Viewing and Projection

# Computer Graphics

# Objectives

- Introduce the classical views
- Compare and contrast image formation by computer with how images have been formed by architects, artists, and engineers
- Learn the benefits and drawbacks of each type of view

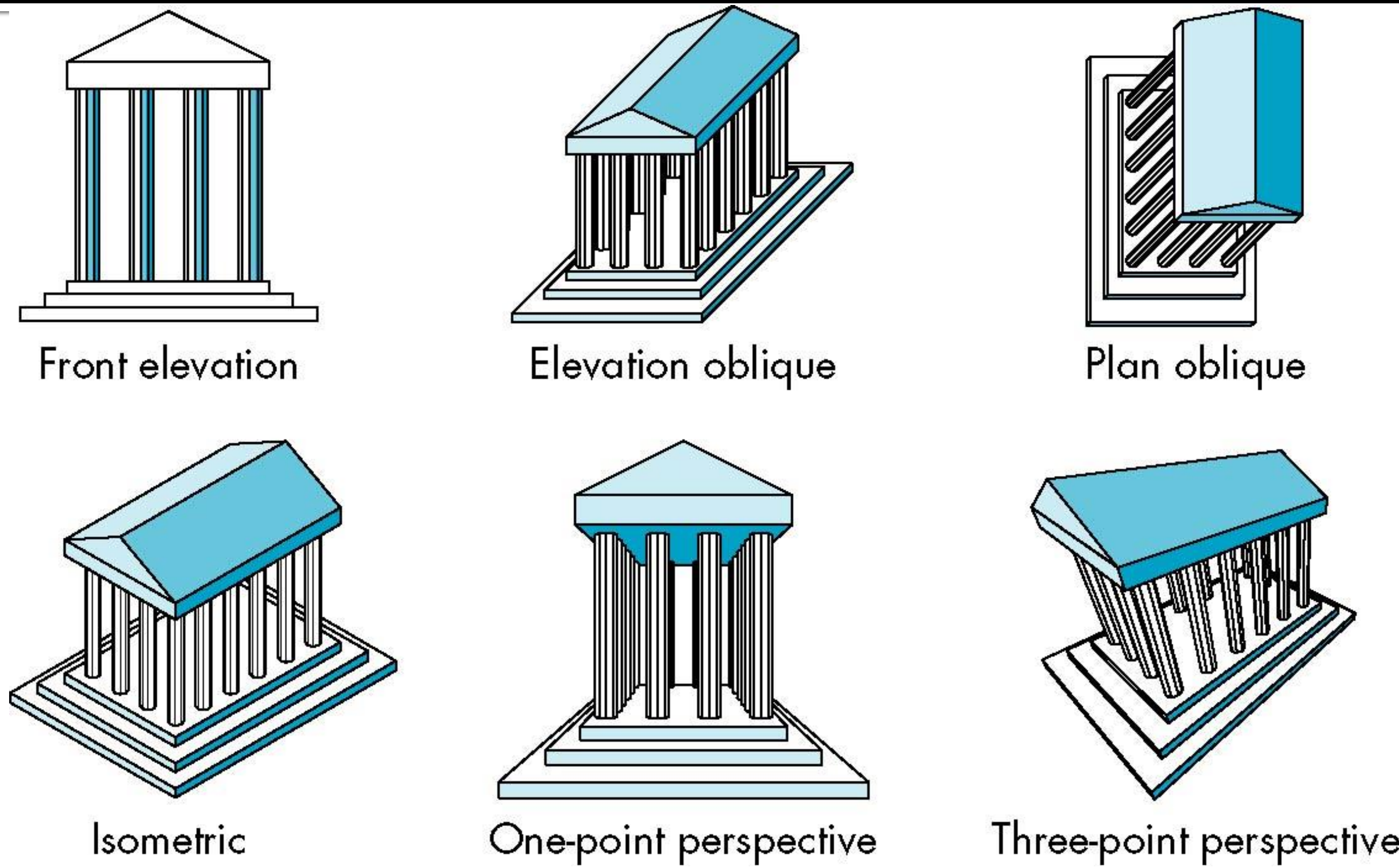
# Classical Viewing

- Viewing requires three basic elements
  - One or more objects
  - A viewer with a projection surface
  - Projectors that go from the object(s) to the projection surface
- Classical views are based on the relationship among these elements
  - The viewer picks up the object and orients it how he would like to see it
- Each object is assumed to be constructed from flat *principal faces*
  - Buildings, polyhedra, manufactured objects

# Planar Geometric Projections

- Standard projections project onto a plane
- Projectors are lines that either
  - converge at a center of projection
  - are parallel
- Such projections preserve lines
  - but not necessarily angles

