Hidden **M**arkov **M**odel

CSE 467: Pattern Recognition

Let,

Bob and Alice are two friends

Bob’s mood changes based on weather

If it is sunny, bob is happy

If it is rainy, bob is grumpy

Alice is in a different place from Bob

Bob says to Alice via phone that he is happy today and Alice predicts today is sunny

Bob says to Alice via phone that he is grumpy today and Alice predicts today is rainy

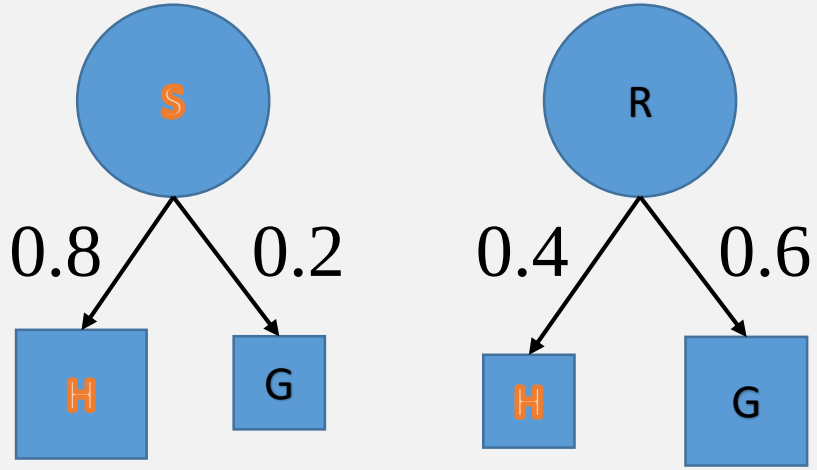
Let,

- Bob and Alice two friends
- Bob’s mood changes based on weather
- If it is sunny bob is happy
- If it is rainy bob is grumpy
- Alice is in a different place from Bob
- Bob says to Alice via phone that he is happy today and Alice predict today is sunny
- Bob says to Alice via phone that he is grumpy today and Alice predict today is rainy

