NATIONAL UNIVERSITY OF SINGAPORE

CS5340 – Uncertainty Modeling in AI

(Semester 1 : AY2018/19)

Time Allowed : 2 Hours

INSTRUCTIONS TO STUDENTS

1. Please write your Student Number only. Do not write your name.
2. This assessment paper contains **FOUR** questions and comprises **SIX** printed pages.
3. Students are required to answer **ALL** questions.
4. Students should write the answers for each question on a new page.
5. Students can bring in **ONE A4 summary sheet**.
6. Non-programmable electronic calculators are allowed.

Question 1

Figure 1.1 shows a Markov Random Field (MRF) with two random variables X_1 and X_2, where $x_i \in \{0,1\}$. Furthermore, let $\phi_1(x_1)$ and $\phi_2(x_2)$ denote the unary potentials, and $\psi_{12}(x_1, x_2)$ denotes the pairwise potential. Given the observations over 14 trials as shown in Table 1.1, find the unknown value of $\psi_{12}(x_1 = 0, x_2 = 0)$ in the potential tables shown in Table 1.2. Show all your workings clearly.

Figure 1.1

Trial Number	Outcomes	
	X_1	X_2
1	0	0
2	1	0
3	1	1
4	1	0
5	0	0
6	0	1
7	1	1
8	0	0
9	1	0
10	1	1
11	0	0
12	0	0
13	1	0
14	1	1

Table 1.1

X_1	$\phi_1(x_1)$
0	2
1	1

X_2	$\phi_2(x_2)$
0	1
1	2

X_1	X_2	$\psi_{12}(x_1, x_2)$
0	0	$\psi_{12}(x_1 = 0, x_2 = 0)$
0	1	1
1	0	2
1	1	2

Table 1.2

(25 marks)

Question 2

Figure 2.1 shows a Bayesian network with five random variables X_1, X_2, X_3, X_4, X_5 , where $x_i \in \{0,1\}$ for $i = 1, 2, 4$, and $x_i \in \{0,1,2\}$ for $i = 3, 5$.

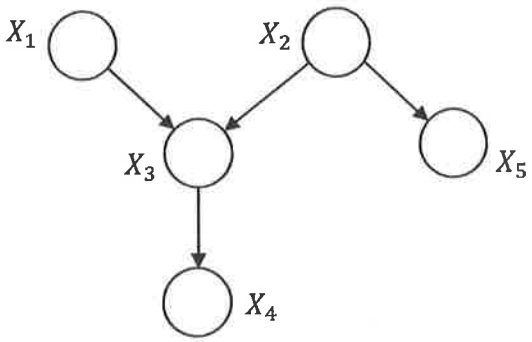


Figure 2.1

(a) Write down all the conditional independences given by the Bayesian network.

(5 marks)

(b) Write down the factorized expression of the joint probability given by the Bayesian network.

(1 mark)

(c) Convert the Bayesian network into a factor graph. Draw the factor graph and write down the expression of each factor clearly in your answer.

(6 marks)

(d) Table 2.1 gives the probability tables of the Bayesian network, find the conditional probability $p(x_1|x_3 = 1, x_2)$. Show all your workings clearly.

X_1	X_2	X_3	$p(x_3\|x_1, x_2)$
0	0	0	0.3
0	0	1	0.4
0	1	0	0.9
0	1	1	0.08
1	0	0	0.05
1	0	1	0.25
1	1	0	0.5
1	1	1	0.3

X_1	$p(x_1)$
0	0.6

X_2	$p(x_2)$
0	0.7

X_3	X_4	$p(x_4\|x_3)$
0	0	0.1
1	0	0.4
2	0	0.99

Table 2.1

(13 marks)

Question 3

Figure 3.1 shows a graphical model with six binary-state latent random variables $Z = \{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6\}$, $z_i \in \{0,1\}$, and six binary-state observed random variables $X = \{X_1, X_2, X_3, X_4, X_5, X_6\}$, $x_i \in \{0,1\}$. Table 3.1 gives the pairwise potentials $\phi(z_i, z_j)$, $\forall\, ij \in \mathcal{E}_Z$ and conditional probability $p(x_i|z_i)$ for $i = 1, \ldots 6$, where $\mathcal{E}_Z$ denotes all the edges between the latent random variables in the graphical model. Find the configuration of Z that maximizes the joint probability $p(X, Z)$.

(**Hint**: convert the graphical model into a factor graph, where the respective pairwise potential and conditional probability are represented as a single factor.)

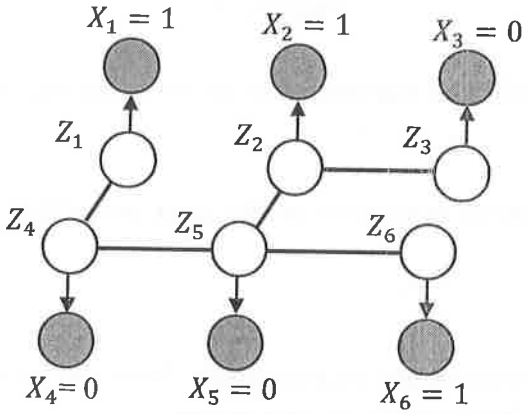


Figure 3.1

Z_i	Z_j	$\phi(z_i, z_j)$
0	0	0
0	1	2
1	0	2
1	1	0

X_i	Z_i	$p(x_i \mid z_i)$
0	0	0.9
0	1	0.05
1	0	0.1
1	1	0.95

Table 3.1

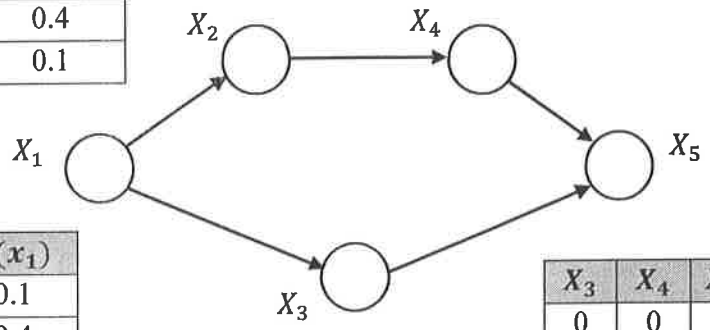
(25 marks)

Question 4

The Bayesian network shown in Figure 4.1 has five random variables X_1, X_2, X_3, X_4, X_5, where $x_i \in \{0,1,2\}$ for $i = 1, 2$ and $x_i \in \{0,1\}$ for $i = 3, 4, 5$.

X_1	X_2	$p(x_2\|x_1)$
0	0	0.1
1	0	0.4
2	0	0.8
0	1	0.2
1	1	0.4
2	1	0.1

X_2	X_4	$p(x_4\|x_2)$
0	0	0.2
1	0	0.5
2	0	0.7



X_1	$p(x_1)$
0	0.1
1	0.4

X_1	X_3	$p(x_3\|x_1)$
0	0	0.3
1	0	0.4
2	0	0.1

X_3	X_4	X_5	$p(x_5\|x_3, x_4)$
0	0	0	0.6
0	1	0	0.3
1	0	0	0.1
1	1	0	0.8

Figure 4.1

(a) Given the following numbers drawn from a uniform distribution $u \sim \mathcal{U}(0,1)$:

$$u = [\ 0.4387 \quad 0.4898 \quad 0.7513 \quad 0.4984 \quad 0.2760\ ],$$

generate one set of samples from the joint distribution $p(x_1, x_2, x_3, x_4, x_5)$ using Gibbs sampling. Use $x_1 = 0$, $x_2 = 0$, $x_3 = 0$, $x_4 = 0$, $x_5 = 0$ as the initialization. Show all your workings clearly.

(15 marks)

(b) Table 4.1 shows 10 sets of samples drawn from Gibbs sampling. Ignoring the burn-in effect and initialization, find the approximation for the following probabilities using the generated samples:

i. $p(x_2)$
ii. $p(x_3, x_5)$
iii. $p(x_3, x_4 = 1, x_5 = 1)$
iv. $p(x_3|x_2 = 1)$

Sample #	X_1	X_2	X_3	X_4	X_5
0	0	0	0	0	0
1	2	0	1	1	0
2	2	0	0	1	0
3	0	0	0	1	1
4	1	1	1	0	0
5	2	2	1	1	0
6	2	0	1	0	1
7	1	2	0	0	0
8	2	1	0	0	0
9	1	0	1	1	0
10	1	0	1	1	1

Table 4.1

(*10 marks*)

--End--