Lecture on

Bayesian Belief Networks

(Basics)

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Lecture contents

- Brief introduction to BBN and example
- Definition of BBN
- Purpose of BBN
- Construction of BBN
- Concluding remarks

A simple example

Problem:

Two colleagues Norman and Martin live in the same city, work together, but come to work by completely different means - Norman usually comes by train while Martin always drives. The railway workers go on strike sometimes.

Goal:

We want to predict whether Norman and Martin will be late for work.

A simple example

Two causal relations:

- Strike of the railway *can* cause Norman to be late
- Strike of the railway *can* cause Martin to be late

IMPORTANT:

These relations are **NOT** absolute!!

Strike of the railway does **NOT** guarantee that Norman and Martin will be late for sure. It **ONLY** increases the probability (chance) of lateness.

What is Bayesian Belief Network (BBN)?

Bayesian Belief Network (**BBN**) is a directed acyclic graph associated with a set of conditional probability distributions.

BBN is a set of nodes connected by directed edges in which:

- ***nodes*** represent discrete or continuous random variables in the problem studied,
- ***directed edges*** represent direct or causal relationships between variables and do not form *cycles*,
- each node is associated with a ***conditional probability distribution*** which quantitatively expresses the strength of the relationship between that node and its parents.

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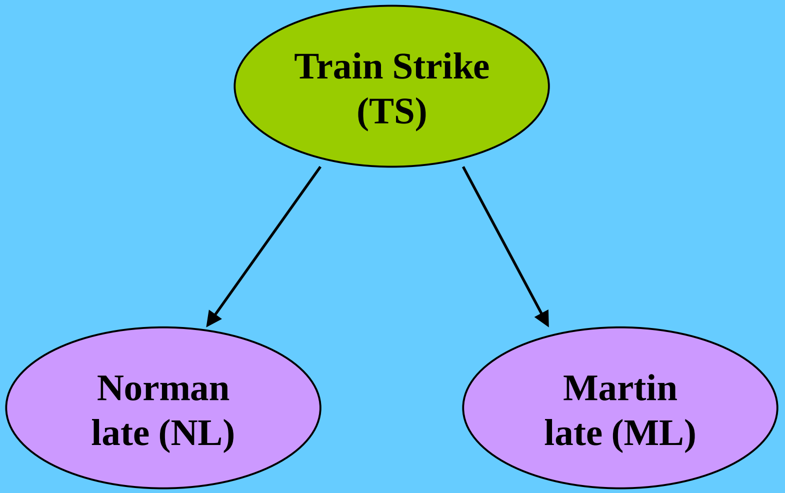
BBN for late-for-work example

3 discrete (Yes/No) variables

Train strike	Probability
Y	0.1
N	0.9



		Train strike	
		Y	N
Norman late	Y	0.8	0.1
	N	0.2	0.9

		Train strike	
		Y	N
Martin late	Y	0.6	0.5
	N	0.4	0.5

What do we need BBNs for?

Bayesian Belief Network:

- gives the intuitive representation of the problem that is easily understood by non-experts
- incorporates uncertainty related to the problem

and also

- allows to calculate probabilities of different scenarios (events) relevant to the problem and to predict consequences of these scenarios

BBN for late-for-work example

Train strike	Probability
Y	0.1
N	0.9

		Train strike	
		Y	N
Norman late	Y	0.8	0.1
	N	0.2	0.9

		Train strike	
		Y	N
Martin late	Y	0.6	0.5
	N	0.4	0.5

$P(ML = Y, NL = Y, TS = N)$

$P(ML = Y, NL = N, TS = N)$

…

$P(ML = m, NL = n, TS = t)$ - joint probability

$P(ML,NL,TS) = P(ML|TS) \cdot P(NL|TS) \cdot P(TS)$ - joint distribution

$P(ML = Y, NL = Y, TS = N) =$
$= P(ML = Y|TS = N) \cdot P(NL = Y|TS = N) \cdot P(TS = N)$
$= 0.5 \cdot 0.1 \cdot 0.9 = 0.045$

ML	NL	TS	Probability
Y	Y	Y	0.048
N	Y	Y	0.032
Y	N	Y	0.012
N	N	Y	0.008
Y	Y	N	0.045
N	Y	N	0.045
Y	N	N	0.405
N	N	N	0.405

BBN for late-for-work example

$P(ML = Y, NL = Y)$ - marginal probability

$$P(ML, NL) = \sum_t P(ML, NL, TS = t)$$ - marginal distribution (marginalization)

ML	NL	TS	Probability
Y	Y	Y	0.048
N	Y	Y	0.032
Y	N	Y	0.012
N	N	Y	0.008
Y	Y	N	0.045
N	Y	N	0.045
Y	N	N	0.405
N	N	N	0.405

$$P(ML = Y, NL = Y) = \sum_t P(ML = Y, NL = Y, TS = t) =$$
$$= P(ML = Y, NL = Y, TS = Y) + P(ML = Y, NL = Y, TS = N)$$
$$= 0.048 + 0.045 = 0.093$$

ML	NL	Probability
Y	Y	0.093
N	Y	0.077
Y	N	0.417
N	N	0.413

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BBN for late-for-work example

$P(NL = Y)$ - marginal probability

$$P(NL) = \sum_t \sum_m P(ML = m, NL, TS = t)$$ - marginal distribution (marginalization)

$$P(NL = Y) = \sum_m \sum_t P(ML = m, NL = Y, TS = t) =$$
$= P(ML = Y, NL = Y, TS = Y) + P(ML = N, NL = Y, TS = N) +$
$+ P(ML = N, NL = Y, TS = Y) + P(ML = Y, NL = Y, TS = N)$
$= 0.048 + 0.045 + 0.032 + 0.045 = 0.17$

OR easier

$$P(NL = Y) = \sum_m P(ML = m, NL = Y) =$$
$= P(ML = Y, NL = Y) + P(ML = N, NL = Y) =$
$= 0.093 + 0.077 = 0.17$

ML	NL	TS	Probability
Y	Y	Y	0.048
N	Y	Y	0.032
Y	N	Y	0.012
N	N	Y	0.008
Y	Y	N	0.045
N	Y	N	0.045
Y	N	N	0.405
N	N	N	0.405

ML	NL	Probability
Y	Y	0.093
N	Y	0.077
Y	N	0.417
N	N	0.413

NL	Probability
Y	0.17
N	0.83

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Building and quantifying BBN

1. Identification and understanding of the problem (purpose, scope, boundaries, variables)
2. Identification of relationships between variables and determination of the graphical structure of the BBN
3. Verification and validation of the structure
4. Quantification of conditional probability distributions
5. Verification and validation of the model – test runs
6. Analysis of the problem via conditionalization process

NOT easy in practice

Additional remarks on BBNs

- there are other BBNs in addition to discrete ones, i.e. continuous BBNs, mixed discrete-continuous BBNs
- BBNs are not only about causal relations between variables, they also capture probabilistic dependencies and can model mathematical (functional) relationships
- BBNs can be really large and complex
- but once the right BBN is built the analysis of the problem is fun and simple using available BBN software, e.g. Netica, Hugin, UniNet