Lecture 3

Uninformed Search

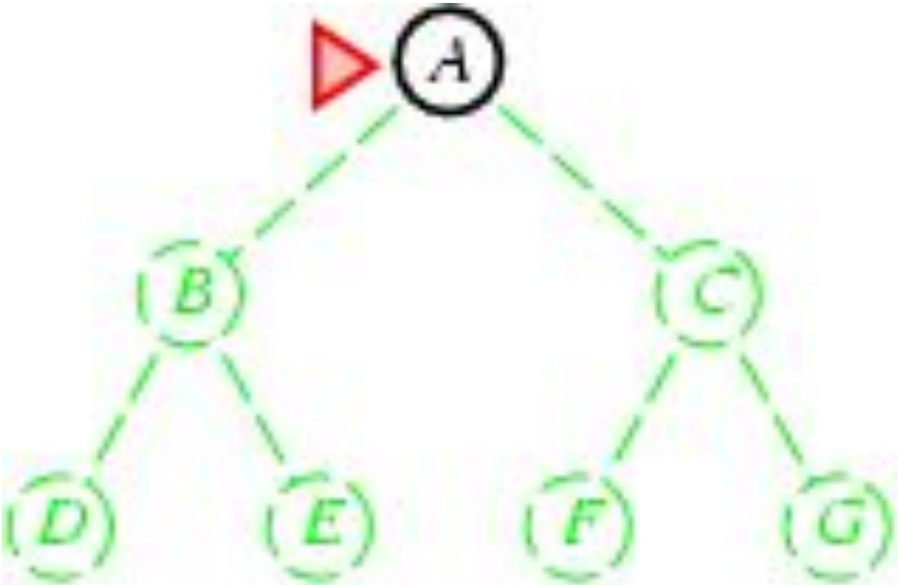
Uninformed search strategies

- Uninformed: While searching you have no clue whether one non-goal state is better than any other. Your search is blind. You don't know if your current exploration is likely to be fruitful.
- Various blind strategies:
- Breadth-first search
- Uniform-cost search
- Depth-first search
- Iterative deepening search

Breadth-first search

- Expand shallowest unexpanded node
- Fringe: nodes waiting in a queue to be explored
- Implementation:
  - *fringe* is a first-in-first-out (FIFO) queue, i.e., new successors go at end of the queue.

Is A a goal state?

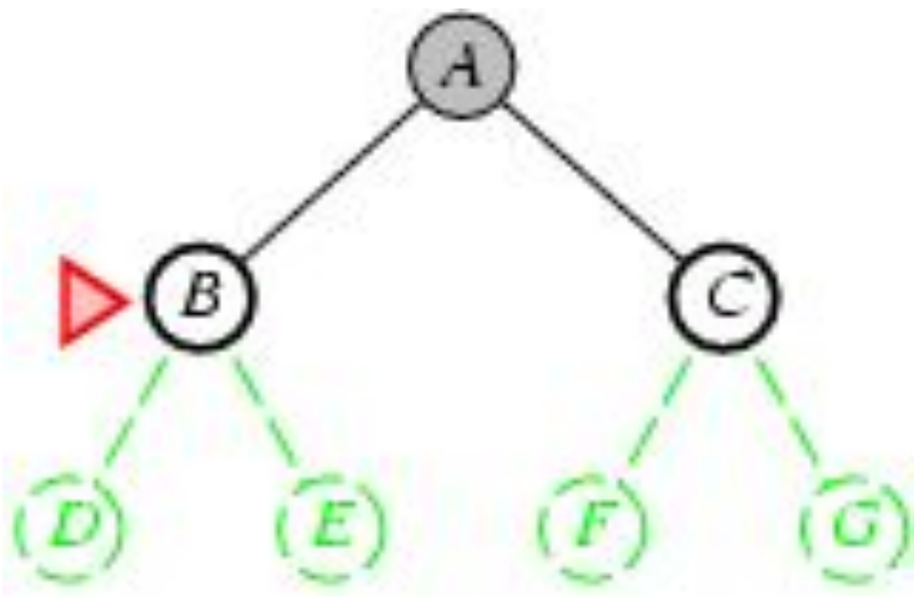


Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end

Expand:
fringe = [B,C]

Is B a goal state?



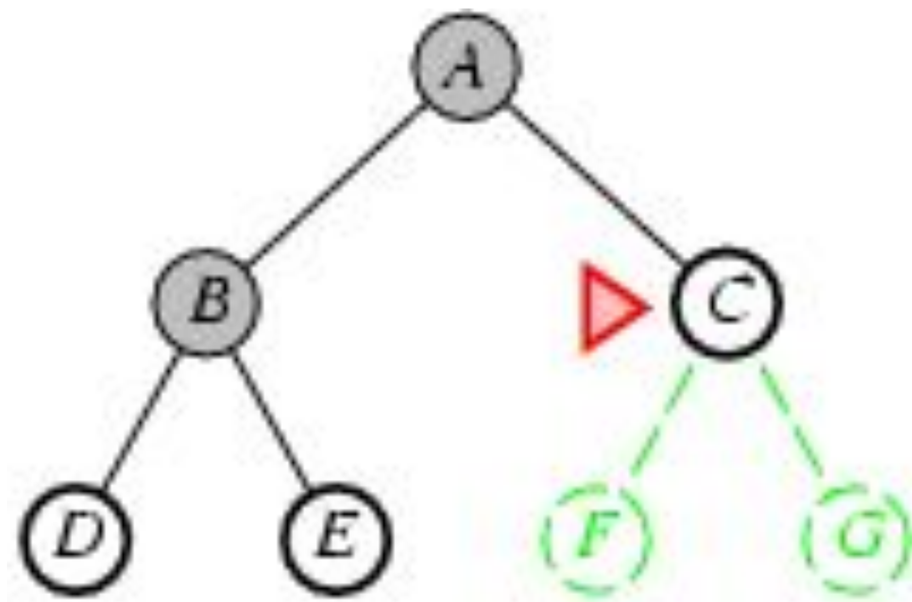
Breadth-first search

- Expand shallowest unexpanded node

- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end

Expand:
fringe=[C,D,E]

Is C a goal state?