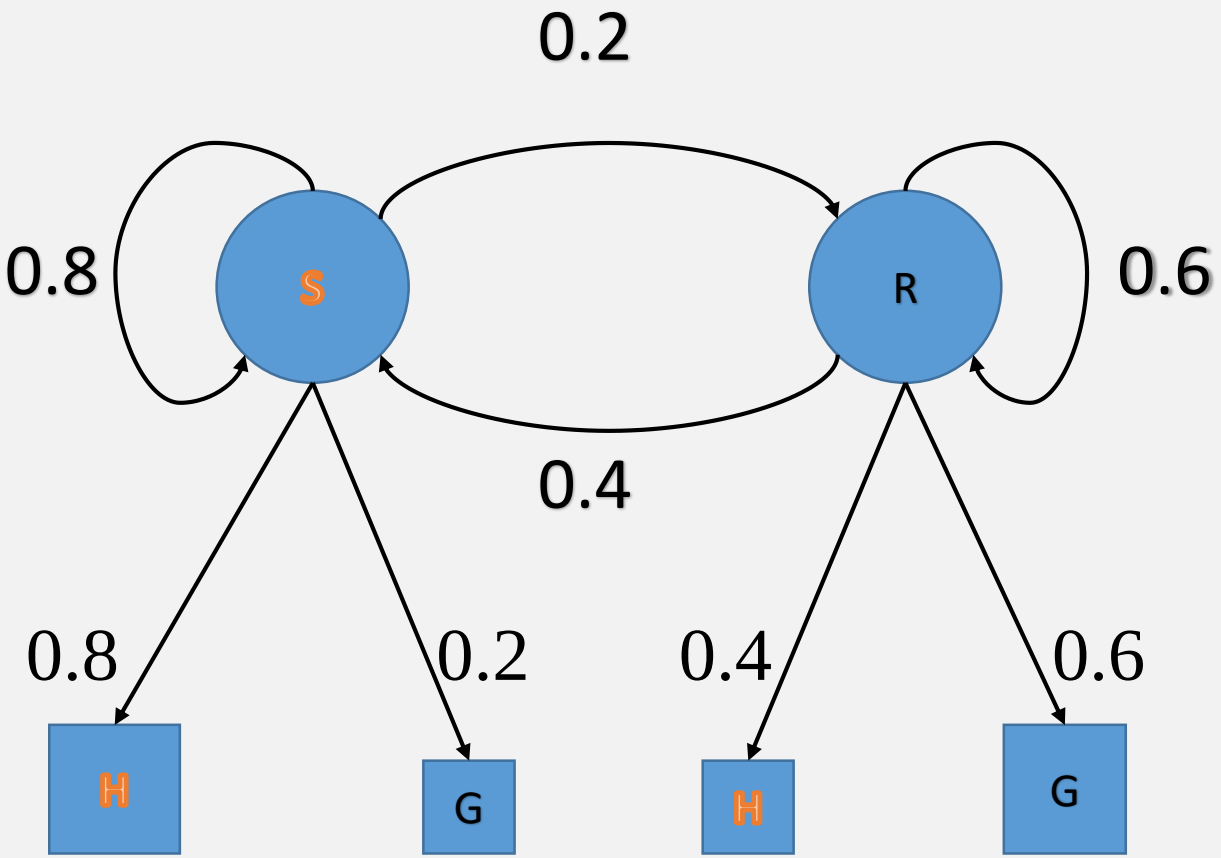
Let,

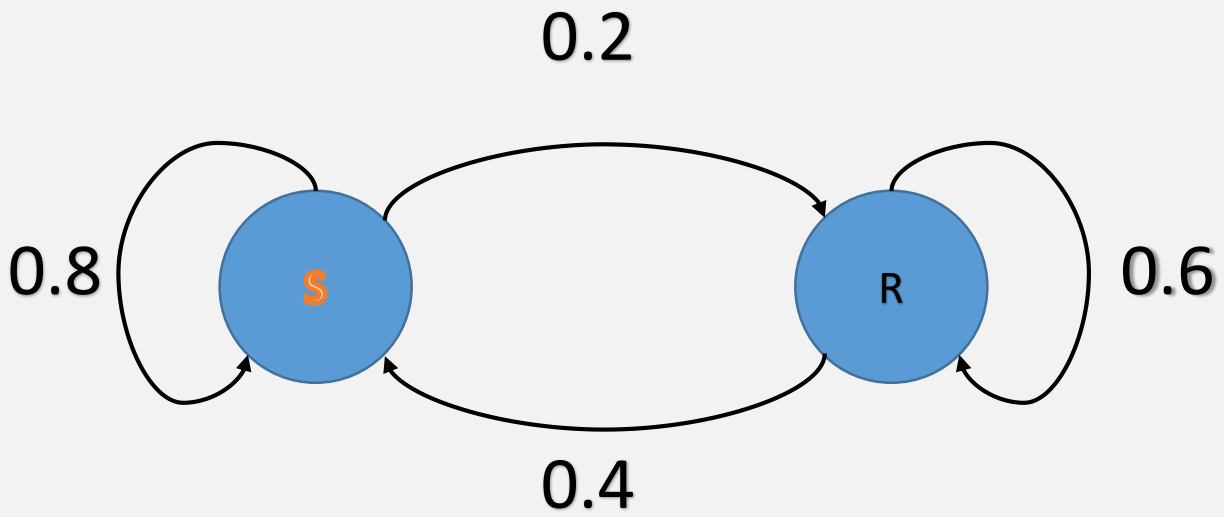
Bob is mostly happy when the weather is sunny, but there is some exception

Bob is mostly grumpy when the weather is rainy, but there is some exception



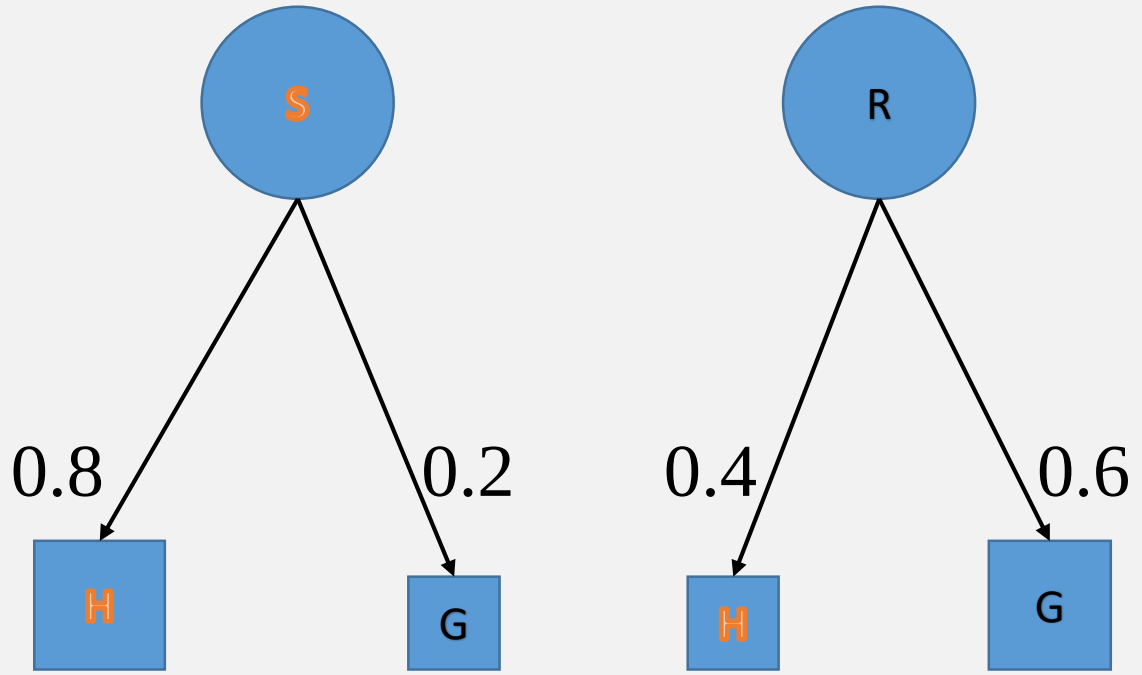
	Sat	Sun	Mon	Tue	Wed	Thurs	Fri
Bob Says	H	G	H	G	H	G	H
Alice Predict	S	R	S	R	S	R	S

