Graphs and Tree

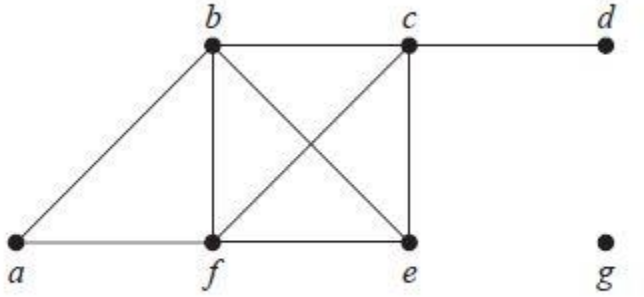
CSE 103

Cut vertex

- Sometimes the removal from a graph of a vertex and all incident edges produces a subgraph with more connected components. Such vertices are called **cut vertices**

degree

- A vertex of degree zero is called **isolated**. Vertex *g* in graph *G* in Example 1 is isolated.
- A vertex is **pendant** if and only if it has degree one. Vertex *d* in graph *G* in Example 1 is pendant.

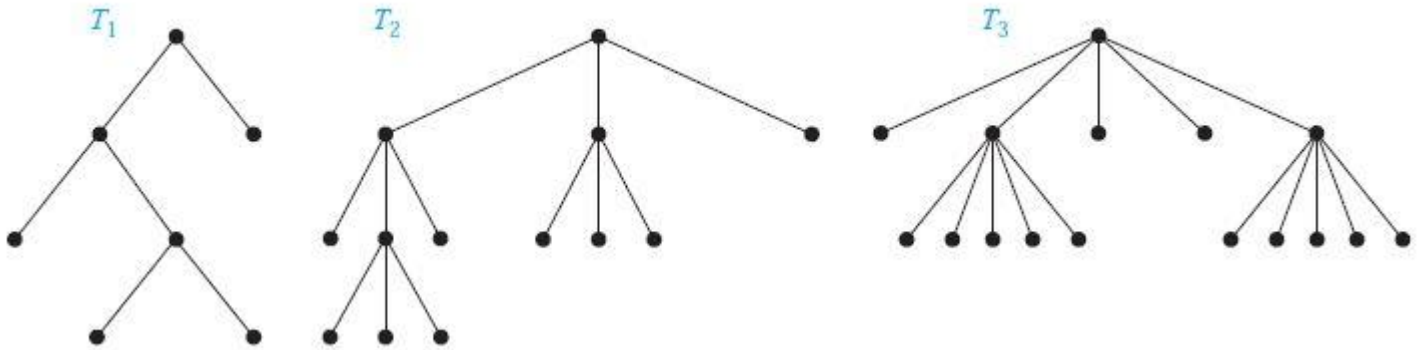


G

Tree

- **DEFINITION 1** A *tree* is a connected undirected graph with no simple circuits.
- **DEFINITION 2** A *rooted tree* is a tree in which one vertex has been designated as the root and every edge is directed away from the root.
- The terminology for trees are **parent, child, siblings**, **ancestors, descendants.**

- **DEFINITION 3** A rooted tree is called an *m-ary tree* if every internal vertex has no more than *m* children. The tree is called a *full m-ary tree* if every internal vertex has exactly *m* children. An *m*-ary tree with *m* = 2 is called a *binary tree*.



Binary search tree

- Form a binary search tree for the words *mathematics, physics, geography, zoology, meteorology, geology, psychology*, and *chemistry* (using alphabetical order).

Binary search tree

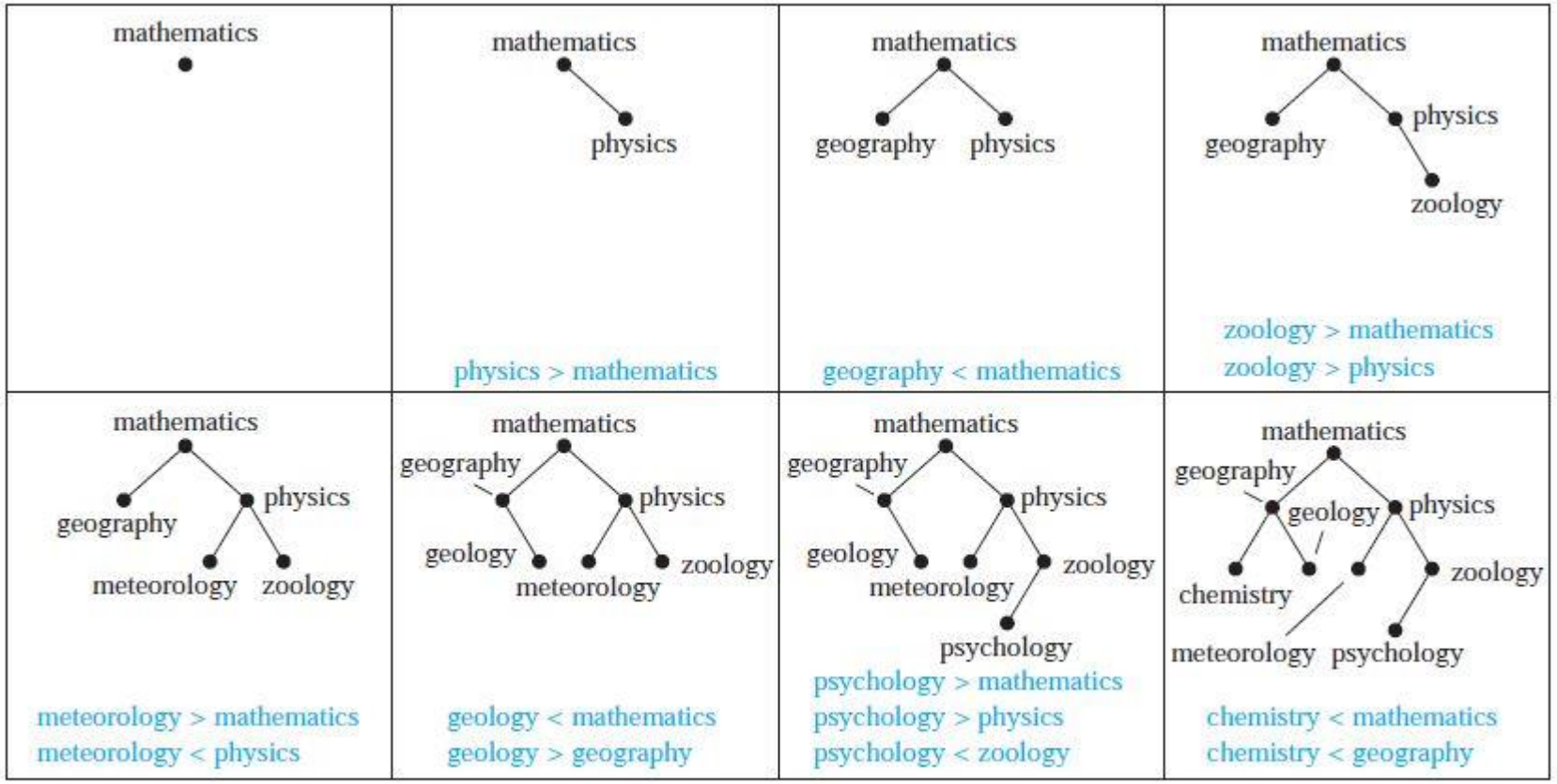


FIGURE 1 **Constructing a Binary Search Tree.**

- **ORDERED ROOTED TREES** An **ordered rooted tree** is a rooted tree where the children of each internal vertex are ordered.
- Example: Binary tree
- **THEOREM 2** A tree with n vertices has $n - 1$ edges.
- **THEOREM 3** A full m-ary tree with i internal vertices contains $n = mi + 1$ vertices.

- **THEOREM 4** A full m-ary tree with
- (i) n vertices has $i = (n - 1)/m$ internal vertices and $l = [(m - 1)n + 1]/m$ leaves,
- (ii) i internal vertices has $n = mi + 1$ vertices and $l = (m - 1)i + 1$ leaves,
- (iii) l leaves has $n = (ml - 1)/(m - 1)$ vertices and $i = (l - 1)/(m - 1)$ internal vertices.

Infix, Prefix, and Postfix Notation

- What is the ordered rooted tree that represents the expression *((x + y)↑2)* + *((x − 4)/3)*? => This is infix form.