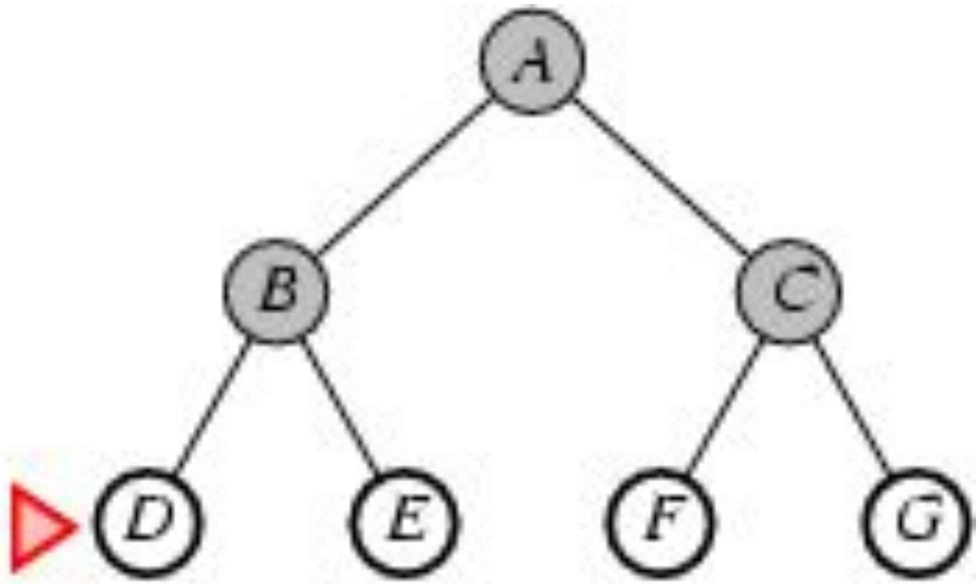


Breadth-first search

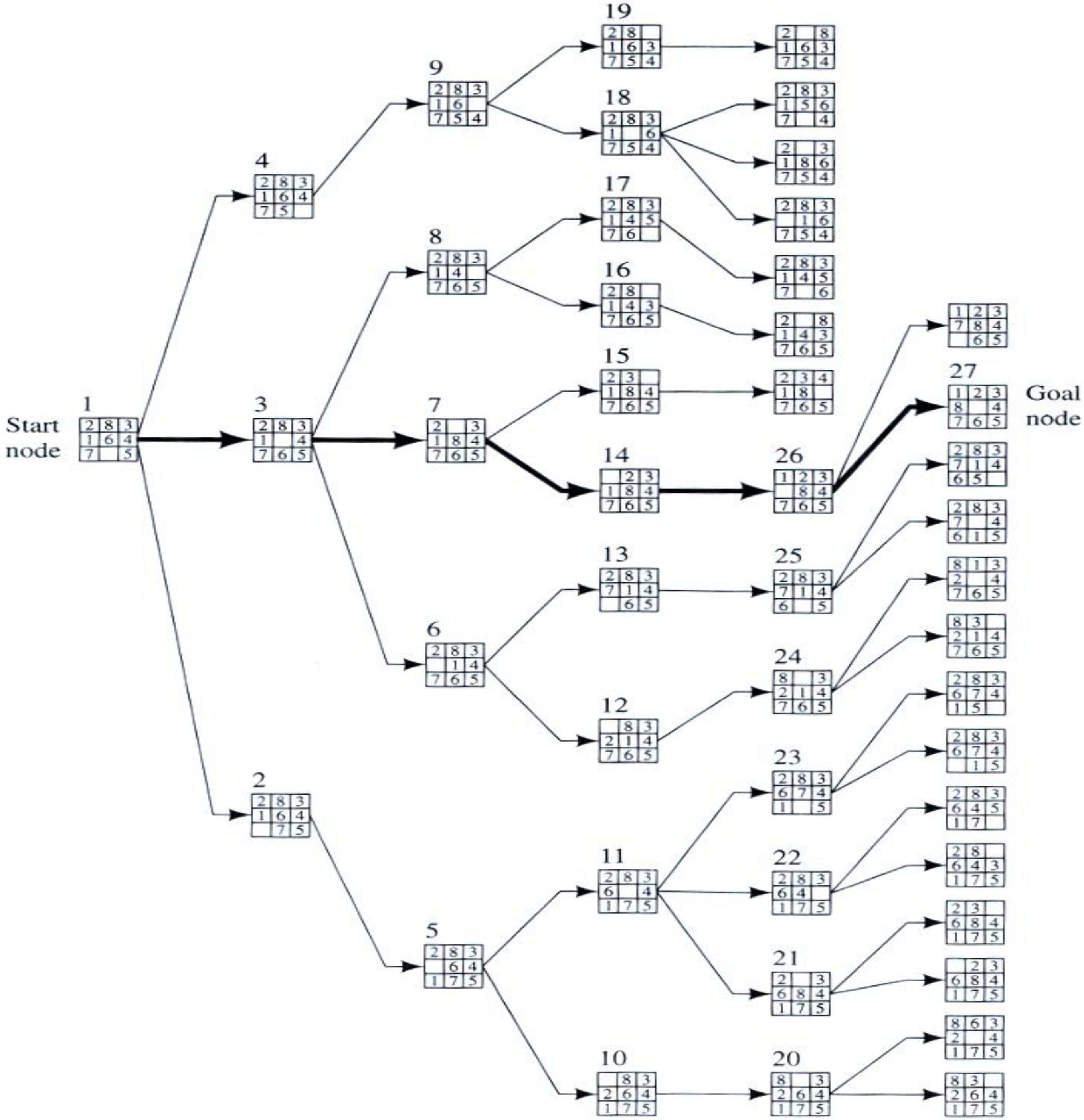
- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end

Expand:
fringe=[D,E,F,G]

Is D a goal state?



Example BFS


Start node
Goal node
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27

Properties of breadth-first search

- Complete? Yes it always reaches goal (if *b* is finite)
- Time? $1+b+b^2+b^3+\ldots+b^d+(b^{d+1}-b)) = O(b^{d+1})$
  (this is the number of nodes we generate)
- Space? $O(b^{d+1})$ (keeps every node in memory, either in fringe or on a path to fringe).
- Optimal? Yes (if we guarantee that deeper solutions are less optimal, e.g. step-cost=1).

- Space is the bigger problem (more than time)

Uniform-cost search

Breadth-first is only optimal if step costs is increasing with depth (e.g. constant). Can we guarantee optimality for any step cost?

Uniform-cost Search: Expand node with smallest path cost g(n).

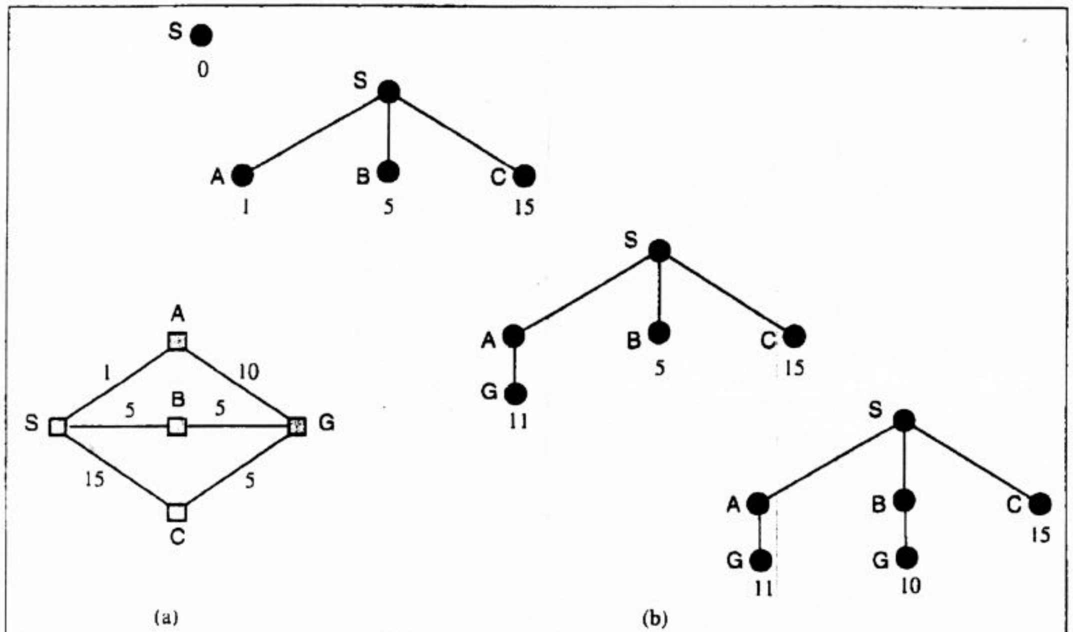


Figure 3.13 A route-finding problem. (a) The state space, showing the cost for each operator. (b) Progression of the search. Each node is labelled with $g(n)$. At the next step, the goal node with $g = 10$ will be selected.

Proof Completeness:

Given that every step will cost more than 0, and assuming a finite branching factor, there is a finite number of expansions required before the total path cost is equal to the path cost of the goal state. Hence, we will reach it.

Proof of optimality given completeness:

Assume UCS is not optimal.
Then there must be a goal state with path cost smaller than the goal state which was found (invoking completeness).
However, this is impossible because UCS would have expanded that node first by definition.
Contradiction.

Uniform-cost search

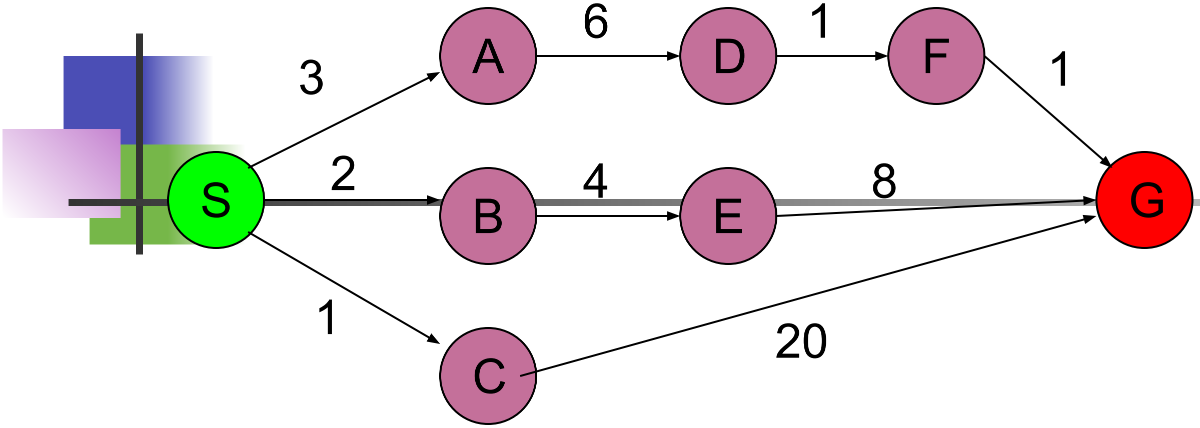
Implementation: *fringe* = queue ordered by path cost
Equivalent to breadth-first if all step costs all equal.

