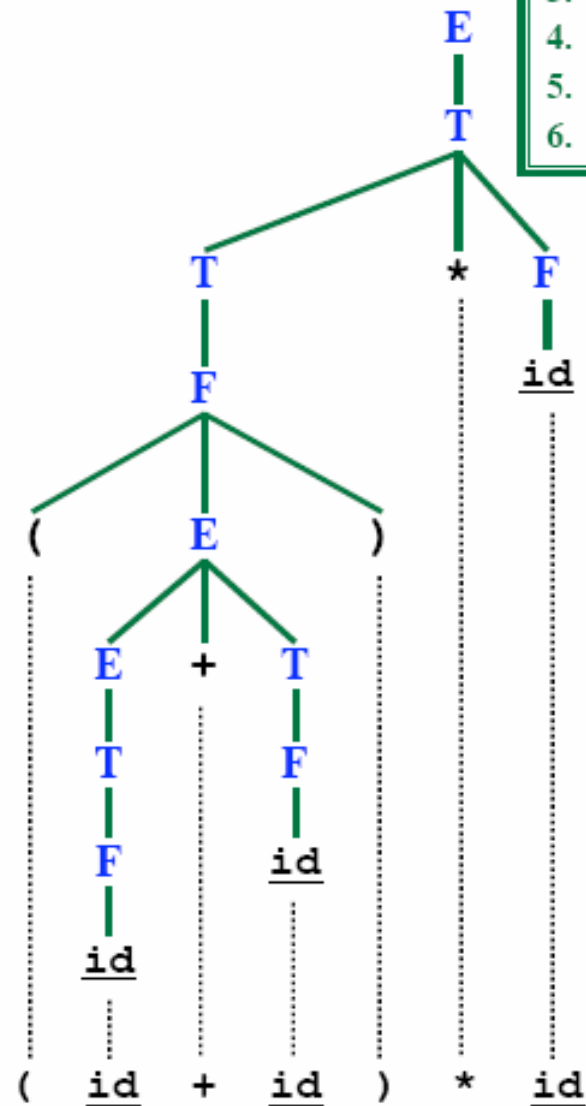
Rules Used:

$F \rightarrow \underline{id}$
 $T \rightarrow F$
 $E \rightarrow T$
 $F \rightarrow \underline{id}$
 $T \rightarrow F$
 $E \rightarrow E + T$
 $F \rightarrow (E)$
 $T \rightarrow F$
 $F \rightarrow \underline{id}$
 $T \rightarrow T * F$
 $E \rightarrow T$

Right-Sentential Forms:

$(\underline{id} + \underline{id}) * \underline{id}$
 $(F + \underline{id}) * \underline{id}$
 $(T + \underline{id}) * \underline{id}$
 $(E + \underline{id}) * \underline{id}$
 $(E + F) * \underline{id}$
 $(E + T) * \underline{id}$
 $(E) * \underline{id}$
 $F * \underline{id}$
 $T * \underline{id}$
 $T * F$
 T
 E

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow \underline{id}$



LR parsing corresponds to rightmost derivation in reverse

Reduction

- A reduction step replaces a specific substring (matching the body of a production)

(id + id) * id
(F + id) * id
(T + id) * id
(E + id) * id
(E + F) * id
(E + T) * id

(E) * id
F * id
T * id
T * F
T
E

- | | |
|----|--------------------------------|
| 1. | $E \rightarrow E + T$ |
| 2. | $E \rightarrow T$ |
| 3. | $T \rightarrow T * F$ |
| 4. | $T \rightarrow F$ |
| 5. | $F \rightarrow (E)$ |
| 6. | $F \rightarrow \underline{id}$ |

- Reduction is the opposite of derivation
- Bottom up parsing is a process of **reducing** a string ω to the start symbol S of the grammar

Handle

- Informally, a **handle** is a substring (in the parsing string) that matches the right side of a production rule.
 - But not every substring matches the right side of a production rule is handle
- A **handle** of a right sentential form $\gamma (\equiv \alpha\beta\omega)$ is a production rule $A \rightarrow \beta$ and a position of γ where the string β may be found and replaced by A to produce
the previous right-sentential form in a rightmost derivation of γ .