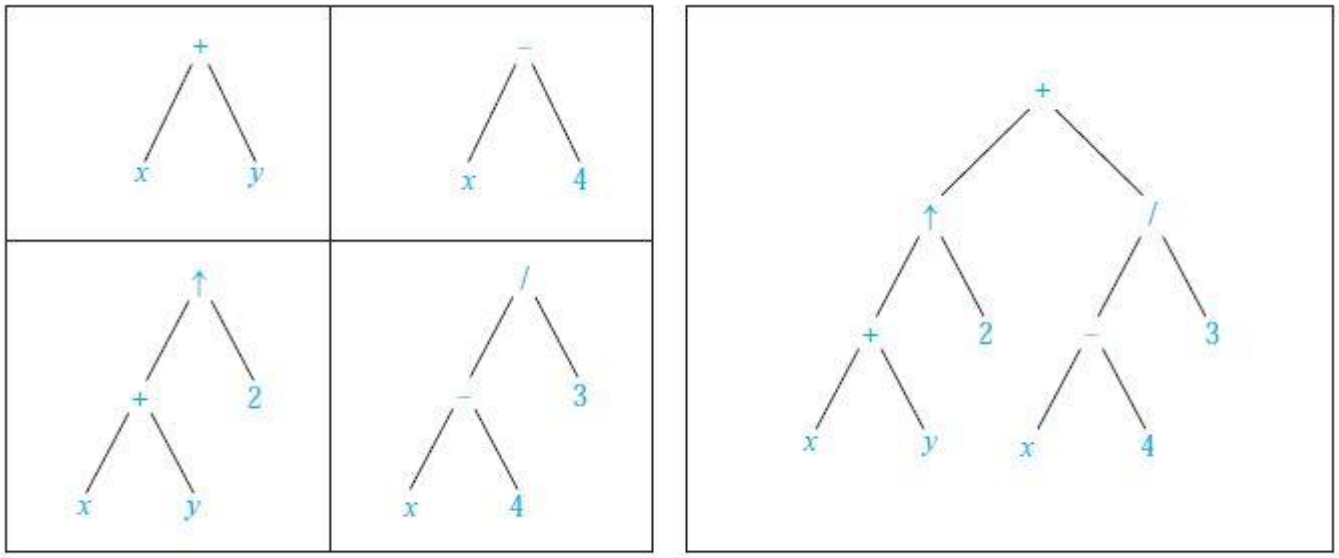


FIGURE 10 A Binary Tree Representing $((x + y) \uparrow 2) + ((x - 4)/3)$.

- What is the prefix form for *((x + y)* ↑ 2*)* + *((x* − 4*)/*3*)*?
- *Solution:* We obtain the prefix form for this expression by traversing the binary tree that represents it in preorder, shown in Figure 10. This produces + ↑ + *x y* 2 / − *x* 4 3.

Infix Expression	Prefix Expression	Postfix Expression
A + B	+ A B	A B +
A + B * C	+ A * B C	A B C * +

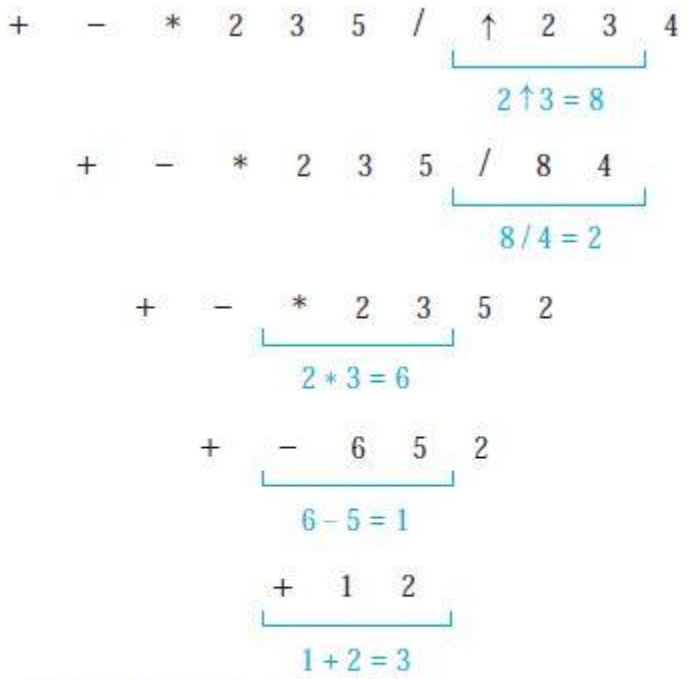


FIGURE 12 **Evaluating a Prefix Expression.**

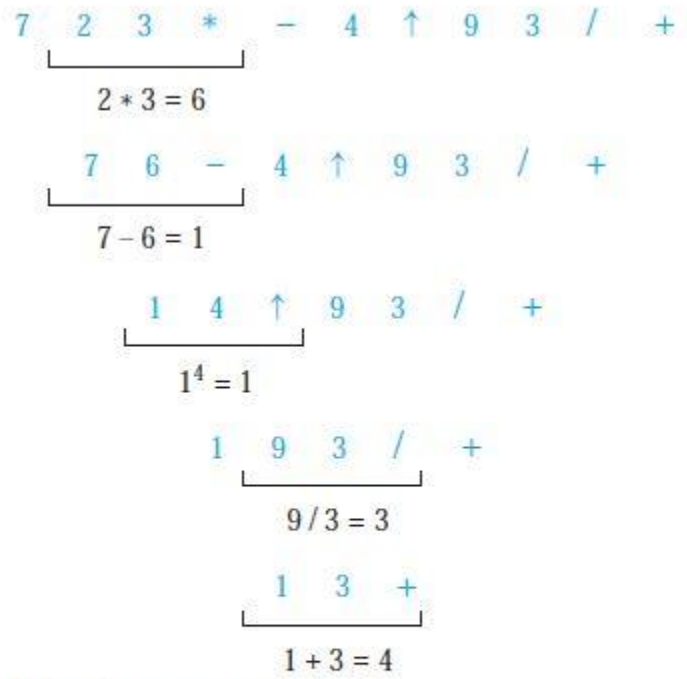


FIGURE 13 **Evaluating a Postfix Expression.**

- What is the value of the prefix expression + − ∗ 2 3 5/↑ 2 3 4?
- What is the value of the postfix expression 7 2 3 ∗ − 4 ↑ 9 3/+?