Complete? Yes, if step cost $\geq \varepsilon$
(otherwise it can get stuck in infinite loops)

Time? # of nodes with *path cost* $\leq$ cost of optimal solution.

Space? # of nodes on paths with path cost $\leq$ cost of optimal solution.

Optimal? Yes, for any step cost $\geq \varepsilon$



straight-line distances

h(S-G)=10
h(A-G)=7
h(D-G)=1
h(F-G)=1
h(B-G)=10
h(E-G)=8
h(C-G)=20

The graph above shows the step-costs for different paths going from the start (S) to the goal (G). On the right you find the straight-line distances.

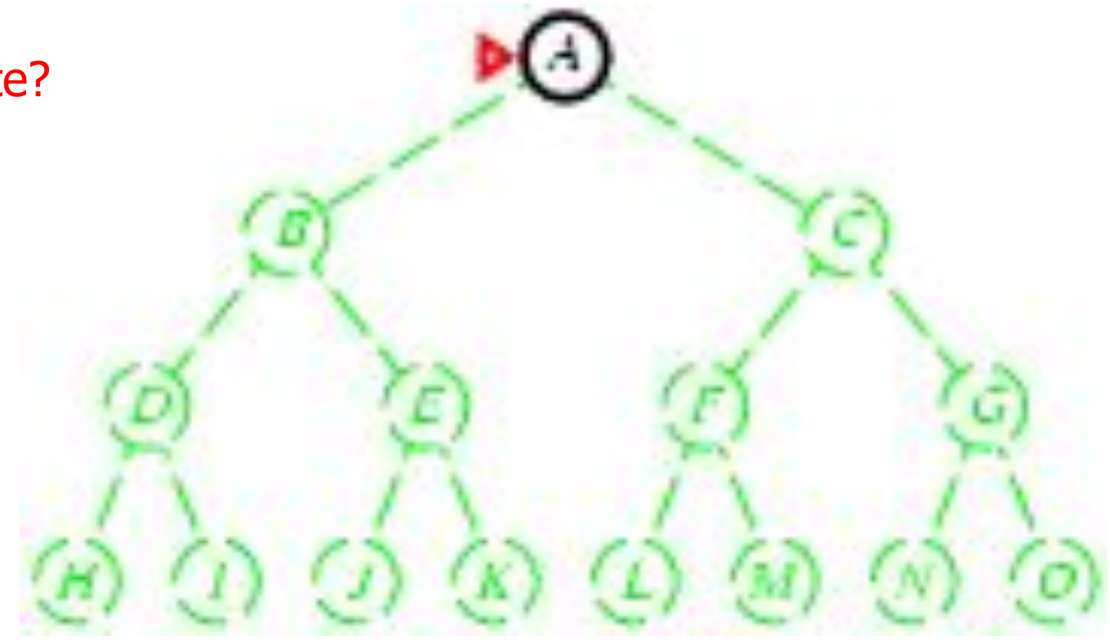
Use uniform cost search to find the optimal path to the goal.

Exercise for at home

Depth-first search

- Expand *deepest* unexpanded node
- Implementation:
  - *fringe* = Last In First Out (LIPO) queue, i.e., put successors at front

Is A a goal state?

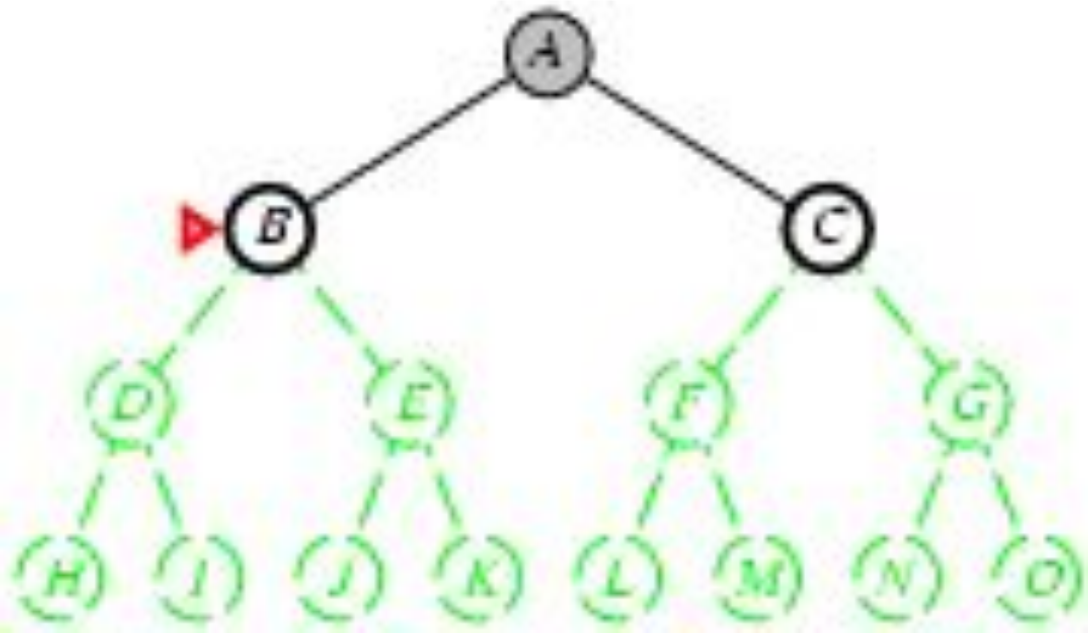


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[B,C]

Is B a goal state?

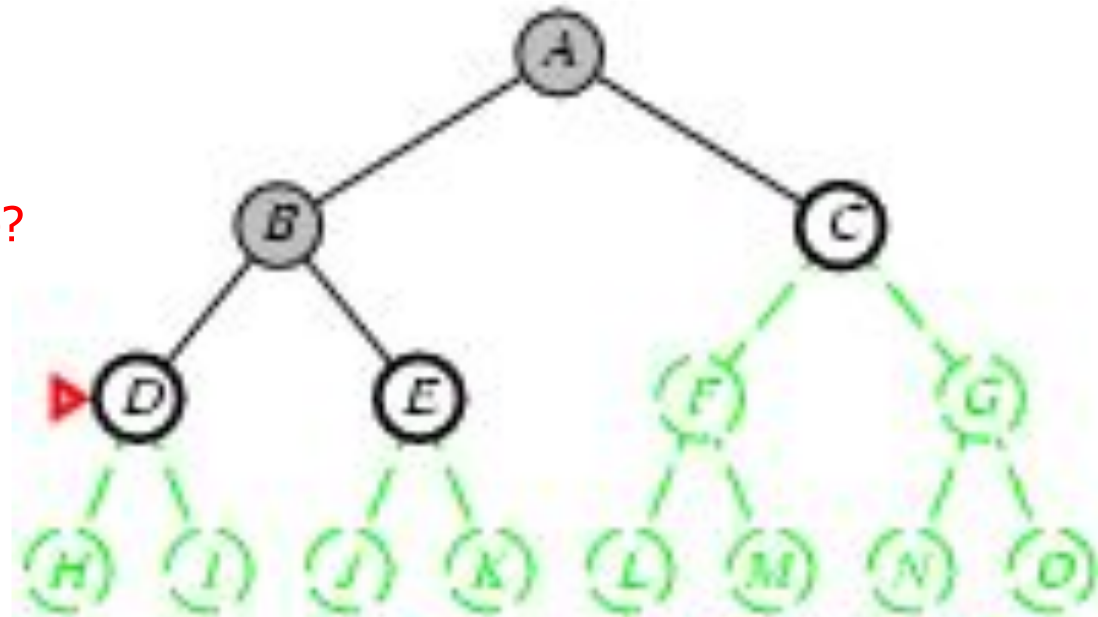


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[D,E,C]

Is D = goal state?

