National University of Singapore School of Computing

Semester 1, AY2023-24

CS4246/CS5446

AI Planning and Decision Making

Tutorial Week 11: Game Theory

Guidelines

You may discuss the content of the questions with your classmates. But everyone should work on and be ready to present ALL the solutions.

Problem 1: Battle of the Sexes

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

	Ballet	Concert
Ballet	1,2	0,0
Concert	0,0	2,1

- a) Explain why are the strategy profiles $\langle Ballet, Ballet \rangle$ and $\langle Concert, Concert \rangle$ Nash equilibria.
- b) Find a Nash equilibrium where both players play mixed strategies.
- c) Compute the expected utility of all three equilibria for the husband. Do the same for the wife.
- d) Compute the utility of both players going to their preferred activity, and the expected utility of for both players when they both select each activity randomly with equal probability.

Problem 2: Pure Maxmin and Minmax Strategies

Consider the following 3×3 two-person, zero-sum matrix game:

	t_1	t_2	t_3
s_1	1	6	0
s_2	2	0	3
s_3	3	2	4

In this question, we assume that only pure strategies are considered.

- a) Find the maxmin strategy and value for the row player.
- b) Find the minmax strategy for the column player against the row player and the minmax value for the row player.
- c) Prove that, in general (i.e., not just for the game above), $\max_s \min_t u(s,t) \leq \min_t \max_s u(s,t)$.