$\boxed{\text{Ex: 03}}$ Given that, $(-1-\sqrt{3})^{1/7} = \sqrt[7]{(-1-\sqrt{3})}$ Evaluate it.

~~different~~

Sol^n: ~~Gn~~ let,

$$(-1-\sqrt{3})^{1/7} = (x+iy)^{1/7}$$

$$= r^{1/7} \cdot e^{i\theta/7}$$

$$\therefore (-1-\sqrt{3})^{1/7} = 2^{1/7} \cdot e^{i\left(\frac{2\pi n + 4\pi/3}{3}\right)}$$

Here,

$x = -1$

$y = -\sqrt{3}$

$(-1) + i(-\sqrt{3})$

$x + iy$

3rd quadrant

NOW, $r = \sqrt{(-1)^2 + (-\sqrt{3})^2} = 2$

$$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right) = \tan^{-1}(\sqrt{3})$$

$$= \frac{\pi}{3} \; 160^\circ$$

$$= \pi + \frac{\pi}{3} = \frac{4\pi}{3} \text{ (Real Argument)}$$

$\therefore$ Ideal Argument $= 2\pi n + \frac{4\pi}{3}$ Here, $n = 0, 1, 2, 3 \ldots$

$$\therefore (-1-\sqrt{3})^{1/7} = 2^{1/7} \cdot e^{i\left(\frac{2\pi n + 4\pi/3}{3}\right)}$$

Ans:

N.B:

$-1+(-4i)$

$-1-4i = x+iy$

$1-4i = x-iy$

$-1+4i = x+iy$

$1+4i = x+iy$

Assignment: 3

A.W: Evaluate followings:

01. Given, $1^{1/3} = (1 + i \times 0)^{1/3}$

Soln: let, $(1+0.i)^{1/3} = (x+iy)^{1/3}$

$= r^{1/3} \cdot e^{i\theta/3}$

$= 1^{1/3} \cdot e^{i\left(\frac{2\pi n}{3}\right)}$

Here,
$x = 1$
$y = 0$

1st quadrant ✓

Now, $r = \sqrt{1+0} = 1$

$\theta = \tan^{-1}(0/1) = \tan^{-1}(0) = 0^\circ$ (Real argument)

Ideal Argument, $\theta = 2\pi n + 0^\circ = 2\pi n$

$\therefore 1^{1/3} = 1^{1/3} \cdot e^{i\left(\frac{2\pi n}{3}\right)}$ Ans.

02. Given, $(-1)^{1/3} = (-1 + 0.i)^{1/3}$

Soln: let, $(-1+0.i)^{1/3} = (x+iy)^{1/3}$

$= r^{1/3} \cdot e^{i\theta/3}$

$= 1^{1/3} \cdot e^{i\left(\frac{2\pi n}{3}\right)}$

Here,
$x = -1$
$y = 0$

2nd quadrant ✓

$(-1) + 0.i$
$x + iy$ ✓

Now, $r = \sqrt{(-1)^2 + 0} = 1$

$\theta = \tan^{-1}(0/-1) = \tan^{-1}(0) = 0^\circ$ (Real Argument)
$= \pi - 0^\circ = \pi$

Ideal Argument, $\theta = 2\pi n + \pi = 2\pi n$

$\therefore (-1)^{1/3} = 1^{1/3} \cdot e^{i\left(\frac{2\pi n + \pi}{3}\right)}$

Here, $n = 0, 1, 3, 4 \ldots$

Ans.

[03.] $i^{1/3}$ (Given)

$= (0+i)^{1/3}$

Soln:

$(0+i)^{1/3} = (x+iy)^{1/3}$

$= r^{1/3} \cdot e^{i\theta/3}$

$\therefore i^{1/3} = 1^{1/3} \cdot e^{i\left(\frac{2\pi n + \pi/2}{3}\right)}$

Here,
$x = 0$
$y = 1$

1st quadrant

Now, $r = \sqrt{0+1} = 1$

$\theta = \tan^{-1}(1/0) = \cancel{\tan}\frac{\pi}{2} / 90^\circ$ ✓

Ideal Argument $= 2\pi n + \frac{\pi}{2}$

Here, $n = 0, 1, 2, 3, \ldots$

Ans:

[04.] Given, $(-i)^{1/3}$

$= (0-i)^{1/3}$

Soln:

$(0-i)^{1/3} = (x+iy)^{1/3}$

$= r^{1/3} \cdot e^{i\theta/3}$

$\therefore (-i)^{1/3} = 1^{1/3} \cdot e^{i\left(\frac{2\pi n + 5\pi/2}{3}\right)}$

Here,
$x = 0$
$y = -1$

4th quadrant

Now, $r = \sqrt{0+(-1)^2} = 1$

$\theta = \tan^{-1}\left(-\frac{1}{0}\right) = 90^\circ / \frac{\pi}{2} = 2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$

Ideal argument $= 2\pi n + \frac{5\pi}{2}$

Here, $n = 0, 1, 2, 3 \ldots$

Ans:

$\boxed{05.}$ Given, $(\sqrt{3}+i\sqrt{3})^{1/3}$

Soln: let,

$$(\sqrt{3}+i\sqrt{3})^{1/3} = (x+iy)^{1/3}$$

$$= r^{1/3} \cdot e^{i\theta/3}$$

$$\therefore (\sqrt{3}+i\sqrt{3})^{1/3} = \sqrt{6}^{1/3} \cdot e^{i\left(\frac{2\pi n + \pi/4}{3}\right)}$$