$$S \Rightarrow \alpha A \omega \Rightarrow \alpha \beta \omega$$

Handle Pruning

- A right-most derivation in reverse can be obtained by **handle-pruning**.

$$S = \gamma_0 \xRightarrow{\text{rm}} \gamma_1 \xRightarrow{\text{rm}} \gamma_2 \xRightarrow{\text{rm}} \dots \xRightarrow{\text{rm}} \gamma_{n-1} \xRightarrow{\text{rm}} \gamma_n = \omega$$

n-th right-sentential form

input string

- Start from γ_n , find a handle $A_n \rightarrow \beta_n$ in γ_n , and replace β_n in by A_n to get γ_{n-1} .
- Then find a handle $A_{n-1} \rightarrow \beta_{n-1}$ in γ_{n-1} , and replace β_{n-1} in by A_{n-1} to get γ_{n-2} .
- Repeat this, until we reach S .

Shift-Reduce Parsing

- Bottom-up parsing is also known as **shift-reduce parsing** because its two main actions are shift and reduce.
- data structures: input-string and stack
- Operations
 - At each **shift** action, the current symbol in the input string is pushed to a stack.
 - At each **reduction** step, the symbols at the top of the stack (this symbol sequence is the right side of a production) will be replaced by the non-terminal at the left side of that production.
 - **Accept**: Announce successful completion of parsing
 - **Error**: Discover a syntax error and call error recovery

Shift Reduce Parsing

$S \rightarrow a T R e$
 $T \rightarrow T b c \mid b$
 $R \rightarrow d$

Remaining input: **a**bbcde

Rightmost derivation:

$S \Rightarrow a T R e$
 $\Rightarrow a T d e$
 $\Rightarrow a T b c d e$
 $\Rightarrow a b b c d e$

Shift Reduce Parsing

$S \rightarrow a T R e$

$T \rightarrow T b c \mid b$

$R \rightarrow d$

Remaining input: **b**cde

➔ Shift a, Shift b

a b

Rightmost derivation:

$S \rightarrow a T R e$

➔ $a T d e$

➔ $a T b c d e$

➔ a b b c d e

Shift Reduce Parsing

$S \rightarrow a T R e$

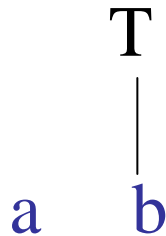
$T \rightarrow T b c \mid b$

$R \rightarrow d$

Remaining input: **b**cde

➔ Shift a, Shift b

➔ Reduce $T \rightarrow b$



Rightmost derivation:

$S \rightarrow a T R e$

➔ $a T d e$

➔ $a T$ **$b c d e$**

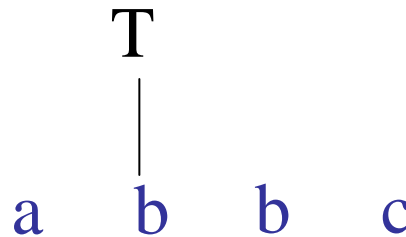
➔ **$a b b c d e$**

Shift Reduce Parsing

$S \rightarrow a T R e$
 $T \rightarrow T b c \mid b$
 $R \rightarrow d$

Remaining input: **de**

- ➔ Shift a, Shift b
- ➔ Reduce $T \rightarrow b$
- ➔ Shift b, Shift c



Rightmost derivation:

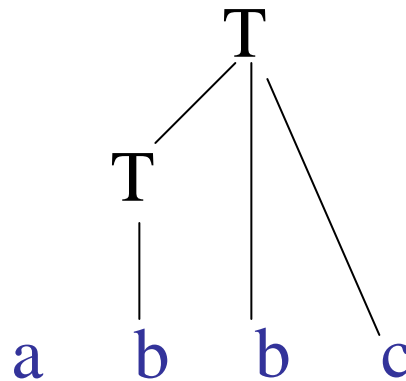
$S \rightarrow a T R e$
➔ $a T d e$
➔ $a T b c$ $d e$
➔ **$a b b c d e$**

Shift Reduce Parsing

$S \rightarrow a T R e$
 $T \rightarrow T b c \mid b$
 $R \rightarrow d$

➔ Shift a, Shift b
➔ Reduce $T \rightarrow b$
➔ Shift b, Shift c
➔ Reduce $T \rightarrow T b c$

Remaining input: **de**



Rightmost derivation:

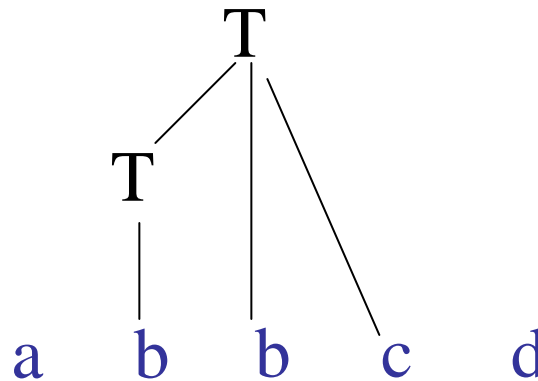
$S \rightarrow a T R e$
➔ a **T** **d** **e**
➔ **a** **T** **b** **c** **d** **e**
➔ **a** **b** **b** **c** **d** **e**

Shift Reduce Parsing

$S \rightarrow a T R e$
 $T \rightarrow T b c \mid b$
 $R \rightarrow d$

➔ Shift a, Shift b
➔ Reduce $T \rightarrow b$
➔ Shift b, Shift c
➔ Reduce $T \rightarrow T b c$
➔ Shift d

Remaining input: **e**



Rightmost derivation:

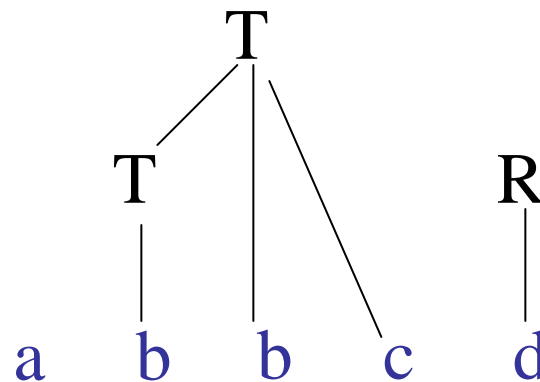
$S \rightarrow a T R e$
➔ $a T d e$
➔ $a T b c d e$
➔ $a b b c d e$

Shift Reduce Parsing

$S \rightarrow a T R e$
 $T \rightarrow T b c \mid b$
 $R \rightarrow d$

- ➔ Shift a, Shift b
- ➔ Reduce $T \rightarrow b$
- ➔ Shift b, Shift c
- ➔ Reduce $T \rightarrow T b c$
- ➔ Shift d
- ➔ Reduce $R \rightarrow d$

Remaining input: **e**



Rightmost derivation:

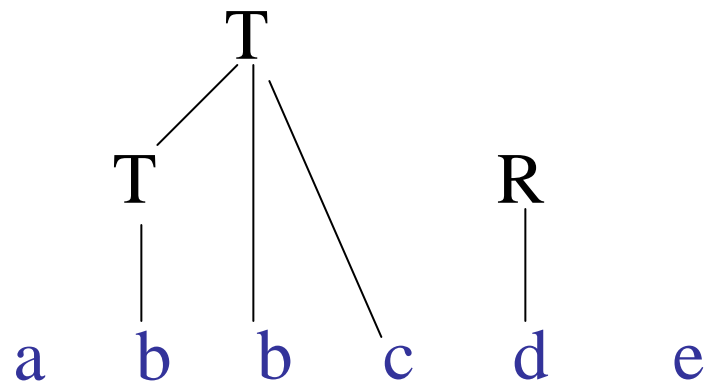
$S \rightarrow \underline{a T R} e$
➔ $a T \underline{d} e$
➔ $a T \underline{b c} d e$
➔ $a \underline{b b c} d e$

Shift Reduce Parsing

$S \rightarrow a T R e$
 $T \rightarrow T b c \mid b$
 $R \rightarrow d$

➔ Shift a, Shift b
➔ Reduce $T \rightarrow b$
➔ Shift b, Shift c
➔ Reduce $T \rightarrow T b c$
➔ Shift d
➔ Reduce $R \rightarrow d$
➔ Shift e

Remaining input:



Rightmost derivation:

$S \rightarrow \underline{a T R e}$
➔ $a T \underline{d} e$
➔ $a T \underline{b c d} e$
➔ $a \underline{b b c d} e$

Shift Reduce Parsing

$S \rightarrow a T R e$

$T \rightarrow T b c \mid b$

$R \rightarrow d$

➔ Shift a, Shift b

➔ Reduce $T \rightarrow b$

➔ Shift b, Shift c

➔ Reduce $T \rightarrow T b c$

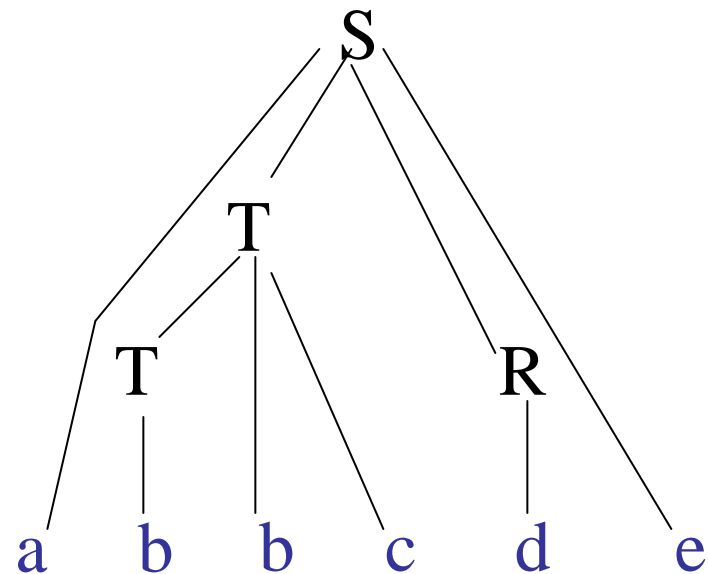
➔ Shift d

➔ Reduce $R \rightarrow d$

➔ Shift e

➔ Reduce $S \rightarrow a T R e$

Remaining input:



Rightmost derivation:

$S \rightarrow a T R e$

➔ $a T d e$

➔ $a T b c d e$

➔ $a b b c d e$

Example Shift-Reduce Parsing

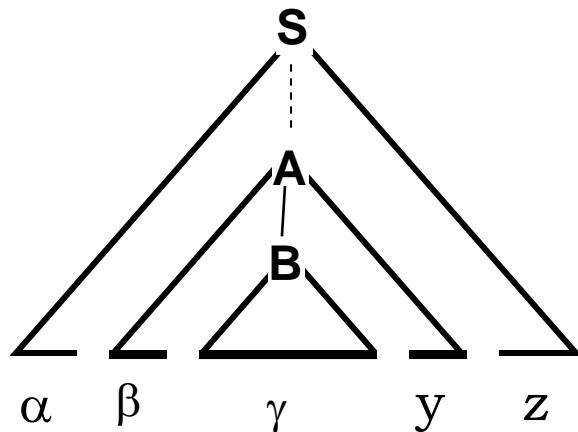
Consider the grammar:

Stack	Input	Action
\$	$id_1 + id_2 \$$	shift
$\$id_1$	$+ id_2 \$$	reduce 6
$\$F$	$+ id_2 \$$	reduce 4
$\$T$	$+ id_2 \$$	reduce 2
$\$E$	$+ id_2 \$$	shift
$\$E +$	$id_2 \$$	shift
$\$E + id_2$		reduce 6
$\$E + F$		reduce 4
$\$E + T$		reduce 1
$\$E$		accept

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow \underline{id}$

Shift-Reduce Parsing

- Handle will always appear on Top of stack, never inside
- Possible forms of two successive steps in any rightmost derivation
- CASE 1:



$$S \xRightarrow{*}_{rm} \alpha Az \Rightarrow \alpha \beta Byz \Rightarrow \alpha \beta \gamma yz$$

STACK

$\$ \alpha \beta \gamma$

After Reducing the handle

$\$ \alpha \beta B$

Shifting from Input

$\$ \alpha \beta B \gamma$

Reduce the handle

$\$ \alpha A$

INPUT

$yz \$$

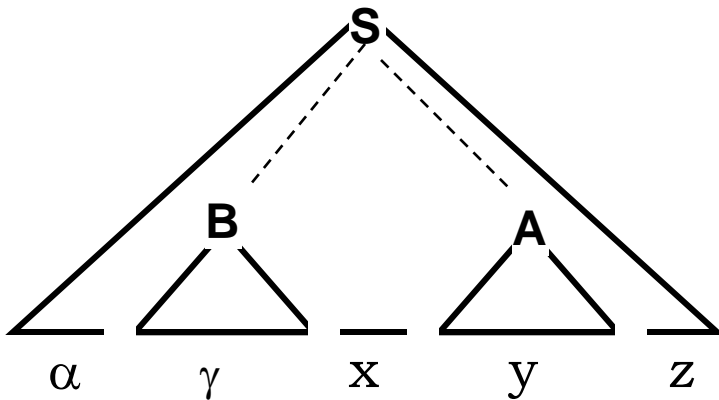
$yz \$$

$z \$$

$z \$$

Shift-Reduce Parsing

- Case 2:



$$S \xRightarrow[\text{rm}]{*} \alpha B x A z \xRightarrow[\text{rm}]{} \alpha B x y z \xRightarrow[\text{rm}]{} \alpha \gamma x y z$$

STACK

\$αγ

After Reducing the handle

\$αB

Shifting from Input

\$αBxy

Reducing the handle

\$αBxA

INPUT

xyz\$

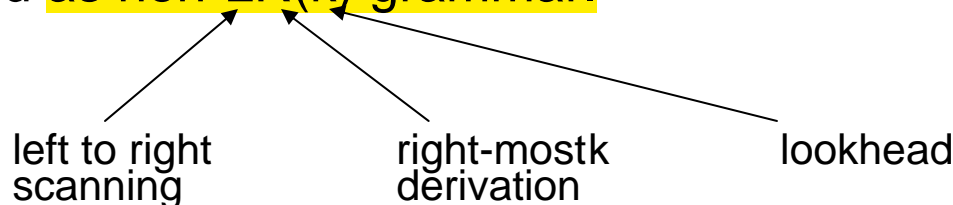
xyz\$

z\$

z\$

Conflicts During Shift-Reduce Parsing

- There are context-free grammars for which shift-reduce parsers cannot be used.
- Stack contents and the next input symbol may not decide action:
 - **shift/reduce conflict**: Whether make a shift operation or a reduction.
 - **reduce/reduce conflict**: The parser cannot decide which of several reductions to make.
- If a shift-reduce parser cannot be used for a grammar, that grammar is called **as non-LR(k) grammar**.



- An ambiguous grammar can never be a LR grammar.

Shift-Reduce Conflict in Ambiguous Grammar

stmt → **if** *expr* **then** *stmt*
 | **if** *expr* **then** *stmt* **else** *stmt*
 | **other**

STACK

....**if** *expr* **then** *stmt*

INPUT

else....\$

- We can't decide whether to shift or reduce?
- But we can adapt to parse certain ambiguous grammar to using shift-reducing parsers
 - We resolve in favor of SHIFT then we have a solution

Reduce Reduce conflict Example : Page 239-240