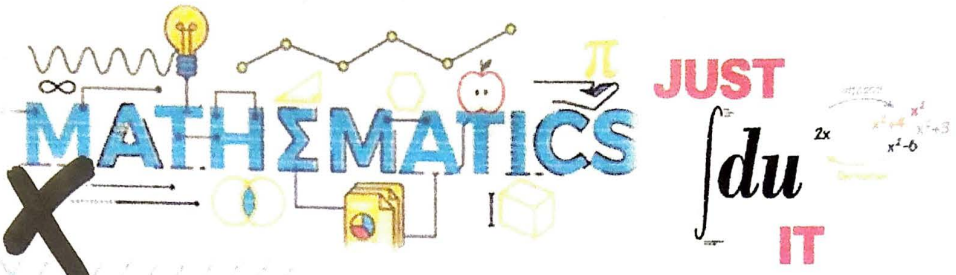

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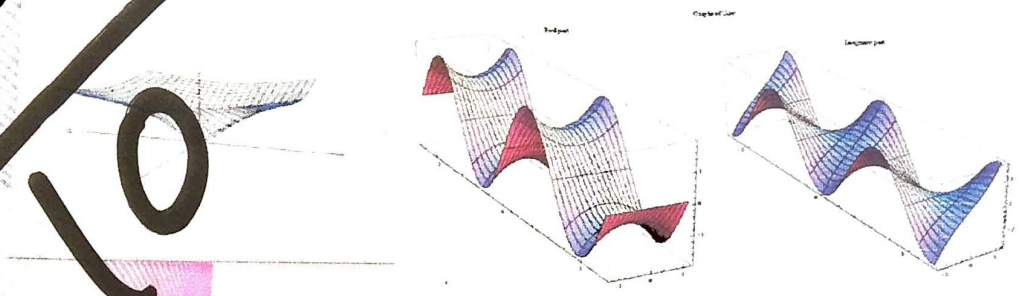

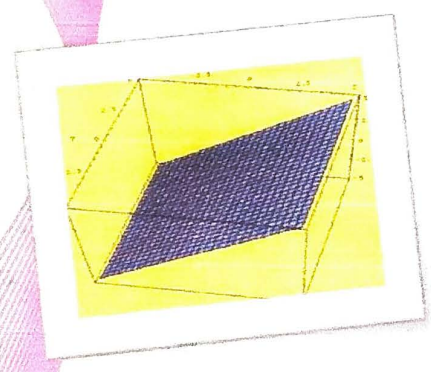

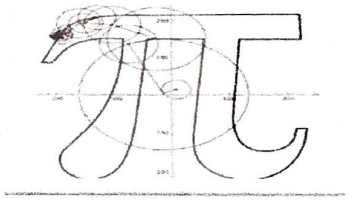

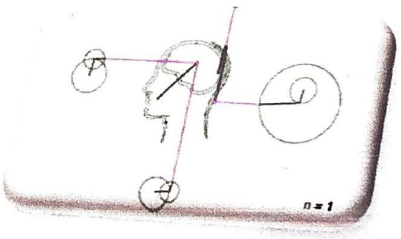

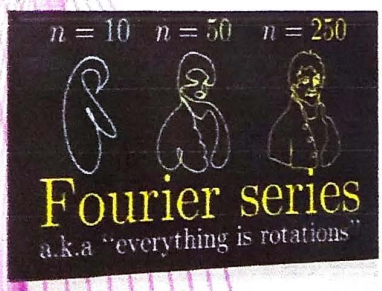

Submitted to:

Name: Md. Faishal Badsha

Lecturer

Department of Mathematics and Statistics

at Bangladesh University of Business and Technology.

Submitted by:

Name: Syeda Nowshin Ibnat

ID: 17183103020

Intake: 39 **Section:** 01

Program: B.Sc. in CSE

Assignment

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Lec-01

Chapter-1

Topic : 1
Number System:

→ Natural Number, N → 1, 2, 3, 4, 5, - - - - ∞

→ Whole Number, W → 0, 1, 2, 3, 4, 5, 6, 7 - - - ∞

→ Real Number, R → -∞ - - - - -3, -2, -1, 0, 1, 2, 3 - - - ∞

$N \subset R$

$W \subset R$

$N \subset W$

Topic : 02

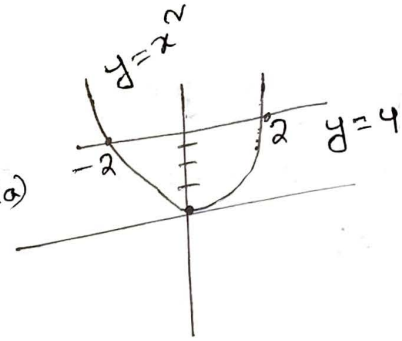
$x^2 = 4$

$x = \pm 2$

Let,

$y = x^2$ (parabola)

