Stationary Target

Measurements

$z_{VISION} = \boldsymbol{Z_v} = [r_x, r_y, r_z, r_\theta]$; $z_{DIST} = [r_z]$; $z_{GPS} = [\boldsymbol{r} + \boldsymbol{b}]$; $z_{GPS_v} = [\dot{\boldsymbol{r}}] = [\dot{r}_x, \dot{r}_y, \dot{r}_z]$

Position

$State:\ \boldsymbol{x}_1^s = [\boldsymbol{r}, \dot{\boldsymbol{r}}, \boldsymbol{b}\,] = [r_x, r_y, r_z, \dot{r}_x, \dot{r}_y, \dot{r}_z, b_x\,, b_y, b_z]$

$Input: \boldsymbol{u}^s = [a_x^{UAV}, a_y^{UAV}, a_z^{UAV}] + [w_x^{UAV}, w_y^{UAV}, w_z^{UAV}]$

$$State\ Space\ Model: \begin{bmatrix} \dot{\boldsymbol{r}} \\ \ddot{\boldsymbol{r}} \\ \dot{\boldsymbol{b}} \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{r}} \\ \boldsymbol{0}_{3,1} \\ \boldsymbol{0}_{2,1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0}_{3,1} \\ \boldsymbol{a}^{UAV} + \boldsymbol{w}_a^{UAV} \\ \boldsymbol{0}_{2,1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0}_{3,1} \\ \boldsymbol{0}_{3,1} \\ \boldsymbol{w_b} \end{bmatrix}$$

$$Model: \begin{bmatrix} \boldsymbol{r}_{k+1} \\ \dot{\boldsymbol{r}}_{k+1} \\ \boldsymbol{b}_{k+1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{r}_k + T_s \dot{\boldsymbol{r}}_k \\ \dot{\boldsymbol{r}}_k \\ \boldsymbol{b}_k \end{bmatrix} + \begin{bmatrix} -0.5T_s^2(\boldsymbol{a}^{UAV} + \boldsymbol{w}_a^{UAV}) \\ -T_s(\boldsymbol{a}^{UAV} + \boldsymbol{w}_a^{UAV}) \\ \boldsymbol{0}_{2,1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0}_{3,1} \\ \boldsymbol{0}_{3,1} \\ \boldsymbol{w_b} \end{bmatrix}$$

Orientation

$State:\ \boldsymbol{x}_2^s = [r_\theta]$ $\quad Model: r_{k+1} = r_k$ $\quad Input: \boldsymbol{u} = [-]$

Orientation implementation

KF_θ	$\boldsymbol{x} = [r_\theta, \dot{r}_\theta]$	$\boldsymbol{u} = [-]$	$\boldsymbol{z} = [z_V]$

$G_k = 0$ $\quad \Phi_k = 1$ $\quad H_k = 1$ $\quad Q_k = 0$

Decoupled position implementation

KF_x, KF_y, KF_z	$\boldsymbol{x} = [r_x, \dot{r}_x, b_x]$	$\boldsymbol{u} = [u_x^S]$	$\boldsymbol{z} = [\boldsymbol{Z}_V, \boldsymbol{Z}_{GPS}, \boldsymbol{Z}_{GPS_v}]$

$$\Phi_k^d = \begin{bmatrix} 1 & T_s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad H_k^d = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad G_k^d = \begin{bmatrix} -0.5T_s^2 \\ -T_s \\ 0 \end{bmatrix}$$

$$Q_k^d = G_k^d \sigma_{a,x}^2 G_k^{d\,T} + Q_{k,bias}^d = \begin{bmatrix} \frac{1}{4}\sigma_{a,x}^2 T_s^4 & \frac{1}{2}\sigma_{a,x}^2 T_s^3 & 0 \\ \frac{1}{2}\sigma_{a,x}^2 T_s^3 & \sigma_{a,x}^2 T_s^2 & 0 \\ 0 & 0 & \sigma_{b,x}^2 \end{bmatrix} \qquad Q_{k,bias}^d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_{b,x}^2 \end{bmatrix}$$

Coupled position implementation

$KFxyz$	$\boldsymbol{x} = [\boldsymbol{r}, \dot{\boldsymbol{r}}, \boldsymbol{b}\,]$	$\boldsymbol{u} = \boldsymbol{u}^s$	$\boldsymbol{z} = [\boldsymbol{Z}_V, \boldsymbol{Z}_{GPS}, \boldsymbol{Z}_{GPS_v}]$

$\Phi_k^c = \Phi_k^d \otimes I_{3,3} \in R^{9,9}$ $\quad H_k^c = H_k^d \otimes I_{3,3} \in R^{9,9}$ $\quad G_k^c = G_k^d \otimes I_{3,3} \in R^{9,3}$

$Q_k^c = G_k^c S_k G_k^{c\,T} + Q_{k,bias}^c$ $\quad R_k = rot\ Body\ to\ VcNED$ $\quad S_k = R_k C_k R_k^T \in R^{3,3}$

$$C_k = \begin{bmatrix} \sigma_{a,x}^2 & 0 & 0 \\ 0 & \sigma_{a,y}^2 & 0 \\ 0 & 0 & \sigma_{a,z}^2 \end{bmatrix} \qquad Q_{k,bias}^c = \begin{bmatrix} \sigma_{b,x}^2 & 0 & 0 \\ 0 & \sigma_{b,z}^2 & 0 \\ 0 & 0 & \sigma_{b,z}^2 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \in R^{9,9}$$

Moving Target

Measurements

$z_{VISION} = \boldsymbol{Z_v} = [r_x, r_y, r_z, r_\theta]$; $\boldsymbol{z_{DIST}} = [r_z]$; $\boldsymbol{z_{GPS}} = [\boldsymbol{r} + \boldsymbol{b}]$

Position

$State: \ \boldsymbol{x_1^m} = [r_x, r_y, r_z, \dot{r}_x, \dot{r}_y, \dot{r}_z, b_x, b_y, b_z, a_x^t, a_y^t, a_z^t]$

$Input: \boldsymbol{u^m} = [a_x^{UAV}, a_y^{UAV}, a_z^{UAV}] + [w_x^{UAV}, w_y^{UAV}, w_z^{UAV}]$

$$State\ Space\ Model: \begin{bmatrix} \dot{\boldsymbol{r}} \\ \ddot{\boldsymbol{r}} \\ \dot{\boldsymbol{b}} \\ \dot{\boldsymbol{a}}^t \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{r}} \\ \boldsymbol{a^t} \\ \boldsymbol{0}_{3,1} \\ \boldsymbol{0}_{3,1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0}_{4,1} \\ -(\boldsymbol{a}^{UAV} + \boldsymbol{w}_a^{UAV}) \\ \boldsymbol{0}_{3,1} \\ \boldsymbol{0}_{3,1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0}_{3,1} \\ \boldsymbol{0}_{3,1} \\ \boldsymbol{w_b} \\ \boldsymbol{w_a^t} \end{bmatrix}$$

$Model:$

$$\begin{bmatrix} \boldsymbol{r}_{k+1} \\ \dot{\boldsymbol{r}}_{k+1} \\ \boldsymbol{b}_{k+1} \\ \boldsymbol{a}_{k+1}^t \end{bmatrix} = \begin{bmatrix} \boldsymbol{r}_k + T_s\, \dot{\boldsymbol{r}}_k + 0.5 T_s^2 \boldsymbol{a}_k^t \\ \dot{\boldsymbol{r}}_k + T_s \boldsymbol{a}_k^t \\ \boldsymbol{b}_k \\ \boldsymbol{a}_k^t \end{bmatrix} + \begin{bmatrix} -0.5 T_s^2 (\boldsymbol{a}^{UAV} + \boldsymbol{w_a}^{UAV}) \\ -T_s(\boldsymbol{a}^{UAV} + \boldsymbol{w_a}^{UAV}) \\ \boldsymbol{0}_{3,1} \\ \boldsymbol{0}_{3,1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0}_{3,1} \\ \boldsymbol{0}_{3,1} \\ \boldsymbol{w_b} \\ \boldsymbol{w_a^t} \end{bmatrix}$$

Orientation

$State: \ \boldsymbol{x_2^m} = [r_\theta, \dot{r}_\theta]$

$Input: \boldsymbol{u} = [-]$

$$Model: \begin{bmatrix} r_{k+1} \\ \dot{r}_{k+1} \end{bmatrix} = \begin{bmatrix} r_k + T_s \dot{r}_k \\ \dot{r}_k \end{bmatrix} + \begin{bmatrix} 0 \\ w_\theta \end{bmatrix}$$

$$\Phi_k = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix} \qquad G_k = 0$$

$$Q_k = \sigma_v^2 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad H_k = [1, 0]$$

Decoupled position Implementation

KF_x, KF_y, KF_z	$\boldsymbol{x} = [r_x, \dot{r}_x, b_x, a_x^t]$	$\boldsymbol{u} = [a_x^d]$	$\boldsymbol{z} = [z_V, Z_{GPS}]$

$$\Phi_k^d = \begin{bmatrix} 1 & T_s & 0 & 0.5T_s^2 \\ 0 & 1 & 0 & T_s \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad G_k^d = \begin{bmatrix} -0.5T_s^2 \\ -T_s \\ 0 \\ 0 \end{bmatrix} \qquad H_k^d = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$Q_k^d = \begin{bmatrix} \frac{T_s^4}{4}(\sigma_a^{UAV})^2 & \frac{T_s^3}{2}(\sigma_a^{UAV})^2 & 0 & 0 \\ \frac{T_s^3}{2}(\sigma_a^{UAV})^2 & T_s^2(\sigma_a^{UAV})^2 & 0 & 0 \\ 0 & 0 & \sigma_b^2 & 0 \\ 0 & 0 & 0 & (\sigma_a^t)^2 \end{bmatrix}$$

$$Q_k^d = G_k^d (\sigma_a^{UAV})^2 {G_k^d}^T + Q_{k,bias}^d + Q_{k,acc}^d$$

$$Q_{k,bias}^d + Q_{k,acc}^d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & (\sigma_b)^2 & 0 \\ 0 & 0 & 0 & (\sigma_a^t)^2 \end{bmatrix}$$

Coupled position Implementation

KF_{xyz}	$\boldsymbol{x} = [\boldsymbol{r}, \dot{\boldsymbol{r}}, \boldsymbol{b}, \boldsymbol{a^t}]$	$\boldsymbol{u} = \boldsymbol{u^m}$	$\boldsymbol{z} = [z_V, Z_{GPS}]$

$$\Phi_k^c = \Phi_k^d \otimes I_{3,3} \in R^{12,12} \qquad G_k^c = G_k^d \otimes I_{3,3} \in R^{12,3} \qquad H_k^c = H_k^d \otimes I_{3,3} \in R^{12,12}$$

$$Q_k^c = G_k^c S_k^{UAV} {G_k^c}^T + Q_{k,bias}^c \qquad R_k = rot\ Body\ to\ VcNED \qquad S_k^{UAV} = R_k C_k^{UAV} R_k^T \in R^{3,3}$$

$$C_k^{UAV} = \begin{bmatrix} (\sigma_{a,x}^{UAV})^2 & 0 & 0 \\ 0 & (\sigma_{a,y}^{UAV})^2 & 0 \\ 0 & 0 & (\sigma_{a,z}^{UAV})^2 \end{bmatrix}$$

$$Q_{k,bias}^c = \begin{bmatrix} \sigma_{b,x}^2 & 0 & 0 \\ 0 & \sigma_{b,z}^2 & 0 \\ 0 & 0 & \sigma_{b,z}^2 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \in R^{12,12}$$