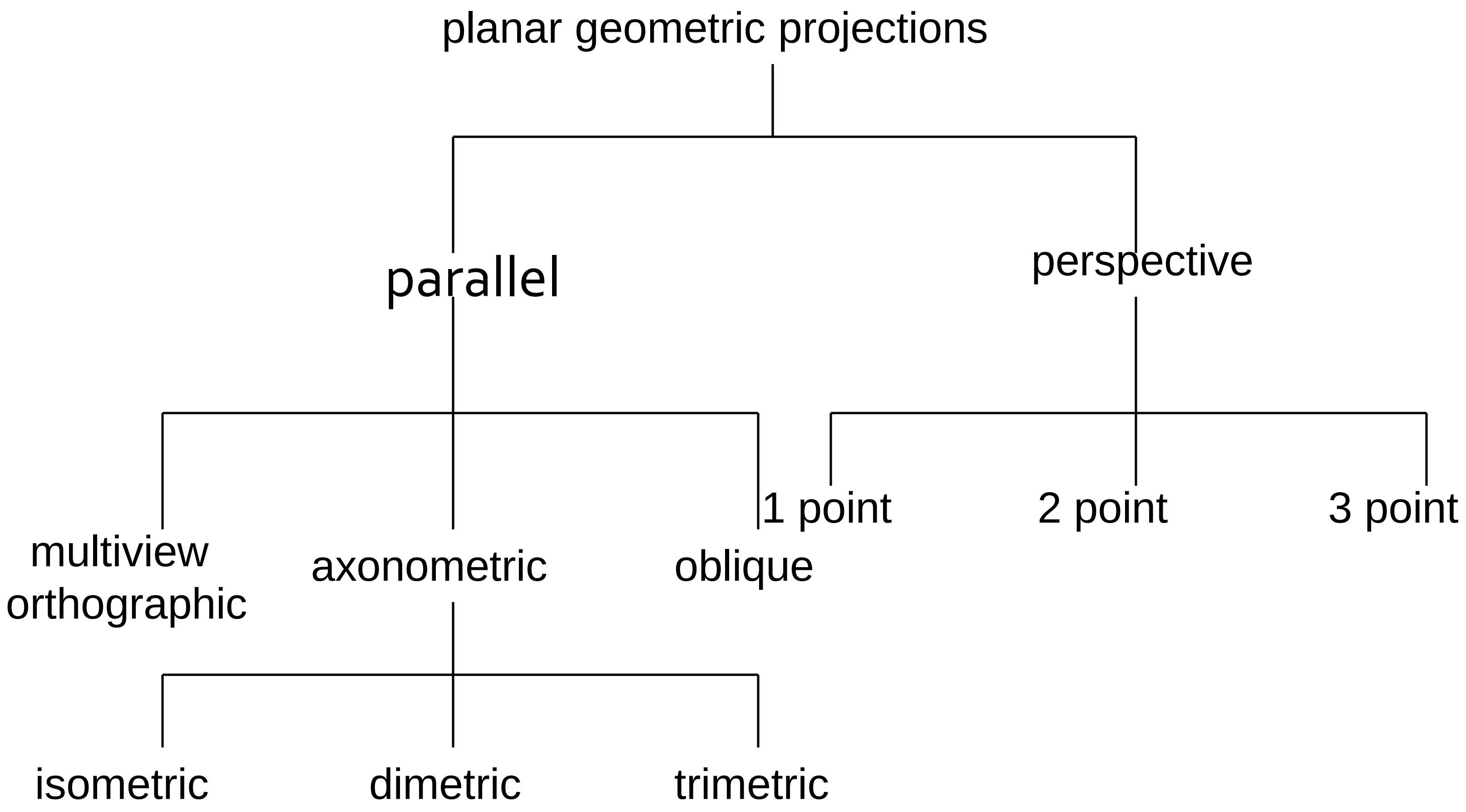
# Classical Projections



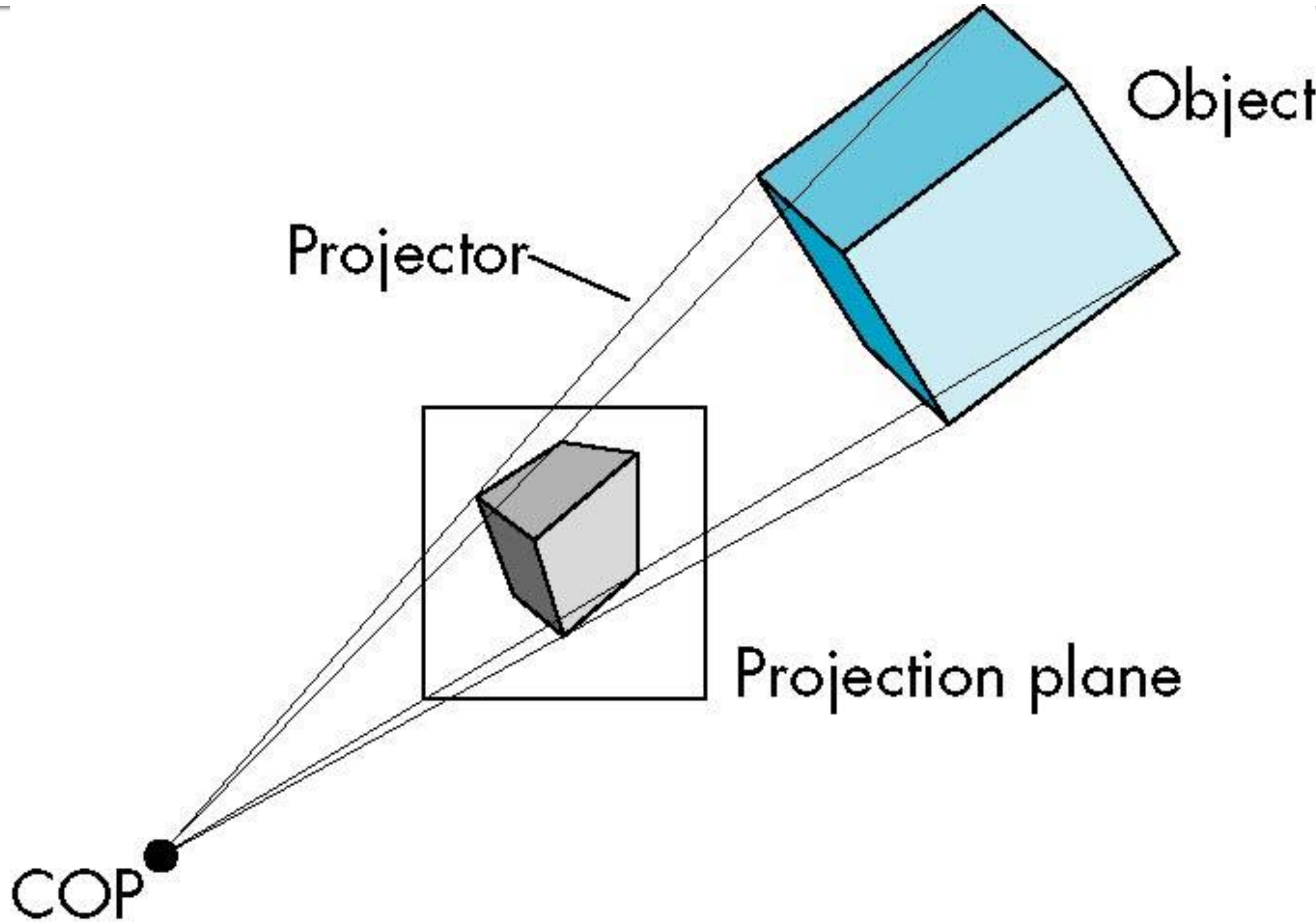
# Perspective vs Parallel

- Computer graphics treats all projections the same and implements them with a single pipeline
- Classical viewing developed different techniques for drawing each type of projection
- Fundamental distinction is between parallel and perspective viewing even though mathematically parallel viewing is the limit of perspective viewing

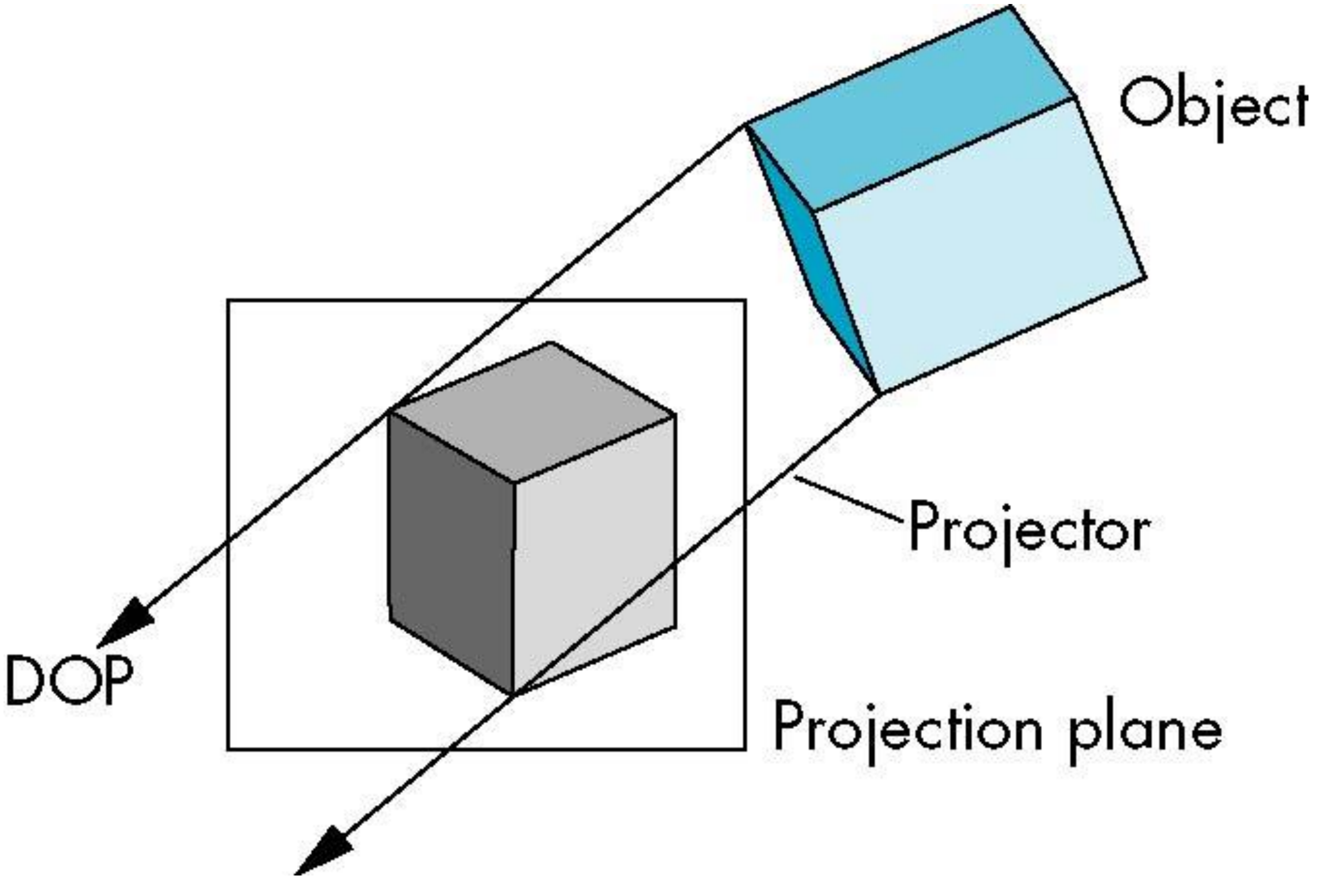
# Taxonomy of Planar Geometric Projections



