Template Matching

Q: Find Minimum Edit Distance → DREAM → Ref. Word; DRREUM → Test word

Insert = 1
change = 1
Delete = 1

Sol^n:

No change (diagonal value)

Ref. Word (j)

Test word (i)

	Null	D	R	E	A	M
Null	0	1	2	3	4	5
D	1	0	0+1 = 1 (min)	2	3 (2+1)	4
R	2	1	0	1	2	3
R	3	2	1	1	2	3
E	4	3	2	1	2	3
U	5	4	3	2	2	3
M	6	5	4	3	3	2

Adjacent min; change add

Final Outcome

if (r == c)
d(i,j) = d(i-1),(j-1)

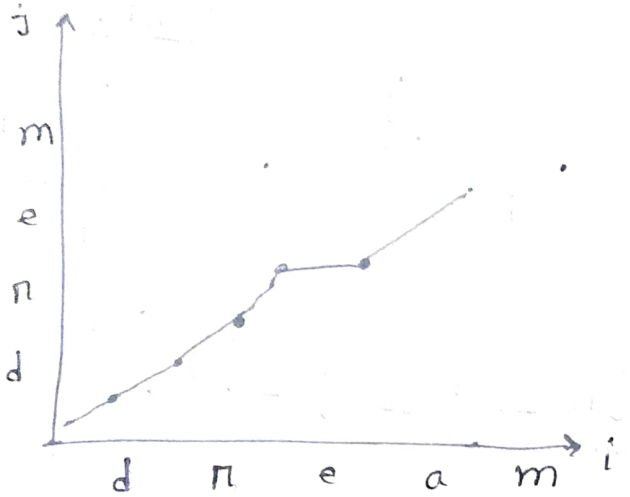
Else if (r != c)
d(i,j) = min (d(i-1),(j)), d(i-1),(j-1), d((i),(j-1))}

INFORM
INFORMATION → cost - 5
INFORMING → cost - 3

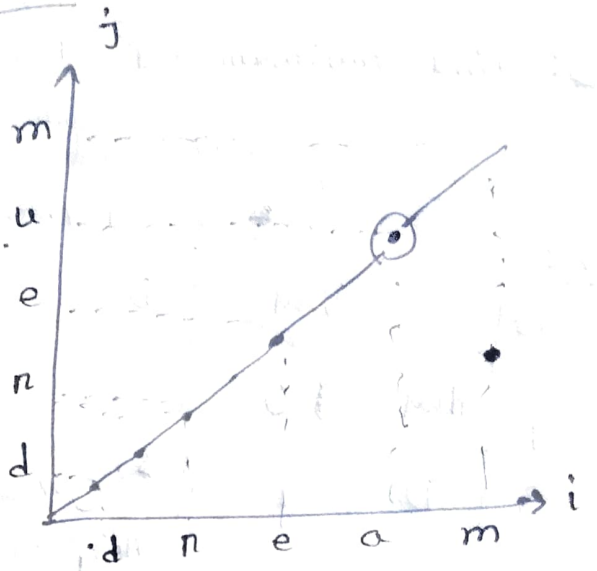
So, for suggestion ~~to~~ Informing will come first then information will come. cause 3 < 5.

Q type → there will be a given Ref. word and test word given.

Graph Draw



Imention, D = 1



Change, D = 1

Fuzzy C Means

Q: Input Number of objects = 6, Number of clusters = 2

X	Y	C1	C2
1	6	0.8	0.2
2	5	0.9	0.1
3	8	0.7	0.3
4	4	0.3	0.7
5	7	0.5	0.5
6	9	0.2	0.8

Soln:

Step-1: Initialize the membership matrix.

Step-2: Find the constraint using the equation.

Formula

$$c_j = \left[\frac{\sum_i [u_j(x_i)]^m x_i}{\sum_i [u_j(x_i)]^m}, \frac{\sum_i [u_j(y_i)]^m y_i}{\sum_i [u_j(y_i)]^m} \right]$$

distance from cluster. power

Here, ~~u_{ij} distance~~

~~m is a~~

Here, m is a fuzzification parameter. The range of m is always [1.25, 2]

$$= \left[\frac{u_1(x_1)^m x_1 + u_1(x_2)^m x_2 + u_1(x_3)^m x_3 + u_1(x_4)^m x_4 + u_1(x_5)^m x_5 + u_1(x_6)^m x_6}{u_1(x_1)^m + u_1(x_2)^m + u_1(x_3)^m + u_1(x_4)^m + u_1(x_5)^m + u_1(x_6)^m}, \right.$$

$$\left. \frac{u_1(y_1)^m y_1 + u_1(y_2)^m y_2 + u_1(y_3)^m y_3 + u_1(y_4)^m y_4 + u_1(y_5)^m y_5 + u_1(y_6)^m y_6}{u_1(y_1)^m + u_1(y_2)^m + u_1(y_3)^m + u_1(y_4)^m + u_1(y_5)^m + u_1(y_6)^m} \right]$$

$$c_1 = \left[\frac{1*0.8^2 + 2*0.9^2 + 3*0.7^2 + 4*0.3^2 + 5*0.5^2 + 6*0.2^2}{0.8^2 + 0.9^2 + 0.7^2 + 0.3^2 + 0.5^2 + 0.2^2}, \right.$$

