# ECE 231 Laboratory Exercise 4

# Thévenin and Norton Theorems

Laboratory Group (Names) ______________ ______________ _______________

## OBJECTIVES

- Learn various ways to measure **Thévenin's** voltage and resistance.
- Validate the maximum power theorem.

## BACKGROUND

**Thévenin's theorem (1883)** states that any linear circuit can be replaced by a single voltage source and a single series resistance. In 1926 Norton's Theorem was shown to be equal to Thévenin's Theorem, see Figure 1. You might wonder why the 57 year delay between the theorems. Batteries were easy to construct and incorporate into a circuit. No one knew how to make a good constant current source. We do not have current sources available in the lab to verify Norton's theorem, but it can be calculated using Ohm's Law. Constructing constant current sources is beyond the scope of this course.

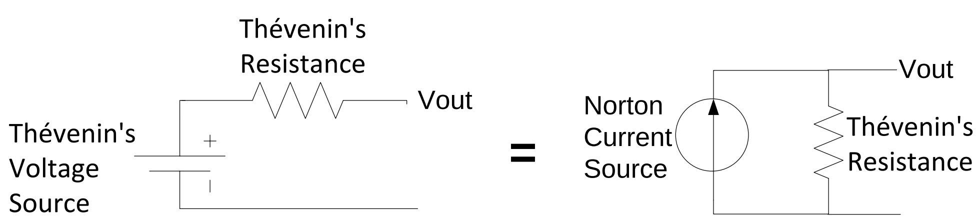


**Figure 1. Thévenin's and Norton's equivalent circuits for a Linear Circuit**

## Procedure for Finding the Thèvenin Equivalent Circuit Mathematically

### A. Circuits with independent sources only. No Dependent sources.

Step 1. Find $R_{\text{Thèvenin}}$

1. Deactivate all of the independent sources by shorting all batteries or DC supplies and opening all current sources.
2. The equivalent resistance between the terminals for which you would like to know the Thèvenin resistance is found by combining all of the resisters into one equivalent resistance between the appropriate terminals. These are usually designated “a” and “b.”

3. A load resistor which is equivalent to the Thèvenin resistance will result in maximum power being dissipated in the load resistor and of ½ the input voltage will be across the load.

$$Power_{\text{maximum}} = \frac{V_{th}^2}{4R_{th}}$$

Step 2. Find V $_{\text{Thèvenin}}$ between terminals "a" and "b."

1. Use the original circuit and nodal analysis to find the voltage between terminals "a" and "b."

## B. Circuits with independent and dependent sources.

Step 1. Find R $_{\text{Thèvenin}}$

1. Deactivate all of the independent sources by shorting all batteries or DC supplies and opening all current sources.
2. Connect a 1 amp your current source between terminals "a" and "b."
3. Find the voltage between terminals "a" and "b" using nodal analysis. This voltage will be the Thèvenin resistance by the use of Ohm's law.

$$\text{Resistance} = \frac{Voltage}{Current} = \frac{V_{ab}}{1} = R_{ab} = R_{Thevenin}$$

Step 2. Find V $_{\text{Thèvenin}}$ between terminals "a" and "b."

1. Use the original circuit and nodal analysis to find the voltage between terminals "a" and "b."

## C. Circuits with dependent sources only. No independent sources.

These circuits <u>cannot</u> output any power as such they reduce to a Thèvenin resistance only.

Step 1. Find R $_{\text{Thèvenin}}$

1. Connect a 1 amp your current source between terminals "a" and "b."
2. Find the voltage between terminals "a" and "b." This voltage will be the Thèvenin resistance by the use of Ohm's law.

$$\text{Resistance} = \frac{Voltage}{Current} = \frac{V_{ab}}{1} = V_{ab}$$

3. Find this voltage using nodal analysis.

Thévenin's and Norton's Theorems are expressed mathematically by equation 1.

$$r_{Thevenin} = \frac{v_{thevenin}}{i_{Norton}} = \frac{v_{open\ circuit}}{i_{short\ circuit}} = \frac{v_{open\ circuit} - v_{loaded}}{\frac{v_{loaded}}{R_{load}}} = half\ power\ load \qquad (1)$$

Measuring $V_{open\ circuit}$ just requires a single voltmeter measurement by definition.

Thévenin's and Norton's Theorems are expressed mathematically by equation 1.

**CAUTION**

**Do not attempt to measure $I_{short\ circuit}$ by shorting your circuit under test. This can be hazardous to both you and the circuit, especially when testing industrial power circuits.**

Determining the short circuit current is extremely important in the design of power distribution systems. When you examine the circuit breakers on your home power panel you will notice that the manufacturer has the Short Circuit capacity prominently displayed on the circuit breaker. It will be either 5000 A or 10,000 A. For industrial plants it can go as high as 200,000 A. Installing a circuit breaker with a smaller short circuit rating than that which can be supplied by the utility company can result in an explosion and fire. The short circuit capacity of a circuit determines the fuse size you use to protect electronic circuits.

Small current sources are frequently used in many electronic circuits and integrated circuits; however, they are rarely used in industrial power circuits. They are also commonly used to drive light emitting diodes (LEDs).

The maximum power theorem states that the maximum power will be delivered to a load when the load resistance is equal to the Thévenin's resistance. This is the basis for selecting the resistance of a speaker system for a stereo. This assures that in the design stereo systems that maximize the power will be delivered from the amplifier to the speakers.

## PROCEDURE

### Part 1

1. Choose three resistors for R1, R2, and R3. Measure their resistance values using the multimeter. Do not use the color code to determine the resistance value. Choose three resistors that are reasonably close in value. Do not pick, for example, 10K, 300, and 100 ohms. The 10 K resistor will make it difficult to get good experimental results. You should realize by now that the resistor color codes are not an accurate way to determine resistor values.
2. Construct the circuit shown in Figure 2 on the protoboard. Using a multimeter, measure the voltage between points "a" and "b" with NO LOAD connected. Record your measurement in column 6. This is $R_{Thèvenin}$ by definition.
3. Remove the 10 Vdc power source and connect a jumper between "1" and "2." This is the same as shorting the supply voltage mathematically. Now measure the resistance between "a" and "b" using your multimeter. By definition this is $R_{Thèvenin}$. Record this value in Table 1. Column 1.
4. Calculate $R_{Thèvenin}$ by combining the series and parallel resistors with the source disabled (shorted). Record this value in Table 1. Column 2. Now compare your measured value and calculated values in order to perform an error analysis. Enter this value in column 3.

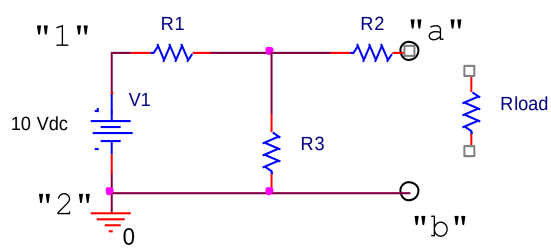


**Figure 2. Linear resistor network.**

5. We are now going to determine $R_{Thevenin}$ in another way. Connect a load to your circuit constructed in step 2. For best results the load resistance should be in the same range as your estimated $R_{Thevenin}$.
6. Now measure the output voltage between "a" and "b" in order to make the appropriate calculation. Divide this voltage by Rload. This will be the current going through the Thévenin equivalent circuit.
7. Simply apply Ohm's Law to find $r_{Thevenin}$..

$$r_{Thevenin} = \frac{voltage\ across\ r_{Thevenin}}{ir_{Thevenin}} = \frac{v_{open\ circuit} - v_{loaded}}{\frac{v_{loaded}}{R_{load}}} \qquad (2)$$

8. How does this $R_{Thevenin}$ compare to the value determined in column 2. Calculate % difference between columns 2 and 4 then enter this value in Table 1, column 5.

**Table 1. Measured and calculated data.**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| **$R_{Thevenin}$** Measured with sources removed (shorted) | **$R_{Thevenin}$** Calculated 1 with sources removed | **% Error** between measured and calculated | **$R_{Thevenin}$** Calculated 2 using Ohm's Law and Rload | **% Difference** between calculated 1 and calculated 2 | **$V_{ab}$** Measured | **$V_{ab}$** Calculated | **% Error** Thévenin voltage measured and calculated |
| | | | | | | | |

Part 2

1. Now construct the network shown in Figure 2, but replace $R_{load}$ with a potentiometer connected between "a" and "b." The equivalent circuit is shown in Figure 3. We will now determine $r_{thevenin}$ using the potentiometer.
2. Measure the voltage between "a" and "b" as the potentiometer is adjusted.
3. Adjust the potentiometer wiper until the voltmeter reads $V_{Thevenin}/2$ NOT $V_{source}/2$. The potentiometer is now set at the maximum power load which is equal to $r_{thevenin}$
4. Calculate the maximum power delivered to the load using equation (2).
5. Measure the value of the potentiometer and determine how close it is to the value of $r_{thevenin}$ determined above.

$$Maximum\ power = \frac{\left(\frac{V_{Thevenin}}{2}\right)^2}{R_{load}} = \frac{V_{ab}^2}{R_{potentiometer}} \qquad (2)$$

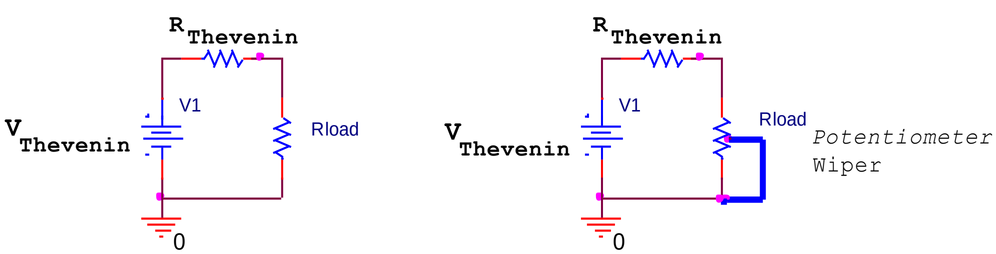


**Figure 3. Maximum power network.**

6. Now prove that this is the load for maximum power. Prove it by measuring the voltage Vab across the potentiometer after the potentiometer is rotated 1 turn CW. Then measure the potentiometer resistance at this position. Calculate the power delivered to the potentiometer using equation (3).

$$power = \frac{(v_{potentiometer})^2}{R_{potentiometer}} \qquad (3)$$

7. Repeat step 4, but this time rotate the potentiometer 2 turns CCW (1 turn to get back to the maximum power resistance then one additional turn). Calculate the power delivered to the potentiometer using equation (3). Compare results.

$P_{1\ turn\ CW}$= ____________ $P_{max}$= ___________ $P_{2turns\ CCW}$= _____________

This value must be less than $P_{max}$ This value must be less than $P_{max}$

## Conclusion

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