

PPL - ASSIGNMENT 4

חלק 1 אסטרטגיה

(a) $\{f : [T1 \rightarrow T2], g : [T1 \rightarrow T2], a : T1\} \vdash (f (g a)) : T2$ 1

false - f מקבל את g ומקבל את a . כלומר, g מקבל את a ופזא T_1 ומחזיר T_2 . אז f מקבל T_2 אזה פזא T_2 זקק עקב T_1 .

(b) $\{f : [T1 \times T2 \rightarrow T3]\} \vdash (\text{lambda } (x) (f x 100)) : [T2 \rightarrow T3]$

false - f מקבל את x ו-100. 100 פזא Number, אז עזי T_2 , פזא עזא פזא Number.

(c) $\{f : [T1 \rightarrow T2]\} \vdash ((\text{lambda } (x) (f x))) : [T1 \rightarrow T2]$

true - סמננה מקבל פזאזר א מציפוס T_1 אז f מקבל T_1 ומחזיר T_2 , כנעל.

(d) $\{f : [T1 \times T2 \rightarrow T3], y : T2\} \vdash (\text{lambda } (x) (f x y)) : [T1 \rightarrow T3]$

true - אז y יפיה מציפוס T_2 אז f מקבל פזאזרים מציפוס T_1, T_2 ויחזיר T_3 .

(a) $((\text{lambda } (f x1) (\text{if } x1 (f 1 x1) (f 3 x1))) + \#t)$ 2

renaming : $((\text{lambda } (f x) (\text{if } x (f 1 x) (f 3 x))) + \#t)$

assignment sub-expressions :

Expression	variable
$((\text{lambda } (f x) (\text{if } x (f 1 x) (f 3 x))) + \#t)$	T_0
$((\text{lambda } (f x) (\text{if } x (f 1 x) (f 3 x))))$	T_1
$(f x)$	T_2
$(\text{if } x (f 1 x) (f 3 x))$	T_3
$(f 1 x)$	T_4
$(f 3 x)$	T_5
f	T_f
x	T_x
1	T_{num1}
3	T_{num3}
+	T_+
$\#t$	$T_{\#t}$

:type equations

Expression	Equation
$((\text{lambda } (f \ x) (\text{if } x (f \ 1 \ x) (f \ 3 \ x))) + *t)$	$T_i = [T_+ \times T_{*t} \rightarrow T_o]$
$((\text{lambda } (f \ x) (\text{if } x (f \ 1 \ x) (f \ 3 \ x))))$	$T_i = [T_f \times T_x \rightarrow T_3]$
$(f \ x)$	$T_f = [T_x \rightarrow T_2]$
$(\text{if } x (f \ 1 \ x) (f \ 3 \ x))$	$T_x = T_f$
$(f \ 1 \ x)$	$T_f = [\text{Number} \times T_x \rightarrow T_4]$
$(f \ 3 \ x)$	$T_f = [\text{Number} \times T_x \rightarrow T_5]$

Expression	Equation
1	$T_{\text{num}1} = \text{Number}$
3	$T_{\text{num}3} = \text{Number}$
+	$T_+ = [\text{Number} \times \text{Number} \rightarrow \text{Number}]$
*t	$T_{*t} = \text{Boolean}$

:solve the equations

Equation	Substitution
1. $T_i = [T_+ \times T_{*t} \rightarrow T_o]$	$\{\}$
2. $T_i = [T_f \times T_x \rightarrow T_3]$	
3. $T_f = [T_x \rightarrow T_2]$	
4. $T_x = T_f$	
5. $T_f = [\text{Number} \times T_x \rightarrow T_4]$	
6. $T_f = [\text{Number} \times T_x \rightarrow T_5]$	
7. $T_{\text{num}1} = \text{Number}$	
8. $T_{\text{num}3} = \text{Number}$	
9. $T_+ = [\text{Number} \times \text{Number} \rightarrow \text{Number}]$	
10. $T_{*t} = \text{Boolean}$	

Equation	Substitution
2. $T_i = [T_f \times T_x \rightarrow T_3]$	$\{ T_i = [T_f \times T_{*t} \rightarrow T_0] \}$
3. $T_f = [T_x \rightarrow T_2]$	
4. $T_x = T_f$	
5. $T_f = [\text{Number} \times T_x \rightarrow T_4]$	
6. $T_f = [\text{Number} \times T_x \rightarrow T_5]$	
7. $T_{num1} = \text{Number}$	
8. $T_{num3} = \text{Number}$	
9. $T_+ = [\text{Number} \times \text{Number} \rightarrow \text{Number}]$	
10. $T_{*t} = \text{Boolean}$	

Equation	Substitution
3. $T_f = [T_x \rightarrow T_2]$	$\{ T_i = [T_f \times T_x \rightarrow T_3] ,$ $T_+ = T_f ,$ $T_{*t} = T_x ,$ $T_0 = T_3 \}$
4. $T_x = T_f$	
5. $T_f = [\text{Number} \times T_x \rightarrow T_4]$	
6. $T_f = [\text{Number} \times T_x \rightarrow T_5]$	
7. $T_{num1} = \text{Number}$	
8. $T_{num3} = \text{Number}$	
9. $T_+ = [\text{Number} \times \text{Number} \rightarrow \text{Number}]$	
10. $T_{*t} = \text{Boolean}$	

Equation	Substitution
4. $T_x = T_f$	$\{ T_i = [[T_x \rightarrow T_2] \times T_x \rightarrow T_3] ,$ $T_+ = [T_x \rightarrow T_2]$ $T_{*t} = T_x ,$ $T_0 = T_3 \}$
5. $T_f = [\text{Number} \times T_x \rightarrow T_4]$	

6. $T_f = [\text{Number} \times T_x \rightarrow T_5]$	
7. $T_{num1} = \text{Number}$	
8. $T_{num3} = \text{Number}$	
9. $T_+ = [\text{Number} \times \text{Number} \rightarrow \text{Number}]$	
10. $T_{*t} = \text{Boolean}$	

Equation	Substitution
5. $T_f = [\text{Number} \times T_x \rightarrow T_4]$	$\{ T_i = [[T_x \rightarrow T_2] \times T_x \rightarrow T_3] ,$ $T_+ = [T_x \rightarrow T_2]$ $T_{*t} = T_x ,$ $T_0 = T_3 \}$
6. $T_f = [\text{Number} \times T_x \rightarrow T_5]$	
7. $T_{num1} = \text{Number}$	
8. $T_{num3} = \text{Number}$	
9. $T_+ = [\text{Number} \times \text{Number} \rightarrow \text{Number}]$	
10. $T_{*t} = \text{Boolean}$	

Equation	Substitution
6. $T_f = [\text{Number} \times T_x \rightarrow T_5]$	$\{ T_i = [[T_x \rightarrow T_2] \times T_x \rightarrow T_3] ,$ $T_+ = [T_x \rightarrow T_2]$ $T_{*t} = T_x ,$ $T_0 = T_3 \}$
7. $T_{num1} = \text{Number}$	
8. $T_{num3} = \text{Number}$	
9. $T_+ = [\text{Number} \times \text{Number} \rightarrow \text{Number}]$	
10. $T_{*t} = \text{Boolean}$	

Equation	Substitution
7. $T_{num1} = \text{Number}$ 8. $T_{num3} = \text{Number}$	$\{ T_i = [[T_x \rightarrow T_2] \times T_x \rightarrow T_3] ,$ $T_+ = [T_x \rightarrow T_2]$ $T_{*t} = T_x ,$ $T_0 = T_3 \}$
9. $T_+ = [\text{Number} \times \text{Number} \rightarrow \text{Number}]$	

10. $T_{\#t} = \text{Boolean}$	
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Equation	Substitution
9. $T_+ = [\text{Number} \times \text{Number} \rightarrow \text{Number}]$	$\{ T_1 = [[T_x \rightarrow T_2] \times T_x \rightarrow T_3],$ $T_+ = [T_x \rightarrow T_2]$ $T_{\#t} = T_x,$ $T_0 = T_3,$ $T_{\text{num}1} = \text{Number},$ $T_{\text{num}3} = \text{Number} \}$
10. $T_{\#t} = \text{Boolean}$	

Equation	Substitution
10. $T_{\#t} = \text{Boolean}$	$\{ T_1 = [[\text{Number} \times \text{Number} \rightarrow \text{Number}] \times [\text{Number} \times \text{Number}] \rightarrow T_3],$ $T_+ = [\text{Number} \times \text{Number} \rightarrow \text{Number}],$ $T_{\#t} = [\text{Number} \times \text{Number}],$ $T_0 = T_3,$ $T_{\text{num}1} = \text{Number},$ $T_{\text{num}3} = \text{Number},$ $T_2 = \text{Number} \}$

קיבלנו סתירה מנני ל- $T_{\#t}$ מנצח דקדק Boolean מנצח $\text{Number} \times \text{Number}$ -! ,
כדור קיבלנו לכאור סיכום.

(b) $((\text{lambda } (f1 \ x1 \ y1) (f1 \ x1 \ y1))) * 1 \ 3)$

$((\text{lambda } (f \ x \ y) (f \ x \ y))) * 1 \ 3) \Rightarrow ((\text{lambda } (f \ x \ y) (f \ x \ y))) * 1 \ 3)$:renaming

:assignment sub-expressions

Expression	variable
$((\text{lambda } (f \ x \ y) (f \ x \ y))) * 1 \ 3)$	T_0
$((\text{lambda } (f \ x \ y) (f \ x \ y)))$	T_1
$(f \ x \ y)$	T_2
f	T_f
x	T_x
y	T_y
$*$	T_*
1	$T_{\text{num}1}$

3	T_{num3}
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:type equations

Expression	Equation
$((\text{lambda } (f \ x \ y) (f \ x \ y)) \neq 1 \ 3)$	$T_i = [T_* \times T_{num1} \times T_{num3} \rightarrow T_o]$
$((\text{lambda } (f \ x \ y) (f \ x \ y)))$	$T_i = [T_f \times T_x \times T_y \rightarrow T_z]$
$(f \ x \ y)$	$T_f = [T_x \times T_y \rightarrow T_z]$

Expression	Equation
*	$T_* = [\text{Number} \times \text{Number} \rightarrow \text{Number}]$
1	$T_{num1} = \text{Number}$
3	$T_{num3} = \text{Number}$

:solve the equations

Equation	Substitution
1. $T_i = [T_* \times T_{num1} \times T_{num3} \rightarrow T_o]$	$\{\}$
2. $T_i = [T_f \times T_x \times T_y \rightarrow T_z]$	
3. $T_f = [T_x \times T_y \rightarrow T_z]$	
4. $T_* = [\text{Number} \times \text{Number} \rightarrow \text{Number}]$	
5. $T_{num1} = \text{Number}$	
6. $T_{num3} = \text{Number}$	

Equation	Substitution
2. $T_i = [T_f \times T_x \times T_y \rightarrow T_z]$	$\{T_i = [T_* \times T_{num1} \times T_{num3} \rightarrow T_o]\}$
3. $T_f = [T_x \times T_y \rightarrow T_z]$	
4. $T_* = [\text{Number} \times \text{Number} \rightarrow \text{Number}]$	
5. $T_{num1} = \text{Number}$	
6. $T_{num3} = \text{Number}$	

Equation	Substitution
3. $T_f = [T_x * T_y \rightarrow T_2]$	$\{T_i = [T_f * T_{num1} * T_{num3} \rightarrow T_2],$ $T_f = T_x,$ $T_x = T_{num1},$ $T_y = T_{num3},$ $T_o = T_2 \}$
4. $T_* = [Number * Number \rightarrow Number]$	
5. $T_{num1} = Number$	
6. $T_{num3} = Number$	

Equation	Substitution
4. $T_* = [Number * Number \rightarrow Number]$	$\{T_i = [T_* * T_{num1} * T_{num3} \rightarrow T_2],$ $T_f = T_*,$ $T_x = T_{num1},$ $T_y = T_{num3},$ $T_o = T_2 \}$
5. $T_{num1} = Number$	
6. $T_{num3} = Number$	

Equation	Substitution
5. $T_{num1} = Number$ 6. $T_{num3} = Number$	$\{T_i = [[Number * Number \rightarrow Number] * T_{num1} * T_{num3} \rightarrow T_2]$ $T_* = [Number * Number \rightarrow Number],$ $T_x = T_{num1},$ $T_y = T_{num3},$ $T_o = T_2 \}$

Equation	Substitution
	$\{T_i = [[Number * Number \rightarrow Number] * Number * Number \rightarrow T_2]$ $T_* = [Number * Number \rightarrow Number],$ $T_{num1} = Number$ $T_{num3} = Number$ $T_o = T_2 \}$

3.6n

if type <var>(tenv) = type <val>(tenv) : define typing rule 3.1

then type <(define (var : var.texp) val)>(tenv) = void