NATIONAL UNIVERSITY OF SINGAPORE

CS5340 – Uncertainty Modeling in AI

(Semester 1 : AY2017/18)

Time Allowed : 2 Hours

INSTRUCTIONS TO STUDENTS

1. Please write your Student Number only. Do not write your name.
2. This assessment paper contains **FOUR** questions and comprises **SEVEN** printed pages.
3. Students are required to answer **ALL** questions.
4. Students should write the answers for each question on a new page.
5. Students are allowed to bring in **ONE A4 summary sheet.**
6. Non-programmable electronic calculators are allowed.

Question 1

a)

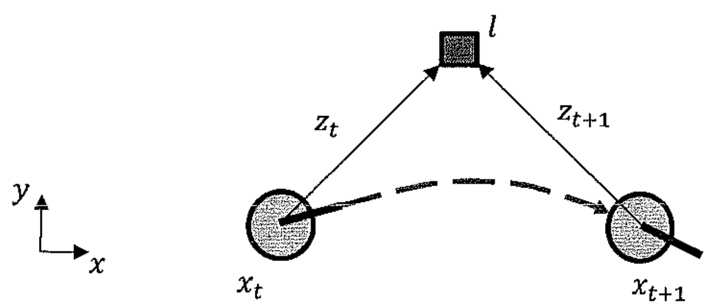


Fig. 1.1

Fig. 1.1 shows a mobile robot that traverses from pose x_t to x_{t+1} over time t to $t+1$. The robot is equipped with an 1-dimensional range sensor that returns the distances z_t and z_{t+1} of a landmark structure l in the environment from the poses x_t and x_{t+1} respectively. Let u_t denotes the control command given by the user to move the robot from x_t to x_{t+1}.

(i) Taking $\{u_t, l, x_t, x_{t+1}, z_t, z_{t+1}\}$ as random variables, state whether each of these random variables is an observed or latent/hidden random variable. Explain your answers.

(3 marks)

(ii) Given the following conditional independencies:

$$l \perp u_t \mid \emptyset, \quad x_t \perp l \mid u_t, \quad x_{t+1} \perp \{l, u_t\} \mid x_t,$$

$$z_t \perp \{u_t, x_{t+1}\} \mid \{x_t, l\}, \quad z_{t+1} \perp \{u_t, x_t, z_t\} \mid \{l, x_{t+1}\}.$$

Write the factorized probability and draw the Bayesian network that represents the joint distribution $p(u_t, l, x_t, x_{t+1}, z_t, z_{t+1})$ assuming the following topological ordering of the random variables:

$$\{u_t, l, x_t, x_{t+1}, z_t, z_{t+1}\}.$$

Show all your workings clearly.

(14 marks)

(iii) Write the following probability distribution $p(z_t, z_{t+1} \mid l)$ in terms of the factorized probability obtained in (ii). Simplify your answer.

(2 marks)

b)

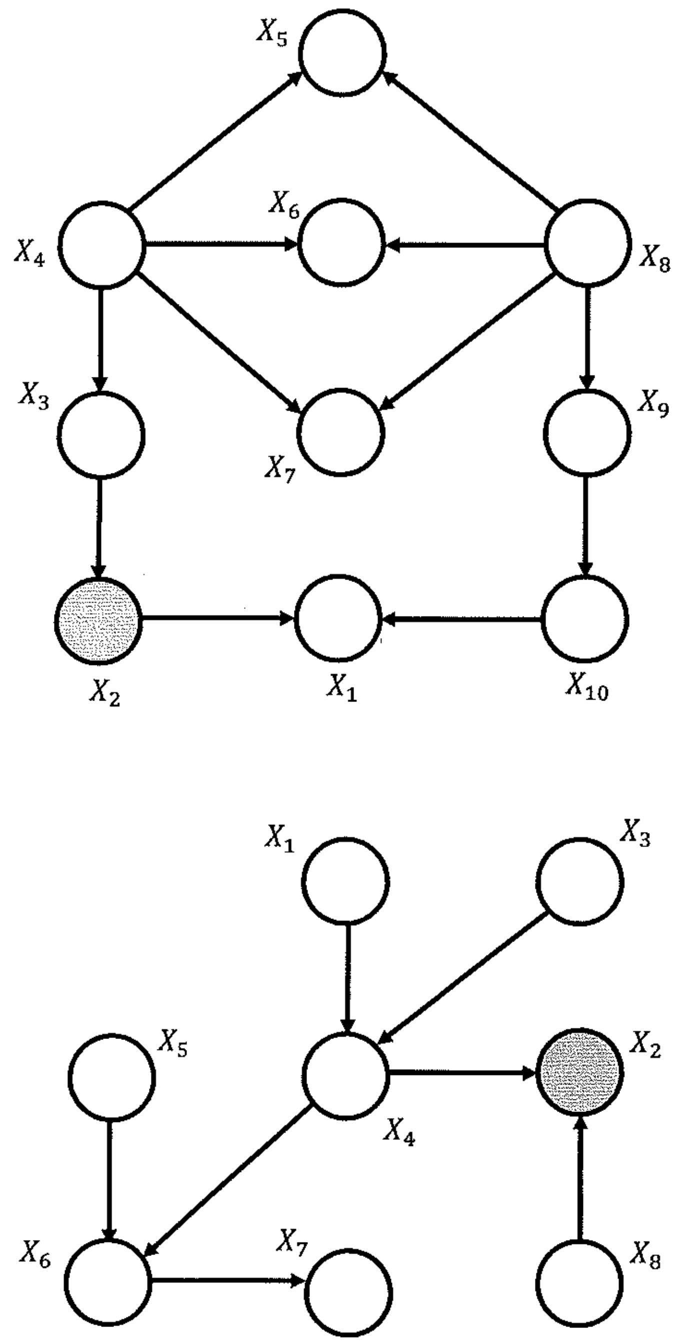


Fig. 1.2

For each of the Bayesian networks shown in Fig. 1.2, determine the largest set of nodes X_B such that $X_1 \perp X_B \mid X_2$. Explain your answers.

(6 marks)

Question 2

Evaluate (give the distribution tables) the following probabilities:

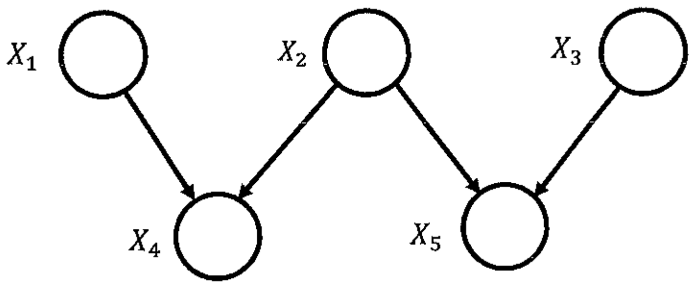
$$p(x_1 \mid x_5), \quad p(x_2 \mid x_4), \quad p(x_3 \mid x_2), \quad p(x_4 \mid x_3), \quad p(x_5)$$

for the Bayesian network shown in Fig. 2.1, where each random variable takes a binary state, i.e. $x_i \in \{T, F\}$. Show all your workings clearly.

$p(x_1)$
0.2

$p(x_2)$
0.44

$p(x_3)$
0.2



x_1	x_2	$p(x_4 \mid x_1, x_2)$
T	T	0.35
T	F	0.6
F	T	0.01
F	F	0.95

x_2	x_3	$p(x_5 \mid x_2, x_3)$
T	T	0.35
T	F	0.6
F	T	0.01
F	F	0.95

Fig. 2.1

(25 marks)

Question 3

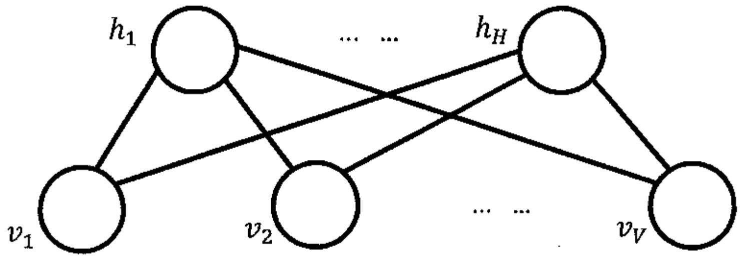


Fig. 3.1

The restricted Boltzmann machine is a Markov Random Field (MRF) defined on a bipartite graph as shown in Fig. 3.1. It consists of a layer of visible variables $\boldsymbol{v} = [v_1, \dots, v_V]^T$ and hidden variables $\boldsymbol{h} = [h_1, \dots, h_H]^T$, where all variables are binary taking states {0,1}. The joint distribution of the MRF is given by:

$$p(\boldsymbol{v}, \boldsymbol{h}) = \frac{1}{Z(\boldsymbol{W}, \boldsymbol{a}, \boldsymbol{b})} \exp(\boldsymbol{v}^T \boldsymbol{W} \boldsymbol{h} + \boldsymbol{a}^T \boldsymbol{v} + \boldsymbol{b}^T \boldsymbol{h}),$$

where $\theta = \{\boldsymbol{W}_{V\times H}, \boldsymbol{a}_{V\times 1}, \boldsymbol{b}_{H\times 1}\}$ are the parameters of the potential functions, and $Z(.)$ is the partition function.

a) Given that:

$$p(h_i = 1 \mid \boldsymbol{v}) = \sigma\left(b_i + \textstyle\sum_j W_{ji} v_j\right),$$

where $\sigma(x) = \frac{e^x}{1+e^x}$ is the sigmoid activation function. Show that the distribution of hidden units conditioned on the visible units factorizes as:

$$p(\boldsymbol{h} \mid \boldsymbol{v}) = \textstyle\prod_i p(h_i \mid \boldsymbol{v}).$$

Show all your workings clearly.

(10 marks)

b) Assuming that the restricted Boltzmann machine consists of only 2 visible and 1 hidden variables, and the joint distribution of the MRF is given by:

h	v_1	v_2	$\exp(\boldsymbol{v}^T \boldsymbol{W} h + \boldsymbol{a}^T \boldsymbol{v} + bh)$
0	0	0	1.00
0	0	1	2.13
0	1	0	4.65
0	1	1	9.90
1	0	0	3.65
1	0	1	8.66
1	1	0	4.22
1	1	1	10.01

Find the unknown parameters, i.e. $\theta = \{\boldsymbol{W}_{2\times 1}, \boldsymbol{a}_{2\times 1}, b\}$.

(15 marks)

Question 4

a)

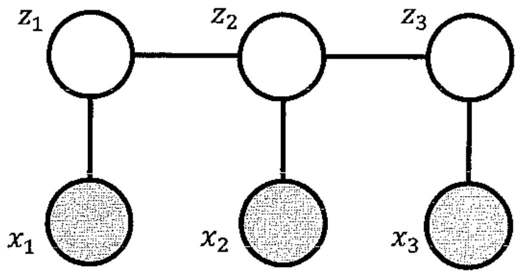


Fig. 4.1

Fig. 4.1 shows a Markov Random Field (MRF) representation of a Hidden Markov Model (HMM) over three time steps. The hidden variables z_1, z_2, z_3 are discrete random variables that take three possible states $z_n \in \{F, H, M\}$, and x_1, x_2, x_3 are the observed variables that take on real values $x_n \in \mathbb{R}$. The joint distribution is given by:

$$p(z_1, z_2, z_3, x_1, x_2, x_3) = \frac{1}{Z} \prod_{n=2}^{3} \psi_t(z_n, z_{n-1}) \prod_{n=1}^{3} \psi_e(x_n, z_n),$$

where Z is the partition function, and the transition potential $\psi_t(z_n, z_{n-1})$ and the emission potentials $\psi_e(x_n, z_n)$ are given by:

$\psi_t(z_n, z_{n-1})$	$z_n = F$	$z_n = H$	$z_n = M$
$z_{n-1} = F$	2.0	3.0	5.0
$z_{n-1} = H$	1.0	6.0	3.0
$z_{n-1} = M$	4.5	2.0	2.5

z_1	$\psi_e(x_1, z_1)$
F	1.0
H	8.0
M	1.0

z_2	$\psi_e(x_2, z_2)$
F	7.0
H	1.0
M	2.0

z_3	$\psi_e(x_3, z_3)$
F	2.0
H	3.0
M	5.0

Decode the message that corresponds to the states of the hidden variables that give the maximal probability. Show all your workings clearly.

(15 marks)

b) Give the junction tree of the Bayesian network shown in Fig. 4.2 using the following elimination order: $\{x_7, x_6, x_5, x_4, x_3, x_2, x_1\}$. Show all your workings clearly.

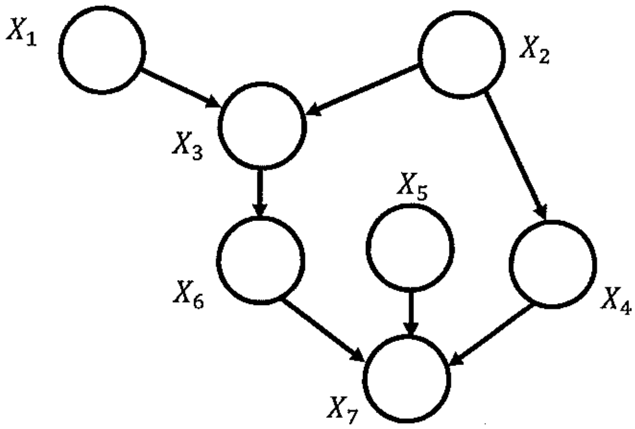


Fig. 4.2

(*10 marks*)

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