$y = x$

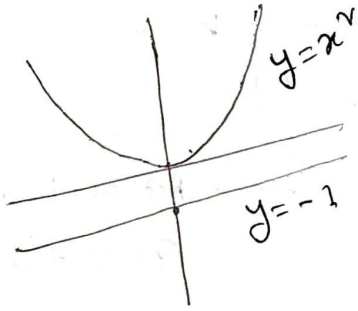


$x^2 = -1$

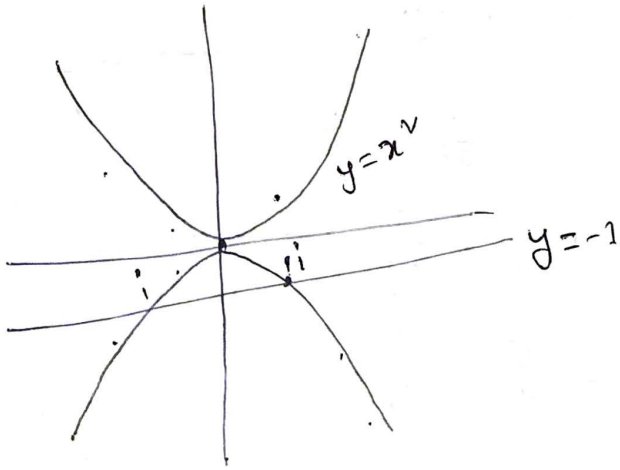
$x = \sqrt{-1}$

$y = x^2$

$y = -1$



Imagine that there is an imaginary plane.



Here,

$i \cdot i = i^2 = -1$

$\therefore i = \sqrt{-1}$

$\therefore x = \sqrt{i^2}$

$= \pm i$

Topic 3:

complex number:

$z = a + ib$ → Real
→ imaginary
→ Real of z

All numbers are complex numbers:

- → Only Real
- → " complex
- → purely imaginary
- → " Real

$a = 0$

$\boxed{z = ib}$ purely imaginary

→ as well as complex

$z = 0 + ib$

$z = a + ib$ | $b = 0$

$\boxed{z = a}$

purely real → As well as imaginary

$z = a + i.0$

Topic 4:

Eulers Formula:

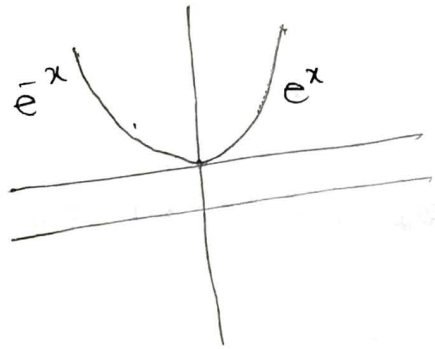
Q: State and prove eulers unity.

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \cdots$$

$$e^1 = 1 + \frac{1^1}{1!} + \frac{1^2}{2!} + \frac{1^3}{3!} + \frac{1^4}{4!} + \frac{1^5}{5!} + \cdots$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

Statement: $e^{i\pi} = -1$



$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \cdots \longrightarrow (1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \longrightarrow (2)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \longrightarrow (3)$$

From equation (1)

$$e^{ix} = 1 + \frac{ix}{1!} + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \cdots$$

$$\Rightarrow e^{ix} = 1 + \frac{ix}{1!} + \frac{i^2x^2}{2!} + \frac{i^3x^3}{3!} + \frac{i^4x^5}{5!} + \frac{i^6x^6}{6!} + \cdots$$

$$\Rightarrow e^{ix} = 1 + \frac{ix}{1!} - \frac{x^2}{2!} - \frac{ix^3}{3!} + \frac{x^4}{4!} + \frac{ix^5}{5!} - \frac{x^6}{6!} + \cdots$$

$$\Rightarrow e^{ix} = \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\right) + \left(\frac{ix}{1!} - \frac{ix^3}{3!} + \frac{ix^5}{5!} - \frac{ix^7}{7!} + \cdots\right)$$

$$\Rightarrow e^{ix} = \cos x + i\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right)$$

$$\Rightarrow \boxed{e^{ix} = \cos x + i \sin x} \longrightarrow \text{Eulers Formula}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

put, $\theta = \pi$

$e^{i\pi} = \cos\pi + i\sin\pi$

$e^{i\pi} = -1 + 0$

$\therefore \boxed{e^{i\pi} = -1} \rightarrow$ Eulers statement

$e^{i\pi} + 1 = 0$

Topic: 5

De movier's theorem:

$(\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)$

For $n = 2$

$(\cos\theta + i\sin\theta)^2 = \cos^2\theta + 2\cdot(\cos\theta)\cdot(i\sin\theta) + (i\sin\theta)^2$

$= \cos^2\theta + i\, 2\sin\theta\cos\theta + i^2\sin^2\theta$

$= \cos^2\theta + i\, 2\sin\theta\cos\theta - \sin^2\theta$

$= \cos^2\theta - \sin^2\theta + i\cdot\sin 2\theta$

$= \cos 2\theta + i\sin 2\theta$

Some formulas:

1) $2\cos^2\theta = 1 + \cos 2\theta$

2) $2\sin^2\theta = 1 - \cos 2\theta$

3) $\sin 2\theta = 2\sin\theta\cos\theta$

4) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

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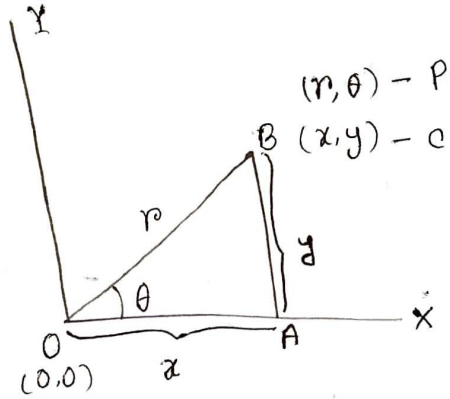
Topic : 01

Coordinate system:



(r, θ) – Polar coordinate

(x, y) – cartesian coordinate

ΔOAB

$\sin\theta = \frac{y}{r} \left(\frac{AB}{OB}\right)$ $\quad \therefore y = \cancel{r\cos\theta}\ r\sin\theta$

$\cos\theta = \frac{x}{r} \left(\frac{OA}{OB}\right)$ $\quad \therefore x = r\cos\theta$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$

$\therefore \theta = \tan^{-1}(y/x)$

Now, $y^2 = r^2 \sin^2\theta$

$x^2 = r^2 \cos^2\theta$

$x^2 + y^2 = r^2(\sin^2\theta + \cos^2\theta)$

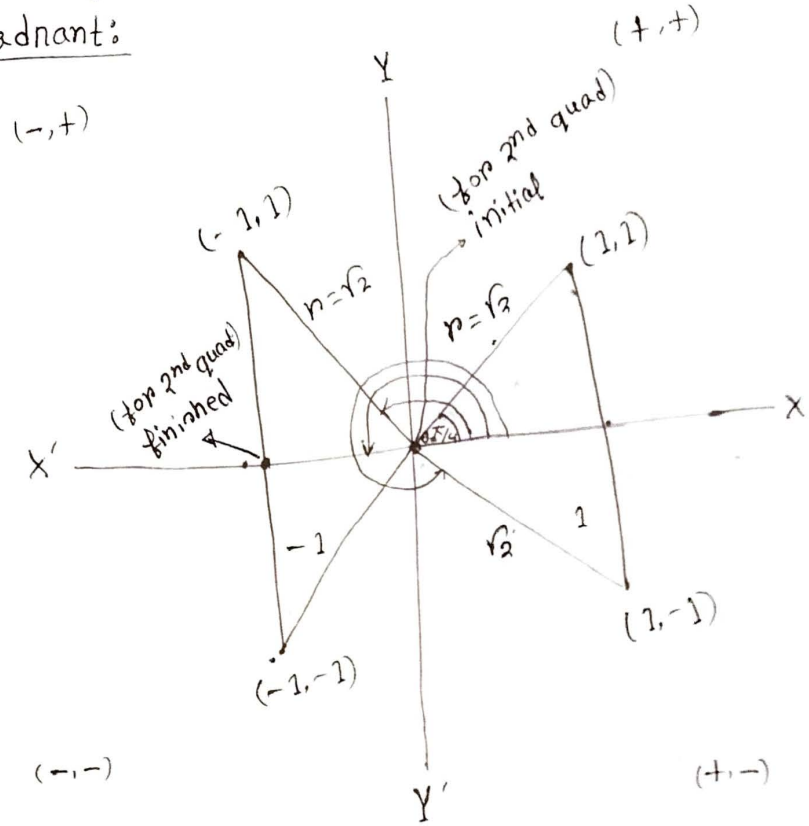
$\therefore r = \sqrt{x^2 + y^2}$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$$\begin{cases} r = \sqrt{x^2 + y^2} \\ \theta = \tan^{-1}(y/x) \end{cases}$$

Topic : 02

Quadrant:



at 1st quadrant:

$x = 1,\ y = 1$

$r = \sqrt{x^2 + y^2} = \sqrt{1+1} = \sqrt{2}$

$\theta = \tan^{-1}(y/x) = \tan^{-1}(1/1) = \tan^{-1}(1) = \pi/4 \ / 45^\circ$

at 2nd quadrant:

$x = -1,\ y = 1$

$r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

$\theta = \tan^{-1}(1/-1) = \tan^{-1}(-1) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

at 3rd quadrant:

$x = -1,\ y = -1$

$r = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$

$\theta = \tan^{-1}(-1/-1) = \tan^{-1}(1) = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

N.B

"+" initial

"−" finished

at 4th quadrant:

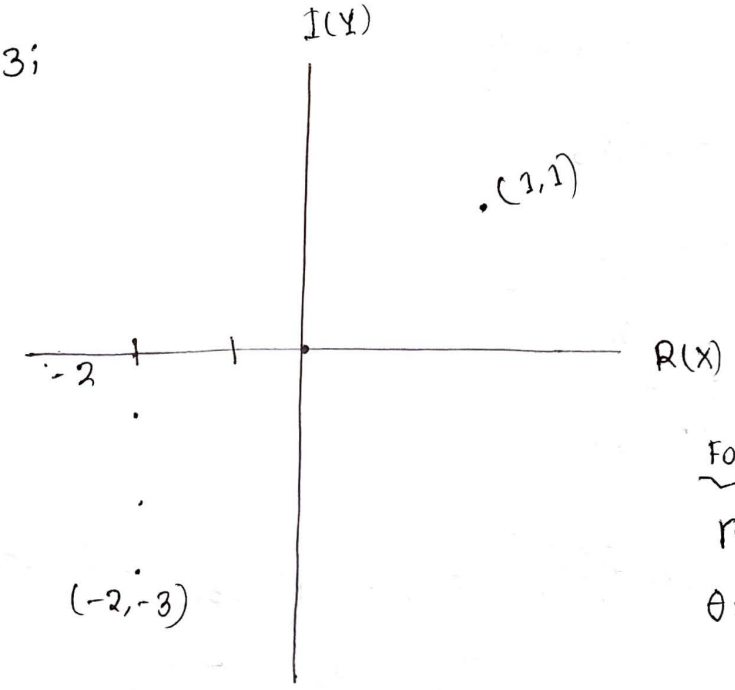
$x = 1,\ y = -1$

$r = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$

$\theta = \tan^{-1}(-1/1) = \tan^{-1}(-1) = 2\pi - \frac{\pi}{4} = \frac{8\pi - \pi}{4} = \frac{7\pi}{4}$

$z = 1 + i$

$z = -2 - 3i$



X = Real

Y = imaginary

For complex numbers:

r = modulas

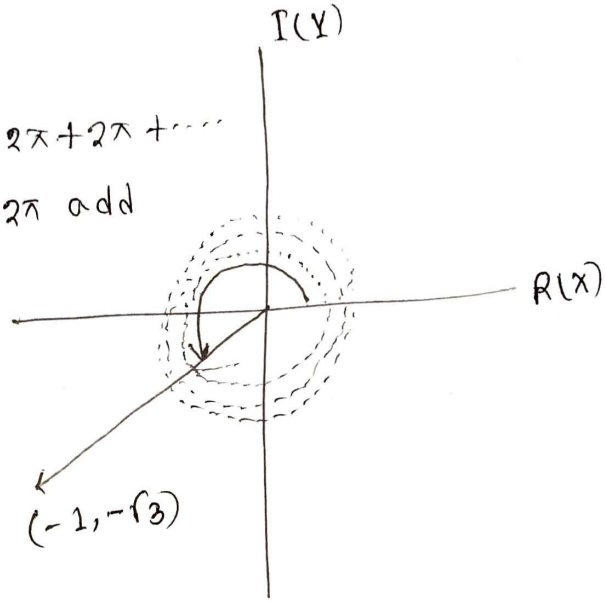
θ = Angument

Exercise

Ex: 1 Find the modulas and argument of $z = -1 - i\sqrt{3}$.

