Compute the Linear Discriminant projection for the following two dimensional dataset.

– Samples for class ω1 : X1 = (x1, x2) = {(4,2), (2,4), (2,3), (3,6), (4,4)}

– Sample for class ω2 : X2 = (x1, x2) = {(9,10), (6,8), (9,5), (8,7), (10,8)}

The classes mean are:

$$\mu_1 = \frac{1}{N}\sum_{x\in\omega 1} x = \frac{1}{5}\begin{pmatrix}4&2&2&3&4\\2&4&3&6&4\end{pmatrix} = \begin{pmatrix}3\\3.8\end{pmatrix}$$

$$\mu_2 = \frac{1}{N}\sum_{x\in\omega 2} x = \frac{1}{5}\begin{pmatrix}9&6&9&8&10\\10&8&5&7&8\end{pmatrix} = \begin{pmatrix}8.4\\7.6\end{pmatrix}$$

$$N = number\ of\ elements\ = 5$$

$For\ class\ \omega 1,$

$$(x-\mu_1) =$$

$$\begin{pmatrix}4-3 & 2-3 & 2-3 & 3-3 & 4-3\\ 2-3.8 & 4-3.8 & 3-3.8 & 6-3.8 & 4-3.8\end{pmatrix}$$

$$=\begin{pmatrix}1 & -1 & -1 & 0 & 1\\ -1.8 & 0.2 & -0.8 & 2.2 & 0.2\end{pmatrix}$$

$$\left(x-\mu_1\right)^T = \begin{bmatrix}1 & -1.8\\ -1 & 0.2\\ -1 & -0.8\\ 0 & 2.2\\ 1 & 0.2\end{bmatrix}$$

$Now, \quad S1 = 1/(N-1)\{(x-\mu_1).(x-\mu_1)^T\}$

$$= \left(\frac{1}{5-1}\right)\left\{\begin{pmatrix} 1 & -1 & -1 & 0 & 1 \\ -1.8 & 0.2 & 0.8 & 2.2 & 0.2 \end{pmatrix}.\begin{pmatrix} 1 & -1 & -1 & 0 & 1 \\ -1.8 & 0.2 & 0.8 & 2.2 & 0.2 \end{pmatrix}^T\right\}$$

$$= \left(\frac{1}{4}\right)\begin{pmatrix} 4 & -1 \\ -1 & 8.8 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -0.25 \\ -0.25 & 2.2 \end{pmatrix}$$

Like as $S1$,

$$S2 = \begin{pmatrix} 2.3 & -0.05 \\ -0.05 & 3.3 \end{pmatrix}$$

$$Sw = S1 + S2 = \begin{pmatrix} 1 & -0.25 \\ -0.25 & 2.2 \end{pmatrix} + \begin{pmatrix} 2.3 & -0.05 \\ -0.05 & 3.3 \end{pmatrix}$$

$$= \begin{pmatrix} 3.3 & -0.3 \\ -0.3 & 5.5 \end{pmatrix}$$

$$w^* = S_W^{-1}(\mu_1 - \mu_2) = \begin{pmatrix} 3.3 & -0.3 \\ -0.3 & 5.5 \end{pmatrix}^{-1}\left[\begin{pmatrix} 3 \\ 3.8 \end{pmatrix} - \begin{pmatrix} 8.4 \\ 7.6 \end{pmatrix}\right]$$

$$= \begin{pmatrix} 0.3045 & 0.0166 \\ 0.0166 & 0.1827 \end{pmatrix}\begin{pmatrix} -5.4 \\ -3.8 \end{pmatrix}$$

$$= \begin{pmatrix} 0.9088 \\ 0.4173 \end{pmatrix}$$