



Decision Analysis

CS4246/CS5446

AI Planning and Decision Making

This Lecture
Will Be
Recorded!



Please join:
pollev.com/anarayan



Recap: Types of Decision Theory

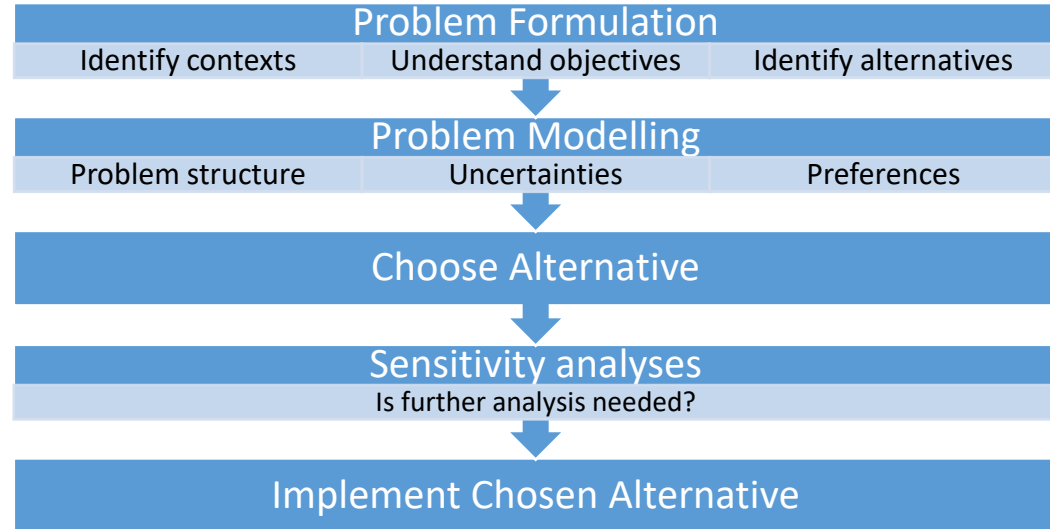
- Normative decision theory
 - Describes how ideal, rational agents should behave
- Descriptive decision theory
 - Describes how actual agents (humans) really behave
- Prescriptive decision theory
 - Prescribes guidelines for agents to behave rationally

Quiz

Quiz answer

Recap: Prescriptive Decision Theory

- DA Process
- DA modeling
 - Decision networks
 - Decision trees
- Modeling preferences
 - Fundamental objectives
- Solving decision models
 - Chance values expectation
 - Decision value maximization
- Sensitivity analysis



Quiz

Quiz answer



Topics

- The decision analysis framework
 - Formulating decision models
 - Decision networks: Influence diagrams (15.5)
 - Decision trees
 - Analyzing decision networks
 - Sensitivity analysis and robust decision models (15.6.6)
- Information value theory (15.6)
 - Expected value of perfect information (15.6.1-15.6.3)
 - Implementing an information gathering agent (15.6.4)



Value of Information

Information and Decision Making

- Information is gathered to reduce uncertainty in decision making
 - Consult experts
 - Conduct surveys
 - Perform mathematical or statistical analyses
 - Do research
 - Read books, journals, newspapers
 - Learn from past data
- Relevant questions:
 - How to evaluate or measure the “value” or usefulness of the information?
 - What does it mean for a knowledge source to provide perfect information?
 - Shall we invest effort or pay \$X for additional information to help problem solving?



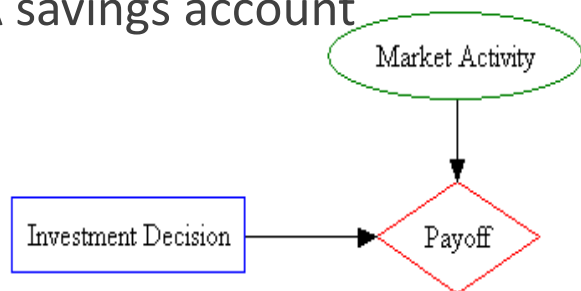
Value of Information

- Costly information gathered to reduce uncertainty
 - How to place value on information in a problem?
 - How to decide whether or not to gather more information?
- Main ideas:
 - Information has value to the extent that it is likely to cause a change of plan and to the extent that the new plan is significantly better than the old one
 - Use conditional probabilities and Bayes' Theorem to model the expected value of information

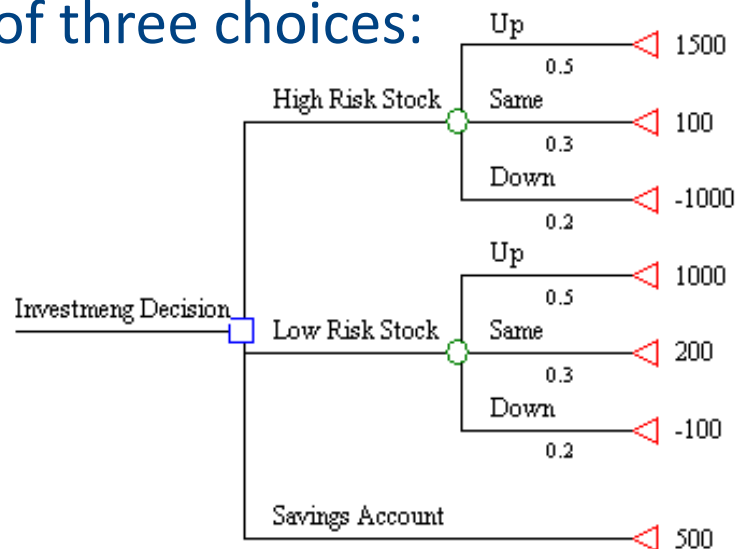
Example: Stock Investment

- An investor may invest in one of three choices:

- A high-risk stock
- A low-risk stock
- A savings account



Influence diagram



Decision tree

Expected Value of Information

- Expected value of information (EVI)
 - Indicates if information is worth gathering
 - Lower bound: zero expected value
 - Upper bound: expected value of perfect information
- Information has:
 - no value or zero expected value if the same choice will be made before and after obtaining information
 - positive expected value if it leads to a different choice
 - maximum expected value if information is perfect
- EVI is defined in terms of the decision context
 - Different people in different situations may place different values on the same information

Stock Investment Example (cont.)

Should investor consult an expert?

- If the expert always provides perfect information:
 - $P(\text{Exp says "Up"} \mid \text{Market Up}) = 1$
 - $P(\text{Exp says "Down"} \mid \text{Market Up}) = 0$
 - $P(\text{Exp says "Up"} \mid \text{Market Down}) = 0$
- To show there is no uncertainty after hearing the expert, apply Bayes' theorem:

$P(\text{Market Up} \mid \text{Exp says "Up"})$

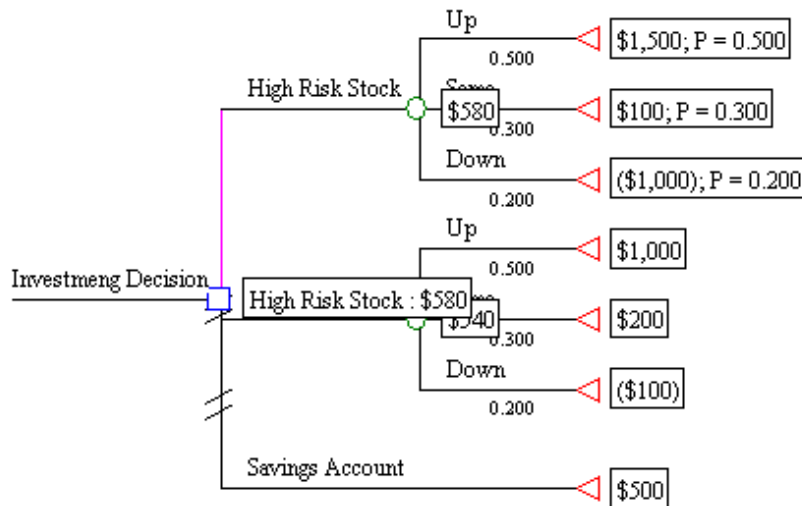
$$= \frac{P(\text{Exp says "Up"} \mid \text{Market Up}) P(\text{Market Up})}{[P(\text{Exp says "Up"} \mid \text{Market Up})P(\text{Market Up}) + P(\text{Exp says "Up"} \mid \text{Market Down})P(\text{Market Down})]}$$

$= 1$

- Note:
 - The posterior probability is equal to 1 regardless of the prior probability
 - How do real problems differ from the above situation?

Stock Investment Example (cont.)

- The optimal choice is the high-risk stock with EMV \$580
- Assumption: optimistic about market (Up, 0.5 prob)
- How much would the investor be willing to pay for information about the market activity?

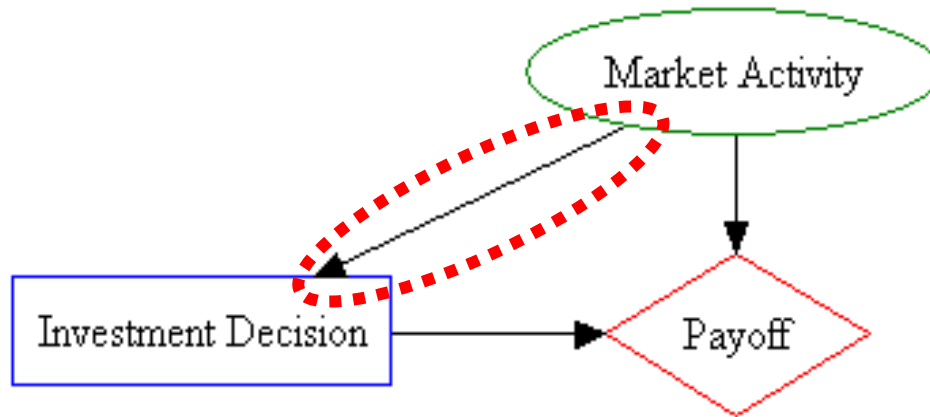


Expected Value of Perfect Information

- Max amount that the decision maker is willing to pay for perfect information
- To find expected value of perfect information (EVPI):
 - Modify the decision model to indicate perfect information
 - Solve the model and find the EMV (\$1000)
 - $EVPI = EMV(\text{with perfect information}) - EMV(\text{original})$
 $= \$1000 - \$580 = \$420$

Modifying Influence Diagram for EVPI

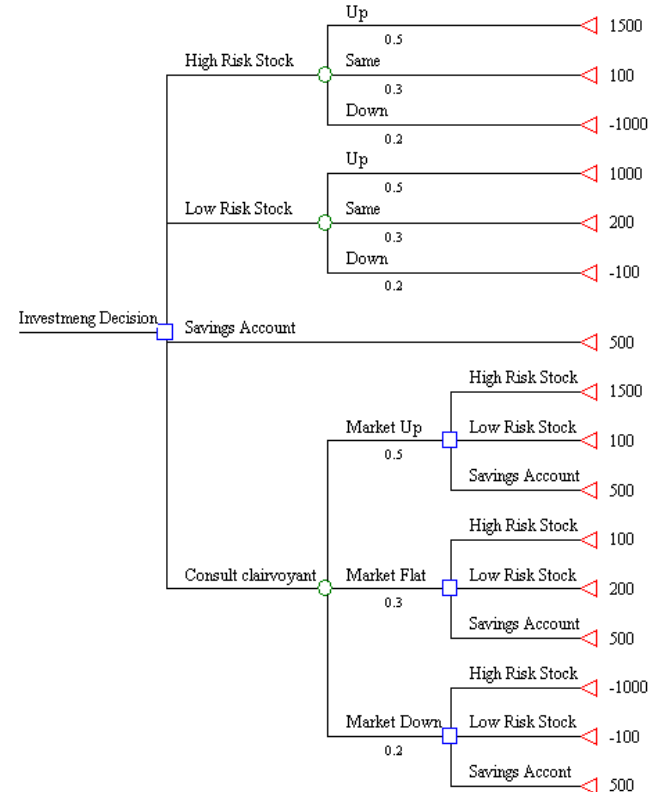
- Impose order on the decision and uncertain event nodes
- The uncertainty nodes for which perfect information is available comes before the decision node



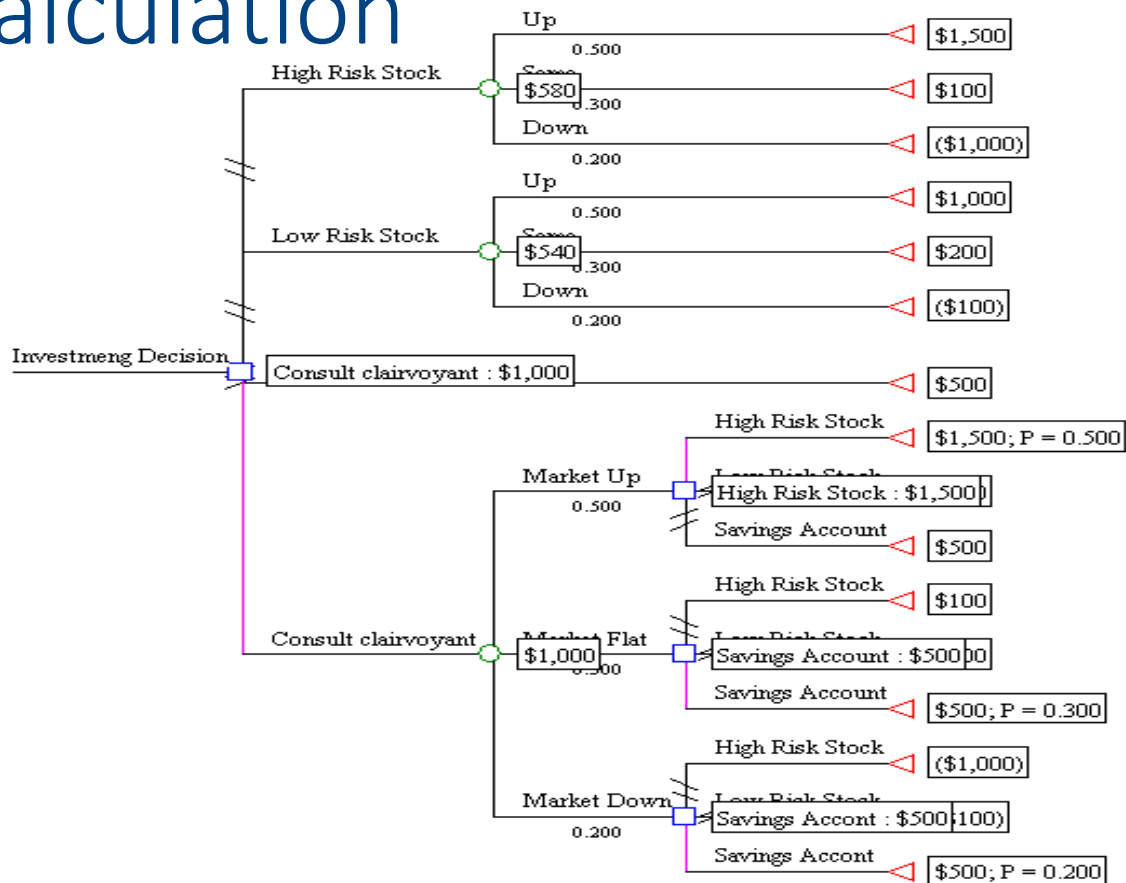
Influence diagram with perfect information

Modifying Decision Tree for EVPI

- Reorder the decision and the uncertain event nodes
- The uncertainty nodes for which perfect information is available comes before the decision node



EVPI Calculation



(Expected) Value of Perfect Information

Assume exact evidence about variable E_j ; compute value of perfect information (VPI)

- Given expected utility with current best action α :

$$EU(\alpha) = \max_a \sum_{s'} P(\text{Result}(a) = s') U(s')$$

- Value of the best new action after $E_j = e_j$ is obtained

$$EU(\alpha_{e_j} | e_j) = \max_a \sum_{s'} P(\text{Result}(a) = s' | e_j) U(s')$$

- Variable E_j can take multiple values e_j , so on averaging:

$$VPI(E_j) = \sum_{e_j} P(E_j = e_j) EU(\alpha_{e_j} | E_j = e_j) - EU(\alpha)$$

Properties of Value of information

- Expected value of information is always non-negative

$$\forall j \ VPI(E_j) \geq 0$$

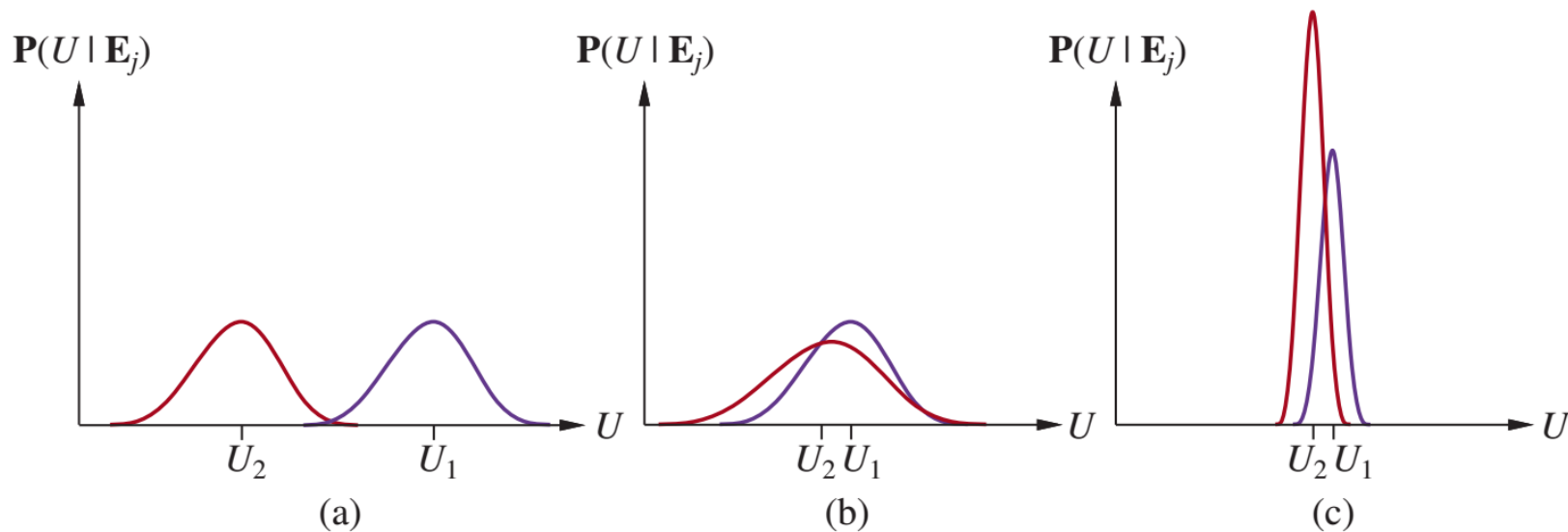
- VPI is not additive

$$VPI(E_j, E_k) \neq VPI(E_j) + VPI(E_k)$$

- VPI is order independent

$$VPI(E_j, E_k) = VPI(E_j) + VPI(E_k|E_j) = VPI(E_k) + VPI(E_j|E_k) = VPI(E_k, E_j)$$

When to Gather More Information?



Source: RN Figure 15.8

Information Gathering, Decision-Theoretic Agent

- Agent should gather information before taking actions, if possible
 - Cost associated with getting the information, so how to choose which variable to get more information about?

function INFORMATION-GATHERING-AGENT(*percept*) **returns** an *action*

persistent: *D*, a decision network

integrate *percept* into *D*

$j \leftarrow$ the value that maximizes $VPI(E_j) / C(E_j)$

if $VPI(E_j) > C(E_j)$

then return *Request*(E_j)

else return the best action from *D*

Unit gain per unit cost
OR
Information value

Source: RN Figure 15.9



Summary

- **Decision analysis**
 - A prescriptive framework for guiding systematic, rational decision making
 - Involve formulation of explainable decision models and solutions
 - Extensive applications in practice
 - Theoretical foundations and methodological bases for AI decision systems – decision-theoretic AI
- **Challenges and opportunities for AI**
 - Human-machine collaboration in decision making
 - Uncertainty modeling with expert judgment and observational data
 - Preference modeling in complex, changing and uncertain conditions
 - Responsible AI – ethical, governance, and regulatory conditions

Homework

- Readings:

- RN: 15.6.1-15.6.4 (Value of information)

- *Optional:*

- Howard, R.A., [Decision Analysis: Practice and Promise](#). Management Science, 1988. 34(6): p. 679-695. [Accessible through NUS Library e-Resources]

- Reviews:

- RN: 13.2-13.5; 14.2-14.4 (Conditional probability and Bayesian networks)
- Charniak, E., Bayesian networks without tears: making Bayesian networks more accessible to the probabilistically unsophisticated. AI Mag., 1991. 12(4): p. 50–63.

References

- Decision analysis: (Journal articles publicly available online or through NUS Library e-Resources)
 - Howard, Ronald, A. (1966). "[Decision Analysis: Applied Decision Theory](#)" (PDF). Proceedings of the Fourth International Conference on Operational Research, Wiley-Interscience.
 - Howard, R.A., [Decision Analysis: Practice and Promise](#). Management Science, 1988. 34(6): p. 679-695.
- Decision analysis: (Reference books and e-books)
 - Clemen, R.T. and T. Reilly, *Making Hard Decisions with DecisionTools*. 2013: Cengage Learning.
 - "Theoretical probability models," Chapter 9
 - "Using data," Chapter 10
 - "Value of information," Chapter 12
 - Howard, R.A. and A.E. Abbas, *Foundations of Decision Analysis*. 2016: Pearson.
 - Abbas, A.E., *Foundations of Multiattribute Utility*. 2018, Cambridge: Cambridge University Press.
 - Fenton, N. and M. Neil, *Risk Assessment and Decision Analysis with Bayesian Networks*. 2nd ed. 2019: CRC Press, Inc.
- Website:
 - Decision Analysis Section of the Institute For Operations Research and Management Science (INFORMS):
 - <http://decision-analysis.society.informs.org/index.html>

Computing Tools

- Commercial Products – mostly derived from academic research work
 - DPL – Decision Programming Language
 - Palisade PrecisionTree
 - Treeage Pro
 - Netica
 - Hugin Expert
 - Supertree
 - ...
- Commercial Products With FREE academic version
 - Bayesfusion: GeNIe and SMILE
 - ...
- Also check out survey at:
 - INFORMS Decision Analysis Software Survey (2020)
 - <https://pubsonline.informs.org/doi/10.1287/orms.2020.06.04/full/>
 - <https://pubsonline.informs.org/magazine/orms-today/2020-decision-analysis-software-survey>