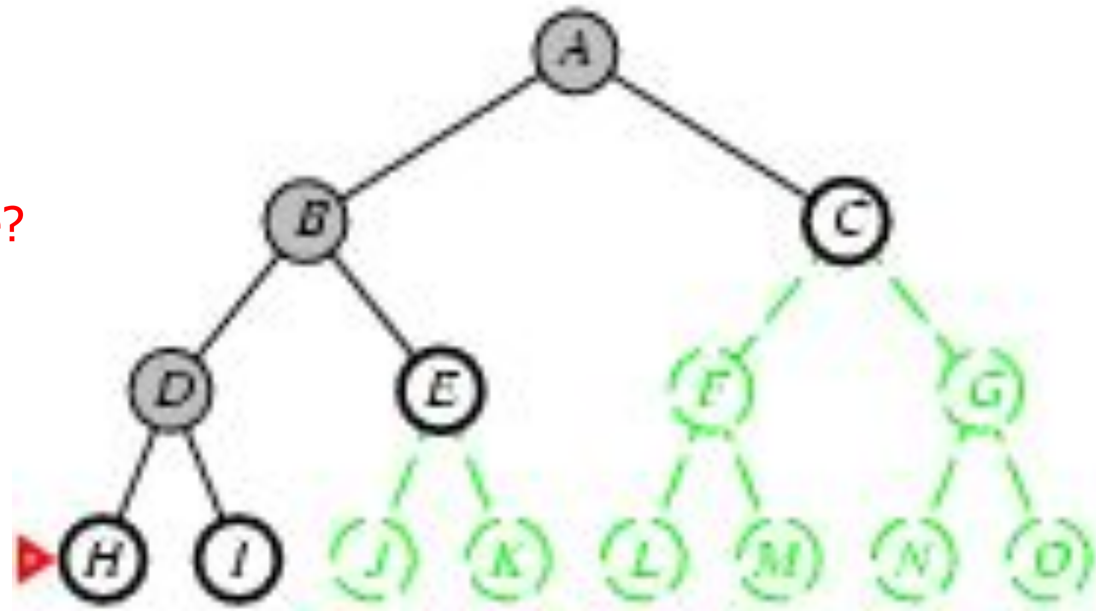


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[H,I,E,C]

Is H = goal state?

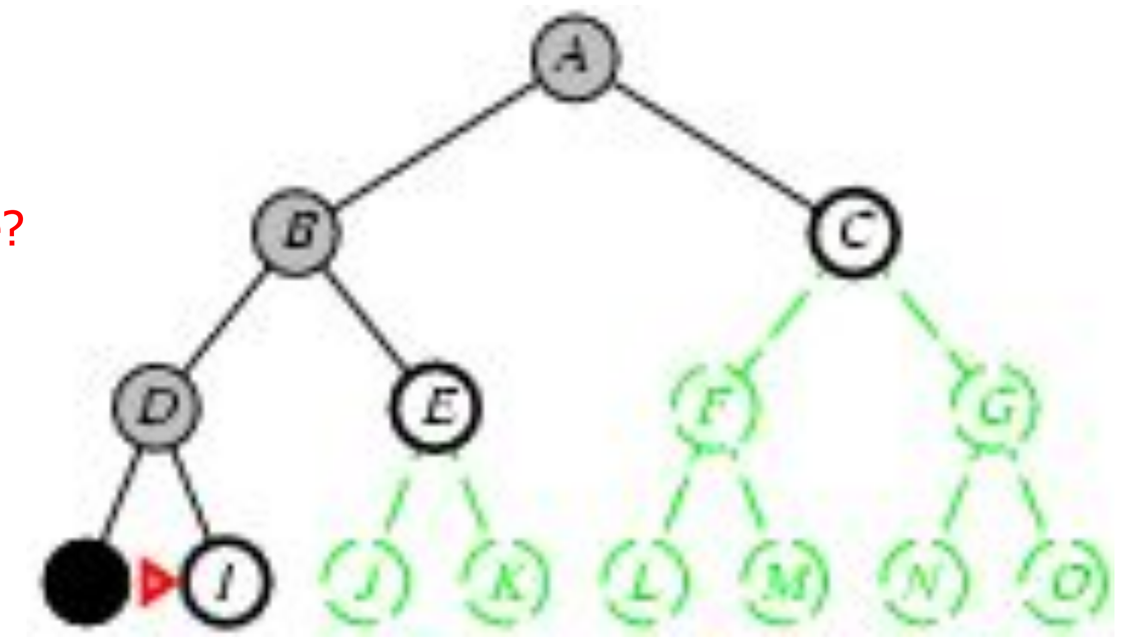


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[I,E,C]

Is I = goal state?

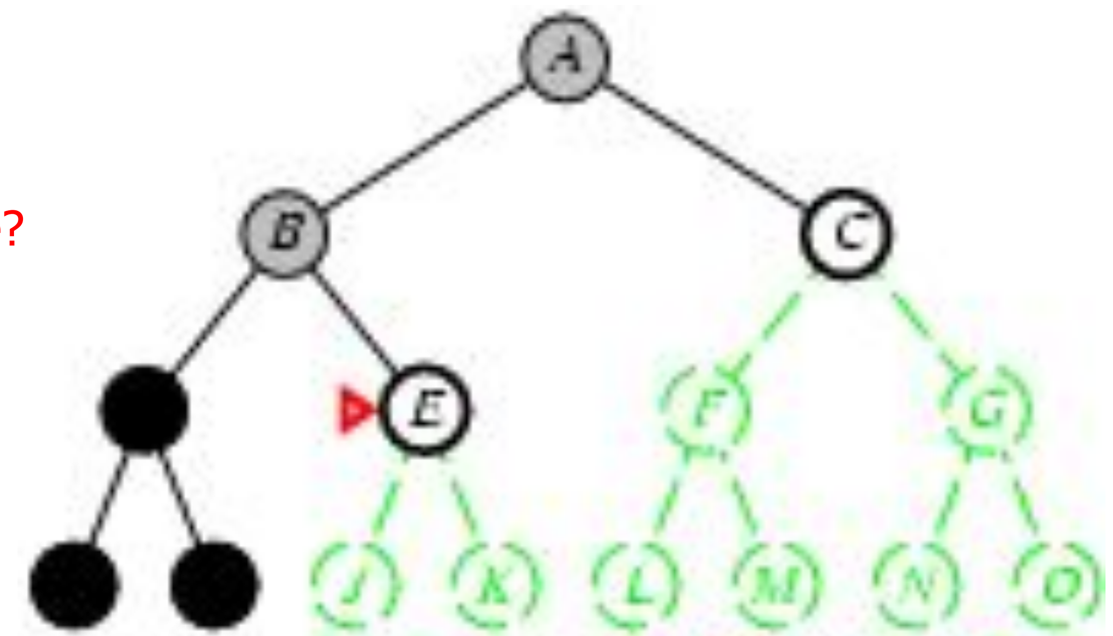


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[E,C]

Is E = goal state?

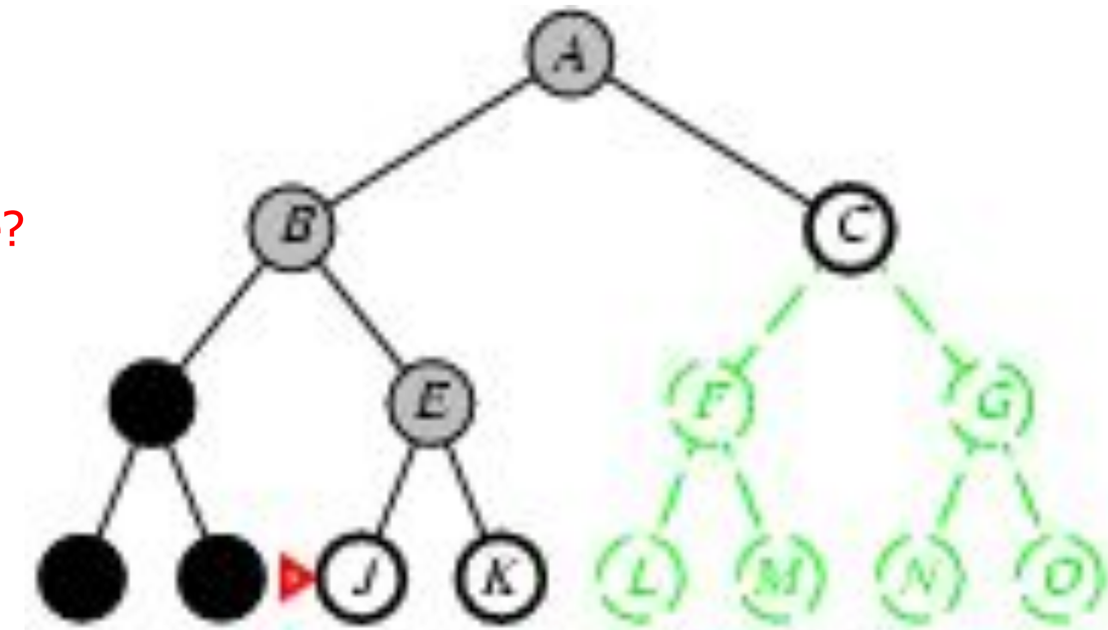


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[J,K,C]

Is J = goal state?