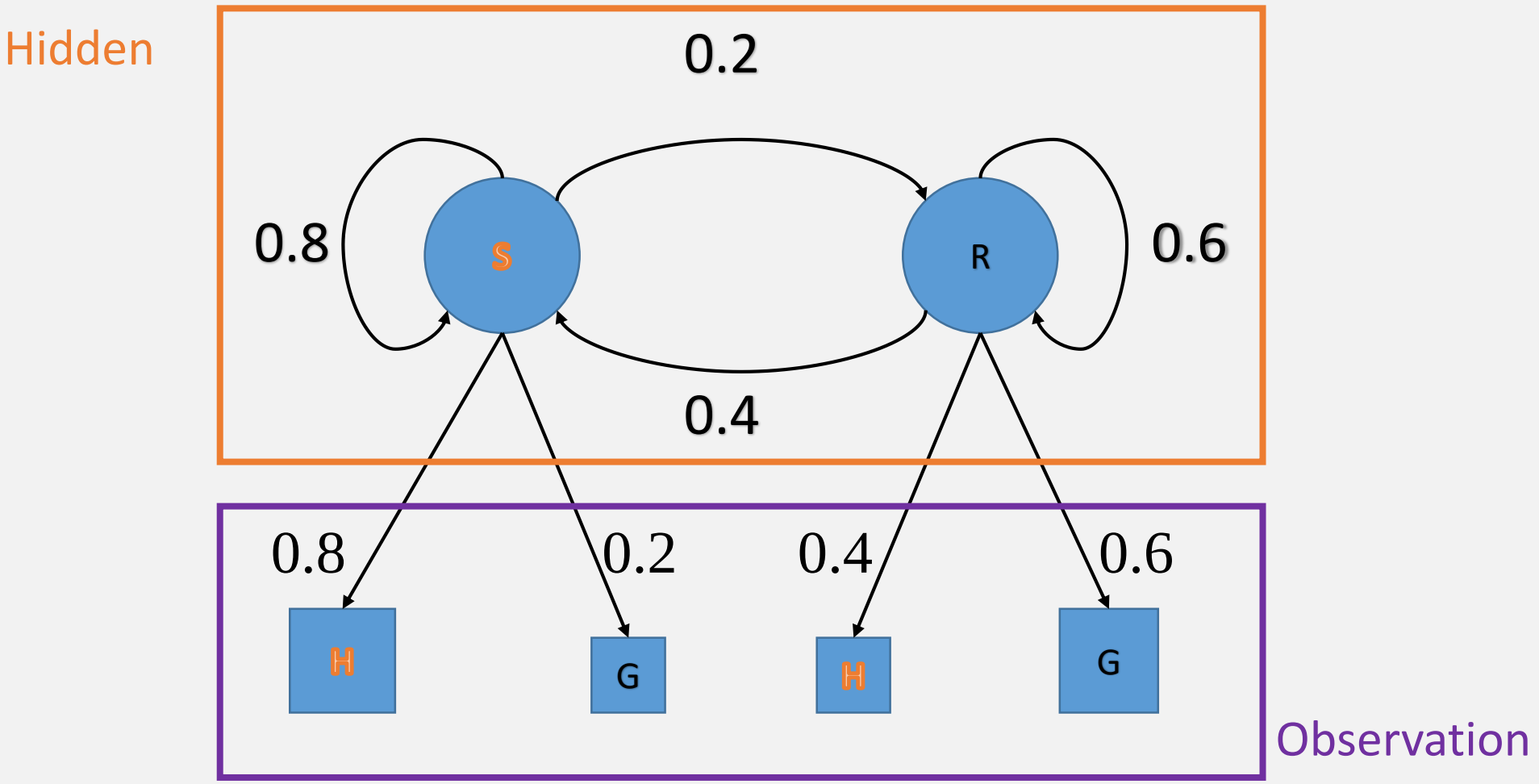

0.2
0.8
S
R
0.6
0.4
0.8
0.2
0.4
0.6
H
G
H
G



Transition Probability

Emission Probability




Hidden
0.2
0.8
S
R
0.6
0.4
0.8
0.2
0.4
0.6
H
G
H
G
Observation

What is the difference between MM and HMM?

4 Questions:

- How to calculate the probabilities?
- What is the probability that a random day is sunny or rainy?
- If Bob is happy today, what is the probability that it's sunny or rainy today?
- If for three days Bob is happy, grumpy, happy, what was the weather?

How to calculate the probabilities?

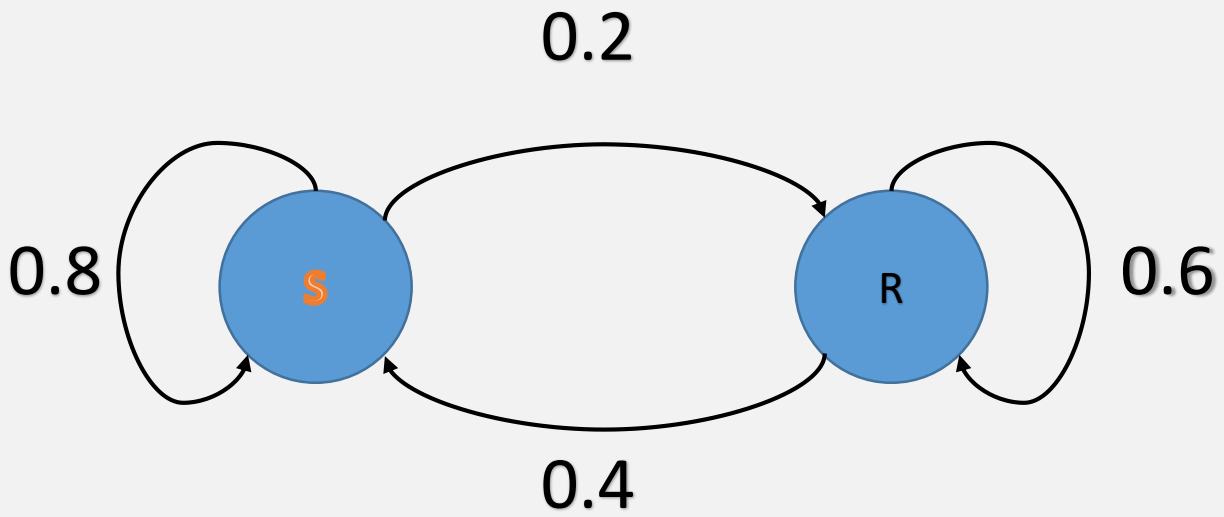
From the previous data:

S	S	S	S	R	R	R	S	S	S	S	R	R	S	S	S
G	H	H	H	G	G	H	G	H	H	H	G	H	H	H	

Total Sunny (S) = 10

Total Rainy (R) = 5

	S	R
S	8/10	2/10
R	2/5	3/5



Transition Probability

How to calculate the probabilities?

From the previous data:

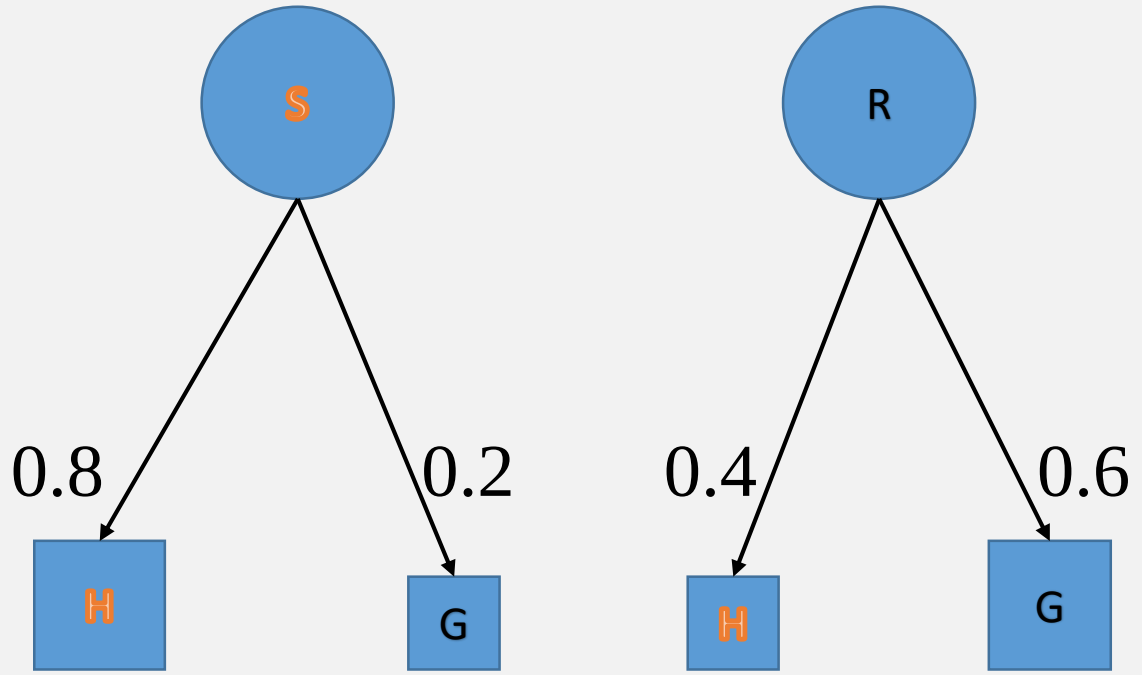
S	S	S	S	R	R	R	S	S	S	S	R	R	S	S	
G	H	H	H	G	G	H	G	H	H	H	G	H	H	H	

Total Sunny (S) = 10

Total Rainy (R) = 5

	H	G
S	8/10	2/10
R	2/5	3/5

Emission Probability



What is the probability that a random day is sunny or rainy?

From the previous data:

Total Sunny (S) = 10

Total Rainy (R) = 5

So, P(S) = 10/15 = 2/3

P(R) = 5/15 = 1/3

If Bob is happy today, what is the probability that it's sunny or rainy today?

Recall Bayes Theorem,

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

We have to find out P(S|H) and P(R|H).

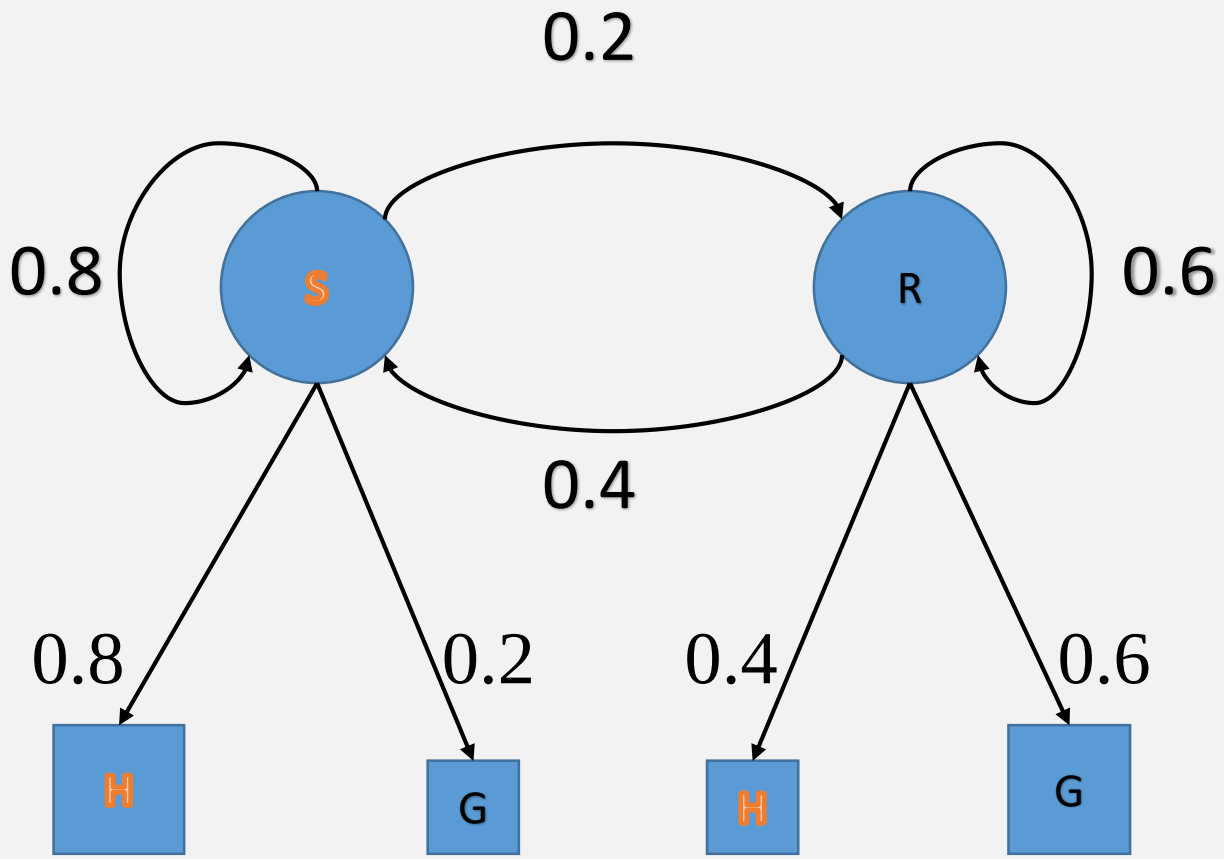
We can write, $P(S|H) = \frac{P(H|S)P(S)}{P(H)}$

$P(H|S) = ?$

$P(S) = ?$

$P(H) = ?$

If Bob is happy today, what is the probability that it's sunny or rainy today?



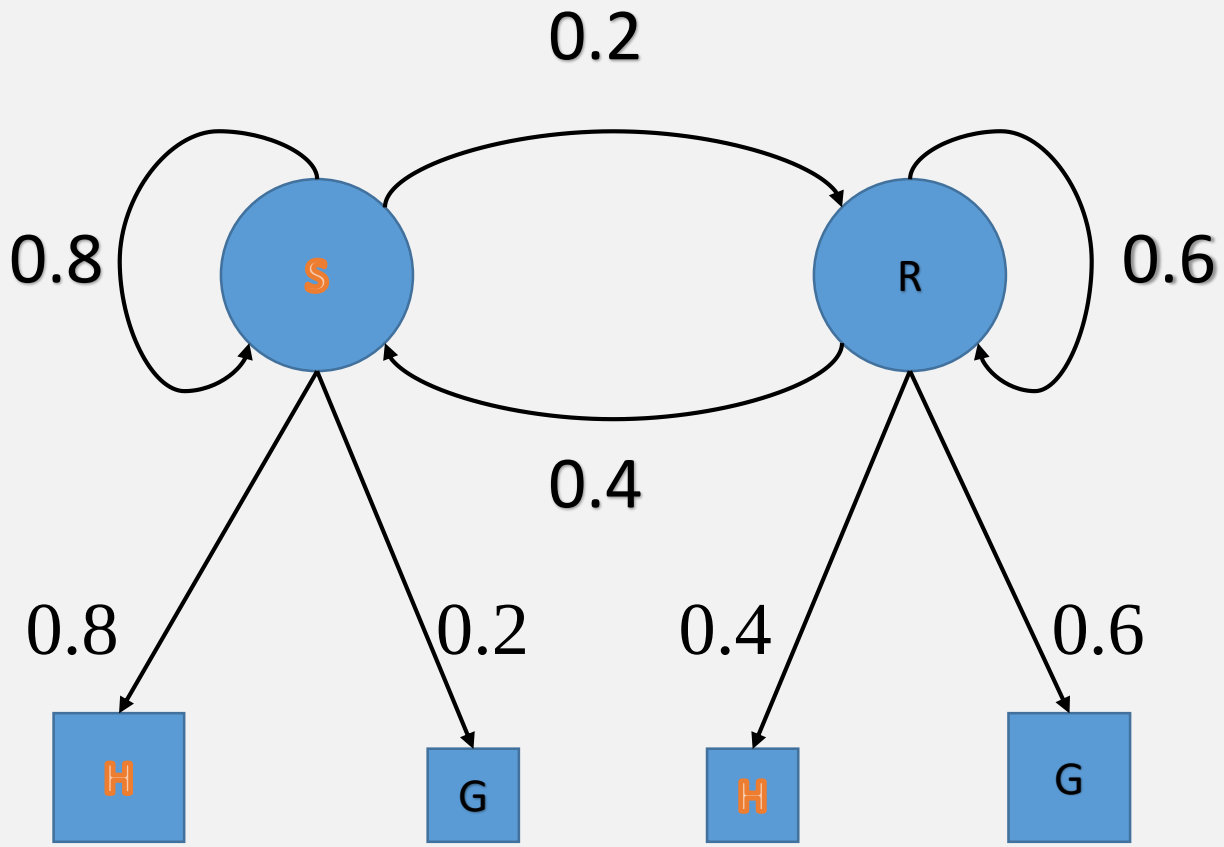
$P(H|S) = 0.8$

$P(S) = 2/3$

$P(H) = ?$

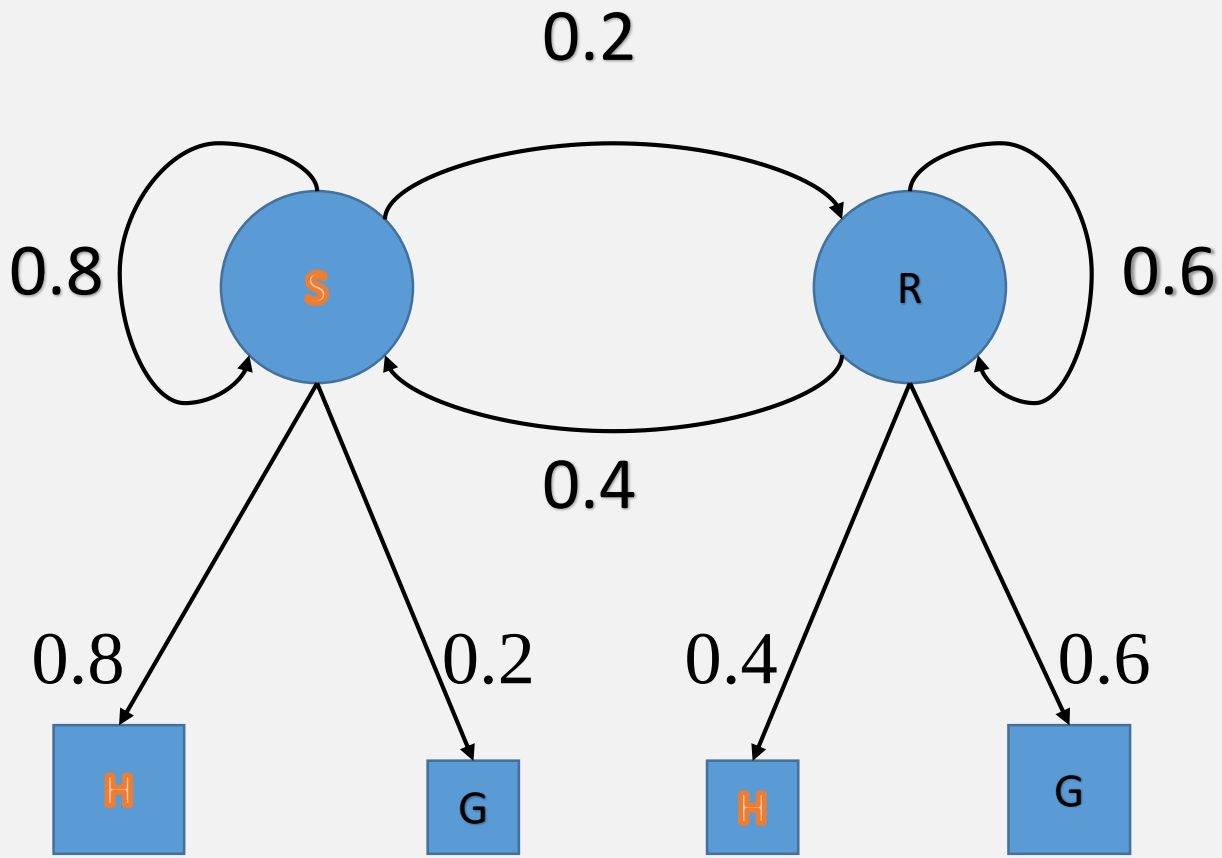
Easy to find out!

If Bob is happy today, what is the probability that it's sunny or rainy today?



From dataset,
$P(H)=10/15 = 2/3$

If Bob is happy today, what is the probability that it's sunny or rainy today?



