Problem 6

VC dimension of:

a) unions of two rectangles with edges vertical/horizontal (not angled)

3

For any set of 3 points:

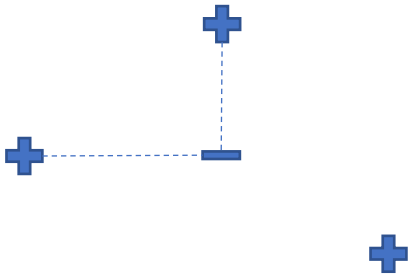
1. if they have the same label +: use one of the rectangles to include all of them

2. if they have the same label -: do not include any of them with either rectangles

3. if they have different labels ++-: we can always include the two + with two rectangles separately, while neither of the includes the -

4. if they have different labels +--: include the + with one rectangle, and do not include the two – with either rectangles

For a set of 4 points:



Neither of the two + can be included in one rectangle without including the -, so there's no possible combination of two rectangles to correctly classify this.

b) circles

2

Any set of 2 points:

1. if they have the same label +: use the circle to include all of them

2. if they have the same label -: do not include any of them with the circle

3. if they have different labels +-: we can always include the + with the circle, without including the -

For a set of 3 points:

There’s no circle that can include both + without including the -.

c) triangles

2

Any set of 2 points:

1. if they have the same label +: use the triangle to include all of them

2. if they have the same label -: do not include any of them with the triangle

3. if they have different labels +-: we can always include the + with the triangle, without including the -

For a set of 3 points:

There’s no circle that can include both + without including the -.

d) multidimensional "sphere" given by $f(x) = \text{sign}\,[(x - c)(x - c) - b]$ in the Euclidean space with m dimensions $\mathbb{R}\,m$.

2

For any set of two points:

1. if they have the same label -: use the sphere to include all of them

2. if they have the same label +: do not include any of them with the sphere

3. if they have different labels +-: we can always include the - with the sphere, without including the +

For a set of three points:

Let x1 and x2 be two -s, and let x3 be the middle point of x1 and x2, and x3 is labeled +.

There's no possible sphere that could include both x1 and x2, without including x3.