Lecture - 2
Math - 1

Bayesian classifier

Bayesian classifier uses ~~Bayesian classif.~~ Bayes' Theorem of Probability for classification.

Ex:

Frequency Table

		Play Golf	
		Yes	No
Outlook	Sunny	3	2
	Overcast	4	0
	Rainy	2	3

		Play Golf	
		Yes	NO
Temp	Hot	2	2
	Mild	4	2
	cool	3	1

		Play Golf	
		Yes	NO
Humi-dity	High	3	4
	Nonmal	6	1

		Play Golf	
		Yes	NO
Windy	True	3	3
	False	6	2

Likelihood Table

LT-1

		Play Golf		
		Yes	NO	
Outlook	Sunny	3/9	2/5	5/14
	Overcast	4/9	0/5	4/14
	Rainy	2/9	3/5	5/14
		9/14	5/14	

LT-2

		Play Golf		
		Yes	NO	
Temp	Hot	2/9	2/5	4/14
	Mild	4/9	2/5	6/14
	cool	3/9	1/5	4/14
		9/14	5/14	

LT-3

		Play Golf		
		Yes	NO	
Humi-dity	High	3/9	4/5	7/14
	Nonmal	6/9	1/5	7/14
		9/14	5/14	

LT-4

		Play Golf		
		Yes	NO	
Windy	True	3/9	3/5	6/14
	False	6/9	2/5	8/14
		9/14	5/14	

From "LT-1" we found:

$P(c) = P(Yes) = 9/14 = 0.64$

$P(\cancel{c}) = P(NO) = 5/14 = 0.36$

~~P(Given~~

Calculating $P(x|c)$; ~~$= P(Yes|Sunny)$~~

$= P(Sunny|Yes) = \frac{3}{9} = 0.33$

$\boxed{\text{Posterion Probability}}$ $P(c|x) = P(Yes|Sunny)$ (Given)*

$= 0.33 \times 0.64\ (9/14) \div 0.36$

$= 0.6$

And. $P(x|c) = P(Sunny|NO) = 2/5 = 0.4$

$\boxed{\text{Posterion Probability}}$ $P(c|x) = \cancel{P(Sunny|NO)}\ P(NO|Sunny)$

$= \left(\frac{2}{5} \times \frac{5}{14}\right) / \frac{5}{14} = 0.4$

~~0.004~~

Ex:

Outlook	Temp	Humidity	Windy	Play
Rainy	cool	High	True	?

Soln: $P(X|Yes)$ ~~$P(Yes|X)$~~ $= P(Rainy|Yes) \times P(cool|Yes) \times$

$P(High|Yes) \times P(True|Yes) \times P(Yes)$

$= 2/9 \times 3/9 \times 3/9 \times 3/9 \times 9/14$

$= 0.00529$

Again,

$P(X|NO)$ ~~P(NO|X)~~ $= P(Rainy|NO) \times P(cool|NO) \times P(High|NO) \times P(False|NO) \times P(NO)$

$$= 3/5 \times \frac{1}{5} \times \frac{4}{5} \times \frac{3}{5} \times \frac{5}{14}$$

$$= 0.02057$$

Here, we can see NO > Yes ($0.02057 > 0.00529$)

So, X belongs to class ("Play Golf = "NO")

Ans:

Lecture - 4
Math

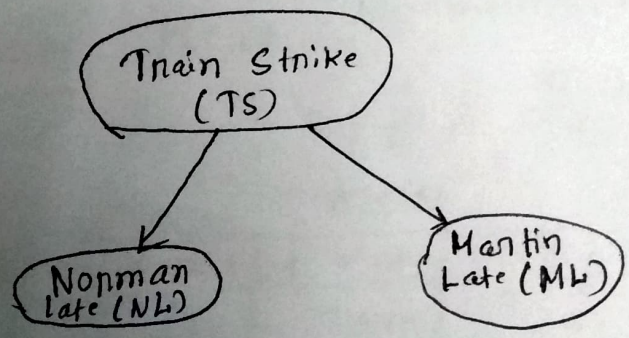
Bayesian Belief Network

Ex: Nonman – Train (via) } Predict → whether Nonmen
Martin – Drives } and Martin will be late for work.

Train Strike	Probability
Y	0.1
N	0.9

		Train State	
		Y	N
Nonman late	Y	0.8	0.1
	N	0.2	0.9

		Train State	
		Y	N
Martin Late	Y	0.6	0.5
	N	0.4	0.5



Formula: $P(ML, NL, TS) = P(ML \mid TS) \cdot P(NL \mid TS) \cdot P(TS)$
(joint distribution)

$P(ML, NL) = \sum_{t} P(ML, NL, TS = t)$
(Marginal Distribution)

Now, $P(ML=Y, NL=Y, TS=N) = P(ML=Y \mid TS=N) \cdot P(NL=Y \mid TS=N) \cdot P(TS=N)$

$= 0.5 \times 0.1 \times 0.9$

$= 0.045$

$P(ML=Y, NL=Y) = P(ML=Y, NL=Y, TS=Y) + P(ML=Y, NL=Y, TS=N)$

$= (0.6 \times 0.8 \times 0.1) + (0.5 \times 0.1 \times 0.9)$

$= 0.048 + 0.045$

$= 0.093$

Again, $P(NL=Y) = P(ML=Y, NL=Y, TS=Y) + P(ML=N, NL=Y, TS=N) + P(ML=N, NL=Y, TS=Y) + P(ML=Y, NL=Y, TS=N)$

$= 0.048 + 0.045 + 0.032 + 0.045$

$= 0.17$

or, $P(NL=Y) = P(ML=Y, NL=Y) + P(ML=N, NL=Y)$

$= 0.093 + 0.077$

$= 0.17$

Ans:

Lecture - 3
Math

Linear Discriminant Analysis (LDA)

Ex: compute the linear discription projection:

Samples → Class ω_1 : $X_1 = (x_1, x_2) = \{(4,2), (2,4), (2,3), (3,6), (4,4)\}$

class ω_2 : $X_2 = (x_1, x_2) = \{(9,10), (6,8), (9,5), (8,7), (10,8)\}$

The classes mean are:

$$\mu_1 = \frac{1}{N} \sum_{x \in \omega_1} x = \frac{1}{5} \begin{pmatrix} 4 & 2 & 2 & 3 & 4 \\ 2 & 4 & 3 & 6 & 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 3.8 \end{pmatrix}$$

$$\mu_2 = \frac{1}{N} \sum_{x \in \omega_2} x = \frac{1}{5} \begin{pmatrix} 9 & 6 & 9 & 8 & 10 \\ 10 & 8 & 5 & 7 & 8 \end{pmatrix} = \begin{pmatrix} 8.4 \\ 7.6 \end{pmatrix}$$

Here, N = Num of elements = 5

For class ω_1,

$$(x - \mu_1) = \begin{pmatrix} 1 & -1 & -1 & 0 & 1 \\ -1.8 & 0.2 & -0.8 & 2.2 & 0.2 \end{pmatrix}$$

Now, $(x-\mu_1)^T = \begin{bmatrix} 1 & -1.8 \\ -1 & 0.2 \\ -1 & -0.8 \\ 0 & 2.2 \\ 1 & 0.2 \end{bmatrix}$

Now,

$$S_1 = 1/(N-1)\{(x-\mu_1)\cdot(x-\mu_1)^T\}$$

$$= \frac{1}{5-1}\left\{\begin{pmatrix} 1 & -1 & -1 & 0 & 1 \\ -1.8 & 0.2 & -0.8 & 2.2 & 0.2 \end{pmatrix}\cdot\right.$$

$$\begin{bmatrix} 1 & -1.8 \\ -1 & 0.2 \\ -1 & -0.8 \\ 0 & 2.2 \\ 1 & 0.2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} \cancel{1\times(-1)\times(-1)\times(-1)\times(-1)\times} \\ 1\times 1 + (-1)\times(-1) + (-1)\times(-1) + 0 + 1 \\ 1.8\times 1.8 + 0.2\times 0.2 + 0.8\times 0.8 + 2.2\times 2.2 \\ + 0.2\times 0.2 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} \cancel{1\times(-1)} \\ 1+1+1+0+1 & -1.8-0.2+0.8+0+0.2 \\ -1.8-0.2+0.8+0+0.2 & 3.24+0.04+0.64+4.84+0.04 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 4 & -1 \\ -1 & 8.8 \end{pmatrix} = \begin{pmatrix} 1 & -0.25 \\ -0.25 & 2.2 \end{pmatrix}$$

For class ω_2,

$$(x-\mu_2) = \begin{pmatrix} 0.6 & -2.4 & 0.6 & -0.4 & 1.6 \\ 2.4 & 0.4 & -2.6 & -0.6 & 0.4 \end{pmatrix}.$$

and $$(x-\mu_2)^T = \begin{bmatrix} 0.6 & 2.4 \\ -2.4 & 0.4 \\ 0.6 & -2.6 \\ -0.4 & -0.6 \\ 1.6 & 0.4 \end{bmatrix}$$

Now, $$S_2 = \frac{1}{N-1}\left\{(x-\mu_2)\cdot(x-\mu_2)^T\right\}$$

$$= \frac{1}{5-1}\begin{pmatrix} 0.6\times0.6+2.4\times2.4+0.6\times0.6+0.4\times0.4+1.6\times1.6 & 0.6\times2.4-2.4\times0.4-0.6\times2.6+0.6\times0.4+1.6\times0.4 \\ 2.4\times0.6-0.4\times2.4-2.6\times0.6+0.6\times0.4+0.4\times1.6 & 2.4\times2.4+0.4\times0.4+2.6\times2.6+0.6\times0.6+0.4\times0.4 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 9.2 & -0.2 \\ -0.2 & 13.2 \end{pmatrix}$$

$$= \begin{pmatrix} 2.3 & -0.05 \\ -0.05 & 3.3 \end{pmatrix}$$

$$S_W = S_1 + S_2 = \begin{pmatrix} 3.3 & -0.3 \\ -0.3 & 5.5 \end{pmatrix}$$

$$W^* = S_W^{-1}(\mu_1 - \mu_2) = \begin{pmatrix} 3.3 & -0.3 \\ -0.3 & 5.5 \end{pmatrix}^{-1} \left[\begin{pmatrix} 3 \\ 3.8 \end{pmatrix} - \begin{pmatrix} 8.4 \\ 7.6 \end{pmatrix} \right]$$

Formula:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}$$

inverse matrix

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{18.15 - 0.09} \begin{pmatrix} 5.5 & 0.3 \\ 0.3 & 3.3 \end{pmatrix} \cdot \begin{pmatrix} -5.4 \\ -3.8 \end{pmatrix}$$

(18.06)

$$= \frac{1}{18.06} \begin{pmatrix} 0.3045 & 0.0166 \\ 0.0166 & 0.1827 \end{pmatrix} \cdot \begin{pmatrix} -5.4 \\ -3.8 \end{pmatrix}$$

$$= \begin{pmatrix} -1.6443 - 0.0631 \\ -0.0896 - 0.6943 \end{pmatrix}$$

$$= \begin{pmatrix} -1.7074 \\ -0.7839 \end{pmatrix}$$

Ans: