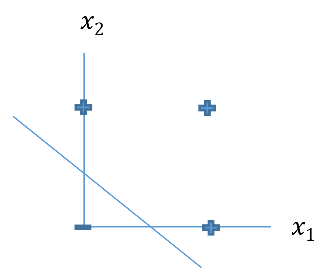
CS4248
AY 2022/23 Semester 1
Tutorial 4 Solutions

1. (a)

x_1	x_2	$x_1 \vee x_2$
0	0	0
0	1	1
1	0	1
1	1	1

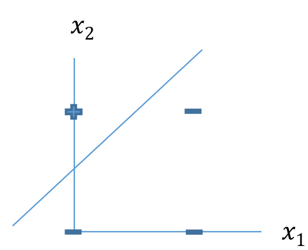


Line equation: $-\frac{1}{2} + x_1 + x_2 = 0$

$w_0 = -\frac{1}{2}$ $w_1 = 1$ $w_2 = 1$

(b)

x_1	x_2	$\neg x_1 \wedge x_2$
0	0	0
0	1	1
1	0	0
1	1	0



Line equation: $-\frac{1}{2} - x_1 + x_2 = 0$

$w_0 = -\frac{1}{2}$ $w_1 = -1$ $w_2 = 1$

2.

$$s_1 = w_1 \cdot i_1 + w_2 \cdot i_2 + w_3$$

$$h_1 = \tanh(s_1) = \frac{e^{s_1} - e^{-s_1}}{e^{s_1} + e^{-s_1}}$$

$$o_1 = w_4 \cdot h_1 + w_7$$

$$o_2 = w_5 \cdot h_1 + w_8$$

$$o_3 = w_6 \cdot h_1 + w_9$$

$$L = (o_1 - t_1)^2 + (o_2 - t_2)^2 + (o_3 - t_3)^2$$

(a)

$$\frac{\partial L}{\partial w_3} = \frac{\partial L}{\partial h_1} \cdot \frac{\partial h_1}{\partial s_1} \cdot \frac{\partial s_1}{\partial w_3} = \left(\frac{\partial L}{\partial o_1} \cdot \frac{\partial o_1}{\partial h_1} + \frac{\partial L}{\partial o_2} \cdot \frac{\partial o_2}{\partial h_1} + \frac{\partial L}{\partial o_3} \cdot \frac{\partial o_3}{\partial h_1}\right) \cdot \frac{\partial h_1}{\partial s_1} \cdot \frac{\partial s_1}{\partial w_3}$$

$$= [2(o_1 - t_1) \cdot w_4 + 2(o_2 - t_2) \cdot w_5 + 2(o_3 - t_3) \cdot w_6] \cdot (1 - h_1^2) \cdot 1$$

(b)

$$s_1 = 0.25 \cdot 0.2 + 0.25 \cdot 0.8 + 0.1 = 0.35$$

$$h_1 = 0.336$$

$$o_1 = 0.1 \cdot 0.336 + 0.3 = 0.3336$$

$$o_2 = -0.1 \cdot 0.336 - 0.2 = -0.2336$$

$$o_3 = 0.2 \cdot 0.336 - 0.3 = -0.2328$$

$$L = (0.3336 - 1)^2 + (-0.2336 - 0)^2 + (-0.2328 - 0)^2 = 0.553$$

(c)

$$w_3 = 0.1 - 0.5 \cdot \frac{\partial L}{\partial w_3}$$

$$\frac{\partial L}{\partial w_3} = [2(0.3336 - 1) \cdot 0.1 + 2(-0.2336 - 0) \cdot (-0.1) + 2(-0.2328 - 0) \cdot 0.2] \cdot (1 - 0.336^2) = -0.159$$

$$w_3 = 0.1795$$

3. (a) Yes. Justification:

Let $\boldsymbol{xW} + \boldsymbol{b} = \boldsymbol{y}$

$[\widetilde{\boldsymbol{h_1}}]_i = m_i \cdot \tanh(\boldsymbol{y}_i)$

$[\widetilde{\boldsymbol{h_2}}]_i = \tanh(m_i \cdot \boldsymbol{y}_i)$

When $m_i = 1$, $[\widetilde{\boldsymbol{h_1}}]_i = \tanh(\boldsymbol{y}_i) = [\widetilde{\boldsymbol{h_2}}]_i$

When $m_i = 0$, $[\widetilde{\boldsymbol{h_1}}]_i = 0 = \tanh(0 \cdot \boldsymbol{y}_i) = [\widetilde{\boldsymbol{h_2}}]_i$

(b) No. Justification:

$$[\widetilde{\boldsymbol{h_1}}]_i = m_i \cdot \frac{1}{1 + e^{-y_i}}$$

$$[\widetilde{\boldsymbol{h_2}}]_i = \frac{1}{1 + e^{-(m_i \cdot y_i)}}$$

When $m_i = 0$,

$$[\widetilde{\boldsymbol{h_1}}]_i = 0$$
$$[\widetilde{\boldsymbol{h_2}}]_i = \frac{1}{2} \neq [\widetilde{\boldsymbol{h_1}}]_i$$