# Perspective Projection

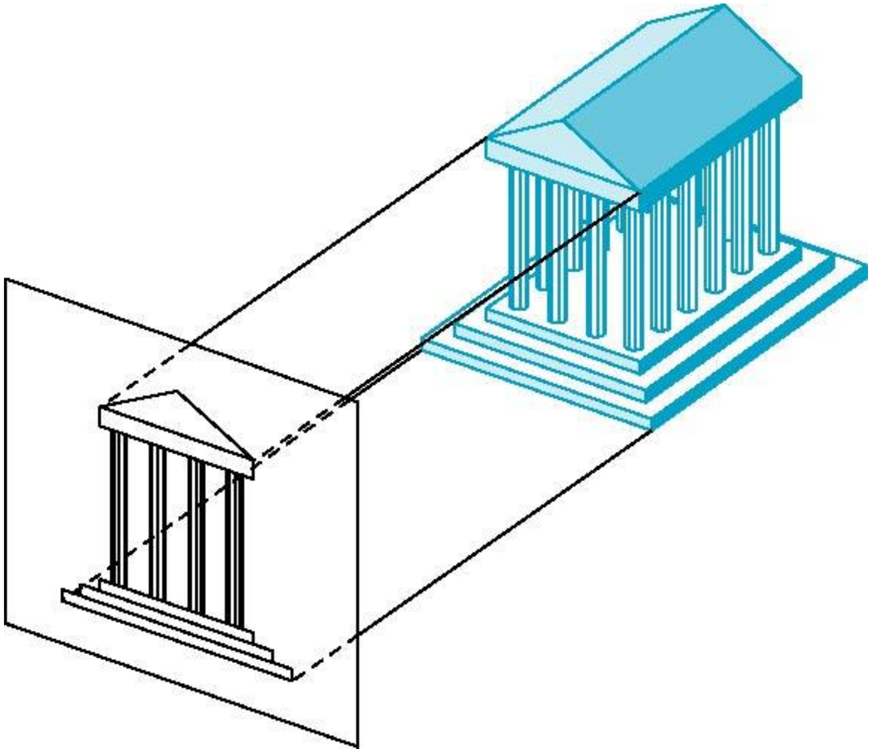


# Parallel Projection



# Orthographic Projection

Projectors are orthogonal to projection surface



# Multiview Orthographic Projection

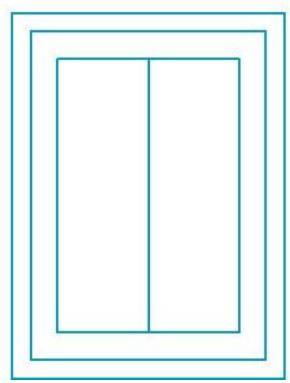
- Projection plane parallel to principal face
- Usually form front, top, side views

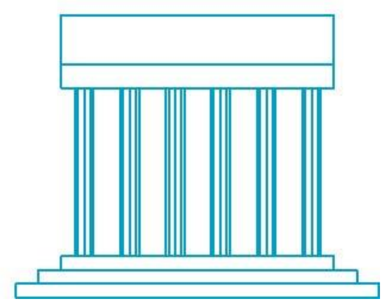
isometric (not multiview orthographic view)

front

in CAD and architecture, we often display three multiviews plus isometric

top





side

# Advantages and Disadvantages

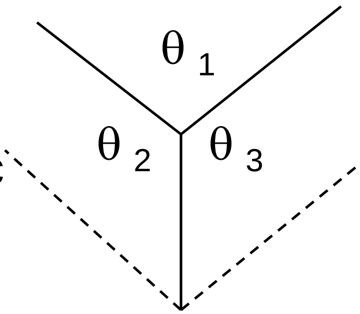
- Preserves both distances and angles
  - Shapes preserved
  - Can be used for measurements
    - Building plans
    - Manuals
- Cannot see what object really looks like because many surfaces hidden from view
  - Often we add the isometric