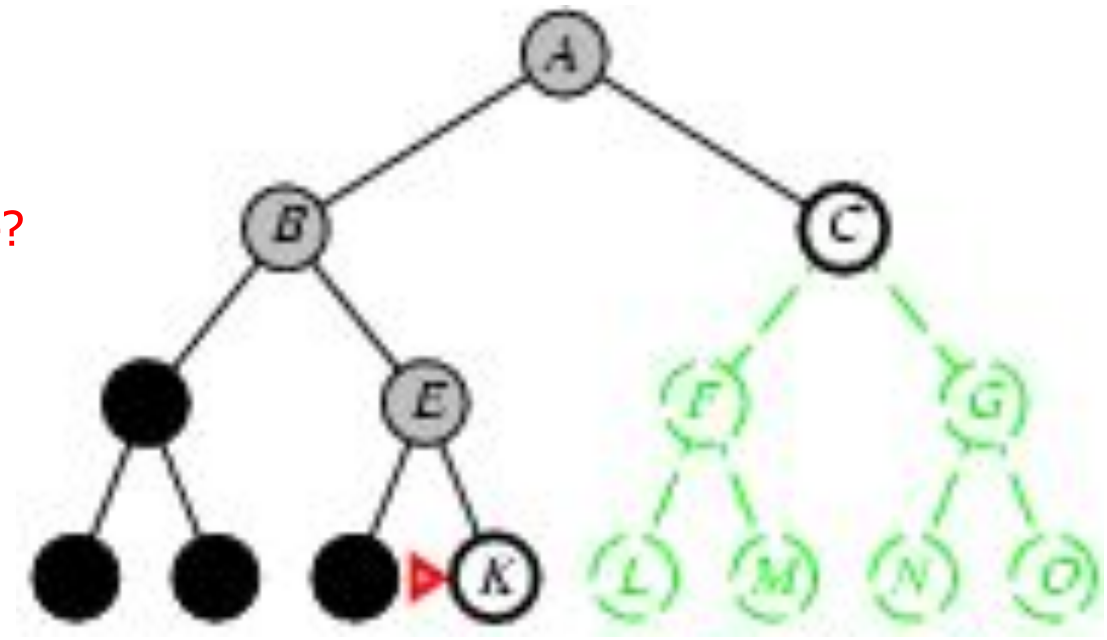


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[K,C]

Is K = goal state?

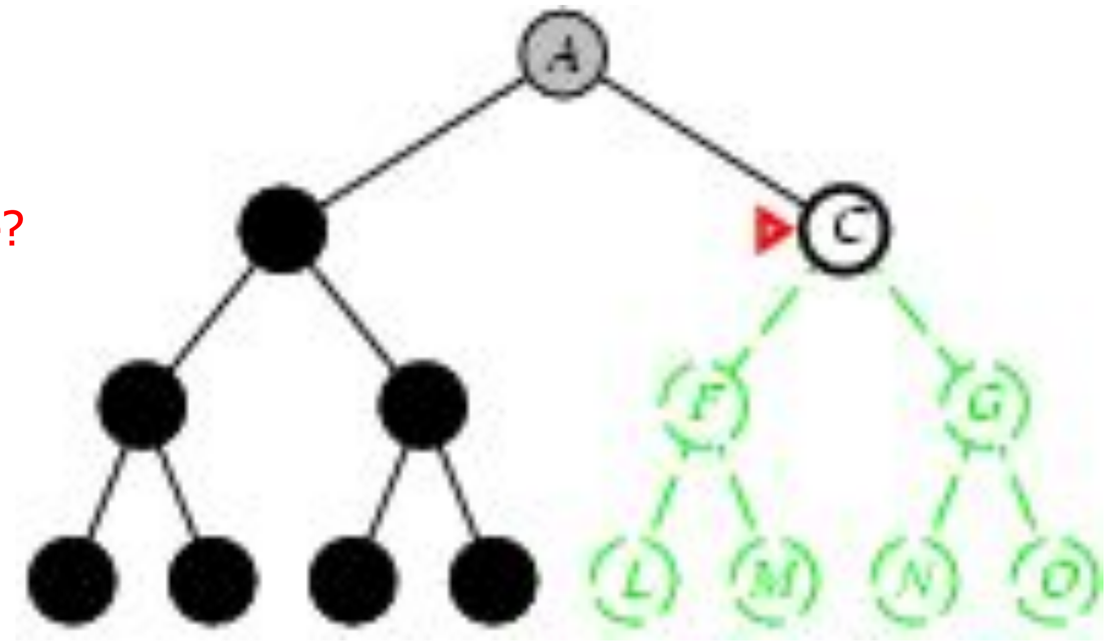


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[C]

Is C = goal state?

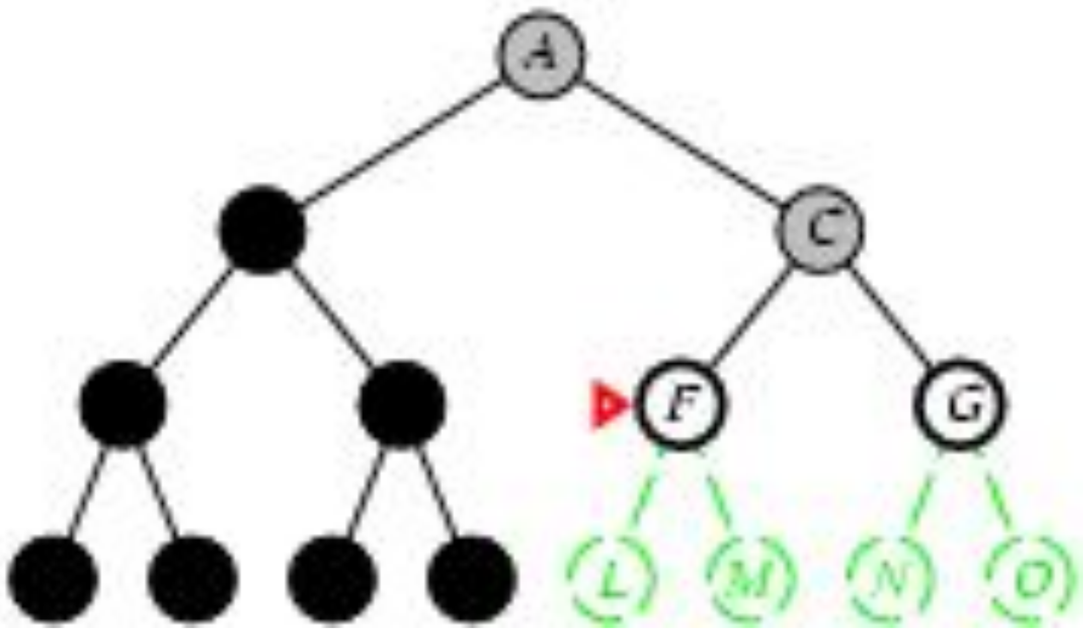


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[F,G]

Is F = goal state?

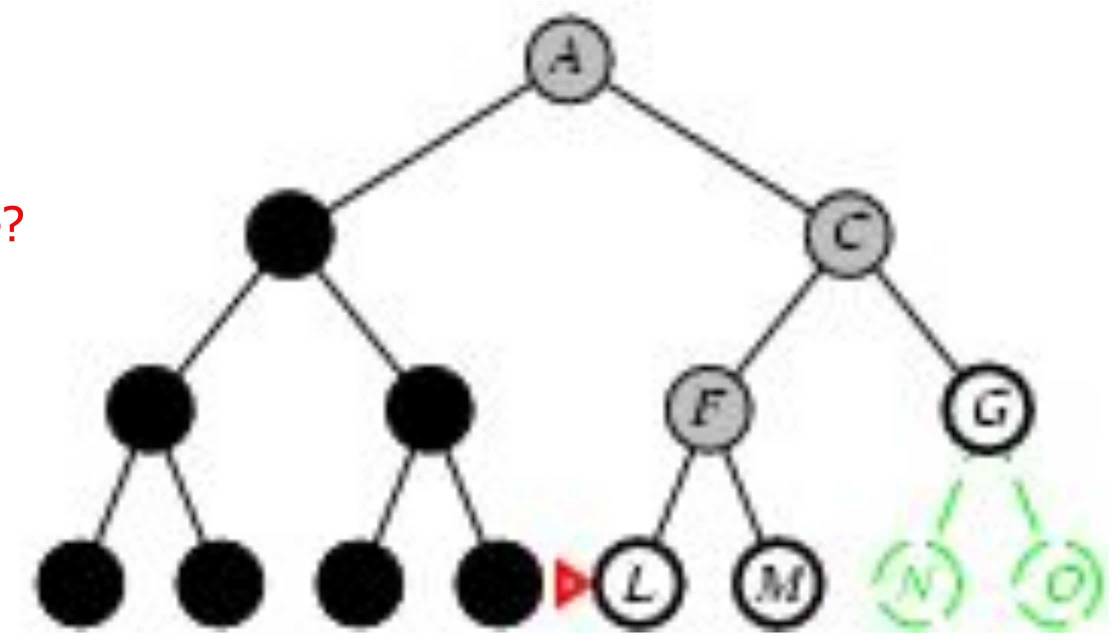


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[L,M,G]

Is L = goal state?

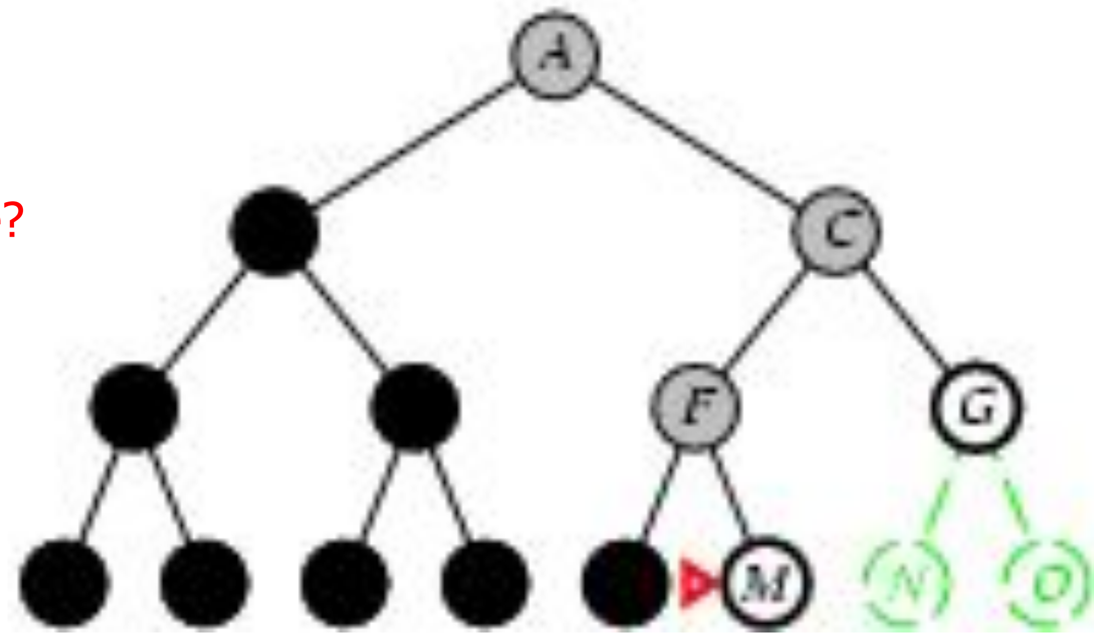


Depth-first search

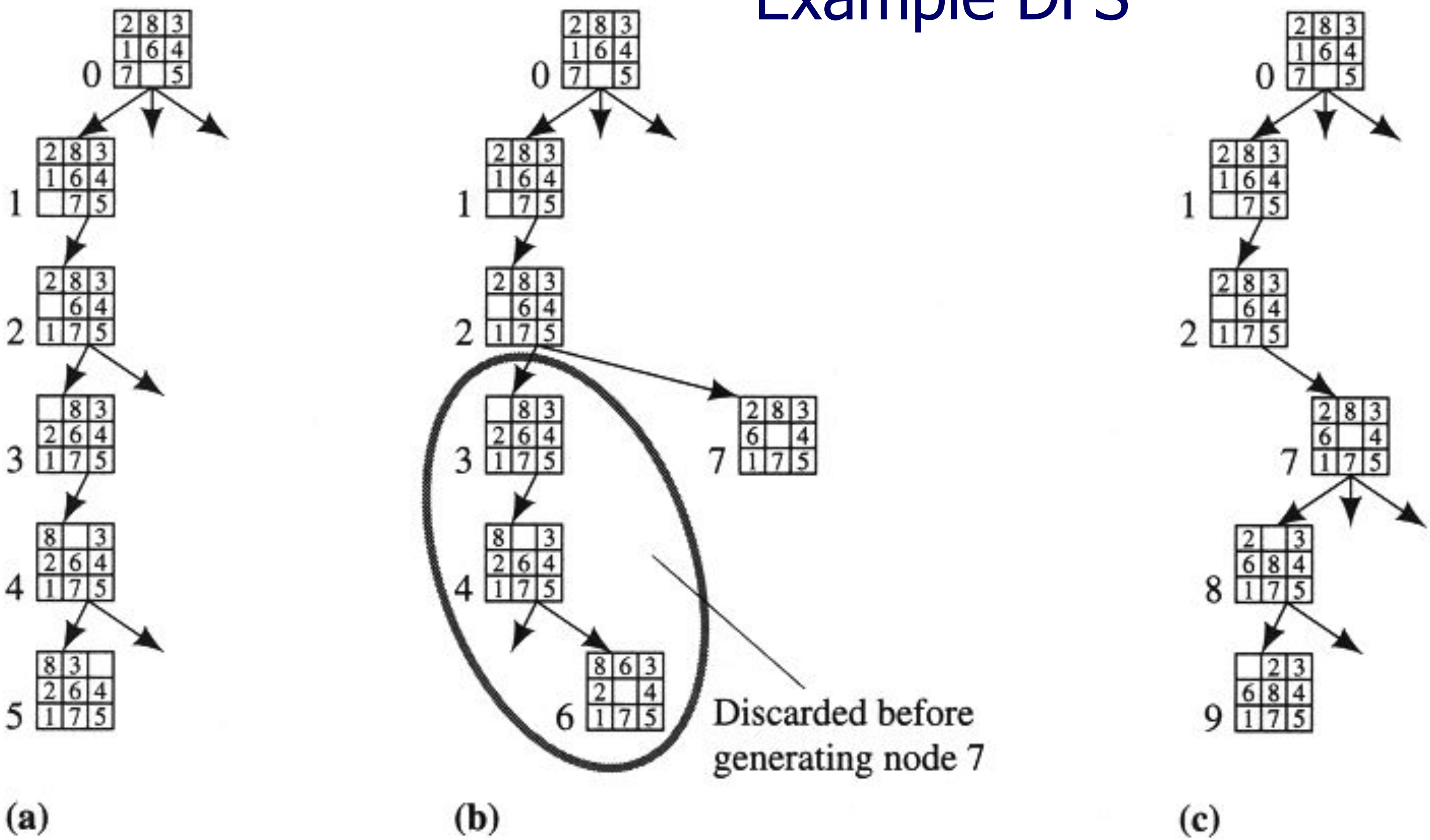
- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[M,G]

Is M = goal state?

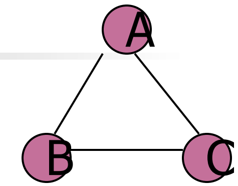


Example DFS



Generation of the First Few Nodes in a Depth-First Search

Properties of depth-first search



- Complete? No: fails in infinite-depth spaces

  Can modify to avoid repeated states along path
- Time? $O(b^m)$ with m=maximum depth
- terrible if *m* is much larger than *d*
  - but if solutions are dense, may be much faster than breadth-first
- Space? $O(bm)$, i.e., linear space! (we only need to remember a single path + expanded unexplored nodes)
- Optimal? No (It may find a non-optimal goal first)

Iterative deepening search

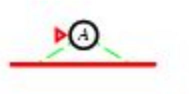
- To avoid the infinite depth problem of DFS, we can
  decide to only search until depth L, i.e. we don't expand beyond depth L.
  ☐ Depth-Limited Search

- What of solution is deeper than L? ☐ Increase L iteratively.
  ☐ Iterative Deepening Search

- As we shall see: this inherits the memory advantage of Depth-First
  search, and is better in terms of time complexity than Breadth first search.

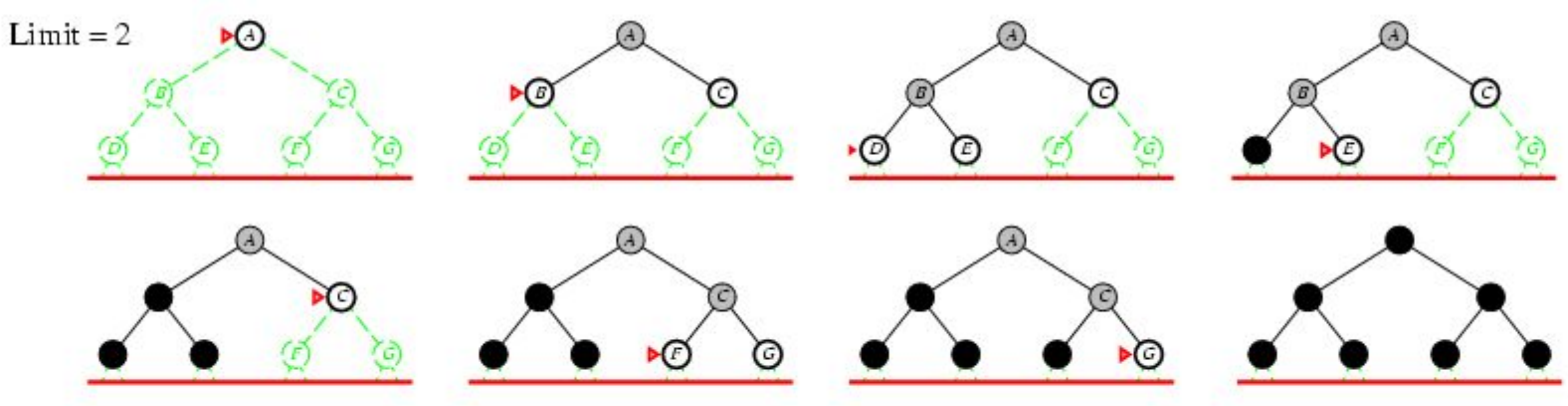
Iterative deepening search $L=0$

Limit = 0

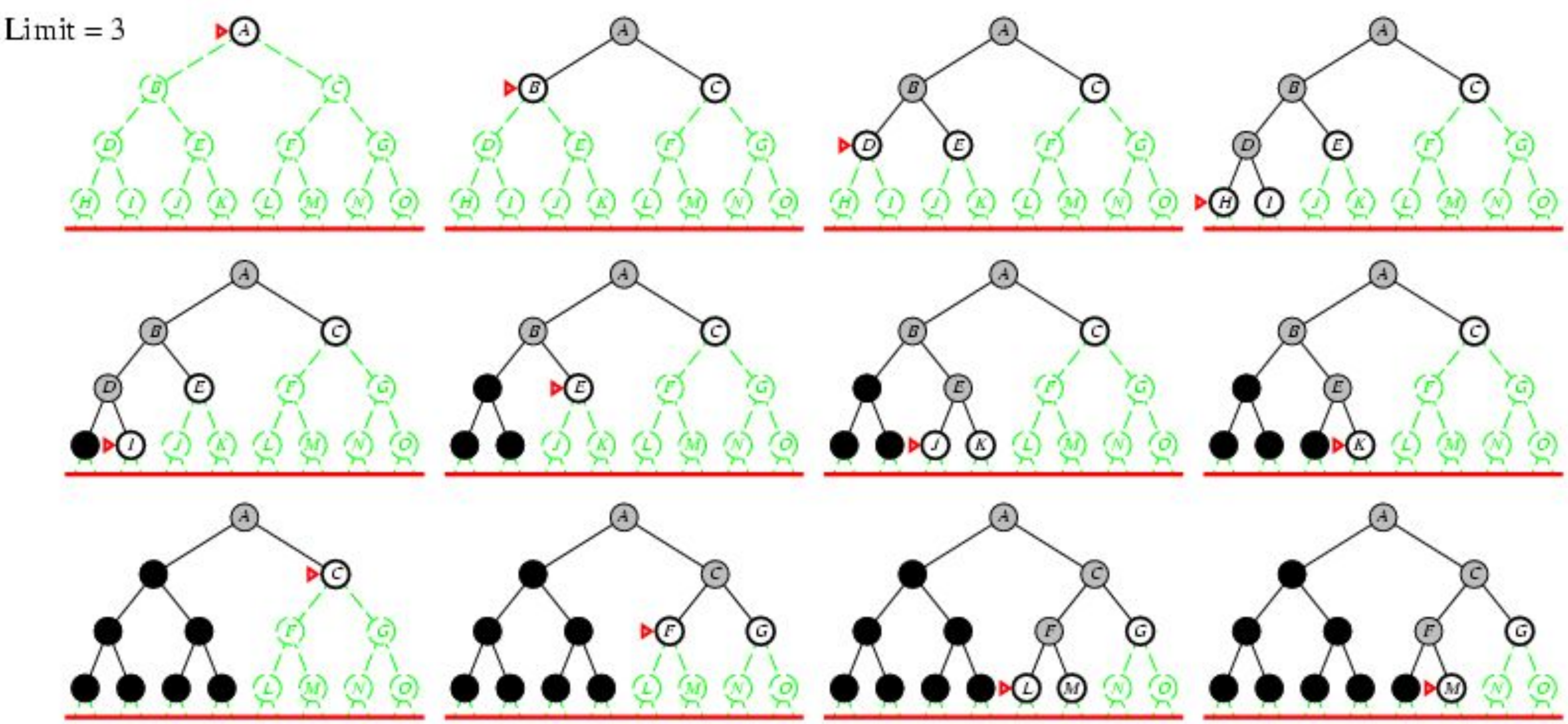


Iterative deepening search $L=1$

Iterative deepening search L=2



Iterative Deepening Search L=3



Iterative deepening search

- Number of nodes generated in a depth-limited search to depth *d* with branching factor *b*:

$$N_{DLS} = b^0 + b^1 + b^2 + \ldots + b^{d-2} + b^{d-1} + b^d$$

- Number of nodes generated in an iterative deepening search to depth *d* with branching factor *b*:

$$N_{IDS} = (d+1)b^0 + d\, b^1 + (d-1)b^2 + \ldots + 3b^{d-2} + 2b^{d-1} + 1b^d =$$

$$O(b^d) \neq O(b^{d+1})$$

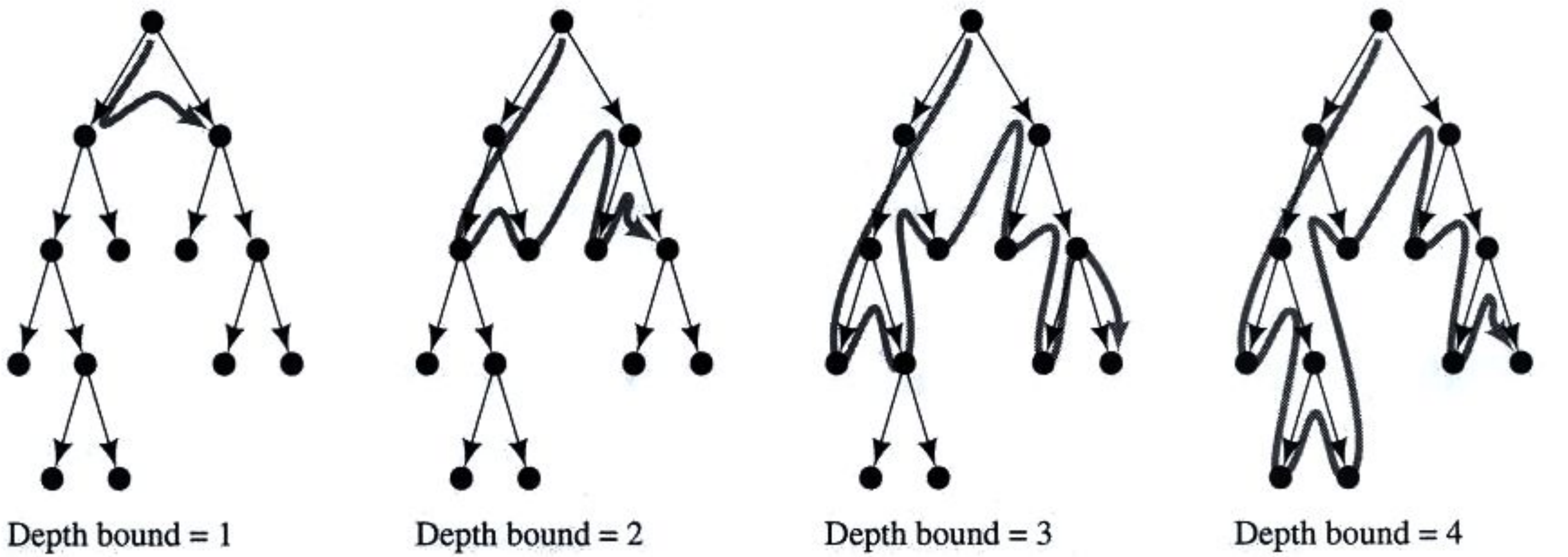
BFS

- For $b = 10$, $d = 5$,
  - $N_{DLS} = 1 + 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 = 111{,}111$
  - $N_{IDS} = 6 + 50 + 400 + 3{,}000 + 20{,}000 + 100{,}000 = 123{,}450$
  - N_{BFS} = ………………………………………………………………………… = 1,111,100

Properties of iterative deepening search

- Complete? Yes
- Time? $O(b^d)$
- Space? $O(bd)$
- Optimal? Yes, if step cost = 1 or increasing function of depth.

Example IDS



Stages in Iterative-Deepening Search

Bidirectional Search

- Idea
  - simultaneously search forward from S and backwards from G
  - stop when both "meet in the middle"
  - need to keep track of the intersection of 2 open sets of nodes
- What does searching backwards from G mean
  - need a way to specify the predecessors of G
    - this can be difficult,
    - e.g., predecessors of checkmate in chess?
  - which to take if there are multiple goal states?
  - where to start if there is only a goal test, no explicit list?

Bi-Directional Search

Complexity: time and space complexity are: $O(b^{d/2})$

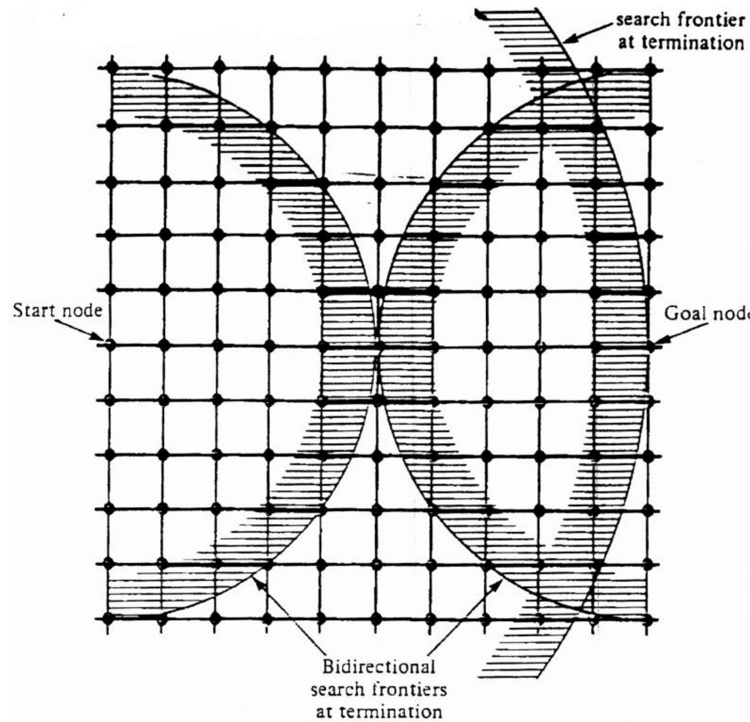


Fig. 2.10 Bidirectional and unidirectional breadth-first searches.

Summary of algorithms

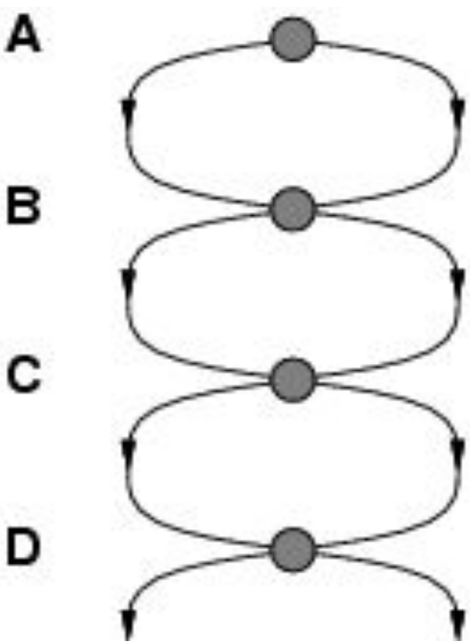
Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes

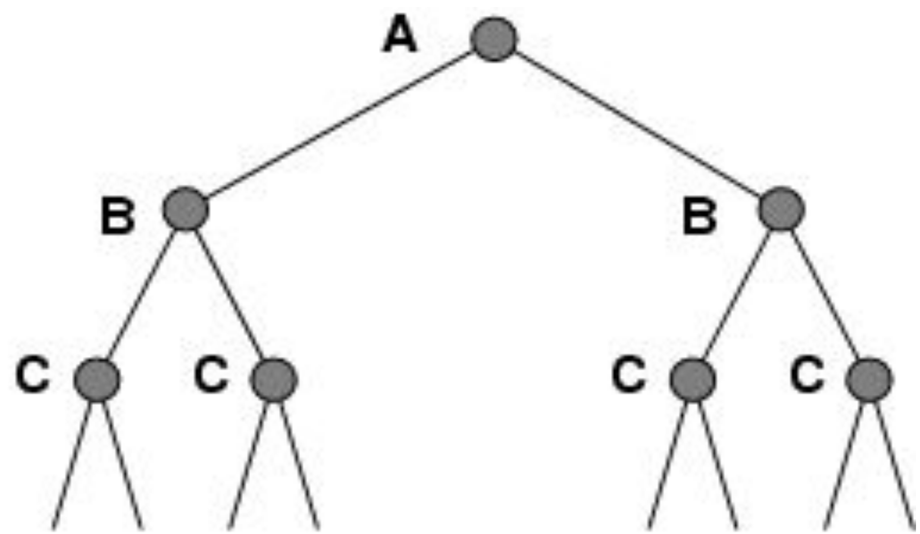
even complete
if step cost is not
increasing with depth.

preferred
uninformed
search strategy

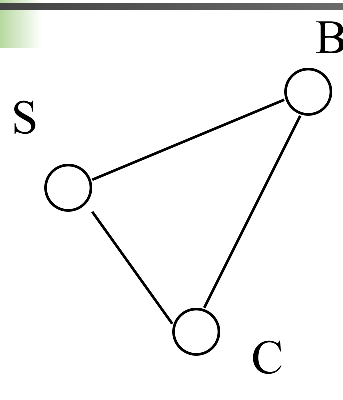
Repeated states

- Failure to detect repeated states can turn a linear problem into an exponential one!

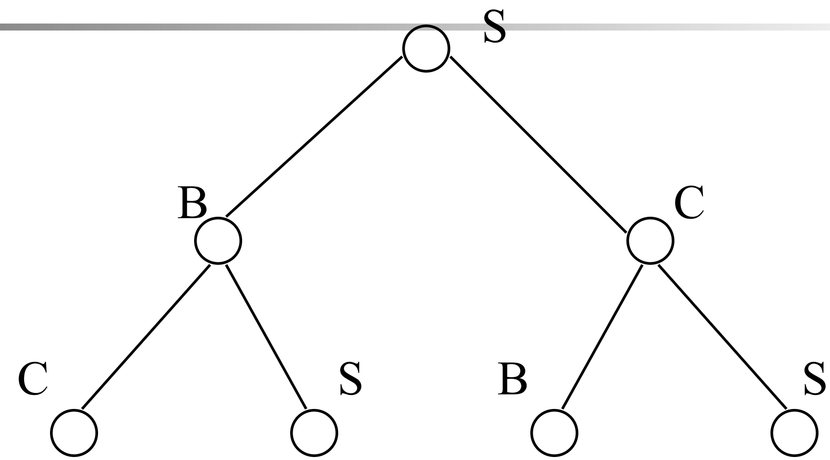




Solutions to Repeated States



State Space



Example of a Search Tree

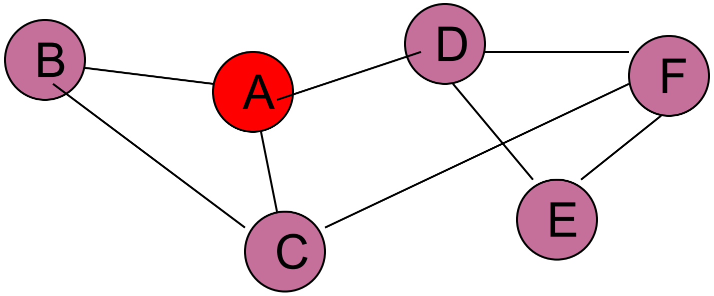
- Graph search ← optimal but memory inefficient
  - never generate a state generated before
    - must keep track of all possible states (uses a lot of memory)
    - e.g., 8-puzzle problem, we have 9! = 362,880 states
    - approximation for DFS/DLS: only avoid states in its (limited) memory: avoid looping paths.
      (this does not avoid the problem on the previous slide)
    - Graph search optimal for BFS and UCS, not for DFS.

Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored
- Variety of uninformed search strategies
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms

Exercise

2. Consider the graph below:



a) [2pt] Draw the first 3 levels of the full search tree with root node given by A.
b) [2pt] Give an order in which we visit nodes if we search the tree breadth first.
c) [2pt] Express time and space complexity for general breadth-first search in terms of the branching factor, b, and the depth of the goal state, d.
d) [2pt] If the step-cost for a search problem is *not* constant, is breadth first search always optimal? (Explain).