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Mid Term Answer Sheet

Course Code: CSE 223

Course Title: Numerical Analysis

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Answer to the question no 1(a)

Soln: Given,

F(x)= x^3 – x – 3 absolute error, E_a < 0.001

Here, x=0

∴ f(x)= 0 – 0 – 3= –3

f’(x)= 3x^3 – 1= 3×0 – 1= – 1

x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$	$x_{n+1} - x_n$
0	-3	-1	-3	-3
-3	-27	26	-1.9615	1.0385
-1.9615	-8.5853	10.5424	-1.1471	0.8144
-1.1471	-3.3623	2.9475	-0.0064	1.1407
-0.0064	-2.9936	-0.9999	-3.0003	-2.9939

∴ Using Newton-Rapson method the root is= -3.0003

Ans.

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Answer to the question no 1(b)

Soln: Given,

$$x\text{^}3 + 3x\text{^}2 + 12\,x + 8 = 0$$

Here, f(x)= $x\text{^}3 + 3x\text{^}2 + 12\,x + 8$

$f(x_1) = f(0) = 0 + 0 + 0 + 8 = 8$

$f(x_2) = f(-1) = (-1)\text{^}3 + 3 \times (-1)\text{^}2 + 12 \times (-1) + 8 = -2$

$f(x_1) \times f(x_2) = 8 \times (-2) = -16 < 0$

x_1	x_2	$f(x_1)$	$f(x_2)$	$x_0 = \frac{x_1 + x_2}{2}$	$f(x_0)$	$x_2 - x_1$
0	-1	8	-2	-0.5	2.625	-1
-0.5	-1	2.625	-2	-0.75	0.2656	-0.5
-0.75	-1	0.2656	-2	-0.875	-0.8730	-0.25
-0.75	-0.875	0.2656	-0.8730	-0.8125	-0.3059	-0.125
-0.75	-0.8125	0.2656	-0.3059	-0.7813	-0.0212	-0.0625

∴ Using Bisection method root is= -0.7813 Ans.

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Answer to the question no 2(a)

Soln: Round off error occurs when a fixed number of digits are used to represent exact numbers. Rounding a number can be done in two ways. One is known as chopping the other is known as symmetric rounding.

Chopping

True x $= 538.68673$

$= 0.53868673 \times 10^3$

$= (0.5386 + 0.00008673)\ 10^3$

$= [0.5386 + (0.8673 \times 10^{-4})]\ 10^3$

This can be expressed in general form as

True x $= (f_x + g_x \times 10^{-d})\ 10^E$

$= f_x \times 10^E + g_x \times 10^{E-d}$

$= 0.5386 \times 10^3 + 0.8673 \times 10^{-4} \times 10^3$

Approximate x $= f_x \times 10^E$

$= 0.5386 \times 10^3$

Error $= g_x \times 10^{E-d}$

$= 0.8673 \times 10^{-1}$

Now, $0.5386\times 10^3 + 0.8673 \times 10^{-1}$

$= 538.68673$

$=$ x

<u>Symmetric Round off</u>

If $g_x < 0.5$ then approximate, x $= f_x \times 10^E$

IF $g_x > 0.5$ then approximate, x $= (f_x + 10^{-d})\times 10^E$

$= (0.5386 + 10^{-4}) \times 10^3$

$= (0.5386 + 0.0001) \times 10^3$

$= 0.5387 \times 10^3$

Error $= (g_x - 1) \times 10^{E-d}$

$= (0.8673 - 1) \times 10^{3-4}$

$= -0.1327 \times 10^{-1}$

Here,

$0.5387 \times 10^3 + (-0.1327 \times 10^{-1})$

$= 538.68673$

$=$ x Ans.

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Answer to the question no 2(b)

Numerical errors are introduced during the process of implementation of a numerical method. They come in two forms, round off errors and truncation errors.

Round off Error

Round off error occur when a fixed number of digits are used to represent exact numbers. Rounding errors often become obvious in

Percentages.

For example, if we have three types of cost to make a widget, and we want to see which cost is the largest portion of the total cost, we have the following list:

Cost A: 4.567%

Cost B: 85.654%

Cost C: 9.779 %

Total: 100.000%

Now, if we round those percentages to the nearest whole number, we get:

Cost A: 5%

Cost B: 86%

Cost C: 10 %

Total: 101%

So, here we can see the total is 101%. Which is not mathematically possible. The difference is due to rounding.

Truncation Error

Truncation errors arise from using an approximation in place of an exact mathematical procedure.

For example, The use of a number of discrete steps in the solution of a differential equation. Considering the following infinite series:

$$\sin \mathrm{x} = \mathrm{x} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

When we calculate the sine of an angle using this series, we cannot use all the terms in the series for computation. We usually terminate the process after a certain term calculated. The term “truncated” introduce an error which is called truncation error.

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Answer to the question no 3(a)

Secant method derivation:

Secant method, like the false position and bisection methods, uses two initial estimates but does not require that they must bracket the root.

For example, the secant method can use the points x1 and x2 as starting values, although they do not bracket the root.

Slope of the secant line passing through x_1 and x_2 is given by

$$\frac{\mathrm{f}(x_1)}{x_1 - x_3} = \frac{\mathrm{f}(x_2)}{x_2 - x_3}$$

$$\mathrm{f}(x_1)\,(x_2 - x_3) = \mathrm{f}(x_2)\,(x_1 - x_3)$$

or $$x_3[\,\mathrm{f}(x_2) - \mathrm{f}(x_1)] = \mathrm{f}(x_2)x_1 - \mathrm{f}(x_1)x_2$$

then $$x_3 = \frac{\mathrm{f}(x_2)x_1 - \mathrm{f}(x_1)x_2}{\mathrm{f}(x_2) - \mathrm{f}(x_1)} \text{ -------------(1)}$$

By adding and subtracting $\mathrm{f}(x_2)x_2$ to the numerator and rearranging the terms we get:

$$x_3 = \frac{x_2\mathrm{f}(x_2) - x_2\mathrm{f}(x_1) - x_2\mathrm{f}(x_2) + x_1\mathrm{f}(x_2)}{\mathrm{f}(x_2) - \mathrm{f}(x_1)}$$

$$= \frac{x_2\{\mathrm{f}(x_2) - \mathrm{f}(x_1)\} - \mathrm{f}(x_2)\,(x_2 - x_1)}{\mathrm{f}(x_2) - \mathrm{f}(x_1)}$$

$$= \frac{x_2\{f(x_2)-f(x_1)\}}{f(x_2)-f(x_1)} - \frac{f(x_2)\,(x_2 - x_1)}{f(x_2)-f(x_1)}$$

$$= x_2 - \frac{f(x_2)\,(x_2 - x_1)}{f(x_2)-f(x_1)}$$

$$\therefore x_3 = x_2 - \frac{f(x_2)\,(x_2 - x_1)}{f(x_2)-f(x_1)} \text{ ----------------(2)}$$

Equation (2) is the secant formula. If the secant line represents the linear interpolation polynomial of the function f(x) (with the interpolating points x_1 and x_2) then x_3, which intercepts the x-axis, represents the approximate root of f(x).

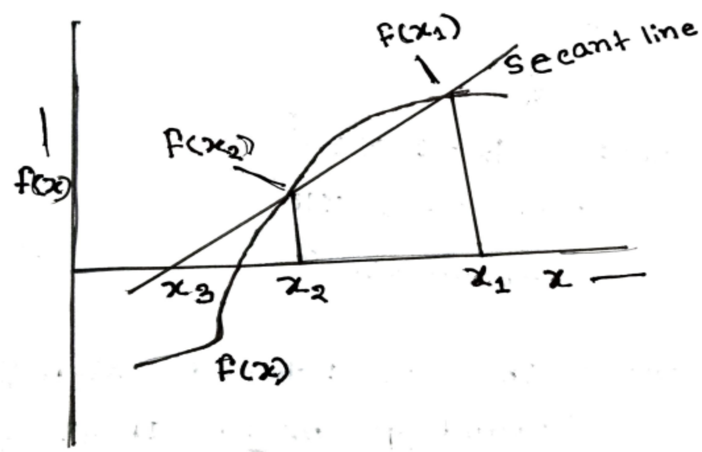

Figure: Graphical depiction of secant method

The approximate value of the root can be refined by repeating this procedure by replacing x_1, and x_2, by x_2, and x_3, respectively, in Eq. (2)

That is, next approximate value is given by

$$x_4 = x_3 - \frac{f(x_3)\,(x_3 - x_1)}{f(x_3) - f(x_1)}$$

This procedure is continued till the desired level of accuracy is obtained.

We can express the secant formula in general form as follows:

$$x_{i+1} = x_i - \frac{f(x_i)\,(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

This formula is use to find root of nonlinear equation.

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How secant method differs from false position method:

No.	Secant method	False position method
1	Number of iteration is need more than false position.	Number of iteration is less than secant method.
2	Need more time to find root than false position method.	Take less time to find root than secant method.
3	Maybe diverge.	Always Converge.
4	If converges, it does faster than false position.	Slower convergence than secant in case the secant converges.
5	$x_1\ replace\ x_2$ $x_2\ replace\ x_3$	x_0 replaces whichever of the original values yielded a function value with the same sign as f(x_0).
6	Formula: $x_3 = \frac{\mathrm{f}x_2\, x_1 - \mathrm{f}x_1 x_2}{\mathrm{f}x_2 - \mathrm{f}x_1}$	Formula: $x_0 = \frac{\mathrm{f}(x_2)x_1 - \mathrm{f}(x_1)x_2}{\mathrm{f}(x_2) - \mathrm{f}(x_1)}$

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Answer to the question no 3(b)

List of various methods available for finding roots of nonlinear equation is given below:

1. Direct analytical method
2. Graphical method
3. Trial and error method
4. Iterative method

We can group the iterative method in two categories:

i. Bracketing method.
   a. Bisection method
   b. False position method
ii. Open End method
   a. Newton Rapson method
   b. Secant method
   c. Muller's method
   d. Fixed point method
   e. Bairtow's method