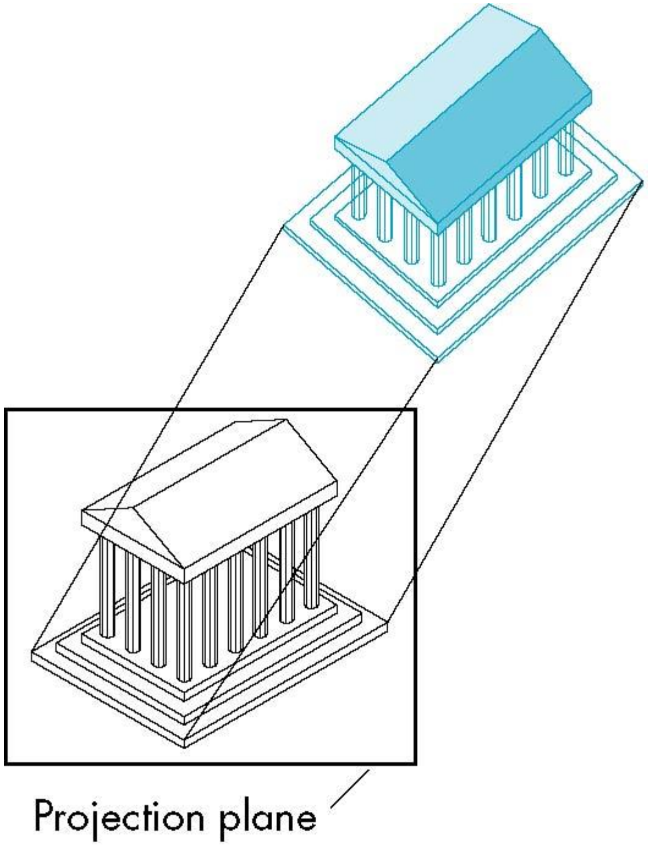
# Axonometric Projections

Allow projection plane to move relative to object

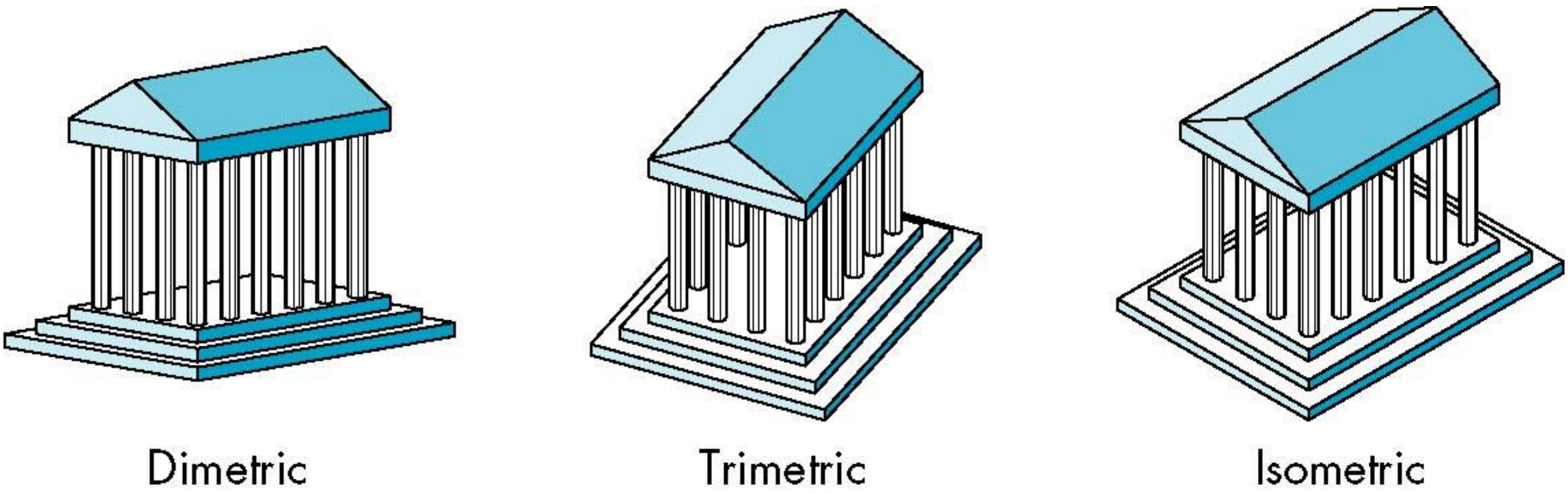
classify by how many angles of a corner of a projected cube are the same

