

## Assignment 2

### Uncertainties

**Name**

**Matriculation Number**

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#### *Instructions*

- Due date: **14 April 2023 noon**.
  - Type up your solutions or write **neatly**.
  - Submit your work online to Canvas under `Assignments/Homework 2`. Only pdf files are accepted. Name your submission file `MatricNo-HW2.pdf`.
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Problem	Max Points	Points
1	10	
2	10	
3	15	
4	20	
5	10	
6	15	
7	10	
<b>Total</b>	<b>90</b>	

1. Unmanned aerial vehicles (UAVs) have gained popularity in many applications, including photography, surveillance, and delivery. One basic UAV capability is hovering, *i.e.*, staying in a fixed 3-D pose for an extended duration. For this, the UAV needs a sensor looking downwards to measure the distance to the ground. Choose a suitable distance sensor for
  - (a) a light-weight UAV,
  - (b) a full-size outdoor UAV.

Consider among the following options: camera, LIDAR, ultrasonic sensor, and GPS. The primary considerations include sensor accuracy, weight, cost, . . . . State your assumptions clearly and justify your choice.
2. Consider the *Girl-and-Tiger* example discussed in the class. Suppose that at time  $t = 0$ , the probabilities for the tiger to be behind the left door and the right door are  $(0.5, 0.5)$ . Answer the following questions using the state-transition and the observation models in the lecture slides.
  - (a) If the girl always chooses to listen and gets a sequence of observations,  $(TL, TR, TR, \dots)$ , what is the probability distribution of the tiger's location at time  $t = 1, 2, 3$ , respectively?
  - (b) If we additionally allow the tiger to jump to the opposite room with probability 0.1 at each time step, what is the probability distribution over the tiger's position at time  $t = 1, 2, 3$ , after receiving the same sequence of observations?
3. Particle filtering uses a finite set of samples to approximate a continuous probability distribution. This may lead to various difficulties in practice. In this problem, we explore a common one called *particle collapse*. Consider the extreme situation in which a robot stays stationary and does not move at all. It also receives no observations. The probability distribution on the initial robot state is uniform. Suppose that we use 2 particles sampled uniformly at random from the initial probability distribution. Naturally the weights for the two particles are  $(1/2, 1/2)$ .
  - (a) After one step of particle filtering, what is the probability that we have 2 particles representing 2 distinct locations? What is the probability that we have 2 particles representing the same location?  
*Hint.* Consider the resampling step carefully.
  - (b) Answer the same questions above, after  $N$  steps of particle filtering.
  - (c) The outcome in the previous parts is highly undesirable. Why?
  - (d) Suppose that we use  $K$  particles with  $K$  much larger than 2. Would it help improve the situation? Why or why not?
  - (e) What would you do to alleviate the issue of particle collapse? Explain your idea.
4. In the class, we have covered three filtering algorithms: histogram filter, particle filter, and Kalman filter. Choose a suitable filtering algorithm for each task below. Justify your choice and if needed, state your assumptions.
  - (a) A self-driving car tracks its lateral position w.r.t. the center of the lane. It processes the the front-view camera images to detect lanes.
  - (b) A self-driving car wants to merge into a neighbouring lane. It monitors a human-driven vehicle in the target lane to decide on the suitable actions. It assumes that the human driver's intention may be DECELERATE or ACCELERATE, *i.e.*, to give way or not to give way, respectively. The self-driving car tries to infer the human's intention according to the observed vehicle behavior.
  - (c) Our new robot Spot is lost in COM1. It tries to re-localize itself on a COM1 map, using 2D LIDAR readings.
5. A household *NurseBot* is tasked to care for a baby and must decide on whether to feed the baby if the baby cries. Crying is a noisy indicator of hunger. There is an 80% chance the baby cries when hungry, and there is a 15% chance that the baby cries when not hungry. If the NurseBot feeds the baby, then the baby stops being hungry at the next time step. If the baby is not hungry and not fed, then it becomes hungry at the next time step for 5% of the time. If the baby is hungry, it continues being so until fed. The cost of feeding the baby is 1. The cost of the baby being hungry is 2. These costs are additive: if the NurseBot feeds the baby when it is hungry, then the total cost is 3. To find an optimal strategy for feeding the baby, we model the task as a POMDP.

(a) Specify the states, actions, observations.

(b) Specify the state-transition probabilities in a table according to the format below. Each row corresponds to a start state and an action, and each column corresponds to an end state.

	$s_1$	$\dots$	$s_N$
$(s_1, a_1)$			
$(s_1, a_2)$			
$\dots$			
$(s_N, a_K)$			

(c) Specify the observation probabilities, using a similar table format.

	$z_1$	$\dots$	$z_M$
$s_1$			
$\dots$			
$s_N$			

(d) Specify the reward function, using a similar table format as well.

	$a_1$	$\dots$	$a_K$
$s_1$			
$\dots$			
$s_N$			

6. Consider the grid environment below. The robot is initially located at  $A$ ,  $B$ , or  $C$  with equal probabilities, and the robot's goal is to reach the destination  $T$  and stay there. In each time step, the robot may stay put or move from its current grid cell to a neighboring cell deterministically. If the robot tries to move to a neighboring cell occupied by obstacles, it stays unmoved in its current cell. There are no observations.

A					B
		T			
					C

(a) Of the three probabilistic models that we have discussed in the class, MDP, HMM, and POMDP, which one is most suitable for this task and why?

(b) Describe a policy for the robot to reach  $T$  with probability 1.

7. Apply the QMDP algorithm to solve the following navigation task. A mobile robot navigates in a  $3 \times 3$  grid world. At each time step, the robot moves to any of the four neighboring cells. The robot is almost blind. It can only sense whether it hits the wall or reaches the goal at the top-left cell. There is no uncertainty in robot transitions. When the robot hits the wall, the action fails, and the robot stays in the same cell. When the robot reaches the goal, the task terminates. Each action has a cost of 2:  $R_A(a) = -2$ . The reward for a state-action pair is the addition of the state and the action rewards:  $R(s, a) = R_S(s) + R_A(a)$ . The tables

below list the state-reward function  $R_s$ , the MDP value function  $V_{\text{MDP}}$ , which assumes that the robot state is fully observable, and a belief  $b$  over the robot's position. Use the QMDP algorithm to find the best action for the robot at  $b$ .

10	0	0
0	-20	0
0	0	0

$R_s$

10	8	6
8	-20	4
6	4	2

$V_{\text{MDP}}$

0	0	0
0	0	0.3
0	0.1	0.6

$b$

- (a) Calculate the Q-value  $Q_{\text{MDP}}(b, a)$  for all four actions, UP, DOWN, LEFT, RIGHT.
- (b) What is the best action at  $b$ ?