01.12.19

Final Starts

Lec-01

Furier → One dimensional (sound) (main)

→ Two dimensional (image)

→ Three " (video)

basic signals → sin & cos wave

Signal: Signal produces a information

Signal is something that include information. For example room temperature include information that is how cold it is and how hot it is.

Another example market price include information about the price over the type.

Sound of Sin wave

tune { Sin x → Sin 2x → Sin 3x

Sin x + sin 2x + sin 3x

new shape → voice

it will not match with the 3 signals individualy

System: System is something that can process signal or information.

Signal processing: The act of processing signal using a system is called signal processing.

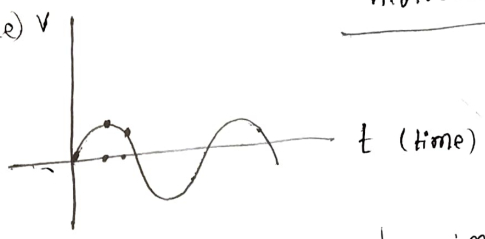
There are three types of Signals:

- 1D (sound) signal
- 2D (image) "
- 3D (video) "
  - → (continuous images)

1D signal : ~~As~~ Such as voice
↳ the most common

The most common way to represent the signal is value vs time In fact each point can be identified by a unique time.

Assume it as a voice



time & 1 value → 1D
time & 2 " → 2D
time & 3 " → 3D
~~or more~~

$f(t) = x(t)$

$t = 2 \quad x(2)$

$f(t) = \sin(t)$

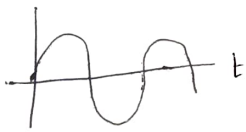
$f(t) = \sin(t)$

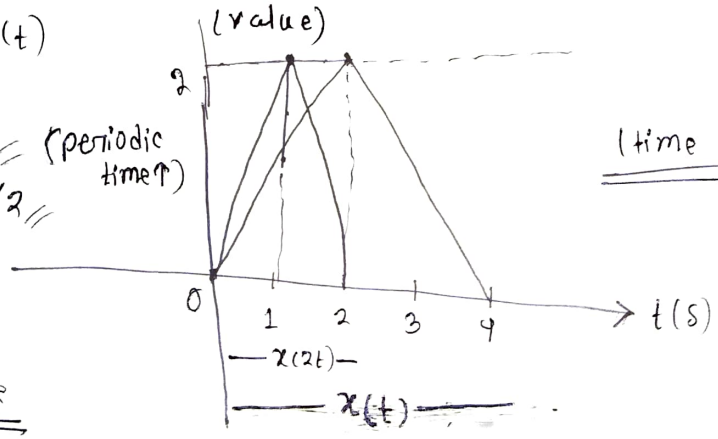
$x(t) = T$ (periodic time T)

$x(2t) = T/2$

$f(t) = T$

$f(2t) = \frac{T}{2}$





(time scaling)

Amplitude

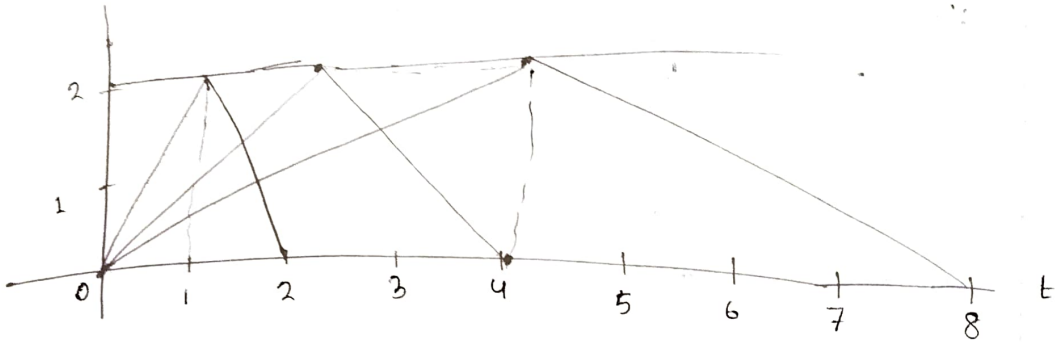
x - axis → domain

y - axis → amplitude (বিস্তার)

$x(t) = T$ (main signal)

$x(t/2) = 2T$

(X) mul → scaling
(+) add → shifting
(−) sub



$$\overbrace{a\overbrace{x(\overbrace{bt+c}^{1})}^{2}}^{3}+d$$

4 → Time Scaling (2nd)

2 → Time Shifting (1st)

1

Amplitude Scaling (3rd)

3

4 → Amplitude Shifting (4th)

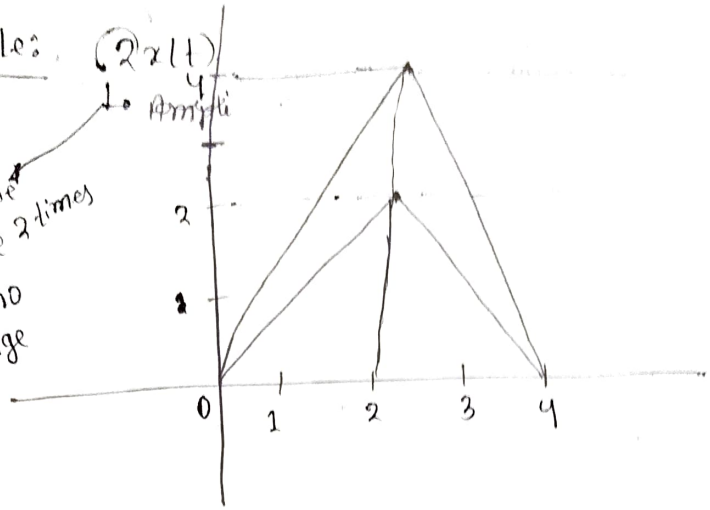
different type of signals

- $x(2t)$
- $2x(2t)$
- $2x(2t+1)-22$

Example: $2x(t)$

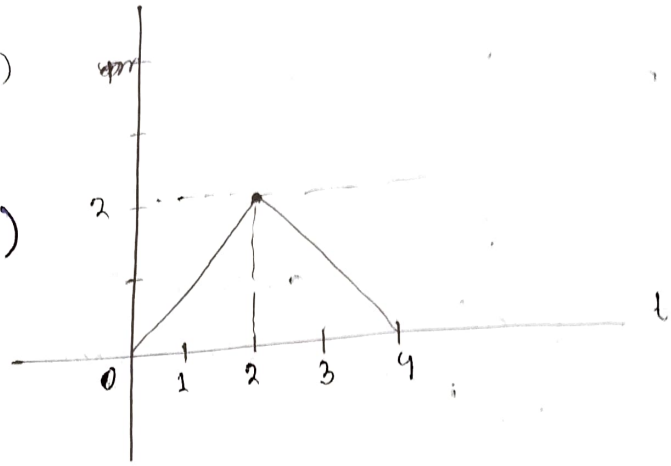
i) 1. Amplitude

Amplitude will be 2 times
time no change



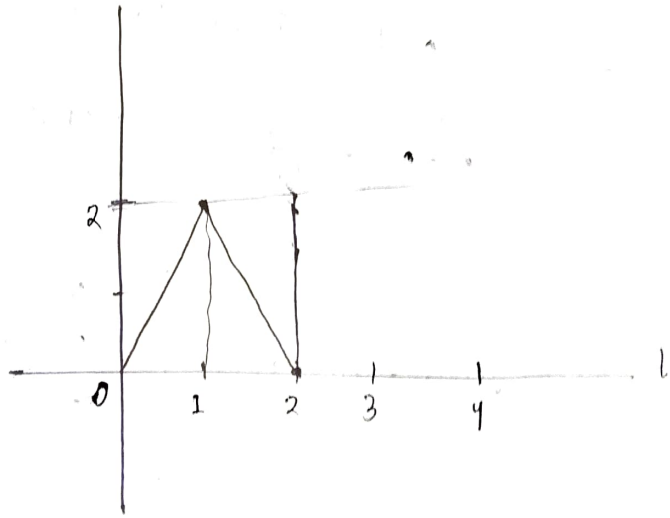
ii) $2x(2t)$

Step: 1
(Time Scaling)



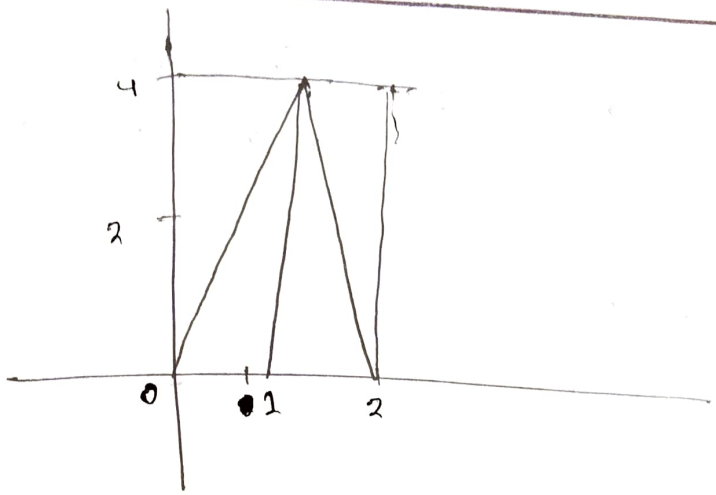
~~Step: 2~~
(~~Amplitude Scaling~~)