Here,
$x = \sqrt{3}$
$y = \sqrt{3}$

1st quadrant. ✓

Now, $r = \sqrt{(\sqrt{3})^2 + (\sqrt{3})^2} = \sqrt{3+3} = \sqrt{6}$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \tan^{-1}(1) = \frac{\pi}{4} / 45^\circ$ (Real Argument)

Ideal Argument $= 2\pi n + \frac{\pi}{4}$ Here, $n = 0, 1, 2, 3 \cdots$

Ans:

$\boxed{06.}$ Given, $(1-i\sqrt{2})^{1/3}$

Soln:

$$(1-i\sqrt{2})^{1/3} = (x+iy)^{1/3}$$

$$= r^{1/3} \cdot e^{i\theta/3}$$

$$\therefore (1-i\sqrt{2})^{1/3} = \sqrt{3}^{1/3} \cdot e^{i\left(\frac{2\pi n + 305.3^\circ}{3}\right)}$$

Here,
$x = 1$
$y = -\sqrt{3}$

4th quadrant. ✓

Now, $r = \sqrt{(1)^2 + (-\sqrt{2})^2} = \sqrt{3}$

$\theta = \tan^{-1}\left(\frac{-\sqrt{2}}{1}\right) = \tan^{-1}(-\sqrt{2}) = -54.7^\circ$

$= 360^\circ - 54.7^\circ$

$= 305.3^\circ$

Ideal Argument, $\theta = 2\pi n + 305.3^\circ$ Here, $n = 0, 1, 2, 3, \cdots$

Ans:

07. Given, $(6i)^{1/3}$
$= (0+6i)^{1/3}$

Soln: let,
$(0+6i)^{1/3} = (x+iy)^{1/3}$
$= r^{1/3} \cdot e^{i\theta/3}$
$\therefore (6i)^{1/3} = 6^{1/3} \cdot e^{i\left(\frac{2\pi n + \pi/2}{3}\right)}$

Here,
$x = 0$
$y = 6$
1st quadrant.

Now,
$r = \sqrt{0+36} = 6$
$\theta = \tan^{-1}\left(\frac{6}{0}\right) = \frac{\pi}{2} / 90^\circ$

$\therefore$ Ideal Angument, $\theta = 2\pi n + \frac{\pi}{2}$ Here, $n = 0, 1, 2, 3, \ldots$

Ans:

08. Given, $(2i)^{1/3} = (0+2i)^{1/3}$

Soln: let,
$(0+2i)^{1/3} = (x+iy)^{1/3}$
$= r^{1/3} \cdot e^{i\theta/3}$
$\therefore (2i)^{1/3} = 2^{1/3} \cdot e^{i\left(\frac{2\pi n + \pi/2}{3}\right)}$

Here,
$x = 0$
$y = 2$
1st quadrant.

Now, $r = \sqrt{0+4} = 2$
$\theta = \tan^{-1}\left(\frac{2}{0}\right) = \frac{\pi}{2} / 90^\circ$

$\therefore$ Ideal Angument, $\theta = 2\pi n + \frac{\pi}{2}$ Here, $n = 0, 1, 2, 3 \ldots$

Ans:

09. Given, $(-2i)^{1/4} = (0-2i)^{1/4}$

Soln:

$$(0-2i)^{1/4} = (x-iy)^{1/4}$$

$$= r^{1/4} e^{-i\theta/4}$$

$$\therefore (-2i)^{1/4} = 2^{1/4} e^{-i\left(\frac{2\pi n + \pi/2}{4}\right)}$$

Here,
$x = 0$
$y = -2$

4th quadrant. ✓

Now, $r = \sqrt{0+(-2)^2} = 2$

$\theta = \tan^{-1}\left(\frac{-2}{0}\right) = \frac{\pi}{2} / 90^\circ$

Ideal Angument, $\theta = 2\pi n + \frac{\pi}{2}$

Here, $n = 0, 1, 2, 3 \ldots$

Ans:

10. Given, $(81i)^{1/17} = (0+81i)^{1/17}$

Soln:

$$(0+81i)^{1/17} = (x+iy)^{1/17}$$

$$= r^{\frac{1}{17}} \cdot e^{i\theta/17}$$

$$\therefore (81i)^{\frac{1}{17}} = 81^{1/17} \cdot e^{i\left(\frac{2\pi n + \pi/2}{17}\right)}$$

Here,
$x = 0$
$y = 81$

1st quadrant. ✓

Now, $r = \sqrt{0+(81)^2} = 81$

$\theta = \tan^{-1}\left(\frac{81}{0}\right) = \frac{\pi}{2} / 90^\circ$

Ideal Angument, $\theta = 2\pi n + \frac{\pi}{2}$

Here, $n = 0, 1, 2, 3 \ldots$

Ans:

11. Given, $(64i)^{1/9} = (0+64i)^{1/9}$

Soln:

$$(0+64i)^{1/9} = (x+iy)^{1/9}$$

$$= r^{1/9} \cdot e^{i\theta/9}$$

$$= 64^{1/9} \cdot e^{i\left(\frac{2\pi n + \pi/2}{9}\right)}$$

Here,
$x = 0$
$y = 64$

1st quadrant. ✓

Now, $r = \sqrt{0+(64)^2} = 64$

$\theta = \tan^{-1}\left(\frac{64}{0}\right) = \frac{\pi}{2} / 90^\circ$

$\therefore$ Ideal Angument, $\theta = 2\pi n + \frac{\pi}{2}$

Here, $n = 0, 1, 2, 3 \ldots$

Ans:

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Lec-

Exercise

Type : 01

Express $\left(\frac{1+2i}{2+i}\right)^2$ in the form of $A+iB$. Also find its modulus and argument.

$A+iB$ → complex number ideal format

Soln: Given,

$$\left(\frac{1+2i}{2+i}\right)^2 = \frac{(1+2i)^2}{(2+i)^2}$$

$$= \frac{(1)^2+2.1.2i+(2i)^2}{(2)^2+2.2.i+(i)^2}$$

$$= \frac{1+4i-4}{4+4i-1}$$

$$= \frac{-3+4i}{3+4i}$$ → conjugate (অনুবন্ধী)

$$= \frac{-3+4i}{3+4i} \times \frac{3-4i}{3-4i}$$

$$= \frac{(-3+4i)\times(3-4i)}{(3)^2-(4i)^2}$$

$$= \frac{-9+12i+12i-16i^2}{9-16i^2}$$

$$= \frac{-9+24i+16}{9+16}$$

$$= \frac{7+24i}{25}$$

$$= \frac{7}{25}+i\frac{24}{25} \quad (A+iB)$$

Now, Modulas, $r = \sqrt{x^2+y^2} = \sqrt{\left(\frac{7}{25}\right)^2+\left(\frac{24}{25}\right)^2} = 1$

and Argument, $\theta = \tan^{-1}(y/x)$

$= \tan^{-1}\left(\frac{24}{25} \times \frac{25}{7}\right)$

$= \tan^{-1}\left(\frac{24}{7}\right) = 73.7^\circ$ (Real argument)

$\therefore$ Ideal argument, $\theta = 2\pi n + 73.7^\circ$ Ans:

Assignment: 4

(A.W)

Find two mudulas and argugment of the following:

$\boxed{i.}$ Given that, $\frac{-2}{1+i\sqrt{3}}$

$= \frac{-2(1-i\sqrt{3})}{(1+i\sqrt{3})(1-i\sqrt{3})}$

$= \frac{-2+i2\sqrt{3}}{(1)^2-(i\sqrt{3})^2}$

$= \frac{-2+i2\sqrt{3}}{1+3}$

$= -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ $\boxed{(-x)+iy}$ w

Now, ~~Argument~~, Modulas, $r = \sqrt{x^2+y^2} = \sqrt{(-1/2)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$

$= 1$

And Angument, $\theta = \tan^{-1}(y/x) = \tan^{-1}\{\{\frac{\sqrt{3}}{2} \times (-2)\}$

$= \tan^{-1}(-\sqrt{3})$

$= \frac{2\pi}{3}$

$= \pi - \frac{2\pi}{3}$ (Situated in 2nd quadrant)

$= \frac{\pi}{3}$ (Real argument)

$\therefore$ Ideal Angument, $\theta = 2\pi n + \frac{\pi}{3}$

Ans:

$\boxed{ii.}$ Given that,

$$\frac{1-i}{1+i}$$

$$= \frac{(1-i)(1-i)}{(1+i)(1-i)}$$ (Multiply with conjugate)

$$= \frac{1-i-i+i^2}{1-i^2}$$

$$= \frac{-2i}{2}$$

$$= -i \quad (0 - 1.i)$$

$x - iy$

Now, Modulas, $r = \sqrt{x^2+y^2} = \sqrt{0+(-1)^2} = 1$

And Angument, $\theta = \tan^{-1}(y/x) = \tan^{-1}(-1/0)$

$= \frac{\pi}{2}$

$= 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$ (Real argu)

$\therefore$ Ideal argument, $\theta = 2\pi n + \frac{3\pi}{2}$

32 Ans

iii. $\left(\frac{2+i}{3-i}\right)^2$ ← (Given that)

$$= \frac{(2+i)^2}{(3-i)^2}$$

$$= \frac{(2)^2+2\cdot2\cdot i+(i)^2}{(3)^2-2\cdot3\cdot i+(i)^2}$$

$$= \frac{4+4i-1}{9-6i-1}$$

$$= \frac{3+4i}{8-6i}$$

$$= \frac{(3+4i)(8+6i)}{(8-6i)(8+6i)}$$

$$= \frac{24+18i+32i+24i^2}{(8)^2-(6i)^2}$$

$$= \frac{24+50i-24}{64+36}$$

$$= \frac{50i}{100}$$

$$= \frac{1}{2}i \quad \left(0+i\frac{1}{2} = x+iy\right)$$

Now, Modulas, $r=\sqrt{x^2+y^2} = \sqrt{0+(1/2)^2} = \frac{1}{2}$

And Argument, $\theta = \tan^{-1}(y/x) = \tan^{-1}(1/2/0)$

$$= \frac{\pi}{2} / 90^\circ$$

$$= 2\pi + \frac{\pi}{2} = \frac{5\pi}{2} \text{ (Real argument)}$$

$\therefore$ Ideal argument, $\theta = 2\pi n + \frac{5\pi}{2}$ Ans:

Proofs

(1) # prove that $|z|^2 = |-z|^2 = |\bar{z}|^2 = |-\bar{z}|^2 = z\bar{z}$

Proof:

i) let, $z = x+iy$

$|z| = \sqrt{x^2+y^2}$

$|z|^2 = (\sqrt{x^2+y^2})^2$

$\therefore |z|^2 = x^2+y^2$

ii) $-z = -x-iy$

$\Rightarrow |-z| = |-(x+iy)|$

$\Rightarrow |-z| = |x+iy|$

$\Rightarrow |-z|^2 = (\sqrt{x^2+y^2})^2$

$\therefore |-z|^2 = x^2+y^2$

iii) $|\bar{z}|^2$

$\bar{z} = x-iy$

$|\bar{z}| = |x-iy|$

$= |x+i(-y)|$

$= \sqrt{(x)^2+(-y)^2}$

$|\bar{z}|^2 = (\sqrt{x^2+y^2})^2$

$\therefore |\bar{z}|^2 = x^2+y^2$

iv) $|-\bar{z}|^2$

$\bar{z} = x-iy$

$\Rightarrow -\bar{z} = -x+iy$

$\Rightarrow |-\bar{z}| = |(-x)+iy|$

$= \sqrt{(-x)^2+(y)^2}$

$\Rightarrow |-\bar{z}|^2 = (\sqrt{x^2+y^2})$

$\therefore |-z|^2 = x^2+y^2$

v) We know,

$z = x+iy$

$\bar{z} = x-iy$

$z\cdot\bar{z} = (x+iy)(x-iy)$

$\therefore z\cdot\bar{z} = x^2 - ixy + ixy - i^2y^2 = x^2+y^2$

$\therefore |z|^2 = |-z|^2 = |\bar{z}|^2 = |-z|^2 = z\bar{z}$

(proved)

② # Prove that $\overline{z_1+z_2} = \overline{z_1}+\overline{z_2}$

Proof: let, $z_1 = x_1+iy_1$

$z_2 = x_2+iy_2$

$\overline{z_1} = x_1-iy_1$

$\overline{z_2} = x_2-iy_2$

Now, $z_1+z_2 = (x_1+iy_1)+(x_2+iy_2)$

$= (x_1+x_2)+i(y_1+y_2)$

$= (\underline{x} - i\,\underline{y})$

$\therefore \overline{z_1+z_2} = (x_1+x_2)-i(y_1+y_2)$

$= (x_1-iy_1)+(x_2-iy_2)$

$= \overline{z_1}+\overline{z_2}$

$\therefore \overline{z_1+z_2} = \overline{z_1}+\overline{z_2}$

(proved)

③ # Prove that $\overline{\overline{z}} = z$

Proof: Let, $z = x+iy$

$\overline{z} = x-iy$

$\overline{\overline{z}} = x+iy = z$

$\therefore \overline{\overline{z}} = z$

(proved)

④ # Prove that $\overline{z_1+\overline{z_2}} = \overline{z_1}+z_2$

Proof:

let,

$z_1 = x_1+iy_1$

$z_2 = x_2+iy_2$

$\overline{z_1} = x_1-iy_1$

$\overline{z_2} = x_2-iy_2$

Now, $z_1+\overline{z_2} = (x_1+iy_1)+(x_2-iy_2)$

$= (x_1+x_2)+i(y_1-y_2)$

$\therefore \overline{z_1+\overline{z_2}} = (x_1+x_2)-i(y_1-y_2)$

Again,

$\overline{z_1+\overline{z_2}} = x_1-iy_1+x_2+iy_2$

$\boxed{\overline{z_1}+\overline{\overline{z_2}} \neq \overline{\overline{z_2}}=z}$

$= (x_1-iy_1)+(x_2+iy_2)$

$= \overline{z_1}+z_2$

$\therefore \overline{z_1+\overline{z_2}} = \overline{z_1}+z_2$

(proved)

Formulas:

① $|z|^2 = z\overline{z}$

② $\overline{\overline{z}} = z$

③ $\overline{z_1+z_2} = \overline{z_1}+\overline{z_2}$

④ $\overline{z_1+\overline{z_2}} = \overline{z_1}+z_2$

⑤ $\overline{z_1 z_2} = \overline{z_1}\cdot\overline{z_2}$

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① # Prove that $|z_1 z_2| = |z_1| \cdot |z_2|$

Proof: We know,

$$|z|^2 = z\bar{z}$$

let, $|z_1 \cdot z_2|^2 = (z_1 z_2) \cdot (\bar{z}_1 \cdot \bar{z}_2)$

$$= (z_1 \cdot \bar{z}_1) \cdot (z_2 \cdot \bar{z}_2)$$

$$= |z_1|^2 \cdot |z_2|^2$$

$$|z_1 \cdot z_2|^2 = (|z_1| \cdot |z_2|)^2$$

$$\therefore |z_1 z_2| = |z_1| \cdot |z_2|$$

For Question: prove that $|z_1 z_2 z_3 \ldots\ldots z_n| = |z_1||z_2||z_3| \ldots |z_n|$

proof will be same, just have to write: Similary it will be prove for that.

$\boxed{\text{N.B}}$

$z = x + iy$

$\bar{z} = x - iy$

$z + \bar{z} = x + iy + x - iy$

$= 2x$ → this 'x' is z's real part

$$\boxed{z + \bar{z} = 2Re(z)}$$

3* for exm (must)

② # Prove that $|z_1+z_2| \le |z_1|+|z_2|$

Proof:

We know, $|z|^2 = z\bar{z}$

$$|z_1+z_2|^2 = (z_1+z_2)(\overline{z_1+z_2})$$

$$= (z_1+z_2)(\bar{z}_1+\bar{z}_2)$$

$$= z_1\bar{z}_1 + z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_2$$

$$= |z_1|^2 + z_1\bar{z}_2 + z_2\bar{z}_1 + |z_2|^2$$

$\boxed{\bar{\bar{z}} = z}$

$$= |z_1|^2 + z_1\bar{z}_2 + \bar{\bar{z}}_2\,\bar{z}_1 + |z_2|^2$$

$\boxed{\bar{z}_1 \cdot \bar{z}_2 = \overline{z_1 \cdot z_2}}$

$$= |z_1|^2 + \underbrace{z_1\bar{z}_2}_{\text{Real } z} + \underbrace{\overline{z_1\bar{z}_2}}_{\text{conjugate } \bar{z}} + |z_2|^2$$

$$= |z_1|^2 + 2\,\mathrm{Real}(z_1\bar{z}_2) + |z_2|^2 \quad [\because z+\bar{z} = 2\mathrm{Re}(z)]$$

$$\Rightarrow |z_1+z_2|^2 \le |z_1|^2 + 2\cdot|z_1\bar{z}_2| + |z_2|^2$$

$2\mathrm{Re}(z)$

$9 = 5 + 3 + 1$

$= 5 + |z| + 1$

$= 5 + \sqrt{10} + 1$

$= 5 + 3.16 + 1$

$9.19 > 9$

$z = 3+i$

$\mathrm{Re}(z) = 3$

$|z| = \sqrt{3^2+1^2} = \sqrt{10}$

$z = x+iy$

$\mathrm{Re}(z) = x$

$|z| = \sqrt{x^2+y^2}$

$\boxed{\mathrm{Re}(z) \le |z|}$

$\Rightarrow |z_1+z_2|^2 \leq |z_1|^2 + 2.|z_1||\overline{z_2}| + |z_2|^2$

$\Rightarrow |z_1+z_2|^2 \leq |z_1|^2 + 2|z_1||z_2| + |z_2|^2$ $[\because |z| = |\overline{z}|]$

$3 = |-3| = 3$

$\Rightarrow |z_1+z_2|^2 \leq (|z_1|+|z_2|)^2$

$\therefore |z_1+z_2| \leq |z_1|+|z_2|$ (proved)

For Question: Prove that $|z_1+z_2+z_3+\cdots+z_n| \leq |z_1|+|z_2|+|z_3|+\cdots+|z_n|$

Similarly proved for that

3* for exm (must)

(3) # Prove that $|z_1-z_2| \geq |z_1|-|z_2|$

OR, Q: Prove that $|z_1-z_2-z_3-\cdots-z_n| \geq |z_1|-|z_2|-|z_3|-\cdots-|z_n|$

Side note:

$z_1 = -4$

$z_2 = -6$

$|-4-6| = 10$

$4-6 = -2$

Soln: We know,

$$|z|^2 = z\overline{z}$$

$$\therefore |z_1-z_2|^2 = (z_1-z_2)(\overline{z_1-z_2})$$

$$= (z_1-z_2)(\overline{z_1}-\overline{z_2})$$

$$= z_1\overline{z_1} - z_1\overline{z_2} - z_2\overline{z_1} + z_2\overline{z_2}$$

$$= |z_1|^2 - (z_1\overline{z_2} + z_2\overline{z_1}) + |z_2|^2$$

$$= |z_1|^2 - (z_1\overline{z_2} + \overline{\overline{z_2}}\,\overline{z_1}) + |z_2|^2$$

$$= |z_1|^2 - (z_1\overline{z_2} + \overline{z_1\overline{z_2}}) + |z_2|^2$$

$$= |z_1|^2 - 2Re(z_1-\overline{z_2}) + |z_2|^2$$

$$\Rightarrow |z_1-z_2|^2 = |z_1|^2 - 2Re(z_1\cdot\overline{z_2}) + |z_2|^2$$

$9 = 5 + \overset{2Re(z)}{3} + 1$

$$\Rightarrow |z_1-z_2|^2 \geq |z_1|^2 - 2|z_1\cdot\overline{z_2}| + |z_2|^2$$

$$\Rightarrow |z_1-z_2|^2 \geq |z_1|^2 - 2|z_1|.|z_2| + |z_2|^2$$

$$\Rightarrow |z_1-z_2|^2 \geq (|z_1|-|z_2|)^2$$

$\therefore |z_1-z_2| \geq |z_1|-|z_2|$ (proved)

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Shapes / Figures

Lec-06

Describe geometrically the region of the followings.

01.

$|z-4| > |z|$

Soln: let,

$z = x+iy$

then $|(x+iy)-4| > |x+iy|$

$\Rightarrow |(x-4)+iy| > |x+iy|$

$\Rightarrow \sqrt{(x-4)^2+y^2} > \sqrt{x^2+y^2}$

$\Rightarrow (x-4)^2+y^2 > x^2+y^2$

$\Rightarrow x^2-2.x.4+(4)^2+y^2 > x^2+y^2$

$\Rightarrow \underline{x^2}-8x+16+\underline{y^2} > \underline{x^2}+\underline{y^2}$ → considered as equal sign

$\Rightarrow -8x+16 > 0$

$\Rightarrow -8x > -16$

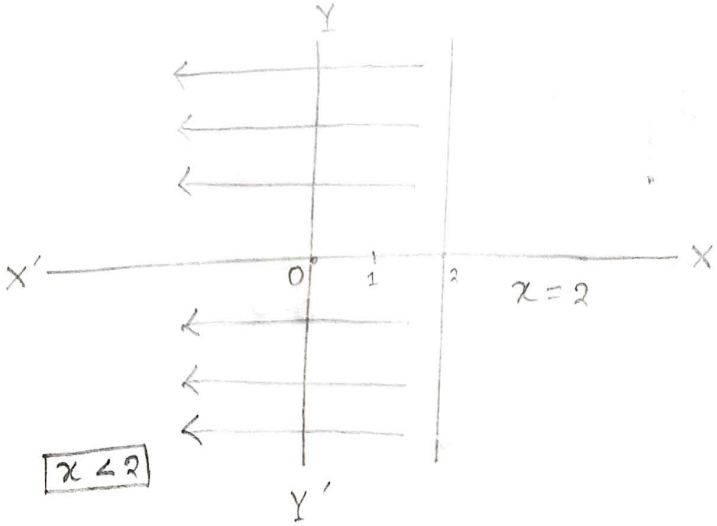
$\Rightarrow 8x < 16$

$\Rightarrow x < \frac{16}{8}$

$\therefore x < 2$

$x < 4$

$x > -4$ sign change



$\boxed{x < 2}$

$x < 2$ is the set of all point lie in left side of $x = 2$ except $x = 2$.

$\boxed{02.}$ Given, $|z - i| = |z + i|$

<u>Soln:</u> let, $z = x + iy$

then $|(x+iy) - i| = |(x+iy) + i|$

$\Rightarrow |x + (iy - i)| = |x + (iy + i)|$

$\Rightarrow \sqrt{x^2 + (iy - i)^2} = \sqrt{x^2 + (iy + i)^2}$

$\Rightarrow x^2 + i^2y^2 - 2 \cdot iy \cdot i + i^2 = x^2 + i^2y^2 + 2 \cdot iy \cdot i + i^2$

$\Rightarrow x^2 - y^2 + 2y - 1 = x^2 - y^2 - 2y - 1$

$\Rightarrow 2y + 2y = 0$

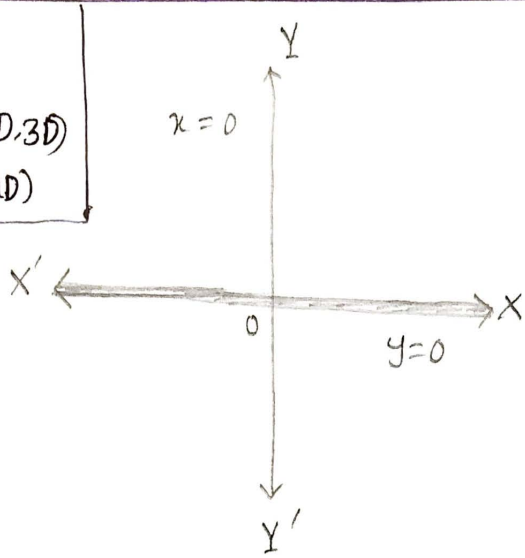
$\Rightarrow 4y = 0$

$\therefore y = 0$

N.B:

Axes → Ex: xy (2D, 3D)

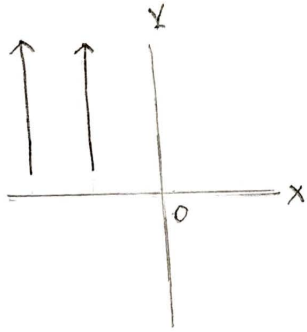
Axis → Ex: x (1D)



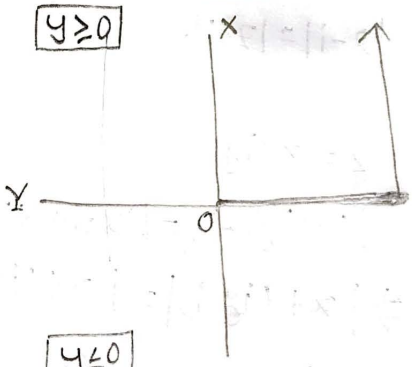
$y=0$ is the set of all points which lie on exactly x axis

Some example of figures w

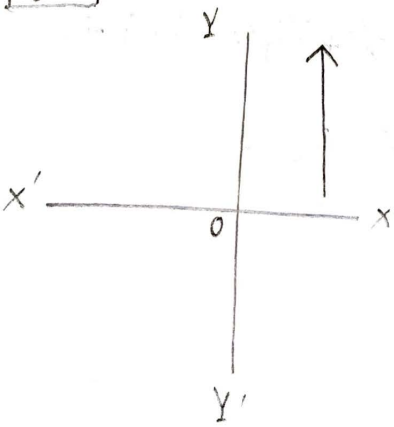
$y<0$



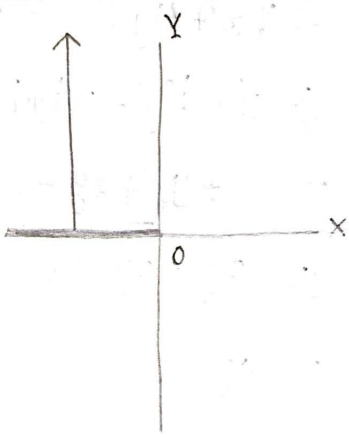
$y \geq 0$



$y>0$



$y \leq 0$



$\boxed{03.}$ # $Im(z) > 1$

Soln: let, $z = x + iy$

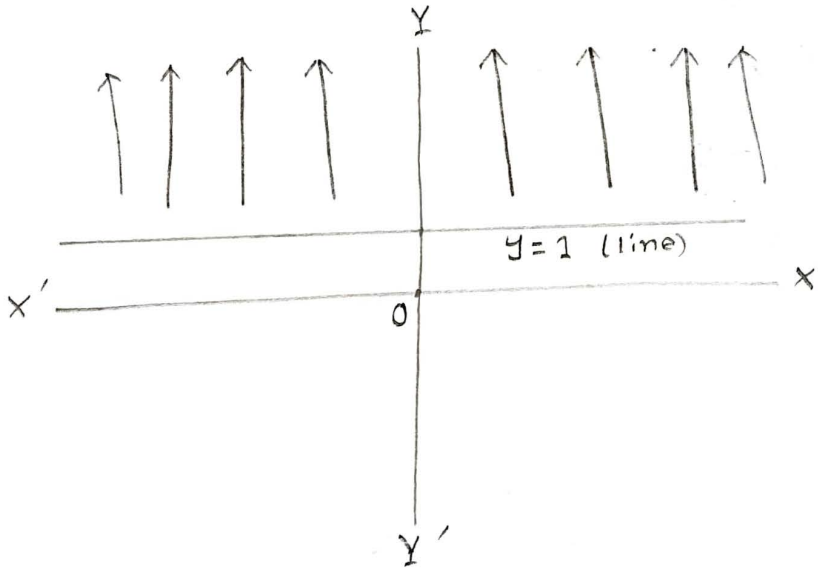
then $Im(x+iy) > 1$

$\therefore y > 1$

Notes:

$\begin{cases} x = a & \text{always parallel } y \text{ axis} \\ y = b & \text{" " } x \text{ axis} \end{cases}$

$\begin{cases} x = 0 \rightarrow y \text{ axis.} \\ y = 0 \rightarrow x \text{ "} \end{cases}$



$y > 1$ is the set of all points lie in above of $y = 1$ except $y = 1$.

A.W : 5

Given, $Re(\bar{z}-1)=3$

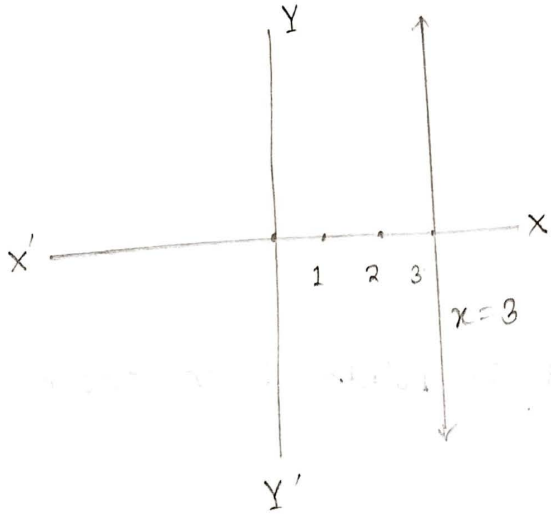
Soln: let,
$z = x+iy$
$\bar{z} = x-iy$

then $Re(x-iy-1)=3$

$\Rightarrow Re((x-1)-iy)=3$

$\Rightarrow x-1=3$

$\therefore x=3$



$x=3$ is the set of all points which lie on exactly o
$x=3$ and parallel to y axis.

Examples for circle

$\boxed{0.4}$ Given, $|z+3i| > 4$

Soln: $|(x+iy)+3i| > 4$

$\Rightarrow |x+i(y+3)| > 4$

$\Rightarrow \sqrt{x^2+(y+3)^2} > 4$

$\Rightarrow x^2+(y+3)^2 > 4^2 \longrightarrow (x-h)^2+(y-k)^2 = r^2$

$\Rightarrow x^2+y^2+2\cdot y\cdot 3+9 > 16$

$\Rightarrow x^2+y^2+6y+9-16 > 0$

$\Rightarrow x^2+y^2+6y-7 > 0$

which in form of $x^2+y^2+2gx+2fy+c=0$

$x^2+y^2+2\cdot(0)\cdot x+2\cdot(3)\cdot y+(-7) > 0$

Here, $2g=0$ $\therefore g=0$ $2f=6$ $\therefore f=3$ $c=-7$

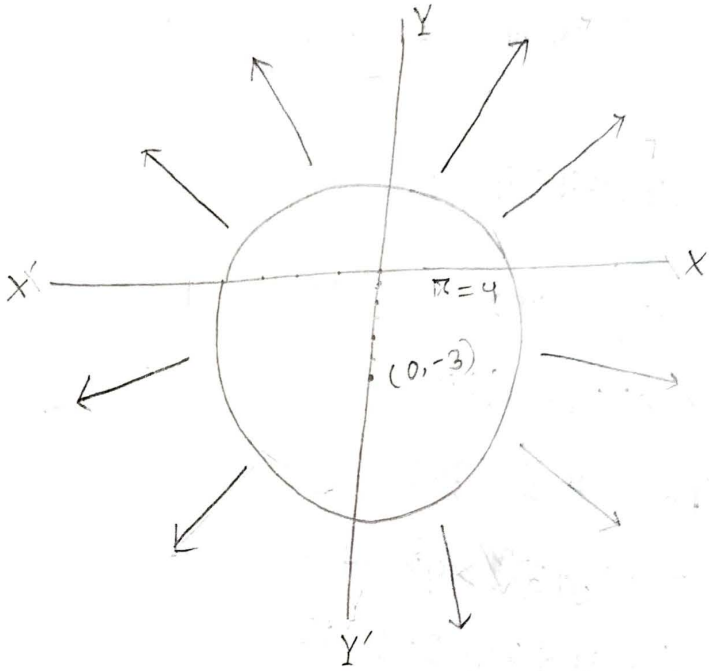
$\therefore$ center $=(-g,-f)=(0,-3)$

and Radius, $r=\sqrt{g^2+f^2-c}$

$=\sqrt{0+3^2+7}$

$=\sqrt{16}$

$=4$

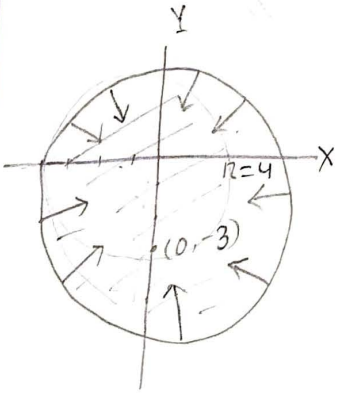


$x^2+(y+3)^2>4^2$

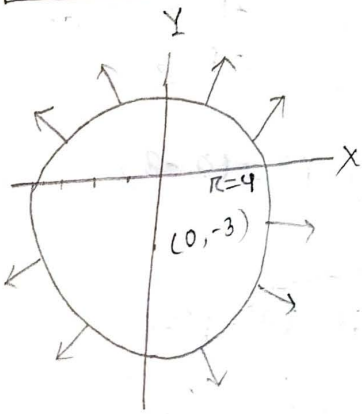
$(x-0)^2+(y+3)^2>4^2$

& $x^2+(y+3)^2>4^2$ in the set of all points lie on the exterion of the circle.

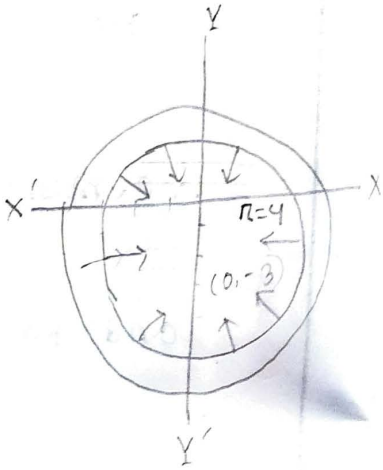
Some figures



equ $\leq r^2$



equ $\geq r^2$



equ $< r^2$

Formulas:

① $(x-h)^2+(y-k)^2=r^2$ center(h,k), radius r

② $x^2+y^2+2gx+2fy+c=0$ $(-g,-f)$, $\sqrt{g^2+f^2-c}$

$\boxed{0.5}$ Given,

$|z-2+i| \leq 1$

Soln: $|(x+iy)-2+i| \leq 1$ $[\because z=x+iy]$

$\Rightarrow |(x-2)+i(y+1)| \leq 1$

$\Rightarrow \sqrt{(x-2)^2+(y+1)^2} \leq 1$

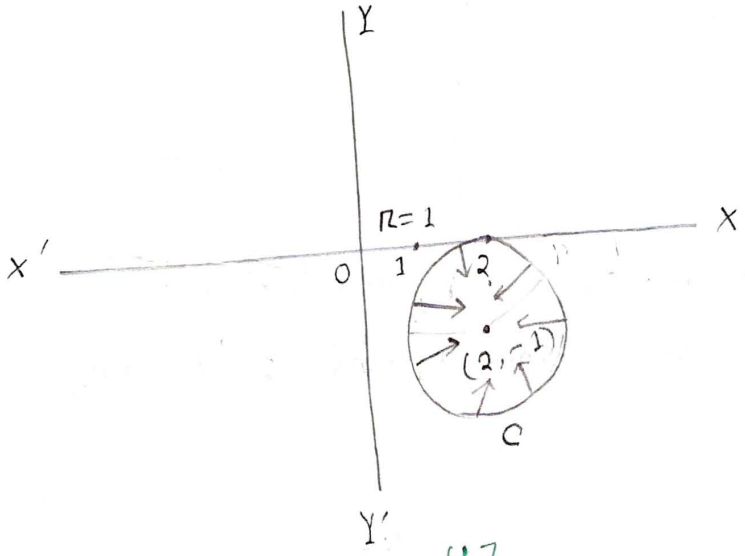
$\Rightarrow (x-2)^2+(y+1)^2 \leq 1$

$\Rightarrow (x-2)^2+\{y-(-1)\}^2 \leq 1$] comparing

$(x-h)^2+(y-k)^2=r^2$

Here, center $=(2,-1)$

radius, $r=1$



$(x-2)^2+(y+1)^2 \leq 1$ is the set of all points which lie on the surface and inner portion of the circle C.

2* for exm

$\boxed{0.6}$ Given,

$$1 < |z-2i| \leq 2$$

Soln:

$$1 < |z-2i|$$

$$\Rightarrow 1 < |x+iy-2i|$$

$$\Rightarrow 1 < |x+i(y-2)|$$

$$\Rightarrow 1 < \sqrt{x^2+(y-2)^2}$$

$$\Rightarrow 1 < x^2+(y-2)^2$$

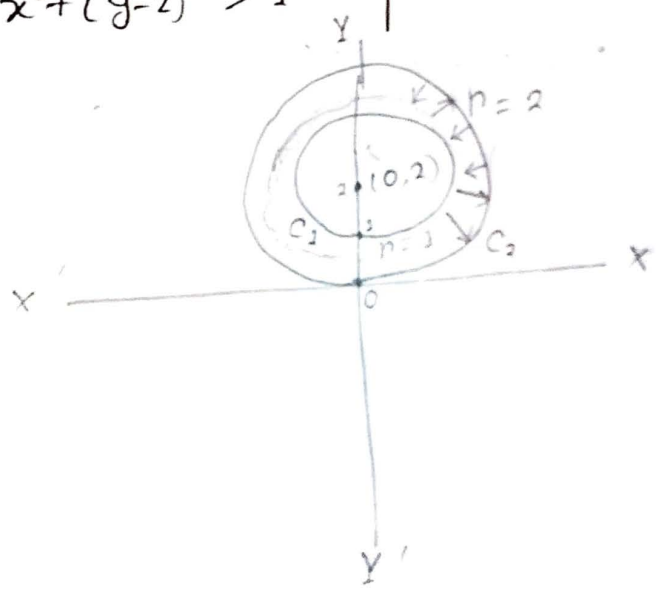
$$\Rightarrow x^2+(y-2)^2 > 1$$

$$|z-2i| \leq 2$$

$$\Rightarrow x^2+(y-2)^2 \leq 2^2$$

$$(x-h)^2+(y-k)^2 = r^2$$

center $= (0,2)$



This is the set of all points which is lie between the circle C_1 and C_2 including circle C_2.