CS5340 Lab 3: Hidden Markov Models

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N: No. of random variables. **K**: No. of states. **Batches**: No. of inputs

e_step()

Get emission probabilities P(X|Z) of size N*K for all batches using the given initial parameters (mean and standard deviation) for each state K.

- Calculate alpha using the forward step. Normalise alpha using the scaling factor and store it for beta calculation:

$$\alpha(\mathbf{z}_1) = p(\mathbf{x}_1, \mathbf{z}_1) = p(\mathbf{z}_1)p(\mathbf{x}_1|\mathbf{z}_1) = \prod_{k=1}^{K} \{\pi_k p(\mathbf{x}_1|\phi_k)\}^{z_{1k}}$$

$$\boxed{c_n \widehat{\alpha}(\mathbf{z}_n) = p(\mathbf{x}_n|\mathbf{z}_n) \sum_{\mathbf{z}_{n-1}} \widehat{\alpha}(\mathbf{z}_{n-1}) p(\mathbf{z}_n|\mathbf{z}_{n-1})} \quad c_n = \sum_{z_n} \tilde{\alpha}(\mathbf{z}_n)$$

- Calculate beta using the backward step. Initialize Bn = 1. Normalise beta using the scaling factor:

$$\boxed{c_{n+1}\widehat{\beta}(\mathbf{z}_n) = \sum_{\mathbf{z}_{n+1}} \widehat{\beta}(\mathbf{z}_{n+1}) p(\mathbf{x}_{n+1}|\mathbf{z}_{n+1}) p(\mathbf{z}_{n+1}|\mathbf{z}_n) = \tilde{\beta}(\mathbf{z}_n)}$$

- Calculate gamma_list and joint posterior distribution:

$$\boxed{\begin{aligned} \gamma(\mathbf{z}_n) &= \widehat{\alpha}(\mathbf{z}_n)\widehat{\beta}(\mathbf{z}_n) \\ \xi(\mathbf{z}_{n-1}, \mathbf{z}_n) &= c_n^{-1}\, \widehat{\alpha}(\mathbf{z}_{n-1}) p(\mathbf{x}_n|\mathbf{z}_n) p(z_n|z_{n-1}) \widehat{\beta}(\mathbf{z}_n) \end{aligned}}$$

m_step()

Calculate the parameters according to the formula:

$$\pi_k = \frac{\sum_{m=1}^{M} \gamma(z_{1k}^m)}{\sum_{m=1}^{M} \sum_{k=1}^{K} \gamma(z_{1k}^m)}$$

$$A_{jk} = \frac{\sum_{m=1}^{M} \sum_{n=2}^{N} \xi(z_{n-1,j}^m, z_{nk}^m)}{\sum_{m=1}^{M} \sum_{n=2}^{N} \sum_{k=1}^{K} \xi(z_{n-1,j}^m, z_{nk}^m)}$$

$$\mu_k = \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} \gamma(z_{nk}^m) x_n^m}{\sum_{m=1}^{M} \sum_{n=1}^{N} \gamma(z_{nk}^m)}$$

$$\sigma_k^2 = \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} \gamma(z_{nk}^m)(x_n^m - \mu_k)^2}{\sum_{m=1}^{M} \sum_{n=1}^{N} \gamma(z_{nk}^m)}$$

fit_hmm()

1. Create Theta_old using initialised parameters.
2. Perform the e_step() followed by the m_step() to get theta_current.
3. Check if any of the parameters have changed less than the threshold.
4. If no, update Theta_old to Theta_current and repeat 2-3 until the change in any of the parameters is less than the threshold → convergence.