$\frac{T}{2}$

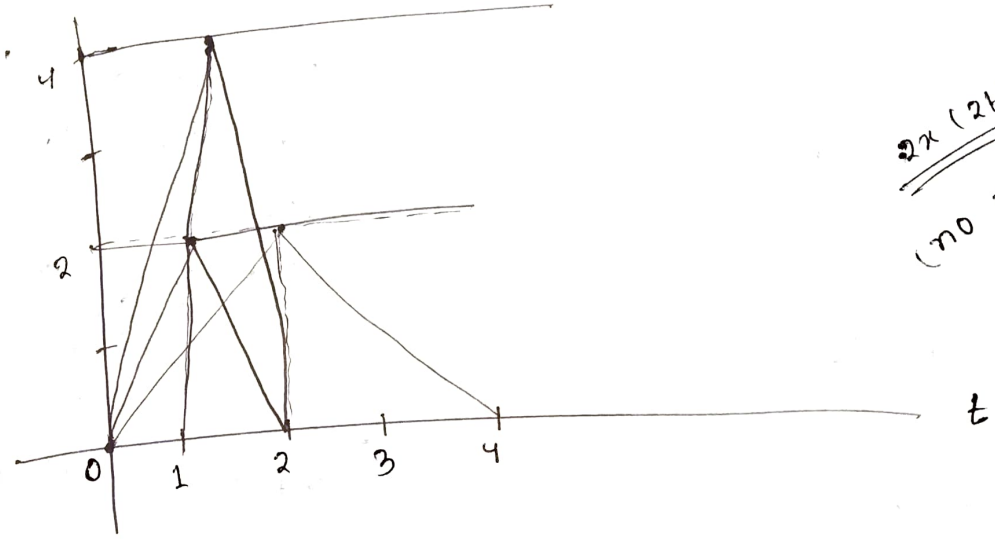


Step:2

(Amplitude Scaling)



End



$2x(2t)$

(no need)

$$\overset{3}{a}\times(\overset{2}{b}t+\overset{1}{c})+\overset{4}{d}$$

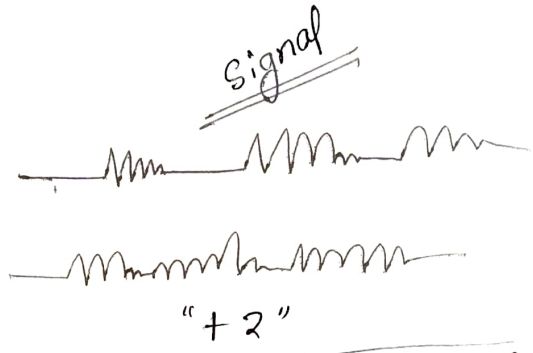
(inside bracket → time
outside bracket → Amplitude)

Ex:

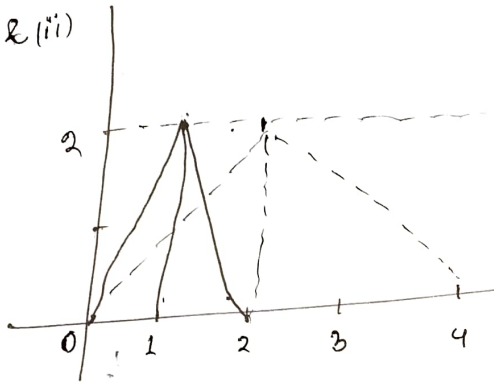
i) $x(t) = T$

ii) $x(2t) = T/2$

iii) $x(2t+2)$



(i) & (ii)

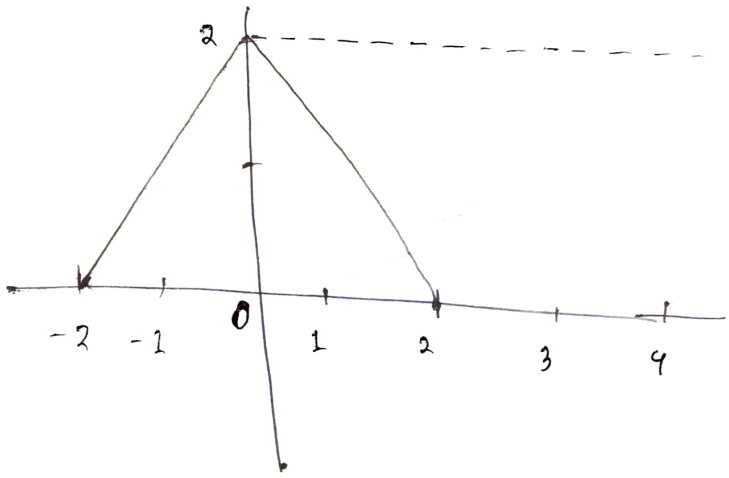


time shifting "+"
left (starts) (decrease)
time shifting "−"
right (starts)
(increase)

Shiffting → origin changes
Scaling → no change of origin

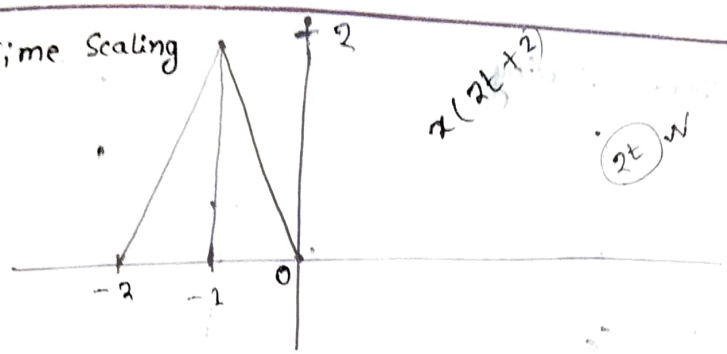
(iii) $x(2t+2)$

Step 01: Time Shifting



+2

Step 02: Time Scaling



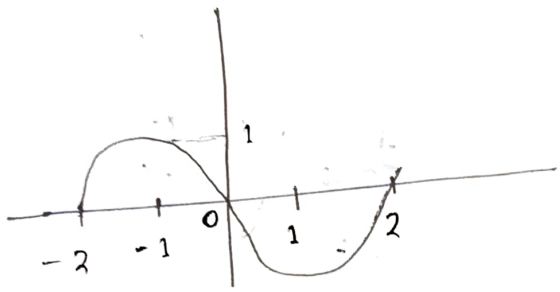
$x(2t+2)$

2t

Ex: $2x(2t+2)+2$

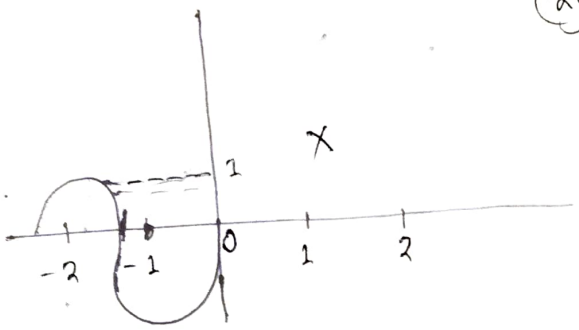
Step 01: Time Shifting

+2

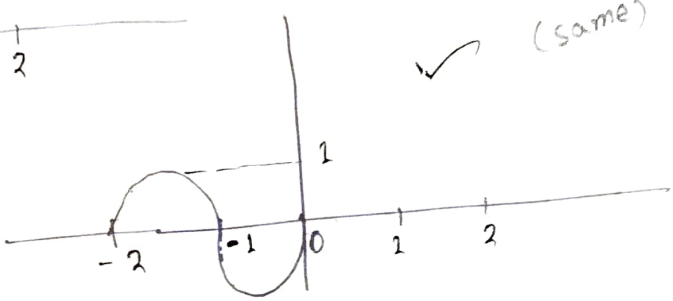


Step 02: Time Scaling

2t



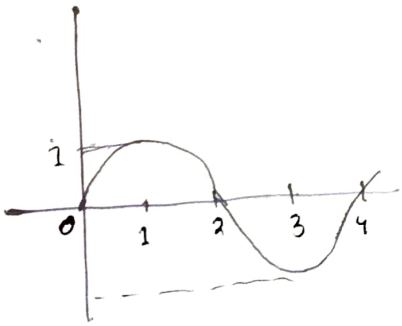
Same



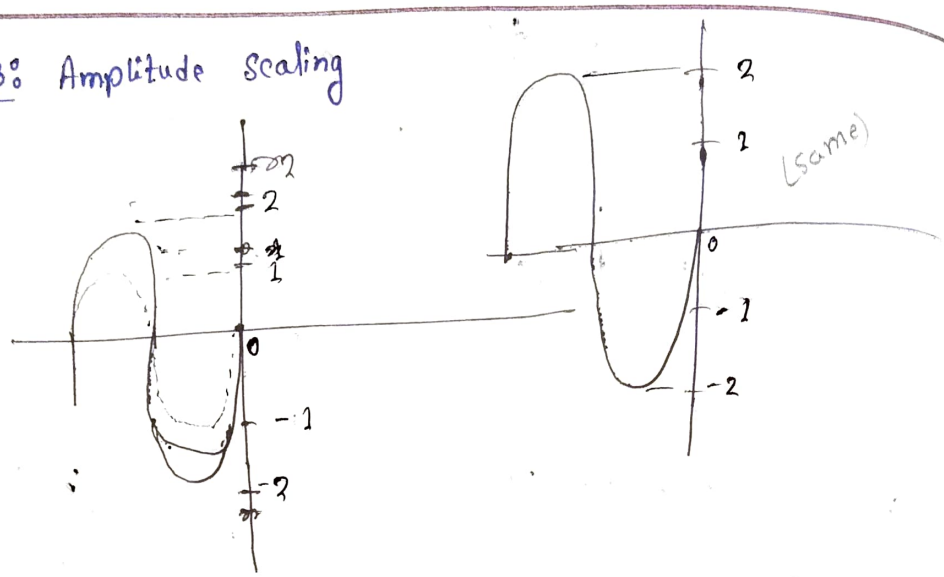
(same)

basic signal

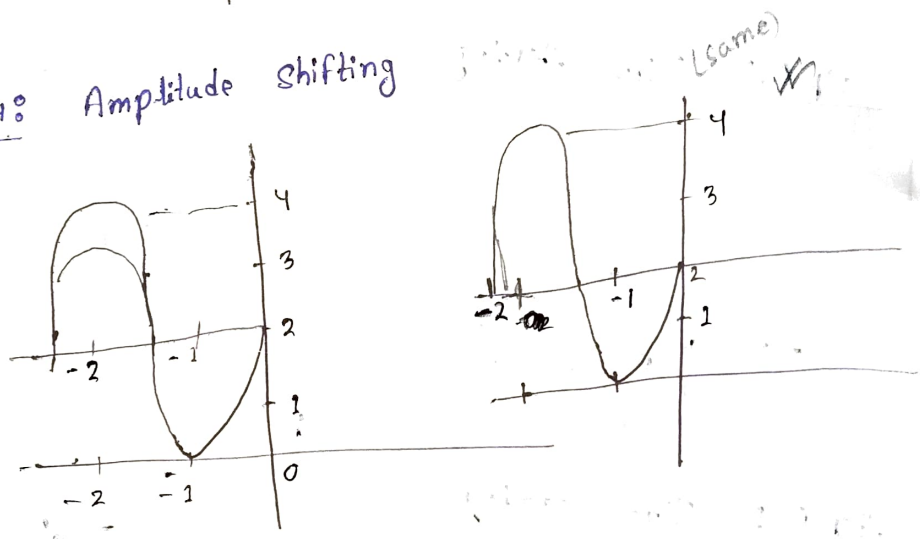
write a basic sion signal with amplitude 1 and time period or duration 4



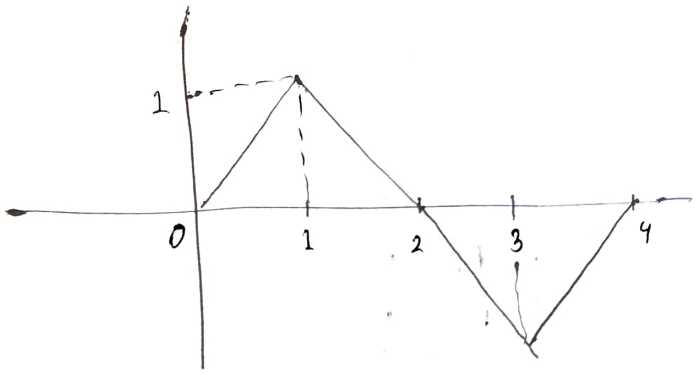
Step 03: Amplitude Scaling



Step 04: Amplitude Shifting

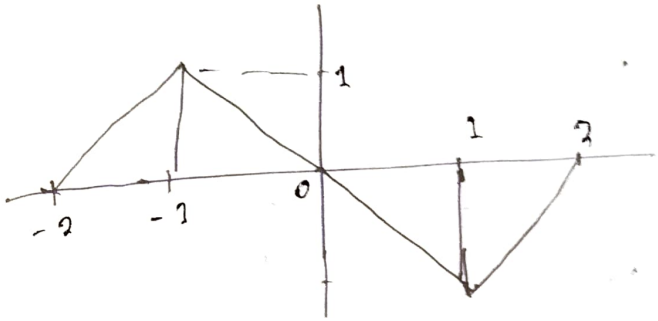


Ex: $x(t)$



$-3x(-t+2)+1$

Step 01: Time Shifting

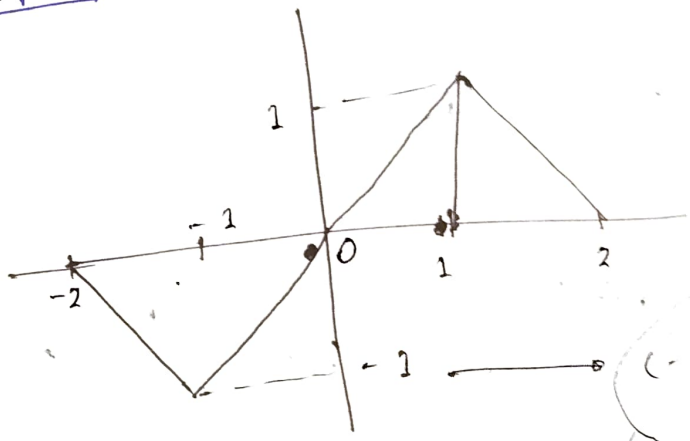


time → x axis change

Amplitude

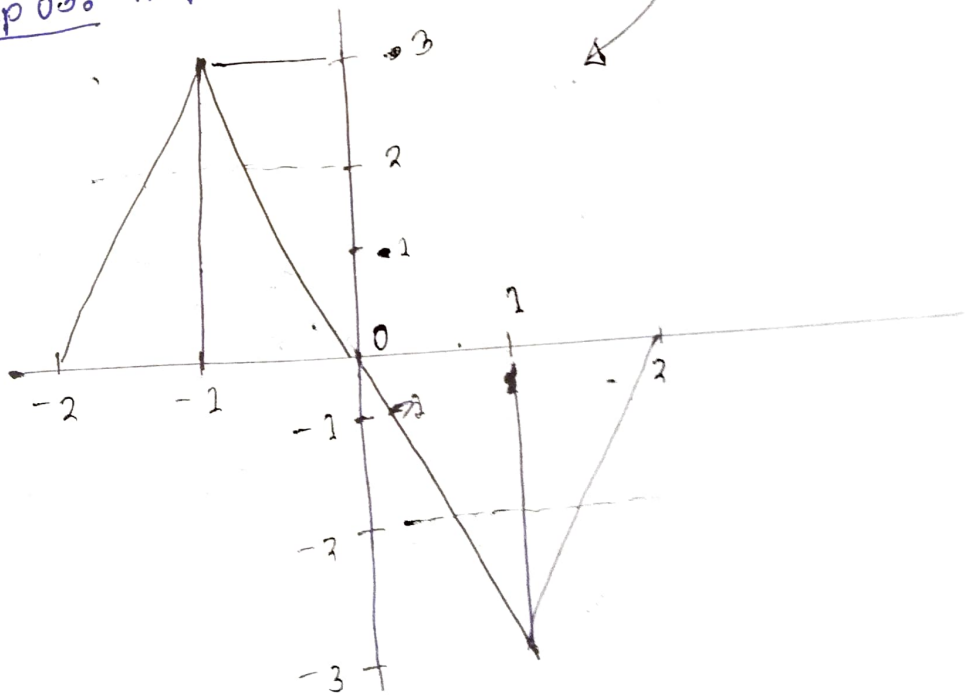
↳ y axis change

Step 02: Time Scaling

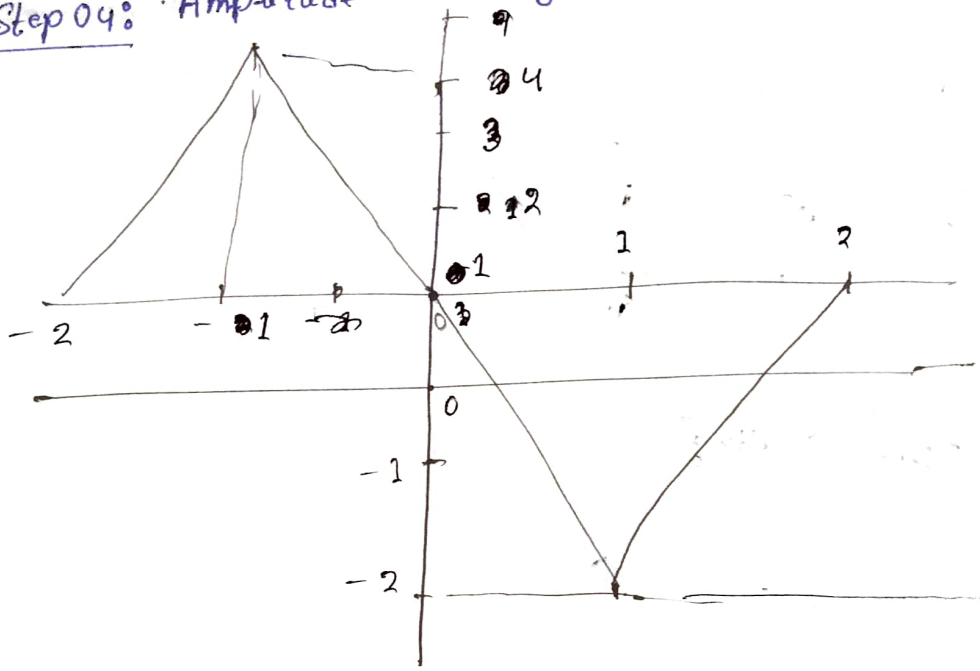


$(-1 \times -3) = 3$

Step 03: Amplitude Scaling

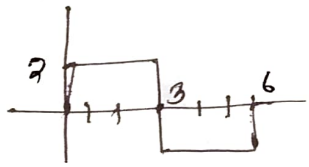


Step 04: Amplitude Shifting



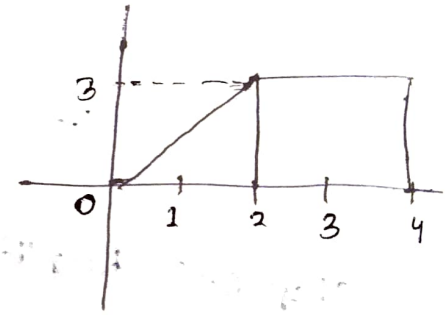
A.W.

#1



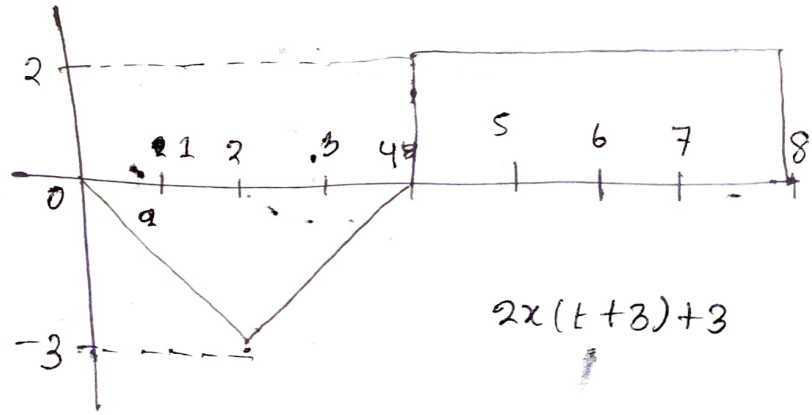
$2x(3t-3)+2$

#2



$3x(-2t+2)-2$

#3

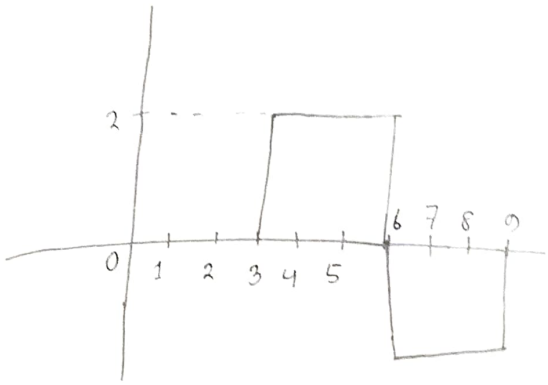


$2x(t+3)+3$

Solution 2

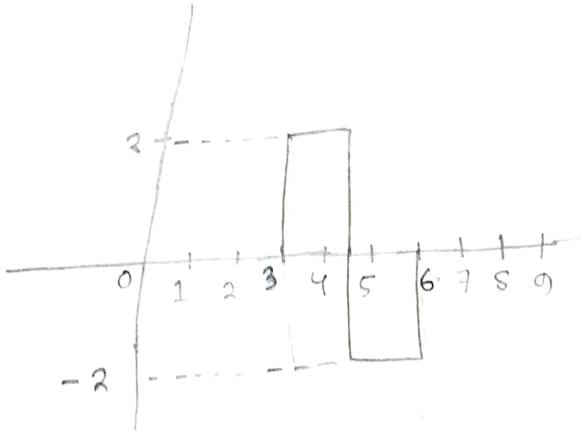
$2x(3t-3)+2$

Step: 01 time Shifting

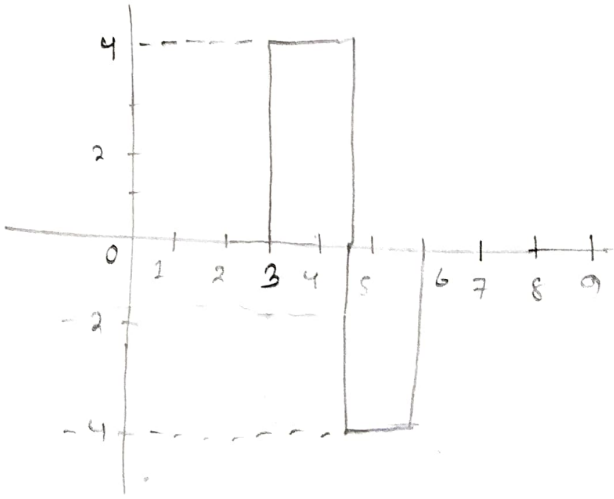


Step: 02 time Scaling

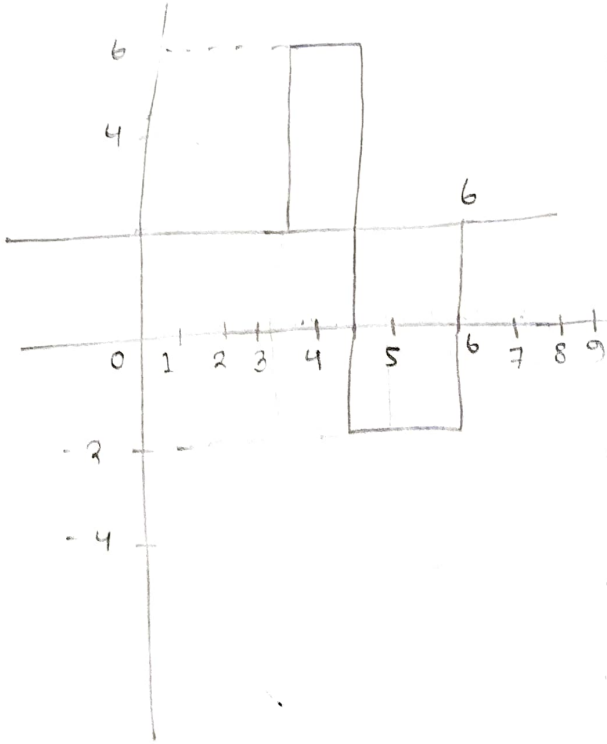
$\frac{6}{3} = 2$



Step: 03 Amplitude Scaling



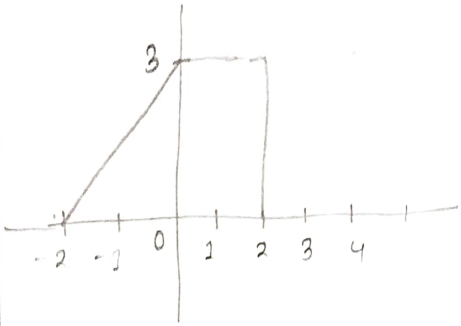
Step: 04 Amplitude Shifting



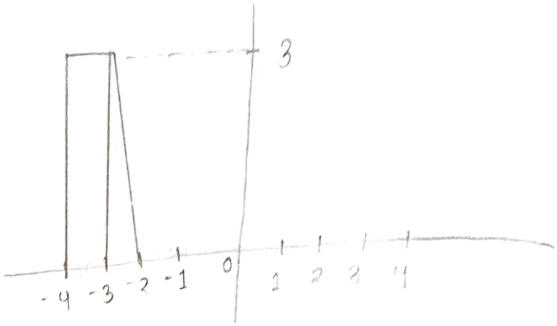
$\frac{4}{-2} = -2$

$3x(-2t+2)-2$

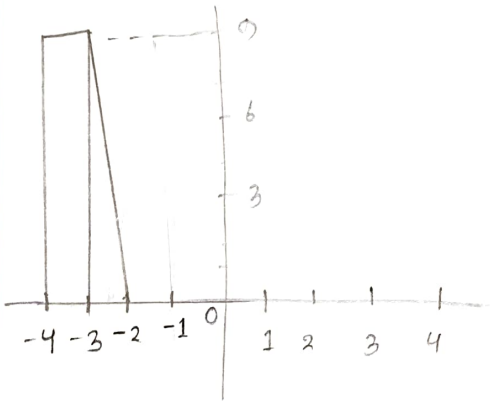
Step:01 Time shifting



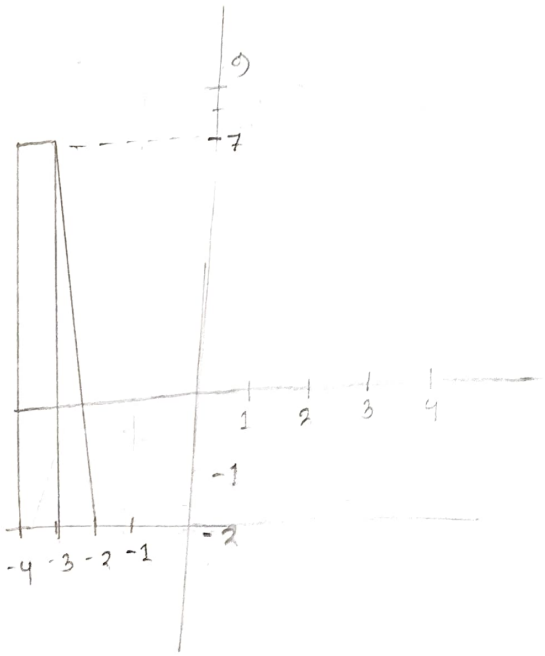
Step:02 Time scaling



Step:03 Amplitude scaling

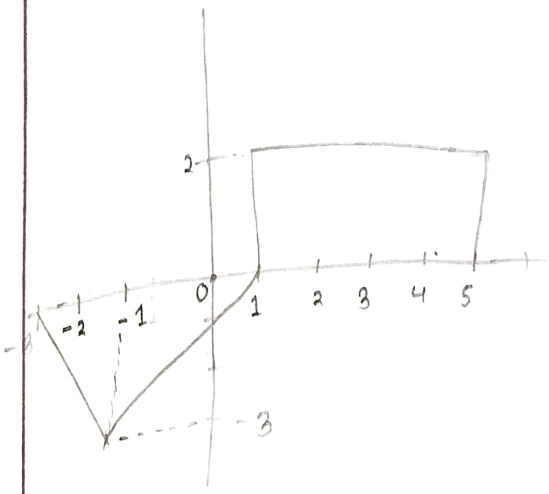


Step:04 Amplitude shifting

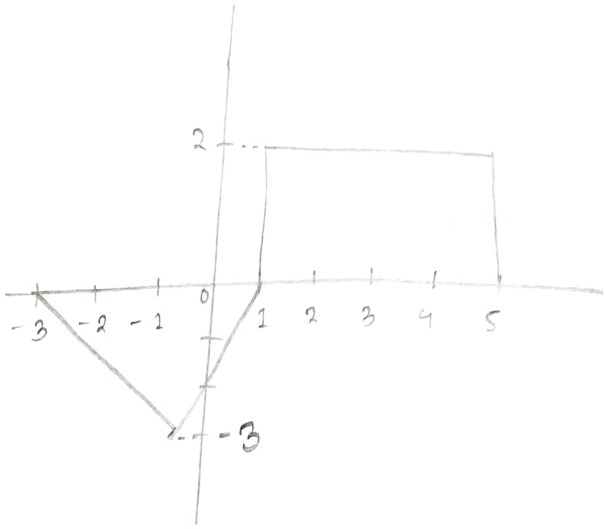


$2x(t+3)+3$

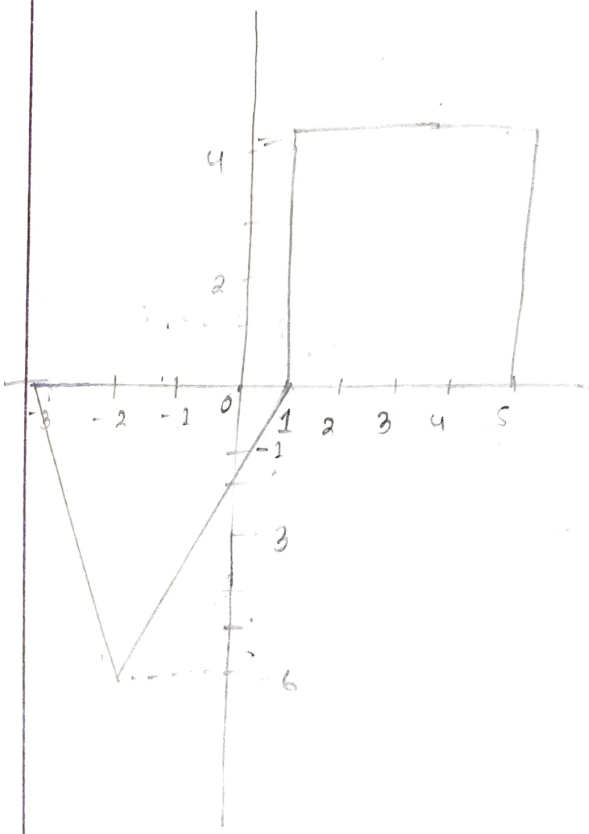
Step 01 Time shifting



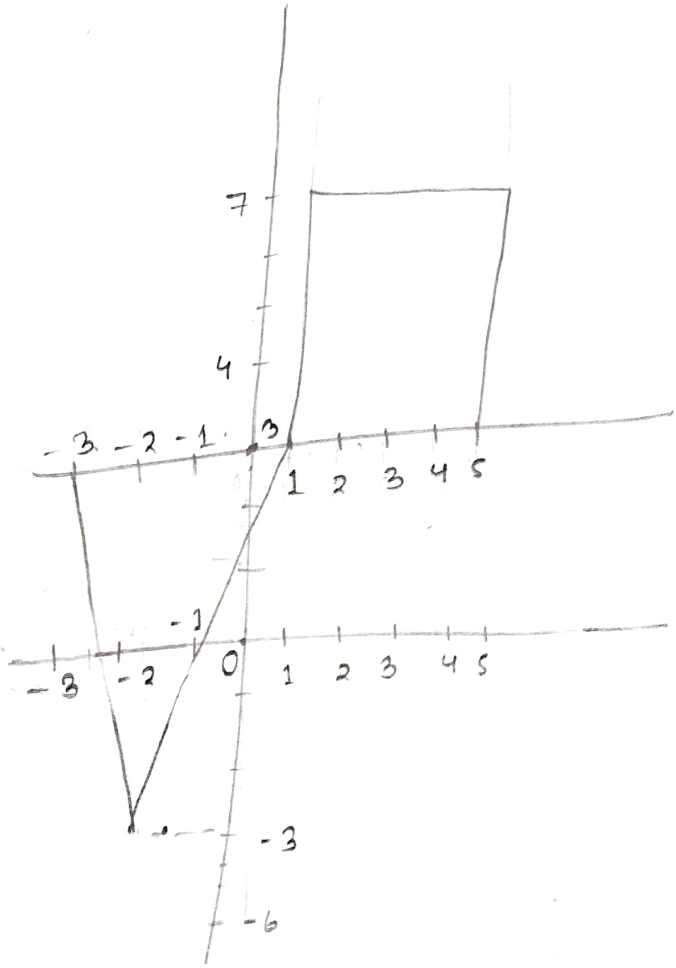
Step 02 Time Scaling



Step 03 Amplitude scaling



Step 04 Amplitude shifting



05-12-19

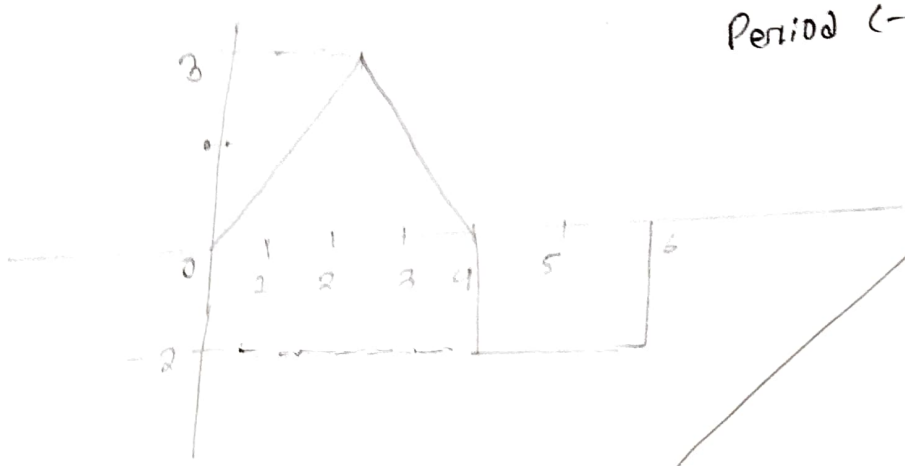
Lec-05

Example:01 $x(t)$

$-2x(-2t+2)-3$

Period (+) ve = Origin →

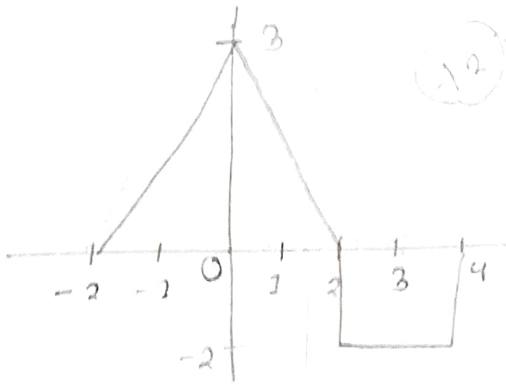
Period (−) ve = ← Origin



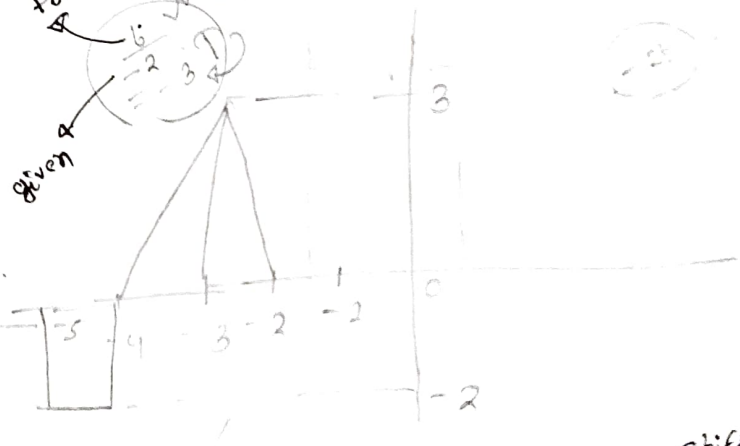
total period

given

Step 01: Time Shifting

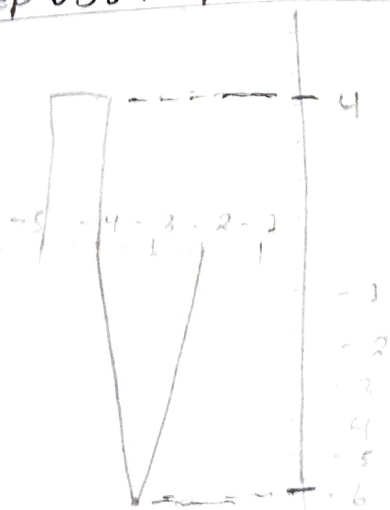


Step 02: Time Scaling



Step 03: Amplitude Scaling

$-2\times3 = -6$

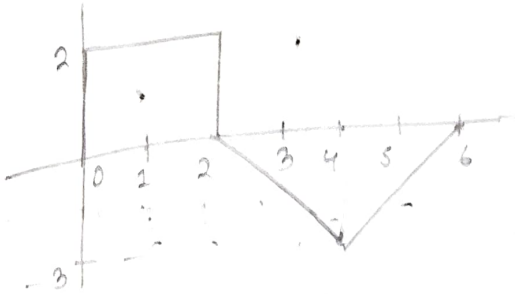


$(-2\times3 = -6)$ W

$(-2)\times(-2) = 4$ W

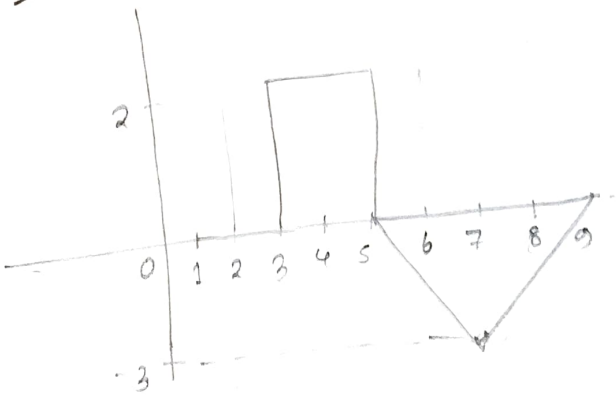
Step 04: Amplitude shifting

Example: 02 $2x(-2t-3)+4$

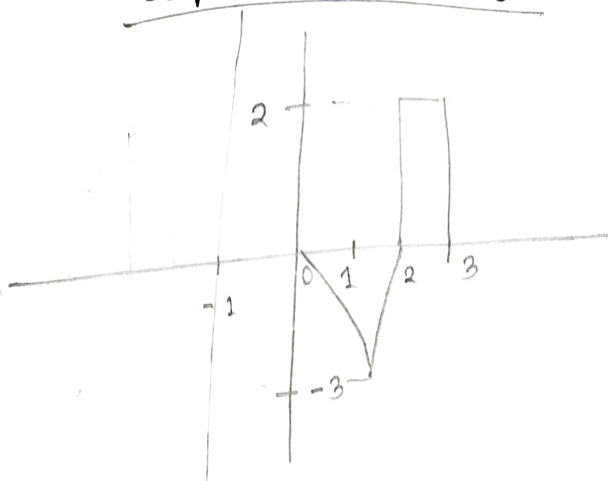


$\frac{6}{-2}$ -3

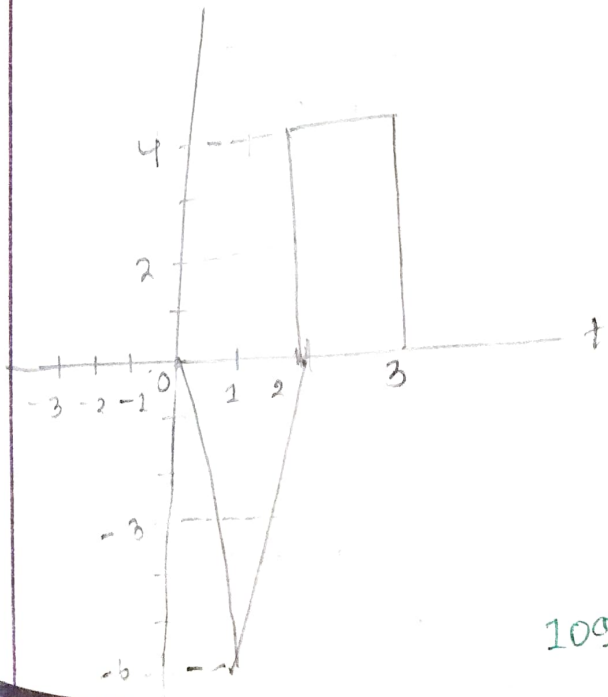
Step 01: Time Shifting



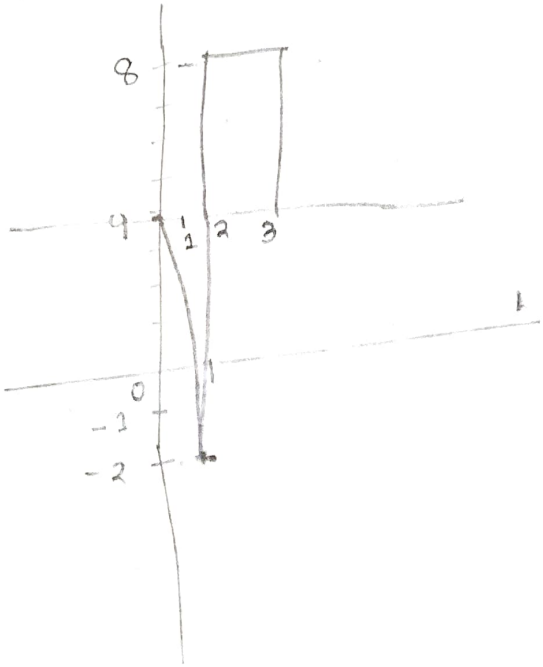
Step 02: Time Scaling



Step 03: Amplitud Scaling



Step 04: Amplitude Shifting

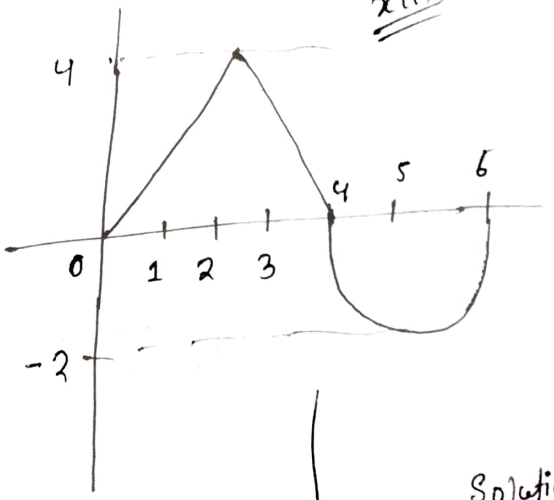


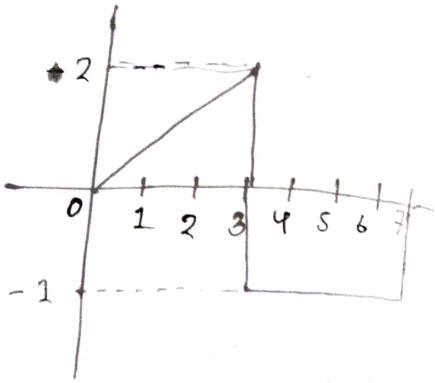
A.W

① # $-2x(-t+3)-1$

② # $-3x(2t+3)-4$

$x(t)$

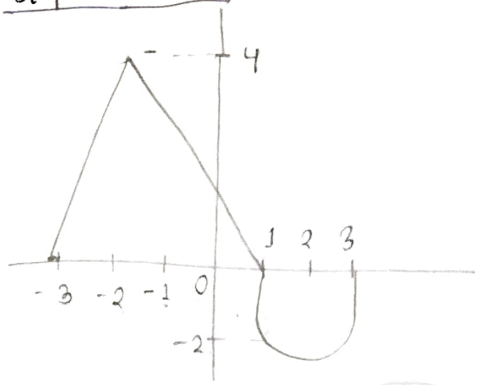




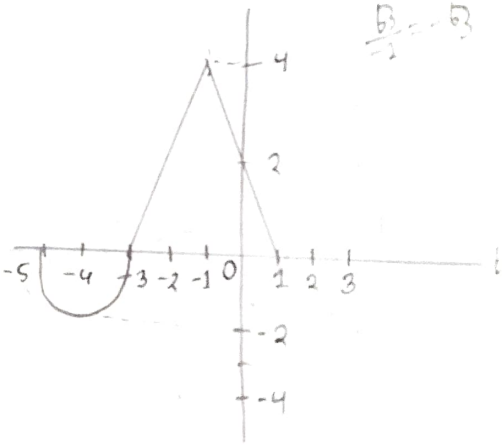
Solution

①

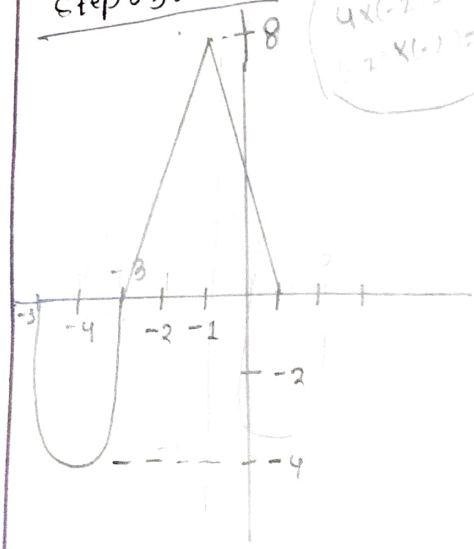
Step 01: T. Sh



Step 02: T. Se



Step 03: A. Se

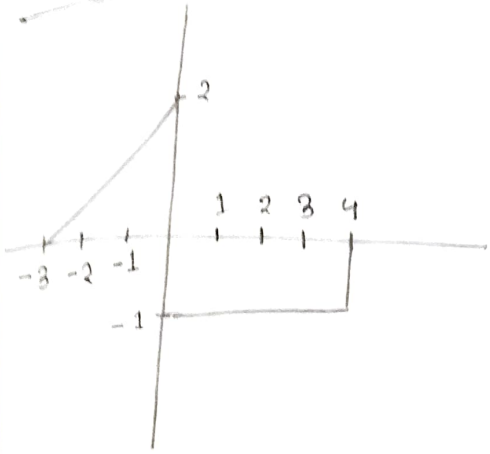


Step: A. Sh

(2) # $-3x(2t+3)-4$

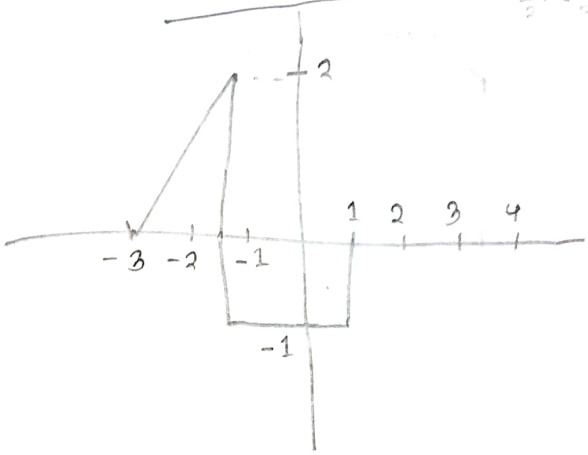
(i) → T.Sc

St-1: T.Sh

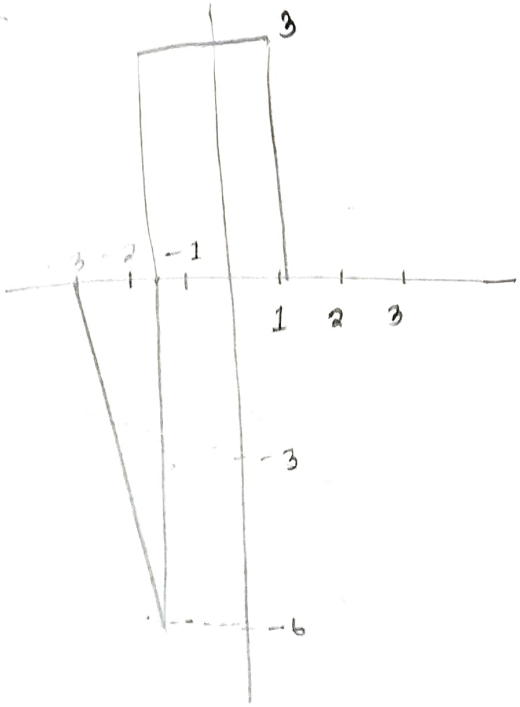


St-2: T.Sc

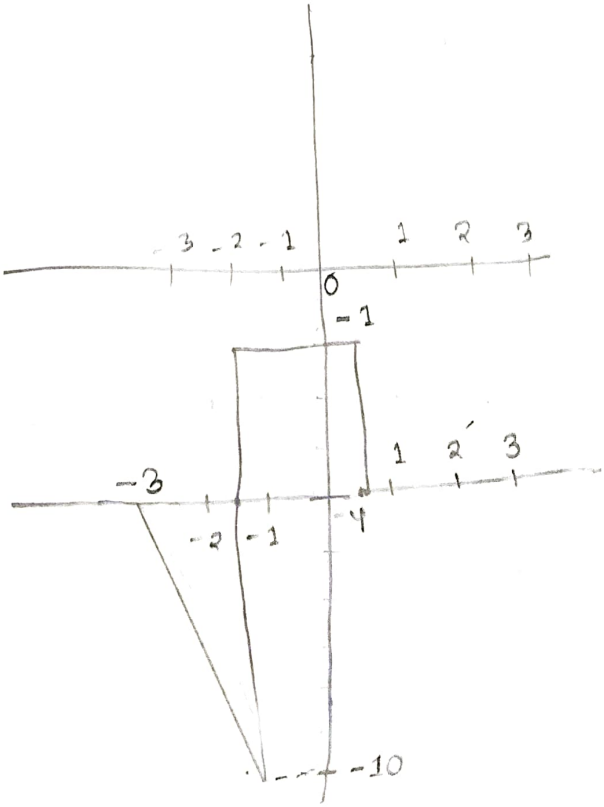
$\frac{7}{2} = 3.5$



St-3: A.Sc



St-4: A.Sh



Periodic Function

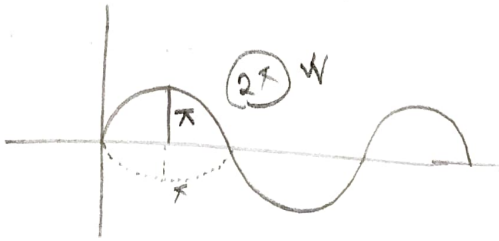
A function $f(x)$ is said to be periodic if there exist a positive integer 'T' such that $f(x+T) = f(x)$

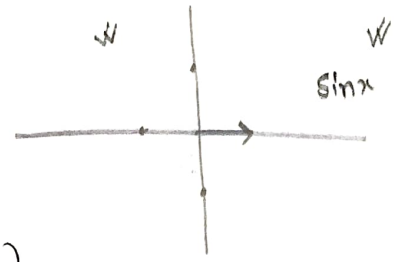
Example: $\sin x$, $\cos x$, $\tan x$ etc.

Q:
Prove that $\sin x$ is a periodic signal having period 2π.

Soln: let, $f(x) = \sin x$

$$f(x+2\pi) = \sin(x+2\pi)$$
$$= \sin(2\pi + x)$$
$$= \sin(4 \cdot \tfrac{\pi}{2} + x)$$
$$= \sin x$$
$$= f(x)$$





Again, $f(x+2\pi+2\pi) = \sin(x+2\pi+2\pi)$

$$= \sin(4\pi + x)$$
$$= \sin(8 \cdot \tfrac{\pi}{2} + x)$$
$$= \sin x$$

$$\therefore f(x+4\pi) = f(x)$$

So, we can write $f(x+2\pi) = f(x+4\pi) = f(x+6\pi) = \cdots = f(x)$

So $\sin x$ is a periodic function having period 2π.

Q:
Prove that $\sin 2x$ is a periodic function.

Soln: Given,

$$f(x) = \sin 2x$$

We know,

the period of $\sin x = 2\pi$

So, the " " $\sin 2x = \frac{2\pi}{2} = \pi$

$$\therefore f(x+\pi) = [\sin 2(x+\pi)]$$
$$= \sin(2x+2\pi)$$
$$= \sin(4 \cdot \tfrac{\pi}{2} + 2x)$$
$$= \sin 2x$$
$$= f(x)$$

π add (unlimited)

$\downarrow$

~~sin~~ $f(x)$

$\downarrow$ Period

$f(x) = \sin 3x = \left(\frac{2\pi}{3}\right)$

$f(x + \frac{2\pi}{3}) = \left[\sin 3\left(x + \frac{2\pi}{3}\right)\right]$

N.B:

$\sin x = 2\pi$ $\quad$ $\sin x + \sin 2x =$ different signal

$\sin 2x = \pi$

$\downarrow$

$\underbrace{2+5}_{\text{2 diff values}} = 8$ (diff value)

If the period of $f(x)$ is T then the period of $f(ax)$ is T/a. Now, if the period of $f_1(x)$ is T_1 and if the periode of $f_2(x)$ is T_2, then the period of $f_1(x) + f_2(x)$ is the L.C.M of the period of T_1 and T_2.

Q:
Find the period of $\sin x + \sin 2x$ and prove that it is periodic function.

Soln: Here,

$f_1(x) = \sin x$

Period $= 2\pi$

$f_2(x) = \sin 2x$

Period $= \pi$

let, $f(x) = \sin x + \sin 2x$

$$f(x+2\pi) = \sin(x+2\pi) + \sin 2.(x+2\pi)$$
$$= \sin(2\pi+x) + \sin(4\pi+2x)$$
$$= \sin x + \sin 2x$$
$$= f(x)$$

Again, $f(x+2\pi+2\pi) = \sin(x+2\pi+2\pi) + \sin. 2(x+2\pi+2\pi)$

$$= \sin(4\pi+x) + \sin(8\pi+2x)$$
$$= \sin x + \sin 2x$$
$$= f(x)$$

So, we can write $f(x+2\pi) = f(x+4\pi) = f(x+6\pi) = \cdots = f(x)$

So, it is a periodic function.

A.W-3

① # Prove that $\cos x$ is a period function having period 2π.

Soln:

let, $f(x) = \cos x$

$$f(x+2\pi) = \cos(x+2\pi)$$
$$= \cos(2\pi + x)$$
$$= \cos(4 \cdot \frac{\pi}{2} + x)$$
$$= \cos x$$
$$= f(x)$$

Again, $f(x+2\pi+2\pi) = \cos(x+2\pi+2\pi)$
$$= \cos(4\pi + x)$$
$$= \cos(8 \cdot \frac{\pi}{2} + x)$$
$$= \cos x$$
$$= f(x)$$

So, we can write $f(x+2\pi) = f(x+4\pi) = f(x+6\pi) = \cdots = f(x)$

So, $\cos x$ is a period function having period 2π.

(Proved)

② # Prove that $\tan x$ is a periodic function having period π.

Soln:

let, $f(x) = \tan x$

$$f(x+\pi) = \tan(x+\pi)$$
$$= \tan(\pi + x)$$
$$= \tan x$$
$$= f(x)$$

Again, $f(x+\pi+\pi) = \tan(x+\pi+\pi)$
$$= \tan(2\pi + x)$$
$$= \tan(4 \cdot \tfrac{\pi}{2} + x)$$
$$= \tan x$$
$$= f(x)$$

So, we can write $f(x+\pi) = f(x+2\pi) = f(x+3\pi) = \cdots = f(x)$

So $\tan x$ is a periodic function having period π.

(Proved)

③ # Prove that $\sin 5x$ is a periodic function.

Soln: We know,

Period of $\sin x$ is 2π

Now let, $f(x) = \sin 5x$

so, the period of $\sin 5x = \frac{2\pi}{5}$

$$\therefore f\left(x + \frac{2\pi}{5}\right) = \sin 5 \cdot \left(x + \frac{2\pi}{5}\right)$$
$$= \sin(5x + 2\pi)$$
$$= \sin(2\pi + 5x) \quad / \quad \sin\left(4 \cdot \frac{\pi}{2} + 5x\right)$$
$$= \sin 5x = f(x)$$

Again, $f(x+\frac{2\pi}{5}+\frac{2\pi}{5}) = \sin 5\left(x+\frac{4\pi}{5}\right)$

$= \sin(5x+4\pi)$

$= \sin(8\cdot\frac{\pi}{2}+5x)$

$= \sin 5x$

So, $\sin 5x$ is a periodic function.

Proved

(4) Prove that $\cos 9x$ is a periodic function.

Soln: Given, $f(x) = \cos 9x$

We know, the period of $\cos x = 2\pi$

So, " " " $\cos 9x = \frac{2\pi}{9}$

$\therefore f(x+\frac{2\pi}{9}) = \cos 9\cdot\left(x+\frac{2\pi}{9}\right)$

$= \cos(9x+2\pi)$

$= \cos(4\cdot\frac{\pi}{2}+9x)$

$= \cos 9x$

$= f(x)$

So, $\cos 9x$ is a periodic function.

Proved

⑤ # Prove that $\sin 6x$ is a periodic function.

Soln: Given,

$$f(x) = \sin 6x$$

We know,

the period of $\sin x = 2\pi$

So, " " of $\sin 6x = \frac{2\pi}{6} = \frac{\pi}{3}$

$$\therefore f\left(x+\frac{\pi}{3}\right) = \sin 6\cdot\left(x+\frac{\pi}{3}\right)$$
$$= \sin(2\cdot\pi + 6x)$$
$$= \sin 6x$$
$$= f(x)$$

So, $\sin 6x$ is a periodic function.

Proved

⑥ # Prove that $\tan 3x$ is a periodic function.

Soln: Given, $f(x) = \tan 3x$

We know, the period of ~~sin~~ $\tan x = \pi$

So, " " of $\tan 3x = \frac{\pi}{3}$

$$\therefore f\left(x+\frac{\pi}{3}\right) = \tan 3\left(x+\frac{\pi}{3}\right)$$
$$= \tan(\pi + 3x)$$
$$= \tan 3x$$

So, $\tan 3x$ is a periodic function.

(Proved)

⑦ # Prove that $\cot 7x$ is a periodic function.

Soln: Given, $f(x) = \cot 7x$

We know,

the period of $\cot x = \pi$

So, " " of $\cot 7x = \frac{\pi}{7}$

$$\therefore f\left(x+\frac{\pi}{7}\right) = \left[\cot 7\cdot\left(x+\frac{\pi}{7}\right)\right]$$

$$= \cot(\pi + 7x)$$

$$= \cot 7x$$

$$= f(x)$$

So, $\cot x$ is a periodic function. (Proved)

⑧ # Prove that $\sin 2x + \sin x + \cos x$ is a periodic function.

Soln: Here,

$f_1(x) = \sin 2x$

Period $= \frac{2\pi}{2} = \pi$ $[\because \sin x = 2\pi]$

$f_2(x) = \sin x$

Period $= 2\pi$

$f_3(x) = \cos x$

Period $= 2\pi$

L.C.M

$$\begin{array}{r|l} \pi & \pi, 2\pi, 2\pi \\ \hline & 1, 2, 2 \end{array}$$

$= 4\pi$

let. $f(x) = \sin 2x + \sin x + \cos x$

$$f(x+4\pi) = \sin 2\cdot(x+4\pi) + \sin(x+4\pi) + \cos(x+4\pi)$$

$$= \sin(8\pi + 2x) + \sin(4\pi + x) + \cos(4\pi + x)$$

$$= \sin(4\cdot 2\pi + 2x) + \sin(2\cdot 2\pi + x) + \cos(2\cdot 2\pi + x)$$

$$= \sin 2x + \sin x + \cos x$$

So, Given function is a periodic function.

(Proved)

Periods

$$\begin{cases} \sin x = 2\pi \\ \cos x = 2\pi \\ \tan x = \pi \\ \cot x = \pi \\ \sec x = 2\pi \\ \operatorname{cosec} x = 2\pi \end{cases}$$

09.12.19

Odd & Even Function

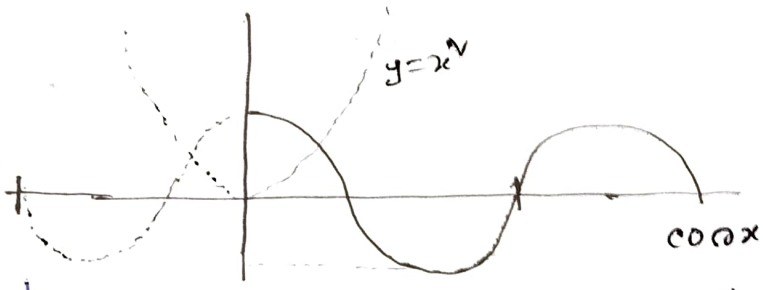
Lec-05

Even function (+):

A function is said to be even if $f(-x) = f(x)$.

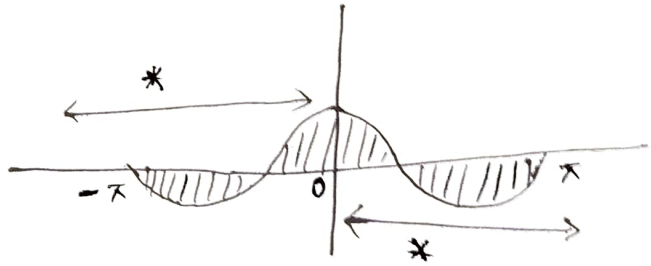
Example: Any algebric even power function (x^2, x^4, x^6 ... etc.), $\cos x, \sec x, |x|$ etc.

An even function is symmetric about Y axis.



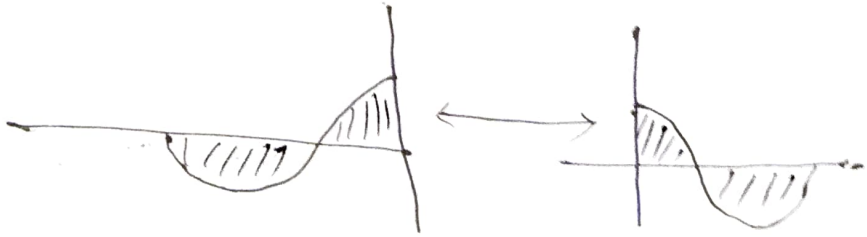
For even function $\int_{-\pi}^{\pi} f(x)\,dx = 2\int_{0}^{\pi} f(x)\,dx$

$y = \cos x$



$\int_{-\pi}^{\pi} f(x)\,dx$

* This two area are same so we can write 2. One area $= 2\int_{0}^{\pi} f(x)\,dx$



Q: Prove that x^2 is an even function.

Soln: let, $f(x) = x^2$

$f(-x) = (-x)^2$

$= x^2 = f(x)$

$\therefore f(-x) = f(x)$

so, $f(x) = x^2$ is an even function. (Proved)

Q: Prove that $\cos x$ is an even function.

Soln: let, $f(x) = \cos x$

$f(-x) = \cos(-x)$

$= \cos x = f(x)$

$\therefore f(-x) = f(x)$

so, $f(x) = \cos x$ is an even function. (Proved)

Odd function (-):

A function $f(x)$ is said to be odd if $f(-x) = -f(x)$

Example: any algebric odd power function ($x, x^3, x^5, \ldots$ etc.), $\sin x$, $\tan x$ etc.

Odd function is symmetric about origin.

N.B: For odd power function $\int_{-\pi}^{\pi} f(x)\,dx = 0$

Ex:1

$$\int_{-\pi}^{\pi} \sin x \, dx = \left[-\cos x\right]_{-\pi}^{\pi} = -\left[\cos\pi - \cos(-\pi)\right]$$

$$= -\left[\cos\pi - \cos\pi\right]$$

$$= 0$$

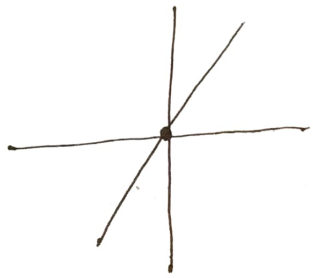
Ex:2

$$\int_{-\pi}^{\pi} x^3 dx = \left[\frac{x^4}{4}\right]_{-\pi}^{\pi} = \frac{1}{4}\left[\pi^4 - (-\pi)^4\right]$$

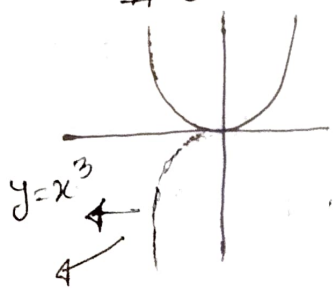
$$= \frac{1}{4}(\pi^4 - \pi^4)$$

$$= 0$$

$y = x$



$y = x^2$



it depends on origin

Q: Prove that x^3 is an odd function.

Soln: let, $f(x) = x^3$

$f(-x) = (-x)^3$

$f(-x) = -x^3$

$\therefore f(-x) = -f(x)$

So, $f(x) = x^3$ is an odd function.

(Proved)

Prove that tanx is an odd function.

Soln: let, $f(x) = \tan x$

$\Rightarrow f(-x) = \tan(-x)$

$\Rightarrow f(-x) = -\tan x$

$\therefore f(-x) = -f(x)$

So, $f(x) = \tan x$ is an odd function.

$\cos(-\theta) = \cos\theta$

$\sec(-\theta) = \sec\theta$

Other's are "$-\theta$"

Ex: $x^2 \cdot \sin x = \text{odd}$

$+ \cdot - = -$

even $\cdot$ odd $=$ odd

$+ \cdot + = +$

$- \cdot - = +$

~~Proved~~

Odd function

$\int_{-\pi}^{\pi} f(x)\,dx = 0$

for math

5*

x^2 $\sin x$ odd

$+ \cdot - = -$

x^2 $\cos x$ even

$+ \cdot + = +$

x^3 $\tan x$ even

$- \cdot - = +$

$\sin x$ x^4 odd

$- \cdot + = -$

even $\rightarrow$ { differentiation, Integration } $\rightarrow$ Odd

Odd $\rightarrow$ { differentiation, Integration } $\rightarrow$ even

Examples:

$x^4 = 4x^3$

$\int x^4\,dx = \frac{x^5}{5}$