National University of Singapore
School of Computing

Semester 1, AY2023-24

CS4246/CS5446 AI Planning and Decision Making

Tutorial Week 5: Rational Decision Making

Guidelines

You may discuss the content of the questions with your classmates. But everyone should work on and be ready to present ALL the solutions.

Problem 1: Allais Paradox

The Allais paradox (Allais, 1953) is a well-known problem potentially suggesting that humans are "predictably irrational" (Ariely, 2009) [1]. People are given a choice between lotteries A and B and then between C and D, which have the following prizes:

A: 80% chance of \$4000	C: 20% chance of \$4000
B: 100% chance of \$3000	D: 25% chance of \$3000

Most people consistently prefer B over A (i.e., taking the sure payoff), and C over D (taking the higher EMV).

a) Show that the normative analysis (i.e., describing how a rational agent should act) disagrees. [**Hint**: Set U(\$0) = 0; show that the preferences between A, B and C, D are opposites, hence a contradiction.]

Solution:

Inequality from preferring B to A:

$$0.8U(\$4000) < U(\$3000)$$

Inequality from preferring C to D:

$$0.2U(\$4000) > 0.25U(\$3000)$$

Now multiply by 4:

$$0.8U(\$4000) > U(\$3000)$$

There is a contradiction between the 2 preferences.

In that case, then B$\succ$A implies that $U(\$3000) > 0.8U(\$4000)$, whereas C$\succ$D implies exactly the reverse. In other words, there is no utility function that is consistent with these choices.

[1] For a possible explanation of such a paradox, refer to page 620 of AIMA textbook.

b) Prove that the judgments B $\succ$ A and C $\succ$ D in the above Allais paradox violate the axiom of substitutability. [**Hint**: You may wish to consider using the axiom of decomposability.]

Solution:
We know A$\prec$B and C$\succ$D. By the axiom of decomposability,

$$C \sim [0.25, A; 0.75; \$0] \text{ and } D \sim [0.25, B; 0.75, \$0]. \tag{1}$$

Using the above and C$\succ$D, we have $[0.25, A; 0.75; \$0] \succ [0.25, B; 0.75, \$0]$. This implies that EU(A)>EU(B) $\implies$ A $\succ$B.

From A$\prec$B and substitutability, we would have $[0.25, A; 0.75, \$0] \prec [0.25, B; 0.75, \$0]$.

This is a contradiction, hence substitutability is violated.

Problem 2: Preference Modelling

Alex is given the choice between two games:

- **Game 1**: a fair coin is flipped and if it comes up heads, Alex receives \$100. If the coin comes up tails, Alex receives nothing.
- **Game 2**: a fair coin is flipped twice. Each time the coin comes up heads, Alex receives \$50, and Alex receives nothing for each coin flip that comes up tails.

Alex prefers Game 2 to Game 1. Argue that Alex would prefer to receive \$50 compared to being allowed to participate in Game 1.

Solution:
Since Alex prefers Game 2 to Game 1:

$$\begin{aligned}
&0.5U(\$0) + 0.5U(\$100) < 0.25U(\$0) + 0.5U(\$50) + 0.25U(\$100) \\
&\implies 0.25U(\$0) + 0.25U(\$100) < 0.5U(\$50) \\
&\implies 0.5U(\$0) + 0.5U(\$100) < U(\$50)
\end{aligned}$$

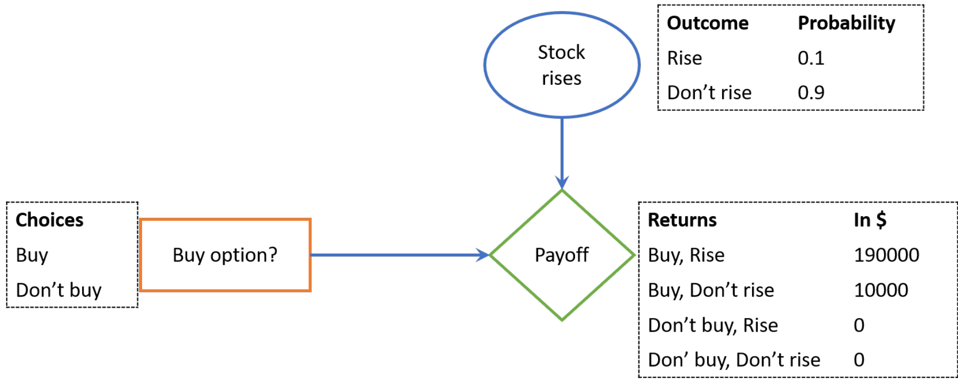
The left hand side is the expected utility of participating in Game 1 while the right hand side is the expected utility of receiving \$50. Since the LHS is less than the RHS, Alex would prefer \$50 over participating in game 1.

Problem 3: Basic Risky Decision

Richie Bean is trying to strike it big in the stock market during the economic downturn. He is considering buying some options to a very risky stock on a diamond mine in Africa. There is only a 10% chance that the stock price will rise if he exercises his options, but the payoff is \$200,000. It costs \$10,000 to buy and exercise the options. The alternative is not to buy at all, in which case Mr. Bean's profit is zero.

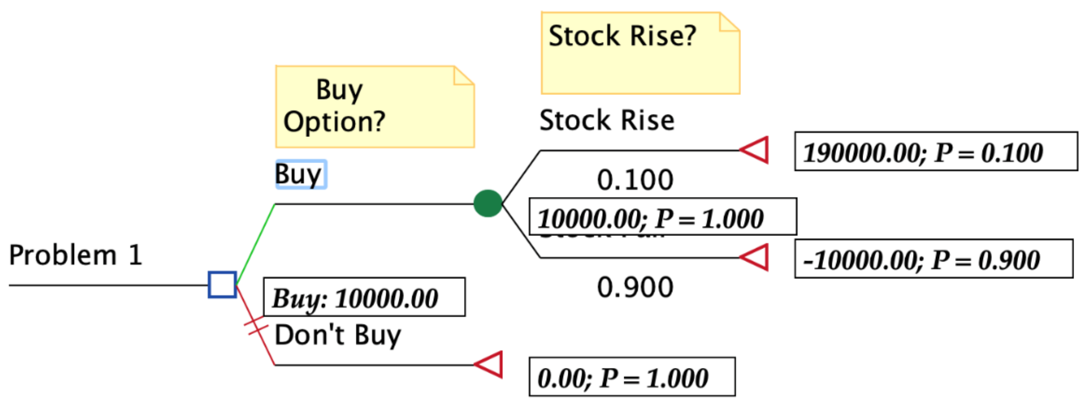
a. Draw an influence diagram to represent Mr. Bean's problem. Clearly indicate all the options/outcomes and numbers. Should he buy the options? Use the solution approaches mentioned in the lecture to substantiate your answer.

Solution:



b. Draw an decision tree to represent Mr. Bean's problem. Clearly indicate all the options/outcomes and numbers. Should he buy the options? Show all the details in your decision tree.

Solution:



EMV for buy \$10000, don't buy \$0.