Practice

Prove that $f(x) = x^3 \sin 2x$ is an even function.

Soln: let, $f(x) = x^3 \sin 2x$

$$f(-x) = (-x)^3 \sin 2(-x)$$

$$= \cancel{x^3 \sin}$$

$$= -x^3 . \{-\sin(2x)\}$$

$$= x^3 \sin 2x$$

$\therefore f(x) = f(-x)$

So, $f(x) = x^3 \sin 2x$ is an even function.

(Proved)

Prove that $f(x) = x^2 \tan x \cos 2x$ is an odd function.

Soln: let, $f(x) = x^2 \tan x \cos 2x$

$$f(-x) = (-x^2) \tan(-x) \cos 2(-x)$$

$$= x^2 (-\tan x) . (\cos 2x)$$

$$= -x^2 \tan x \cos 2x$$

$\therefore f(x) = f(-x)$

So, $f(x) = x^2 \tan x \cos 2x$ is an odd function

(Proved)

Find the period of the function $f(x) = \sin x + \cos 2x + \tan x$ and also show that it is periodic function.

<u>Soln:</u> let,

$f_1(x) = \sin x \rightarrow$ Period $= 2\pi$

$f_2(x) = \cos 2x \rightarrow$ Period $= \pi$

$f_3(x) = \tan x \rightarrow$ Period $= \pi$

$\pi \,|\, 2\pi, \pi, \pi$

$2, 1, 1$

2π

let, $f(x) = \sin x + \cos 2x + \tan x$

$= \cancel{2\pi +}$

$= \sin(2\pi + x) + \cos 2(2\pi + x) + \tan(2\pi + x)$

$= \sin x + \cos 2x + \tan x$

$= F(x)$

Again,

$f(x + 2\pi) = \sin(x + 2\pi + 2\pi) + \cos(2x + 4\pi + 2\pi) + \tan(x + 2\pi + 2\pi)$

$= \sin x + \cos 2x + \tan x$

$F(x + 2\pi + 2\pi) = \sin(x + 2\pi + 2\pi + 2\pi) + \cos(2x + 2\pi + 2\pi + 2\pi) + \tan(x + 2\pi + 2\pi + 2\pi)$

$= \sin x + \cos 2x + \tan x$

So, it is a periodic function

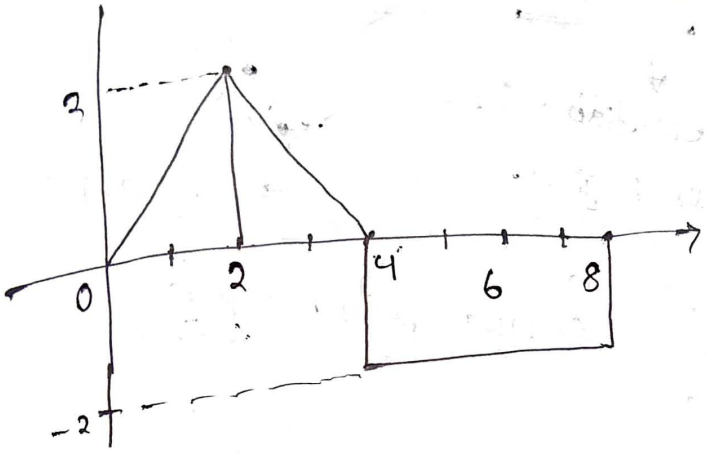
(Proved)

$x(t)$

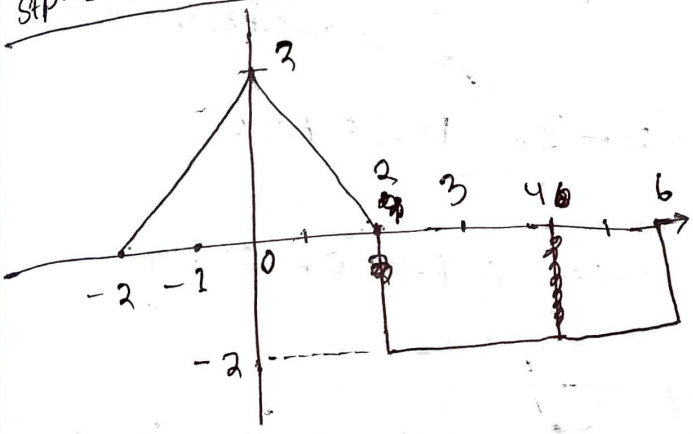
Q: $2x(2t+2)-3$

T.C

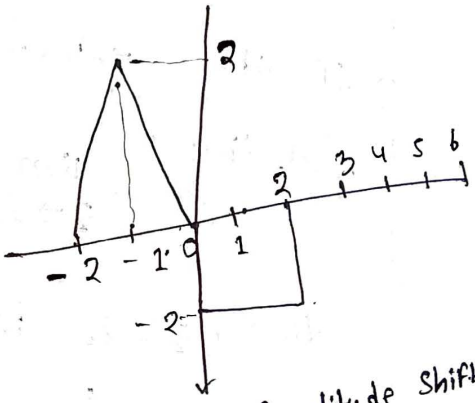
T.S



Step-1 time shifting



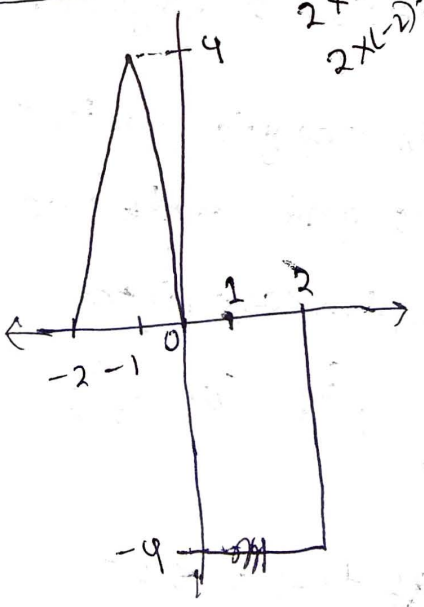
Step-2 time Scaling



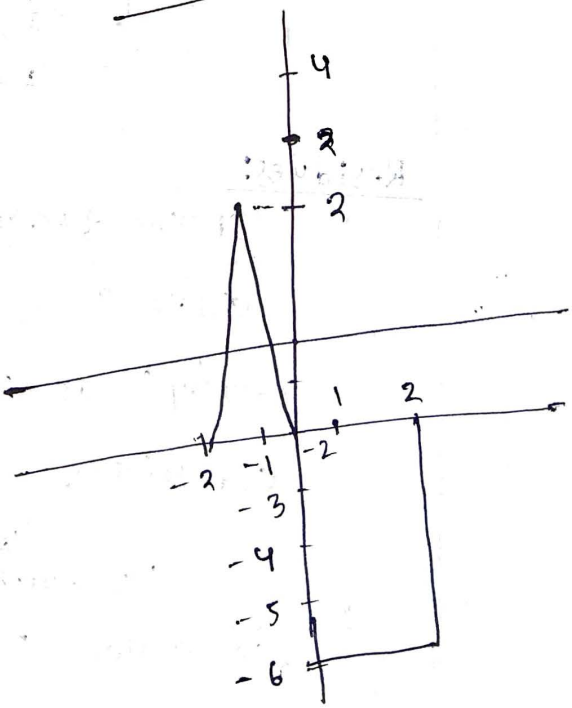
Step-3 Amplitude scaling

$2\times2=4$

$2\times(-2)=-4$



Step-4 Amplitude Shifting



CH: 04 (Singularities, Residue and some theorem)

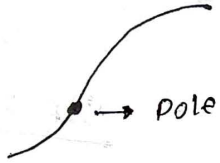
Singular point: A point at which an analytic function $f(z)$ fails to be $\boxed{\text{analytic}}$ is called a singular point.

$\downarrow$

Differentiable in a certain region.

$\hookrightarrow$ singular point

Pole:



singular points = pole

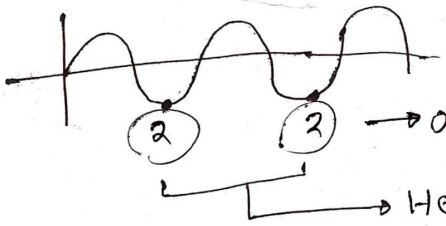
If there exists a ~~poin~~ positive integer n such that

$$\lim_{z \to z_0} (z - z_0)^{n} f(z) = A \neq 0$$

$n \rightarrow$ pole number (order of pole)

then, $z = z_0$ is called a pole of order n.

$z_0 = 2$



$\rightarrow$ order of pole is $= 2$,

$\rightarrow$ Hence 2 singular point

Residues:

If the function $F(z)$ is analytic within a circle C of radious r and center a except at $z = a$, then the ~~coefficent~~ coefficient a_{-1} of $\frac{1}{z-a}$ of $\frac{1}{z-a}$

$\hookrightarrow$ (residue)

(a suffix of -1)

is called residue of $f(z)$ at $z = a$.

Pole and Residues $\rightarrow \lim_{z \to a} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} \left[(z-a)^m \cdot f(z) \right]$

Examples:

Calculate the residues of the function

Ex:1

$$f(z) = \frac{z^2}{z^2+a^2}$$

Soln: Given, $f(z) = \frac{z^2}{z^2+a^2}$

The pole of $f(z)$ are obtained by solving,

$$z^2+a^2=0$$
$$\Rightarrow z^2 = -a^2$$
$$\Rightarrow z^2 = (ai)^2$$
$$\Rightarrow z^2 = \pm ai$$
$$\therefore z = \pm ai$$

Here, $z = +ai$ and $z = -ai$ both are the single pole of $f(z)$.

Residues at $z = ai$ is

$$= \lim_{z \to ai} \frac{1}{(1-1)!} \cdot \frac{d^{(1-1)}}{dz^{(1-1)}} \left[(z-a)^1 \cdot f(z) \right]$$

$$= \lim_{z \to ai} \left[(z-ai) \cdot \frac{z^2}{z^2+a^2} \right]$$

$$= \lim_{z \to ai} \left[(z-ai) \cdot \frac{z^2}{z^2-(ai)^2} \right]$$

$$= \lim_{z \to ai} \left[(z-ai) \cdot \frac{z^2}{(z+ai)(z-ai)} \right]$$

$$= \lim_{z \to ai} \left[\frac{z^2}{z+ai} \right]$$

$$= \frac{(ai)^2}{ai+ai} = \frac{-a^2}{2ai} = \frac{-a}{2i}$$

So, residue at point $z = ai$ is $\boxed{-\frac{a}{2i}}$

Again residue at $z = -ai$

$$= \lim_{z \to -ai} \frac{1}{(1-1)!} \frac{d^{(1-1)}}{dz^{(1-1)}} \left[(z+ai)^1 \cdot z \right]$$

$$= \lim_{z \to -ai} \left[(z+ai) \frac{z^2}{(z^2+a^2)} \right]$$

$$= \lim_{z \to -ai} \left[(z+ai) \frac{z^2}{\{z^2-(-ai)^2\}} \right]$$

$$= \lim_{z \to -ai} \left[(z+ai) \frac{z^2}{(z+ai)(z-ai)} \right]$$

$$= \lim_{z \to -ai} \frac{z^2}{z-ai}$$

$$= \frac{(-ai)^2}{-ai-ai} = \frac{a^2 i^2}{-2ai} = \frac{-a^2}{-2ai} = \frac{a}{2i}$$

So, residue at $z = -ai$ is $\boxed{\frac{a}{2i}}$

Ans:

Evaluate

$$\oint \frac{z^2}{z^2+a^2} dz$$

Ex:3 $\oint \frac{z^2-2z}{(z+1)^2(z^2+4)} dz$ ~~Find the~~

Find the residue of the ~~algorithm~~ Function

$$\frac{z^2-2z}{(z+1)^2(z^2+4)}$$

Soln: Given,

$$F(z) = \frac{z^2 - 2z}{(z+1)^2 (z^2+4)}$$

The pole of $f(z)$ are obtained by solving equations or,

For pole,

$$(z+1)^2 (z^2+4) = 0$$

$$(z+1)^2 = 0$$

$$\Rightarrow (z+1)(z+1) = 0$$

$$\therefore z = -1, -1$$

$$z^2 + 4 = 0$$

$$\Rightarrow z^2 = -4$$

$$\Rightarrow z^2 = (2i)^2$$

$$\therefore z = \pm 2i$$

~~Now~~, here $z = -1$ is a double pole and $z = 2i$ and $z = -2i$ are single pole of $F(z)$.

Now, residues at $z = -1$ is

$$= \lim_{z \to -1} \frac{1}{(2-1)!} \cdot \frac{d^{(2-1)}}{dz^{(2-1)}} \cdot \left[(z+1)^2 F(z) \right]$$

$$= \lim_{z \to -1} \frac{1}{1!} \cdot \frac{d}{dz} \left[(z+1)^2 \cdot \frac{z^2 - 2z}{(z+1)^2 (z^2+4)} \right]$$

$$= \lim_{z \to -1} \frac{d}{dz} \left(\frac{z^2 - 2z}{z^2 + 4} \right)$$

$$\boxed{\frac{u}{v} = \frac{v \frac{d}{dz} u - u \frac{d}{dz} v}{v^2}}$$

$$= \lim_{z \to -1} \frac{(z^2+4) \frac{d}{dz}(z^2 - 2z) - (z^2 - 2z) dz (z^2+4)}{(z^2+4)^2}$$

$$= \lim_{z \to -1} \frac{(z^2+4)(2z-2) - (z^2-2z)(2z)}{(z^2+4)^2}$$

$$= \lim_{z \to -1} \frac{2z^3 + 2z^2 + 8z - 8 - 2z^3 + 4z^2}{(z^2+4)^2}$$

$$= \lim_{z \to -1} \frac{2z^2 + 8z - 8}{(z^2+1)^2}$$

$$= \frac{2(-1)^2 + 8(-1) - 8}{\{(-1)^2+4\}^2} = \frac{-14}{25}$$

Again ~~the~~ residue at point $z = +2i$

$$= \lim_{z \to 2i} \frac{1}{(1-1)!} \cdot \frac{d^{(-1-1)}}{dz^{(1+1)}} \left[(z-2i)^2 \cdot f(z) \right]$$

$$= \lim_{z \to 2i} \left[(z-2i) \frac{(z^2-2z)}{(z+1)^2(z^2+4)} \right]$$

$$= \lim_{z \to 2i} \left[(z-2i) \frac{z^2-2z}{(z+1)^2\{z^2-(2i)^2\}} \right]$$

$$= \lim_{z \to 2i} \left[(z-2i) \frac{z^2-2z}{(z+1)^2(z-2i)(z+2i)} \right]$$

$$= \lim_{z \to 2i} \frac{z^2-2z}{(z+1)^2(z+2i)}$$

$$= \frac{(2i)^2 - 2\cdot(2i)}{(2i+1)^2(2i+2i)} = \frac{4i^2-4i}{(2i+1)^2 \cdot 4i} = \frac{4i(i-1)}{(2i+1)^2 \cdot 4i}$$

$$= \frac{4i-1}{(2i+1)^2}$$

Ans:

Show that $\oint \frac{e^{tz}}{(z^2+1)^2} dz = \pi i (\sin t - t\cos t)$

where C is the circle $|z| = 3$ and $t > 0$.

Soln: Let, $f(z) = \frac{e^{tz}}{(z^2+1)^2}$

For pole $(z^2+1)^2 = 0$

$(z^2+1)(z^2+1) = 0$

$z^2+1 = 0$

$\therefore z = \pm i$

$z^2+1 = 0$

$\Rightarrow z^2 = -1$

$\Rightarrow z^2 = -i^2$

$\therefore z = \pm i$

Here, $z = i$ is the pole of order 2 and also $z = -i$ is the pole of order 2.

Now, residues at point ~~$z=i$~~ $z = \pm i$

$$= \lim_{z \to i} \frac{1}{(2-1)!} \frac{d^{2-1}}{dz^{2-1}} \left[(z-i)^2 \cdot f(z)\right]$$

$$= \lim_{z \to i} \frac{d}{dz} \left[(z-i)^2 \cdot \frac{e^{tz}}{(z^2+1)^2}\right]$$

$$= \lim_{z \to i} \frac{d}{dz} \left[(z-i)^2 \cdot \frac{e^{tz}}{(z-i)^2(z+i)^2}\right]$$

$$= \lim_{z \to i} \frac{d}{dz} \left[\frac{e^{tz}}{(z+i)^2}\right]$$

$$= \lim_{z \to i} \frac{(z+i)^2 \frac{d}{dz}(e^{tz}) - e^{tz} \frac{d}{dz}(z+i)^2}{\{(z+i)^2\}^2}$$

$$= \lim_{z \to i} \frac{(z+i)^2 (e^{tz} \cdot t) - e^{tz} \cdot 2(z+i) \cdot 1}{\{(z+i)^4\}}$$

$$= \lim_{z \to i} \frac{te^{tz}(z+i)^2 - 2e^{tz}(z+i)}{(z+i)^4}$$

$$= \lim_{z \to i} \frac{e^{tz}(z+i)\{t(z+i)-2\}}{(z+i)^4}$$

$$= \lim_{z \to i} \frac{e^{tz}(z+i)\{t(z+i)-2\}}{(z+i)^4}$$

$$= \lim_{z \to i} \frac{e^{tz}\{t(z+i)-2\}}{(z+i)^3}$$

$$= \frac{e^{ti}\{t(i+i)-2\}}{(i+i)^3}$$

$$= \frac{e^{ti}(2ti-2)}{(2i)^3}$$

$$= \frac{e^{ti}\cdot 2(ti-1)}{-8i} = \frac{e^{ti}(ti-1)}{-4i} = \frac{i\cdot e^{ti}(ti-1)}{-4i^2}$$

$$= \frac{e^{ti}(ti^2-i)}{}$$

$$= \frac{e^{ti}(t-i)}{4}$$

$$= \frac{-e^{ti}(t+i)}{4}$$

So, residue at point $z=i$ is $= \dfrac{-e^{ti}(t+i)}{4}$

So, residue at point $z=-i$ is

$$= \lim_{z \to -i} \frac{1}{(2-1)!}\cdot\frac{d^{(2-1)}}{dz^{(2-1)}}\left[(z+i)^2 f(z)\right]$$

$$= \lim_{z \to -i} \frac{d}{dz}\left[(z+i)^2\cdot\frac{e^{tz}}{(z^2+1)^2}\right]$$

$$= \lim_{z \to -i} \frac{d}{dz}\left[(z+i)^2\cdot\frac{e^{tz}}{(z^2-i^2)^2}\right]$$

$$\boxed{\oint f(z) = 2\pi i \cdot (\text{sum of residue})}$$

$$= \lim_{z \to -i} \frac{d}{dz}\left[(z+i)^2 \cdot \frac{e^{tz}}{(z+i)^2 (z-i)^2}\right]$$

$$= \lim_{z \to -i} \frac{d}{dz}\left[\frac{e^{tz}}{(z-i)^2}\right]$$

$$= \lim_{z \to -i} \frac{(z-i)^2 \frac{d}{dz} e^{tz} - e^{tz} \frac{d}{dz}(z-i)^2}{\{(z-i)^2\}^2}$$

$$= \lim_{z \to -i} \frac{(z-i)^2 \cdot (e^{tz} \cdot t) - e^{tz} \cdot 2(z-i) \cdot 1}{\{(z-i)^4\}}$$

$$= \lim_{z \to -i} \frac{e^{tz} \cdot (z-i)^2 - 2e^{tz}(z-i)}{(z-i)^4}$$

$$= \lim_{z \to -i} \frac{e^{tz}(z-i)\{t(z-i) - 2\}}{(z-i)^4}$$

$$= \lim_{z \to -i} \frac{e^{tz} \{t(-i-i) - 2\}}{(-i-i)^3}$$

$$= \lim_{z \to -i} \frac{e^{-ti}\{t(-2i) - 2\}}{(-2i)^3}$$

$$= \lim_{z \to -i} \frac{e^{-ti}\{t(-2i - 2)\}}{-8i^3}$$

$$= \frac{-e^{-ti}(ti+1)}{-4i^2}$$

$$= \frac{-e^{-ti}(-t+i)}{4i^3}$$

$$= \frac{-e^{-ti} - 2t(-i+2)}{-4i^3} = -\frac{e^{-ti}(ti^2+i)}{4}$$

$$= \frac{-e^{-ti}(t-i)}{4}$$

$$\oint_C f(z)\,dz = 2\pi i$$

$$\Rightarrow \oint \frac{e^{tz}}{(z^2+1)^2}\,dz = 2\pi i\left[\frac{-e^{ti}(t+i)}{4} - \frac{e^{-ti}(t-i)}{4}\right]$$

$$= \frac{-2\pi i}{4}\left[e^{ti}(t+i) + e^{-ti}(t-i)\right]$$

$$= \frac{-\pi i}{2}\left[(t\cdot e^{ti} + i\cdot e^{ti}) + (i\cdot e^{ti} + e^{-ti})\right]$$

$$= \frac{-\pi i}{2}\left[t(e^{ti} + e^{-ti}) + i(e^{ti} - e^{-ti})\right]$$

$$= \frac{-\pi i}{2}\left[(t\,2\cos t + i\cdot 2i\sin t)\right]$$

$$= \frac{-\pi i}{2}\left[2(t\cos t + i^2\sin t)\right]$$

$$= -\pi i\,(t\cos t - \sin t)$$

$$= \pi i\,(\sin t - t\cos t)$$

(Showed)

07.01.2020
Tue-day

Lec-15

Last math of this course

Ex: Show that $\oint_c \frac{e^z}{(z^2+\pi^2)^2} \cdot dz = \frac{i}{\pi}$ where C is the circle $|z| = 2$.

Soln: Given, $\oint_c \frac{e^z}{(z^2+\pi^2)^2} dz$

$\oint$ close integration

let, $f(z) = \frac{e^z}{(z^2+\pi^2)^2}$

$\oint_c f(z) \cdot dz = 2\pi i$ (Sum of residues)

For, pole

$(z^2+\pi^2)^2 = 0$

$\Rightarrow (z^2+\pi^2)(z^2+\pi^2) = 0$

$\Rightarrow z^2+\pi^2 = 0$

$\Rightarrow z^2 = -\pi^2$

$\Rightarrow z^2 = (\pi i)^2$

$\therefore z = \pm \pi i$

$z^2+\pi^2 = 0$

$\Rightarrow z^2 = -\pi^2$

$\Rightarrow z^2 = (\pi i)^2$

$\therefore z = \pm \pi i$

N.B

$(0 \pm \pi i)$ $(0 \pm \pi i)$

$\sqrt{0+(\pi)^2} = \sqrt{x^2+y^2}$

$= \sqrt{\pi^2} = \pi$

$= 3.1424$

$z = \pi i$ is a pole of order 2 and also $z = -\pi i$ is pole of order 2 and all poles lie in given circle.

Residue at $z = \pi i$

$$= \lim_{z \to \pi i} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{(m-1)}} \left[(z-\pi i)^m f(z) \right]$$

Here, m = Order $\longrightarrow$ for same points Order = 2

$$= \lim_{z \to \pi i} \frac{1}{(2-1)!} \frac{d}{dz}\left[(z-\pi i)^2 \cdot \frac{e^z}{(z^2+\pi^2)^2}\right]$$

$$= \lim_{z \to \pi i} \frac{d}{dz}\left[(z-\pi i)^2 \cdot \frac{e^z}{\{z^2-(\pi i)^2\}^2}\right]$$

$$= \lim_{z \to \pi i} \frac{d}{dz}\left[(z-\pi i)^2 \cdot \frac{e^z}{\{(z+\pi i)(z-\pi i)\}^2}\right]$$

$$= \lim_{z \to \pi i} \frac{d}{dz}\left[\frac{e^z}{(z+\pi i)^2}\right]$$

$$= \lim_{z \to \pi i} \frac{d}{dz}\left[\frac{e^z}{(z+\pi i)^2}\right]$$

$$= \lim_{z \to \pi i} \left[\frac{(z+\pi i)^2 \cdot \frac{d}{dz} e^z - e^z \cdot \frac{d}{dz}(z+\pi i)^2}{(z+\pi i)^4}\right]$$

$$= \lim_{z \to \pi i} \left[\frac{(z+\pi i)^2 \cdot e^z - e^z \cdot 2(z+\pi i)}{(z+\pi i)^4}\right]$$

$$= \lim_{z \to \pi i} \frac{e^z(z+\pi i)\left[(z+\pi i)-2\right]}{(z+\pi i)^4}$$

$$= \lim_{z \to \pi i} \frac{e^z\left[(z+\pi i)-2\right]}{(z+\pi i)^3}$$

$$= \frac{e^{\pi i}\left[(\pi i+\pi i)-2\right]}{(\pi i+\pi i)^3}$$

$$= \frac{e^{\pi i}(2\pi i-2)}{(2\pi i)^3} = \frac{e^{\pi i}\,2(\pi i-1)}{8\pi^3 i^3} = \frac{e^{\pi i}(\pi i-1)}{-4\pi^3 i}$$

$$= \frac{e^{\pi i} \cdot i(\pi i - 2)}{-4\pi^3 i^2}$$

$$= \frac{e^{\pi i}(\pi i^2 - i)}{4\pi^3}$$

$$= \frac{e^{\pi i} \cdot (-\pi - i)}{4\pi^3} = \frac{-e^{\pi i}(\pi + i)}{4\pi^3}$$

Residue at $z = -\pi i$

$$= \lim_{z \to -\pi i} \frac{1}{(m-1)!} \cdot \frac{d^{(m-1)}}{dz^{(m-1)}} \left[(z+\pi i)^m f(z) \right]$$

$$= \lim_{z \to -\pi i} \frac{1}{(2-1)!} \cdot \frac{d}{dz} \left[(z+\pi i)^2 \cdot \frac{e^z}{(z^2+\pi^2)^2} \right]$$

$$= \lim_{z \to -\pi i} \frac{d}{dz} \left[(z+\pi i)^2 \frac{e^z}{\{z^2 - (\pi i)^2\}^2} \right]$$

$$= \lim_{z \to -\pi i} \frac{d}{dz} \left[\frac{e^z}{(z-\pi i)^2} \right]$$

$$= \lim_{z \to -\pi i} \left[\frac{(z-\pi i)^2 \frac{d}{dz} e^z - e^z \frac{d}{dz}(z-\pi i)^2}{(z-\pi i)^4} \right]$$

$$= \lim_{z \to -\pi i} \left[\frac{(z-\pi i)^2 \cdot e^z - e^z 2(z-\pi i)}{(z-\pi i)^4} \right]$$

$$= \lim_{z \to -\pi i} \frac{e^z (z-\pi i)\left[(z-\pi i) - 2\right]}{(z-\pi i)^4}$$

$$= \lim_{z \to -\pi i} \frac{e^{z}[(z-\pi i)-2]}{(z-\pi i)^{3}}$$

$$= \frac{e^{-\pi i}[-2\pi i-\pi i-2]}{(-\pi i-\pi i)^{3}}$$

$$= \frac{e^{-\pi i}[-2\pi i-2]}{(-2\pi i)^{3}}$$

$$= \frac{-2e^{-\pi i}[\pi i+1]}{-8\pi^{3}i^{3}}$$

$$= \frac{e^{-\pi i}[\pi i+1]}{4\pi^{3}i}$$

$$= \frac{e^{-\pi i}\cdot i(\pi i+1)}{-4\pi^{3}i^{2}}$$

$$= \frac{e^{-\pi i}(\pi i^{2}+i)}{4\pi^{3}}$$

$$= \frac{e^{-\pi i}(-\pi+i)}{4\pi^{3}}$$

$$= \frac{-e^{-\pi i}(\pi-i)}{4\pi^{3}}$$

$$\therefore \oint_{C} \frac{e^{z}}{(z^{2}+\pi^{2})^{2}}\,dz = 2\pi i \text{ (sum of residues)}$$

$$= 2\pi i\left[-\frac{e^{-\pi i}(\pi+i)}{4\pi^{3}} + \frac{-e^{-\pi i}(\pi-i)}{4\pi^{3}}\right]$$

The End

(class Lectures end here)

$$= \frac{2\pi i}{4\pi^3}\left[-e^{\pi i}(\pi+i) - e^{-\pi i}(\pi-i)\right]$$

$$= \frac{i}{2\pi^2}\left[-\pi e^{\pi i} - i e^{\pi i} - \pi e^{-\pi i} + i e^{-\pi i}\right]$$

$$= \frac{i}{2\pi^2}\left[-\pi(e^{\pi i}+e^{-\pi i}) - i(e^{\pi i}-e^{-\pi i})\right]$$

$$= \frac{i}{2\pi^2}\left[-\pi(2\cos\pi) - i\,2i(\sin\pi)\right]$$

$$= \frac{i}{2\pi^2}\left[-\pi\cdot 2(-1)\right]$$

$$= \frac{i}{2\pi^2}\cdot 2\pi$$

$$= \frac{i}{\pi}$$

A.W (1) Prove that $\frac{1}{2\pi i}\oint_c \frac{e^{zt}}{z^2(z^2+2z+2)}\,dz = \frac{1}{2}(t-2)+\frac{1}{2}e^{-t}\cos t$

where c is the circle $|z| = 3$

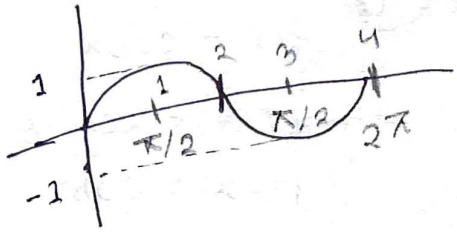
(2) Evaluate $\oint_c \frac{e^{3z}}{(z+\pi i)}\,dz$ where c is the circle $|z+2| = 4$

(this 2 are not that important)

Practice

Lec-16

Signal processing

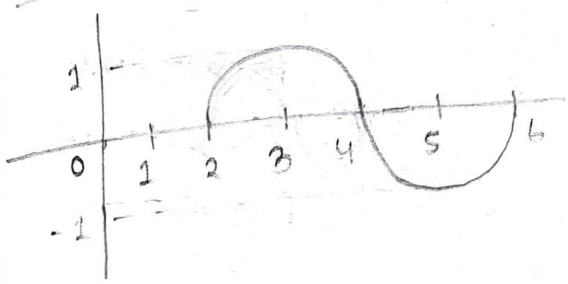


$\sin(x)$.

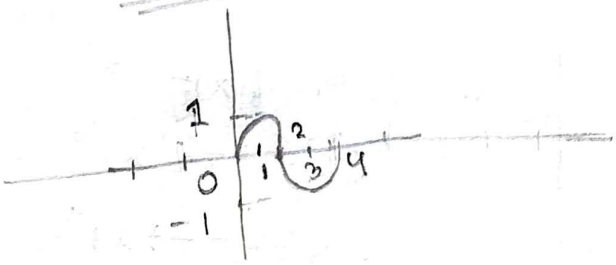
$-2\sin(-2x-2)-3$

$\frac{4}{2} = -2$

① → T. Sc



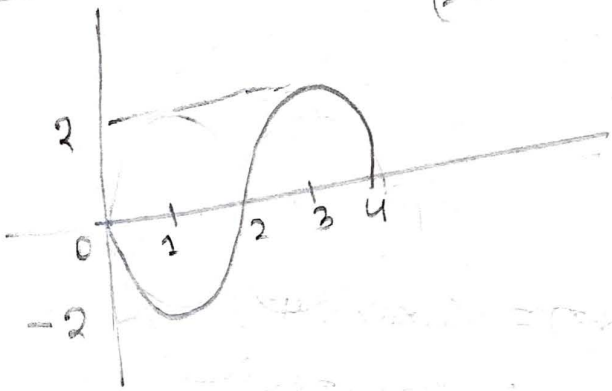
② → T. Sh



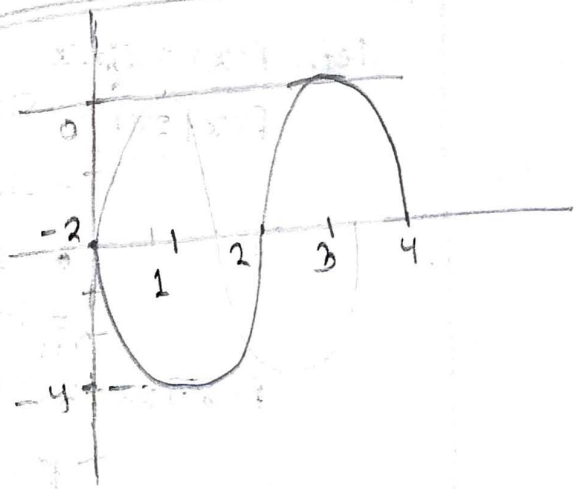
③ → A. Sc

$1 \times -2 = -2$

$(-1) \times (-2) = 2$



④ → A. Sh



A.W Solution

Soln: Let, $F(z) = \dfrac{e^{zt}}{z^2(z^2+2z+2)}$

For pole $z^2(z^2+2z+2)=0$

$z^2=0$

$z=0,0$

Finnt 4 math must

$z^2+2z+2=0$

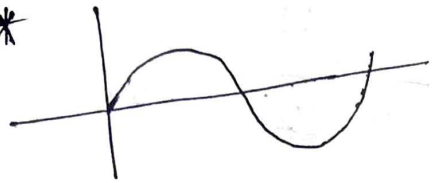
$$z = \frac{-2 \pm \sqrt{(2)^2-4.1.2}}{2.1}$$

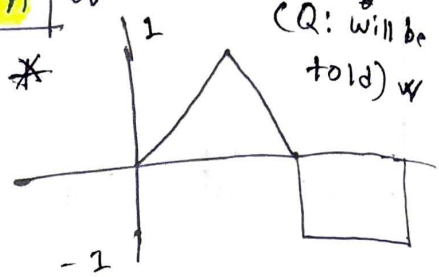
$$= \frac{-2 \pm \sqrt{4-8}}{2}$$

$$= \frac{-2 \pm \sqrt{4i^2}}{2}$$

Question pattern

(Q: will be told)

1. 



→ 5 marks

combined

2. Odd / even (must - 1) → 3/4 marks (product function → $x^3 \sin^2 x$)

3. Fourier → 6/7 marks

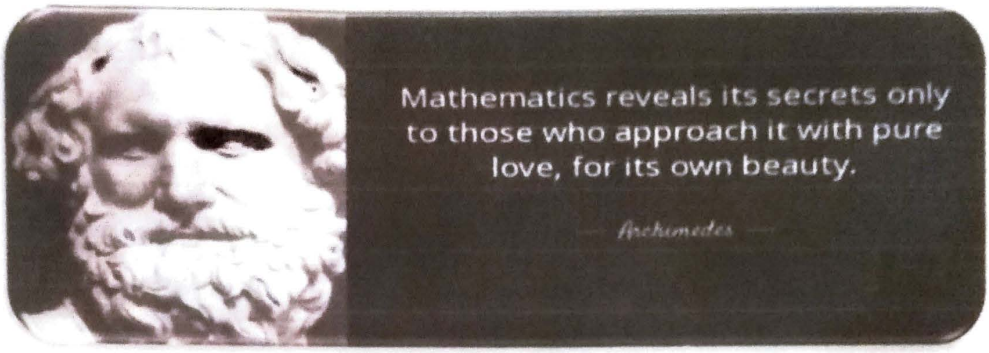
4. Periodic function → must "1" (4/5 marks)

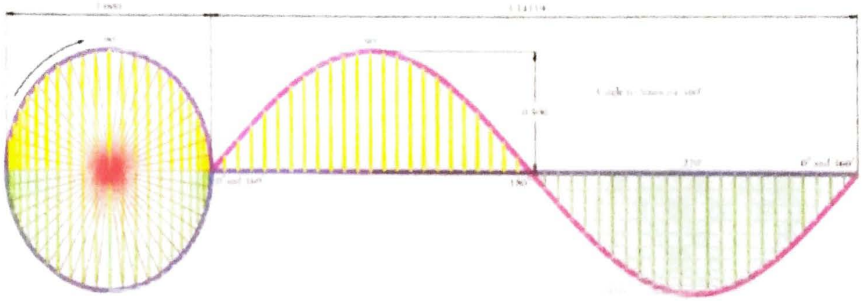
Fourier → 20 marks

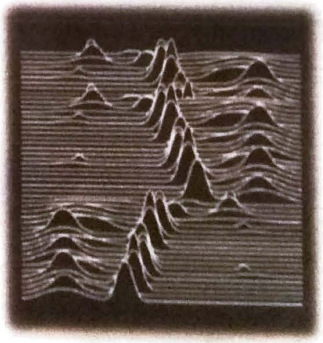
1 math → 20 marks

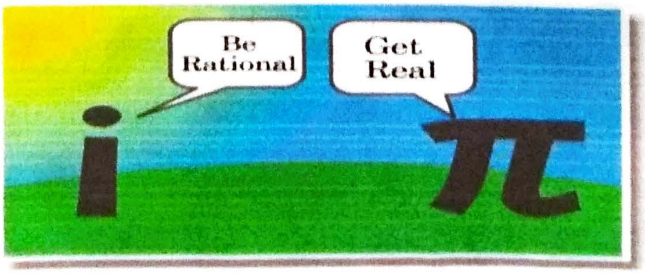
Residue → 1/2 (minmum) → 4 maths (3*)

insπre


Mathematics reveals its secrets only to those who approach it with pure love, for its own beauty.
— Archimedes —



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