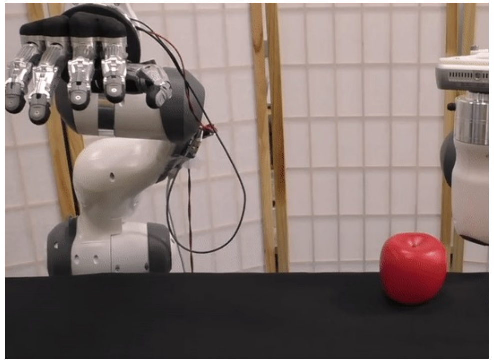
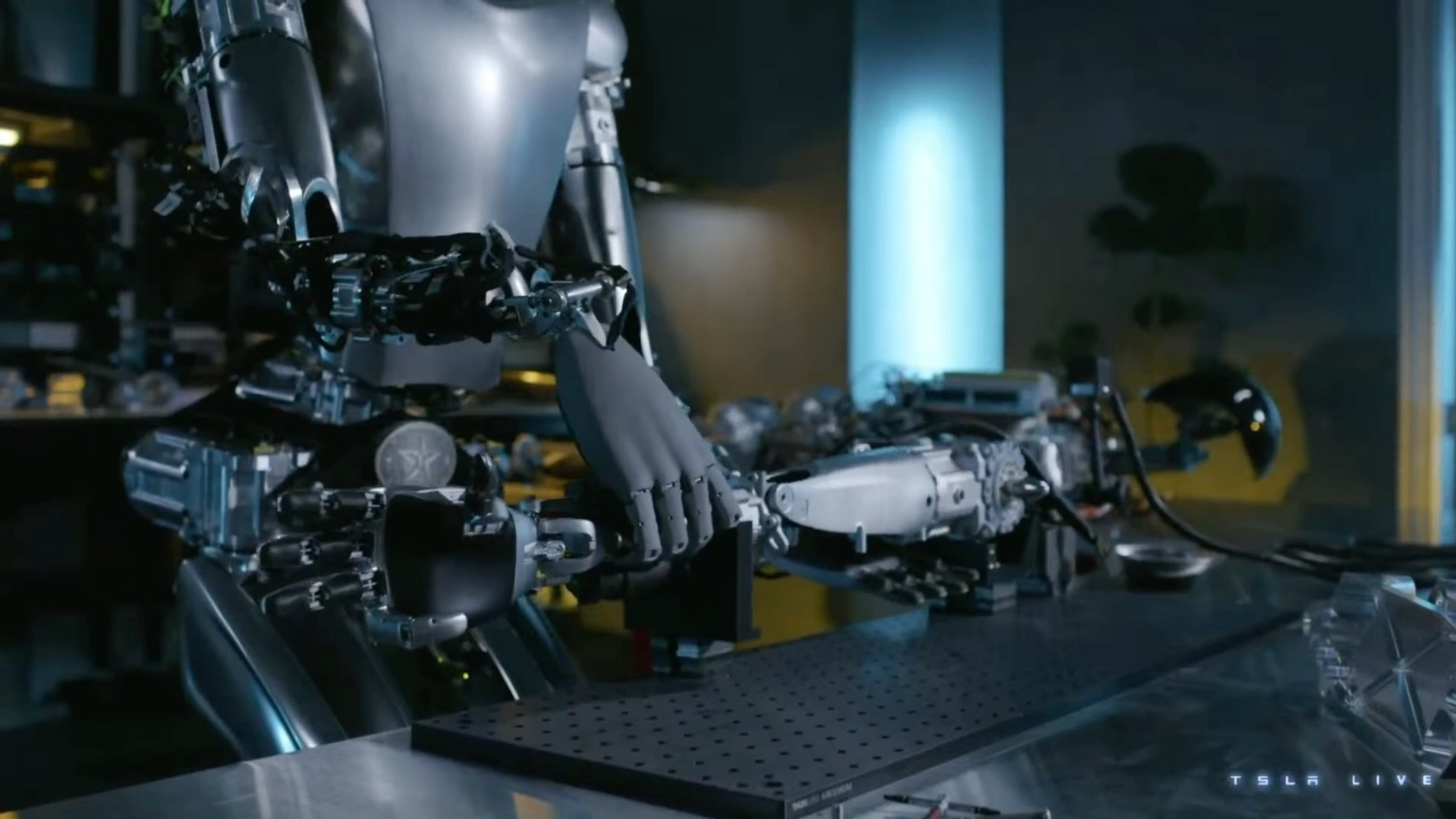
Introduction to Robotic Grasping

Lin Shao

April 11, 2023




TSLA LIVE

Today's Plan

- What is a Grasp?
  - Why is grasping challenging?
- How to model a Grasp?
- How to evaluate a Grasp?
    - Form Closure
    - Force Closure
- How to generate a Grasp?
- Learning to Grasp

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What is a grasp?

- A grasp is an act of restraining an object's motion through application of forces and torques at a set of contact points

Allegro Hand

Why is grasping hard?

- The configuration of Robotic hands

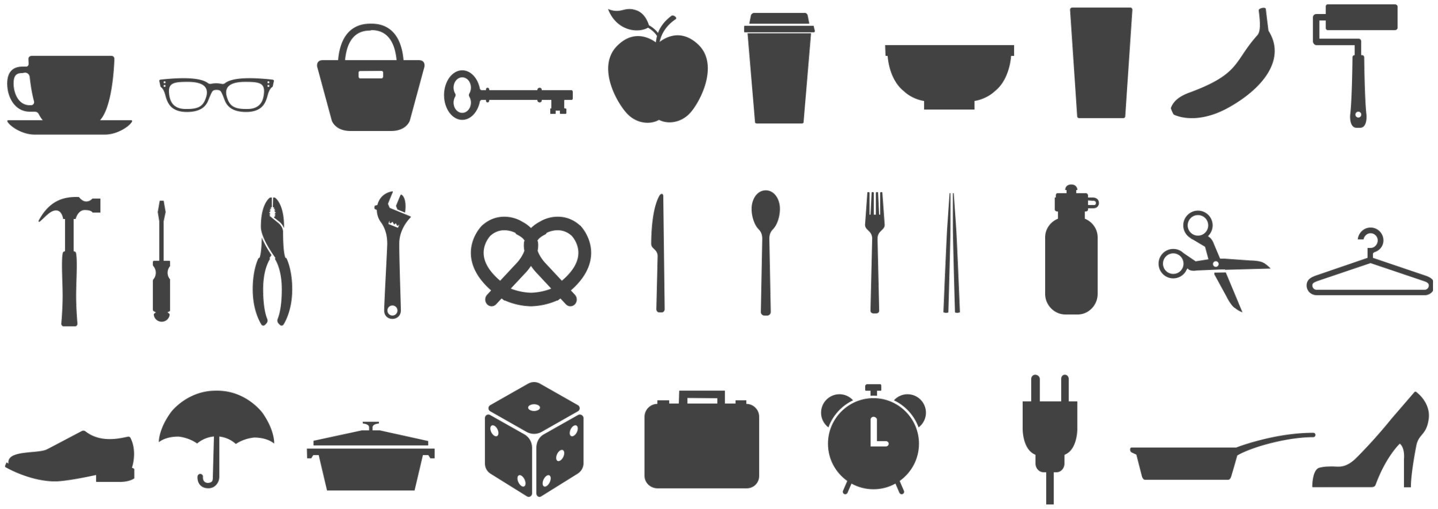
may be high-dimensional.

  - Allegro Hand has 4 fingers
    with 3 joints each for a total of 12 dimensions.
  - there are an additional 6 degrees of freedom
    in the wrist pose (position and orientation)
  - 18 dimension

Allegro Hand

Why is grasping hard?

- the large diversity of object geometry



Allegro Hand

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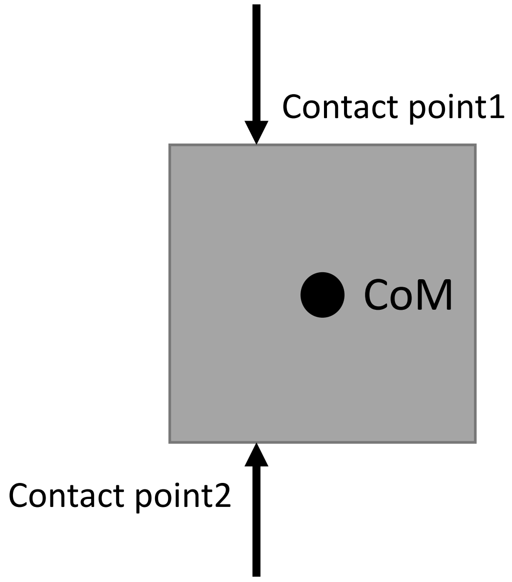
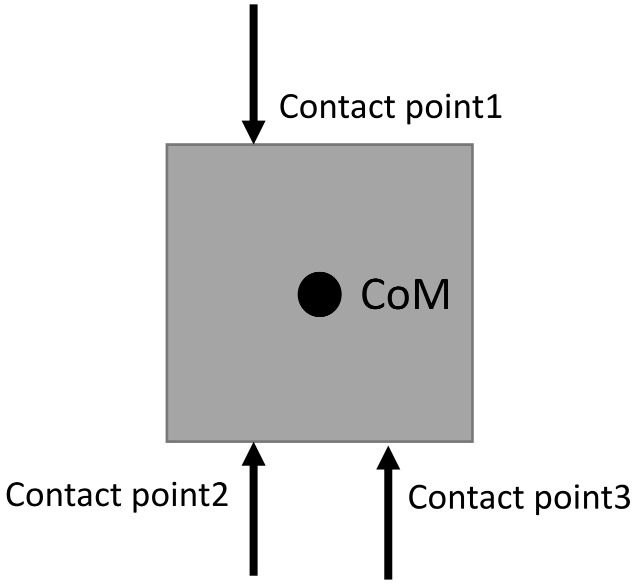
Let’s model a grasp analytically!

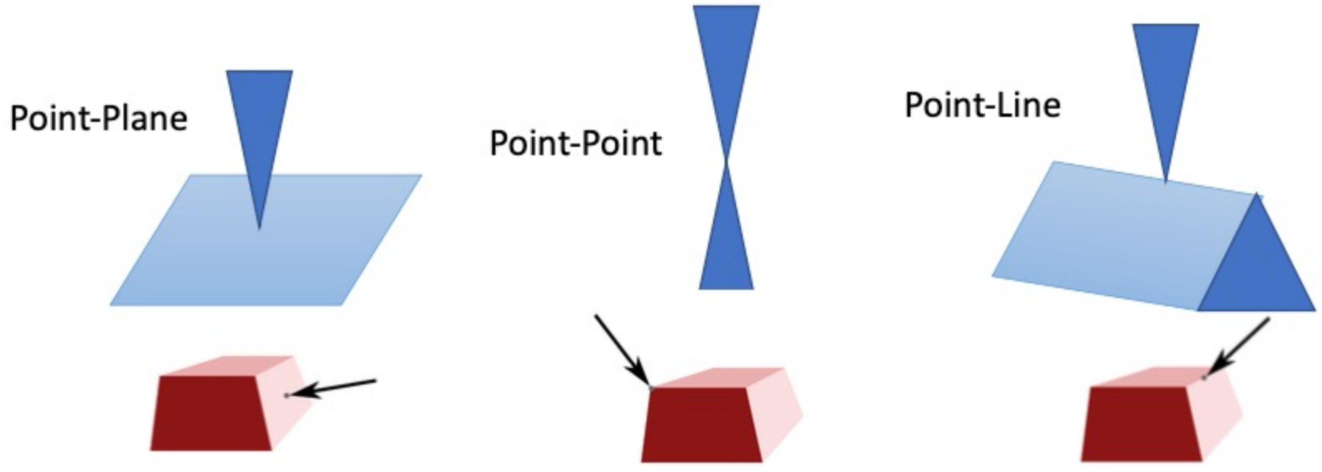
- a grasp may be parameterized in several ways
- the quality of the grasp is defined by the resulting contacts between the gripper and the object.





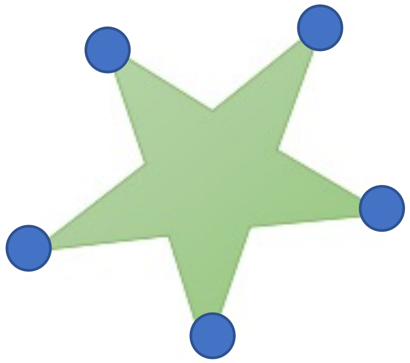
Contact types

- Point: point on plane(stable), point on point or line(unstable)
- Line: line on plane or nonparallel line (stable), line on parallel line (unstable)
- Plane: plane on plane (stable)



Everything as a Point Contact

- Line contact -> 2 points
- Plane contact -> convex hull of points
- Any distribution of normal forces across a region can be represented as a weighted sum of point forces along that region’s convex hull

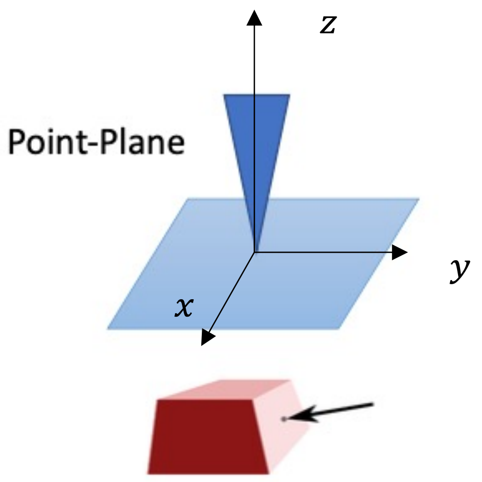


Point-on-Plane Contact Models

- Contact Models specify the admissible forces and torques that can be transmitted through a particular contact
- Local reference frame at contact point, z aligned with normal pointing inward

$$f = f_{normal} + f_{tangent},$$

$$f_{normal} = [0,0,f_z], f_{tangent} = [f_x, f_y, 0], f_z \geq 0$$



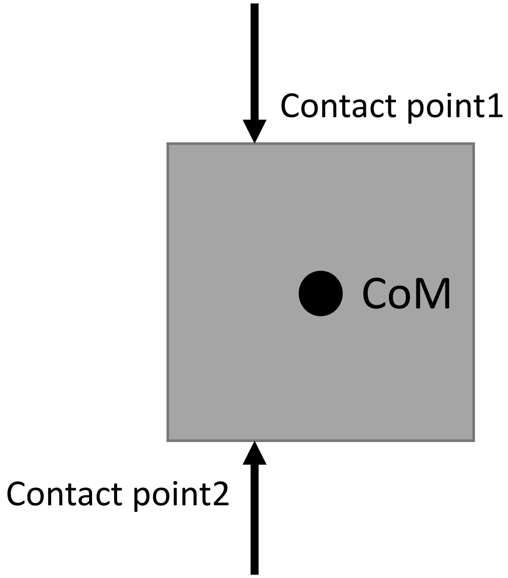
- Three common contact models, each defining a set of

admissible forces that can be applied through the contact:
  - Frictionless Point Contact
  - Point Contact with Friction
  - Soft-finger Contact

Frictionless Point Contact

- Forces can only be applied in direction normal to the surface of the object
- Contact Model is more common in form closure grasps

$$F = \{f_{normal} | f_z \geq 0\}$$



Point Contact with Friction

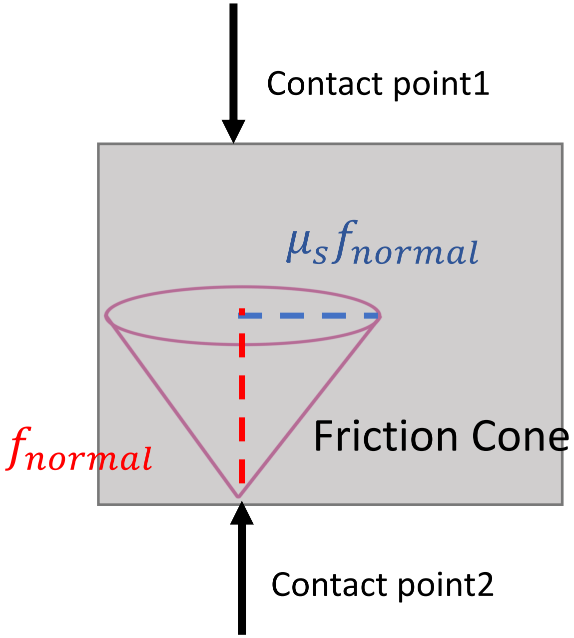
- A point contact with friction can apply more than just a normal force
- Coulomb friction, Static Coefficient of friction μ_s
- The admissible forces (i.e. forces that don't lead to slipping) are typically defined by a *friction cone*

$f = f_{normal} + f_{tangent},$

f is the contact force

f_{normal} is the normal force

$f_{tangent}$ is the friction force



A point contact is stable if its force satisfies the following equation

$$F = \{f \,|\, f_z \geq 0, \|f_{tangent}\| \leq \mu_s \|f_{normal}\|\}$$

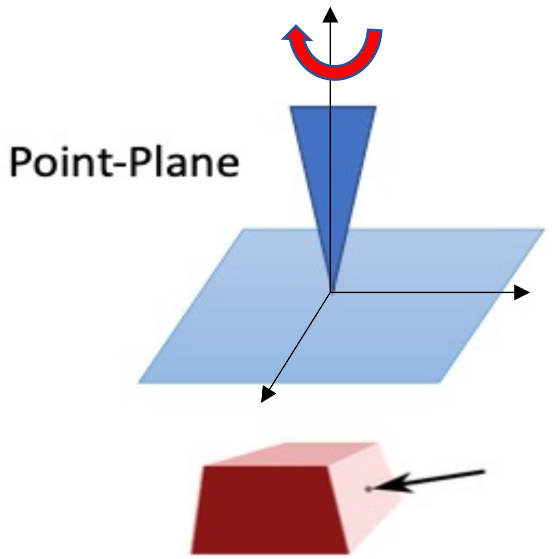
Soft-finger Contact Model

- includes a friction cone
- Also allows for torque around the normal

$$f = f_{normal} + f_{tangent},$$

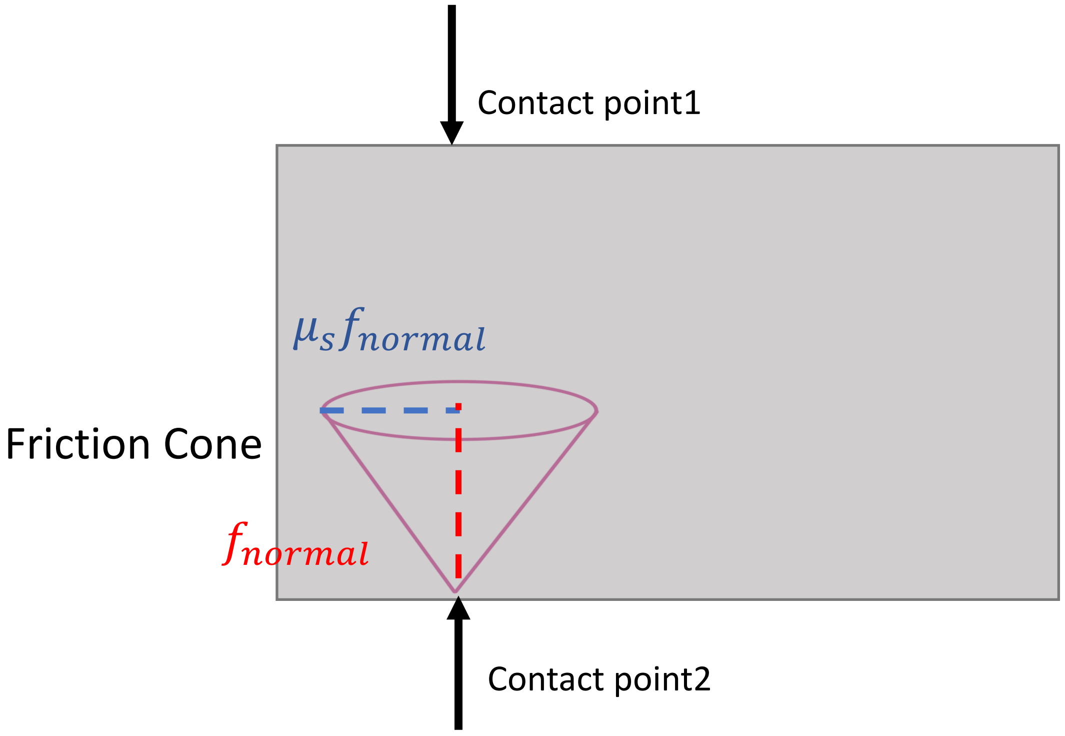
$$F = \{f \,|\, f_z \geq 0, \|f_{tangent}\| \leq \mu_s \|f_{normal}\|, \tau_{normal} \leq \gamma f_z\}$$

γ Torsional friction coefficient

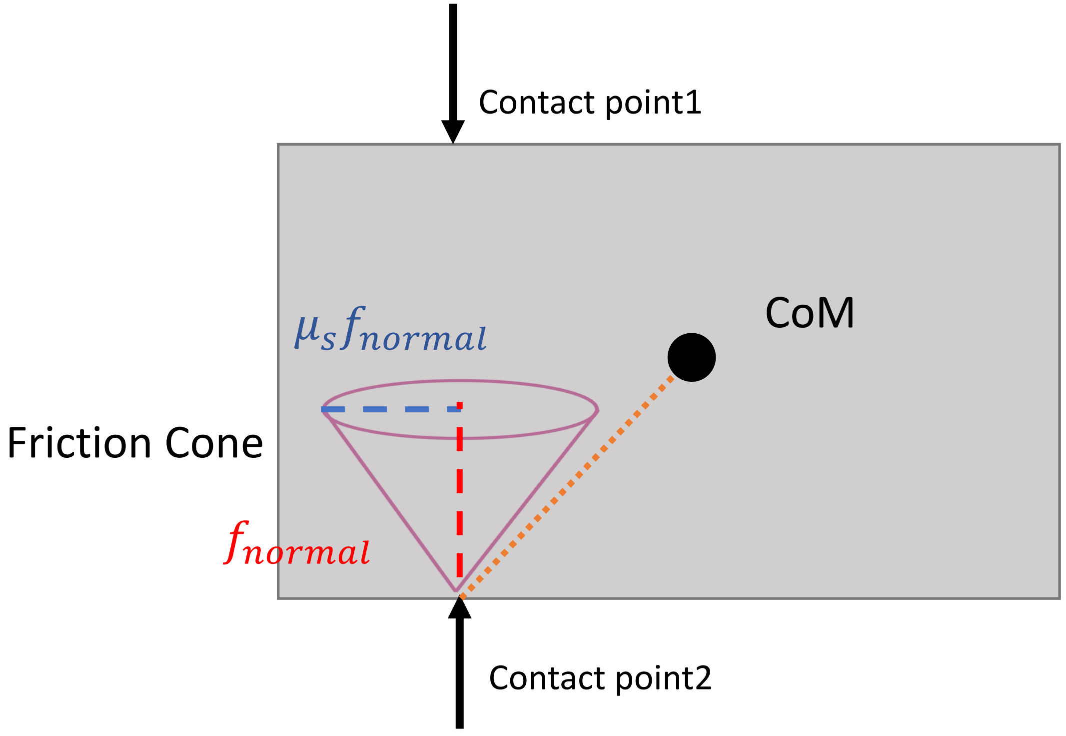


Three Contact Models

- Frictionless Point Contact
- Point Contact with Friction
- Soft-finger Contact



Local Analysis



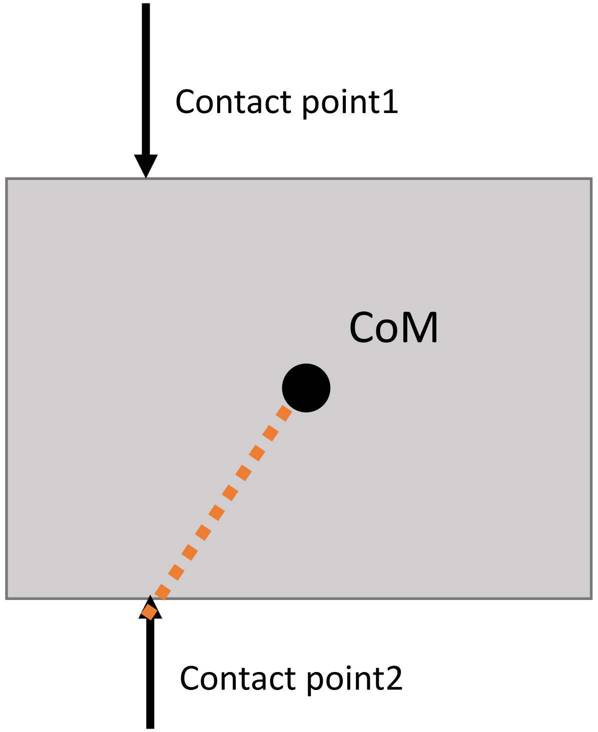
Global Analysis of Grasp

Wrench

- A wrench is a vector that describes forces and torques applied at a contact that act on the object center of mass
- Stacked as 6D vector written wrt to frame in object

$$w = \begin{bmatrix} f \\ \tau \end{bmatrix} \in \mathbb{R}^6 \text{ , w} = \begin{bmatrix} f \\ (d \times f) \end{bmatrix}$$

d :vector defining position of the contact with respect to CoM



Grasp

- A grasp can be defined as the set of all possible wrenches that can be achieved through the contact points

$$w = \sum_{i=1}^{k} G_i f_i = \begin{bmatrix} G_1 & \dots & G_k \end{bmatrix} \begin{bmatrix} f_1 \\ \vdots \\ f_k \end{bmatrix} = G \begin{bmatrix} f_1 \\ \vdots \\ f_k \end{bmatrix}$$

- G_i wrench basis matrix describes transformation from local contact reference frame to global object-centric reference frame
- $\begin{bmatrix} G_1 & \dots & G_k \end{bmatrix}$: grasp map

Grasp Wrench Space

- *The grasp wrench space* W *for a grasp with k contact points is* the set of all possible wrenches that can be applied to an object through admissible forces at k contact points

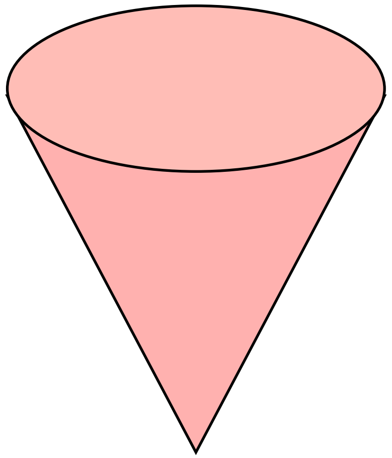
$$w = \left\{ w \mid w = \sum_{i=1}^{k} G_i f_i \,, f_i \in FrictionCone_i, i = 1, \ldots, k \right\}$$

The grasp wrench space is defined by the output of all possible applied force combination.

- For 3D objects, grasp wrench space is 6D
  - 3D for force, 3D for torque
  - For 2D object, it’s 3D

Friction Cones

- A point contact remains in the contact mode while its contact force lies inside the *friction cone*

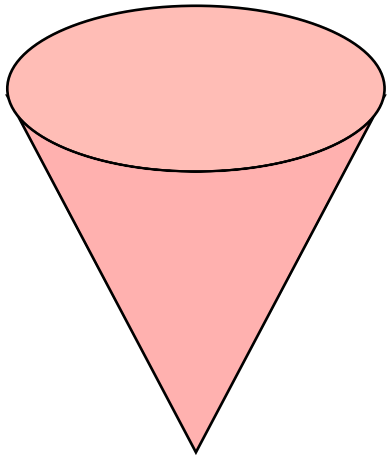


$f = f_{normal} + f_{tangent},$

$\|f^{tangent}\| \leq \mu \|f^{normal}\|$

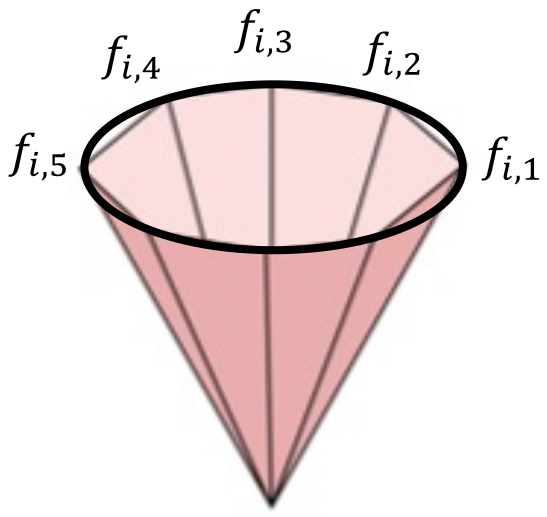
Linearized Friction Cone

- A pyramidal inner-approximation of the friction cone is often more useful from a computational standpoint, since its definition only requires a *finite* set of vectors



$$f_i = f_{normal} + f_{tangent},$$

$$\left\| f^{tangent} \right\| \leq \mu \left\| f^{normal} \right\|$$



Friction Cone = $\{f_{i,1}, f_{i,2}, \dots, f_{i,m}\}$

$$f_{i,j} = f_{i,j}^{normal} + f_{i,j}^{tangent}$$

$$\left\| f_{i,j}^{tangent} \right\| = \mu \left\| f_{i,j}^{normal} \right\|$$

Grasp Wrench Space from Linearized Friction Cone

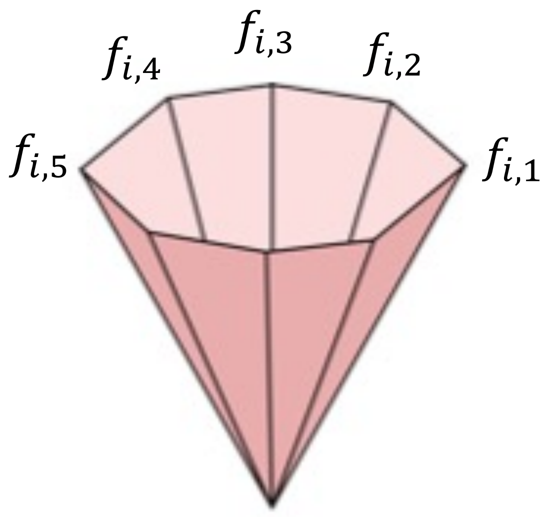
- Linearized Friction Cone whose edges are defined by the set of m forces:
  $$\{f_{i,1}, f_{i,2}, \dots, f_{i,m}\}$$
- Any forces can be written as a positive combination of these vectors

$$f_i = \sum_{j=1}^{m} \alpha_{i,j} f_{i,j}$$

$$a_{i,j} \geq 0, \sum_{j=1}^{m} \alpha_{i,j} \leq 1$$

$$f_{i,j} = f_{i,j}^{normal} + f_{i,j}^{tangent}$$

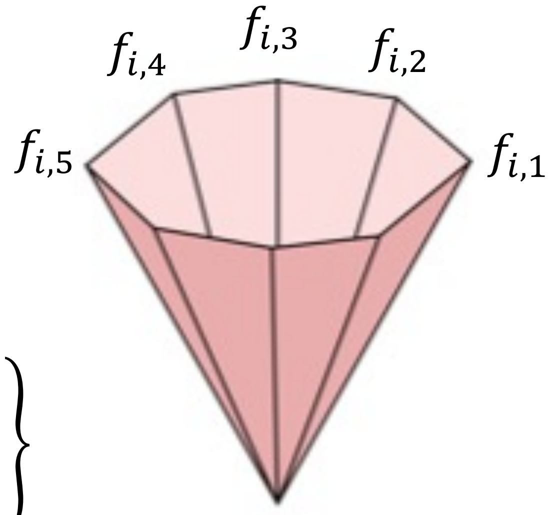
$$\left\| f_{i,j}^{tangent} \right\| = \mu \left\| f_{i,j}^{normal} \right\|, \left\| f_{i,j}^{tangle} \right\| = 1$$



Grasp Wrench Space from Linearized Friction Cone

- *The grasp wrench space* W *for a grasp with k contact points* the set of all possible wrenches that can be applied to an object through admissible forces at k contact points

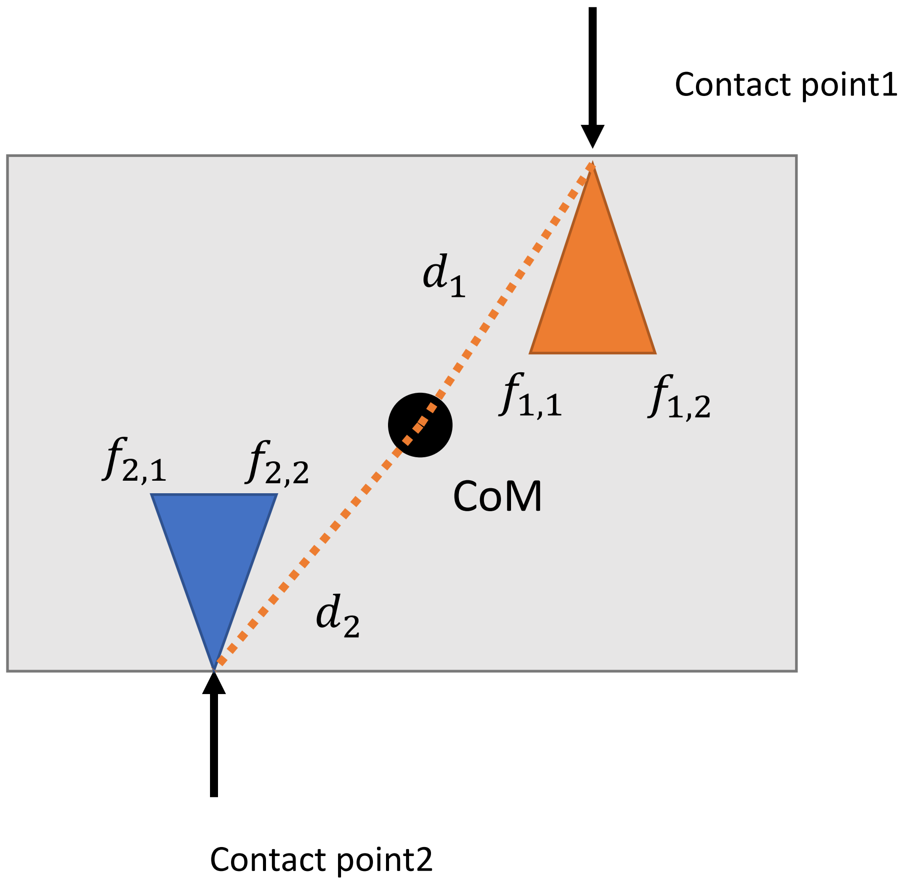
$$w = \left\{ w \middle| w = \sum_{i=1}^{k} w_i = \sum_{i=1}^{k} G_i f_i \,, f_i \in F_i, i = 1, \dots, k \right\}$$



$$w = \left\{ w \middle| w = \sum_{i=1}^{k} w_i \,, w_i = \sum_{j}^{m} \alpha_{i,j} w_{i,j} \,, w_{i,j} = \begin{bmatrix} f_{i,j} \\ (d_{i,j} \times f_{i,j}) \end{bmatrix}, a_{i,j} \geq 0, \sum_{j=1}^{m} \alpha_{i,j} \leq 1 \; i = 1, \dots, k \right\}$$

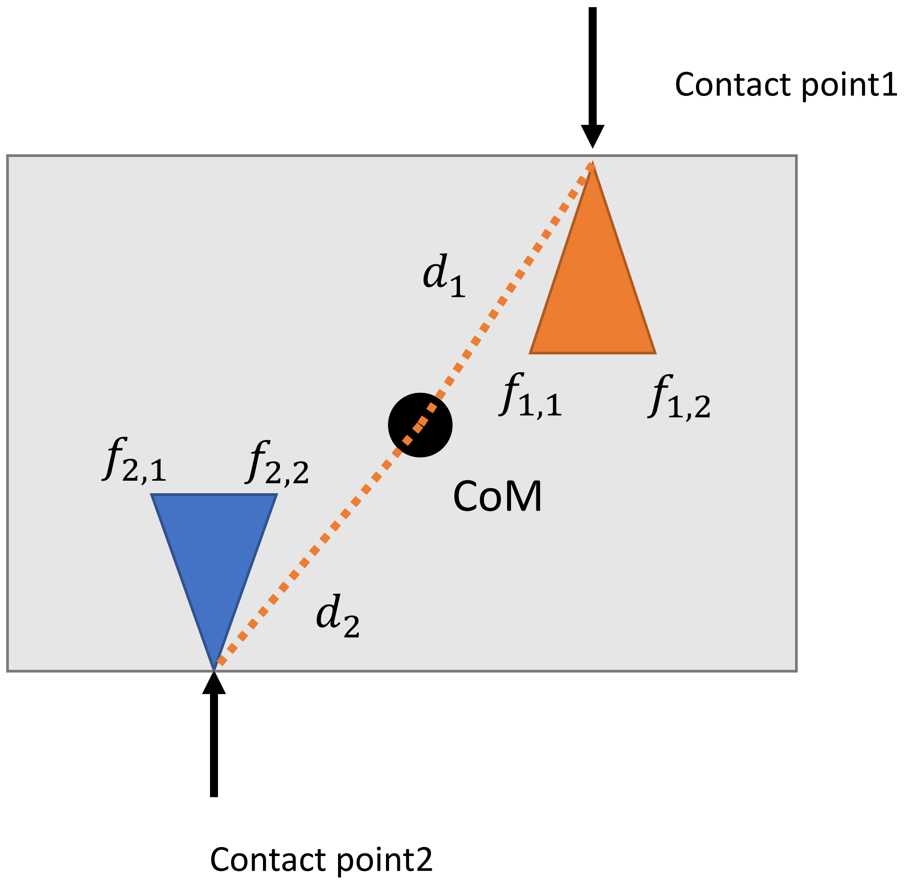
Grasp Wrench Space from Friction Cone

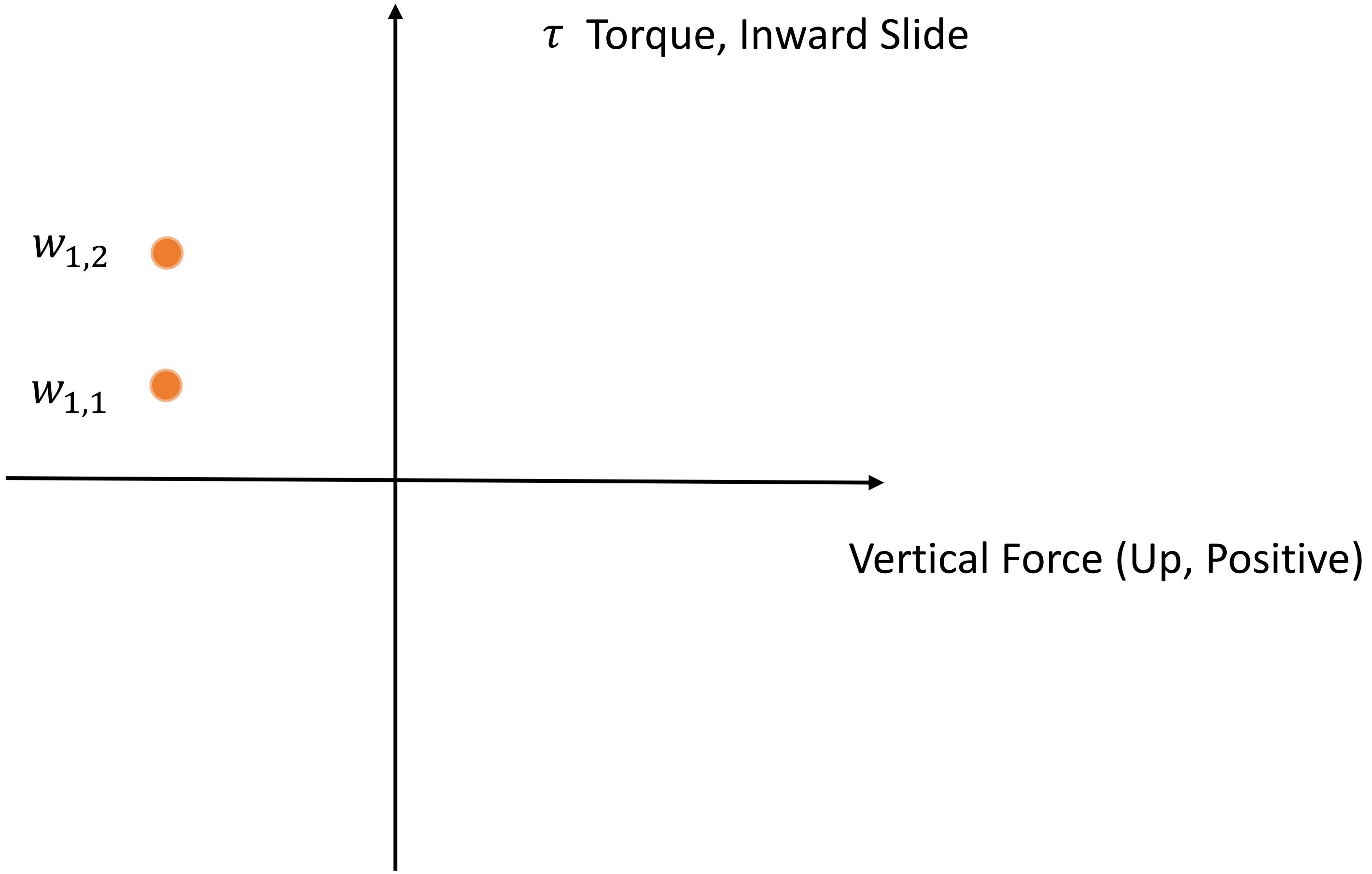
2D Object: Grasp Wrench Space is 3D



Grasp Wrench Space from Friction Cone

2D Object: Grasp Wrench Space is 3D





Grasp Wrench Space from Friction Cone

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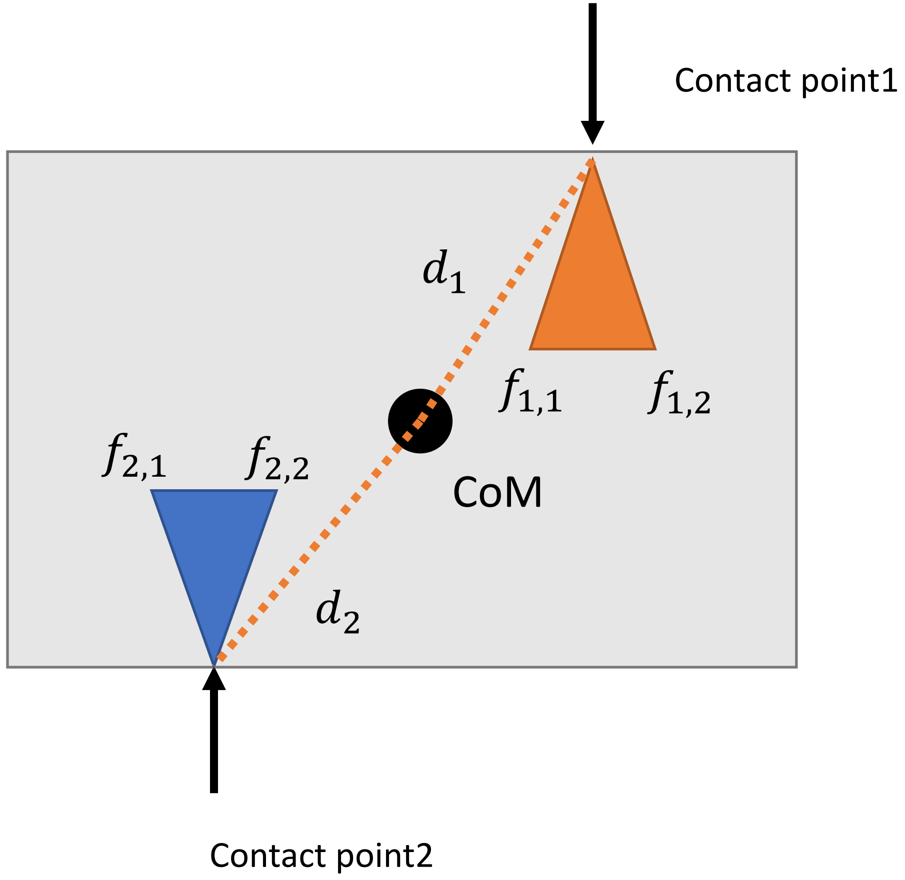

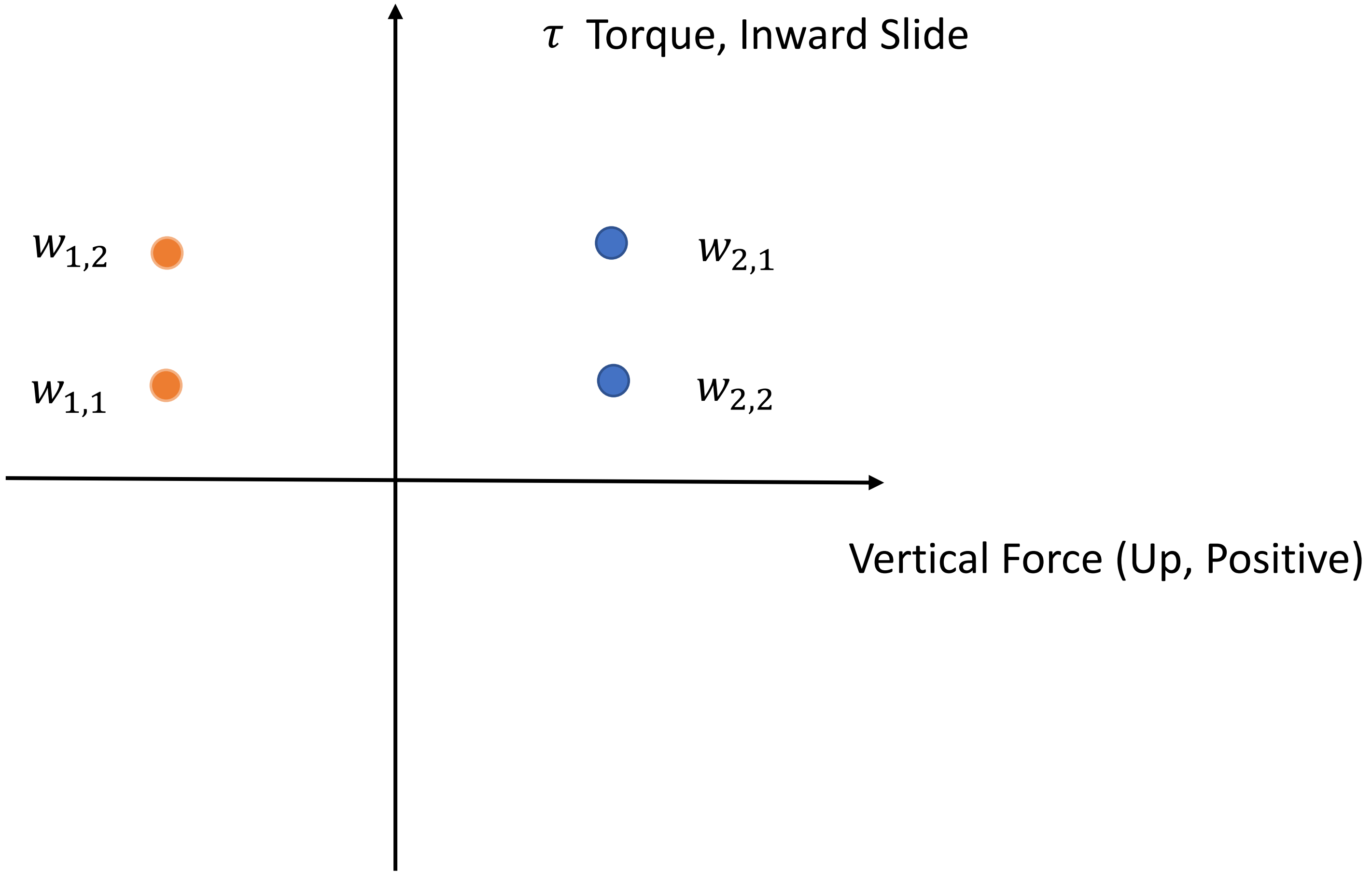

Grasp Wrench Space from Friction Cone

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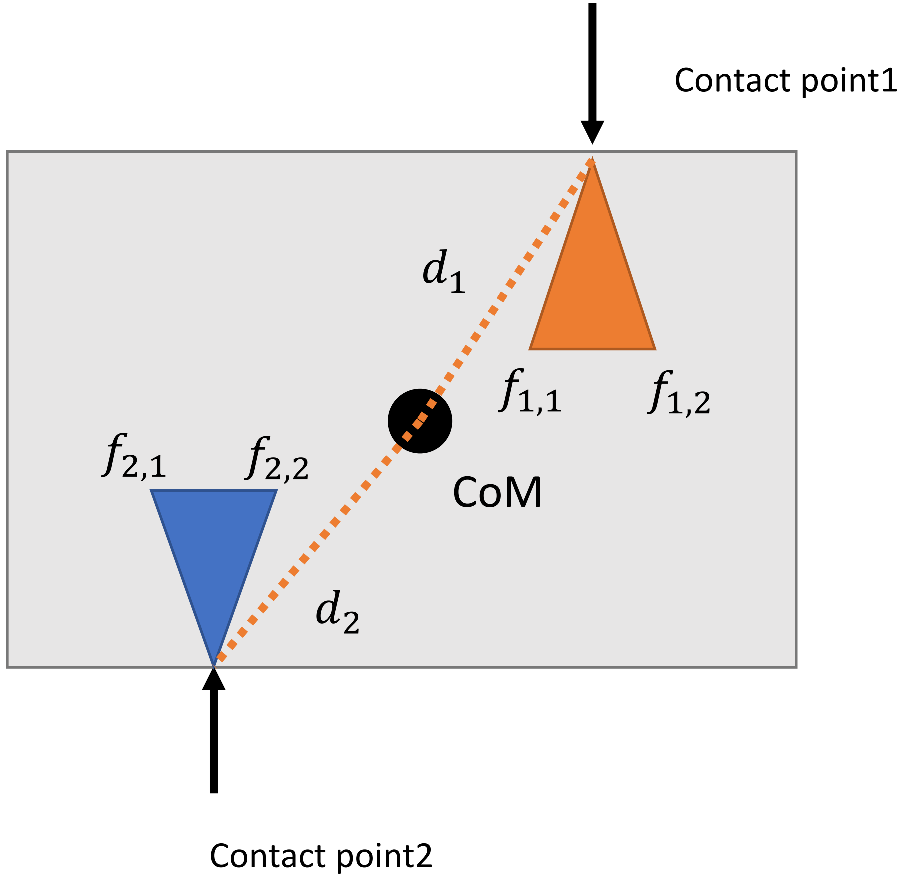
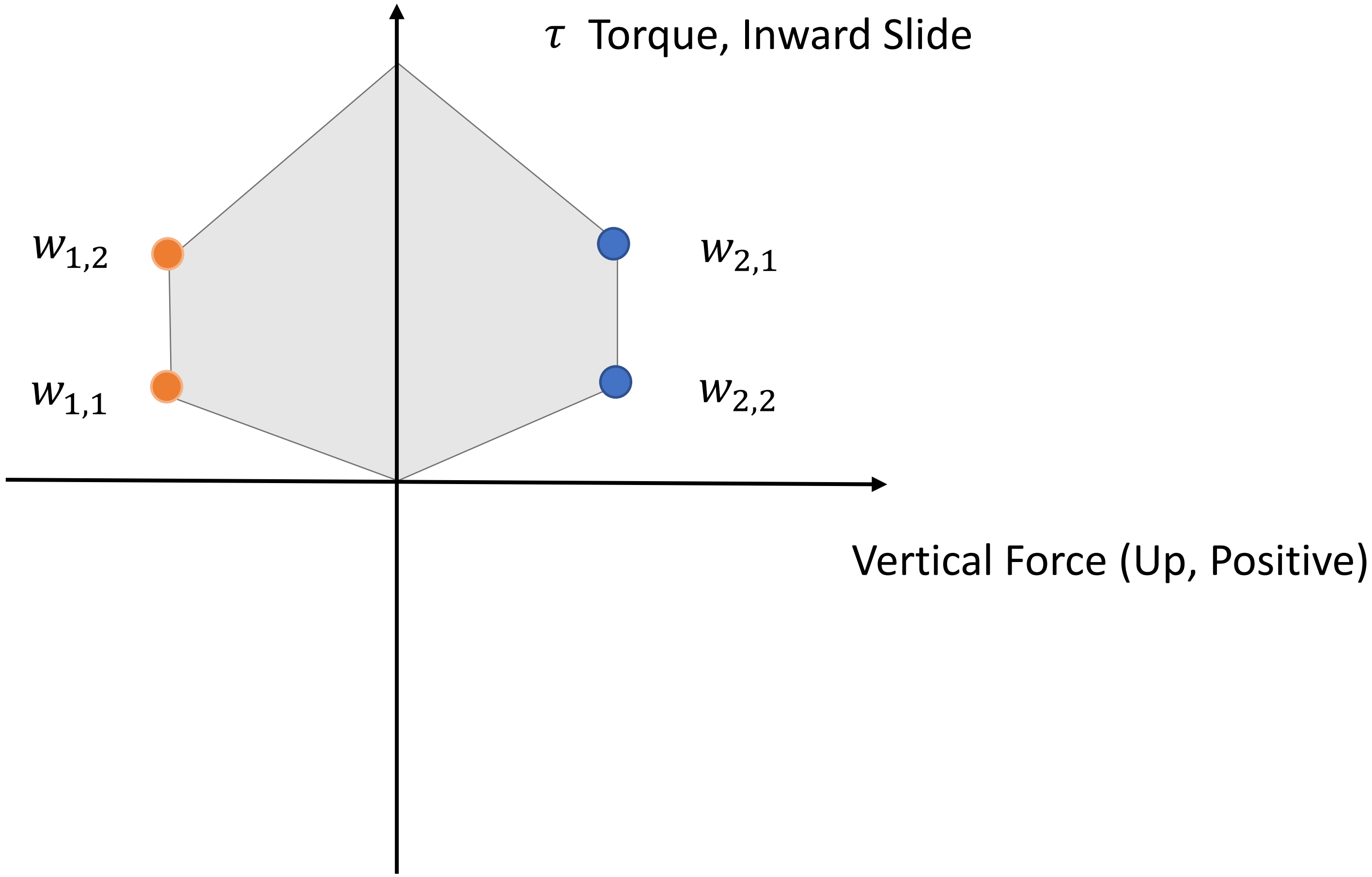
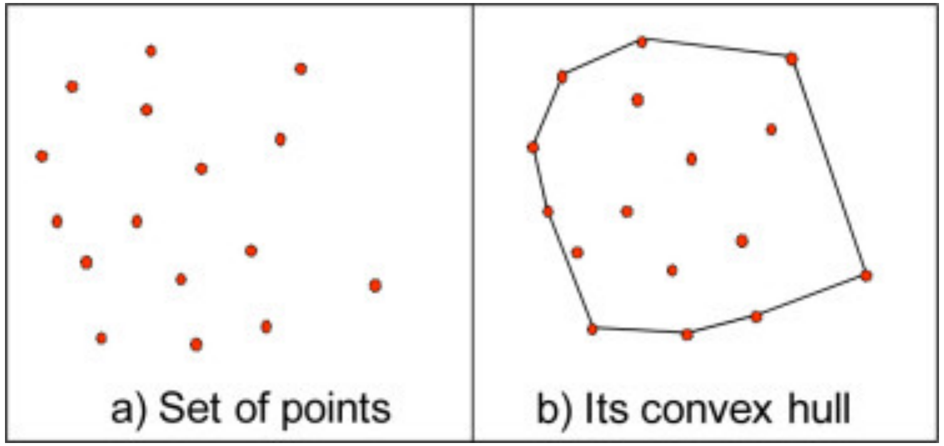




Convex Hull

- Convex hull of a shape is smallest convex set that contains it.



Grasp Wrench Hull

- The grasp wrench space W defines the set of all possible wrenches that can be applied to an object by a grasp,
- unfortunately computing this set can be quite cumbersome in practice
- grasp wrench hull

$$w = \left\{ w \middle| w = \sum_{i=1}^{k} \sum_{j}^{m} \alpha_{i,j} w_{i,j} \, , w_{i,j} = \begin{bmatrix} f_{i,j} \\ (d_i \times f_{i,j}) \end{bmatrix}, \sum_{i=1}^{k} \sum_{j}^{m} \alpha_{i,j} = 1 \, , a_{i,j} \geq 0, \right\}$$

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Grasp Wrench Space

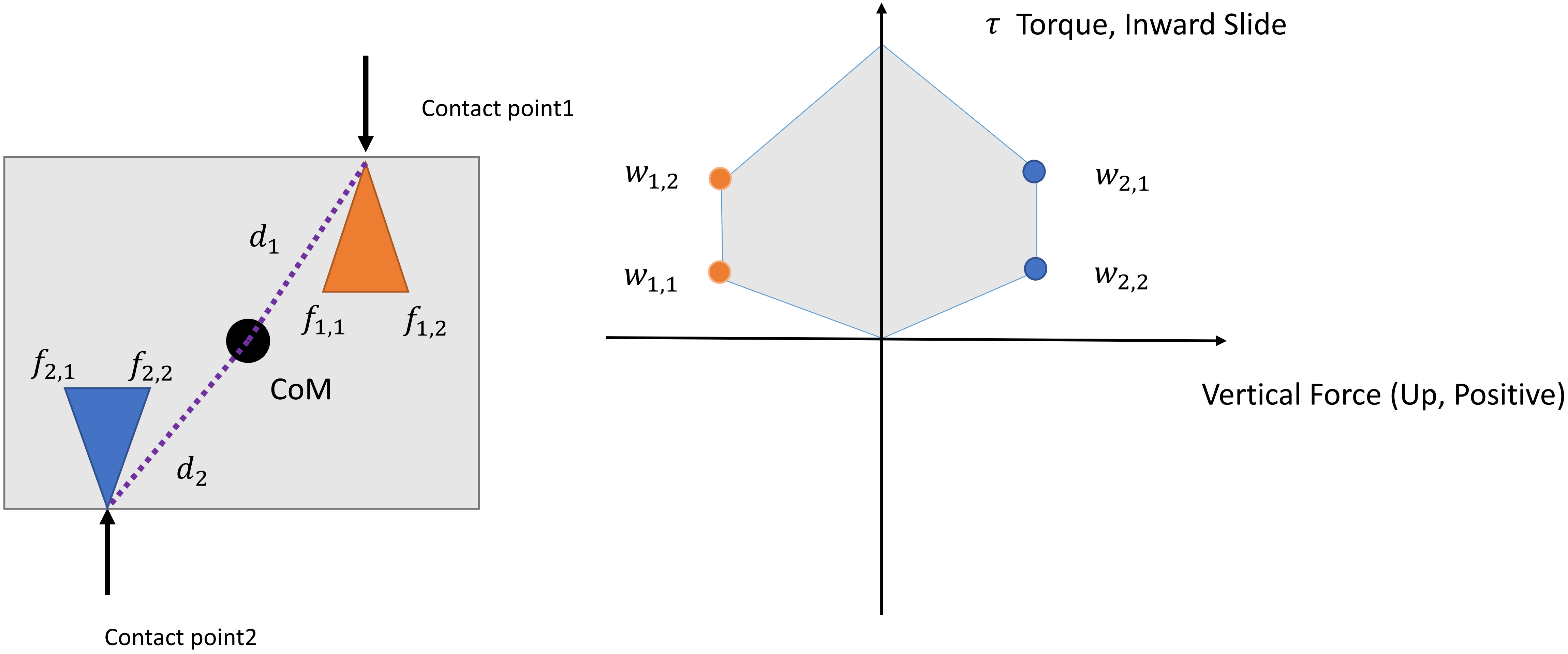
Grasp Wrench Hull

- Grasp Wrench Hull = Convex Hull of all the wrenches from the contact points

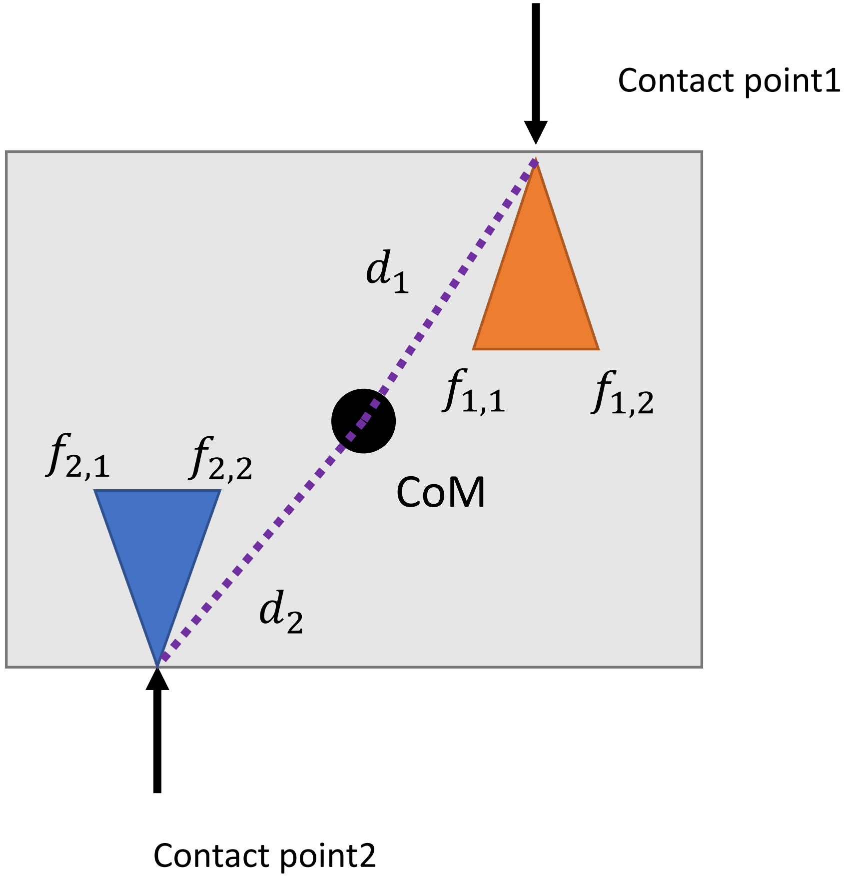
$$w = \left\{ w | w = \sum_{i=1}^{k} \sum_{j}^{m} \alpha_{i,j} w_{i,j} \, , w_{i,j} = \begin{bmatrix} f_{i,j} \\ (d_i \times f_{i,j}) \end{bmatrix} , \sum_{i=1}^{k} \sum_{j}^{m} \alpha_{i,j} = 1 \right\}$$

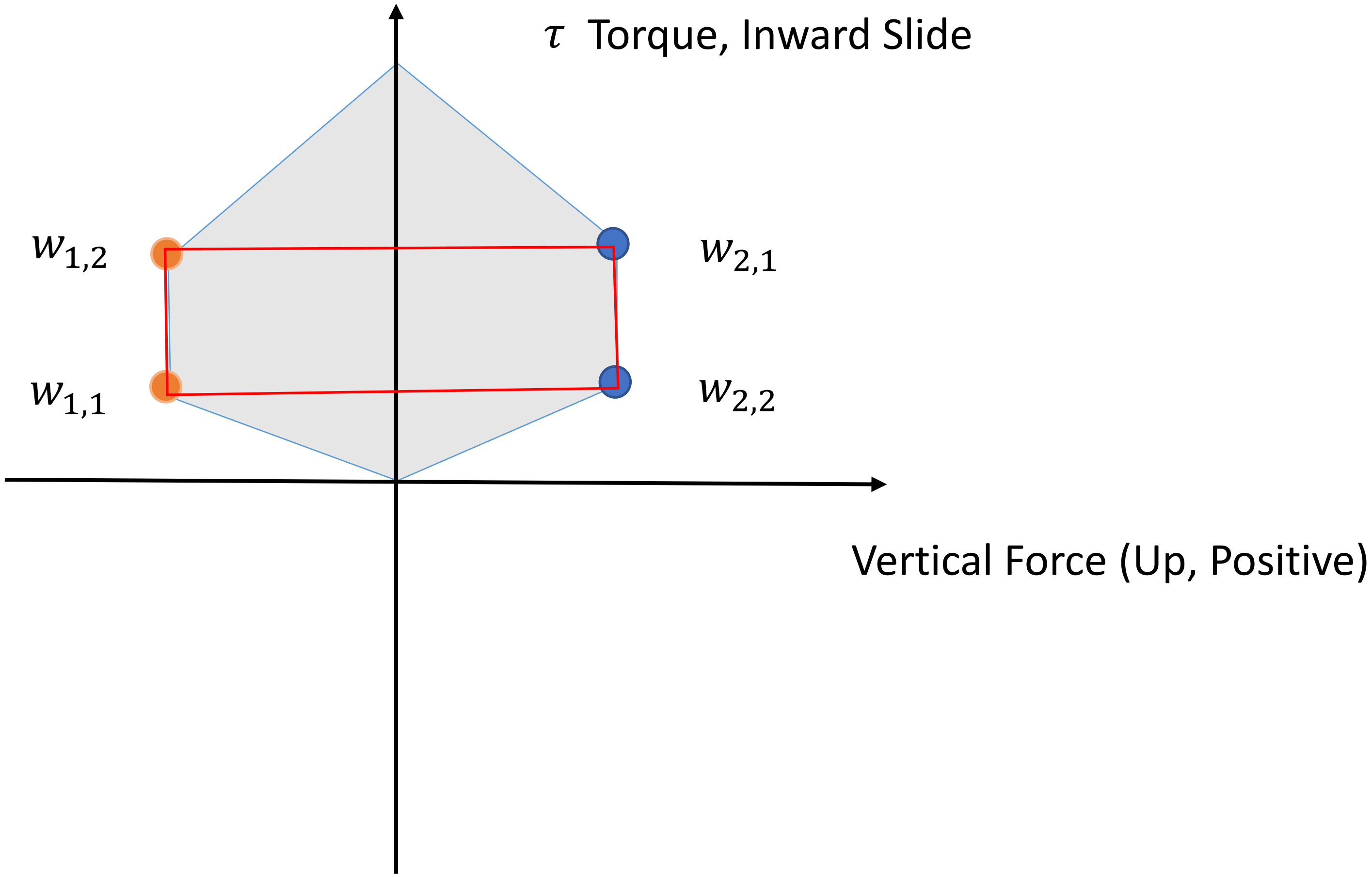
$$w = \left\{ w | w = ConvexHull(\{ w_{i,j} \}), w_{i,j} = \begin{bmatrix} f_{i,j} \\ (d_i \times f_{i,j}) \end{bmatrix} \right\}$$

Grasp Wrench Space from Friction Cone



Grasp Wrench Hull





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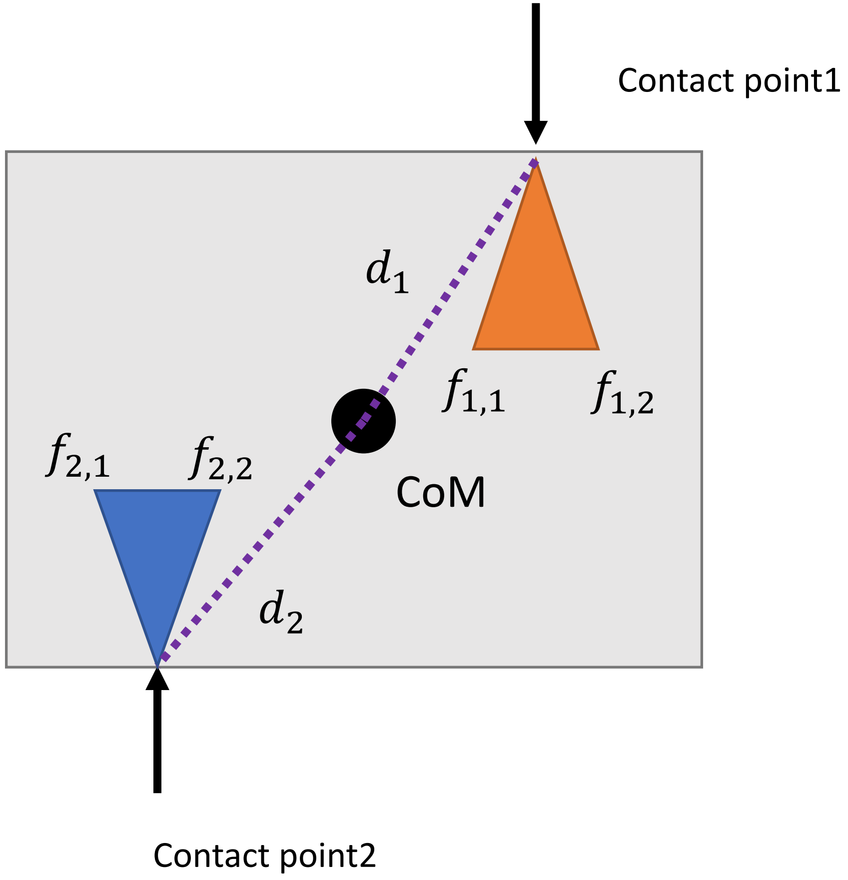

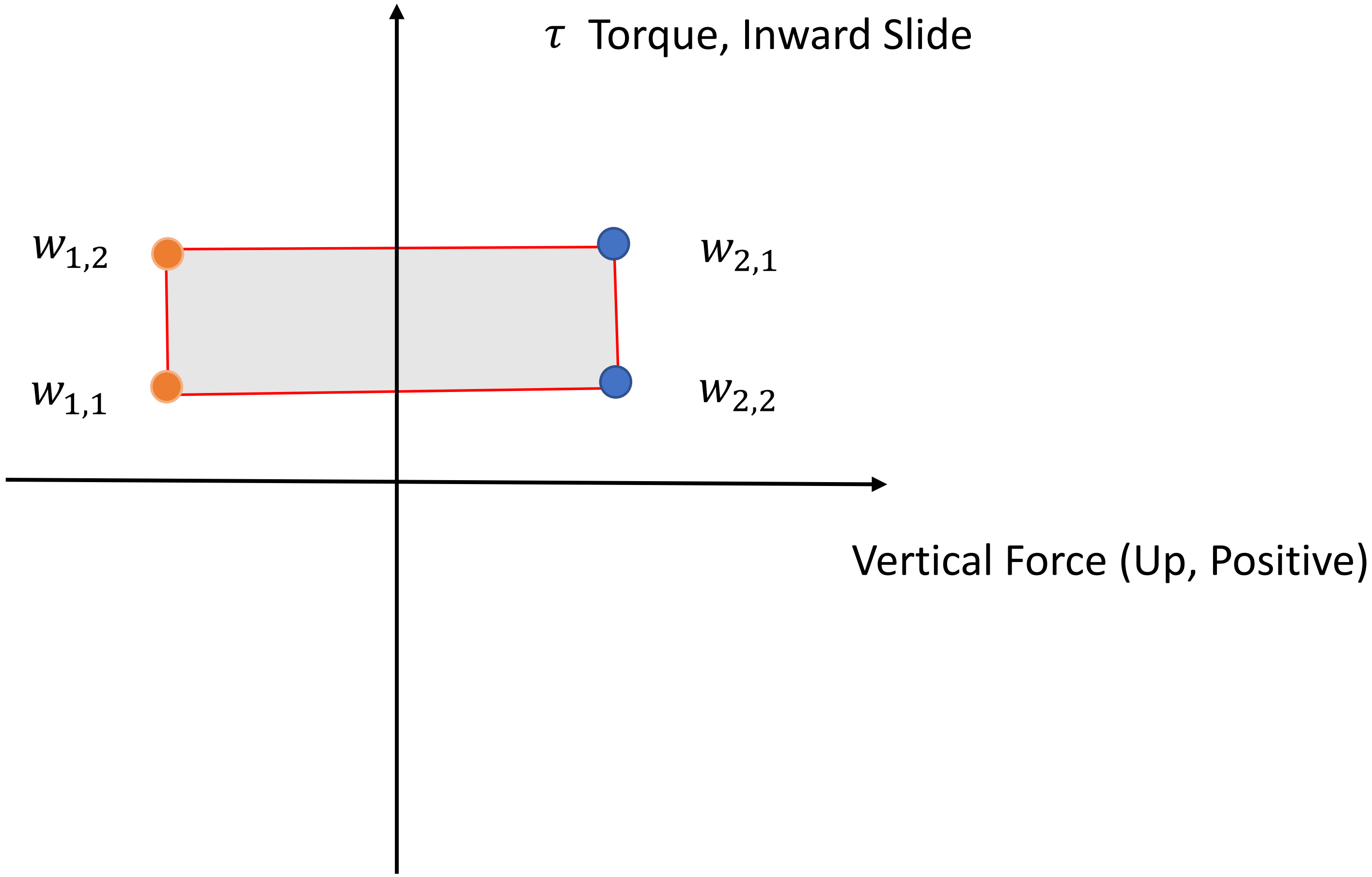

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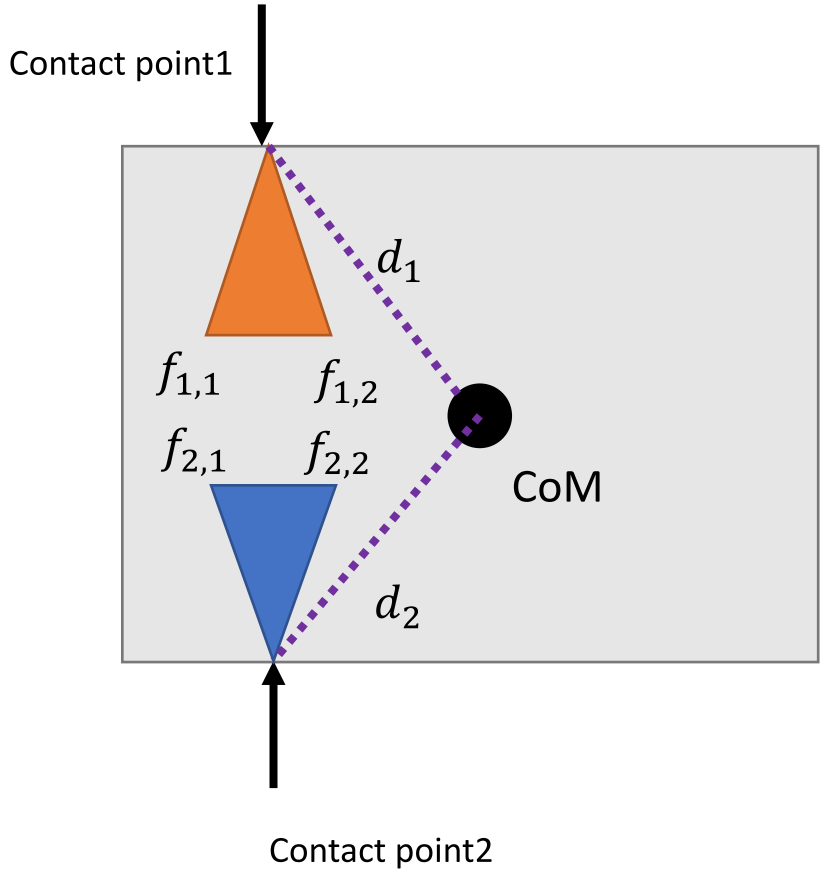
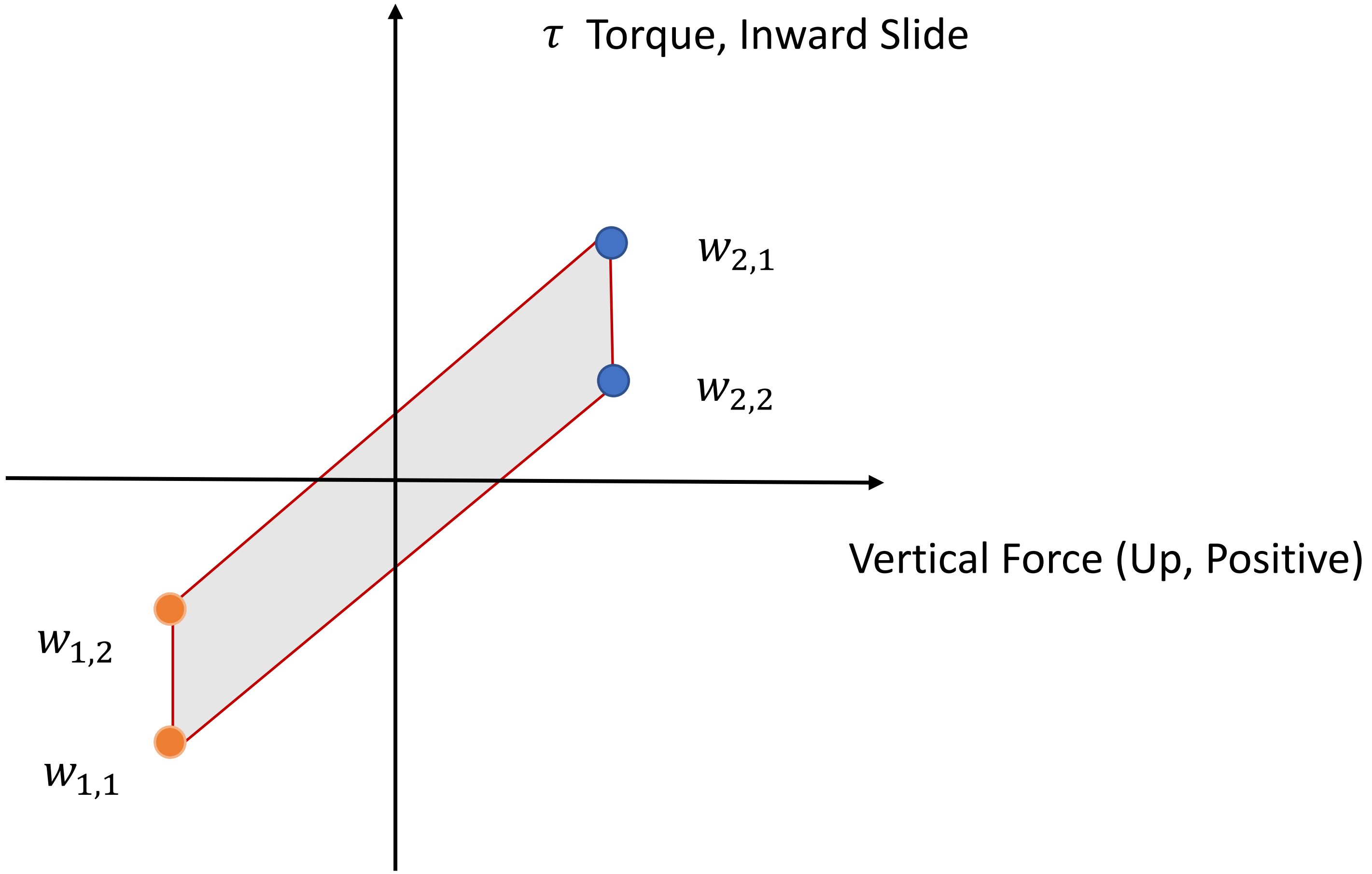
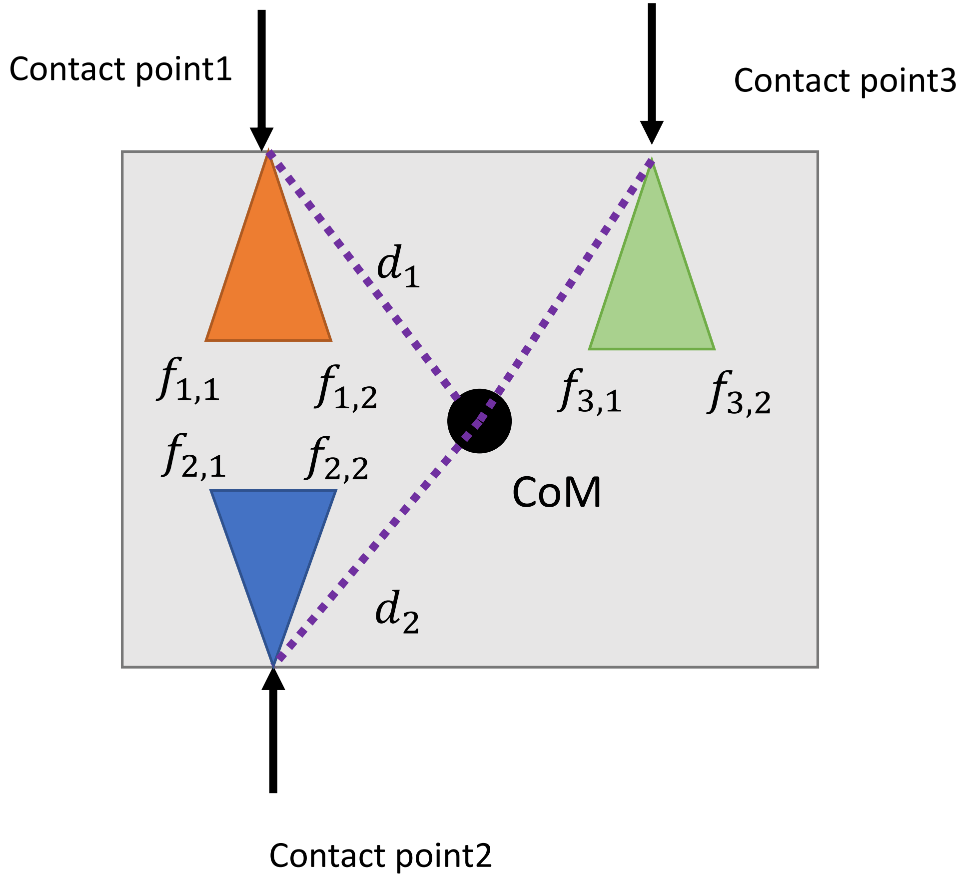
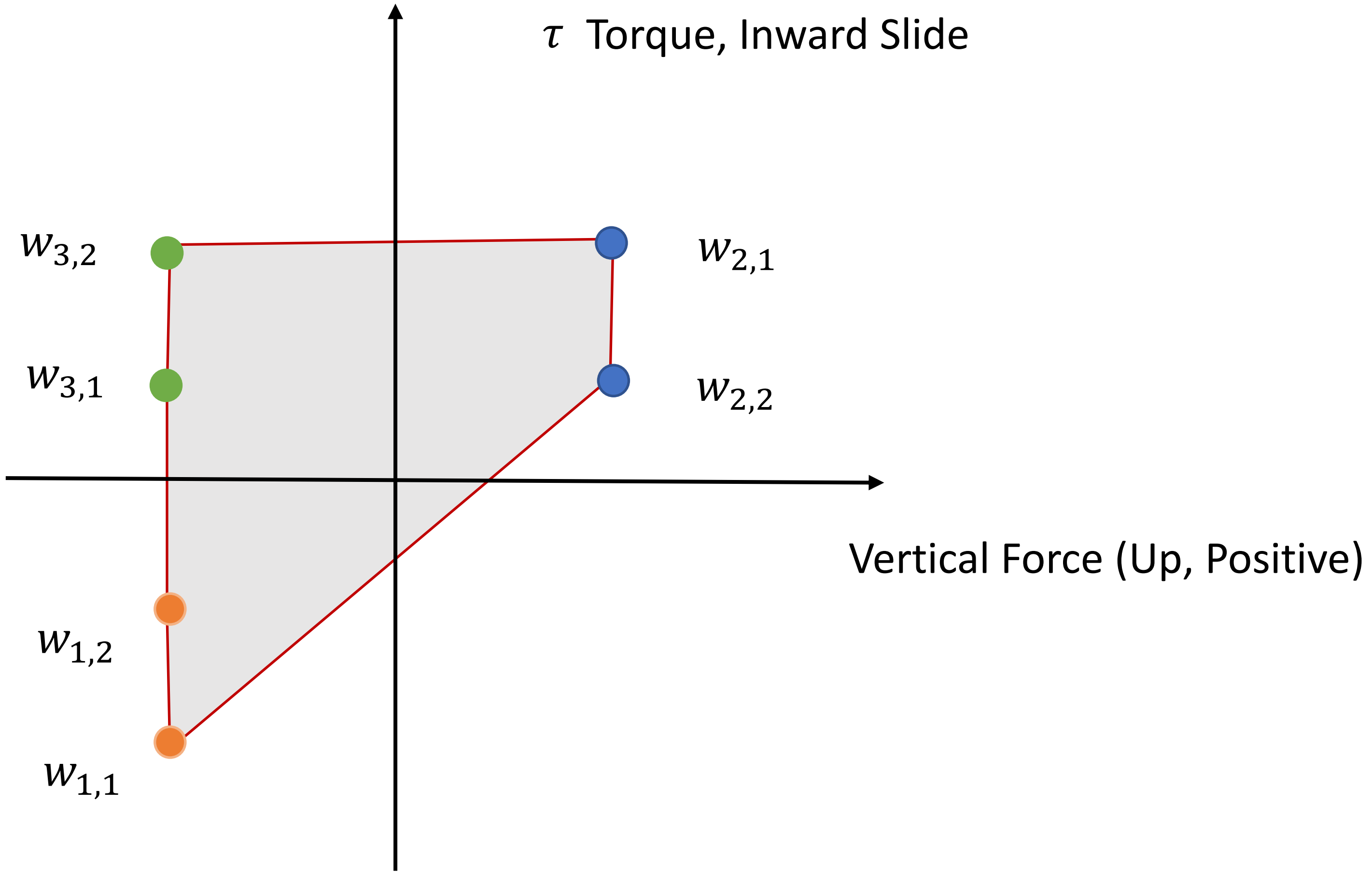




Grasp Wrench Hull





Modeling of a Grasp

- Wrench Grasp Hull
- Wrench Grasp Space
- Wrench
- Friction Cone
- Point Contact

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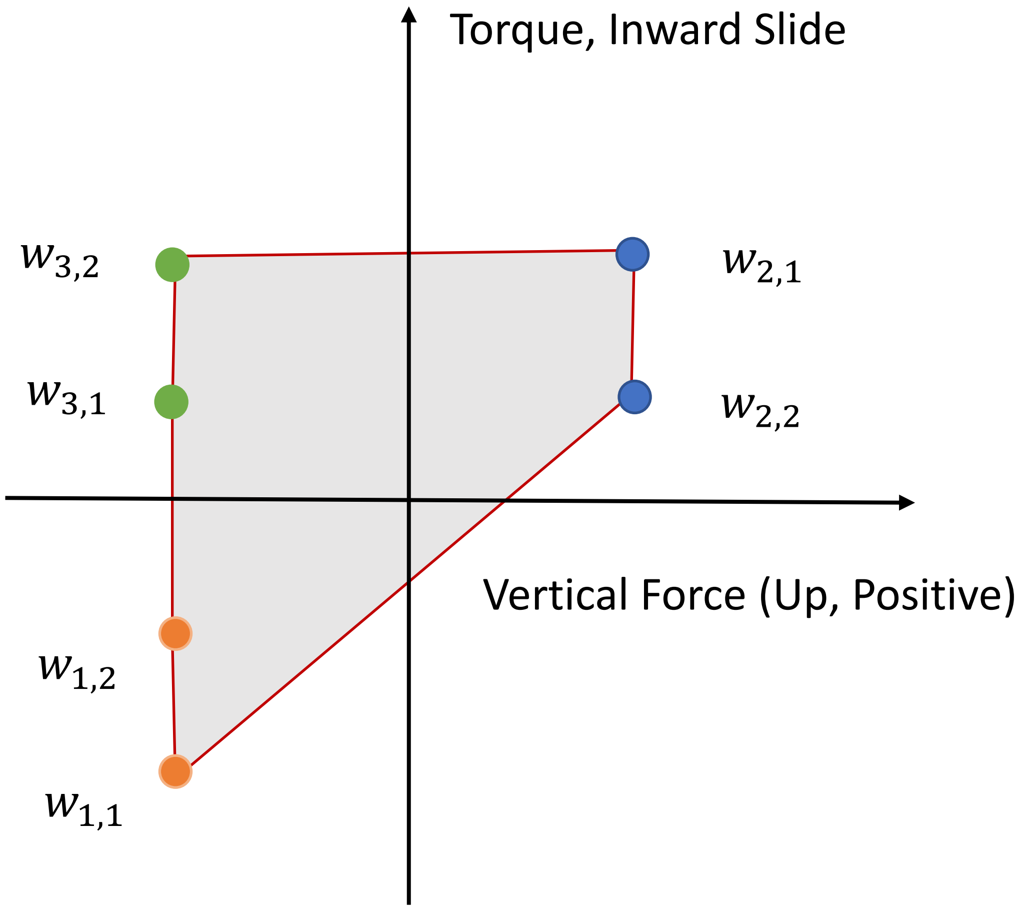
How to evaluate a potential grasp?

- What are good characteristics?
  - Grasp Maintenance:
    - Contact forces applied by the hand are such that they prevent contact separation and unwanted contact sliding
  - Closure:
    - Grasps that can be maintained for every possible disturbance

Force Closure

- A grasp is a force-closure grasp if for any external wrench w^{ext} there exist contact forces $f_c \in F_C$ such that
  $$G f_c = -w^{ext}$$
- i.e., if able to apply sufficient force at each contact,

every external wrench can be compensated for.



Grasp Analysis-Force Closure

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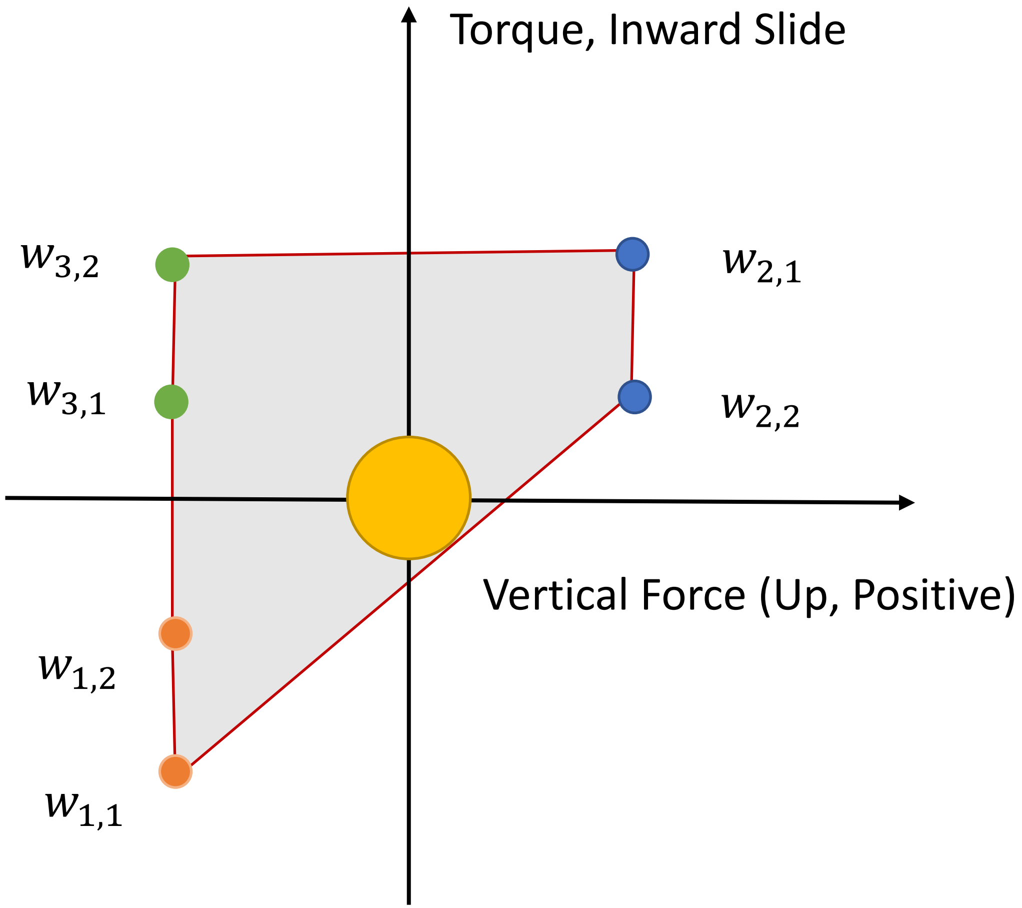
  $$G f_c = -w^{ext}$$
- i.e., if able to apply sufficient force at each contact, every external wrench can be compensated for.

A grasp is in force closure if the origin of the wrench space is contained in the interior of the wrench hull

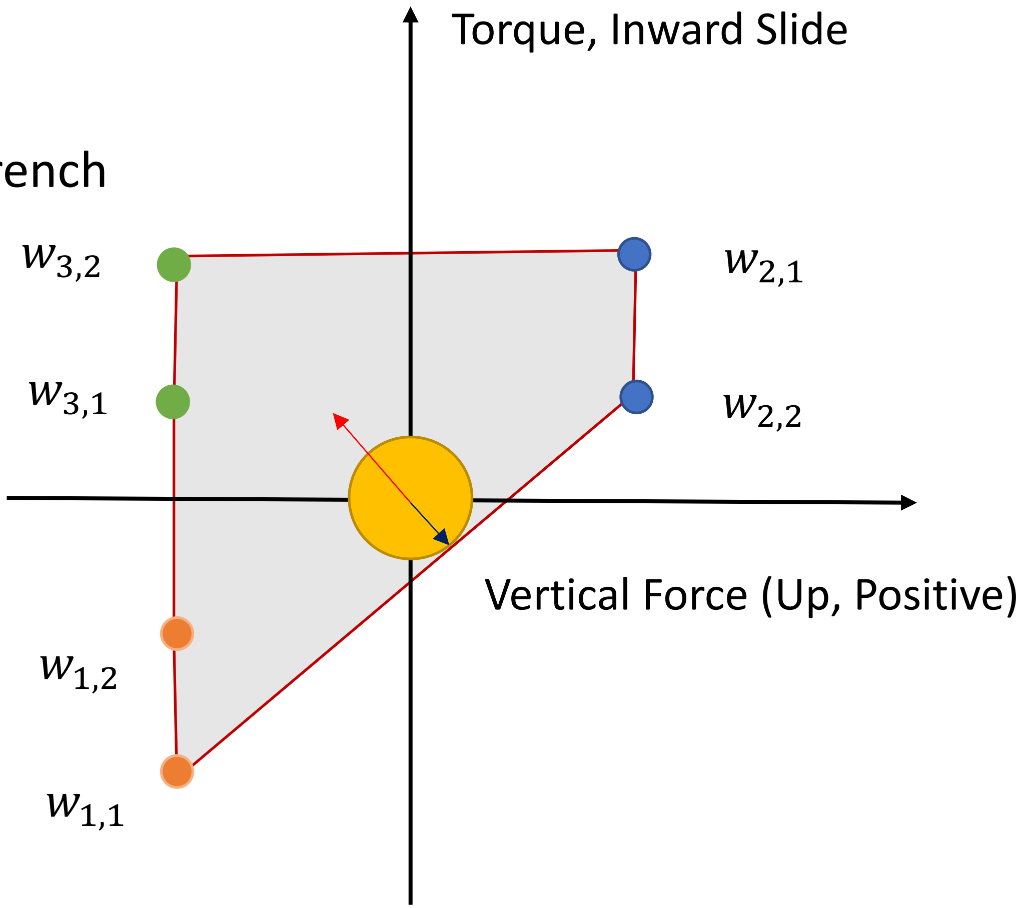


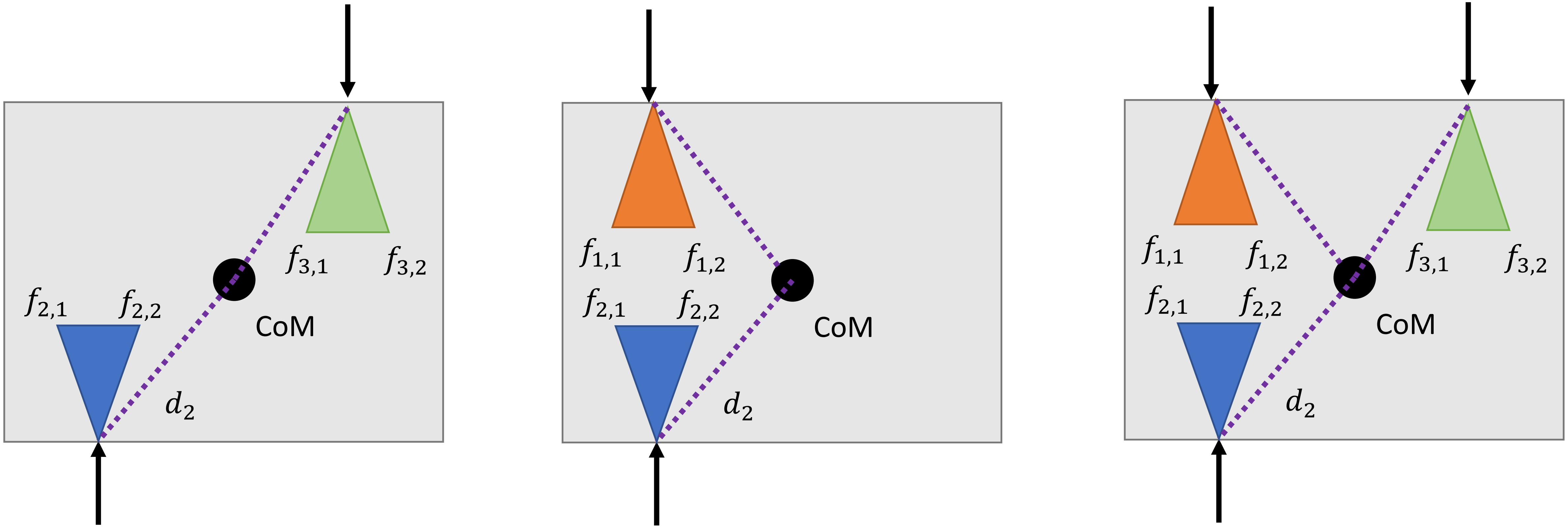
Grasp Analysis-Force Closure

- If Force closure, there is the largest ball centered at the origin that is completely contained in the grasp wrench hull

- The radius represents the magnitude of the *smallest* external wrench that pushes the grasp to the limits.

- The direction from the origin to where the ball touches the the boundary of wrench hull identifies the (opposite) direction in which the grasp is least able to resist external wrenches.

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CoM
d_2
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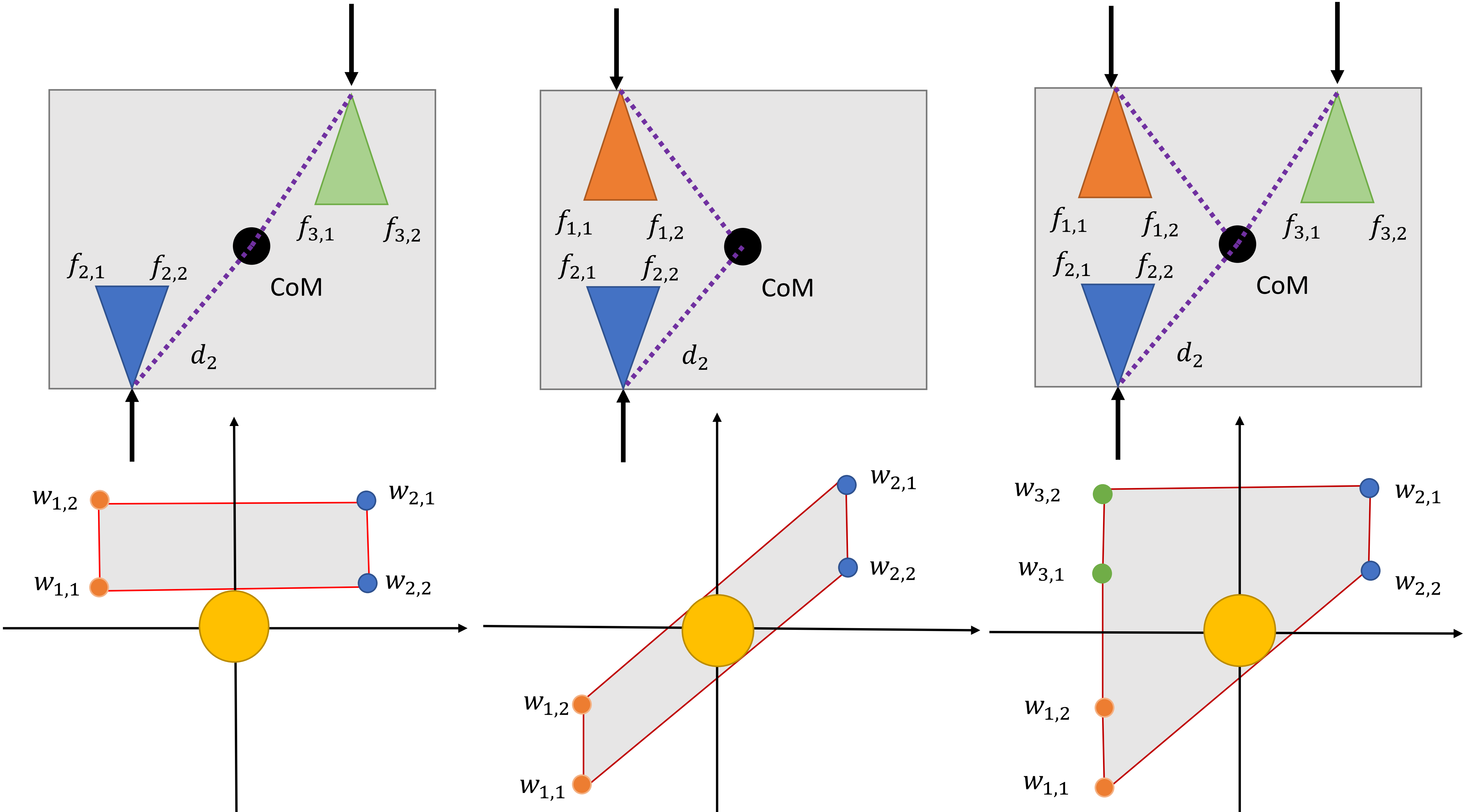

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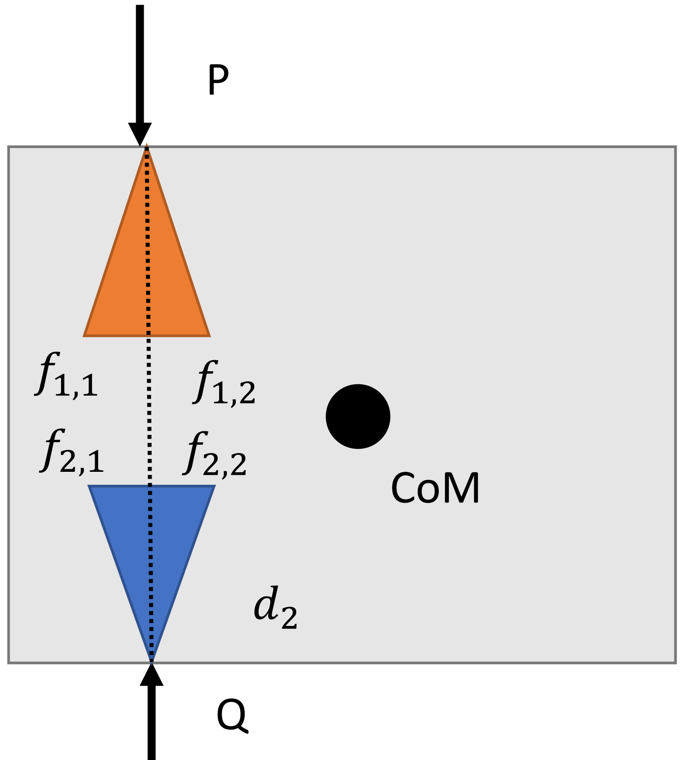
$w_{1,1}$

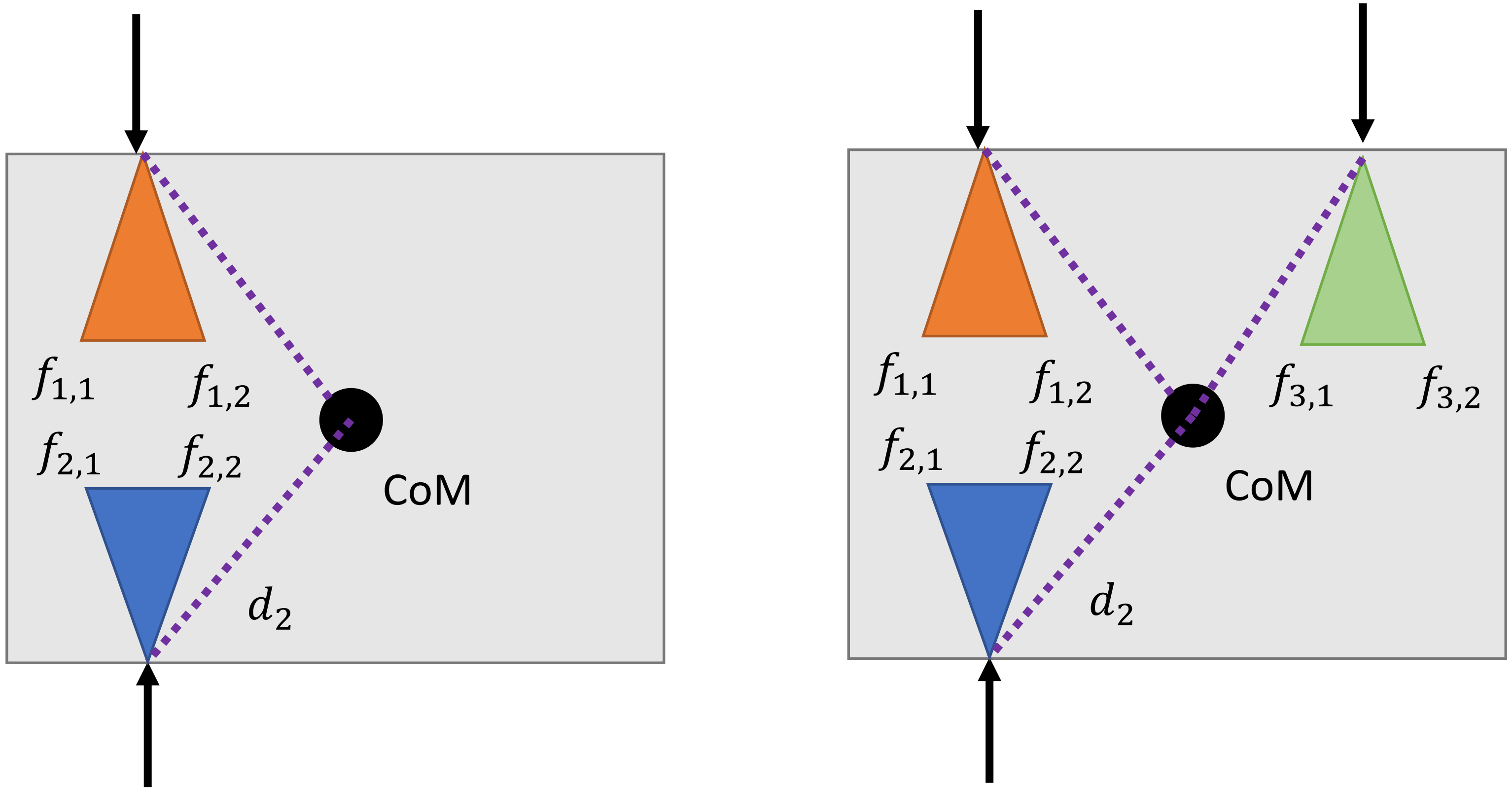
Force Closure for the Antipodal Grasp

- Two point contacts with friction at P and Q form a force closure grasp if and only if the segment PQ, or QP, points strictly into and out of the two friction cones respectively at P and Q.

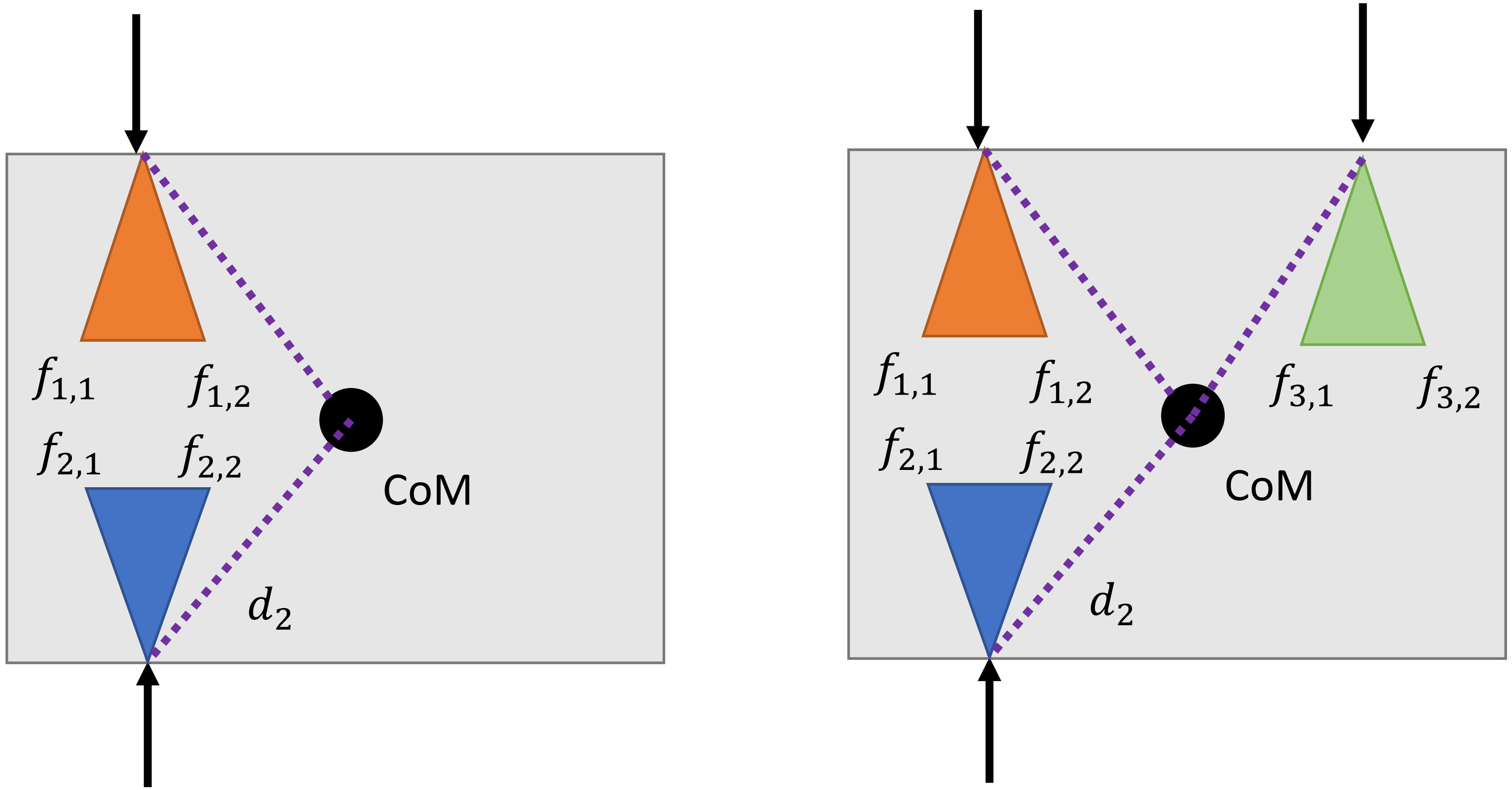
Constructing Stable Force-Closure Grasps. Nguyen. IJRR,1988



Both grasps are stable



Are these grasps equally good?

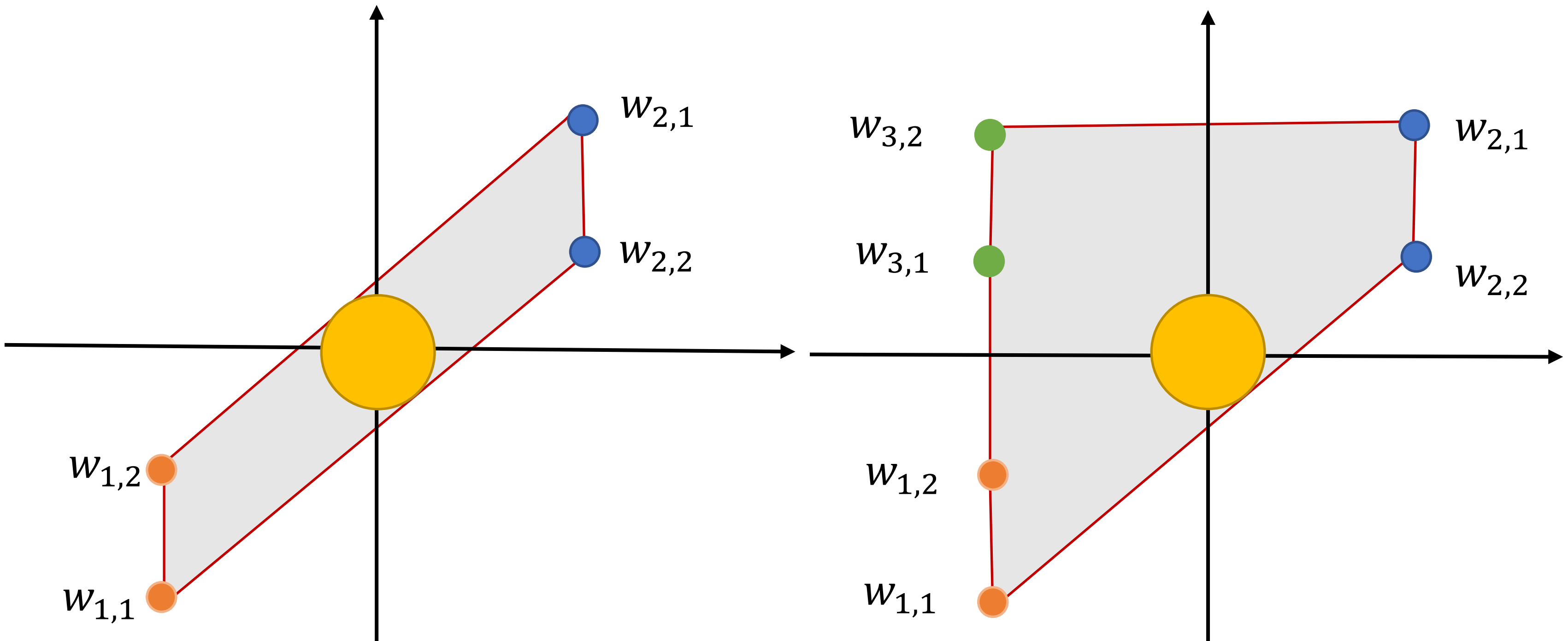


Grasp Quality

- Quality is how well a grasp can resist disturbances?
- Worst-case scenario
  - How efficiently can a grasp resist disturbance wrenches at its weakest point
- Weakest means the direction (in wrench space) at which the sum normal force is converted to the desired wrench least efficiently

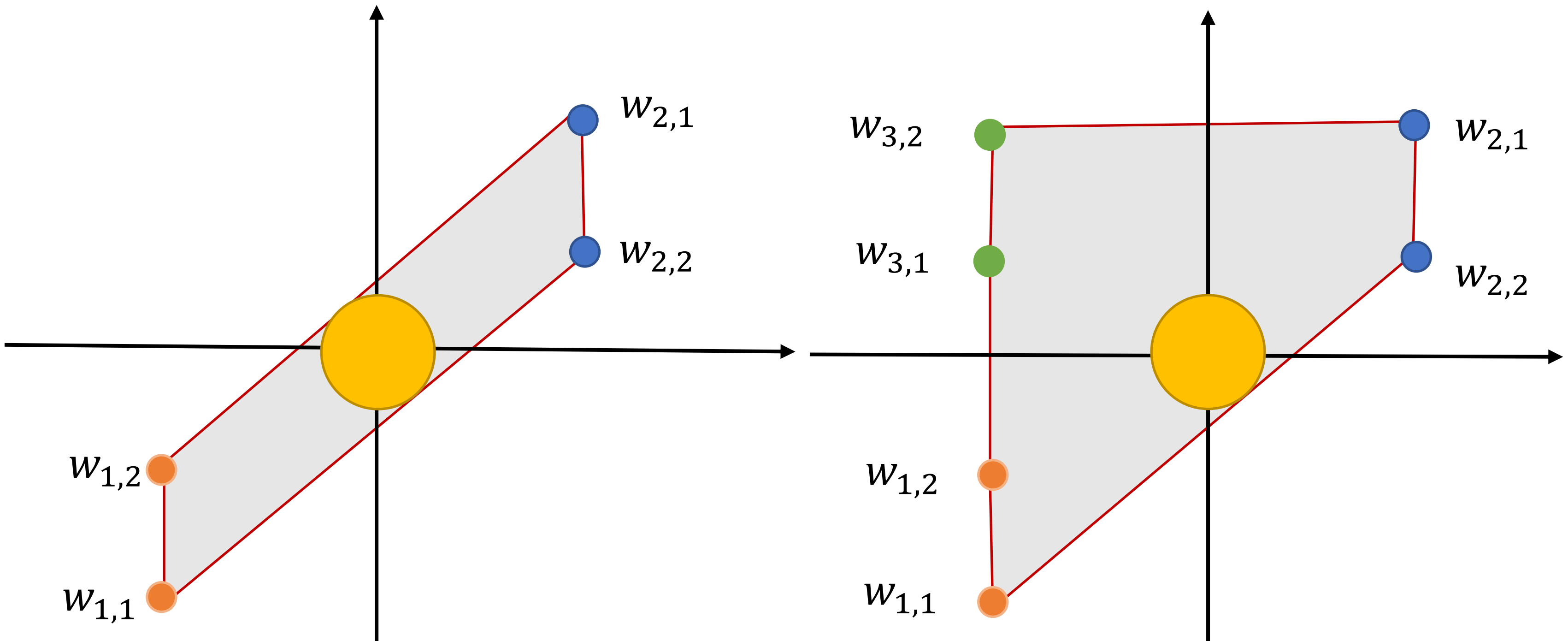
Worst Case Scenario

- The point on the wrench hull that is closest to the origin is the weakest point
- Disturbances in the opposite direction are hardest to resist
- Metric ε = The radius of the largest ball that can be enclosed in the wrench hull



Average Case Scenario

- How efficiently can a grasp resist a disturbance wrench on average?
- Metric ν = Volume of the convex hull in wrench space
- The three-point contact has more volume, so it is more stable on average



Force Closure

- Grasp can be maintained under any object wrench
- Forces can be applied at the contact points to withstand the external wrench
- Friction forces help balance the wrench
- Fewer contacts needed compared to Form Closure

Form Closure

- Joint angles locked
- Palm fixed in space
- Impossible to move the object
- No wiggle room
- Power grasps, enveloping grasps

Form Closure vs Force Closure

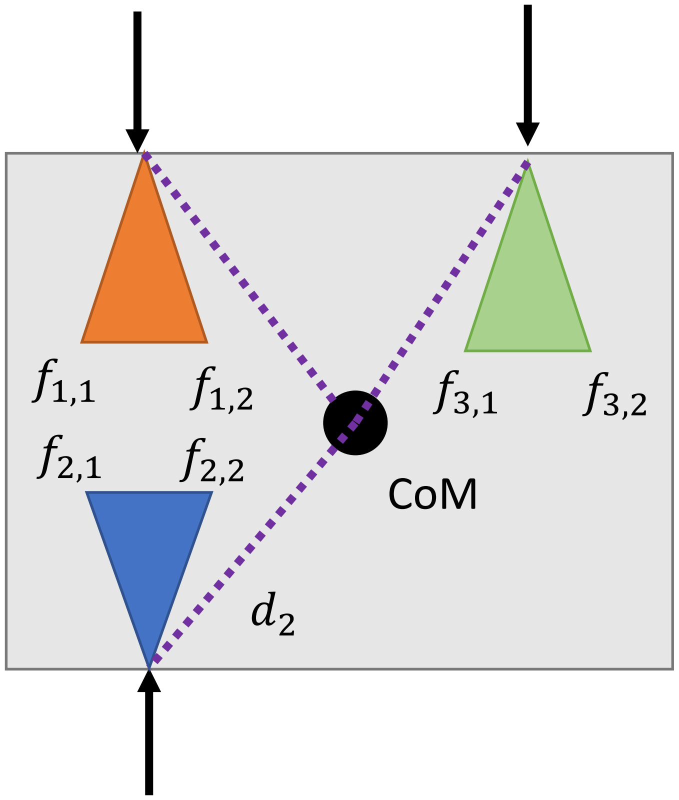
- Both are in contact configuration that resists all external disturbances
- Note: Every form closure grasp is also in force closure
- Why do I need less contact points to be in Force closure?

Today’s Plan

- What is a Grasp?
    - Why is grasping challenging?
- Analytical Approach to Modeling a Grasp
    - Modeling of a Grasp
    - Stability Analysis of a Grasp
        - Form Closure
        - Force Closure
- Generating Grasp Force
- Learning to Grasp

Grasp Force Optimization

- In force closure, you can theoretically resist any wrench
- But what forces do you need to apply at each contact to generate the desired wrench?



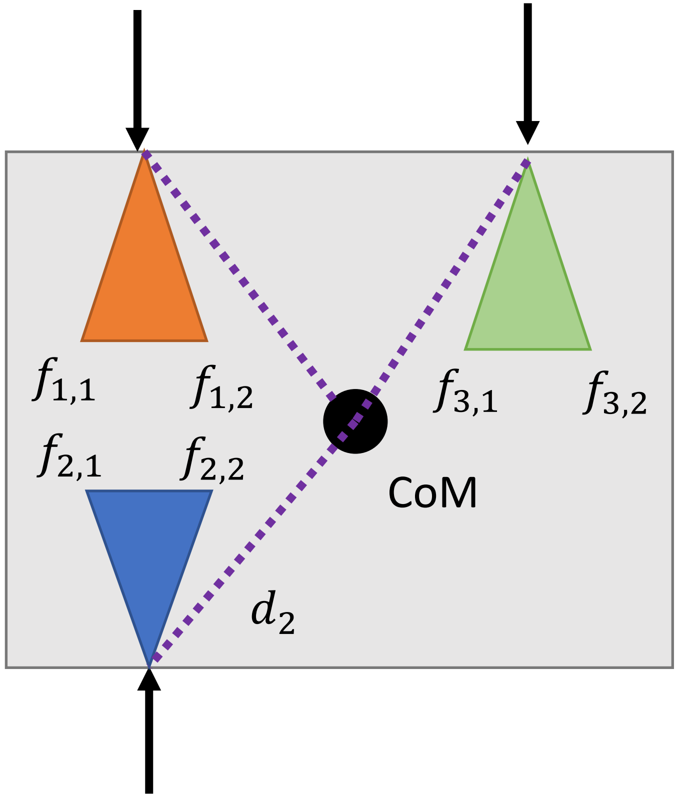
Grasp Force

- M contact points c^i, $\mathrm{i} = 1, \dots, \mathrm{M}$
- f^i is the contact force applied at contact point c^i
- Local coordinate system where x, y are tangent to surface, and z is aligned with surface normal pointing inward
- $f^i = (f_x^i, f_y^i, f_z^i)$
- Friction cone

$$\sqrt{[f_x^i]^2 + [f_y^i]^2} \le \mu_i f_z^i$$

- Friction Cone Constraint $K_i = \left\{ x \in \mathcal{R}^3 \middle| \sqrt{[x_1\ ]^2 + [x_2\ ]^2} \le \mu_i x_3 \right\}$
- Compact notation

$$f^i \in K_i, i = 1, \dots, M$$

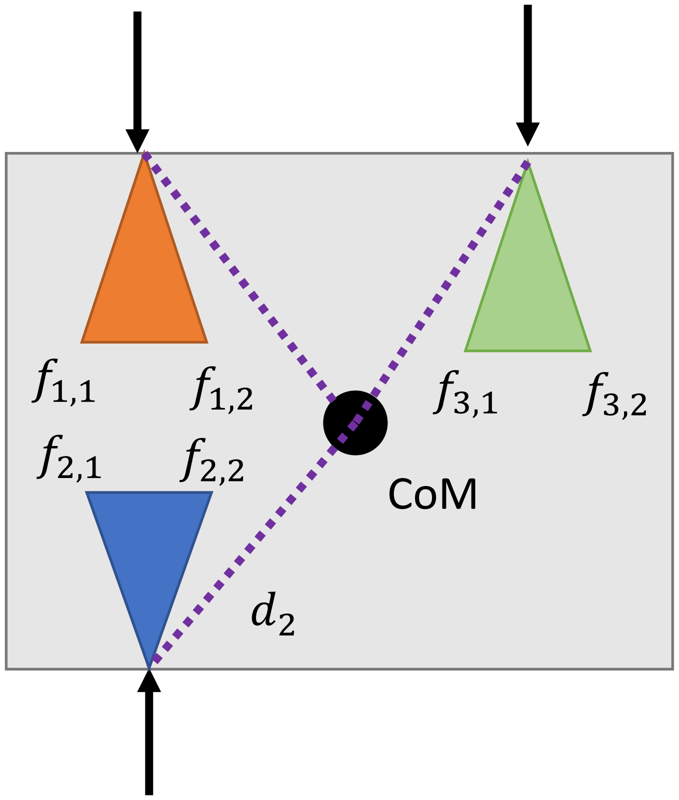


Equilibrium Constraints – Force

- $Q \in SO(3)$ transforms forces from local to global coordinate system
- $Q^i f^i$: force applied to object
- Applied forces need to generate a force

that compensates external torque $f^{ext} \in \mathcal{R}^3$

$$\sum_{i-1}^{M} Q^i f^i + f^{ext} = 0$$

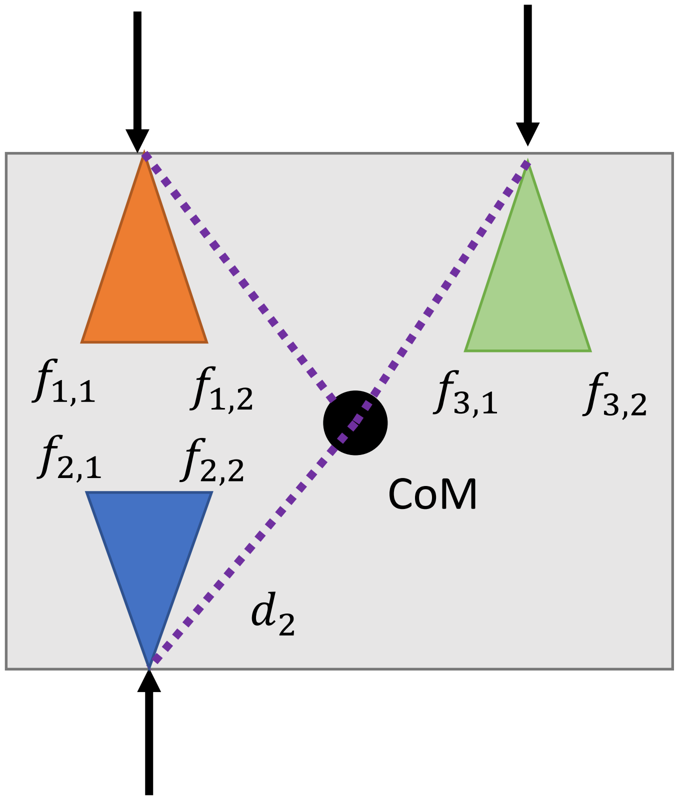


Equilibrium Constraints – Torque

- $Q \in SO(3)$ transforms forces from local to global coordinate system
- $d^i \times Q^i f^i$: torque applied to object
- Applied forces need to generate a torque

that compensates external torque $\tau^{ext} \in \mathcal{R}^3$

$$\sum_{i-1}^{M} d^i \times Q^i f^i + \tau^{ext} = 0$$



Matrix Notation of Cross Product

- $d^i \times Q^i f^i = S^i Q^i f^i$

where

$$S^i = \begin{bmatrix} 0 & -d_z^i & d_y^i \\ d_z^i & 0 & d_y^i \\ -d_y^i & d_x^i & 0 \end{bmatrix}$$

Equilibrium Constraints – Torque

- $Q \in SO(3)$ transforms forces from local to global coordinate system
- $d^i \times Q^i f^i$: torque applied to object
- Applied forces need to generate a torque

that compensates external force $\tau^{ext} \in \mathcal{R}^3$

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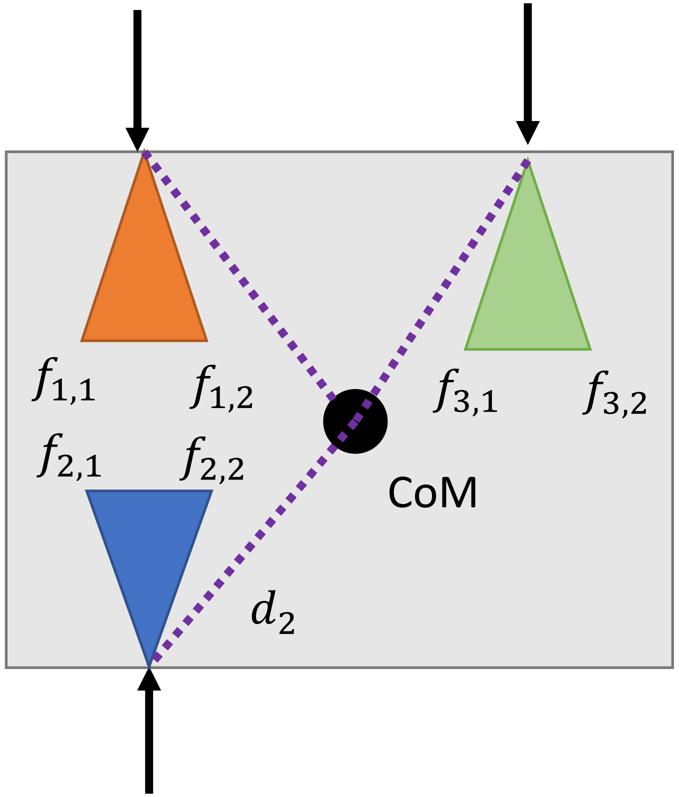


Equilibrium Constraints – Wrench

- Contact force vector

$$f = (f^1, \dots, f^M) \in \mathcal{R}^{3M}$$

- Contact Matrices

$$G^i = \begin{bmatrix} Q^i \\ S^i Q^i \end{bmatrix} \in \mathcal{R}^{6\times 3}$$

- Grasp matrix

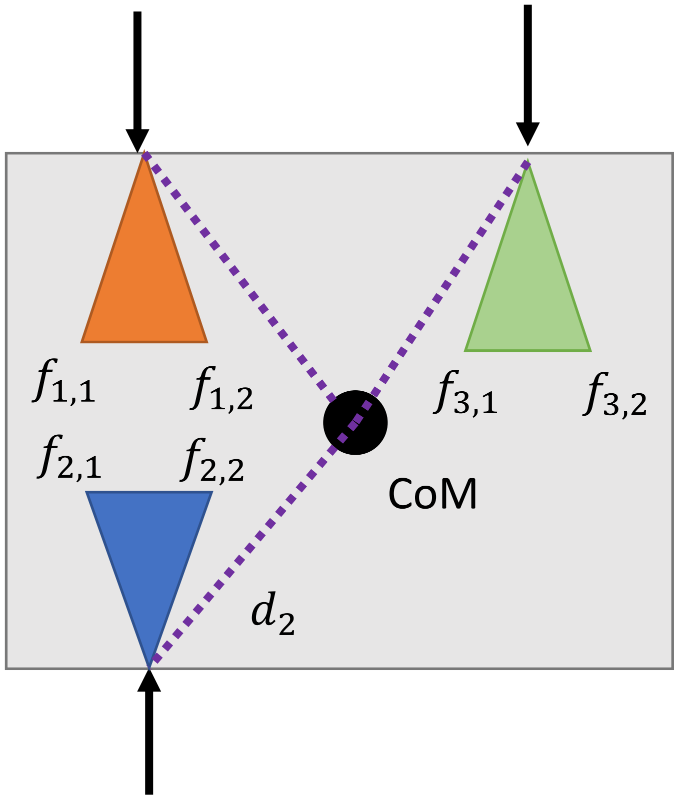
$$G = [G^i, \dots., G^i] \in \mathcal{R}^{6\times 3M}$$

- External Wrench

$$w^{ext} = \begin{bmatrix} f^{ext} \\ \tau^{ext} \end{bmatrix} \in \mathcal{R}^{6}$$

- Equilibrium conditions

$$Gf + w^{ext} = 0$$



Constraints -Hardware

- Hardware constraints (max torque, kinematic limits).

$$f \in C^{other}$$

Optimization Problem

- Objective function

$$F = max\{\|f^1\|, \dots, \|f^M\|\}$$

- Optimization problem

$$= \max_{i-1,..,M} \sqrt{[f_x^i]^2+[f_y^i]^2+[f_z^i]^2}$$

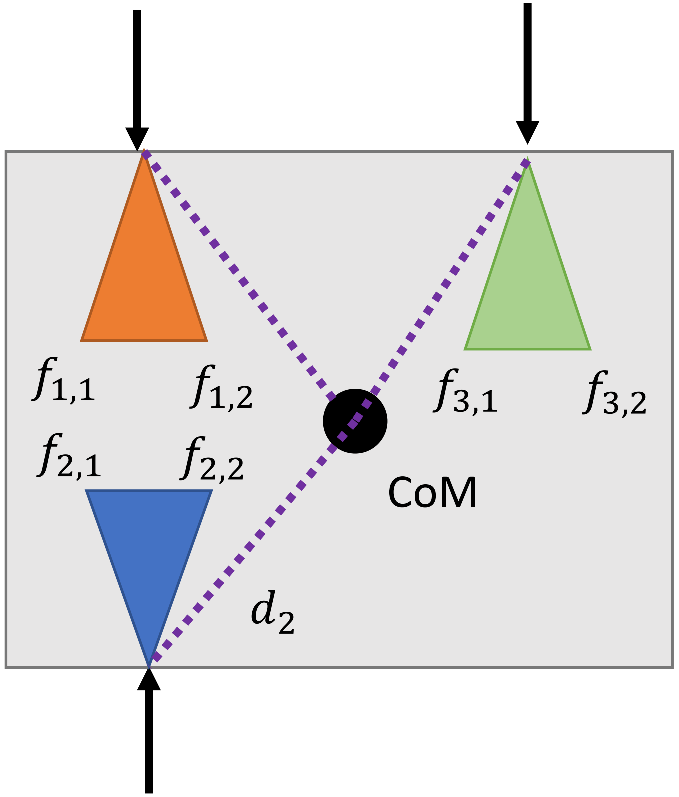
$$\min F$$

$$\text{st.}\, f^i \in K_i$$

$$Gf + w^{ext} = 0$$

$$f \in C^{other}$$

- Second-order cone program
 because friction cones are quadratic.



Fast Computation of Optimal Contact Forces.
Stephen P. Boyd and Ben Wegbreit. Transactions on Robotics. 2007.

Today's Plan

- What is a Grasp?
  - Why is grasping challenging?
- How to model a Grasp?
- How to evaluate a Grasp?
    - Form Closure
    - Force Closure
- How to generate a Grasp?
- Learning to Grasp

Learning to grasp?

- One-Stage Approach:

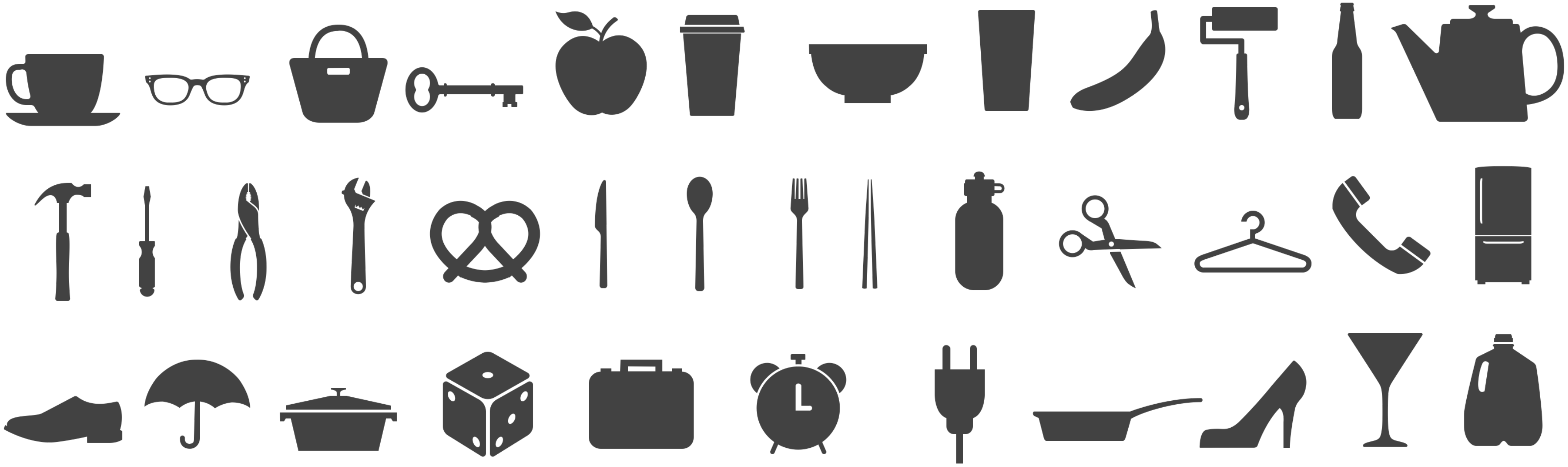
  Directly tell which regions to grasp.

- Two-Stage Approach:

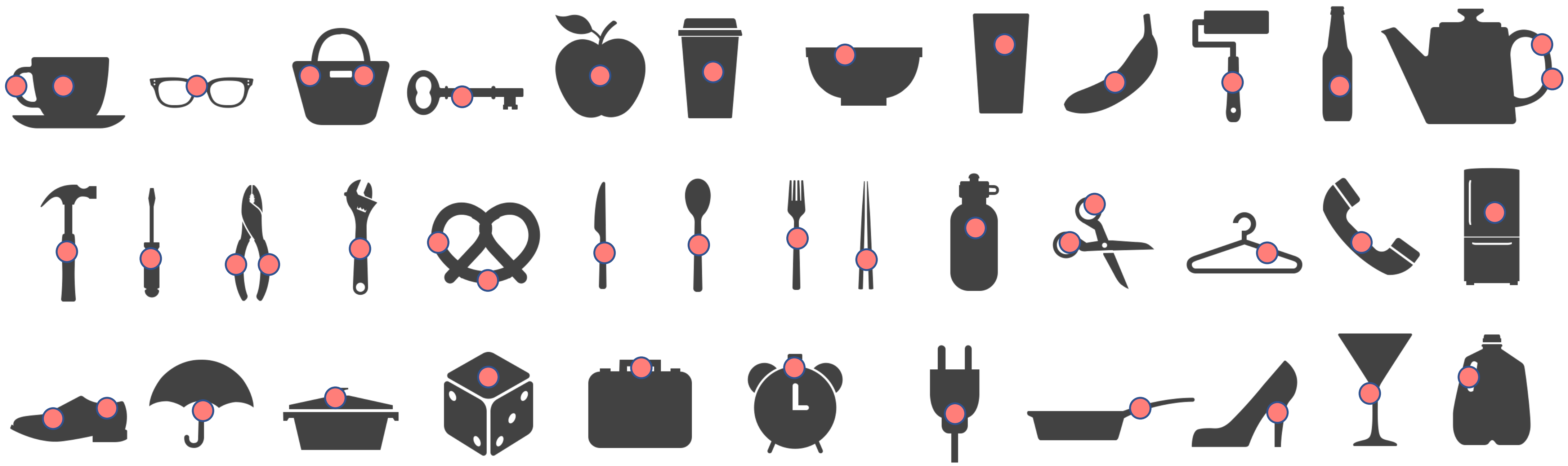
  Evaluating whether a region is a good grasp or not?

  Grasp Evaluation + Grasp Proposal

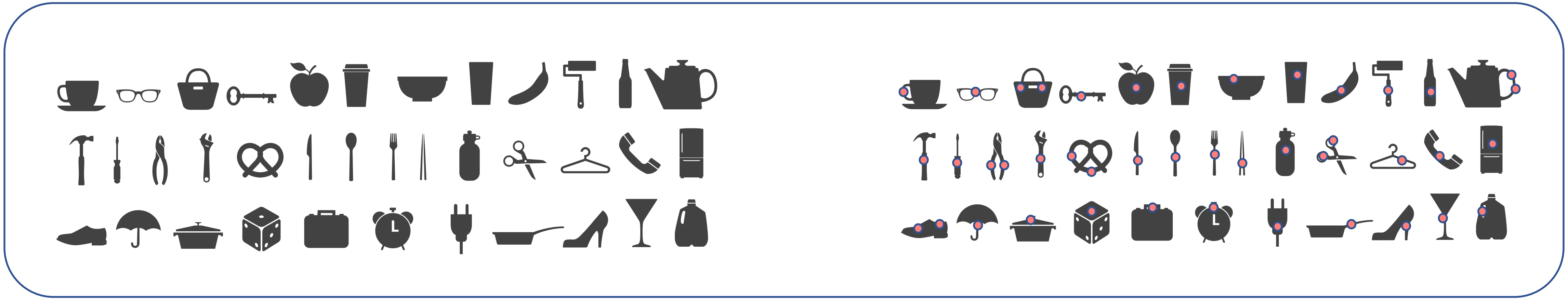
Grasp Detection as a Classification Problem

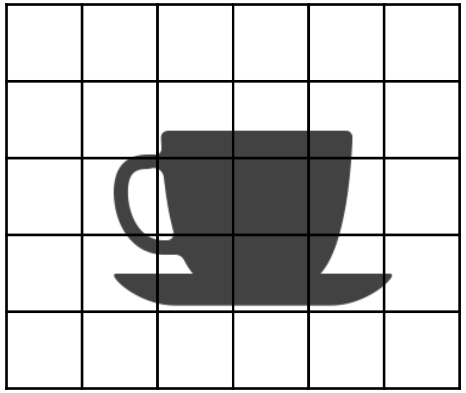


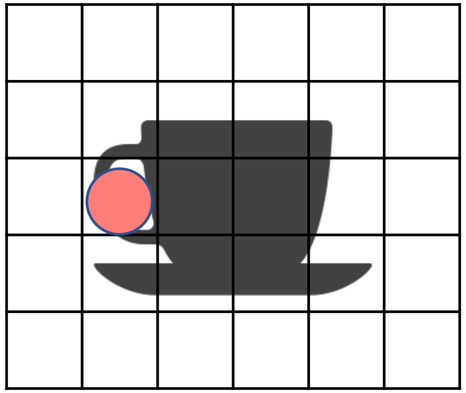
Grasp Detection as a Classification Problem



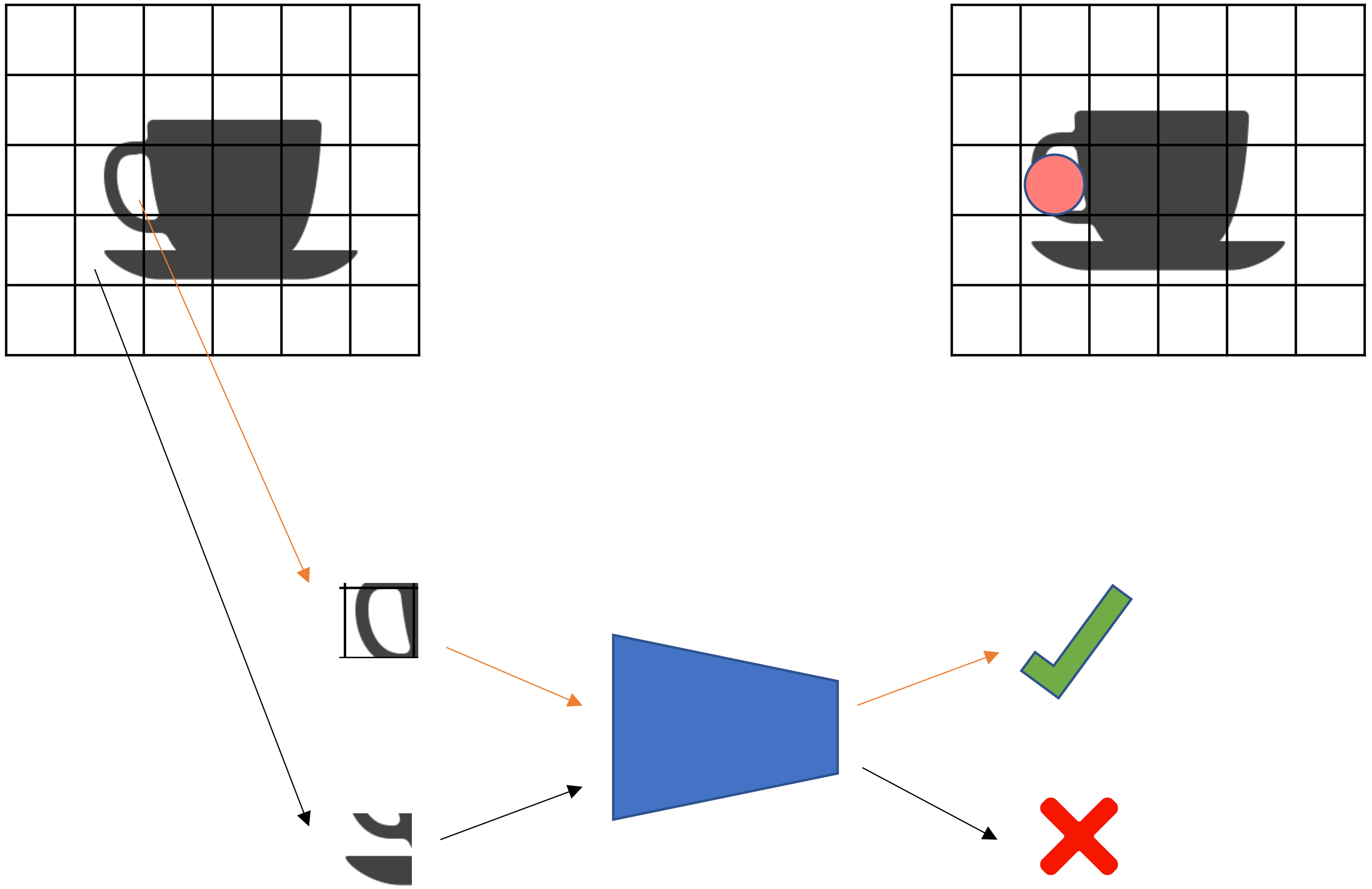
Grasp Detection as a Classification Problem



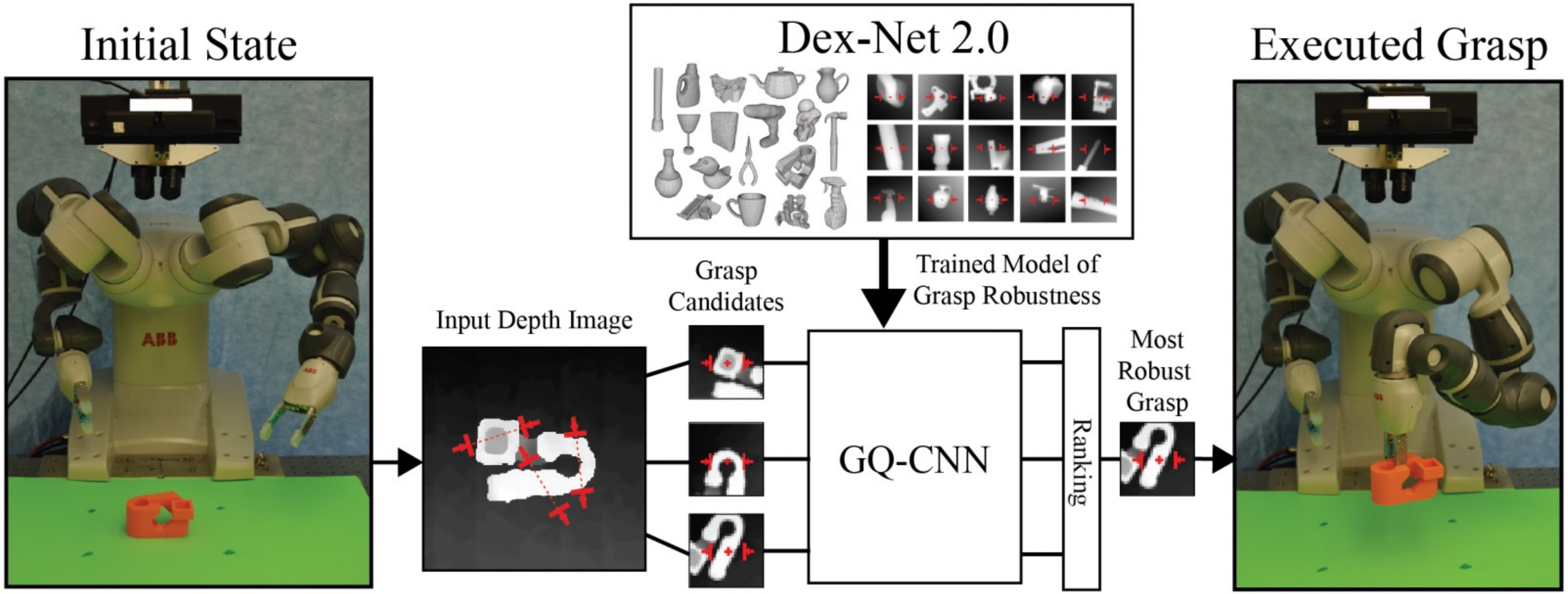




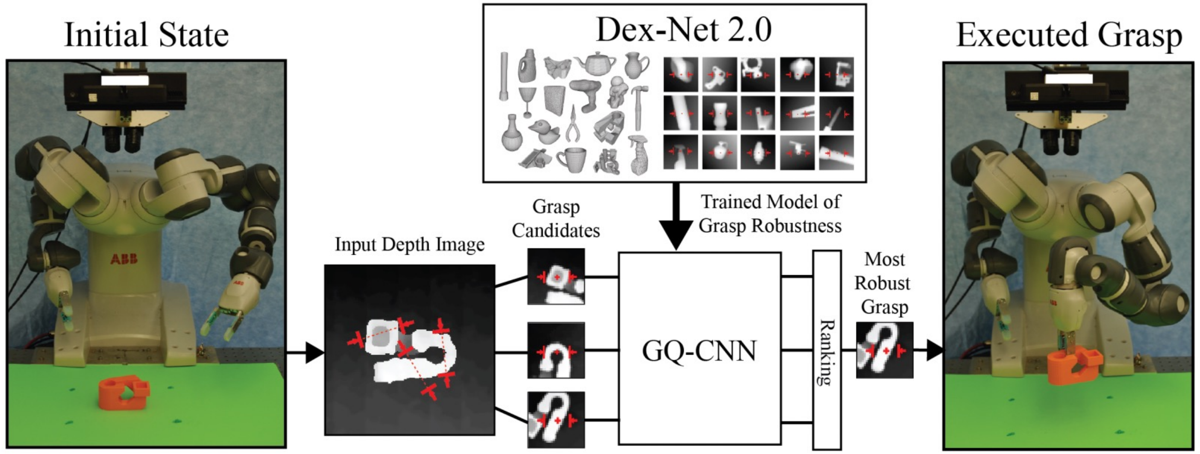
Grasp Detection as a Classification Problem



Dex-Net 2.0: Deep Learning to Plan Robust Grasps with Synthetic Point Clouds and Analytic Grasp Metrics



Dex-Net 2.0: Deep Learning to Plan Robust Grasps with Synthetic Point Clouds and Analytic Grasp Metrics

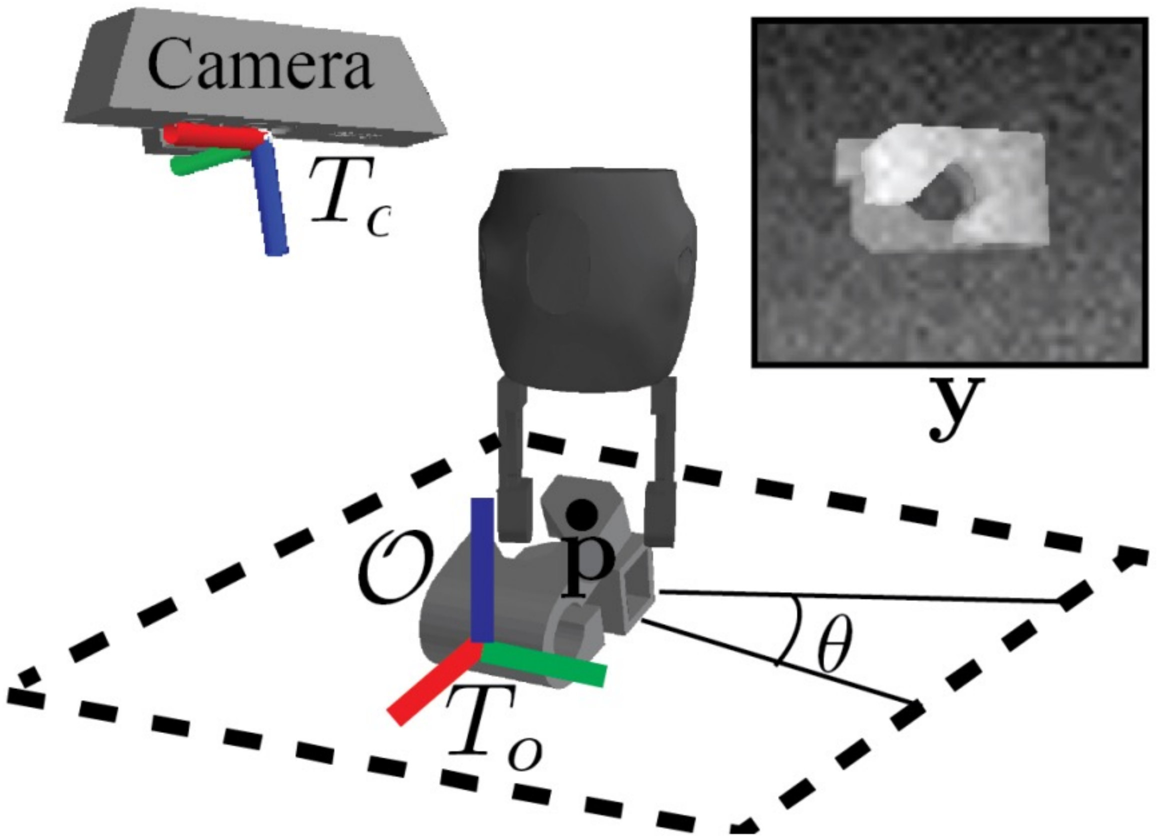


Two-Fingered Parallel-Jaw Gripper

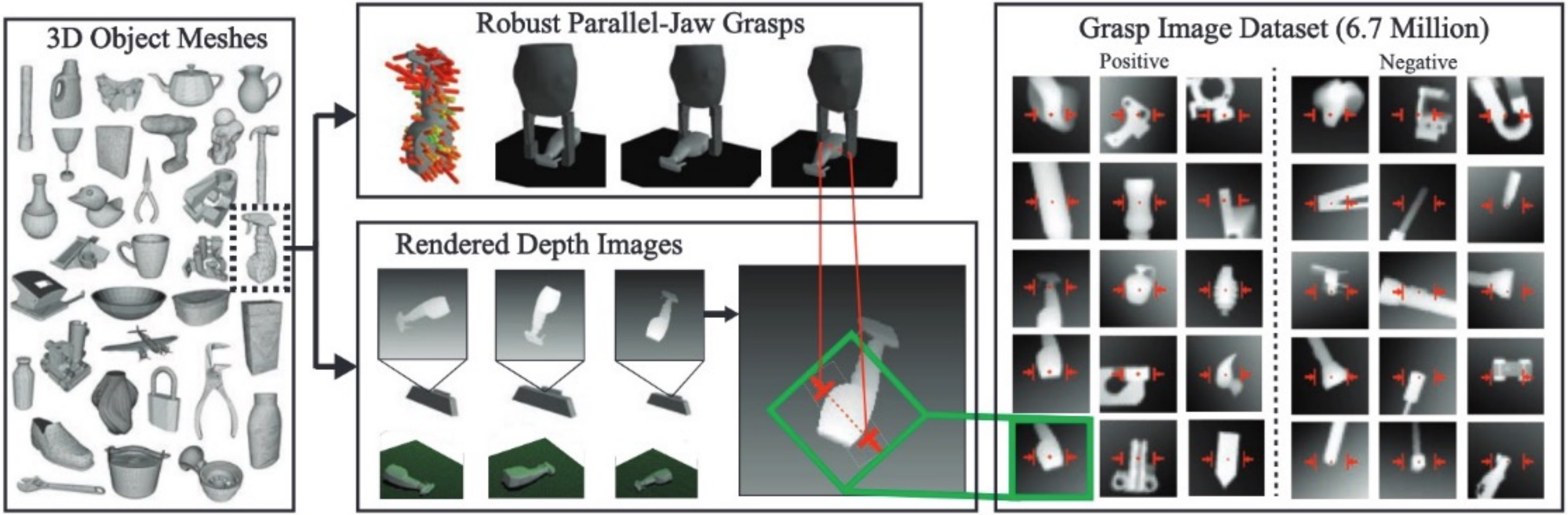
Grasp Parameterization

No need to consider the grasp force

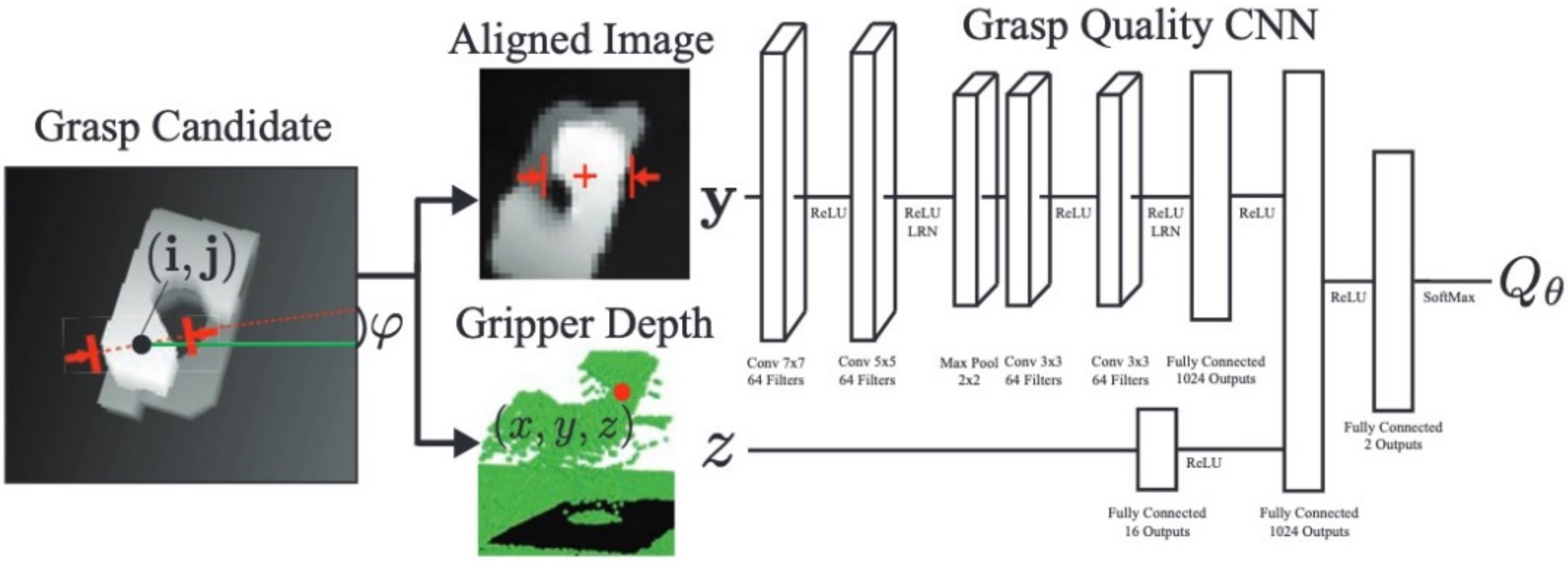
Grasp Annotation



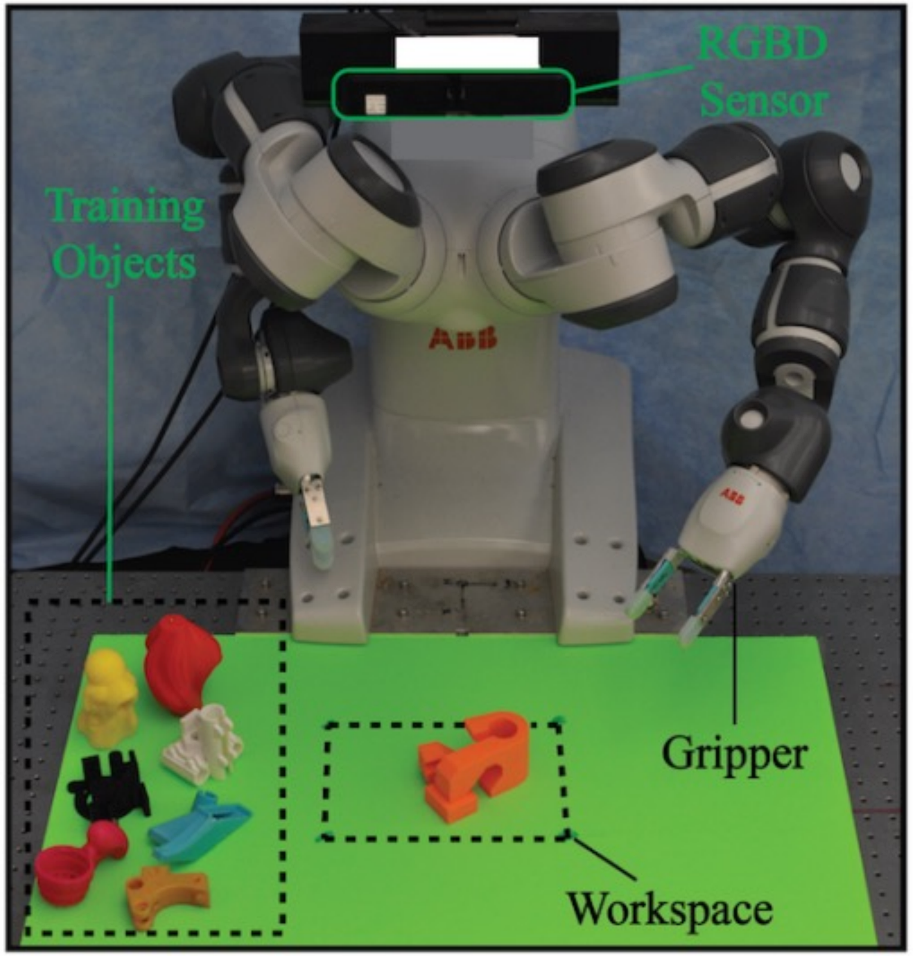
Dataset Generation



Grasp Classification Network

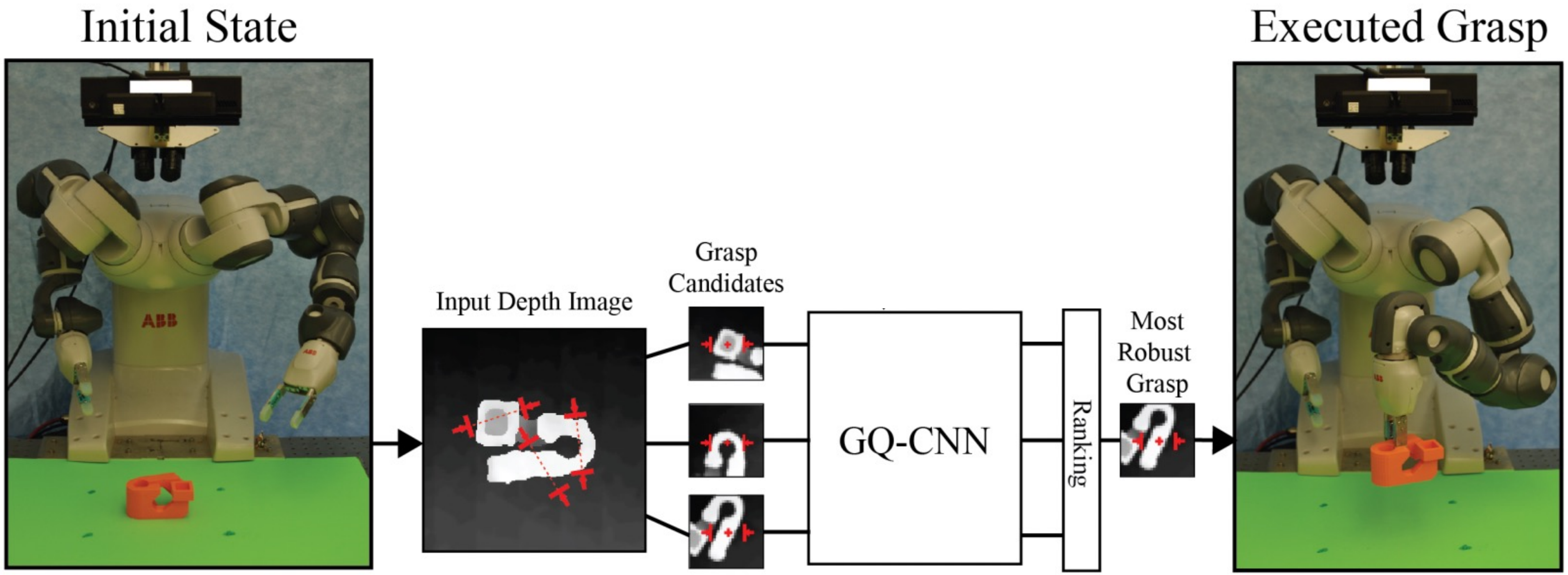


Experimental Setup

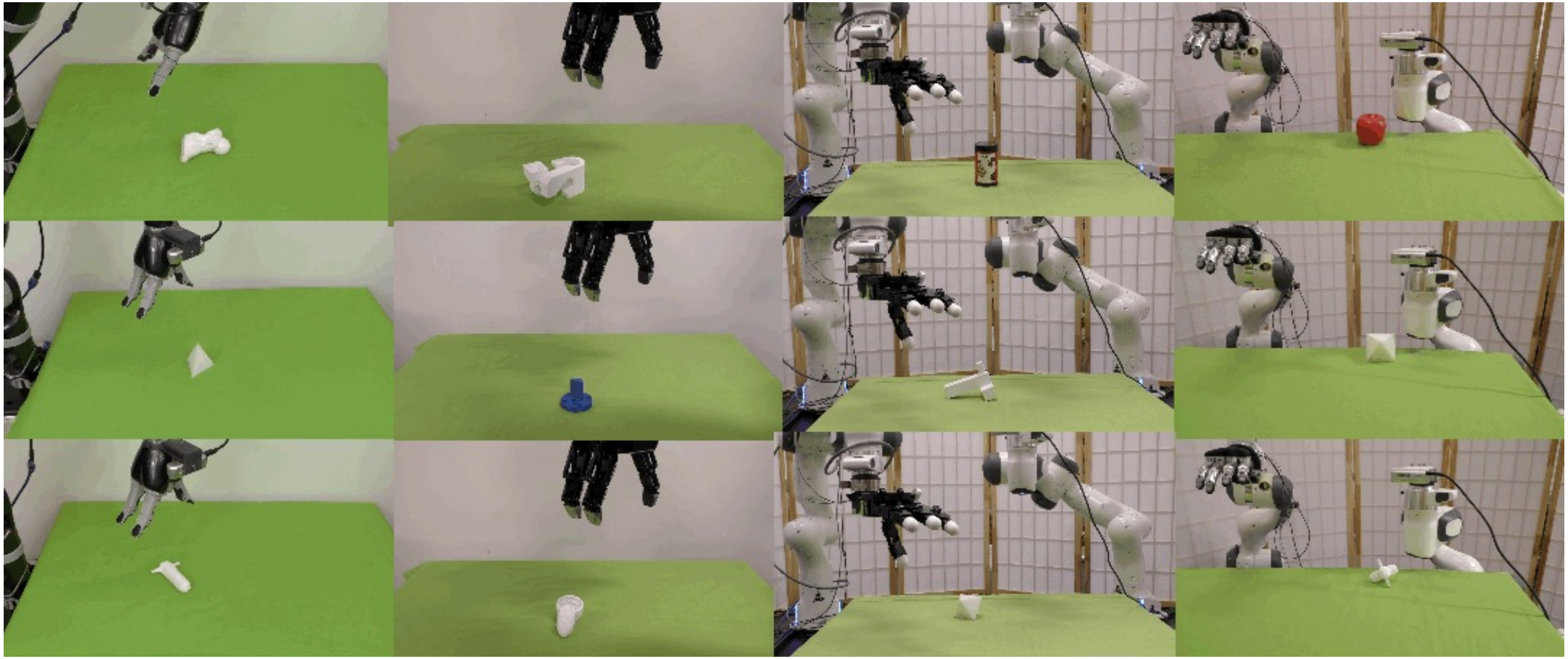


Training Objects (Adversarial)

Test Objects


Initial State
Input Depth Image
Grasp
Candidates
GQ-CNN
Ranking
Most
Robust
Grasp
Executed Grasp

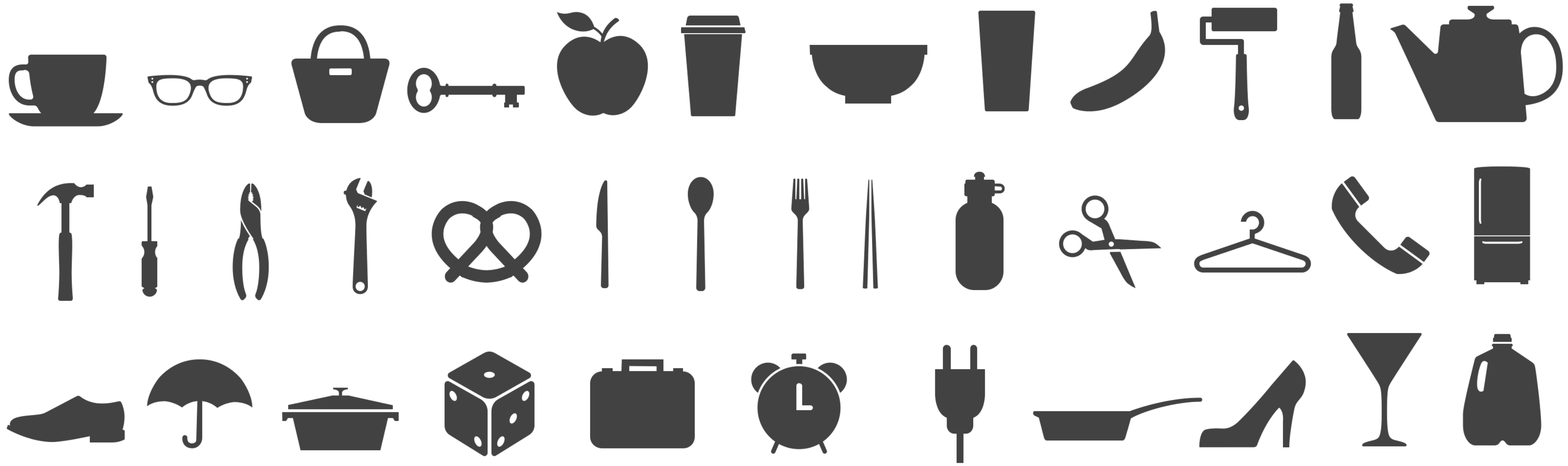
Videos



We propose UniGrasp for grasping any object with any gripper

UniGrasp outputs contact points

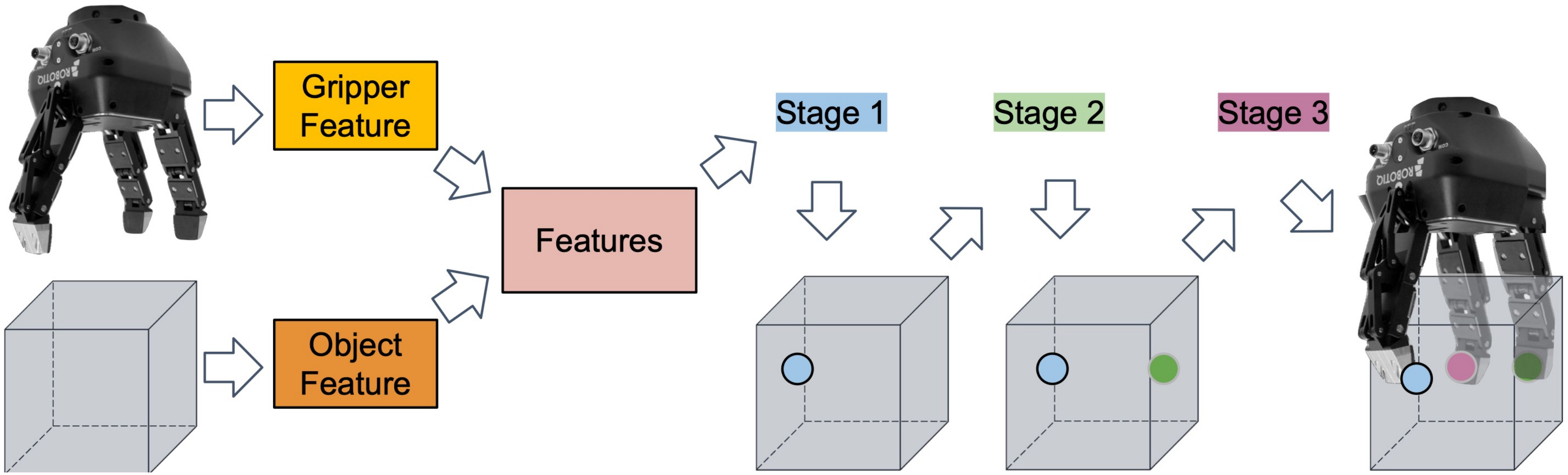
contact points



UniGrasp

Input: Object point cloud, Robot hand specification

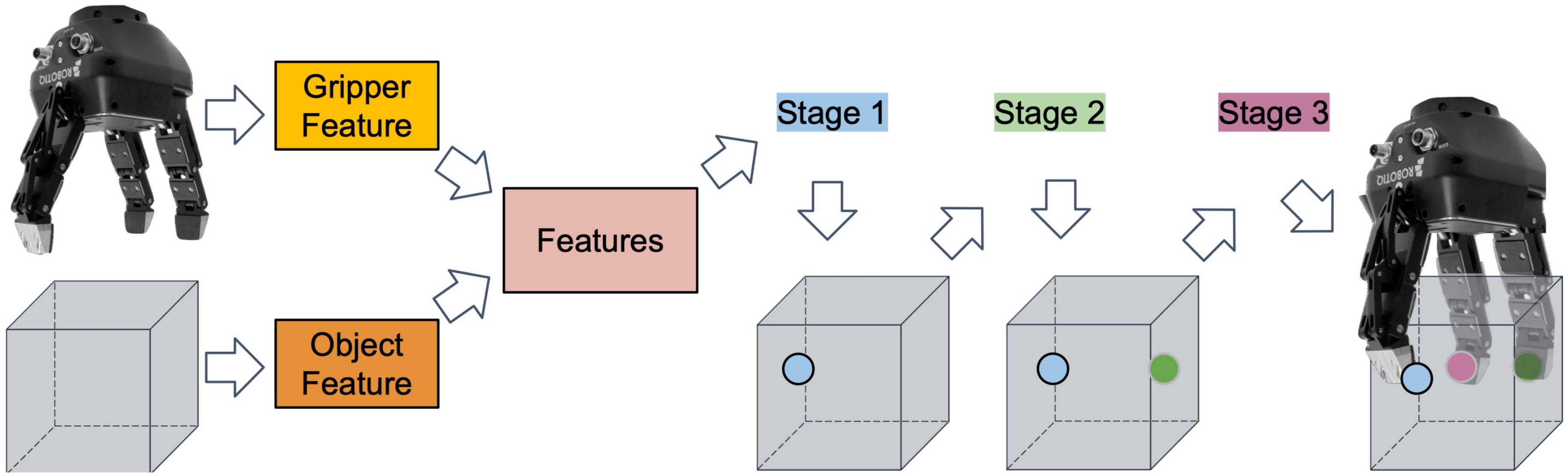
Approach:



Output: Contact points (reachable, force closure)

Input: Object point cloud, Robot hand specification

Approach:



Output: Contact points (reachable, force closure)

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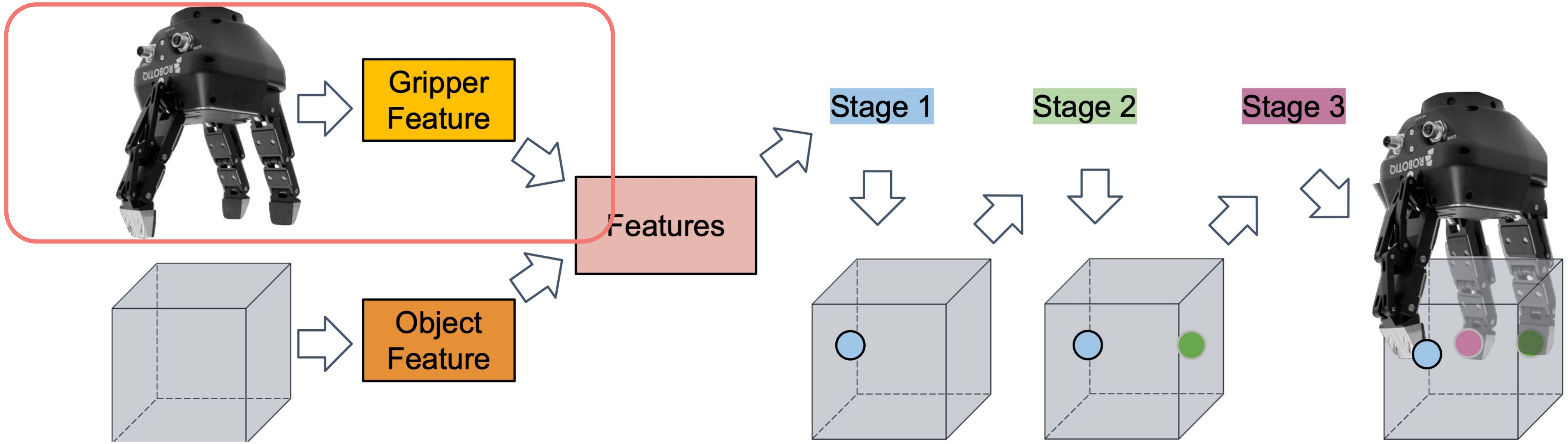
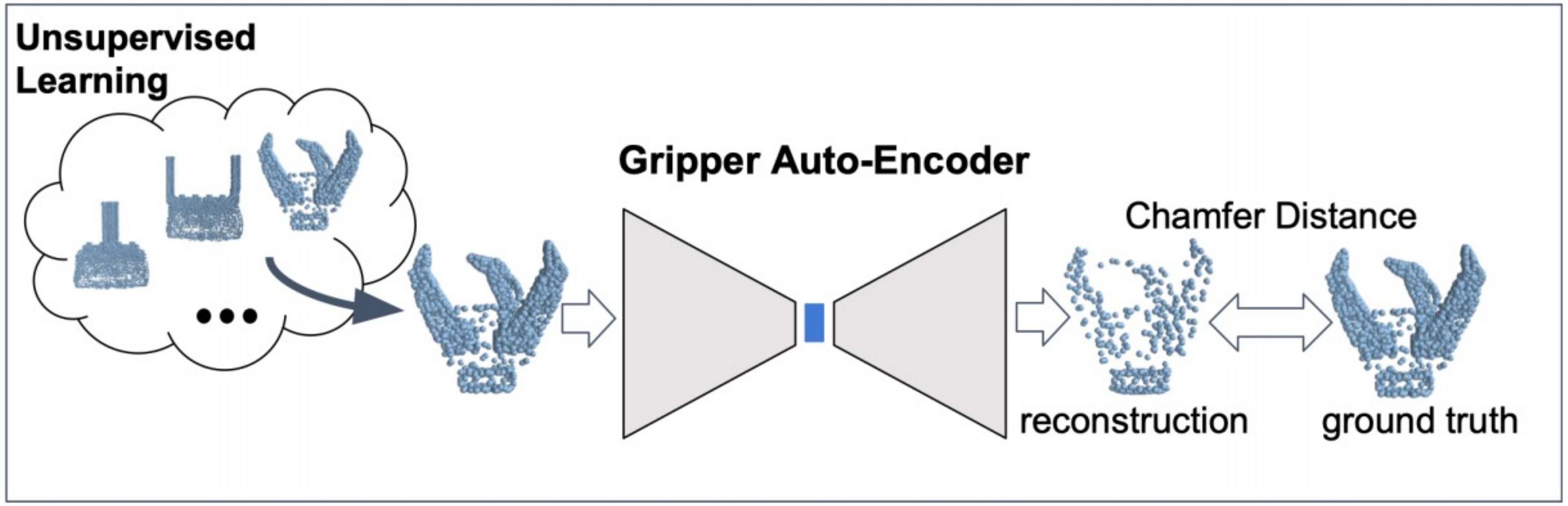
Approach:

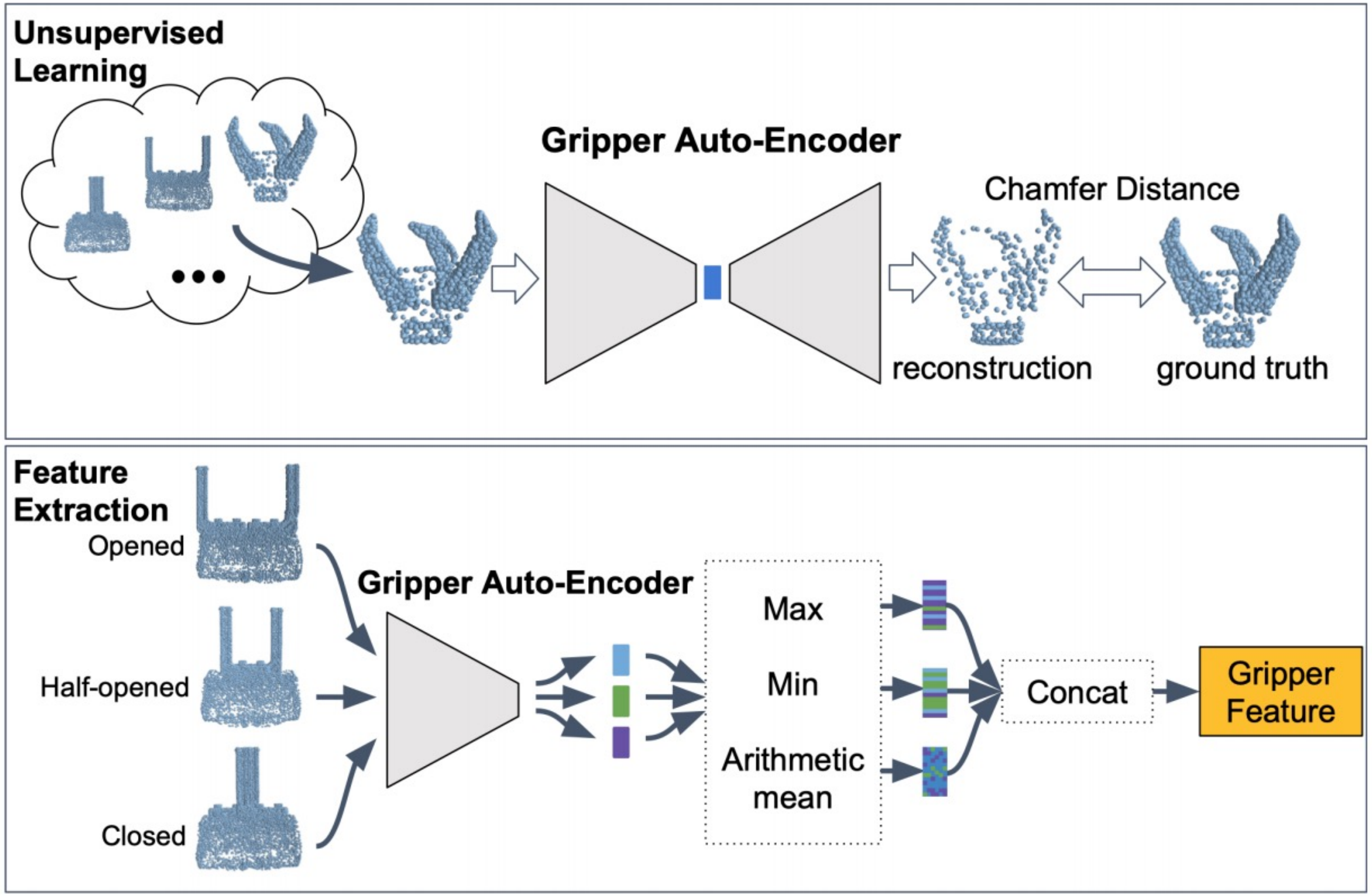


Output: Contact points (reachable, force closure)

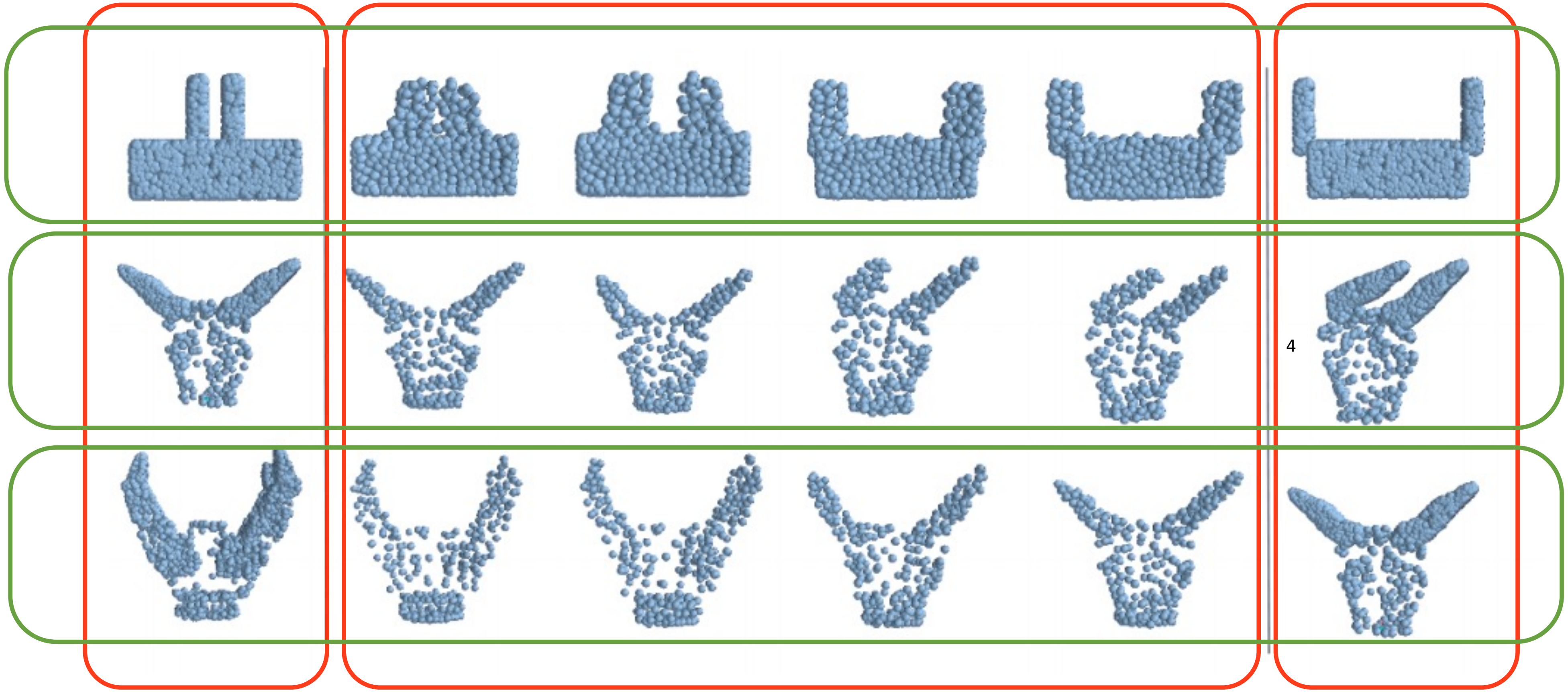
Robot Hand Representation



Robot Hand Representation



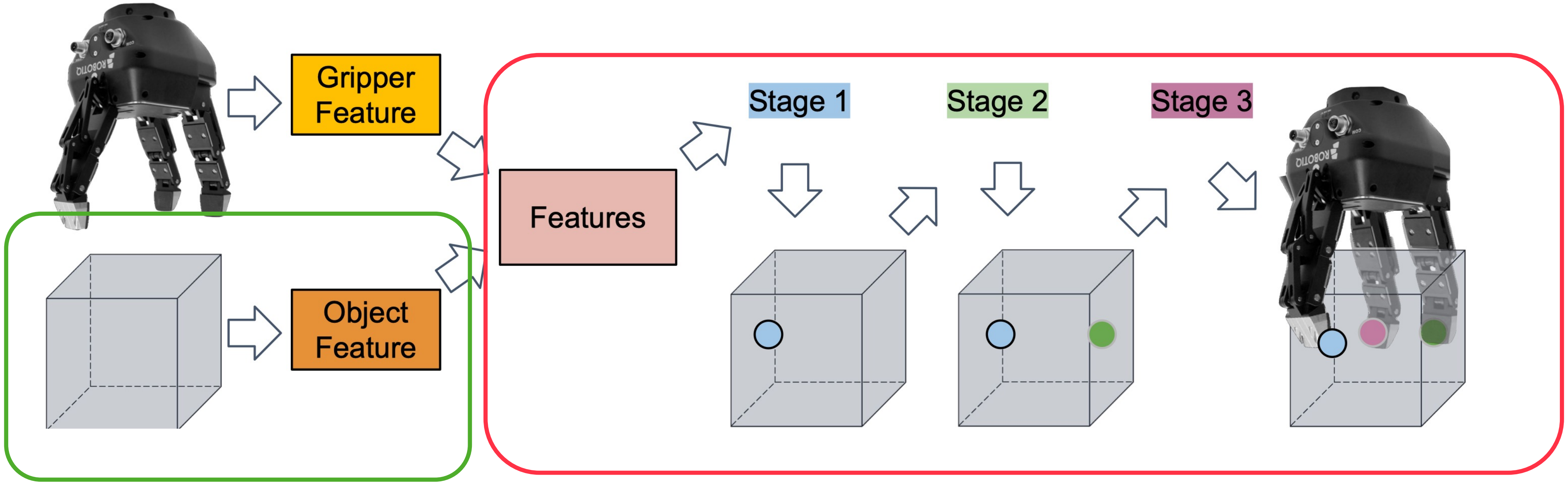
Interpolation in Hand Feature Space



Input Point Clouds Interpolation Input Point Clouds

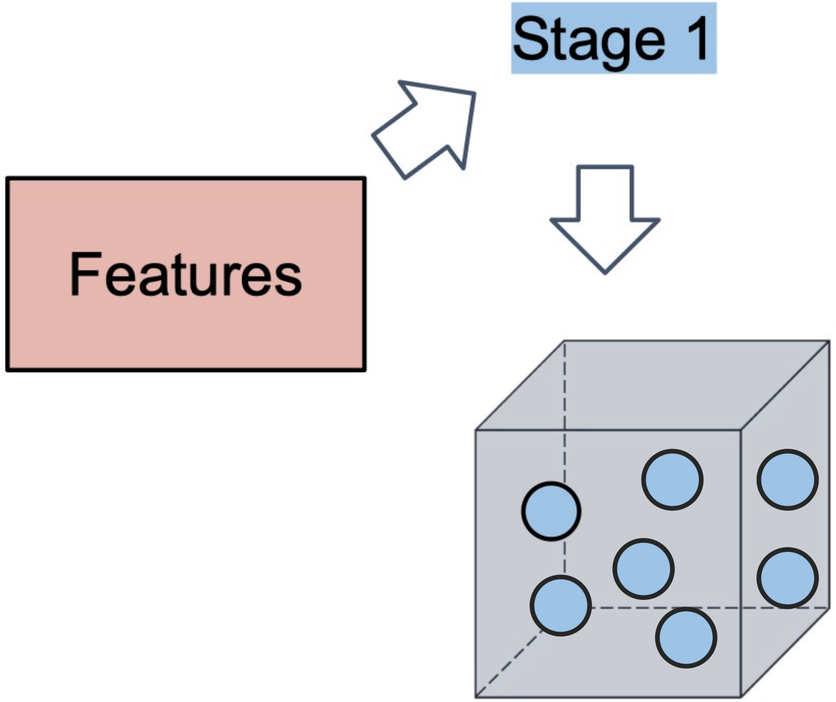
Input: Object point cloud, Gripper point cloud

Approach:

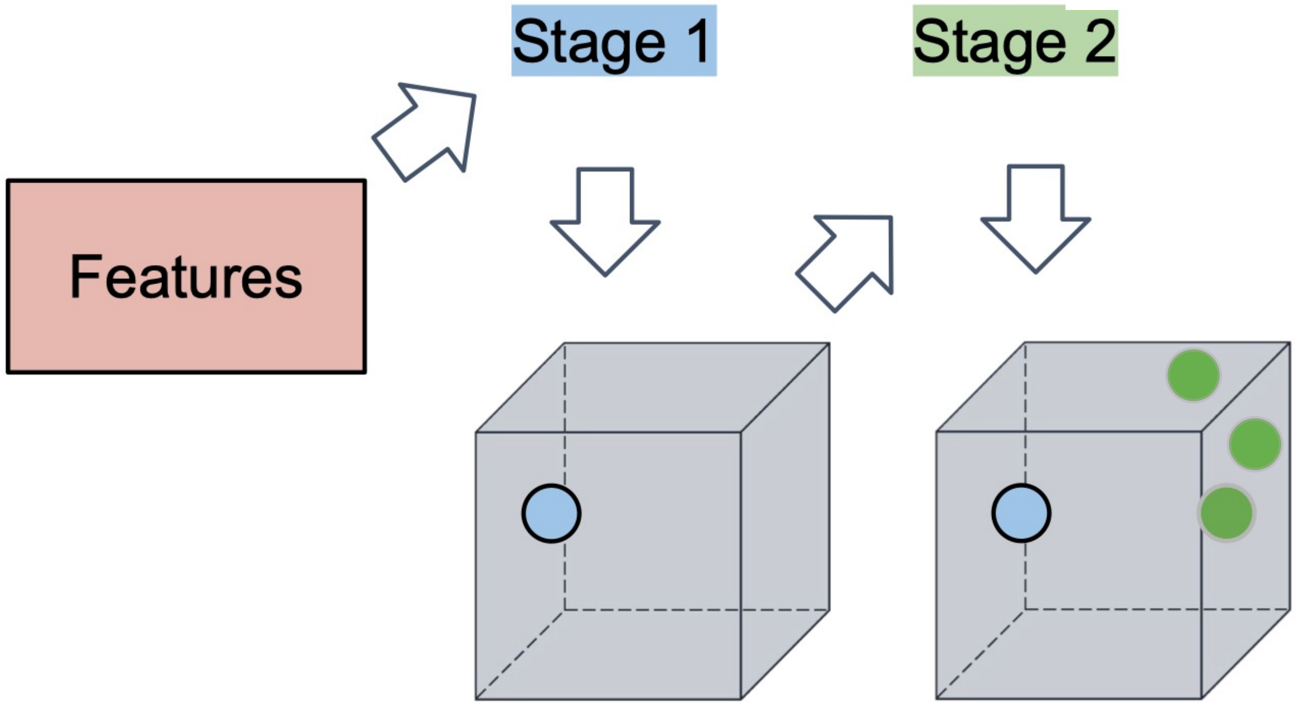


Output: Contact points (reachable, force closure)

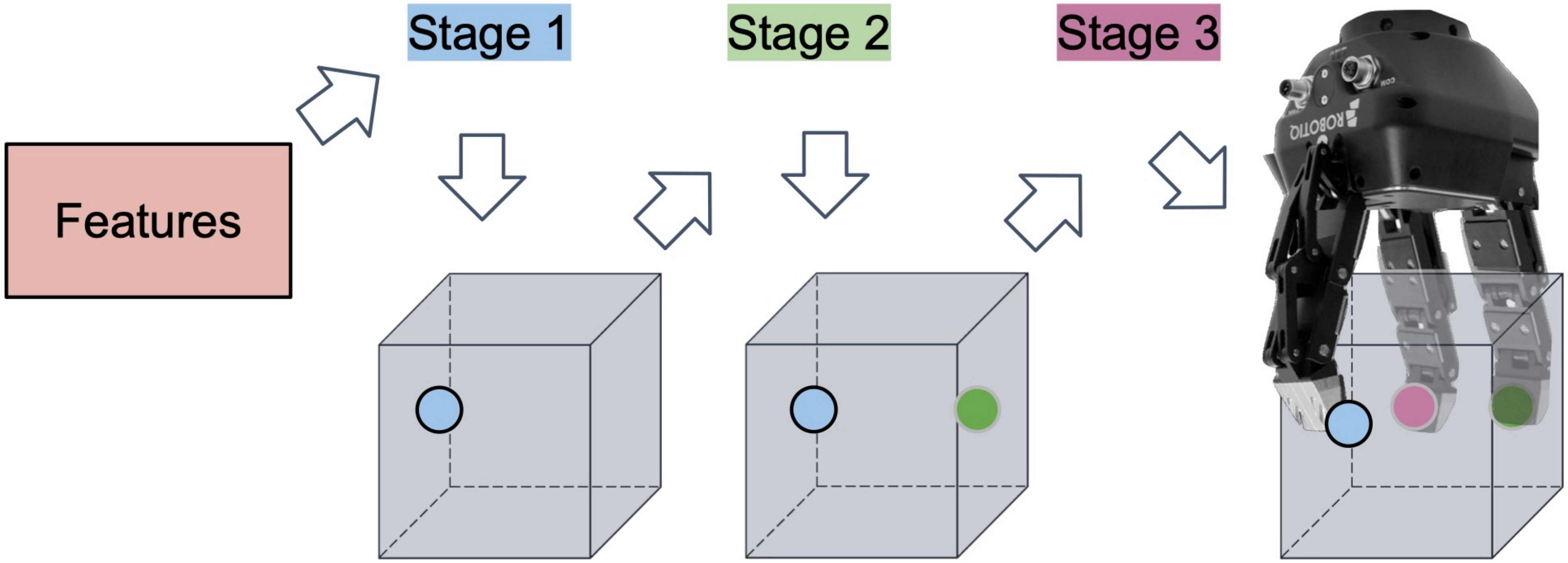
Point Set Selection Network



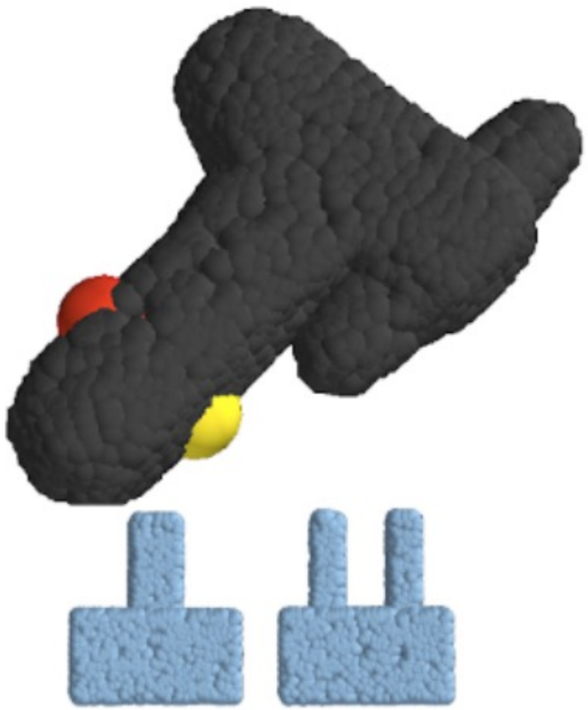
Point Set Selection Network



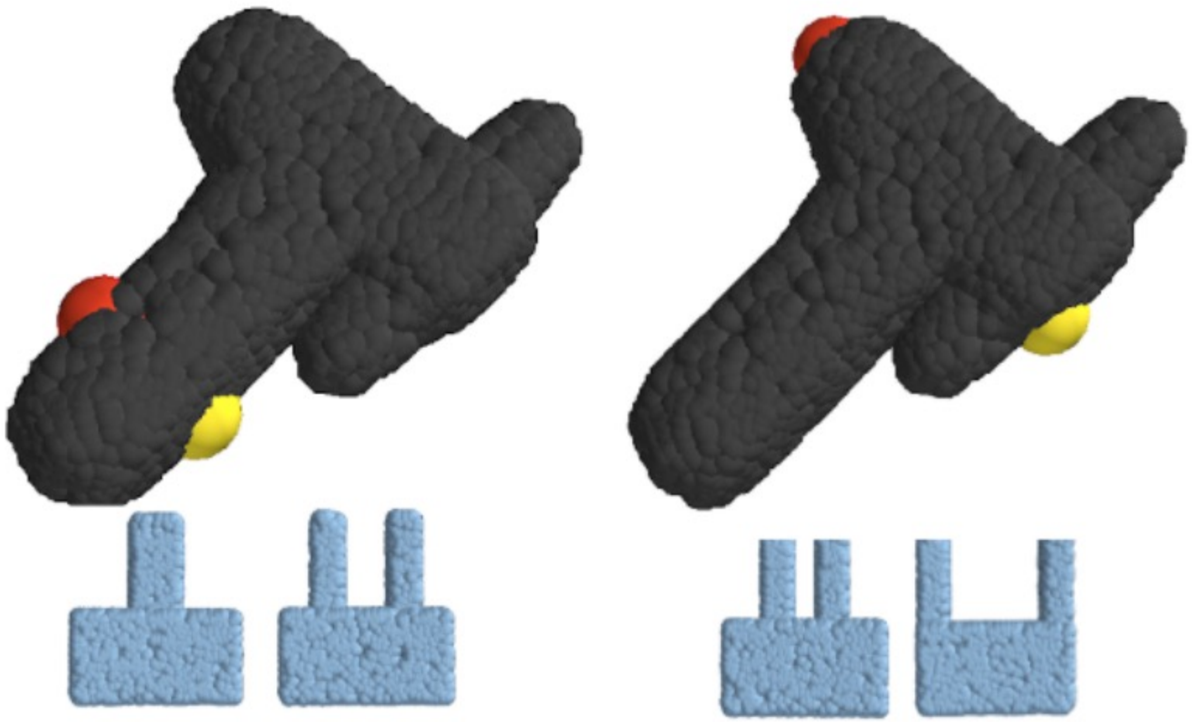
Point Set Selection Network



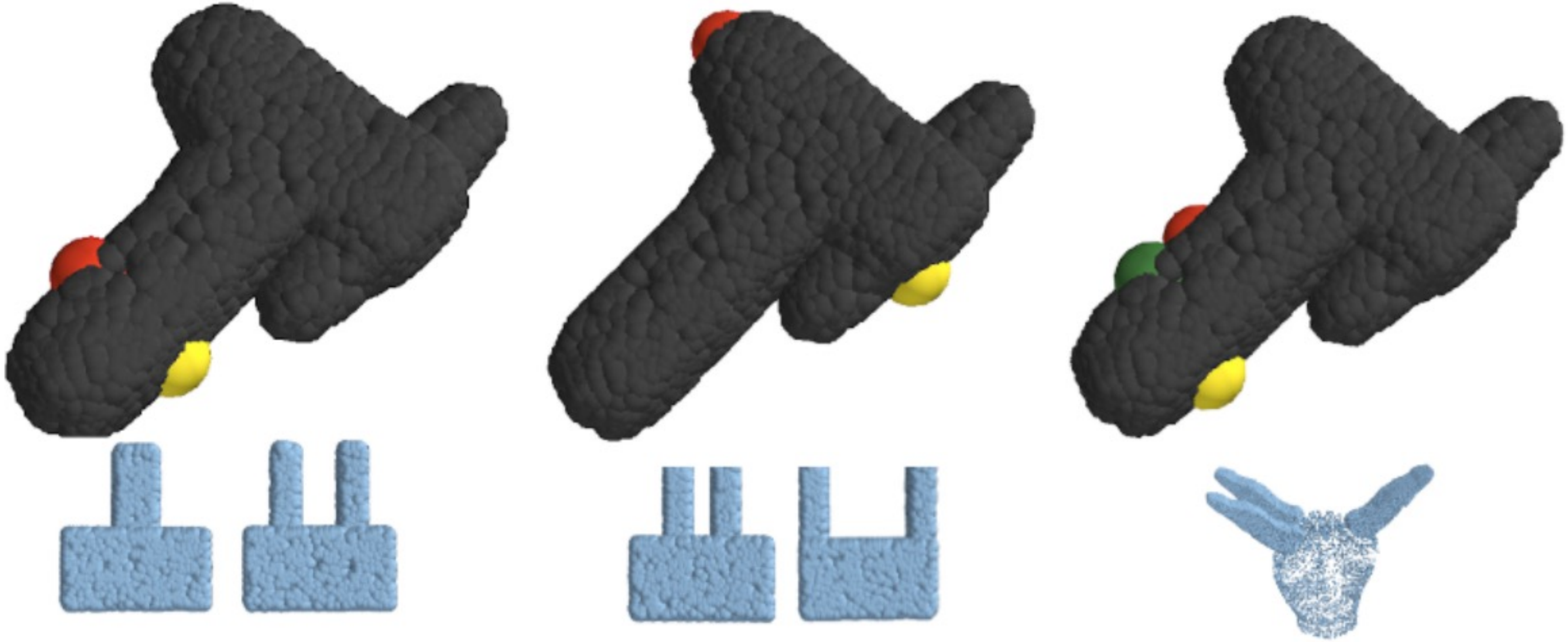
Output for Same Object and Different Grippers



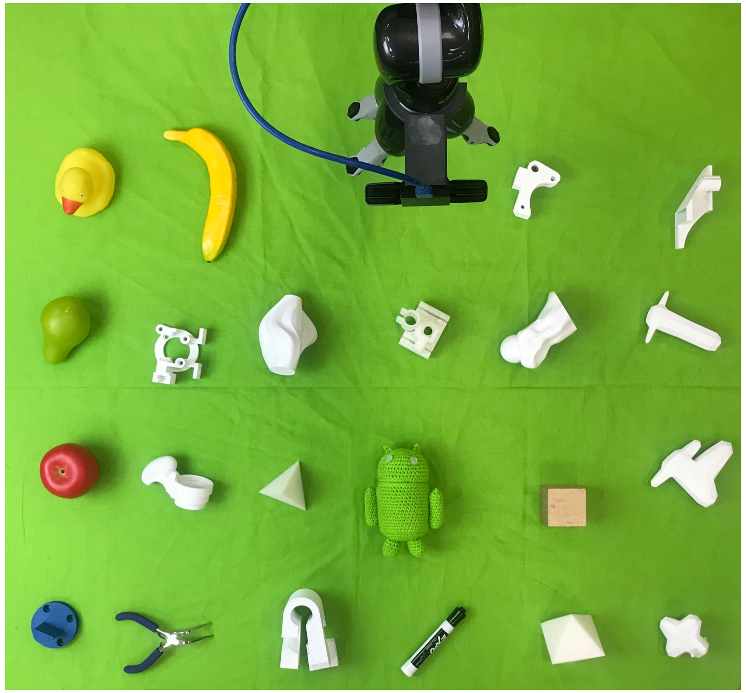
Output for Same Object and Different Grippers



Output for Same Object and Different Grippers

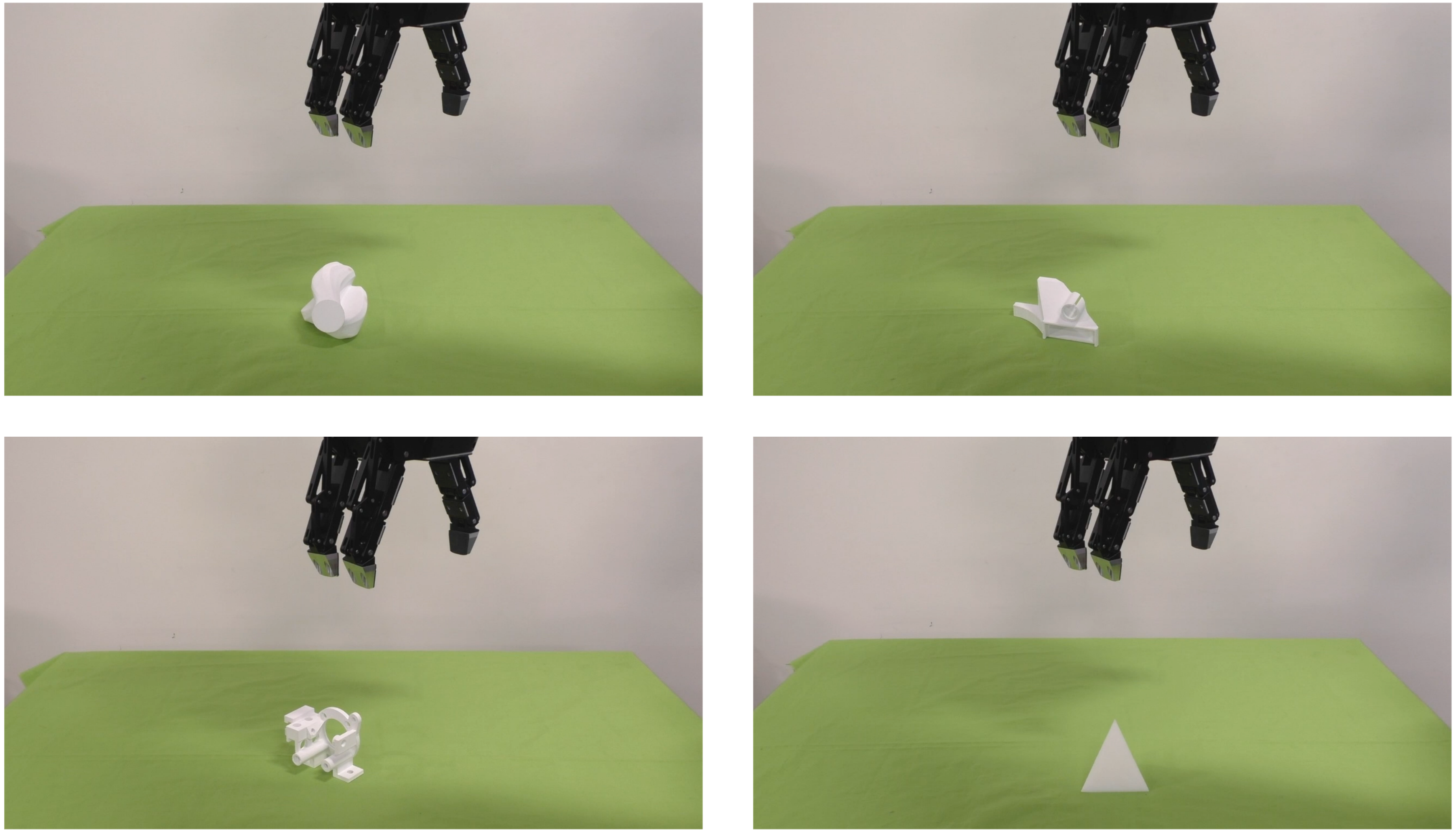


Evaluation: Grasping novel objects with various grippers



Evaluation: Grasping novel objects with a known Robotiq-3F

three contact points for three fingers

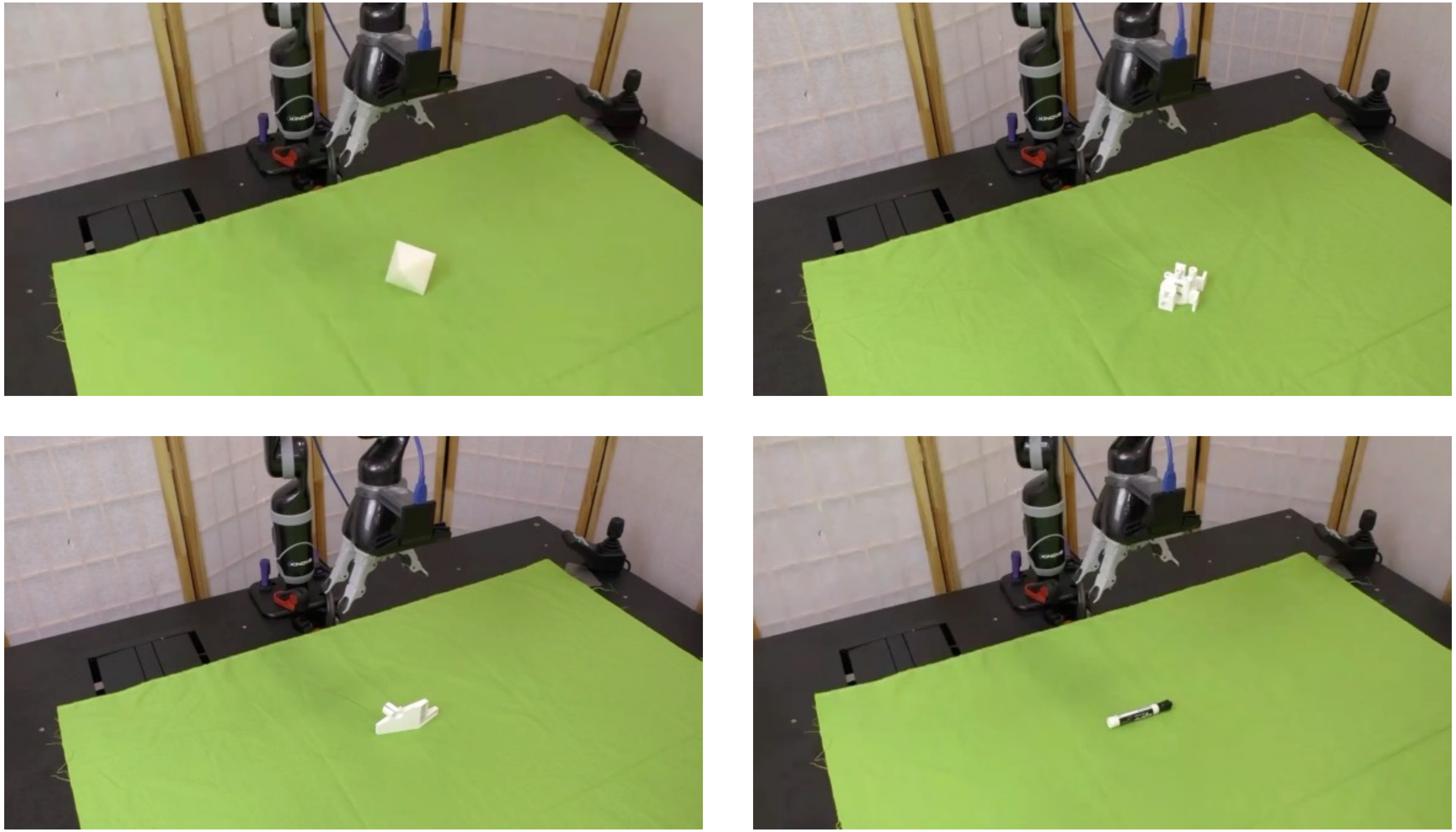


~95.38% grasp success rate on 65 trials

Grasping novel objects with **novel** grippers.

Evaluation: Grasping novel objects with a novel gripper

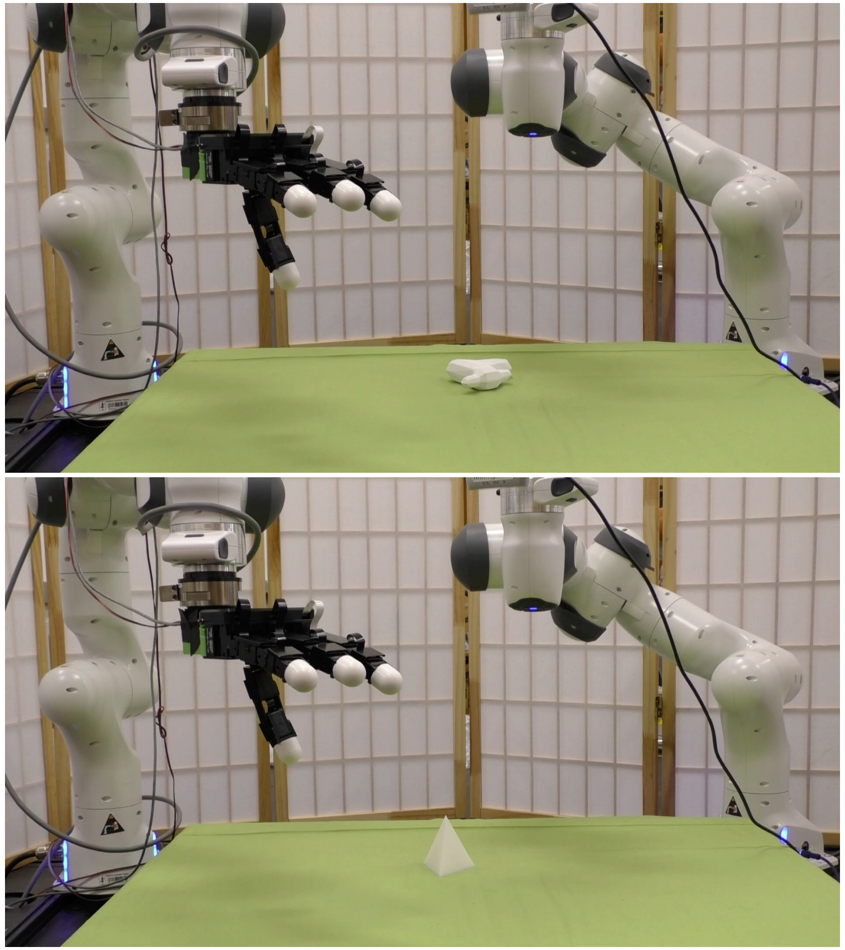
two contact points for two fingers



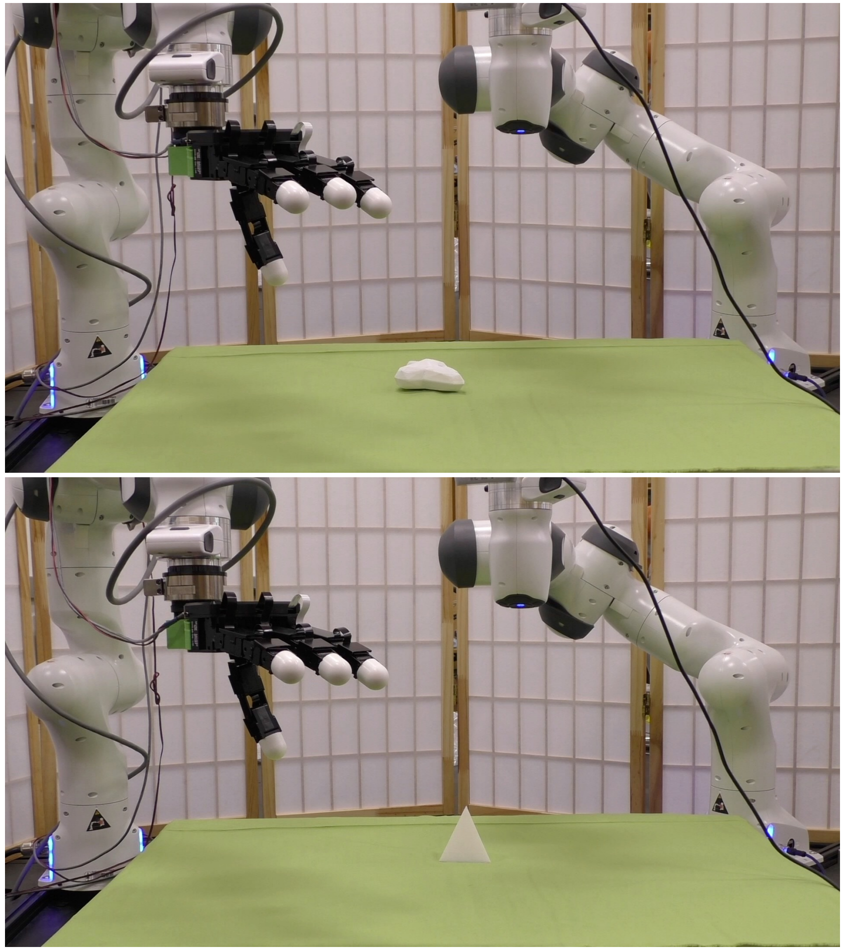
~93.33% grasp success rate on 60 trials

Evaluation: Grasping novel objects with a novel gripper

three contact points for thumb index middle finger

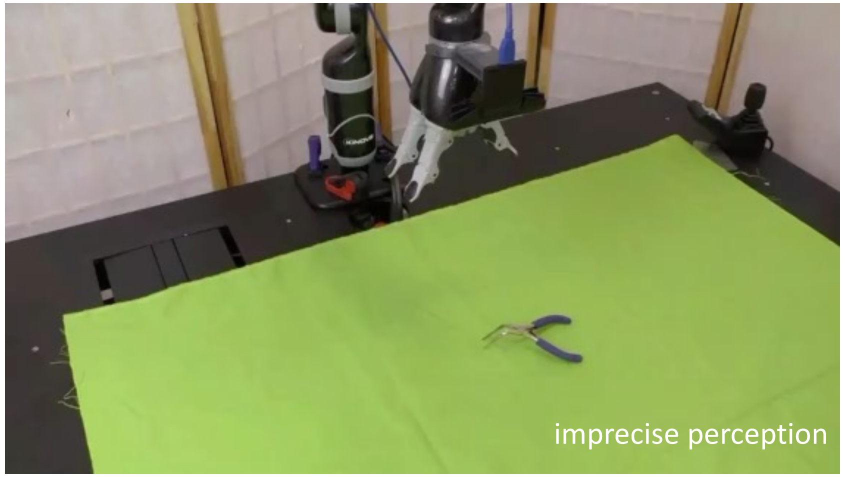


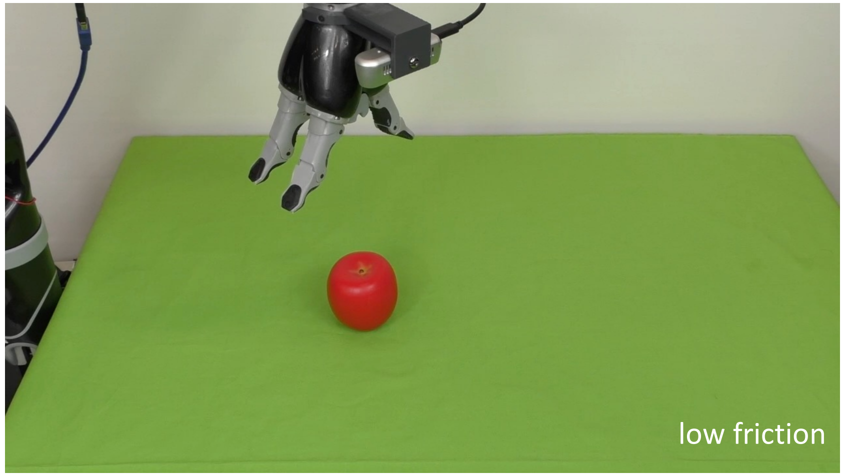
Baseline: ~40.00% grasp success rate on 60 trials

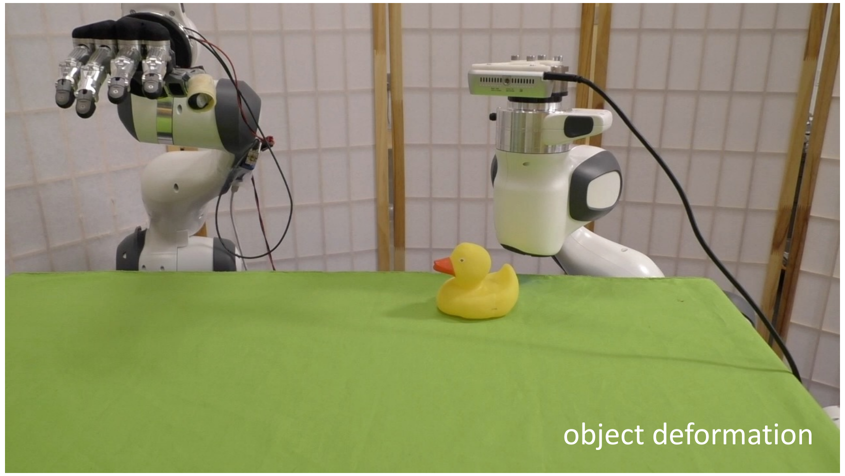


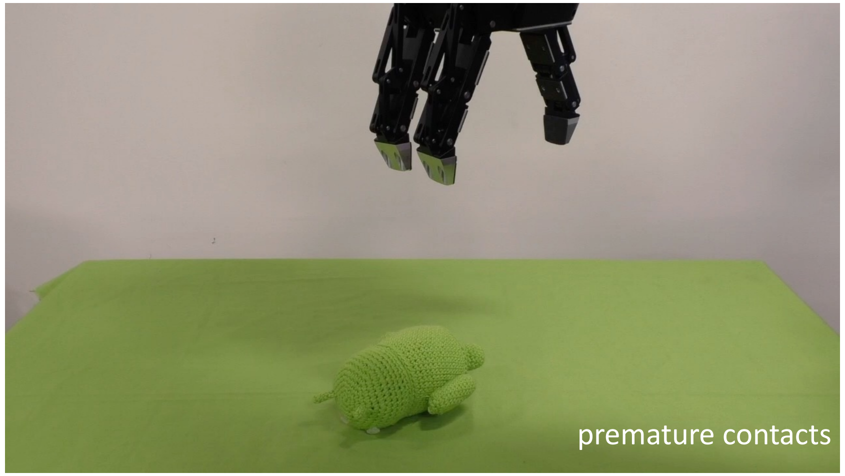
UniGrasp: ~90.00% grasp success rate on 60 trials

Failure cases due to









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References

- Principles of Robot Autonomy II
  http://web.stanford.edu/class/cs237b
- *Fast Computation of Optimal Contact Forces.* Stephen P. Boyd and Ben Wegbreit. Transactions on Robotics. 2007.
- *DexNet2.0: Deep Learning to Plan Robust Grasps with Synthetic Point Clouds and Analytic Grasp Metrics. Mahler et al. RSS, 2017*
- *UniGrasp:Learning a Unified Model to Grasp with Multifingered Robotic Hands*. Shao et al. RAL.

Thank you!