none: trimetric
two: dimetric
three: isometric



# Types of Axonometric Projections

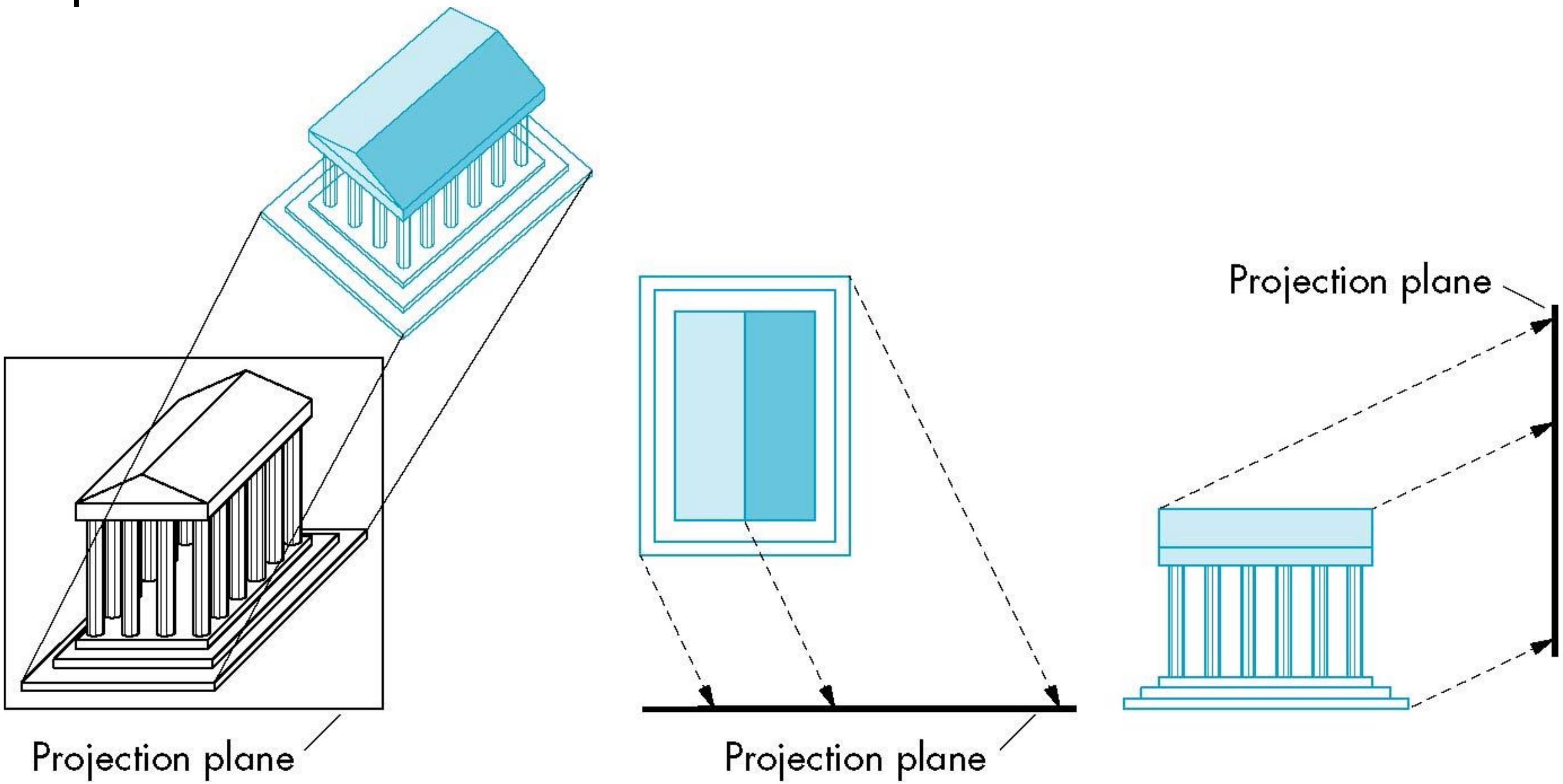


# Advantages and Disadvantages

- Lines are scaled (*foreshortened*) but can find scaling factors
- Lines preserved but angles are not
  - Projection of a circle in a plane not parallel to the projection plane is an ellipse
- Can see three principal faces of a box-like object
- Some optical illusions possible
  - Parallel lines appear to diverge
- Does not look real because far objects are scaled the same as near objects
- Used in CAD applications

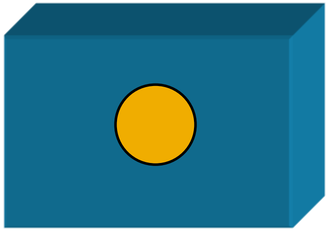
# Oblique Projection

Arbitrary relationship between projectors and projection plane



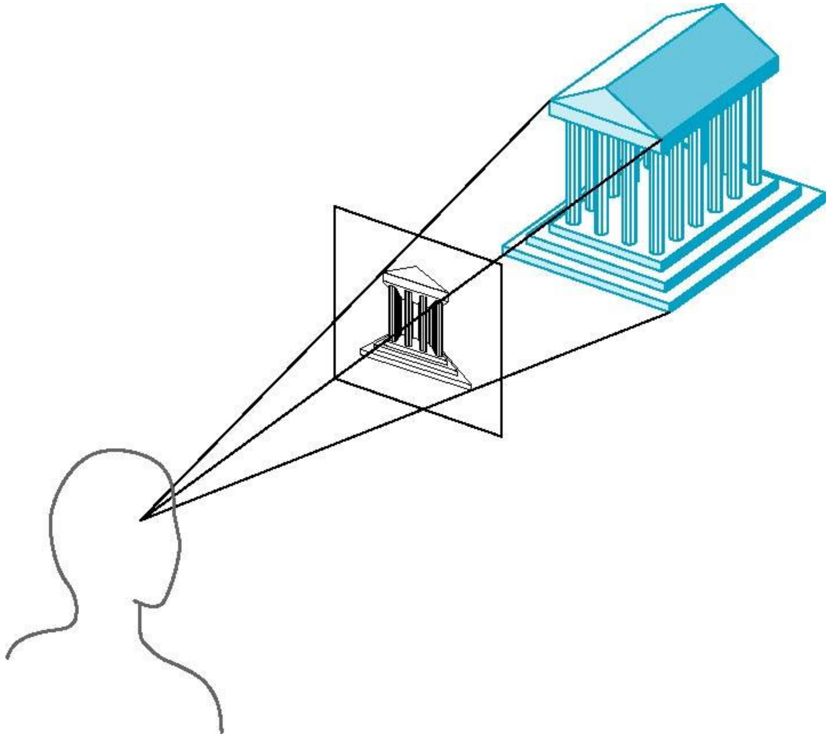
# Advantages and Disadvantages

- Can pick the angles to emphasize a particular face
  - Architecture: plan oblique, elevation oblique
- Angles in faces parallel to projection plane are preserved while we can still see “around” side



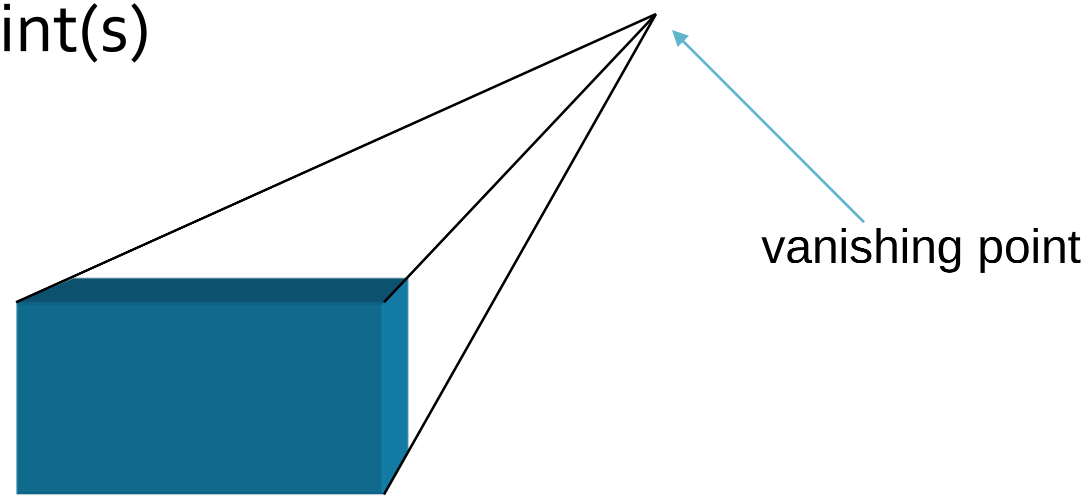
# Perspective Foreshortening

Projectors converge at center of projection



# Vanishing Points

- Parallel lines (not parallel to the projection plan) on the object converge at a single point in the projection (the *vanishing point*)
- Drawing simple perspectives by hand uses these vanishing point(s)



# Three-Point Perspective

- No principal face parallel to projection plane
- Three vanishing points for cube

# Two-Point Perspective

- One principal direction parallel to projection plane
- Two vanishing points for cube

# One-Point Perspective

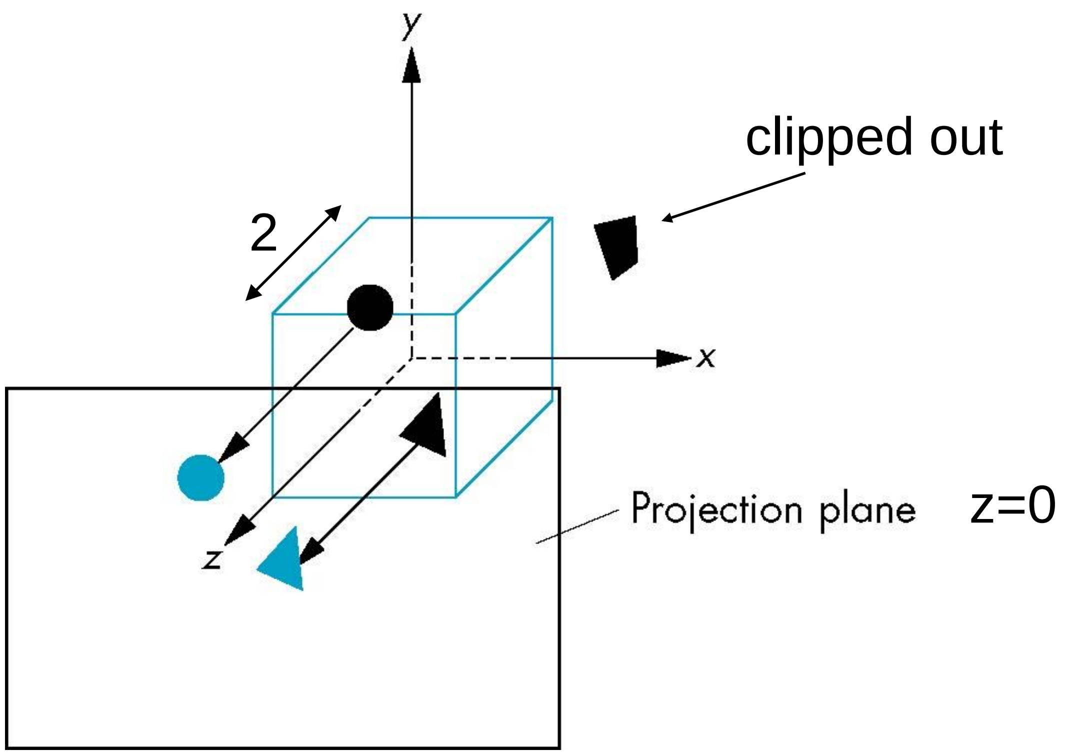
- One principal face parallel to projection plane
- One vanishing point for cube

# Advantages and Disadvantages

- Objects further from viewer are projected smaller than the same sized objects closer to the viewer (*diminution*)
  - Looks realistic
- Equal distances along a line are not projected into equal distances (*nonuniform foreshortening*)
- Angles preserved only in planes parallel to the projection plane
- More difficult to construct by hand (perspective projection) than parallel projections (but not more difficult by computer)

# Default Projection

Default projection is orthogonal



# Homogeneous Coordinate Representation

default orthographic projection

$$x_p = x$$
$$y_p = y$$
$$z_p = 0$$
$$w_p = 1$$

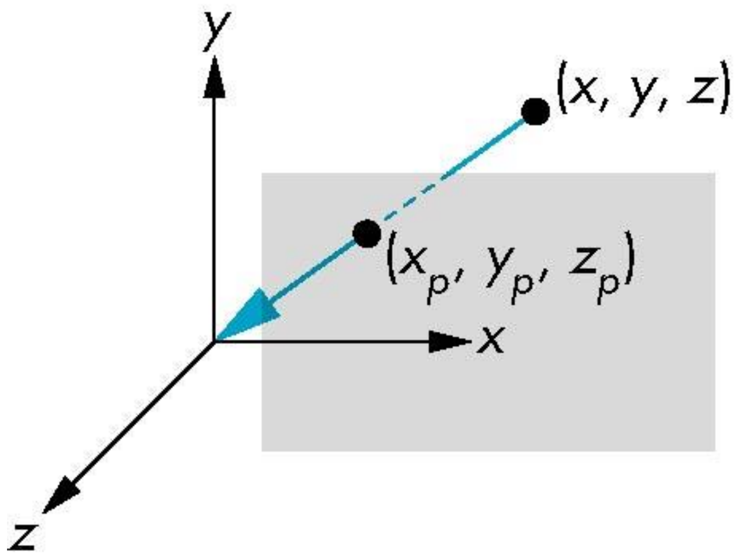
$$\mathbf{p}_p = \mathbf{M}\mathbf{p}$$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In practice, we can let **M** = **I** and set the *z* term to zero later

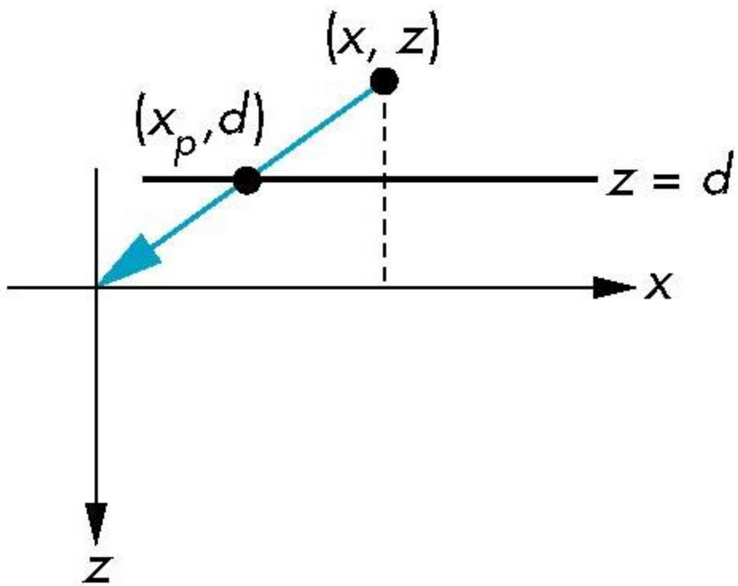
# Simple Perspective

- Center of projection at the origin
- Projection plane $z = d,\ d < 0$

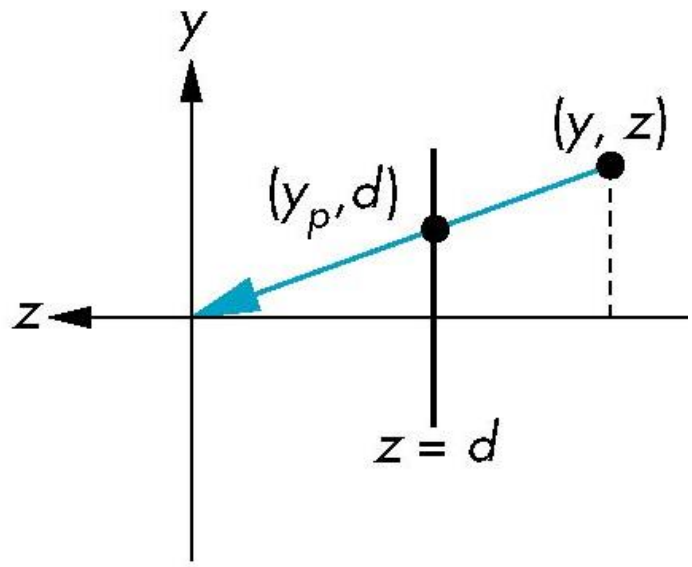


# Perspective Equations

Consider top and side views




$$\frac{x}{z/d}$$




$$x_p = \frac{x}{z/d} \qquad y_p = \frac{y}{z/d} \qquad z_p = d$$

# Homogeneous Coordinate Form

consider $\mathbf{q} = \mathbf{Mp}$ where

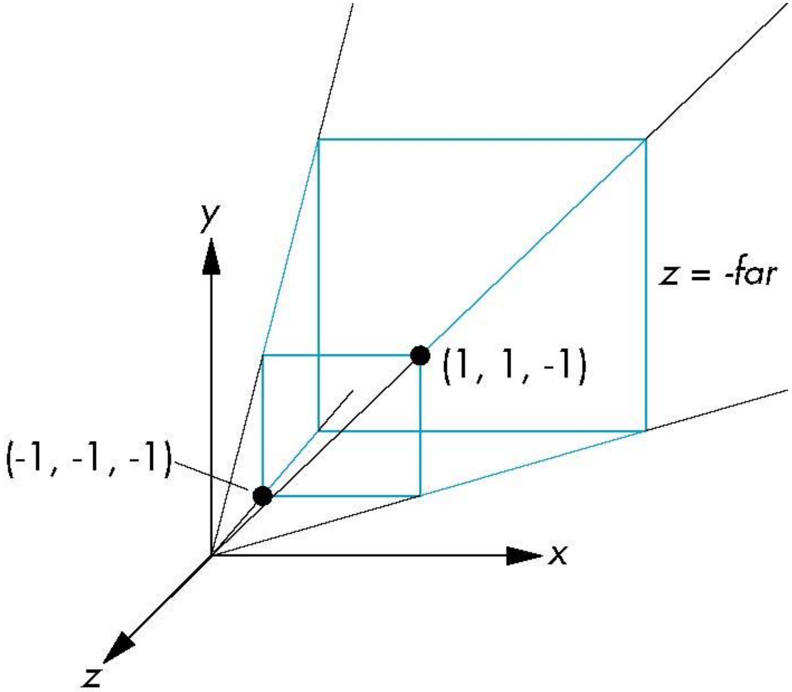
$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$

$$\mathbf{q} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad \Rightarrow \quad \mathbf{p} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

# Simple Perspective

Consider a simple perspective with the COP at the origin, the near clipping plane at $z$ = -1, and a 90 degree field of view determined by the planes

$x = \pm z, y = \pm z$



# Perspective Matrices

Simple projection matrix in homogeneous coordinates

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$