$$\left. \frac{6*0.8^2 + 5*0.9^2 + 8*0.7^2 + 4*0.3^2 + 7*0.5^2 + 9*0.2^2}{0.8^2 + 0.9^2 + 0.7^2 + 0.3^2 + 0.5^2 + 0.2^2} \right]$$

$$c_1 = \frac{5.58}{2.32}, \frac{14.28}{2.32}$$

$\therefore c_1 = (2.4, 6.1)$

Again, $$c_2 = \left[\frac{1*0.2^2 + 2*0.2^2 + 3*0.3^2 + 4*0.7^2 + 5*0.5^2}{0.2^2 + 0.1^2 + 0.3^2 + 0.7^2 +} \right.$$

$$\frac{+ 6*0.8^2}{0.5^2 + 0.8^2}, \frac{6*0.2^2 + 5*0.1^2 + 8*0.3^2 +}{0.2^2 + 0.1^2 + 0.3^2 + 0.7^2 +}$$

$$\left. \frac{4*0.7 + 7*0.5^2 + 9*0.8^2}{0.5^2 + 0.8^2} \right]$$

$$c_2 = \frac{7.38}{1.52}, \frac{10.48}{1.52}$$

$\therefore c_2 = (4.8, 6.8)$

Step-3: Find Distance.

$$D_i = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Centroid 1:

$(\underset{x_1}{1}, \underset{y_1}{6})(\underset{x_2}{2.4}, \underset{y_2}{6.1}) = \sqrt{(1.4)^2 + (0.1)^2} = \sqrt{1.96 + 0.01} = \sqrt{1.97} = 1.40$

$(2,5)(2.4, 6.1) = \sqrt{(2-2.4)^2 + (5-6.1)^2} = \sqrt{0.16 + 1.21} = \sqrt{1.37} = 1.17$

$(3,8)(2.4, 6.1) = \sqrt{0.36 + 3.61} = \sqrt{3.97} = 1.99$

$(4,4)(2.4, 6.1) = \sqrt{2.56 + 4.41} = \sqrt{6.97} = 2.64$

$(5,7)(2.4, 6.1) = \sqrt{6.76 + 0.81} = \sqrt{7.57} = 2.75$

$(6,9)(2.4, 6.1) = \sqrt{12.96 + 8.41} = \sqrt{21.37} = 4.62$

Centroid 2:

$(1,6)(4.8, 6.8) = \sqrt{14.44 + 0.64} = \sqrt{15.08} = 3.88$

$(2,5)(4.8, 6.8) = \sqrt{7.84 + 3.24} = \sqrt{11.08} = 3.32$

$(3,8)(4.8, 6.8) = \sqrt{3.24 + 1.44} = \sqrt{4.68} = 2.16$

$(4,4)(4.8, 6.8) = \sqrt{0.64 + 7.84} = \sqrt{8.48} = 2.91$

$(5,7)(4.8, 6.8) = \sqrt{0.04 + 0.04} = \sqrt{0.08} = 0.28$

$(6,9)(4.8, 6.8) = \sqrt{1.44 + 4.84} = \sqrt{6.28} = 2.50$

Cluster 1		Cluster 2		
Datapoint	Distance	Datapoint	Distance	Distance
(1,6)	1.40	(1,6)	~~(1,6)~~	3.88
(2,5)	1.17	(2,5)	~~(2,5)~~	3.32
(3,8)	1.09	(3,8)	~~(3,8)~~	2.16
(4,4)	2.64	(4,4)	~~(4,4)~~	2.91
(5,7)	2.75	(5,7)	~~(5,7)~~	0.28
(6,9)	4.62	(6,9)	~~(6,9)~~	2.50

Step-4: Update the membership value.

$$u_j(x_i) = \frac{\left[\frac{1}{d_{ji}}\right]^{1/m-1}}{\sum_{k=1}^{c}\left[\frac{1}{d_{ki}}\right]^{1/m-1}}$$

here, $m=2$, i - first data point, j - first cluster
c = cluster

Cluster 1:
Now,

$$u_{11} = \frac{\left(\frac{1}{d_{11}}\right)^{\frac{1}{2-1}}}{\sum_{k=1}^{c}\left(\frac{1}{d_{ki}}\right)^{\frac{1}{2-1}}}$$

Here,
$c = 2$
(cause there are 2 clusters.)

$$u_{11} = \frac{\left(\frac{1}{d_{11}}\right)^1}{\left(\frac{1}{d_{11}}\right)^1 + \left(\frac{1}{d_{21}}\right)^1}$$

$$= \frac{\frac{1}{1.4}}{\left(\frac{1}{1.4}\right)^1 + \left(\frac{1}{3.88}\right)^1} = \frac{0.71}{0.71 + 0.25}$$

$$= 0.7$$

$$u_{12} = \frac{\frac{1}{d_{12}}}{\left(\frac{1}{d_{12}}\right)+\left(\frac{1}{d_{22}}\right)} = \frac{\frac{1}{1.17}}{\frac{1}{1.17}+\frac{1}{3.32}} = \frac{0.56}{0.86} = 0.6$$

$$u_{13} = \frac{\frac{1}{d_{13}}}{(1/d_{13})+(1/d_{23})} = \frac{1/1.99}{(1/1.99)+(1/2.16)} = \frac{0.50}{0.96}$$

$$= 0.5$$

$$u_{14} = \frac{(1/d_{14})}{(1/d_{14})+(1/d_{24})} = \frac{1/2.64}{(1/2.64)+(1/2.91)} = \frac{0.37}{0.37+0.34}$$

$$= 0.5$$

$$u_{15} = \frac{(1/d_{15})}{(1/d_{15})+(1/d_{25})} = \frac{1/2.75}{(1/2.75)+(1/0.28)} = \frac{0.36}{3.03} = 0.1$$

$$u_{16} = \frac{(1/d_{16})}{(1/d_{16})+(1/d_{26})} = \frac{(1/4.62)}{(1/4.62)+(1/2.50)} = \frac{0.21}{0.61} = 0.3$$

Cluster 2:

$$u_{21} = \frac{(1/d_{21})}{(1/d_{12})+(1/d_{21})} = \frac{1/3.88}{(1/1.40)+(1/3.88)} = \frac{0.25}{0.96} = 0.3$$

$$u_{22} = \frac{(1/d_{22})}{(1/d_{12})+(1/d_{22})} = \frac{(1/3.32)}{(1/1.17)+(1/3.32)} = \frac{0.30}{0.86} = 0.4$$

$$u_{23} = \frac{(1/d_{23})}{(1/d_{13})+(1/d_{23})} = \frac{(1/2.16)}{(1/1.99)+(1/2.16)} = \frac{0.46}{0.96} = 0.5$$

$$u_{24} = \frac{(1/d_{24})}{(1/d_{14})+(1/d_{24})} = \frac{1/2.19}{(1/2.64)+(1/2.19)} = \frac{0.34}{0.71}$$

$$= 0.5$$

$$u_{25} = \frac{(1/d_{25})}{(1/d_{15})+(1/d_{25})} = \frac{(1/0.28)}{(1/2.75)+(1/0.28)} = \frac{3.57}{3.93}$$

$$= 0.9$$

$$u_{26} = \frac{(1/d_{26})}{(1/d_{16})+(1/d_{26})} = \frac{0.4}{0.21+0.4} = \frac{0.4}{0.61} = 0.7$$

Now, the new membership value is:

X	Y	C1	C2
1	6	0.7	0.3
2	5	0.6	0.4
3	8	0.5	0.5
4	4	0.5	0.5
5	7	0.1	0.9
6	9	0.3	0.7

Ans:

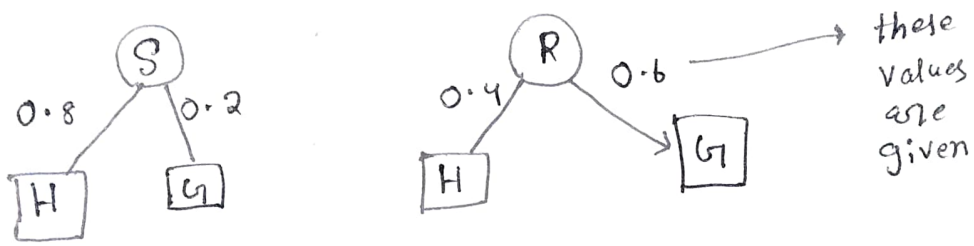
Exam এ যদি Obj 6 টা থাকে তাহলে 1/2 টা point এর updated value নিলেই হবে। সবগুলো করা লাগবে না i.e. $u_{11}, u_{12}, u_{21}, u_{22}$ নিলেই হবে।

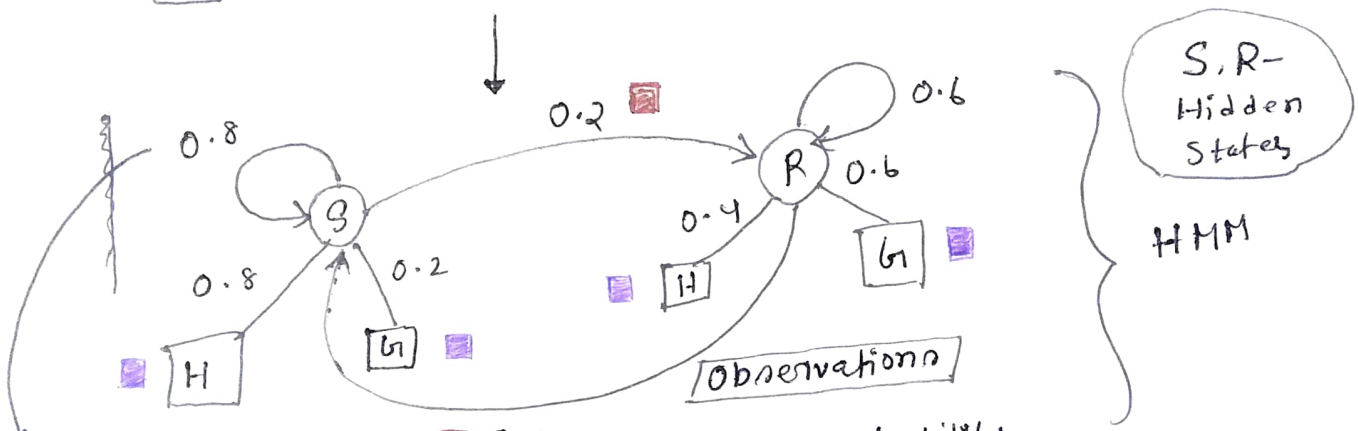
Hidden Markov Model

HMM

Bob → happy
Weather → Sunny

Bob → Grumpy
Weather → Rainy





the Probability of tomorrow being sunny

0.4 → the probability of today rainy ~~sun~~ and tomorrow sunny

transition probabilities
Emission probabilities

Here, Alice can only see Bob's mood. She doesn't get to see the weather. So, those are observations. And the weather is called the hidden states (Alice can't see weather).

Questions

1. How did we find these probabilities ? (How can we calculate it ?)
2. What's the ~~probabilities~~ probability that a random day is sunny or rainy ?
3. If Bob is Happy today, what's the probability that it's sunny or ~~Rainy~~ Rainy today ?
4. If for three days Bob is ~~heppy~~ happy, Grumpy, Happy, what was the weather ?

1 No Ans:

Data:

S	S	S	S	R	R	R	S	S	S	S	R	R	S	S	S
H	H	H	H	G	G	G	H	H	H	H	G	G	H	H	H

S → S (sequence) – 8 times (0.8)

S → R – 2 times (0.2)
↳ probability

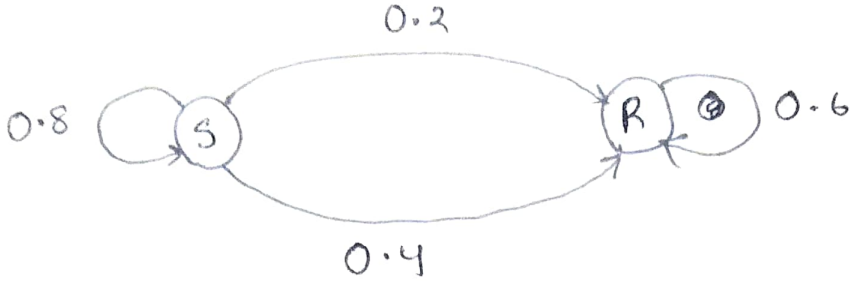
R → R – 3 (0.4)

R → S – 2 (0.6)

S → Sunny
R → Rainy
H → Happy
G → Grumpy

Total Sunny → 10
Total Rainy → 5

	S	R
S	8/10	2/10
R	2/5	3/5



Transitional probability

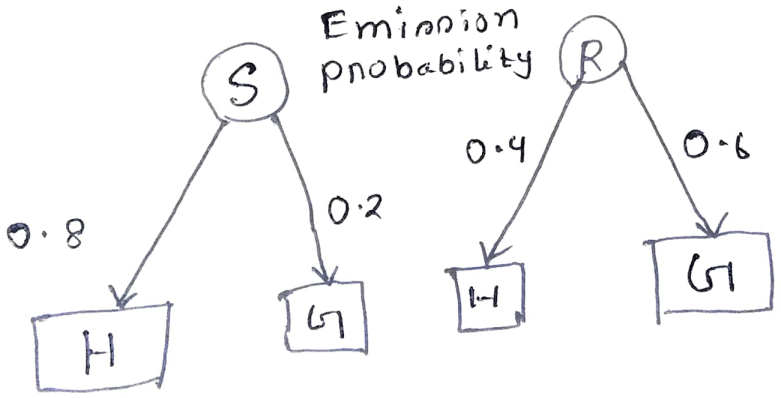
Again,

$$\begin{cases} S \to H & - 8 \\ S \to G & - 2 \end{cases}$$

$$\begin{cases} R \to G & - 3 \\ R \to H & - 2 \end{cases}$$

$$\begin{cases} \text{Total Sunny (S)} - 10 \\ \text{Total Rainy (R)} - 5 \end{cases}$$

	H	G
S	8/10	2/10
R	2/5	3/5

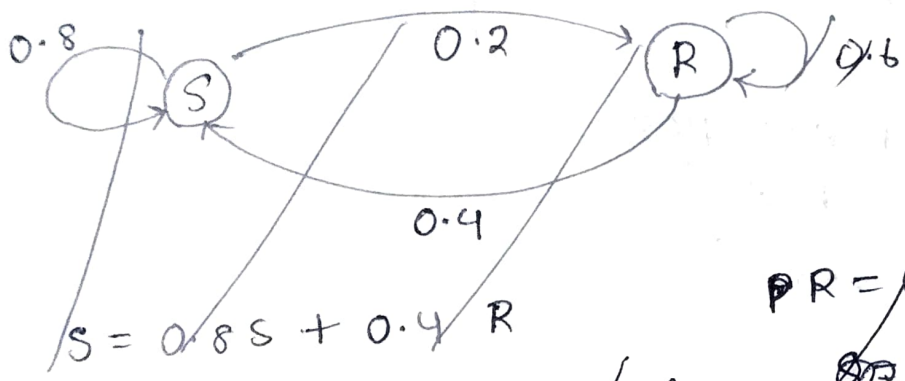


2 NO Ans:

Total Sunny (S) = 10
Total Rainy (R) = 5 → 15

So, $P(S) = 10/15 = 2/3$

$P(R) = 5/15 = 1/3$



$S = 0.8S + 0.4R$

$R = 0.2S + 0.6R$

$S + R = 1$

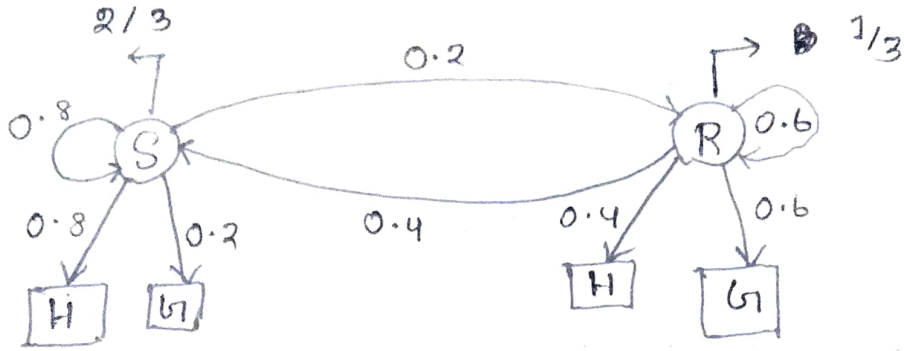
3 NO Ans:

We know, Bayes Theorem $P(A|B) = \dfrac{P(B|A)\,P(A)}{P(B)}$

We have to find out $P(S|H)$ and $P(R|H)$.

We can write, $P(S|H) = \dfrac{P(H|S)\,P(S)}{P(H)}$

$P(H|S) = ?$
$P(S) = ?$
$P(H) = ?$



$P(H|S) = 0.8$

$P(S) = 2/3$

$P(H) = 10/15 = \frac{2}{3}$

Total Happy(H) = 10

Total H + G = 10+5 = 15

Now,

$$P(S|H) = \frac{P(H|S)\,P(S)}{P(H)}$$

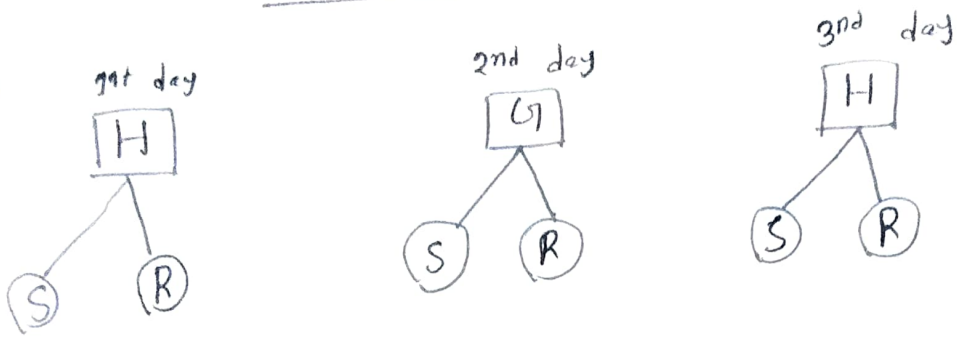
$$= \frac{0.8 \times \frac{2}{3}}{\frac{2}{3}}$$

$$= 0.8 \text{ or } \frac{4}{5}$$

Ans:

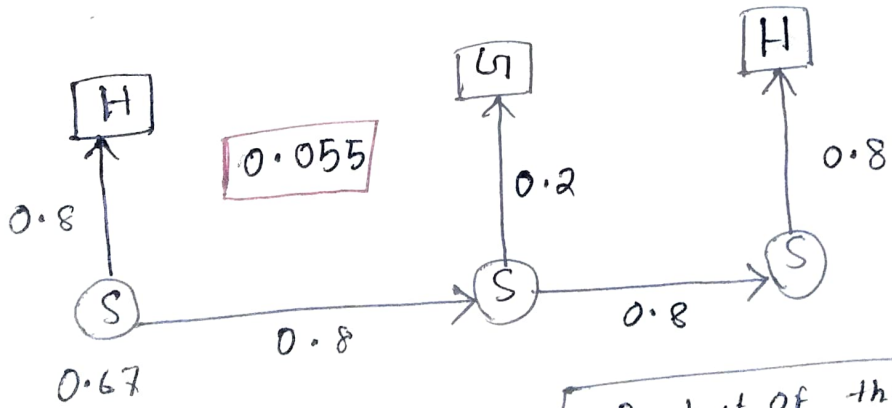
4 No Am:

Soln:



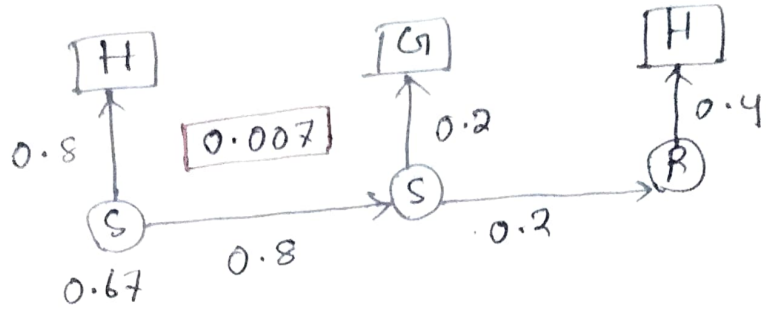
	No.	1st Day	2nd Day	3rd Day
✓	1	S	S	S
✓	2	S	S	R
✓	3	S	R	S
✓	4	S	R	R
✓	5	R	R	R
✓	6	R	S	S
✓	7	R	S	R
✓	8	R	R	S

(1)

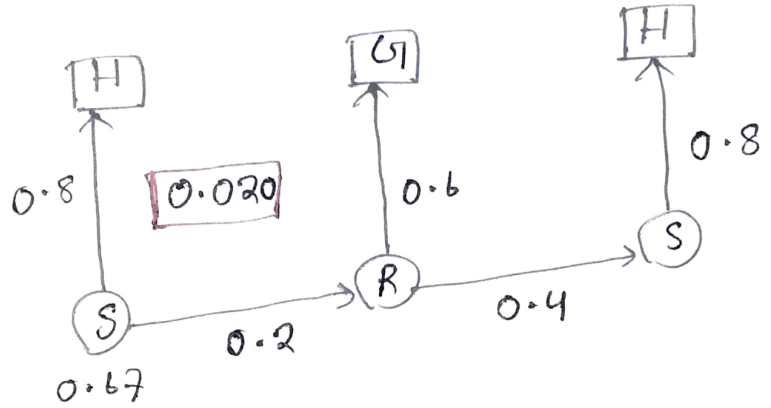


Product of the probability → $0.8 \times 0.67 \times 0.8 \times 0.8 \times 0.8 \times 0.2 = 0.05$

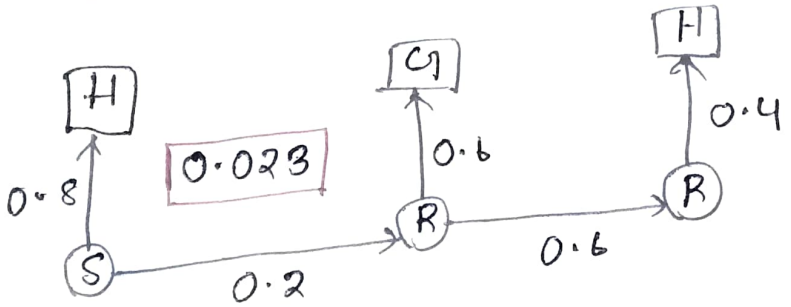
(2)


H
G
H
0.8
0.007
0.2
0.4
S
S
R
0.8
0.2
0.67

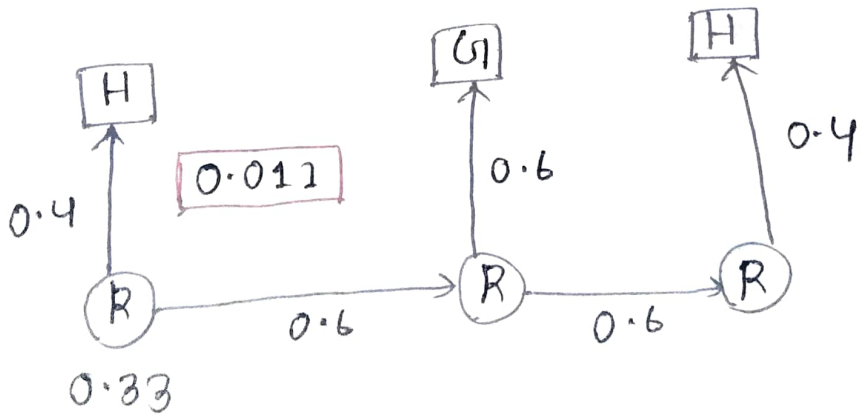
(3)


H
G
H
0.8
0.020
0.6
0.8
S
R
S
0.2
0.4
0.67

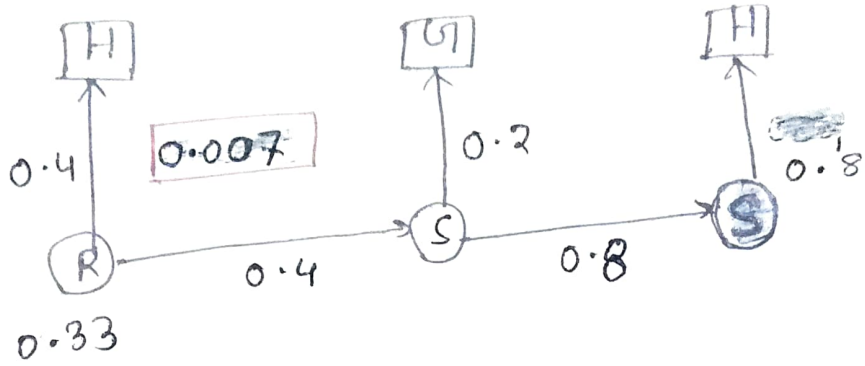
(4)


H
G
H
0.8
0.023
0.6
0.4
S
R
R
0.2
0.6

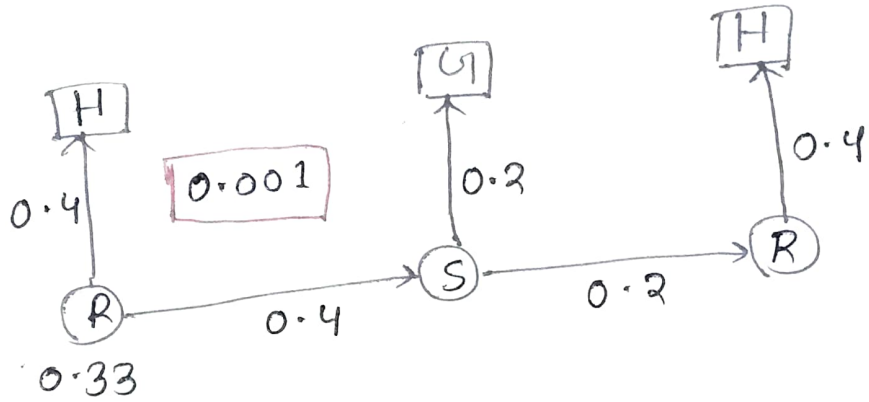
(5)


H
G
H
0.4
0.011
0.6
0.4
R
R
R
0.6
0.6
0.33

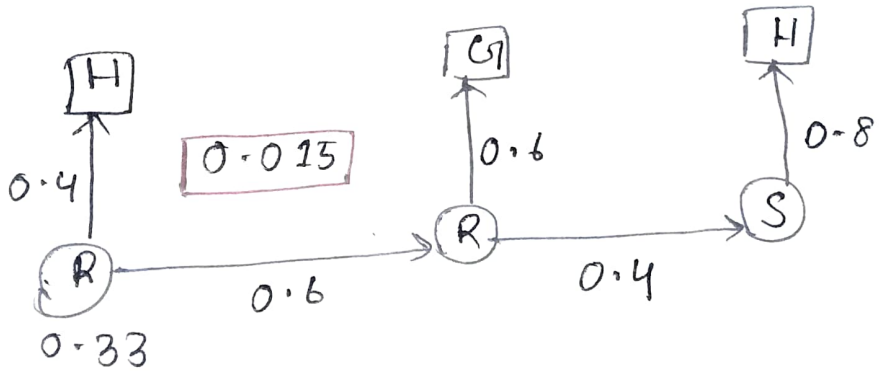
(6)



(7)



(8)



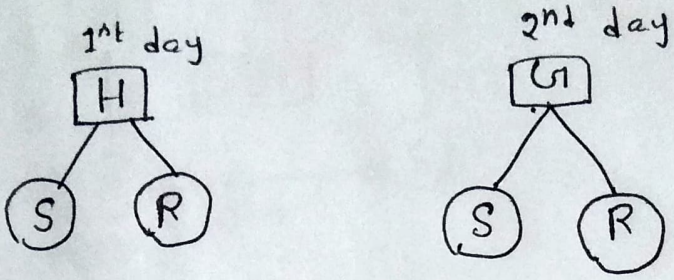
We can see among the 8 combinations 1st one is the highest.

So, if for three days Bob is Happy, Grumpy, Happy then the weather was Sunny, Sunny, Sunny.

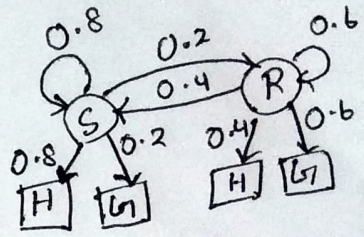
Ans:

■ If for two days Bob is happy, grumpy, what was the weather?

Soln:



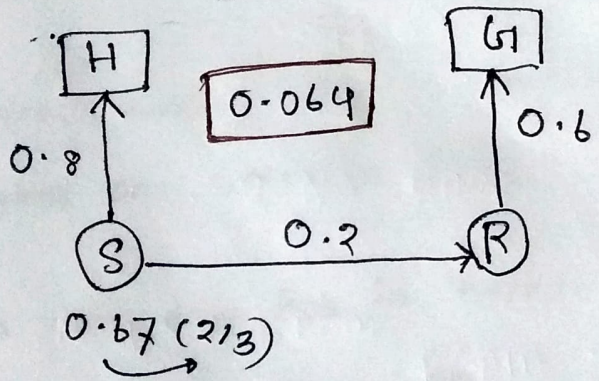
No.	1st Day	2nd Day
1	Sunny	Sunny
2	Sunny	~~Sunny~~ Rainy
3	Rainy	Sunny
4	Rainy	Rainy



