

“Real World” Reinforcement Learning

CS4246/CS5446

AI Planning and Decision Making



Please join:

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This lecture will be
recorded!



Topics

- **Function Approximation (23.4.1 - 23.4.3)**
 - Approximating direct utility estimation
 - Approximating temporal difference learning
 - Deep Reinforcement learning
- **Policy Search (23.5)**
 - REINFORCE
 - Actor-Critic
 - Correlated Sampling

Recall: Reinforcement Learning (RL)

- Based on:

A Markov decision process (MDP) $M \triangleq (S, A, T, R)$ consisting of:

- A set S of states
- A set A of actions
- Missing transition function T ?
- Missing reward function R ?

- Categorization of methods
- Monte Carlo (MC) Learning
 - aka Direct utility estimates
- Adaptive dynamic programming (ADP)
- Temporal difference (TD) learning:
 - Q-learning and SARSA

- Learning (prediction):

- Assume policy
- Solution is an (optimal) utility (value) function: $U: S \rightarrow \mathfrak{R}$ or $V: S \rightarrow \mathfrak{R}$

- Planning (control)

- Assume utility function or Q-function (action-utility function)
- Solution is an (optimal) policy: $\pi: S \rightarrow A$

Quiz

Quiz answer

Scaling

- Number of states grow exponentially with no. of state variables (features)
- Tabular representation (for utility function and Q -function) scales to tens of thousands of states
 - e.g., Number of states in Backgammon & Chess are of the order of 10^{20} & 10^{40}
 - Still considered relatively small as compared to real-world problems!
 - Cannot visit all the states infinitely often
- Approaches to scaling up:
 - **Function approximation** – approximating utility (value) functions
 - **Policy search** – systematic search for good policies



Function Approximation

Linear:

- Approximate Monte Carlo Learning
- Approximate temporal difference (TD) learning

Non-linear:

- Deep neural networks

Function Approximation

- Function approximation constructs compact representation of true utility (value) function and Q -function

- Example: Represent evaluation function for chess as a linear function of features (or basis functions)

$$\hat{U}_{\theta}(s) = \theta_1 f_1(s) + \theta_2 f_2(s) + \cdots + \theta_n f_n(s)$$

- Function approximation uses the n no. of θ parameters to represent a function over a very large number of states
 - RL agent learns θ that best approximate the evaluation (utility) function

Function Approximation

- Function approximation allows **generalization** from small number of states observed in training data to entire state space
 - Example: Backgammon agent learned to play as well as the best human players by observing only $\approx 10^{12}$ states out of 10^{20} states
- Caveat:
 - If n (no. of parameters) is too small, may fail to achieve good approximation

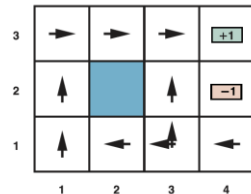


Linear Function Approximation

Approximate Monte Carlo Learning

Approximate temporal difference (TD) learning

Example: Navigation in Grid World



Source: RN Figure 17.2

Compact representation +
generalization

- Use:

$$\hat{U}_{\theta}(x, y) = \theta_0 + \theta_1 x + \theta_2 y$$

- If $(\theta_0, \theta_1, \theta_2) = (0.5, 0.2, 0.1)$, then $\hat{U}(1,1) = 0.8$

- Given a collection of trials:

- Obtain a set of sample values of $\hat{U}_{\theta}(x, y)$
- Find the best fit, in the sense of minimizing the squared error, using standard linear regression

Approximating Monte Carlo Learning

- For MC learning we get a set of training samples

$$((x_1, y_1), u_1), ((x_2, y_2), u_2), \dots, ((x_n, y_n), u_n)$$

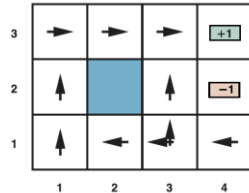
Where u_j is the measured utility of the j^{th} example - observed total reward from state s onward in the j^{th} trial

- This is a supervised learning problem
 - Standard linear regression problem with squared error and linear function
 - Minimize squared-error (loss) function – when partial derivatives wrt to coefficients of linear function are zero

Quiz

Quiz answer

Example: Navigation in Grid World



- Use:

$$\hat{U}_{\theta}(x, y) = \theta_0 + \theta_1 x + \theta_2 y$$

- If $(\theta_0, \theta_1, \theta_2) = (0.5, 0.2, 0.1)$, then $\hat{U}(1,1) = 0.8$

- After a single trial:

- Suppose we run a trial and the total reward obtained starting at $(1,1)$ is 0.4. This suggests that $\hat{U}(1,1)$ currently 0.8, is too large and must be reduced.
- How?

Online Learning

- To minimize the squared error using online learning
 - Update the parameters after each trial.
- For the j^{th} example, take a step in the direction of the gradient of error function:

Half the squared difference of predicted total and actual total

$$\mathcal{E}_j(s) = \frac{(\hat{U}_\theta(s) - u_j(s))^2}{2}$$

ComboBox????????????????
onward in the jth trial

Widrow-Hoff Rule Or
Delta rule
for online least squares

- For parameter θ_i :

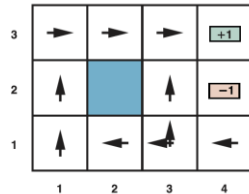
$$\theta_i \leftarrow \theta_i - \alpha \frac{\partial \mathcal{E}_j(s)}{\partial \theta_i} = \theta_i + \alpha (u_j(s) - \hat{U}_\theta(s)) \frac{\partial \hat{U}_\theta(s)}{\partial \theta_i}$$

- Notes:

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- Changing the parameters θ_i in response to an observed transition between two states also changes the values of \hat{U}_θ for every other state! $\hat{U}_\theta(x, y) = \theta_0 + \theta_1 x + \theta_2 y$
- Function approximation allows a reinforcement learner to generalize from its experiences.

Example: Navigation in Grid World



- Use:

$$\hat{U}_{\theta}(x, y) = \theta_0 + \theta_1 x + \theta_2 y$$

- If $(\theta_0, \theta_1, \theta_2) = (0.5, 0.2, 0.1)$, then $\hat{U}(1,1) = 0.8$

- After a single trial:

- Suppose we run a trial and the total reward obtained starting at (1,1) is 0.4.
- Apply delta rule for online least squares to the example where $\hat{U}_{\theta}(x, y)$ is 0.8 and $u_j(1,1)$ is 0.4.
- Parameters θ_0 , θ_1 , and θ_2 are all decreased by 0.4α , which reduces the error for (1,1).

- Applying Delta Rule for linear function approximator:

$$\theta_i \leftarrow \theta_i - \alpha \frac{\partial \mathcal{E}_j(s)}{\partial \theta_i} = \theta_i + \alpha \left(u_j(s) - \hat{U}_{\theta}(s) \right) \frac{\partial \hat{U}_{\theta}(s)}{\partial \theta_i}$$

$$\theta_0 \leftarrow \theta_0 + \alpha \left(u_j(s) - \hat{U}_{\theta}(s) \right)$$

$$\theta_1 \leftarrow \theta_1 + \alpha \left(u_j(s) - \hat{U}_{\theta}(s) \right) x$$

$$\theta_2 \leftarrow \theta_2 + \alpha \left(u_j(s) - \hat{U}_{\theta}(s) \right) y$$

Approximating Temporal Difference Learning

- For TD learning the same idea of online learning can be applied
 - Adjust the parameters to reduce the temporal difference between successive states

- For utilities:

$$\theta_i \leftarrow \theta_i + \alpha \left[\underbrace{R(s, a, s') + \gamma \hat{U}_\theta(s')}_{\text{TD target}} - \hat{U}_\theta(s) \right] \frac{\partial \hat{U}_\theta(s)}{\partial \theta_i}$$

Update parameter to
reduce temporal
difference

- For Q -learning:

$$\theta_i \leftarrow \theta_i + \alpha \left[\underbrace{R(s, a, s') + \gamma \max_a \hat{Q}_\theta(s', a')}_{\text{TD target}} - \hat{Q}_\theta(s, a) \right] \frac{\partial \hat{Q}_\theta(s, a)}{\partial \theta_i}$$

- Notes:

- Also called semi-gradient as the target is not a true value, depends on θ
- For passive TD learning, update rule converges for linear function when using on-policy

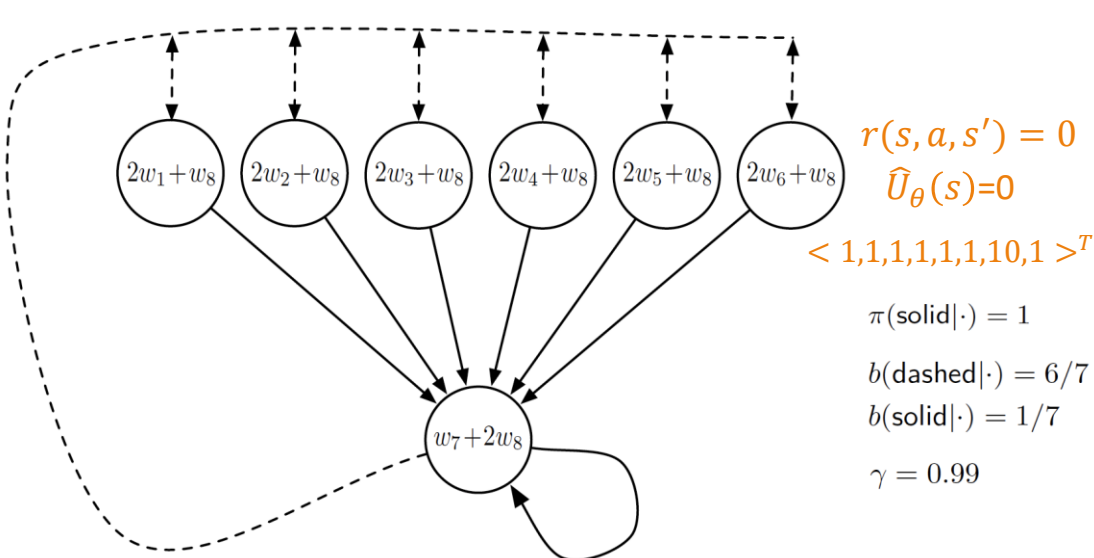


The Deadly Triad

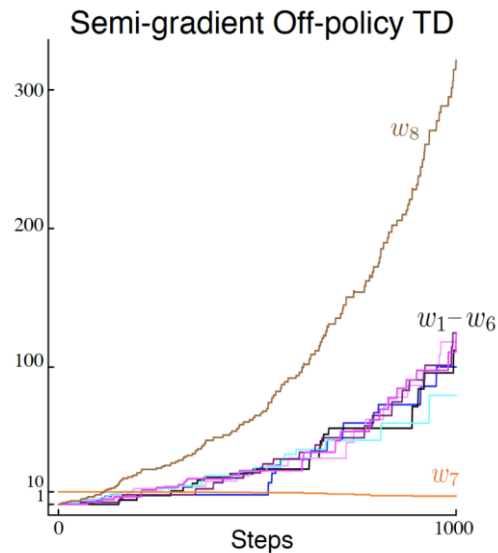
- **Challenges for active learning and non-linear functions:**
 - Instability and divergence may arise when following 3 elements are combined:
- **Function approximation:**
 - Required when state space is large, e.g., using linear function approximation or deep neural nets
- **Bootstrapping:**
 - Using existing estimates as targets, e.g., in TD, rather than complete returns like in MC methods
- **Off-policy training:**
 - Training on transitions other than those produced by the target policy

Example: Baird's Counterexample

- Off-policy TD with linear function approximation diverges



Source: SB Figure 11.1



Source: SB Figure 11.2



Catastrophic Forgetting

- Problems with over-training
 - Forgotten about the “dangerous zones” of the learning regions
- Potential solution: Experience replay
 - Retain “relevant” training examples or trajectories from entire learning process
 - Replay those trajectories to ensure utility or value function is still accurate for parts of state space it no longer visits



Non-Linear Functional Approximation

Deep reinforcement learning

Deep Reinforcement Learning

- Deep neural network in function approximation

- Discovers useful features by itself
- “Transparent” to show selected features if last network layer is linear

$$\hat{U}_{\theta}(s) = \theta_1 f_1(s) + \theta_2 f_2(s) + \dots + \theta_n f_n(s)$$

- Parameters are all weights in all the layers of the network
- Gradients required the same for supervised learning, computed by back propagation

Deep Reinforcement Learning

- Learning the parameters:

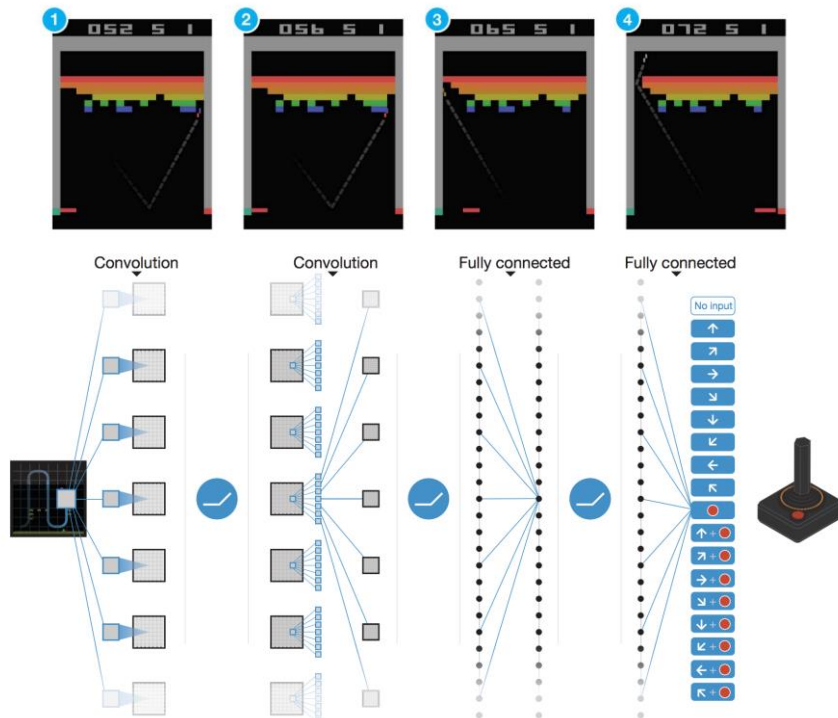
- For utilities:

$$\theta_i \leftarrow \theta_i + \alpha [R(s, a, s') + \gamma \hat{U}_\theta(s') - \hat{U}_\theta(s)] \frac{\partial \hat{U}_\theta(s)}{\partial \theta_i}$$

- For Q -learning:

$$\theta_i \leftarrow \theta_i + \alpha \left[R(s, a, s') + \gamma \max_a \hat{Q}_\theta(s', a') - \hat{Q}_\theta(s, a) \right] \frac{\partial \hat{Q}_\theta(s, a)}{\partial \theta_i}$$

Deep Q -learning for Atari Games¹



Video: <https://youtu.be/TmPfTptdgg>

¹Mnih, V., et al., *Human-level control through deep reinforcement learning*. Nature, 2015. **518**(7540): p. 529-533.

Deep Q-Network (DQN)

- Uses deep neural networks with Q -learning to play 49 Atari games
 - Online Q -learning with non-linear function approximators is unstable and may diverge
- DQN uses experience replay with fixed Q -targets:
 - Take action a_t using ϵ -greedy policy
 - Store $(s_t, a_t, r_{t+1}, s_{t+1})$ in a large buffer D of most recent transitions
 - Sample a random mini-batch (s, a, r, s') from D
 - Set targets to $r + \gamma \max_{a'} Q(s', a', \theta^-)$
 - Do gradient step on the minibatch squared loss w.r.t θ – Optimize MSE btw Q-network and Q-learning targets:
$$\mathcal{L}_i(\theta_i) = E_{s,a,r,s' \sim D_i} \left[\left(r(s, a, s') + \gamma \max_{a'} Q(s', a'; \theta_i^-) - Q(s, a; \theta_i) \right)^2 \right]$$
 - Set θ^- to θ every C steps
- Experience replay and fixed target
 - Help reduce instability by making input less correlated

Deep Q-Network (DQN)

Algorithm 1: deep Q-learning with experience replay.

Initialize replay memory D to capacity N

Initialize action-value function Q with random weights θ

Initialize target action-value function \hat{Q} with weights $\theta^- = \theta$

For episode = 1, M **do**

Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$

For $t = 1, T$ **do**

With probability ϵ select a random action a_t

otherwise select $a_t = \operatorname{argmax}_a Q(\phi(s_t), a; \theta)$

Execute action a_t in emulator and observe reward r_t and image x_{t+1}

Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in D

Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from D

Set $y_j = \begin{cases} r_j & \text{if episode terminates at step } j+1 \\ r_j + \gamma \max_{a'} \hat{Q}(\phi_{j+1}, a'; \theta^-) & \text{otherwise} \end{cases}$

Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ with respect to the network parameters θ

Every C steps reset $\hat{Q} = Q$

End For

End For

- Optimise MSE between Q-network and Q-learning targets

$$\mathcal{L}_i(w_i) = \mathbb{E}_{s,a,r,s' \sim \mathcal{D}_i} \left[\left(r + \gamma \max_{a'} Q(s', a'; w_i^-) - Q(s, a; w_i) \right)^2 \right]$$

Source: Silver, D. Lecture 6 Notes on RL, 2015.

← Take action a_t using ϵ -greedy policy

← Store $(s_t, a_t, r_{t+1}, s_{t+1})$ in a large buffer D

← Sample a random mini-batch (s, a, r, s') from D

← Set targets to $r + \gamma \max_{a'} Q(s', a', \theta^-)$

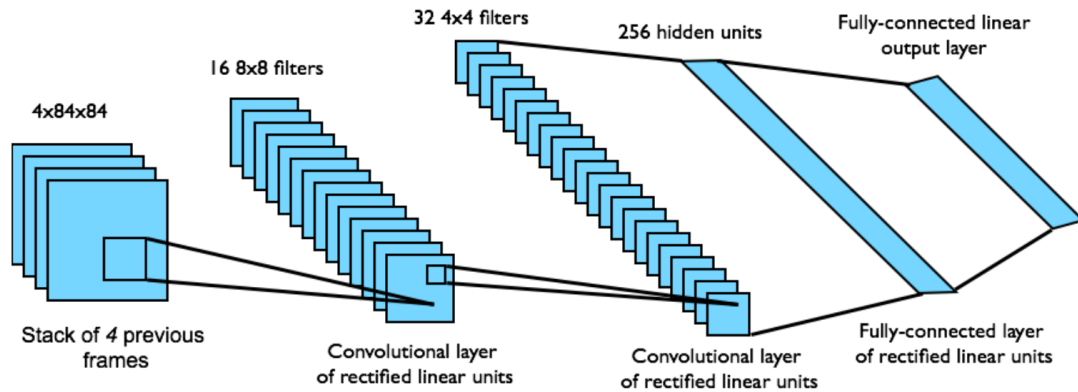
← Do gradient step on minibatch squared loss w.r.t θ

← Set θ^- to θ every C steps

Source: Mnih et al., Nature 2015

DQN in Atari

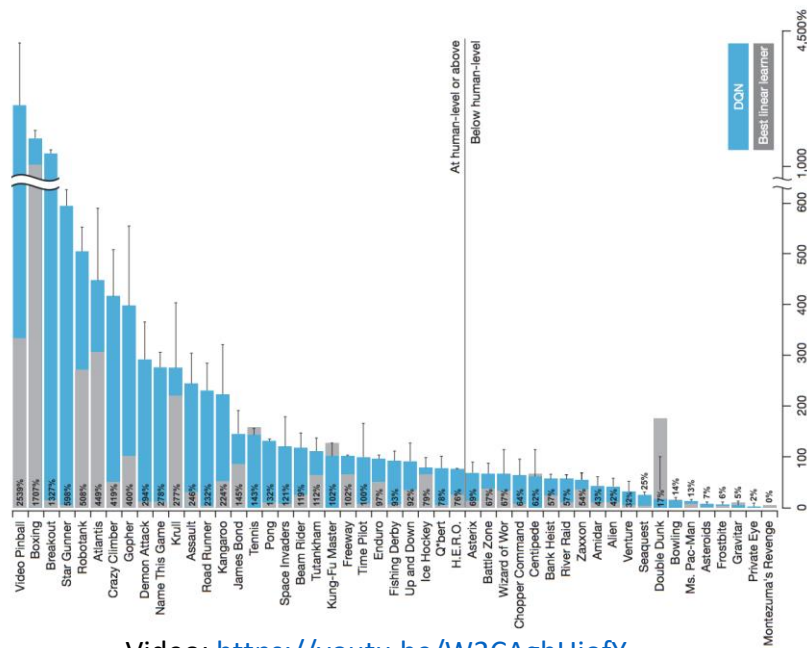
- End-to-end learning of values $Q(s, a)$ from pixels s
- Input state s is stack of raw pixels from last 4 frames
- Output is $Q(s, a)$ for 18 joystick positions
- Reward is change in score for that step



Network architecture and hyperparameters fixed across all games

Source: Silver, D. Lecture 6 Notes on RL, 2015, p 36.

Deep Q -learning for Atari Games¹



Video: <https://youtu.be/W2CAghUiofY>

¹Mnih, V., et al., *Human-level control through deep reinforcement learning*. Nature, 2015. **518**(7540): p. 529-533.

AlphaGo²



Video: <https://youtu.be/WXuK6gekU1Y>

²Silver, D., et al., *Mastering the game of Go with deep neural networks and tree search*. Nature, 2016. **529**: p. 484+.

AlphaGo²

- Go Game
 - Space size of about 10^{170} with branching factor that starts at 361
 - Difficult to define good evaluation function
 - Need function approximation to represent value and policy functions
- AlphaGo² used deep reinforcement learning to beat best human players
 - A Q-function with no look-ahead suffices for Atari games
 - Go requires substantial lookahead.
 - AlphaGo learned both a value function and a Q -function that guided its search by predicting which moves are worth exploring.
 - Q -function implemented as a convolutional neural network – accurate enough by itself to beat most amateur human players without search.

²Silver, D., et al., *Mastering the game of Go with deep neural networks and tree search*. Nature, 2016. **529**: p. 484+.

Deep Reinforcement Learning: Reality

“

Despite its impressive successes, deep RL still faces significant obstacles: it is often difficult to get good performance and the trained system may behave very unpredictably if the environment differs even a little from the training data.

Deep RL is rarely applied in commercial settings. It is, nonetheless, a very active area of research.

”

Russell, Stuart; Norvig, Peter. Artificial Intelligence (Pearson Series in Artificial Intelligence) (p. 858). Pearson Education. 4th Edition.