

CpE 3101L – Introduction to HDL

Unit 7: Finite State Machines (FSMs)

Outline

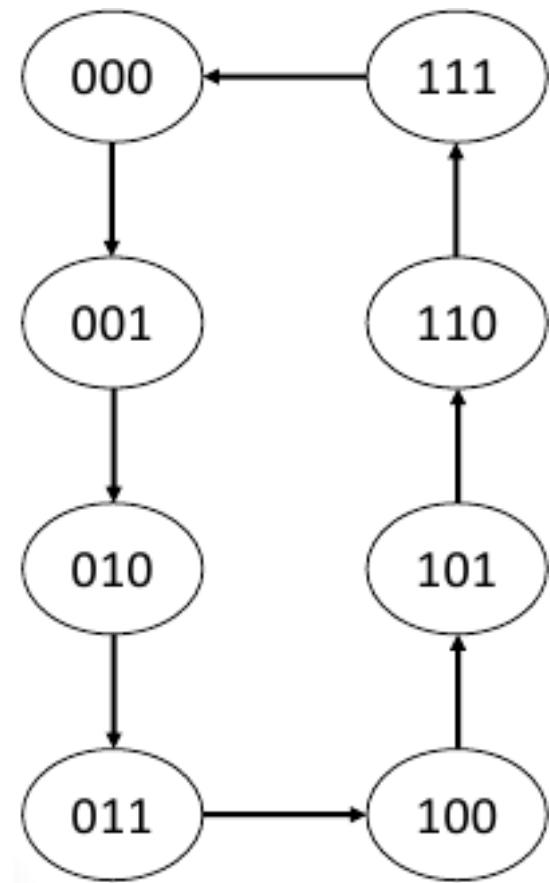
- FSMs in Digital Systems
- Mealy and Moore Models
- State Diagrams as FSM Representations
- Algorithmic State Machines (ASM) and ASM Charts
- Behavioral Modeling of FSMs
- Synthesis of FSMs
- FSM Partitioning Schemes
- State Assignment and Encoding
- Algorithmic State Machine and Datapath (ASMD) Charts
- Design Example with ASMD Chart
- HDL Description of Design Example
- Modular Approach in Complex FSMs

Finite State Machines (FSMs)

- A **state machine** is another term for a **sequential circuit**, which is the basic structure of a digital system.
- The term ***finite*** refers to a **definite, exact, quantifiable number of states** this machine/system has.
- **States** are determined by the **number of memory elements (*flip-flops*)**.
 - Maximum number of states is 2^n where n is the number of flip-flops.

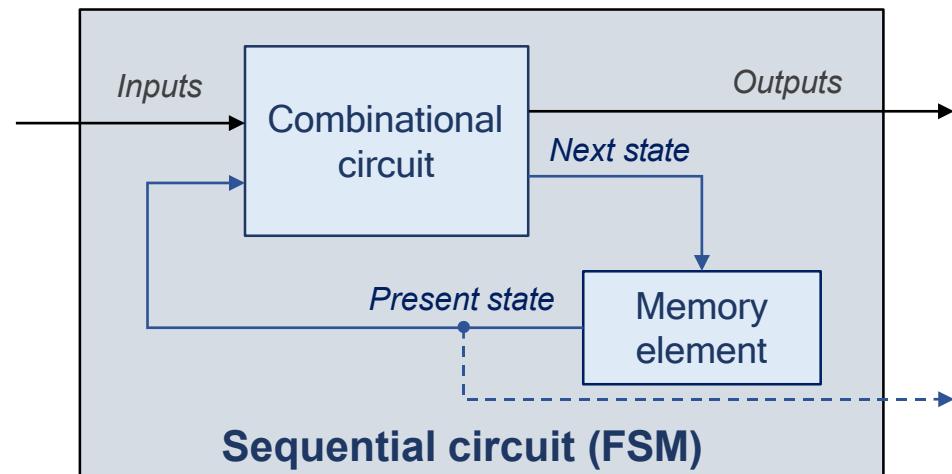
Finite State Machines (FSMs)

- FSMs are usually specified by **state diagrams**.
- Generally composed of **three components/ divisions**.



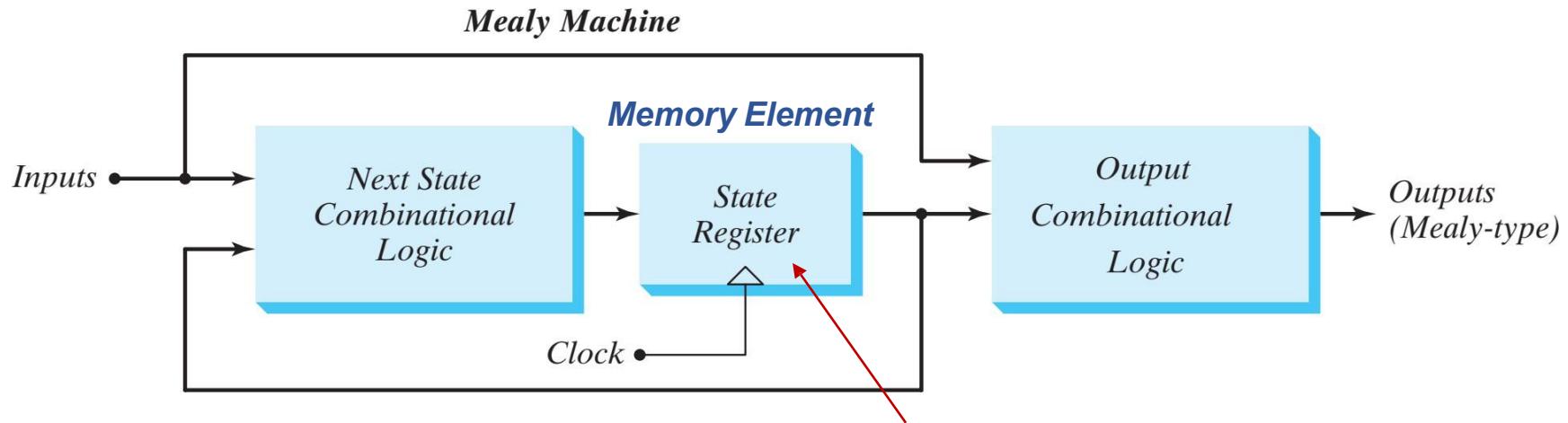
Mealy and Moore Models

- The **most general model** of a **sequential circuit** or an **FSM** has:
 - Inputs
 - Outputs
 - Internal states
- **Two models:**
 - Mealy
 - Moore
- *They differ in the way their output is generated.*



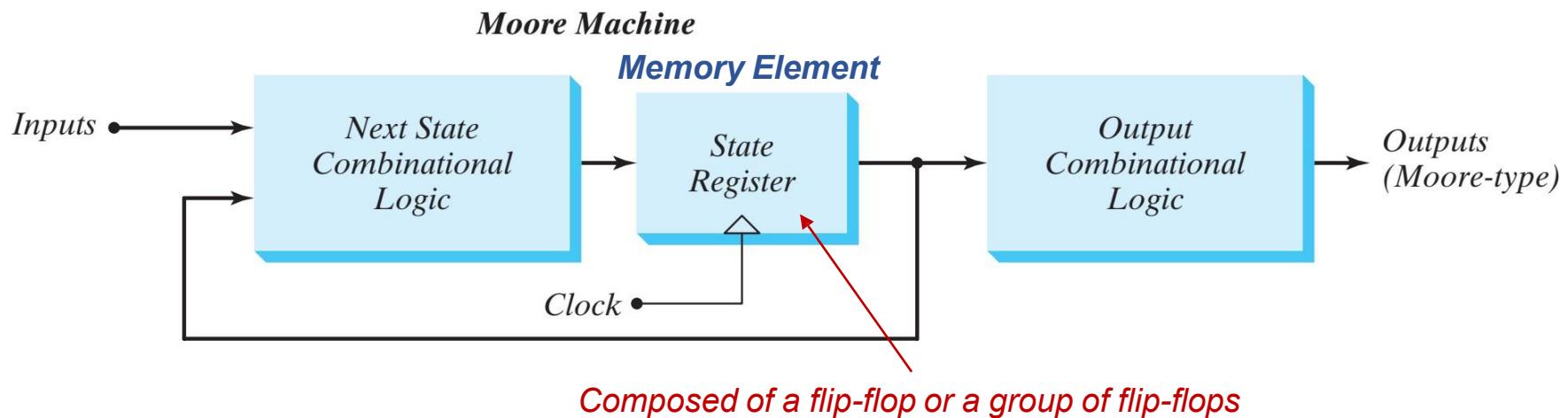
Mealy Model

- Also known as **Mealy FSM** or **Mealy machine**
- The **output** is a function of (*is dependent on*) **both the present state and the input**.
 - **Outputs** of a Mealy FSM **may change if the inputs change at any time** (*harder to synchronize*).
 - **States** cannot change until a triggering clock edge.

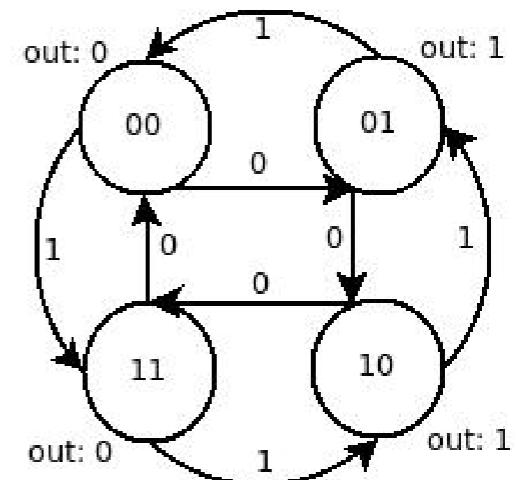
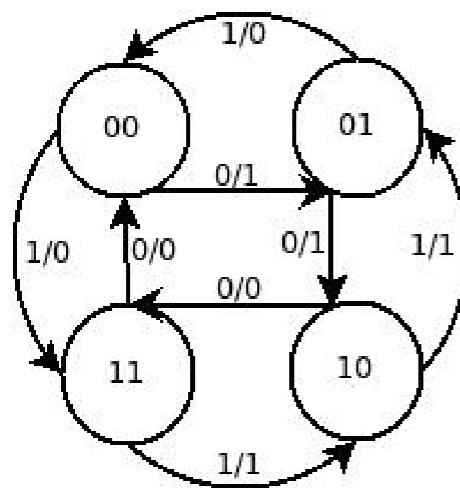
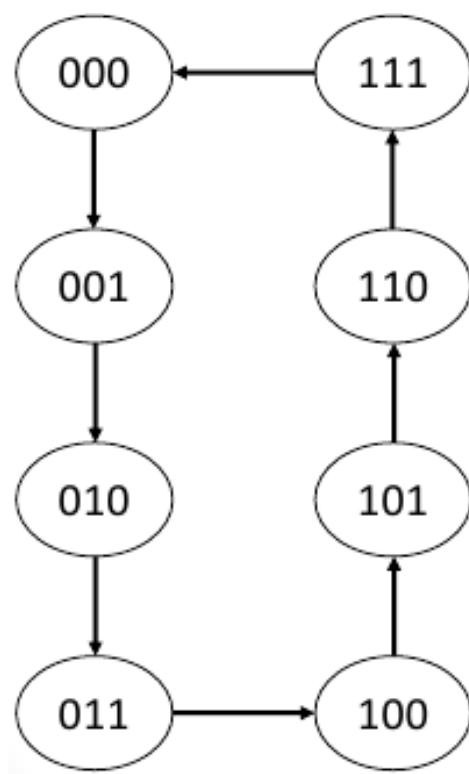


Moore Model

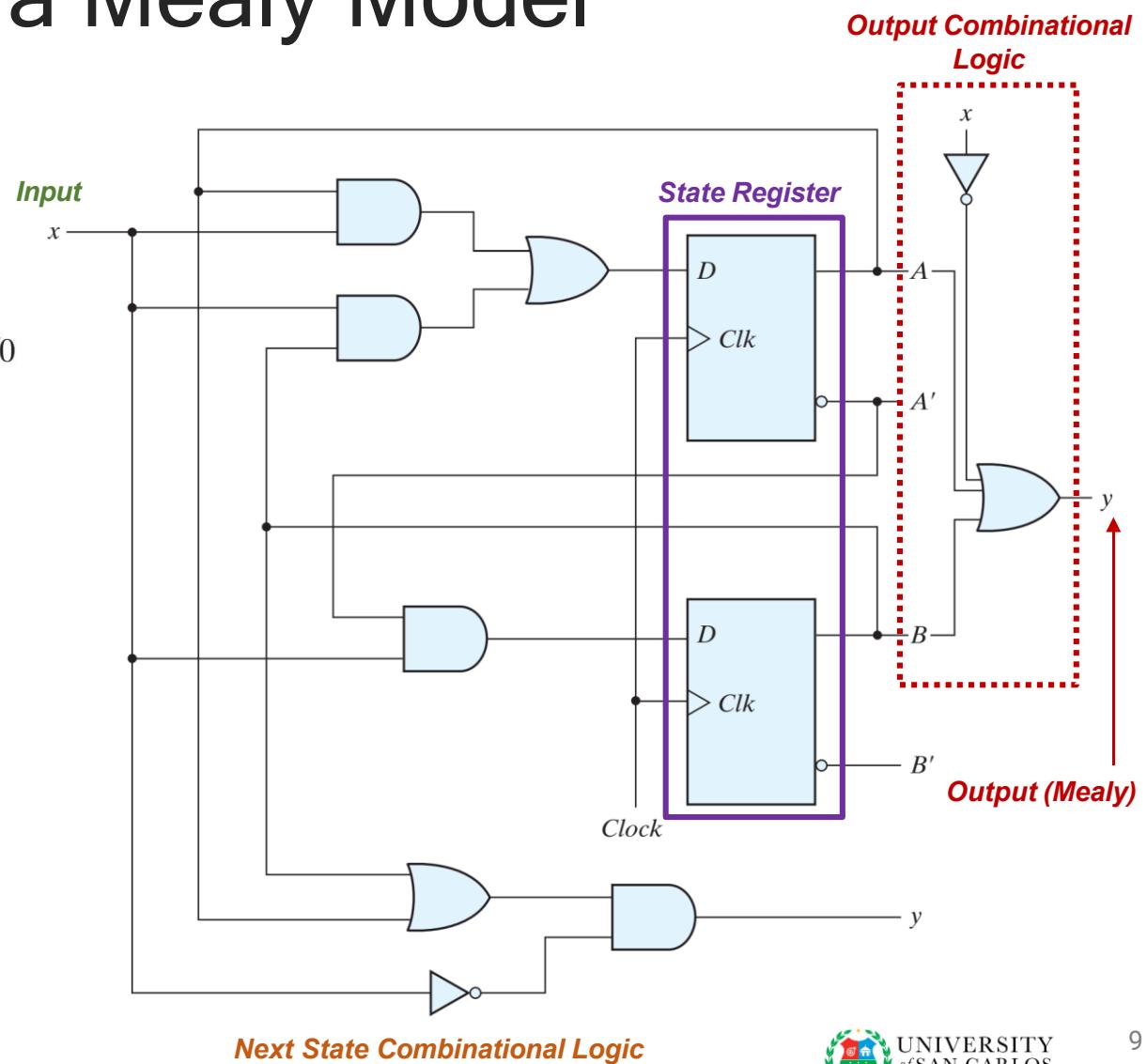
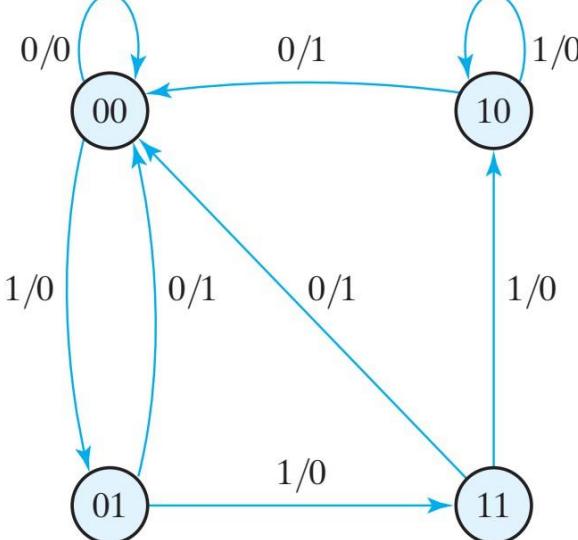
- Also known as **Moore FSM** or **Moore machine**
- The **output** is a **function of (is dependent on) only the present state**.
 - **Outputs** and **states** of the Moore FSM are **synchronized with the clock**.



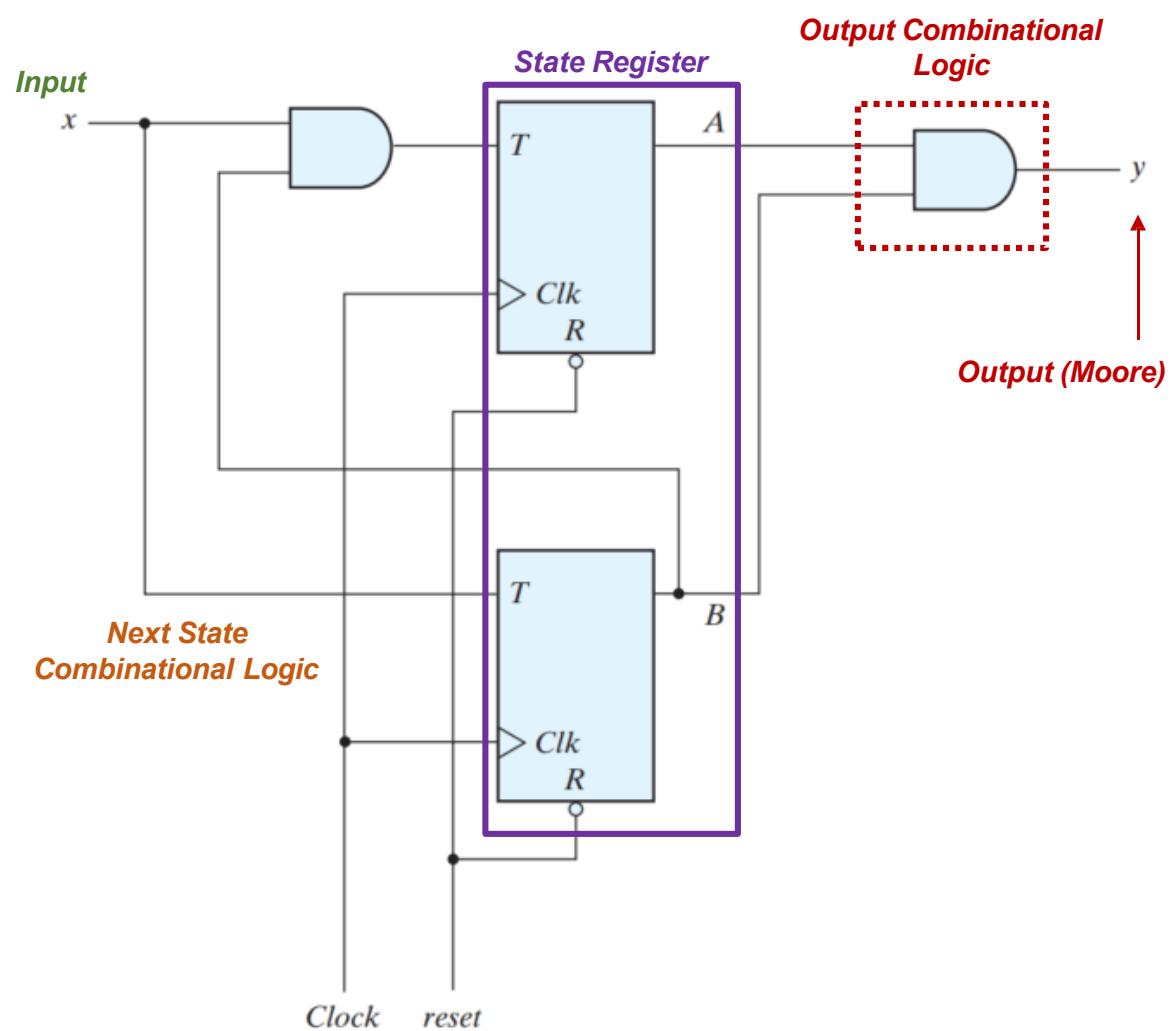
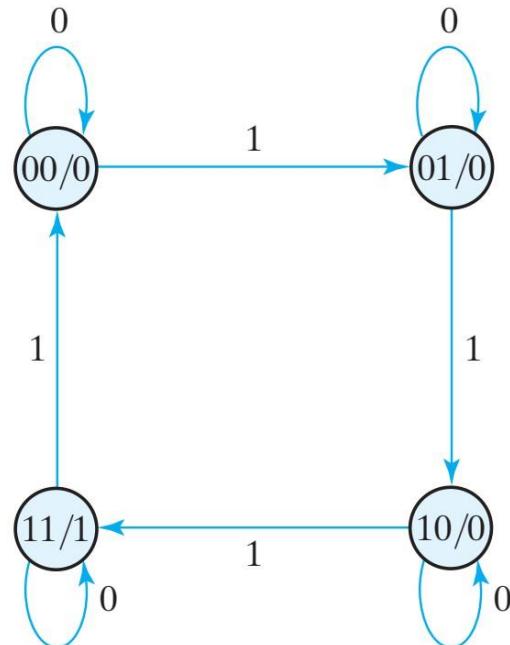
Mealy or Moore?



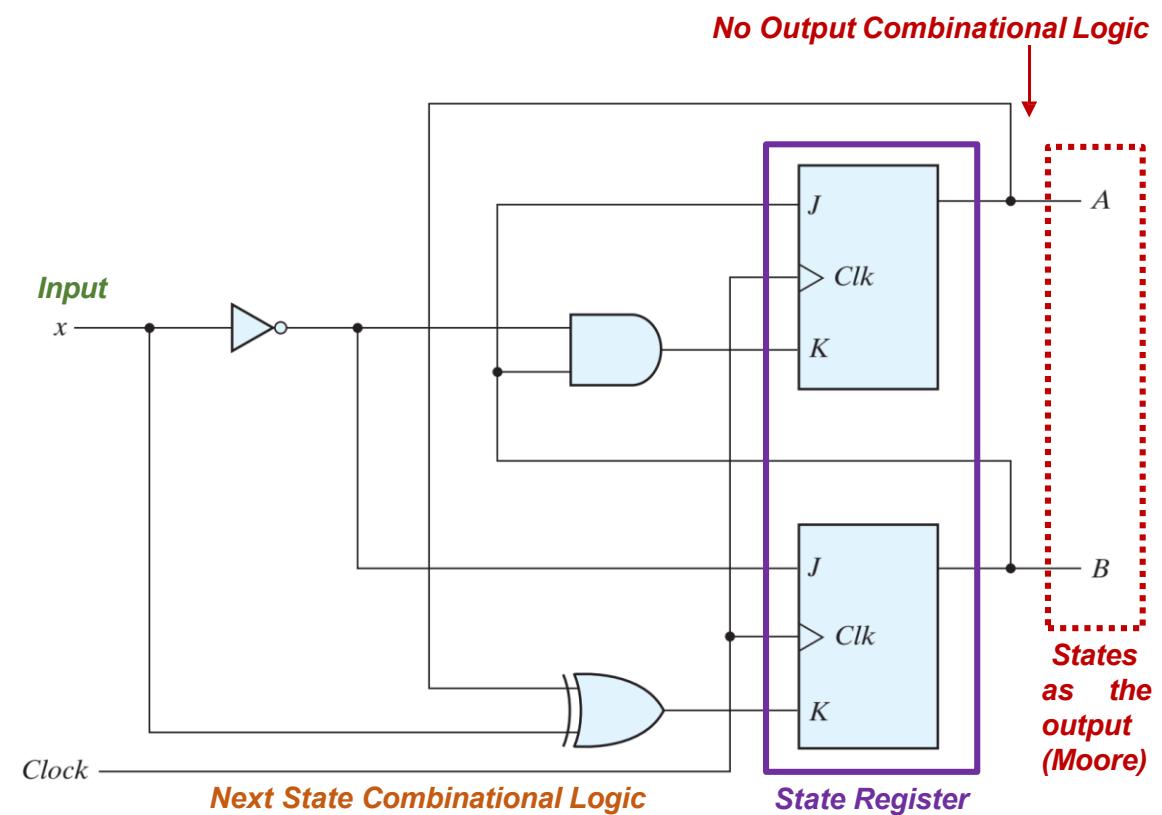
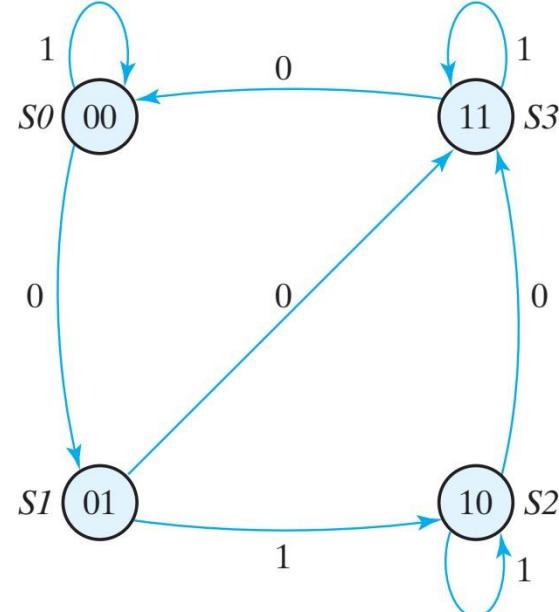
Example of a Mealy Model



Example of a Moore Model



Another Example of a Moore Model

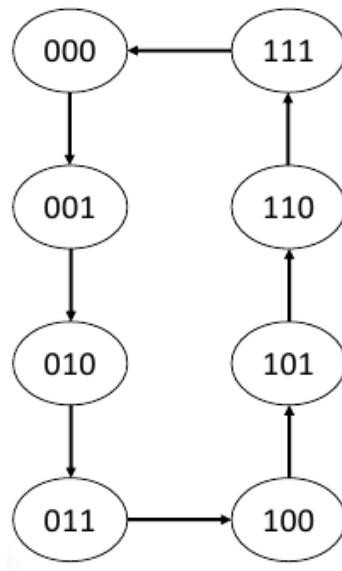


Synthesis of Sequential Circuits

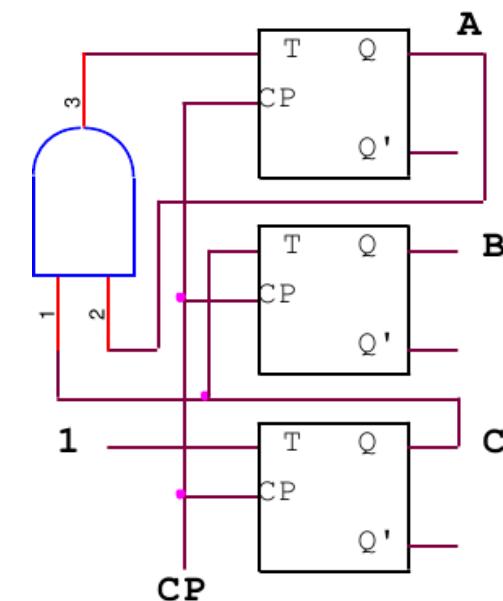
1. Describe circuit into **FSM** and draw **state diagram**.
2. State minimization (*if possible*)
3. Draw **state table** and determine **flip-flop excitation equations**.
4. Construct **circuit** using derived equations.

Traditional (Manual) Method

- Example: **3-Bit Counter (Moore Model)**



Present State			Next State			Flip-flop inputs		
A	B	C	A	B	C	TA	TB	TC
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	1	1



In this case, **state minimization (step #2)** is no longer needed.

BC			
A	00	01	11
0			1
1			1

TA = BC

Bx			
A	00	01	11
0		1	1
1	1	1	

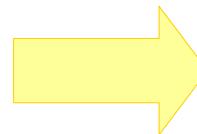
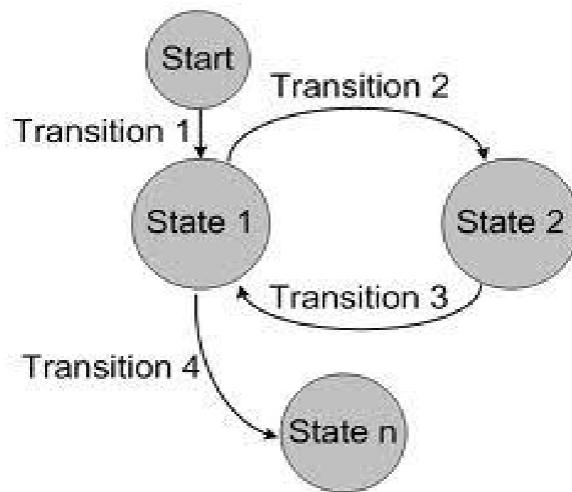
TB = C

Bx				
A	00	01	11	10
0	1	1	1	1
1	1	1	1	1

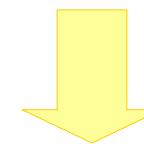
TC = 1

FSM Description in Verilog HDL

- No need to manually determine flip-flop excitations.
- The **HDL synthesis tool** will automatically perform steps for synthesis and can even make optimizations.
- Describe FSM using the **behavioral model**.



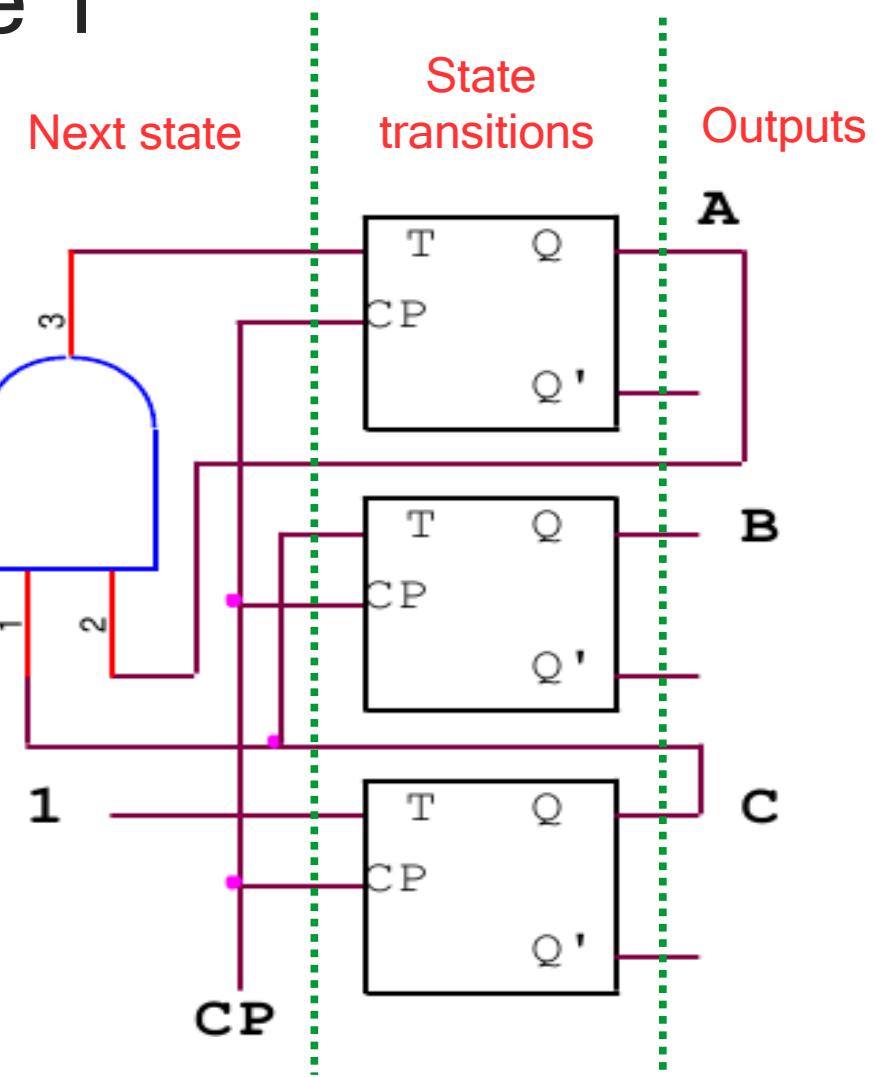
```
always @ (posedge Clk or negedge nReset) begin
    if (!nReset)
        Q <= 1'b0;
    else
        Q <= D;
end
```



```
d_ff u1 (Clk, nReset, x1, x0);
and u2 (x1, x10, x99);
or u3 (x10, x70, x13);
...
```

Partitioning Scheme 1

- There are many ways to describe FSMs in Verilog.
- One partitioning scheme is to **separate the description of:**
 - State transitions
 - Determining the **next states**, and
 - Determining the **outputs**



Partitioning Scheme 1

```

`timescale 1ns/1ps
//3-bit counter (Moore FSM)

module counter_3bit_fsm (Clk, nReset, Count);
    input      Clk, nReset;
    output     [2:0] Count;

    //state transition (state register)
    reg [2:0]      cstate, nstate;
    always @ (posedge Clk)
        if (!nReset)
            cstate <= 0;
        else
            cstate <= nstate;

    //next state assignment
    always @ (cstate)
        case (cstate)
            3'b000 : nstate <= 3'b001;
            3'b001 : nstate <= 3'b010;
            3'b010 : nstate <= 3'b011;
            3'b011 : nstate <= 3'b100;
            3'b100 : nstate <= 3'b101;
            3'b101 : nstate <= 3'b110;
            3'b110 : nstate <= 3'b111;
            default : nstate <= 3'b000;
        endcase
endmodule

```

```

//output assignment
reg [2:0] Count;
always @ (cstate)
    case (cstate)
        3'b000 : Count <= 3'b000;
        3'b001 : Count <= 3'b001;
        3'b010 : Count <= 3'b010;
        3'b011 : Count <= 3'b011;
        3'b100 : Count <= 3'b100;
        3'b101 : Count <= 3'b101;
        3'b110 : Count <= 3'b110;
        default : Count <= 3'b111;
    endcase

    //output assignment (alternate)
    //assign Count = cstate;

endmodule

```

Partitioning Scheme 2

- There are **2 partitions**:

- State transitions
- Combined description of the next state and output assignments

```
'timescale 1ns/1ps
//3-bit counter (Moore FSM)

module counter_3bit_fsm (Clk, nReset, Count);
    input      Clk, nReset;
    output     [2:0] Count;

    //state transition (state register)
    reg      [2:0] cstate, nstate;
    always @ (posedge Clk)
        if (!nReset)
            cstate <= 0;
        else
            cstate <= nstate;

    //next state + output assignment
    reg [2:0] Count;
    always @ (cstate)
        case (cstate)
            3'b000 : begin nstate <= 3'b001; Count <= 3'b000; end
            3'b001 : begin nstate <= 3'b010; Count <= 3'b001; end
            3'b010 : begin nstate <= 3'b011; Count <= 3'b010; end
            3'b011 : begin nstate <= 3'b100; Count <= 3'b011; end
            3'b100 : begin nstate <= 3'b101; Count <= 3'b100; end
            3'b101 : begin nstate <= 3'b110; Count <= 3'b101; end
            3'b110 : begin nstate <= 3'b111; Count <= 3'b110; end
            default : begin nstate <= 3'b000; Count <= 3'b111; end
        endcase
endmodule
```

Partitioning Scheme 3

- There is just **1 partition**:
 - All 3 components (**state transitions, next state, and output assignments**) are combined into a single ***always*** block.

```
 `timescale 1ns/1ps
 //3-bit counter (Moore FSM)

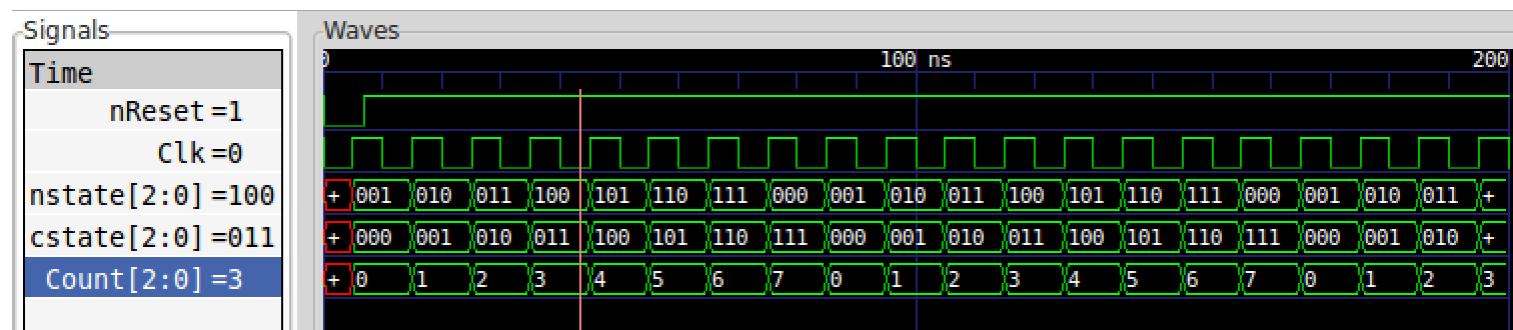
 module counter_3bit_fsm (Clk, nReset, Count);
     input      Clk, nReset;
     output     [2:0] Count;

     //state transition + next state + output assignment
     reg      [2:0] cstate;
     reg      [2:0] Count;

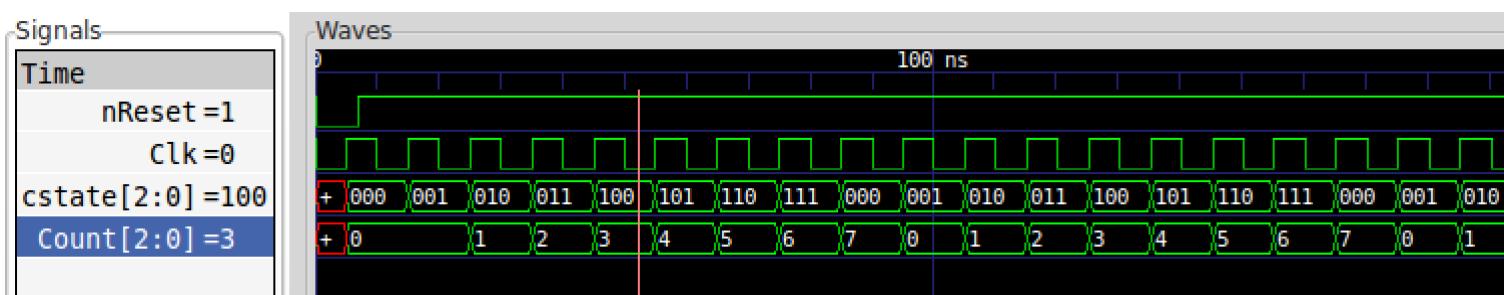
     always @ (posedge Clk)
         if (!nReset) begin
             cstate <= 3'b000;
             Count <= 3'b000;
         end else
             case (cstate)
                 3'b000 : begin cstate <= 3'b001; Count <= 3'b000; end
                 3'b001 : begin cstate <= 3'b010; Count <= 3'b001; end
                 3'b010 : begin cstate <= 3'b011; Count <= 3'b010; end
                 3'b011 : begin cstate <= 3'b100; Count <= 3'b011; end
                 3'b100 : begin cstate <= 3'b101; Count <= 3'b100; end
                 3'b101 : begin cstate <= 3'b110; Count <= 3'b101; end
                 3'b110 : begin cstate <= 3'b111; Count <= 3'b110; end
                 default : begin cstate <= 3'b000; Count <= 3'b111; end
             endcase
         endmodule
```

Compare Waveforms

Partitioning Scheme 1 or 2



Partitioning Scheme 3

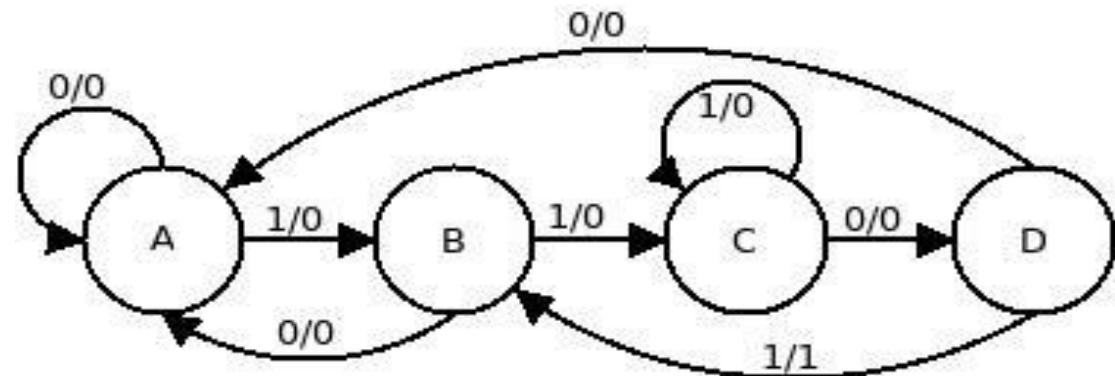


Example 1: Sequence Recognizer Problem

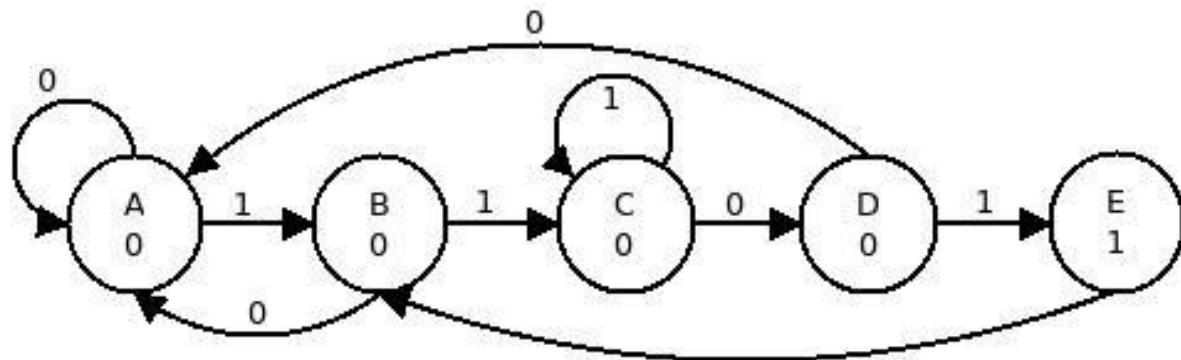
- Design an FSM that recognizes the occurrence of a particular sequence of bits, regardless of where it occurs in a longer sequence. It has to have one input X, output Z, and direct resets on its flip-flops to initialize the state of the circuit to all zeros. The circuit is to recognize the occurrence of the sequence of bits **1101** on X by making Z equal to 1 when the previous inputs of the circuit were 110 and the current input is a 1. Otherwise, Z is 0.

Example 1: State Diagram

- Mealy FSM



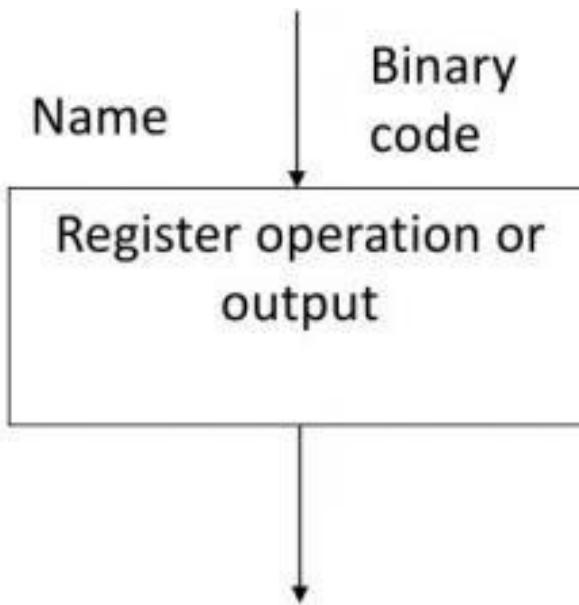
- Moore FSM



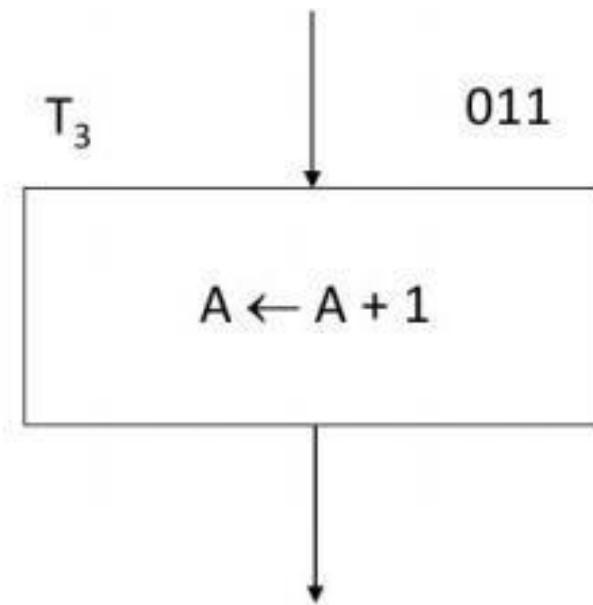
Algorithmic State Machine (ASM) Chart

- A **flowchart** is a convenient way to specify the **sequence of procedural steps and decision paths** for an algorithm.
- A special flow chart that has been developed specifically to **define digital hardware algorithms** is called an **algorithmic state machine (ASM) chart**. It is suitable for describing the sequential operations in a digital system.
- 3 basic elements:
 - State box
 - Decision box
 - Conditional box

State Box



(a) General description



(b) Example

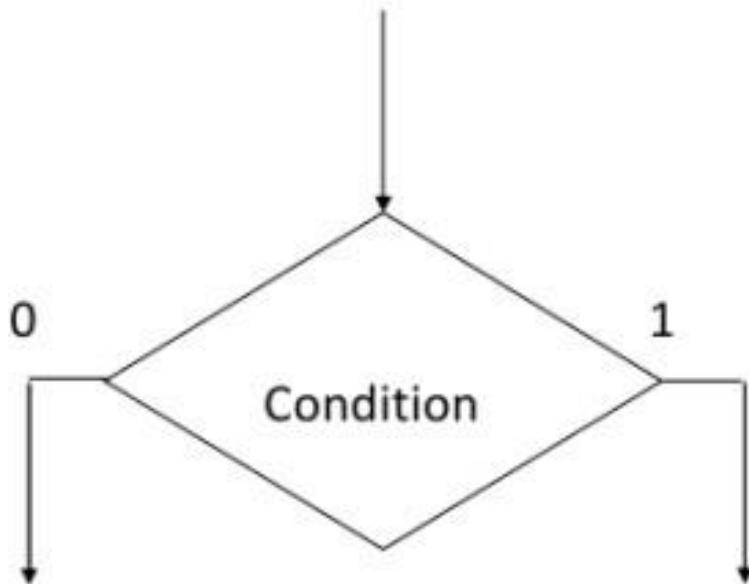
Register-Transfer Language

Symbol	Description	Examples
Letters (and numerals)	Denotes a register	$AR, R2, DR, IR$
Parenthesis	Denotes a part of a register	$R2(1), R2(7:0)$
Arrow	Denotes transfer of data	$R1 \leftarrow R2$
Comma	Separates simultaneous transfers	$R1 \leftarrow R2, R2 \leftarrow R1$
Square brackets	Specifies an address for memory	$DR \leftarrow M[AR]$

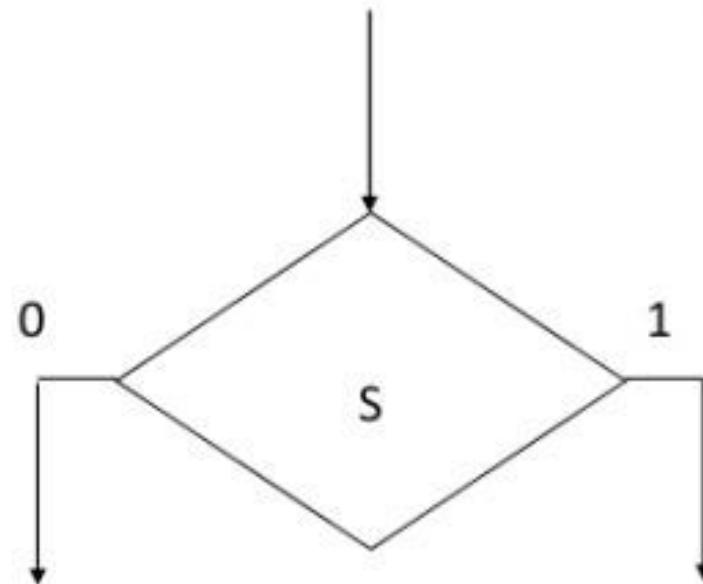
Register-Transfer Language

Symbolic Notation	Description
$A \leftarrow B$	Transfer contents of register B into register A
$R \leftarrow 0$	Clear register R
$F \leftarrow 1$	Set flip-flop F to 1
$A \leftarrow A + 1$	Increment register A by 1 (count-up)
$A \leftarrow A - 1$	Decrement register A by 1 (count-down)
$A \leftarrow A + B$	Add contents of register B to register A

Decision Box

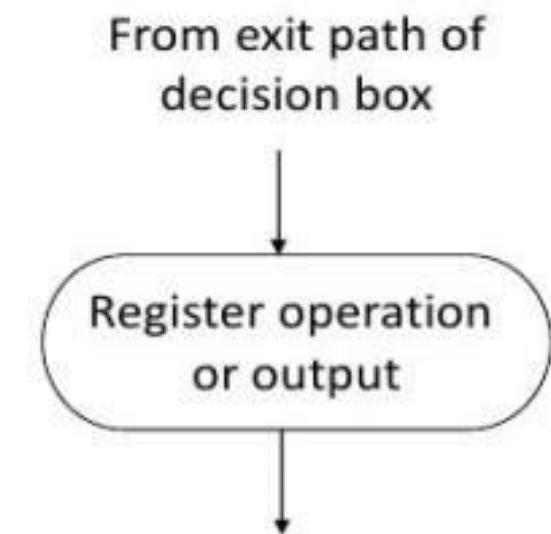


(a) General description

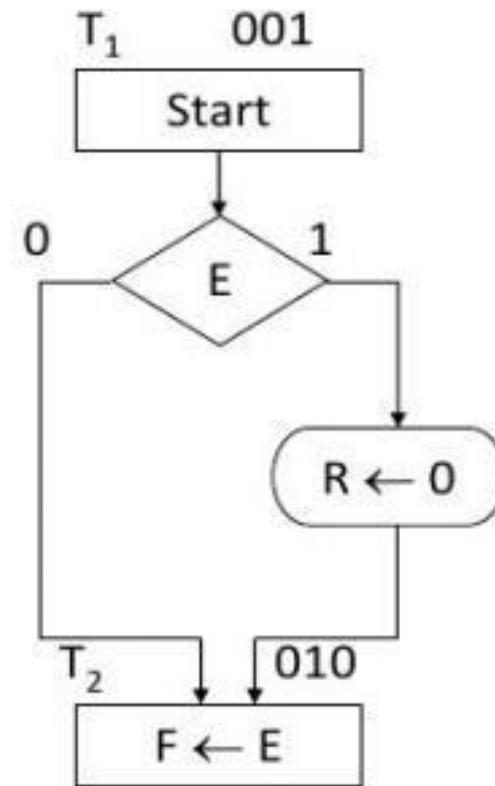


(b) Example

Conditional Box

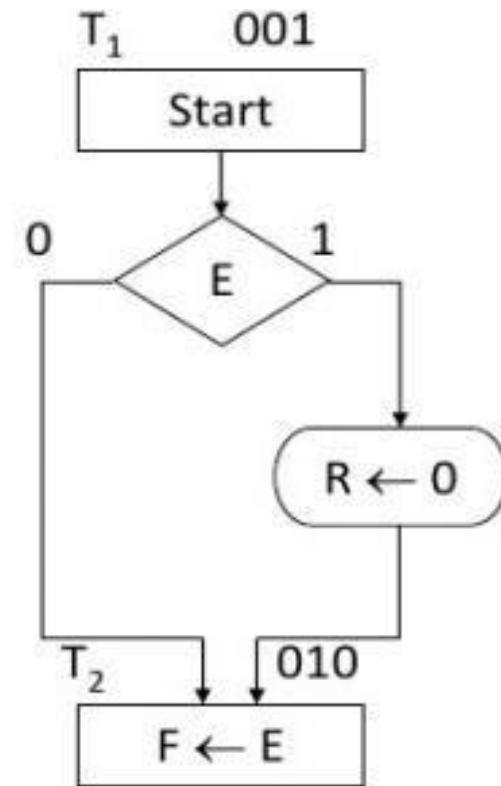
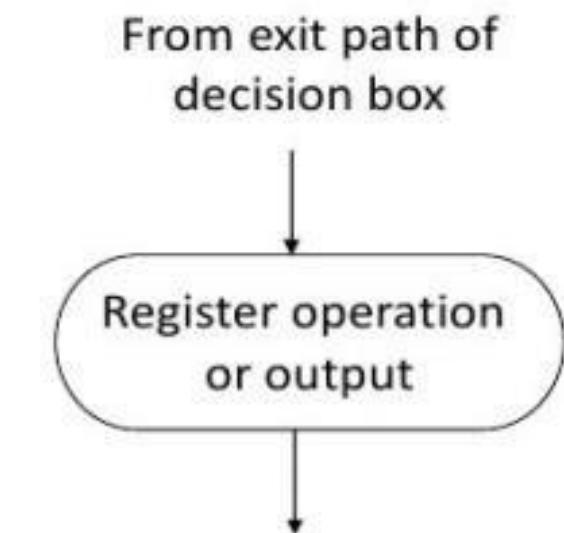


(a) General description



(b) Example with all boxes

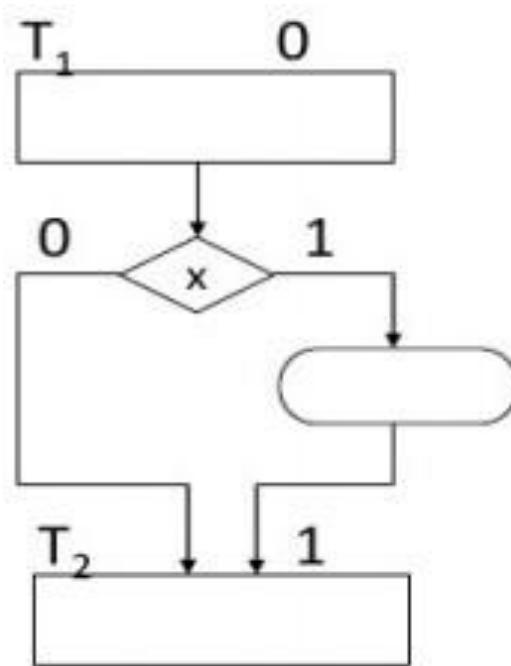
Conditional Box



(b) Example with all boxes

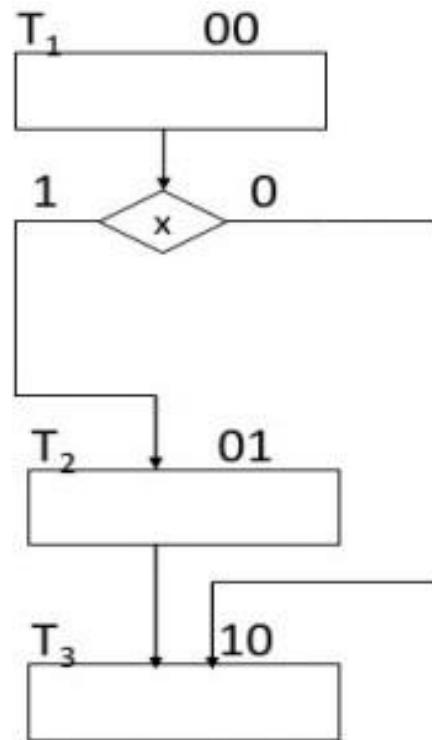
Example 1:

- If $x = 0$, control goes from state T_1 to state T_2 ; if $x = 1$, generate a conditional operation and go from T_1 to T_2 .



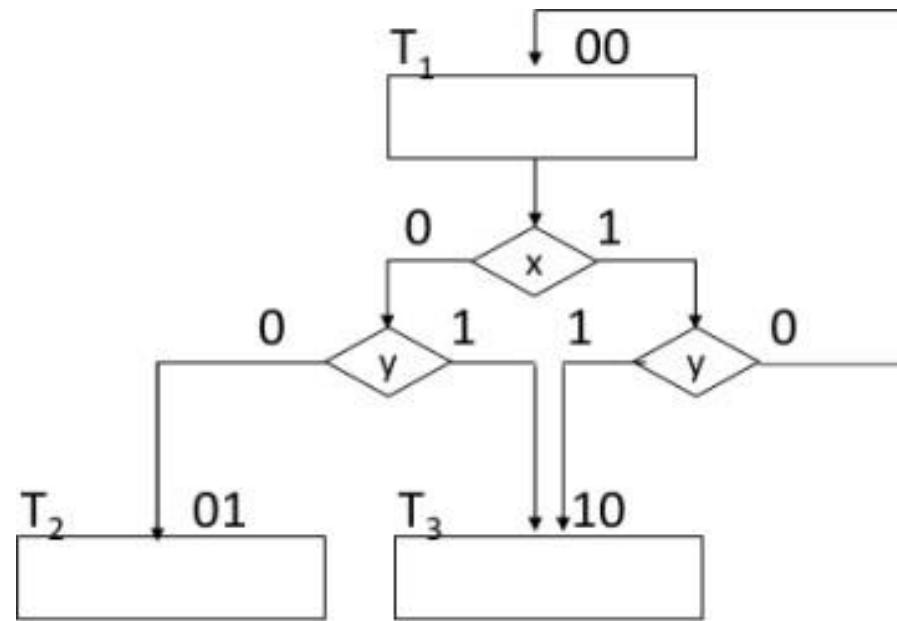
Example 2:

- If $x = 1$, control goes from T_1 to T_2 and then to T_3 ; if $x = 0$, control goes from T_1 to T_3 .

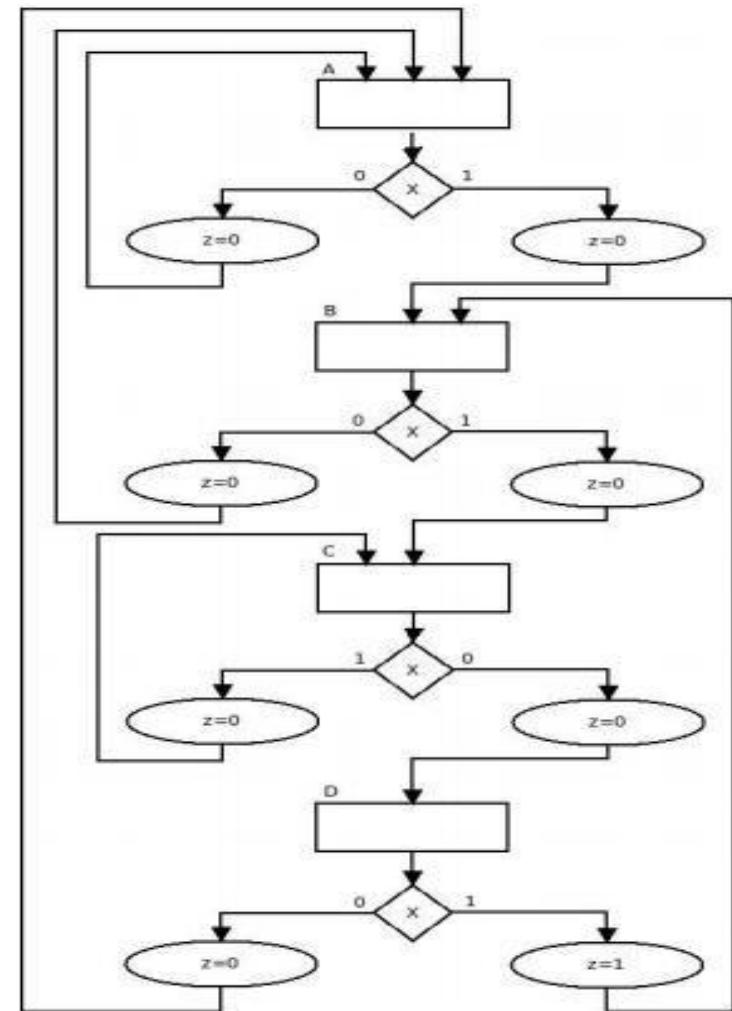
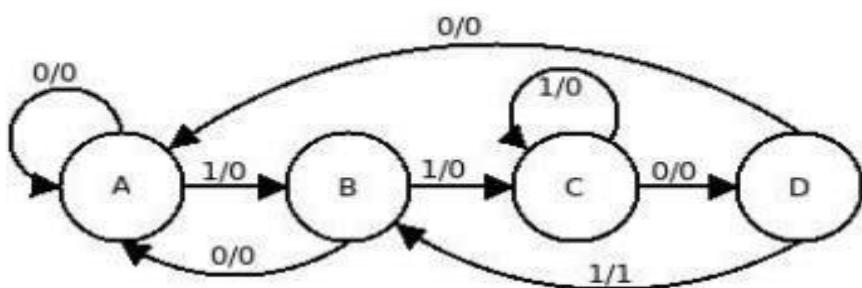


Example 3:

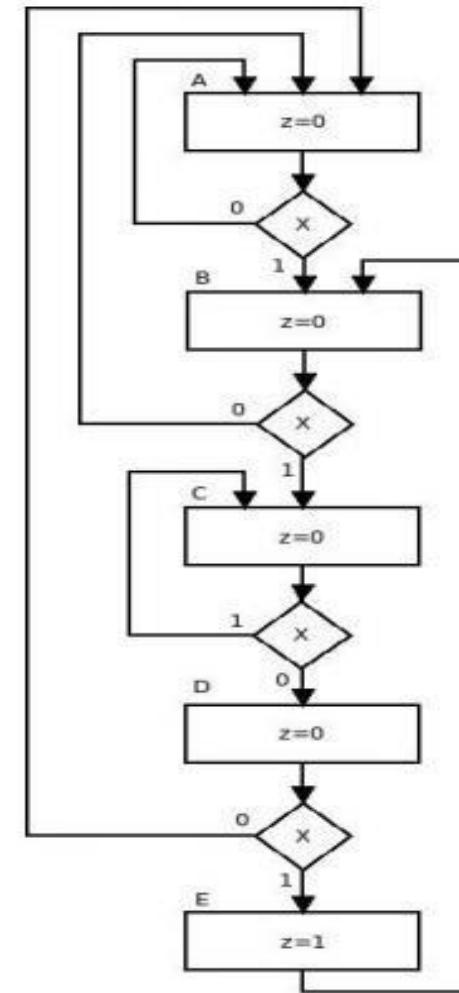
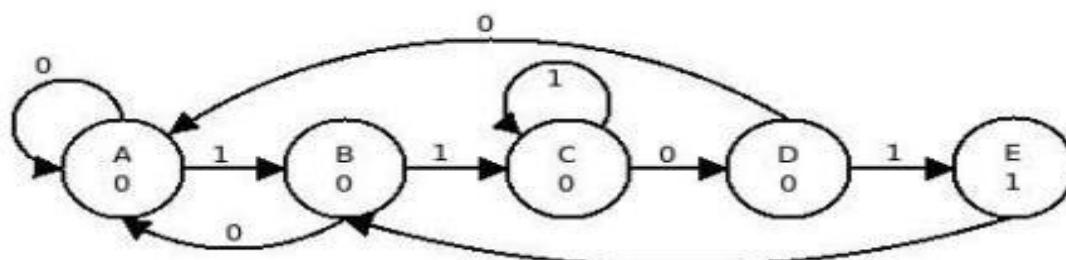
- Start from state T_1 ; then: if $xy = 00$, go to T_2 ; if $xy = 01$, go to T_3 ; if $xy = 10$, go to T_1 ; otherwise, go to T_3 .



Mealy State Diagram vs. ASM Chart



Moore State Diagram vs. ASM Chart



State Assignment and Encoding

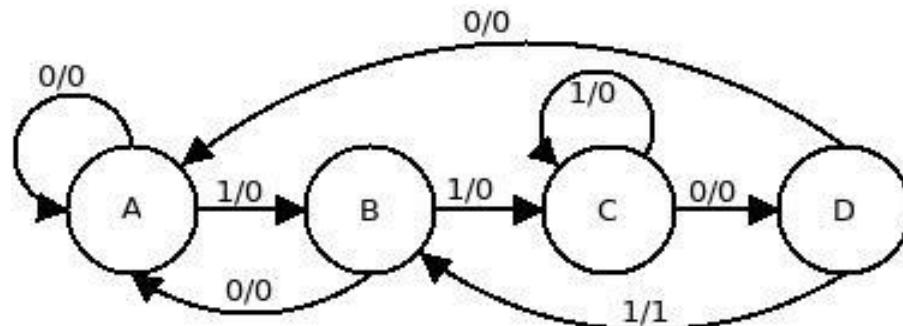
- There are **2 ways**:
- **Hard encoding**
 - Using **binary values** to represent **states**
 - Different schemes: *Sequential (binary), Random, Gray, Johnson, One hot, One cold*
- **Soft encoding**
 - Usually **used in HDLs** through abstract state representation (*the enumerated states*)
 - Use **parameter** or **localparam** keywords to represent states

State Assignment and Encoding

State	Assignment 1	Assignment 2	Assignment 3
a	001	000	000
b	010	010	100
c	011	011	010
d	100	101	101
e	101	111	011

Example 1:

- Mealy Model



```

`timescale 1ns/1ps
//Sequence Recognizer for series: 1101 (Mealy Machine)

module seq_rec1101_mealy (Clk, nReset, X, Z);
    input Clk, nReset, X;
    output Z;

    //state encoding
    parameter A = 2'b00;
    parameter B = 2'b01;
    parameter C = 2'b10;
    parameter D = 2'b11;

    //state transitions
    reg [1:0] cstate, nstate;
    always @ (posedge Clk or negedge nReset)
        if (!nReset)
            cstate <= A;
        else
            cstate <= nstate;
  
```

```

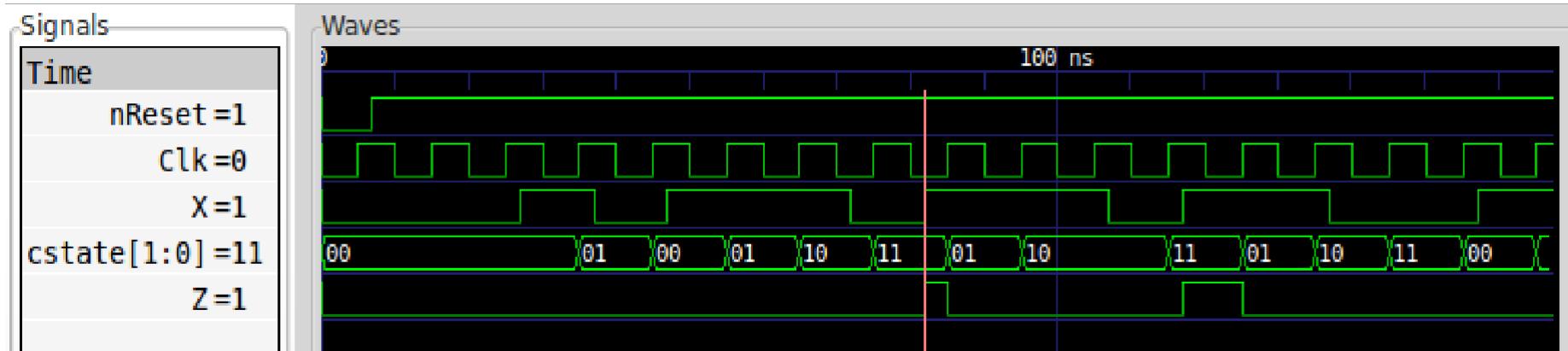
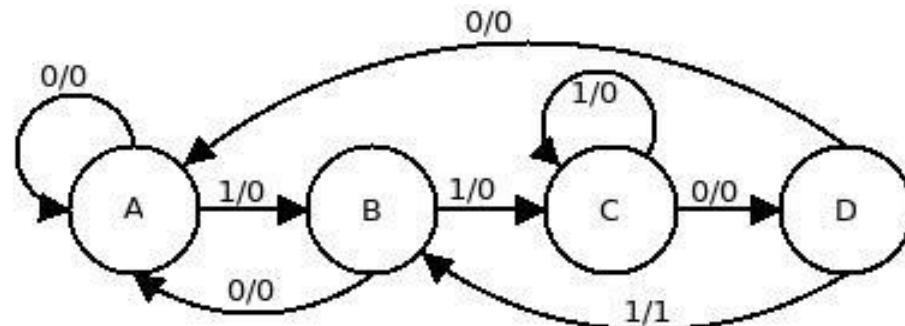
    //next state assignment
    always @ (*)
        case (cstate)
            A : nstate <= (X) ? B : A;
            B : nstate <= (X) ? C : A;
            C : nstate <= (X) ? C : D;
            default : nstate <= (X) ? B : A;
        endcase

        //output assignment
        reg Z;
        always @ (*)
            case (cstate)
                D : Z <= (X) ? 1'b1 : 0;
                default : Z <= 0;
            endcase
  
```

endmodule

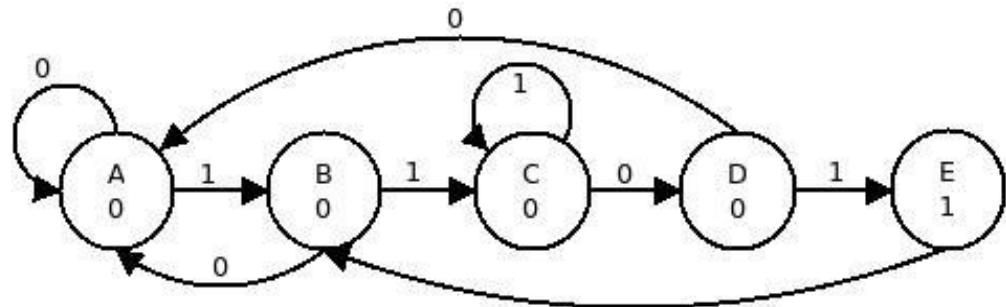
Example 1:

- Mealy Model



Example 1:

- Moore Model



```

`timescale 1ns/1ps
//Sequence Recognizer for series: 1101 (Moore Machine)

module seq_rec1101_moore (Clk, nReset, X, Z);
    input     Clk, nReset, X;
    output    Z;

    //state encoding
    parameter A = 3'b000;
    parameter B = 3'b001;
    parameter C = 3'b010;
    parameter D = 3'b011;
    parameter E = 3'b100;

    //state transitions
    reg [2:0]    cstate, nstate;
    always @ (posedge Clk or negedge nReset)
        if (!nReset)
            cstate <= A;
        else
            cstate <= nstate;
  
```

```

    //next state assignment
    always @ (*)
        case (cstate)
            A : nstate <= (X) ? B : A;
            B : nstate <= (X) ? C : A;
            C : nstate <= (X) ? C : D;
            D : nstate <= (X) ? E : A;
            default : nstate <= B;
        endcase

        //output assignment
        reg Z;
        always @ (*)
            case (cstate)
                E : Z <= 1'bl;
                default : Z <= 0;
            endcase
  
```

endmodule

Example 1: Mealy vs. Moore Output

- **Mealy Model**

```
//next state assignment
always @ (*)
    case (cstate)
        A : nstate <= (X) ? B : A;
        B : nstate <= (X) ? C : A;
        C : nstate <= (X) ? C : D;
        default : nstate <= (X) ? B : A;
    endcase

    //output assignment
    reg Z;
    always @ (*)
        case (cstate)
            D : Z <= (X) ? 1'b1 : 0;
            default : Z <= 0;
    endcase

endmodule
```

- **Moore Model**

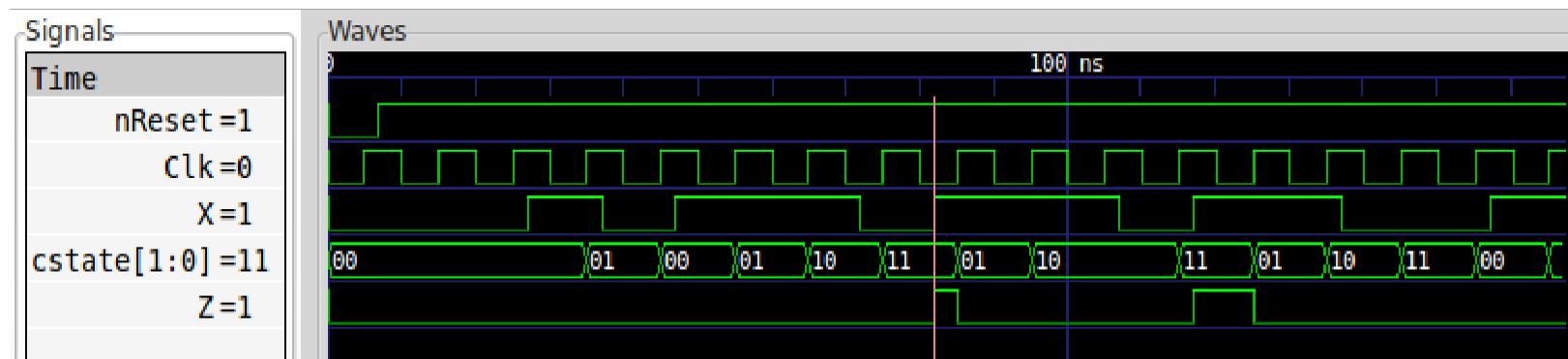
```
//next state assignment
always @ (*)
    case (cstate)
        A : nstate <= (X) ? B : A;
        B : nstate <= (X) ? C : A;
        C : nstate <= (X) ? C : D;
        D : nstate <= (X) ? E : A;
        default : nstate <= B;
    endcase

    //output assignment
    reg Z;
    always @ (*)
        case (cstate)
            E : Z <= 1'b1;
            default : Z <= 0;
    endcase

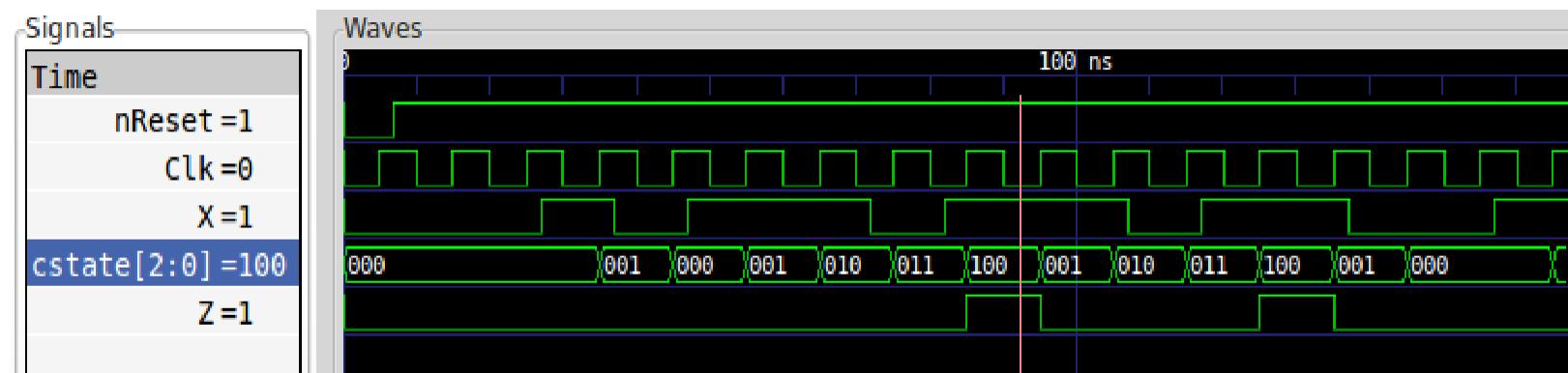
endmodule
```

Example 1: Comparing Waveforms

Mealy Output



Moore Output



Partitioning Further

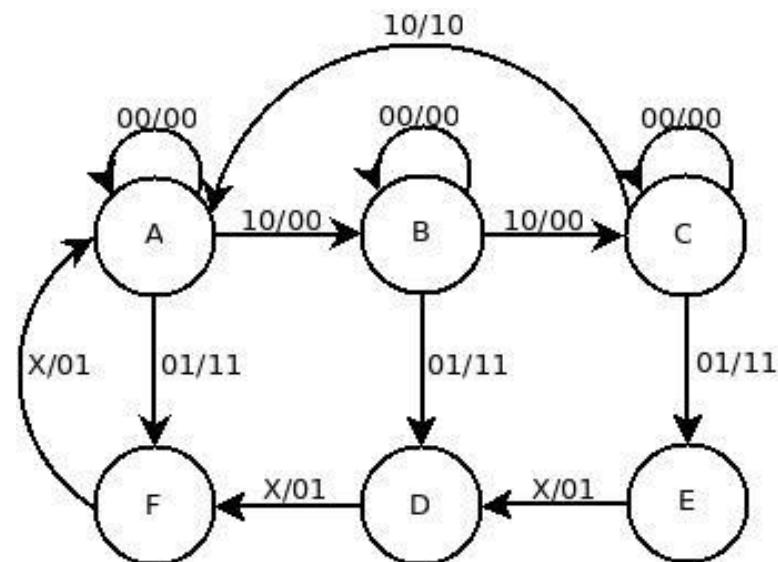
- As the sequential circuit grows in complexity, **multiple output assignments** can be partitioned into multiple separate *always* blocks.

Example 2: Vending Machine (Mealy)

- Implement a **vending machine** which dispenses a **3-peso cost item**.
- Vending machine has **2 inputs**: one for a **1-peso coin** (*input p1*) and another for a **5-peso coin** (*input p5*)
- It has **2 outputs**: **dispensed item** (*output disp*) and a **1-peso change** (*output change*)
- Whenever a coin is inserted, the corresponding input is set to '1' for that clock cycle. Only one coin can be inserted every clock cycle and only from a single denomination.
 - Thus {p1,p5} can only be “00”, “01”, or “10”

Example 2: Vending Machine (Mealy)

- **Possible scenarios:**
 - Insert p5, dispense and change, change
 - Insert p1, p5, dispense and change, change 2x
 - Insert p1 2x, p5, dispense and change, change 3x
 - Insert p1 3x, dispense
- **Assume I/O format:**
 - {p1, p5 / disp, change}



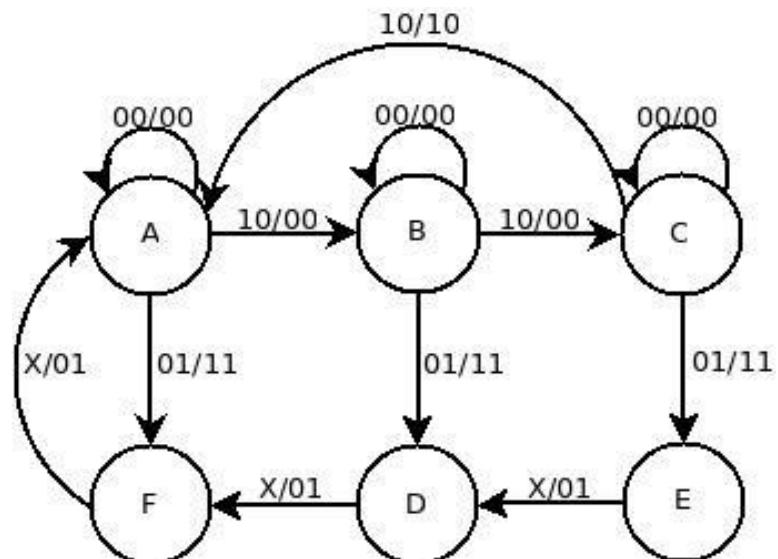
Example 2: Vending Machine (Mealy)

- **State Encoding and State Transition**

```
`timescale 1ns/1ps
module vendo_p3 (clk, nrst, p1, p5, disp, change);
    input    clk, nrst, p1, p5;
    output   disp, change;

    //state encoding
    reg [2:0] cstate, nstate;
    parameter sA = 3'b000;
    parameter sB = 3'b001;
    parameter sC = 3'b010;
    parameter sD = 3'b011;
    parameter sE = 3'b100;
    parameter sF = 3'b101;

    //state transition
    always @ (posedge clk)
        if (!nrst)      cstate <= sA;
        else           cstate <= nstate;
```



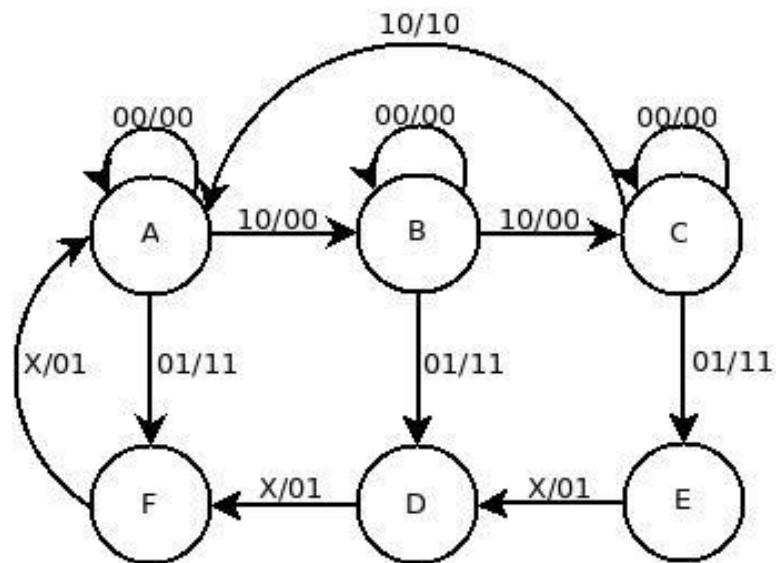
Example 2: Vending Machine (Mealy)

- Next State Logic**

```

//next state logic
always @ (cstate or p1 or p5)
    case (cstate)
        sA:
            case ({p1, p5})
                2'b01      : nstate <= sF;
                2'b10      : nstate <= sB;
                default    : nstate <= sA;
            endcase
        sB:
            case ({p1, p5})
                2'b01      : nstate <= sD;
                2'b10      : nstate <= sC;
                default    : nstate <= sB;
            endcase
        sC:
            case ({p1, p5})
                2'b01      : nstate <= sE;
                2'b10      : nstate <= sA;
                default    : nstate <= sC;
            endcase
        sD:      nstate <= sF;
        sE:      nstate <= sD;
        sF:      nstate <= sA;
        default: nstate <= cstate;
    endcase

```



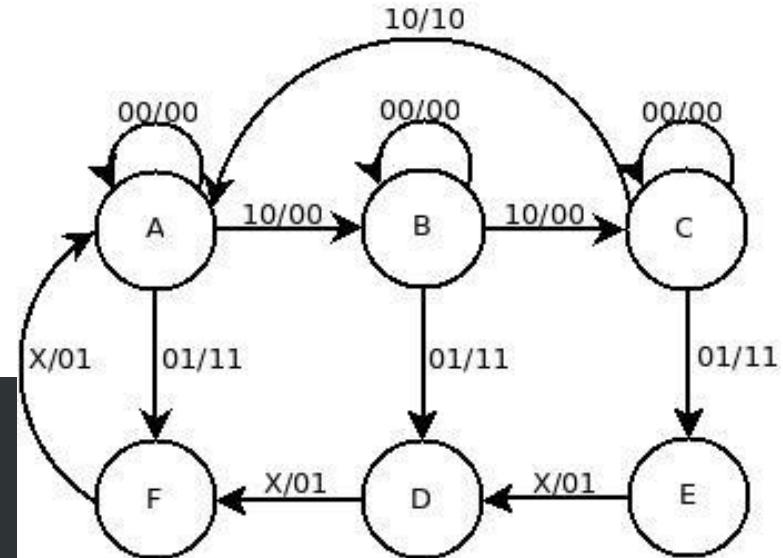
Example 2: Vending Machine (Mealy)

- **Output Logic**

```

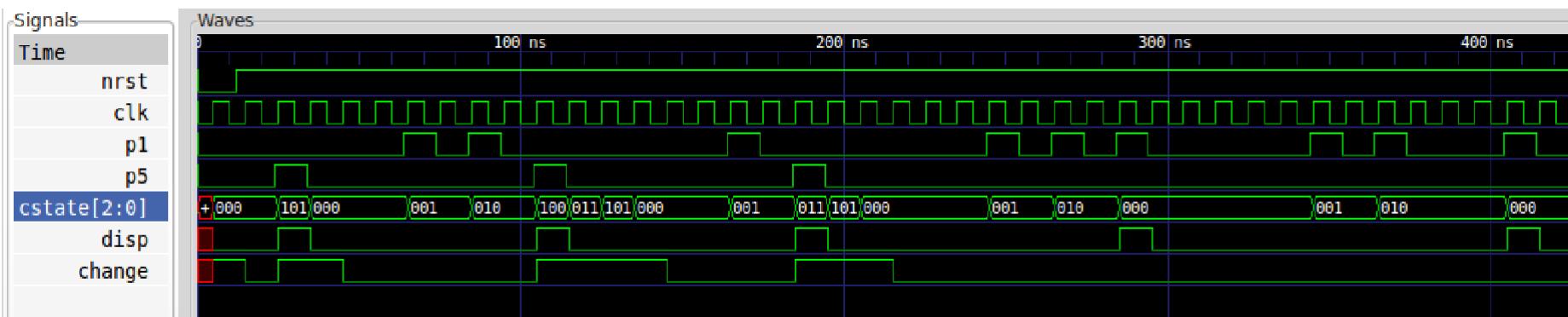
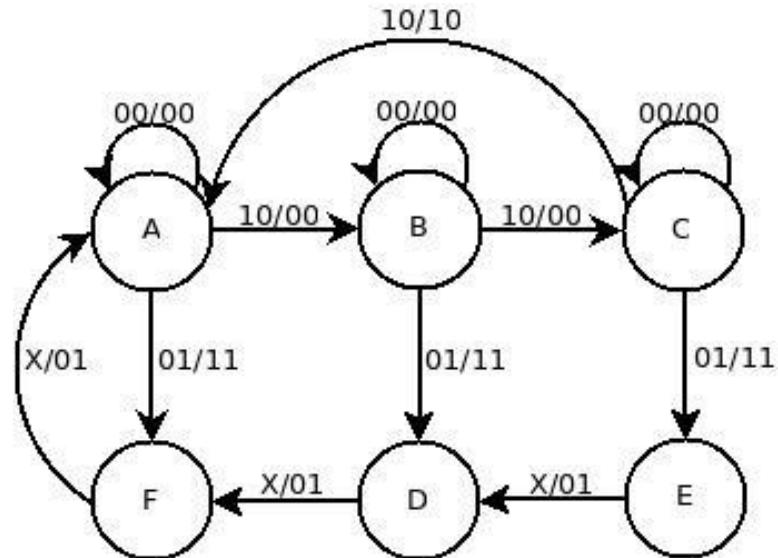
//output logic
reg disp, change;
always @ (posedge clk)
    case (cstate)
        sA, sB:
            case ({p1, p5})
                2'b01      : begin disp <= 1'b1; change <= 1'b1; end
                default    : begin disp <= 0;     change <= 0;     end
            endcase
        sC:
            case ({p1, p5})
                2'b01      : begin disp <= 1'b1; change <= 1'b1; end
                2'b10      : begin disp <= 1'b1; change <= 0;     end
                default    : begin disp <= 0;     change <= 0;     end
            endcase
        default:
            begin disp <= 0;     change <= 1'b1; end
    endcase
endmodule

```



Example 2: Vending Machine (Mealy)

- **Possible scenarios:**
 - Insert p5, dispense and change, change
 - Insert p1, p5, dispense and change, change 2x
 - Insert p1 2x, p5, dispense and change, change 3x
 - Insert p1 3x, dispense

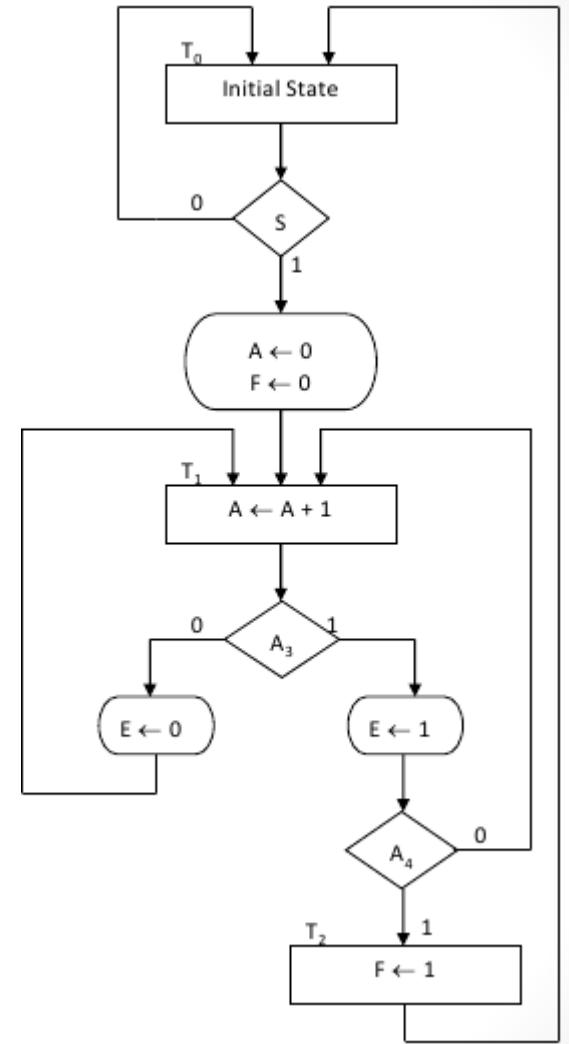
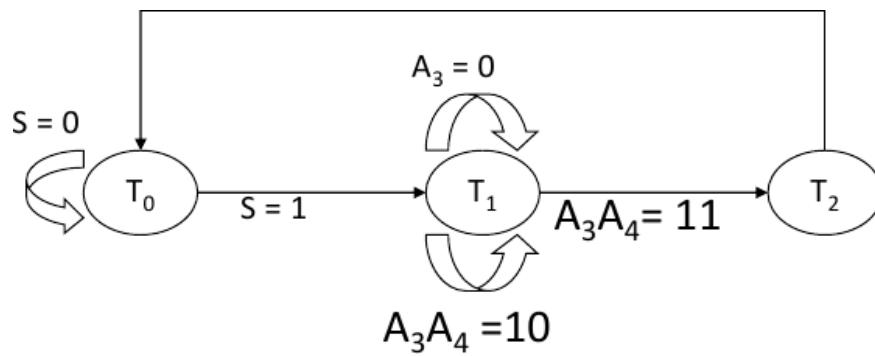


Control Unit as an FSM

Design a controller of a digital system with two flip-flops, E and F, and one 4-bit binary counter, A. The individual flip-flops in A are denoted by A₄, A₃, A₂, and A₁, with A₄ holding the most significant bit of the count. A start signal S initiates the system operation by clearing the counter A and flip-flop F. The counter is then incremented by 1 starting from the next clock pulse and continues to increment until the operations stop. Counter bits A₃ and A₄ determine the sequence of operations:

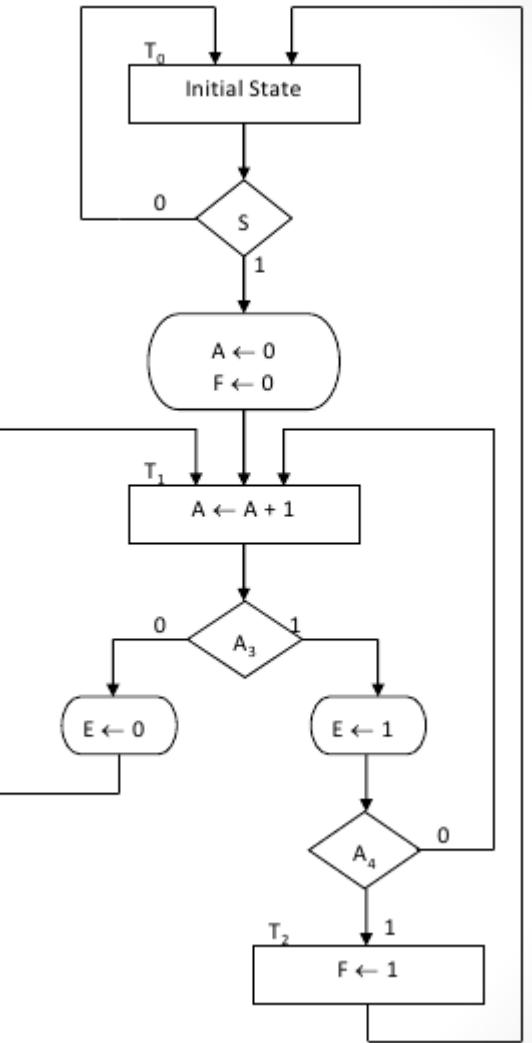
- If A₃ = 0, E is cleared to 0 and the count continues.
- If A₃ = 1, E is set to 1; then if A₄ = 0, the count continues, but if A₄ = 1, F is set to 1 on the next clock pulse and the system stops counting.

ASM Chart and State Diagram

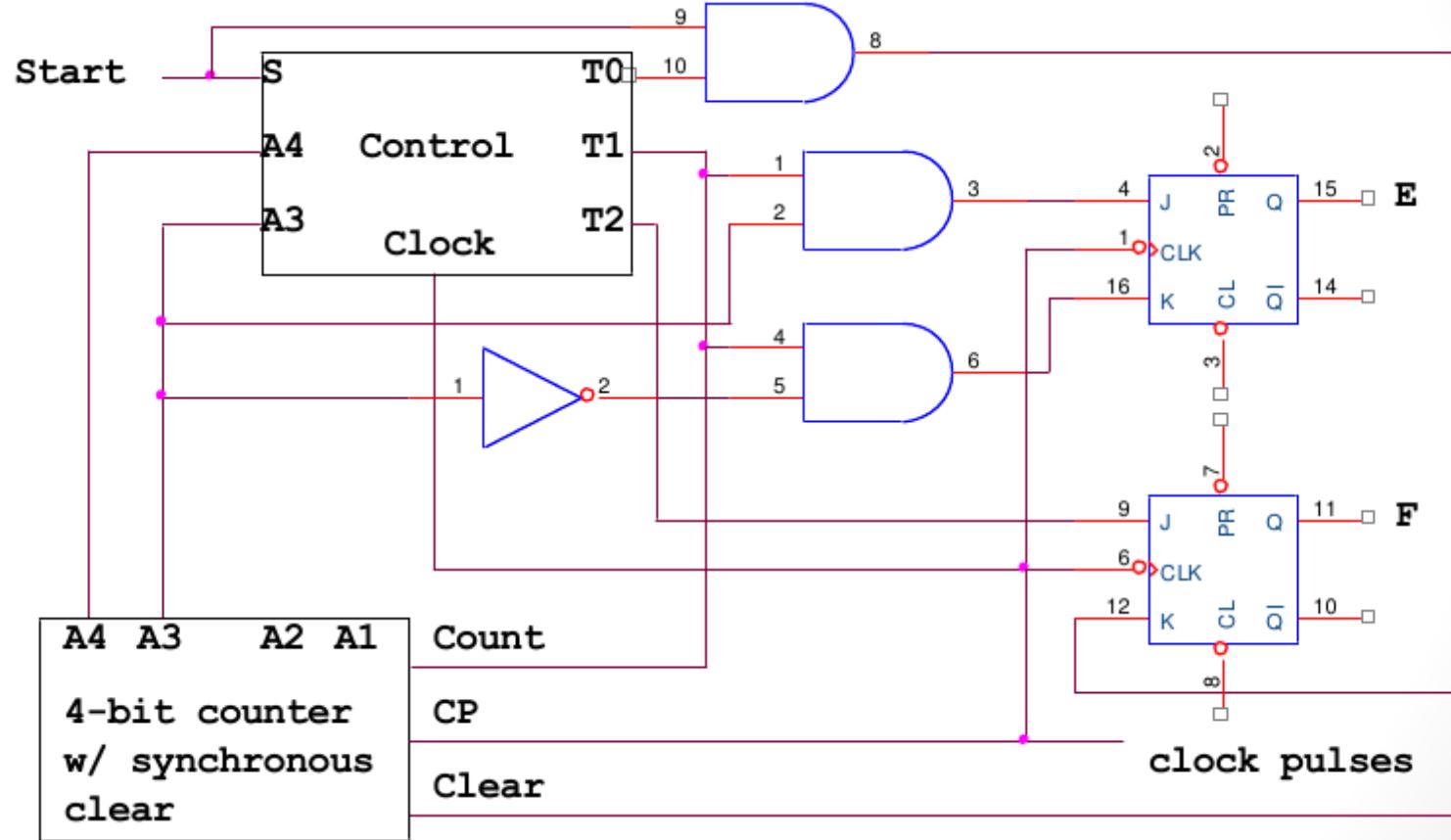


Sequence of Operations

Counter				Flip-flops		Conditions	State
A ₄	A ₃	A ₂	A ₁	E	F		
0	0	0	0	1	0	$A_3 = 0, A_4 = 0$	T_1
0	0	0	1	0	0		
0	0	1	0	0	0		
0	0	1	1	0	0		
0	1	0	0	0	0		
0	1	0	1	1	0		
0	1	1	0	1	0		
0	1	1	1	1	0		
1	0	0	0	1	0		
1	0	0	1	0	0		
1	0	1	0	0	0		
1	0	1	1	0	0		
1	1	0	0	0	0	$A_3 = 1, A_4 = 1$	
1	1	0	1	1	0		T_2
1	1	0	1	1	1		T_0



Control Unit as an FSM



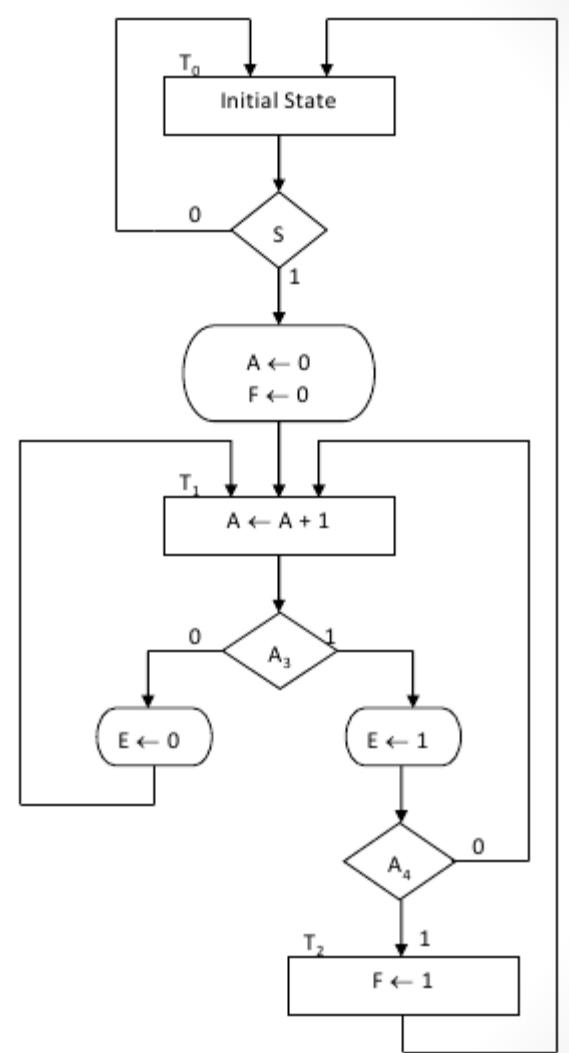
Control Implementation

- Hardwired Control
 - D flip-flop and a Decoder
 - One flip-flop per state
- HDL Implementation

D flip-flop and a Decoder

Symbol	Present State		Inputs			Next State		Outputs		
	G ₁	G ₂	S	A ₃	A ₄	G ₁	G ₂	T ₀	T ₁	T ₂
T ₀	0	0	0	X	X	0	0	1	0	0
T ₀	0	0	1	X	X	0	1	1	0	0
T ₁	0	1	X	0	X	0	1	0	1	0
T ₁	0	1	X	1	0	0	1	0	1	0
T ₁	0	1	X	1	1	1	1	0	1	0
T ₂	1	1	X	X	X	0	0	0	0	1

Note: E and F are not included since they are external outputs (not part of the control logic block)



D flip-flop and a Decoder: K-map of D f/f inputs

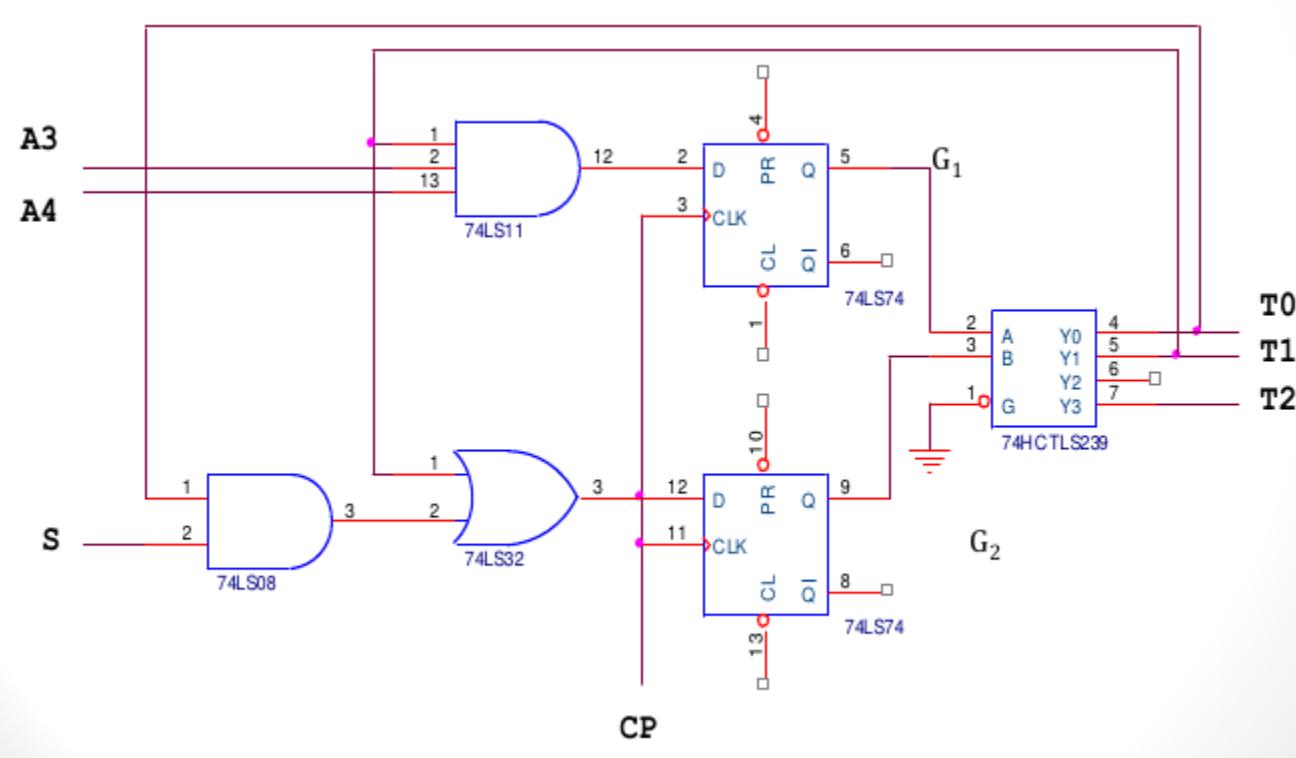
G₁G₂	A₃A₄ when S=0				A₃A₄ when S=1			
	00	01	11	10	10	11	01	00
00	0	0	0	0	0	0	0	0
01	0	0	1	0	0	1	0	0
11	0	0	0	0	0	0	0	0
10	x	x	x	x	x	x	x	x

$$DG_1 = G_1'G_2A_3A_4 = A_3A_4T_1$$

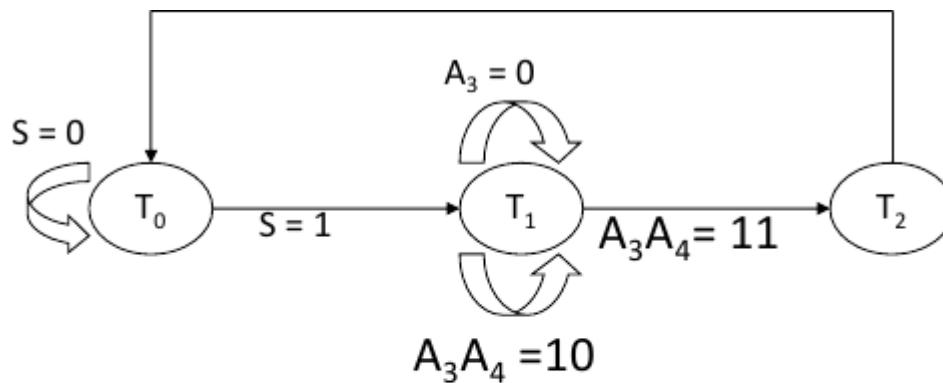
G₁G₂	A₃A₄ when S=0				A₃A₄ when S=1			
	00	01	11	10	10	11	01	00
00	0	0	0	0	1	1	1	1
01	1	1	1	1	1	1	1	1
11	0	0	0	0	0	0	0	0
10	x	x	x	x	x	x	x	x

$$DG_2 = G_1'G_2'S + G_1'G_2 = ST_0 + T_1$$

D flip-flop and a Decoder: Logic diagram



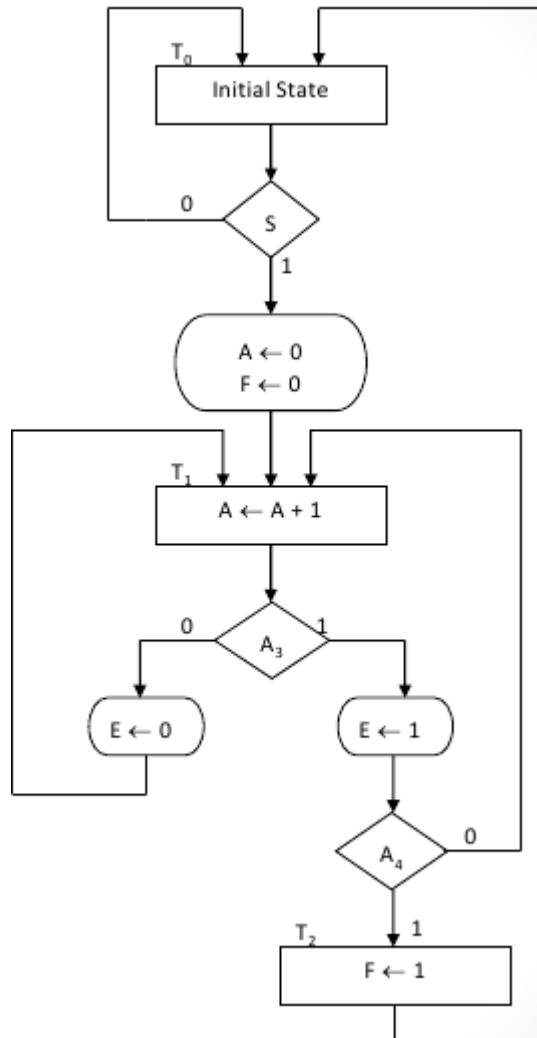
One flip-flop per state



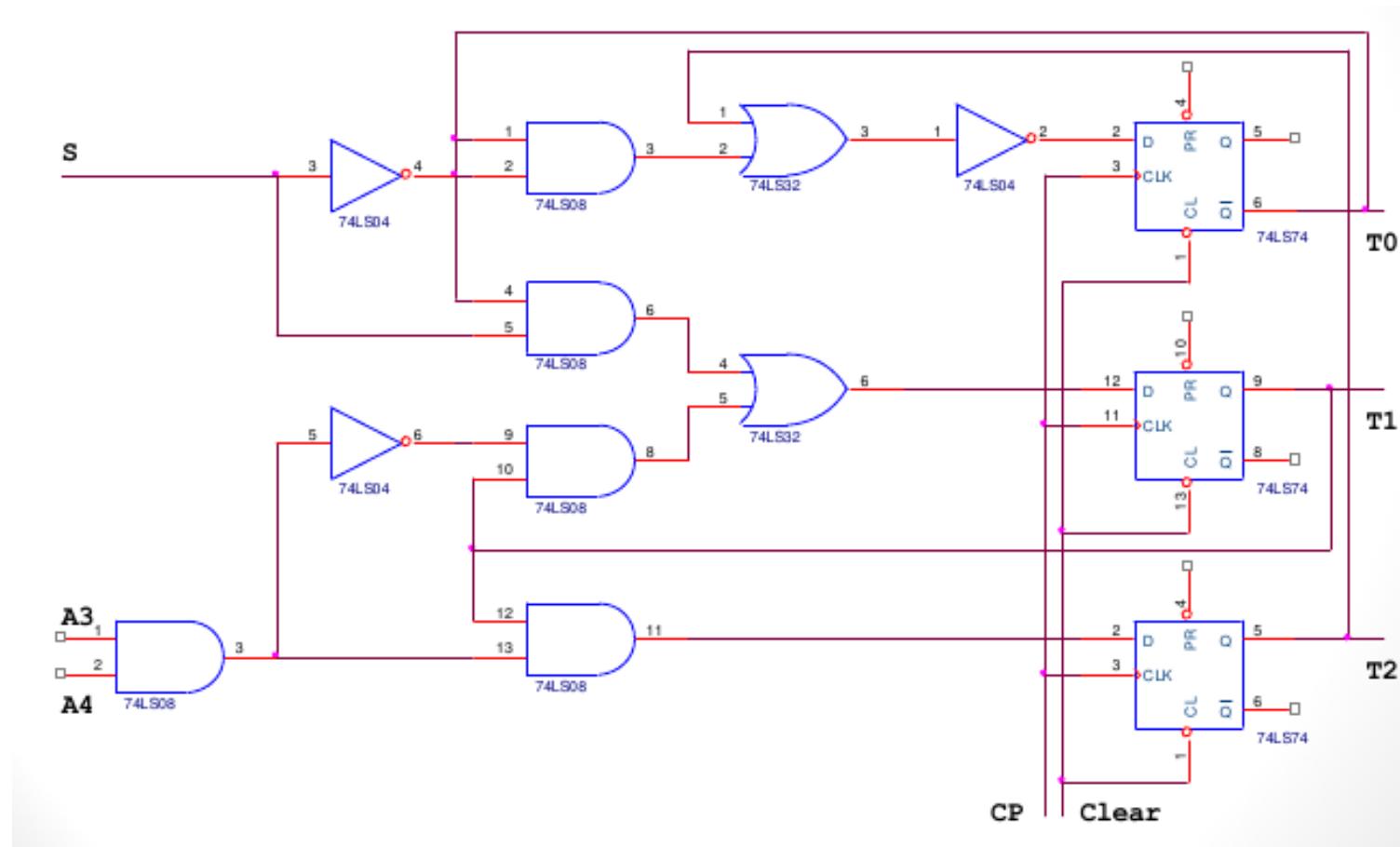
$$DT_0 = S'T_0 + T_2$$

$$DT_1 = ST_0 + A_3'T_1 + A_3A_4'T_1 = ST_0 + (A_3A_4)'T_1$$

$$DT_2 = A_3A_4T_1$$

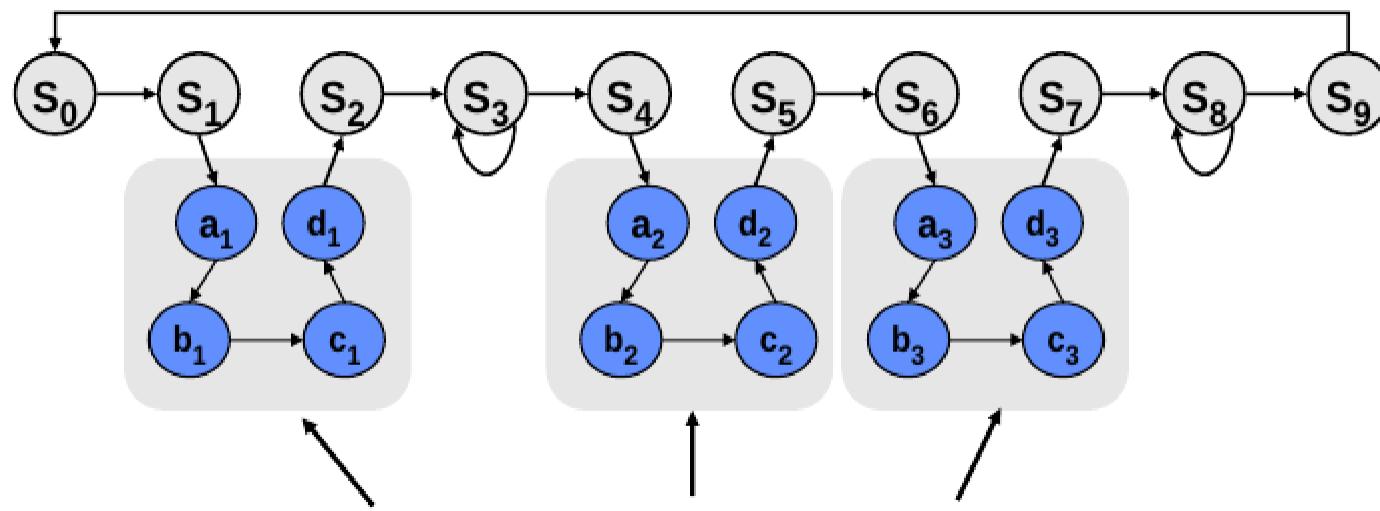


One flip-flop per state: Logic diagram



Towards FSM Modularity

- Consider the following abstract FSM:



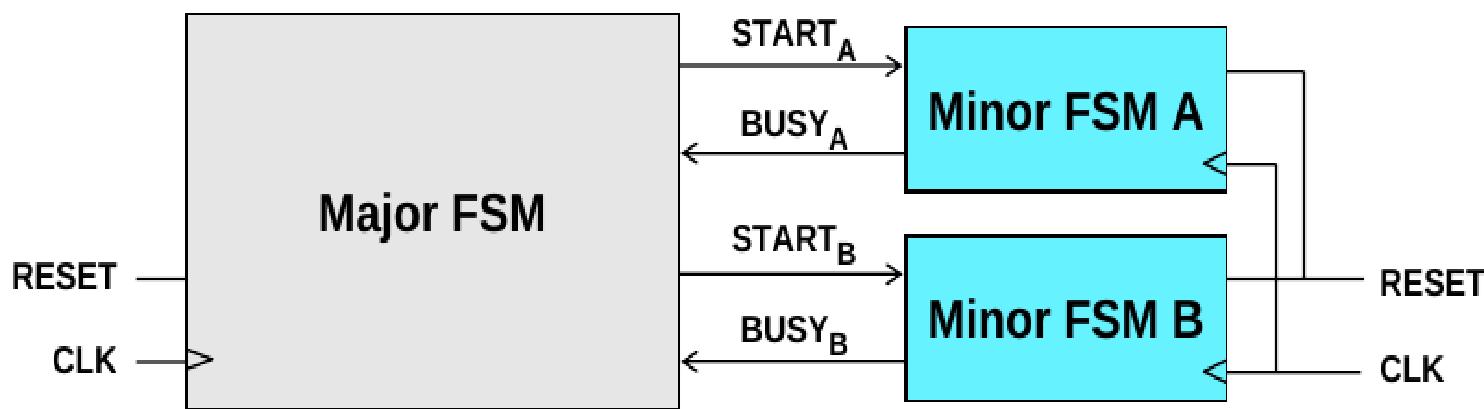
- Suppose that each set of states $a_x \dots d_x$ is a “sub-FSM” that produces exactly the same outputs.

Towards FSM Modularity

- Can we simplify the FSM by removing equivalent states?
 - **No!** The outputs may be the same, but the next-state transitions are not.
- This situation closely resembles a **procedure call** or **function call** in software. How can we apply this concept to FSMs?

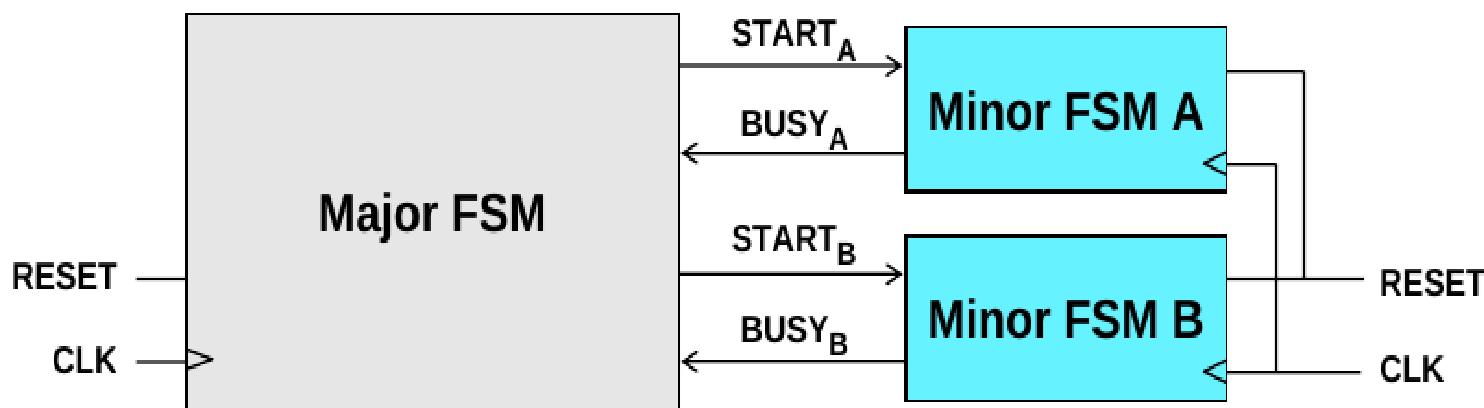
Major/Minor FSM Abstraction

- **Subtasks** are encapsulated in minor FSMs with common reset and clock.
- **Simple communication abstraction:**
 - **START**: tells the minor FSM to begin operation (*call*)
 - **BUSY**: tells the major FSM whether the minor is done (*return*)

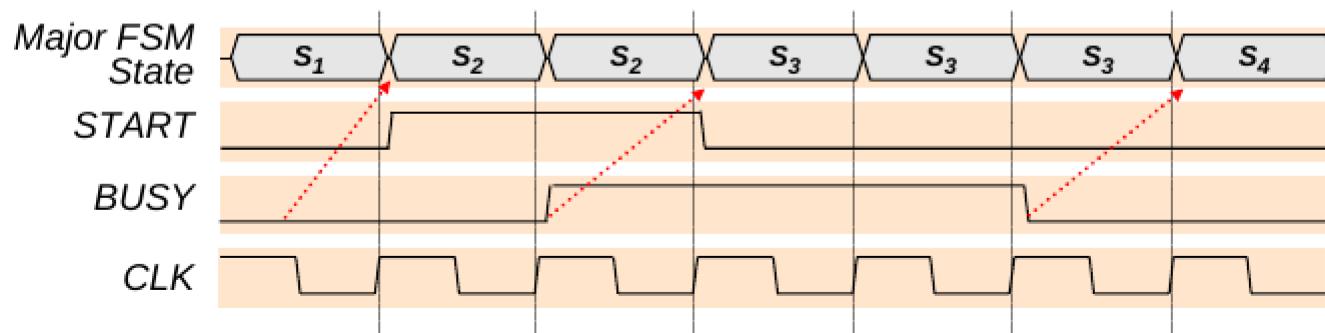
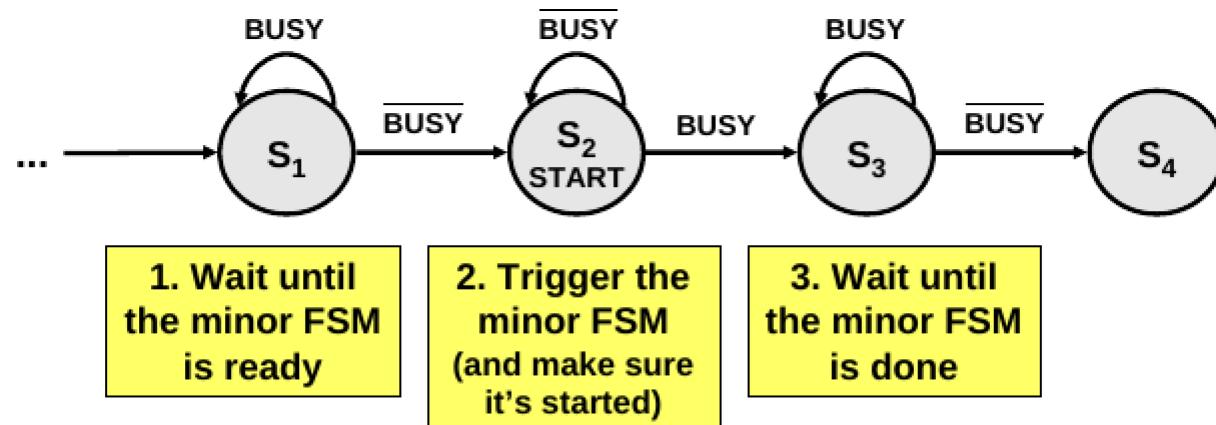


Major/Minor FSM Abstraction

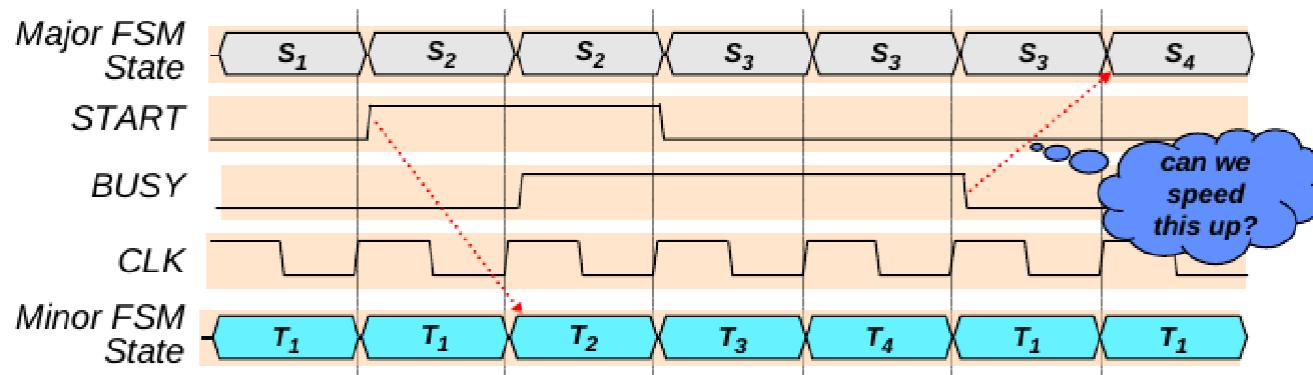
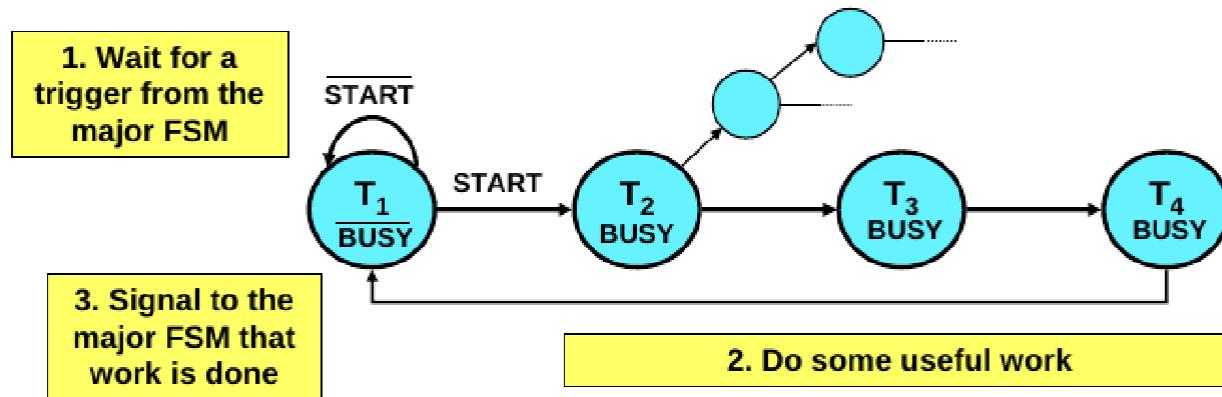
- The major/minor abstraction is great for:
 - Modular designs (*always a good thing*)
 - Tasks that occur often but in different contexts
 - Tasks that require a variable/unknown period of time
 - Event-driven systems



Inside the Major FSM

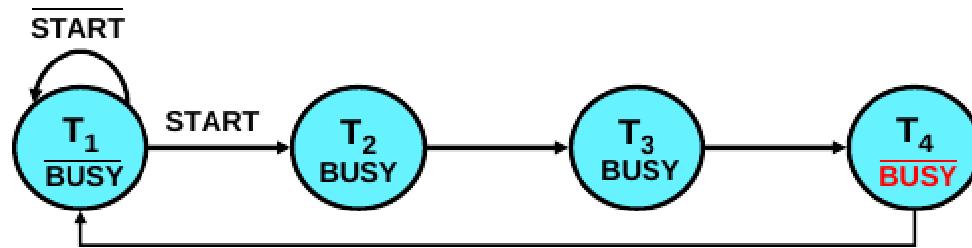


Inside the Minor FSM



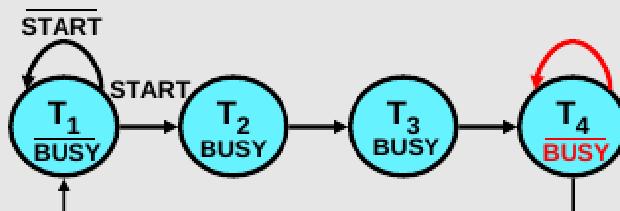
Optimizing the Minor FSM

Good idea: de-assert BUSY one cycle early



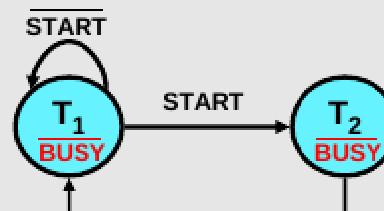
Bad idea #1:

T₄ may not immediately return to T₁

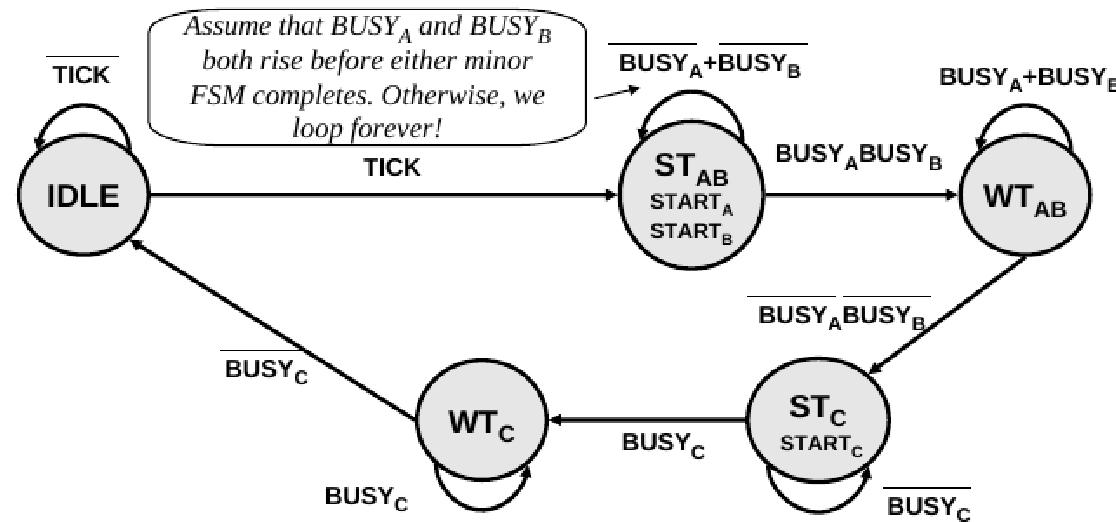
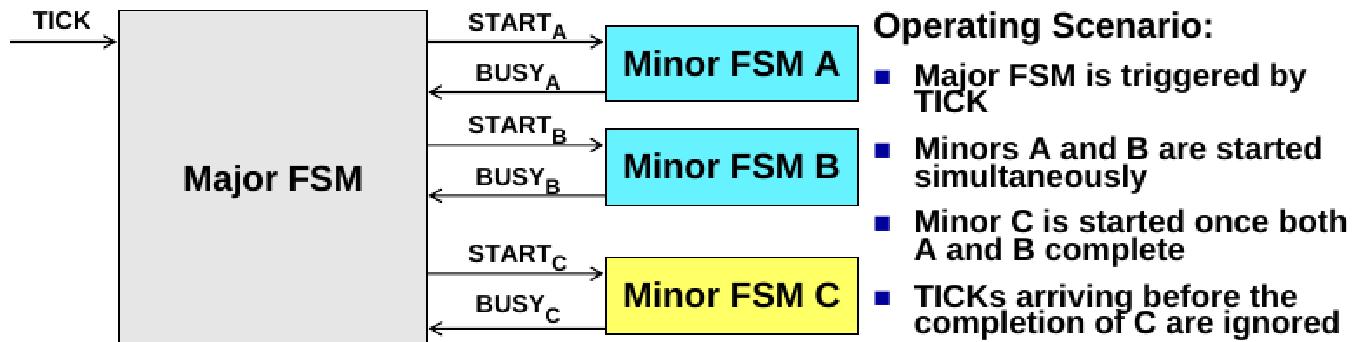


Bad idea #2:

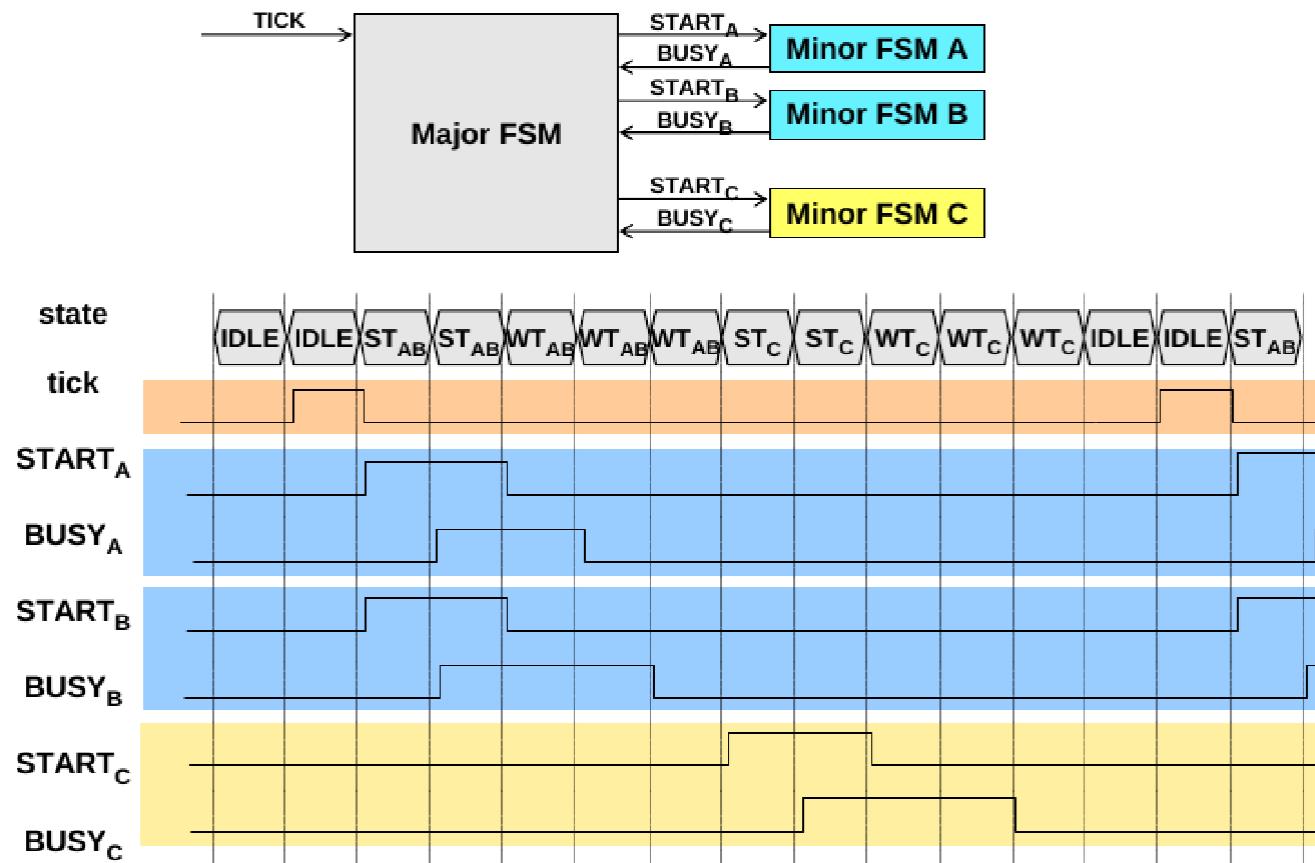
BUSY never asserts!



A Four-FSM Example



Four-FSM Sample Waveform





End of Unit 7