HMM

Now,

2nd combination

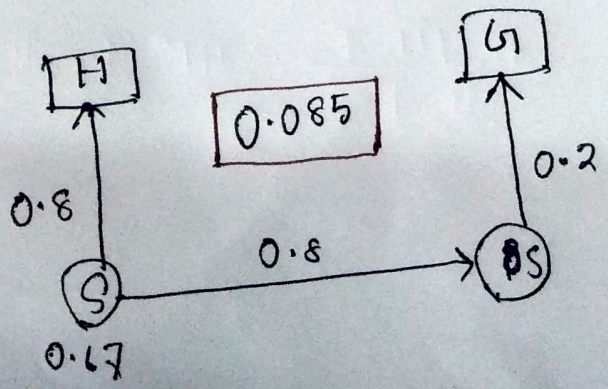


Product of all probability ~~Product~~

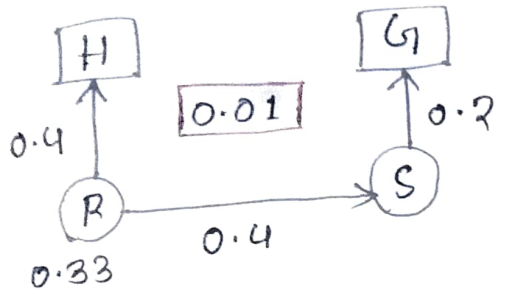
$0.6 \times 0.2 \times 0.8 \times 0.67$

$= 0.064$

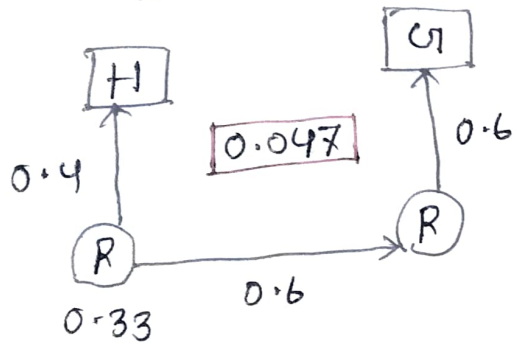
1st combination



3rd combination



4th combination



We can see among the 4 combinations 1st combination is the highest one which is 0.085.

So, if for two days Bob is happy, grumpy then the weather was sunny, sunny.

Ans.

ROC

X	Y	Y(T0)	Y(T3)	Y(T4.5)	Y(T5)	Y(T6.5)	Y(T9.5)
1	0	1	0	0	0	0	0
4	0	1	1	0	0	0	0
5	0	1	1	1	0	0	0
6	1	1	1	1	1	0	0
7	1	1	1	1	1	1	0
9	1	1	1	1	1	1	0

Y(T0)

P/A(T0)	1	0
1	3	0
0	3	0

True positive rate → TPR = TP/(TP+FN)

$= 3/(3+0)$

$= 1$

False positive rate → FPR = FP/(FP+TN)

$= 3/(3+0)$

$= 1$

Y(T3)

P/A (T3)	1	0
1	3	0
0	2	1

TPR $= 3/(3+0)$

$= 1$

FPR $= 2/(2+1)$

$= 0.67$

Y(T 4.5)

P/A (T4.5)	1	0
1	3	0
0	1	2

~~TPR = 3/(2+1)~~ ~~300~~

~~= 0.67~~ = 3

~~FPR = 0~~

TPR = 3/(3+0)

= 1

FPR = 1/(1+2)

= 0.33

Y (T5)

P/A (T5)	1	0
1	3	0
0	0	3

TPR = 3/(3+0)

= 1

FPR = 0/(0+3)

= 0

Y (6.5)

P/A (T6.5)	1	0
1	2	1
0	0	3

TPR = 2/(2+1)

= 0.67

FPR = 0/(0+3)

= 0

Y (9.5)

P/A) (T9.5)	1	0
1	0	3
0	0	3

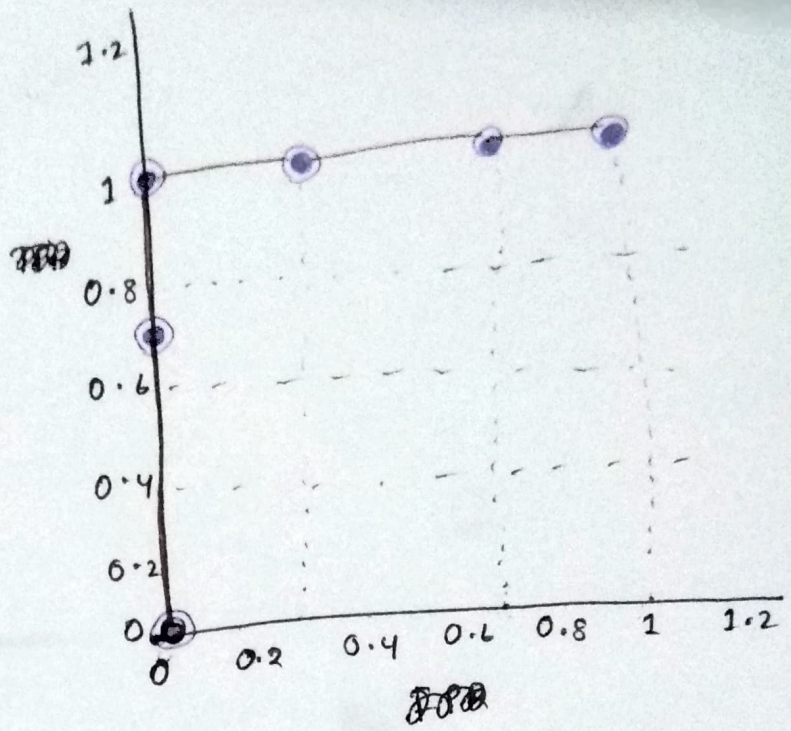
TPR = 0/(0+3)

= 0

FPR = 0/(0+3)

= 0

Threshold	FPR	TPR
0	1	1
3	0.67	1
4.5	0.33	1
5	0	1
6.5	0	0.67
9.5	0	0



Ans: