Parsing

Part II

Writing Grammars

When writing a grammar (or RE) for some language, the following must be true:

1. All strings generated are in the language.
2. Your grammar produces all strings in the language.

Example:

$S \rightarrow (S)\ S \mid \varepsilon$

Generates all strings of balanced parentheses

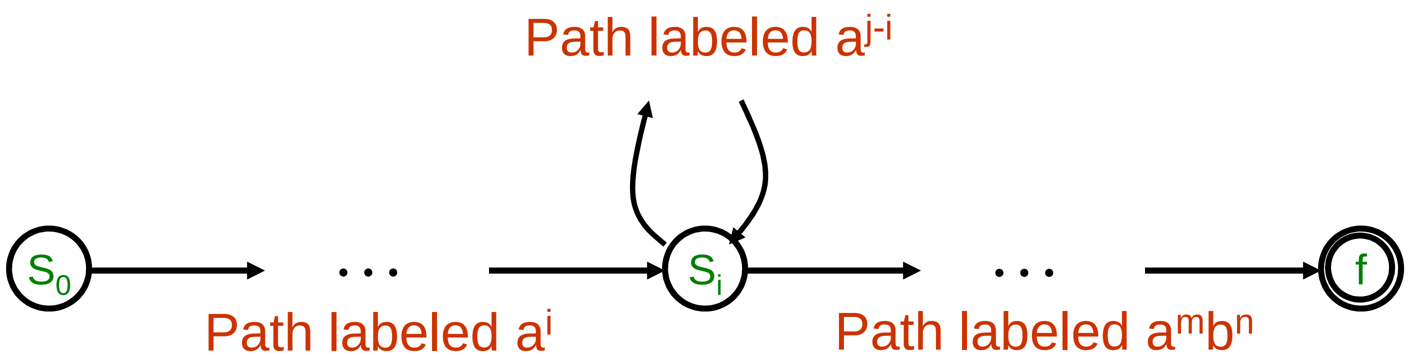
Using induction show that

Every sentence derivable from S is balanced

Every balanced string is derivable from S

- L={ $a^n b^n$ | $n \geq 1$}

- Show that L can be described by a grammar not by a regular expression
- Construct a DFA D with k states to accept L
- For $a^n b^n$ ($n>k$) some state (s_i) of D must be entered twice



$m+j = n$

$m+i \neq n$

Elimination of Ambiguity

Ambiguous Grammar

- A Grammar is ambiguous if there are multiple parse trees for the same sentence
- For the most parsers, the grammar must be unambiguous

Unambiguous grammar

unique selection of the parse tree for a sentence

Disambiguation

- Express Preference for one parse tree over others
  - Add disambiguating rule into the grammar

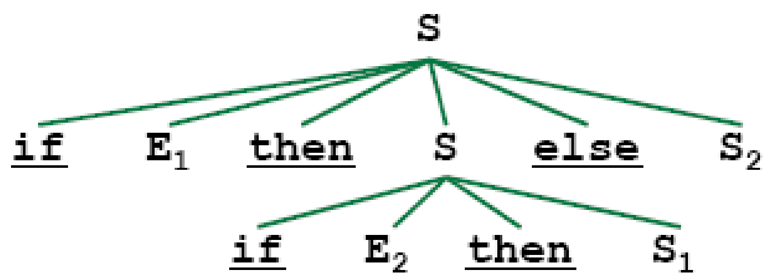
Dangling-else grammar

This grammar is ambiguous!

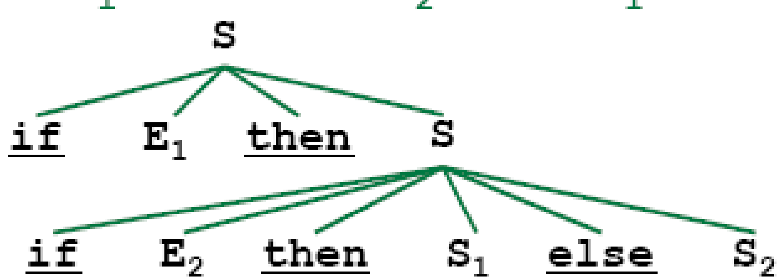
Stmt → **if** Expr **then** Stmt
→ **if** Expr **then** Stmt **else** Stmt
→ ...Other Stmt Forms...

Example String: `if` E_1 `then if` E_2 `then` S_1 `else` S_2

Interpretation #1: `if` E_1 `then (if` E_2 `then` S_1`) else` S_2



Interpretation #2: `if` E_1 `then (if` E_2 `then` S_1 `else` S_2`)`



Dangling-else grammar

Goal: "Match `else`-clause to the closest `if` without an `else`-clause already."

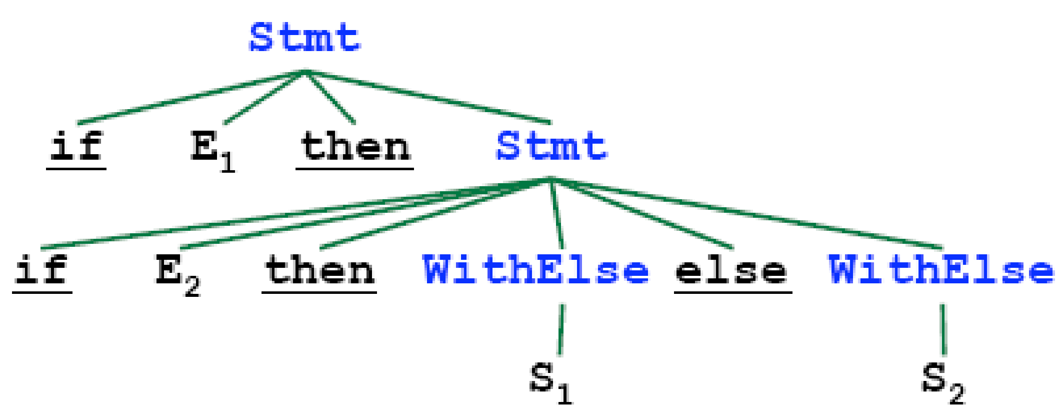
Solution:

Stmt → `if` Expr `then` Stmt
→ `if` Expr `then` WithElse `else` Stmt
→ ...Other Stmt Forms...

WithElse → `if` Expr `then` WithElse `else` WithElse
→ ...Other Stmt Forms...

Any Stmt occurring between `then` and `else` must have an `else`.
i.e., the Stmt must not end with "`then` Stmt".

Interpretation #2: `if` E_1 `then (if` E_2 `then` S_1 `else` S_2`)`



Dangling-else grammar

Goal: "Match `else`-clause to the closest `if` without an `else`-clause already."

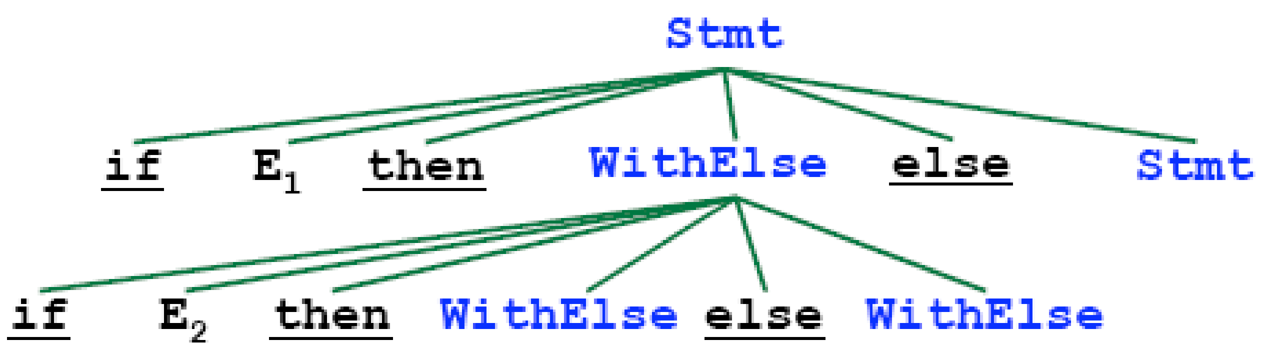
Solution:

Stmt	→	`if` Expr `then` Stmt
	→	`if` Expr `then` WithElse `else` Stmt
	→	...Other Stmt Forms...
WithElse	→	`if` Expr `then` WithElse `else` WithElse
	→	...Other Stmt Forms...

Any Stmt occurring between `then` and `else` must have an `else`.

i.e., the Stmt must not end with "`then` Stmt".

Interpretation #1: `if` E_1 `then` (`if` E_2 `then` S_1) `else` S_2



Left Recursion

Whenever

$$A \Rightarrow^{+} A\alpha$$

Simplest Case: Immediate Left Recursion

Given:

$$A \rightarrow A\alpha \mid \beta$$

Transform into:

$$A \rightarrow \beta A'$$

$$A' \rightarrow \alpha A' \mid \varepsilon$$

where A' is a new nonterminal

More General (but still immediate):

$$A \rightarrow A\alpha_1 \mid A\alpha_2 \mid A\alpha_3 \mid \ldots \mid \beta_1 \mid \beta_2 \mid \beta_3 \mid \ldots$$

Transform into:

$$A \rightarrow \beta_1 A' \mid \beta_2 A' \mid \beta_3 A' \mid \ldots$$

$$A' \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \alpha_3 A' \mid \ldots \mid \varepsilon$$

Immediate Left Recursion Elimination: example

- Grammar

  $E \rightarrow E + T \mid T$

  $T \rightarrow T * F \mid F$

  $F \rightarrow (E) \mid$ **id**

- **Left recursion Eliminated**

  $E \rightarrow T E'$

  $E' \rightarrow + T E' \mid \varepsilon$

  $T \rightarrow F T'$

  $T' \rightarrow * F T' \mid \varepsilon$

  $F \rightarrow (E) \mid$ **id**

Left Recursion in More Than One Step

Example:

$S \rightarrow A\underline{f} \mid \underline{b}$

$A \rightarrow A\underline{c} \mid S\underline{d} \mid \underline{e}$

Is A left recursive? Yes.

Is S left recursive? Yes, but not immediate left recursion. $S \Rightarrow A\underline{f} \Rightarrow S\underline{df}$

Approach:

Look at the rules for S only (ignoring other rules)... No left recursion.

Look at the rules for A...

Do any of A's rules start with S? Yes.

$A \rightarrow S\underline{d}$

Get rid of the S. Substitute in the righthand sides of S.

$A \rightarrow A\underline{fd} \mid \underline{bd}$

The modified grammar:

$S \rightarrow A\underline{f} \mid \underline{b}$

$A \rightarrow A\underline{c} \mid A\underline{fd} \mid \underline{bd} \mid \underline{e}$

Now eliminate immediate left recursion involving A.

$S \rightarrow A\underline{f} \mid \underline{b}$

$A \rightarrow \underline{bd}A' \mid \underline{e}A'$

$A' \rightarrow \underline{c}A' \mid \underline{fd}A' \mid \underline{\varepsilon}$

Left Recursion in More Than One Step

The Original Grammar:

$S \rightarrow A\underline{\mathbf{f}} \mid \underline{\mathbf{b}}$

$A \rightarrow A\underline{\mathbf{c}} \mid S\underline{\mathbf{d}} \mid B\underline{\mathbf{e}}$

$B \rightarrow A\underline{\mathbf{g}} \mid S\underline{\mathbf{h}} \mid \underline{\mathbf{k}}$

So Far:

$S \rightarrow A\underline{\mathbf{f}} \mid \underline{\mathbf{b}}$

$A \rightarrow \underline{\mathbf{bd}}A' \mid B\underline{\mathbf{e}}A'$

$A' \rightarrow \underline{\mathbf{c}}A' \mid \underline{\mathbf{fd}}A' \mid \varepsilon$

Left Recursion in More Than One Step

The Original Grammar:

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$A \rightarrow A\underline{c} \mid S\underline{d} \mid B\underline{e}$

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So Far:

$S \rightarrow A\underline{f} \mid \underline{b}$

$A \rightarrow \underline{bd}A' \mid B\underline{e}A'$

$A' \rightarrow \underline{c}A' \mid \underline{fd}A' \mid \varepsilon$

$B \rightarrow A\underline{g} \mid S\underline{h} \mid \underline{k}$

Look at the B rules next;
Does any righthand side
start with "S"?

Left Recursion in More Than One Step

The Original Grammar:

$$S \to Af \mid b$$
$$A \to Ac \mid Sd \mid Be$$
$$B \to Ag \mid Sh \mid k$$

So Far:

$$S \to Af \mid b$$
$$A \to bdA' \mid BeA'$$
$$A' \to cA' \mid fdA' \mid \varepsilon$$
$$B \to Ag \mid Afh \mid bh \mid k$$

Substitute, using the rules for "S"

$Af\ldots \mid b\ldots$

Left Recursion in More Than One Step

The Original Grammar:

S → A<u>f</u> | <u>b</u>
A → A<u>c</u> | S<u>d</u> | B<u>e</u>
B → A<u>g</u> | S<u>h</u> | <u>k</u>

So Far:

S → A<u>f</u> | <u>b</u>
A → <u>bd</u>A' | B<u>e</u>A'
A' → <u>c</u>A' | <u>fd</u>A' | ε
B → A<u>g</u> | A<u>fh</u> | <u>bh</u> | <u>k</u>

Does any righthand side start with "A"?

Left Recursion in More Than One Step

The Original Grammar:

$S \rightarrow Af \mid b$

$A \rightarrow Ac \mid Sd \mid Be$

$B \rightarrow Ag \mid Sh \mid k$

So Far:

$S \rightarrow Af \mid b$

$A \rightarrow bdA' \mid BeA'$

$A' \rightarrow cA' \mid fdA' \mid \varepsilon$

$B \rightarrow bdA'g \mid BeA'g \mid Afh \mid bh \mid k$

Substitute, using the rules for "A"

$bdA'\ldots \mid BeA'\ldots$

Left Recursion in More Than One Step

The Original Grammar:

S → Af | b

A → Ac | Sd | Be

B → Ag | Sh | k

So Far:

S → Af | b

A → bdA' | BeA'

A' → cA' | fdA' | ε

B → bdA'g | BeA'g | bdA'fh | BeA'fh | bh | k

Substitute, using the rules for "A"

bdA'... | BeA'...

Left Recursion in More Than One Step

The Original Grammar:

$S \rightarrow Af \mid b$

$A \rightarrow Ac \mid Sd \mid Be$

$B \rightarrow Ag \mid Sh \mid k$

So Far:

$S \rightarrow Af \mid b$

$A \rightarrow bdA' \mid BeA'$

$A' \rightarrow cA' \mid fdA' \mid \varepsilon$

$B \rightarrow bdA'g \mid BeA'g \mid bdA'fh \mid BeA'fh \mid bh \mid k$

Finally, eliminate any immediate Left recursion involving "B"

Next Form

$S \rightarrow Af \mid b$

$A \rightarrow bdA' \mid BeA'$

$A' \rightarrow cA' \mid fdA' \mid \varepsilon$

$B \rightarrow bdA'gB' \mid bdA'fhB' \mid bhB' \mid kB'$

$B' \rightarrow eA'gB' \mid eA'fhB' \mid \varepsilon$

Left Recursion in More Than One Step

The Original Grammar:

S → Af | b

A → Ac | Sd | Be | C

B → Ag | Sh | k

C → BkmA | AS | j

If there is another nonterminal, then do it next.

So Far:

S → Af | b

A → bdA' | BeA' | CA'

A' → cA' | fdA' | ε

B → bdA'gB' | bdA'fhB' | bhB' | kB' | CA'gB' | CA'fhB'

B' → eA'gB' | eA'fhB' | ε

Algorithm for Eliminating Left Recursion

Assume the nonterminals are ordered $A_1, A_2, A_3, \ldots$

(In the example: S, A, B)

for each nonterminal A_i (for i = 1 to N) do

  for each nonterminal A_j (for j = 1 to i-1) do

    Let $A_j \rightarrow \beta_1 \mid \beta_2 \mid \beta_3 \mid \ldots \mid \beta_N$ be all the rules for A_j

    if there is a rule of the form

      $A_i \rightarrow A_j\alpha$

    then replace it by

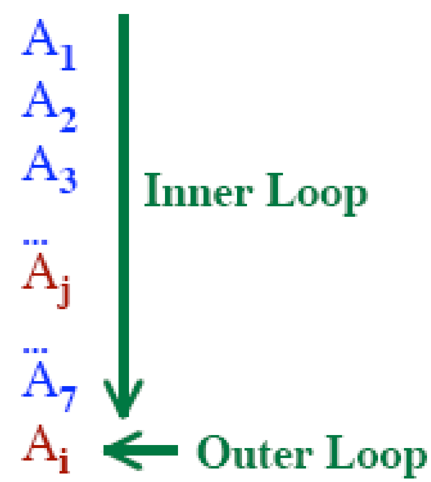
      $A_i \rightarrow \beta_1\alpha \mid \beta_2\alpha \mid \beta_3\alpha \mid \ldots \mid \beta_N\alpha$

    endIf

  endFor

  Eliminate immediate left recursion among the A_i rules

endFor



Left Factoring

Problem:

Stmt → `if` Expr `then` Stmt `else` Stmt

→ `if` Expr `then` Stmt

→ OtherStmt

With predictive parsing, we need to know which rule to use!
(While looking at just the next token)

Solution:

Stmt → `if` Expr `then` Stmt ElsePart

→ OtherStmt

ElsePart → `else` Stmt | ε

General Approach:

Before: $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \alpha\beta_3 \mid \ldots \mid \delta_1 \mid \delta_2 \mid \delta_3 \mid \ldots$

After: $A \rightarrow \alpha C \mid \delta_1 \mid \delta_2 \mid \delta_3 \mid \ldots$

$C \rightarrow \beta_1 \mid \beta_2 \mid \beta_3 \mid \ldots$

Left Factoring

Problem:

$$\underbrace{\text{Stmt}}_{A} \rightarrow \underbrace{\underline{\texttt{if}}\ \text{Expr}\ \underline{\texttt{then}}\ \text{Stmt}}_{\alpha}\ \underbrace{\underline{\texttt{else}}\ \text{Stmt}}_{\beta_1}$$

$$\rightarrow \underbrace{\underline{\texttt{if}}\ \text{Expr}\ \underline{\texttt{then}}\ \text{Stmt}}_{\alpha}\ \underbrace{\varepsilon}_{\beta_2}$$

$$\rightarrow \underbrace{\text{OtherStmt}}_{\delta_1}$$

With predictive parsing, we need to know which rule to use!
(While looking at just the next token)

Solution:

$$\underbrace{\text{Stmt}}_{A} \rightarrow \underbrace{\underline{\texttt{if}}\ \text{Expr}\ \underline{\texttt{then}}\ \text{Stmt}}_{\alpha}\ \underbrace{\text{ElsePart}}_{C}$$

$$\rightarrow \underbrace{\text{OtherStmt}}_{\delta_1}$$

$$\underbrace{\text{ElsePart}}_{C} \rightarrow \underbrace{\underline{\texttt{else}}\ \text{Stmt}}_{\beta_1} \mid \underbrace{\varepsilon}_{\beta_2}$$

General Approach:

Before: $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \alpha\beta_3 \mid \ldots \mid \delta_1 \mid \delta_2 \mid \delta_3 \mid \ldots$

After: $A \rightarrow \alpha C \mid \delta_1 \mid \delta_2 \mid \delta_3 \mid \ldots$

$C \rightarrow \beta_1 \mid \beta_2 \mid \beta_3 \mid \ldots$