Is it possible to find out P(H) from the diagram at left side? If it is not possible to find it out from the diagram, then which diagram can help us?

If Bob is happy today, what is the probability that it's sunny or rainy today?

$$P(S|H) = \frac{P(H|S)P(S)}{P(H)}$$

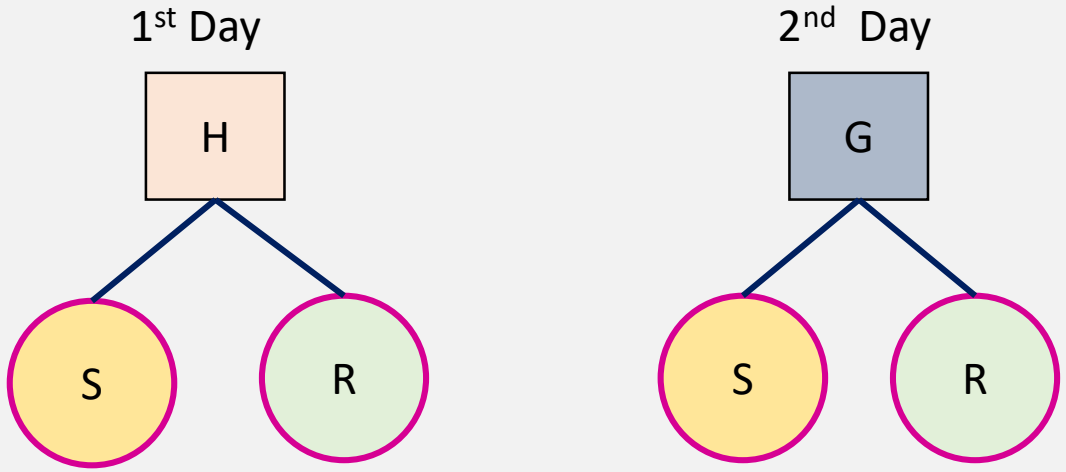
$$= \frac{\frac{4}{5} * \frac{2}{3}}{\frac{2}{3}} = \frac{4}{5}$$

$$P(H|S) = \frac{4}{5}$$

$$P(S) = \frac{2}{3}$$

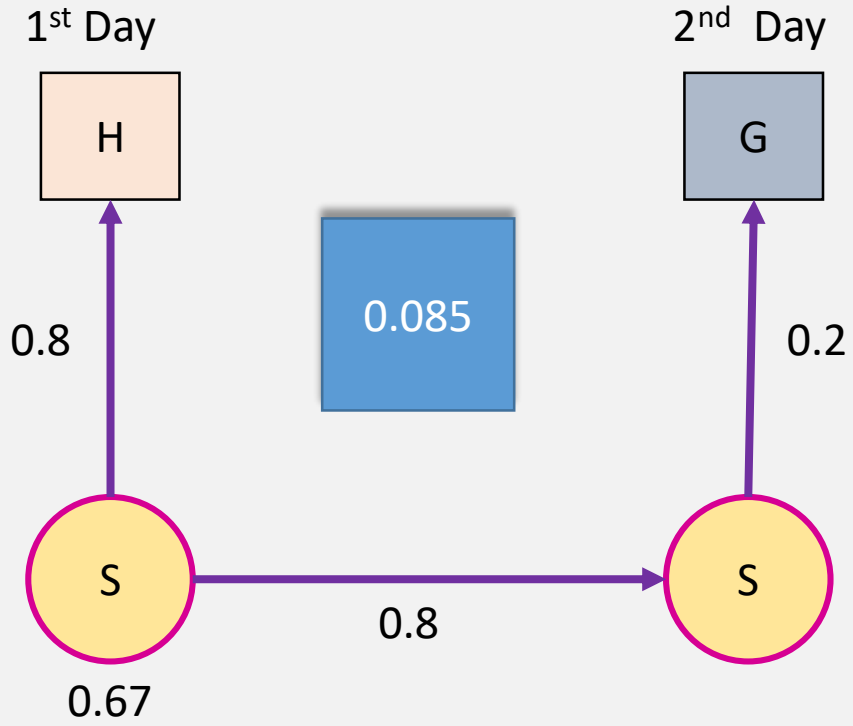
$$P(H) = \frac{2}{3}$$

If for two days Bob is happy, grumpy, what was the weather?



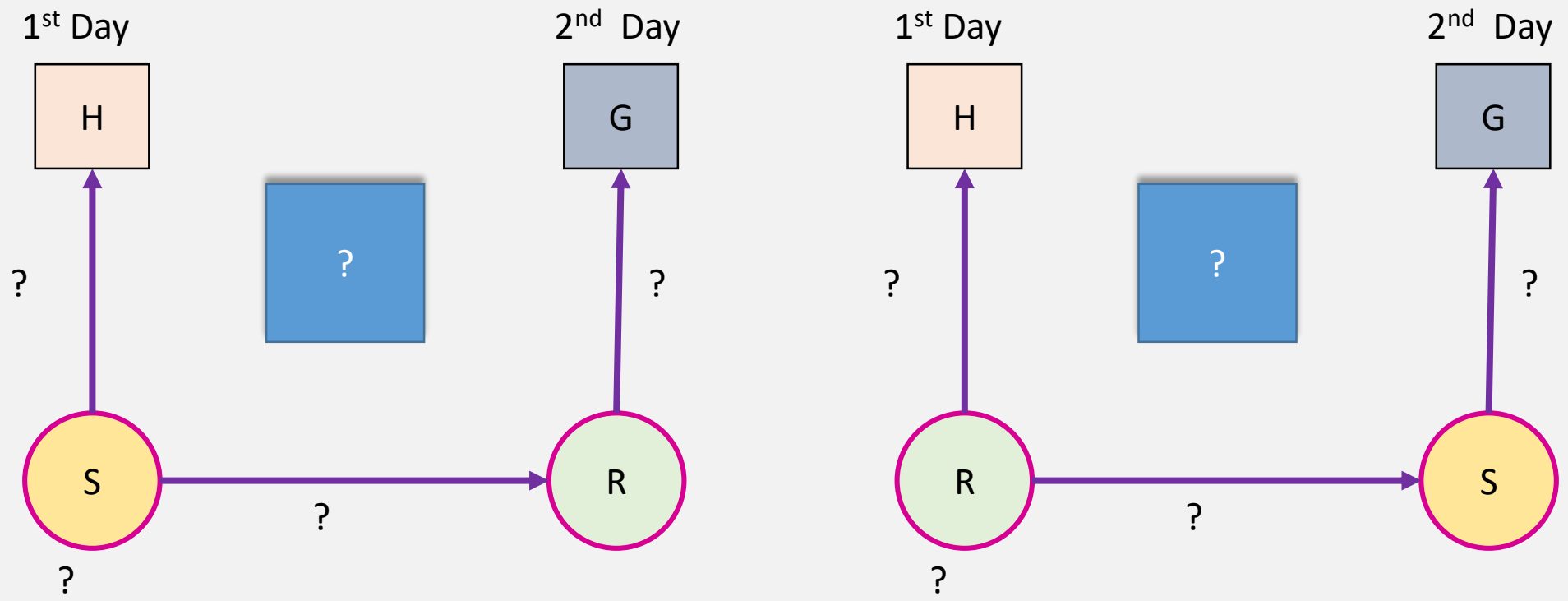
1st Day	2nd Day
Sunny	Sunny
Sunny	Rainy
Rainy	Sunny
Rainy	Rainy

If for two days Bob is happy, grumpy, what was the weather?



Using the conditional probability, the probability of the whole thing (denoted in the blue box) is the product of all the probabilities.

If for two days Bob is happy, grumpy, what was the weather?



Another Combination?

If for three days Bob is happy, grumpy, happy, what was the weather?