Contribution Title

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Abstract. The abstract should summarize the contents of the paper in short terms, i.e. 150-250 words. It is advisable to refrain from incorporating references within the Abstract. The Abstract is intended to be an independent entity, capable of conveying the essence of the paper without reliance on external materials. Moreover, due to the common practice of reading and indexing only the ab-stract of a paper in various contexts, it becomes crucial for it to be self-contained and devoid of abbreviations, footnotes, or references. Essentially, the Abstract must serve as a succinct representation of the entire article, en-capsulating its key elements.

Keywords: First keyword, Second keyword, Third keyword

1 Introduction (Heading 1)

Please note that the first paragraph of a section or subsection is not indented. The first paragraph that follows a table and a figure has an indent and the paragraph that follows any list or equation is not indented. Font of the paper should be Times New Roman.

Subsequent paragraphs, however, are indented. The paper's margins and size must adhere strictly to the specifications provided in the given template.

The inclusion of the corresponding author's email address is mandatory.[1]

2 Related Work (Heading 1)

2.1 Lemmas, Propositions, and Theorems (Heading 2)

The numbers accorded to lemmas, propositions, and theorems, etc., should appear in consecutive order, starting with Lemma 1. Please do not include section counters in the numbering like "Theorem 1.1".

[1] The footnote numeral is set flush left and the text follows with the usual word spacing. Do not put footnotes in the reference list.

2.2 Autonomous Systems

In this section, we will consider the case when the Hamiltonian $H(x)$ is autonomous. For the sake of simplicity, we shall also assume that it is C^1.

We shall first consider the question of nontriviality, within the general framework of (A_∞, B_∞)-subquadratic Hamiltonians. In the second subsection, we shall look into the special case when H is $(0, b_\infty)$-subquadratic, and we shall try to derive additional information.

The General Case: Nontriviality. We assume that H is (A_∞, B_∞)-subquadratic at infinity, for some constant symmetric matrices A_∞ and B_∞, with $B_\infty - A_\infty$ positive definite. Set:

$$\gamma := \text{smallest eigenvalue of} \quad B_\infty - A_\infty \tag{1}$$

$$\lambda := \text{largest negative eigenvalue of} \quad J\frac{d}{dt} + A_\infty \ . \tag{2}$$

Theorem 1 tells us that if $\lambda + \gamma < 0$, the boundary-value problem:

$$\begin{array}{c} \dot{x} = JH'(x) \\ x(0) = x(T) \end{array} \tag{3}$$

has at least one solution $\overline{x}$, which is found by minimizing the dual action functional:

$$\psi(u) = \int_o^T \left[\frac{1}{2} \left(\Lambda_o^{-1} u, u \right) + N^*(-u) \right] dt \tag{4}$$

on the range of Λ, which is a subspace $R(\Lambda)_L^2$ with finite codimension. Here

$$N(x) := H(x) - \frac{1}{2} \left(A_\infty x, x \right) \tag{5}$$

is a convex function, and

$$N(x) \le \frac{1}{2} \left(\left(B_\infty - A_\infty \right) x, x \right) + c \quad \forall x \ . \tag{6}$$

Proposition 1. *Assume $H'(0) = 0$ and $H(0) = 0$. Set:*

$$\delta := \liminf_{x \to 0} 2N(x) \left\| x \right\|^{-2} \ . \tag{7}$$

If $\gamma < -\lambda < \delta$, the solution $\overline{u}$ is non-zero:

$$\overline{x}(t) \ne 0 \quad \forall t \ . \tag{8}$$

Proof. Condition (7) means that, for every $\delta' > \delta$, there is some $\varepsilon > 0$ such that

$$\left\| x \right\| \le \varepsilon \Rightarrow N(x) \le \frac{\delta'}{2} \left\| x \right\|^2 \ . \tag{9}$$

It is an exercise in convex analysis, into which we shall not go, to show that this implies that there is an $\eta > 0$ such that

$$f \|x\| \leq \eta \Rightarrow N^*(y) \leq \frac{1}{2\delta'} \|y\|^2 \ . \tag{10}$$

Fig. 1 is a sample figure.

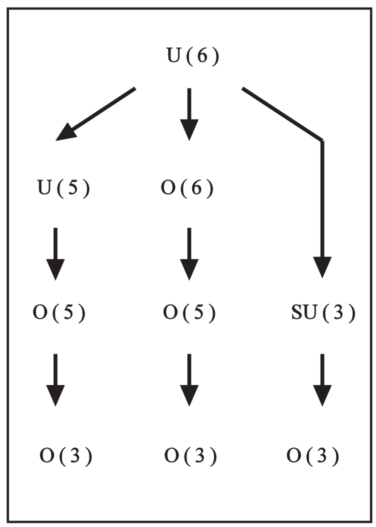

Fig. 1. This is the caption of the figure displaying a white eagle and a white horse on a snow field

Please note that the first paragraph after a figure is indented. Captions that do not constitute a full sentence, do not have a period. The figures included in the paper must be of high resolution, clear, and readable, with appropriate dimensions to ensure the overall appearance and structure of the paper remains intact.

Since u_1 is a smooth function, we will have $\|hu_1\|_\infty \leq \eta$ for h small enough, and inequality (10) will hold, yielding thereby:

$$\psi(hu_1) \leq \frac{h^2}{2} \frac{1}{\lambda} \|u_1\|_2^2 + \frac{h^2}{2} \frac{1}{\delta'} \|u_1\|^2 \ . \tag{11}$$

If we choose δ' close enough to δ, the quantity $\left(\frac{1}{\lambda}+\frac{1}{\delta'}\right)$ will be negative, and we end up with

$$\psi(hu_1) < 0 \qquad \text{for} \;\; h \neq 0 \;\; \text{small} . \tag{12}$$

On the other hand, we check directly that $\psi(0)=0$. This shows that 0 cannot be a minimizer of ψ, not even a local one. So $\overline{u} \neq 0$ and $\overline{u} \neq \Lambda_o^{-1}(0) = 0$. □

Corollary 1. *Assume H is C^2 and (a_∞, b_∞)-subquadratic at infinity. Let ξ_1, $\ldots, \xi_N$ be the equilibria, that is, the solutions of $H'(\xi)=0$. Denote by ω_k the smallest eigenvalue of $H''(\xi_k)$, and set:*

$$\omega := \mathrm{Min}\,\{\omega_1, \ldots, \omega_k\} \; . \tag{13}$$

If:

$$\frac{T}{2\pi} b_\infty < -E\left[-\frac{T}{2\pi}a_\infty\right] < \frac{T}{2\pi}\omega \tag{14}$$

then minimization of ψ yields a non-constant T-periodic solution $\overline{x}$.

We recall once more that by the integer part $E[\alpha]$ of $\alpha \in \mathbb{R}$, we mean the $a \in \mathbb{Z}$ such that $a < \alpha \le a+1$. For instance, if we take $a_\infty = 0$, Corollary 2 tells us that $\overline{x}$ exists and is non-constant provided that:

$$\frac{T}{2\pi} b_\infty < 1 < \frac{T}{2\pi} \tag{15}$$

or

$$T \in \left(\frac{2\pi}{\omega}, \frac{2\pi}{b_\infty}\right) \; . \tag{16}$$

Proof. The spectrum of Λ is $\frac{2\pi}{T}\mathbb{Z} + a_\infty$. The largest negative eigenvalue λ is given by $\frac{2\pi}{T}k_o + a_\infty$, where

$$\frac{2\pi}{T}k_o + a_\infty < 0 \le \frac{2\pi}{T}(k_o+1) + a_\infty \; . \tag{17}$$

Hence:

$$k_o = E\left[-\frac{T}{2\pi}a_\infty\right] \; . \tag{18}$$

The condition $\gamma < -\lambda < \delta$ now becomes:

$$b_\infty - a_\infty < -\frac{2\pi}{T}k_o - a_\infty < \omega - a_\infty \tag{19}$$

which is precisely condition (14). □

Lemma 1. *Assume that H is C^2 on $\mathbb{R}^{2n}\backslash\{0\}$ and that $H''(x)$ is non-degenerate for any $x \neq 0$. Then any local minimizer $\widetilde{x}$ of ψ has minimal period T.*

Proof. We know that $\widetilde{x}$, or $\widetilde{x} + \xi$ for some constant $\xi \in \mathbb{R}^{2n}$, is a T-periodic solution of the Hamiltonian system:

$$\dot{x} = JH'(x) \; . \tag{20}$$

There is no loss of generality in taking $\xi = 0$. So $\psi(x) \geq \psi(\widetilde{x})$ for all $\widetilde{x}$ in some neighbourhood of x in $W^{1,2}\left(\mathbb{R}/T\mathbb{Z};\mathbb{R}^{2n}\right)$.

But this index is precisely the index $i_T(\widetilde{x})$ of the T-periodic solution $\widetilde{x}$ over the interval $(0,T)$, as defined in Sect. 2.6. So

$$i_T(\widetilde{x}) = 0 \; . \tag{21}$$

Now if $\widetilde{x}$ has a lower period, T/k say, we would have, by Corollary 31:

$$i_T(\widetilde{x}) = i_{kT/k}(\widetilde{x}) \geq k i_{T/k}(\widetilde{x}) + k - 1 \geq k - 1 \geq 1 \; . \tag{22}$$

This would contradict (21), and thus cannot happen. □

Notes and Comments. The results in this section are a refined version of [1]; the minimality result of Proposition 14 was the first of its kind.

To understand the nontriviality conditions, such as the one in formula (16), one may think of a one-parameter family x_T, $T \in \left(2\pi\omega^{-1}, 2\pi b_\infty^{-1}\right)$ of periodic solutions, $x_T(0) = x_T(T)$, with x_T going away to infinity when $T \to 2\pi\omega^{-1}$, which is the period of the linearized system at 0.

Sample Heading (Heading 3). Only two levels of headings should be numbered. Lower-level headings remain unnumbered; they are formatted as run-in headings.

Sub-paragraph style is used for the text after the heading 3 and heading 4.

Equation (Heading 3). All equations must be created using the Microsoft Equation Editor. Equations written as plain text or using MathType are not accepted.

Sample Heading (Heading 4). The contribution should contain no more than four levels of headings.

Listing can be used for highlighting important points:

1. In a section, 1st listing will be defined by the help of numerical numbering 1, 2, 3, and so on.
2. In a section, 2nd listing will be defined by the help of letters a), b), c), and so on. In a section, the 3rd listing will be defined with the help of bullets.
   a. Alphabetical list text 1
   b. Alphabetical list text 2
   c. Alphabetical list text 3
      - Bullet list text 1
      - Bullet list text 2

– Bullet list text 3

Tables should be directly typed within the paper rather than inserted as images. They should be placed in such a manner that prevents them from breaking or continuing onto the next page. Please note that the first paragraph after a table is indented. The following Table 1 gives a sample text.

Table 1. This is the example table taken out of *The TEXbook,* p. 246

Year	World population
8000 B.C.	5,000,000
50 A.D.	200,000,000
1650 A.D.	500,000,000
1945 A.D.	2,300,000,000
1980 A.D.	4,400,000,000

Theorem 1 (Ghoussoub-Preiss). *Assume $H(t,x)$ is $(0,\varepsilon)$-subquadratic at infinity for all $\varepsilon > 0$, and T-periodic in t*

$$H(t,\cdot) \qquad \text{is convex} \quad \forall t \tag{23}$$

$$H(\cdot,x) \qquad \text{is} \quad T\text{−periodic} \quad \forall x \tag{24}$$

$$H(t,x) \geq n\left(\|x\|\right) \qquad \text{with} \quad n(s)s^{-1} \to \infty \quad \text{as} \quad s \to \infty \tag{25}$$

$$\forall \varepsilon > 0\ , \quad \exists c\ :\ H(t,x) \leq \frac{\varepsilon}{2}\left\|x\right\|^2 + c\ . \tag{26}$$

Assume also that H is C^2, and $H''(t,x)$ is positive definite everywhere. Then there is a sequence x_k, $k \in \mathbb{N}$, of kT-periodic solutions of the system

$$\dot{x} = JH'(t,x) \tag{27}$$

such that, for every $k \in \mathbb{N}$, there is some $p_o \in \mathbb{N}$ with:

$$p \geq p_o \Rightarrow x_{pk} \neq x_k\ . \tag{28}$$

□

*Example 1 (*External forcing*).* Consider the system:

$$\dot{x} = JH'(x) + f(t) \tag{29}$$

where the Hamiltonian H is $(0, b_\infty)$-subquadratic, and the forcing term is a distribution on the circle:

$$f = \frac{d}{dt}F + f_o \qquad \text{with} \quad F \in L^2\left(\mathbb{R}/T\mathbb{Z};\mathbb{R}^{2n}\right)\ , \tag{30}$$

where $f_o := T^{-1} \int_o^T f(t)dt$. For instance,

$$f(t) = \sum_{k \in \mathbb{N}} \delta_k \xi \ , \tag{31}$$

where δ_k is the Dirac mass at $t = k$ and $\xi \in \mathbb{R}^{2n}$ is a constant, fits the prescription. This means that the system $\dot{x} = JH'(x)$ is being excited by a series of identical shocks at interval T.

Definition 1. *Let $A_\infty(t)$ and $B_\infty(t)$ be symmetric operators in $\mathbb{R}^{2n}$, depending continuously on $t \in [0, T]$, such that $A_\infty(t) \leq B_\infty(t)$ for all t.*

A Borelian function $H : [0, T] \times \mathbb{R}^{2n} \to \mathbb{R}$ is called (A_∞, B_∞)-subquadratic at infinity if there exists a function $N(t, x)$ such that:

$$H(t, x) = \frac{1}{2} \left(A_\infty(t)x, x\right) + N(t, x) \tag{32}$$

$$\forall t \ , \quad N(t, x) \quad \text{is convex with respect to} \ \ x \tag{33}$$

$$N(t, x) \geq n\left(\|x\|\right) \quad \text{with} \ \ n(s)s^{-1} \to +\infty \ \ \text{as} \ \ s \to +\infty \tag{34}$$

$$\exists c \in \mathbb{R} \ : \quad H(t, x) \leq \frac{1}{2} \left(B_\infty(t)x, x\right) + c \quad \forall x \ . \tag{35}$$

If $A_\infty(t) = a_\infty I$ and $B_\infty(t) = b_\infty I$, with $a_\infty \leq b_\infty \in \mathbb{R}$, we shall say that H is (a_∞, b_∞)-subquadratic at infinity. As an example, the function $\|x\|^\alpha$, with $1 \leq \alpha < 2$, is $(0, \varepsilon)$-subquadratic at infinity for every $\varepsilon > 0$. Similarly, the Hamiltonian

$$H(t, x) = \frac{1}{2} k \left\|k\right\|^2 + \left\|x\right\|^\alpha \tag{36}$$

is $(k, k + \varepsilon)$-subquadratic for every $\varepsilon > 0$. Note that, if $k < 0$, it is not convex.

Notes and Comments. The first results on subharmonics were obtained by Rabinowitz in [5], who showed the existence of infinitely many subharmonics both in the subquadratic and superquadratic case, with suitable growth conditions on H'. Again the duality approach enabled Clarke and Ekeland in [2] to treat the same problem in the convex-subquadratic case, with growth conditions on H only.

Recently, Michalek and Tarantello (see [3] and [4]) have obtained lower bound on the number of subharmonics of period kT, based on symmetry considerations and on pinching estimates, as in Sect. 5.2 of this article.

3 Header

In the left header, the author's names are specified. The header accommodates a maximum of two author names; additional authors will be denoted by "et al." For the author's first name, only the initial shall be used, followed by a period (e.g., I. Ekeland et al.).

Conversely, the right header contains the Paper title. It should be limited to four to five words or, alternatively, span half the column width.

For citations of references, we prefer the use of square brackets and consecutive numbers. Citations using labels or the author/year convention are also acceptable. The following bibliography provides a sample reference list with entries for journal articles, an LNCS chapter, a book, proceedings without editors, as well as a URL. The reference list must include a minimum of 20 references, and all references should be cited within the text.

3.1 Reference Citations

Arabic numbers are used for citation, which is sequential either by order of citation or by alphabetical order of the references (preferred), depending on which sequence is used in the list of references. The reference numbers are given in brackets and are not superscript. Please observe the following guidelines:

- Single citation: [9]
- Multiple citation: [4-6, 9]. The numbers should be listed in numerical order.
- If an author's name is used in the text: Miller [9] was the first ...

Please write all references using the Latin alphabet. If the title of the book you are referring to is, e.g., in Russian or Chinese, then please write (in Russian) or (in Chinese) at the end of the transcript or translation of the title. All references cited in the text should be in the list of references and vice versa.

If more than six authors are listed in one particular reference, kindly shorten it and insert "et al." after the sixth author's name.

We strongly encourage you to include DOIs (Digital Object Identifiers) in your references. The DOI is a unique code allotted by the publisher to each online paper or journal article. It provides a stable way of finding published papers and their metadata. The insertion of DOIs increases the overall length of the references section.

Acknowledgments. A third-level heading at the end of the paper is used for general acknowledgments, for example: This study was funded by X (grant number X).

Appendix

If a paper includes an Appendix, it should be placed in front of the references. If it has been placed elsewhere, it will be moved by our typesetters. If there is only one, it is designated "Appendix"; if there are more than one, they are designated "Appendix 1", "Appendix 2", etc.

Appendices should be referred to in the text. The content of an appendix is contained within the sections subordinated to the major heading "Appendix". The language and styling rules for the text also apply to the appendices. The form of numbering of tables, figures, and equations in an appendix should be the same as in the body of the article, continuing the numbering used there.

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