$\frac{4\pi}{3} + 2\pi + 2\pi + 2\pi + \cdots$

Everytime 2π add



ideal argument : $2\pi n + \frac{4\pi}{3}$

Soln:

Here, $z = -1 - i\sqrt{3}$

~~wt~~ we know, $z = x + iy$

So, $x = -1$
$y = -\sqrt{3}$

Modulas, $r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (-\sqrt{3})^2} = \sqrt{4} = 2$

Argument, $\theta = \tan^{-1}(y/x) = \tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right)$

$= \tan^{-1}(\sqrt{3})$

$= \pi + \frac{\pi}{3}$

$= \frac{4\pi}{3}$

$\therefore$ Real argument, $\theta = \frac{4\pi}{3}$.

And,

ideal argument, $\theta = 2\pi n + \frac{4\pi}{3}$ Ans:

$$\boxed{\begin{aligned} &\text{for } n=0, \quad \theta = \frac{4\pi}{3} \\ &n=1, \quad \theta = 2\pi + \frac{4\pi}{3} \\ &n=2, \quad \theta = 2\pi \times 2 + \frac{4\pi}{3} \end{aligned}}$$

$\boxed{\text{Ex:2}}$ Express $-5+5i$ in polar form.

Soln:

let, $-5+5i = x+iy$

$= r\cos\theta + ir\sin\theta$

$= r(\cos\theta + i\sin\theta)$

$\therefore -5+5i = re^{i\theta}$ (from eulers theorem)

We know, $r = \sqrt{x^2+y^2} = \sqrt{(-5)^2+(5)^2} = \sqrt{250} = 5\sqrt{2}$

$\theta = \tan^{-1}(y/x) = \tan^{-1}\left(\frac{5}{-5}\right) = \tan^{-1}(-1)$

$= \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$

Ideal argument $= 2\pi n + \frac{3\pi}{4}$

$\therefore -5+5i = 5\sqrt{2}e^{i(2\pi n + 3\pi/4)}$

Ans:

$\boxed{\text{Ex: 3}}$ Evaluate this $(3-2i)^{1/9}$

<u>Soln:</u>

$$(3-2i)^{1/9} = (x-iy)^{1/9}$$

let,

$x = 3$

$y = -2$

$$= (r\cos\theta - ir\sin\theta)^{1/9}$$

$$= \{r(\cos\theta - i\sin\theta)\}^{1/9}$$

$$= r^{1/9}(\cos\theta - i\sin\theta)^{1/9}$$

$$= r^{1/9}\left(\cos\frac{\theta}{9} - i\sin\frac{\theta}{9}\right)$$ (from de moivre's theorem)

$$= r^{1/9} e^{-i\theta/9} \longrightarrow (1)$$ (From euler's theorem)

Now, $r = \sqrt{x^2+y^2} = \sqrt{(3)^2+(-2)^2} = \sqrt{13}$

$$\theta = \tan^{-1}(y/x) = \tan^{-1}(-2/3)$$

$$= 360^\circ - 33^\circ$$

$$= 327^\circ$$

$\therefore$ Ideal argument, $\theta = 2\pi n + 327^\circ$

$$\therefore (3-2i)^{1/9} = (\sqrt{3})^{1/9} e^{-i\left(\frac{2\pi n + 327^\circ}{9}\right)}$$

Assignment : 1

A.W

(i) Evaluate $(2i)^{1/5}$

Soln:

$$(0+2i)^{1/5} = (x+iy)^{1/5}$$

let, $x=0$, $y=2$

$$= (r\cos\theta + ir\sin\theta)^{1/5}$$

$$= \{r(\cos\theta + i\sin\theta)\}^{1/5}$$

$$= r^{1/5}(\cos\theta + i\sin\theta)^{1/5}$$

$$= r^{1/5}\left(\cos\frac{\theta}{5} + i\sin\frac{\theta}{5}\right)$$

$$= r^{1/5} \cdot e^{i\theta/5} \longrightarrow (1)$$

Now, $r = \sqrt{x^2+y^2} = \sqrt{0+(2)^2} = \sqrt{4} = 2$

$$\theta = \tan^{-1}(y/x) = \tan^{-1}\left(\frac{2}{0}\right)$$

$$= 90^\circ / \frac{\pi}{2}$$

$$= \frac{\pi}{2}$$

$\therefore$ Ideal argument, $\theta = 2\pi n + \frac{\pi}{2}$

$$\therefore (2i)^{1/5} = 2^{1/5} \cdot e^{i\left(\frac{2\pi n + \pi/2}{5}\right)}$$

Ans:

(ii) Evaluate $(-2-3i)^{1/6}$

Soln:

$$(-2-3i)^{1/6} = (x-iy)^{1/6}$$
$$= (r\cos\theta - ir\sin\theta)^{1/6}$$
$$= \{r(\cos\theta - i\sin\theta)\}^{1/6}$$
$$= r^{1/6}(\cos\theta - i\sin\theta)^{1/6}$$
$$= r^{1/6}\left(\cos\frac{\theta}{6} - i\sin\frac{\theta}{6}\right)^{1/6}$$
$$= r^{1/6} e^{-i\theta/6} \longrightarrow (1)$$

let,
$x = -2$
$y = -3$
3rd quadrant

Now, $r = \sqrt{x^2+y^2} = \sqrt{(-2)^2+(-3)^2} = \sqrt{4+9} = \sqrt{13}$

$\theta = \tan^{-1}(y/x) = \tan^{-1}\left(\frac{-3}{-2}\right) = \tan^{-1}(3/2)$
$= 56.3^\circ$
$= 180 + 56.3^\circ$
$= 236.3^\circ$

$\therefore$ Ideal argument, $\theta = 2\pi n + 236.3^\circ$

$$\therefore (-2-3i)^{1/6} = \sqrt{13}^{\,1/6} e^{-i\left(\frac{2\pi n + 236.3^\circ}{6}\right)}$$

Ans:

(iii) Evaluate $\sqrt{(5-2i)}$

Soln:

$$(5-2i)^{1/2} = (x-iy)^{1/2}$$

$$= (r\cos\theta - ir\sin\theta)^{1/2}$$

$$= \{r(\cos\theta - i\sin\theta)\}^{1/2}$$

$$= r^{1/2}(\cos\theta - i\sin\theta)^{1/2}$$

$$= r^{1/2}\left(\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\right)$$

$$= r^{1/2} \cdot e^{-i\theta/2} \longrightarrow (1)$$

Let,
$x = 5$
$y = -2$

NOW, $r = \sqrt{x^2+y^2} = \sqrt{(5)^2+(-2)^2} = \sqrt{25+4} = \sqrt{29}$

$$\theta = \tan^{-1}(y/x) = \tan^{-1}(-2/5)$$

$$= 360^\circ - 21.8^\circ$$

$$= 338.2^\circ$$

$\therefore$ Ideal argument, $\theta = 2\pi n + 338.2^\circ$

$$\therefore (5-2i)^{1/2} = \sqrt{29}^{\,1/2}\, e^{-i\left(\frac{2\pi n + 338.2^\circ}{2}\right)}$$

Ans:

(iv) Expness $(7-6i)$ in polar form.

Soln:

Let, $7-6i = x-iy$

$= r\cos\theta - \cancel{i\sin\theta}\, i r\sin\theta$

$= r(\cos\theta - i\sin\theta)$

$\therefore 7-6i = re^{-i\theta}$

We know, $r = \sqrt{x^2+y^2} = \sqrt{(7)^2+(-6)^2} = \sqrt{49+36} = \sqrt{85}$

$\theta = \tan^{-1}(y/x) = \tan^{-1}\left(\frac{-6}{7}\right)$

$= 360^\circ - 40.6^\circ$

$= 319.4^\circ$

Ideal argument, $\theta = 2\pi n + 319.4^\circ$

$\therefore 7-6i = \sqrt{85}\, e^{-i(2\pi n + 319.4^\circ)}$

Ans:

19.10.19

Topic: 01

Cartesian:

Only for Square root ✓

cartesian → cartesian ✓

Exercise

Ex:1 Find the square root of the following:

Ex: 1 $\sqrt{-48+14i} = ?$

Soln: Given, $-48+14i$

imaginary part:

$14i = \frac{14i}{2} = 7i$.

$7 \times i$ ✗

$7i \times 1$ ✓

N.B

Real part "+" → i × small number

Real part "−" → i × large number

Now, $-48+14i$

$= (7i)^2 + 2.7.i.1 + (1)^2$

$= 49i^2 + 14i + 1$

$= -49 + 14i + 1$

$= -48 + 14i$

$\therefore \sqrt{-48+14i} = \sqrt{(7i+1)^2}$

$= \pm(7i+1)$

Ans:

$\boxed{\text{Ex:02}}$ $-7-24i$ (find the square root)

$\sqrt{-7-24i} = ?$ / $(-7-24i)^{1/2} = ?$

imaginary part:

$24i = \frac{24i}{2} = 12i$

$3i \times 4$ ✗

$3 \times 4i$ ✓

$6i \times 2$ ✓

$6 \times 2i$ ✗

$12i \times 1$ ✓

$12 \times i$ ✗

Now,

$(3)^2 \pm 2 \cdot 3 \cdot 4i + (4i)^2$

$= (3-4i)^2$

$= 9 - 24i + 16i^2$

$= 9 - 24i - 16$

$= -7 - 24i$

$\therefore \sqrt{-7-24i} = \sqrt{(3-4i)^2}$

$= \pm(3-4i)$

$= \pm(4i-3)$

$\boxed{Ex: 03}$ Find the square root of $-60-11i$

in this type imaginary part can't be devided by 2.

Soln: Given,

$$-60-11i$$

$$= \frac{1}{2}(-120-22i)$$

imaginary part:

$$22i = \frac{22i}{2} = 11i$$

$11i \times 1$ ✓

$1i \times 11$ ✗

Now,

$$\frac{1}{2}\{(11i)^2 - 2.11.i.1 + 1^2\}$$

$$= \frac{1}{2}(11i-1)^2$$

$$\therefore \sqrt{-60-11i} = \sqrt{\frac{1}{2}(11i-1)^2} = \pm\frac{1}{\sqrt{2}}(11i-1)$$

Ans:

Assignment: 2

A.W Find the square root of the followings:

$\boxed{01.}$ Given, $7+24i$

imaginary part:

$$24i = \frac{24i}{2} = 12i$$

$3i \times 4$ ✓

$4i \times 3$ ✗

$6i \times 2$ ✗

$6 \times 2i$ ✓

$12i \times 1$ ✗

$12 \times i$ ✓

Now, $\cancel{(3i+4)^2}$

$= (3i)^2 + 2 \cdot 3i \cdot 4 + (4)^2$

$= (3i+4)^2$

$= 9i^2 + 24i + 16$

$= -9 + 24i + 16$

$= 7 + 24i$

$\therefore \sqrt{7+24i} = \sqrt{(3i+4)^2} = \pm (3i+4)$ Ans:

$\boxed{02.}$ Given, $7-24i$

imaginary part :

$24i = \frac{24i}{2} = 12i$

$3i \times 4$ ✓
$4i \times 3$ ✗
$6i \times 2$ ✗
$2i \times 6$ ✓
$12i \times 1$ ✗
$i \times 12$ ✓

Now, $(3i-4)^2$

$= (3i)^2 - 2 \cdot 3i \cdot 4 + (4)^2$

$= 9i^2 - 24i + 16$

$= 7 - 24i$

$\therefore \sqrt{7-24i} = \sqrt{(3i-4)^2} = \pm (3i-4)$

Ans:

$\boxed{03.}$ Given that, $2i$

$= 0 + 2i$

imaginary part:

$2i = \frac{2i}{2} = i$

$i \times 1$ ✓ $1 \times i$ ✓

$(i+1)^2$

Now, $(i)^2 + 2 \cdot i \cdot 1 + (1)^2$

$= -1 + 2i + 1$

$= 2i$

$\therefore \sqrt{2i} = \sqrt{(i+1)^2} = \pm (i+1)$ Ans:

$\boxed{04.}$ Given that,

$-2i$

$= 0 - 2i$

imaginary part:

$2i = \frac{2i}{2} = i$

$i \times 1$

Now, $(i-1)^2$

$= (i)^2 - 2 \cdot i \cdot 1 + (1)^2$

$= -1 - 2i + 1$

$= -2i$

$\therefore \sqrt{-2i} = \sqrt{(i-1)^2} = \pm (i-1)$

Ans:

$\boxed{05.}$ Given that,

$$24+70i$$

imaginary part:

$$70i = \frac{70i}{2} = 35i$$

$7i \times 5$ ✗

$\rightarrow 5i \times 7$ ✓

$35i \times 1$ ✗

$i \times 35$ ✓

Now, $(5i+7)^2$

$= (5i)^2 + 2 \cdot 5i \cdot 7 + (7)^2$

$= -25 + 70i + 49$

$= 24 + 70i$

$\therefore \sqrt{24+70i} = \sqrt{(5i+7)^2} = \pm 5i+7$

Ans:

$\boxed{06.}$ Given that,

$$-24-70i$$

imaginary part:

$$70i = \frac{70i}{2} = 35i$$

$7i \times 5$ ✓

$5i \times 7$ ✗

$35i \times 1$ ✓

$i \times 35$ ✗

Now, $(7i-5)^2$

$= (7i)^2 - 2 \cdot 7i \cdot 5 + (5)^2$

$= -49 - 70i + 25$

$= -24 - 70i$

$\therefore \sqrt{-24-70i} = \sqrt{(7i-5)^2} = \pm(7i-5)$

Ans:

$\boxed{07.}$ Given that,

$$-24+70i$$

imaginary part:

$70i = \frac{70i}{2} = 35i$

$5i \times 7$ ✗

$7i \times 5$ ✓

$35i \times 1$ ✓

$i \times 35$ ✗

Now, $(7i+5)^2$

$= (7i)^2 + 2 \cdot 7i \cdot 5 + (5)^2$

$= -49 + 70i + 25$

$= -24 + 70i$

$\therefore \sqrt{-24+70i} = \sqrt{(7i+5)^2} = 7i+5$ Ans:

$\boxed{08.}$ Given that,

$$60+11i$$

$$= \frac{1}{2}(120+22i)$$

In this example we can't devide imaginary part by 2.

imaginary part:

$22i = \frac{22i}{2} = 11i$

$i \times 11$ ✓

$11i \times 1$ ✗

Now, $\cancel{(i+11)^2}$ $\frac{1}{2}\{(i)^2 + 2 \cdot i \cdot 11 + (11)^2\}$

$= \frac{1}{2}(i+11)^2$

$\therefore \sqrt{60+11i} = \sqrt{\frac{1}{2}(i+11)^2} = \pm\frac{1}{\sqrt{2}}(i+11)$ Ans:

$\boxed{09.}$ Given,

$$-60+11i$$

$$= \frac{1}{2}(-120+322i)$$

imaginary part:

$$22i = \frac{22i}{2} = 11i$$

$11i \times 1$ ✓

$i \times 11$ ✗

Now, $\frac{1}{2}\{(11i)^2 + 2\cdot 11i \cdot 1 + (1)^2\}$

$$= \frac{1}{2}(11i+1)^2$$

$$\therefore \sqrt{-60+11i} = \sqrt{\frac{1}{2}(11i+1)^2} = \pm\frac{1}{\sqrt{2}}(11i+1)$$ Ans:

$\boxed{\text{Topic: 03}}$

Using De movier's

cartesian $\longrightarrow$ polar $\longrightarrow$ cartesian $\left(\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array}\right)$

Find the following two cube root:

$\boxed{\text{Ex: 1}}$ $\sqrt[3]{1} = 1^{1/3} + i\cdot 0$

Here,
$x = 1$
$y = 0$

Soln: $1^{1/3} = (1+0\cdot i)^{1/3} = (x+iy)^{1/3}$

$$= (r\cos\theta + ir\sin\theta)^{1/3}$$

$$= r^{1/3}(\cos\theta + i\sin\theta)^{1/3}$$

$$= r^{1/3}\left(\cos\frac{\theta}{3} + i\sin\frac{\theta}{3}\right)$$

$$= r^{1/3}(e^{i\theta})^{1/3}$$

$$= r^{1/3} \cdot e^{i\theta/3}$$

Now,

$$r = \sqrt{x^2+y^2} = \sqrt{1+0} = 1$$

$$\theta = \tan^{-1}(y/x) = \tan^{-1}(0/1) = \tan^{-1}(0)$$
$$= 0^\circ \text{ (Real Argument)}$$

$\therefore$ Ideal argument, $\theta = 2\pi n + 0^\circ = 2\pi n$

$$\therefore \sqrt[3]{1} = 1^{2/3} \cdot e^{i\left(\frac{2\pi n}{3}\right)}$$ Here, $n = 0, 1, 2, 3 \ldots$

Ans:

$\boxed{\text{Ex: 2}}$ Given, $(-2-2i)^{2/3}$

Soln:

let, $(-2-2i)^{2/3} = (x+iy)^{2/3}$

$$= r^{2/3} \cdot e^{i\theta/3}$$

$$= 2\sqrt{2} \cdot e^{i\left(\frac{2\pi n + 5\pi/4}{3}\right)}$$

Here,
$x = -2$
$y = -2$

N.B:
$(-2) + i(-2)$
$x + iy$

3rd quadrant

Now, $r = \sqrt{(-2)^2 + (-2)^2} = 2\sqrt{2}$

$$\theta = \tan^{-1}(-2/-2) = \tan^{-1}(1)$$
$$= \frac{\pi}{4}$$
$$= \pi + \frac{\pi}{4} \text{ (3rd quadrant initial ✓)}$$
$$= \frac{5\pi}{4} \text{ (Real argument)}$$

$\therefore$ Ideal Argument, $\theta = 2\pi n + \frac{5\pi}{4}$ Here, $n = 0, 1, 2, 3 \ldots$

$$\therefore (-2-2i)^{2/3} = 2\sqrt{2} \cdot e^{i\left(\frac{2\pi n + 5\pi/4}{3}\right)}$$

Ans: