FUNDAMENTAL STELLAR PARAMETERS DERIVED FROM THE EVOLUTIONARY TRACKS

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(Received 13 April, 1981)

Abstract. The surface gravities and radii of stars are calculated for different MK spectral types using the masses of stars determined from their evolutionary tracks in the HR diagram and the most reliable values of effective temperatures and absolute bolometric magnitudes. MK spectral types are calibrated in absolute visual magnitudes using the studies of M_V published since 1965. The calibration of MK types in temperatures is based on the newest investigations including the results both from the ultraviolet and the infrared. The obtained masses, gravities, and the mass-luminosity relationship show reasonable agreement with independent observational data.

1. Introduction

The importance of an exact knowledge of stellar temperatures and surface gravities is continuously increasing as they are indispensable in comparing theoretical calculations of stellar evolutionary tracks and model atmospheres with observational data. The importance of gravities increased considerably after realization that they can replace absolute magnitudes in the HR diagram (Newell, 1973). The gravities are easily determinable in most of photometric systems but one needs a homogeneous set of stars with well-known $\log g$ values to calibrate the photometric diagrams. The calibration of such diagrams by using model stellar atmospheres in most cases gives insufficient accuracy.

The transfer from the $\log L/L_\odot - \log T_e$ to $\log g - \log T_e$ plane and the calibration of photometric quantities could be considerably facilitated by using MK spectral types calibrated in temperatures and surface gravities. During the last decades considerable efforts were directed to calibrate MK types in temperatures. At the same time the calibration in gravities is far from satisfactory. The plots of $\log g$, determined from a comparison of observed line profiles with those given by model atmospheres, against the spectral type for different luminosity classes show considerable dispersion. This may be caused by insufficient accuracy of theoretical profiles predicted by model stellar atmospheres and by the absence of one-to-one dependence between the spectral type and $\log g$ for a given luminosity class, because instead of continuous luminosity scale, one uses discrete luminosity classes. On the other hand, the luminosity of a star is a function, not only of $\log g$ and temperature, but also depends on the mass. In other words, two stars lying at the same point of the HR diagram will have different $\log g$ values, if their masses differ. The overlap of different masses

Astrophysics and Space Science **80** (1981) 353–368. 0004–640X/81/0802–0353$02.40.

in the HR diagram can be minimized by restricting ourselves to the stars within certain limits of chemical composition and by excluding objects in the pre-main-sequence stage as well as in advanced evolutionary stages, such as horizontal or post-horizontal branch stars.

The most appropriate to our mind method to determine mean $\log g$ values for different MK spectral types was used by Kopylov (1967). It is based on the calculation of $\log g$ values for stars in different parts of the $\log L/L_{\odot}$, $\log T_e$ plane, their masses being estimated from the theoretical evolutionary tracks. Before using this method we decided to start with the revision of calibration of MK spectral types in absolute magnitudes.

2. The Calibration of MK Spectral Types in M_V and M_{bol}

The calibrations of MK types in M_V have been reviewed by Keenan and Morgan (1951), Roman (1952), Arp (1958), Keenan (1963), Blaauw (1963), Boulon (1963), Allen (1955, 1963, and 1973), and Schmidt-Kaler (1965). They were based on an increasing number of investigations by different methods. Despite the continuing efforts to increase the accuracy of MK calibration in different parts of the HR diagram, no new summary of calibration results was published during the last ten years. To fill in the gap we collected all the studies of M_V for different MK types made during this period and plotted their results in the diagram M_V, spectral type for different luminosity classes after making the necessary corrections. First, all the determinations of mean M_V for a given spectral type using the material down to limiting apparent magnitude, were transformed into the scale 'per volume of space' adding to their M_V the constant +0.6 for late type giants and +0.5 for the remaining stars. The constants are based on the estimates by Blaauw (1963 and 1973) and Ljuggren and Oja (1965). Secondly, all the determinations of M_V obtained using cluster distances were corrected to agree with the new Hyades distance modulus $V - M_V = 3.2$. This rather conservative modulus was accepted after a critical examination of the most recent studies (Hanson, 1975 and 1980; McAlister, 1977; Anthony-Twarog and Demarque, 1977; Upgren, 1980).

All the literature sources used are listed in Table I together with the adopted corrections. After plotting M_V against the spectral class (ten subclasses were used throughout except for K-type stars) an average smooth curve was drawn through the points and readings were taken at spectral classes which are common in the MK system. The results are presented in Table II. The absolute magnitudes for dwarfs are cut at M5 V as the MK classification system for later dwarfs is not completely defined (Wing, 1973; Wing and Yorka, 1979).

Nearly all the used sources were published after 1965, so that our calibration is almost completely independent of the Schmidt-Kaler's (1965) work, except for early-type luminosity II stars and late-type luminosity IV stars. The zero-age Main Sequence for spectral classes earlier than F0 was taken from theoretical

TABLE I

The literature sources used to derive the mean absolute magnitudes M_V for MK spectral types

References	Method	Correction	Spectral interval
Schmidt-Kaler (1965)	Cluster membership	−0.2	B–A–F II
	Trigon. parallaxes	–	F–G–K IV
Ljuggren and Oja (1965)	Trigon. parallaxes	–	G8–K5 III
Greenstein *et al.* (1970)	Trigon. parallaxes	–	M4–M6 V
Jung (1970)	Statist. parallaxes	+0.5	B–A–F–G8 V, III
		+0.6	G5–K–M III
Jung (1971)	Trigon. parallaxes	+0.5	A–F–G V
		+0.6	G8–K5 III
Woolley *et al.* (1971)	Trigon. parallaxes	–	A0–M5 V
Gliese (1971)	Trigon. parallaxes	–	F5–M2 V
Stothers and Leung (1971)	Cluster membership	−0.2	M0–M4 I
Stothers (1972)	Association membership	−0.2	O–M I
Walborn (1972)	Cluster membership	−0.2	O3–B2 V–I
Walborn (1973)	Cluster membership	−0.2	O3–B0 V–I
Keenan (1973)	Cluster and double star membership	−0.2	F8–M4 I–II
Kholopov (1980)	Cluster sequence fitting	+0.1	ZAMS
Mikami (1978a)	Statist. parallaxes	+0.6	M0–M8 III
Mikami (1978b)	Statist. parallaxes	+0.5	F–G–K V
		+0.6	G–K III
Kennan (1978)	Trigon. parallaxes	+0.6	G8–M2 III
Lesh (1979)	Cluster membership	−0.2	O3–B5 V–III

calculations by Hejlesen (1980) for $X = 0.70$, $Z = 0.03$ transforming the log $L/L_\odot$ data to M_V with bolometric corrections from Table III. The sequence derived from the Hejlesen's data was extrapolated to higher temperatures using the ZAMS points from Ziolkowski (1972) with the same chemical composition. This theoretical sequence is in good agreement with the new observational ZAMS determined by Kholopov (1980) by cluster sequence fitting method after the transformation of used cluster sequences to the Hyades metallicity. It also perfectly envelopes from below the observational diagram (M_V, spectral type) for O–B stars determined by Stothers and Chin (1977). The ZAMS for later spectral types was drawn from the Hejlesen point at F0 to the luminosity V point at G2. For spectral types later than G2 the ZAMS was considered to coincide with the luminosity V sequence.

To transform M_V into M_{bol} we used the bolometric corrections given in Table III. For B–A–F stars they are based on Code (1975) and Code *et al.* (1976) results, and for O-type stars they are taken from Buser and Kurucz (1978) calculations for model atmospheres. For G–K-type stars the *BC* originate from Johnson (1966) and for M-type giants and supergiants from Lee (1970), Flower

TABLE II

Adopted calibration of MK spectral types in absolute magnitudes M_V

Sp	ZAMS	V	IV	III	II	Ib	Iab	Ia
O5	−4.6	−5.6	−5.8	−6.0	−6.3	−6.6	−6.9	−7.2
O6	−4.0	−5.4	−5.7	−5.9	−6.3	−6.6	−6.9	−7.2
O7	−3.9	−5.2	−5.5	−5.8	−6.2	−6.5	−6.8	−7.2
O8	−3.7	−4.9	−5.2	−5.6	−6.1	−6.4	−6.7	−7.2
O9	−3.5	−4.5	−4.9	−5.3	−5.9	−6.3	−6.6	−7.2
B0	−3.1	−4.0	−4.4	−4.9	−5.6	−6.1	−6.5	−7.2
B1	−2.3	−3.3	−3.9	−4.5	−5.2	−5.9	−6.4	−7.2
B2	−1.6	−2.5	−3.1	−3.7	−5.0	−5.9	−6.4	−7.2
B3	−1.0	−1.7	−2.3	−3.0	−4.8	−5.9	−6.4	−7.2
B5	−0.1	−0.8	−1.2	−1.7	−4.6	−5.9	−6.4	−7.2
B6	0.3	−0.5	−0.9	−1.3	−4.4	−5.8	−6.4	−7.2
B7	0.6	−0.2	−0.6	−1.0	−4.2	−5.8	−6.4	−7.2
B8	1.0	0.1	−0.3	−0.7	−3.9	−5.8	−6.4	−7.2
B9	1.4	0.5	0.1	−0.4	−3.6	−5.7	−6.4	−7.2
A0	1.6	0.8	0.4	−0.1	−3.4	−5.5	−6.4	−7.2
A1	1.7	1.1	0.7	0.2	−3.2	−5.3	−6.4	−7.2
A2	1.8	1.3	0.9	0.4	−3.1	−5.2	−6.4	−7.3
A3	1.9	1.5	1.0	0.5	−3.0	−5.1	−6.4	−7.3
A5	2.3	1.9	1.4	0.8	−2.9	−5.0	−6.5	−7.5
A7	2.6	2.3	1.7	1.1	−2.8	−5.0	−6.7	−7.7
F0	3.0	2.8	2.2	1.5	−2.7	−5.0	−6.9	−7.9
F2	3.2	3.1	2.4	1.8	−2.6	−4.9	−7.0	−8.0
F5	3.7	3.6	2.6	2.0	−2.6	−4.8	−7.1	−8.0
F8	4.2	4.1	2.8		−2.5	−4.7	−7.2	−8.1
G0	4.5	4.4	2.9		−2.4	−4.6	−7.2	−8.2
G2		4.7	3.0	1.1:	−2.4	−4.5	−7.2	−8.2
G5		5.1	3.1	1.0	−2.4	−4.4	−7.2	−8.2
G8		5.6	3.2	0.9	−2.5	−4.3	−7.0	−8.1
K0		6.0	3.2	0.8	−2.5	−4.3	−6.8	−7.9
K1		6.2	3.2	0.8	−2.5	−4.3	−6.7	−7.7
K2		6.5		0.7	−2.5	−4.3	−6.6	−7.6
K3		6.7		0.6	−2.5	−4.3	−6.5	−7.5
K4		7.0		0.5	−2.6	−4.4	−6.4	−7.4
K5		7.3		0.3	−2.6	−4.4	−6.2	−7.2
K7		8.1		0.0	−2.7	−4.5	−6.0	−7.0
M0		8.9		−0.6	−2.8	−4.6	−5.8	−6.9
M1		9.4		−0.8	−2.9	−4.6	−5.8	−6.8
M2		10.0		−0.9	−3.0	−4.7	−5.8	−6.7
M3		10.5		−1.0	−3.0	−4.7	−5.8	−6.7
M4		11.5		−0.6	−3.1	−4.7	−5.8	−6.7
M5		13.5		−0.1	−3.1	−4.7	−5.8	−6.7

(1975), and Traat (1976). For M-type dwarfs they are taken from Johnson (1966) and Greenstein *et al.* (1970). The zero-point of the *BC* scale is adjusted to give $BC_{\odot} = -0.07$.

The obtained bolometric absolute magnitudes are listed in Table IV. When deriving M_{bol} for *M* dwarfs we have supplemented the material with the

TABLE III

Adopted temperatures and bolometric corrections for MK spectral types

	log $T_{\rm eff}$			Bol. Correction		
Sp	V	III	I–II	V	III	I–II
O5	4.626		4.618	−4.15		−3.80
O6	4.593		4.585	−3.90		−3.55
O7	4.568		4.556	−3.65		−3.30
O8	4.550		4.535	−3.40		−3.15
O9	4.525		4.512	−3.15		−2.95
B0	4.498		4.431	−2.95		−2.50
B1	4.423		4.371	−2.60		−2.15
B2	4.362		4.307	−2.20		−1.75
B3	4.286		4.243	−1.85		−1.40
B5	4.188		4.137	−1.30		−0.90
B6	4.152		4.100	−1.05		−0.75
B7	4.107		4.068	−0.80		−0.60
B8	4.061		4.041	−0.55		−0.45
B9	4.017		4.013	−0.35		−0.35
A0	3.982		3.991	−0.25		−0.25
A1	3.973		3.978	−0.16		−0.16
A2	3.961		3.964	−0.10		−0.10
A3	3.949		3.949	−0.03		−0.03
A5	3.924		3.919	0.02		0.05
A7	3.903		3.897	0.02		0.09
F0	3.863		3.869	0.02		0.13
F2	3.845		3.851	0.01		0.11
F5	3.813		3.813	−0.02		0.08
F8	3.789	3.782	3.778	−0.03		0.03
G0	3.774	3.763	3.756	−0.05		0.00
G2	3.763	3.740	3.732	−0.07		−0.05
G5	3.740	3.712	3.699	−0.09	−0.22	−0.13
G8	3.720	3.695	3.663	−0.13	−0.28	−0.22
K0	3.703	3.681	3.643	−0.19	−0.37	−0.29
K1	3.695	3.663	3.633		−0.43	−0.35
K2	3.686	3.648	3.623	−0.30	−0.49	−0.42
K3	3.672	3.628	3.613		−0.66	−0.57
K4	3.663	3.613			−0.86	−0.75
K5	3.643	3.602	3.585	−0.62	−1.15	−1.17
K7	3.602			−0.89		
M0	3.591	3.591	3.568	−1.17	−1.25	−1.25
M1	3.574	3.580	3.556	−1.45	−1.45	−1.40
M2	3.550	3.574	3.544	−1.71	−1.65	−1.60
M3	3.531	3.562	3.518	−1.92	−1.95	−2.0
M4	3.512	3.550	3.491	−2.24	−2.4	−2.6
M5	3.491	3.531	3.470	−2.55	−3.1	−3.3
M6		3.512		−4.4	−4.0	

TABLE IV

Bolometric absolute magnitudes M_{bol} for MK spectral types

Sp	ZAMS	V	IV	III	II	Ib	Iab	Ia
O5	−8.7	−9.8	−10.0	−10.2	−10.3	−10.4	−10.7	−11.0
O6	−8.0	−9.3	−9.6	−9.8	−9.9	−10.2	−10.4	−10.8
O7	−7.5	−8.8	−9.1	−9.3	−9.5	−9.8	−10.1	−10.5
O8	−7.2	−8.3	−8.6	−8.9	−9.2	−9.6	−9.8	−10.4
O9	−6.7	−7.6	−8.1	−8.4	−8.9	−9.3	−9.6	−10.2
B0	−6.2	−7.0	−7.4	−7.9	−8.1	−8.6	−9.0	−9.7
B1	−4.9	−5.8	−6.3	−6.8	−7.4	−8.0	−8.6	−9.4
B2	−4.0	−4.7	−5.3	−5.9	−6.8	−7.6	−8.2	−9.0
B3	−2.8	−3.6	−4.1	−4.7	−6.2	−7.3	−7.8	−8.6
B5	−1.4	−2.1	−2.5	−3.0	−5.4	−6.8	−7.3	−8.1
B6	−0.9	−1.6	−2.0	−2.4	−5.2	−6.6	−7.2	−7.9
B7	−0.2	−1.0	−1.4	−1.8	−4.8	−6.4	−7.0	−7.8
B8	0.4	−0.4	−0.8	−1.2	−4.4	−6.2	−6.9	−7.6
B9	1.0	0.1	−0.2	−0.8	−4.0	−6.0	−6.8	−7.5
A0	1.4	0.7	0.2	−0.3	−3.6	−5.7	−6.6	−7.4
A1	1.6	0.9	0.5	−0.1	−3.3	−5.5	−6.6	−7.4
A2	1.7	1.2	0.7	0.1	−3.1	−5.3	−6.5	−7.4
A3	1.9	1.5	1.0	0.4	−3.0	−5.2	−6.4	−7.4
A5	2.3	1.9	1.4	0.8	−2.8	−5.0	−6.4	−7.4
A7	2.6	2.3	1.8	1.1	−2.7	−4.9	−6.5	−7.6
F0	3.0	2.9	2.2	1.6	−2.6	−4.8	−6.7	−7.8
F2	3.2	3.1	2.4	1.8	−2.5	−4.8	−6.8	−7.9
F5	3.7	3.6	2.6	2.0	−2.5	−4.7	−7.0	−7.9
F8	4.2	4.1	2.8		−2.4	−4.6	−7.1	−8.0
G0	4.4	4.4	2.9		−2.4	−4.6	−7.2	−8.1
G2	4.6	4.6	2.9	1.0	−2.4	−4.6	−7.2	−8.2
G5		5.1	3.0	0.8	−2.5	−4.5	−7.3	−8.3
G8		5.5	3.1	0.6	−2.7	−4.5	−7.2	−8.3
K0		5.8	3.0	0.5	−2.8	−4.6	−7.1	−8.2
K1		5.9	3.0	0.4	−2.9	−4.6	−7.1	−8.1
K2		6.0		0.2	−3.0	−4.7	−7.0	−8.0
K3		6.2		−0.1	−3.1	−4.9	−7.0	−8.0
K4		6.4		−0.4				
K5		6.7		−0.9	−3.7	−5.4	−7.0	−8.0
K7		7.3						
M0		7.5		−1.8	−4.0	−5.8	−7.0	−8.1
M1		7.9		−2.4	−4.3	−6.0	−7.2	−8.2
M2		8.3		−2.6	−4.5	−6.2	−7.4	−8.3
M3		8.8		−2.9	−5.1	−6.7	−7.8	−8.7
M4		9.3		−3.1	−5.7	−7.3	−8.4	−9.3
M5		11.0		−3.2	−6.3	−8.0	−9.1	−10.0
M6				−3.6				

determinations of M_{bol} by Veeder (1974) and Mould and Hyland (1976) obtained for individual stars from infrared photometry.

3. The Evolutionary Tracks and the Calibration of MK Spectral Types in Masses, Gravities and Radii

To calibrate the diagram M_{bol}, $\log T_e$ in masses we have used evolutionary tracks and ZAMS models of stars from the sources listed in Table V. Their choice was based on the requirement to embrace a sufficiently wide range of masses and temperatures with almost identical chemical composition. The abundance of elements heavier than helium, Z, was taken to be 0.025–0.03 in most of the used models. This metallicity corresponds to young metal-rich stars of the Hyades type rather than to the Sun, for which we obtain $Z = 0.017$ using the element abundances from Hauge and Engvold (1977) with corrected logarithmic abundance of carbon $A = 8.7$. In the overlapping parts the results of different authors show reasonable agreement. The tracks of the stars of masses 0.5–3 $\mathfrak{M}_\odot$ extend up to the core helium ignition stage in the red giant region. The evolutionary tracks during the central helium burning stage are also included for more massive stars.

For a comparison of theoretical evolutionary tracks with the observed positions of MK types in the HR diagram we need MK spectral type calibration in temperatures. The adopted effective temperatures for different spectral and luminosity classes are given in Table III. For the Main Sequence stars of spectral classes B–A–F–G2 as well as for B supergiants the temperatures closely correspond to the Code *et al.* (1976) scale. For O-type stars T_e are taken from Underhill *et al.* (1978 and 1979) and Conti (1973). For G–K-type dwarfs the temperature scale is taken from Blackwell *et al.* (1980) and Johnson (1966). For K and M-type giants the temperatures are from Ridgway *et al.* (1980). They are

TABLE V

The sources of evolutionary tracks used to derive stellar masses

Source	Chemical composition (X, Z)	Range of masses in $\mathfrak{M}_\odot$
Mengel *et al.* (1979)	0.70, 0.025	0.55–3.5
Sweigart and Gross (1978)	0.70, 0.025	0.7–2.2
Hejlesen (1980)	0.70, 0.03	0.5–10
Paczynski (1970)	0.70, 0.03	0.8–15
Alcock and Paczynski (1978)	0.70, 0.03	2–10
Ziolkowski (1972)	0.70, 0.03	15–60
Chiosi *et al.* (1978)[a]	0.70, 0.02	20–100
Grossman *et al.* (1974)	0.68, 0.03	0.085–0.5

[a]Evolution of stars with mass loss $\alpha = 0.83$.

consistent with Tsuji (1978) and Piccirillo *et al.* (1980) scales from model atmosphere calculations. For G-type giants mean temperatures were determined by us from their $R-I$ taking Johnson (1966) temperature scale. For M-type dwarfs we used averaged values of individual stars from Veeder (1974) and Mould and Hyland (1976). For M-type supergiants we took the Johnson's scale. The mean temperatures of A–K supergiants are based on T_e values of individual stars collected from numerous photometric and spectrophotometric studies.

Code *et al.* (1976) have shown that among the early-type stars the temperature calibration of the index $(B-V)_0$ is the same both for luminosity V and III stars. This leads to the equality of temperatures of the same spectral classes of both luminosities. We considered the temperature scale to be the same for the ZAMS and luminosity V, IV, and III stars of spectral classes from O to F5. On the other hand, we took coinciding temperatures for supergiants of luminosity classes Ia, Iab, Ib, and II, as temperature scales are not yet established for different supergiant luminosity subdivisions separately.

The evolutionary tracks were plotted in the diagram $M_{\rm bol}$, $\log T_e$ together with the points representing different MK types taking $M_{\rm bol}$ from Table IV and T_e from Table III. The theoretical $\log L/L_\odot$ values were transformed into $M_{\rm bol}$ using $M_{\rm bol\odot} = +4.72$. Then using the interpolation techniques, the mass values for each MK spectral type were determined and are given in Table VI. In the case of massive stars in the core helium burning stage with long loops in the HR diagram there is some ambiguity in the mass-luminosity relationship. When making interpolation between the looping evolutionary tracks, the time being spent by the star on every branch of the evolutionary track was taken into consideration.

The knowledge of masses, luminosities, and temperatures for stars of different MK spectral types allows us to calculate their gravities and radii by the formulae

$$\log g = \log \mathfrak{M} + 4 \log T_e + 0.4\, M_{\rm bol} - 12.49$$

and

$$\log R/R_\odot = 8.46 - 2 \log T_e - 0.2\, M_{\rm bol}\,,$$

which are based on the assumption of the following parameters for the Sun: $T_e = 5780$ K, $\log g = 4.44$, and $M_{\rm bol} = +4.72$. The results of $\log g$ and $\log R/R_\odot$ calculations are given in Tables VII and VIII.

4. Comparison with Other Determinations and Discussion

There is a number of other methods to determine masses, gravities, and radii of stars. Let us compare our results with parameter determinations which are independent of the evolution theory.

The masses of binary star components derived from visual binary orbits in conjunction with trigonometric parallaxes and those derived from eclipsing binary orbits in conjunction with double-lined spectra were recently summarized

TABLE VI

Stellar masses $\log \mathfrak{M}/\mathfrak{M}_\odot$ for different MK spectral types derived from the evolutionary tracks

Sp	ZAMS	V	IV	III	II	Ib	Iab	Ia
O5	1.60	1.81	1.85	1.89	1.90	1.92	1.99	
O6	1.48	1.70	1.76	1.80	1.80	1.87	1.91	2.00
O7	1.40	1.59	1.65	1.68	1.71	1.76	1.83	1.92
O8	1.34	1.48	1.54	1.60	1.65	1.72	1.76	1.90
O9	1.28	1.38	1.45	1.49	1.58	1.66	1.72	1.83
B0	1.20	1.30	1.34	1.40	1.40	1.48	1.56	1.70
B1	1.04	1.11	1.18	1.23	1.28	1.38	1.46	1.64
B2	0.92	0.99	1.04	1.08	1.18	1.30	1.38	1.54
B3	0.78	0.84	0.88	0.94	1.11	1.23	1.32	1.45
B5	0.62	0.68	0.72	0.75	1.00	1.18	1.26	1.40
B6	0.56	0.61	0.64	0.68	0.94	1.15	1.26	1.38
B7	0.49	0.53	0.57	0.60	0.91	1.11	1.23	1.36
B8	0.43	0.48	0.49	0.52	0.88	1.08	1.20	1.34
B9	0.36	0.41	0.45	0.49	0.85	1.04	1.20	1.32
A0	0.32	0.35	0.39	0.43	0.81	1.04	1.18	1.30
A1	0.31	0.34	0.36	0.41	0.78	1.00	1.18	1.30
A2	0.29	0.32	0.34	0.39	0.75	0.98	1.15	1.30
A3	0.27	0.30	0.32	0.36	0.75	0.97	1.11	1.30
A5	0.23	0.26	0.29	0.33	0.74	0.95	1.11	1.30
A7	0.20	0.22	0.26	0.30	0.73	0.94	1.15	1.32
F0	0.16	0.16	0.20	0.23	0.72	0.93	1.20	1.38
F2	0.13	0.13	0.16	0.20	0.72	0.93	1.20	1.40
F5	0.08	0.08	0.13	0.18	0.72	0.93	1.26	1.40
F8	0.04	0.04	0.11		0.72	0.93	1.28	1.41
G0	0.02	0.02	0.10		0.72	0.93	1.30	1.43
G2	0.00	0.00	0.10	0.33	0.72	0.93	1.30	1.45
G5		−0.02	0.08	0.39	0.73	0.94	1.32	1.46
G8		−0.04	0.08	0.42	0.76	0.94	1.32	1.46
K0		−0.07	0.11	0.46	0.78	0.96	1.30	1.45
K1		−0.10	0.13	0.46	0.78	0.96	1.30	1.45
K2		−0.10		0.45	0.79	0.98	1.28	1.43
K3		−0.12		0.38	0.80	1.00	1.30	1.43
K4		−0.15		0.36				
K5		−0.19		0.37	0.83	1.08	1.30	1.45
K7		−0.22						
M0		−0.26		0.48	0.83	1.15	1.32	1.46
M1		−0.30		0.54	0.83	1.18	1.34	1.48
M2		−0.35		0.54	0.81	1.18	1.36	1.50
M3		−0.40		0.52	0.84	1.20	1.38	1.56
M4		−0.52		0.51				
M5		(−0.82)		(0.41)				
M6				(0.40)				

TABLE VII

Calibration of MK spectral types in surface gravities (log g)

Sp	ZAMS	V	IV	III	II	Ib	Iab	Ia
O5	4.13	3.90	3.86	3.82	3.76	3.74	3.69	
O6	4.16	3.86	3.80	3.76	3.69	3.64	3.60	3.53
O7	4.18	3.85	3.80	3.74	3.64	3.57	3.52	3.45
O8	4.17	3.87	3.81	3.75	3.62	3.53	3.49	3.39
O9	4.21	3.95	3.82	3.74	3.58	3.50	3.44	3.31
B0	4.22	4.00	3.88	3.74	3.39	3.27	3.19	3.05
B1	4.28	4.00	3.86	3.71	3.31	3.17	3.01	2.87
B2	4.28	4.06	3.88	3.68	3.19	3.00	2.84	2.68
B3	4.31	4.06	3.89	3.71	3.12	2.79	2.68	2.49
B5	4.32	4.10	3.98	3.81	2.90	2.52	2.40	2.22
B6	4.32	4.09	3.96	3.84	2.77	2.42	2.29	2.13
B7	4.35	4.07	3.95	3.82	2.77	2.33	2.21	2.02
B8	4.34	4.07	3.92	3.79	2.79	2.27	2.11	1.97
B9	4.34	4.03	3.94	3.75	2.81	2.20	2.04	1.88
A0	4.32	4.07	3.91	3.75	2.85	2.23	2.01	1.81
A1	4.35	4.10	3.96	3.78	2.88	2.22	1.96	1.76
A2	4.32	4.16	3.98	3.78	2.87	2.23	1.92	1.71
A3	4.34	4.20	4.03	3.83	2.85	2.20	1.86	1.65
A5	4.36	4.22	4.06	3.86	2.81	2.14	1.74	1.53
A7	4.36	4.26	4.10	3.86	2.75	2.08	1.65	1.38
F0	4.32	4.28	4.05	3.83	2.67	2.00	1.51	1.25
F2	4.30	4.26	4.01	3.81	2.63	1.92	1.39	1.15
F5	4.32	4.28	3.93	3.74	2.48	1.81	1.22	1.00
F8	4.39	4.35	3.89		2.38	1.71	1.06	0.83
G0	4.39	4.39	3.84		2.29	1.62	0.95	0.72
G2	4.40	4.40	3.77	3.20	2.20	1.53	0.86	0.61
G5		4.49	3.71	3.07	2.04	1.45	0.71	0.45
G8		4.55	3.64	2.95	1.84	1.30	0.60	0.30
K0		4.57	3.57	2.89	1.74	1.20	0.54	0.25
K1		4.55	3.55	2.78	1.66	1.16	0.54	0.25
K2		4.55		2.63	1.59	1.10	0.48	0.23
K3		4.56		2.36	1.52	1.00	0.46	0.19
K4		4.57		2.16				
K5		4.57		1.93	1.20	0.77	0.35	0.10
K7		4.62						
M0		4.61		1.63	1.01	0.61	0.30	0.00
M1		4.67		1.41	0.84	0.51	0.19	−0.07
M2		4.69		1.31	0.70	0.39	0.09	−0.13
M3		4.71		1.12	0.38	0.10	−0.16	−0.34
M4		4.77		0.98				
M5		5.06		(0.76)				
M6				(0.52)				

TABLE VIII

Stellar radii log $R/R_{\odot}$ for different MK spectral types

Sp	ZAMS	V	IV	III	II	Ib	Iab	Ia
O5	0.95	1.17	1.21	1.25	1.28	1.30	1.36	
O6	0.87	1.13	1.19	1.23	1.27	1.33	1.37	1.45
O7	0.82	1.08	1.14	1.18	1.25	1.31	1.37	1.45
O8	0.80	1.02	1.08	1.14	1.23	1.31	1.35	1.47
O9	0.75	0.93	1.03	1.09	1.22	1.30	1.36	1.48
B0	0.70	0.86	0.94	1.04	1.20	1.32	1.40	1.54
B1	0.59	0.77	0.87	0.97	1.20	1.32	1.44	1.60
B2	0.54	0.68	0.80	0.92	1.21	1.37	1.49	1.65
B3	0.45	0.61	0.71	0.83	1.21	1.43	1.53	1.69
B5	0.36	0.50	0.58	0.68	1.27	1.55	1.65	1.81
B6	0.34	0.48	0.56	0.64	1.30	1.58	1.70	1.84
B7	0.29	0.45	0.53	0.61	1.28	1.60	1.72	1.88
B8	0.26	0.42	0.50	0.58	1.26	1.62	1.76	1.90
B9	0.23	0.41	0.47	0.59	1.23	1.63	1.79	1.93
A0	0.22	0.36	0.46	0.56	1.20	1.62	1.80	1.96
A1	0.19	0.33	0.41	0.53	1.16	1.60	1.82	1.98
A2	0.20	0.30	0.40	0.52	1.15	1.59	1.83	2.01
A3	0.18	0.26	0.36	0.48	1.16	1.60	1.84	2.04
A5	0.15	0.23	0.33	0.45	1.18	1.62	1.90	2.10
A7	0.13	0.19	0.29	0.43	1.21	1.65	1.97	2.19
F0	0.13	0.15	0.29	0.41	1.24	1.68	2.06	2.28
F2	0.13	0.15	0.29	0.41	1.26	1.72	2.12	2.34
F5	0.09	0.11	0.31	0.43	1.30	1.77	2.23	2.41
F8	0.04	0.06	0.33		1.38	1.82	2.32	2.50
G0	0.03	0.03	0.34		1.43	1.87	2.39	2.57
G2	0.01	0.01	0.38	0.78	1.48	1.92	2.44	2.64
G5		−0.04	0.41	0.88	1.56	1.96	2.52	2.72
G8		−0.08	0.43	0.95	1.67	2.03	2.57	2.79
K0		−0.11	0.48	1.00	1.73	2.09	2.59	2.81
K1		−0.11	0.50	1.05	1.77	2.11	2.61	2.81
K2		−0.11		1.12	1.81	2.15	2.61	2.81
K3		−0.12		1.22	1.85	2.21	2.63	2.83
K4		−0.15		1.31				
K5		−0.17		1.44	2.03	2.37	2.69	2.89
K7		−0.20						
M0		−0.22		1.64	2.12	2.48	2.72	2.92
M1		−0.27		1.78	2.21	2.55	2.78	2.99
M2		−0.30		1.83	2.27	2.61	2.85	3.03
M3		−0.36		1.92	2.44	2.76	2.98	3.16
M4		−0.42		1.98				
M5		−0.72		(2.04)				
M6				(2.16)				

by Popper (1980). In Figure 1 we plot the masses from his paper against the masses from our Table VI taken at the corresponding spectral types. The majority of stars belong to the Main Sequence. Both types of masses are in reasonable agreement, only early B-type stars and M3–M5 stars of the Main Sequence show some systematic displacement. However, the importance of this systematic effect may be questioned as other sets of binary star data (Heintze, 1973; Dzervitis, 1973; Lacy, 1979) show somewhat differing results. Moreover, the relation between the masses may be influenced by the errors of spectral types and luminosities of binary star components which often are very uncertain. The influence of spectral types can be eliminated by comparing the mass-luminosity relationships. In Figure 2 we plot this relationship from Tables IV and VI for the Main Sequence stars and from Popper (1980) for the binary star components. The agreement is excellent. In the same Figure the curve giving the mass-luminosity relationship from Heintze (1973) is plotted. It also perfectly fits our results, except for M-type dwarfs.

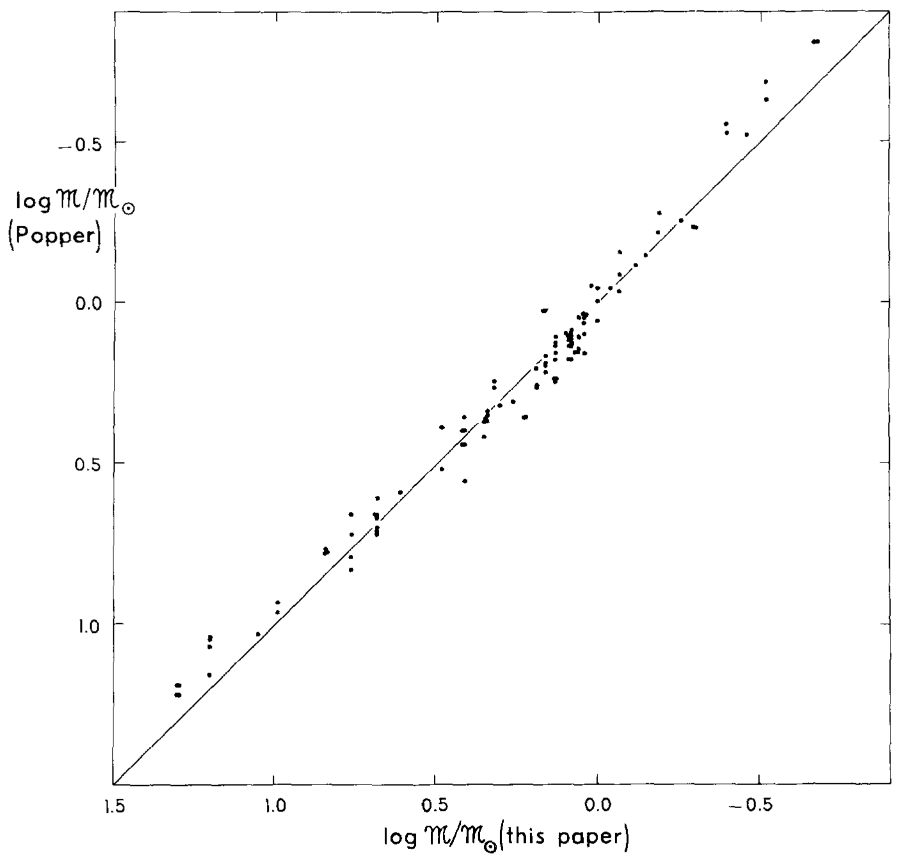

Fig. 1. Stellar masses derived in this paper from evolutionary tracks compared with those obtained by Popper (1980) for binary-star components.

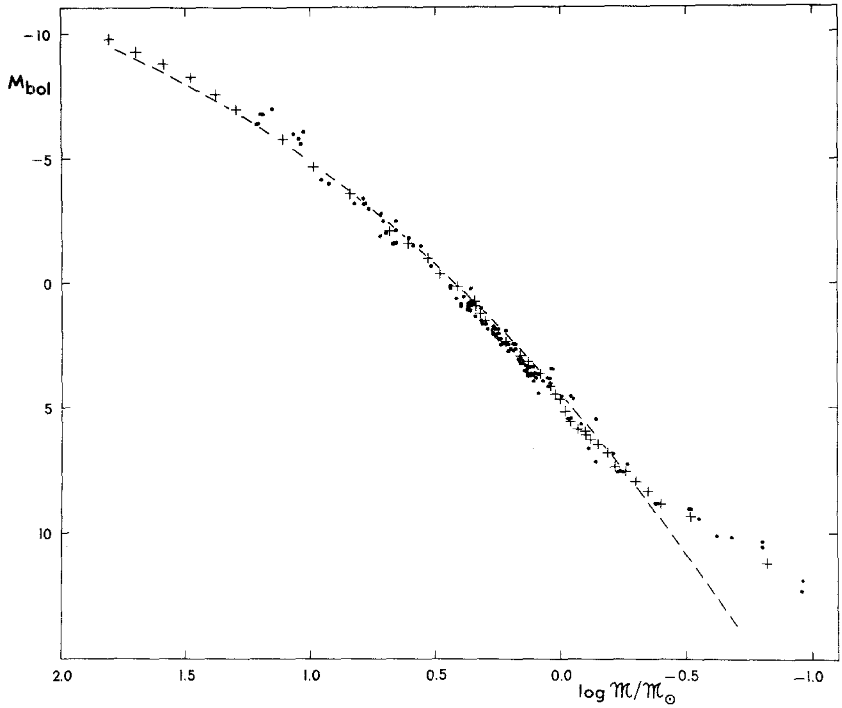

Fig. 2. Mass-luminosity relationship for the luminosity V stars from this paper (plus signs), from Popper (1980) (points) and from Heintze (1973) (broken line).

It is not easy to compare the evolutionary masses of stars of other luminosities with the observational data, due to the absence of reliable data from binaries. However, Piskunov (1977) concludes that some existing determinations of masses of giants and supergiants agree with the evolutionary masses within $\pm 30\%$.

The most extensive list of gravities containing ~70 mostly main-sequence stars determined from eclipsing and visual binaries was published by Heintze (1973). The plot of his $\log g$ against our values for the same spectral types shows dispersion characterized by the mean square error, σ of the order of ± 0.15. Much greater dispersion ($\sigma \approx \pm 0.4$) was found between our values and the spectroscopic $\log g$ determined for 52 F–G–K–M supergiants by Luck (1977a, b and 1979) and Luck and Bond (1980). The $\log g$ values of G–K supergiants from Van Paradijs (1973) show $\sigma = \pm 0.22$. In each of these cases the systematic effects are absent. The same is to be said about $\log g$ values for B-type supergiants for which spectroscopic determinations in combination with model stellar atmospheres give the results practically coinciding with our data (Lamers, 1974; Kovachev and Duerbeck, 1976; van Helden, 1972; Underhill and Fahey, 1973). At the same time $\log g$ values of A and F supergiants determined spectroscopically by Kopylov (1970), Osmer (1972), and Aydin (1972) are systematically too low by ~0.4 if compared with our data. This difference is probably caused by the shortcomings of model stellar atmospheres of A and F supergiants used to determine log g from line profiles. At the same time one has to bear in mind the possibility of real differences between the evolutionary and spectroscopic log g. Vardya (1976) calls attention to the fact that observed log g are affected by acceleration due to rotation, turbulence, radiation pressure, extended atmosphere, pulsation, etc. Most of these factors tend to diminish the spectroscopic log g values and to make them not correspond to the mass. The same is to be said about the stars in advanced evolution stages which violate the mass-luminosity relationship (Barnes and DuPuy, 1975). Such are low mass stars belonging to horizontal and asymptotic branches, metal-poor subdwarfs, subgiants, and giants, hot subdwarfs, and white dwarfs.

The position of ZAMS and evolutionary tracks of stars in the core and the shell hydrogen burning stages are sensitive to the chemical composition and

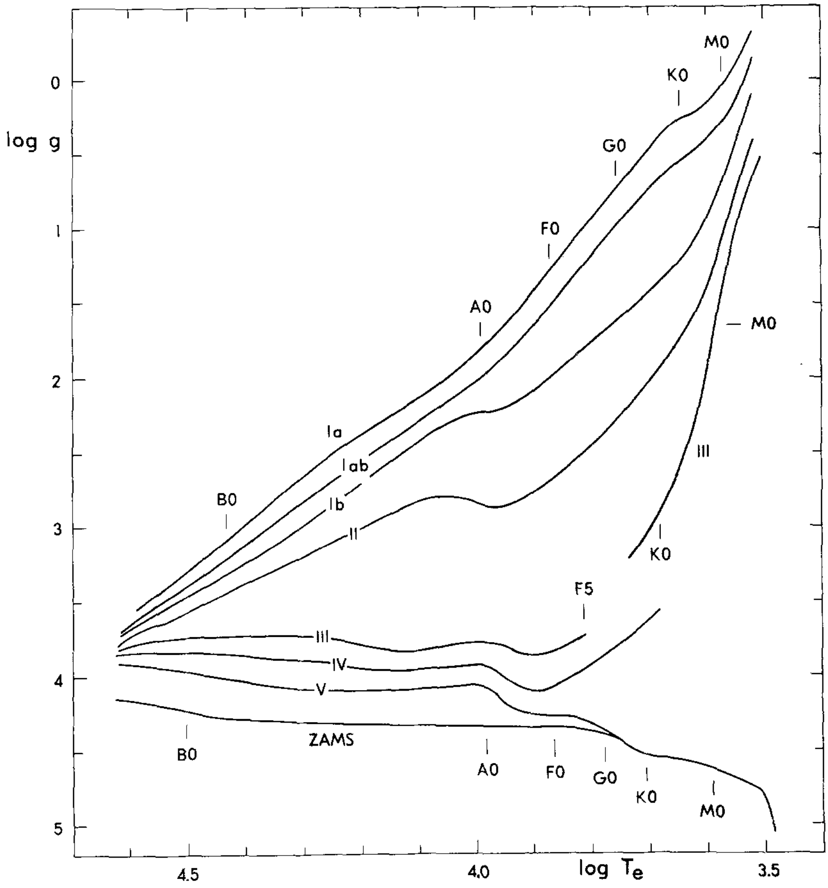

Fig. 3. The diagram log g, log T_e for stars of different luminosities plotted from the data of Tables III and VII.

this effect could be the cause of errors in their evolutionary masses and gravities. Fortunately, the majority of massive disk population stars in the solar neighbourhood have sufficiently uniform chemical composition.

To summarize, we find reasonable agreement between the masses and gravities for disk population stars determined from the theory of evolution and the observational data. This justifies the use of the data from Table VII to calibrate multicolour photometric systems and to determine evolutionary stages of stars from the diagram log g, log T_e plotted in Figure 3. This diagram, being in many respects similar to the traditional HR diagram has an extremely important property: a star can be plotted in it and its evolutionary stage can be estimated without knowing its distance.

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