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Math for Machine Learning

Linear algebra - Week 4

Bases

Span

Orthogonal and orthonormal bases

Orthogonal and orthonormal matrices

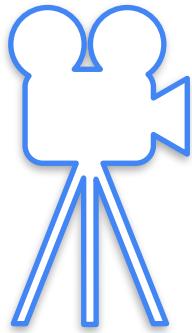


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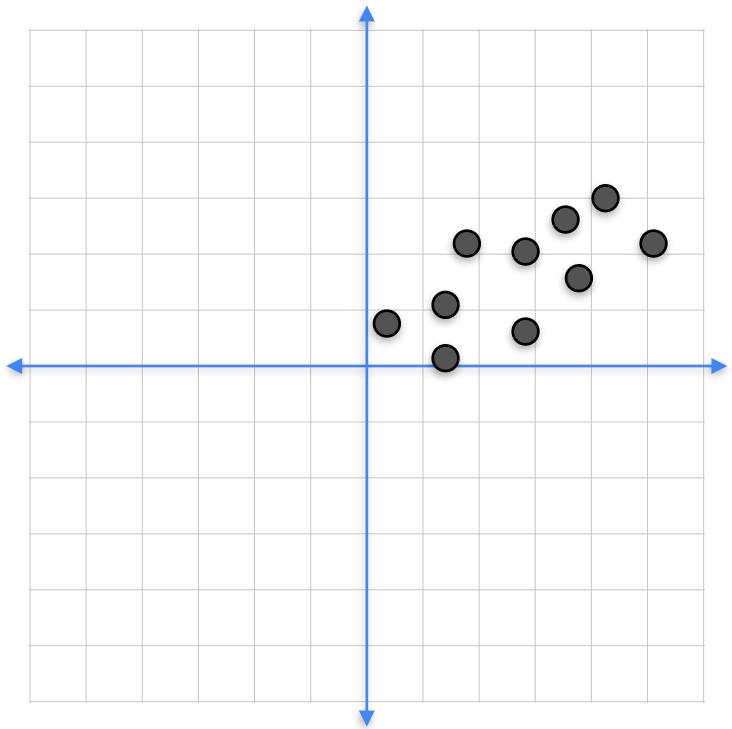
Determinants and Eigenvectors

Machine learning motivation

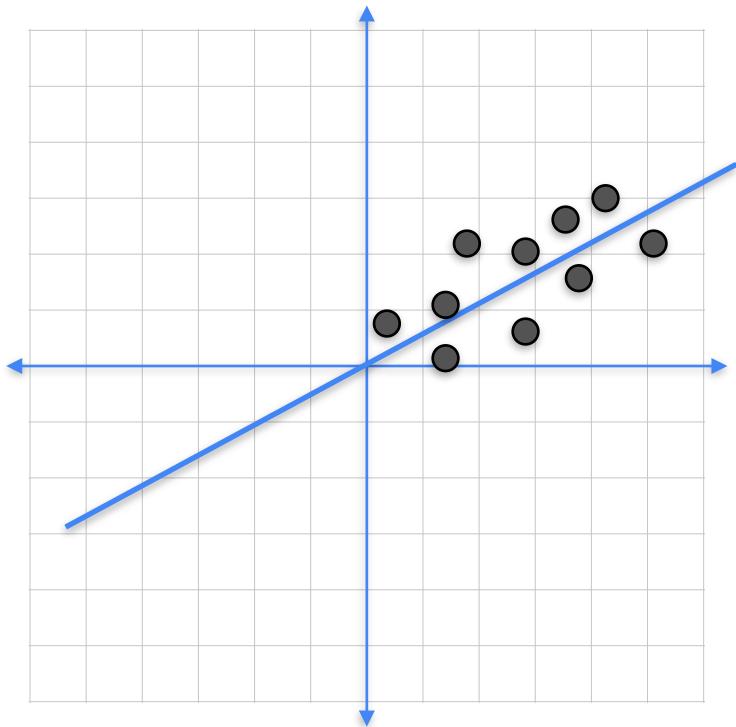
PCA



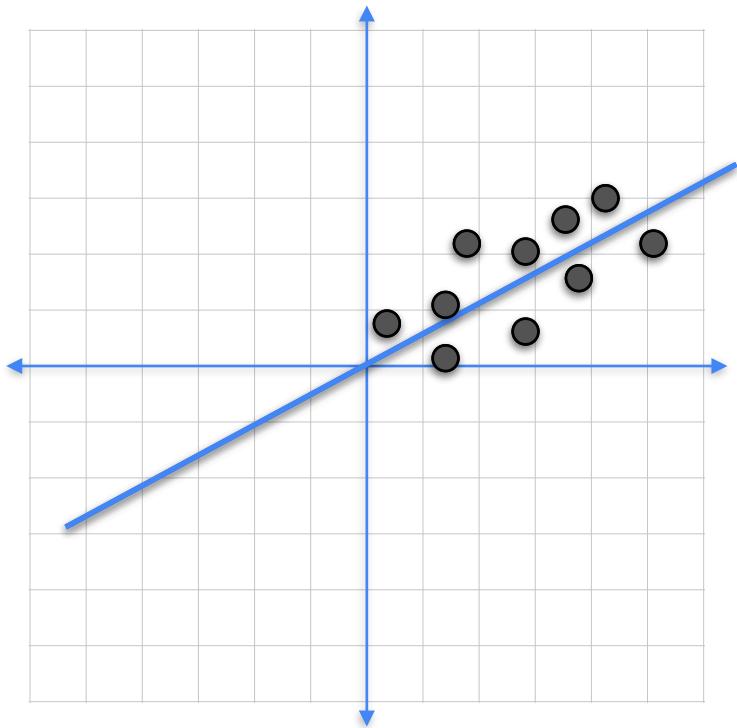
Principal Component Analysis



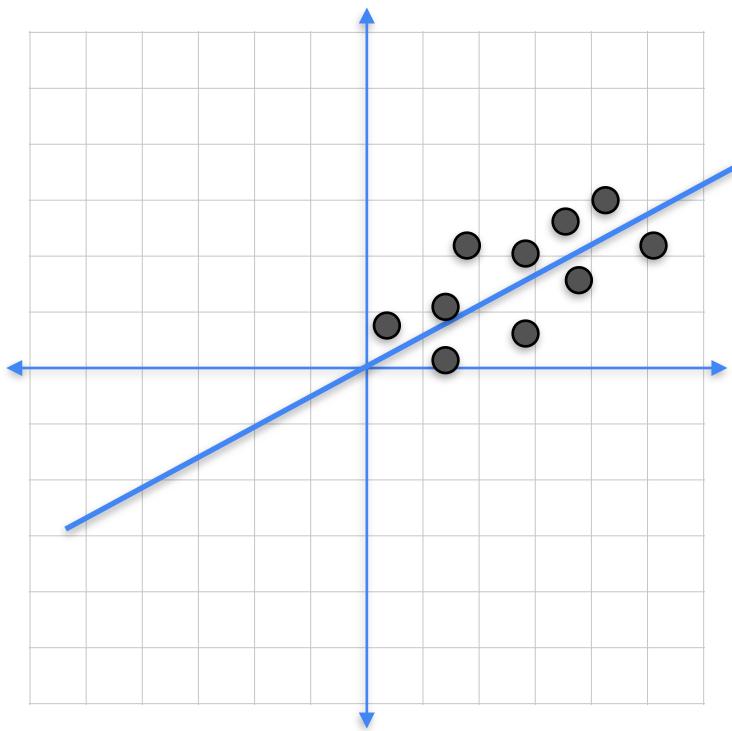
Principal Component Analysis



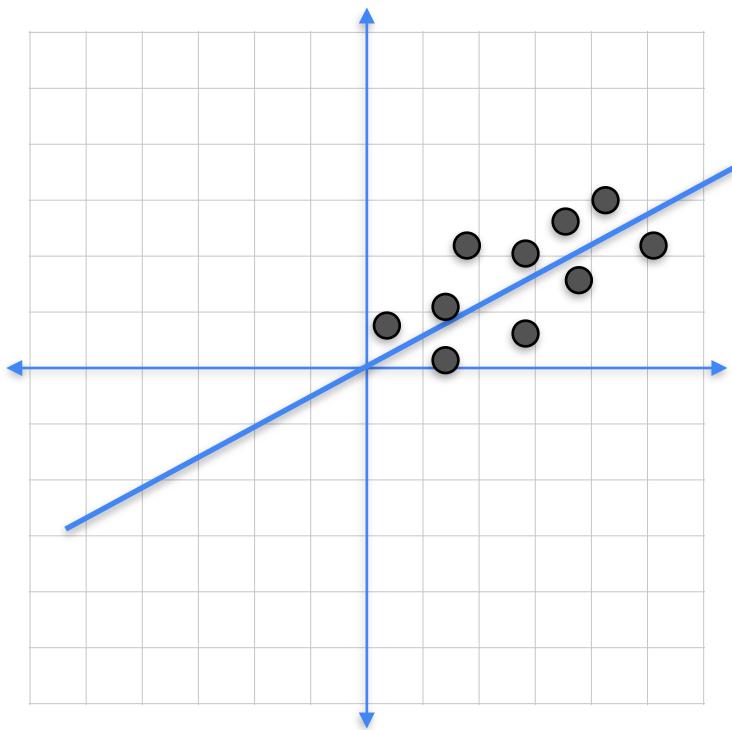
Principal Component Analysis



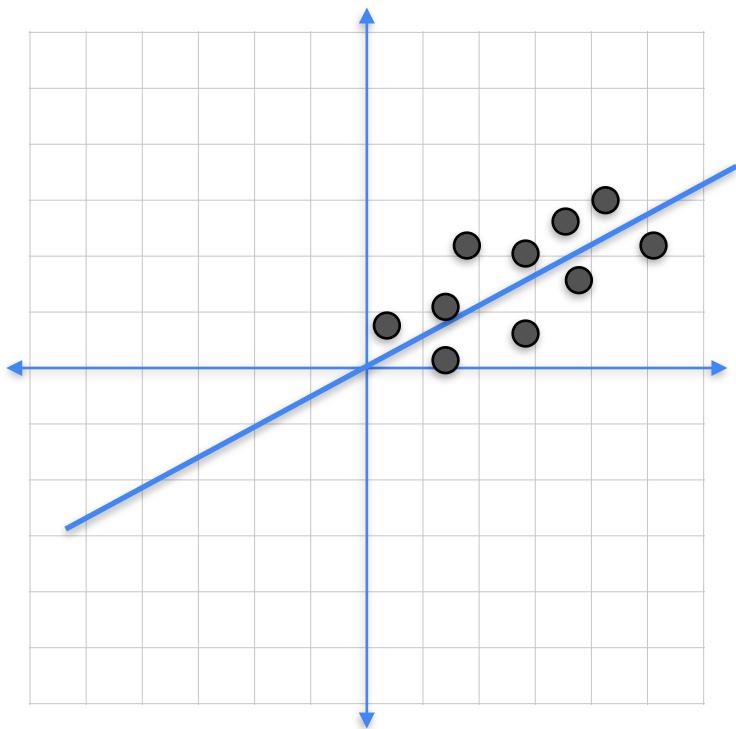
Principal Component Analysis



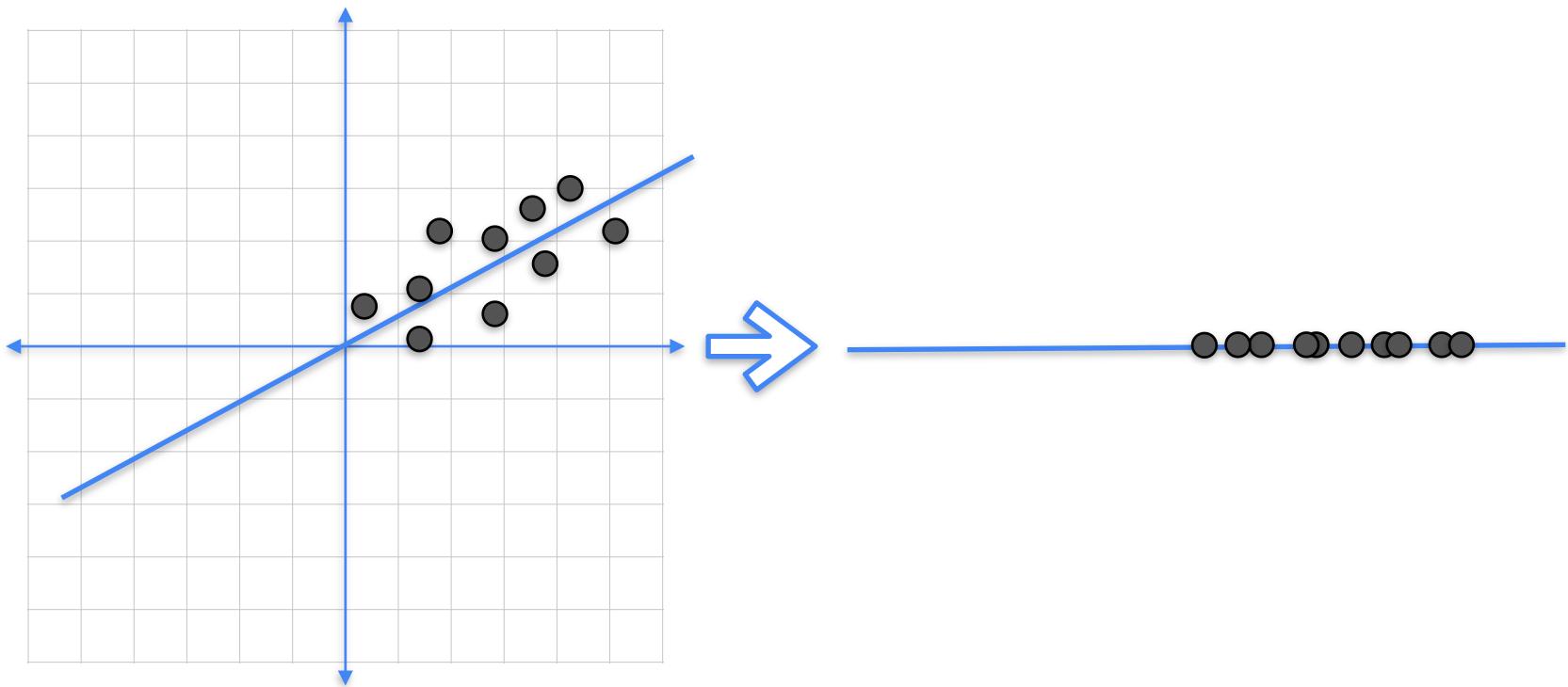
Principal Component Analysis



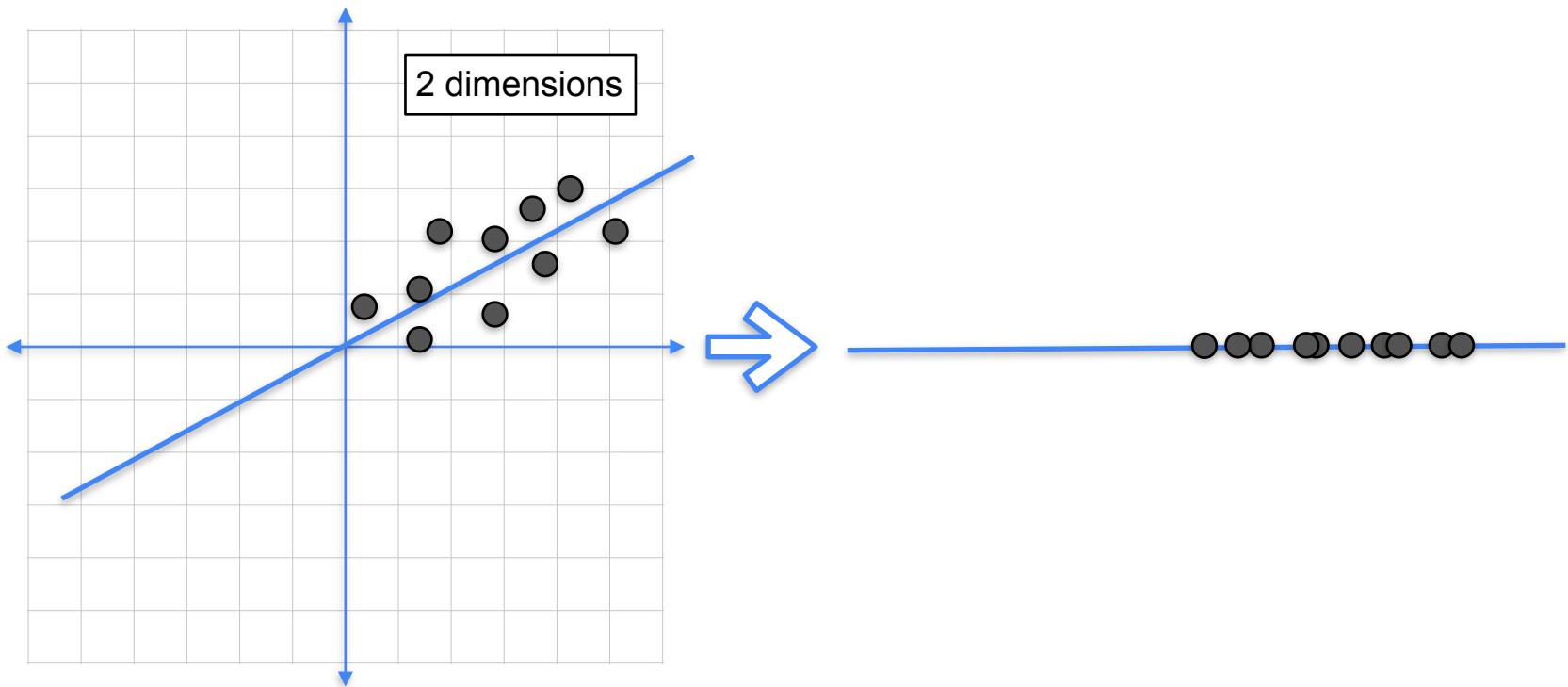
Principal Component Analysis



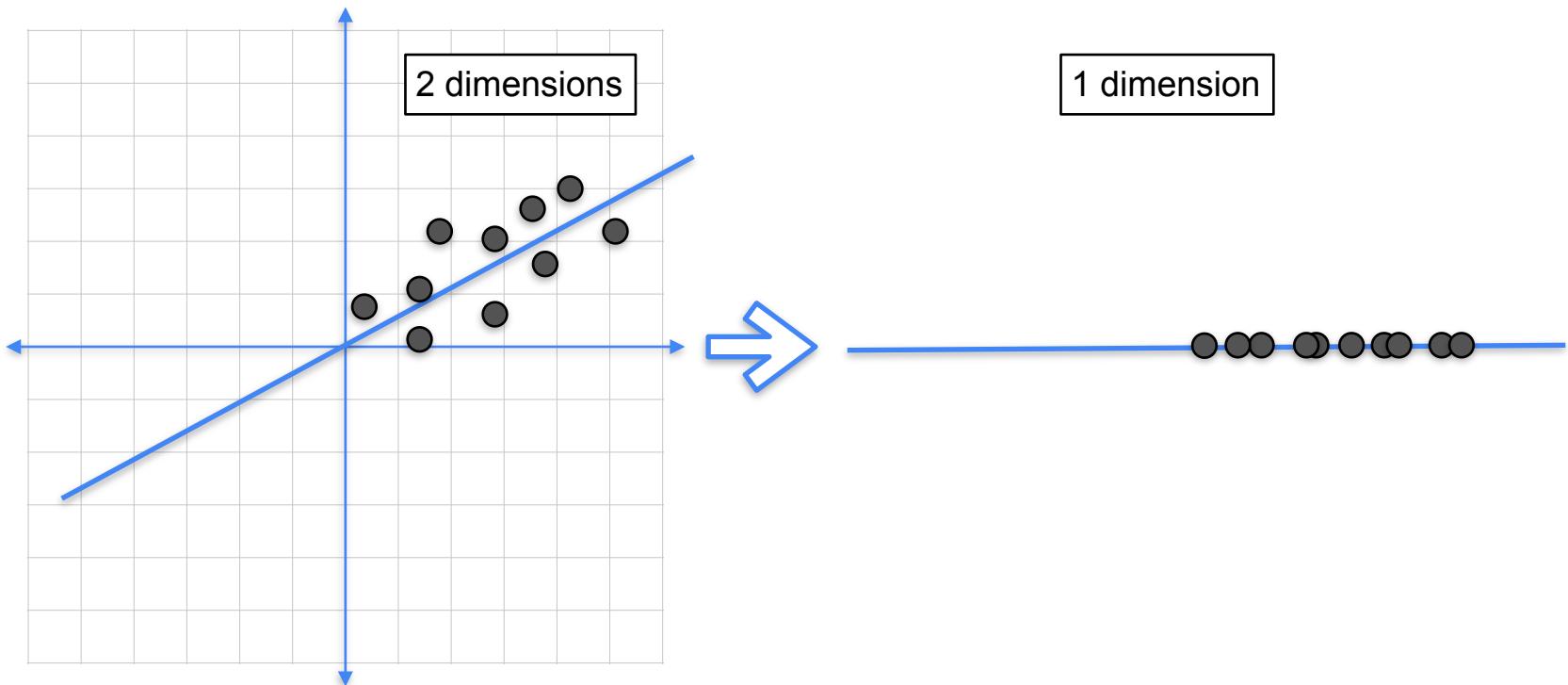
Principal Component Analysis



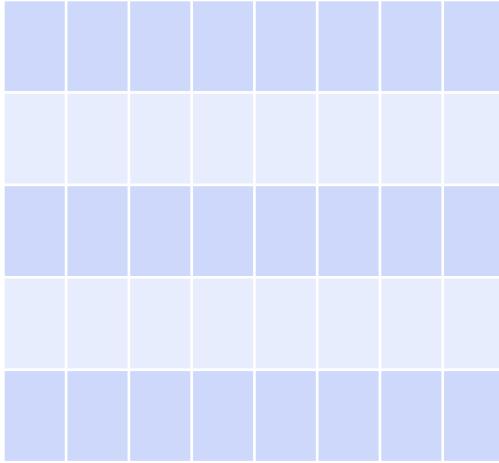
Principal Component Analysis



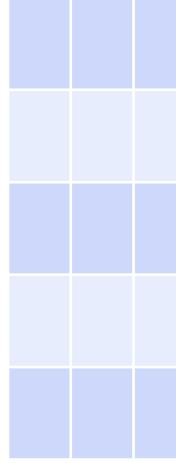
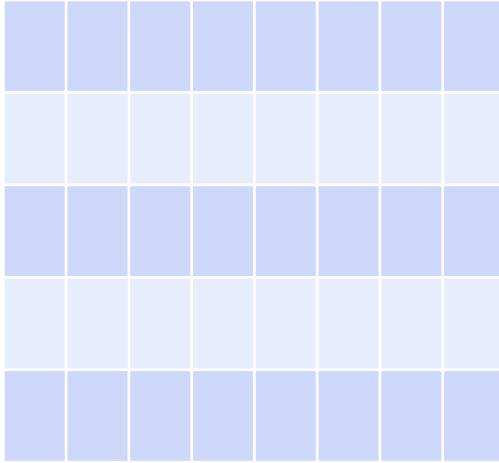
Principal Component Analysis



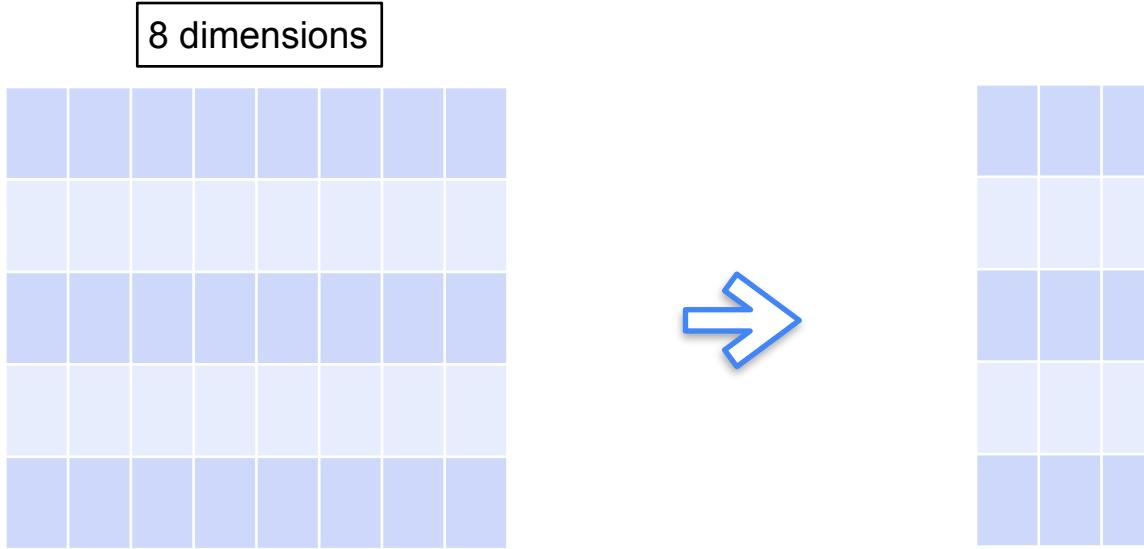
Principal Component Analysis



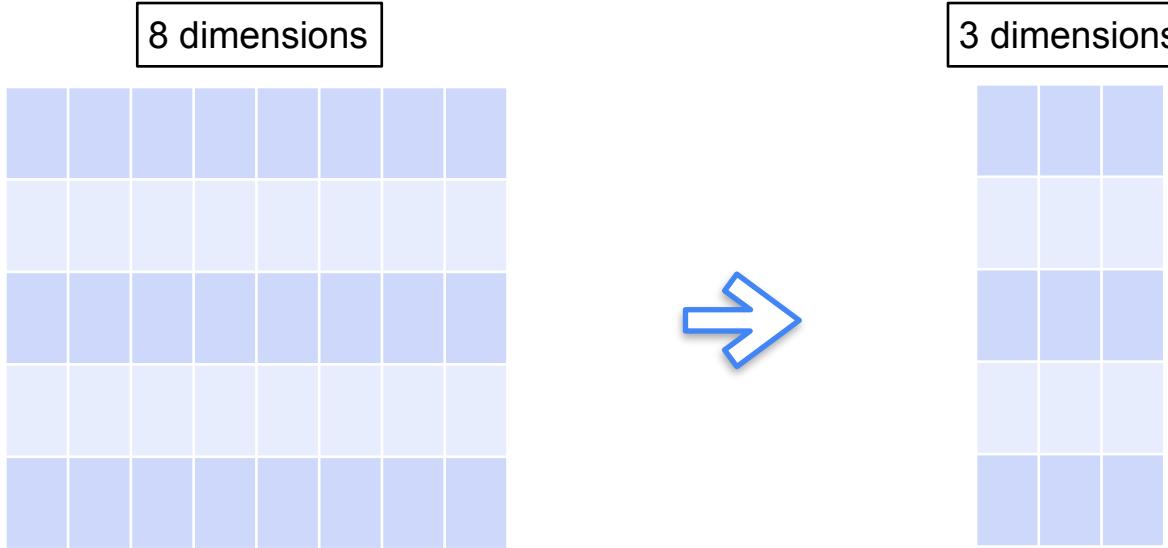
Principal Component Analysis



Principal Component Analysis



Principal Component Analysis



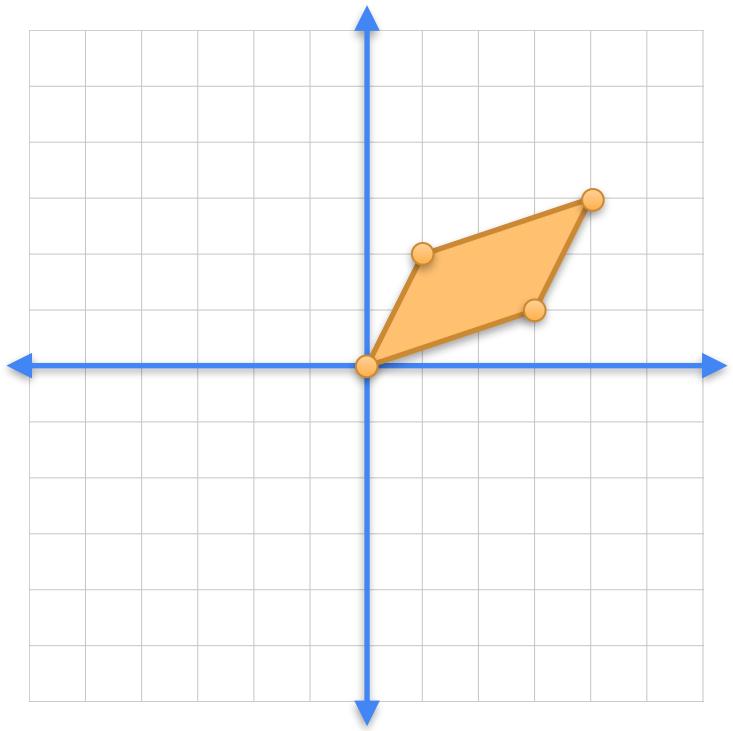
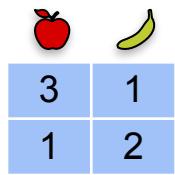
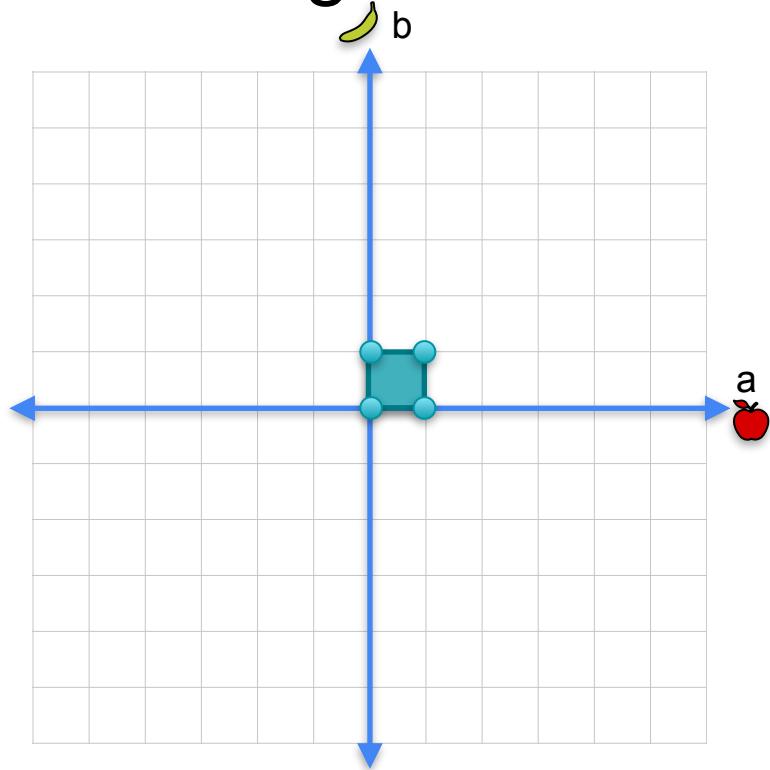


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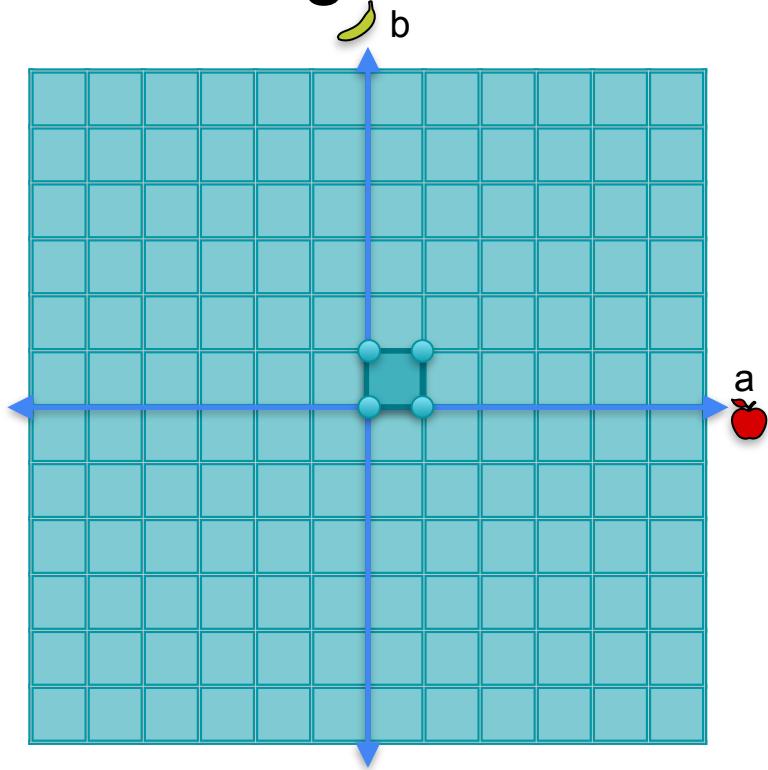
Determinants and Eigenvectors

Singularity and rank of linear transformations

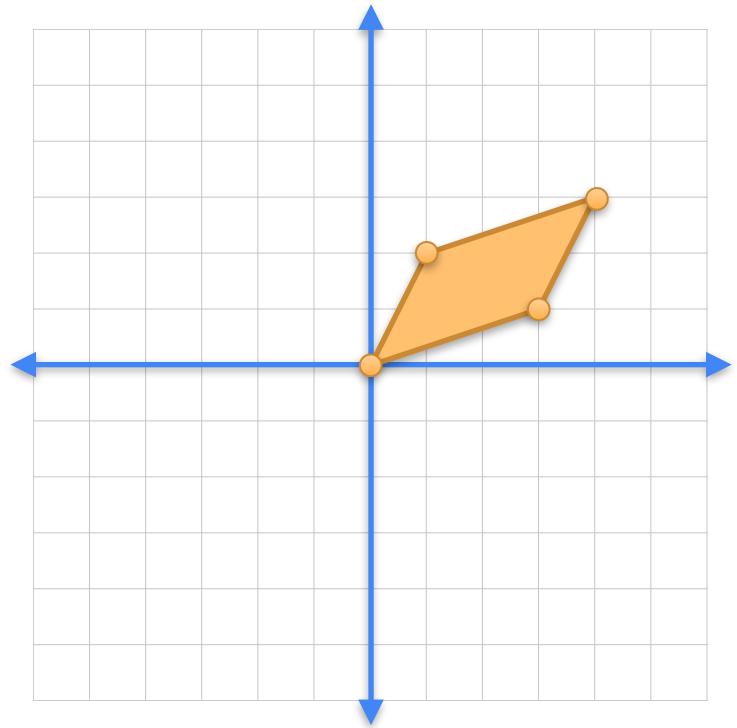
Non-singular transformation



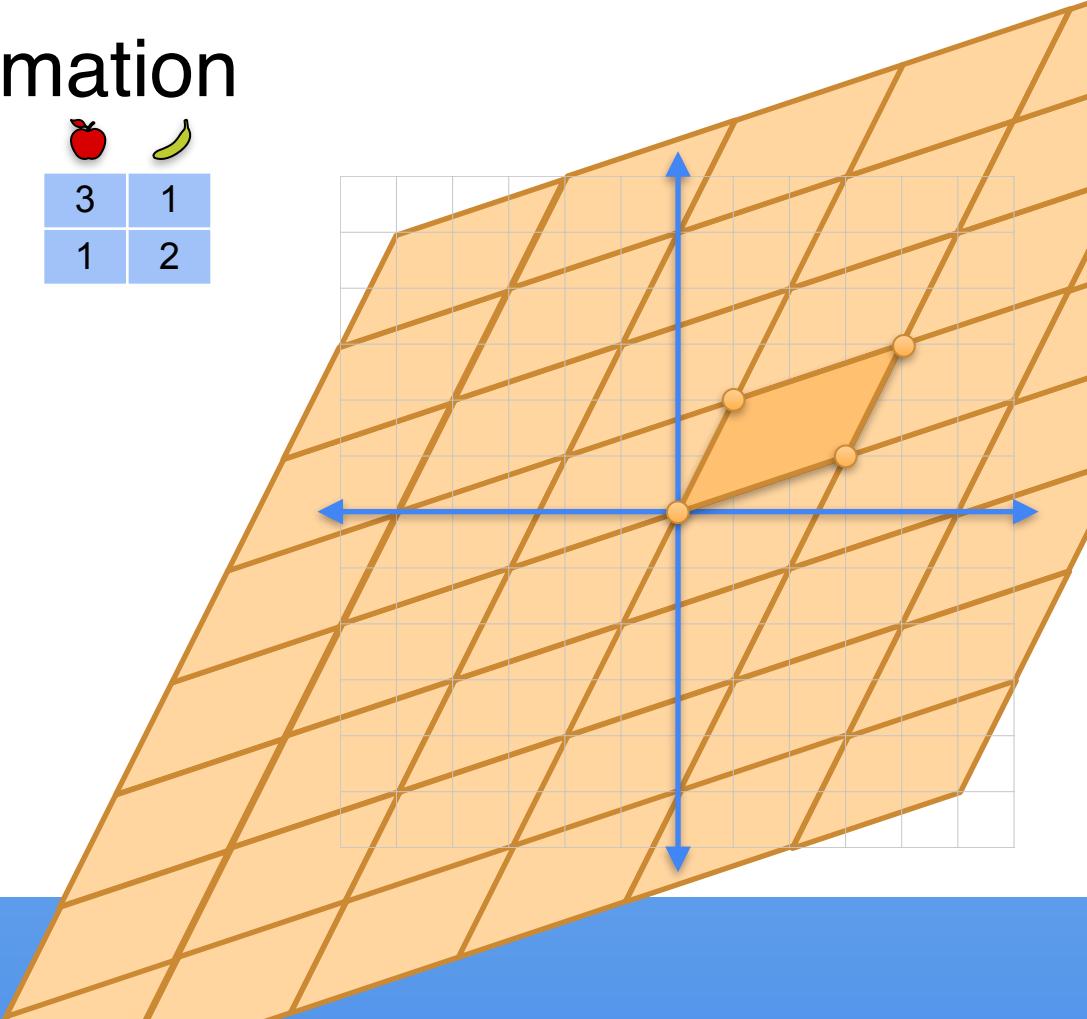
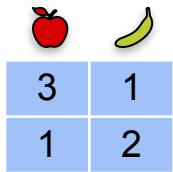
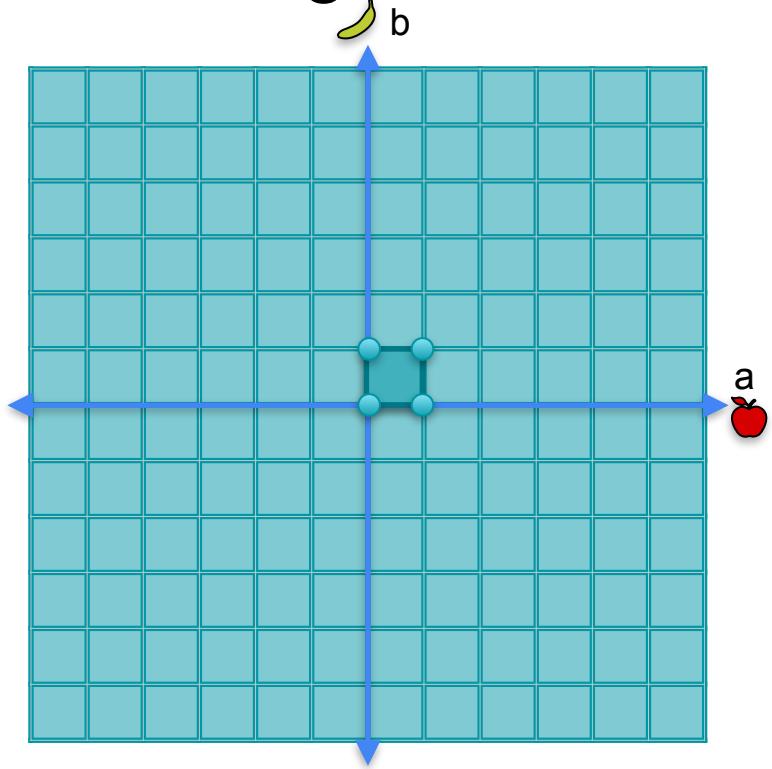
Non-singular transformation



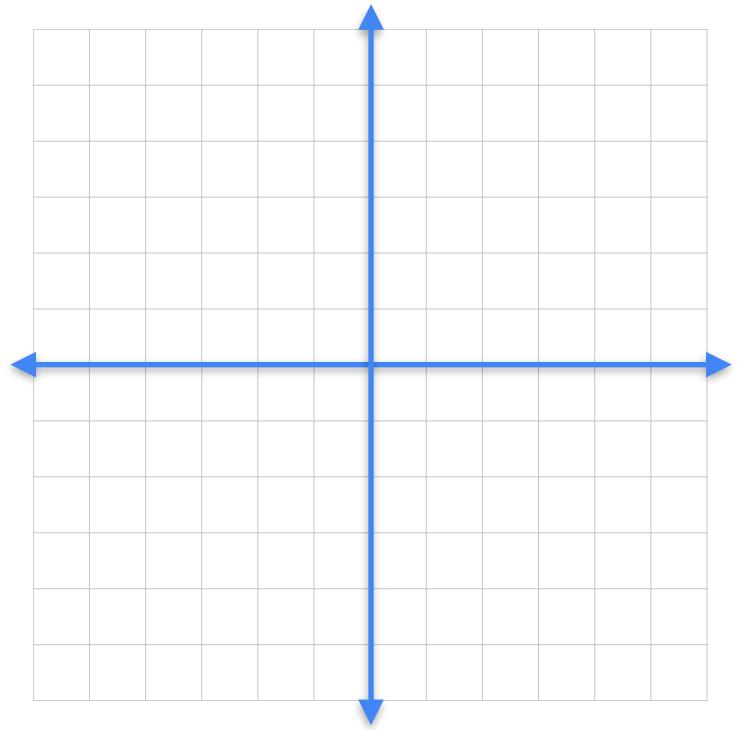
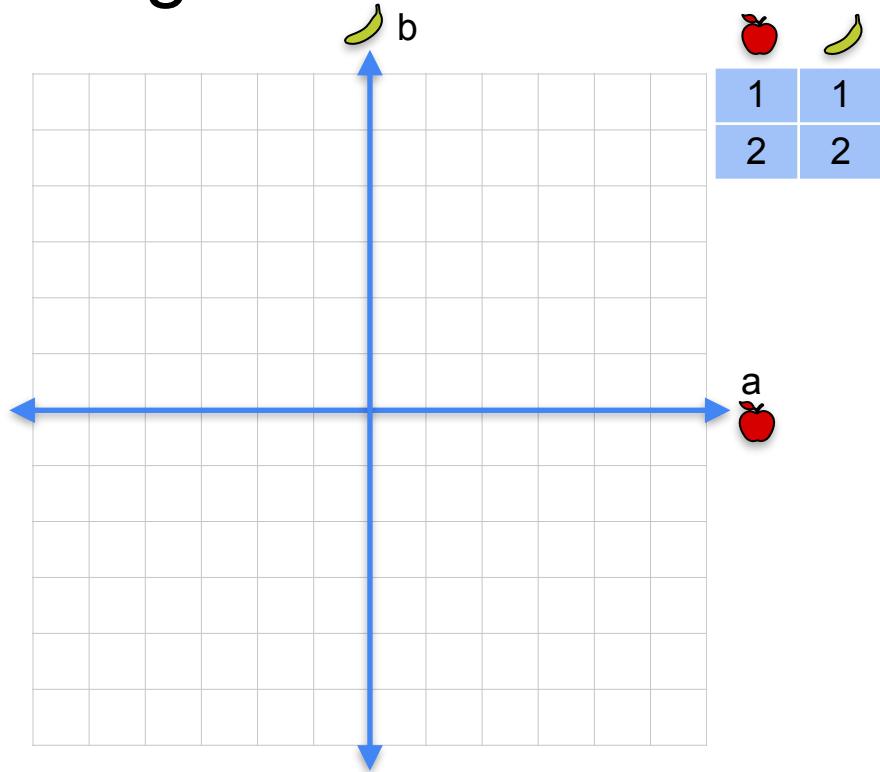
| | | |
|---|-------|--------|
| | apple | banana |
| 3 | 1 | |
| 1 | 2 | |



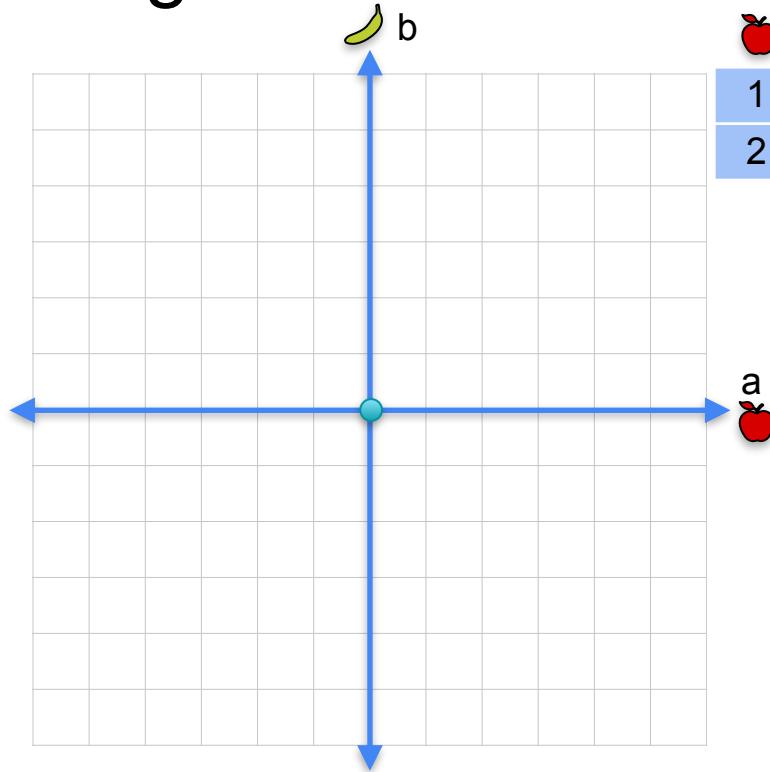
Non-singular transformation



Singular transformation

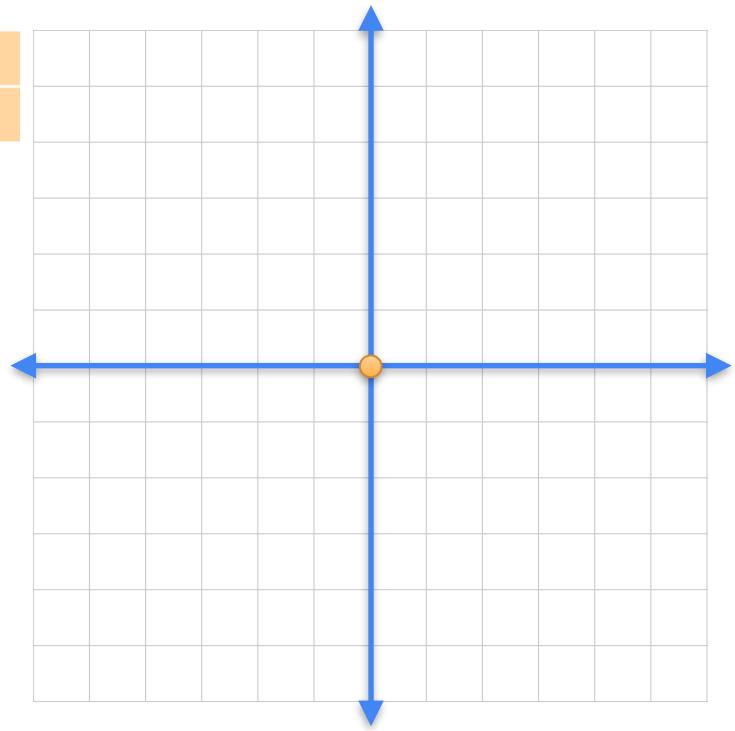


Singular transformation

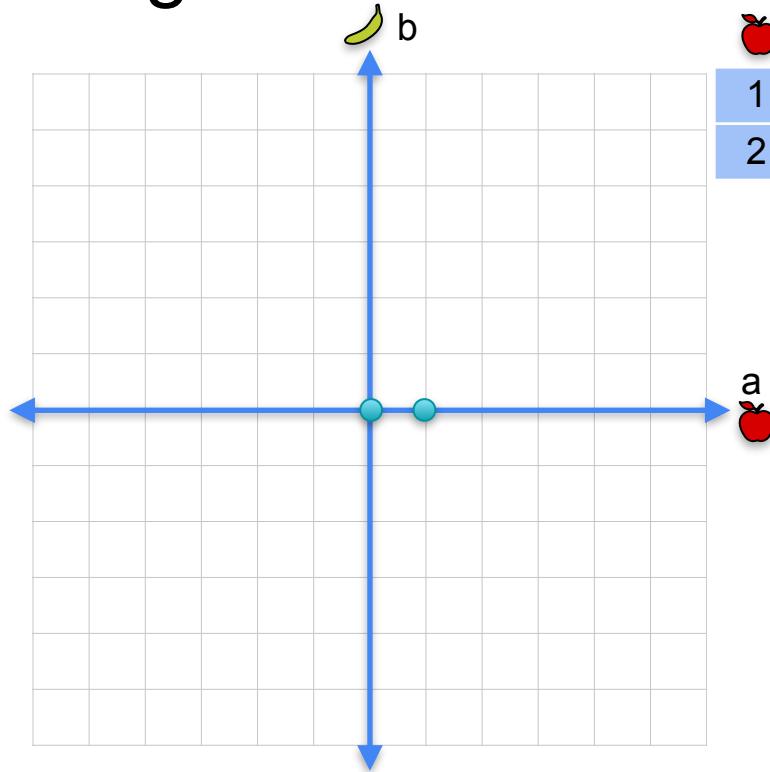


$$\begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} \begin{matrix} 0 \\ 0 \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

$(0,0) \rightarrow (0,0)$

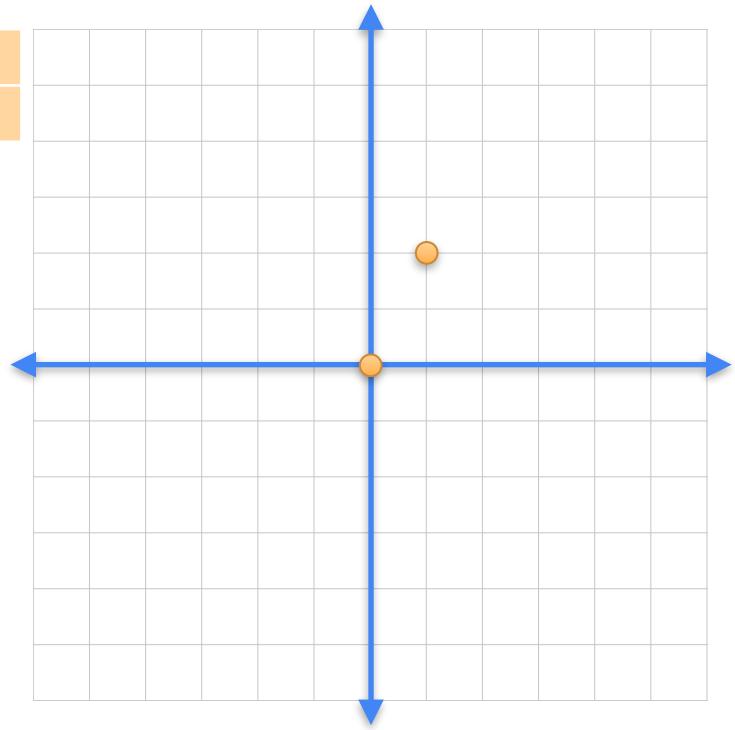


Singular transformation

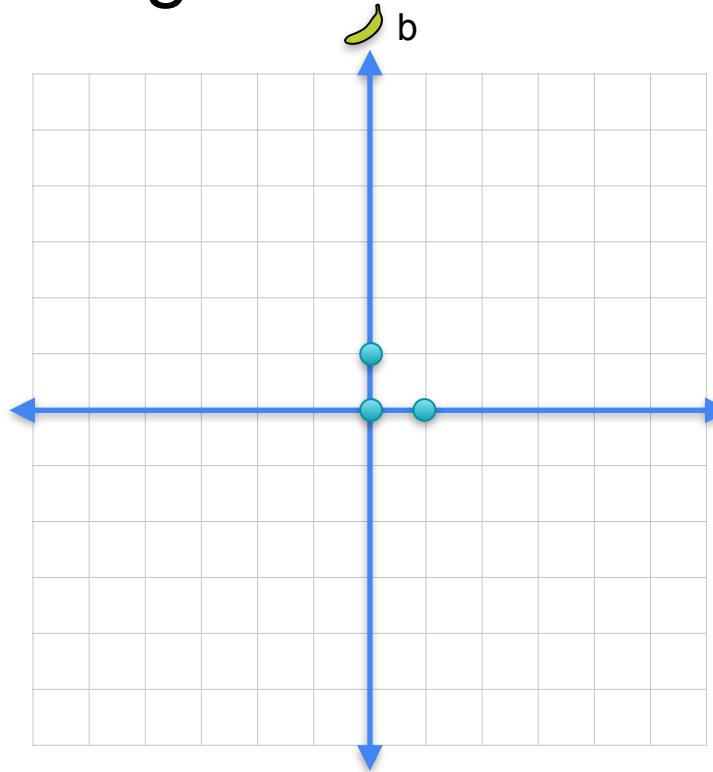


$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{aligned}(0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (1,2)\end{aligned}$$

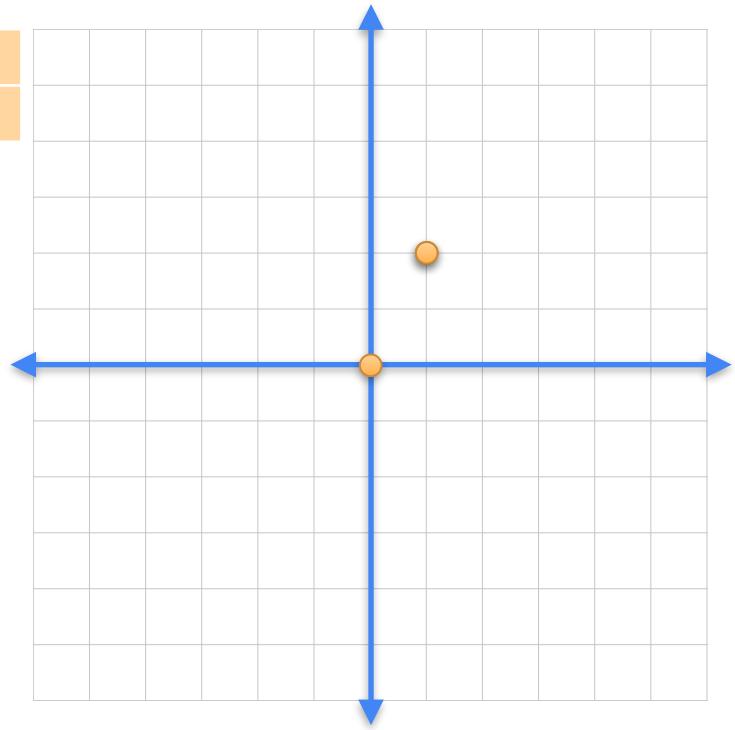


Singular transformation

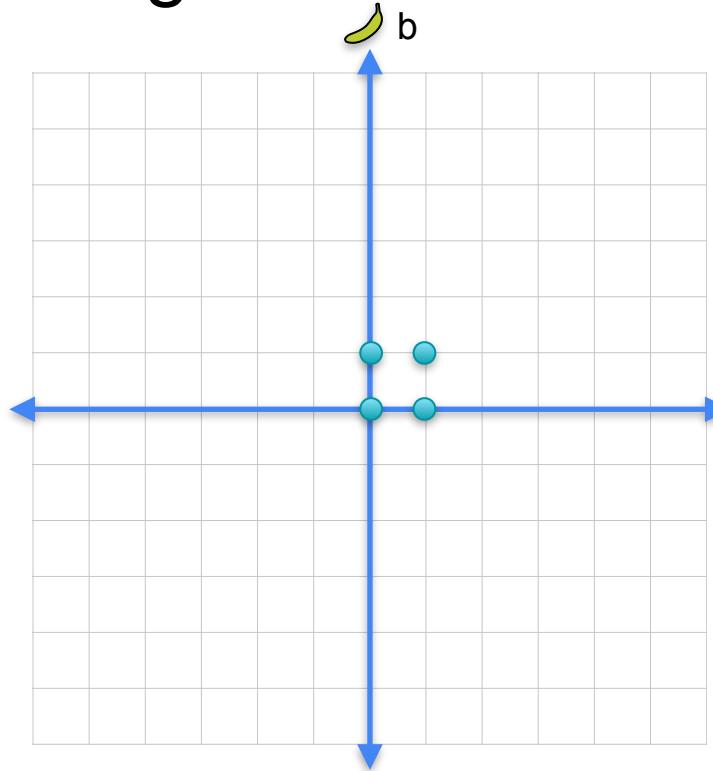


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} & \begin{matrix} 0 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} 1 \\ 2 \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (1,2) \\ (0,1) &\rightarrow (1,2) \end{aligned}$$



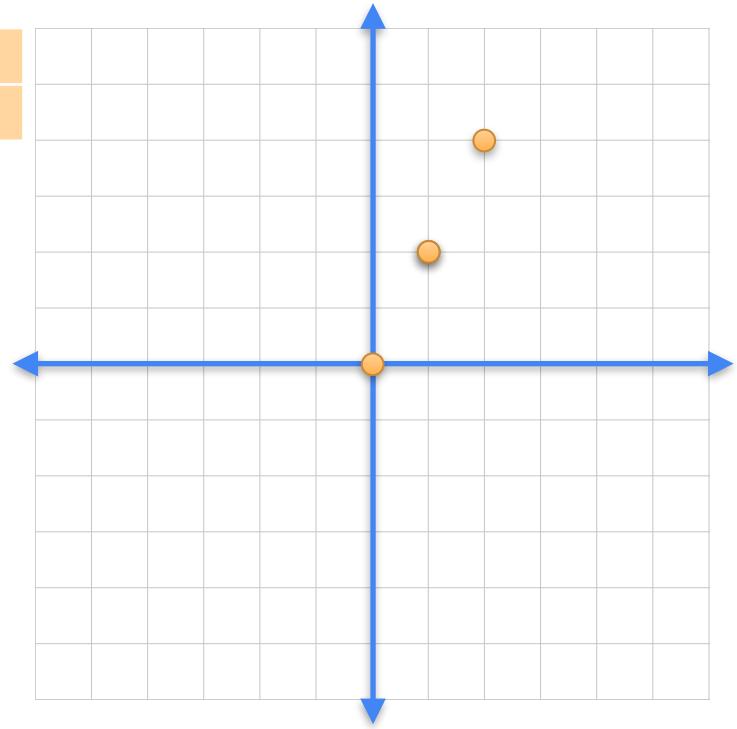
Singular transformation



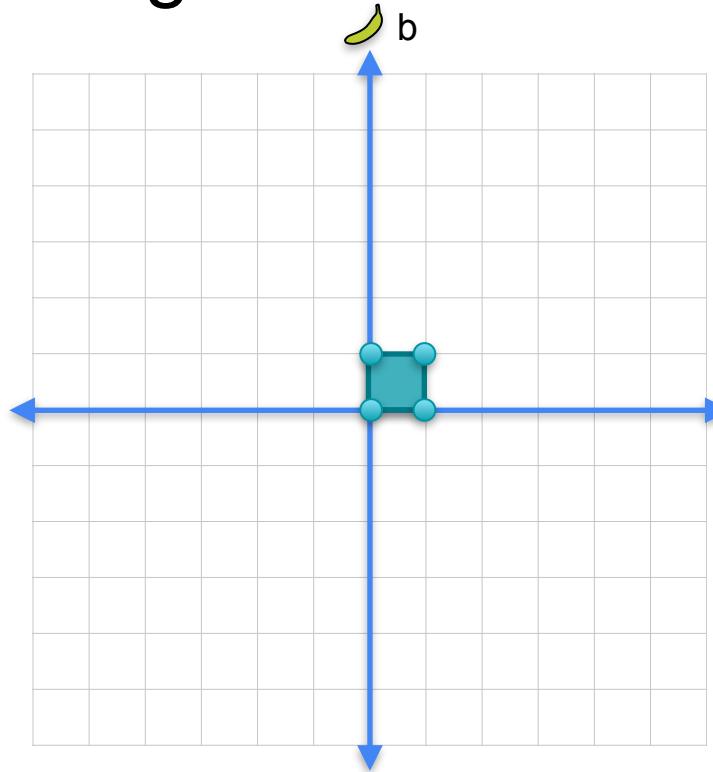
apple banana

$$\begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} \quad \begin{matrix} 1 \\ 1 \end{matrix} = \begin{matrix} 2 \\ 4 \end{matrix}$$

$$\begin{array}{l} (0,0) \rightarrow (0,0) \\ (1,0) \rightarrow (1,2) \\ (0,1) \rightarrow (1,2) \\ (1,1) \rightarrow (2,4) \end{array}$$

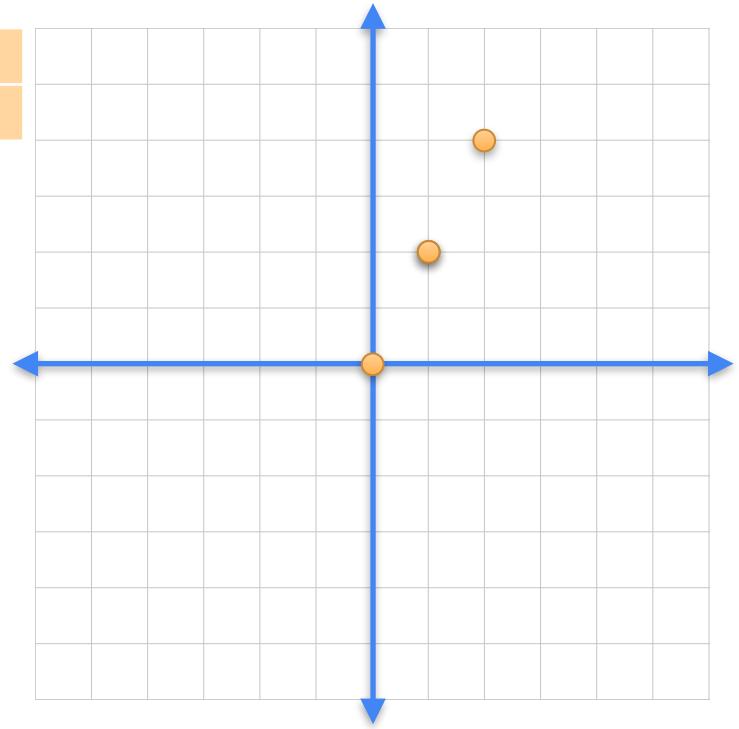


Singular transformation

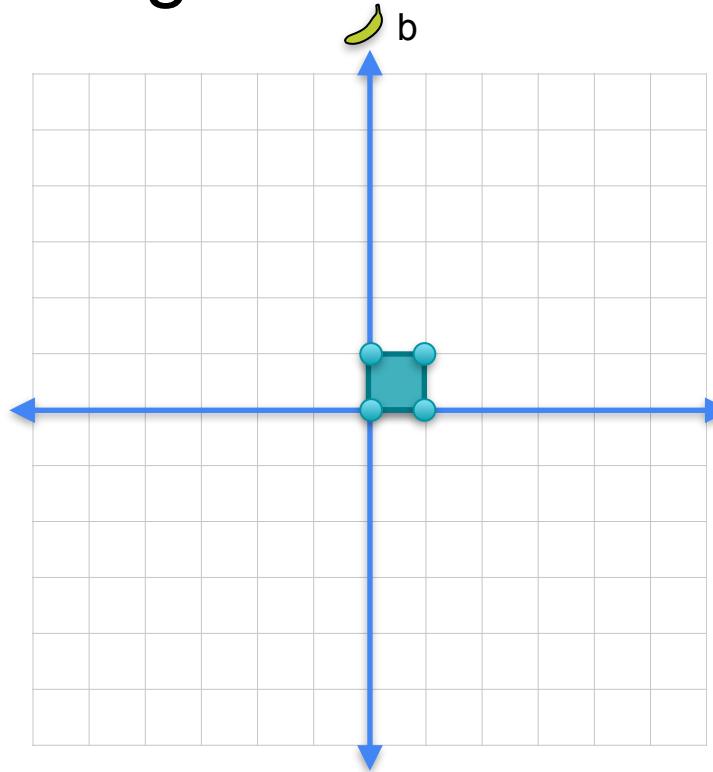


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} = \begin{matrix} 2 \\ 4 \end{matrix} \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$

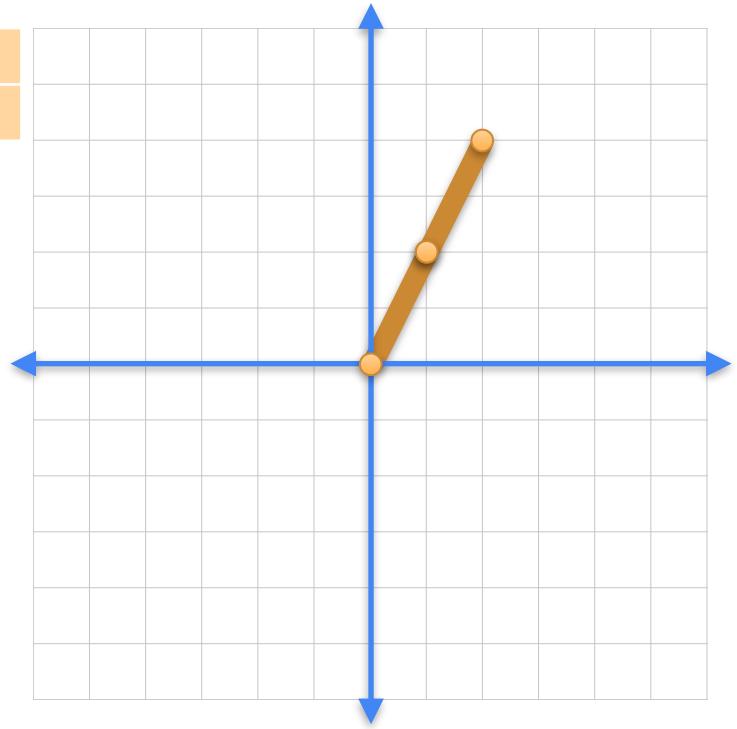


Singular transformation

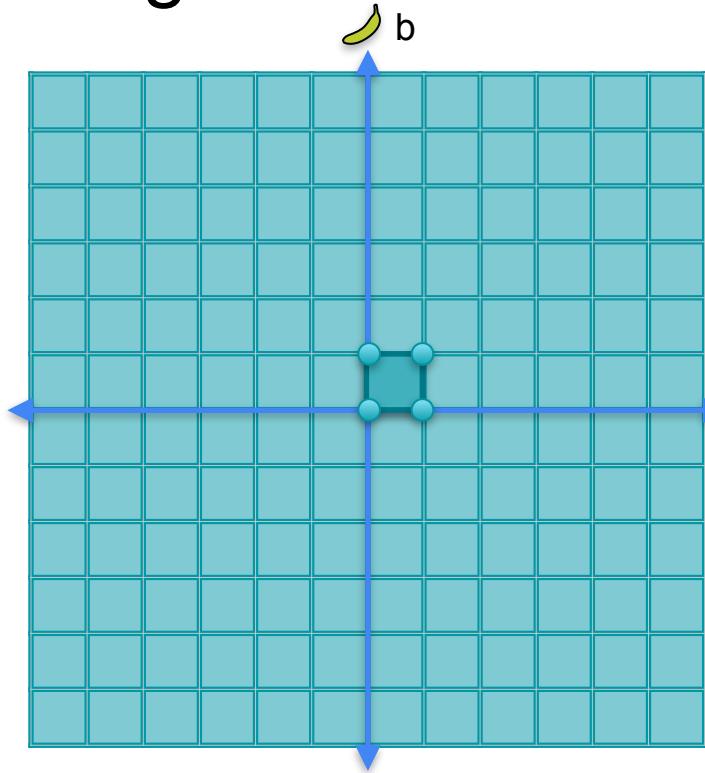


$$\begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} \begin{matrix} 1 \\ 1 \end{matrix} = \begin{matrix} 2 \\ 4 \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$



Singular transformation

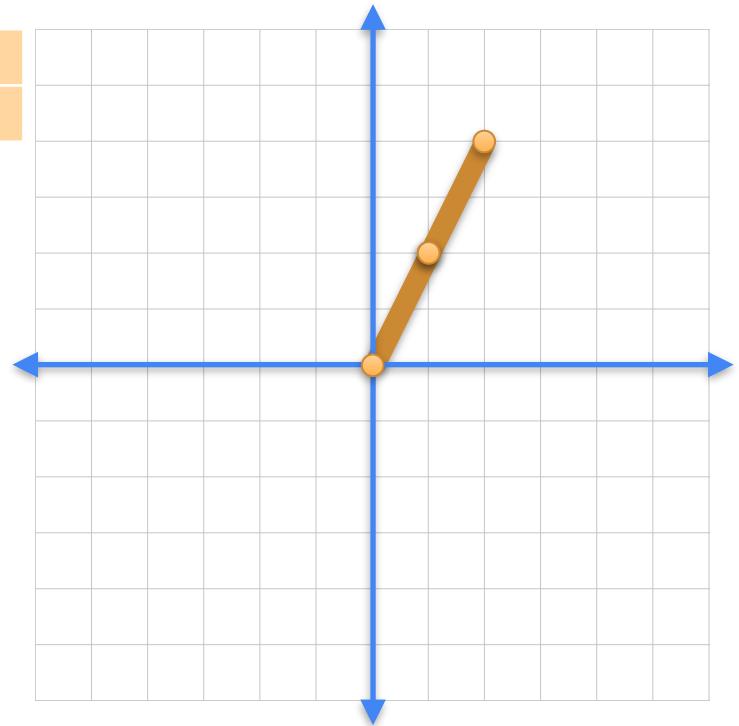


A diagram illustrating a linear transformation. On the left, a 2x2 input matrix is shown with two red apples above it. To its right is a multiplication sign. To the right of the multiplication sign is a 2x1 weight vector represented by a teal column with a red banana icon above it. To the right of the weight vector is an equals sign. To the right of the equals sign is a 2x1 output vector represented by an orange column.

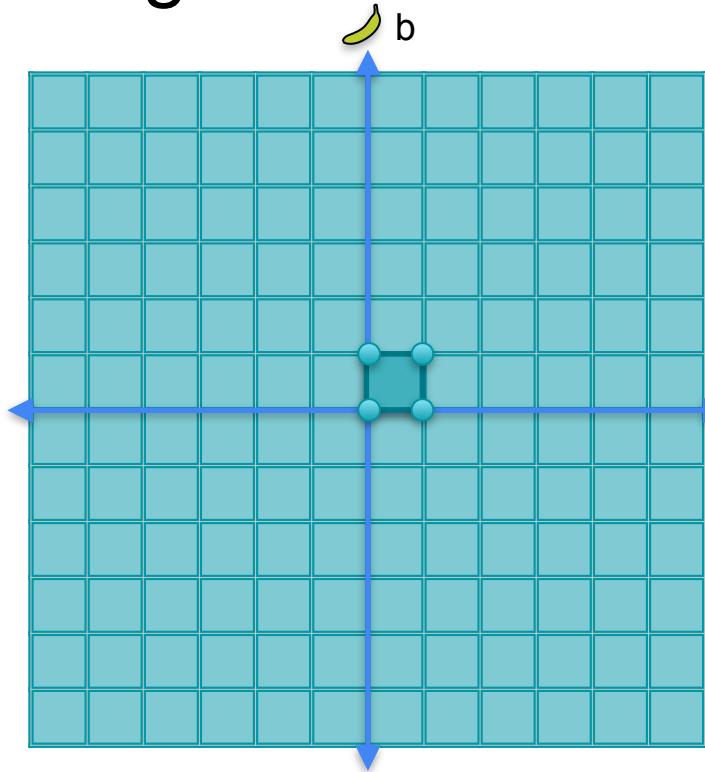
$$\begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} \begin{matrix} 1 \\ 1 \end{matrix} = \begin{matrix} 2 \\ 4 \end{matrix}$$

Mapping of input coordinates to output coordinates:

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (1,2) \\ (0,1) &\rightarrow (1,2) \\ (1,1) &\rightarrow (2,4) \end{aligned}$$



Singular transformation

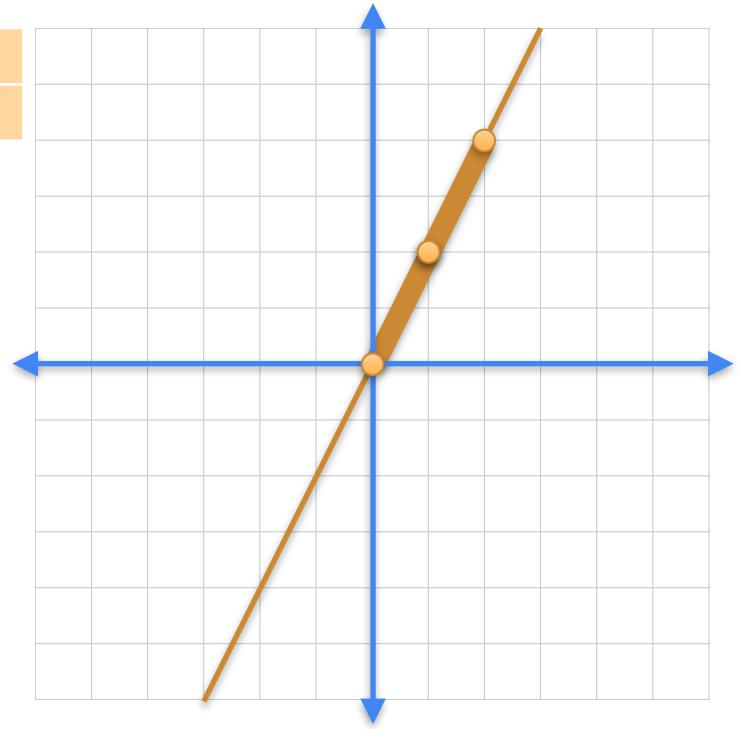


A diagram illustrating a singular transformation. It shows two 2x2 matrices being multiplied by a 2x1 column vector. The first matrix has columns [1, 2] and rows [1, 2]. The second matrix has columns [1, 1] and rows [1, 1]. The result is a 2x1 column vector with entries 2 and 4. To the left of the matrices is a red apple icon, and to the right is a yellow banana icon.

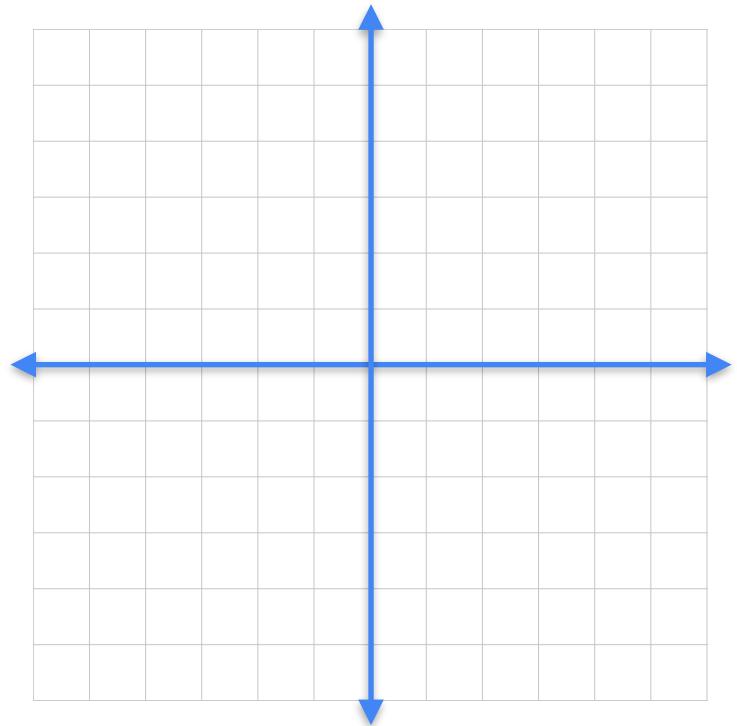
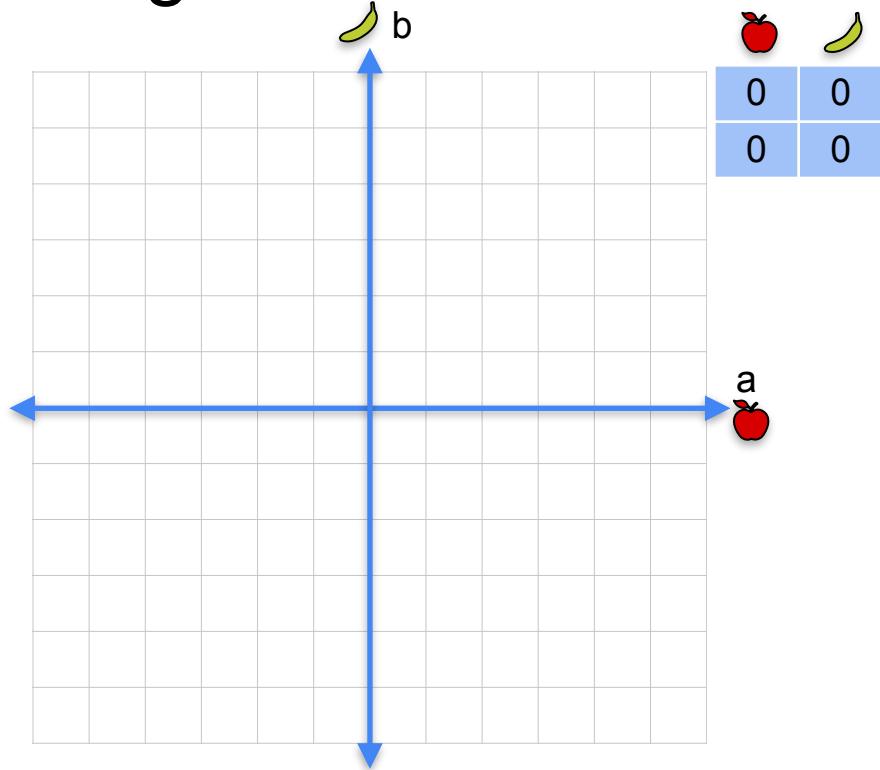
$$\begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} \begin{matrix} 1 \\ 1 \end{matrix} = \begin{matrix} 2 \\ 4 \end{matrix}$$

Mapping of input coordinates to output coordinates:

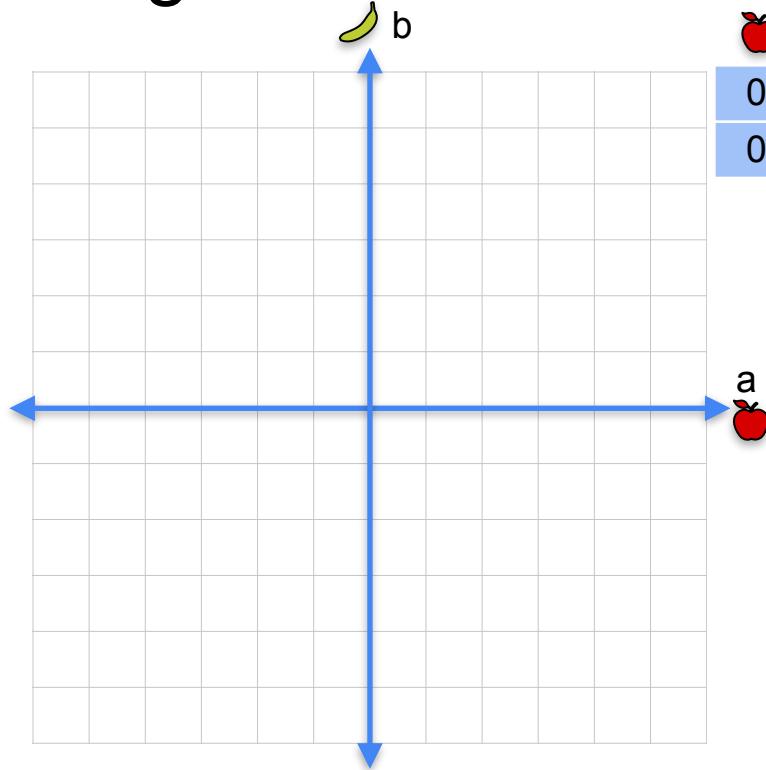
- $(0,0) \rightarrow (0,0)$
- $(1,0) \rightarrow (1,2)$
- $(0,1) \rightarrow (1,2)$
- $(1,1) \rightarrow (2,4)$



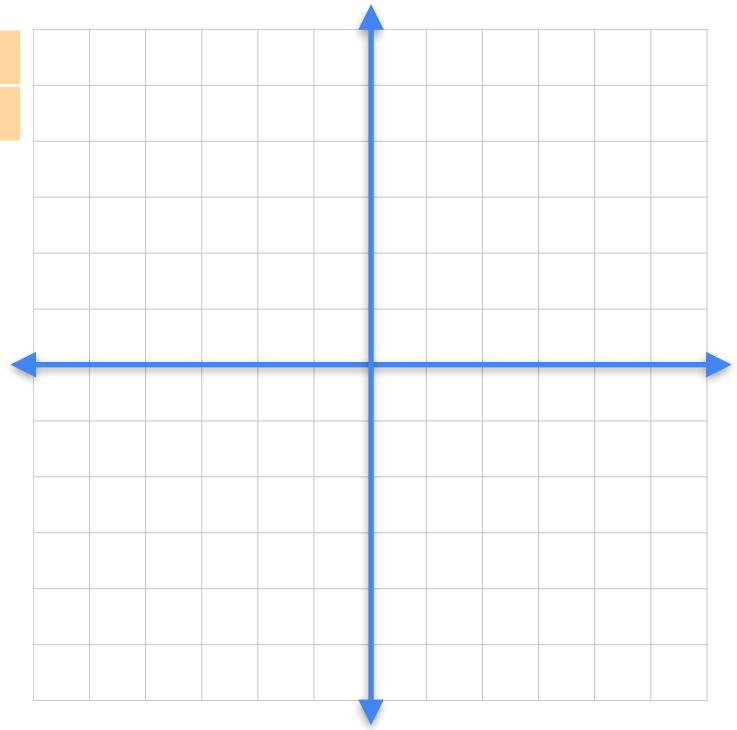
Singular transformation



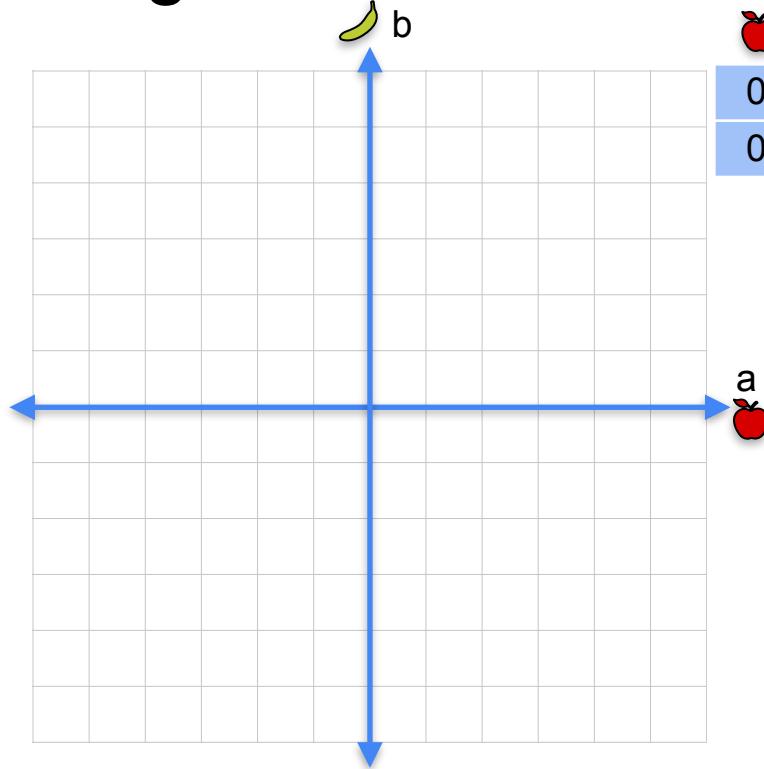
Singular transformation



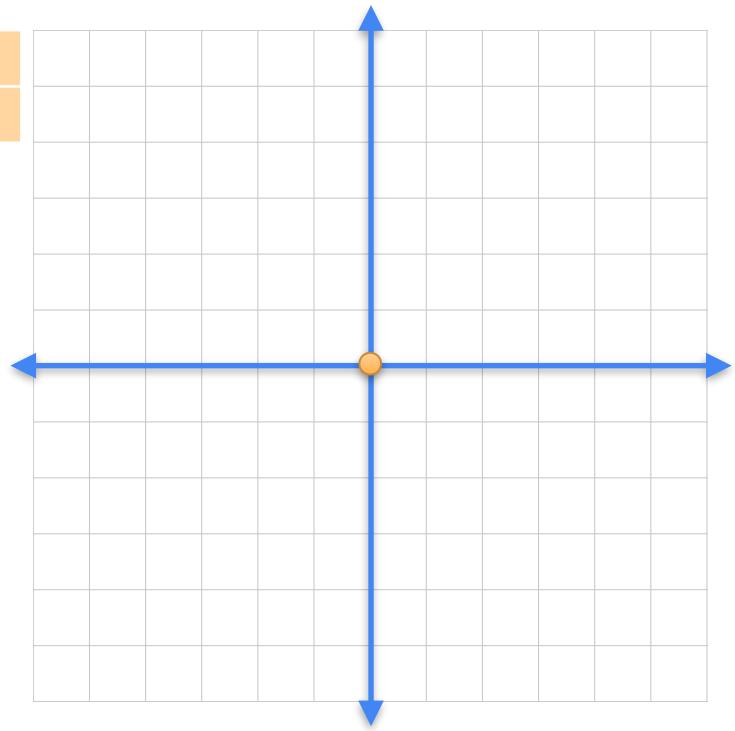
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} a \\ b \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix} \end{matrix}$$



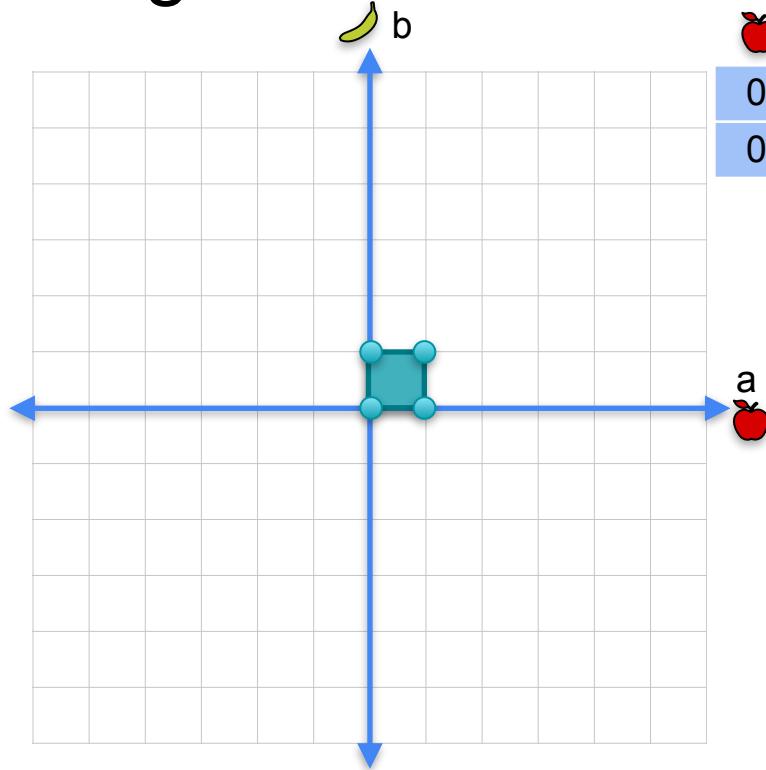
Singular transformation



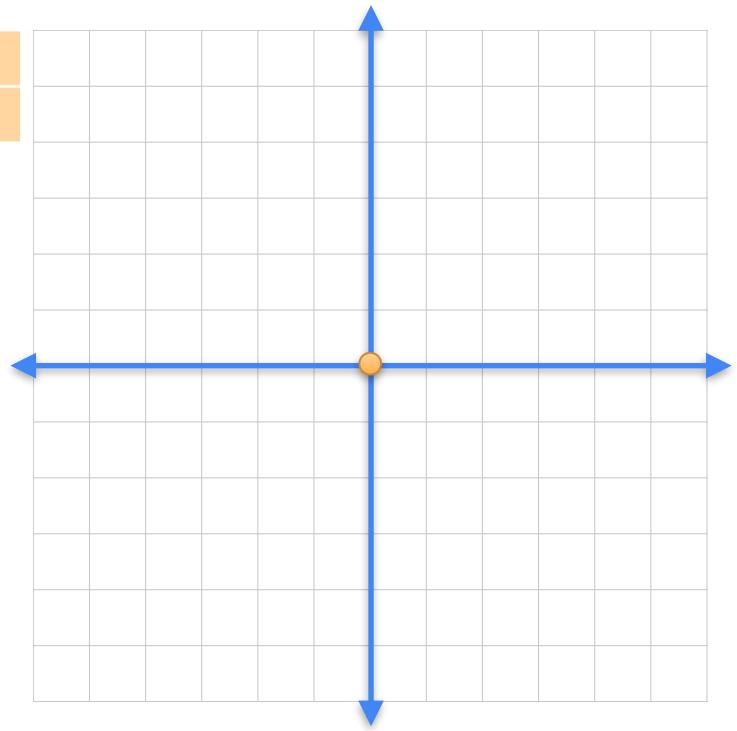
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} a \\ b \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix} \end{matrix}$$



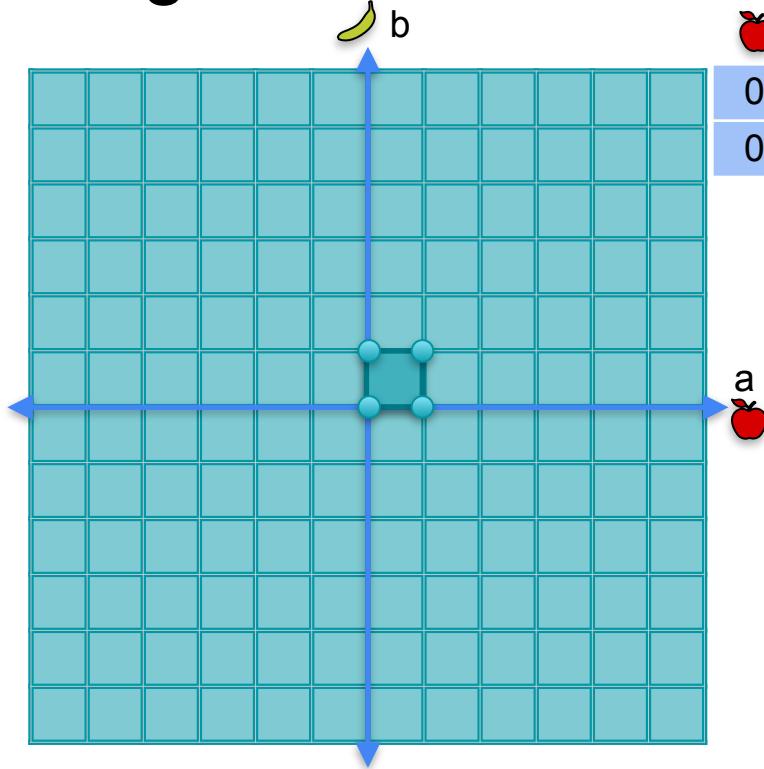
Singular transformation



$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} a \\ b \end{matrix} \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

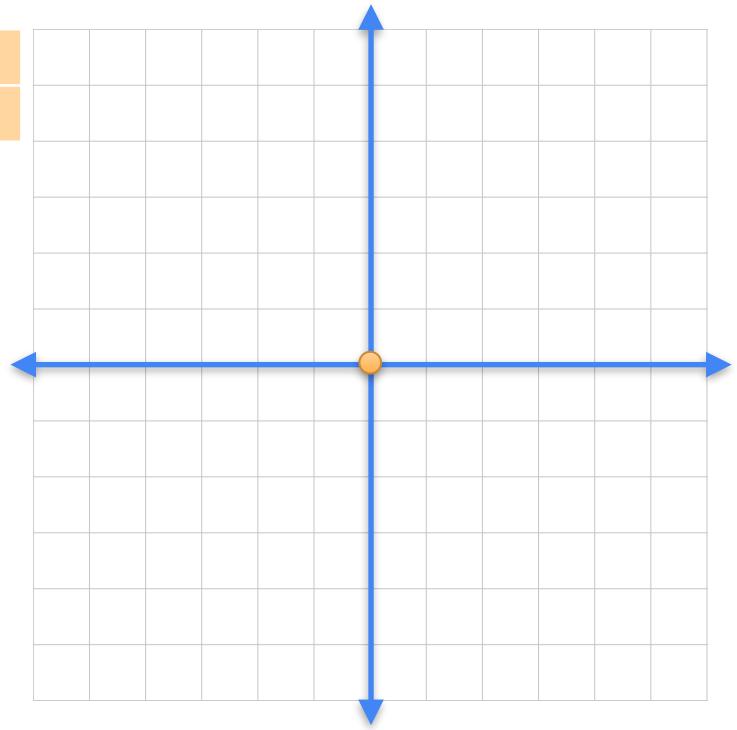


Singular transformation



A diagram illustrating matrix multiplication. On the left, there is a 2x2 matrix with two red apple icons in the top row and two yellow banana icons in the bottom row. To its right is a 2x2 matrix with two teal squares labeled 'a' in the top row and two teal squares labeled 'b' in the bottom row. Between these two matrices is an equals sign (=). To the right of the equals sign is another 2x2 matrix with two orange squares labeled '0' in the top row and two orange squares labeled '0' in the bottom row.

$$\begin{matrix} \text{apple} & \text{banana} \\ 0 & 0 \\ 0 & 0 \end{matrix} \times \begin{matrix} a & b \\ a & b \end{matrix} = \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix}$$

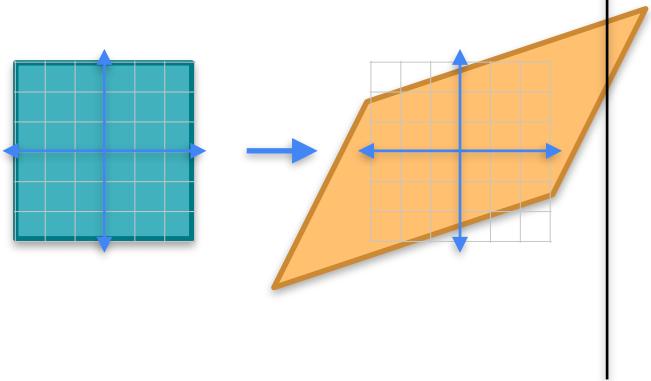


Singular and non-singular transformations

Singular and non-singular transformations

Non-singular

| | | |
|---|---|---|
| |  |  |
| 3 | 1 | |
| 1 | 2 | |



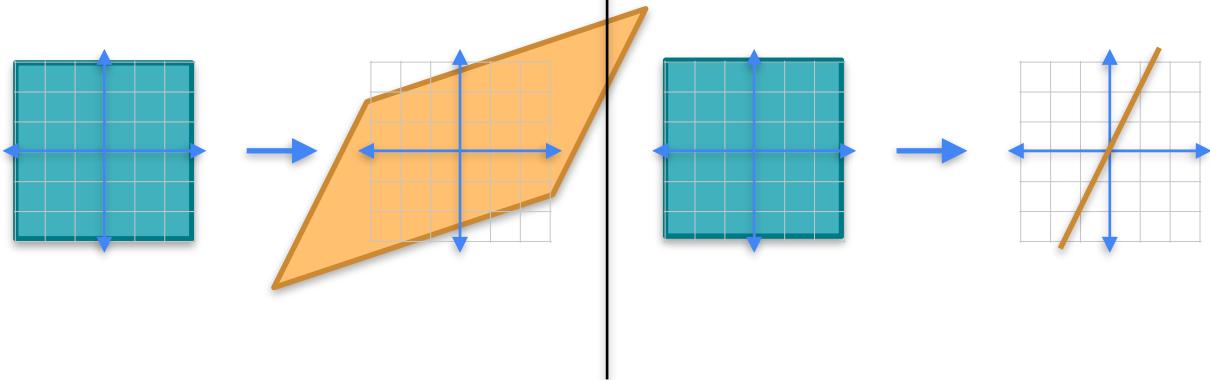
Singular and non-singular transformations

Non-singular

| | |
|---|---|
| | |
| 3 | 1 |
| 1 | 2 |

Singular

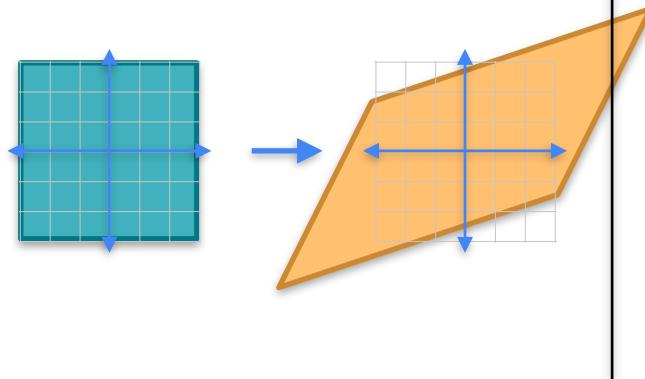
| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |



Singular and non-singular transformations

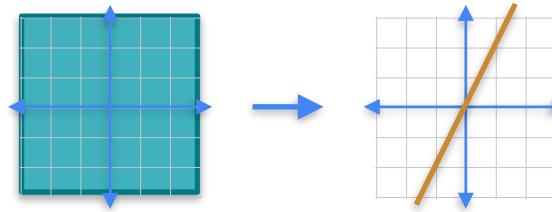
Non-singular

| | |
|---|---|
| | |
| 3 | 1 |
| 1 | 2 |



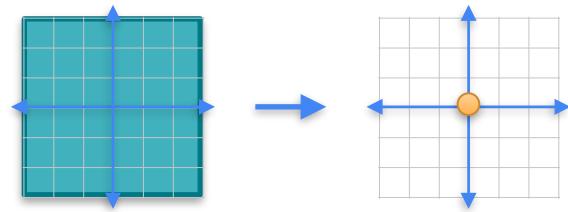
Singular

| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |



Singular

| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |

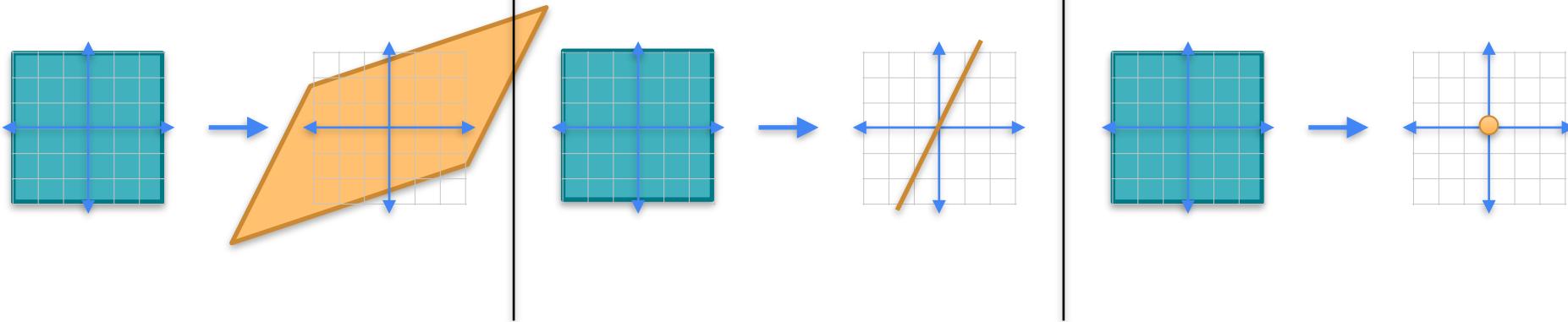


Rank of linear transformations

| | |
|---|---|
| | |
| 3 | 1 |
| 1 | 2 |

| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |

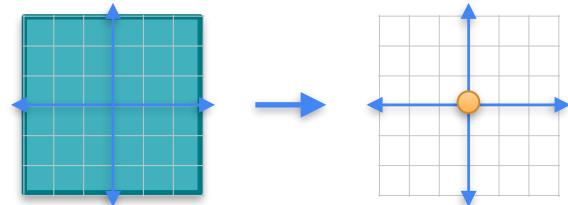
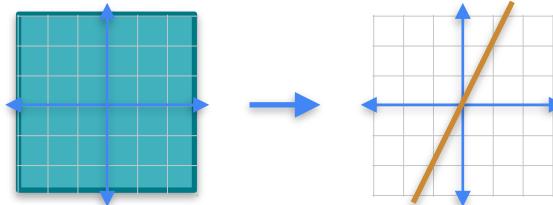
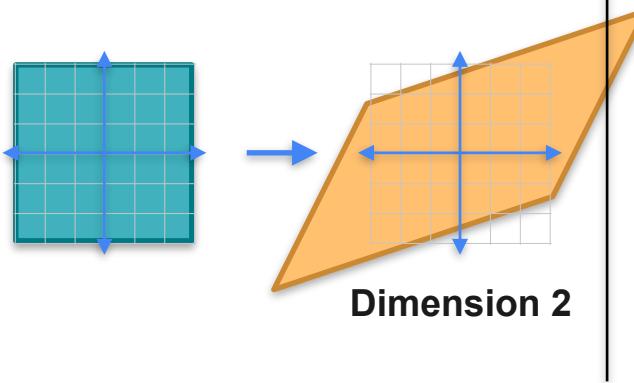


Rank of linear transformations

| | |
|---|---|
| | |
| 3 | 1 |
| 1 | 2 |

| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

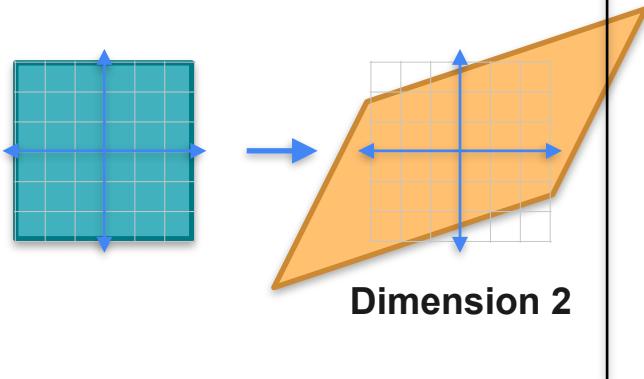
| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |



Rank of linear transformations

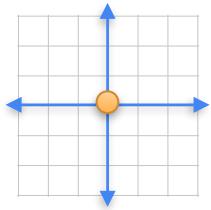
Rank 2

| | | |
|---|---|--|
| | | |
| 3 | 1 | |
| 1 | 2 | |



| | | |
|---|---|--|
| | | |
| 1 | 1 | |
| 2 | 2 | |

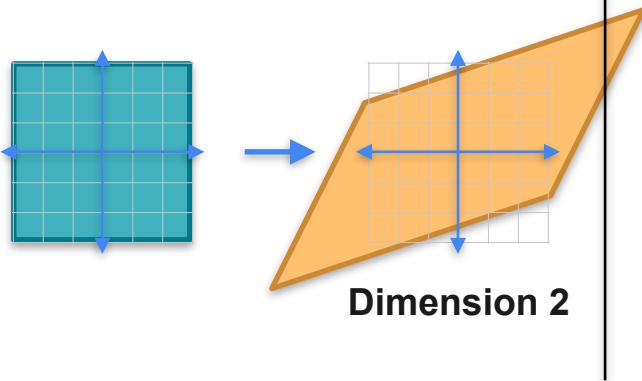
| | | |
|---|---|--|
| | | |
| 0 | 0 | |
| 0 | 0 | |



Rank of linear transformations

Rank 2

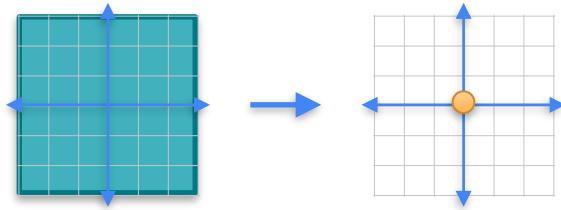
| | | |
|---|---|--|
| | | |
| 3 | 1 | |
| 1 | 2 | |



| | | |
|---|---|--|
| | | |
| 1 | 1 | |
| 2 | 2 | |

Dimension 1

| | | |
|---|---|--|
| | | |
| 0 | 0 | |
| 0 | 0 | |



Rank of linear transformations

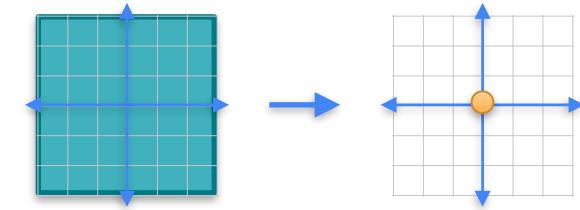
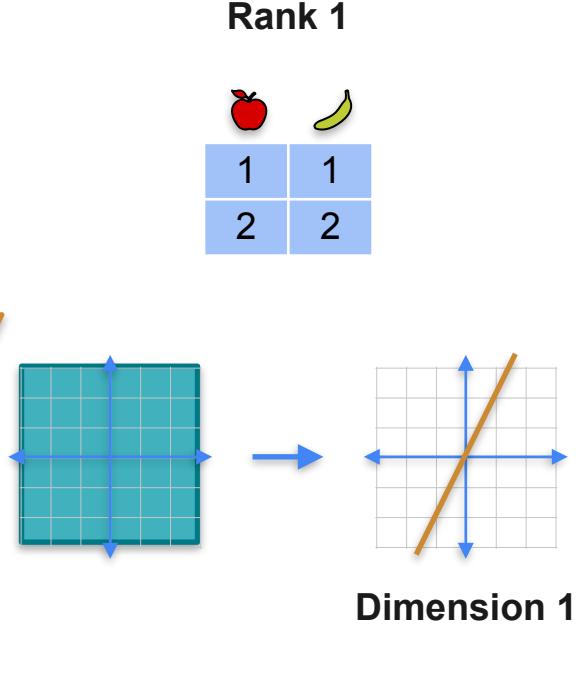
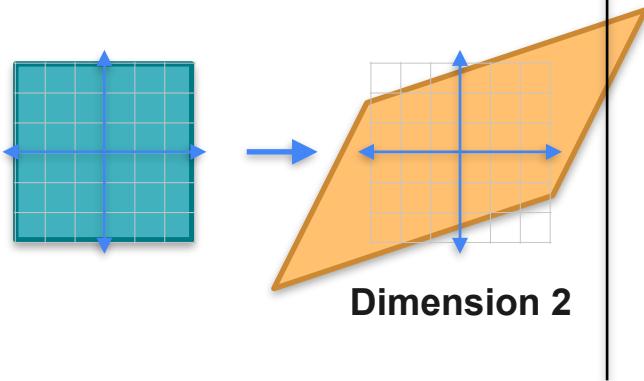
Rank 2

| | |
|---|---|
| | |
| 3 | 1 |
| 1 | 2 |

Rank 1

| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |



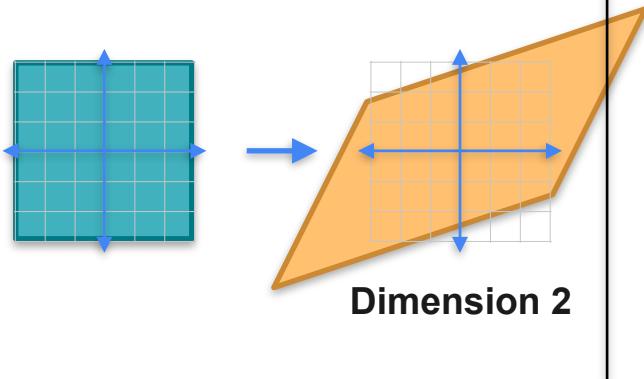
Rank of linear transformations

Rank 2

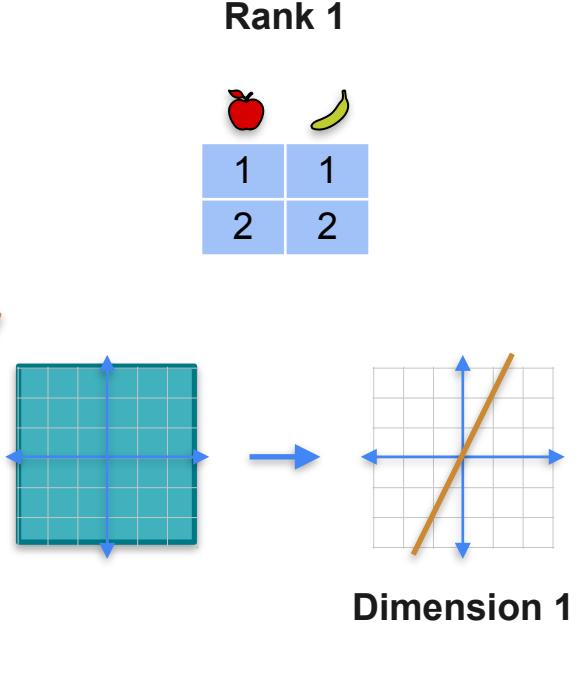
| | |
|---|---|
| | |
| 3 | 1 |
| 1 | 2 |

Rank 1

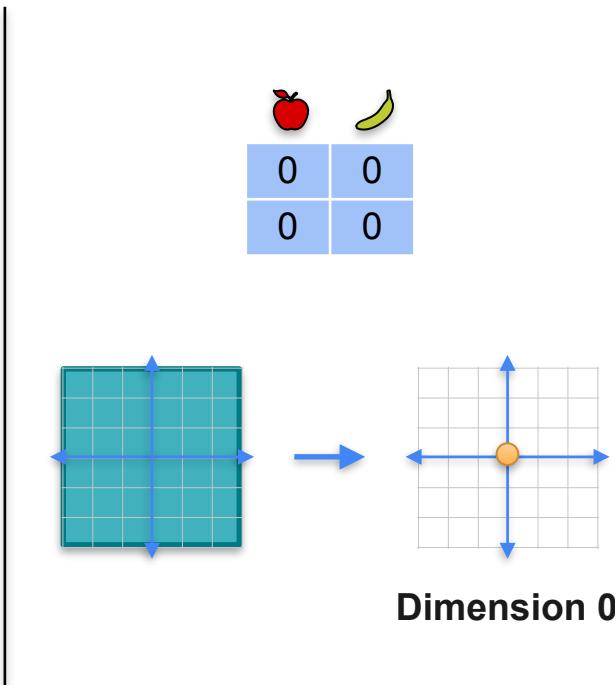
| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |



Dimension 2



Dimension 1



Dimension 0

Rank of linear transformations

Rank 2

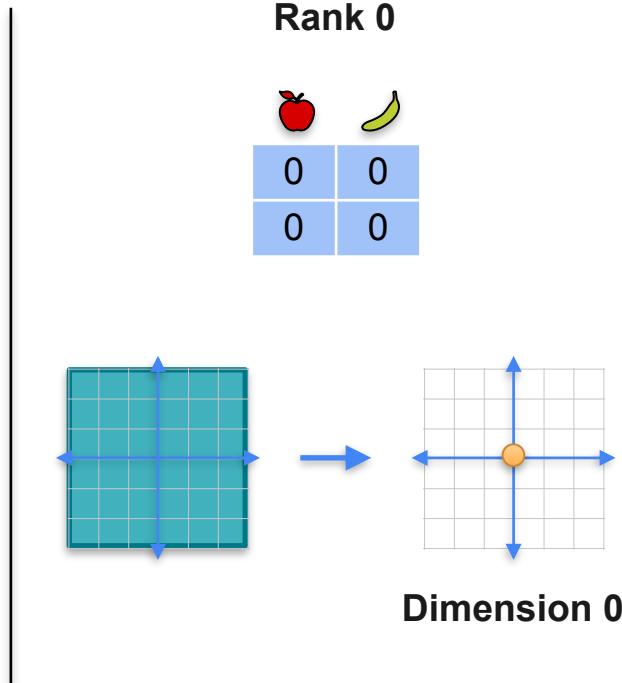
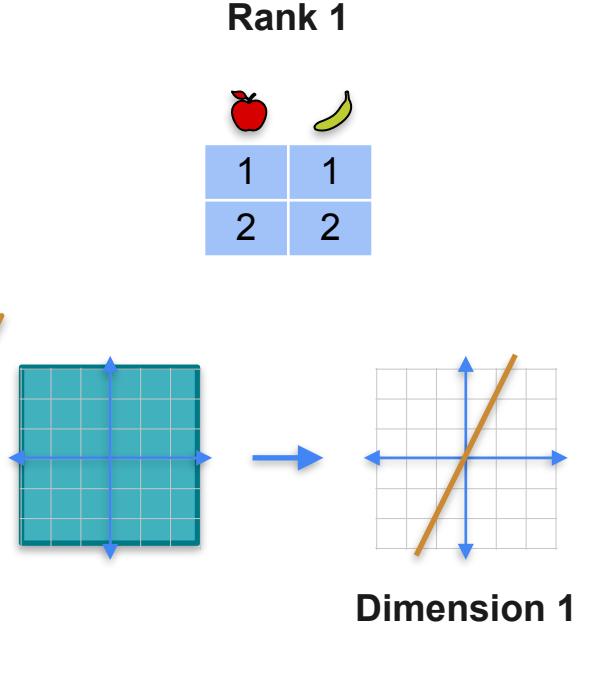
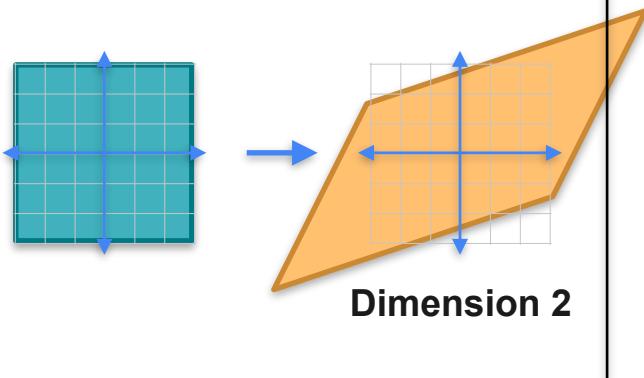
| | |
|---|---|
| | |
| 3 | 1 |
| 1 | 2 |

Rank 1

| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

Rank 0

| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |



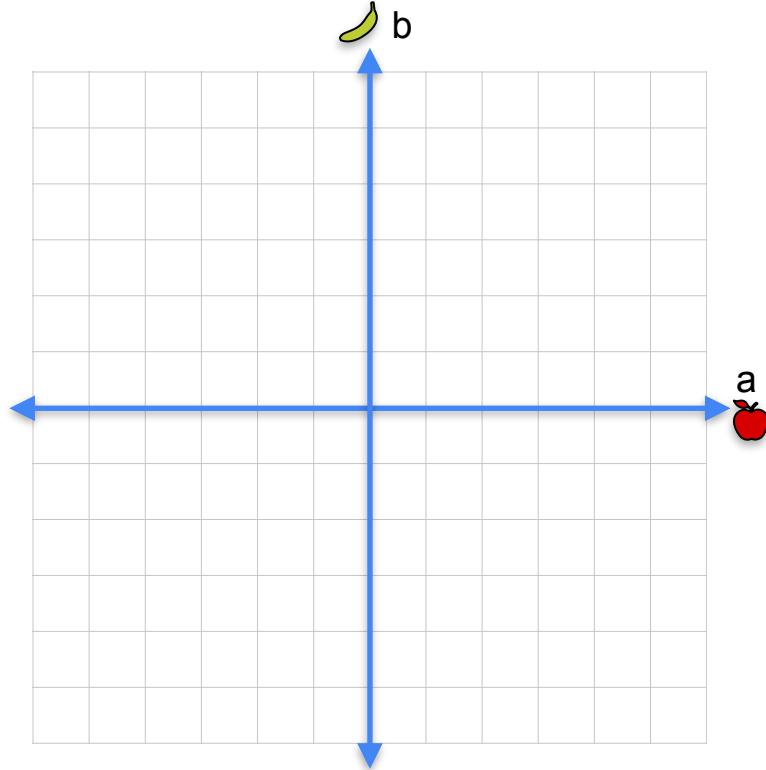


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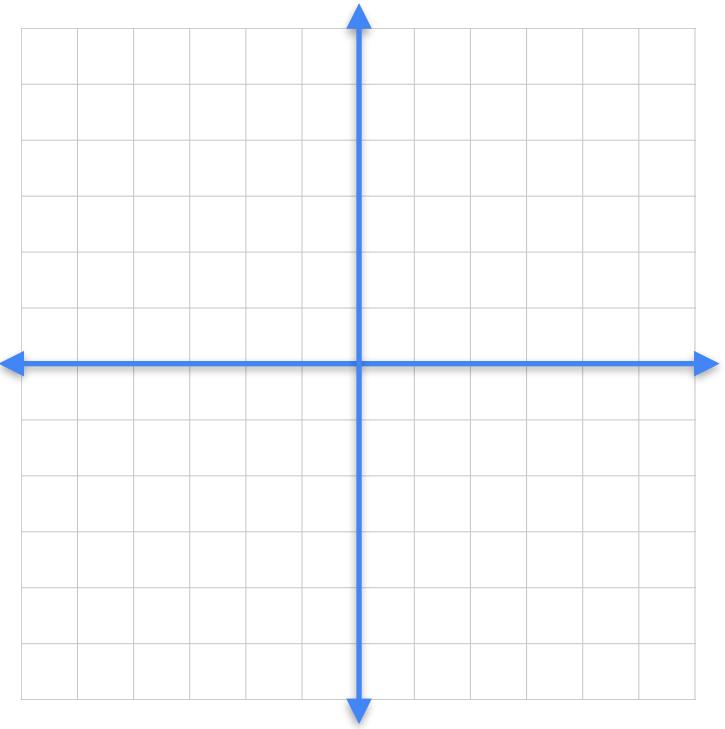
Determinants and Eigenvectors

Determinant as an area

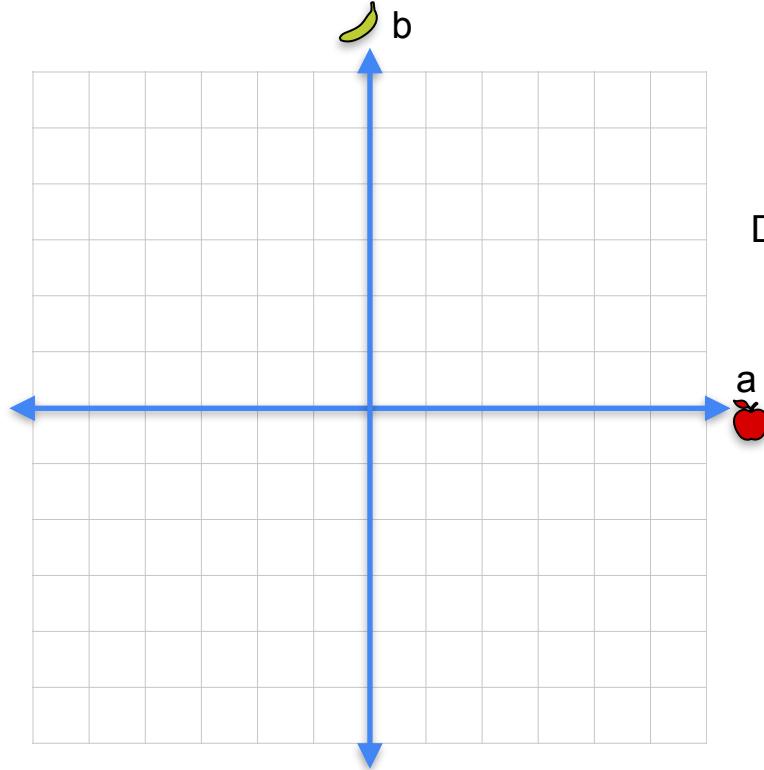
Determinant as an area



| | |
|---|-------|
| | apple |
| 3 | 1 |
| 1 | 2 |



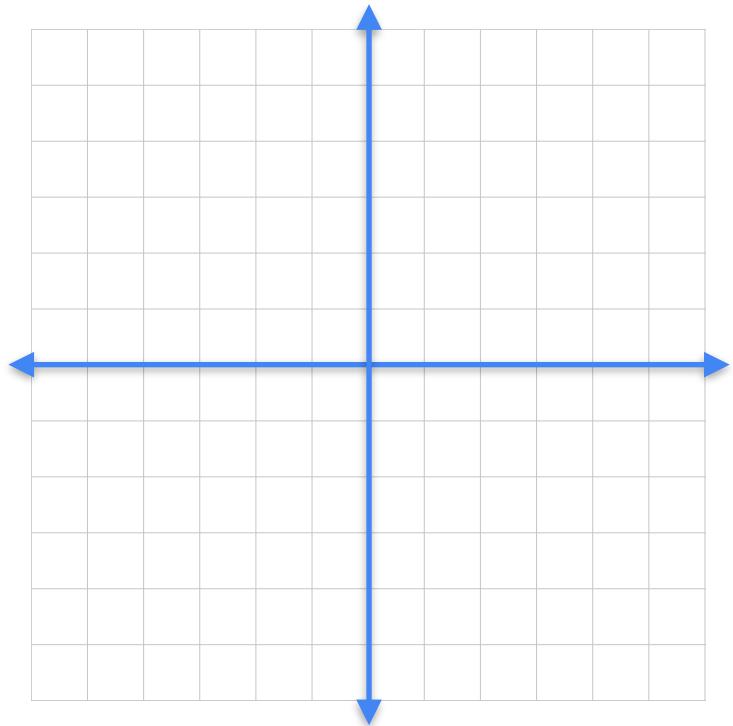
Determinant as an area



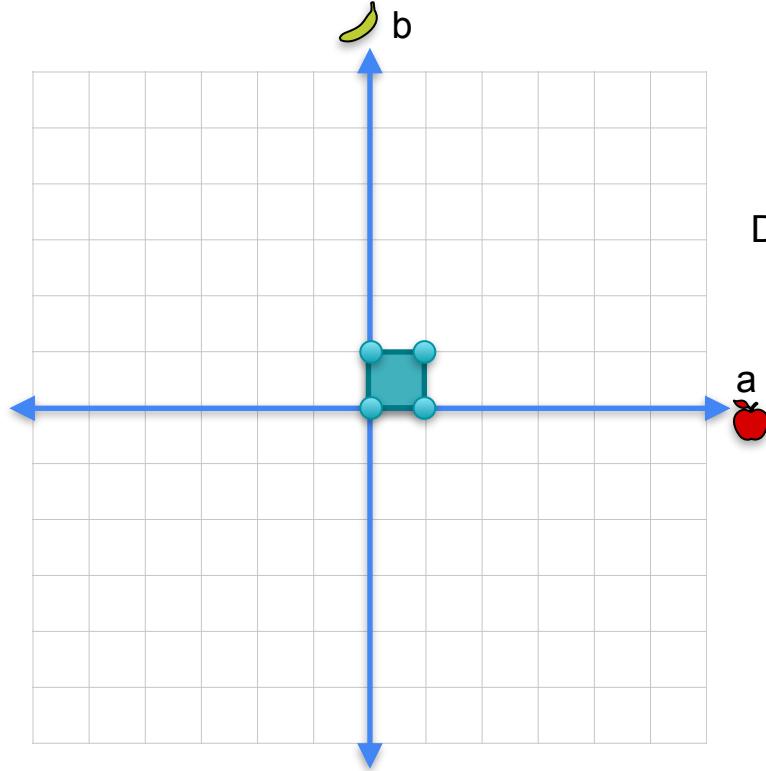
| | |
|---|---|
| 3 | 1 |
| 1 | 2 |

$$\text{Det} = 3 \cdot 2 - 1 \cdot 1$$

$$\text{Det} = 5$$



Determinant as an area

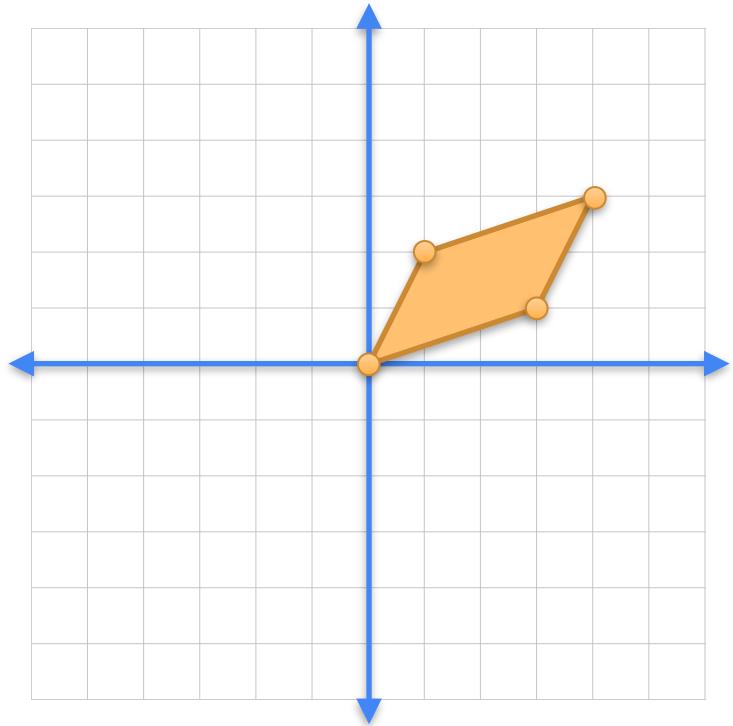


A 2x2 matrix represented as a grid of four squares. The top-left square contains a red apple icon. The top-right square contains a yellow banana icon. The bottom-left square contains a green apple icon. The bottom-right square contains a yellow banana icon.

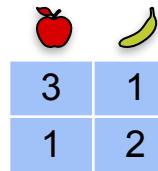
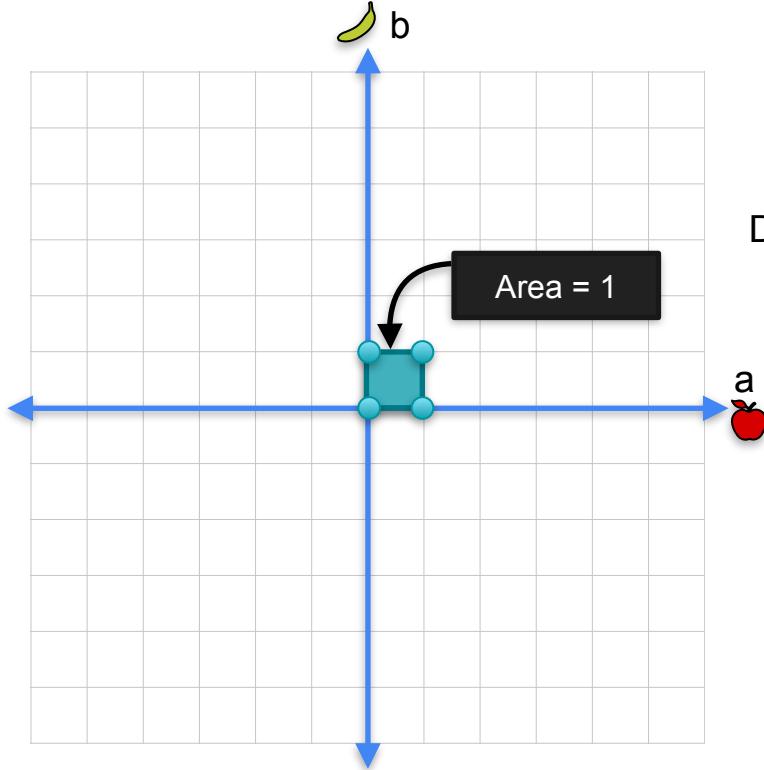
| | |
|-------------|---------------|
| apple | banana |
| green apple | yellow banana |

$$\text{Det} = 3 \cdot 2 - 1 \cdot 1$$

$$\text{Det} = 5$$

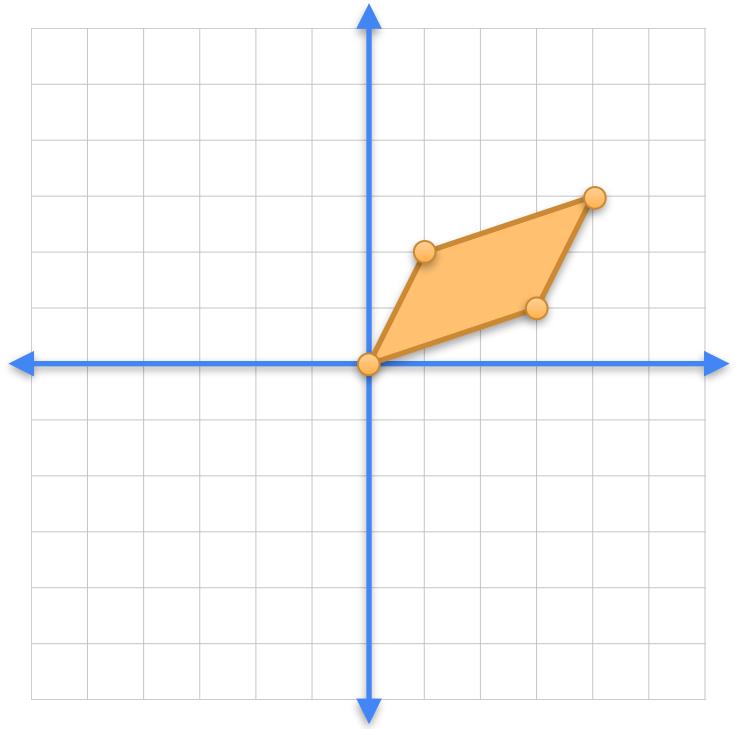


Determinant as an area

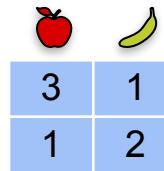
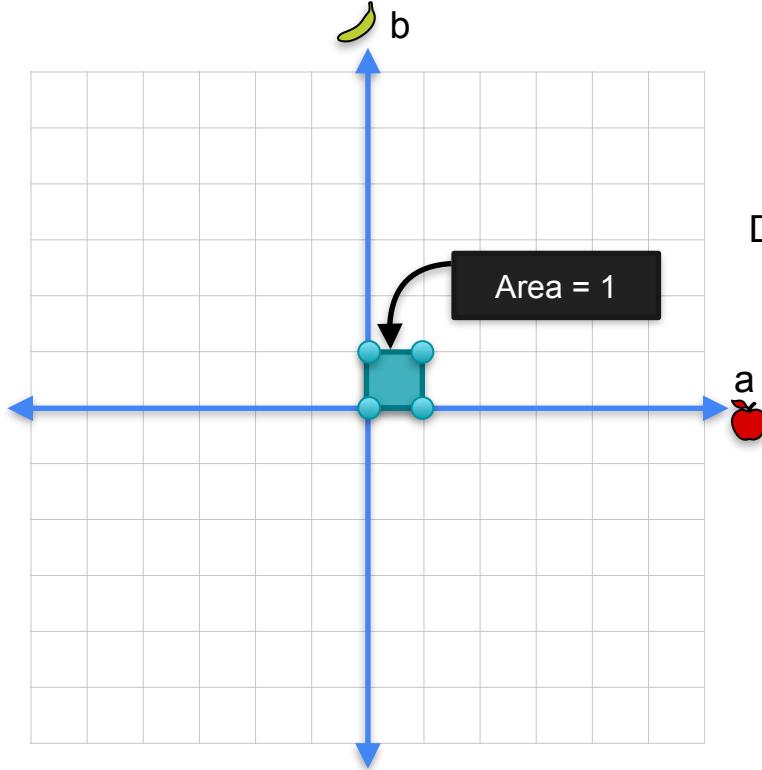


$$\text{Det} = 3 \cdot 2 - 1 \cdot 1$$

$$\text{Det} = 5$$

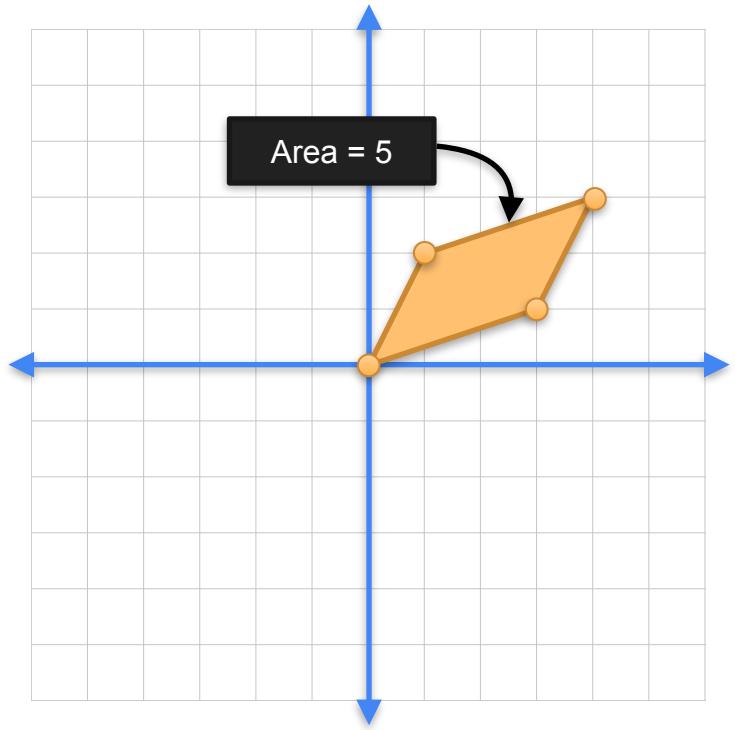


Determinant as an area

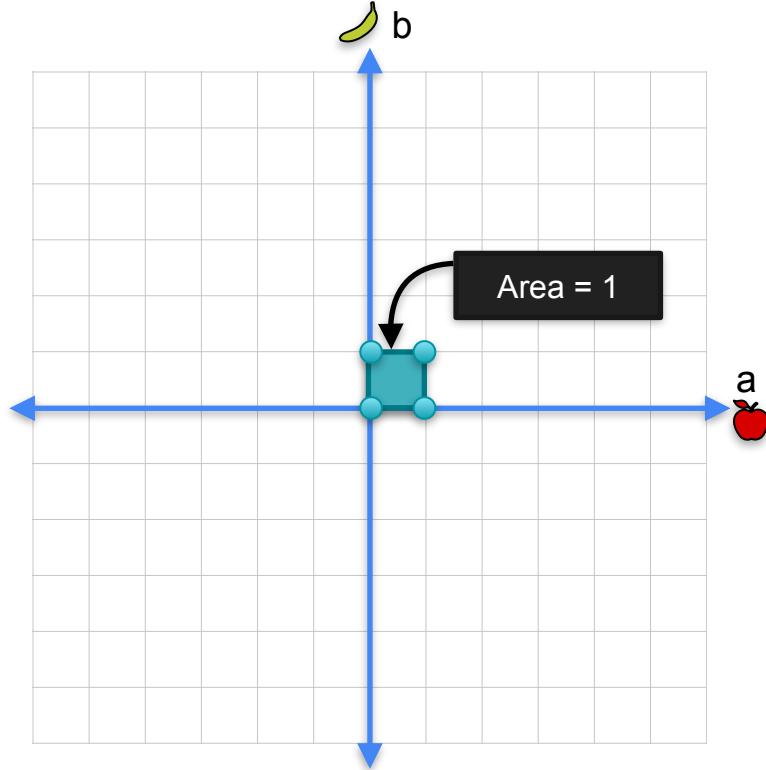


$$\text{Det} = 3 \cdot 2 - 1 \cdot 1$$

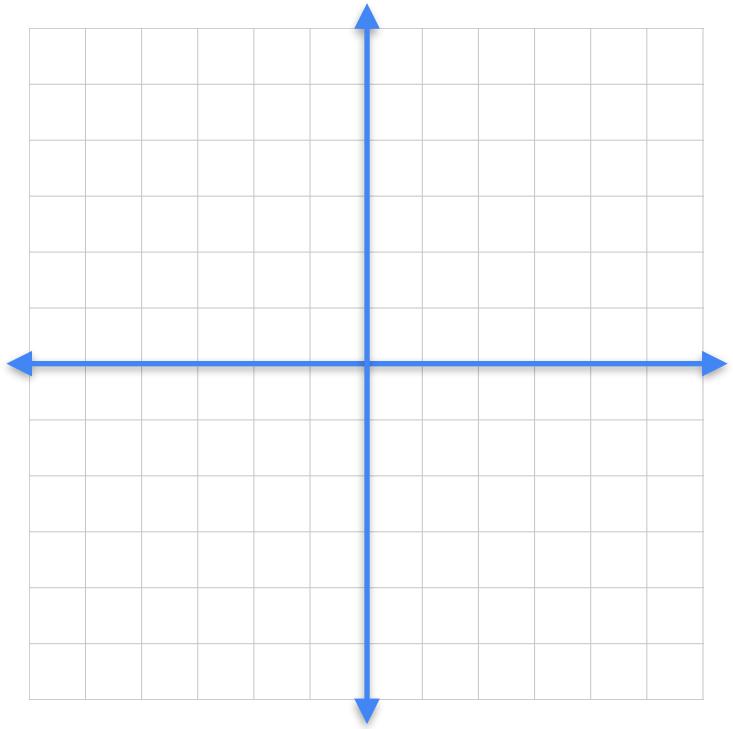
$$\text{Det} = 5$$



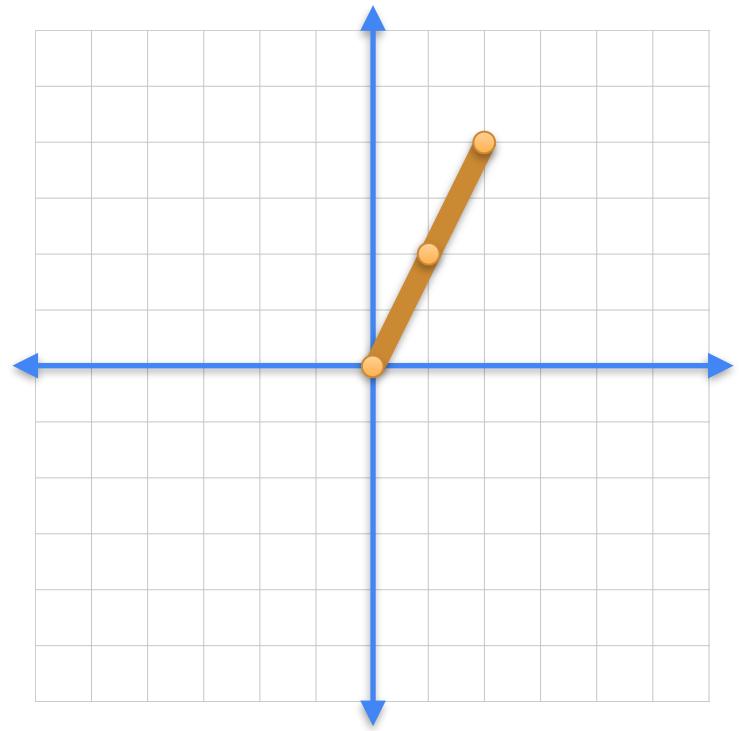
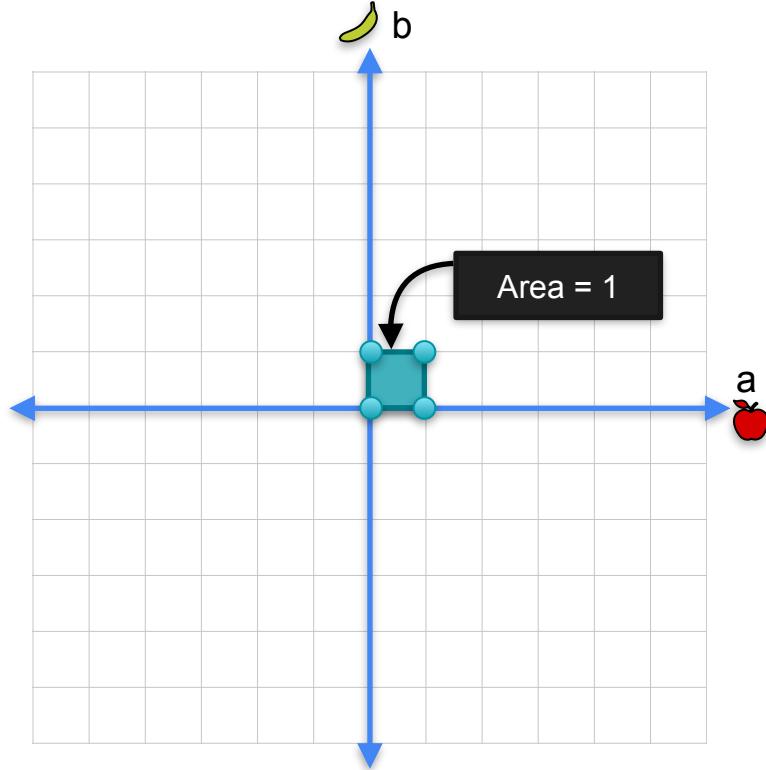
Determinant as an area



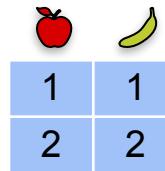
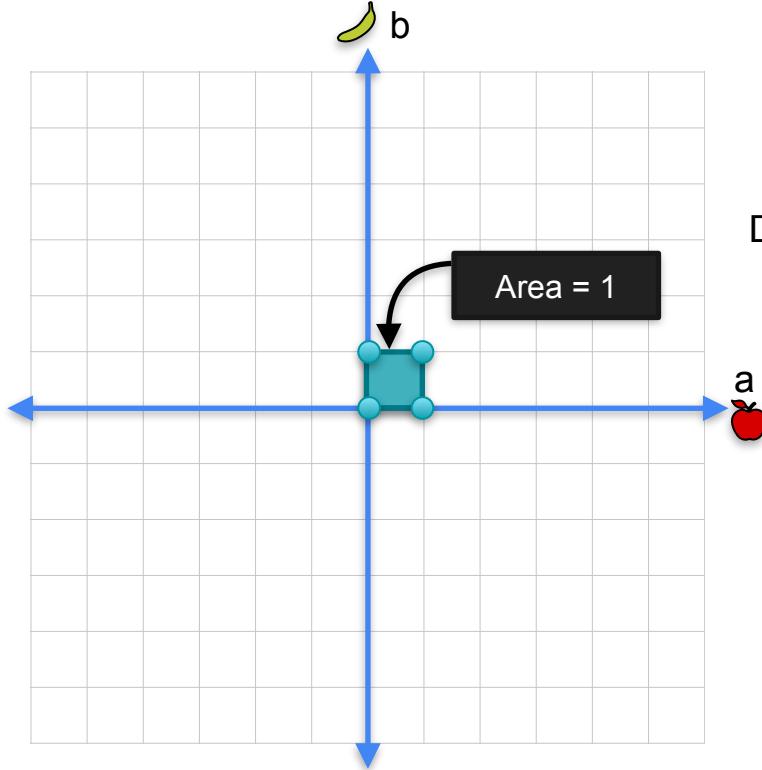
| | |
|---|---|
| 1 | 1 |
| 2 | 2 |



Determinant as an area

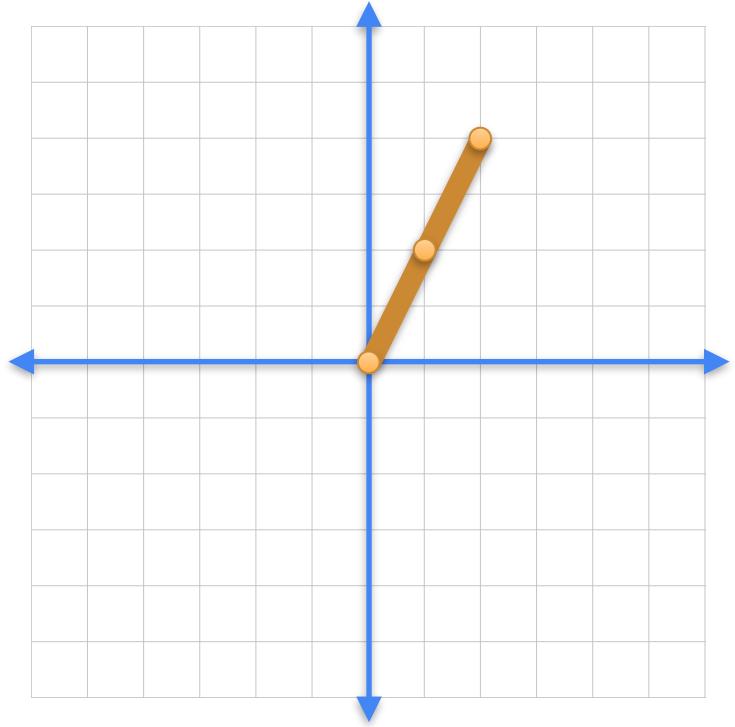


Determinant as an area

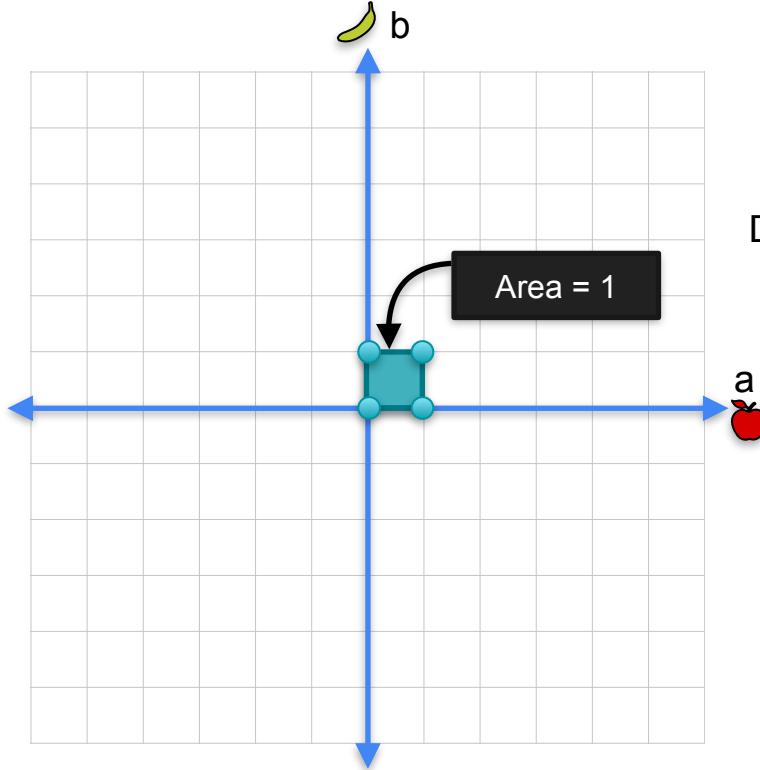


$$\text{Det} = 1 \cdot 2 - 1 \cdot 2$$

$$\text{Det} = 0$$



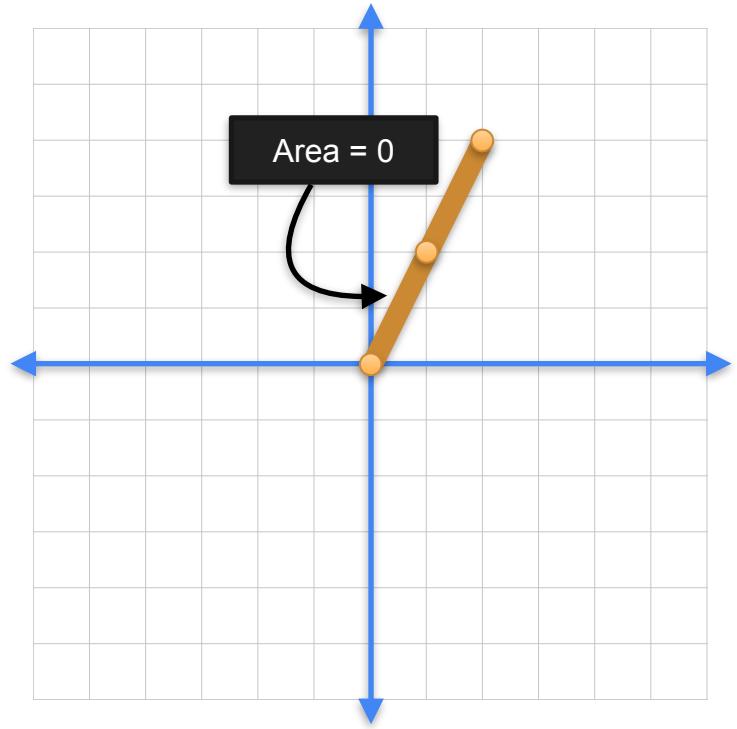
Determinant as an area



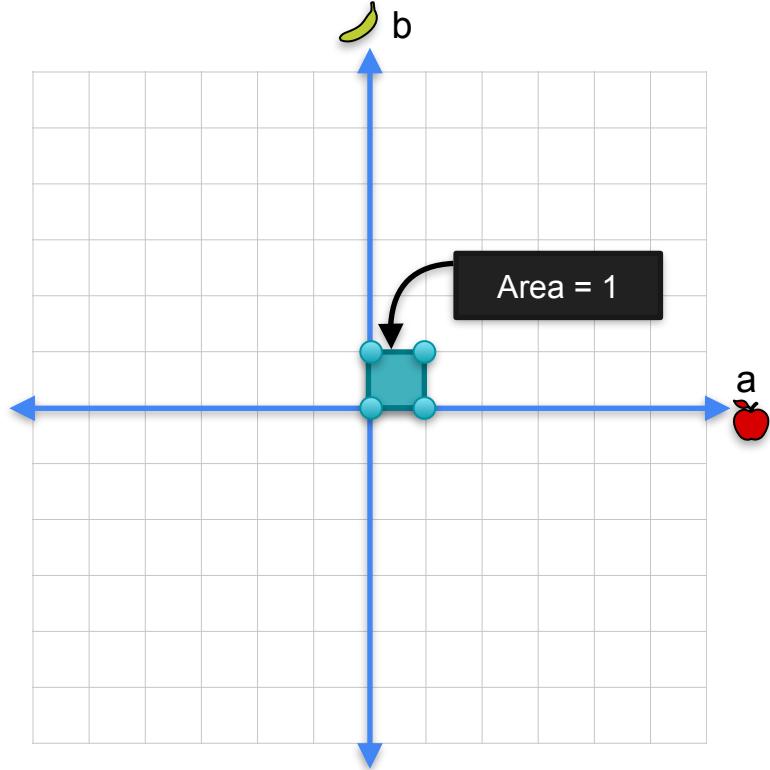
| | |
|---|---|
| 1 | 1 |
| 2 | 2 |

$$\text{Det} = 1 \cdot 2 - 1 \cdot 2$$

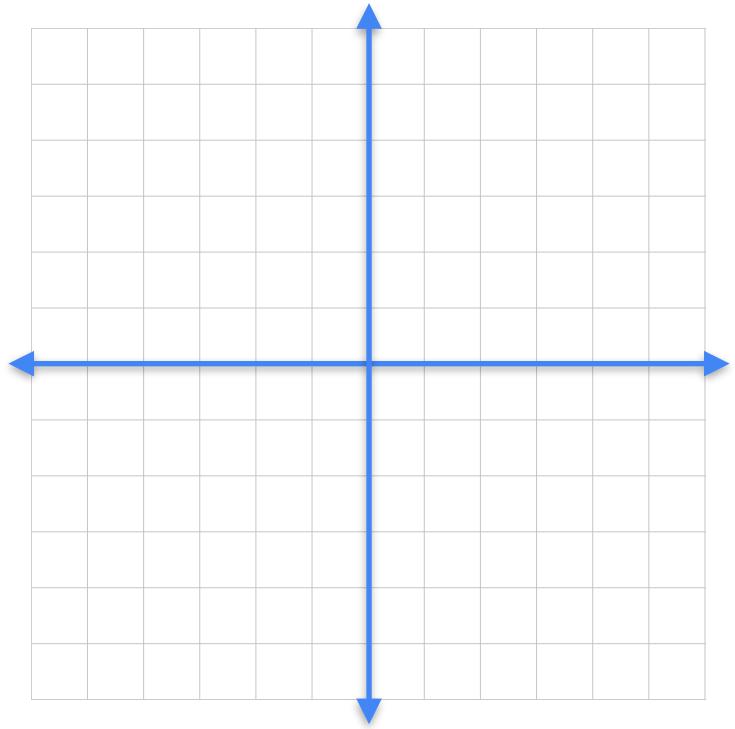
$$\text{Det} = 0$$



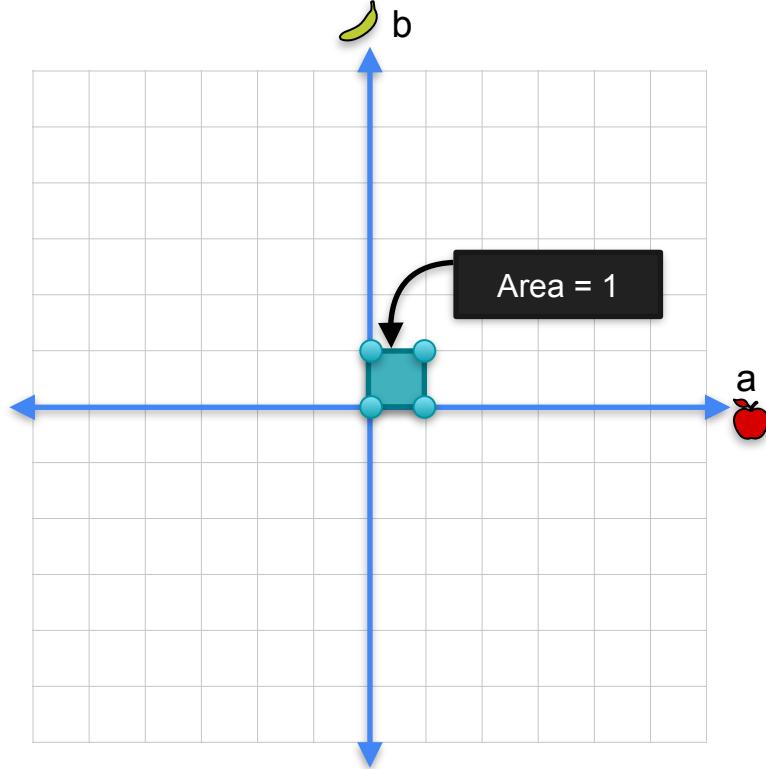
Determinant as an area



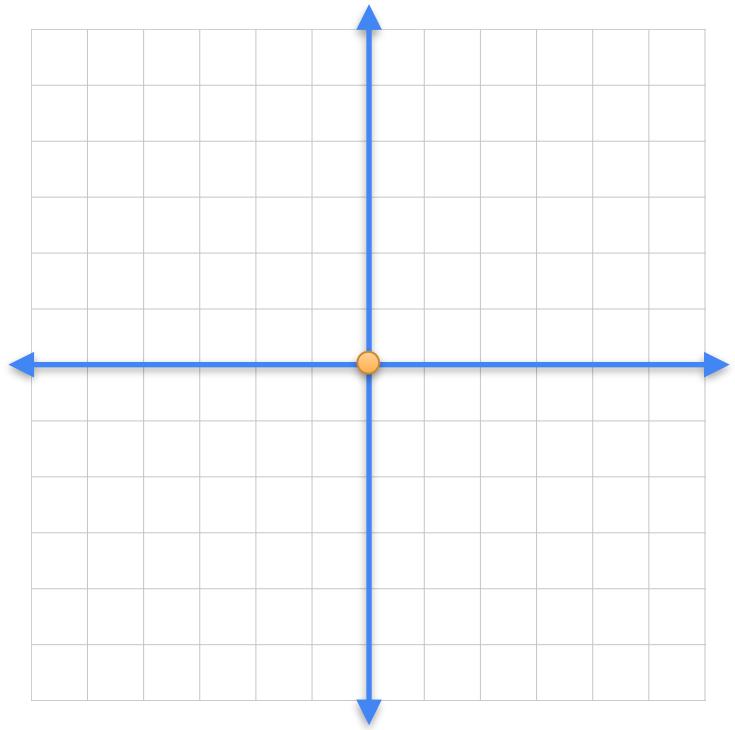
| | | |
|---|-------|--------|
| | apple | banana |
| 0 | 0 | |
| 0 | 0 | |



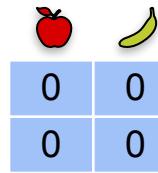
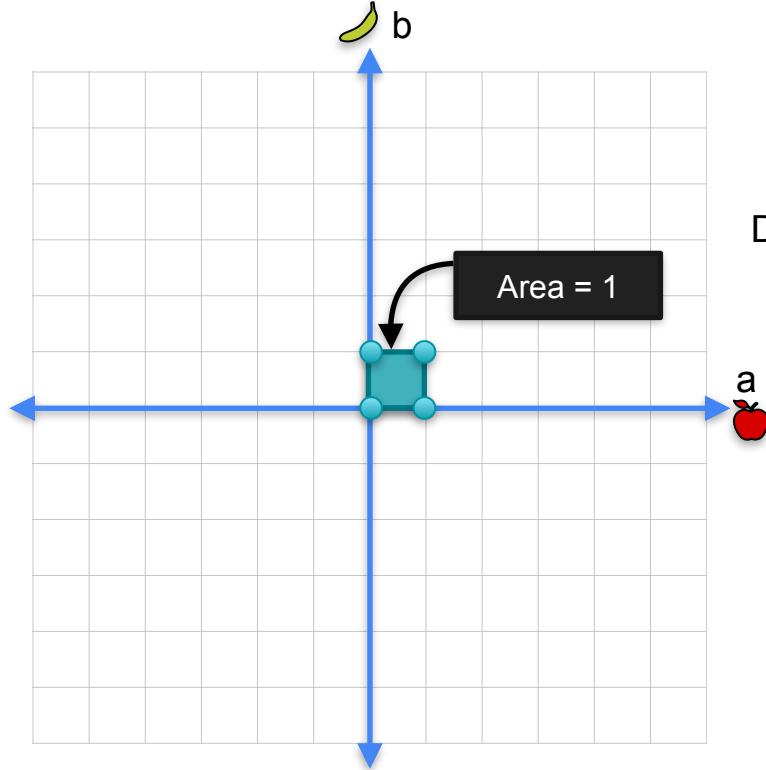
Determinant as an area



| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

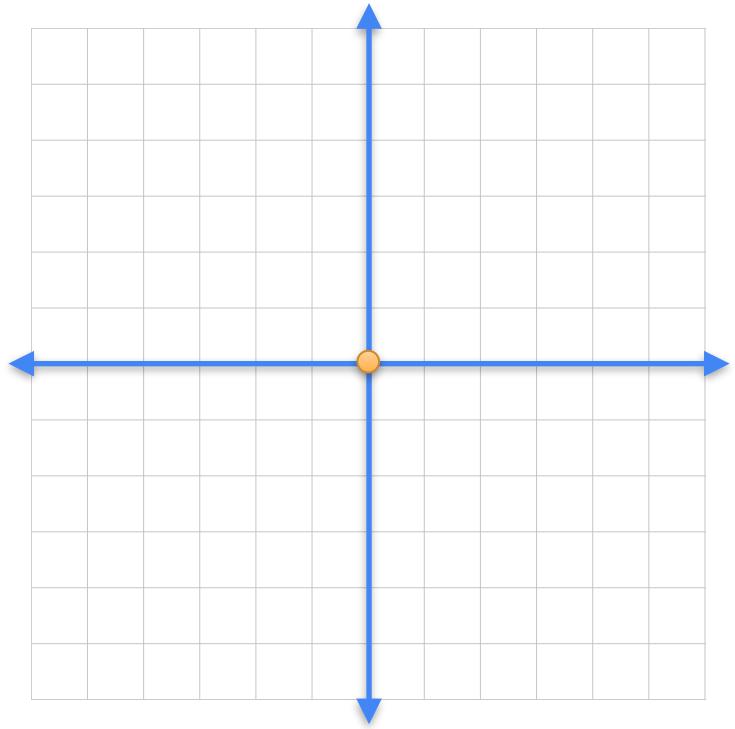


Determinant as an area

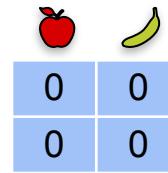
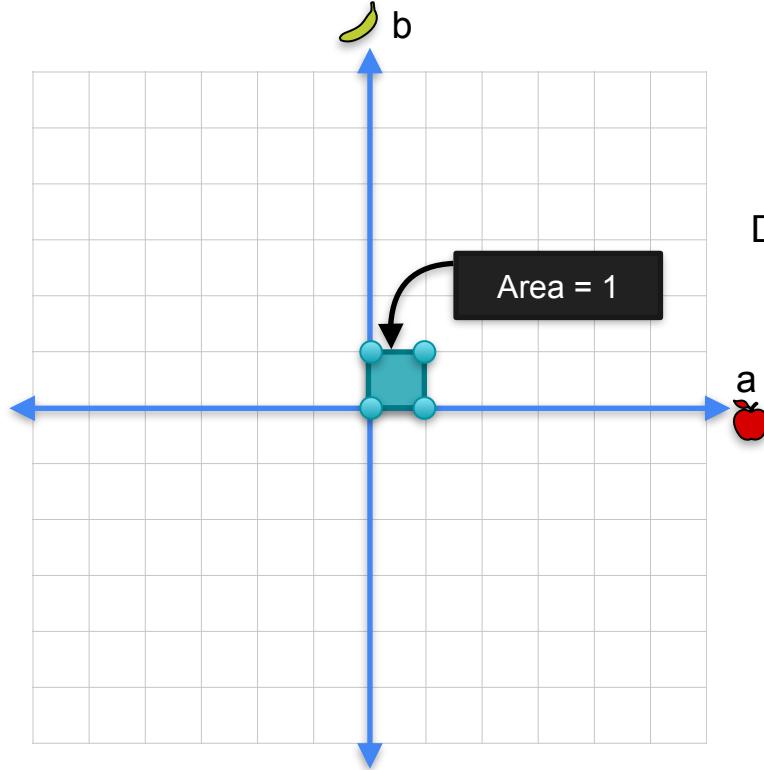


$$\text{Det} = 0 \cdot 0 - 0 \cdot 0$$

$$\text{Det} = 0$$

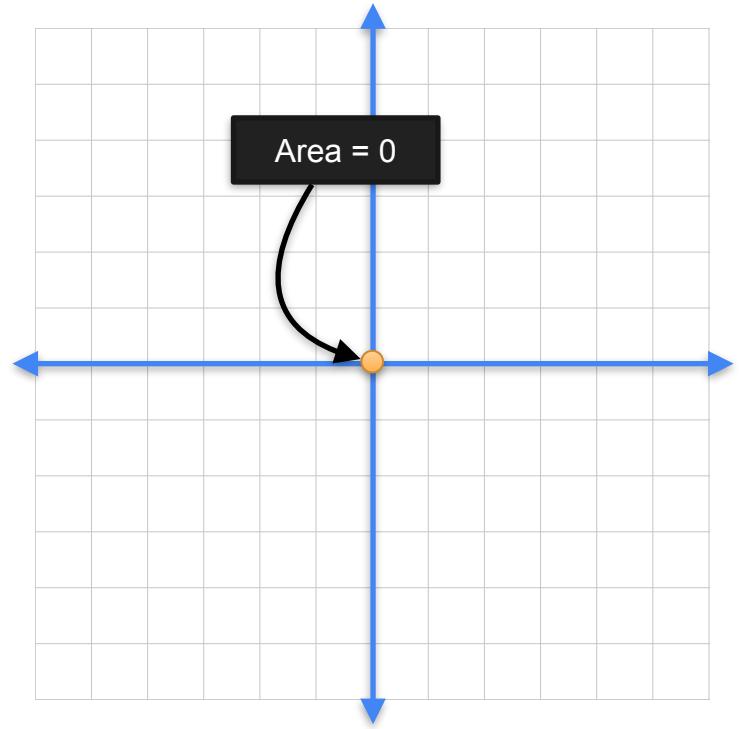


Determinant as an area

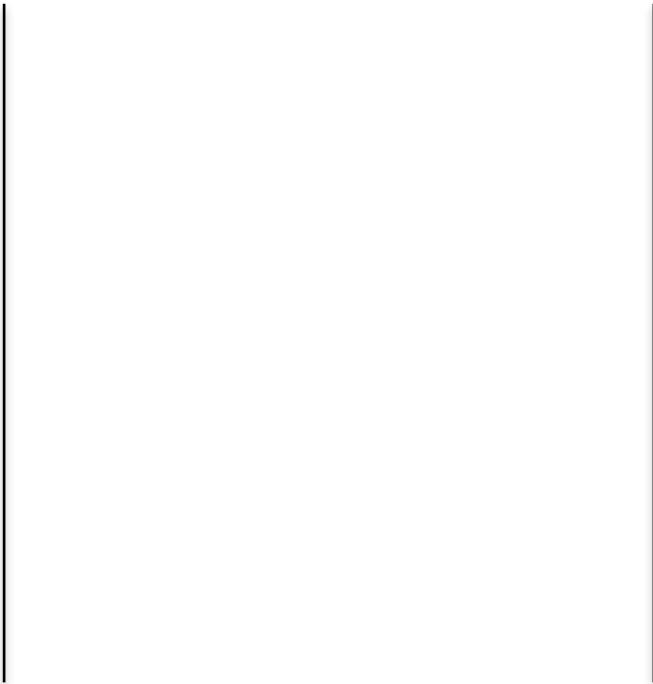


$$\text{Det} = 0 \cdot 0 - 0 \cdot 0$$

$$\text{Det} = 0$$



Determinant as an area



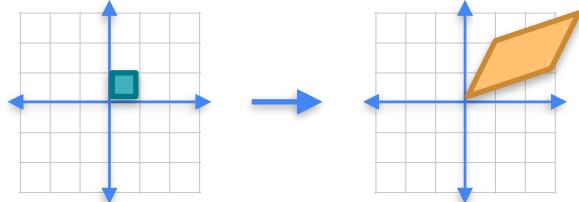
Determinant as an area

Non-singular



| | |
|---|---|
| 3 | 1 |
| 1 | 2 |

$$\text{Determinant} = 5$$



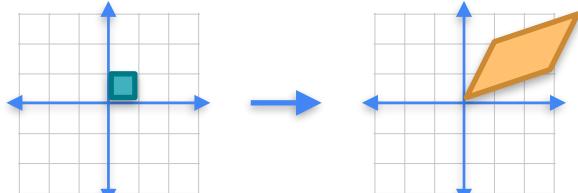
$$\text{Area} = 5$$

Determinant as an area

Non-singular

| | |
|---|---|
| | |
| 3 | 1 |
| 1 | 2 |

Determinant = 5

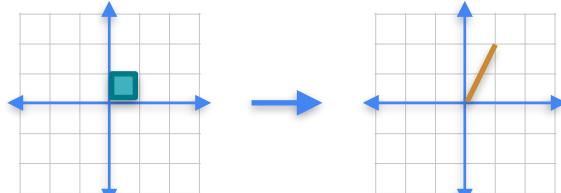


Area = 5

Singular

| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

Determinant = 0



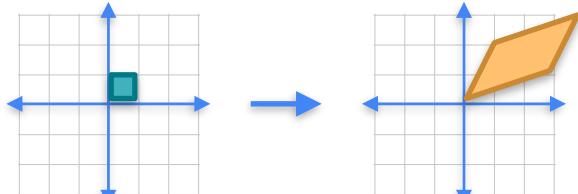
Area = 0

Determinant as an area

Non-singular

| | |
|---|---|
| | |
| 3 | 1 |
| 1 | 2 |

Determinant = 5

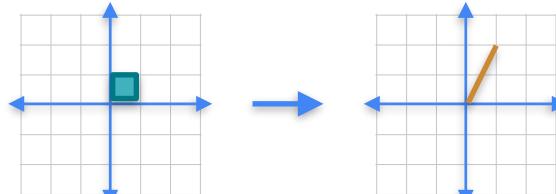


Area = 5

Singular

| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

Determinant = 0

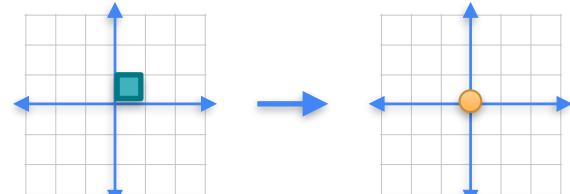


Area = 0

Singular

| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |

Determinant = 0



Area = 0

Negative determinants?

| | |
|---|---|
|  |  |
| 3 | 1 |
| 1 | 2 |

| | |
|---|---|
|  |  |
| 1 | 3 |
| 2 | 1 |

Negative determinants?

| | |
|---|---|
| | |
| 3 | 1 |
| 1 | 2 |

$$\text{Det} = 3 \cdot 2 - 1 \cdot 1$$

$$\text{Det} = 5$$

| | |
|---|---|
| | |
| 1 | 3 |
| 2 | 1 |

Negative determinants?

| | |
|---|---|
|  |  |
| 3 | 1 |
| 1 | 2 |

$$\text{Det} = 3 \cdot 2 - 1 \cdot 1$$

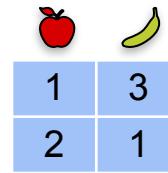
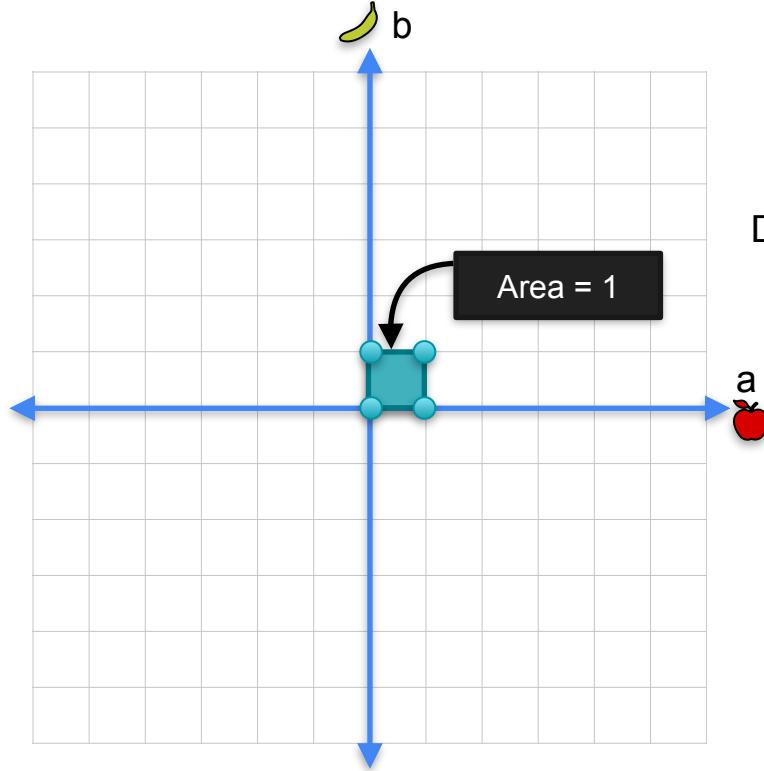
$$\text{Det} = 5$$

| | |
|---|---|
|  |  |
| 1 | 3 |
| 2 | 1 |

$$\text{Det} = 1 \cdot 1 - 3 \cdot 2$$

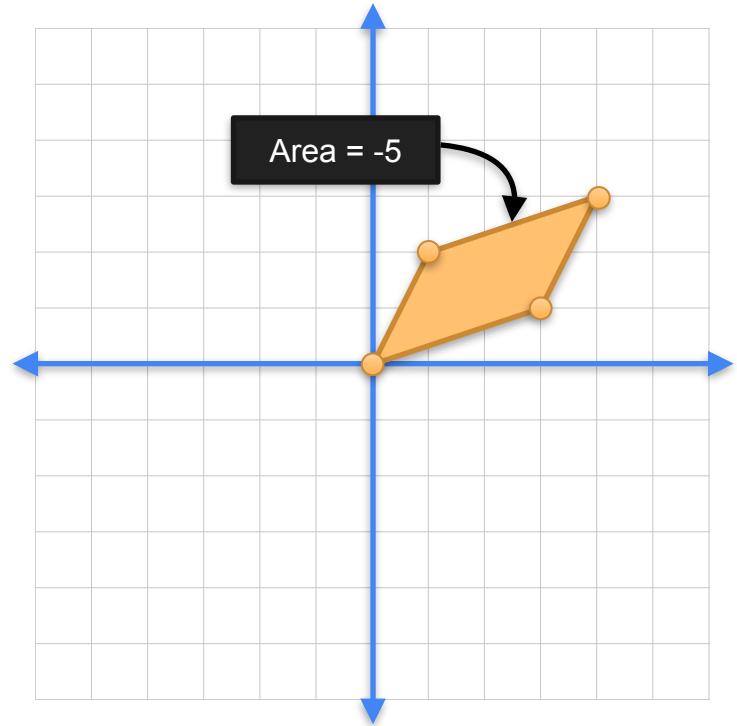
$$\text{Det} = -5$$

Determinant as an area

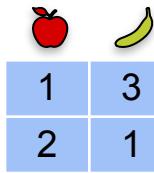
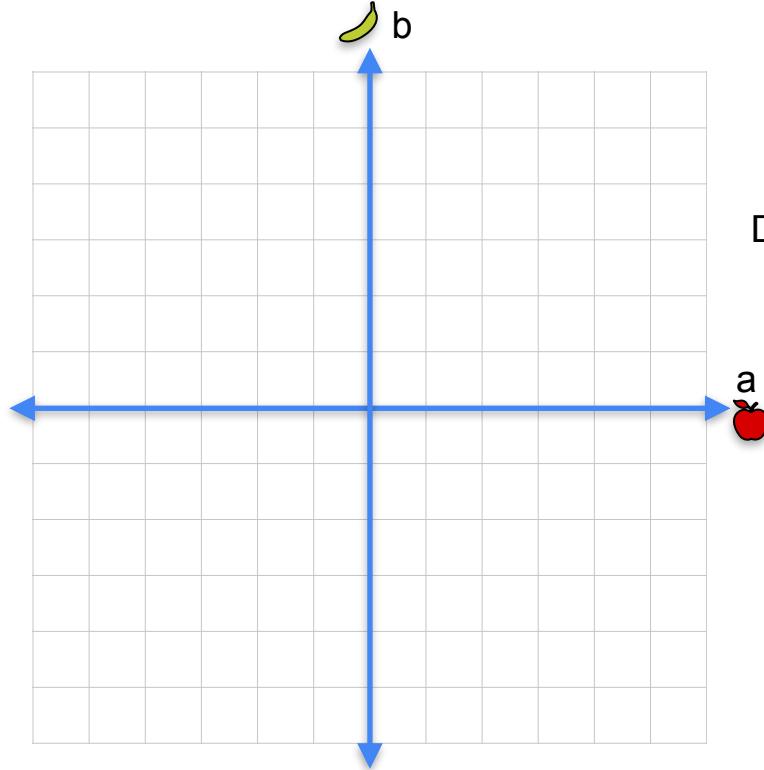


$$\text{Det} = 1 \cdot 1 - 3 \cdot 2$$

$$\text{Det} = -5$$

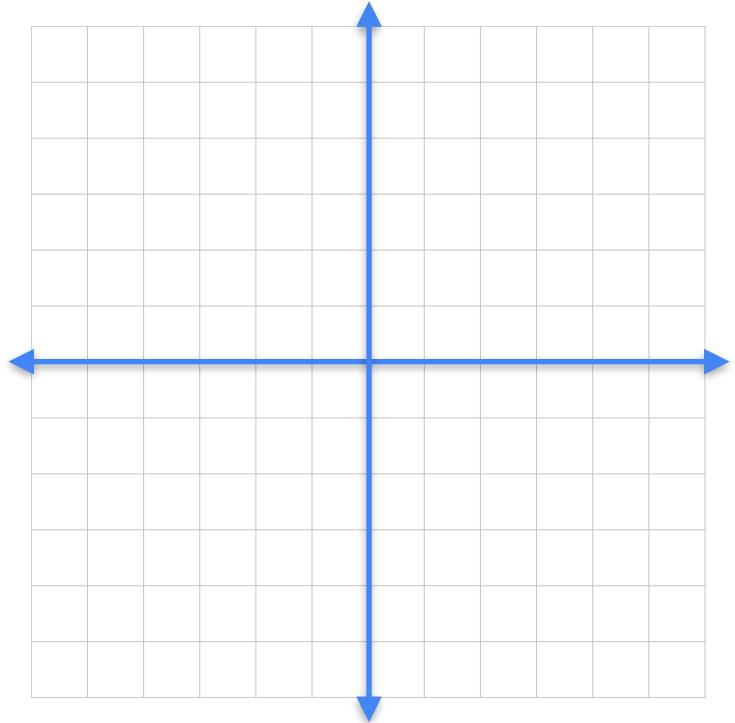


Determinant as an area

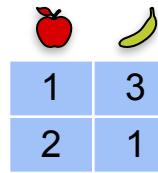
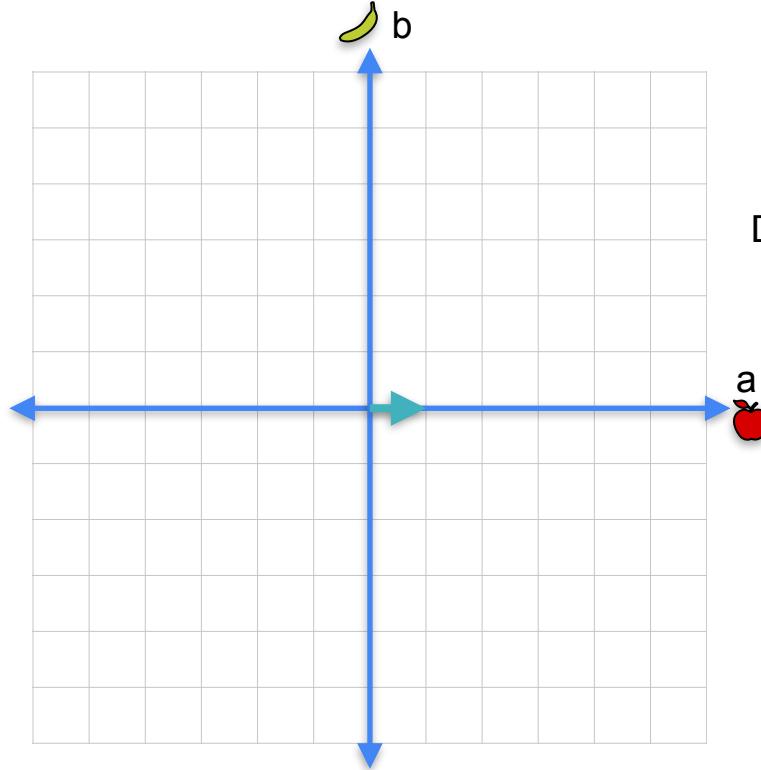


$$\text{Det} = 1 \cdot 1 - 3 \cdot 2$$

$$\text{Det} = -5$$

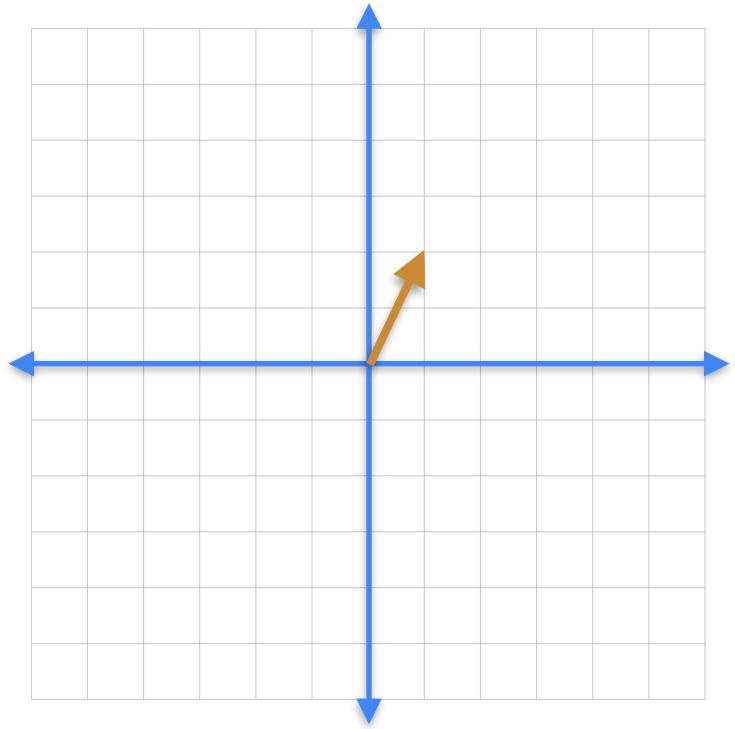


Determinant as an area

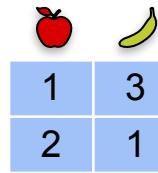
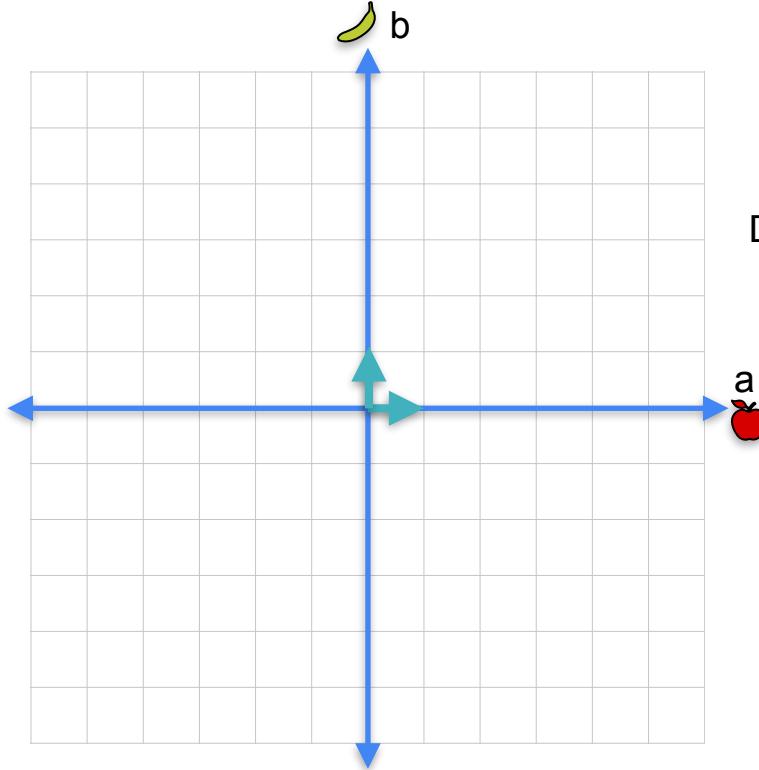


$$\text{Det} = 1 \cdot 1 - 3 \cdot 2$$

$$\text{Det} = -5$$

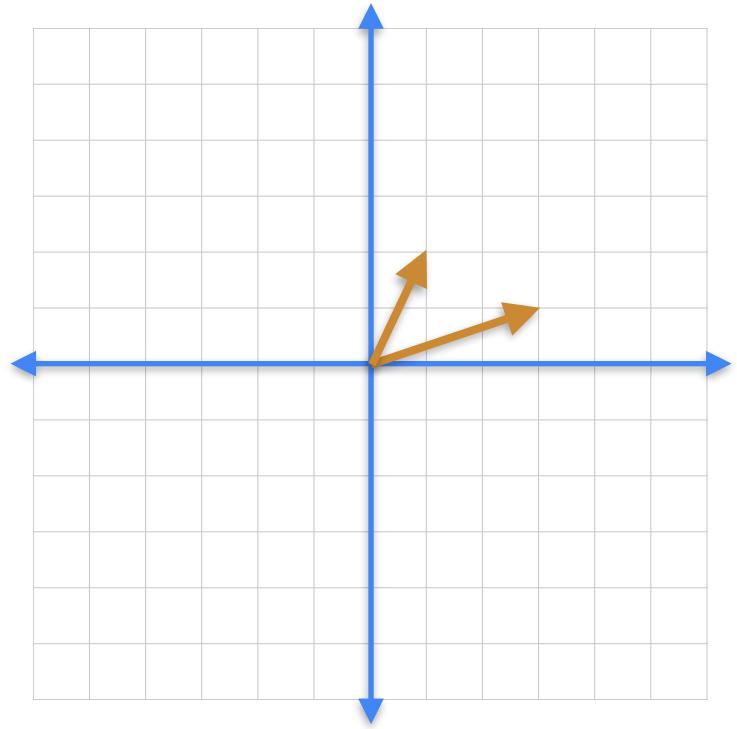


Determinant as an area

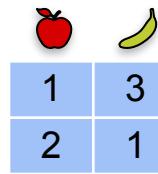
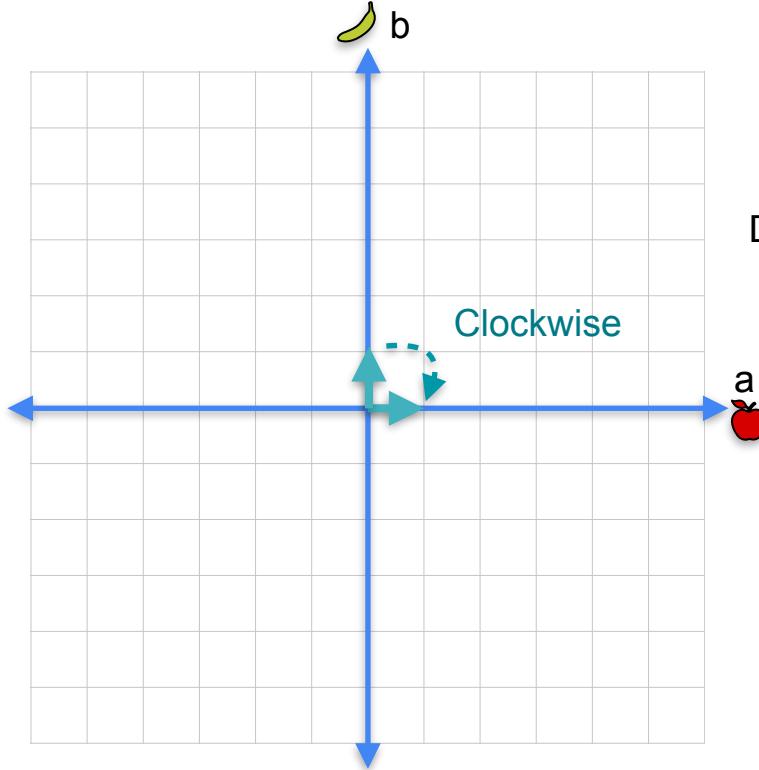


$$\text{Det} = 1 \cdot 1 - 3 \cdot 2$$

$$\text{Det} = -5$$

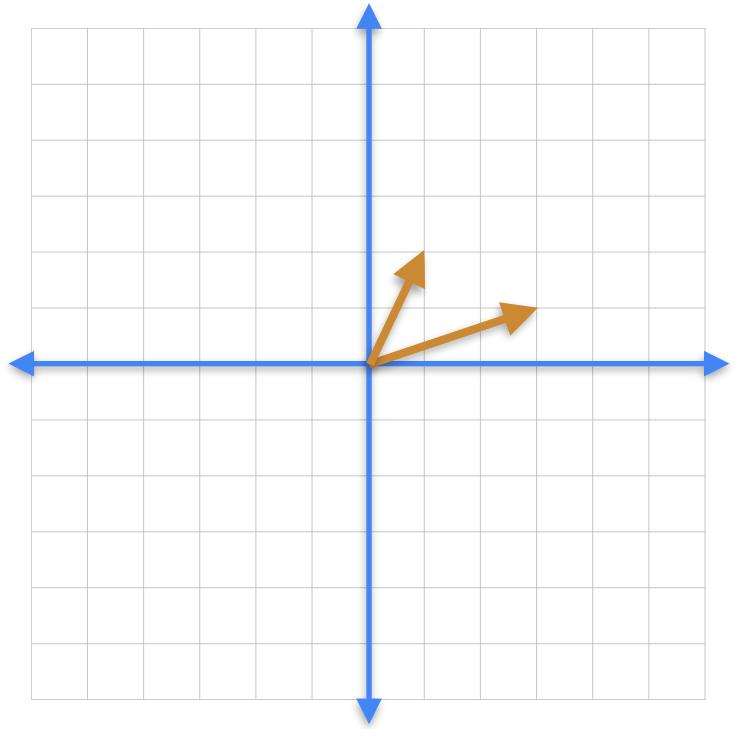


Determinant as an area

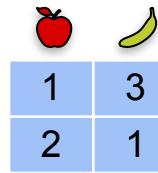
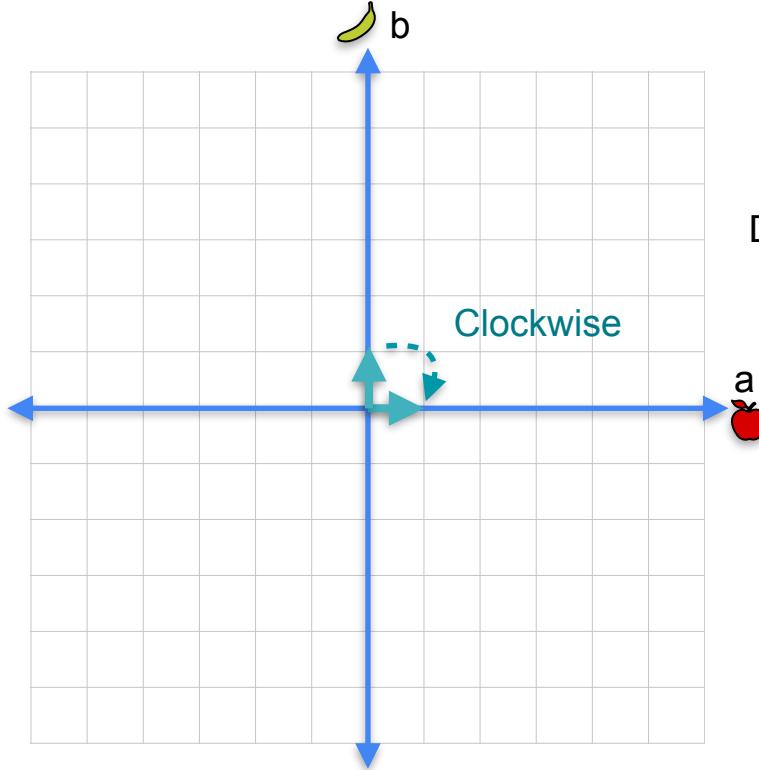


$$\text{Det} = 1 \cdot 1 - 3 \cdot 2$$

$$\text{Det} = -5$$

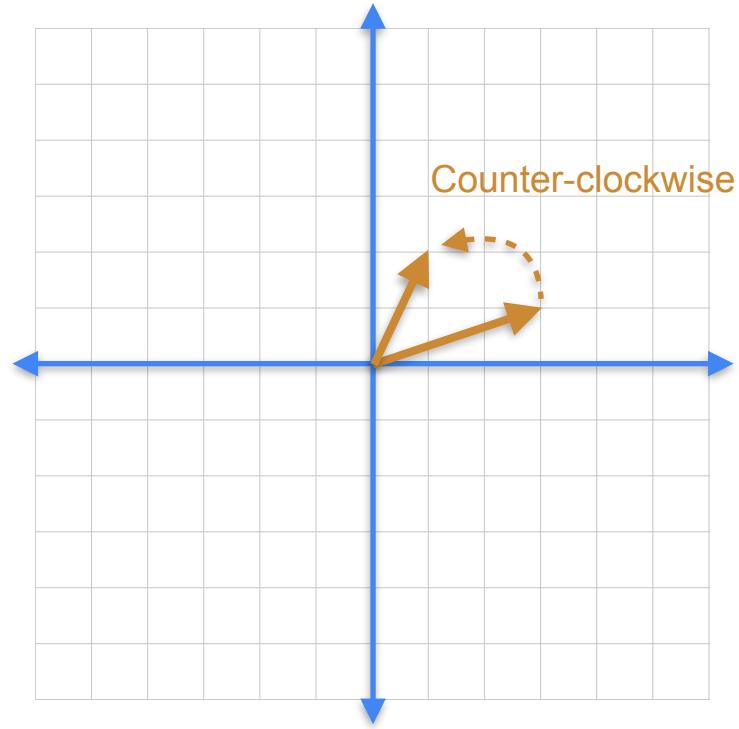


Determinant as an area

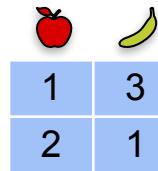
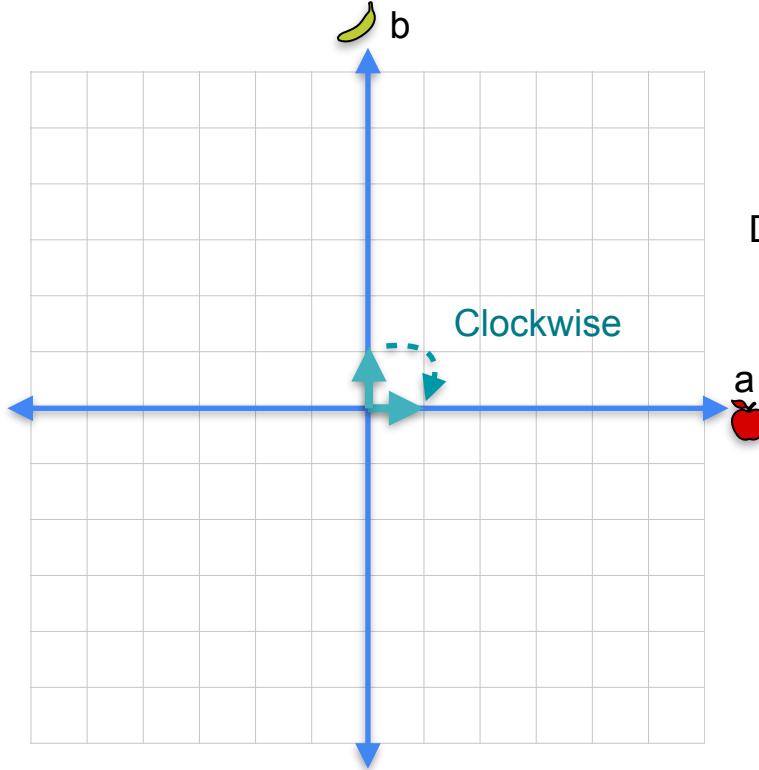


$$\text{Det} = 1 \cdot 1 - 3 \cdot 2$$

$$\text{Det} = -5$$



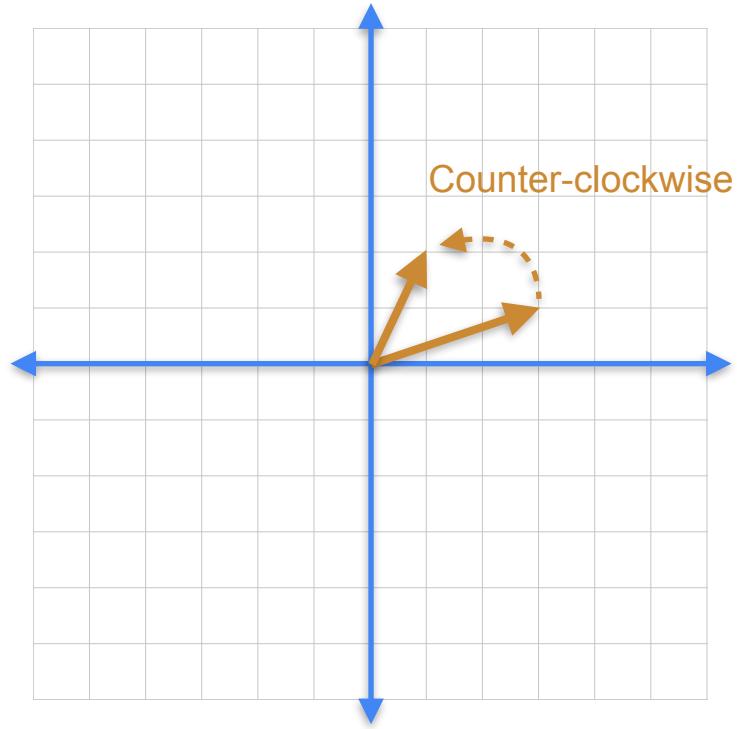
Determinant as an area



$$\text{Det} = 1 \cdot 1 - 3 \cdot 2$$

$$\text{Det} = -5$$

Negative





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Determinants and Eigenvectors

Determinant of a product

Determinant of a product

$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 16 & 8 \\ \hline 7 & 6 \\ \hline \end{array}$$

Determinant of a product

$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 16 & 8 \\ \hline 7 & 6 \\ \hline \end{array}$$

$$\det = 5$$

$$3 \cdot 2 - 1 \cdot 1$$

Determinant of a product

| | |
|---|---|
| 3 | 1 |
| 1 | 2 |

| | |
|---|---|
| 5 | 2 |
| 1 | 2 |

=

| | |
|----|---|
| 16 | 8 |
| 7 | 6 |

$$\det = 5$$

$$3 \cdot 2 - 1 \cdot 1$$

$$\det = 8$$

$$5 \cdot 2 - 2 \cdot 1$$

Determinant of a product

| | |
|---|---|
| 3 | 1 |
| 1 | 2 |

| | |
|---|---|
| 5 | 2 |
| 1 | 2 |

=

| | |
|----|---|
| 16 | 8 |
| 7 | 6 |

$$\det = 5$$

$$3 \cdot 2 - 1 \cdot 1$$

$$\det = 8$$

$$5 \cdot 2 - 2 \cdot 1$$

$$\det = 40$$

$$16 \cdot 6 - 8 \cdot 7$$

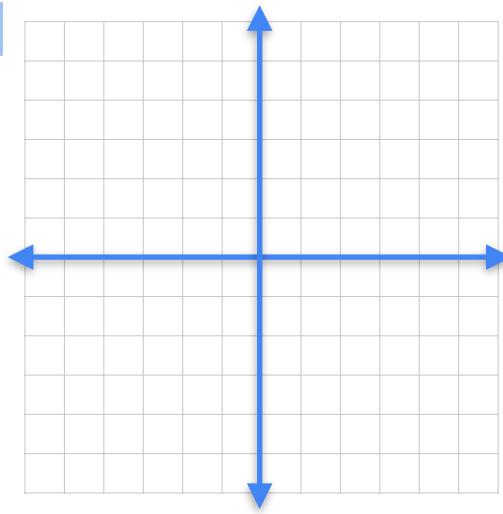
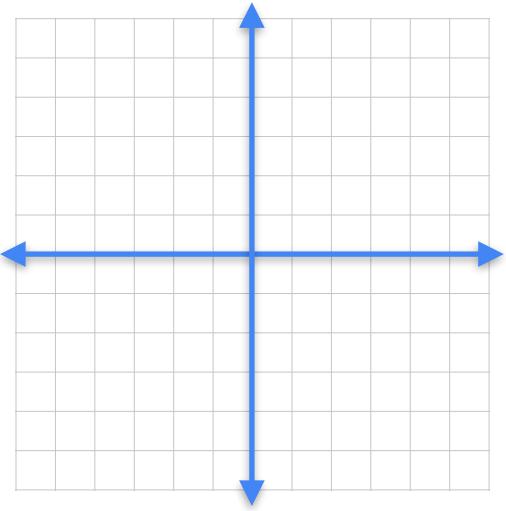
Determinant of a product

$$\det(AB) = \det(A) \det(B)$$

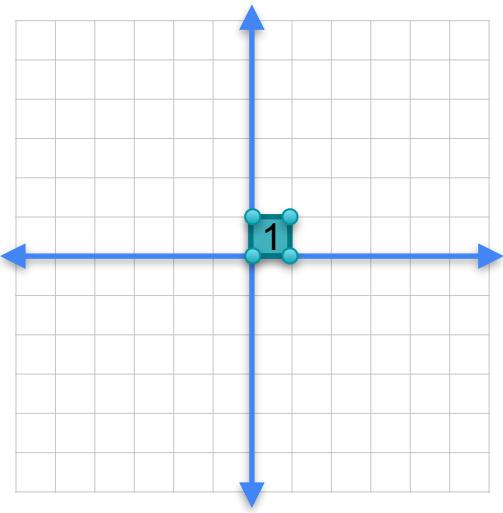
Determinant of a product

| | |
|---|---|
| 3 | 1 |
| 1 | 2 |

Det = 5

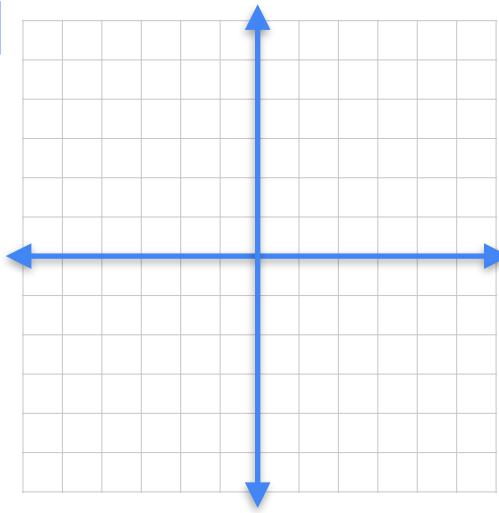


Determinant of a product

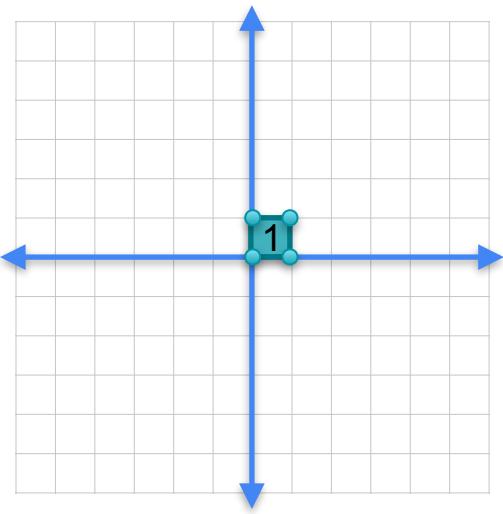


$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array}$$

Det = 5

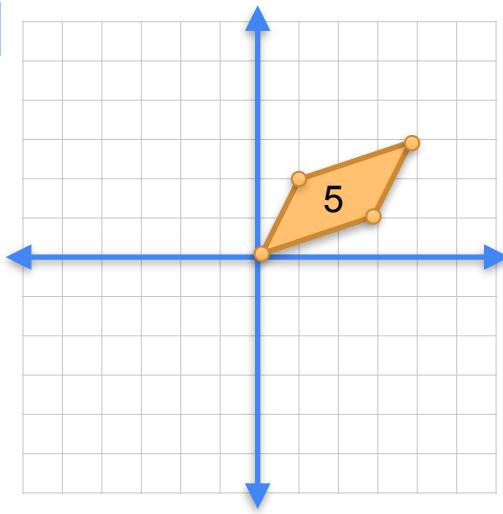


Determinant of a product

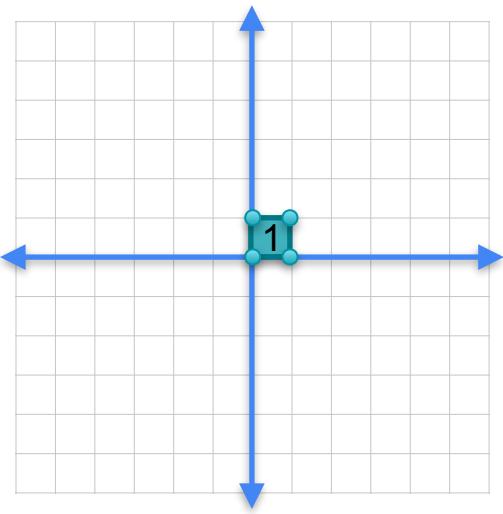


$$\begin{array}{|cc|} \hline 3 & 1 \\ 1 & 2 \\ \hline \end{array}$$

Det = 5

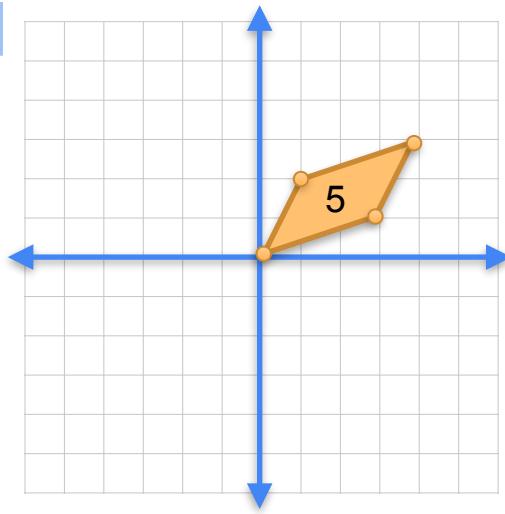


Determinant of a product



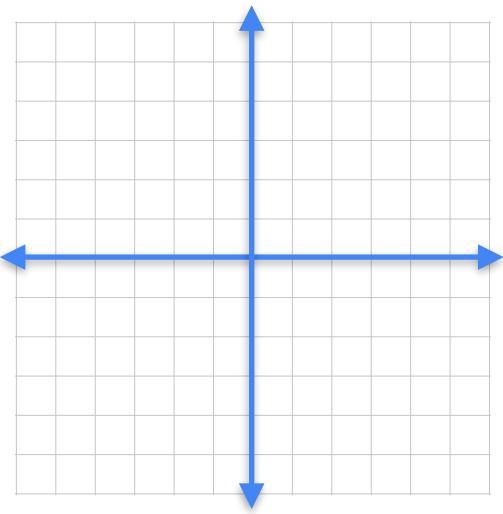
$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array}$$

Det = 5



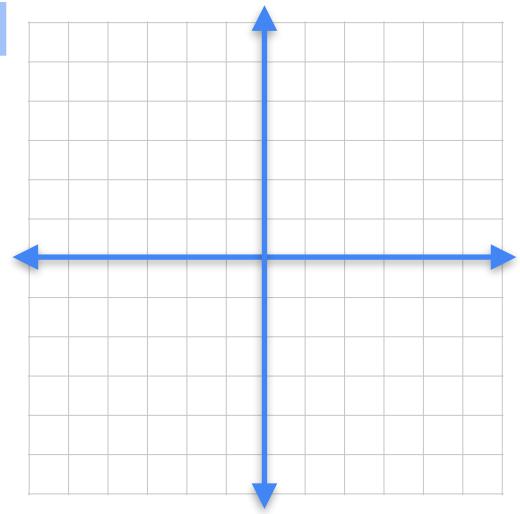
Area blows up by 5

Determinant of a product

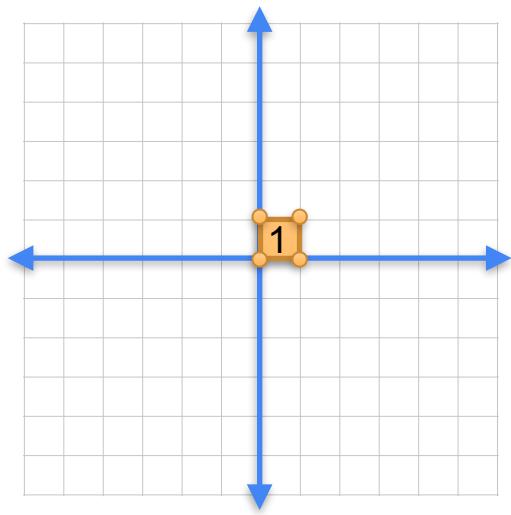


$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline -2 & 1 \\ \hline \end{array}$$

Det = 3

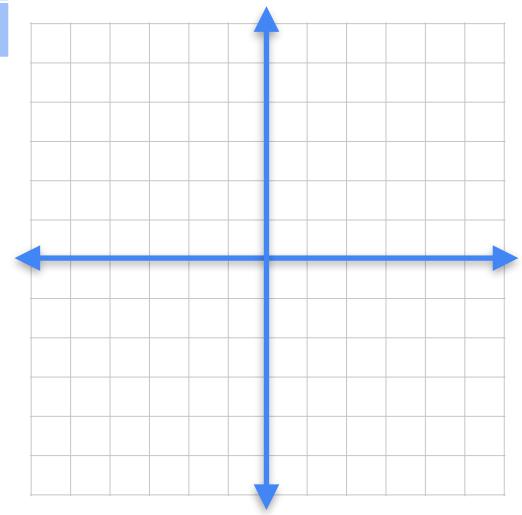


Determinant of a product

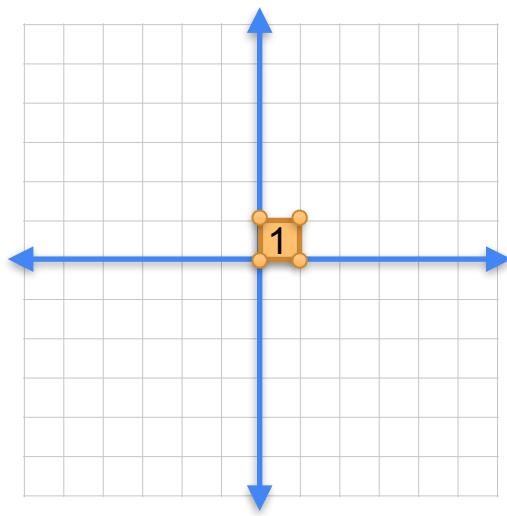


$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline -2 & 1 \\ \hline \end{array}$$

Det = 3

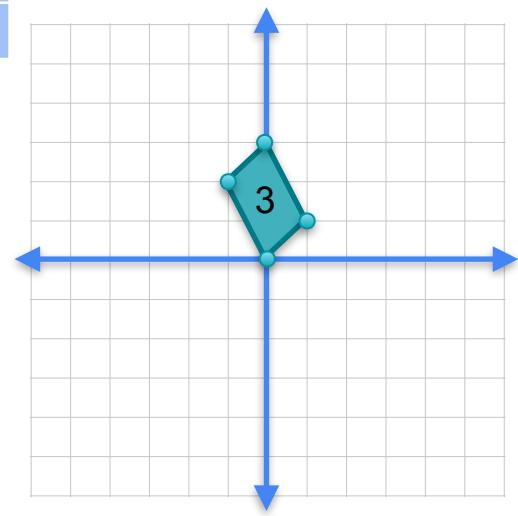


Determinant of a product

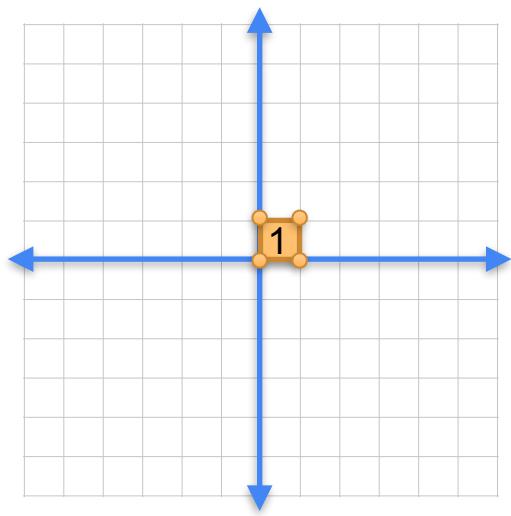


$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline -2 & 1 \\ \hline \end{array}$$

Det = 3

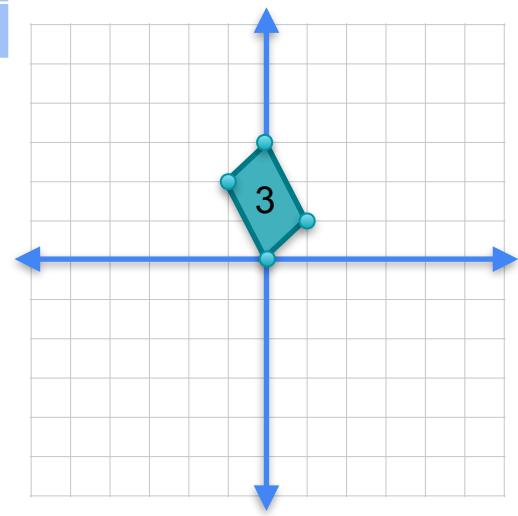


Determinant of a product



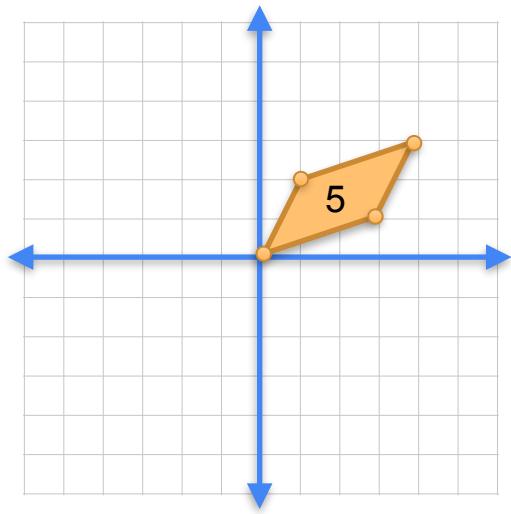
$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline -2 & 1 \\ \hline \end{array}$$

Det = 3



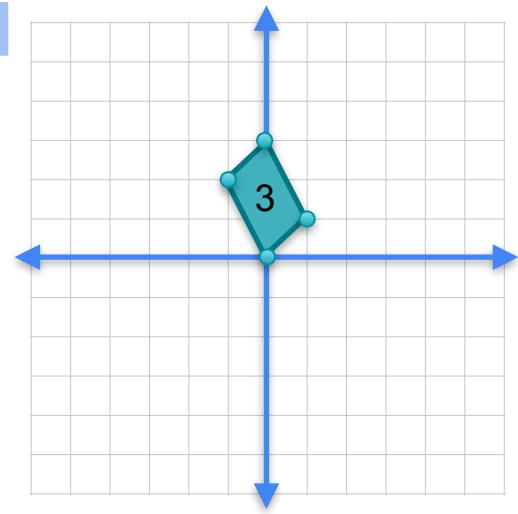
Area blows up by 3

Determinant of a product



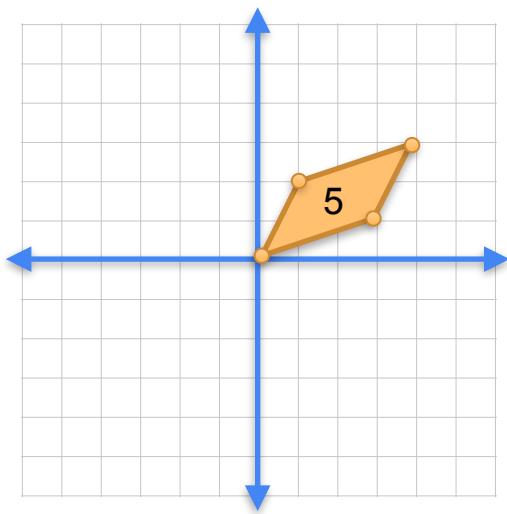
$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline -2 & 1 \\ \hline \end{array}$$

Det = 3



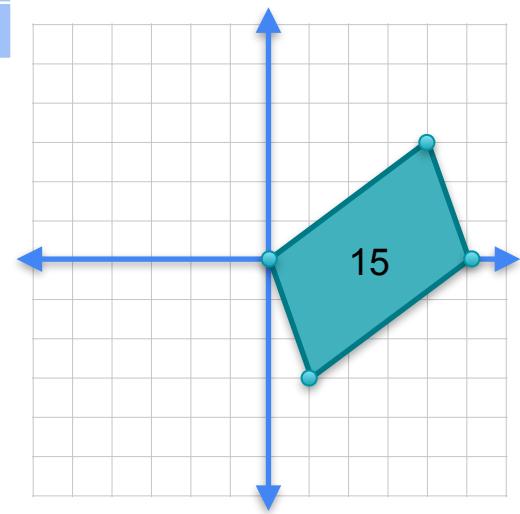
Area blows up by 3

Determinant of a product



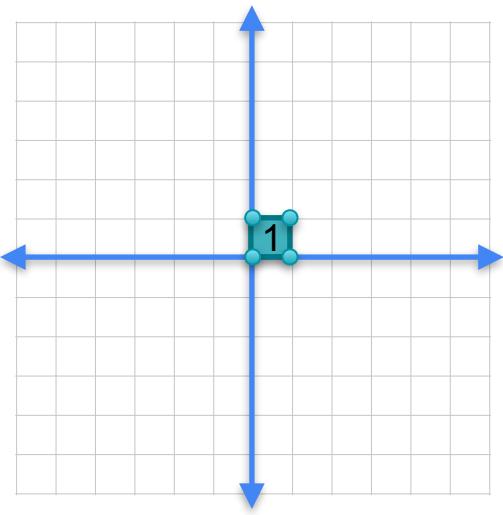
$$\begin{array}{|cc|} \hline 1 & 1 \\ -2 & 1 \\ \hline \end{array}$$

Det = 3



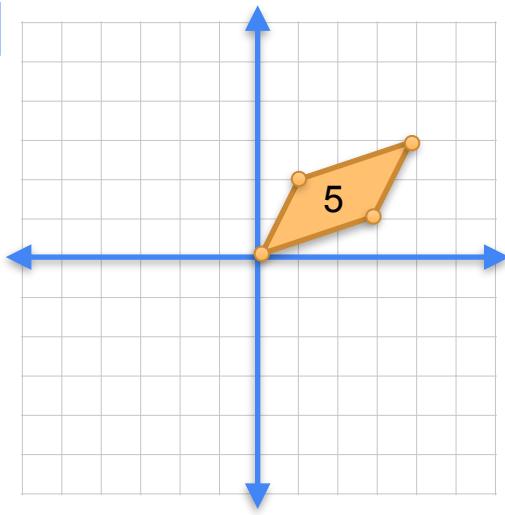
Area blows up by 3

Determinant of a product



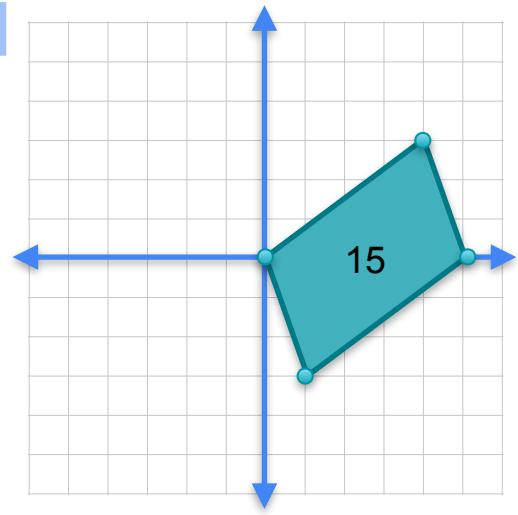
$$\begin{array}{|cc|} \hline 3 & 1 \\ 1 & 2 \\ \hline \end{array}$$

Det = 5

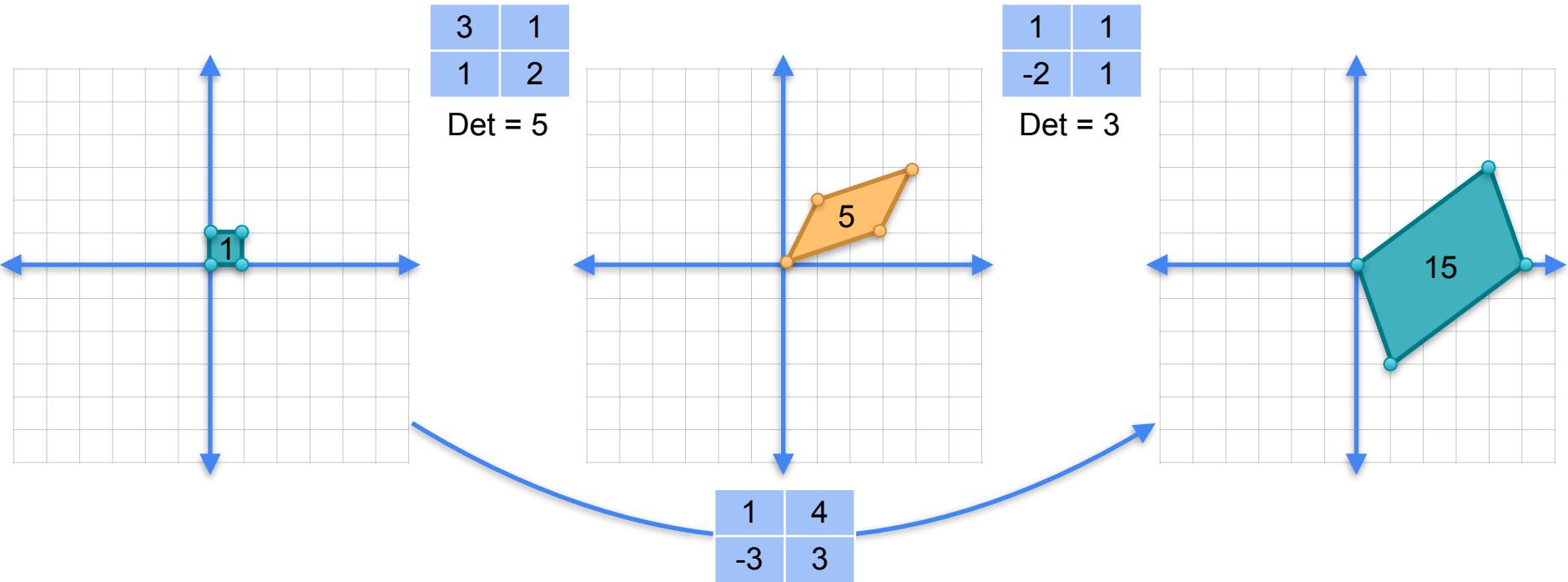


$$\begin{array}{|cc|} \hline 1 & 1 \\ -2 & 1 \\ \hline \end{array}$$

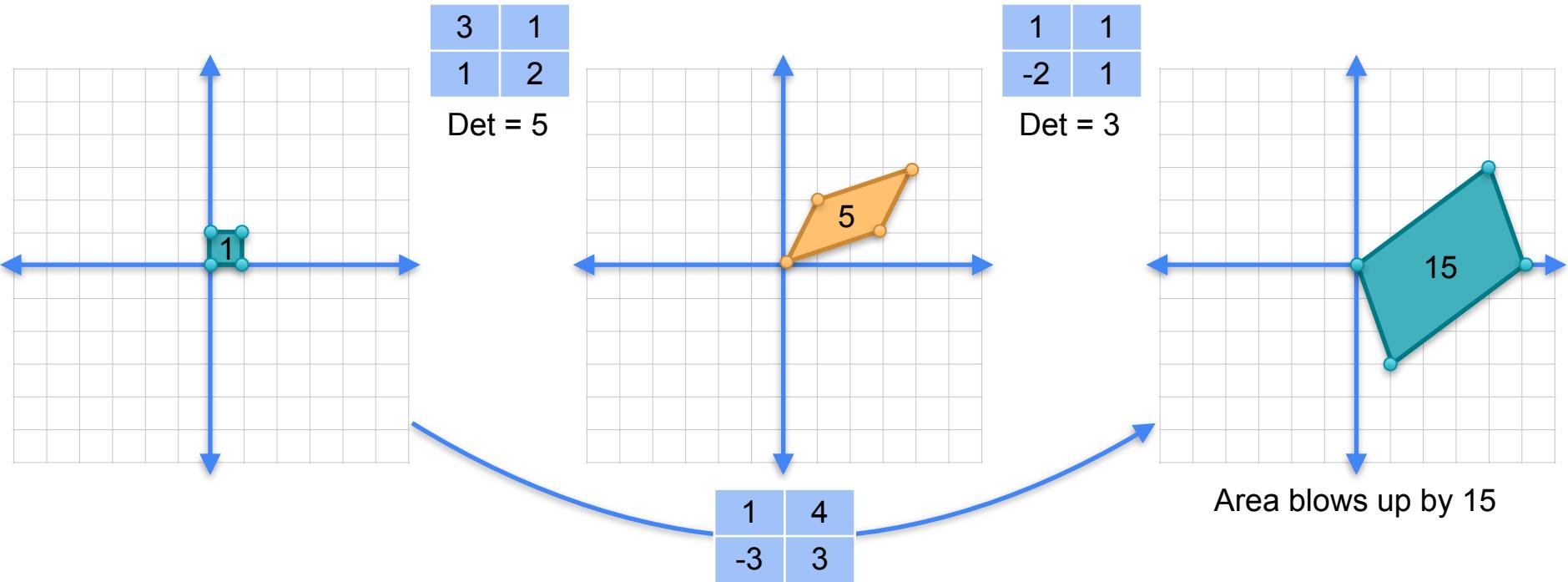
Det = 3



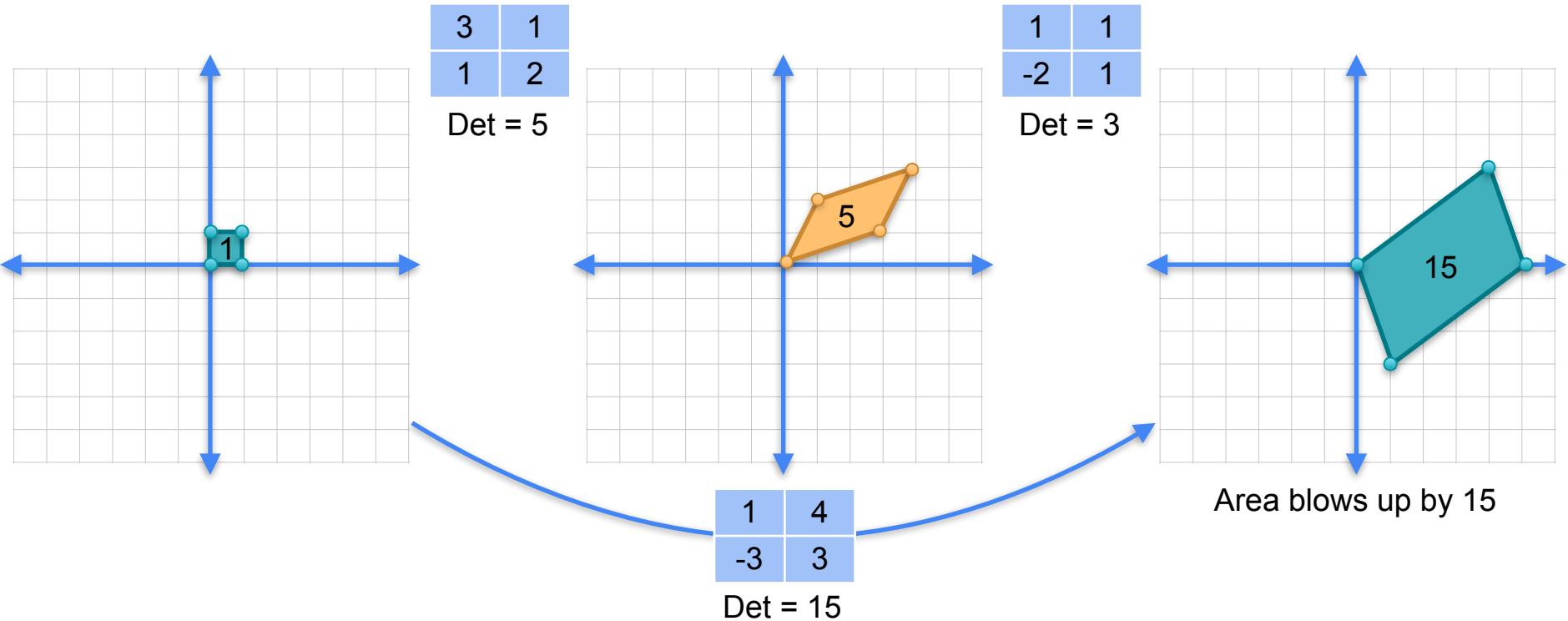
Determinant of a product



Determinant of a product



Determinant of a product



Quiz

- The product of a singular and a non-singular matrix (in any order) is:
 - Singular
 - Non-singular
 - Could be either one

Solution

- If A is non-singular and B is singular, then $\det(AB) = \det(A) \times \det(B) = 0$, since $\det(B) = 0$. Therefore $\det(AB) = 0$, so AB is **singular**.

When one factor is zero

When one factor is zero

5

When one factor is zero

$$5 \cdot 0$$

When one factor is zero

$$5 \cdot 0 = 0$$

When one factor is singular...

| Non-singular | Singular | Singular |
|--|--|--|
| $\begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix}$ | $\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix}$ | $\begin{matrix} 4 & 8 \\ 3 & 6 \end{matrix}$ |
| $\text{Det} = 5$ | $\text{Det} = 0$ | $\text{Det} = 0$ |

If one factor is singular...

| | |
|---|---|
| 3 | 1 |
| 1 | 2 |

$$\text{Det} = 5$$

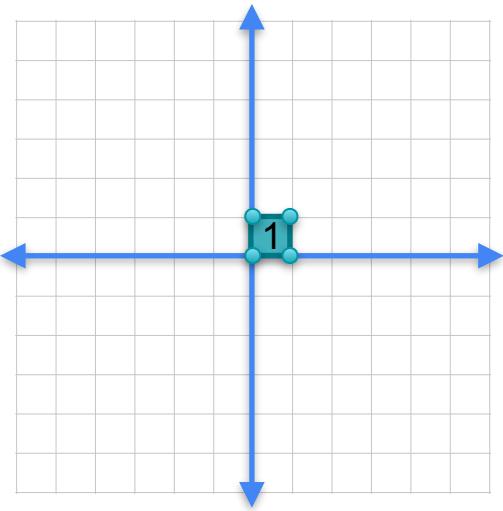
| | |
|---|---|
| 1 | 2 |
| 1 | 2 |

$$\text{Det} = 0$$

| | |
|---|---|
| 4 | 8 |
| 3 | 6 |

$$\text{Det} = 0$$

If one factor is singular...



| | |
|---|---|
| 3 | 1 |
| 1 | 2 |

Det = 5

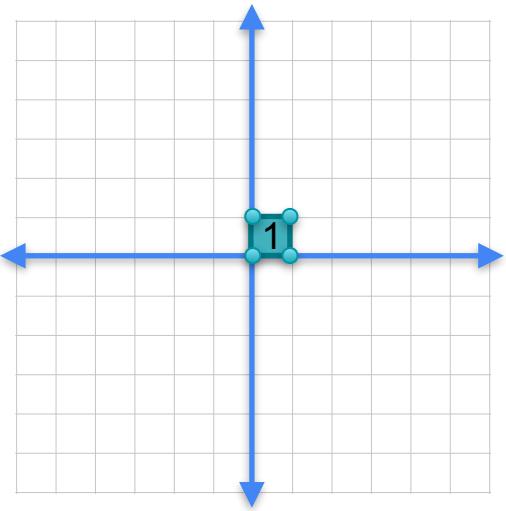
| | |
|---|---|
| 1 | 2 |
| 1 | 2 |

Det = 0

| | |
|---|---|
| 4 | 8 |
| 3 | 6 |

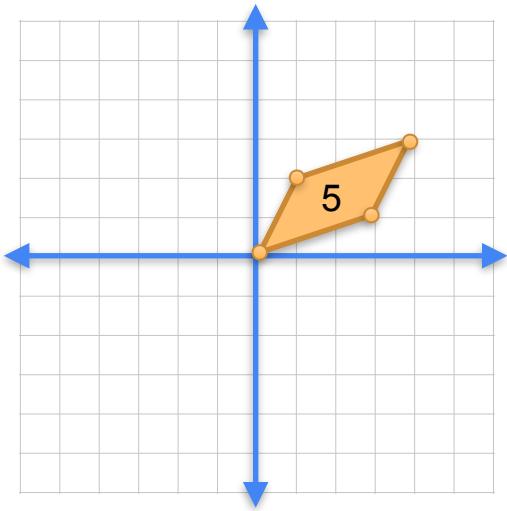
Det = 0

If one factor is singular...



| | |
|---|---|
| 3 | 1 |
| 1 | 2 |

Det = 5



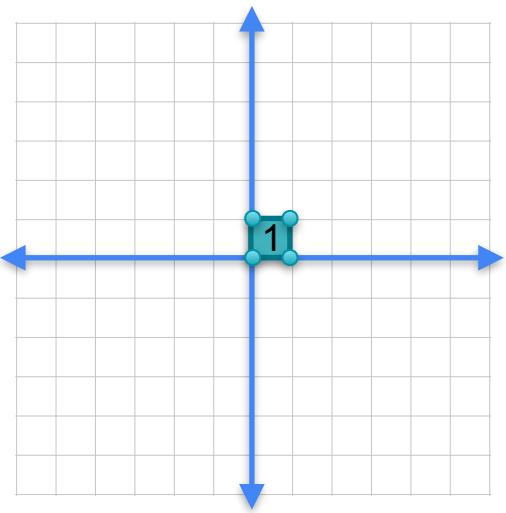
| | |
|---|---|
| 1 | 2 |
| 1 | 2 |

Det = 0

| | |
|---|---|
| 4 | 8 |
| 3 | 6 |

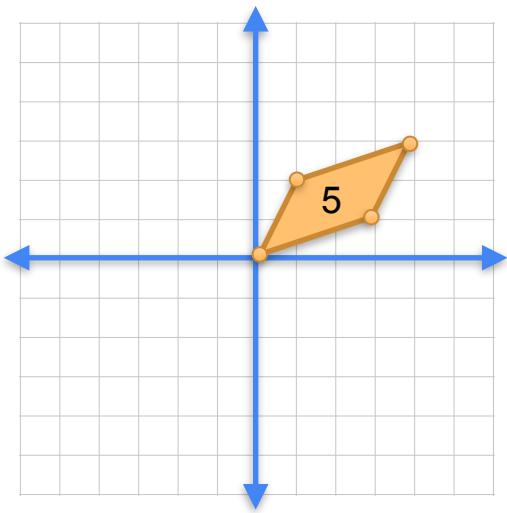
Det = 0

If one factor is singular...



| | |
|---|---|
| 3 | 1 |
| 1 | 2 |

Det = 5

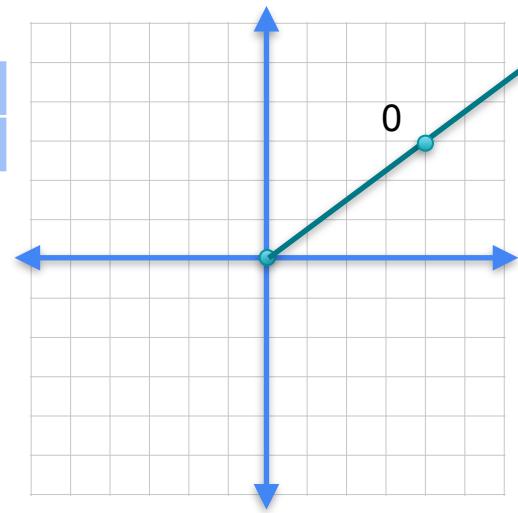


| | |
|---|---|
| 1 | 2 |
| 1 | 2 |

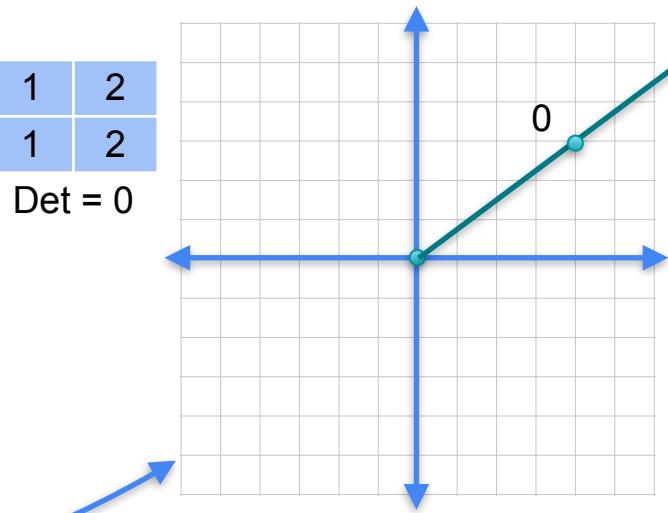
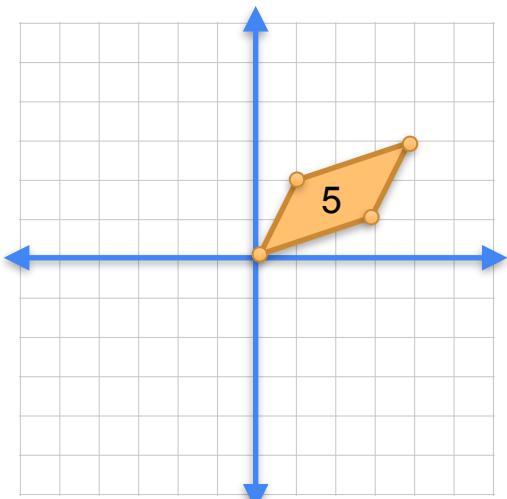
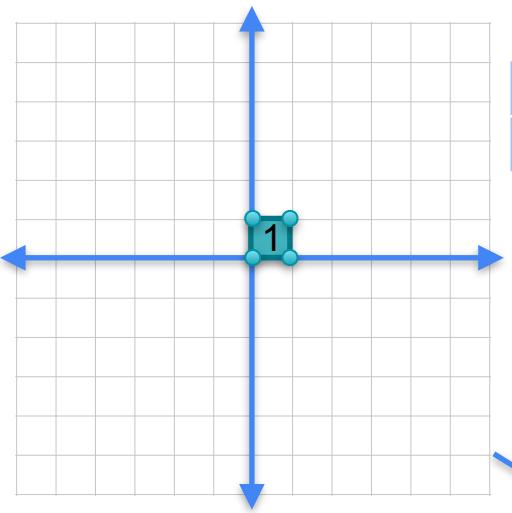
Det = 0

| | |
|---|---|
| 4 | 8 |
| 3 | 6 |

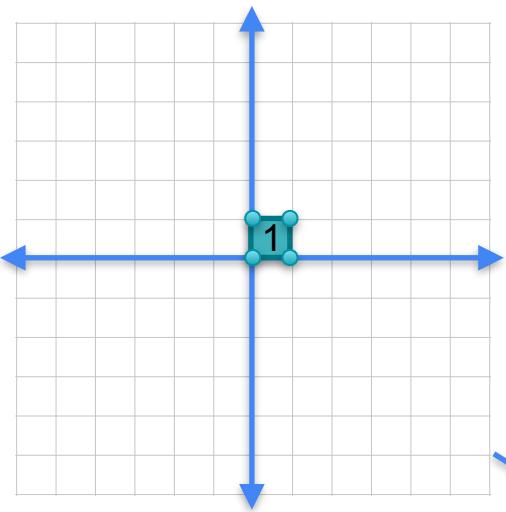
Det = 0



If one factor is singular...

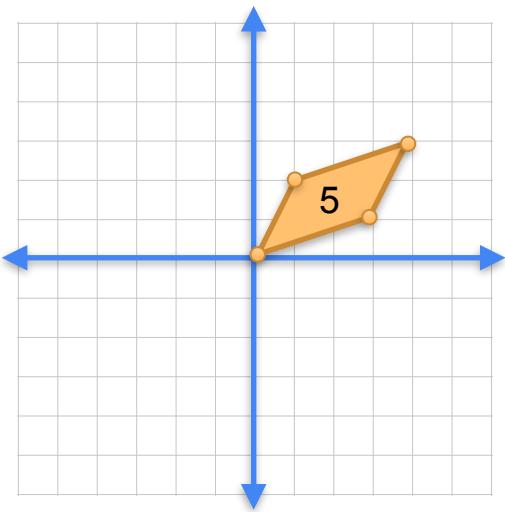


If one factor is singular...



| | |
|---|---|
| 3 | 1 |
| 1 | 2 |

Det = 5

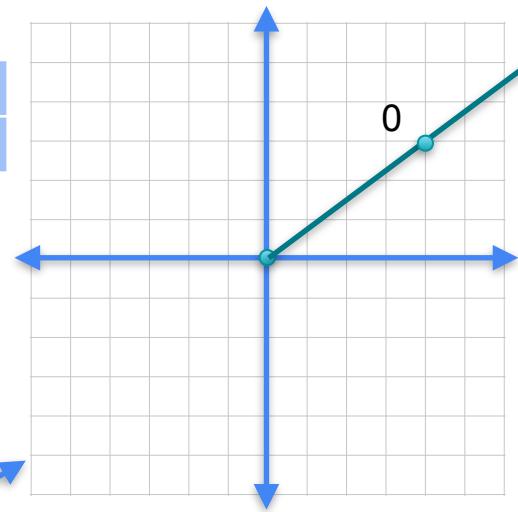


| | |
|---|---|
| 1 | 2 |
| 1 | 2 |

Det = 0

| | |
|---|---|
| 4 | 8 |
| 3 | 6 |

Det = 0



Area blows up by 0



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Determinants and Eigenvectors

Determinant of inverse

Quiz

- Find the determinants of the following matrices

| | |
|------|------|
| 0.4 | -0.2 |
| -0.2 | 0.6 |

| | |
|--------|-------|
| 0.25 | -0.25 |
| -0.125 | 0.625 |

Solution

$$\text{Det} \begin{array}{|c|c|} \hline 0.4 & -0.2 \\ \hline -0.2 & 0.6 \\ \hline \end{array} = (0.4)(0.6) - (-0.2)(-0.2) = 0.2$$

$$\text{Det} \begin{array}{|c|c|} \hline 0.25 & -0.25 \\ \hline -0.125 & 0.625 \\ \hline \end{array} = (0.25)(0.625) - (-0.125)(-0.25) = 0.125$$

Determinant of an inverse

Determinant of an inverse

$$\begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix}^{-1} = \begin{matrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{matrix}$$

Determinant of an inverse

$$\begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix}^{-1} = \begin{matrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{matrix}$$

$$\det = 5$$

Determinant of an inverse

$$\begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix}^{-1} = \begin{matrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{matrix}$$

$$\det = 5$$

$$\det = 0.2$$

Determinant of an inverse

$$\begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix}^{-1} = \begin{matrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{matrix}$$

$$\det = 5$$

$$\det = 0.2$$

$$5^{-1} = 0.2$$

Determinant of an inverse

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\det = 5$$

$$\det = 0.2$$

$$5^{-1} = 0.2$$

Determinant of an inverse

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\det = 5$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\det = 8$$

$$5^{-1} = 0.2$$

Determinant of an inverse

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\det = 5$$

$$\det = 0.2$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\det = 8$$

$$\det = 0.125$$

$$5^{-1} = 0.2$$

Determinant of an inverse

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\det = 5$$

$$5^{-1} = 0.2$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\det = 8$$

$$\det = 0.125$$

$$8^{-1} = 0.125$$

Determinant of an inverse

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\det = 5$$

$$5^{-1} = 0.2$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\det = 8$$

$$8^{-1} = 0.125$$

$$\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = \begin{vmatrix} ? & ? \\ ? & ? \end{vmatrix}$$

$$\det = 0.125$$

Determinant of an inverse

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\det = 5$$

$$5^{-1} = 0.2$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\det = 8$$

$$8^{-1} = 0.125$$

$$\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = \begin{vmatrix} ? & ? \\ ? & ? \end{vmatrix}$$

$$\det = 0$$

Determinant of an inverse

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\det = 5$$

$$5^{-1} = 0.2$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\det = 8$$

$$8^{-1} = 0.125$$

$$\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = \begin{vmatrix} ? & ? \\ ? & ? \end{vmatrix}$$

$$\det = 0$$

$$\det = ???$$

Determinant of an inverse

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\det = 5$$

$$5^{-1} = 0.2$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\det = 8$$

$$8^{-1} = 0.125$$

$$\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = \begin{vmatrix} ? & ? \\ ? & ? \end{vmatrix}$$

$$\det = 0$$

$$0^{-1} = ???$$

Determinant of an inverse

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

Why?

Why?

Why is this?

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

Why?

$$\det(AB) = \det(A) \det(B)$$

Why is this?

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

Why?

$$\det(AB) = \det(A) \det(B)$$

Why is this?

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

$$\det(AA^{-1}) = \det(A) \det(A^{-1})$$

Why?

$$\det(AB) = \det(A) \det(B)$$

Why is this?

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

$$\det(AA^{-1}) = \det(A) \det(A^{-1})$$

$$\det(I) = \det(A) \det(A^{-1})$$

Why?

$$\det(AB) = \det(A) \det(B)$$

Why is this?

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

$$\begin{aligned} \det(AA^{-1}) &= \det(A) \det(A^{-1}) \\ \det(I) &= \det(A) \det(A^{-1}) \\ 1 &= \frac{1}{\det(A)} \end{aligned}$$

Why?

$$\det(AB) = \det(A) \det(B)$$

Why is this?

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

$$\det(AA^{-1}) = \det(A) \det(A^{-1})$$
$$\det(I) = \det(A) \det(A^{-1})$$

The diagram illustrates the derivation of the formula $\det(A^{-1}) = \frac{1}{\det(A)}$. It shows two equations: $\det(AA^{-1}) = \det(A) \det(A^{-1})$ and $\det(I) = \det(A) \det(A^{-1})$. Blue arrows point from the term $\det(A) \det(A^{-1})$ in the second equation to the term $\det(A) \det(A^{-1})$ in the first equation, indicating they are equal. Another blue arrow points from the number 1 in the second equation to the denominator $\det(A)$ in the first equation, indicating they are reciprocals.

Determinant of the identity matrix

$$\det \begin{array}{|cc|} \hline 1 & 0 \\ 0 & 1 \\ \hline \end{array} = 1 \cdot 1 - 0 \cdot 0 = 1$$

Determinant of the identity matrix

$$\det \begin{array}{|cc|} \hline 1 & 0 \\ 0 & 1 \\ \hline \end{array} = 1 \cdot 1 - 0 \cdot 0 = 1$$

$$\det(I) = 1$$

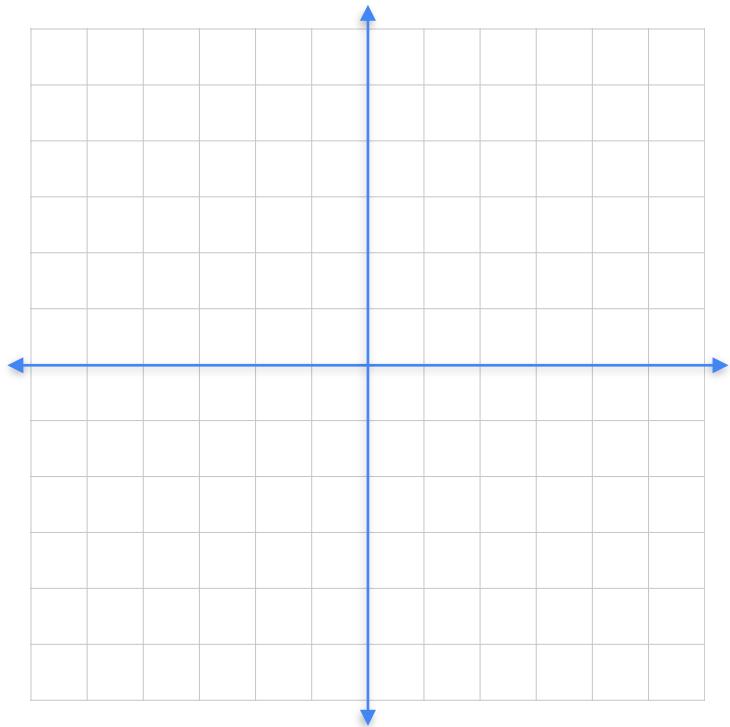


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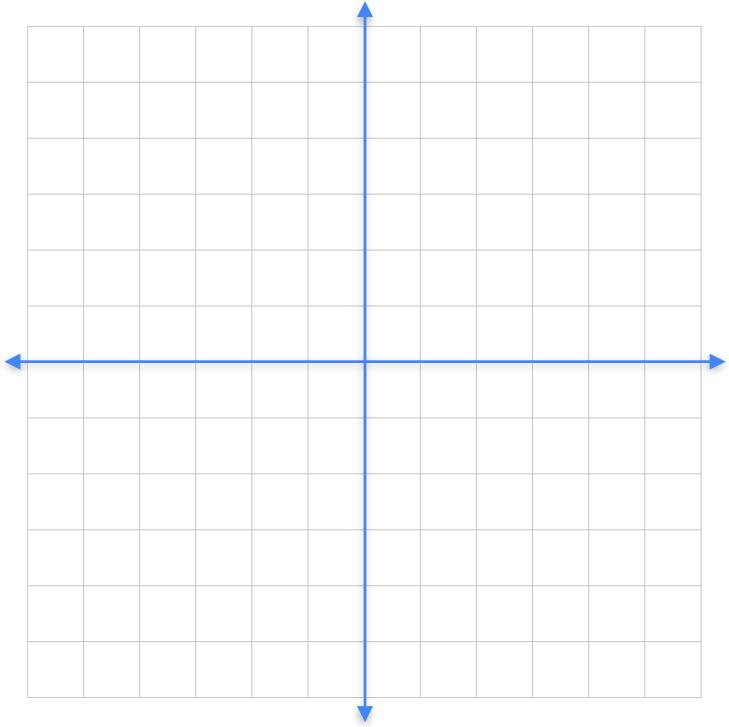
Determinants and Eigenvectors

Bases

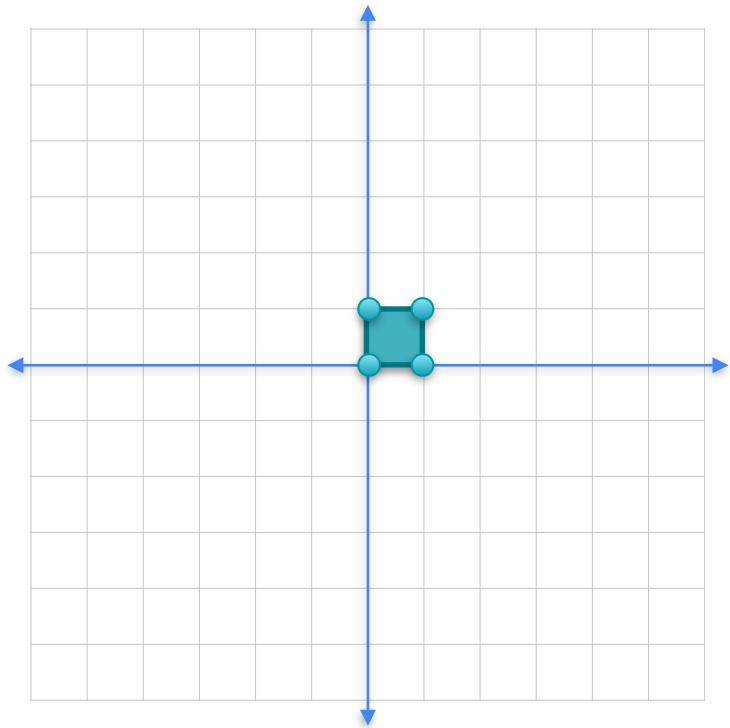
Bases



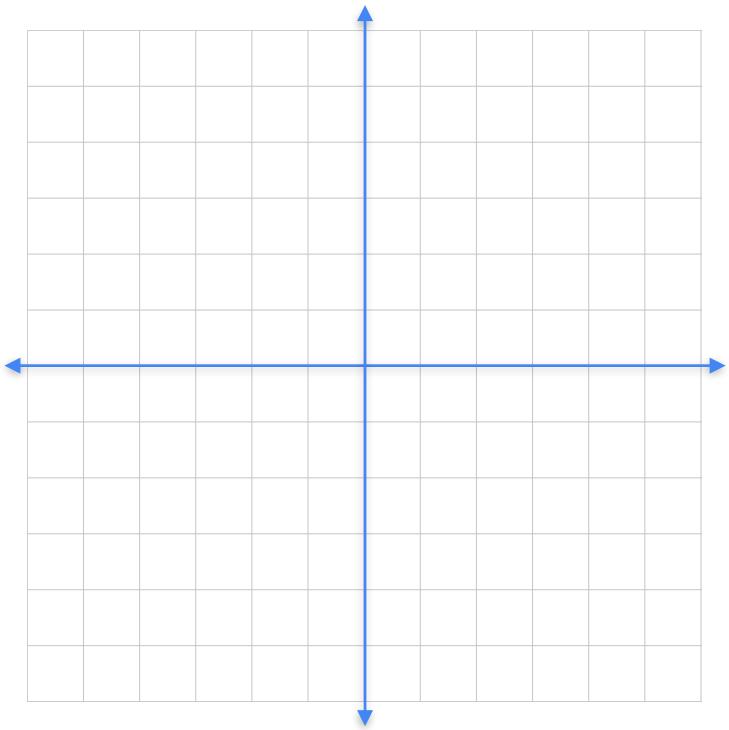
| | |
|---|---|
| 3 | 1 |
| 1 | 2 |



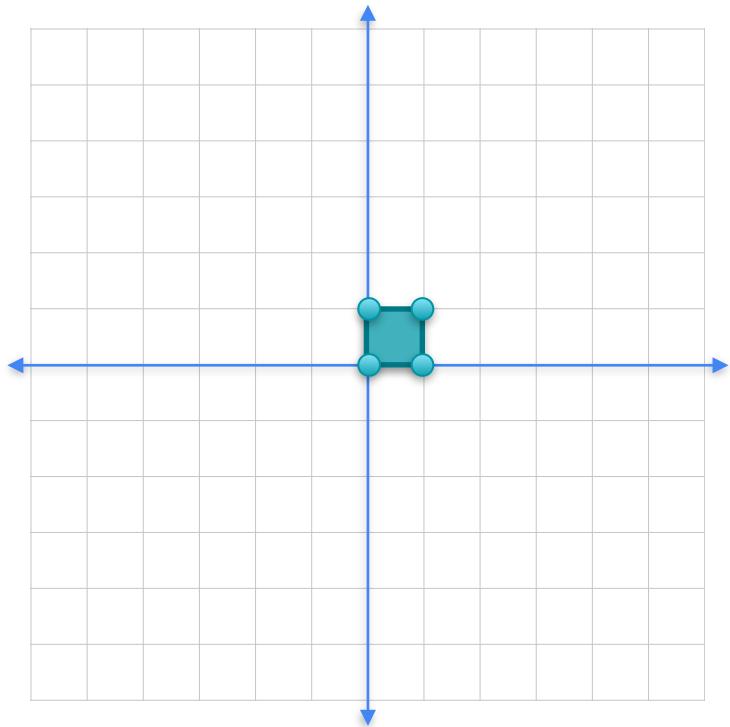
Bases



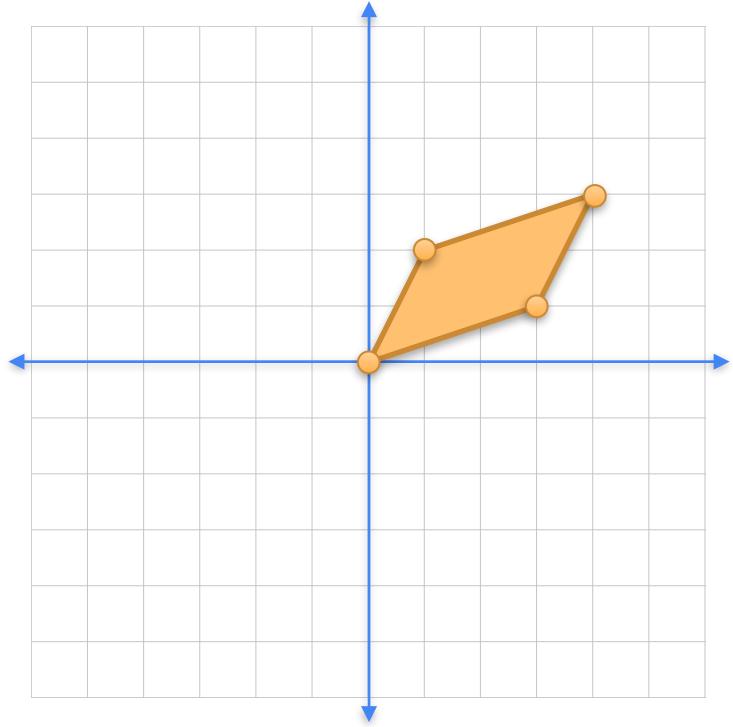
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|---|---|
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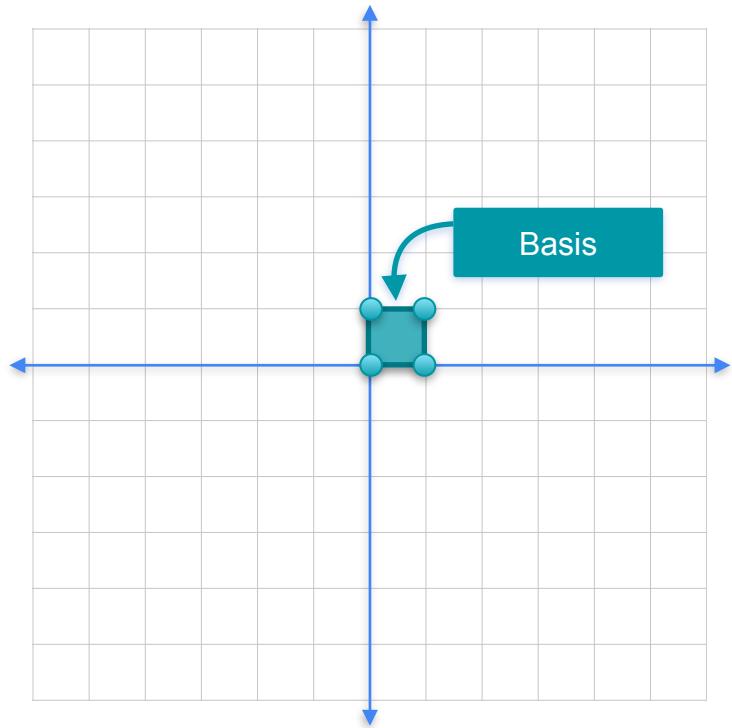
Bases



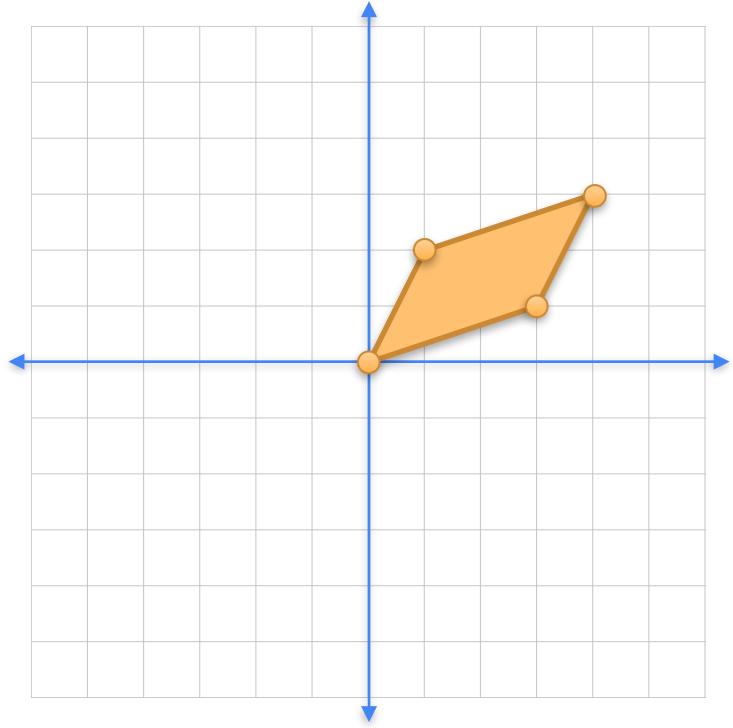
| | |
|---|---|
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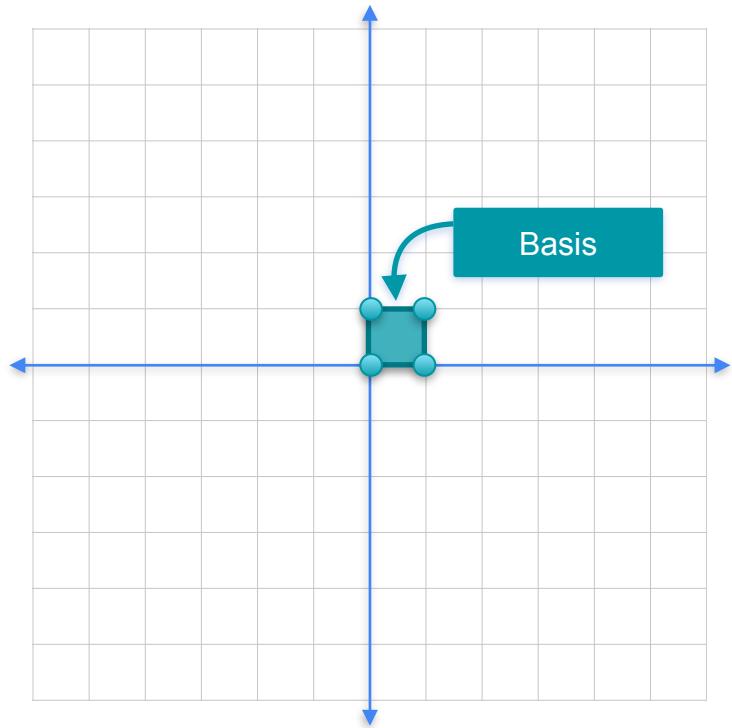
Bases



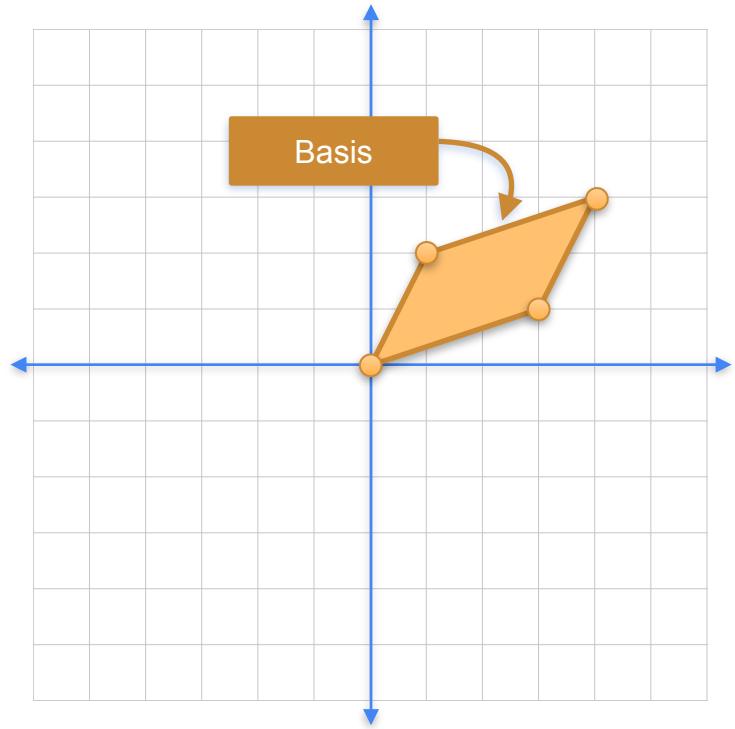
| | |
|---|---|
| 3 | 1 |
| 1 | 2 |



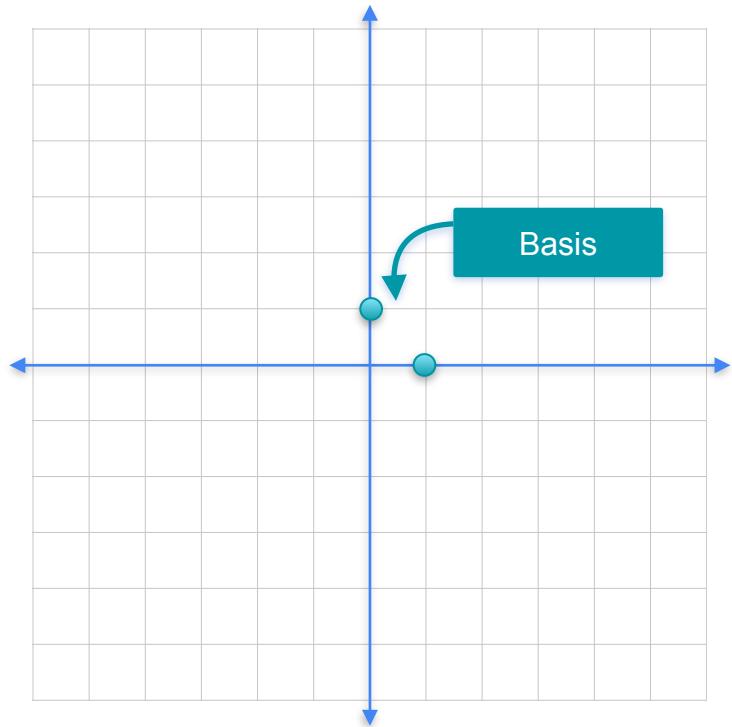
Bases



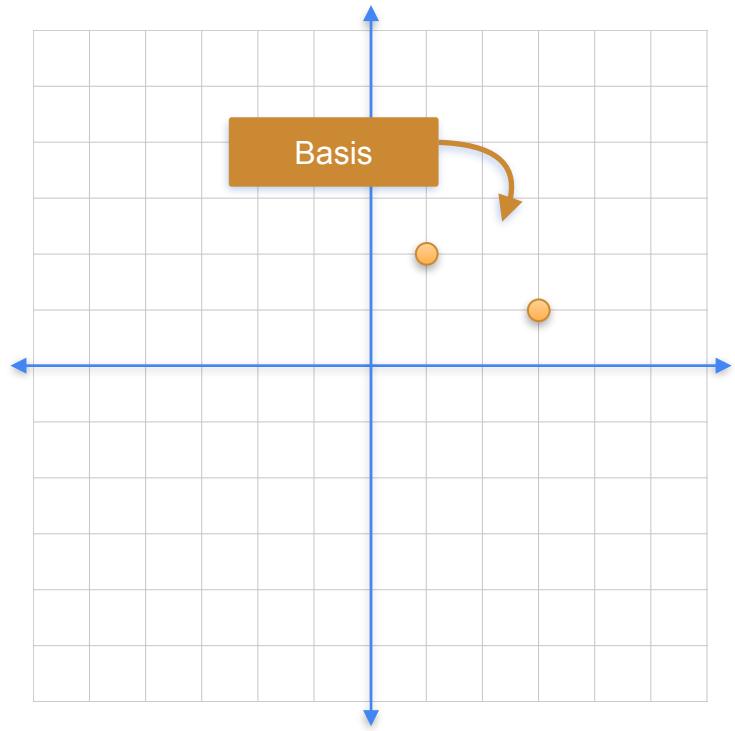
| | |
|---|---|
| 3 | 1 |
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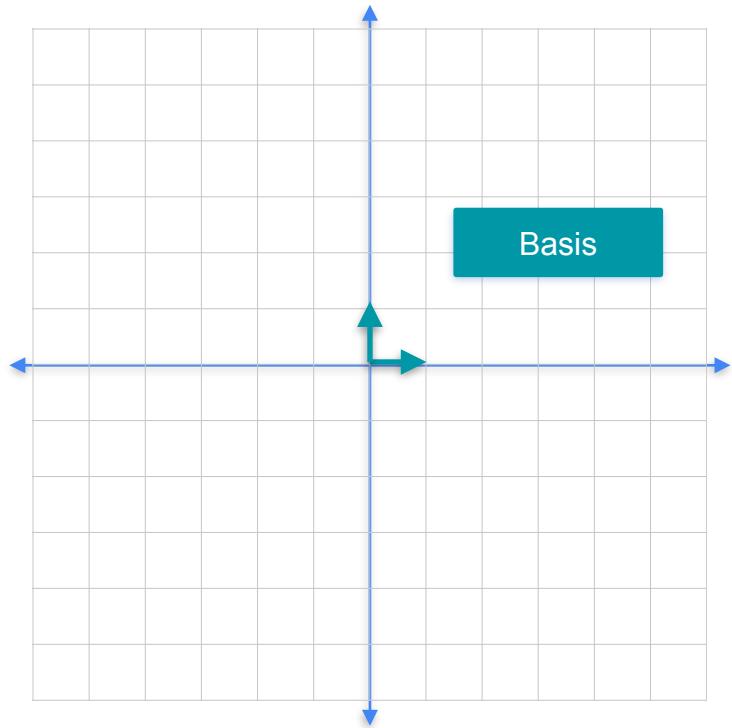
Bases



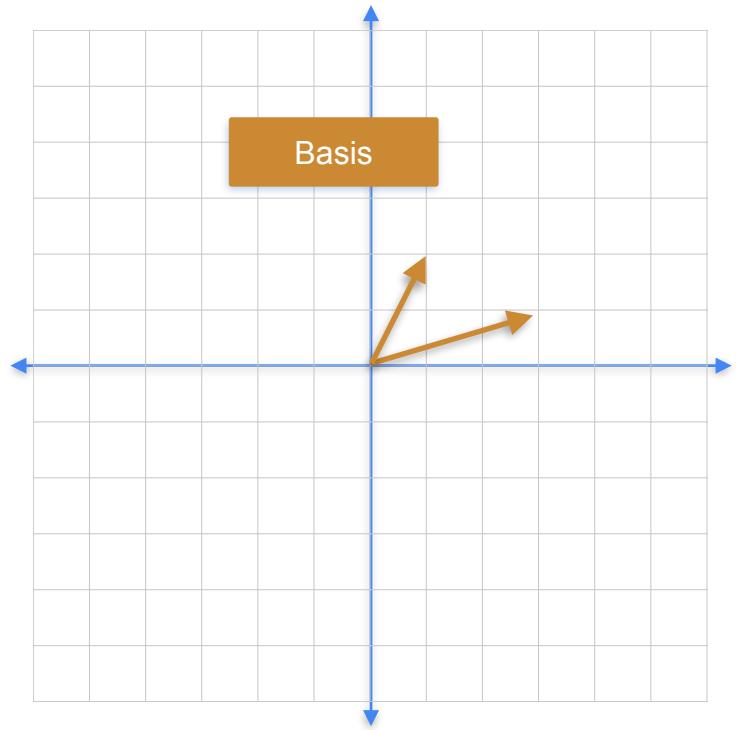
| | |
|---|---|
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| 1 | 2 |



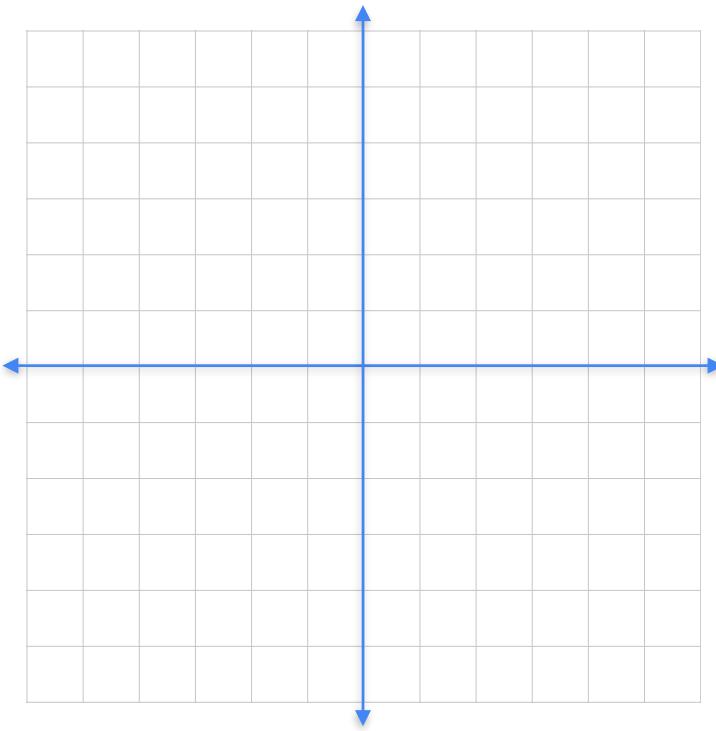
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| | |
|---|---|
| 3 | 1 |
| 1 | 2 |



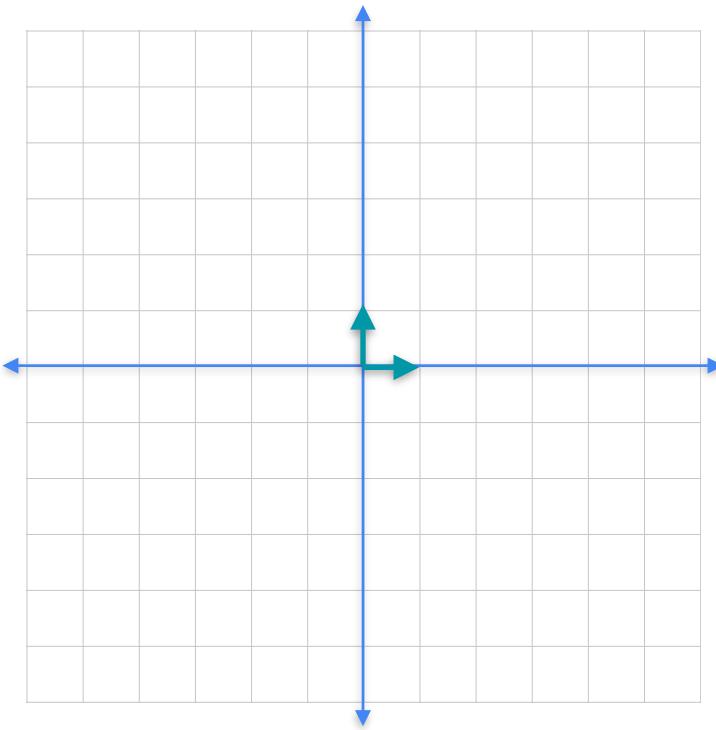
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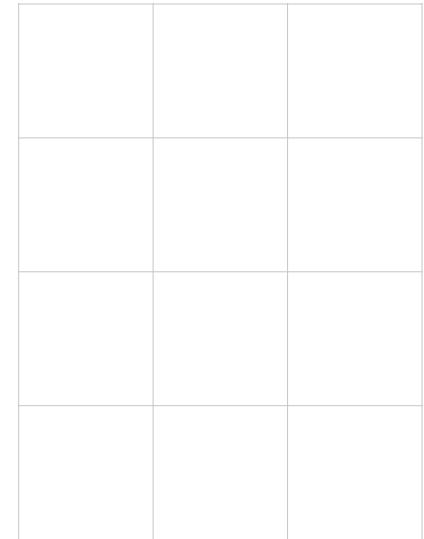
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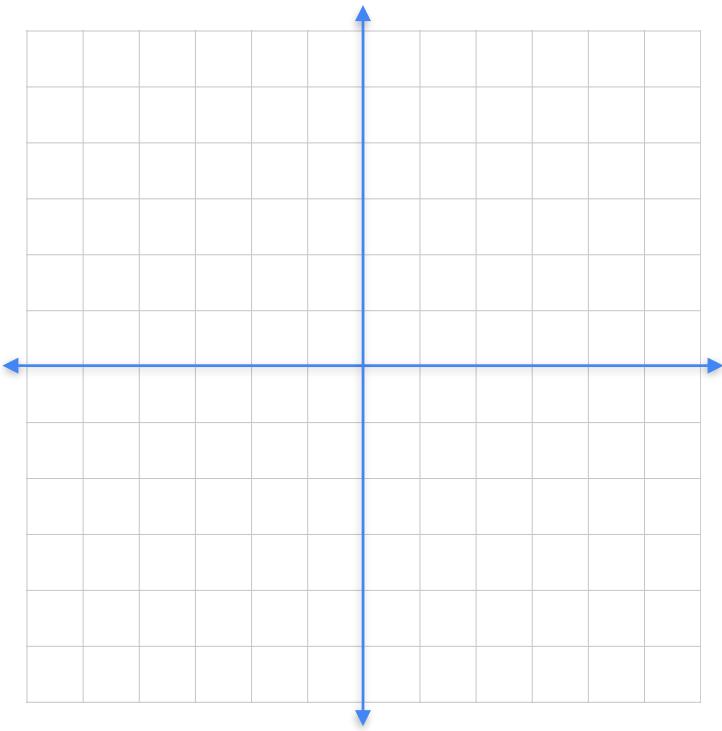
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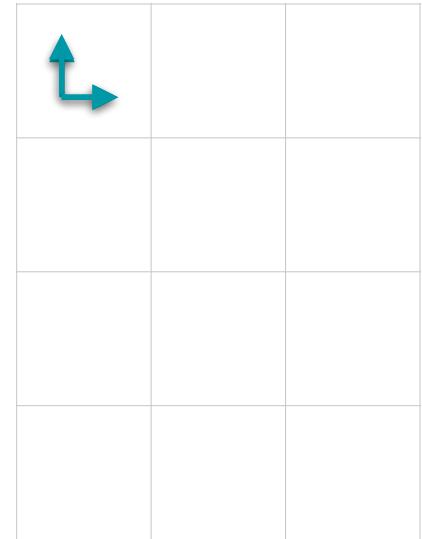
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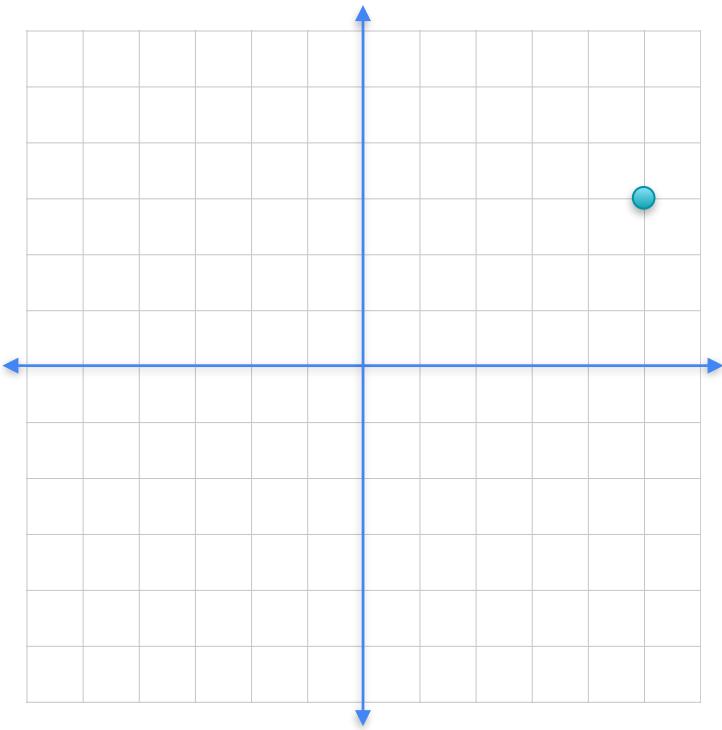
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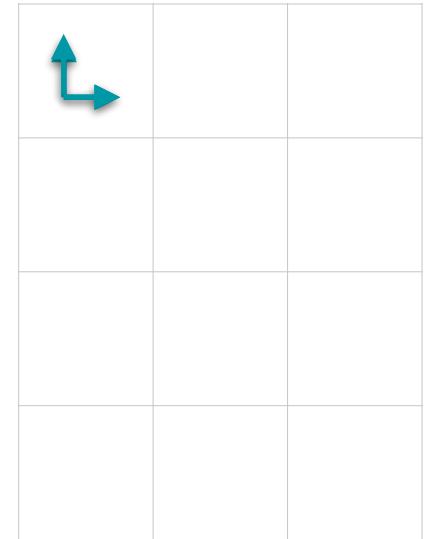
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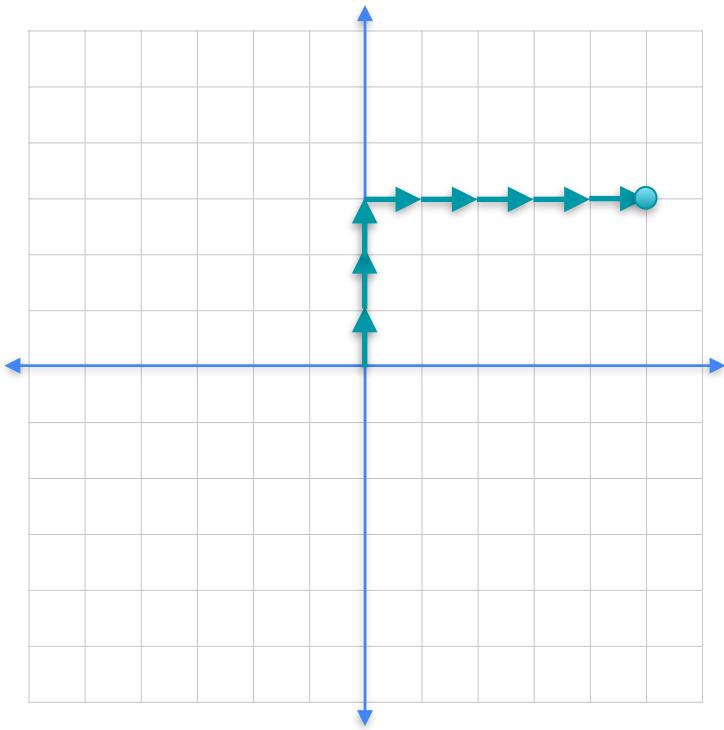
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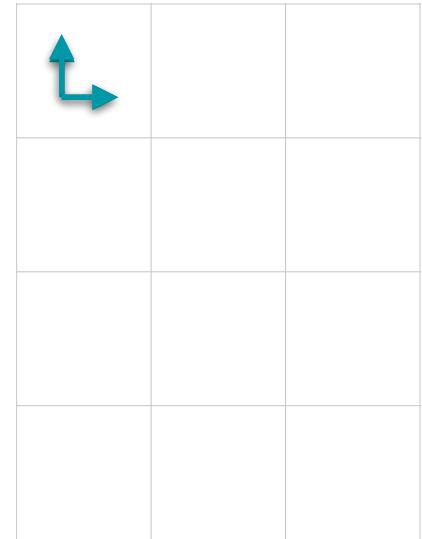
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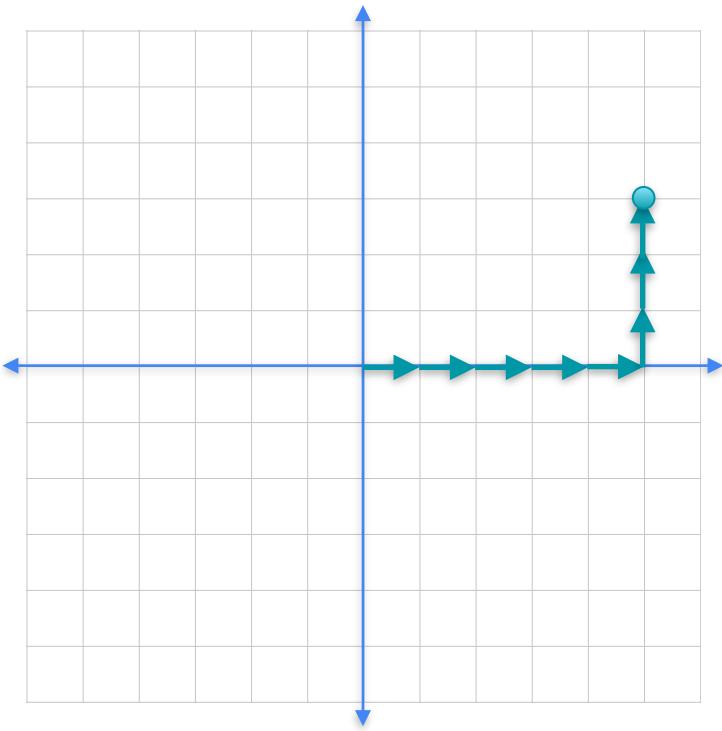
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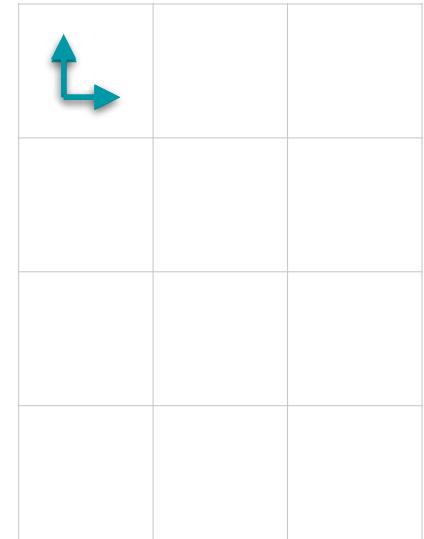
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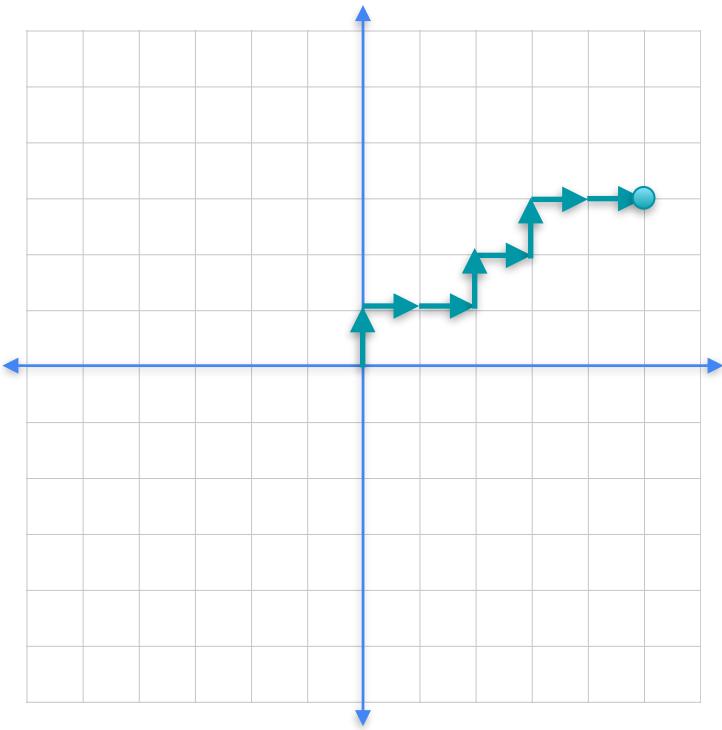
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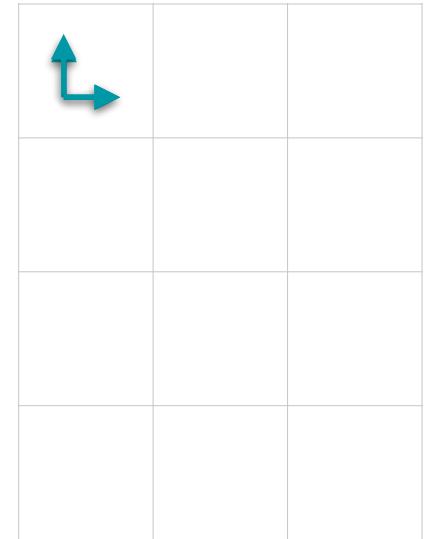
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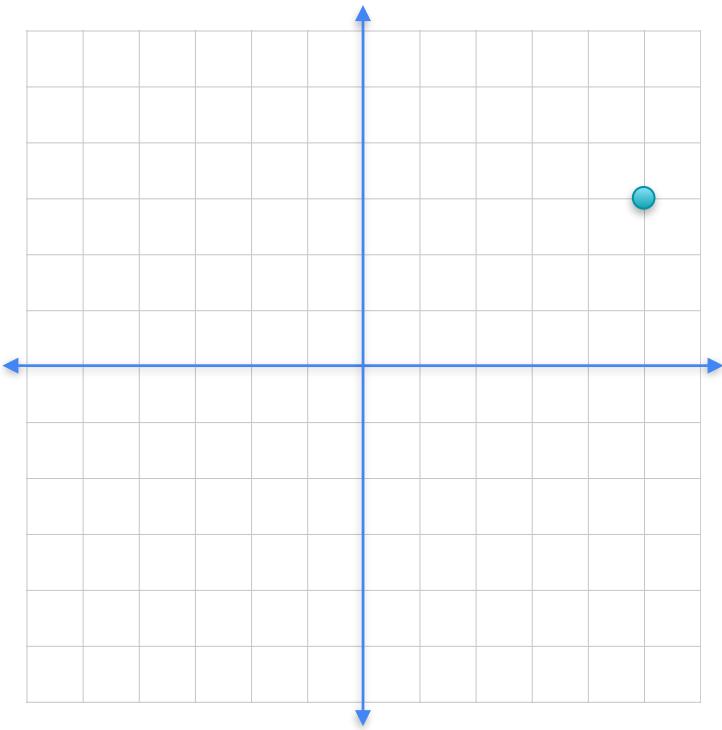
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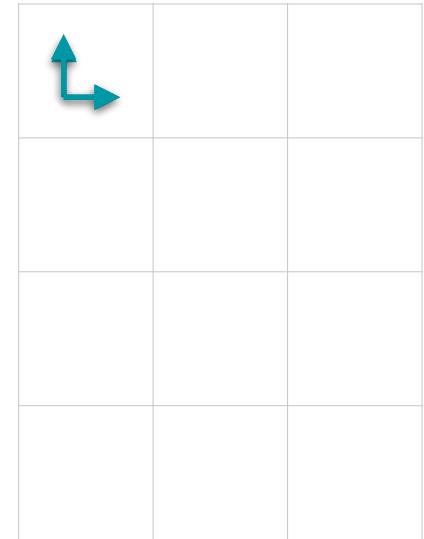
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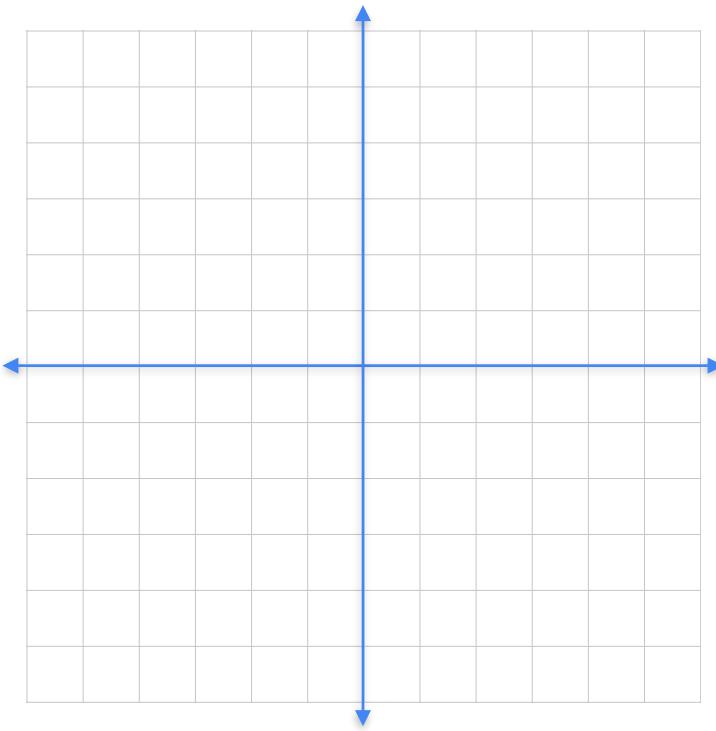
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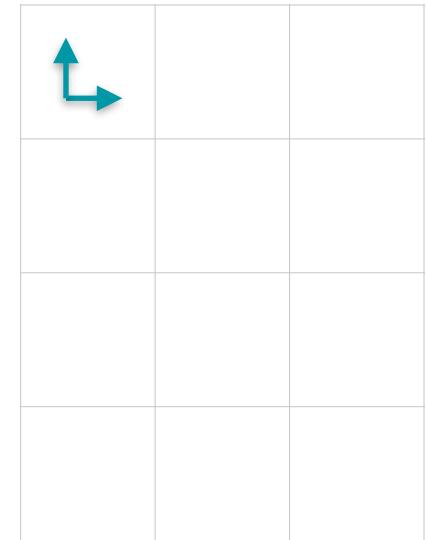
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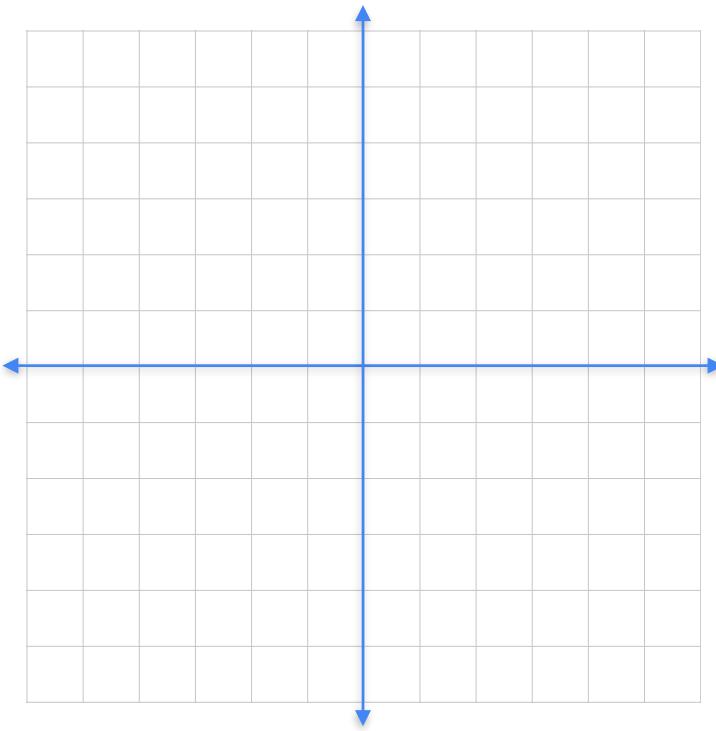
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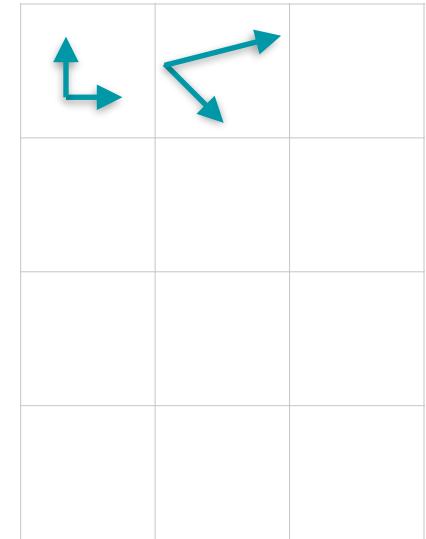
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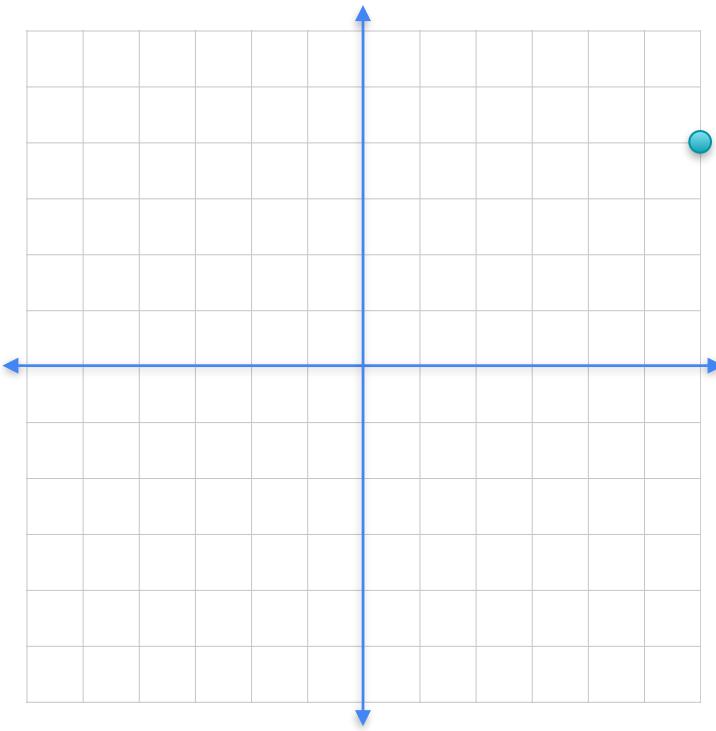
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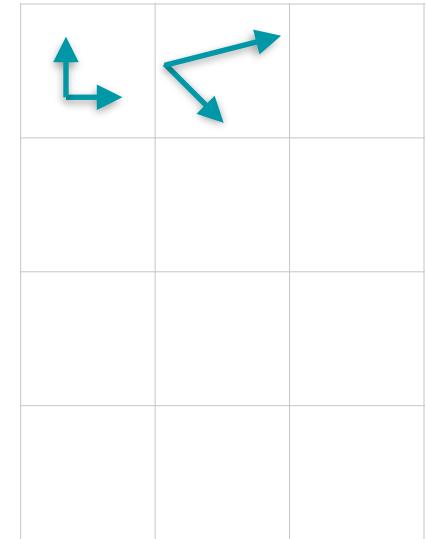
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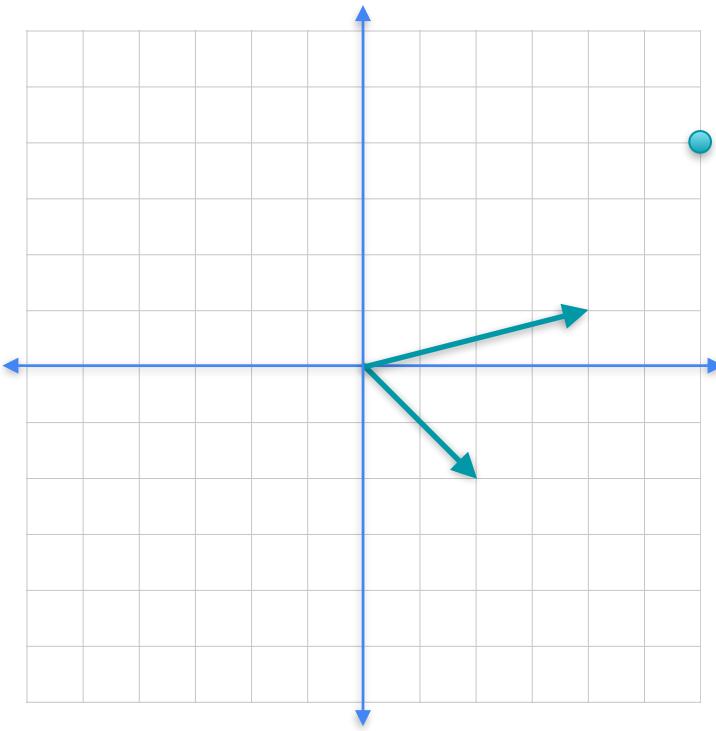
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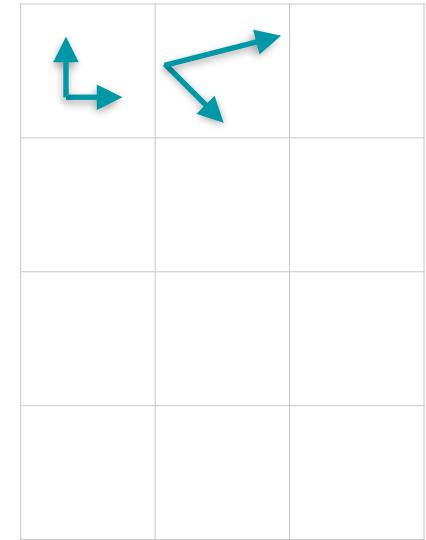
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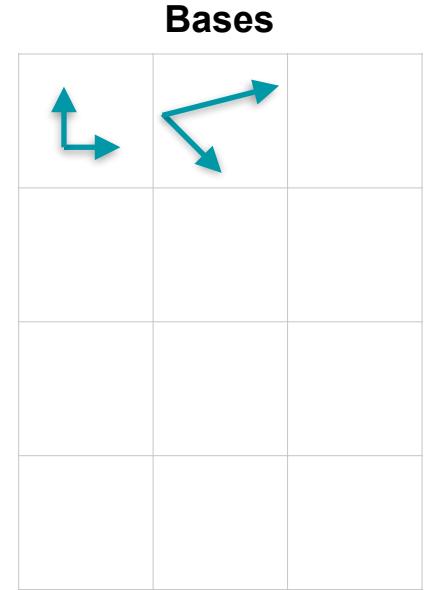
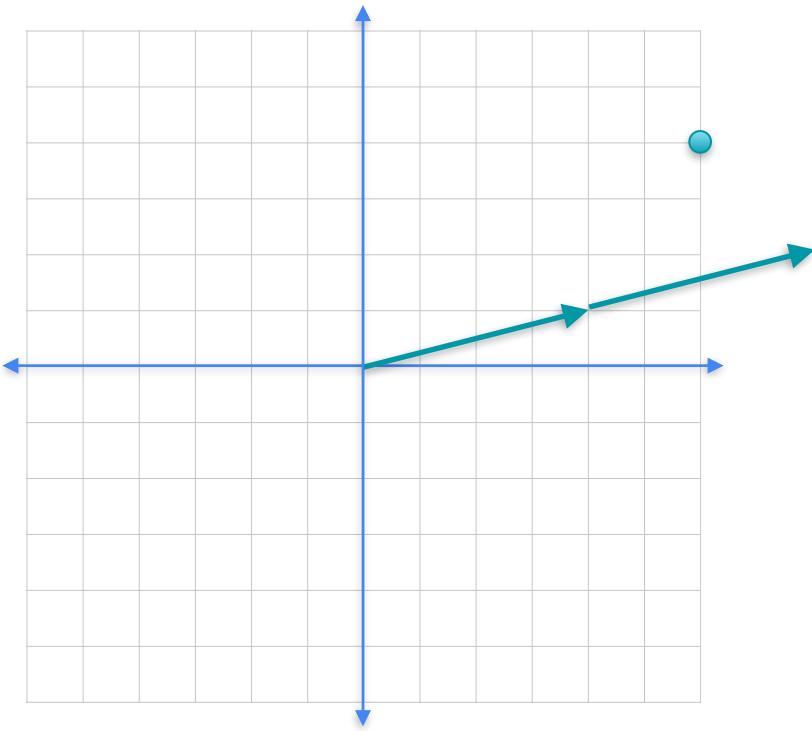
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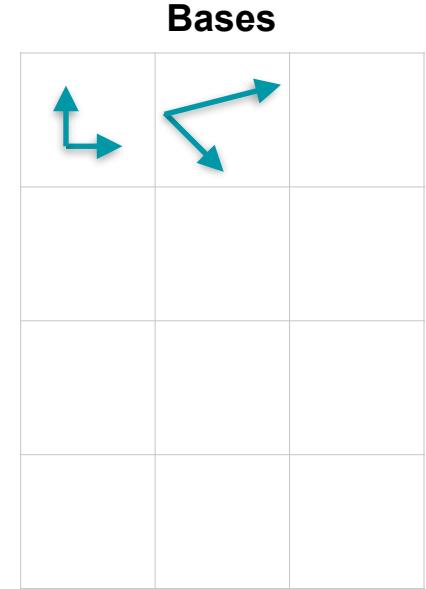
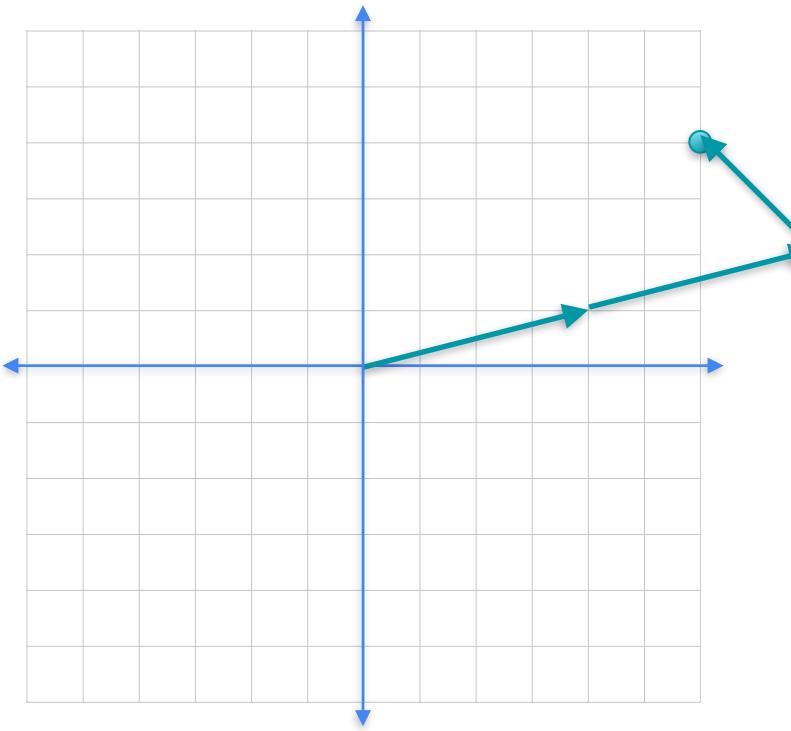
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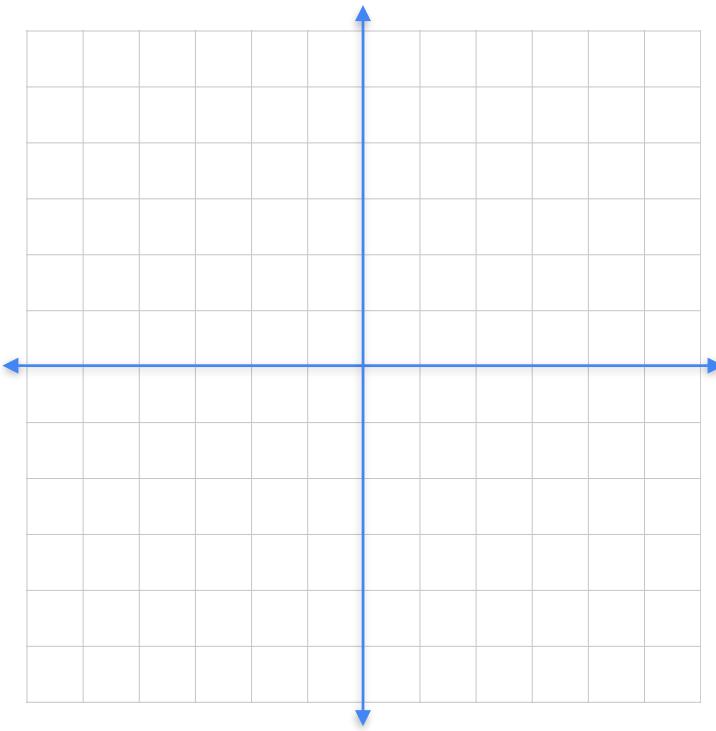
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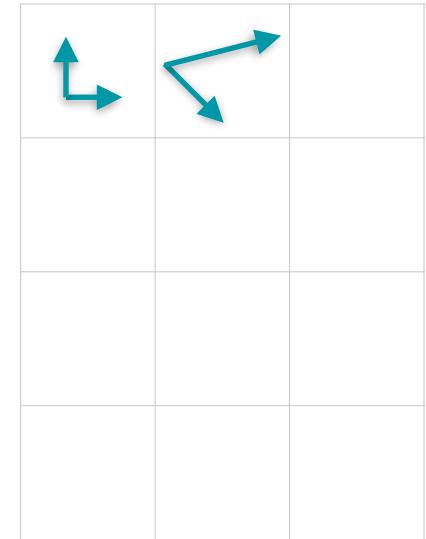
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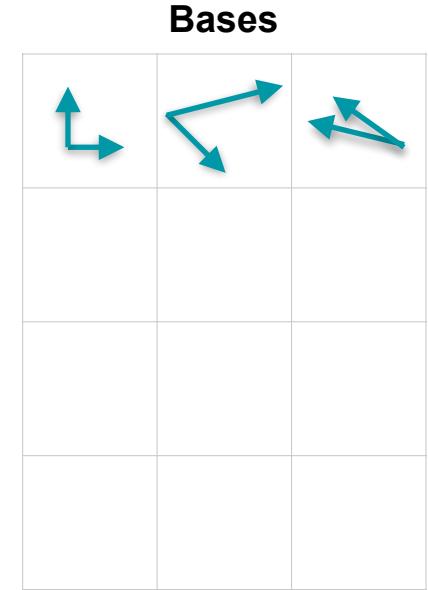
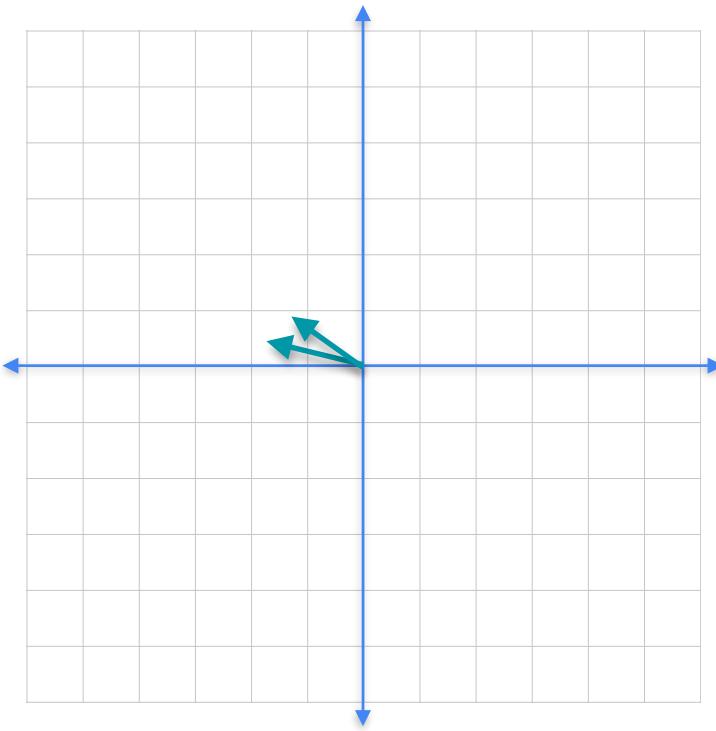
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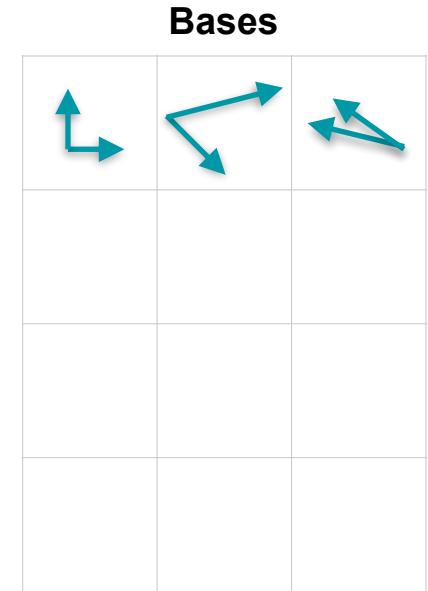
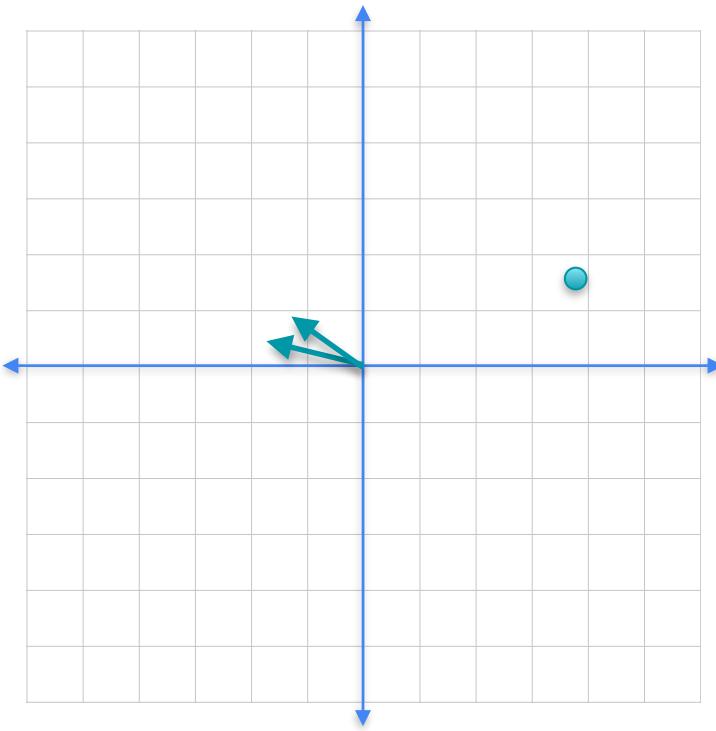
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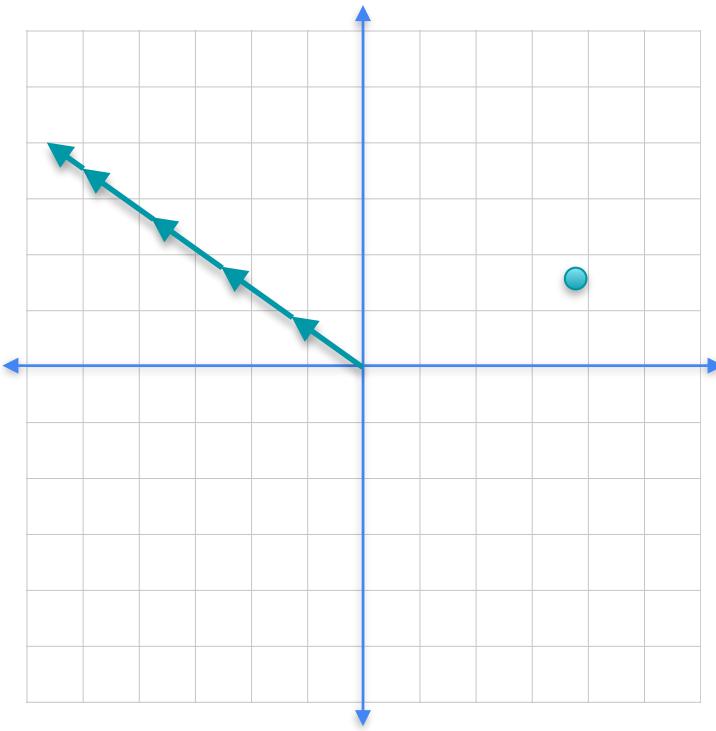
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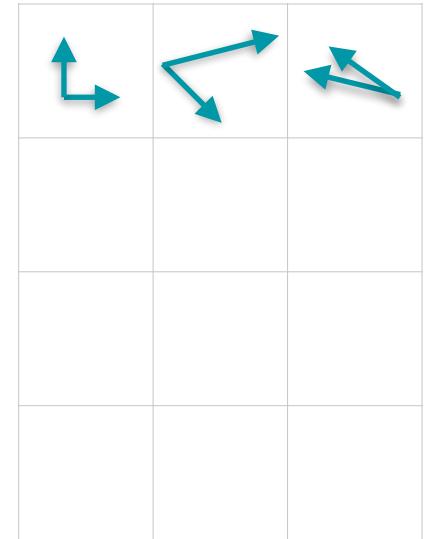
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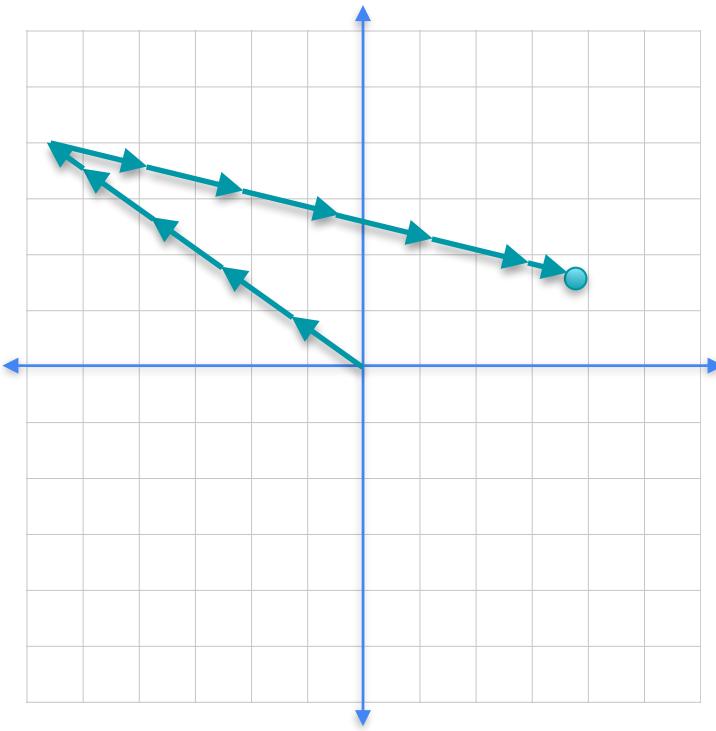
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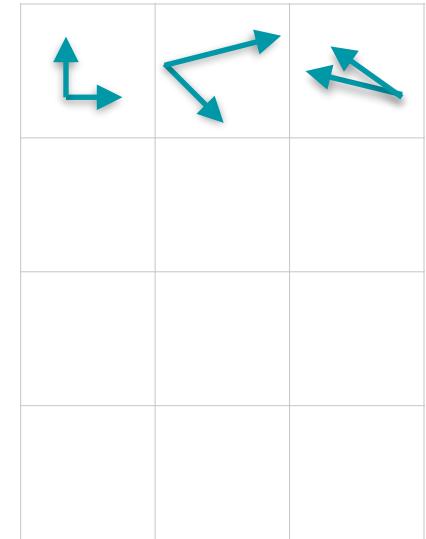
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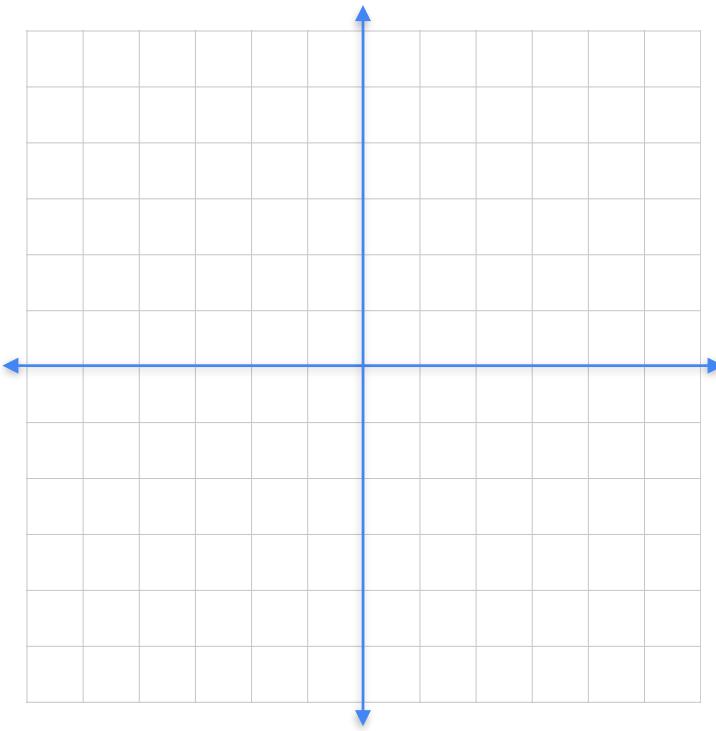
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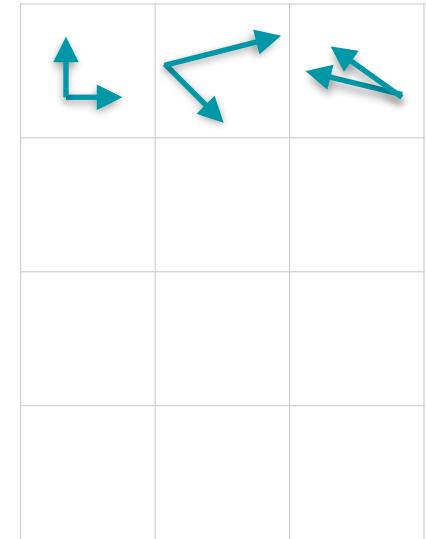
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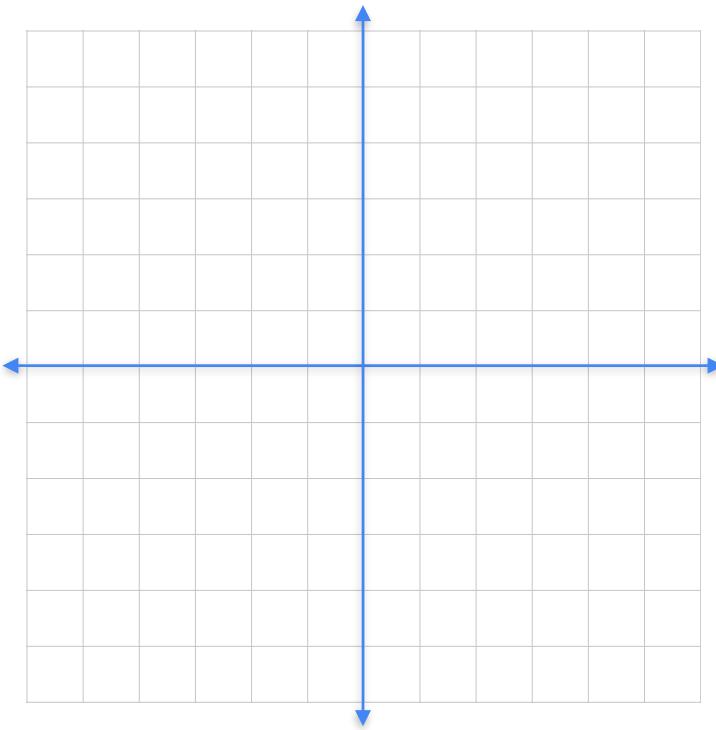
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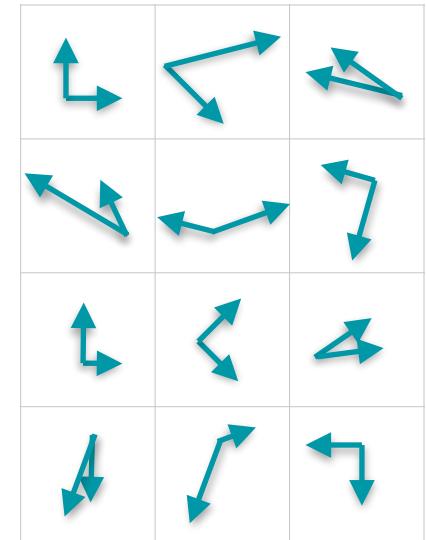
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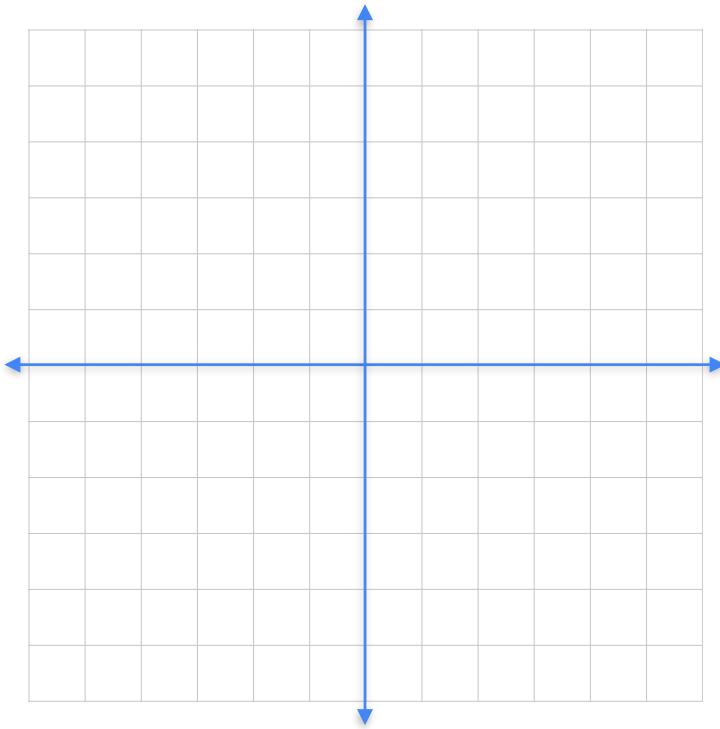
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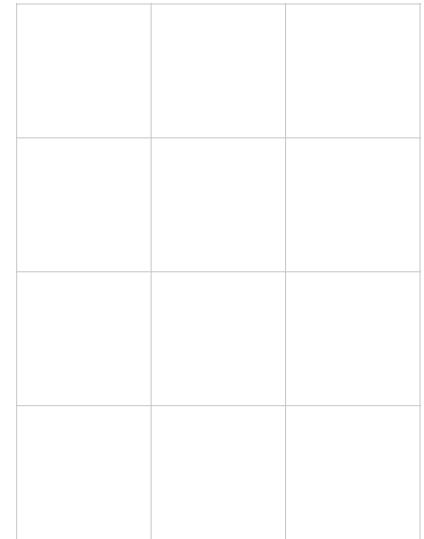
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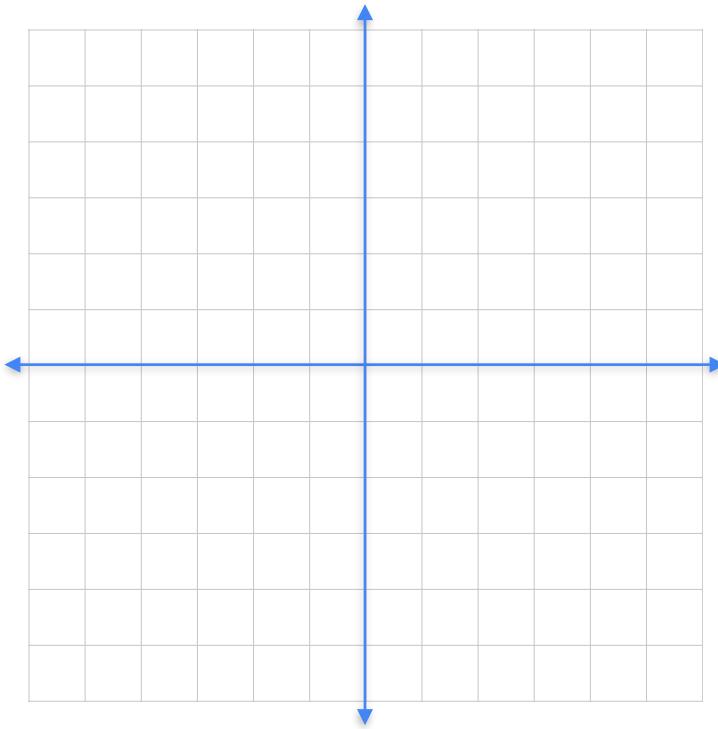
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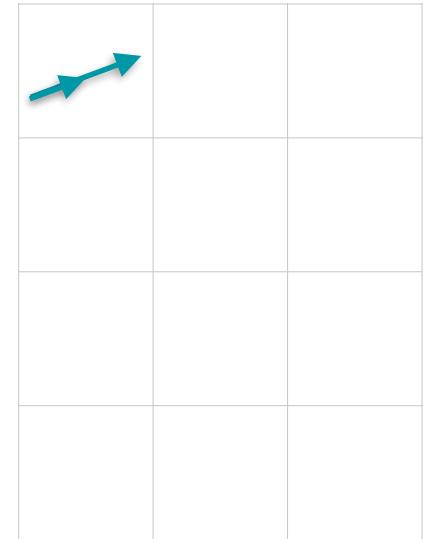
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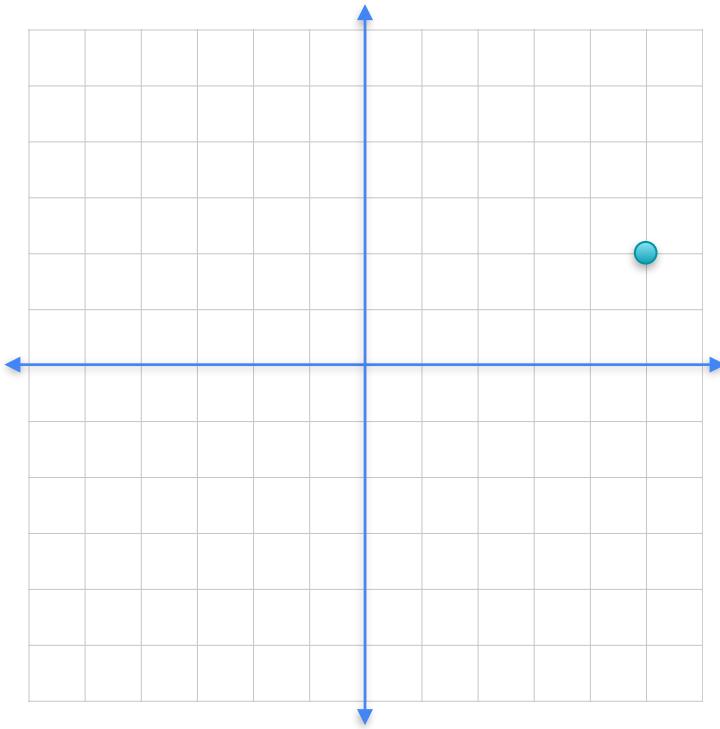
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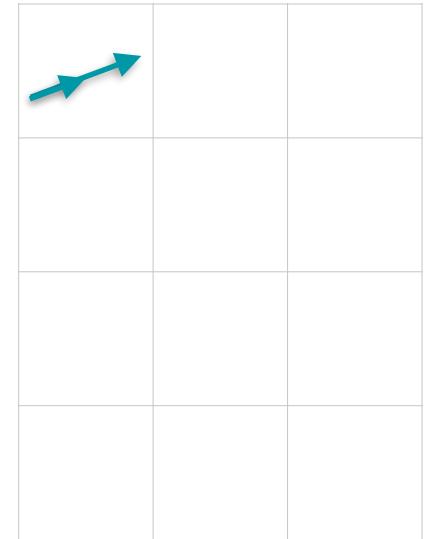
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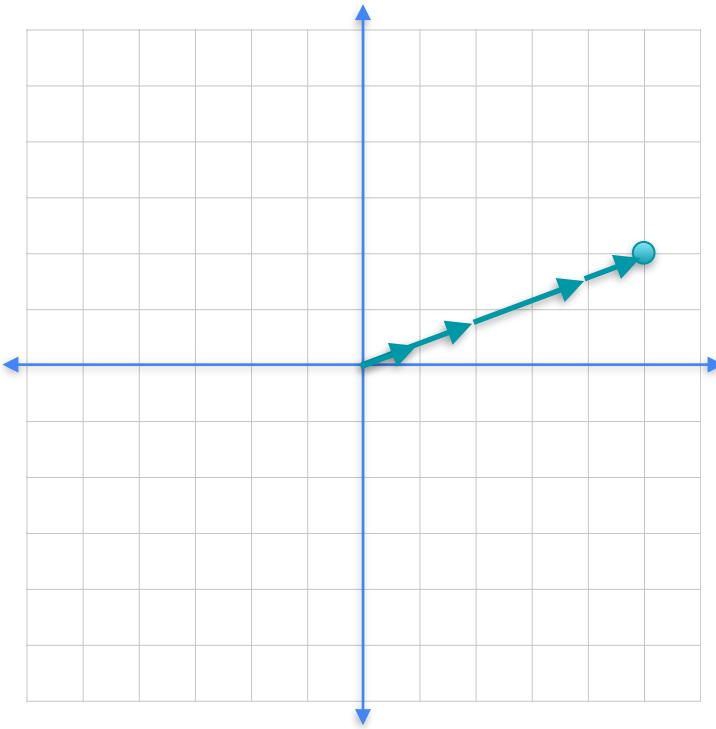
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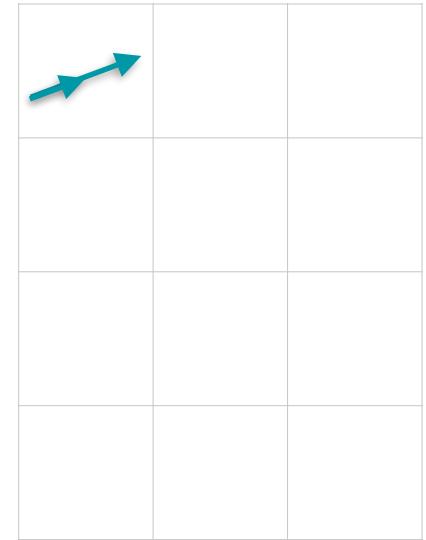
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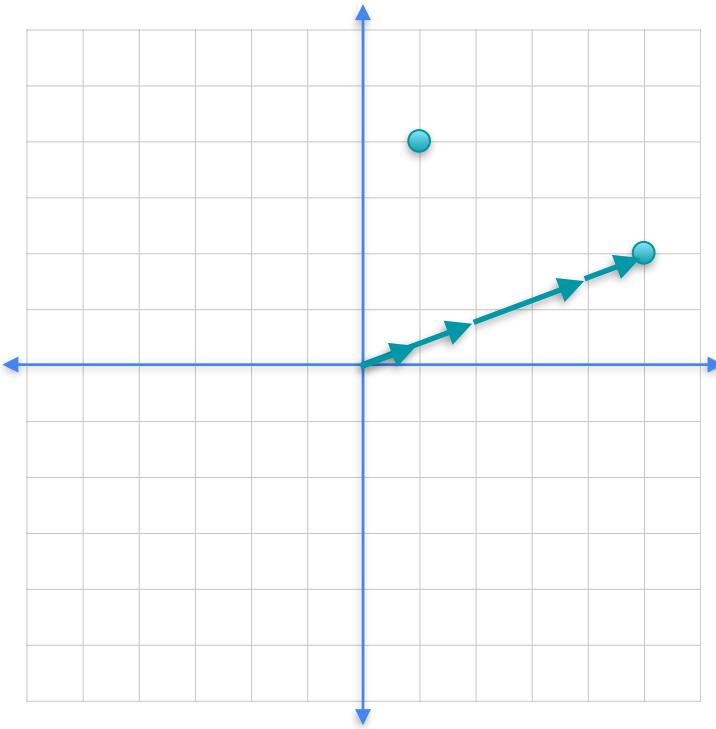
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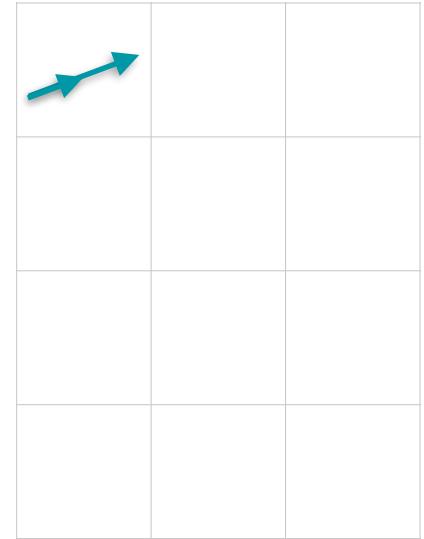
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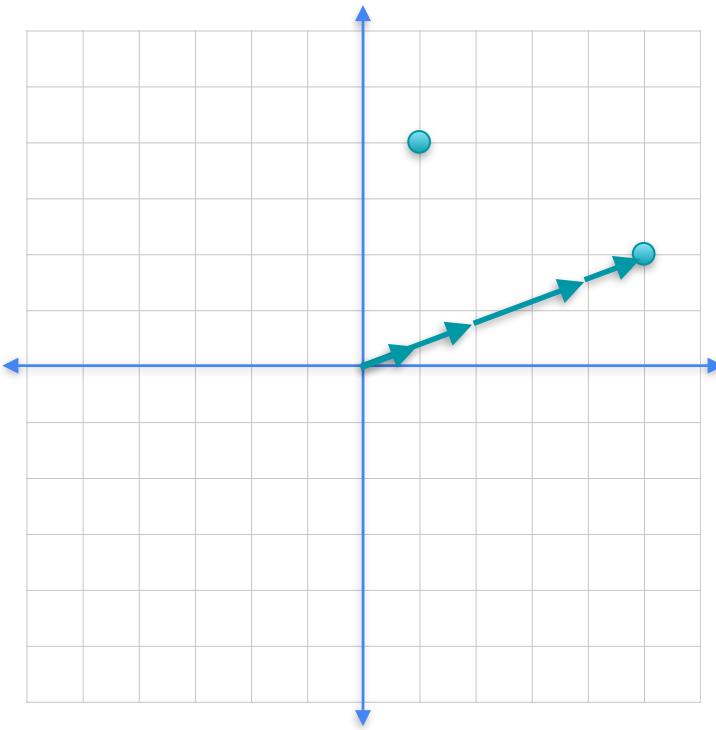
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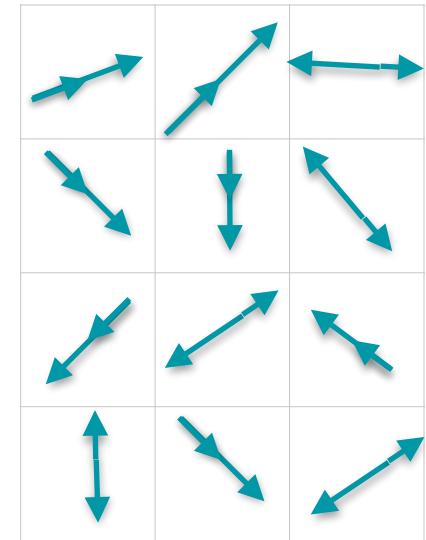
Not bases



What is not a basis?



Not bases



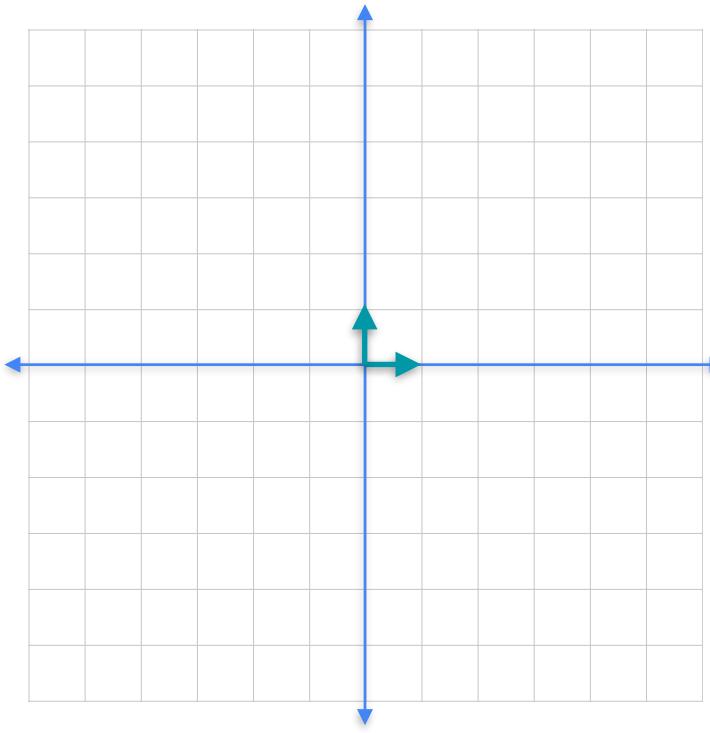


DeepLearning.AI

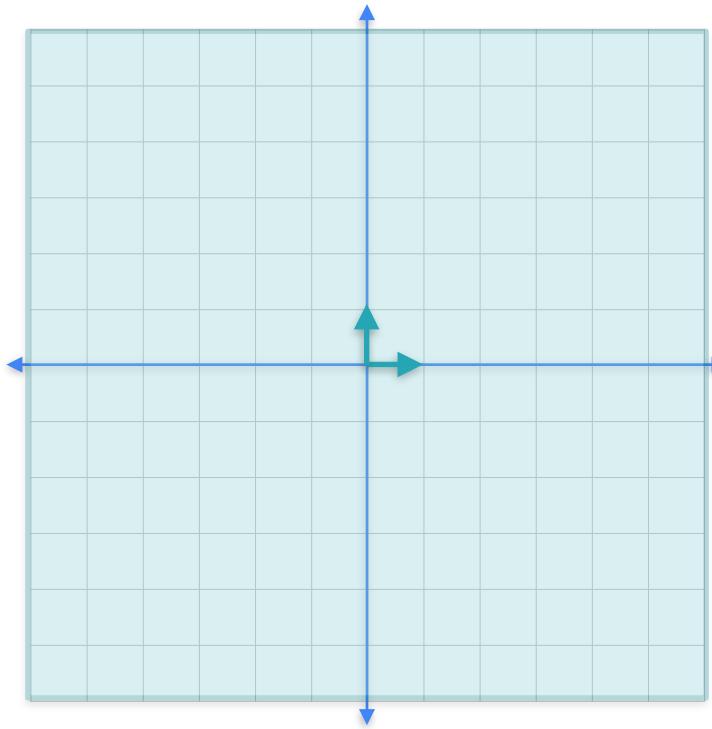
Determinants and Eigenvectors

Span

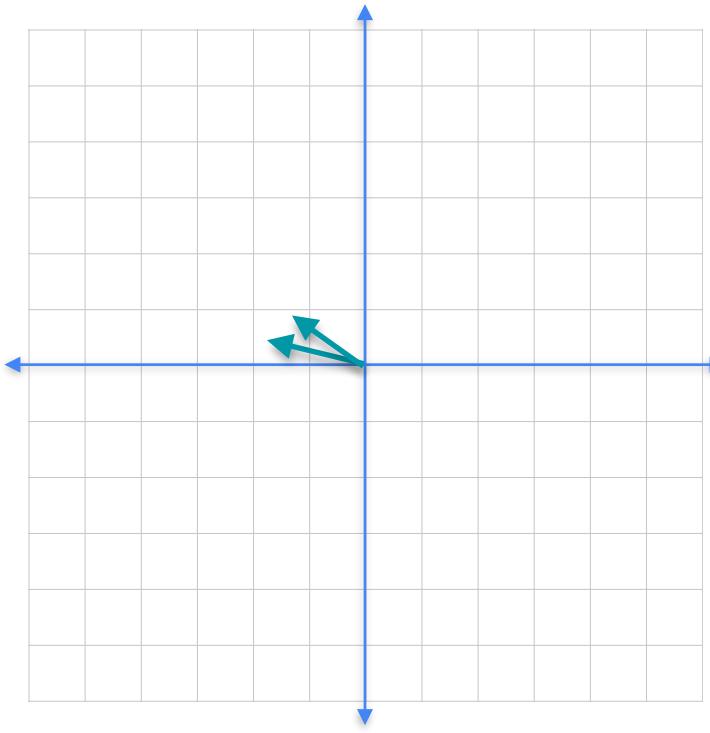
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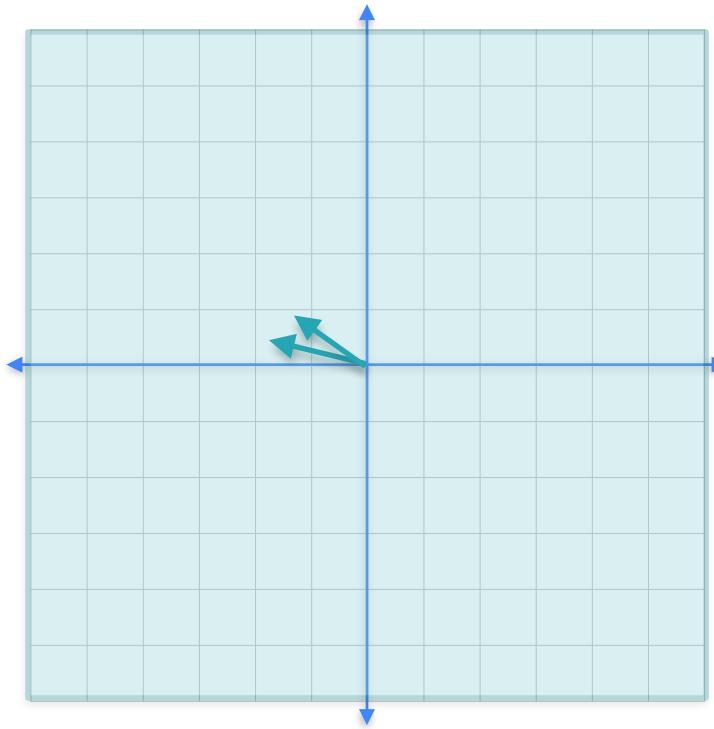
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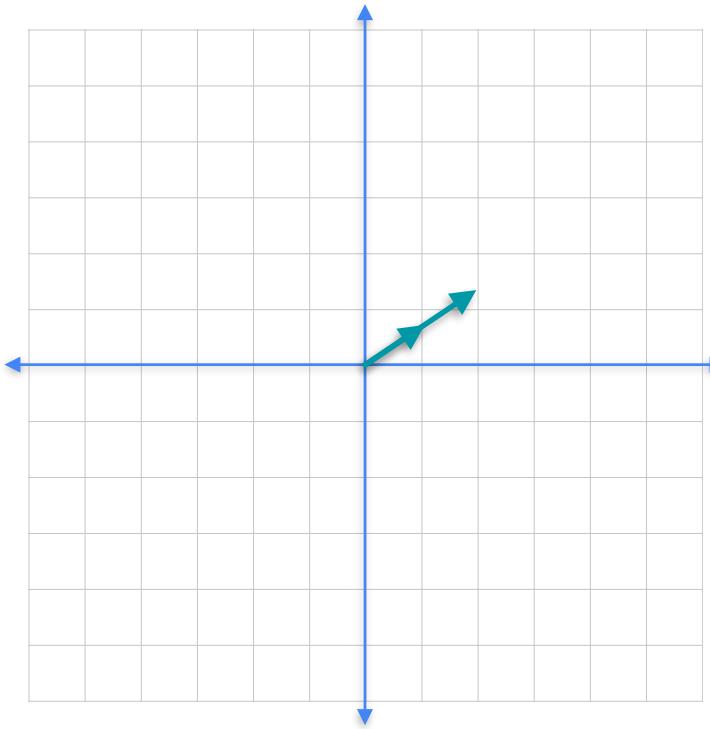
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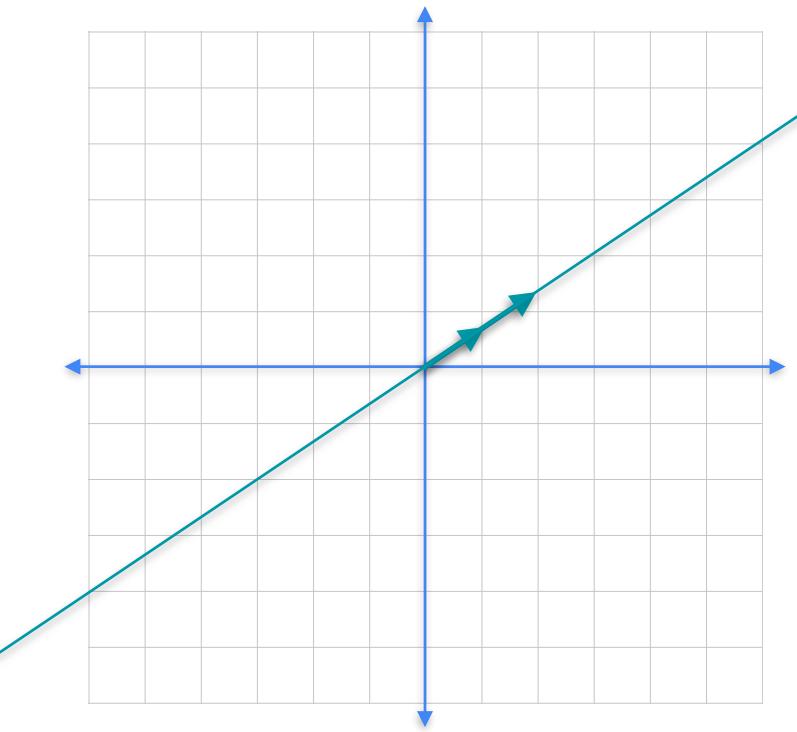
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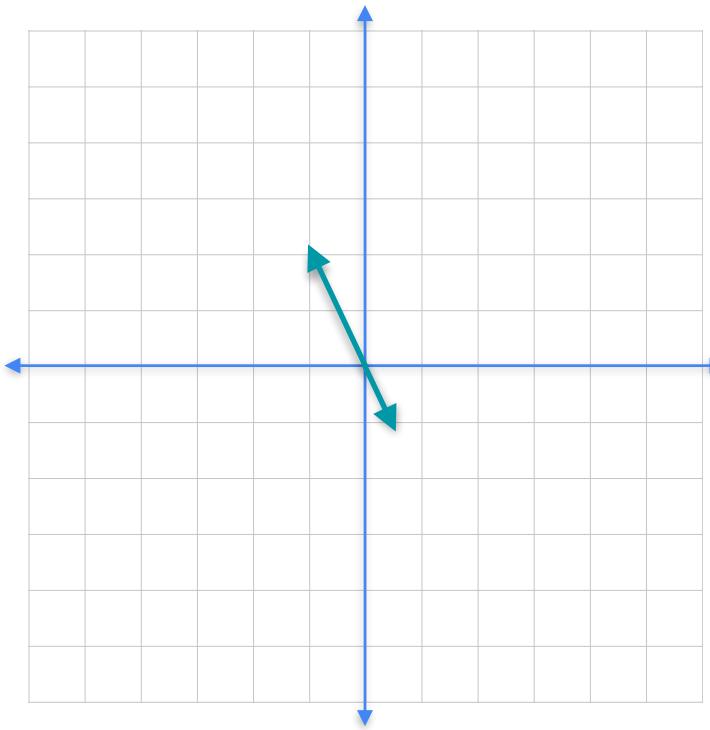
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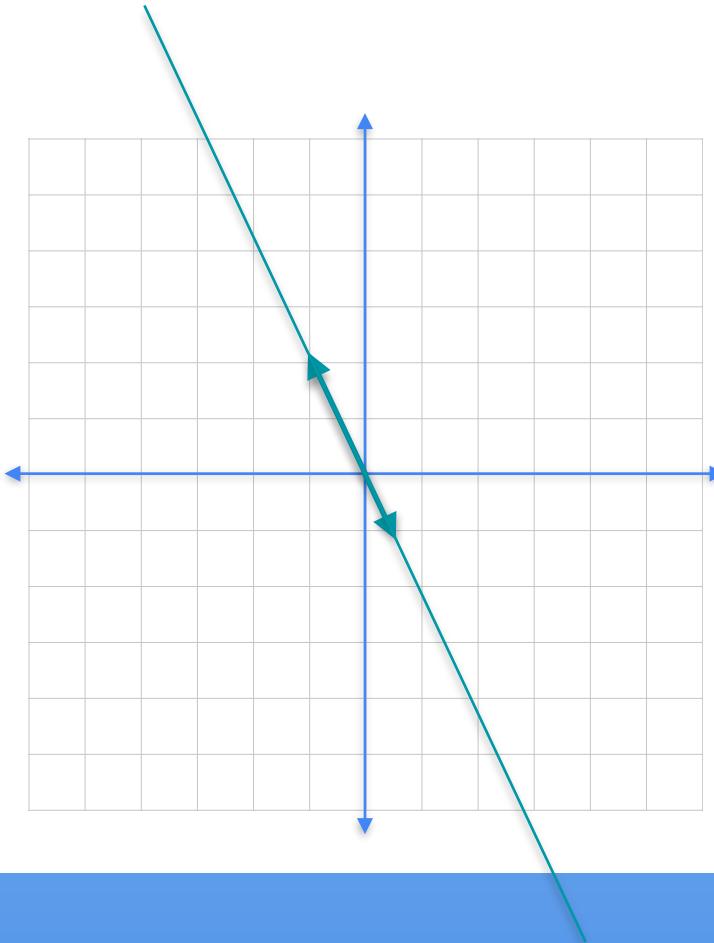
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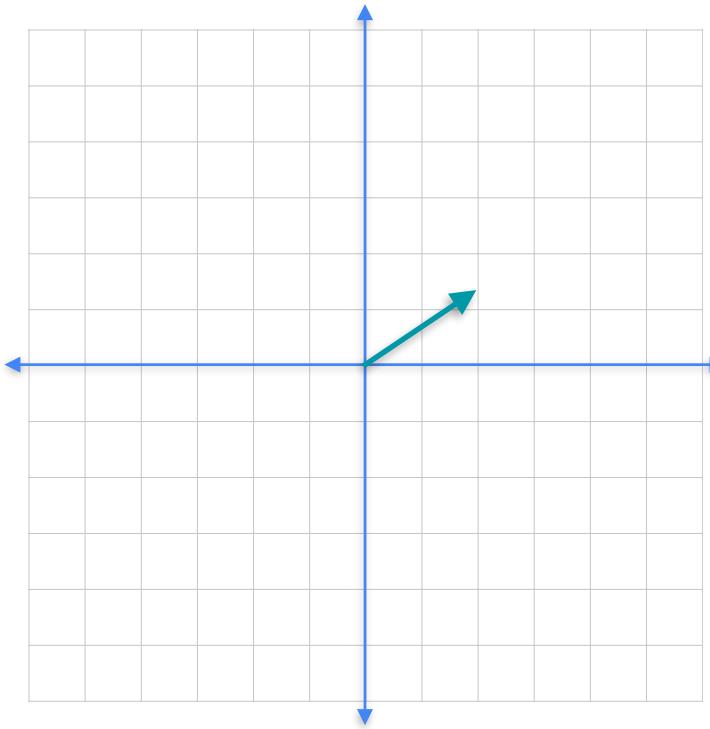
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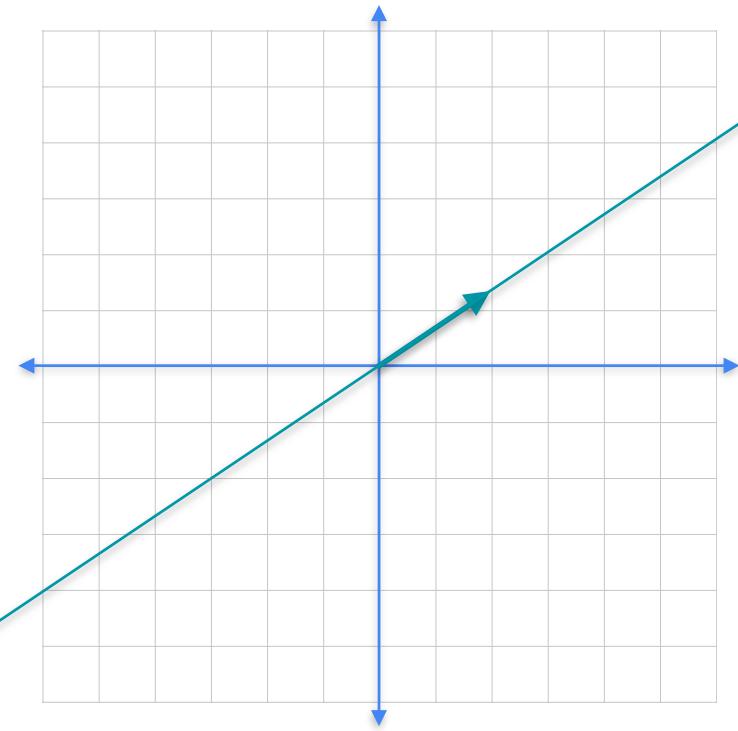
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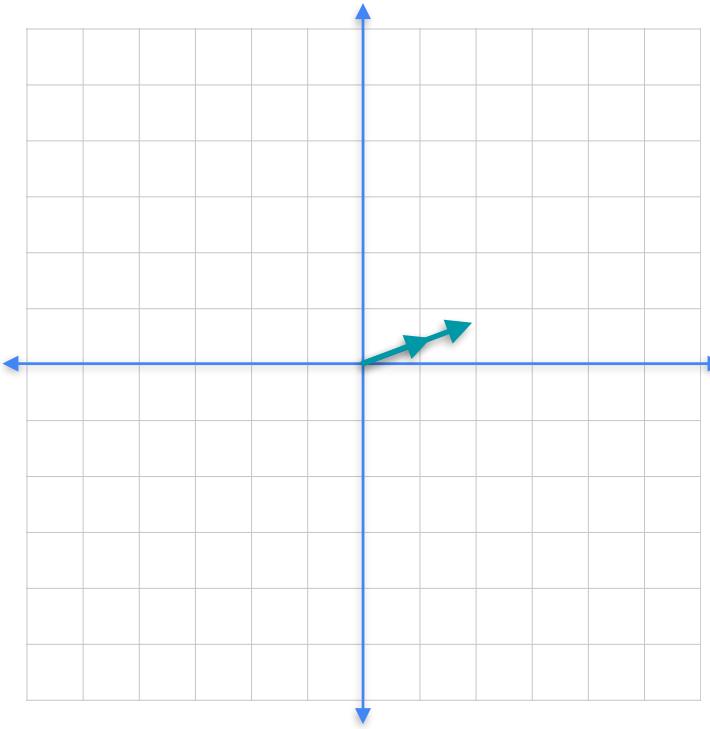
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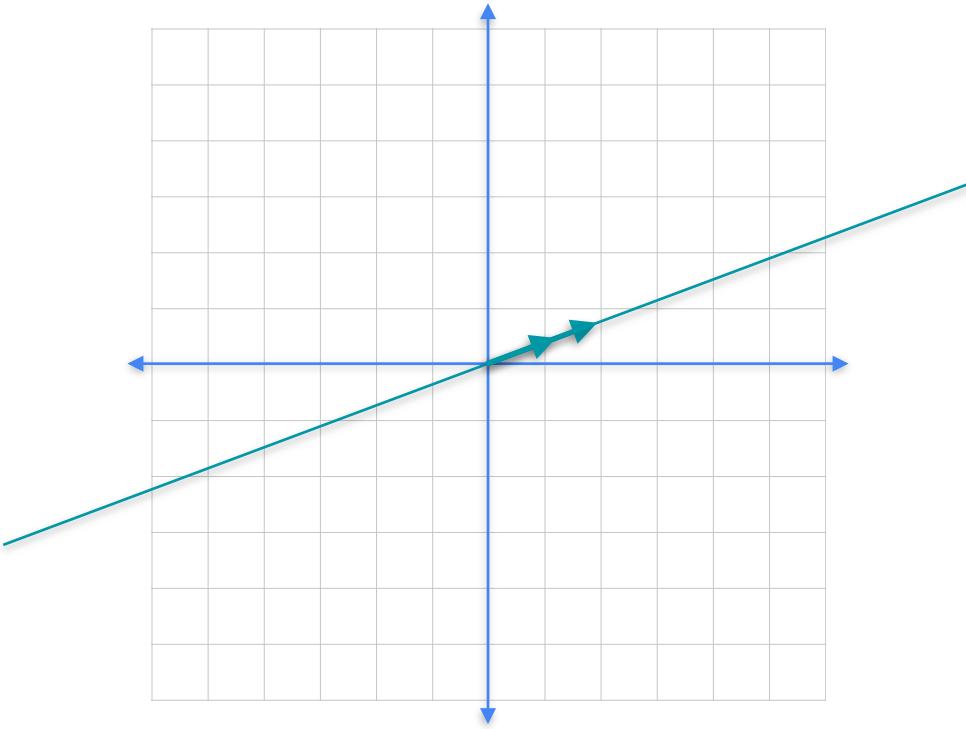
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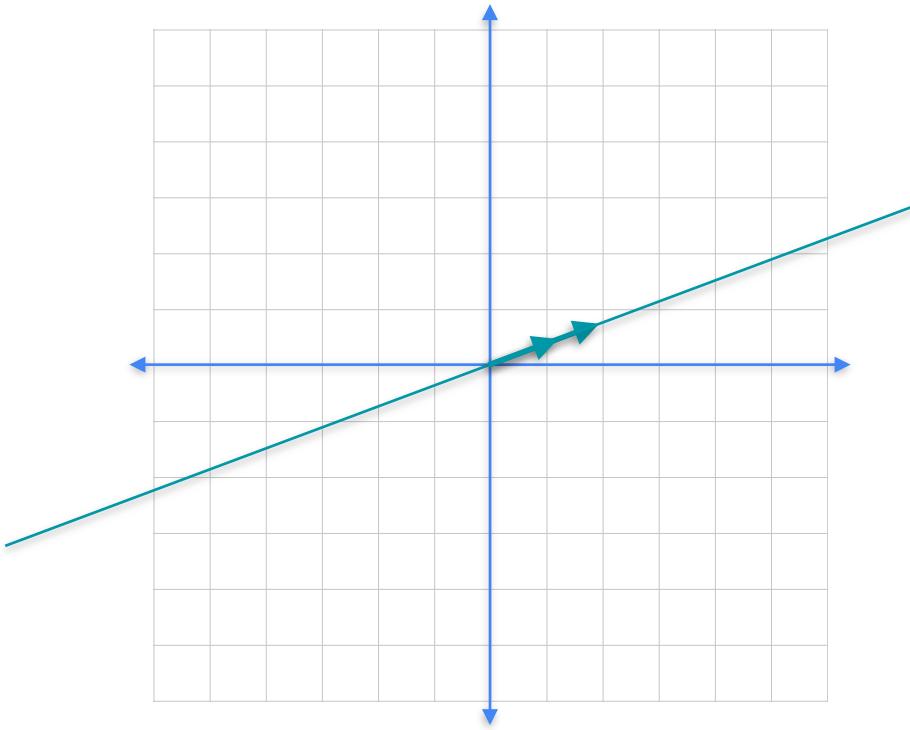
Is this a basis?



Is this a basis?



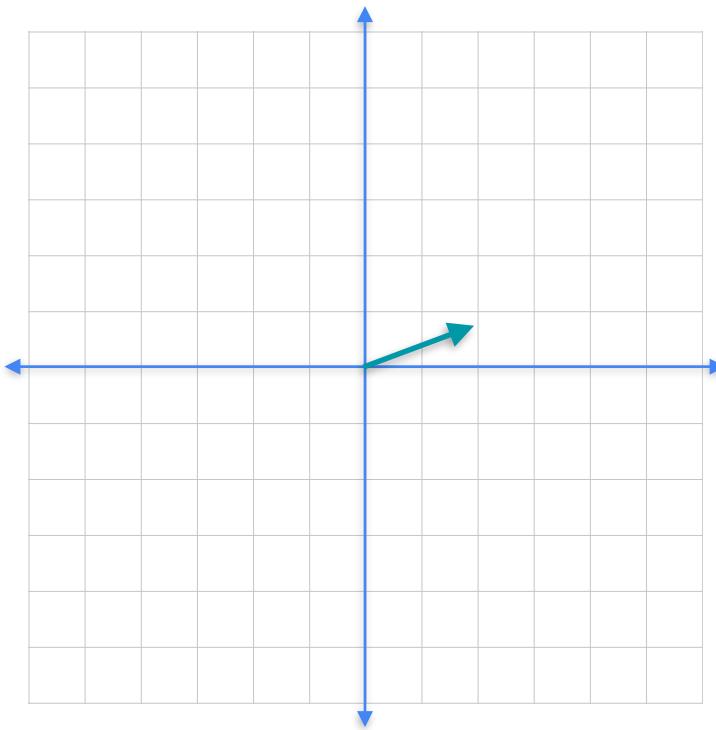
Is this a basis?



No

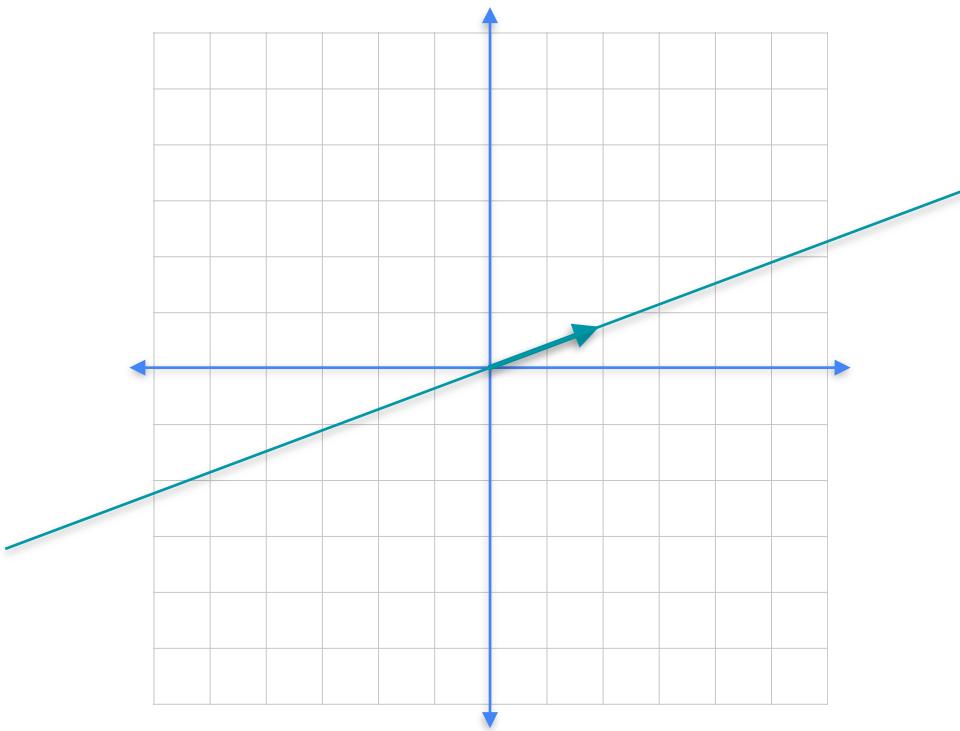
Is this a basis for something?

Bases

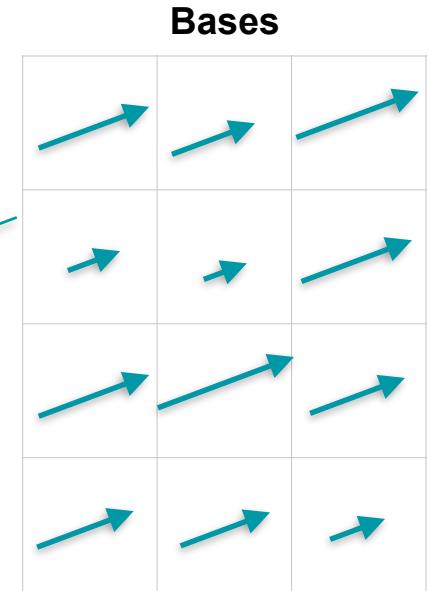
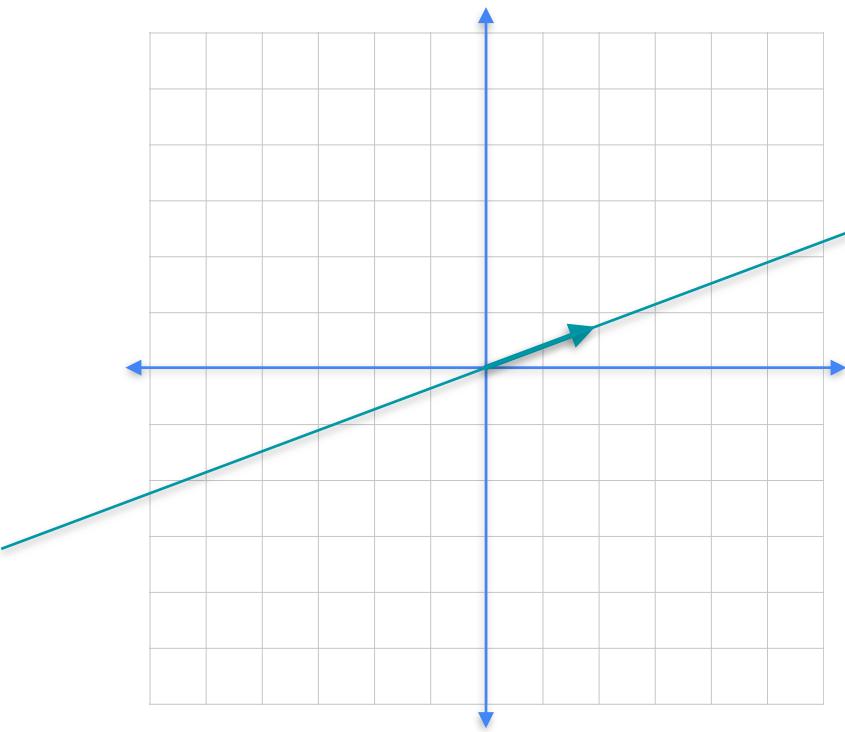


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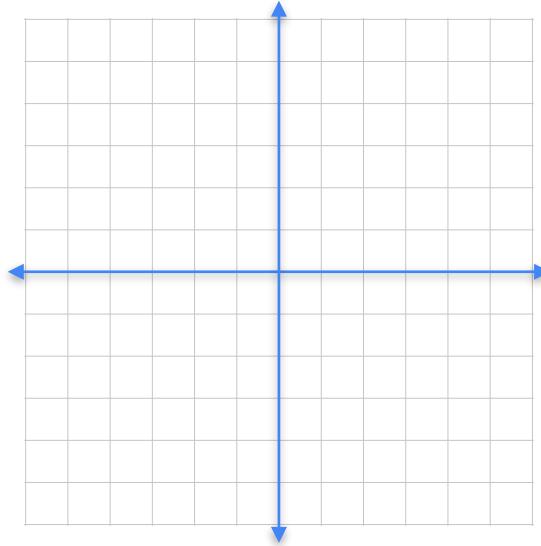
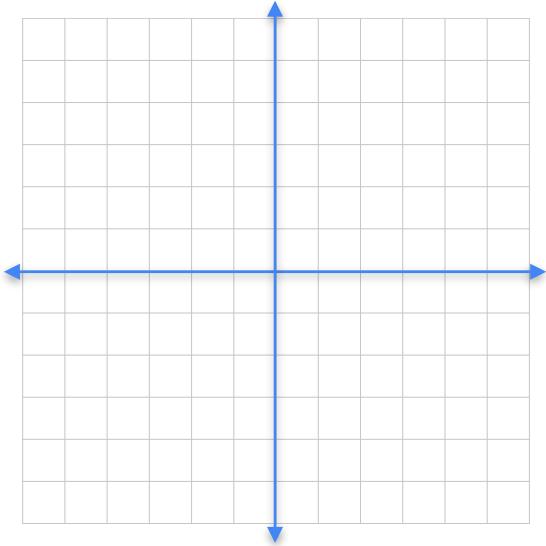
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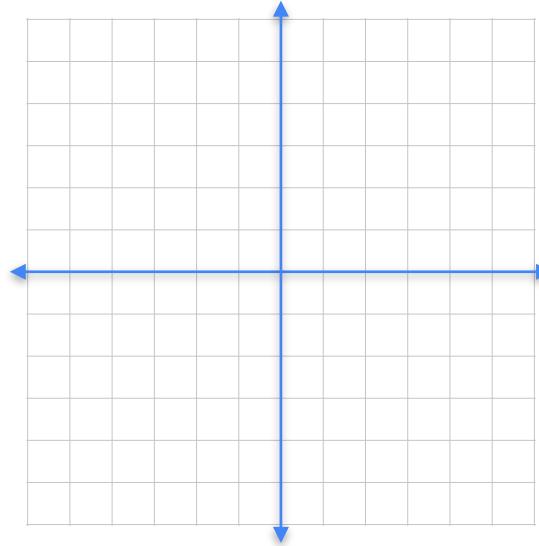
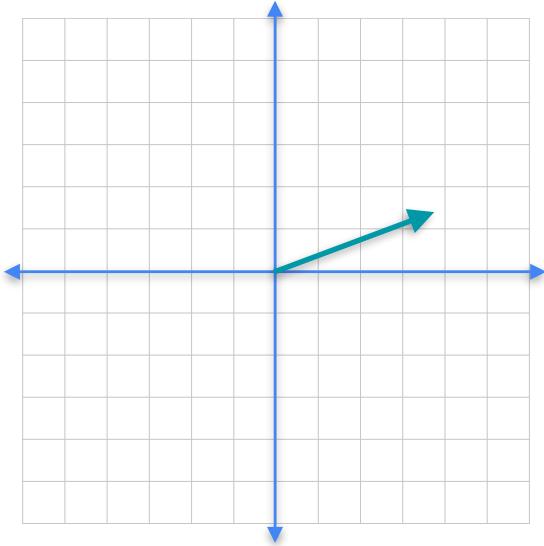
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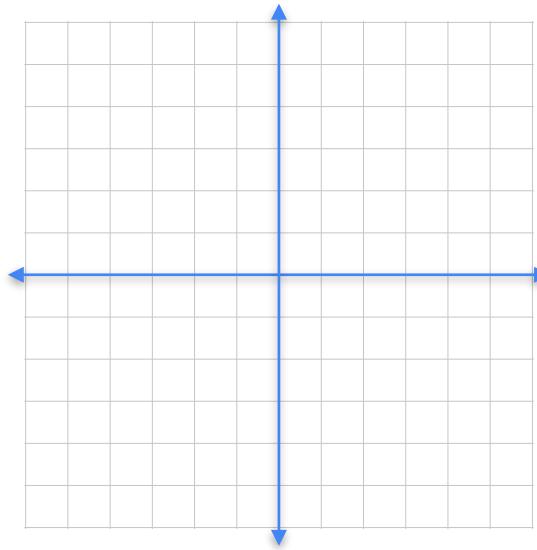
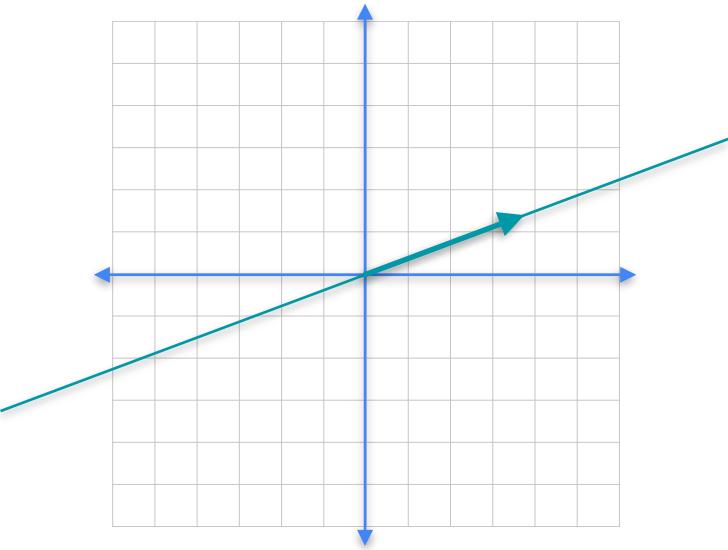
A basis is a minimal spanning set



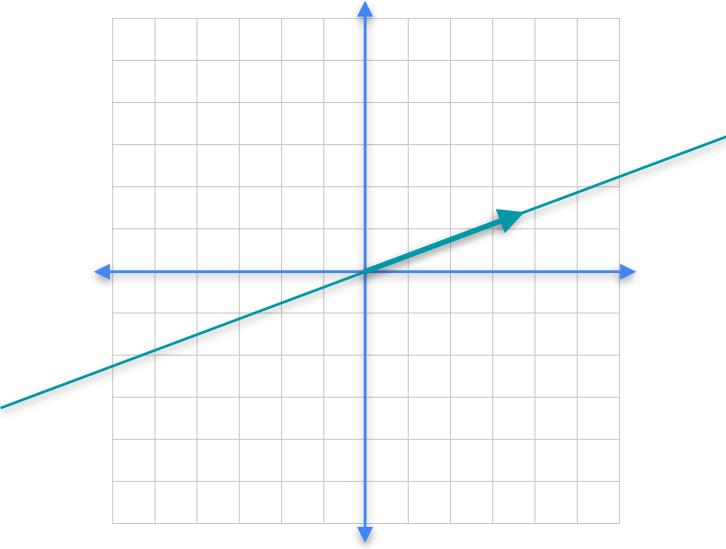
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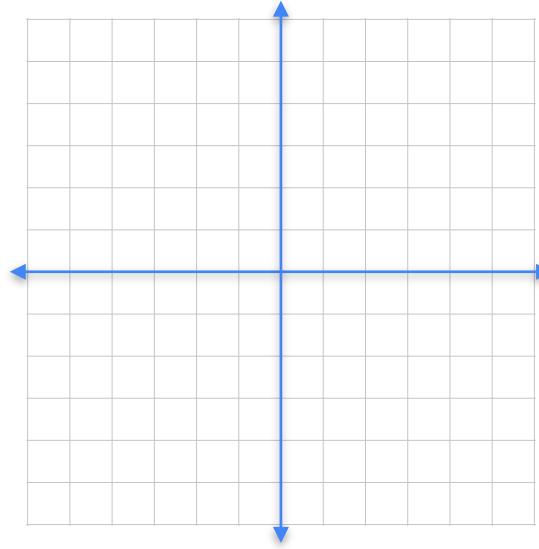
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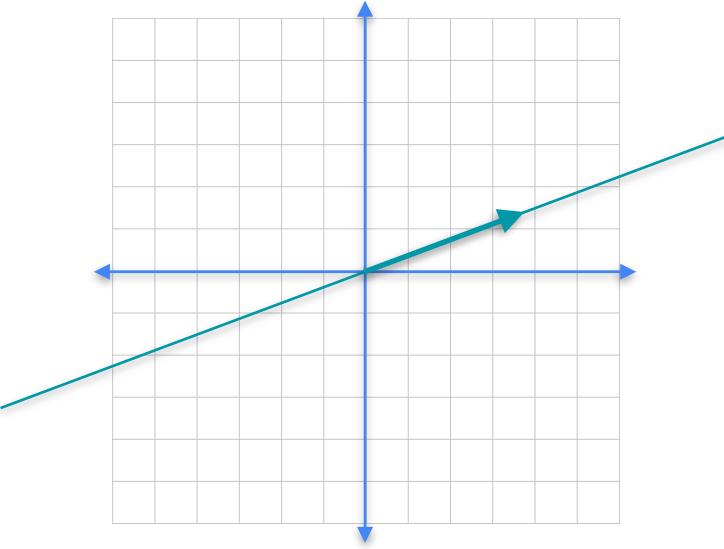
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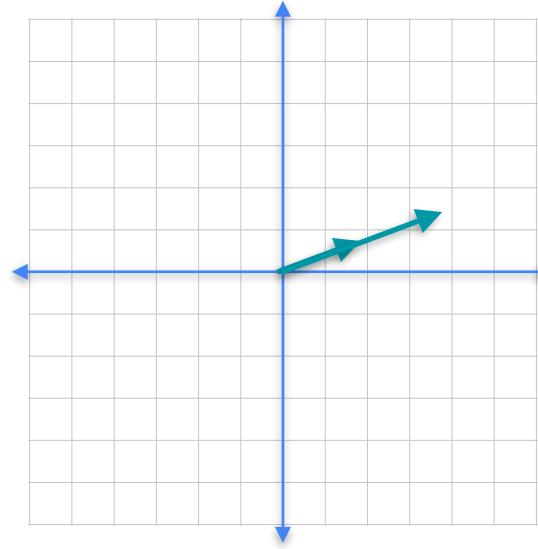
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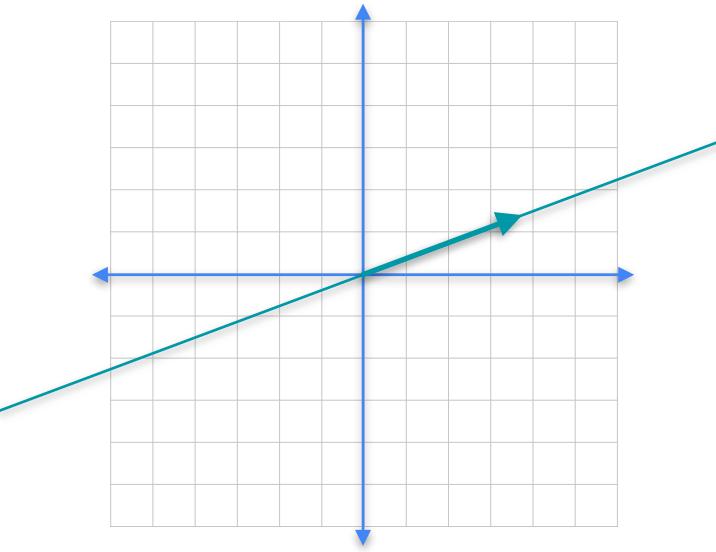
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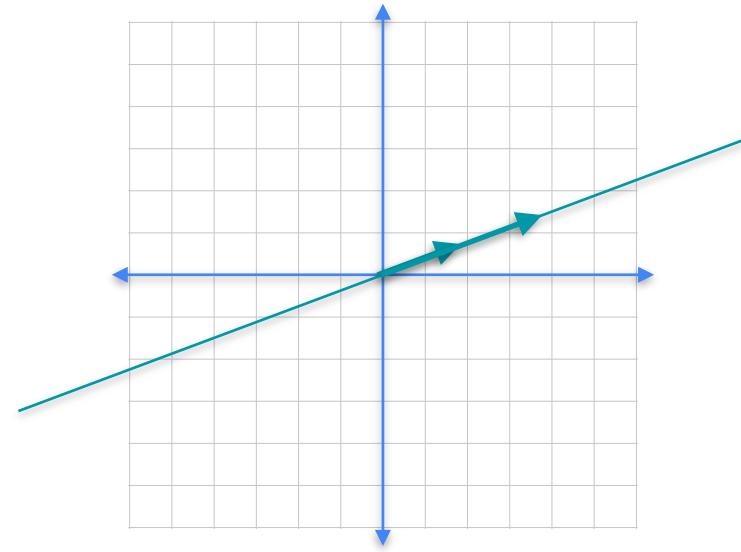
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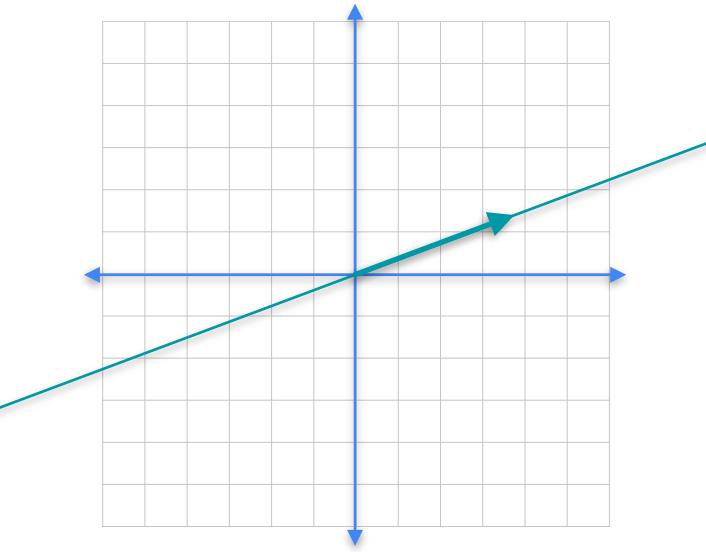
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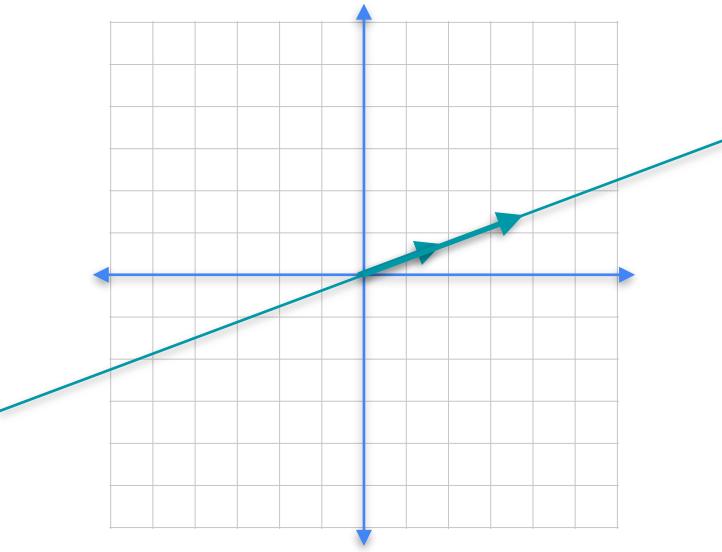
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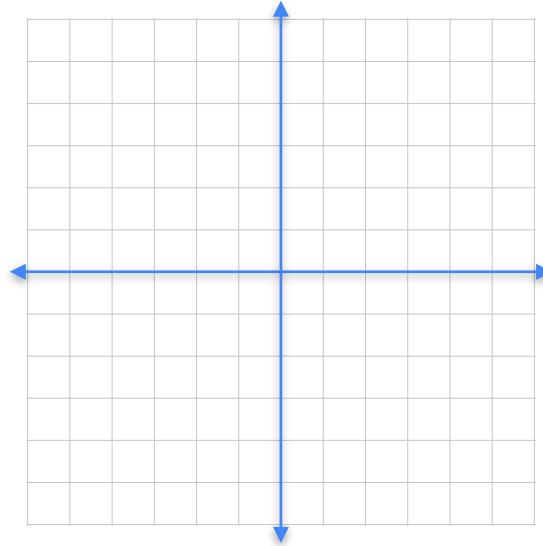
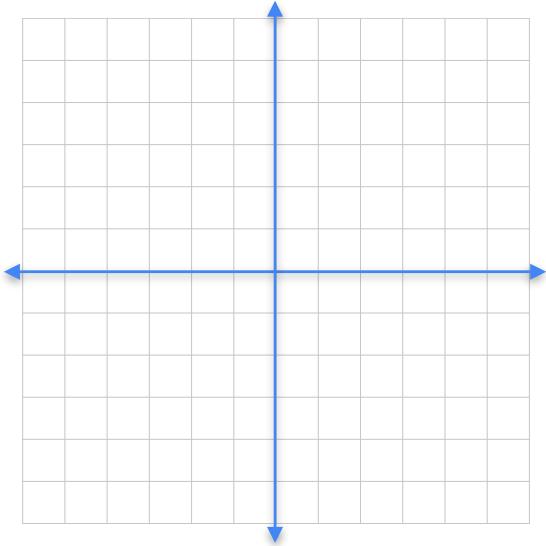


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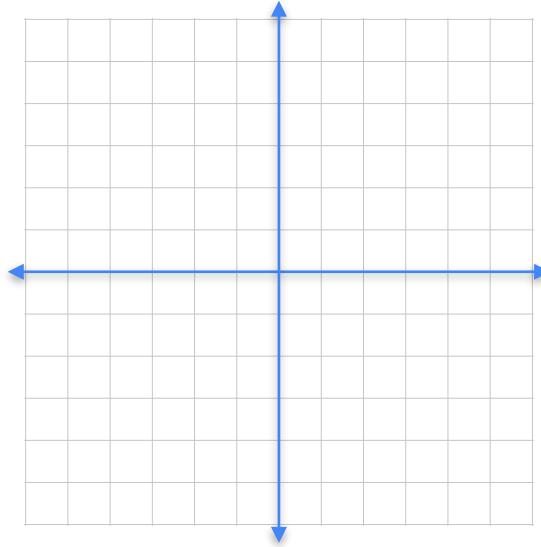
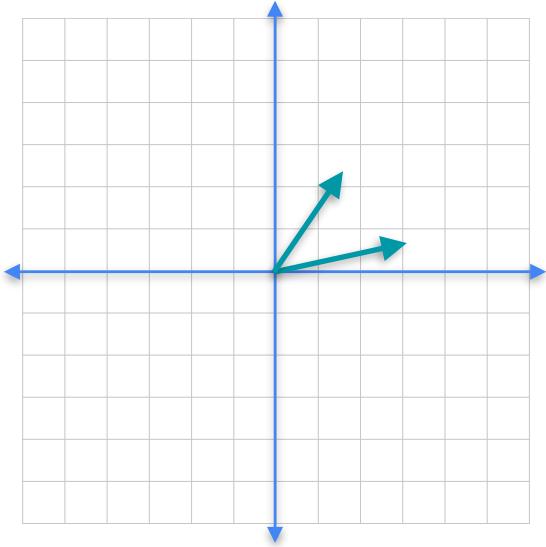


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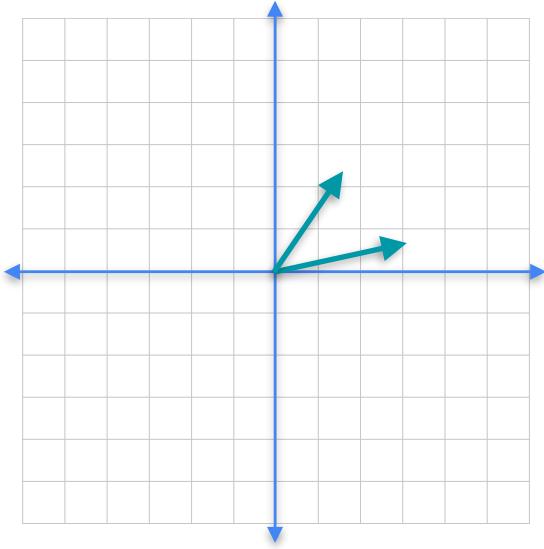
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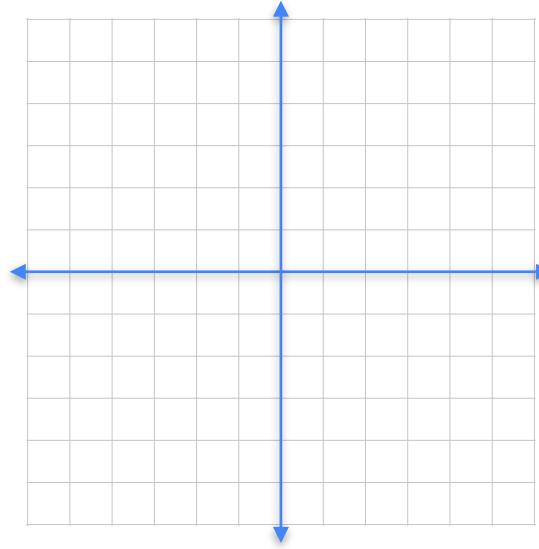
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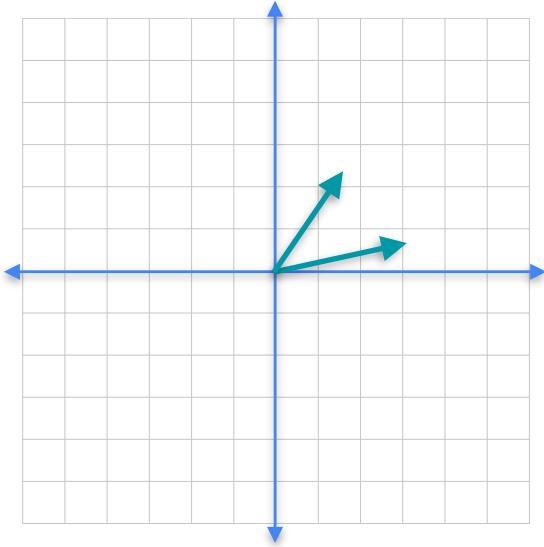
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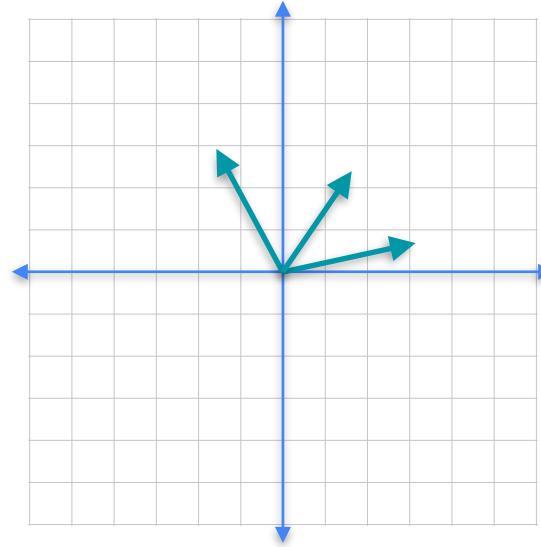
Basis



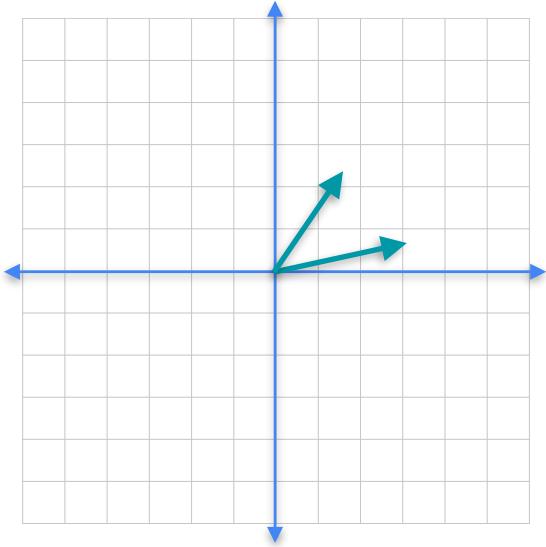
A basis is a minimal spanning set



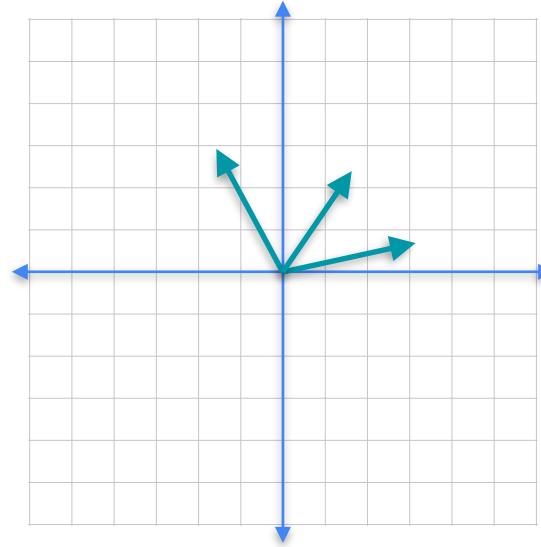
Basis



A basis is a minimal spanning set

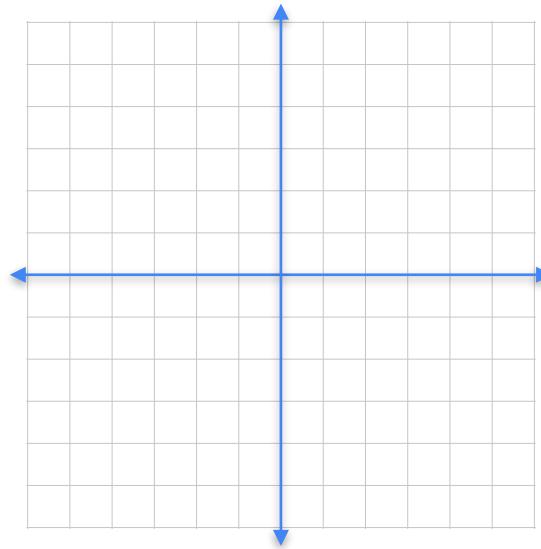
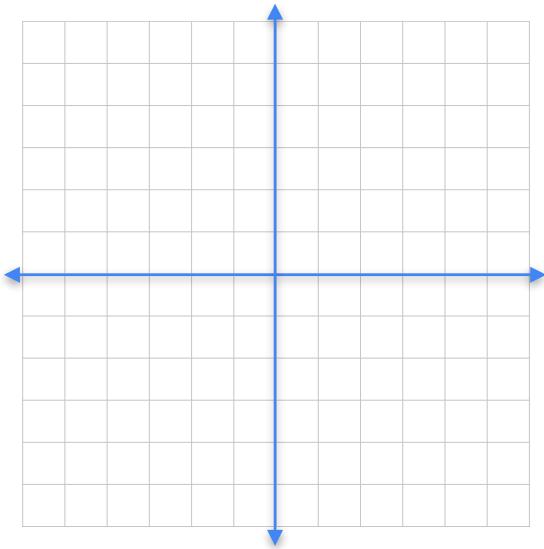


Basis

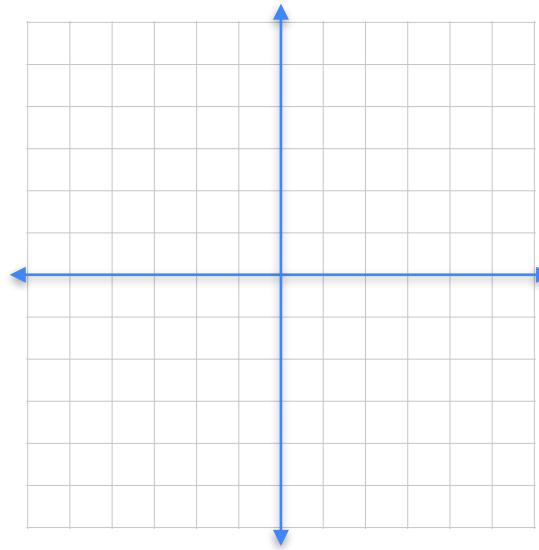
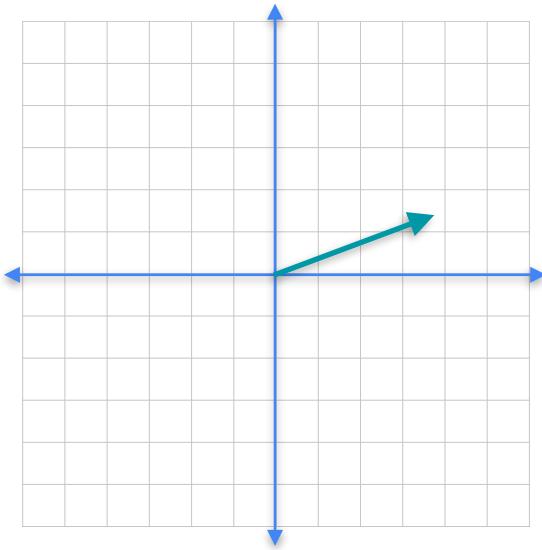


Not a basis

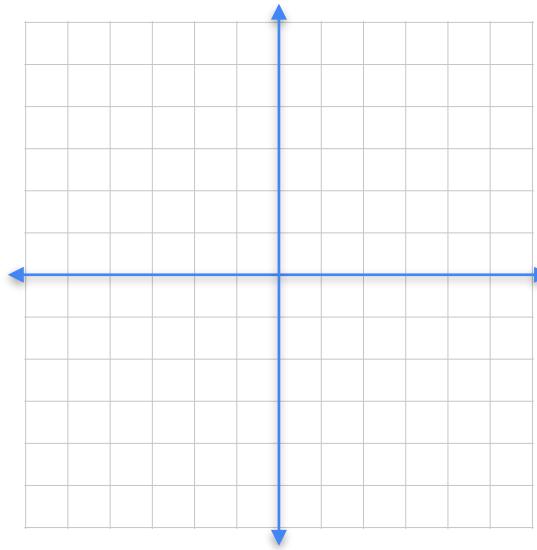
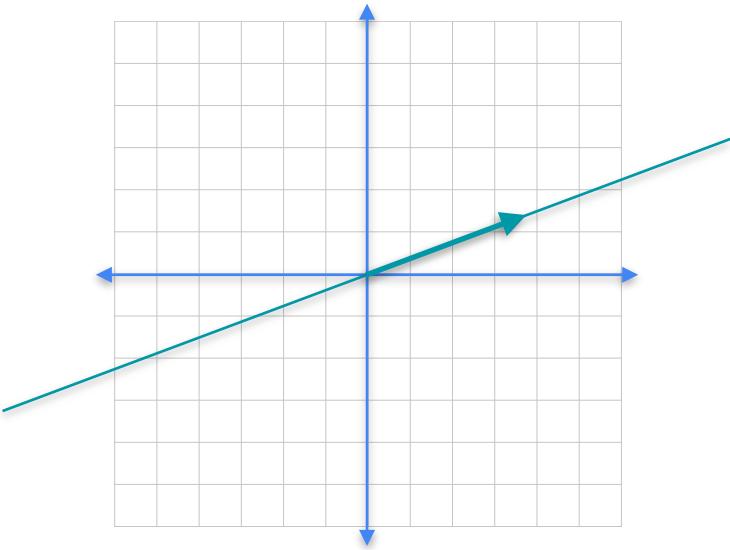
Number of elements in the basis is the dimension



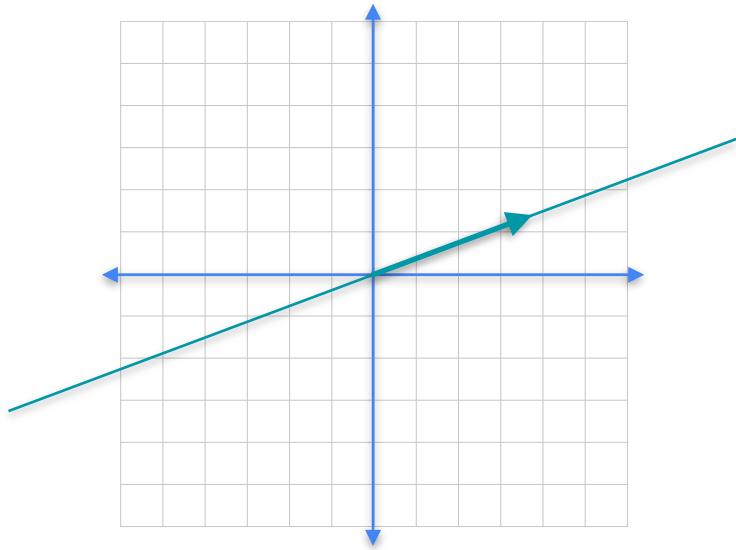
Number of elements in the basis is the dimension



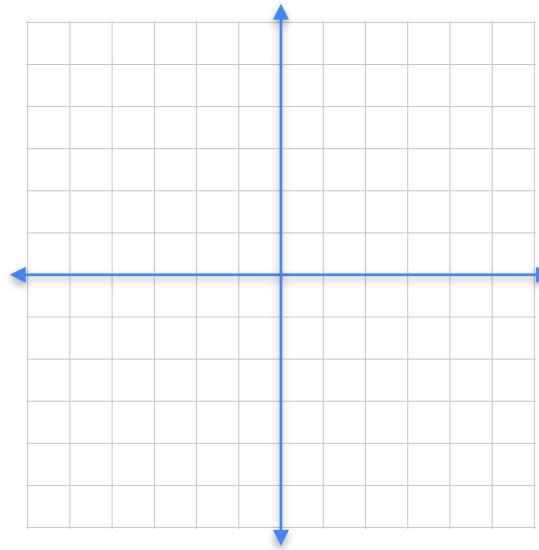
Number of elements in the basis is the dimension



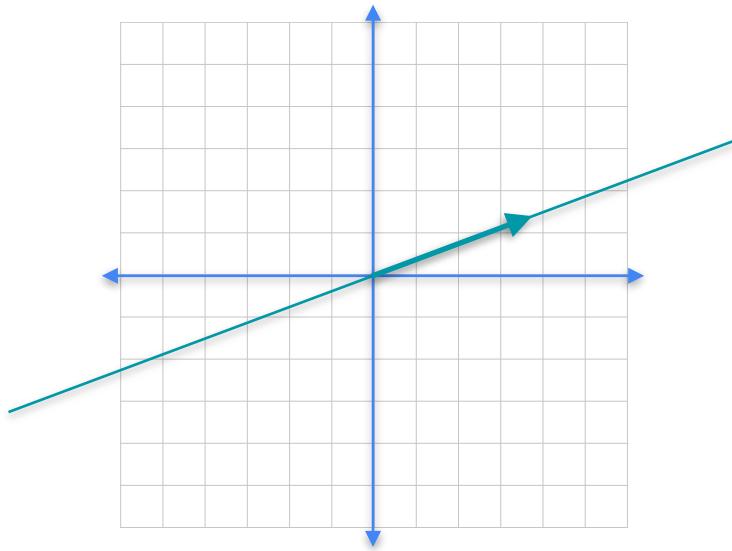
Number of elements in the basis is the dimension



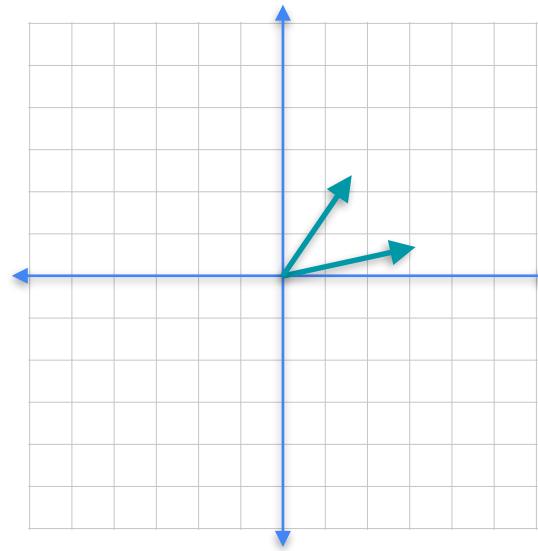
1 element
Dimension = 1



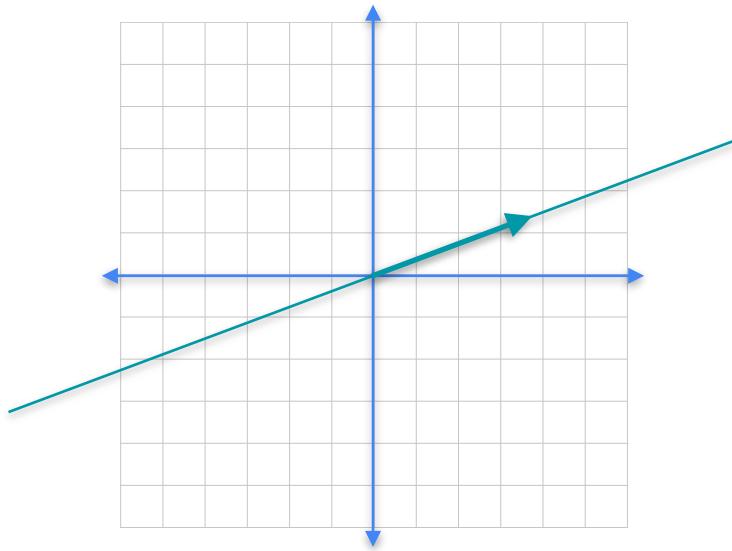
Number of elements in the basis is the dimension



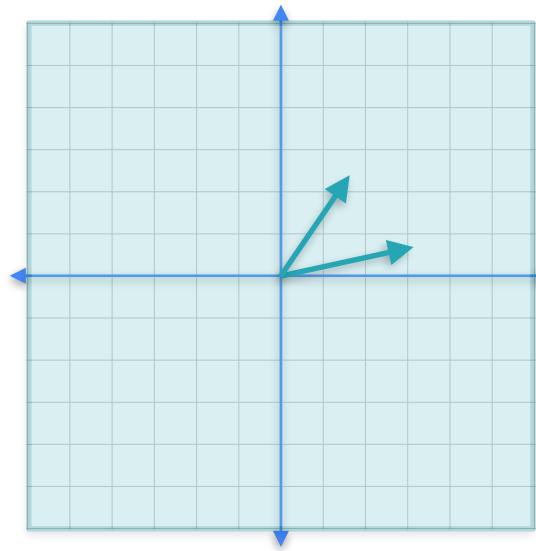
1 element
Dimension = 1



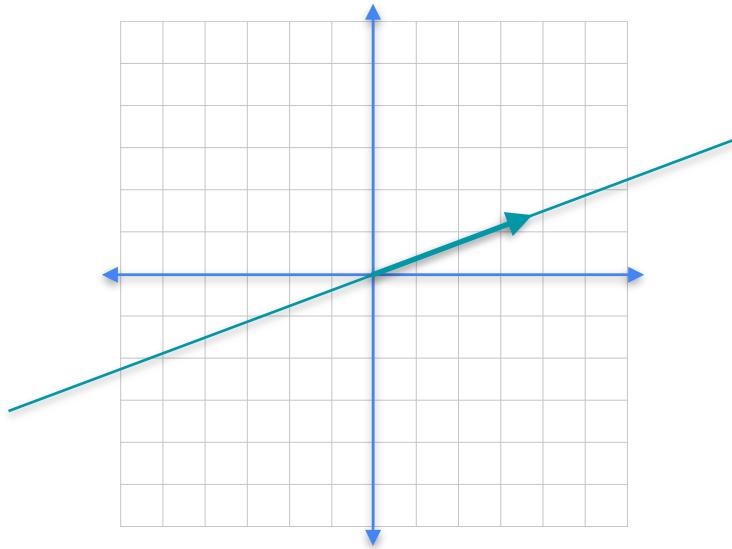
Number of elements in the basis is the dimension



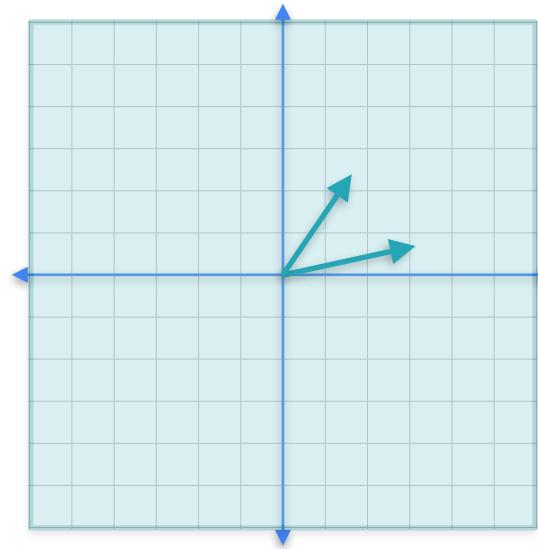
1 element
Dimension = 1



Number of elements in the basis is the dimension



1 element
Dimension = 1



2 elements in the basis
Dimension = 2

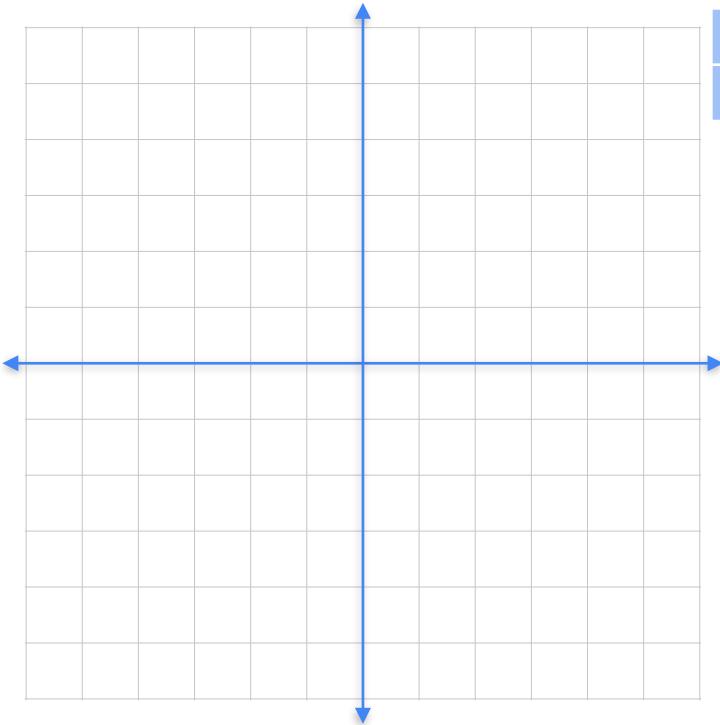


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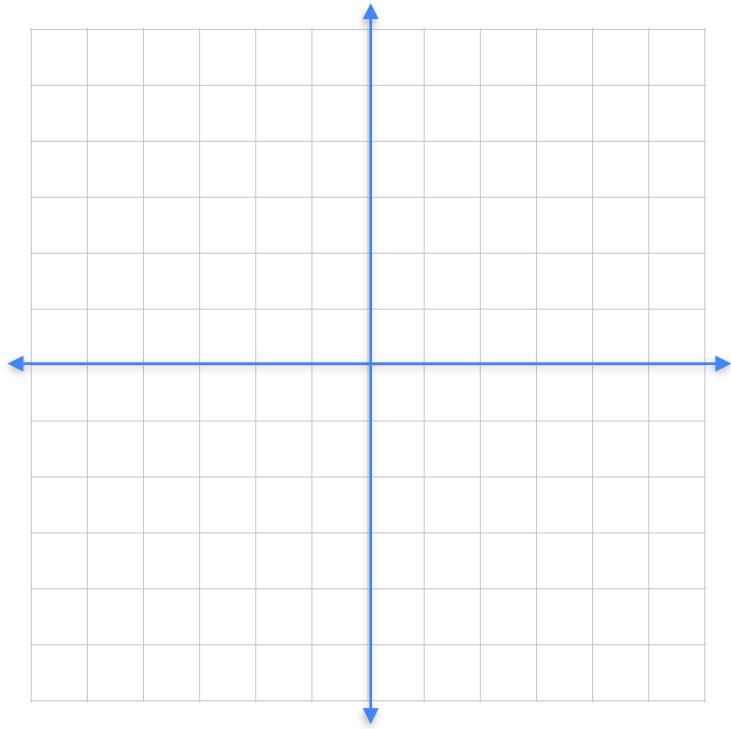
Determinants and Eigenvectors

Eigenbasis

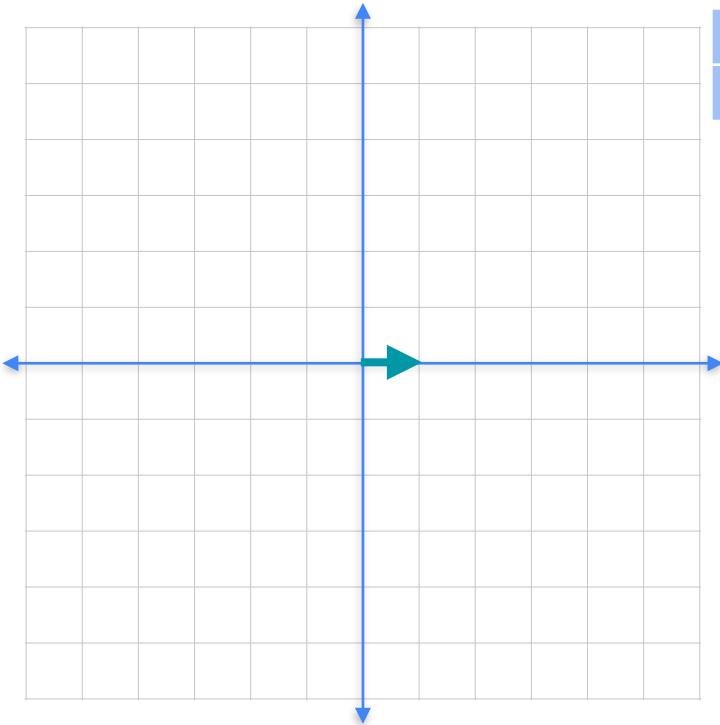
Basis



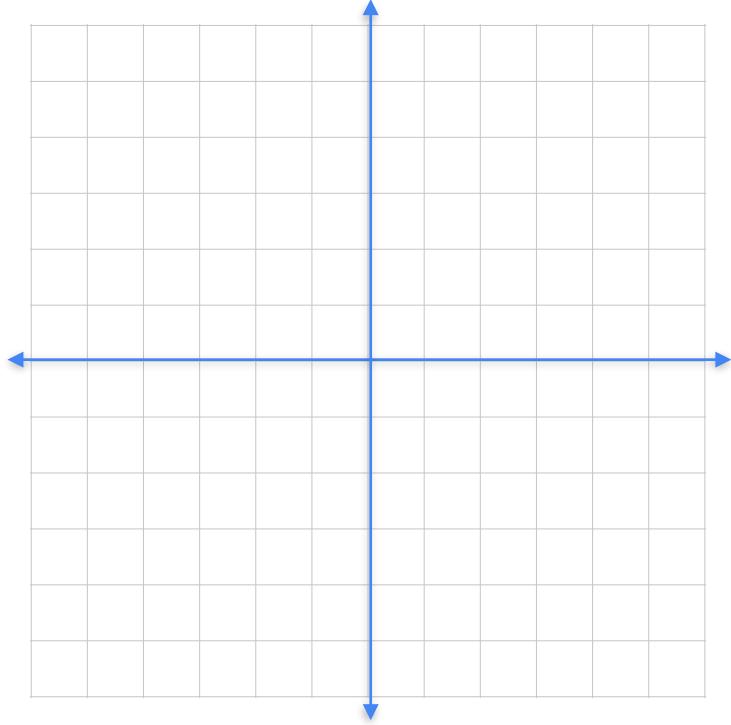
| | |
|---|---|
| 2 | 1 |
| 0 | 3 |



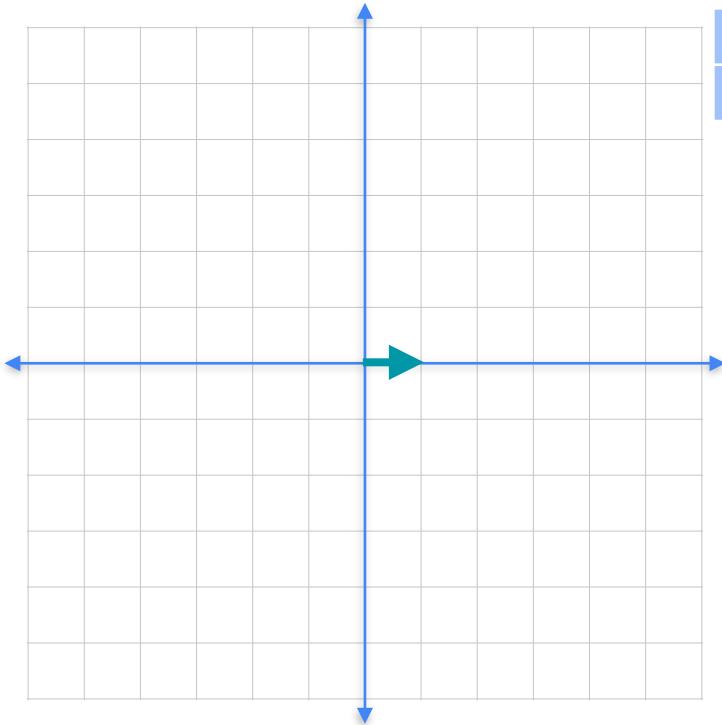
Basis



| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

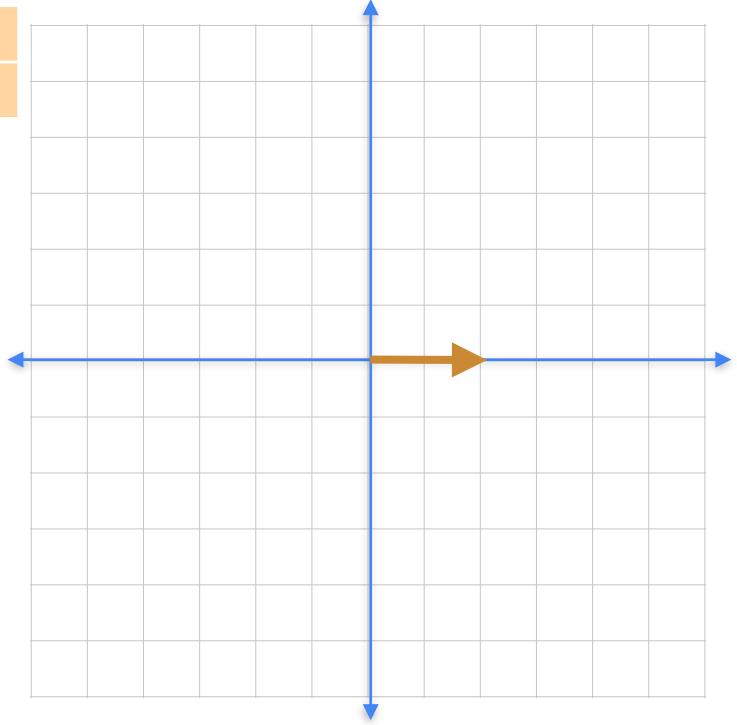


Basis

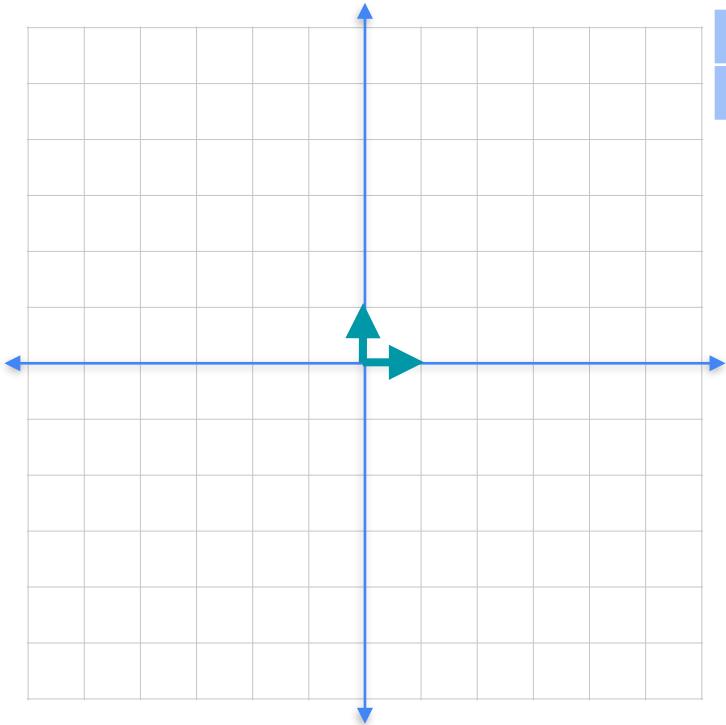


$$\begin{matrix} 2 & 1 & 1 \\ 0 & 3 & 0 \end{matrix} = \begin{matrix} 2 \\ 0 \end{matrix}$$

$$(1,0) \rightarrow (2,0)$$

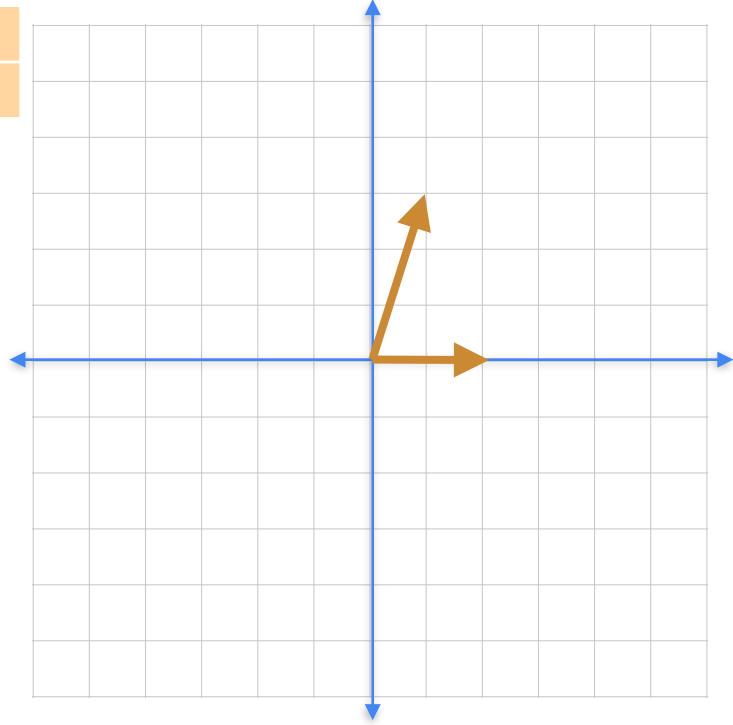


Basis

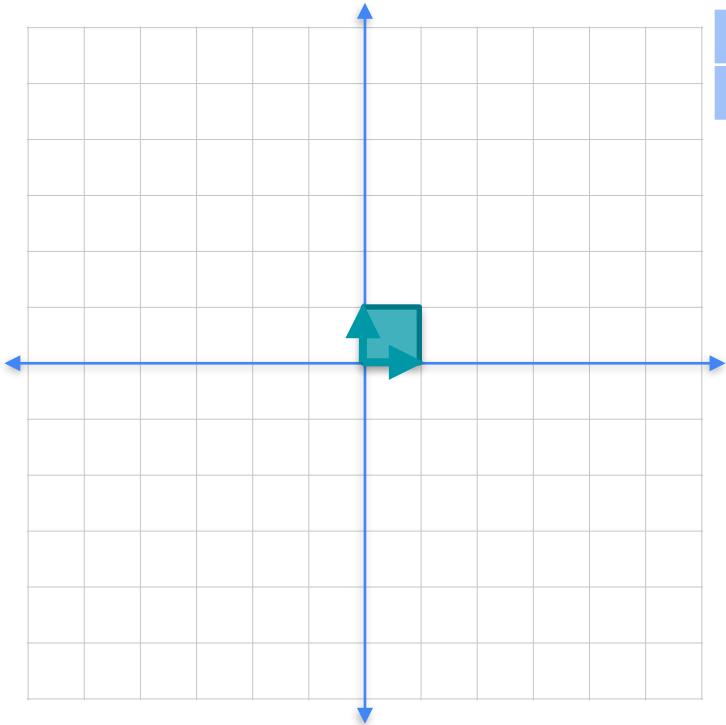


$$\begin{matrix} 2 & 1 & 0 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 1 \\ 3 \end{matrix}$$

$$\begin{aligned} (1,0) &\rightarrow (2,0) \\ (0,1) &\rightarrow (1,3) \end{aligned}$$

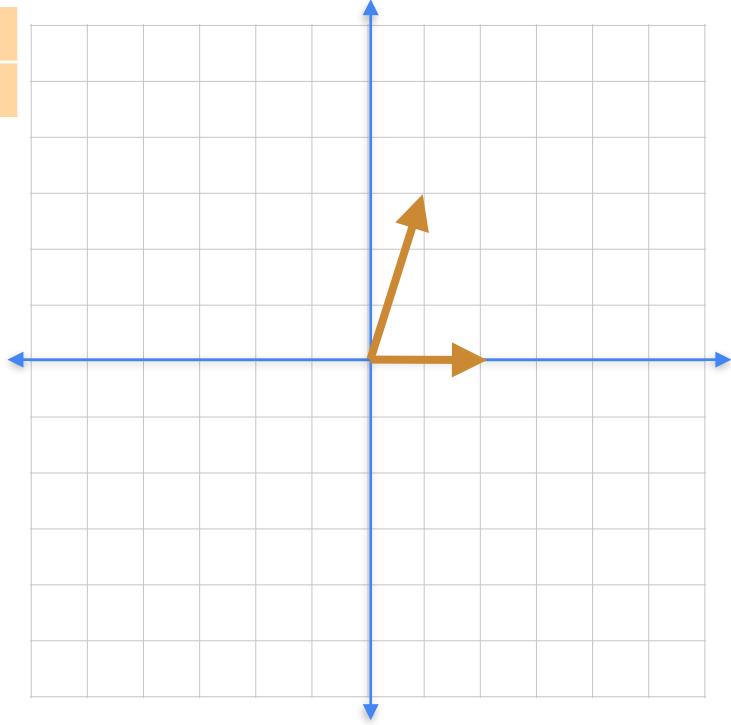


Basis

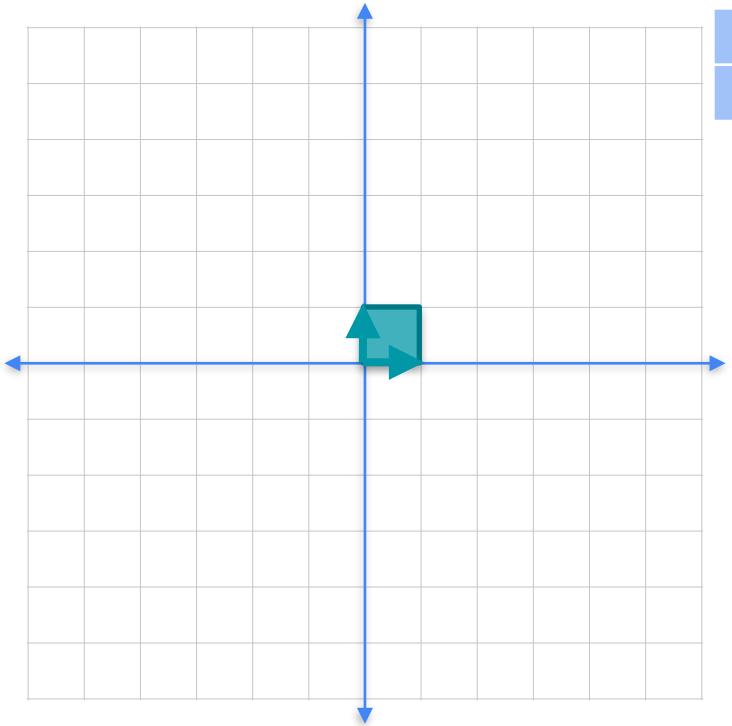


$$\begin{matrix} 2 & 1 & 0 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 1 \\ 3 \end{matrix}$$

$$\begin{aligned} (1,0) &\rightarrow (2,0) \\ (0,1) &\rightarrow (1,3) \end{aligned}$$

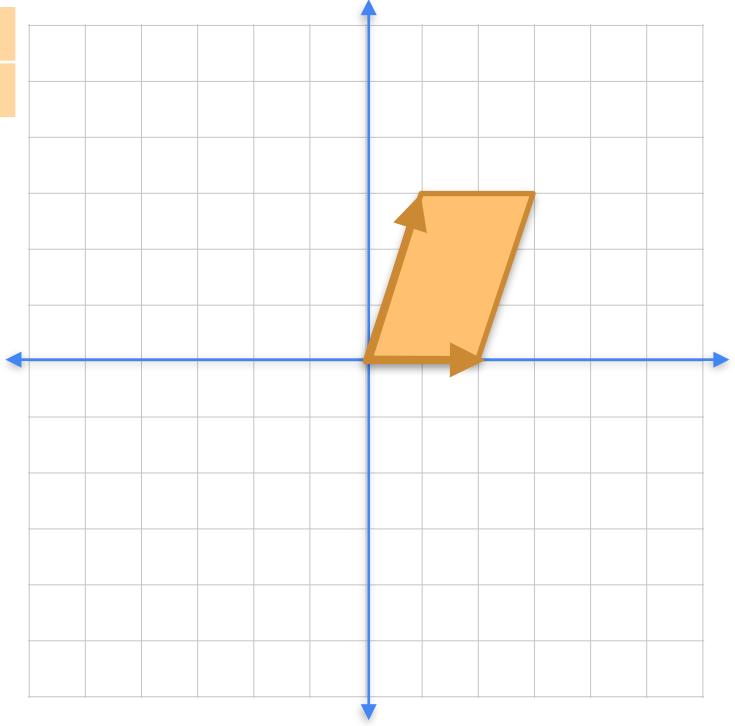


Basis

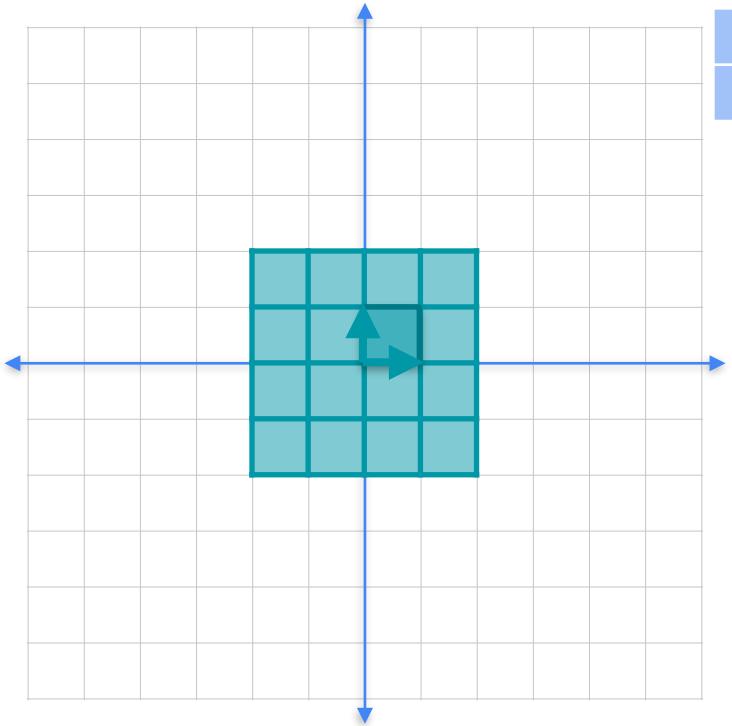


$$\begin{matrix} 2 & 1 & 0 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 1 \\ 3 \end{matrix}$$

$$\begin{aligned} (1,0) &\rightarrow (2,0) \\ (0,1) &\rightarrow (1,3) \end{aligned}$$

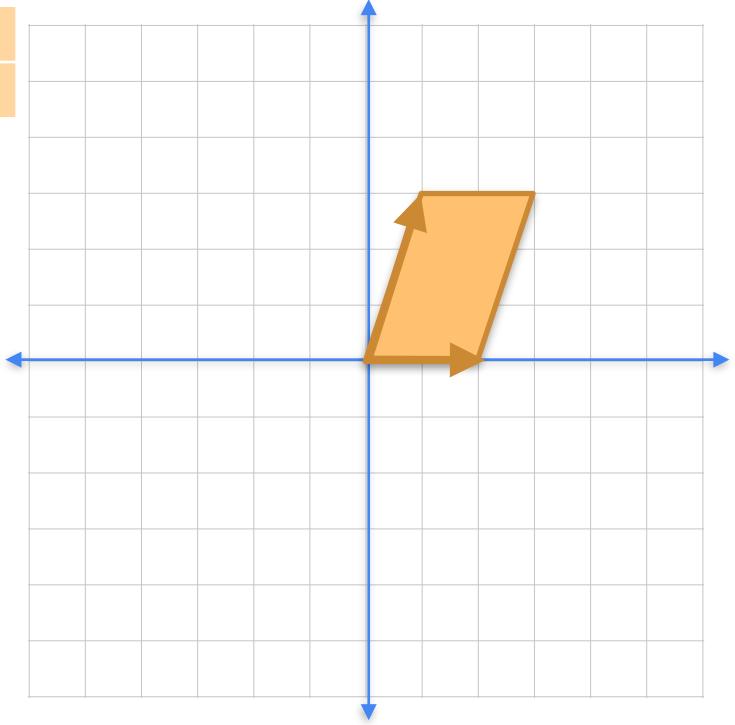


Basis

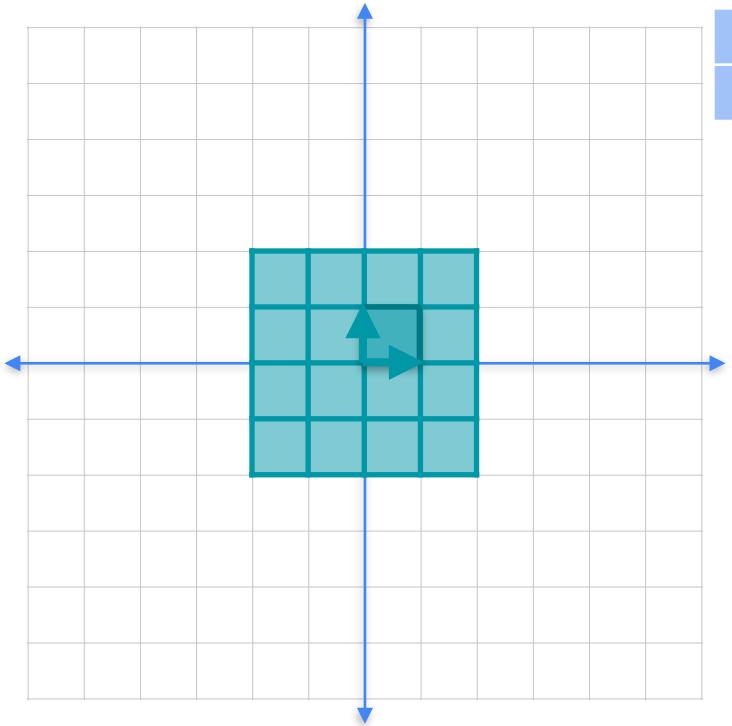


$$\begin{matrix} 2 & 1 & 0 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 1 \\ 3 \end{matrix}$$

$$\begin{aligned} (1,0) &\rightarrow (2,0) \\ (0,1) &\rightarrow (1,3) \end{aligned}$$

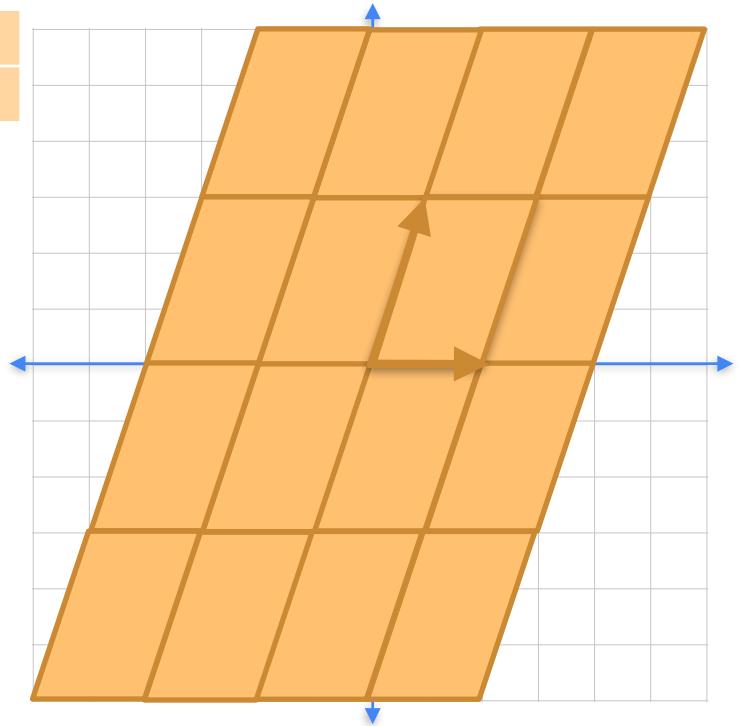


Basis

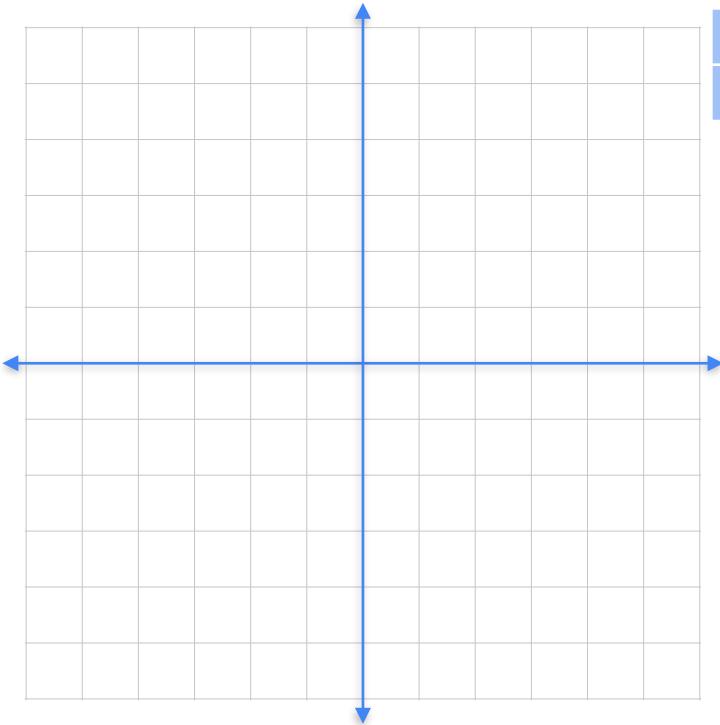


$$\begin{matrix} 2 & 1 & 0 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 1 \\ 3 \end{matrix}$$

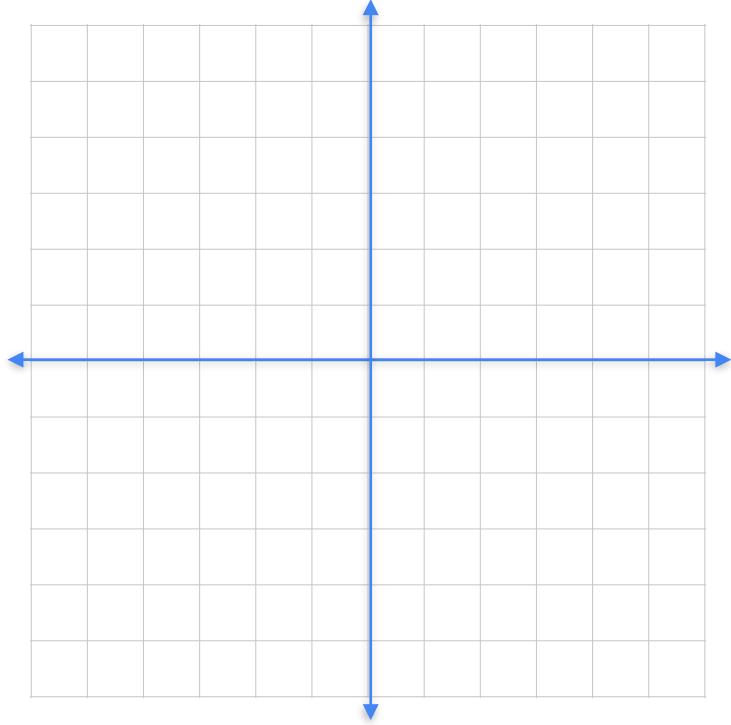
$$\begin{aligned} (1,0) &\rightarrow (2,0) \\ (0,1) &\rightarrow (1,3) \end{aligned}$$



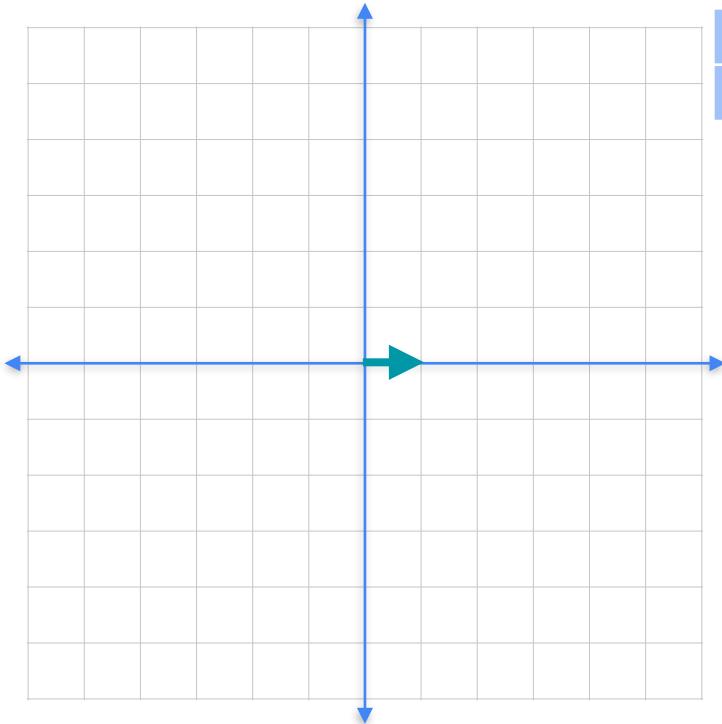
Eigenbasis



| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

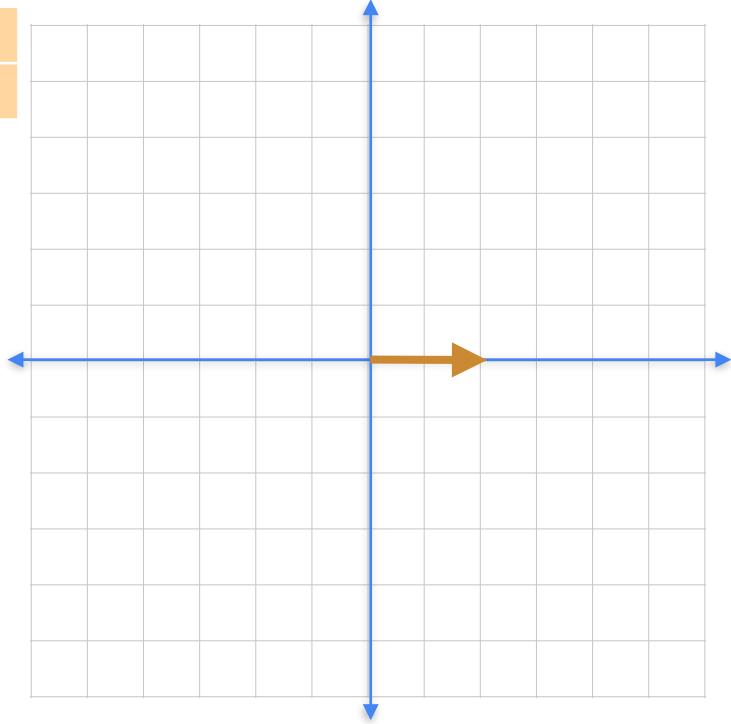


Eigenbasis

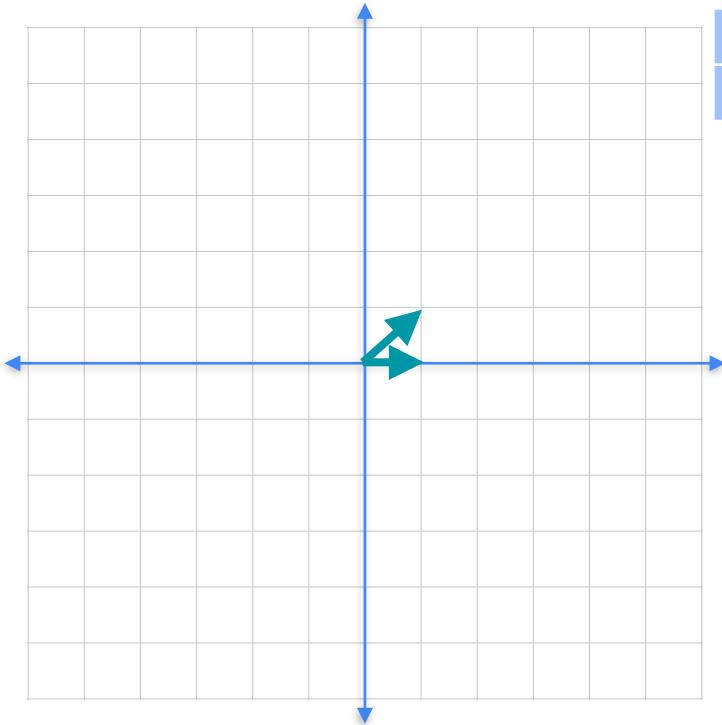


$$\begin{matrix} 2 & 1 \\ 0 & 3 \end{matrix} \begin{matrix} 1 \\ 0 \end{matrix} = \begin{matrix} 2 \\ 0 \end{matrix}$$

$$(1,0) \rightarrow (2,0)$$

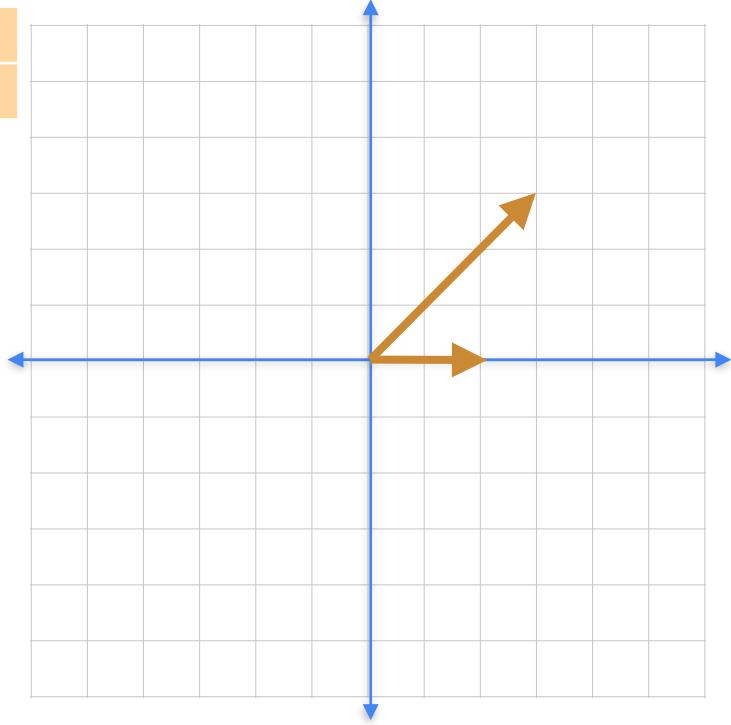


Eigenbasis

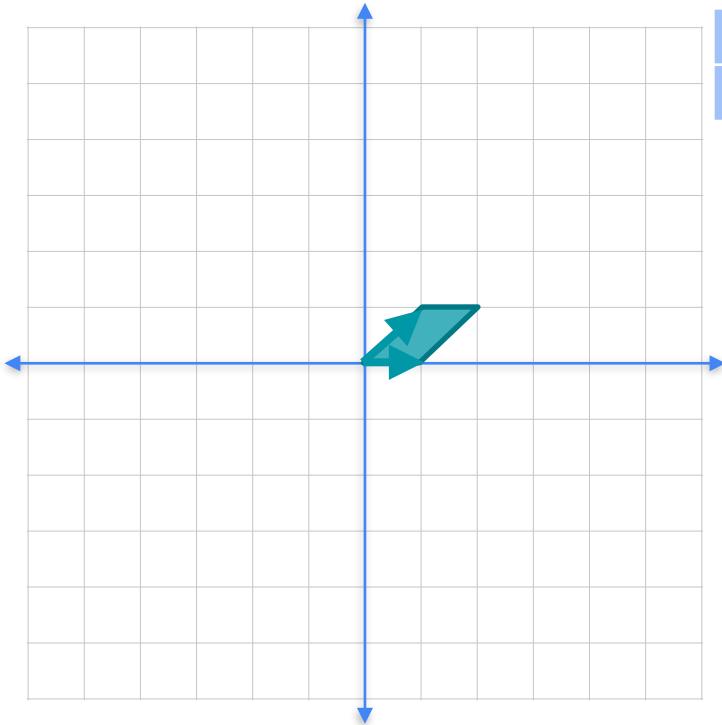


$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$\begin{aligned}(1,0) &\rightarrow (2,0) \\ (1,1) &\rightarrow (3,3)\end{aligned}$$

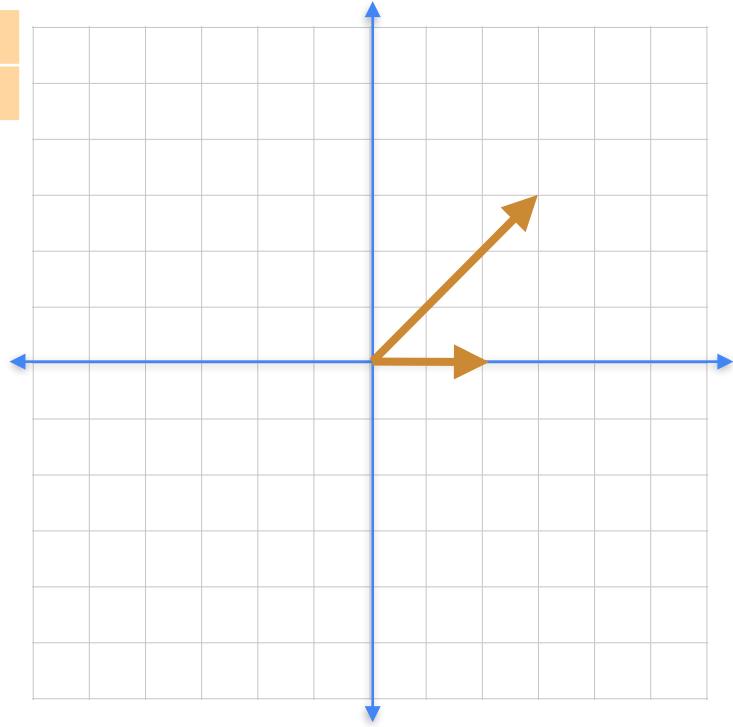


Eigenbasis

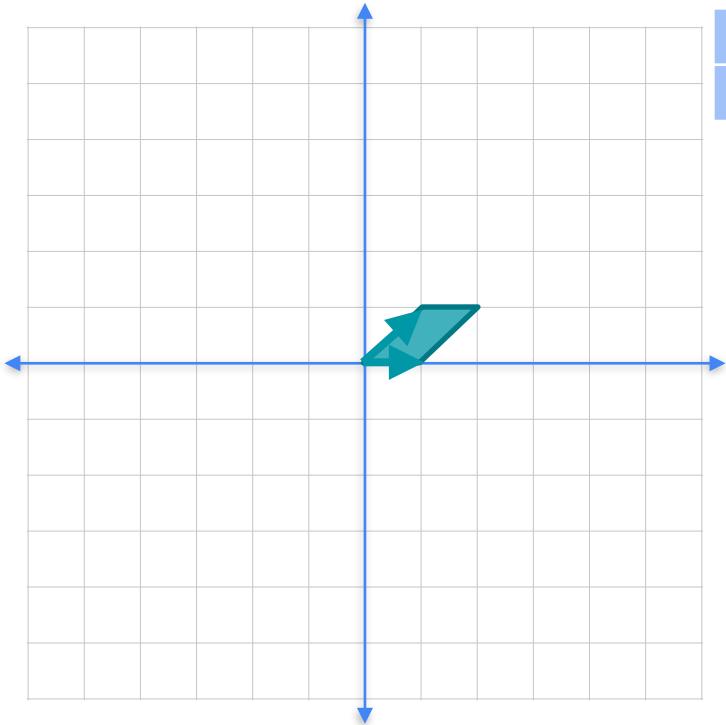


$$\begin{matrix} 2 & 1 & 1 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 3 & \\ 3 & \end{matrix}$$

$$\begin{aligned} (1,0) &\rightarrow (2,0) \\ (1,1) &\rightarrow (3,3) \end{aligned}$$

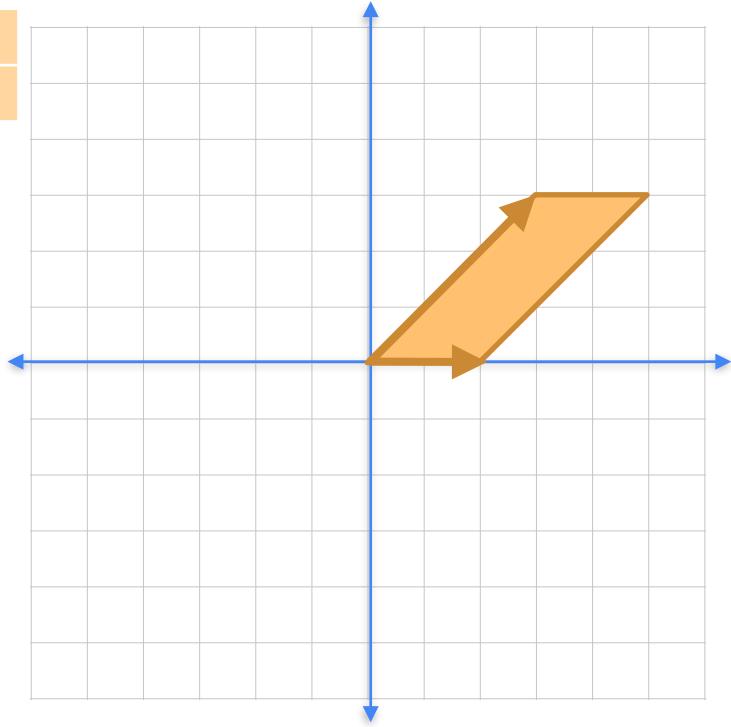


Eigenbasis

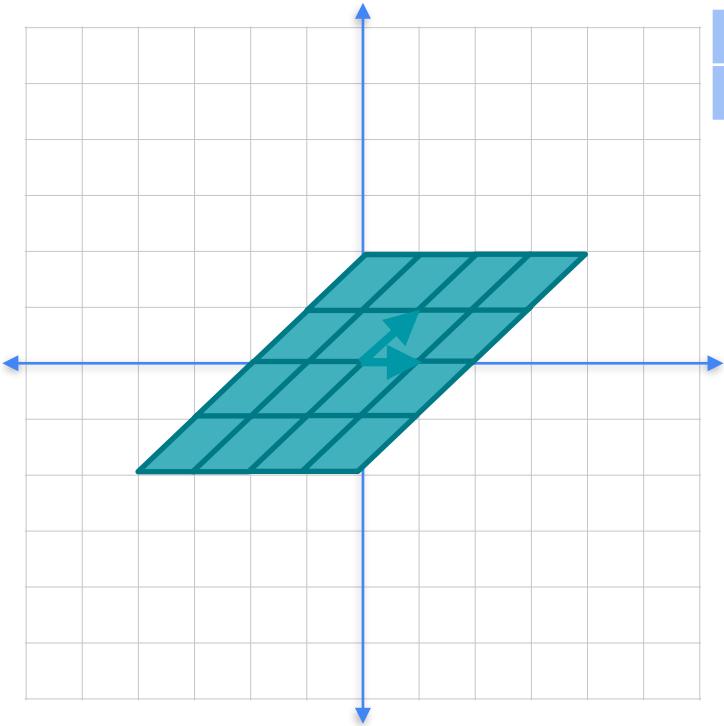


$$\begin{matrix} 2 & 1 & 1 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 3 \\ 3 \end{matrix}$$

$$\begin{aligned} (1,0) &\rightarrow (2,0) \\ (1,1) &\rightarrow (3,3) \end{aligned}$$

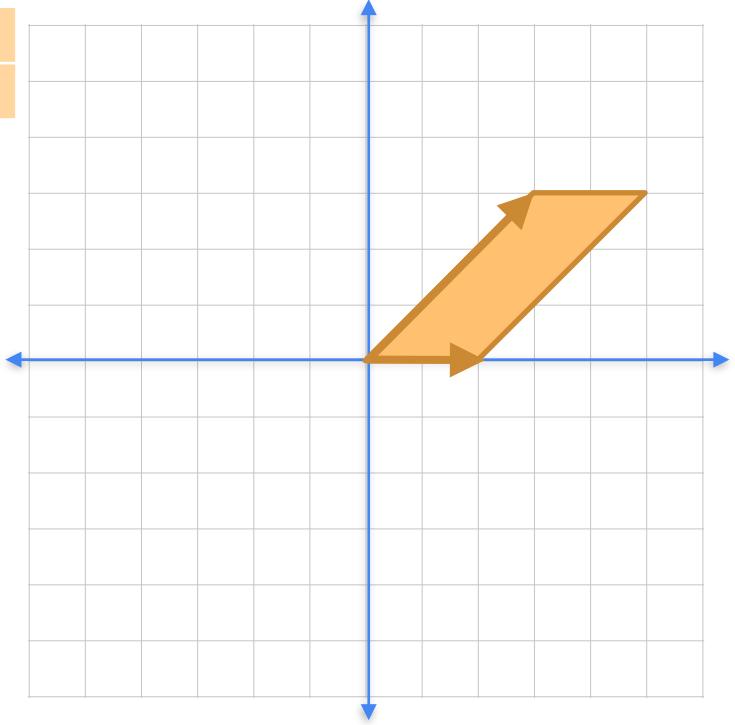


Eigenbasis

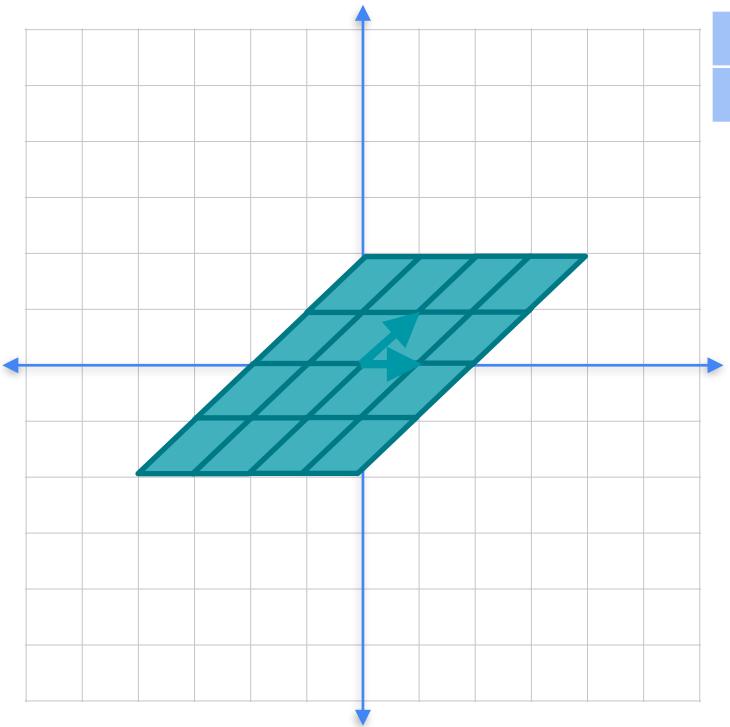


$$\begin{matrix} 2 & 1 & 1 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 3 \\ 3 \end{matrix}$$

$$\begin{aligned} (1,0) &\rightarrow (2,0) \\ (1,1) &\rightarrow (3,3) \end{aligned}$$

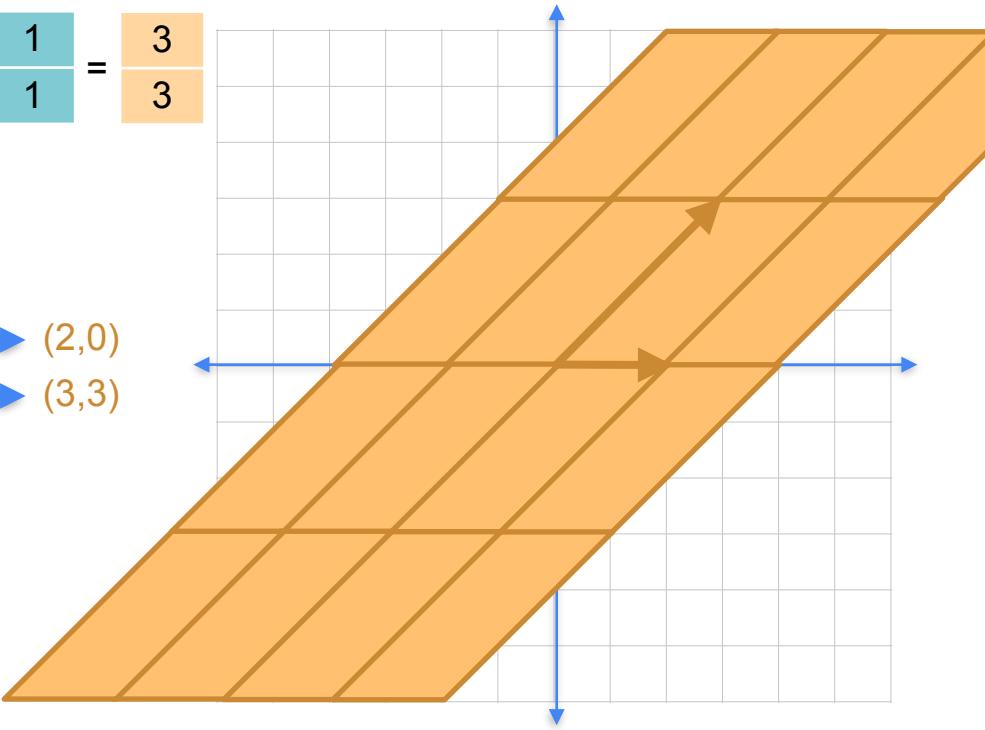


Eigenbasis

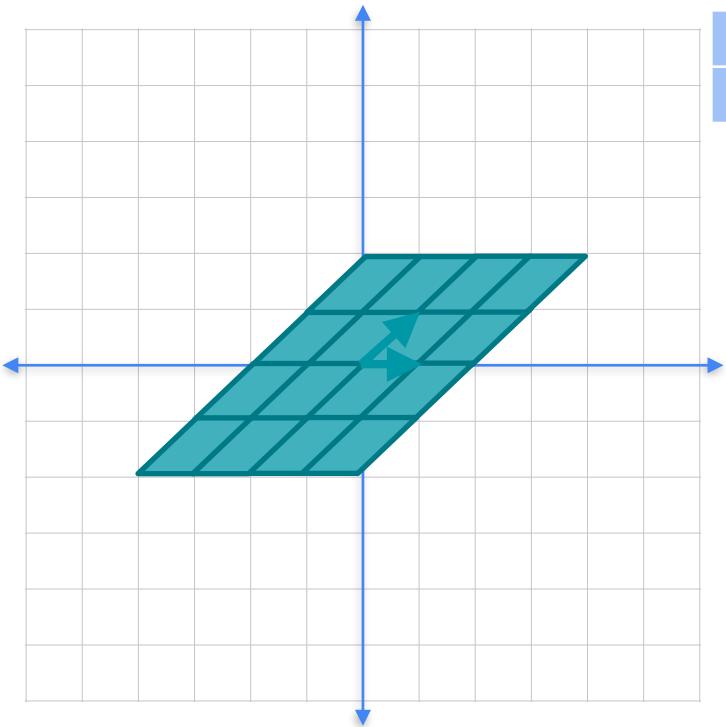


$$\begin{matrix} 2 & 1 & 1 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 3 \\ 3 \end{matrix}$$

$$\begin{aligned} (1,0) &\rightarrow (2,0) \\ (1,1) &\rightarrow (3,3) \end{aligned}$$

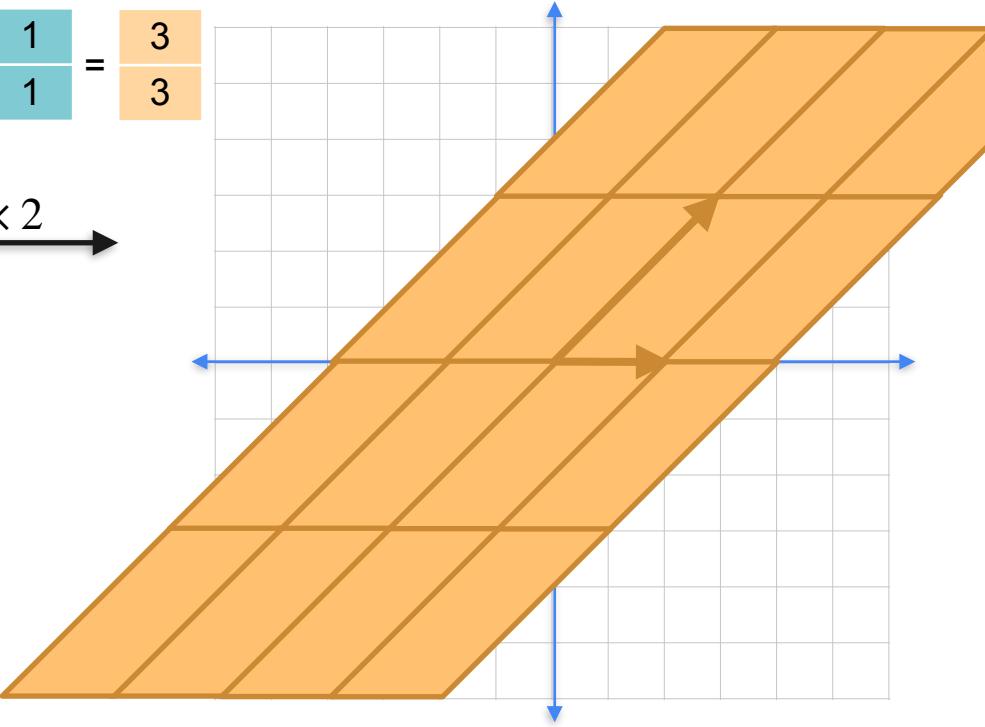


Eigenbasis

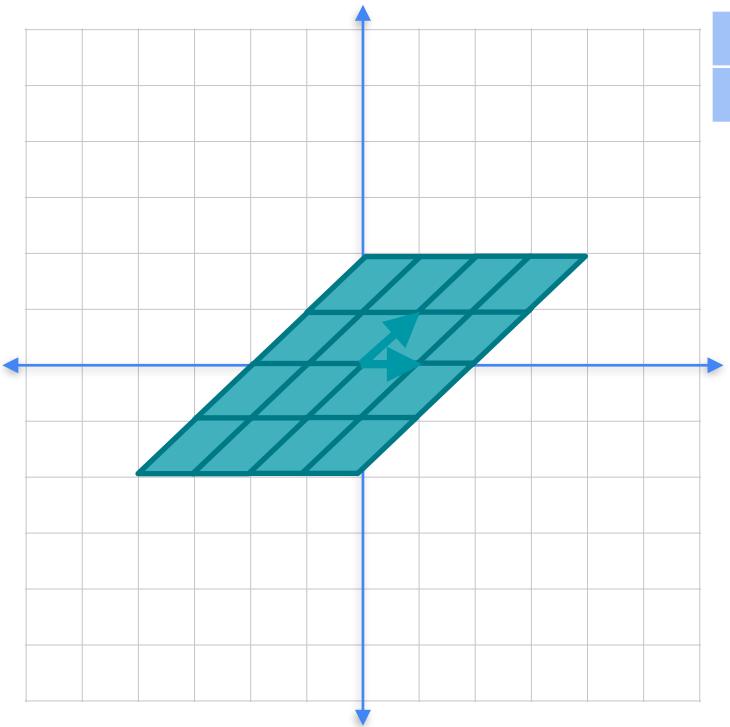


$$\begin{matrix} 2 & 1 & 1 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 3 & 3 \\ 3 & 3 \end{matrix}$$

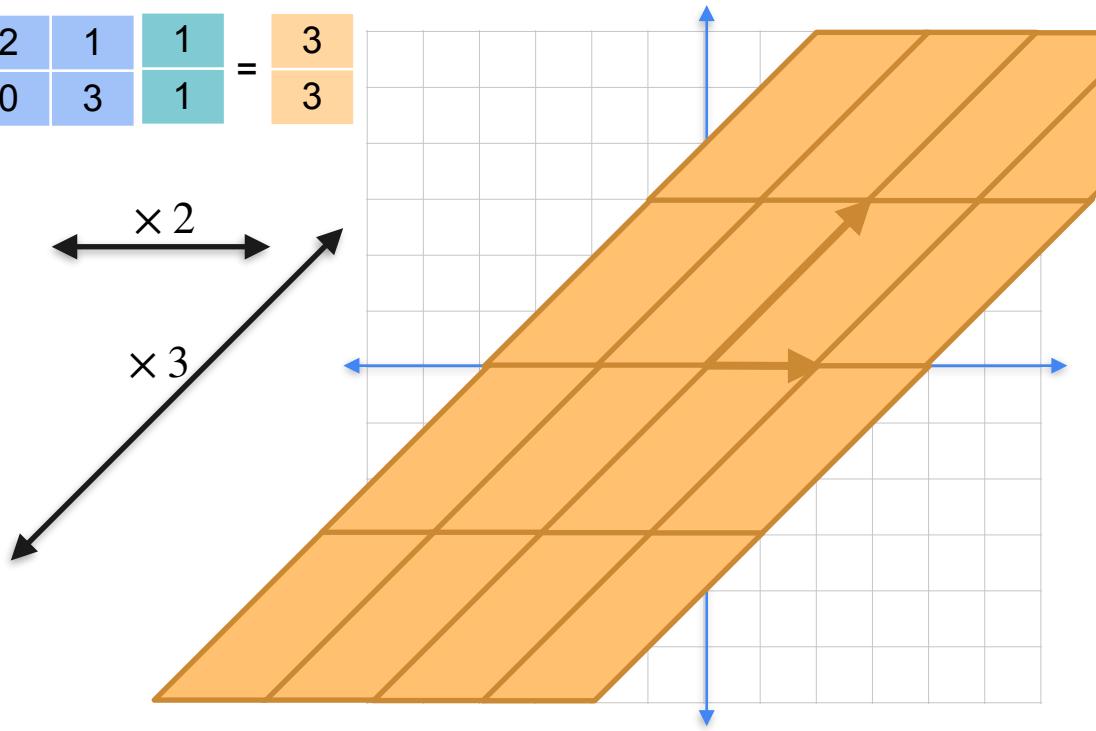
$\times 2$



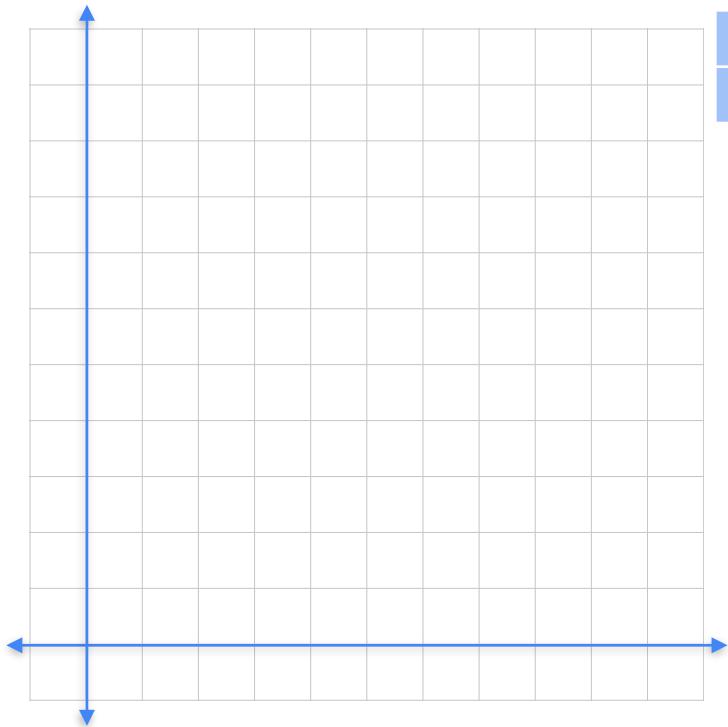
Eigenbasis



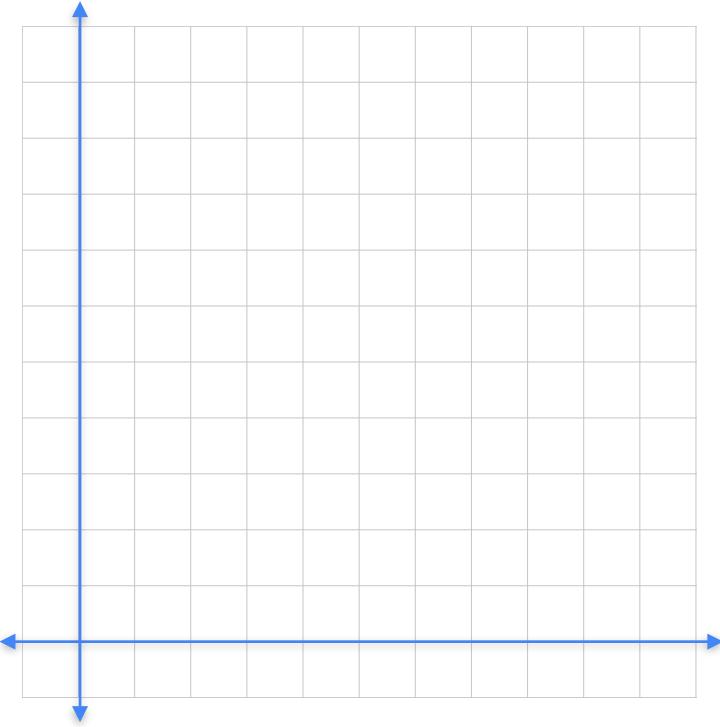
$$\begin{matrix} 2 & 1 & 1 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 3 & \\ 3 & \end{matrix}$$



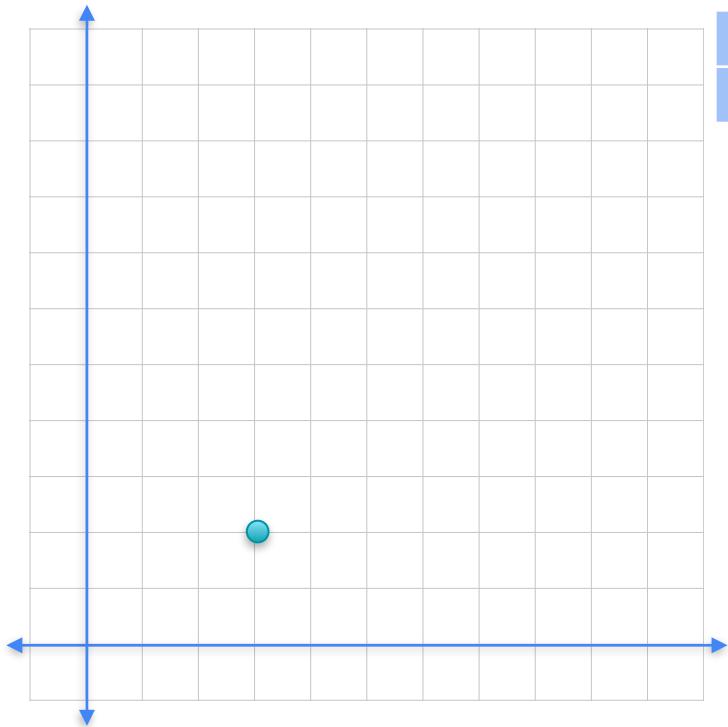
Eigenbasis



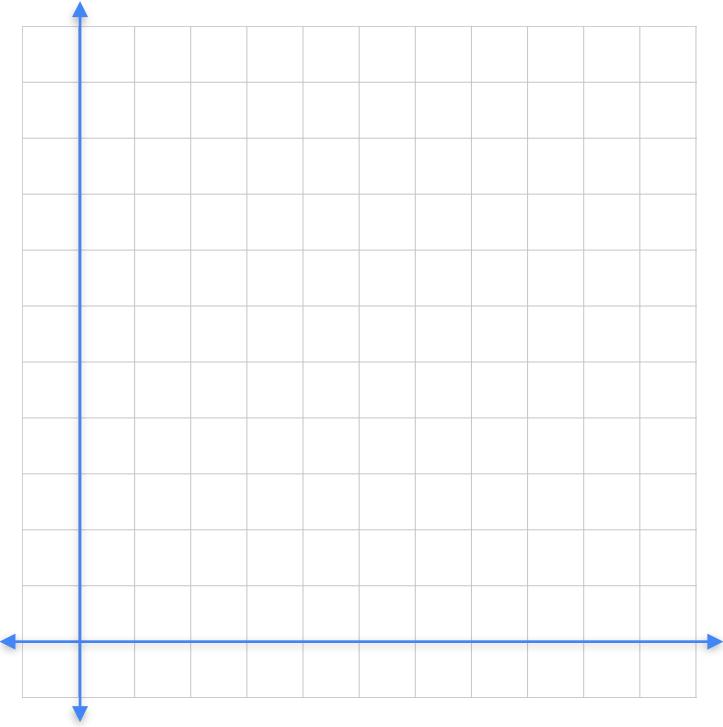
| | |
|---|---|
| 2 | 1 |
| 0 | 3 |



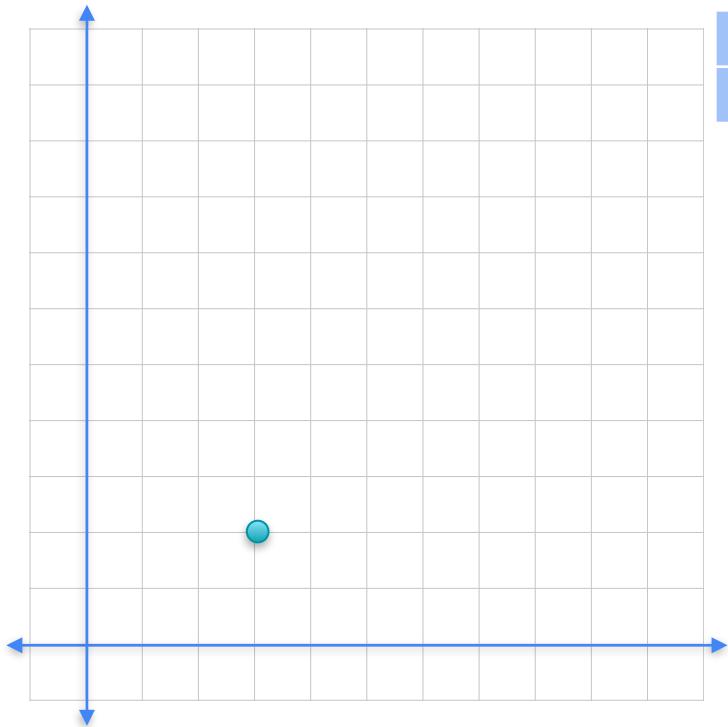
Eigenbasis



| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

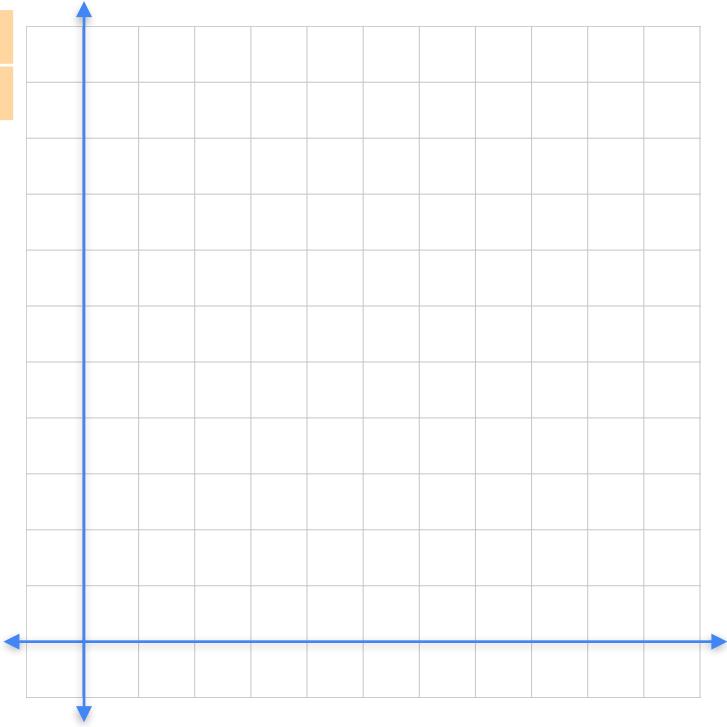


Eigenbasis

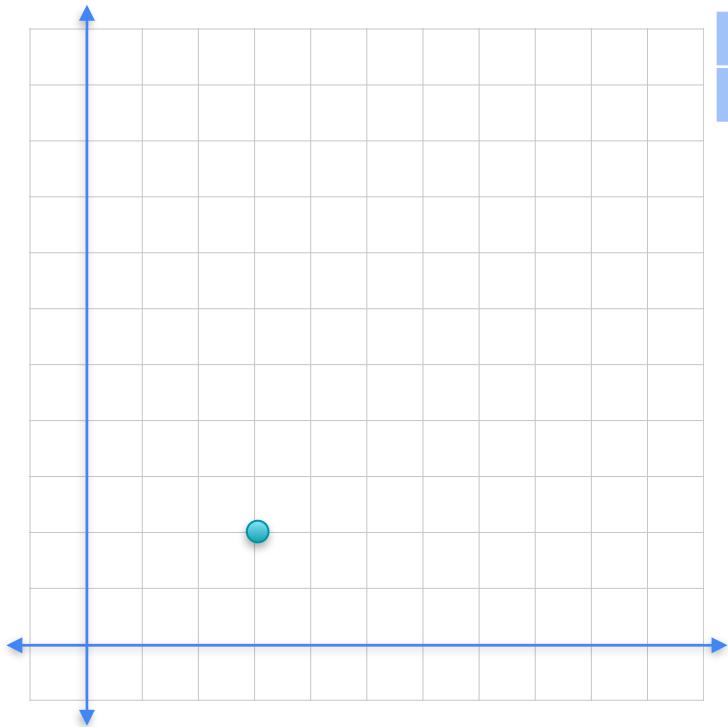


$$\begin{pmatrix} 2 & 1 & 3 \\ 0 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$$

$(3,2) \rightarrow (8,6)$

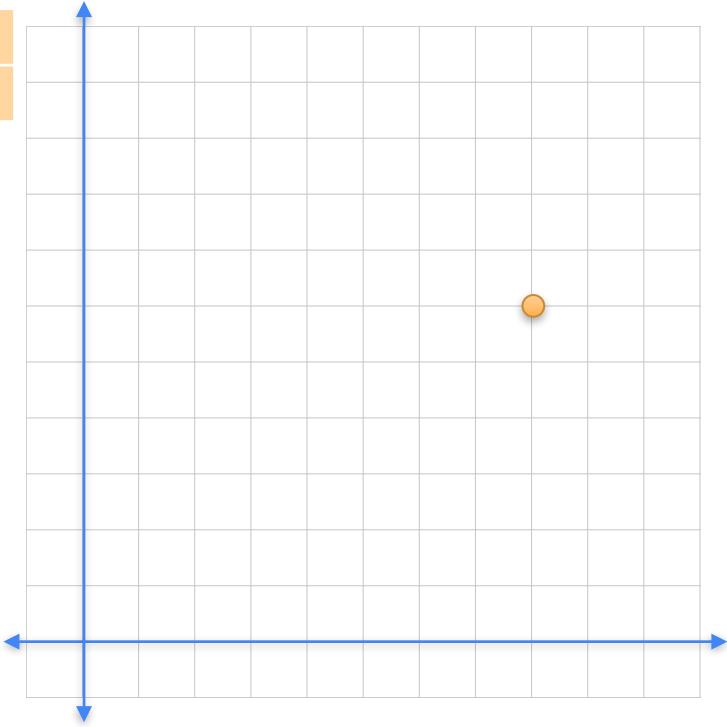


Eigenbasis

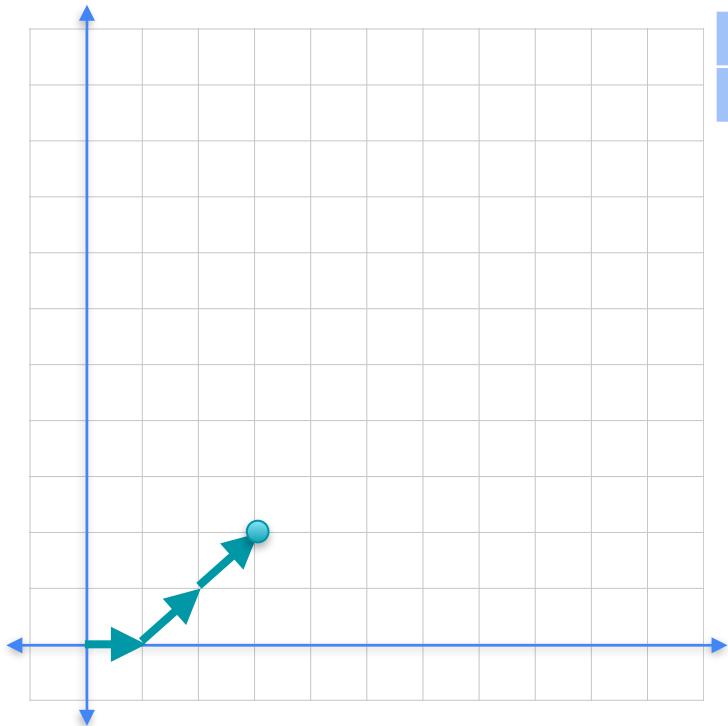


$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

$(3,2) \rightarrow (8,6)$

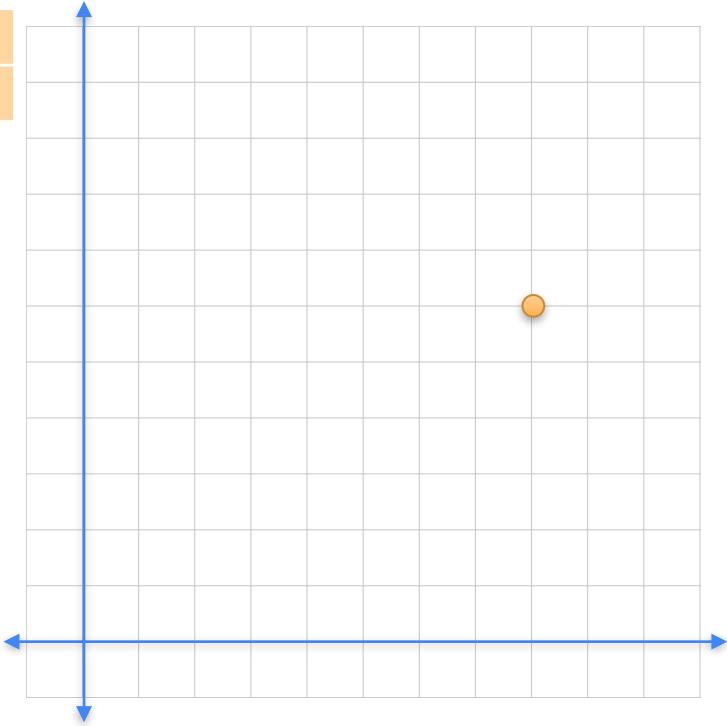


Eigenbasis

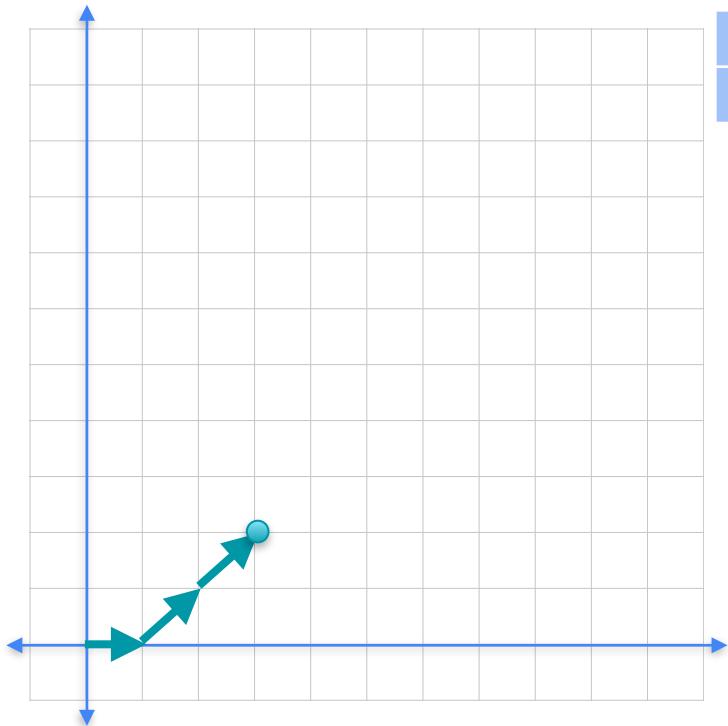


$$\begin{matrix} 2 & 1 & 3 \\ 0 & 3 & 2 \end{matrix} = \begin{matrix} 8 \\ 6 \end{matrix}$$

$(3,2) \rightarrow (8,6)$

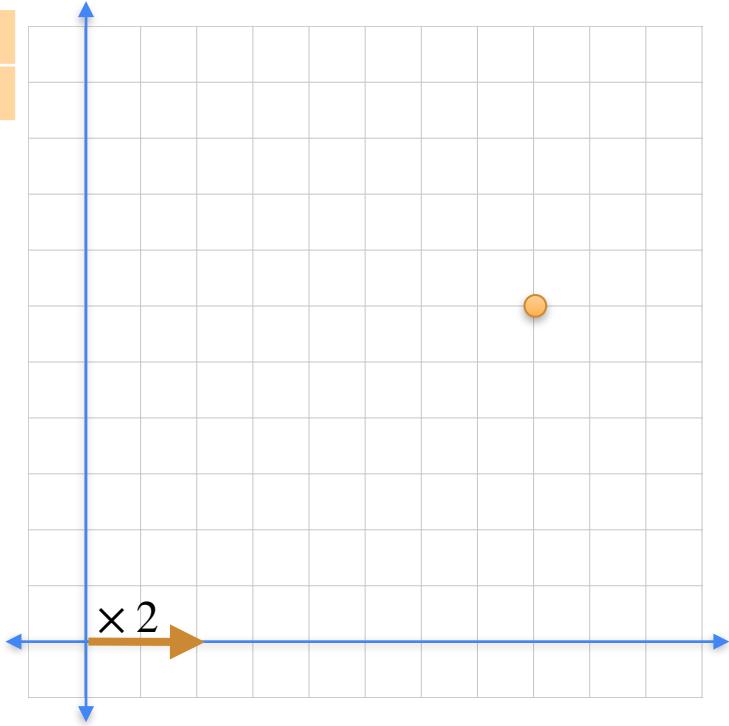


Eigenbasis

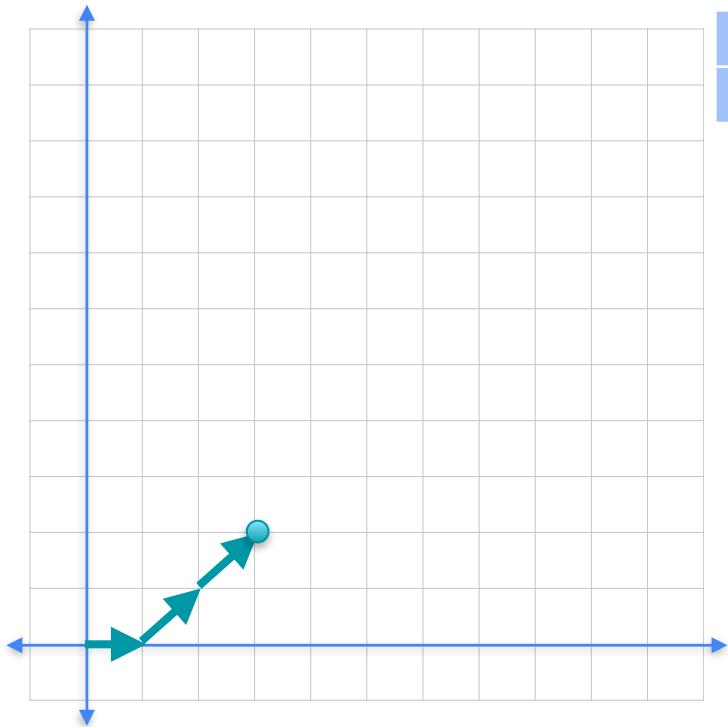


$$\begin{matrix} 2 & 1 & 3 \\ 0 & 3 & 2 \end{matrix} = \begin{matrix} 8 \\ 6 \end{matrix}$$

$(3,2) \rightarrow (8,6)$

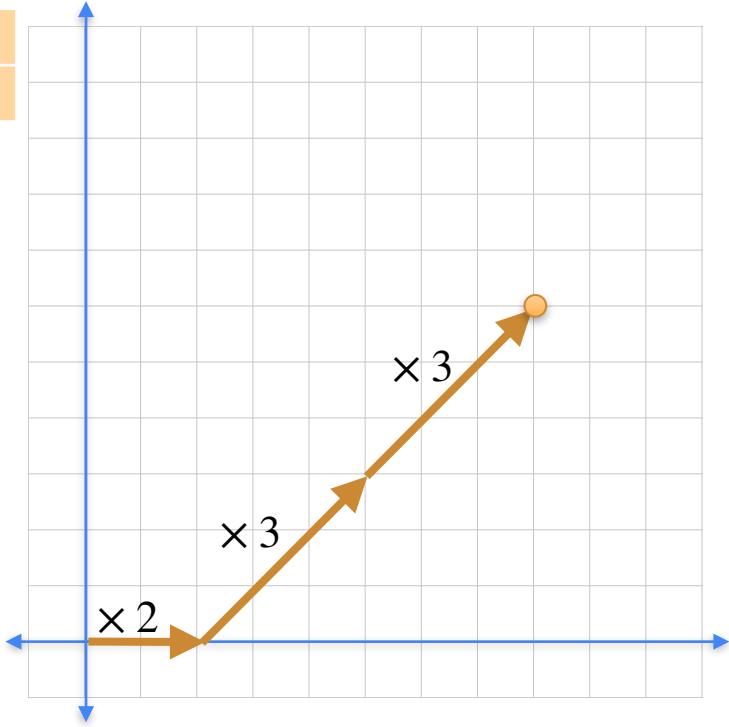


Eigenbasis



$$\begin{matrix} 2 & 1 & 3 \\ 0 & 3 & 2 \end{matrix} = \begin{matrix} 8 \\ 6 \end{matrix}$$

$$(3,2) \rightarrow (8,6)$$



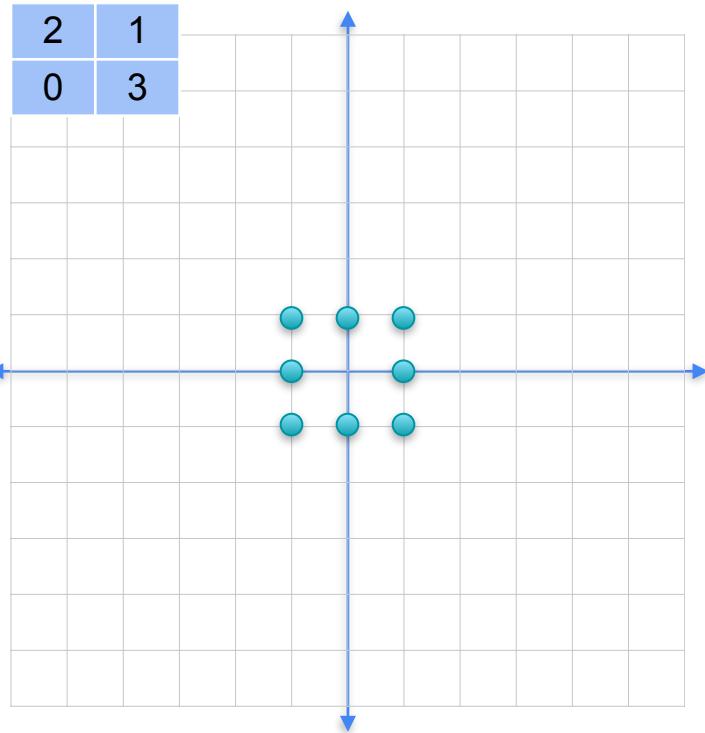


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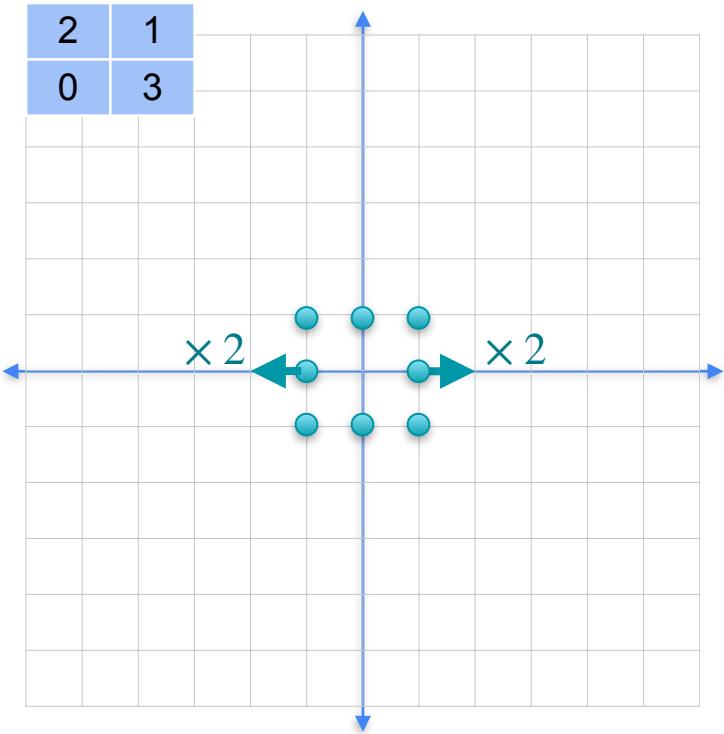
Determinants and Eigenvectors

Eigenvalues and eigenvectors

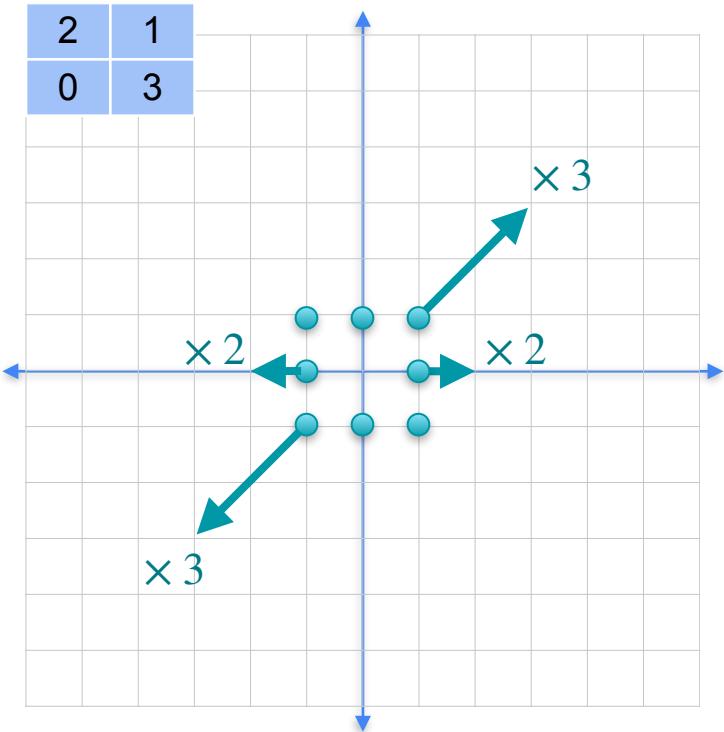
Finding eigenvalues



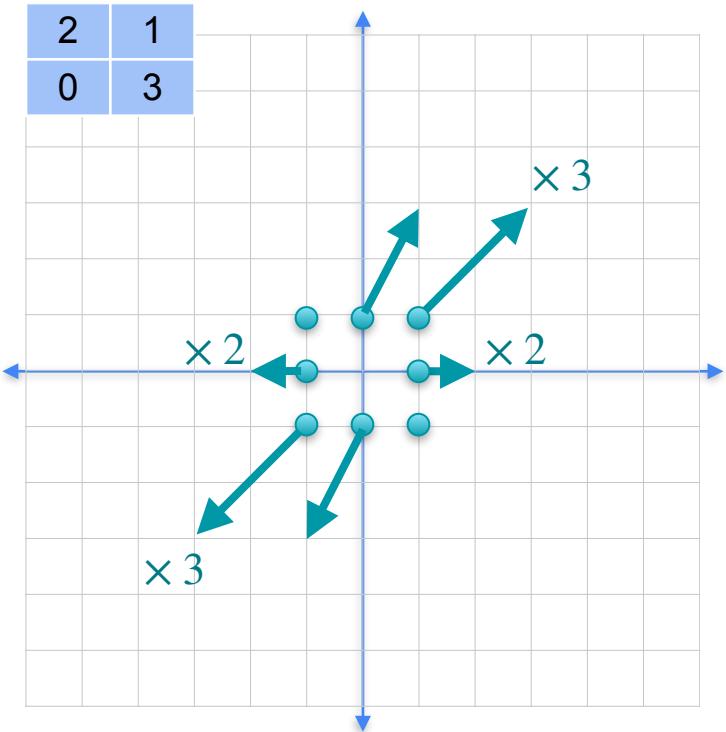
Finding eigenvalues



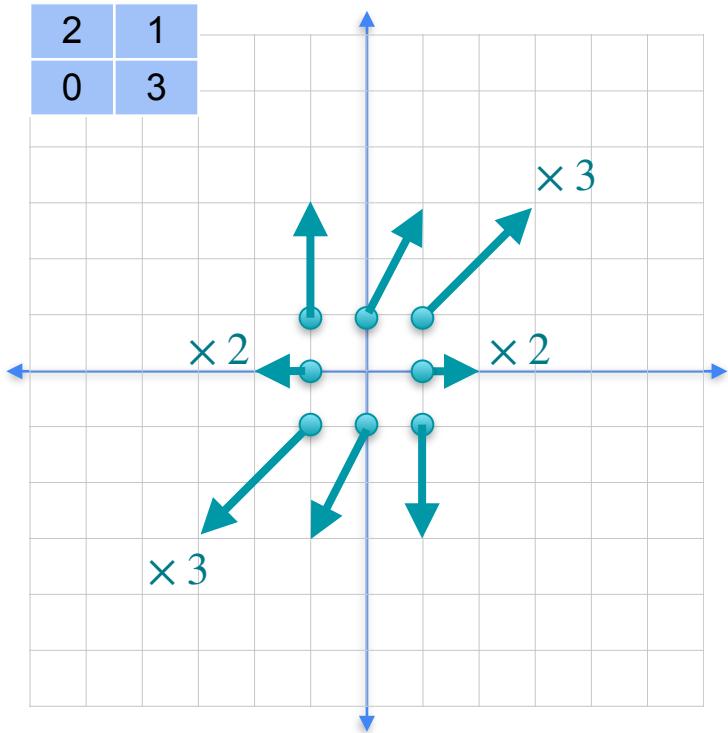
Finding eigenvalues



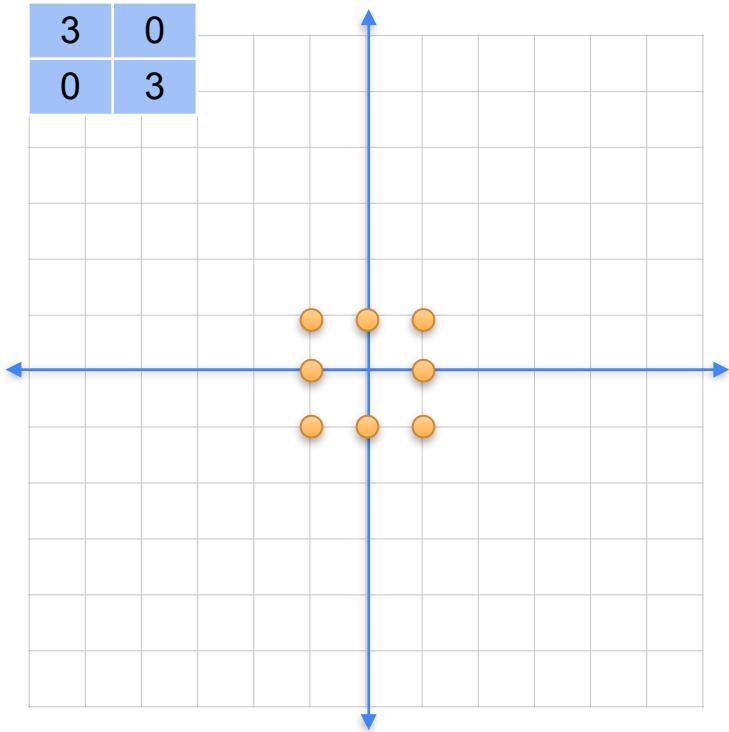
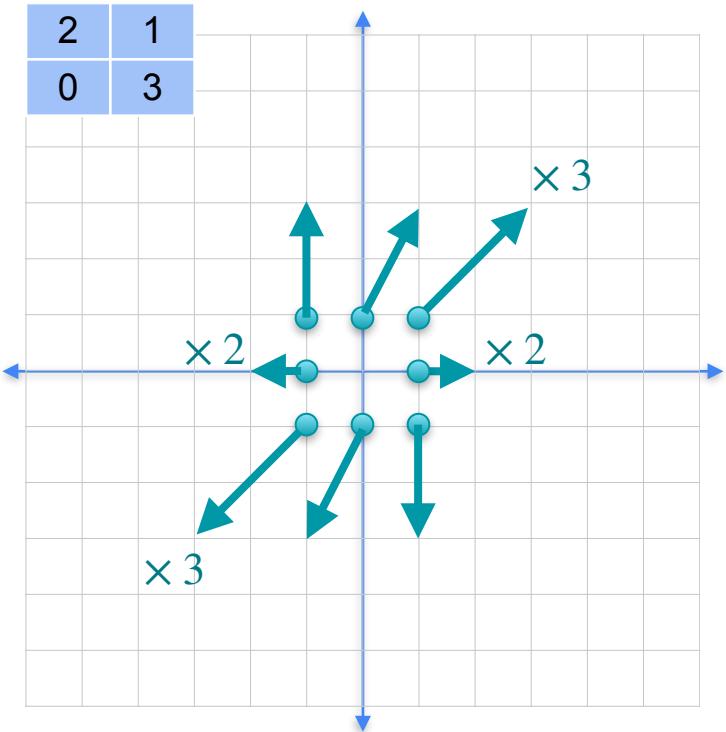
Finding eigenvalues



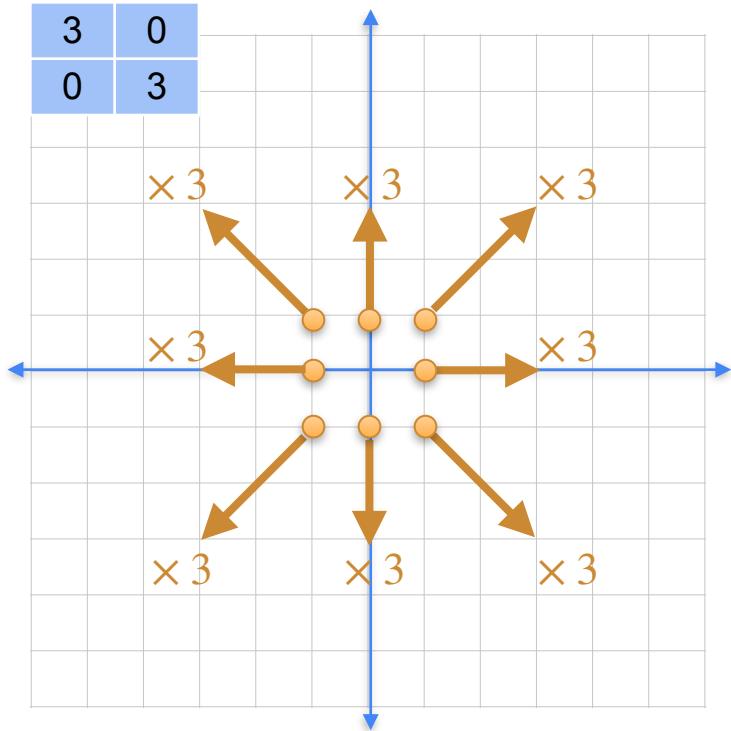
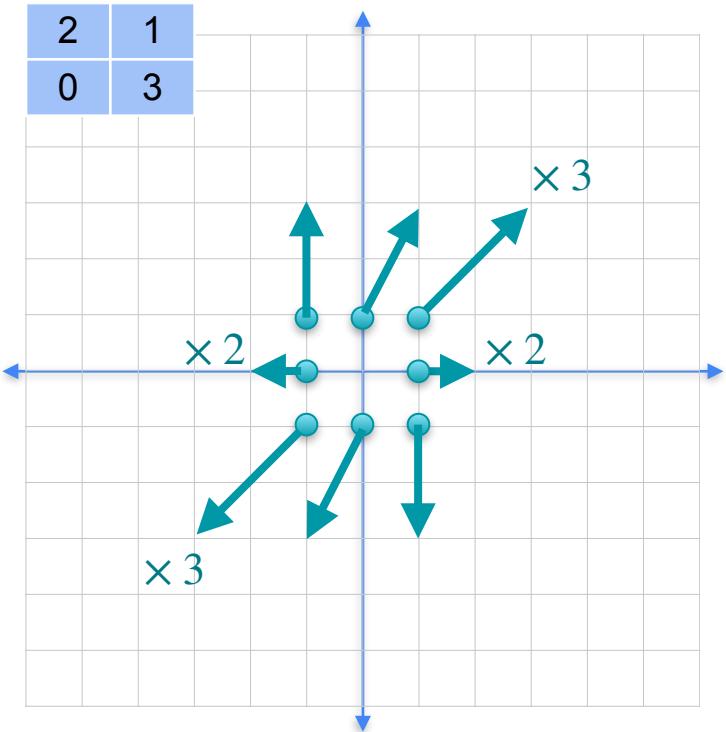
Finding eigenvalues



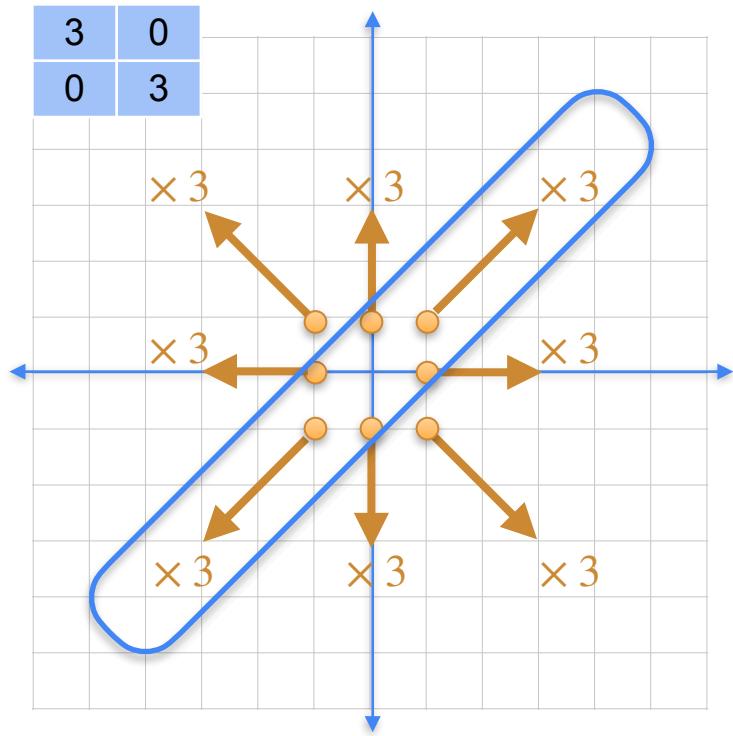
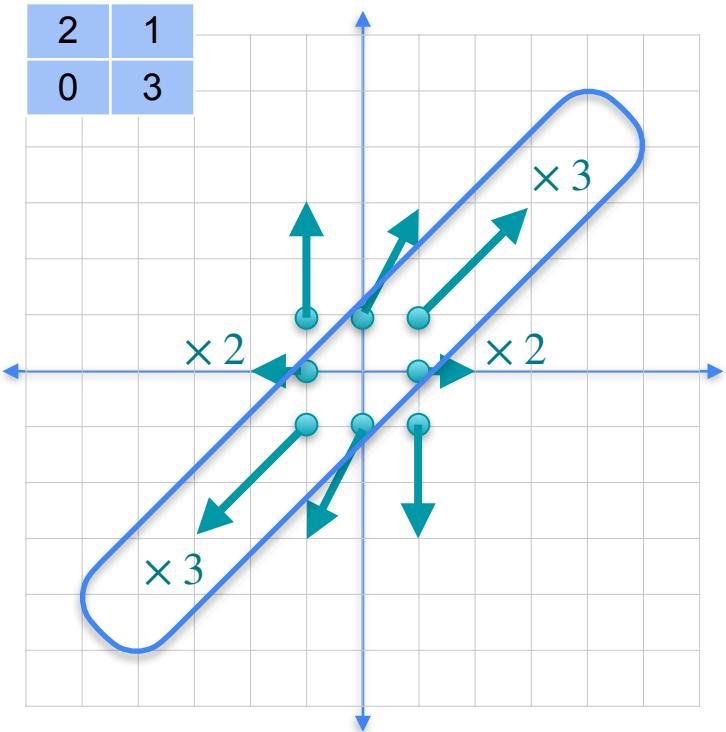
Finding eigenvalues



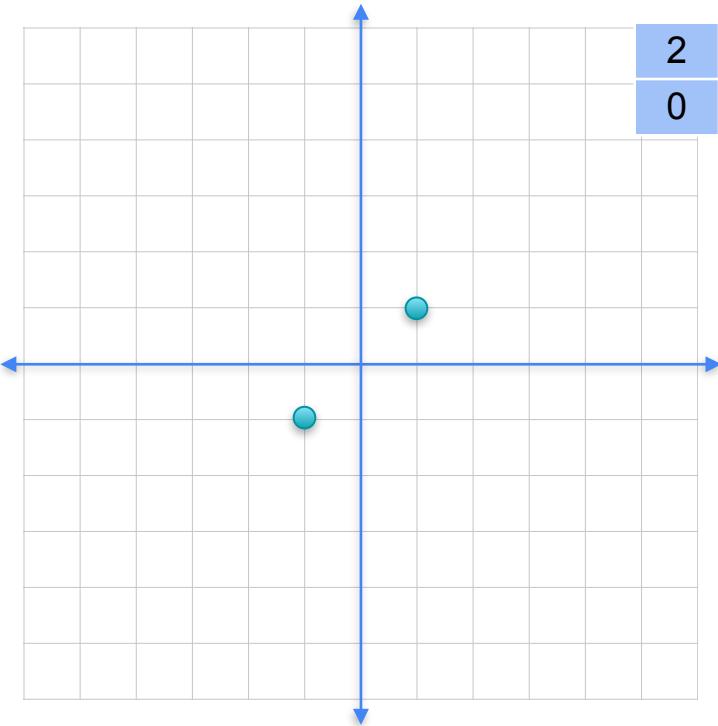
Finding eigenvalues



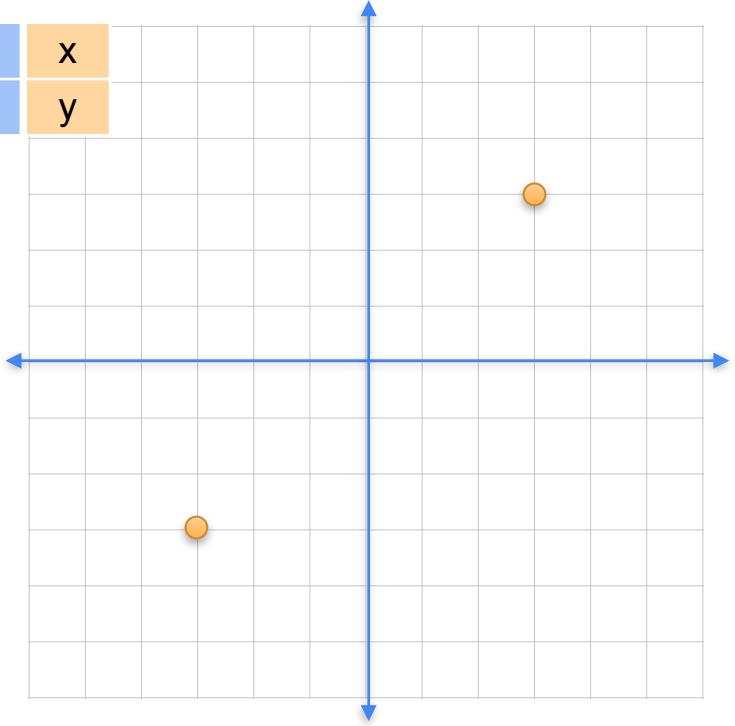
Finding eigenvalues



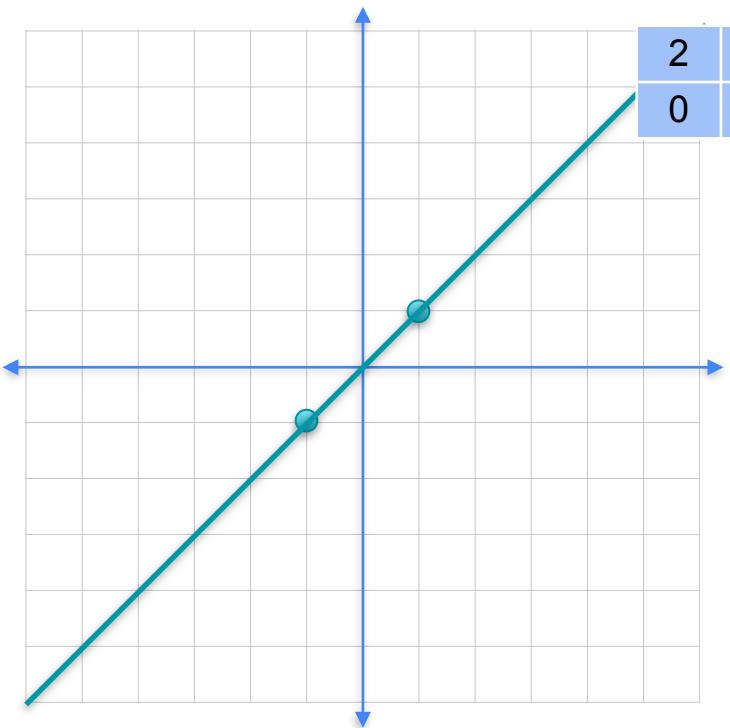
Finding eigenvalues



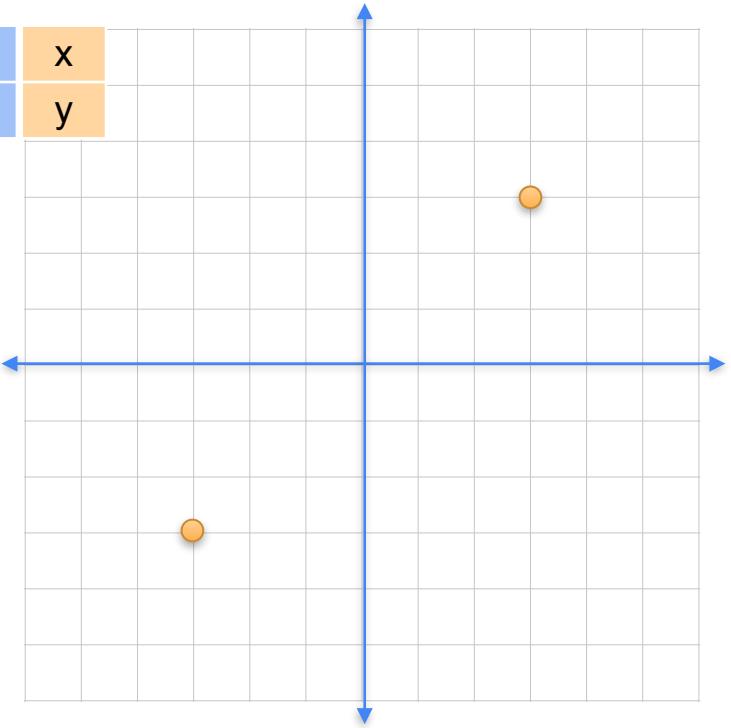
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



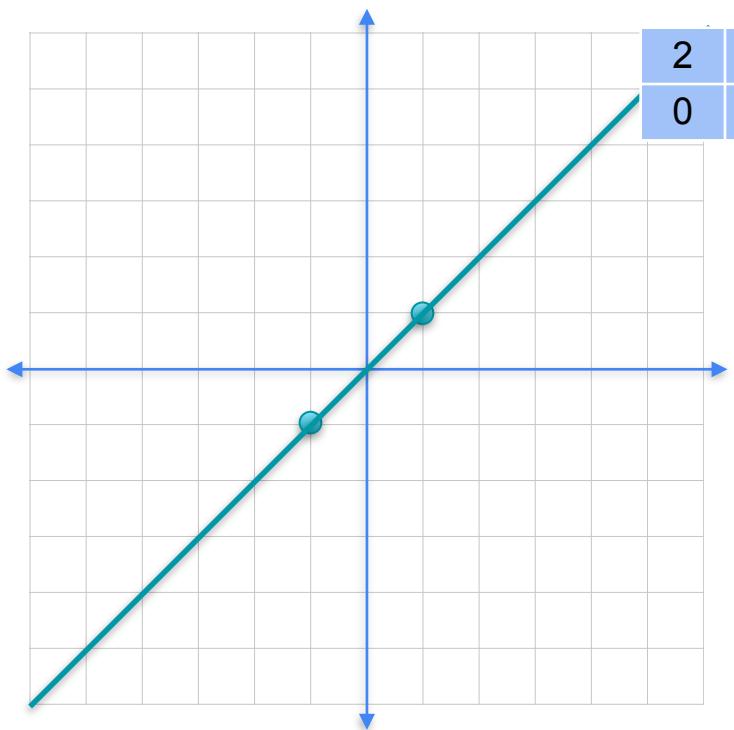
Finding eigenvalues



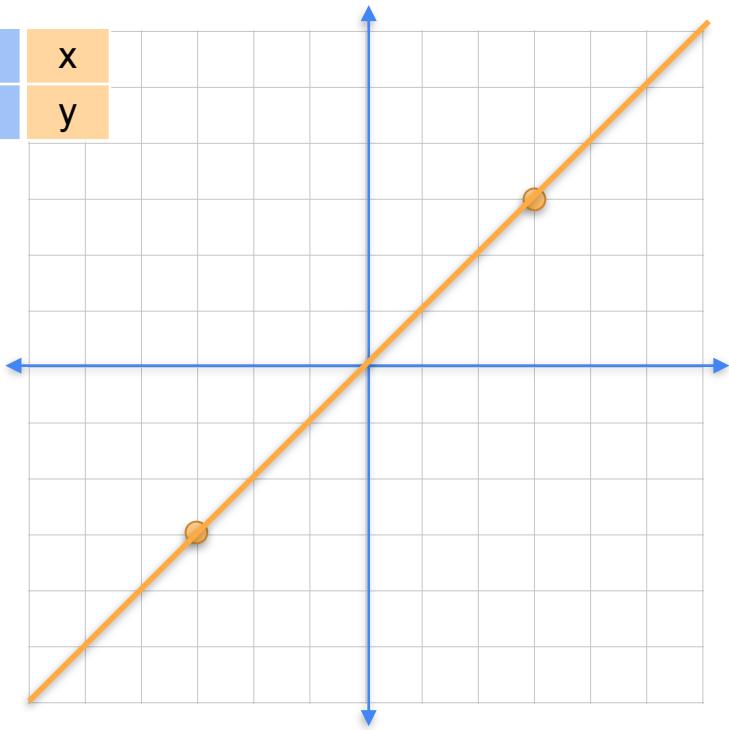
$$\begin{bmatrix} 2 & 1 & x \\ 0 & 3 & y \end{bmatrix} = \begin{bmatrix} 3 & 0 & x \\ 0 & 3 & y \end{bmatrix}$$



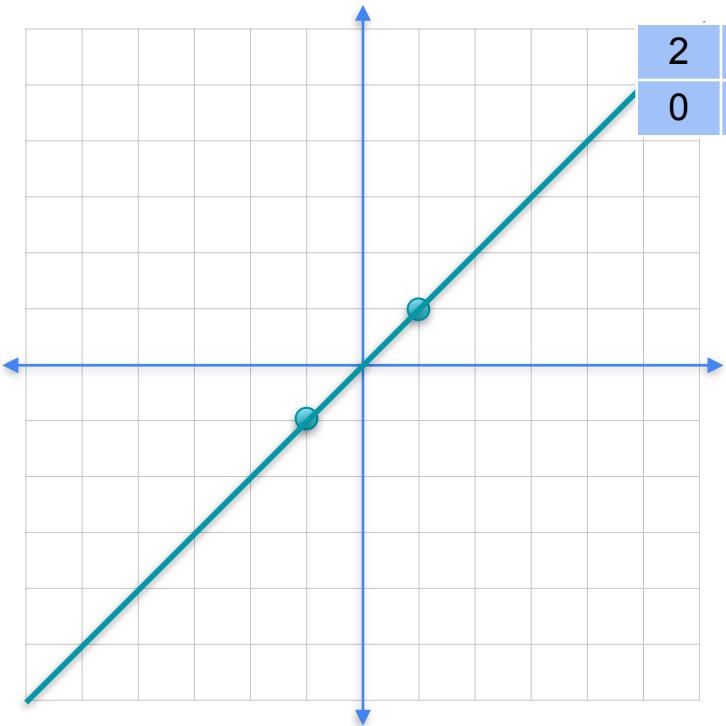
Finding eigenvalues



$$\begin{bmatrix} 2 & 1 & x \\ 0 & 3 & y \end{bmatrix} = \begin{bmatrix} 3 & 0 & x \\ 0 & 3 & y \end{bmatrix}$$

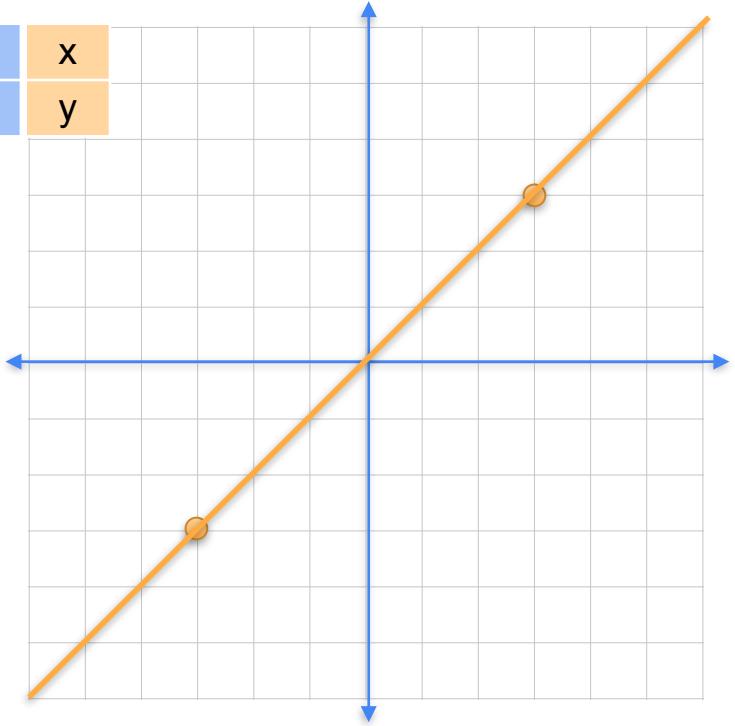


Finding eigenvalues

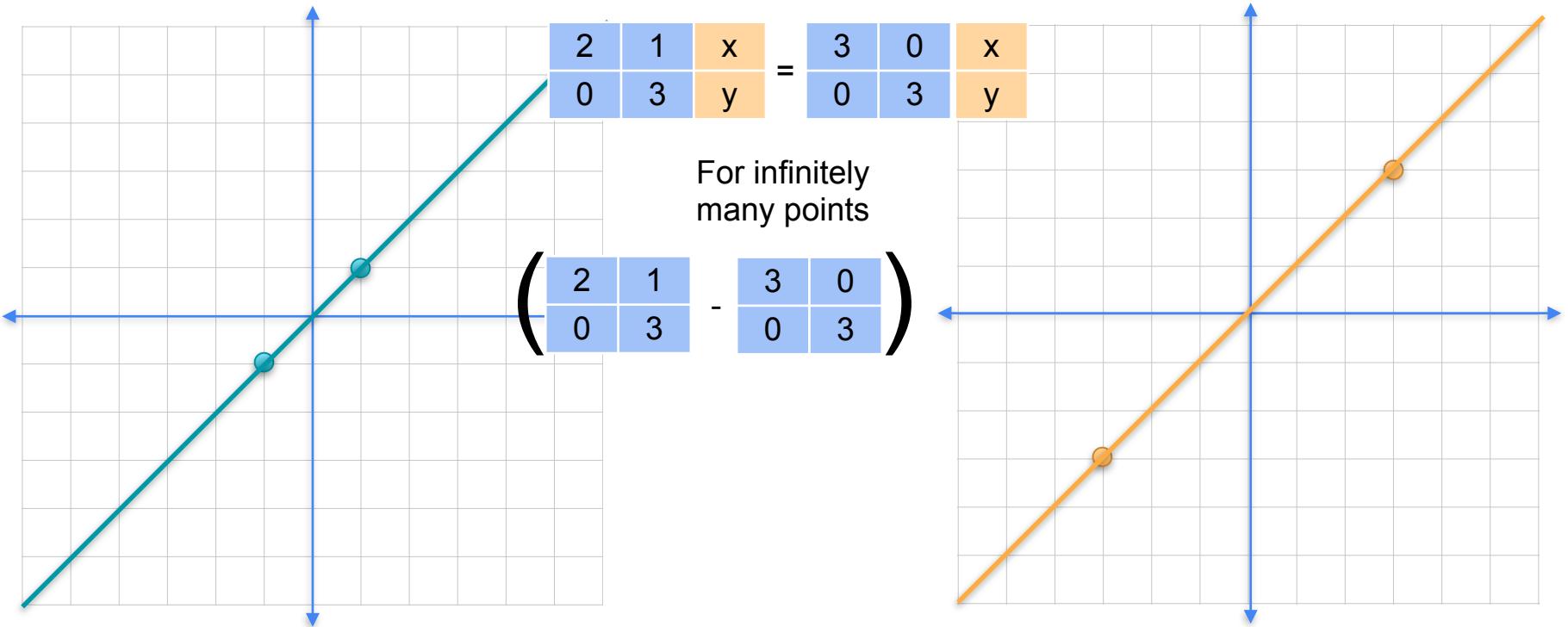


$$\begin{bmatrix} 2 & 1 & x \\ 0 & 3 & y \end{bmatrix} = \begin{bmatrix} 3 & 0 & x \\ 0 & 3 & y \end{bmatrix}$$

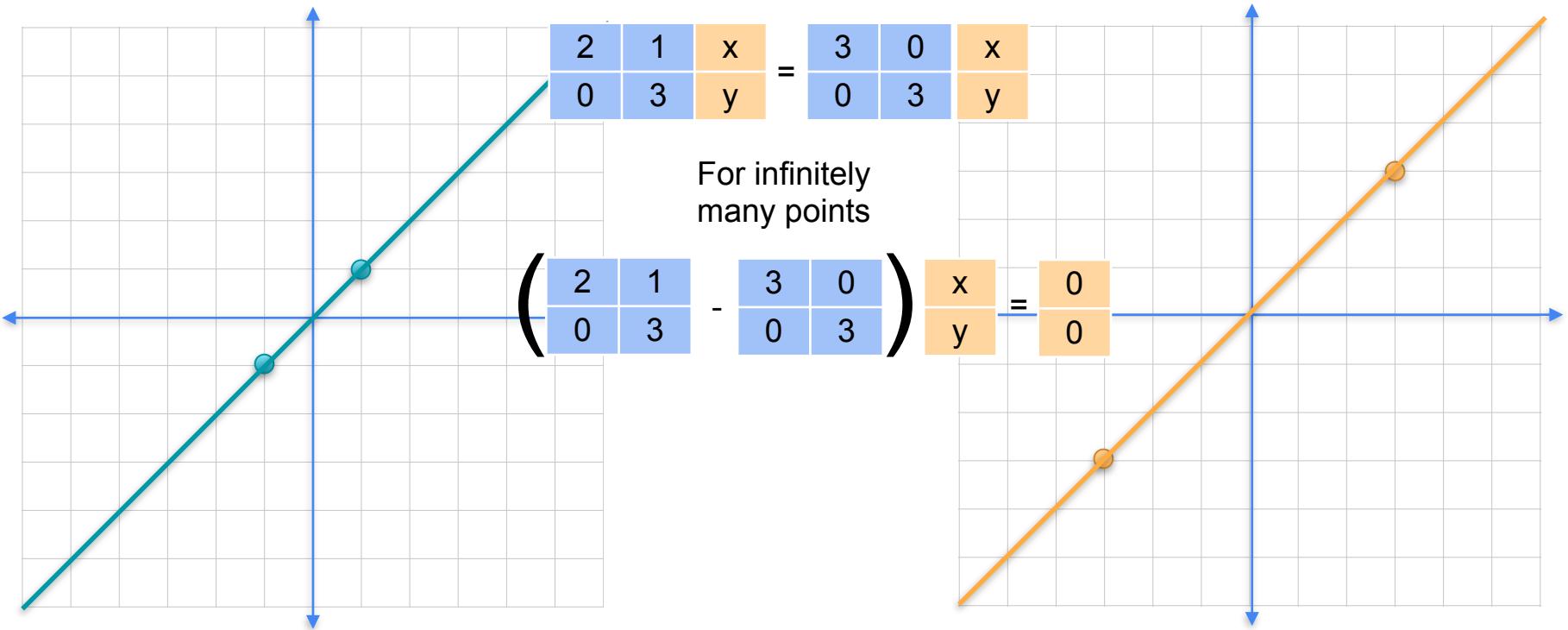
For infinitely
many points



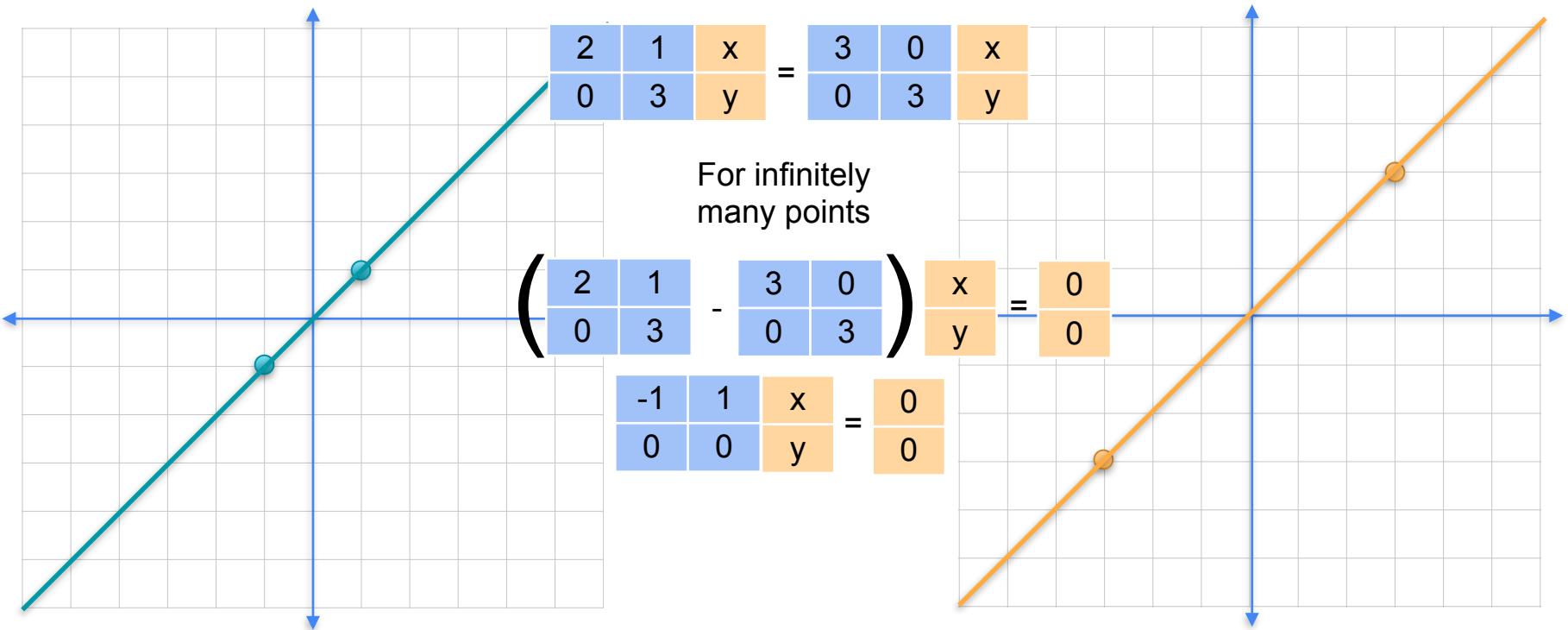
Finding eigenvalues



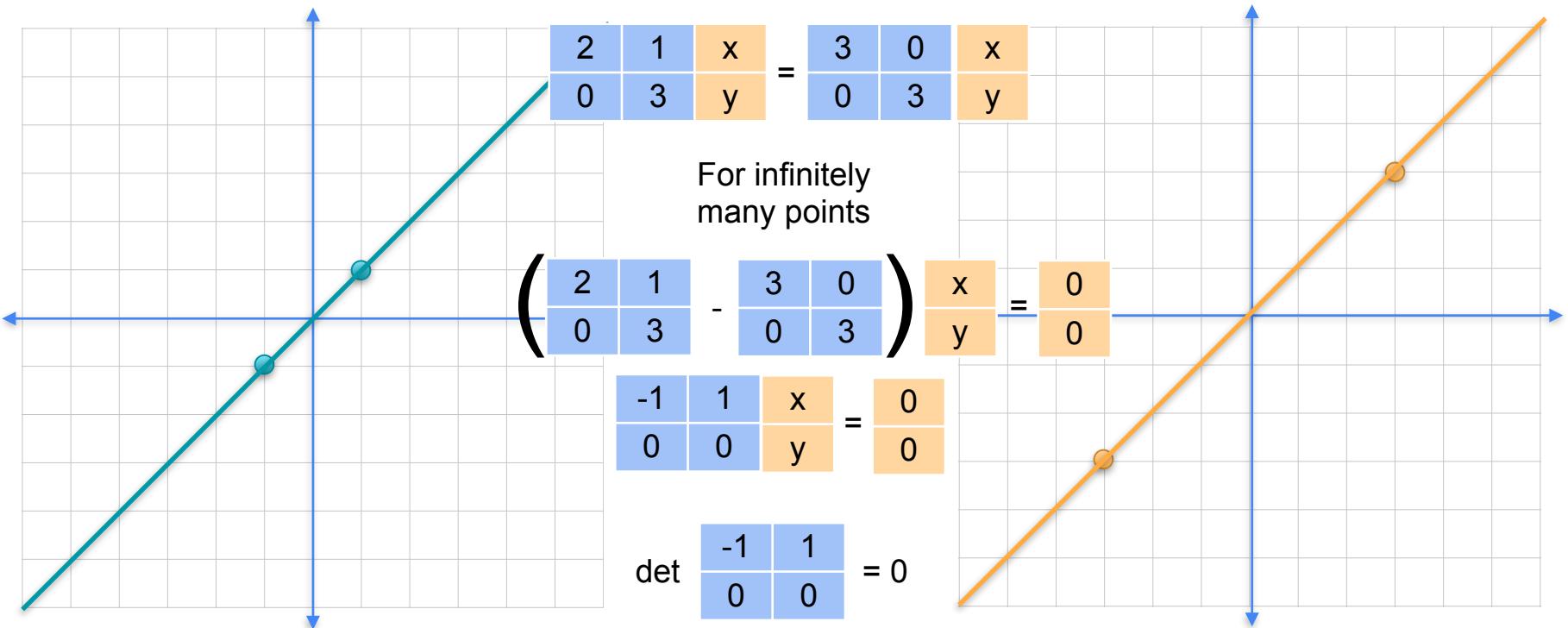
Finding eigenvalues



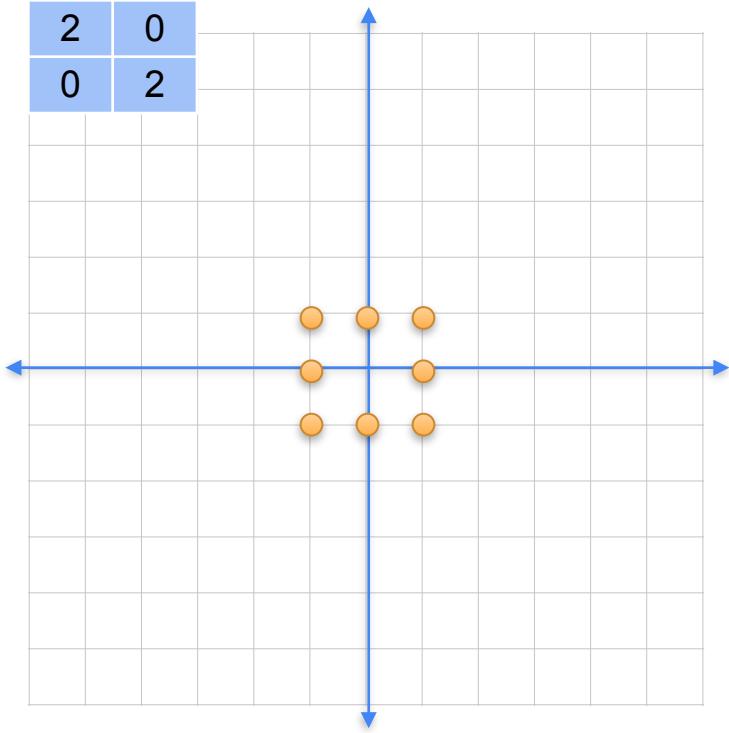
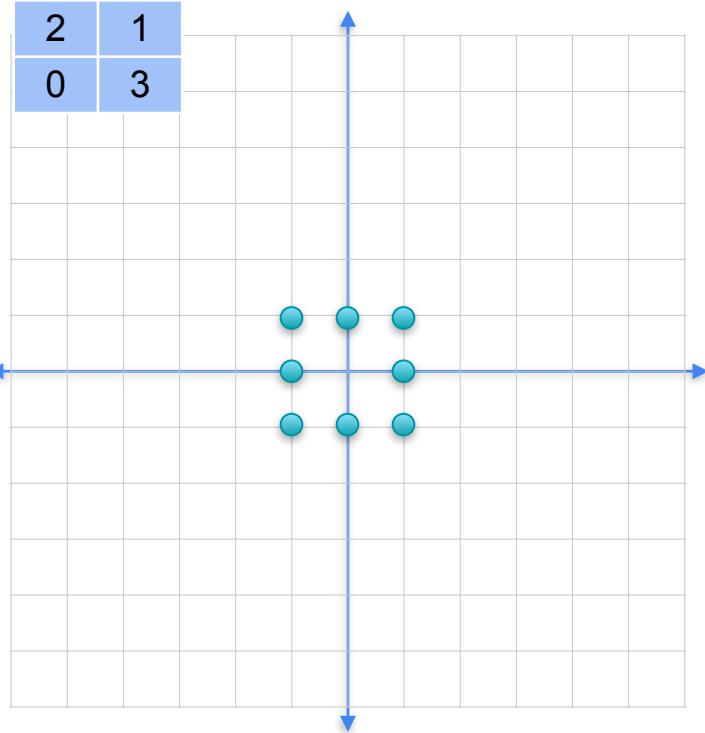
Finding eigenvalues



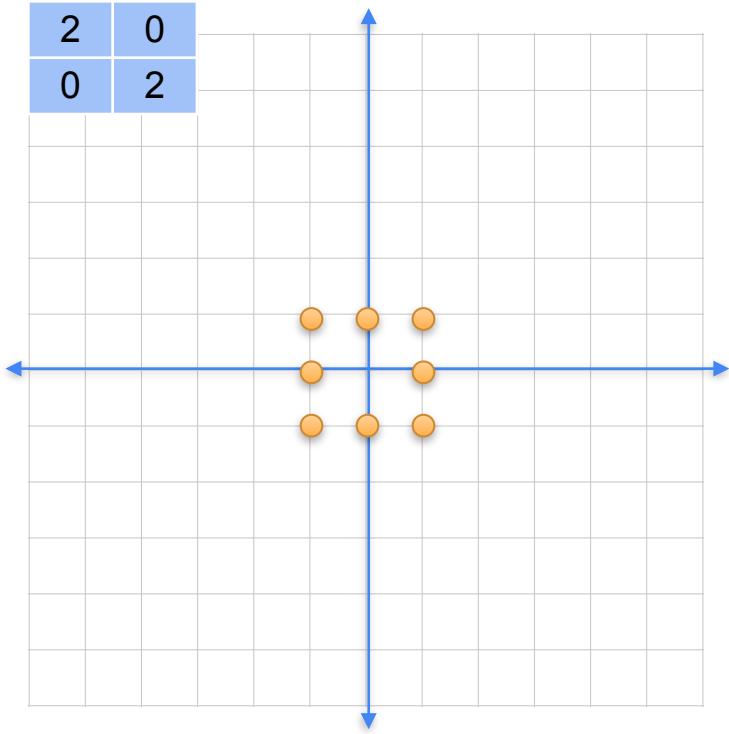
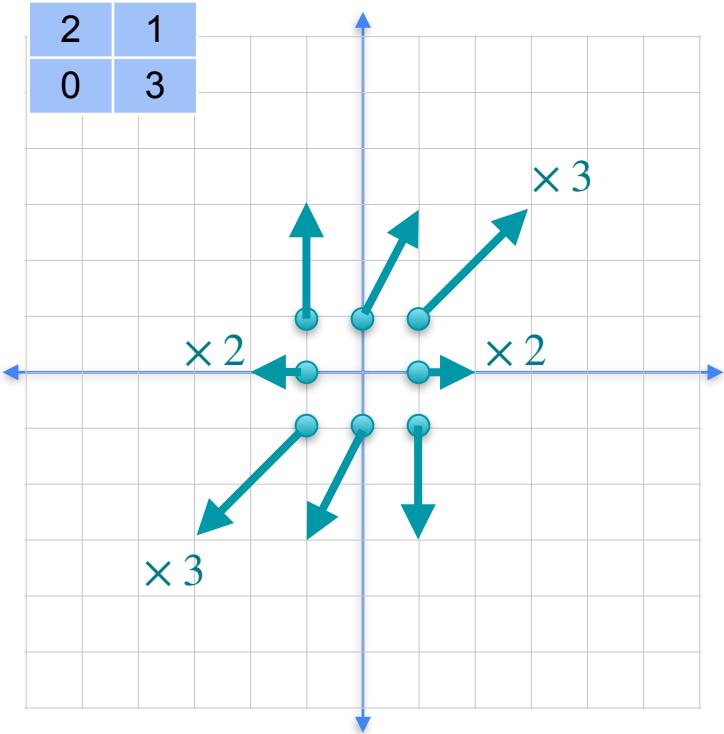
Finding eigenvalues



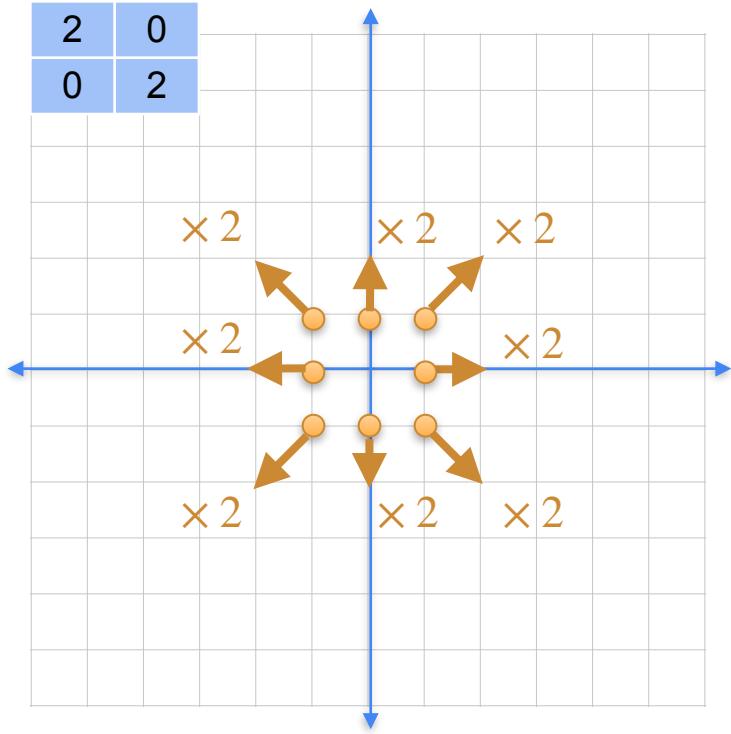
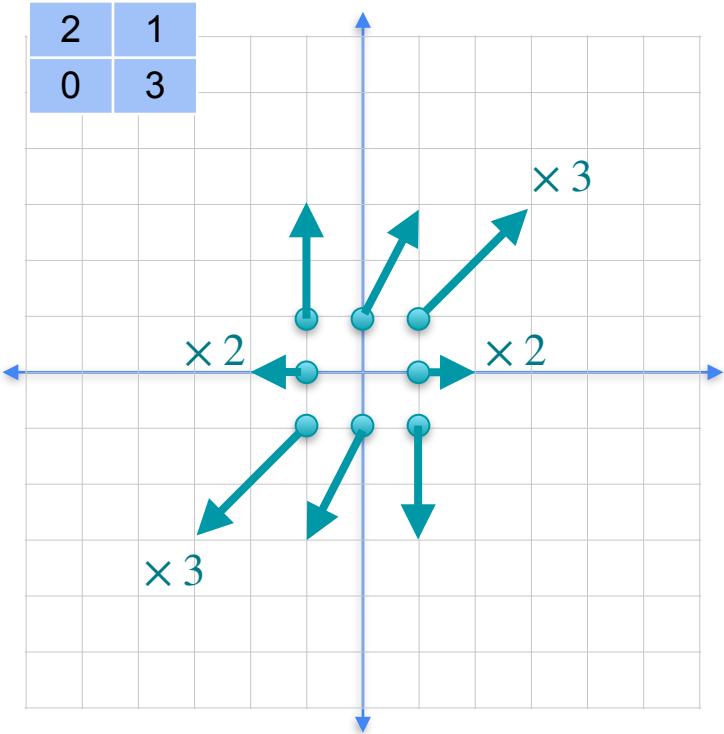
Finding eigenvalues



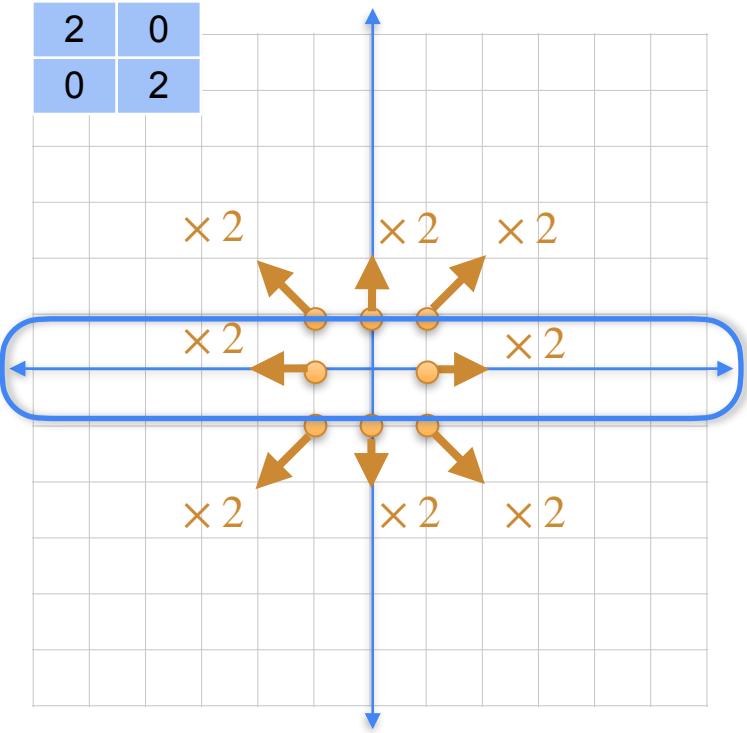
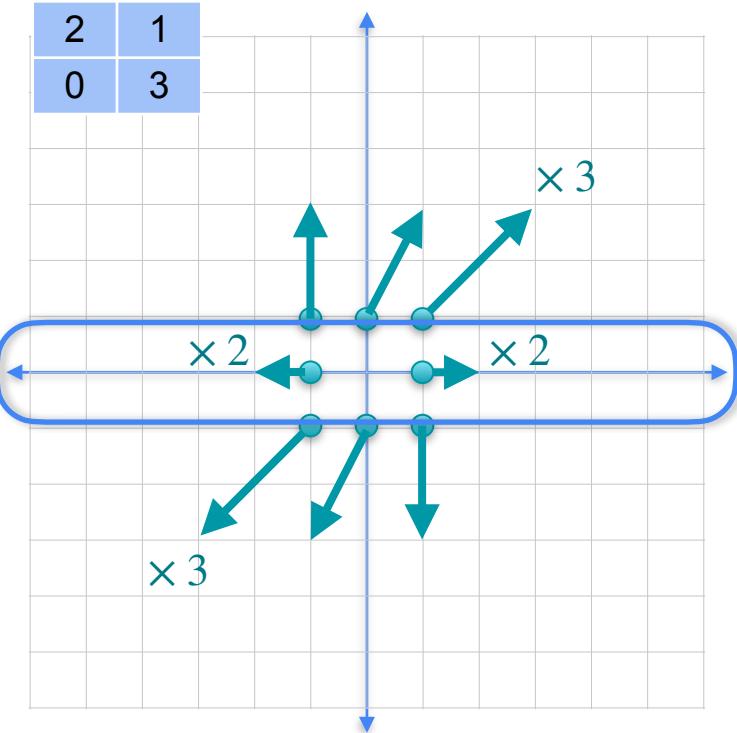
Finding eigenvalues



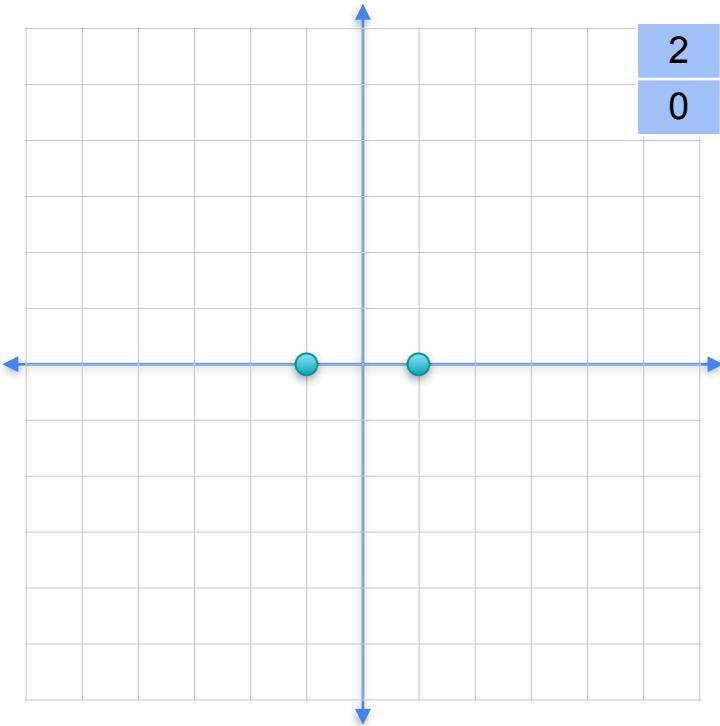
Finding eigenvalues



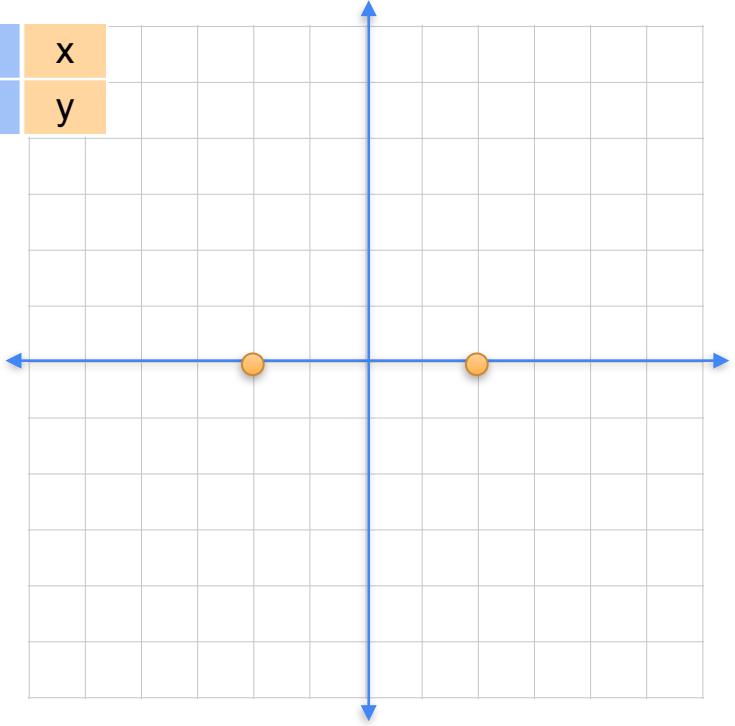
Finding eigenvalues



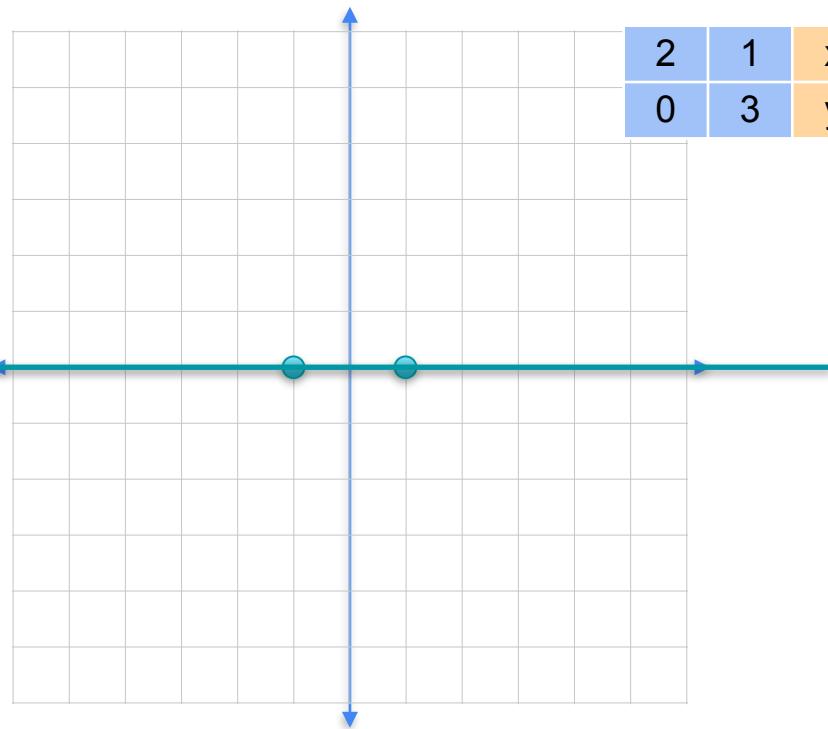
Finding eigenvalues



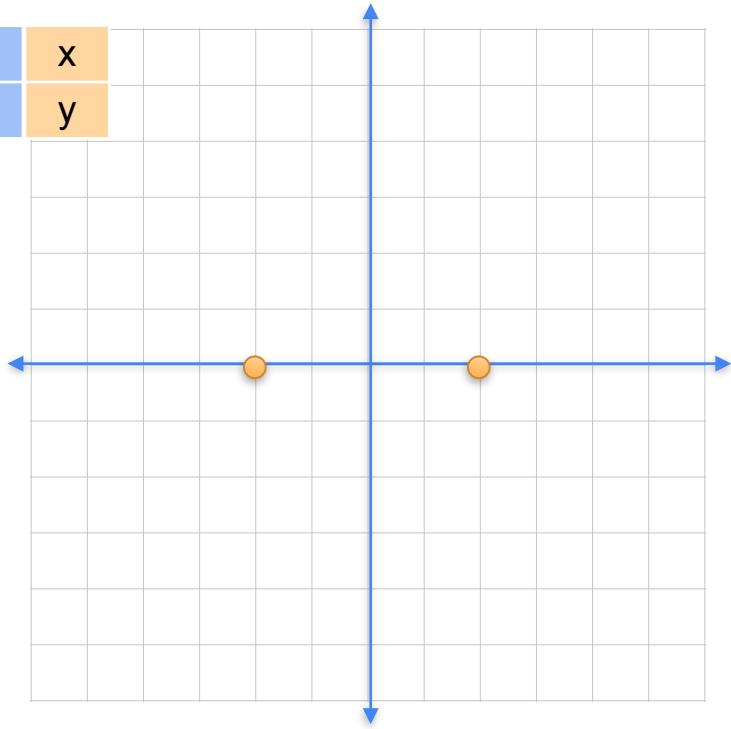
$$\begin{bmatrix} 2 & 1 & x \\ 0 & 3 & y \end{bmatrix} = \begin{bmatrix} 2 & 0 & x \\ 0 & 2 & y \end{bmatrix}$$



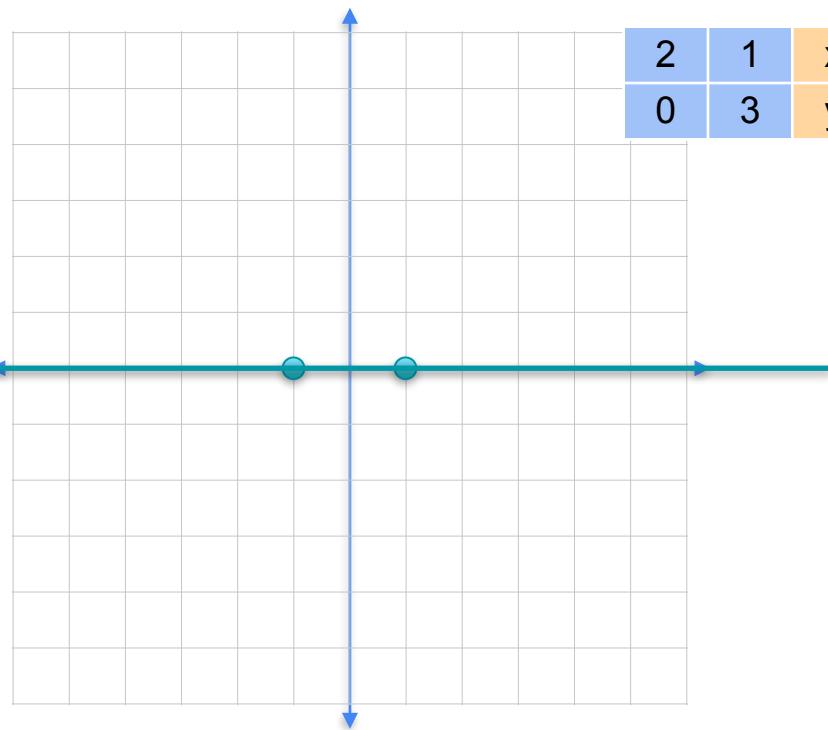
Finding eigenvalues



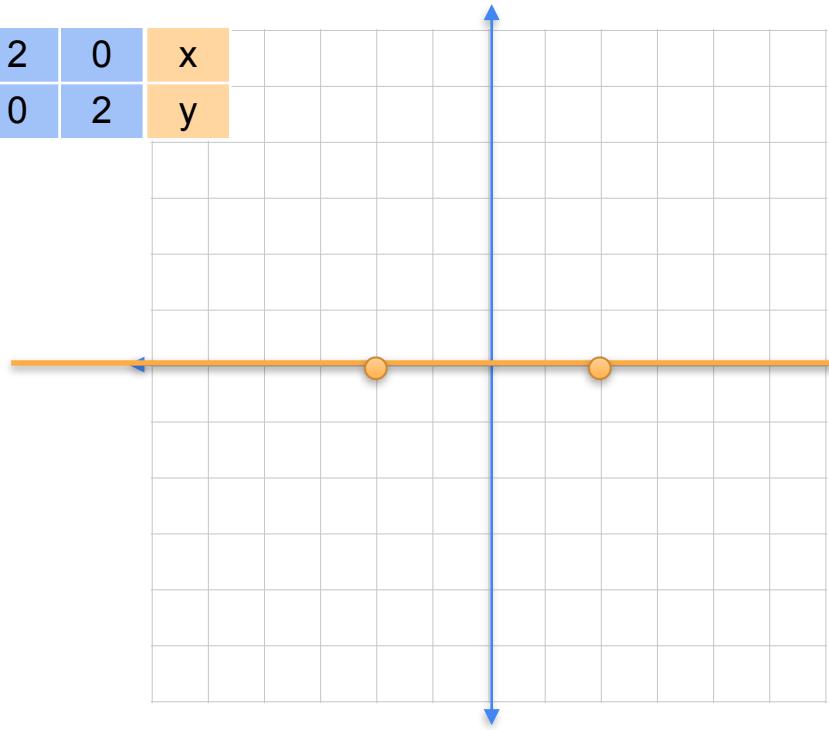
$$\begin{bmatrix} 2 & 1 & x \\ 0 & 3 & y \end{bmatrix} = \begin{bmatrix} 2 & 0 & x \\ 0 & 2 & y \end{bmatrix}$$



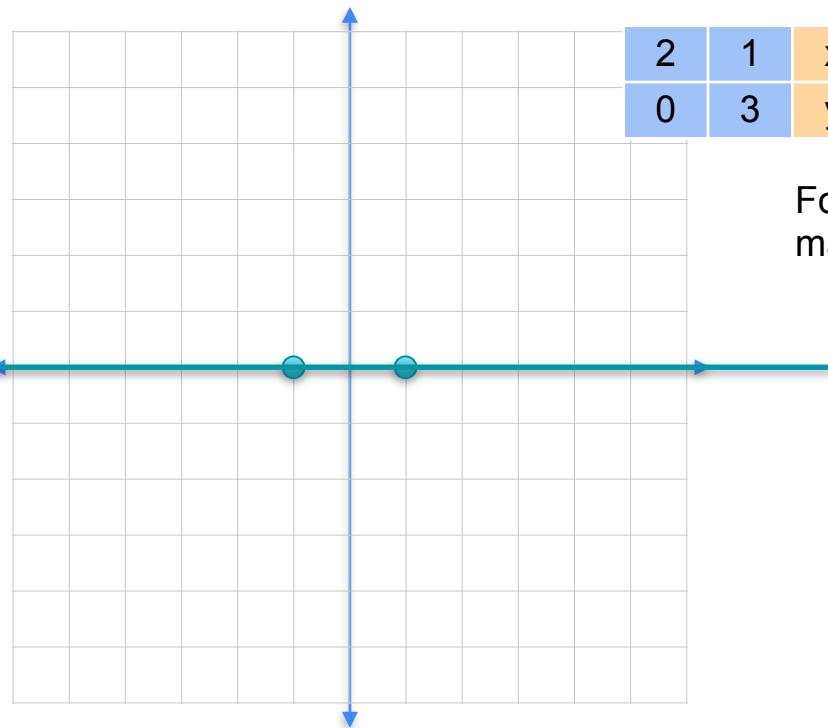
Finding eigenvalues



$$\begin{bmatrix} 2 & 1 & x \\ 0 & 3 & y \end{bmatrix} = \begin{bmatrix} 2 & 0 & x \\ 0 & 2 & y \end{bmatrix}$$

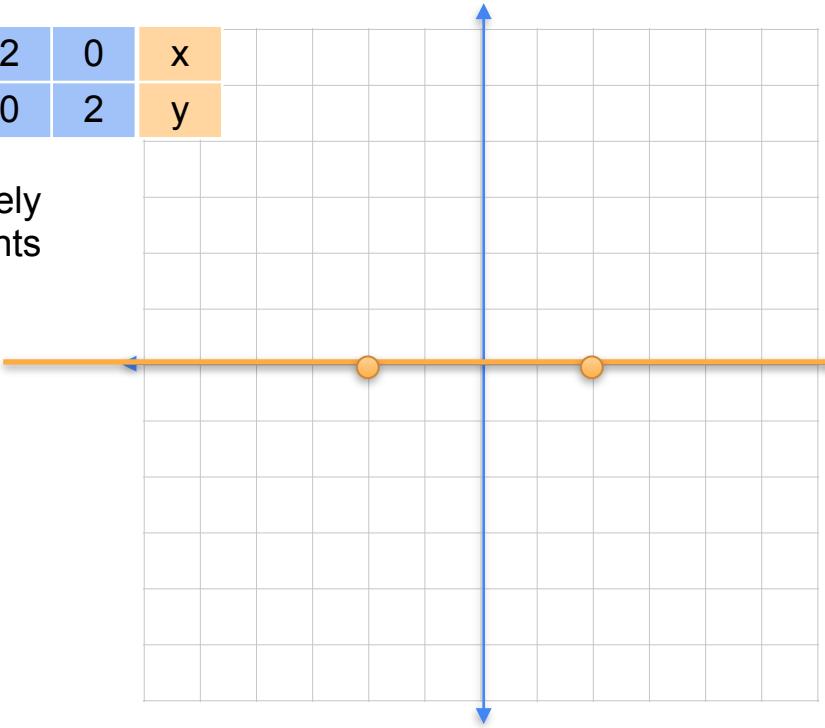


Finding eigenvalues

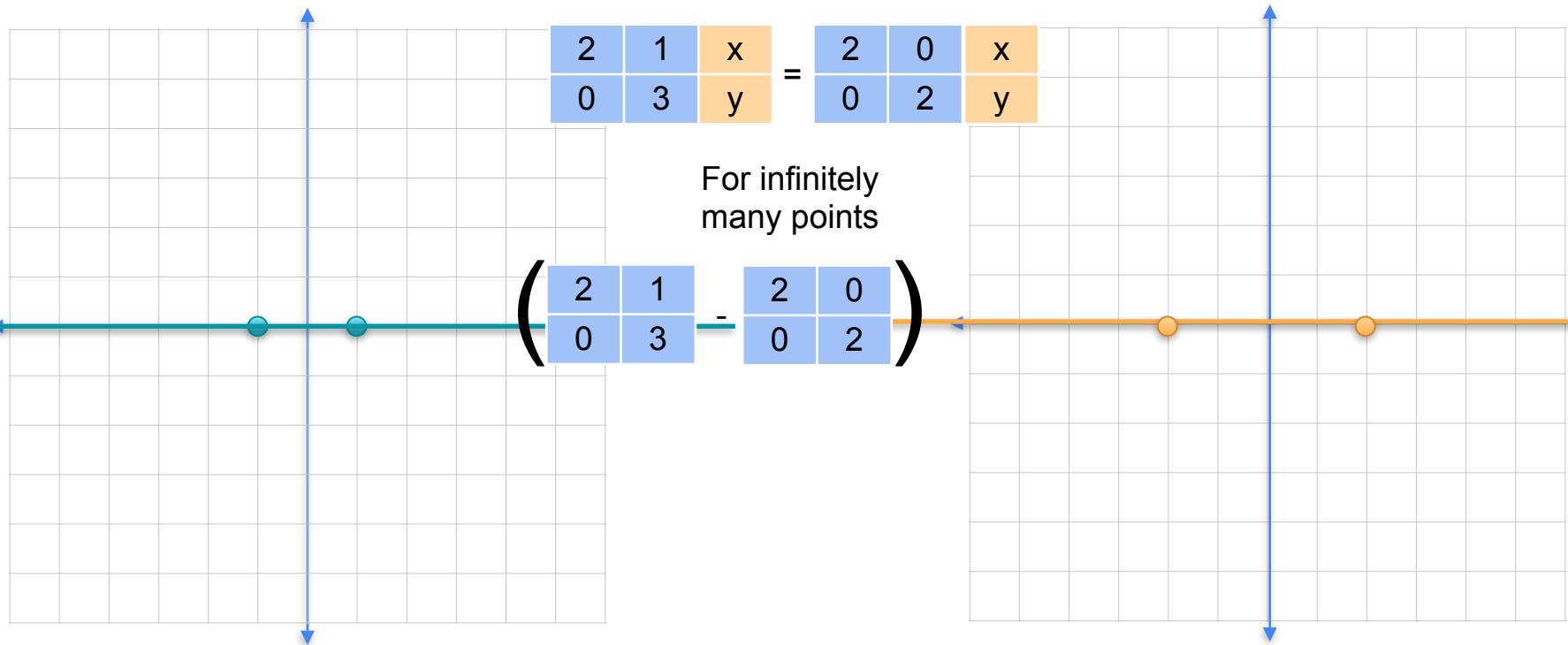


$$\begin{bmatrix} 2 & 1 & x \\ 0 & 3 & y \end{bmatrix} = \begin{bmatrix} 2 & 0 & x \\ 0 & 2 & y \end{bmatrix}$$

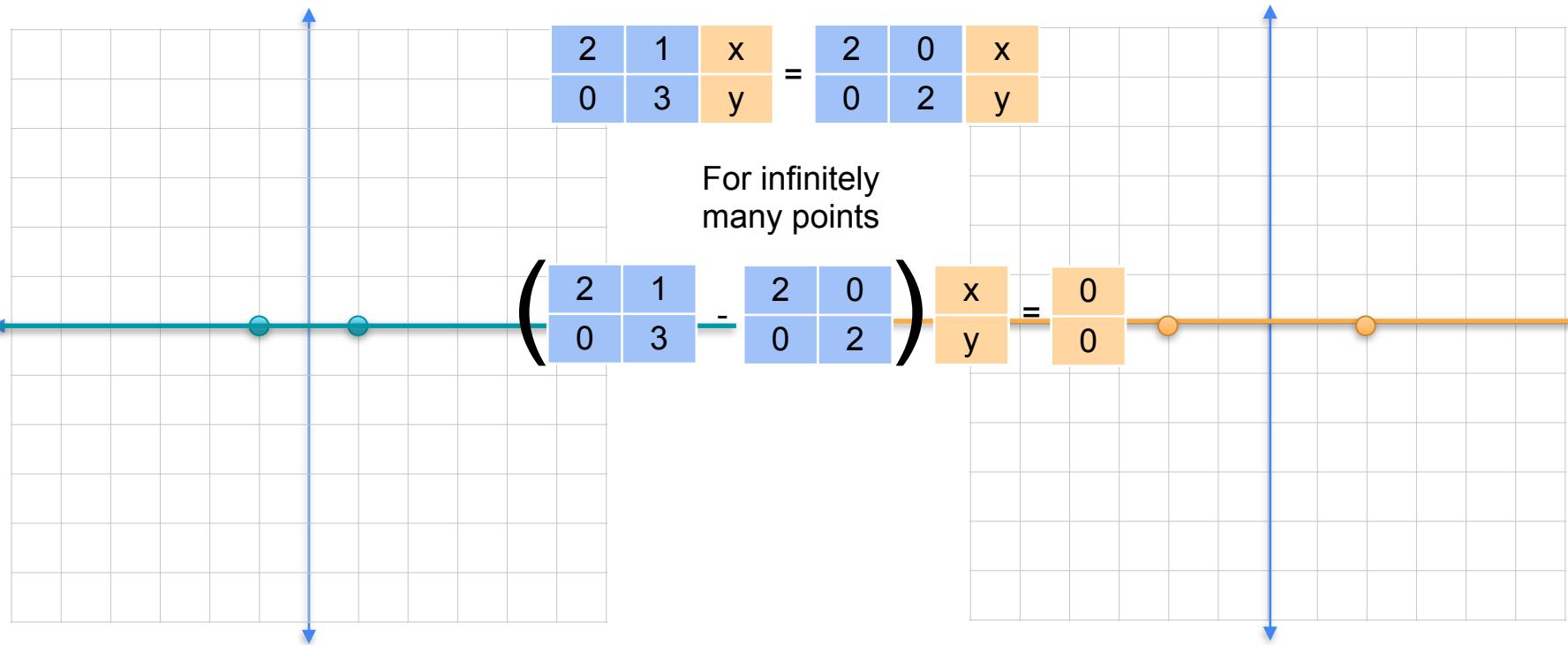
For infinitely
many points



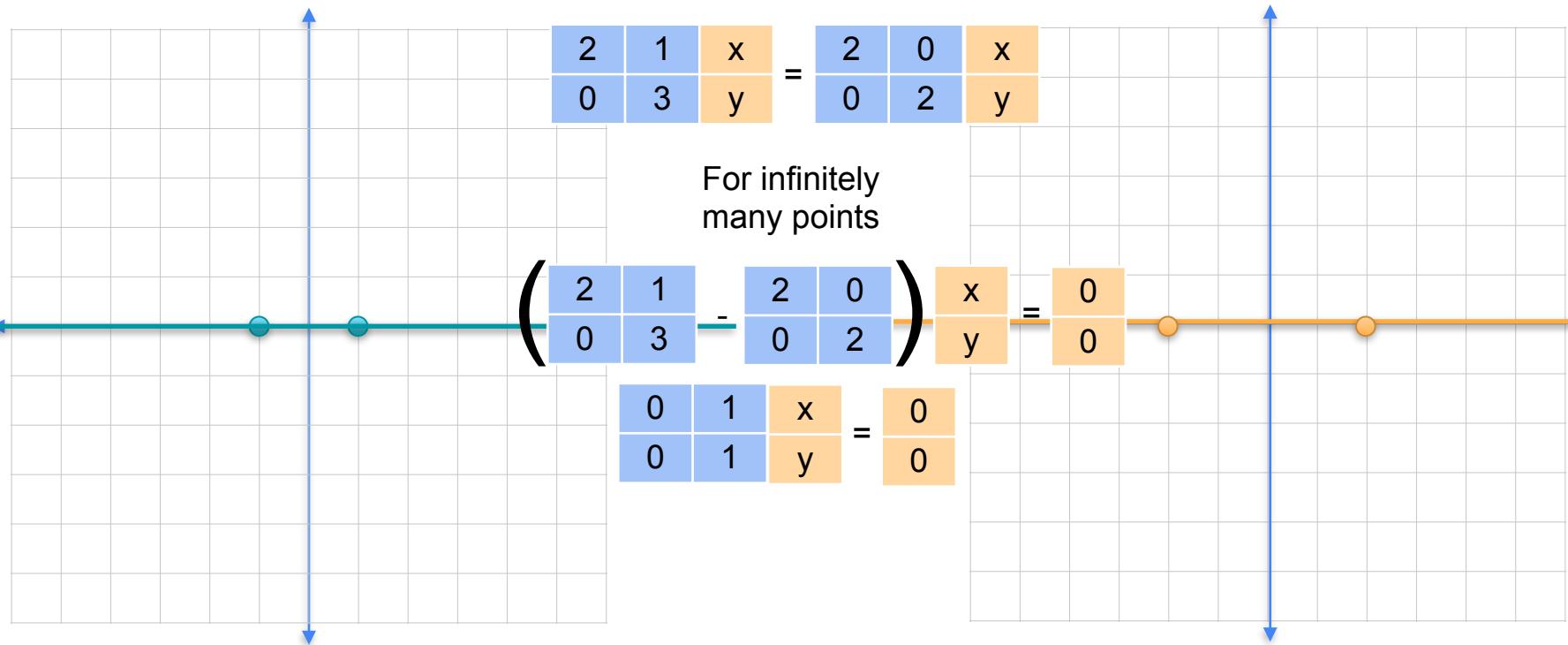
Finding eigenvalues



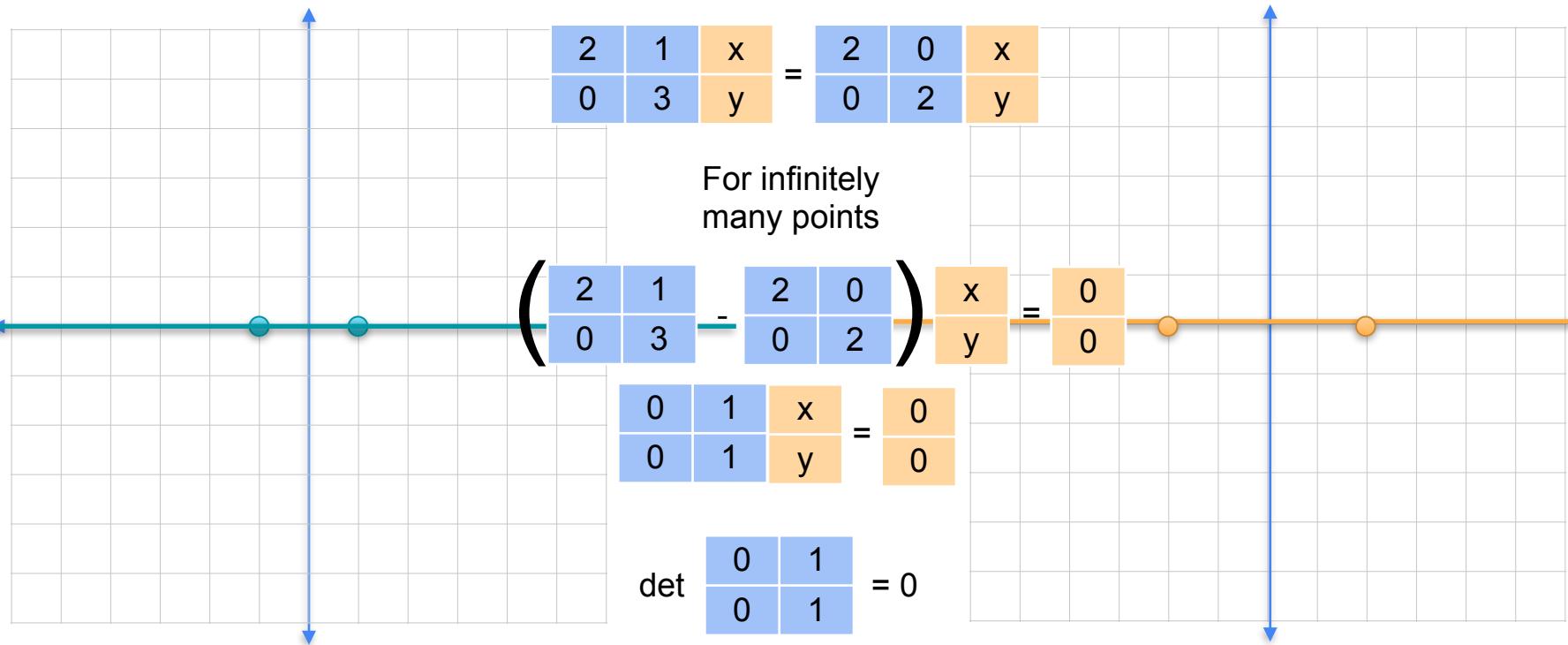
Finding eigenvalues



Finding eigenvalues



Finding eigenvalues



Finding eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

Finding eigenvalues

If λ is an eigenvalue:

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

Finding eigenvalues

If λ is an eigenvalue:

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

| | |
|-----------|-----------|
| λ | 0 |
| 0 | λ |

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{matrix} 2 & 1 & x \\ 0 & 3 & y \end{matrix} = \begin{matrix} \lambda & 0 & x \\ 0 & \lambda & y \end{matrix}$$

For infinitely many (x,y)

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

For infinitely many (x,y)

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 & 0 \\ \hline \end{array}$$

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

For infinitely many (x,y)

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 & 0 \\ \hline \end{array}$$

Has infinitely many solutions

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

For infinitely many (x,y)

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

Has infinitely many solutions

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

For infinitely many (x,y)

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

Has infinitely many solutions

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

$$(2 - \lambda)(3 - \lambda) - 1 \cdot 0 = 0$$

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

For infinitely many (x,y)

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

Has infinitely many solutions

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

Characteristic polynomial

$$(2 - \lambda)(3 - \lambda) - 1 \cdot 0 = 0$$

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

For infinitely many (x,y)

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

Has infinitely many solutions

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

Characteristic polynomial

$$(2 - \lambda)(3 - \lambda) - 1 \cdot 0 = 0$$

$$\lambda = 2$$

$$\lambda = 3$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{matrix} 2 & 1 & x \\ 0 & 3 & y \end{matrix} = 2 \begin{matrix} x \\ y \end{matrix}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
$$0x + 3y = 2y$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$

$$2x + y = 2x$$

$$x = 1$$

$$0x + 3y = 2y$$

$$y = 0$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$

$$2x + y = 2x$$

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$$\begin{array}{|c|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
$$0x + 3y = 2y$$
$$x = 1$$
$$y = 0$$
$$\begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 3 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
$$0x + 3y = 2y$$
$$x = 1$$
$$y = 0$$
$$\begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 3 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 3x$$
$$0x + 3y = 3y$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
$$0x + 3y = 2y$$
$$x = 1$$
$$y = 0$$
$$\begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 3 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 3x$$
$$0x + 3y = 3y$$
$$x = 1$$
$$y = 1$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
$$0x + 3y = 2y$$
$$x = 1$$
$$y = 0$$
$$\begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 3 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 3x$$
$$0x + 3y = 3y$$
$$x = 1$$
$$y = 1$$
$$\begin{array}{|c|} \hline 1 \\ \hline 1 \\ \hline \end{array}$$

Quiz

- Find the eigenvalues and eigenvectors of this matrix:

| | |
|---|---|
| 9 | 4 |
| 4 | 3 |

Solution

- Eigenvalues: 11, 1
- Eigenvectors: (2,1), (-1,2)

| | |
|---|---|
| 9 | 4 |
| 4 | 3 |

- The characteristic polynomial is

$$\det \begin{vmatrix} 9-\lambda & 4 \\ 4 & 3-\lambda \end{vmatrix} = (9 - \lambda)(3 - \lambda) - 4 \cdot 4 = 0$$

- Which factors as $\lambda^2 - 12\lambda + 11 = (\lambda - 11)(\lambda - 1)$

- The solutions are $\lambda = 11$
 $\lambda = 1$



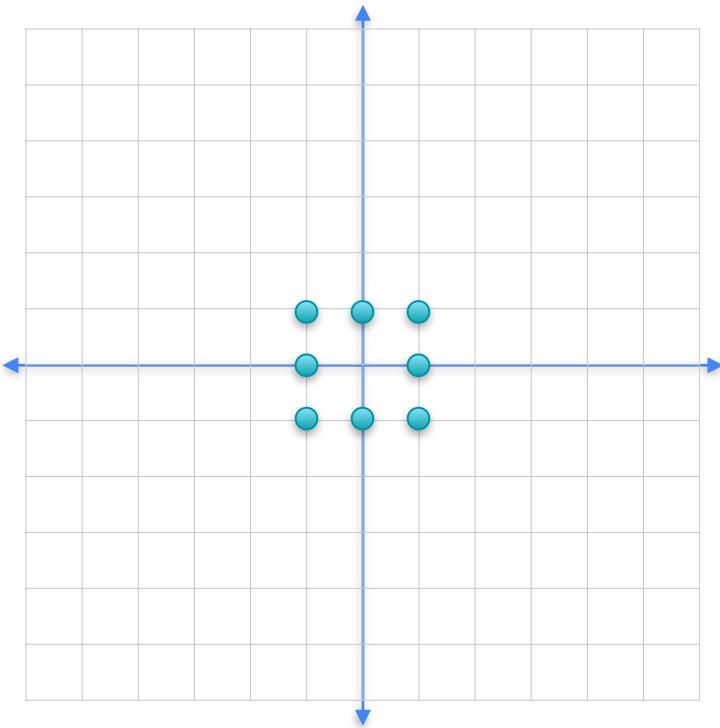
DeepLearning.AI

Determinants and Eigenvectors

Conclusion

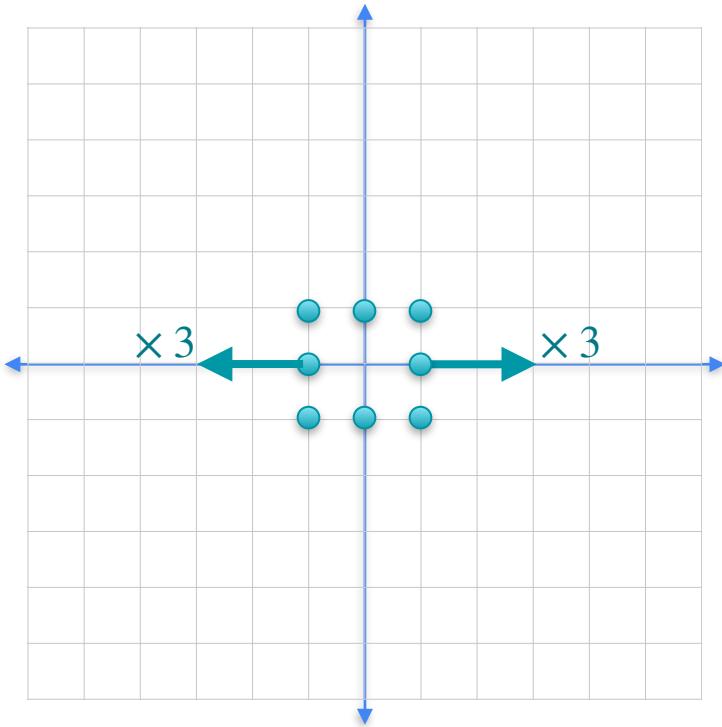
Find eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |



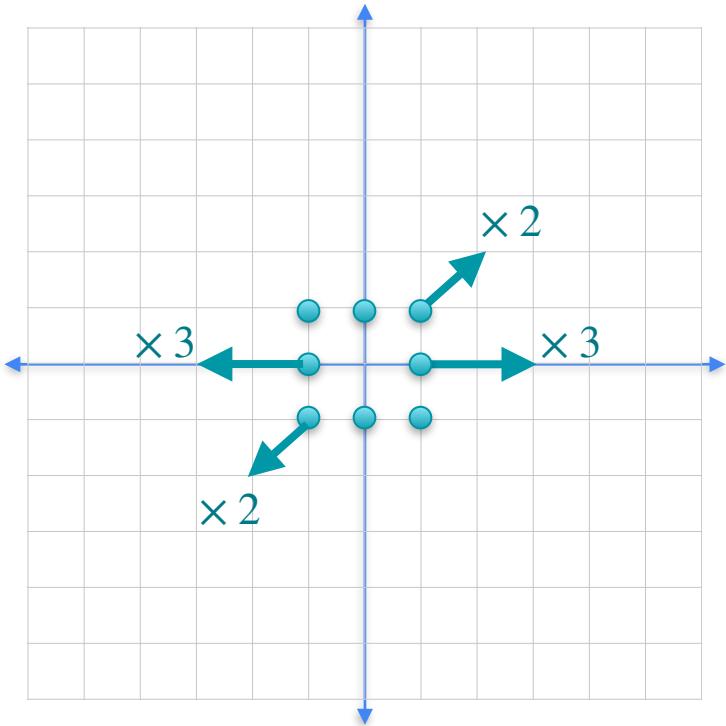
Find eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |



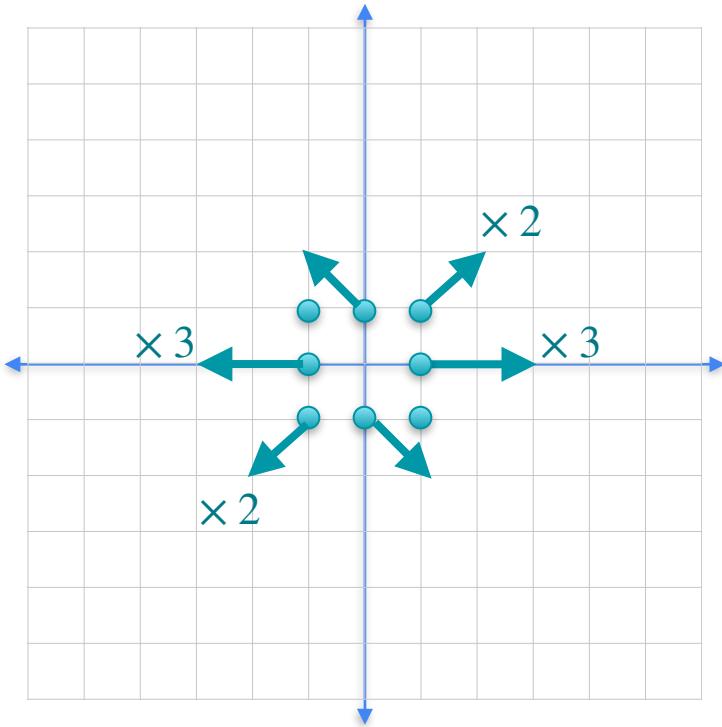
Find eigenvalues

| | |
|---|---|
| 2 | 1 |
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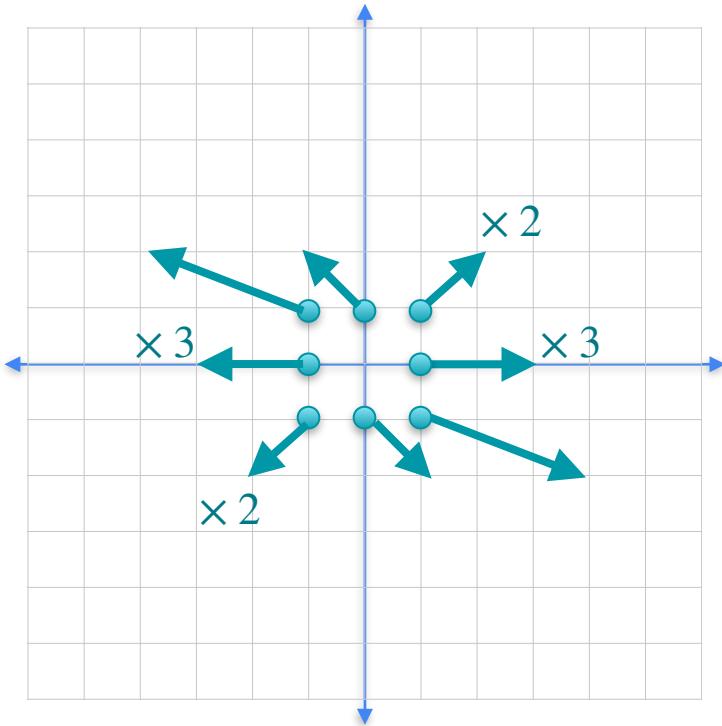
Find eigenvalues

| | |
|---|---|
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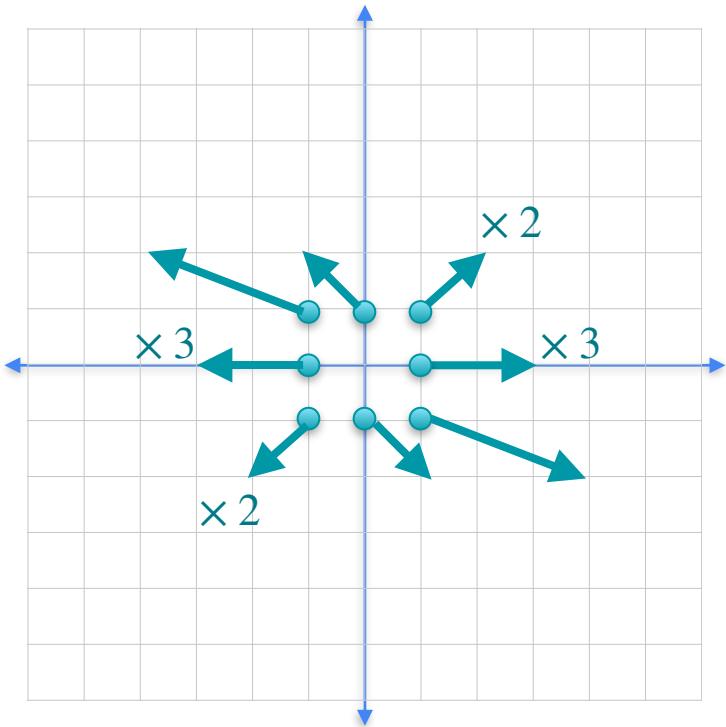
Find eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

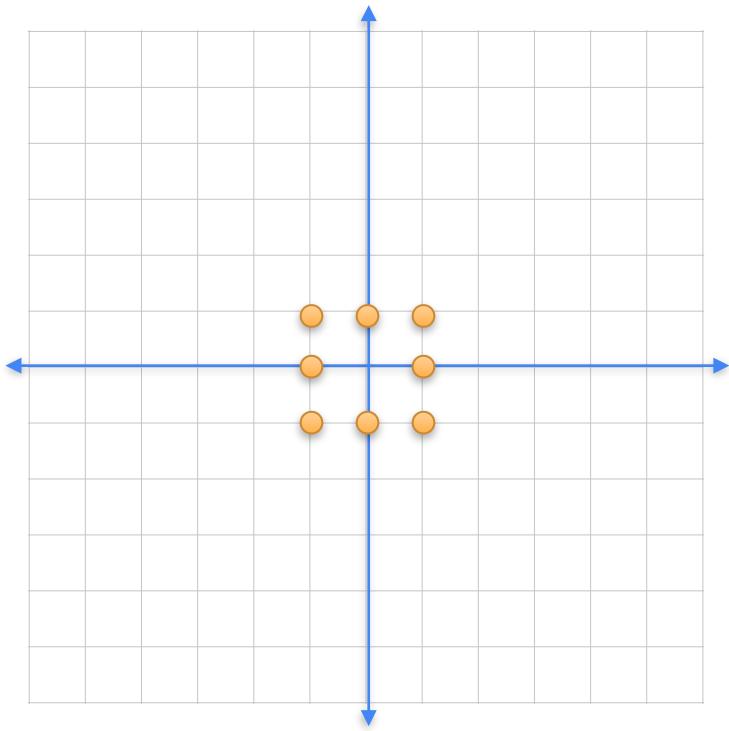


Find eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

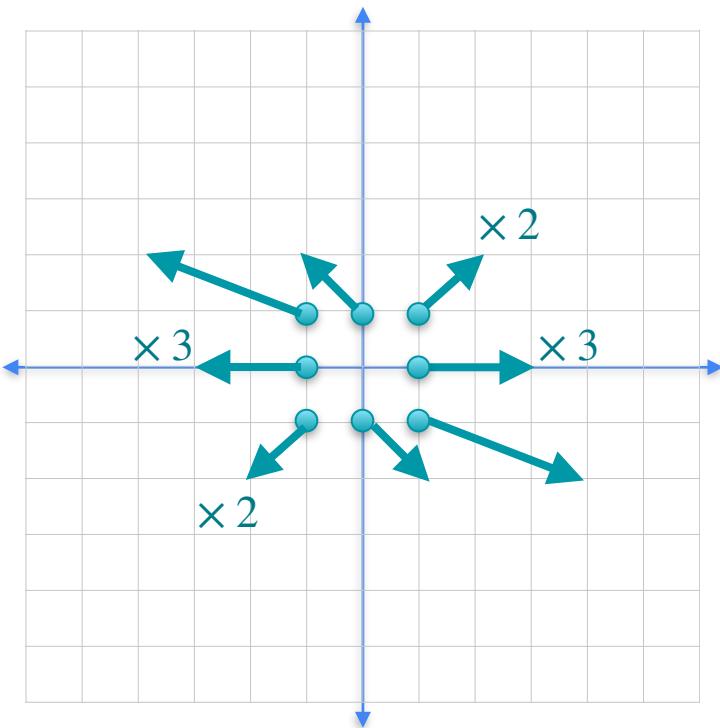


| | |
|---|---|
| 3 | 0 |
| 0 | 3 |

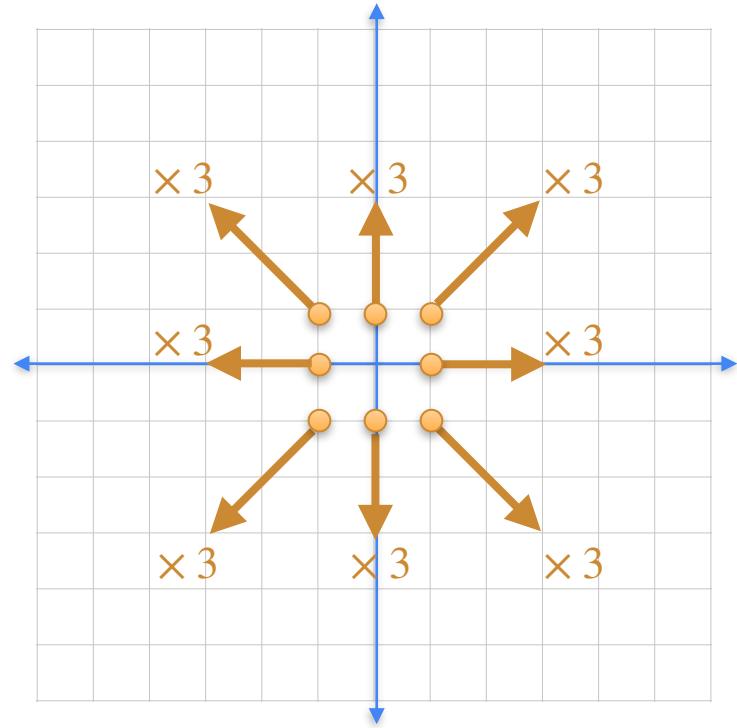


Find eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

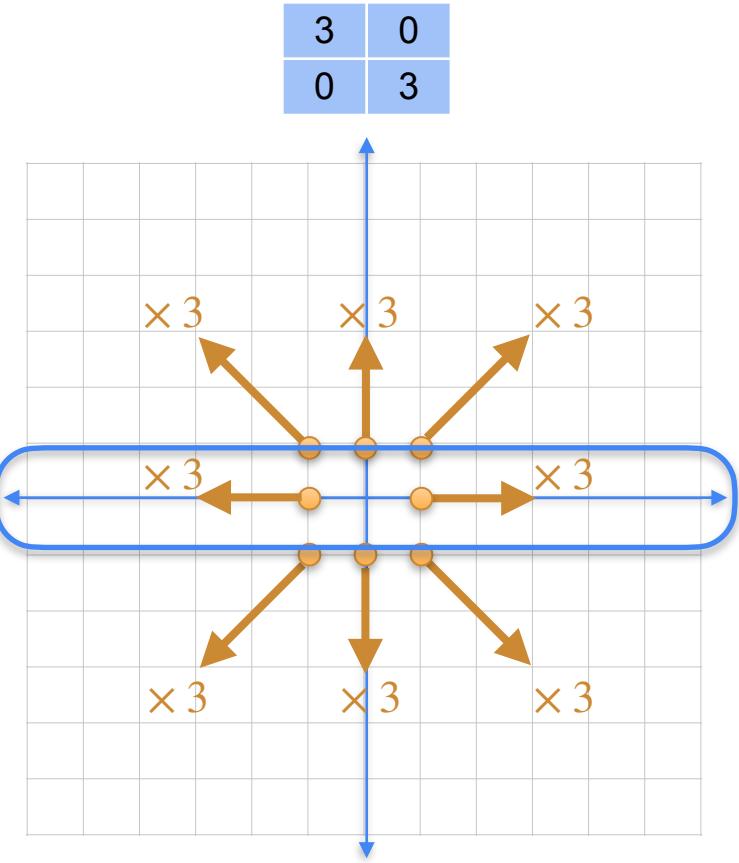
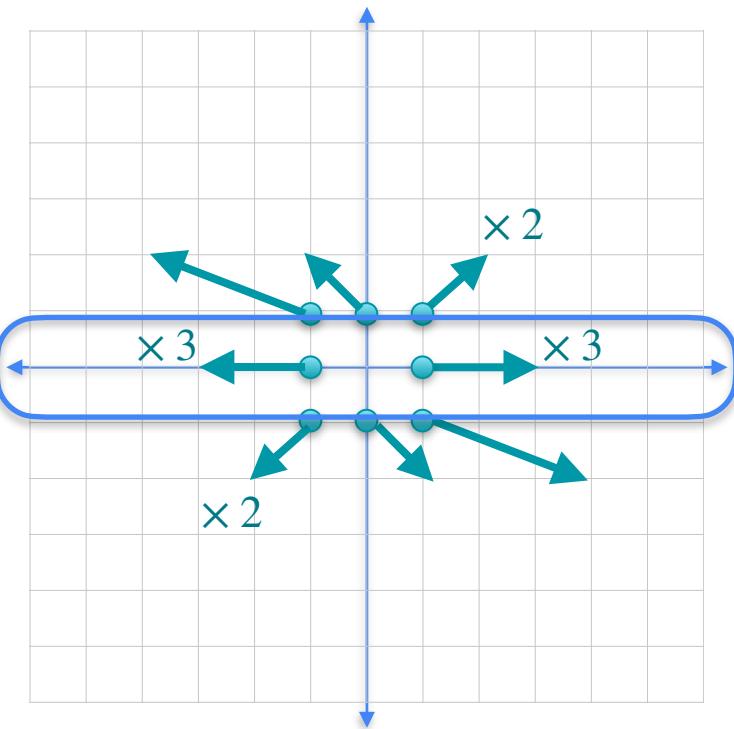


| | |
|---|---|
| 3 | 0 |
| 0 | 3 |

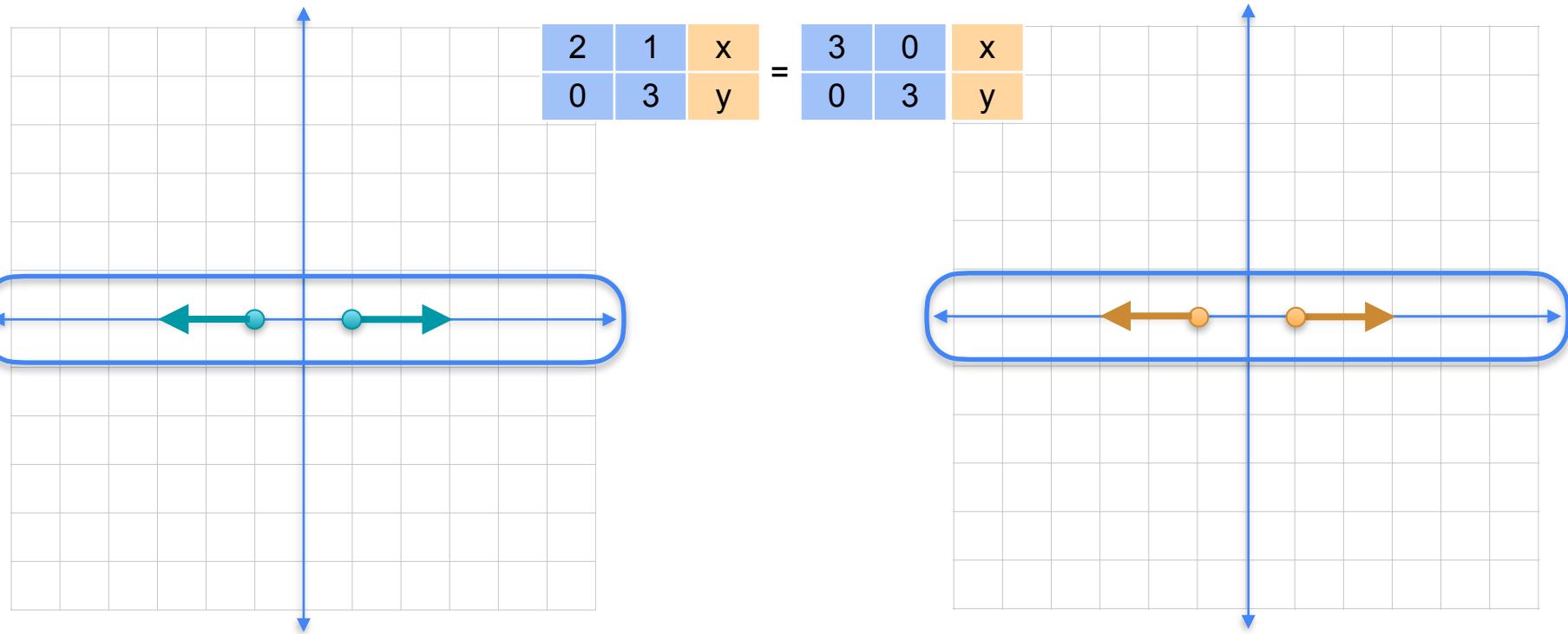


Find eigenvalues

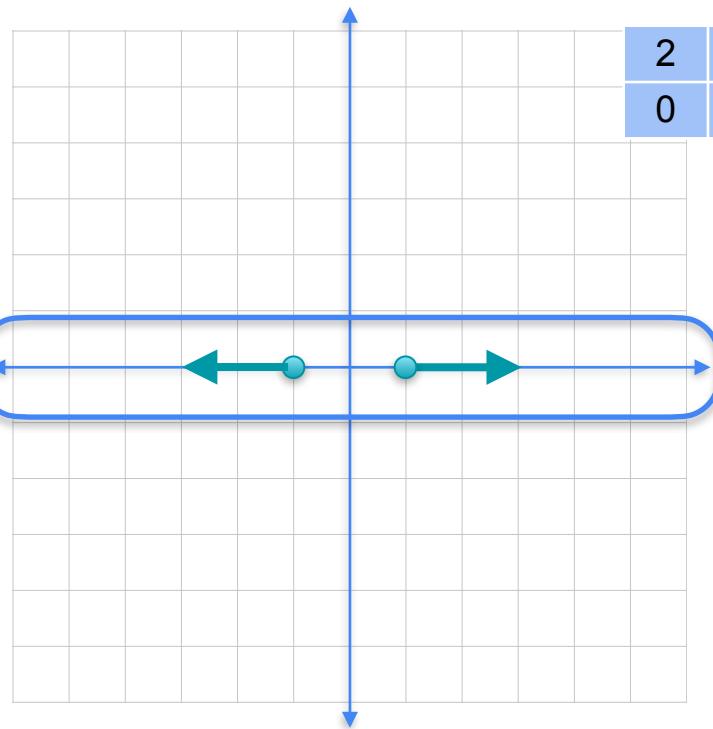
| | |
|---|---|
| 2 | 1 |
| 0 | 3 |



Finding eigenvalues

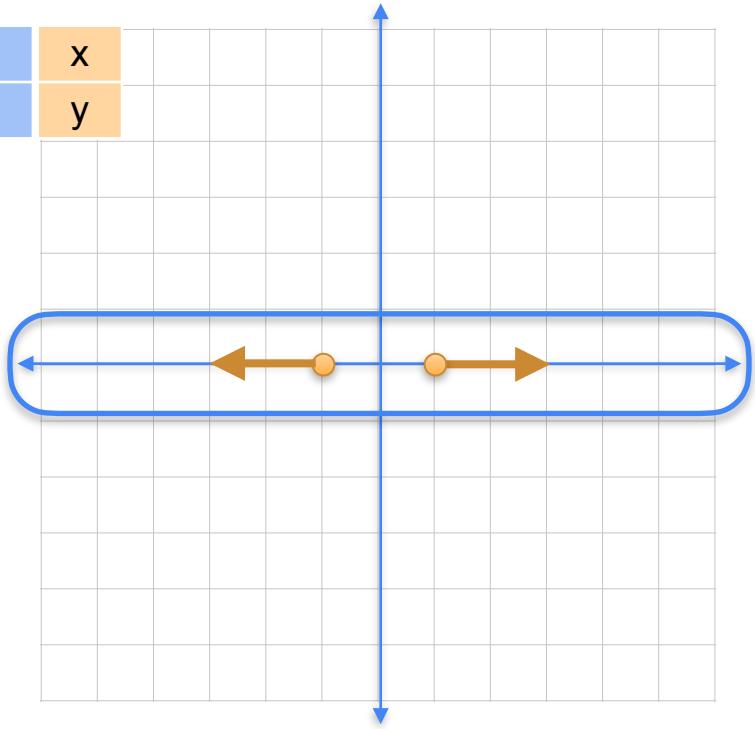


Finding eigenvalues

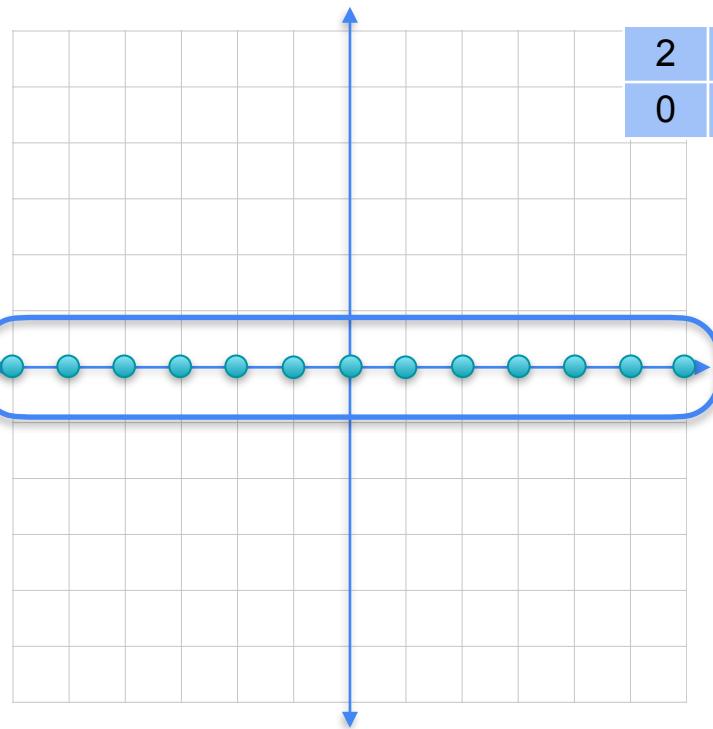


$$\begin{bmatrix} 2 & 1 & x \\ 0 & 3 & y \end{bmatrix} = \begin{bmatrix} 3 & 0 & x \\ 0 & 3 & y \end{bmatrix}$$

For infinitely
many points

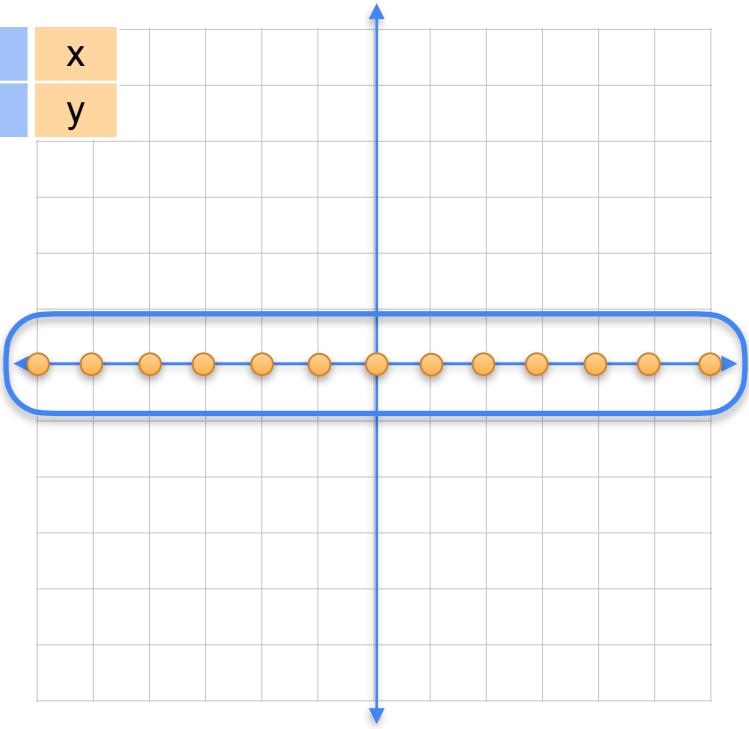


Finding eigenvalues

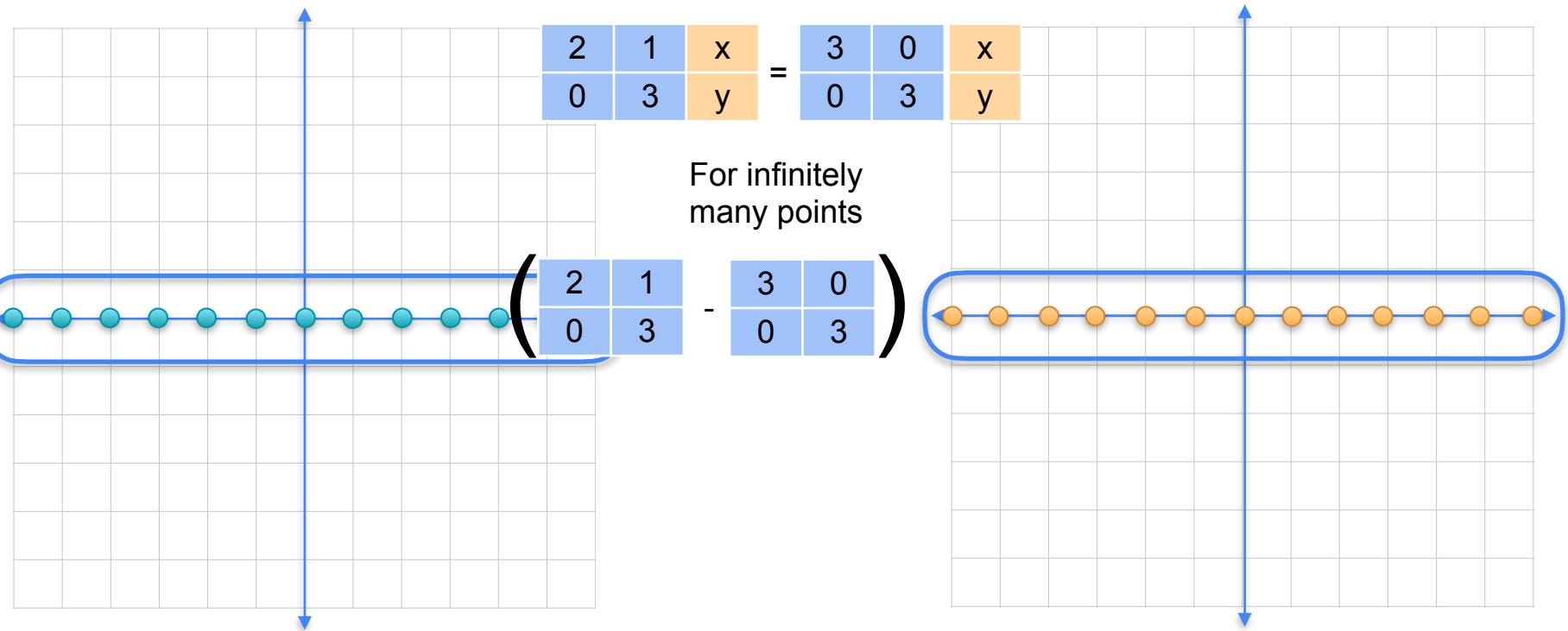


$$\begin{bmatrix} 2 & 1 & x \\ 0 & 3 & y \end{bmatrix} = \begin{bmatrix} 3 & 0 & x \\ 0 & 3 & y \end{bmatrix}$$

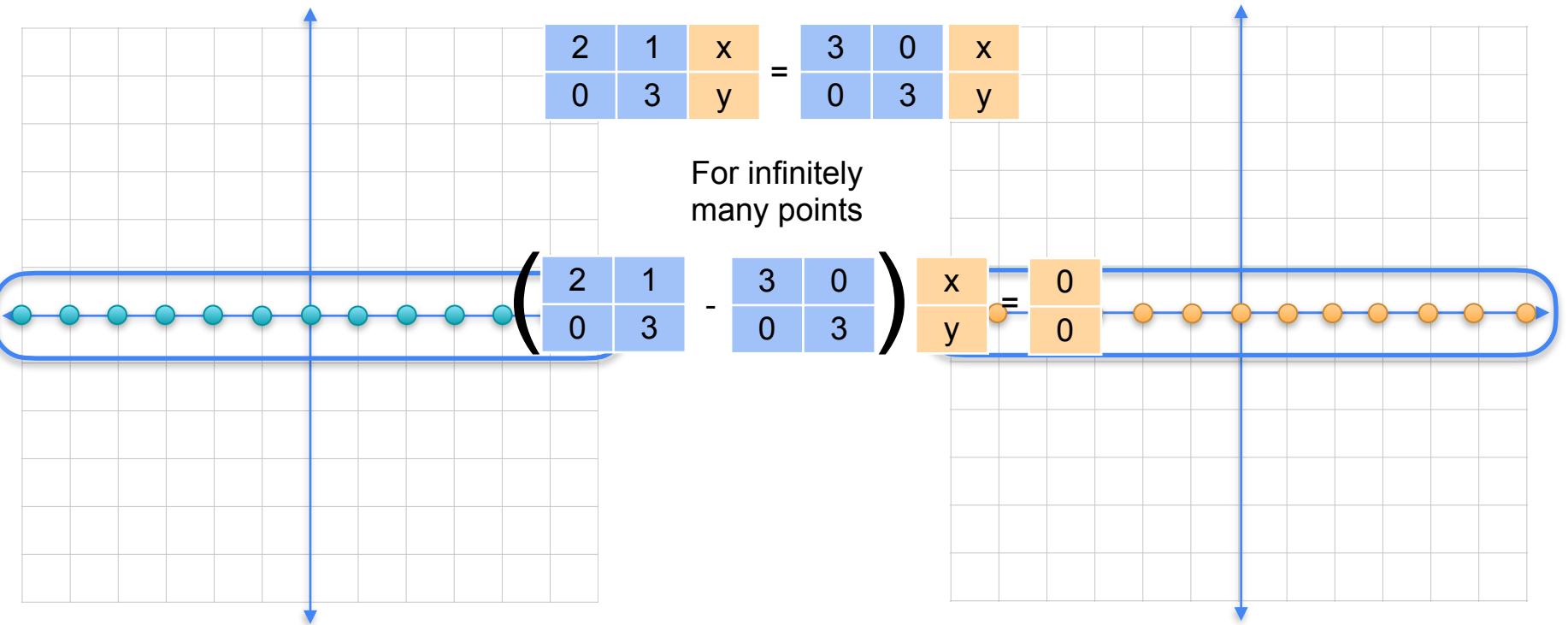
For infinitely
many points



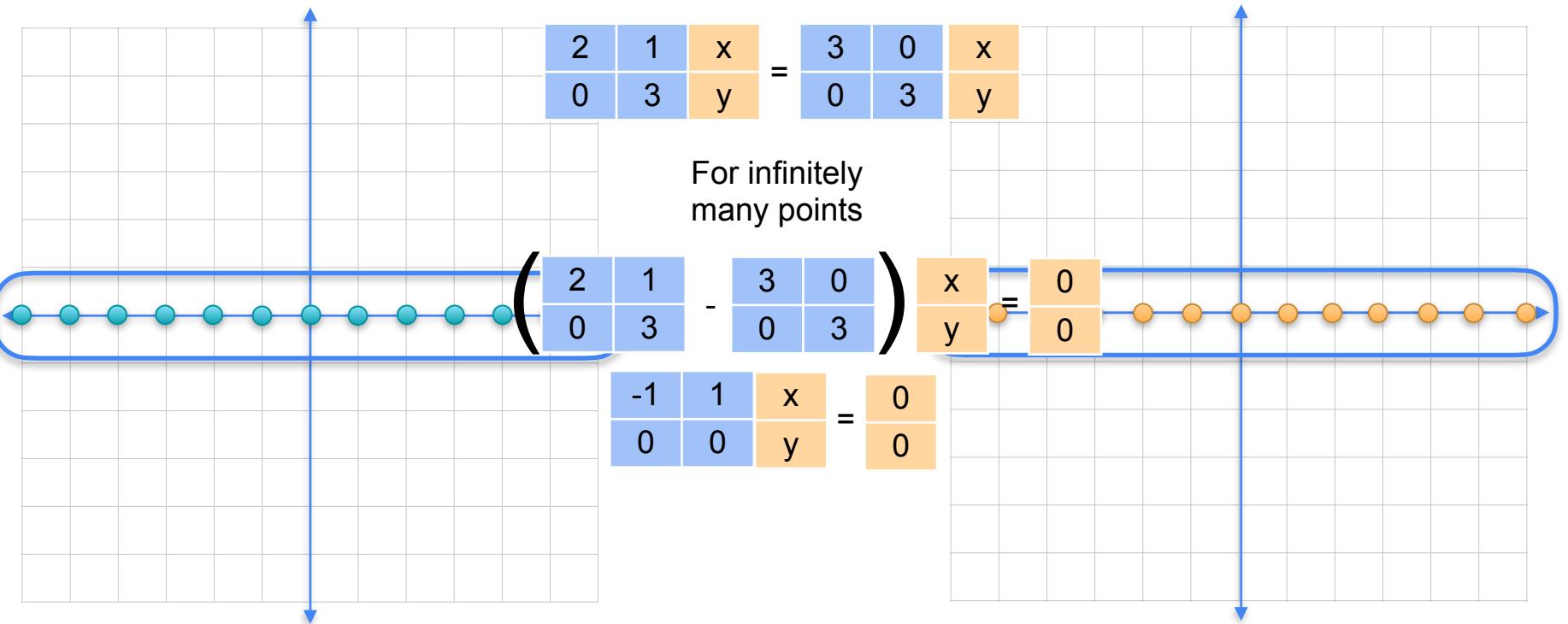
Finding eigenvalues



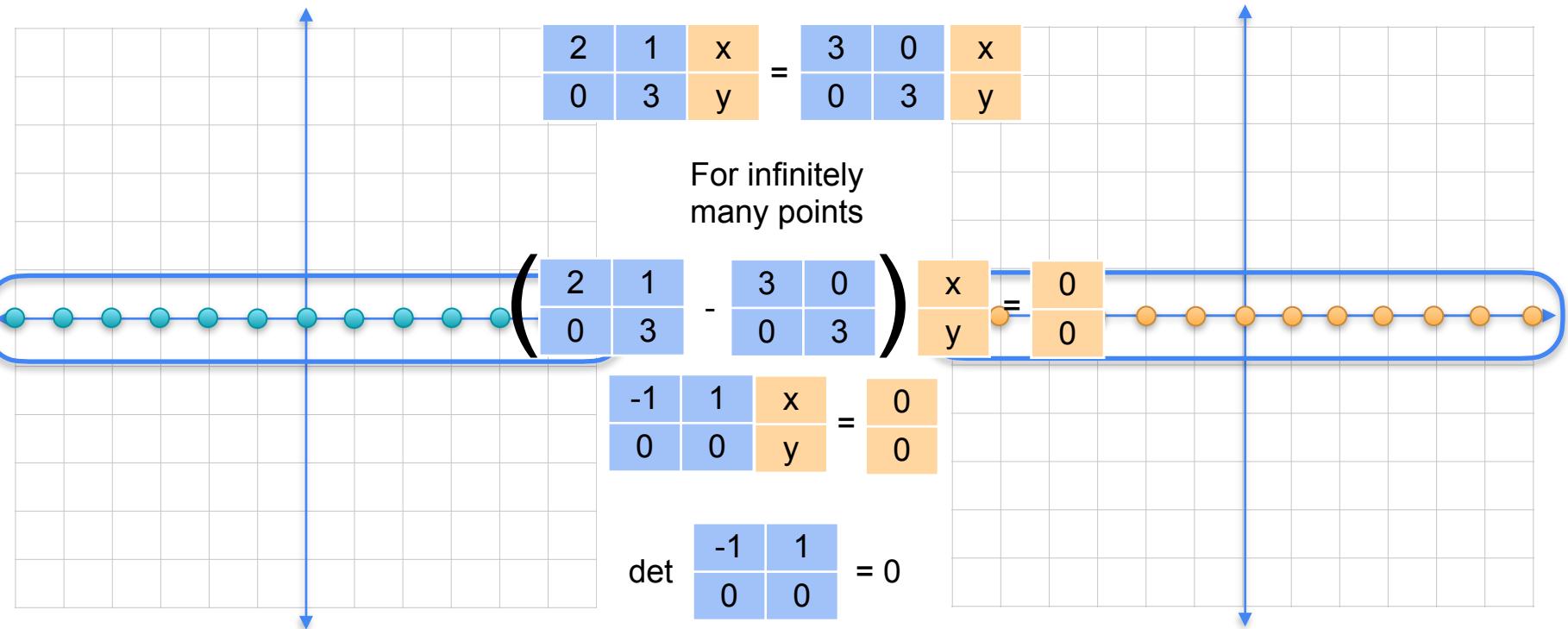
Finding eigenvalues



Finding eigenvalues

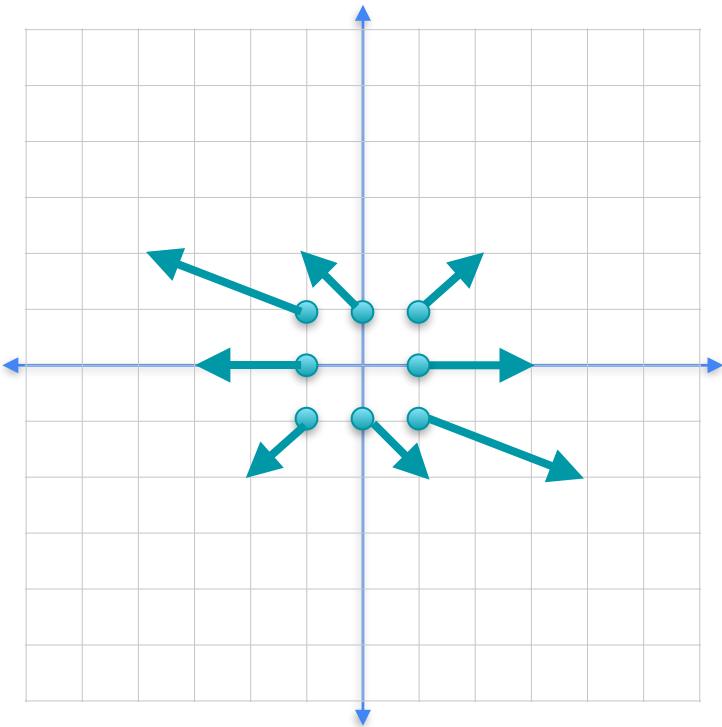


Finding eigenvalues

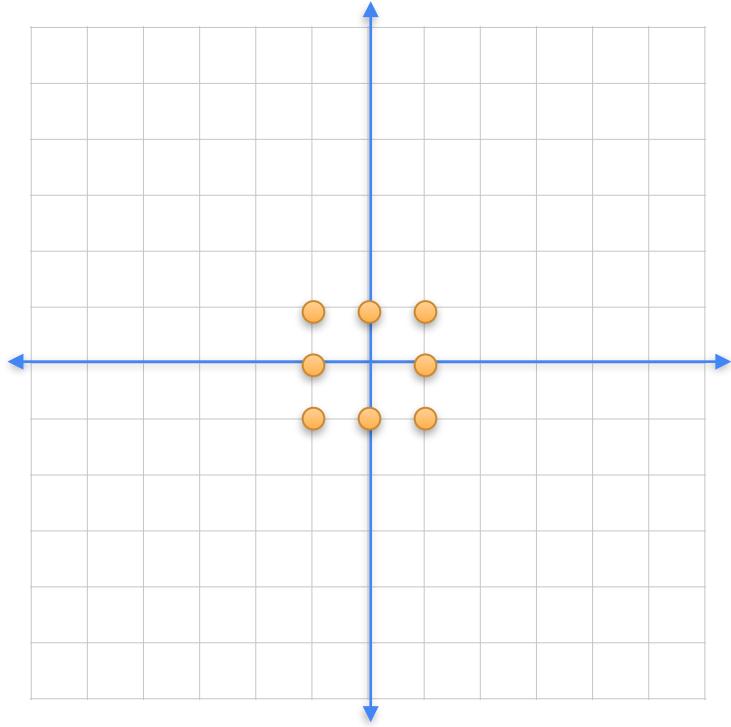


Finding eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

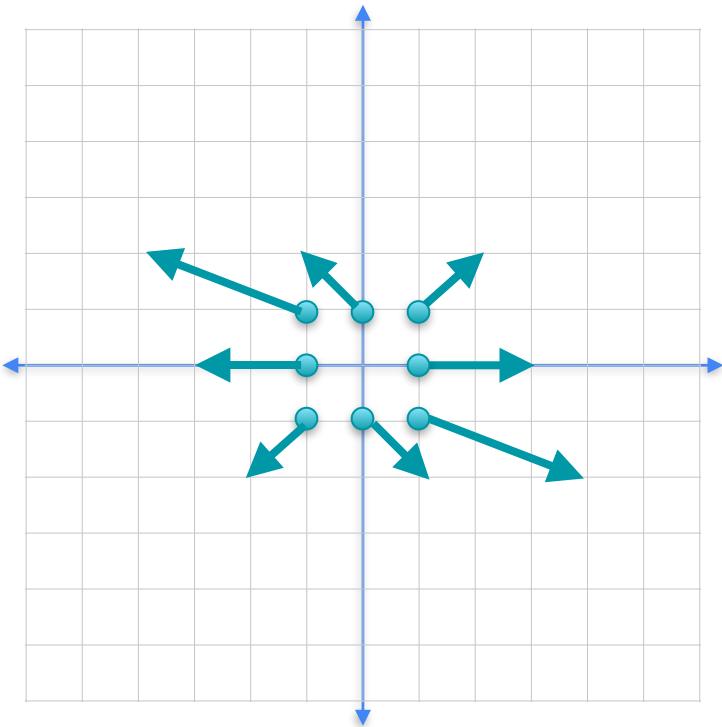


| | |
|---|---|
| 2 | 0 |
| 0 | 2 |

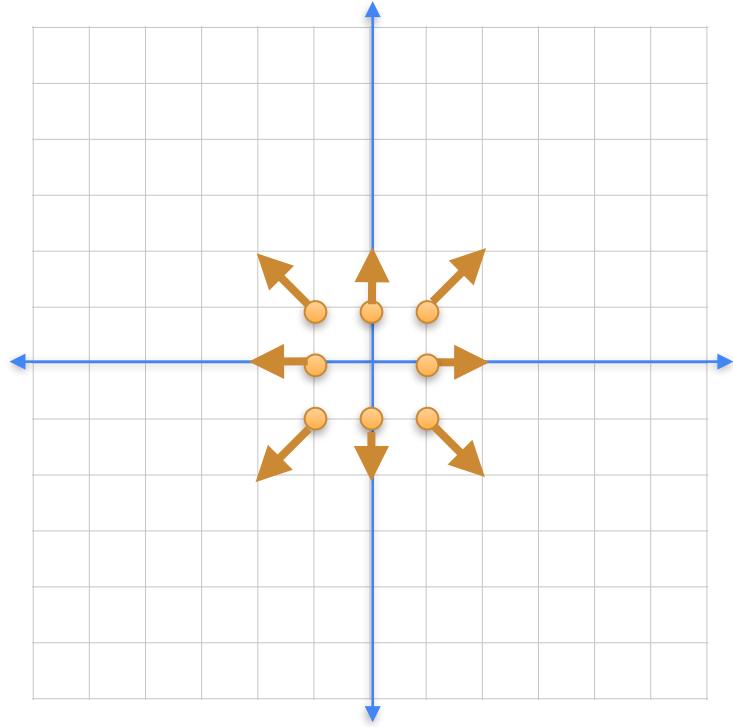


Finding eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

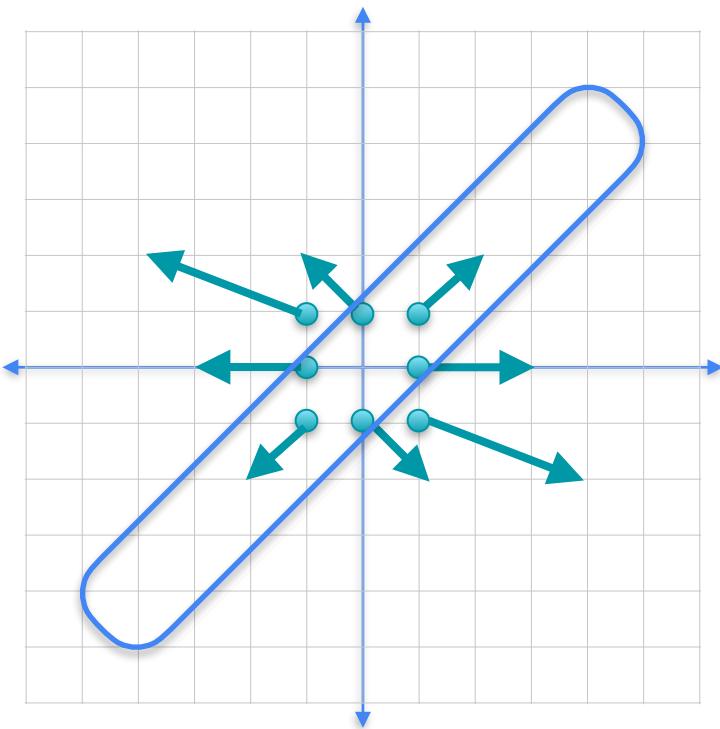


| | |
|---|---|
| 2 | 0 |
| 0 | 2 |

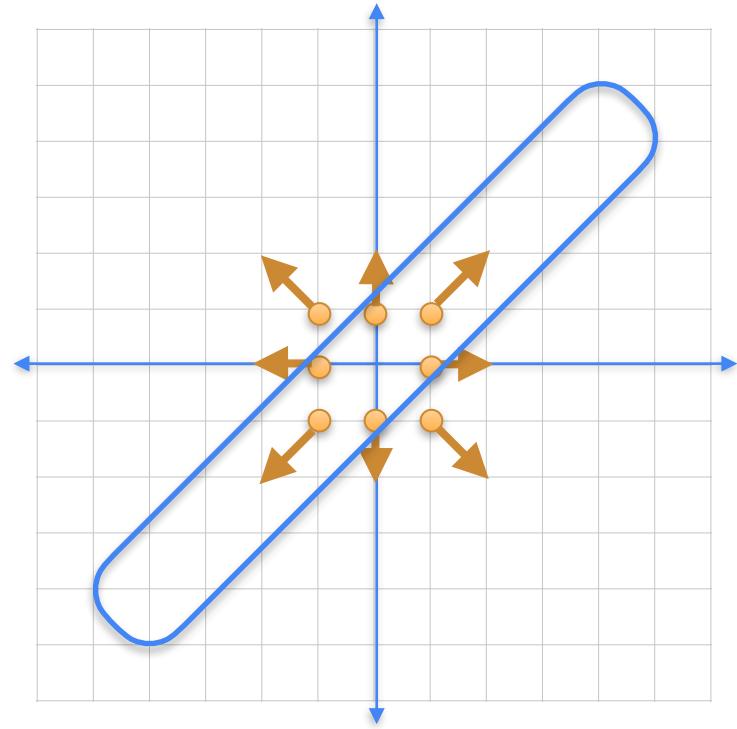


Find eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

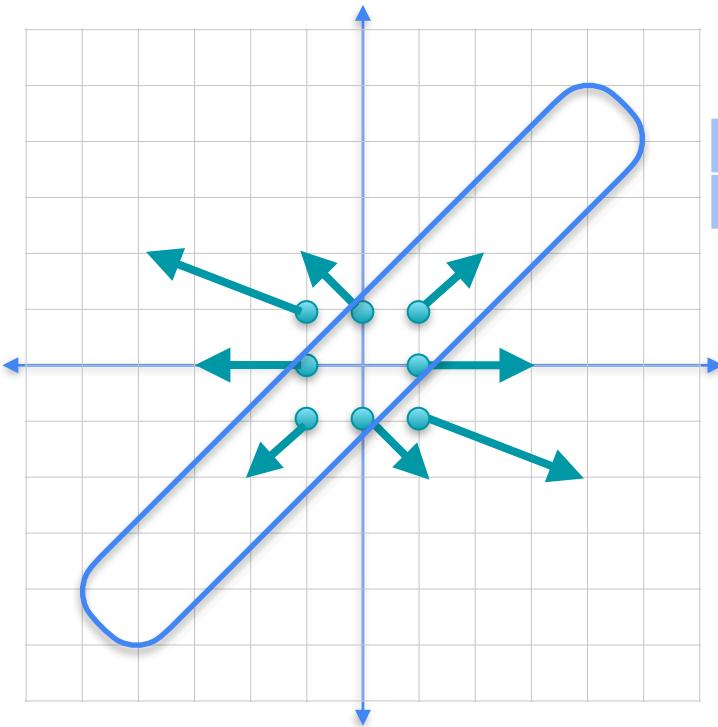


| | |
|---|---|
| 2 | 0 |
| 0 | 2 |



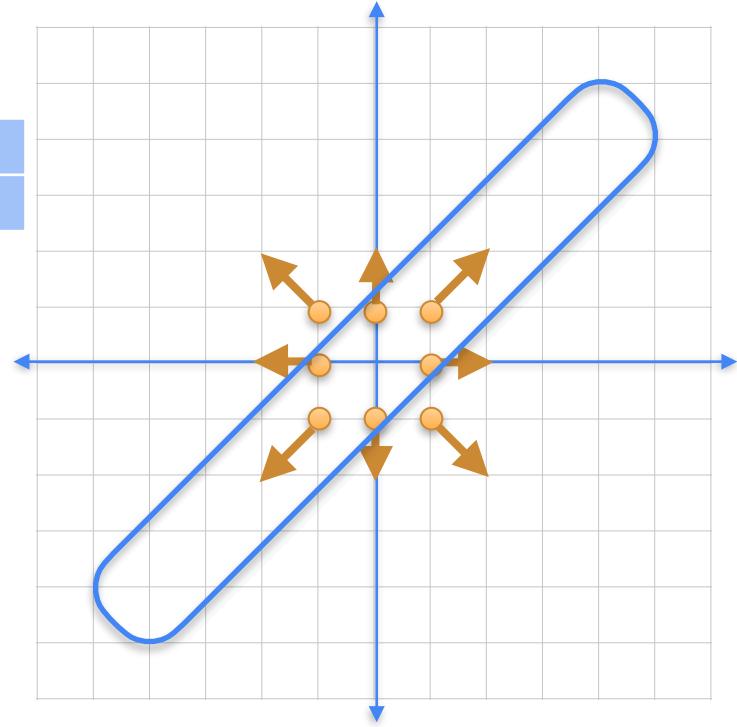
Finding eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |



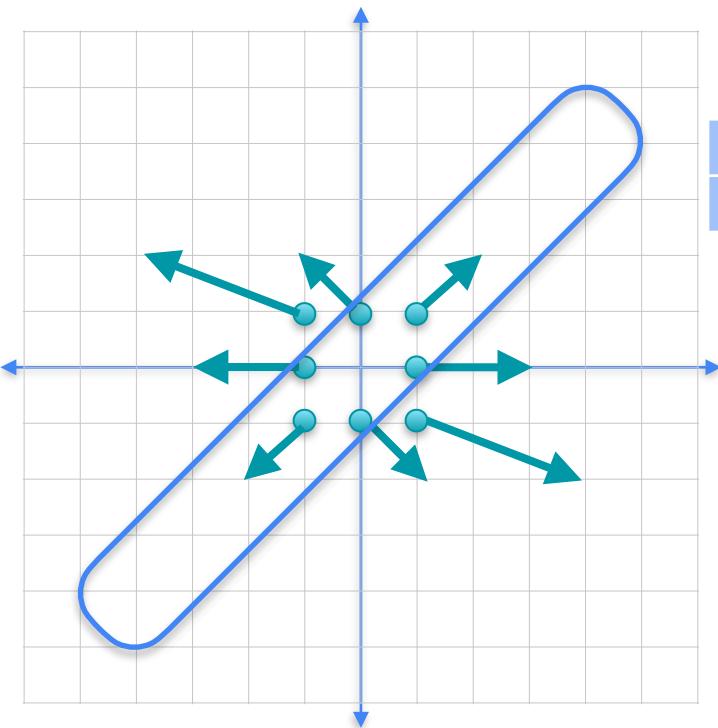
| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

| | |
|---|---|
| 2 | 0 |
| 0 | 2 |



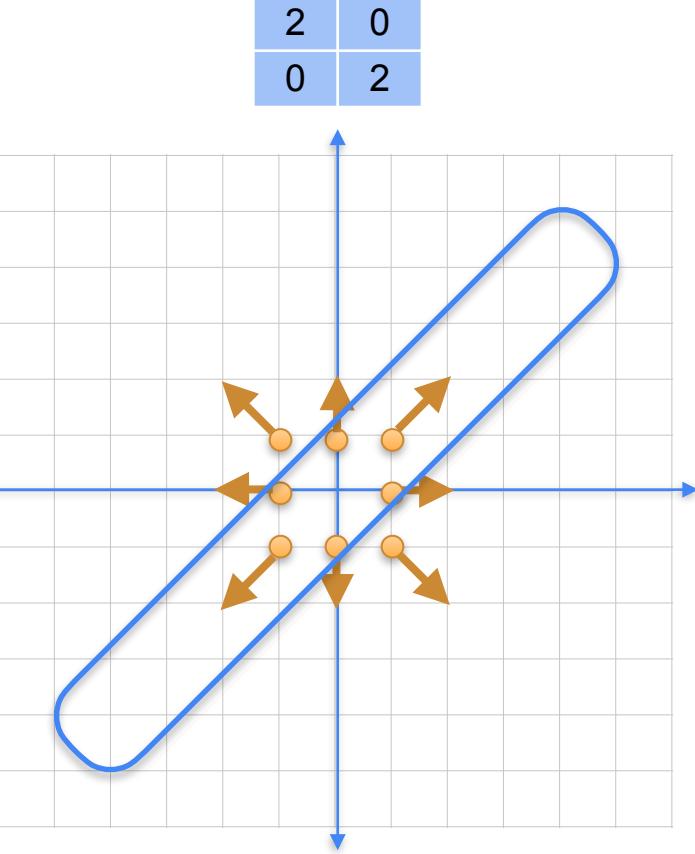
Finding eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |



| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

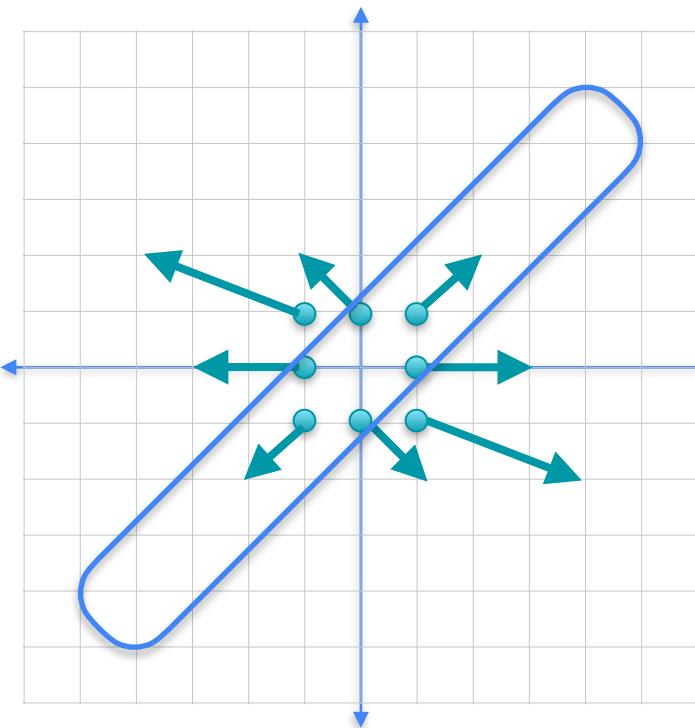
| | |
|---|---|
| 0 | 1 |
| 0 | 1 |



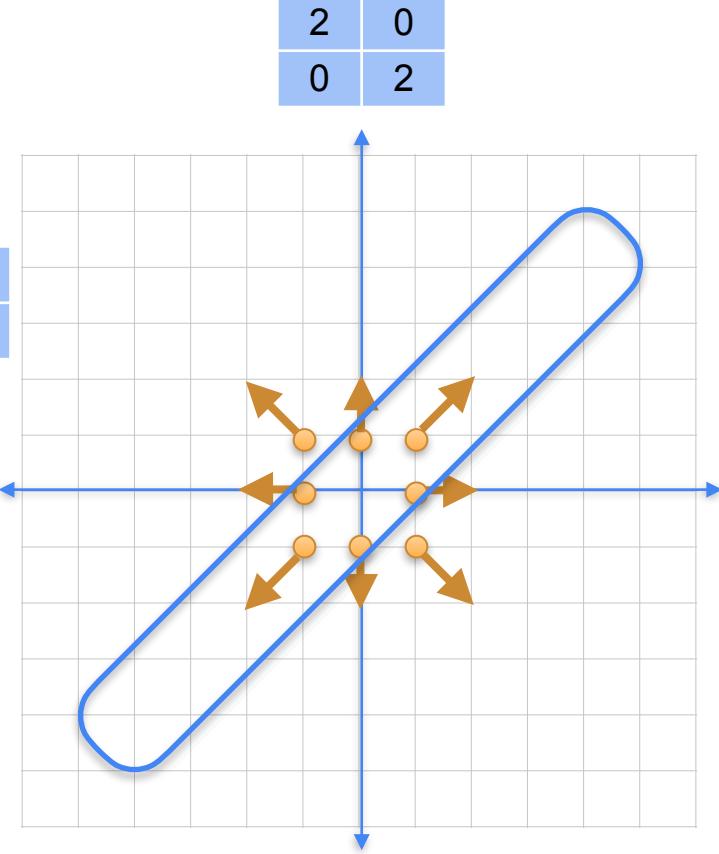
| | |
|---|---|
| 2 | 0 |
| 0 | 2 |

Finding eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

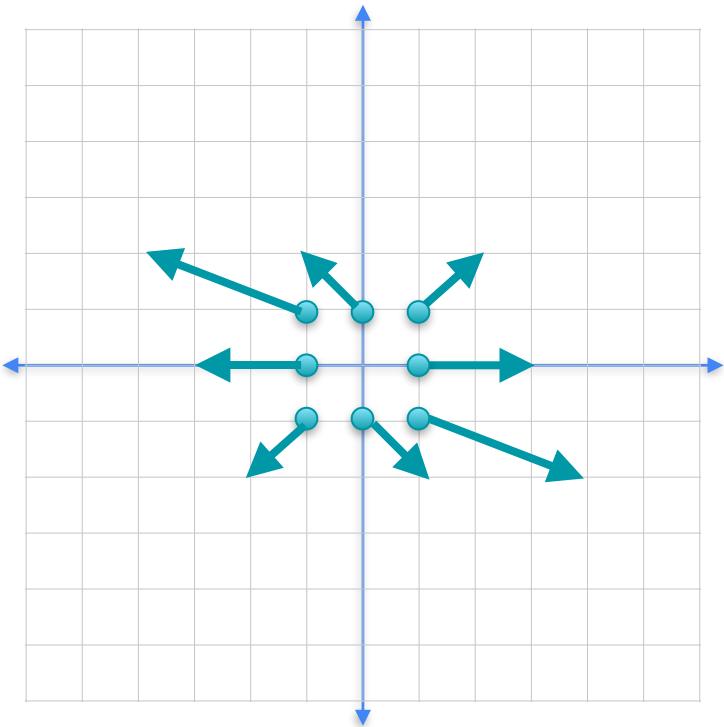


$$\begin{matrix} 2 & 1 \\ 0 & 3 \end{matrix} - \begin{matrix} 2 & 0 \\ 0 & 2 \end{matrix} = \det \begin{matrix} 0 & 1 \\ 0 & 1 \end{matrix} = 0$$

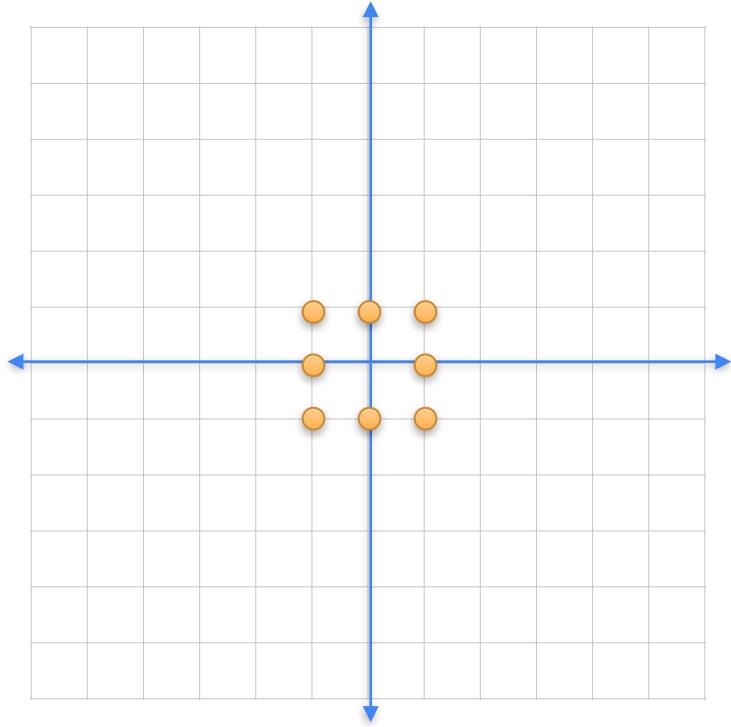


Finding eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

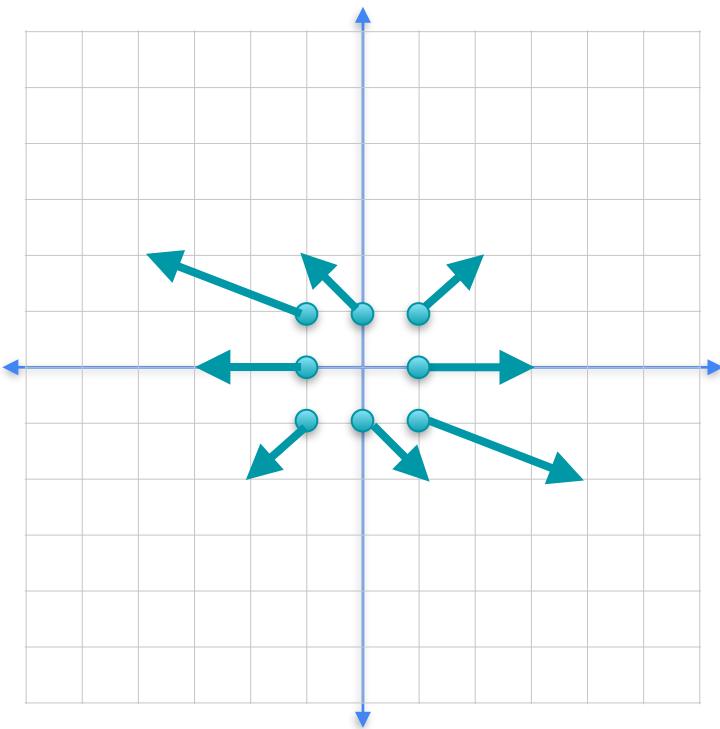


| | |
|---|---|
| 4 | 0 |
| 0 | 4 |

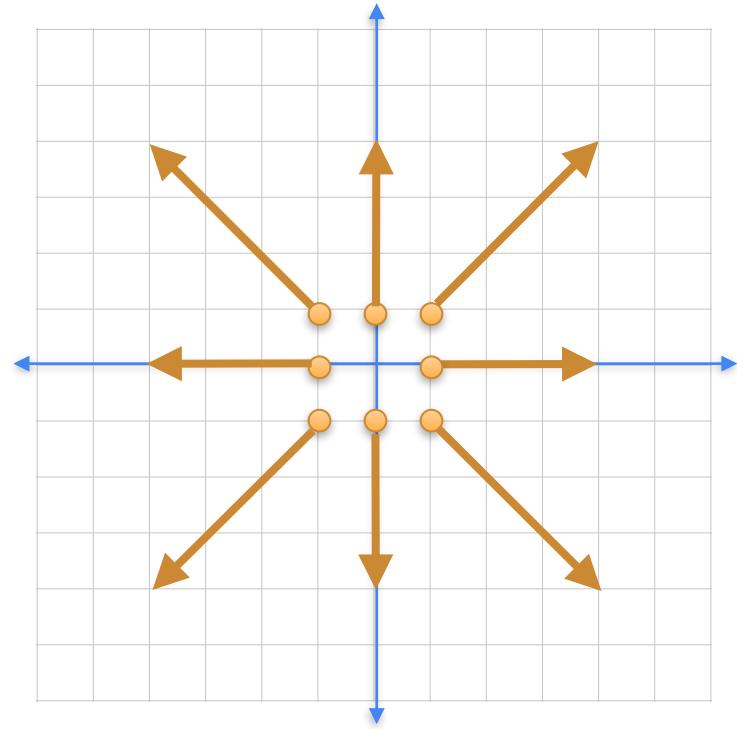


Find eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

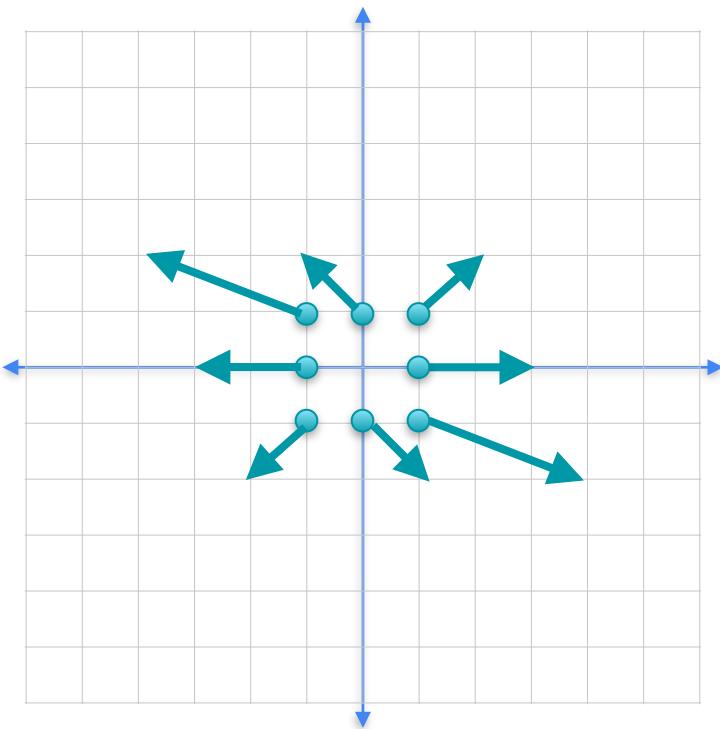


| | |
|---|---|
| 4 | 0 |
| 0 | 4 |

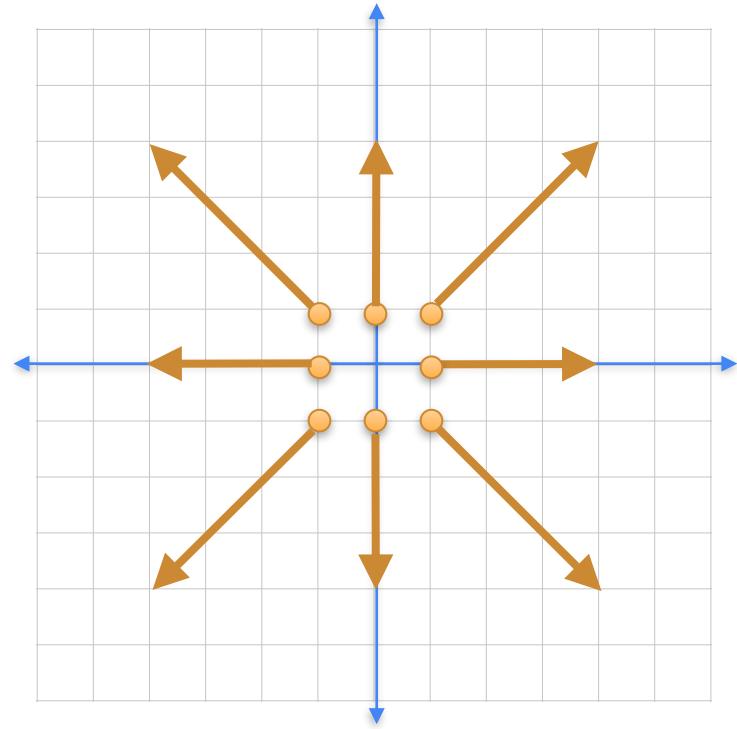


Find eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

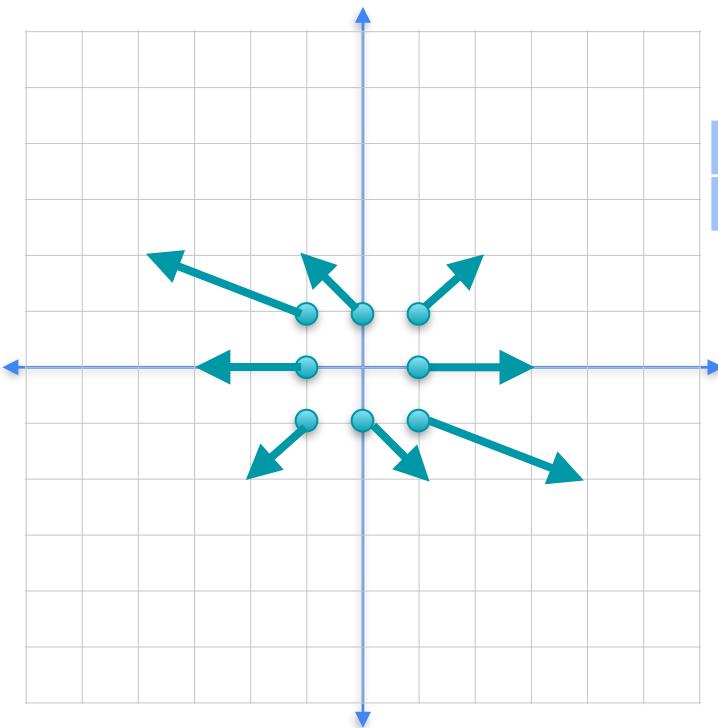


| | |
|---|---|
| 4 | 0 |
| 0 | 4 |



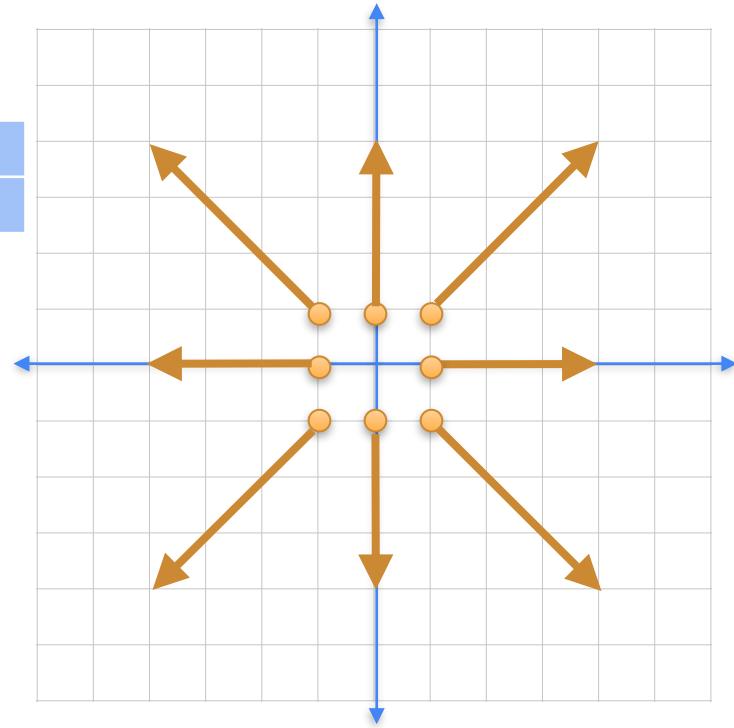
Finding eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |



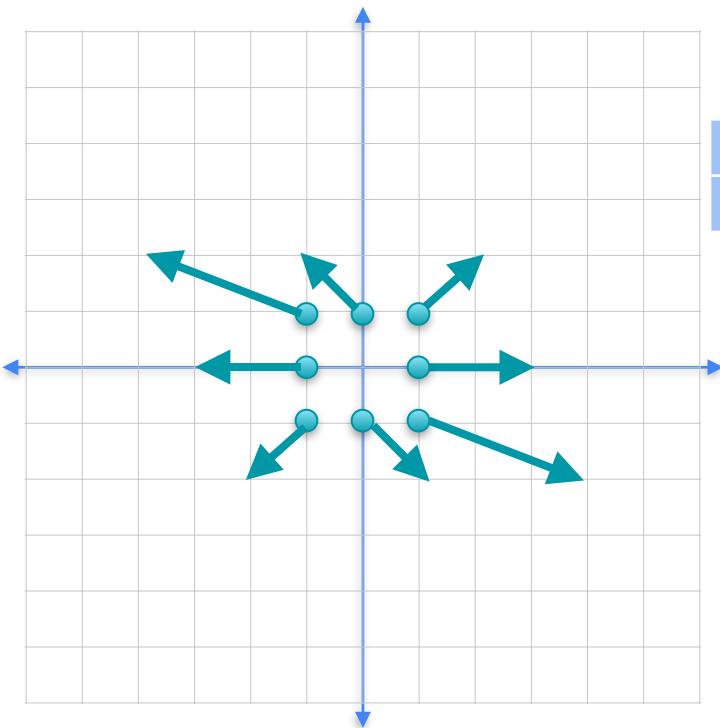
| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

| | |
|---|---|
| 4 | 0 |
| 0 | 4 |



Finding eigenvalues

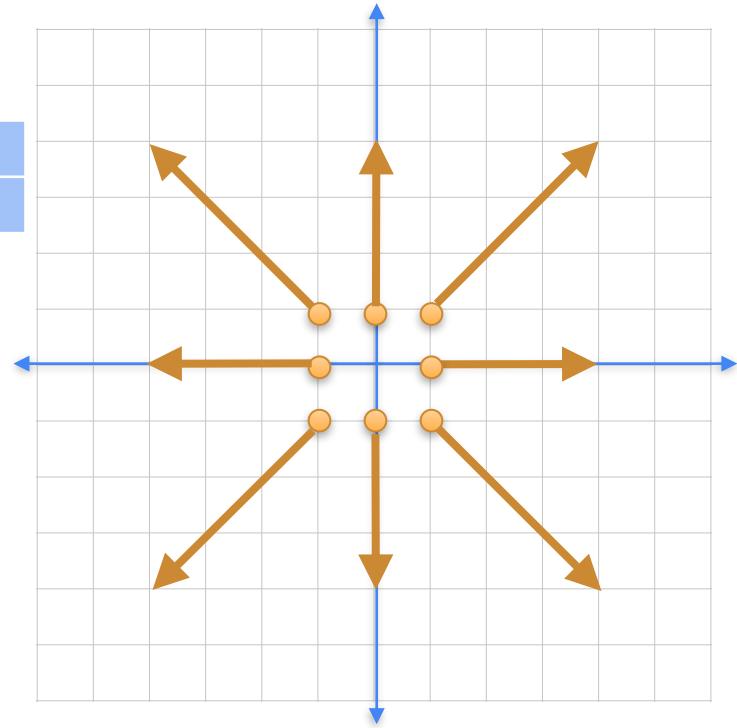
| | |
|---|---|
| 2 | 1 |
| 0 | 3 |



| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

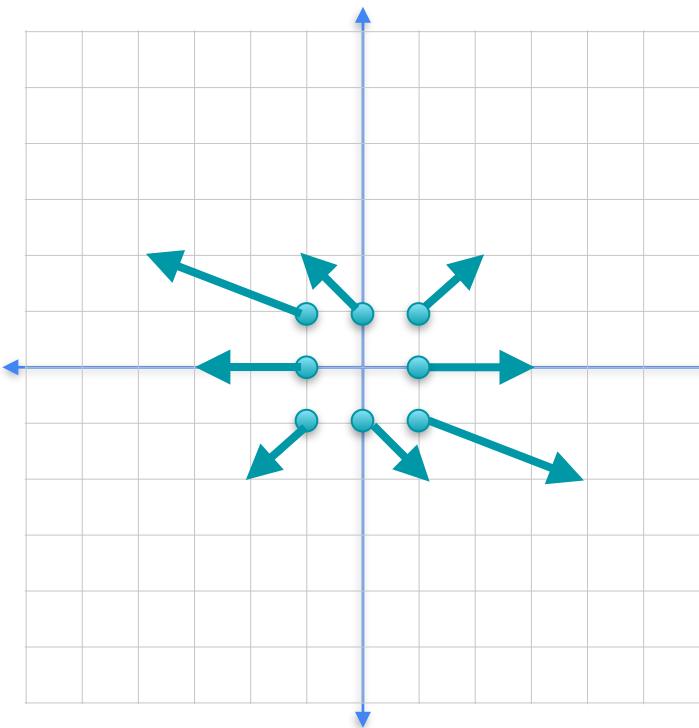
| | |
|----|----|
| -2 | 1 |
| 0 | -1 |

| | |
|---|---|
| 4 | 0 |
| 0 | 4 |



Finding eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |



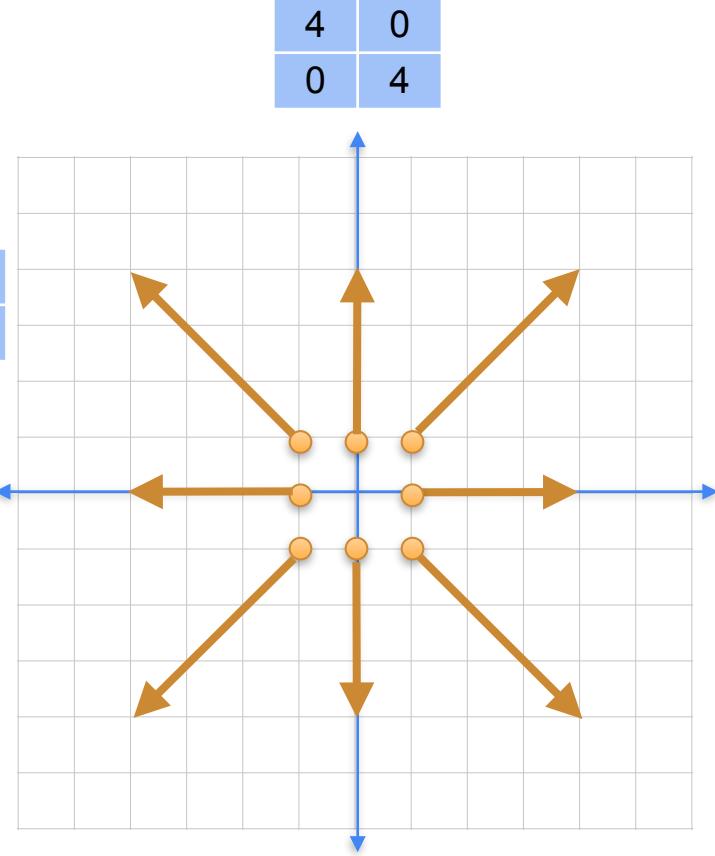
| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

| | |
|---|---|
| 4 | 0 |
| 0 | 4 |

det

| | |
|----|----|
| -2 | 1 |
| 0 | -1 |

$\neq 0$



Finding eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

Finding eigenvalues

| | |
|---|---|
| 2 | 1 |
| 0 | 3 |

| | |
|-----------|-----------|
| λ | 0 |
| 0 | λ |

Finding eigenvalues

$$\begin{matrix} 2 & 1 & x \\ 0 & 3 & y \end{matrix} = \begin{matrix} \lambda & 0 & x \\ 0 & \lambda & y \end{matrix}$$

Finding eigenvalues

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 & 0 \\ \hline \end{array}$$

Finding eigenvalues

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

Finding eigenvalues

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

$$(2 - \lambda)(3 - \lambda) - 1 \cdot 0 = 0$$

Finding eigenvalues

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

Characteristic polynomial $(2 - \lambda)(3 - \lambda) - 1 \cdot 0 = 0$

Finding eigenvalues

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

Characteristic polynomial

$$(2 - \lambda)(3 - \lambda) - 1 \cdot 0 = 0 \quad \begin{array}{l} \lambda = 2 \\ \lambda = 3 \end{array}$$

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

Characteristic polynomial

$$(2 - \lambda)(3 - \lambda) - 1 \cdot 0 = 0 \quad \begin{array}{l} \lambda = 2 \\ \lambda = 3 \end{array}$$

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

For infinitely many (x,y)

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

Characteristic polynomial

$$(2 - \lambda)(3 - \lambda) - 1 \cdot 0 = 0$$

$$\begin{aligned}\lambda &= 2 \\ \lambda &= 3\end{aligned}$$

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

For infinitely many (x,y)

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

Has infinitely many solutions

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

Characteristic polynomial

$$(2 - \lambda)(3 - \lambda) - 1 \cdot 0 = 0$$

$$\lambda = 2$$

$$\lambda = 3$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{matrix} 2 & 1 & x \\ 0 & 3 & y \end{matrix} = 2 \begin{matrix} x \\ y \end{matrix}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
$$0x + 3y = 2y$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$

$$2x + y = 2x$$

$$x = 1$$

$$0x + 3y = 2y$$

$$y = 0$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$

$$2x + y = 2x$$

$$x = 1$$

$$0x + 3y = 2y$$

$$y = 0$$

$$\begin{array}{|c|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
$$0x + 3y = 2y$$
$$x = 1$$
$$y = 0$$
$$\begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 3 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
$$0x + 3y = 2y$$
$$x = 1$$
$$y = 0$$
$$\begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 3 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 3x$$
$$0x + 3y = 3y$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
$$0x + 3y = 2y$$
$$x = 1$$
$$y = 0$$
$$\begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 3 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 3x$$
$$0x + 3y = 3y$$
$$x = 1$$
$$y = 1$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
$$0x + 3y = 2y$$
$$x = 1$$
$$y = 0$$
$$\begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 3 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 3x$$
$$0x + 3y = 3y$$
$$x = 1$$
$$y = 1$$
$$\begin{array}{|c|} \hline 1 \\ \hline 1 \\ \hline \end{array}$$

Quiz

- Find the eigenvalues and eigenvectors of this matrix:

| | |
|---|---|
| 9 | 4 |
| 4 | 3 |

Solution

- Eigenvalues: 11, 1
- Eigenvectors: (2,1), (-1,2)

| | |
|---|---|
| 9 | 4 |
| 4 | 3 |

- The characteristic polynomial is

$$\det \begin{vmatrix} 9-\lambda & 4 \\ 4 & 3-\lambda \end{vmatrix} = (9 - \lambda)(3 - \lambda) - 4 \cdot 4 = 0$$

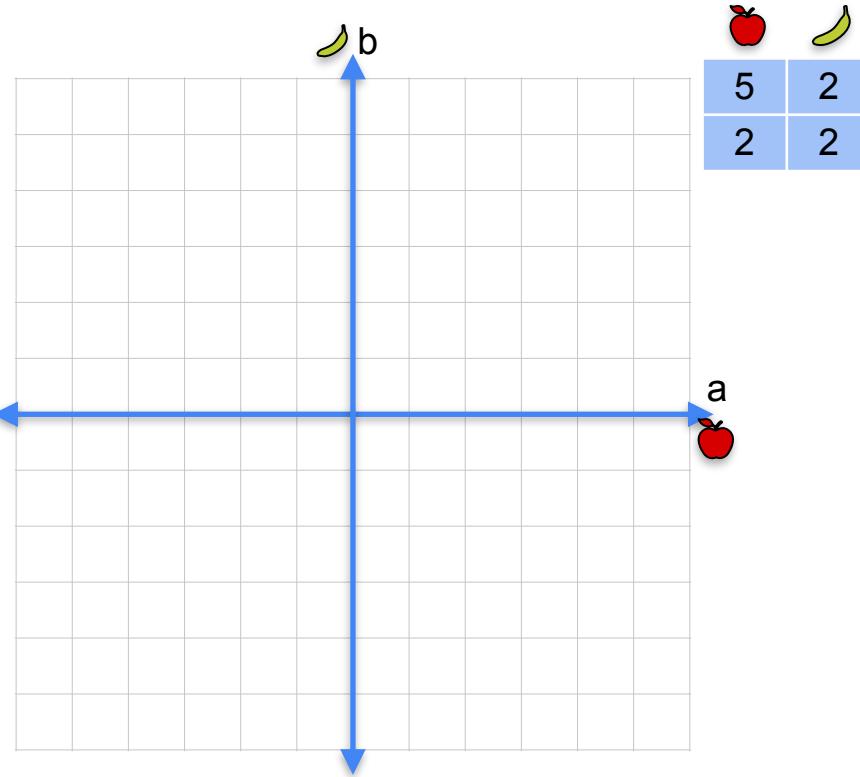
- Which factors as $\lambda^2 - 12\lambda + 11 = (\lambda - 11)(\lambda - 1)$

- The solutions are $\lambda = 11$
 $\lambda = 1$

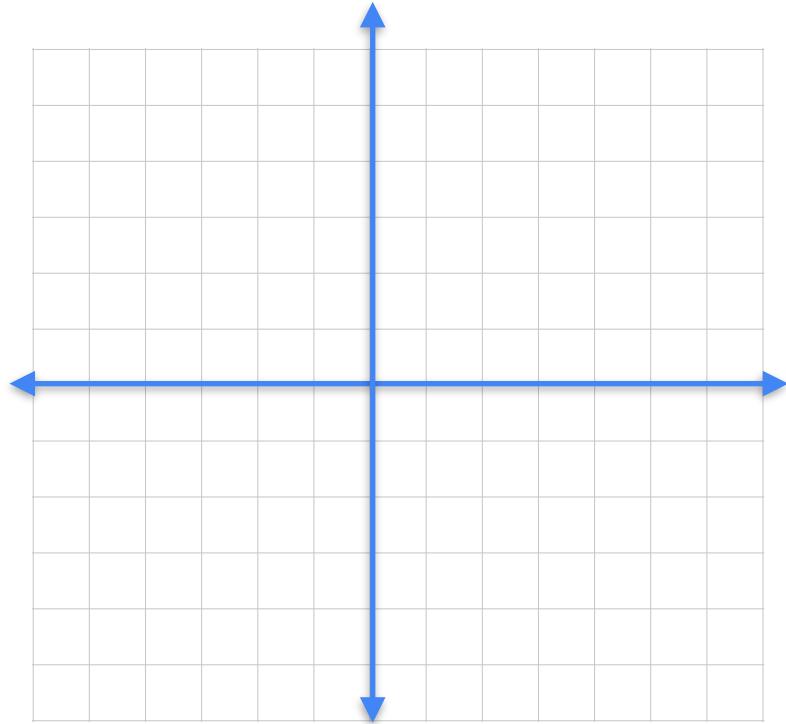
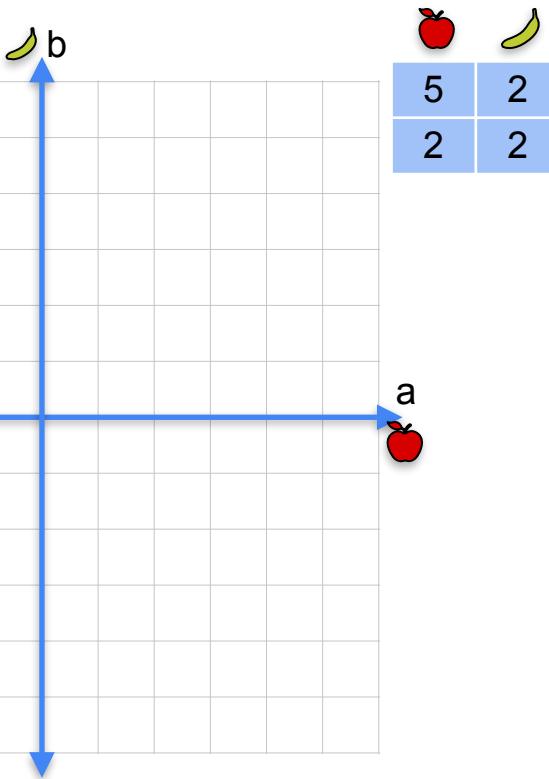
Matrices as linear transformations

| | |
|---|---|
|  |  |
| 5 | 2 |
| 2 | 2 |

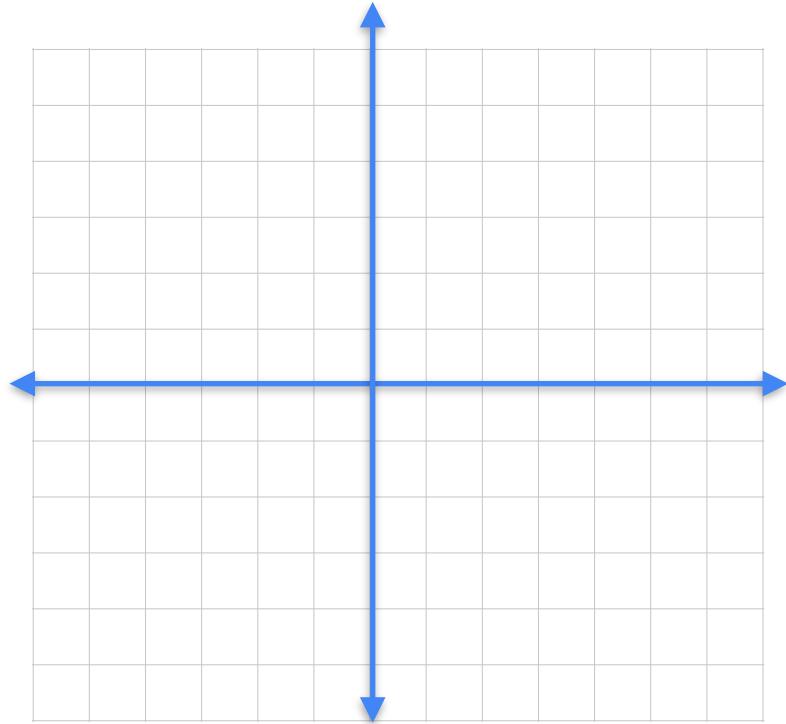
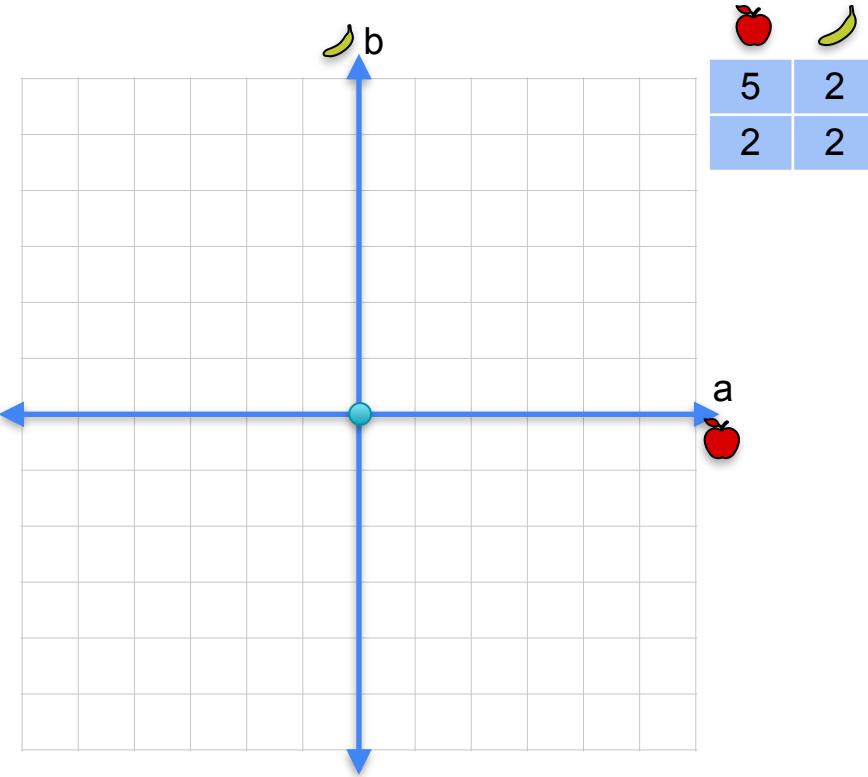
Matrices as linear transformations



Matrices as linear transformations

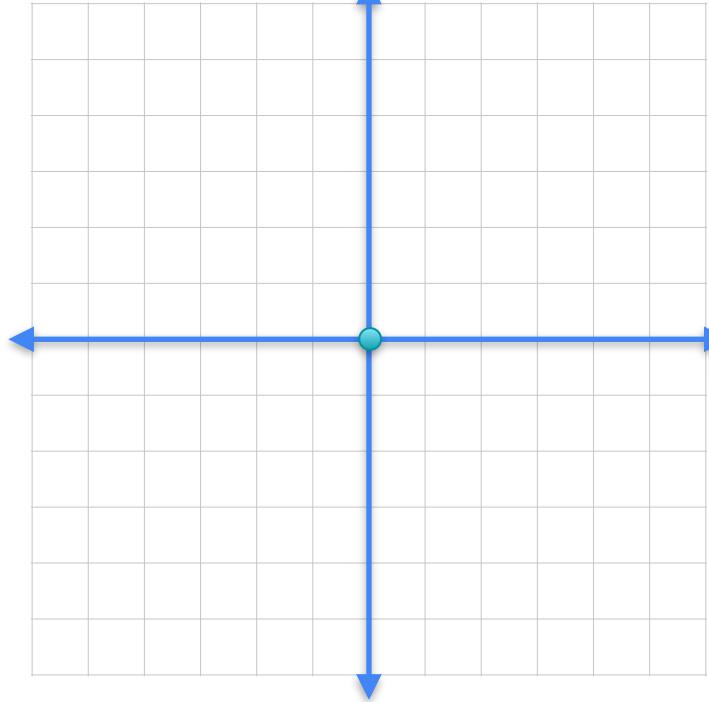


Matrices as linear transformations



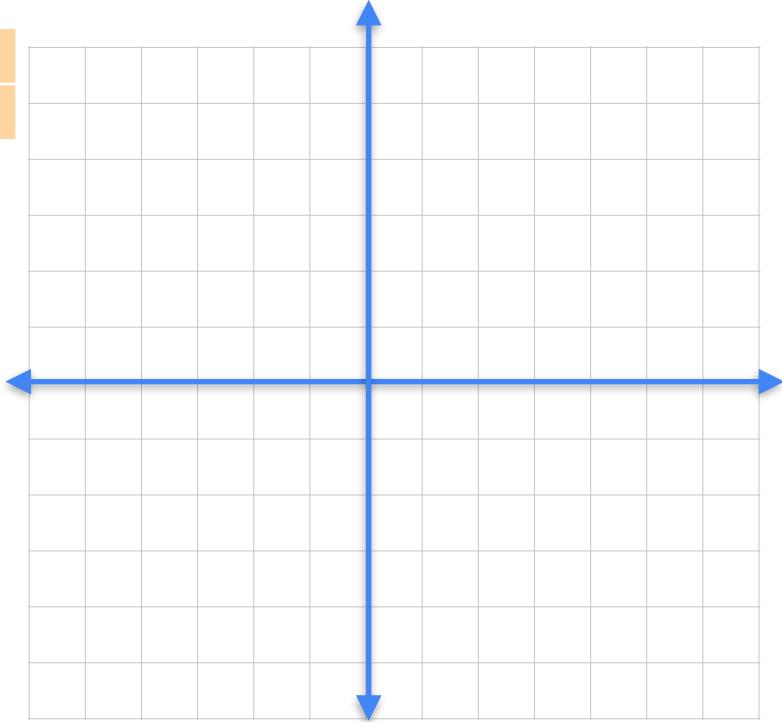
Matrices as linear transformations

b



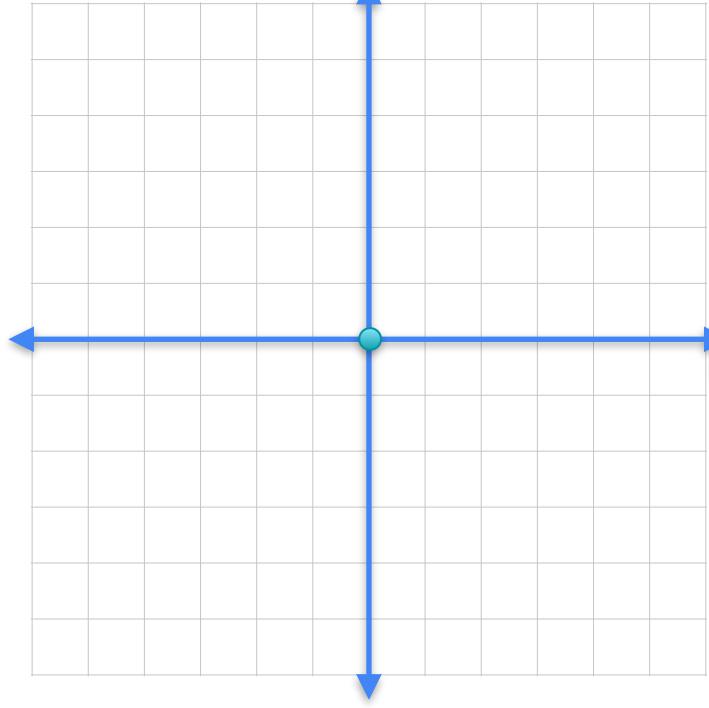
$$\begin{matrix} \text{apple} & \text{banana} \\ 5 & 2 \\ 2 & 2 \end{matrix} \times \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$(0,0) \rightarrow (0,0)$



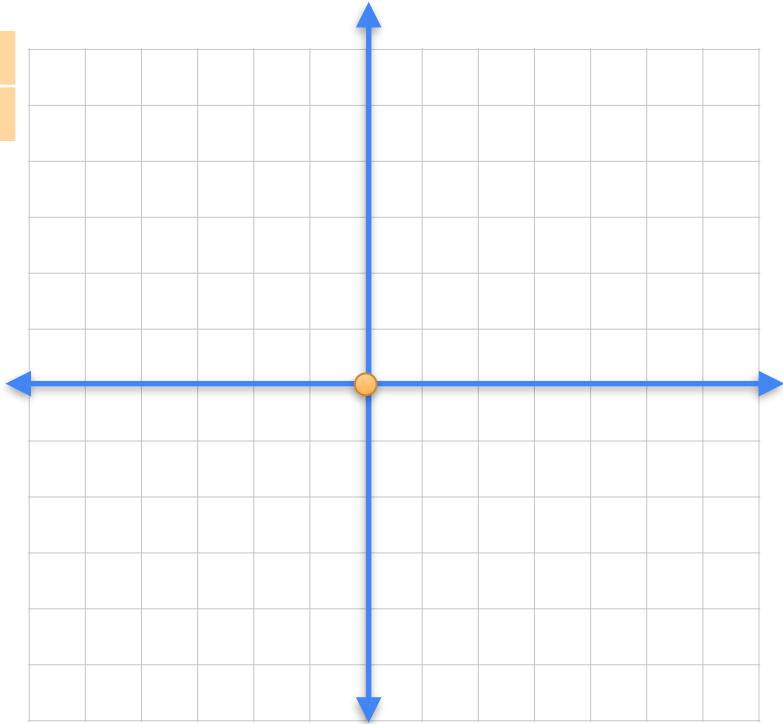
Matrices as linear transformations

b



$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 \\ 2 & 2 \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

$(0,0) \rightarrow (0,0)$



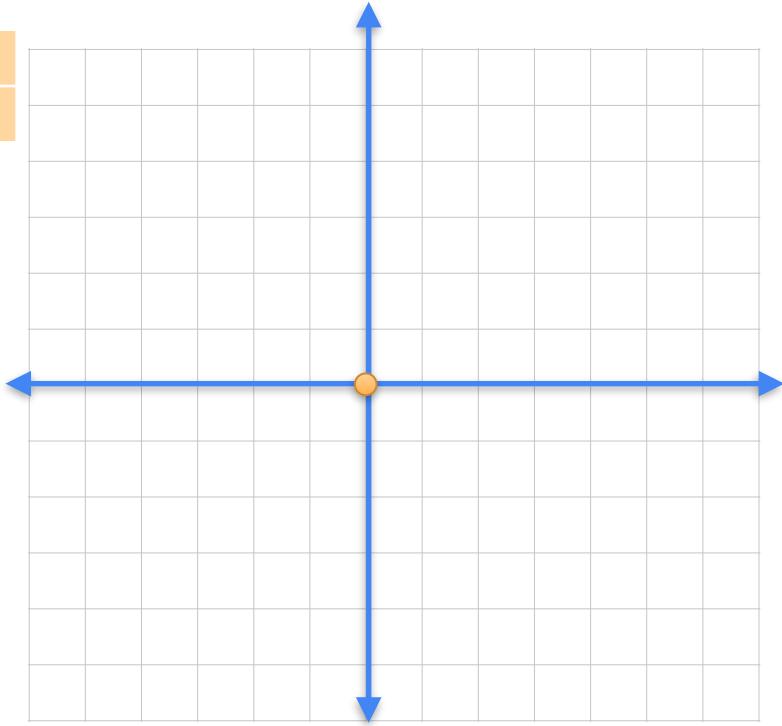
Matrices as linear transformations

b

a

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 \\ 2 & 2 \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

$(0,0) \rightarrow (0,0)$



Matrices as linear transformations

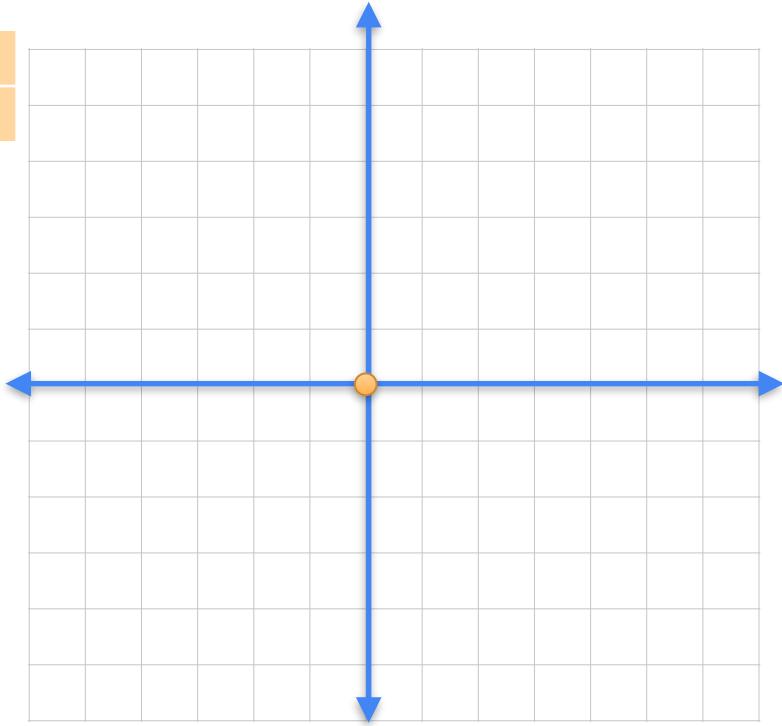
b

a

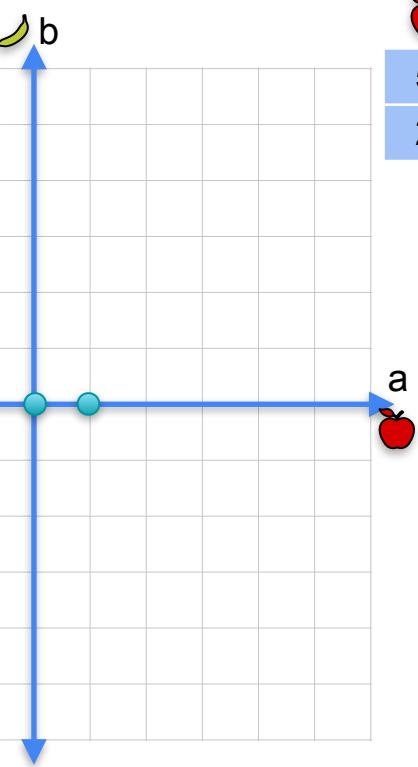
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 \\ 2 & 2 \end{matrix} & \begin{matrix} 1 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$(0,0) \rightarrow (0,0)$$

$$(1,0) \rightarrow (3,1)$$

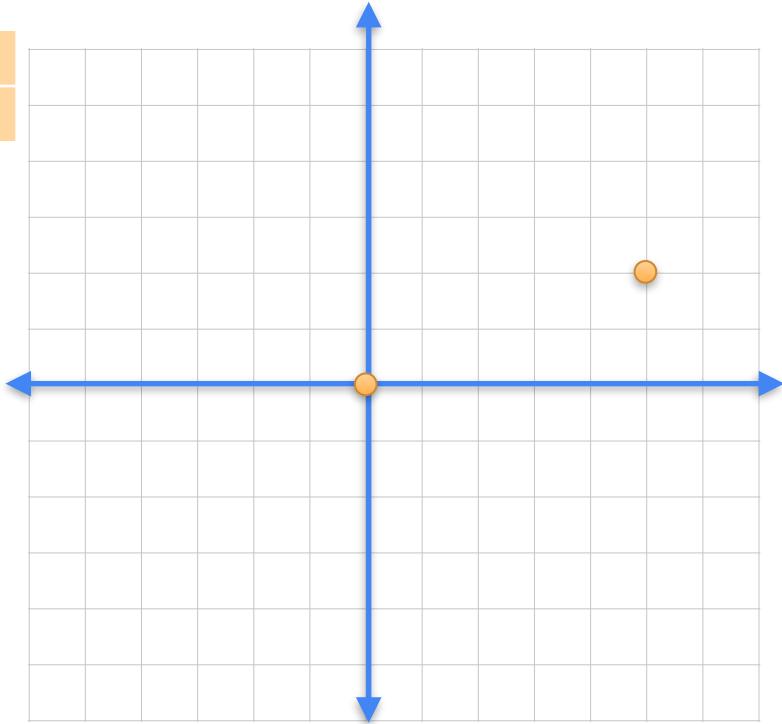


Matrices as linear transformations

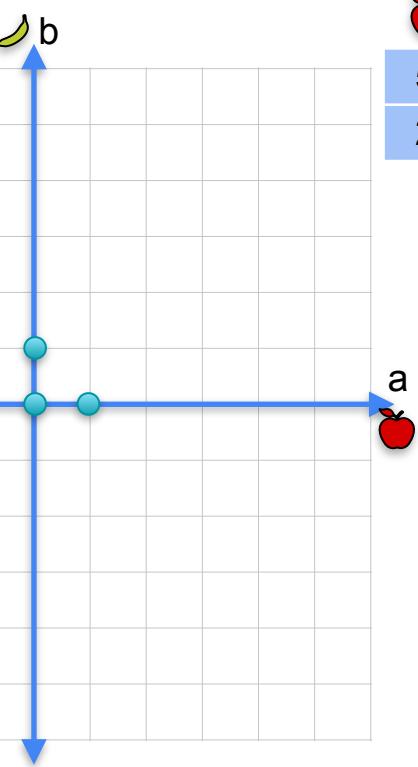


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 \\ 2 & 2 \end{matrix} & \begin{matrix} 1 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \end{aligned}$$

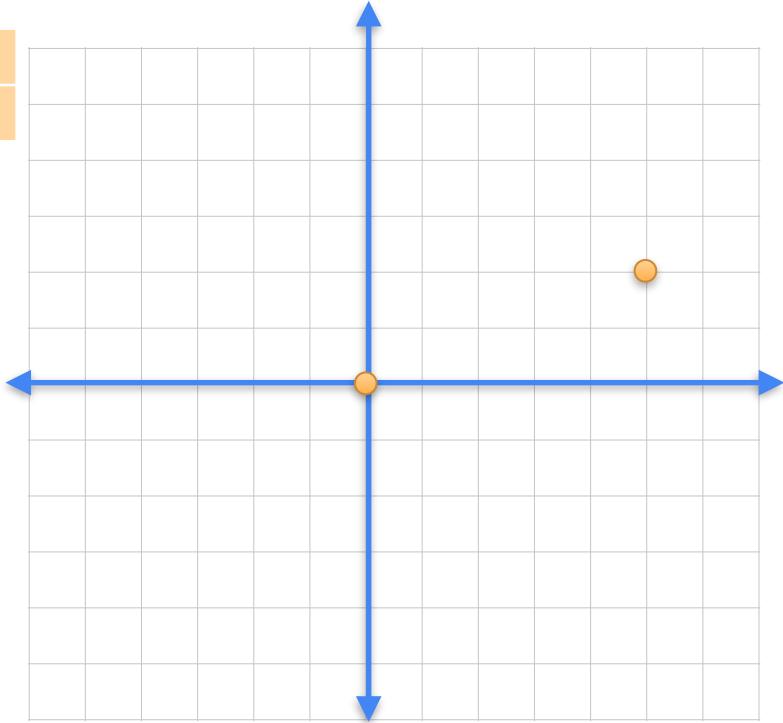


Matrices as linear transformations



$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 \\ 2 & 2 \end{matrix} & \begin{matrix} 1 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \end{aligned}$$



Matrices as linear transformations

b

a

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 & 0 \\ 2 & 2 & 1 \end{matrix} & = \begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 2 & 2 \\ 2 & 2 \end{matrix} \end{matrix} \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \\ (0,1) &\rightarrow (1,2) \end{aligned}$$

o

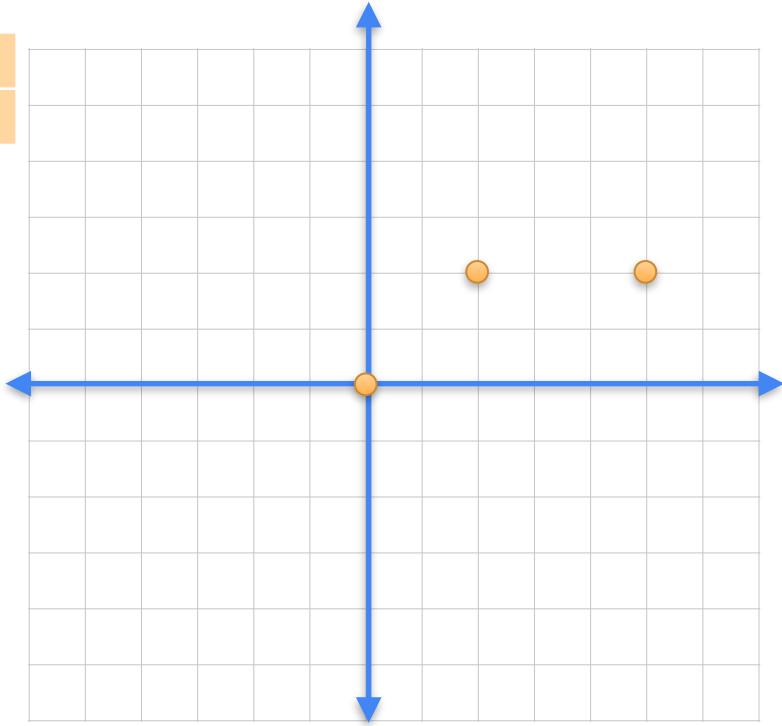
Matrices as linear transformations

b

a

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 & 0 \\ 2 & 2 & 1 \end{matrix} & = \begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 2 & 2 \\ 2 & 2 \end{matrix} \end{matrix} \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \\ (0,1) &\rightarrow (1,2) \end{aligned}$$



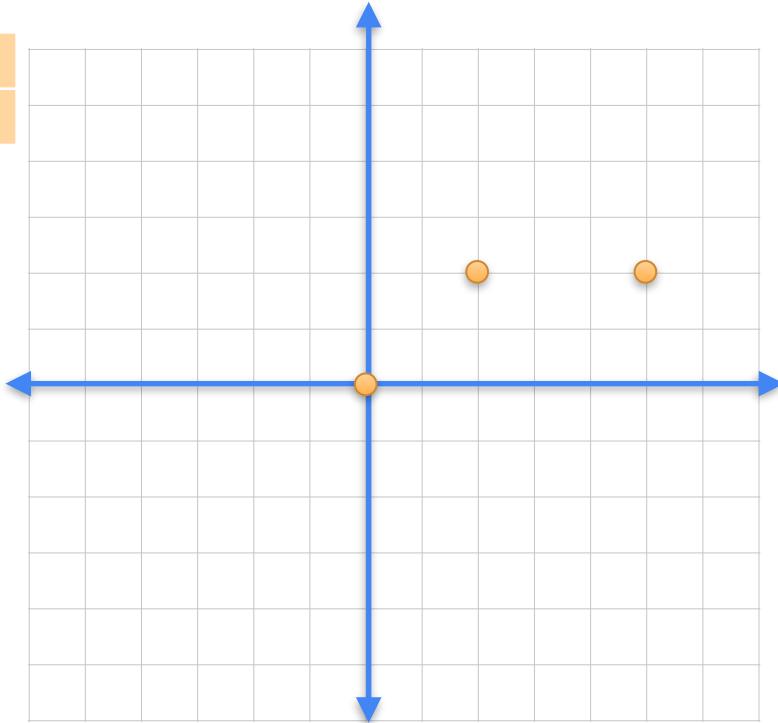
Matrices as linear transformations

b

a

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 & 0 \\ 2 & 2 & 1 \end{matrix} & = \begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 2 & 2 \\ 2 & 2 \end{matrix} \end{matrix} \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \\ (0,1) &\rightarrow (1,2) \end{aligned}$$



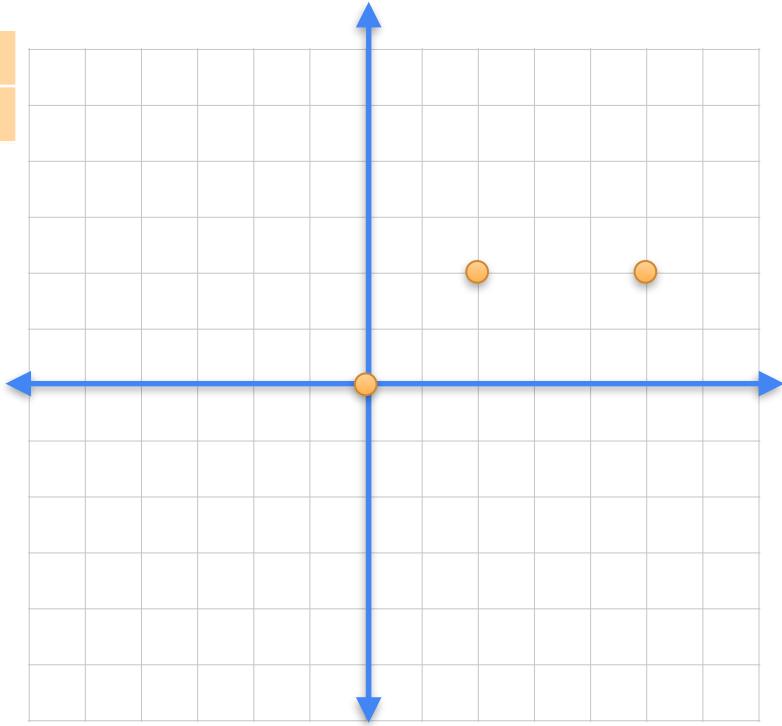
Matrices as linear transformations

b

a

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 & 1 \\ 2 & 2 & 1 \end{matrix} & = \begin{matrix} 7 & 2 \\ \text{apple} & \text{banana} \end{matrix} \end{matrix}$$

- $(0,0) \rightarrow (0,0)$
- $(1,0) \rightarrow (3,1)$
- $(0,1) \rightarrow (1,2)$
- $(1,1) \rightarrow (4,3)$



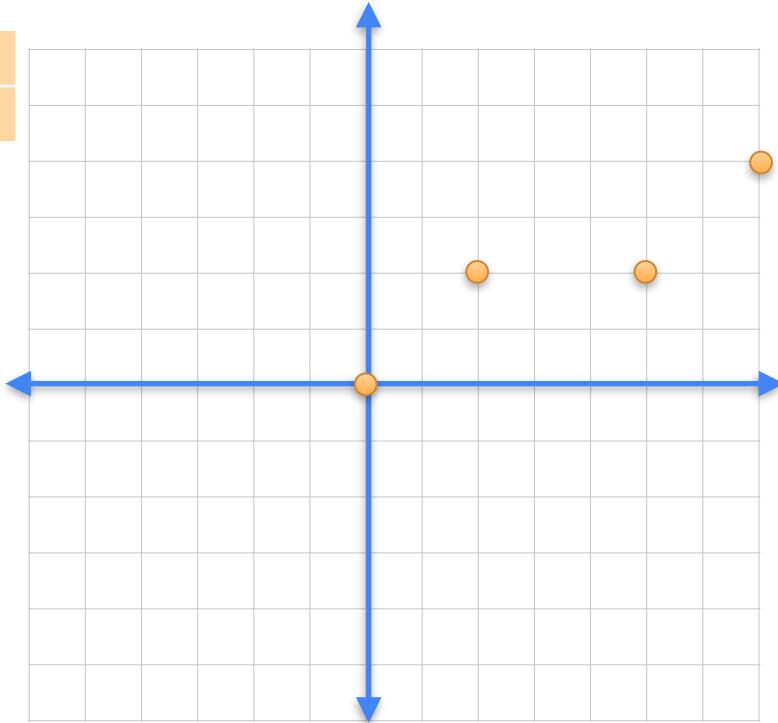
Matrices as linear transformations

b

a

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 & 1 \\ 2 & 2 & 1 \end{matrix} & = \begin{matrix} 7 & 2 \\ \text{apple} & \text{banana} \end{matrix} \end{matrix}$$

- $(0,0) \rightarrow (0,0)$
- $(1,0) \rightarrow (3,1)$
- $(0,1) \rightarrow (1,2)$
- $(1,1) \rightarrow (4,3)$



Matrices as linear transformations

b

a

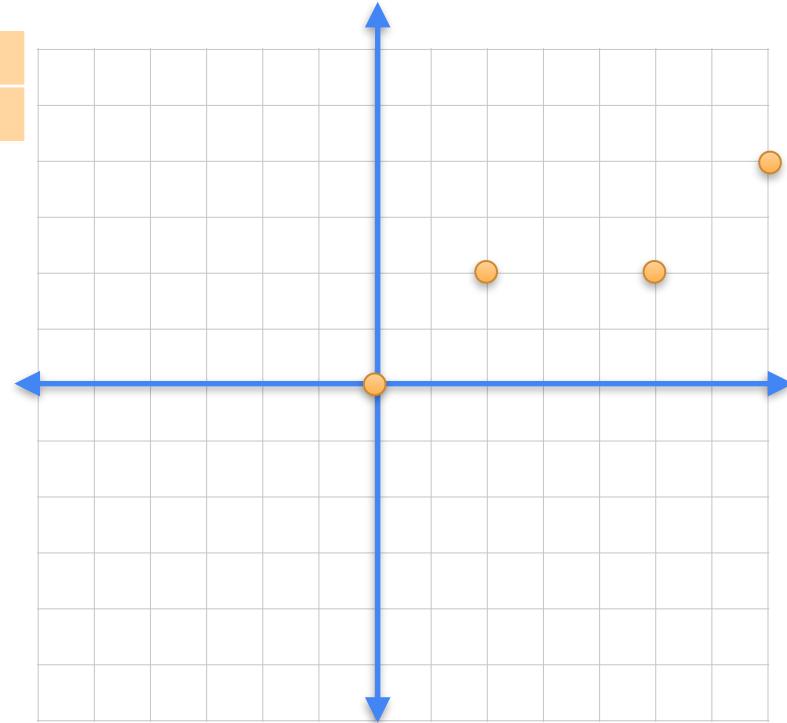
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 \\ 2 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} 7 \\ 2 \end{matrix}$$

$$(0,0) \rightarrow (0,0)$$

$$(1,0) \rightarrow (3,1)$$

$$(0,1) \rightarrow (1,2)$$

$$(1,1) \rightarrow (4,3)$$



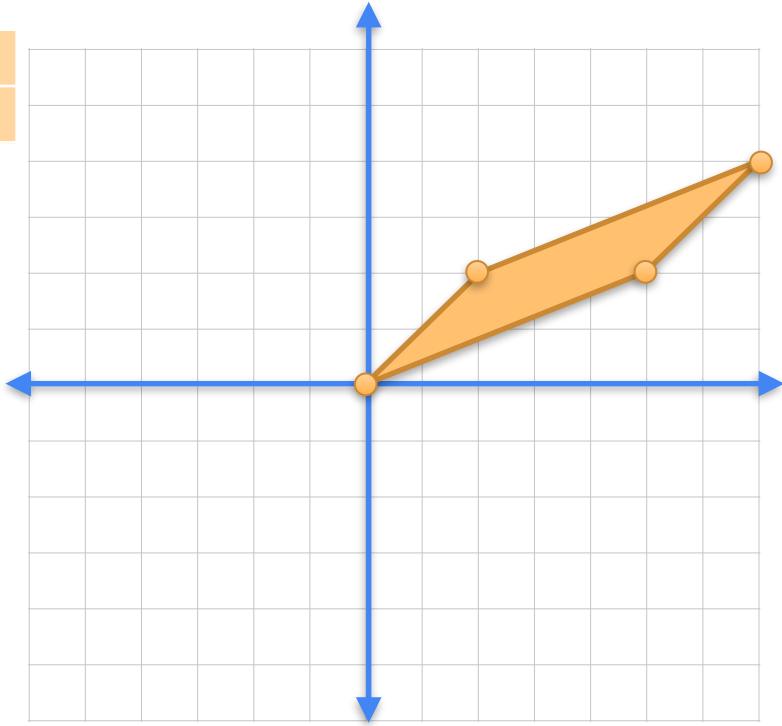
Matrices as linear transformations

b

a

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 & 1 \\ 2 & 2 & 1 \end{matrix} & = \begin{matrix} 7 & 2 \\ \text{apple} & \text{banana} \end{matrix} \end{matrix}$$

- $(0,0) \rightarrow (0,0)$
- $(1,0) \rightarrow (3,1)$
- $(0,1) \rightarrow (1,2)$
- $(1,1) \rightarrow (4,3)$

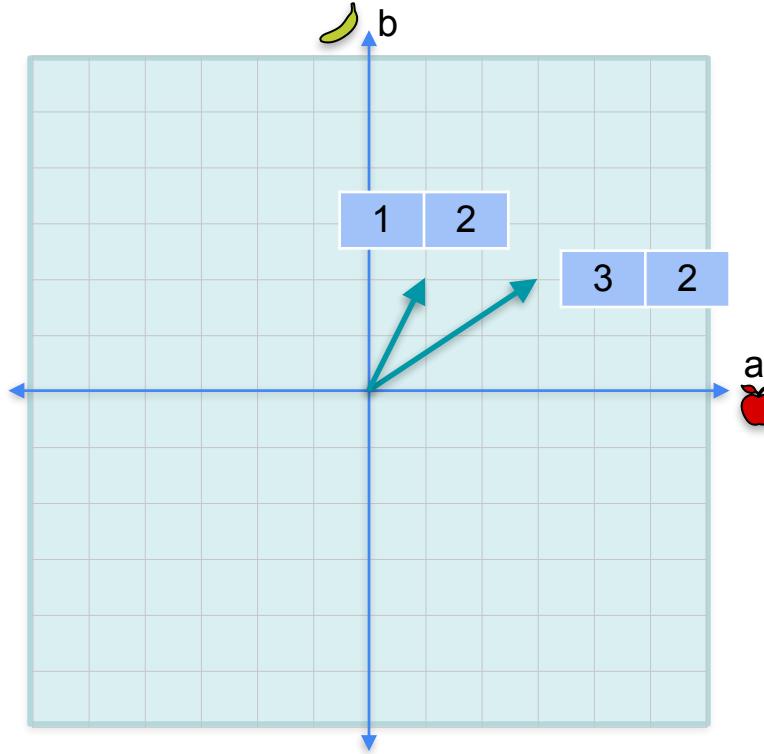


Row span of a matrix

| | |
|---|---|
|  |  |
| 3 | 2 |
| 1 | 2 |

Rows

| | |
|---|---|
| 3 | 2 |
| 1 | 2 |

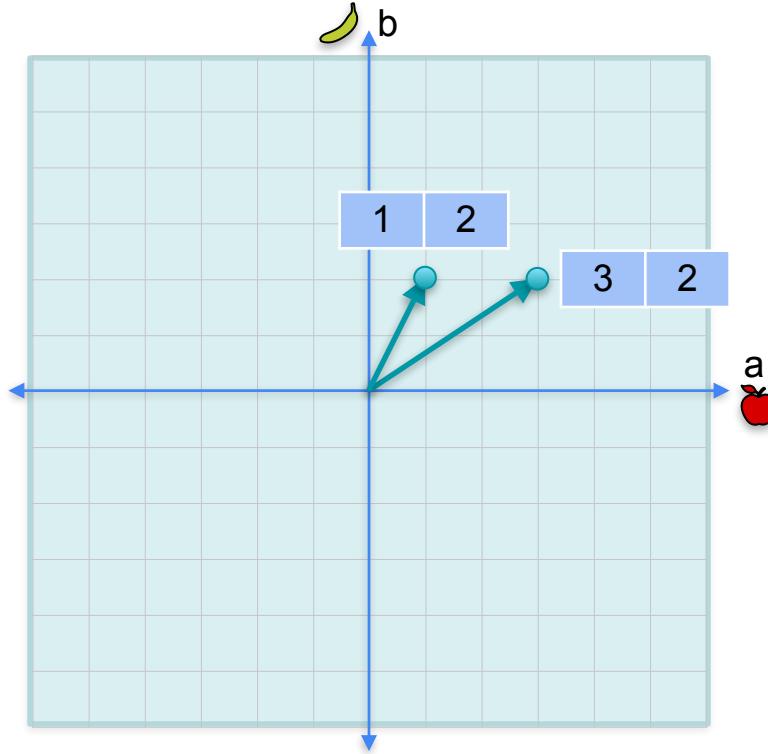


Row span of a matrix

| | |
|---|---|
|  |  |
| 3 | 2 |
| 1 | 2 |

Rows

| | |
|---|---|
| 3 | 2 |
| 1 | 2 |

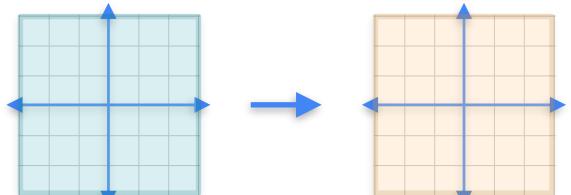


Span of the rows

Non-singular

| | |
|---|---|
| | |
| 3 | 1 |
| 1 | 2 |

Rank = 2

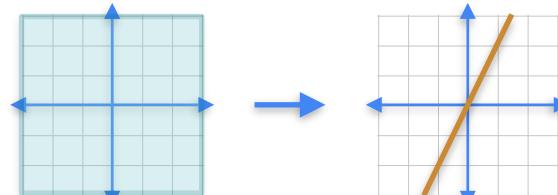


The whole plane

Singular

| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

Rank = 1

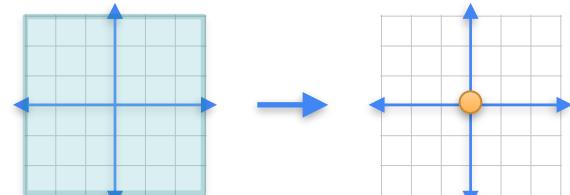


A line

Singular

| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |

Rank = 0



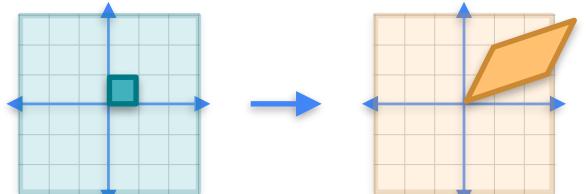
A point

Basis vectors

Non-singular

| | |
|---|---|
| | |
| 3 | 1 |
| 1 | 2 |

Rank = 2

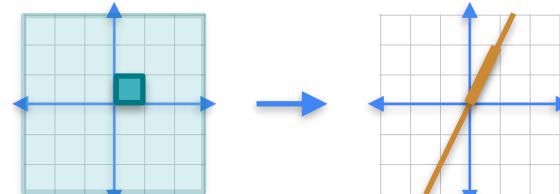


The whole plane

Singular

| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

Rank = 1

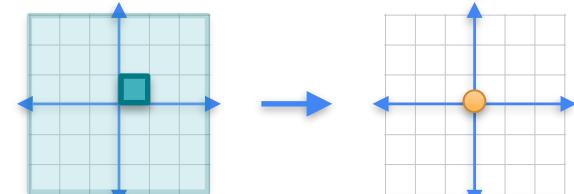


A line

Singular

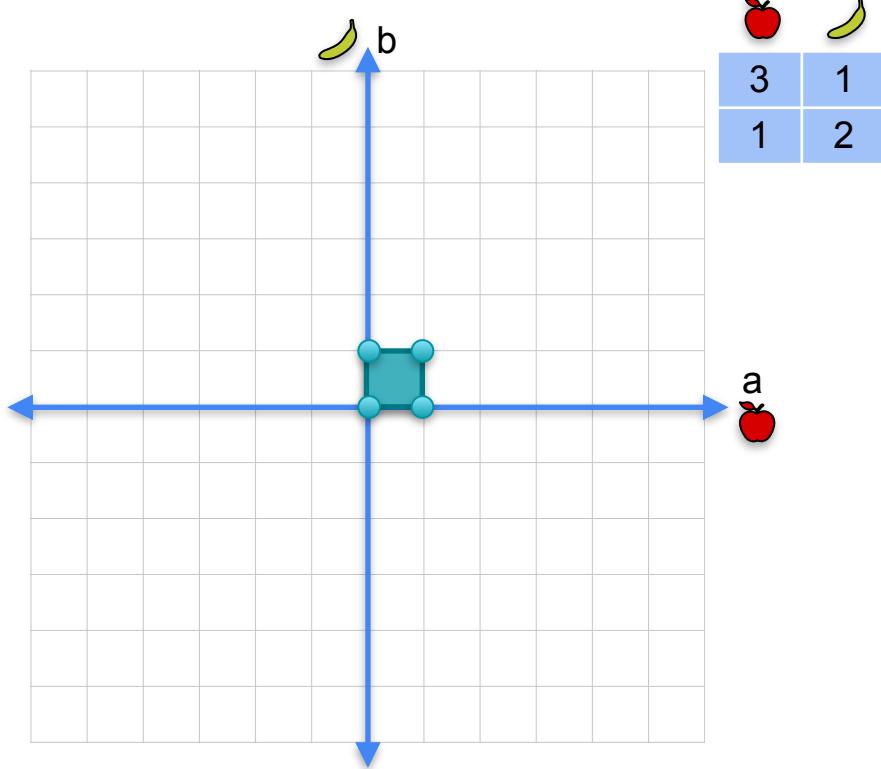
| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |

Rank = 0

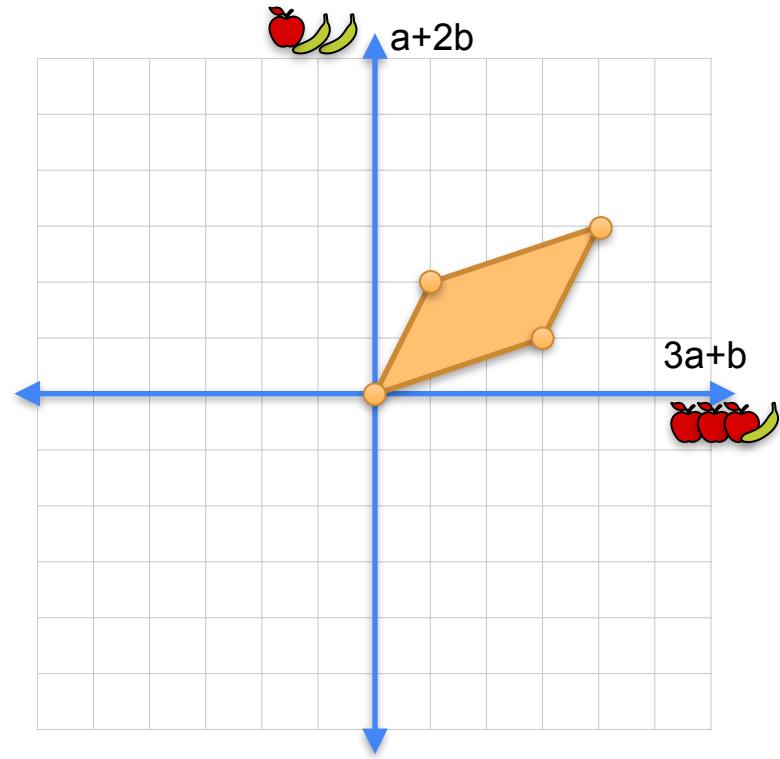


A point

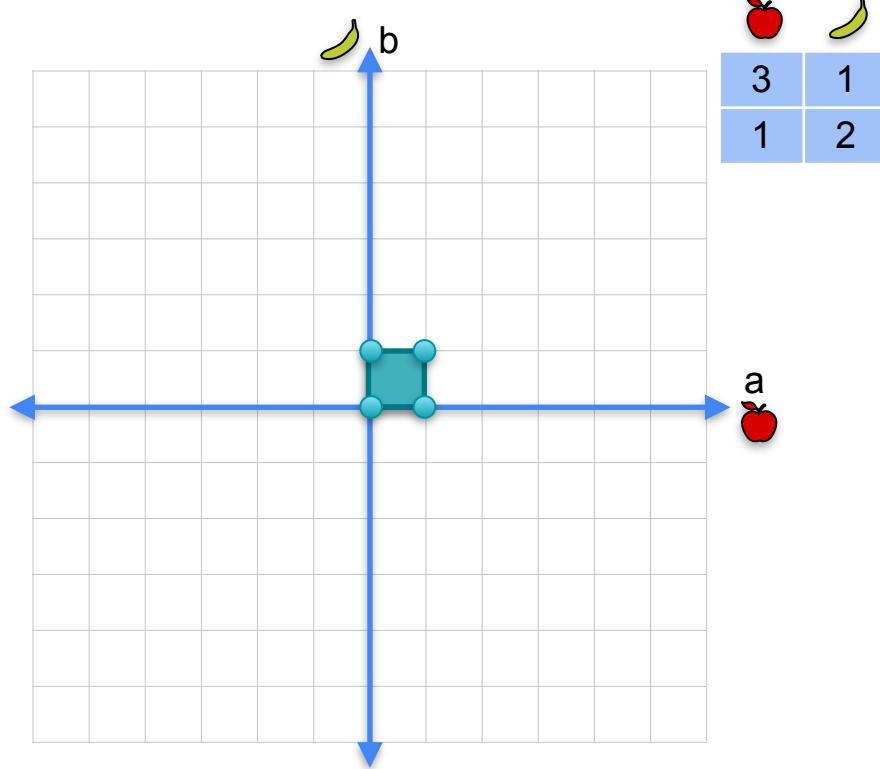
Linear transformation



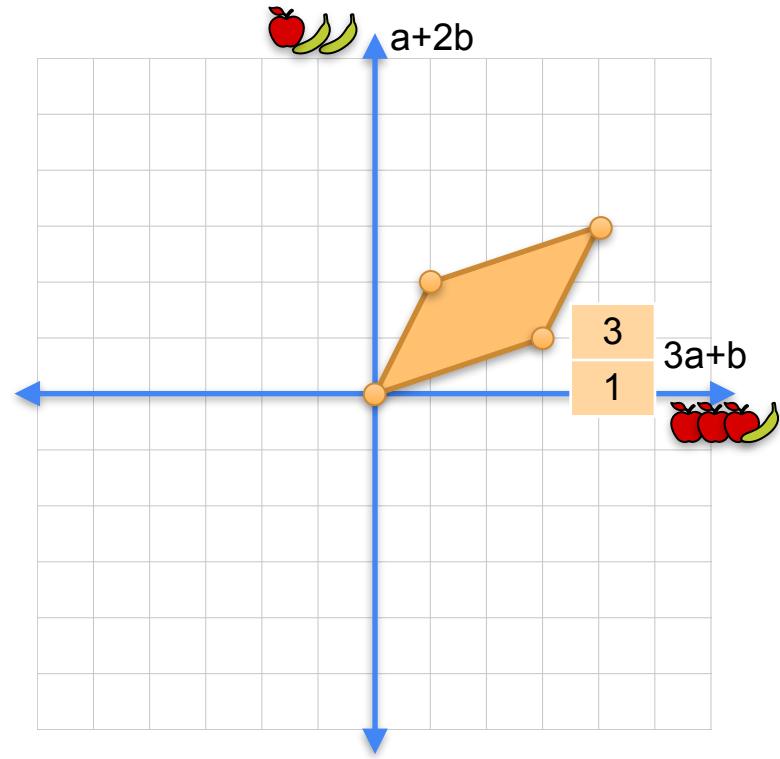
=



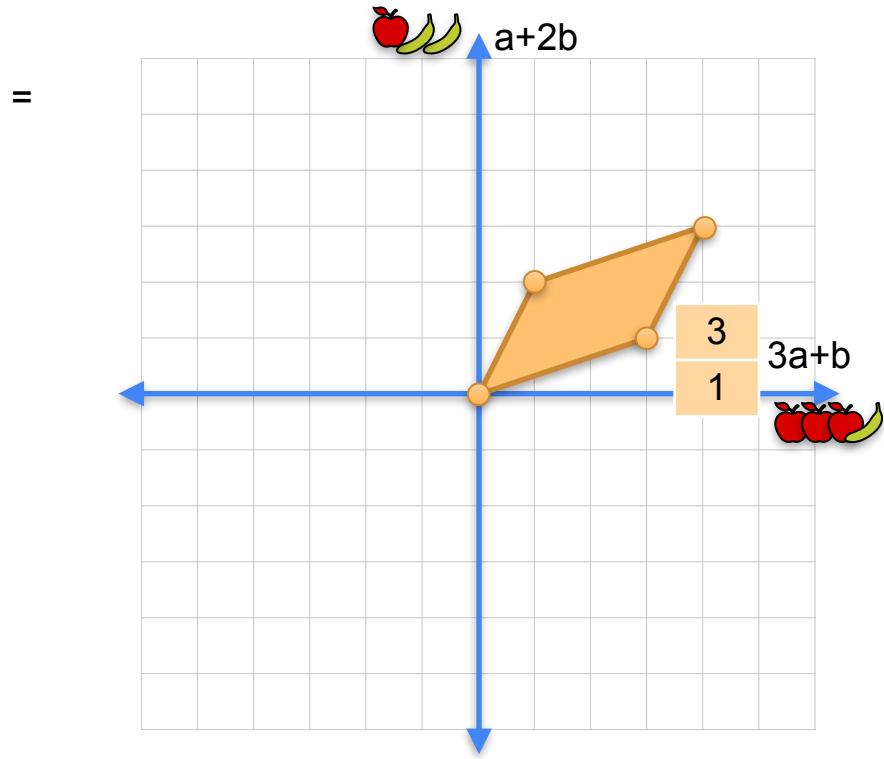
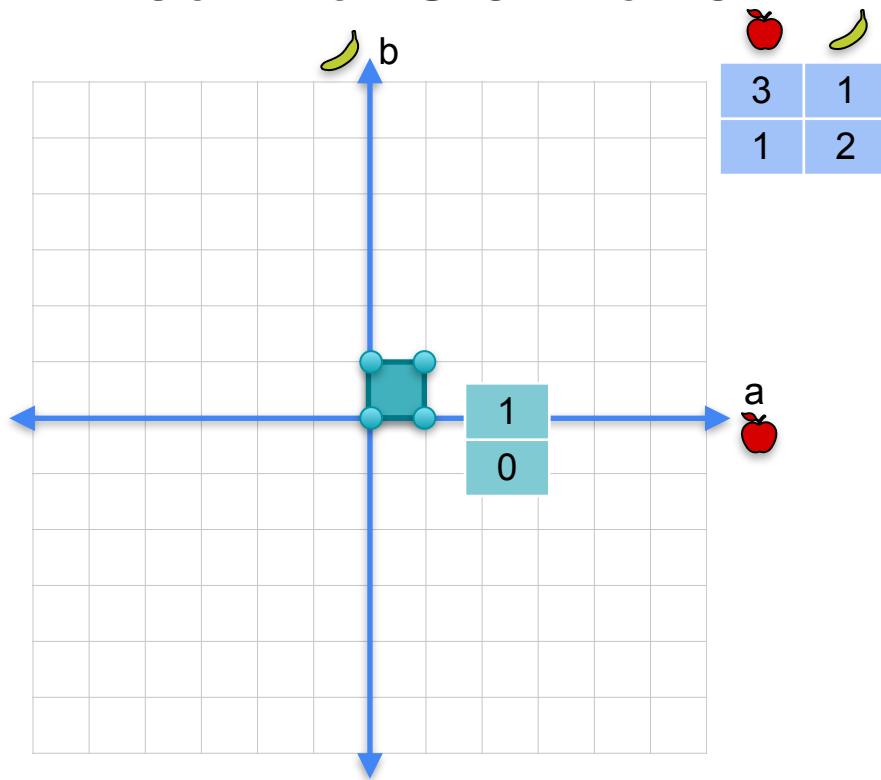
Linear transformation



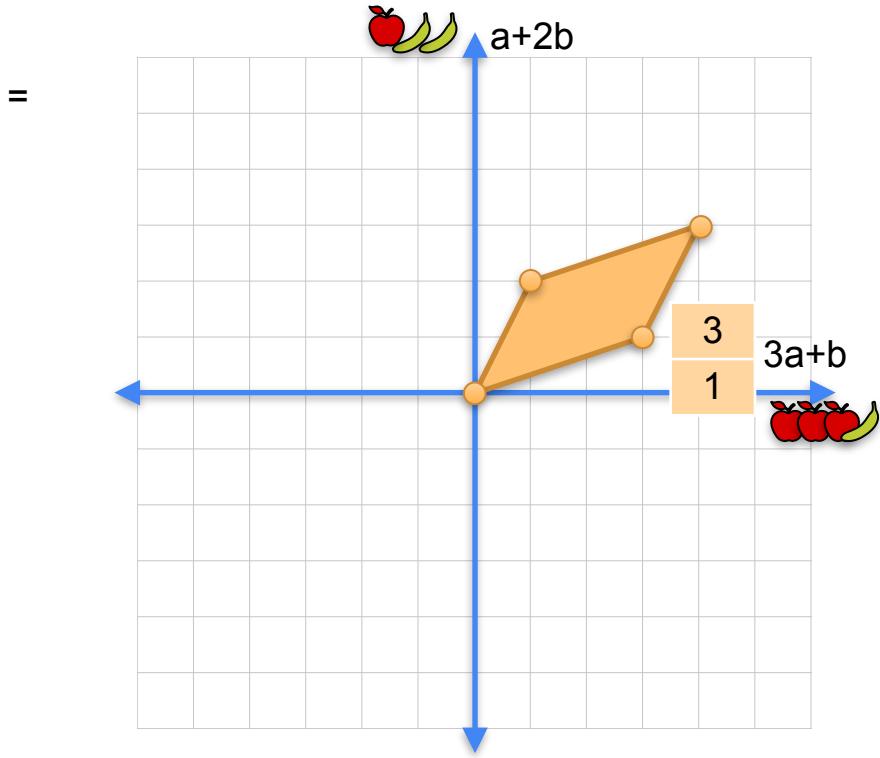
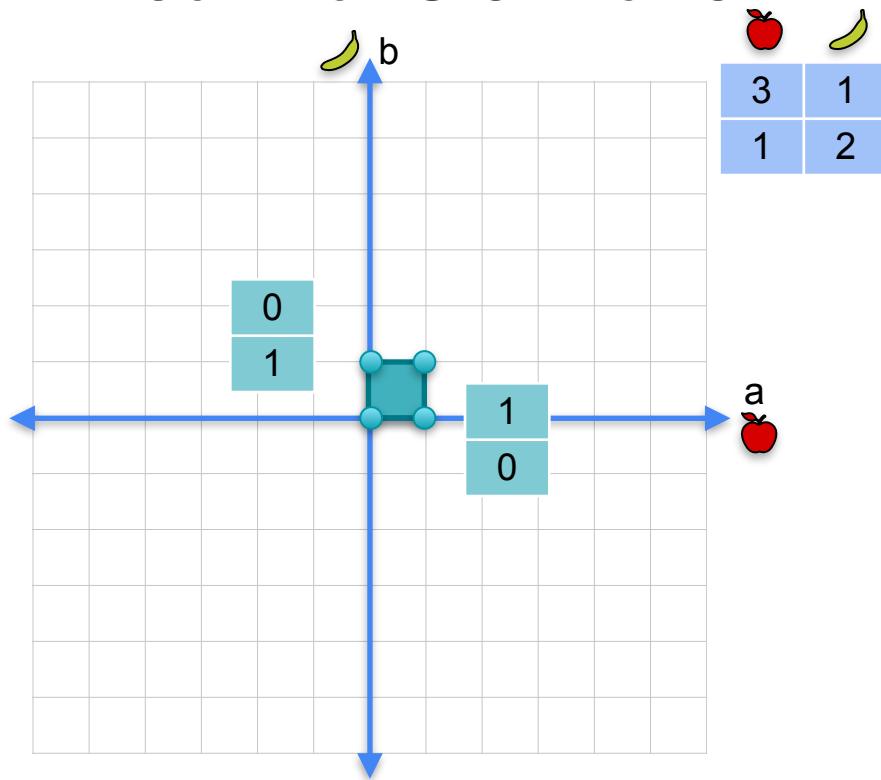
=



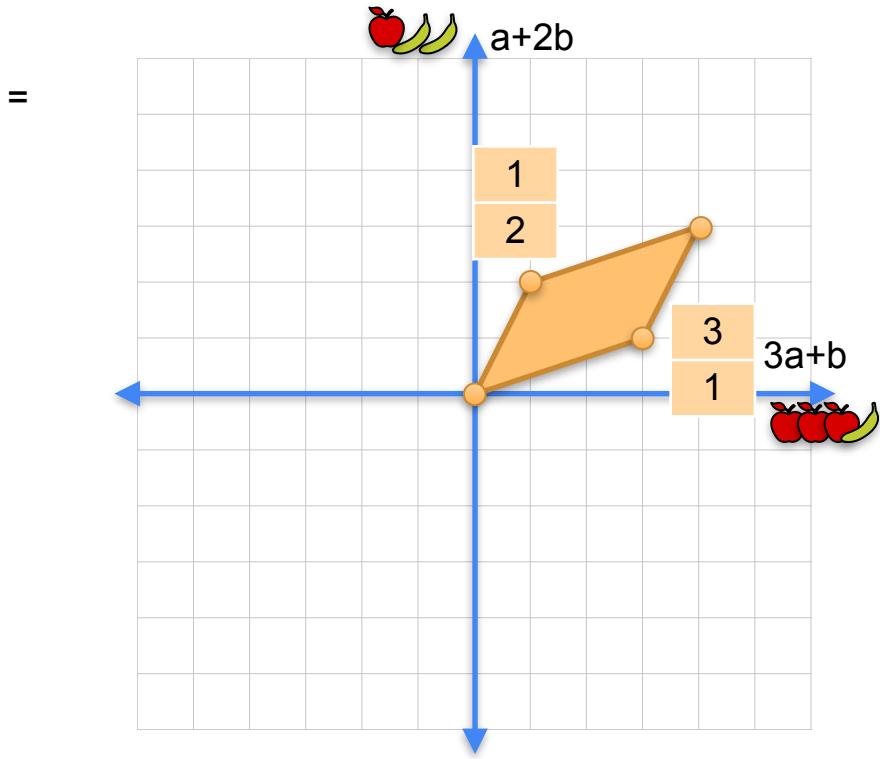
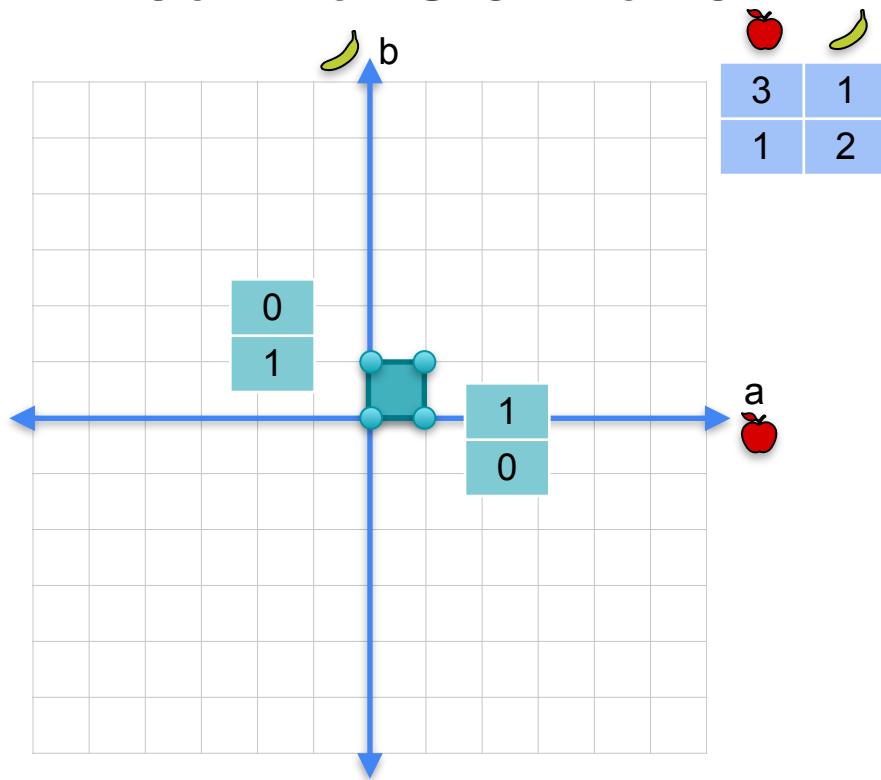
Linear transformation



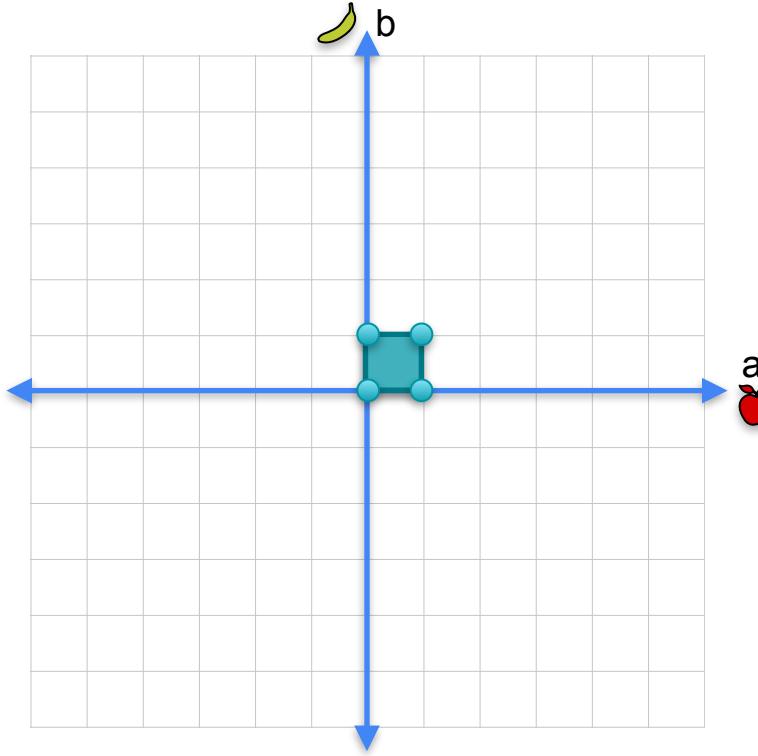
Linear transformation



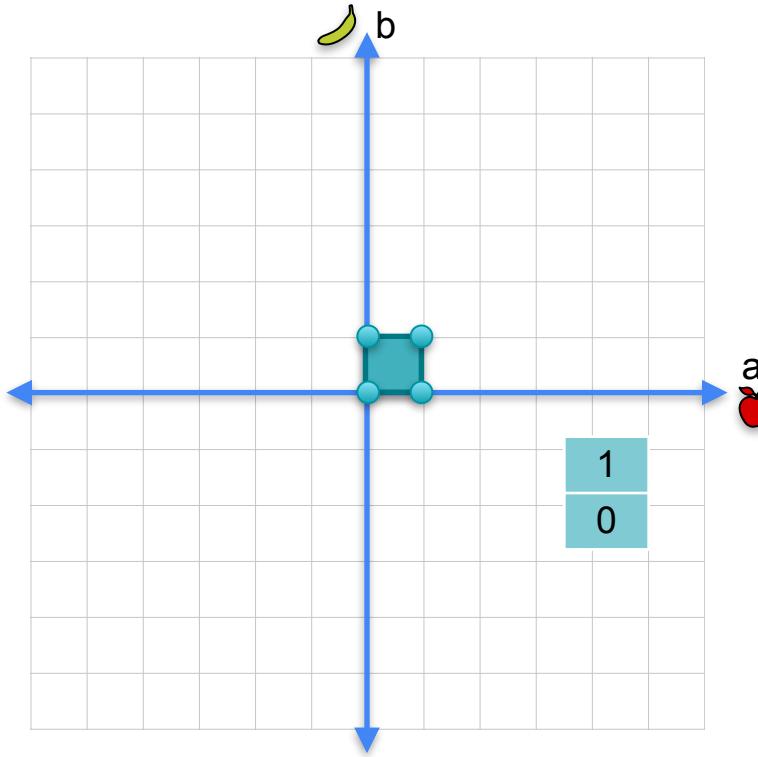
Linear transformation



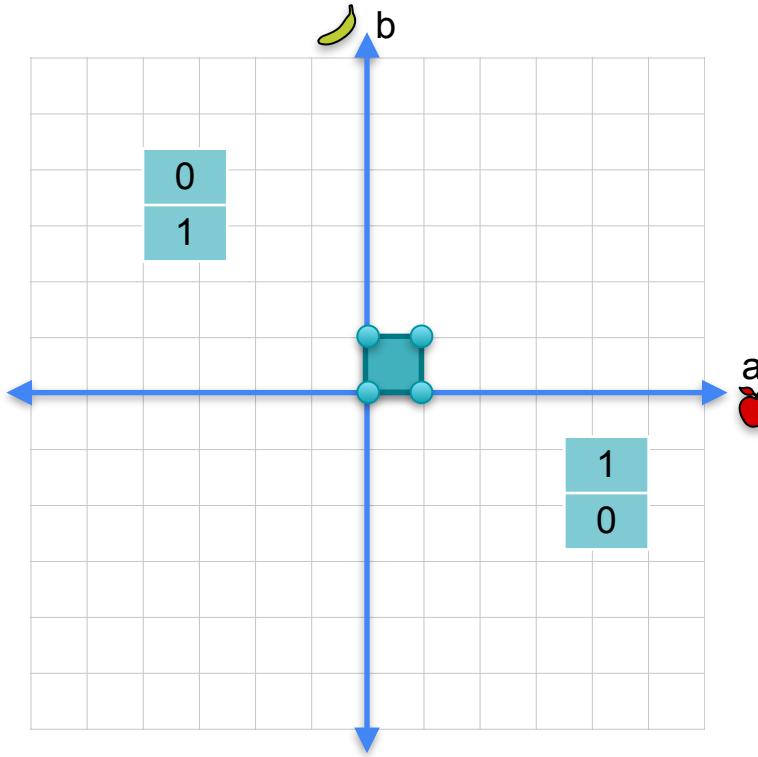
Linear transformation



Linear transformation



Linear transformation



3D



DeepLearning.AI

Math for Machine Learning

Linear algebra - Week 4

Vectors

Matrices

Dot product

Matrix multiplication

Linear transformations

A matrix and its corresponding system of equations

A matrix and its corresponding system of equations

|  |  |
|---|---|
| 1 | 1 |
| 1 | 2 |

A matrix and its corresponding system of equations

| | |
|---|---|
|  |  |
| 1 | 1 |
| 1 | 2 |

| | |
|---|---|
|  |  |
| 1 | 1 |
| 2 | 2 |

A matrix and its corresponding system of equations

|  |  |
|---|---|
| 1 | 1 |
| 1 | 2 |

|  |  |
|--|---|
| 1 | 1 |
| 2 | 2 |

|  |  |
|---|---|
| 0 | 0 |
| 0 | 0 |

A matrix and its corresponding system of equations

System 1

| |  |  |
|---|---|---|
| •  +  = 0 | 1 | 1 |
| •  + 2  = 0 | 1 | 2 |

|  |  |
|--|---|
| 1 | 1 |
| 2 | 2 |

|  |  |
|---|---|
| 0 | 0 |
| 0 | 0 |

A matrix and its corresponding system of equations

System 1

| | 1 | 1 |
|---|---|---|
| + | 1 | 2 |
| + | 0 | 0 |

System 2

| | 1 | 1 |
|---|---|---|
| + | 2 | 2 |
| + | 0 | 0 |

A matrix and its corresponding system of equations

System 1

| • + = 0 | 1 | 1 |
|-------------------|---|---|
| • + 2 = 0 | 1 | 2 |

System 2

| • + = 0 | 1 | 1 |
|--------------------|---|---|
| • 2 + 2 = 0 | 2 | 2 |

System 3

| • 0 + 0 = 0 | 0 | 0 |
|--------------------|---|---|
| • 0 + 0 = 0 | 0 | 0 |

A matrix and its corresponding system of equations

System 1

| | 1 | 1 |
|--|---|---|
| | 1 | 2 |

System 2

| | 1 | 1 |
|--|---|---|
| | 2 | 2 |

System 3

| | 0 | 0 |
|--|---|---|
| | 0 | 0 |

The only two numbers a,
b, such that

- $a+b = 0$
 - and
 - $a+2b = 0$
- are:
 $a=0$ and $b=0$

A matrix and its corresponding system of equations

System 1

| • $a + b = 0$ | | |
|----------------|--|--|
| • $a + 2b = 0$ | | |

System 2

| • $a + b = 0$ | | |
|-----------------|--|--|
| • $2a + 2b = 0$ | | |

System 3

| • $0a + 0b = 0$ | | |
|-----------------|--|--|
| • $0a + 0b = 0$ | | |

The only two numbers a,
b, such that

- $a+b = 0$
 - and
 - $a+2b = 0$
- are:
 $a=0$ and $b=0$

Any pair $(x, -x)$ satisfies that

- $a+b = 0$
 - and
 - $a+2b = 0$
- For example:
 $(1, -1), (2, -2), (-8, 8)$, etc.

A matrix and its corresponding system of equations

System 1

| • $a + b = 0$ | 1 | 1 |
|----------------|---|---|
| • $a + 2b = 0$ | 1 | 2 |

The only two numbers a,
b, such that

- $a+b = 0$
 - and
 - $a+2b = 0$
- are:
 $a=0$ and $b=0$

System 2

| • $a + b = 0$ | 1 | 1 |
|-----------------|---|---|
| • $2a + 2b = 0$ | 2 | 2 |

Any pair $(x, -x)$ satisfies that

- $a+b = 0$
- and
- $a+2b = 0$

For example:
 $(1,-1), (2,-2), (-8,8)$, etc.

System 3

| • $0a + 0b = 0$ | 0 | 0 |
|-----------------|---|---|
| • $0a + 0b = 0$ | 0 | 0 |

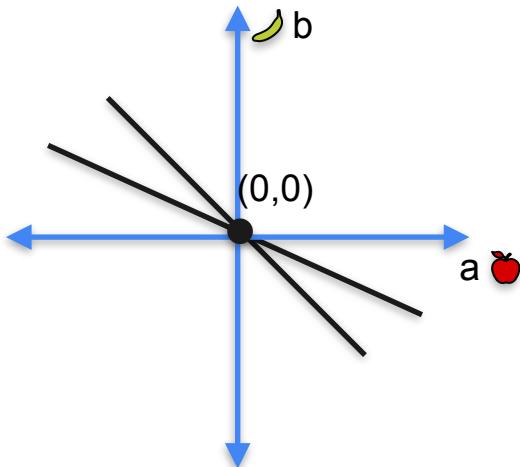
Any pair of numbers satisfies that

- $0a+0b = 0$
 - and
 - $0a+0b = 0$
- For example:
 $(1,2), (3,-9), (-90,8.34)$, etc.

The set of solutions of a system of equations

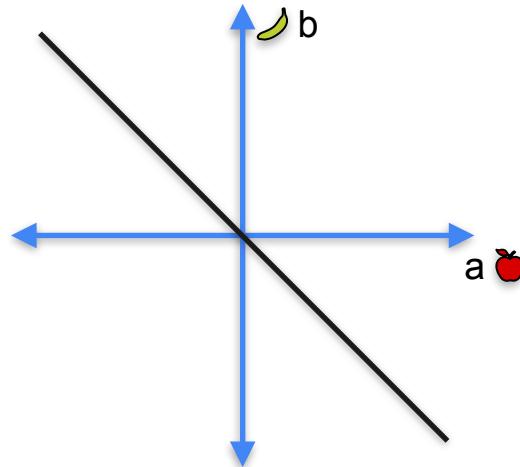
System 1

- $a + b = 0$
 
- $a + 2b = 0$
 



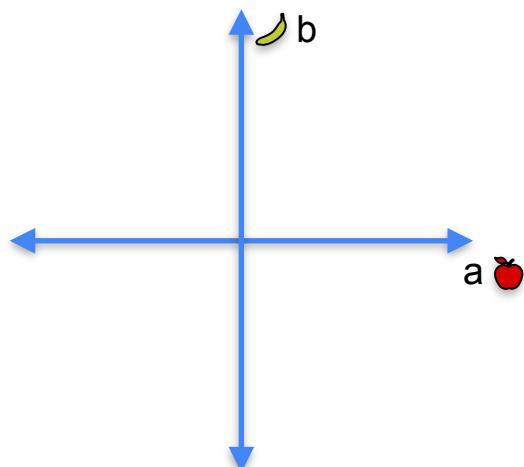
System 2

- $a + b = 0$
 
- $2a + 2b = 0$
 



System 3

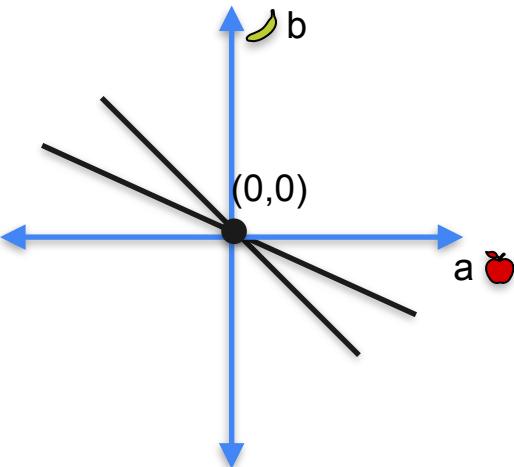
- $0a + 0b = 0$
 
- $0a + 0b = 0$
 



The set of solutions of a system of equations

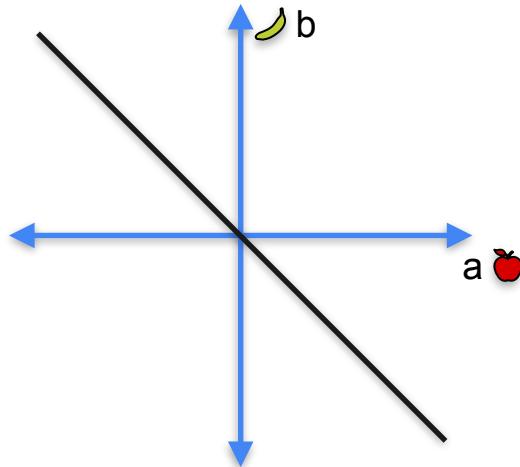
System 1

- $a + b = 0$
 - $a + 2b = 0$
- Solution**
- $a = 0$
 - $b = 0$



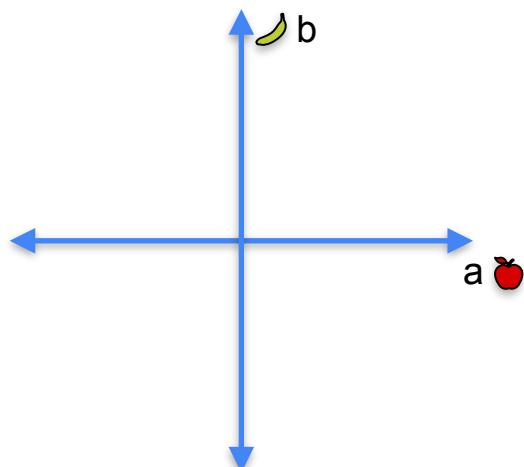
System 2

- $a + b = 0$
- $2a + 2b = 0$



System 3

- $0a + 0b = 0$
- $0a + 0b = 0$

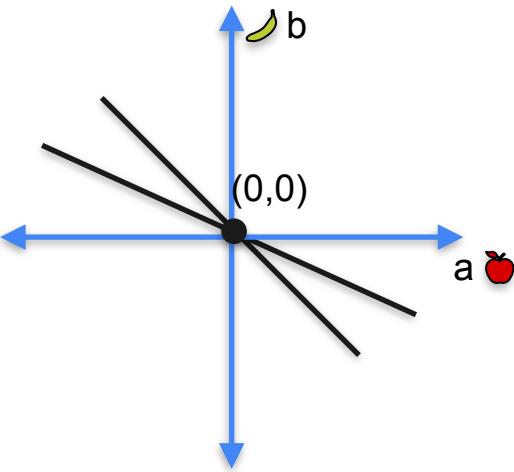


The set of solutions of a system of equations

System 1

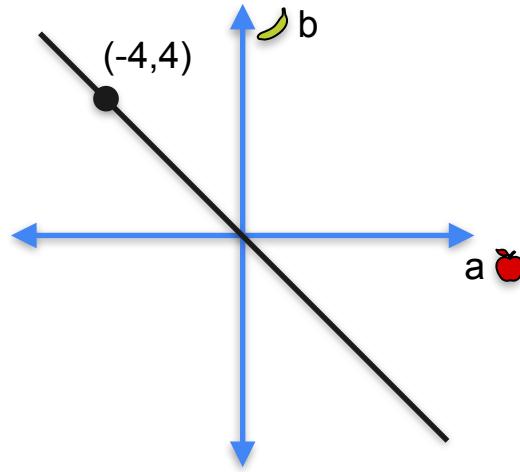
- + = **0**
- + 2 = **0**

Solution
• $a = 0$
• $b = 0$



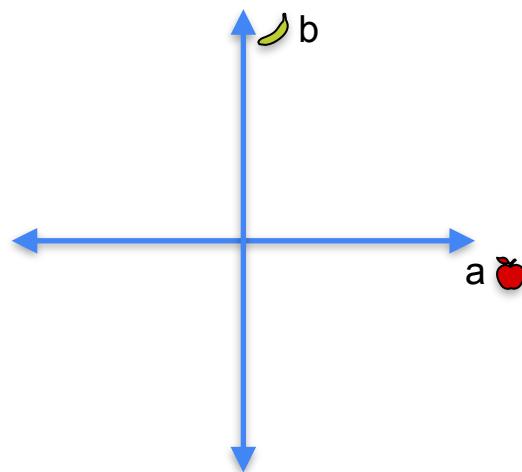
System 2

- + = **0**
- 2 + 2 = **0**



System 3

- $0a + 0b = 0$
- $0a + 0b = 0$

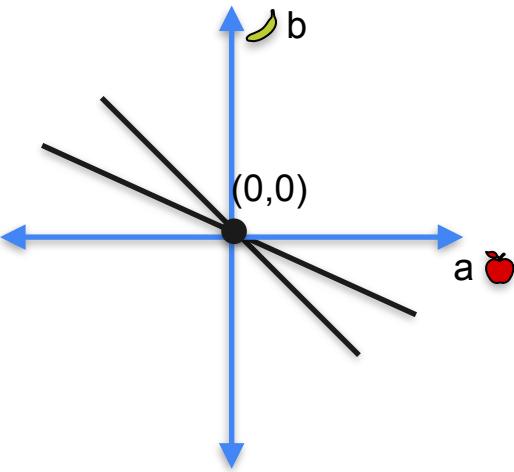


The set of solutions of a system of equations

System 1

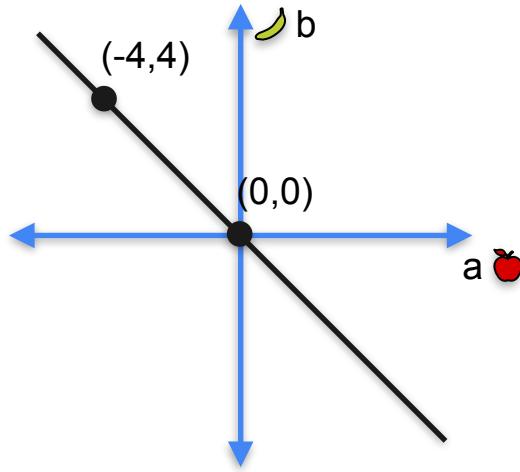
- + = **0**
- + 2 = **0**

Solution
• $a = 0$
• $b = 0$



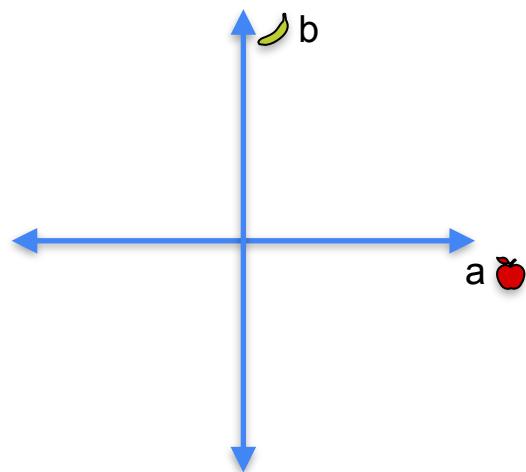
System 2

- + = **0**
- 2 + 2 = **0**



System 3

- $0a + 0b = 0$
- $0a + 0b = 0$

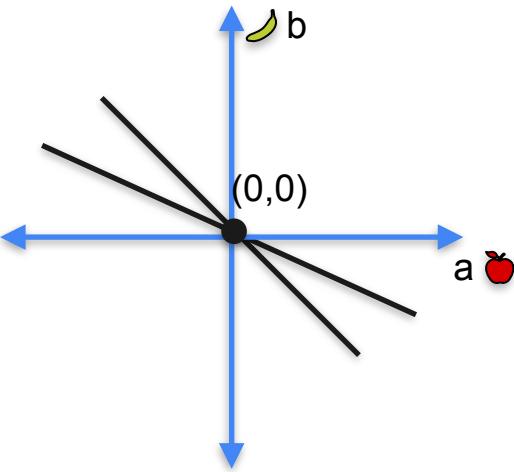


The set of solutions of a system of equations

System 1

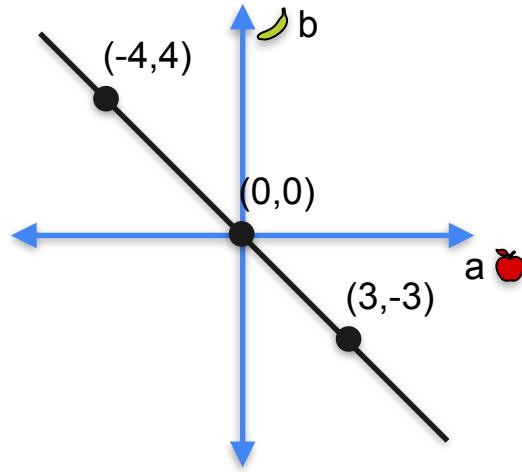
- + = **0**
- + 2 = **0**

Solution
• $a = 0$
• $b = 0$



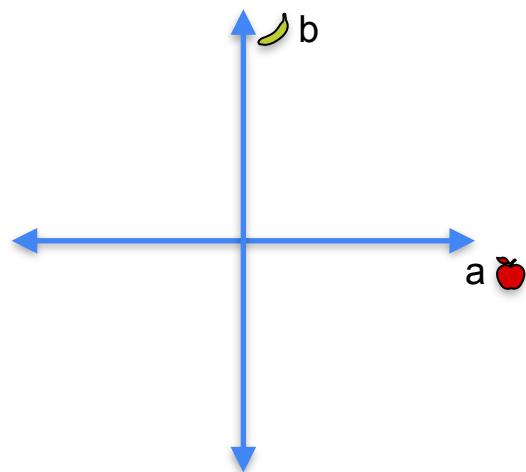
System 2

- + = **0**
- 2 + 2 = **0**



System 3

- $0a + 0b = 0$
- $0a + 0b = 0$

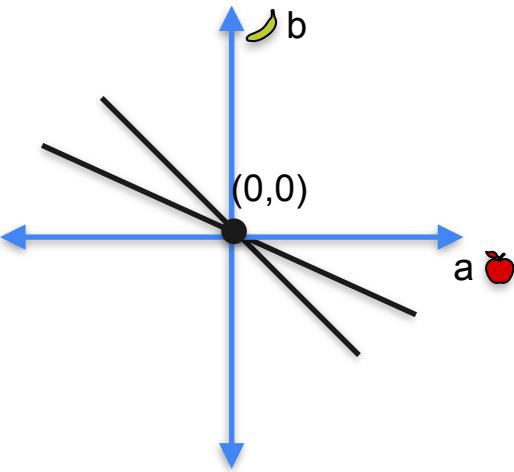


The set of solutions of a system of equations

System 1

- + = **0**
- + 2 = **0**

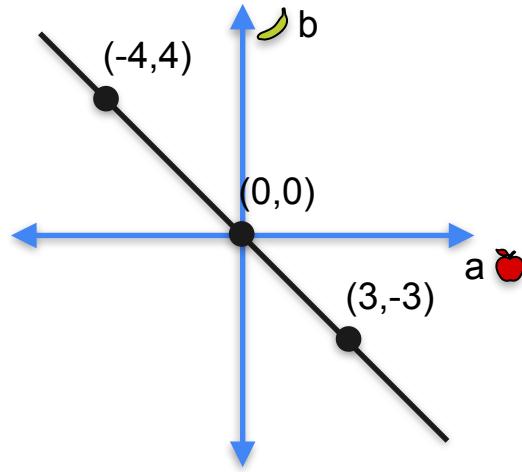
Solution
• $a = 0$
• $b = 0$



System 2

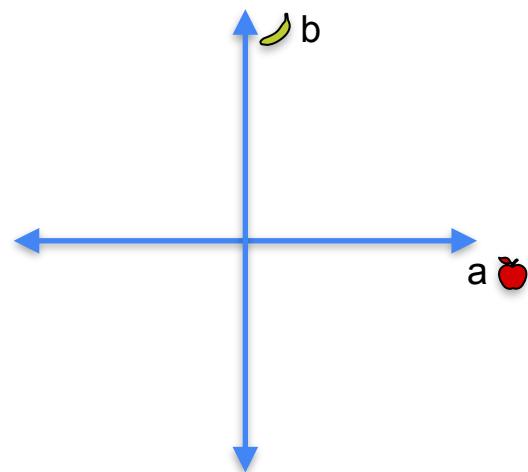
- + = **0**
- 2 + 2 = **0**

Solutions
• any a
• $b = -a$



System 3

- $0a + 0b = 0$
- $0a + 0b = 0$

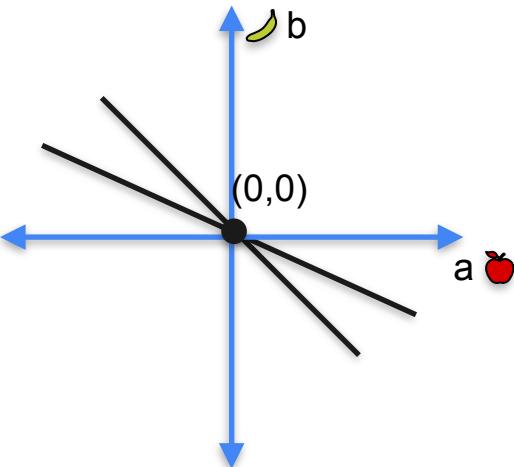


The set of solutions of a system of equations

System 1

- + = **0**
- + 2 = **0**

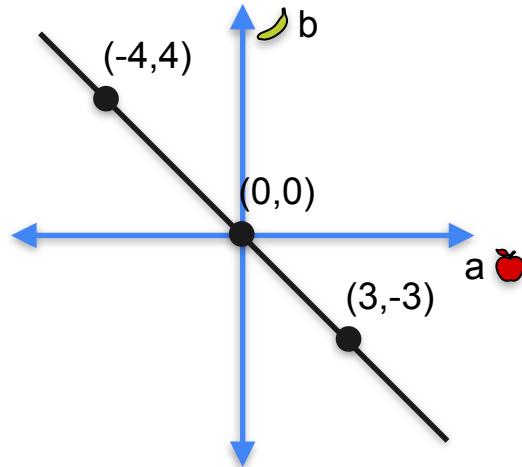
Solution
• $a = 0$
• $b = 0$



System 2

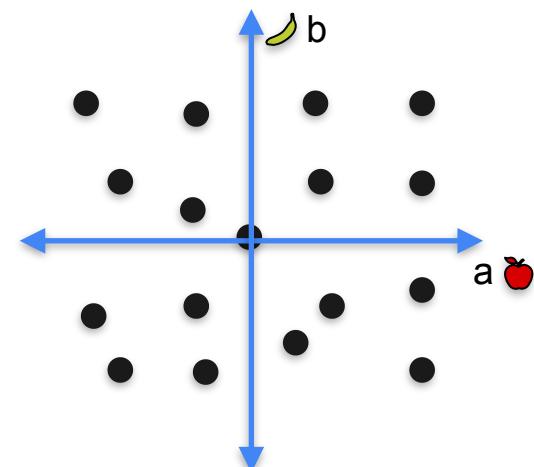
- + = **0**
- 2 + 2 = **0**

Solutions
• any a
• $b = -a$



System 3

- $0a + 0b = 0$
- $0a + 0b = 0$

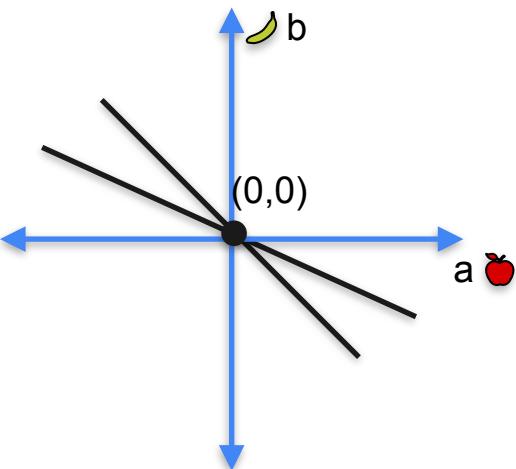


The set of solutions of a system of equations

System 1

- + = **0**
- + 2 = **0**

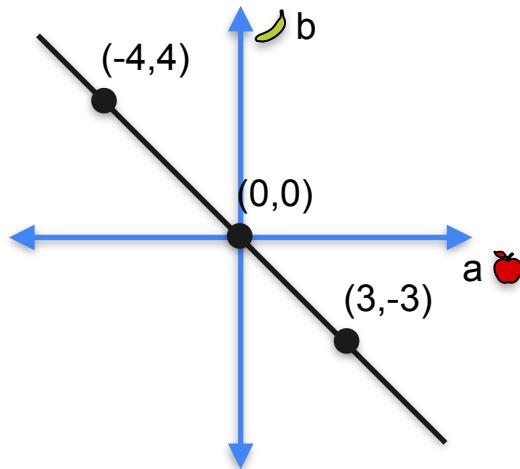
Solution
• $a = 0$
• $b = 0$



System 2

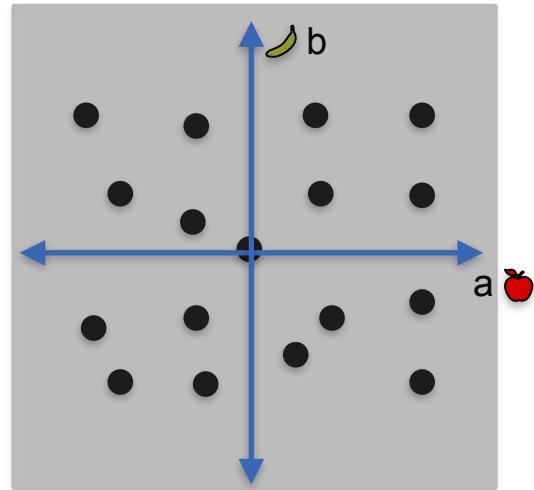
- + = **0**
- 2 + 2 = **0**

Solutions
• any a
• $b = -a$



System 3

- $0a + 0b = 0$
- $0a + 0b = 0$

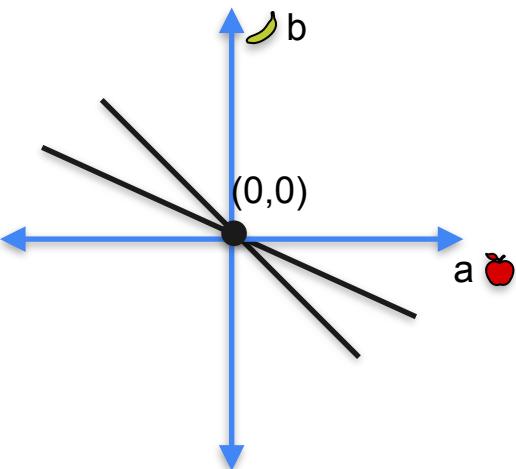


The set of solutions of a system of equations

System 1

- + = **0**
- + 2 = **0**

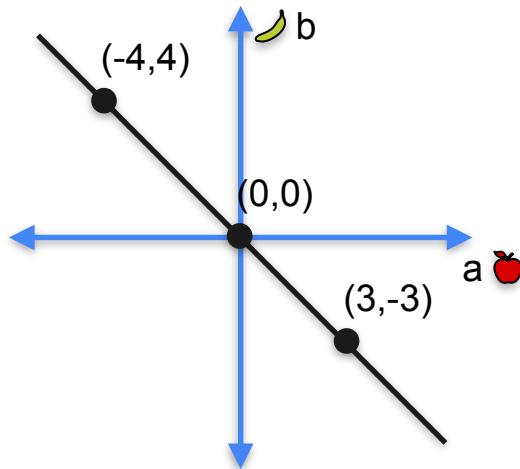
Solution
• $a = 0$
• $b = 0$



System 2

- + = **0**
- 2 + 2 = **0**

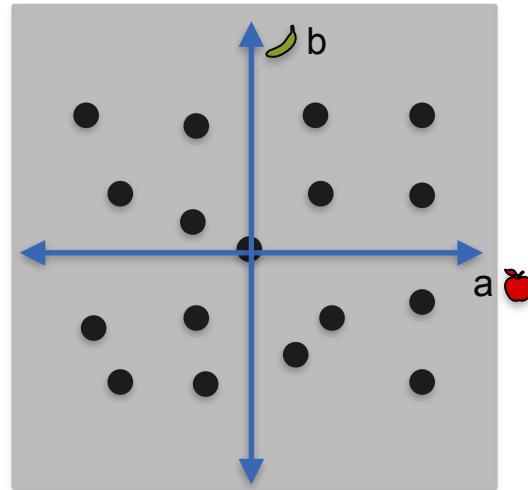
Solutions
• any a
• $b = -a$



System 3

- $0a + 0b = 0$
- $0a + 0b = 0$

Solutions
• any a
• any b



The null space of a matrix

| | |
|---|---|
| | |
| 1 | 1 |
| 1 | 2 |

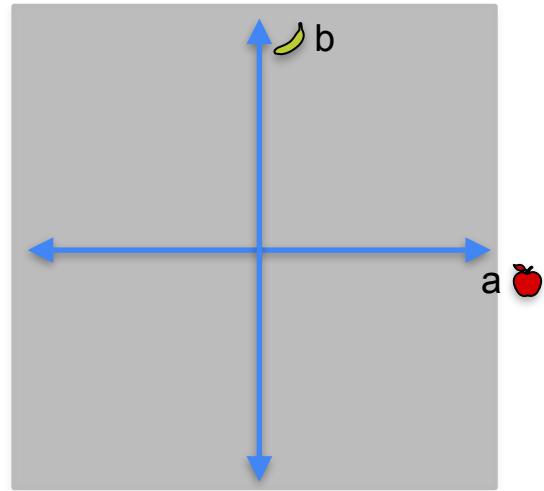
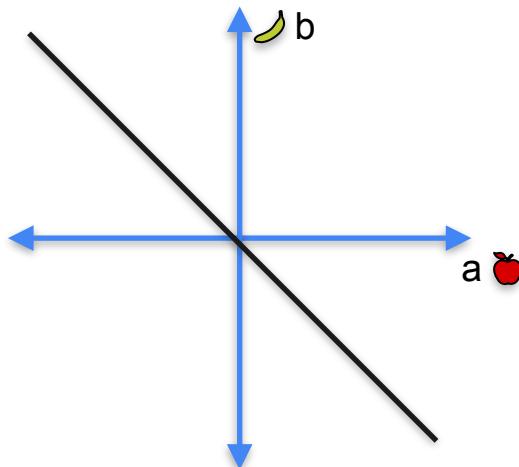
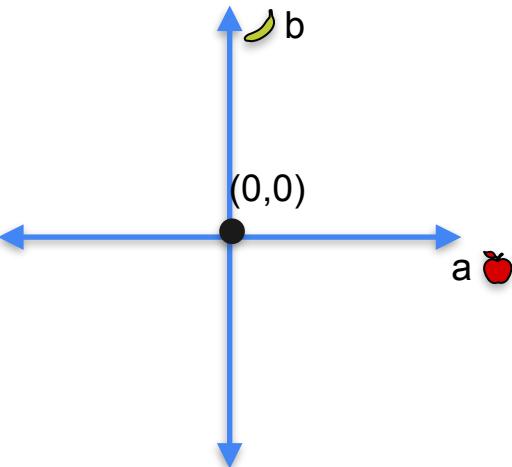
- Null space**
- $a = 0$
 - $b = 0$

| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

- Null space**
- any a
 - $b = -a$

| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |

- Null space**
- any a
 - any b



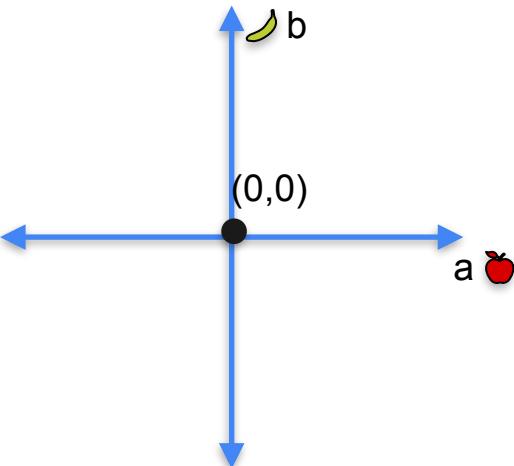
The null space of a matrix

| | |
|---|---|
| | |
| 1 | 1 |
| 1 | 2 |

Null space

- $a = 0$
- $b = 0$

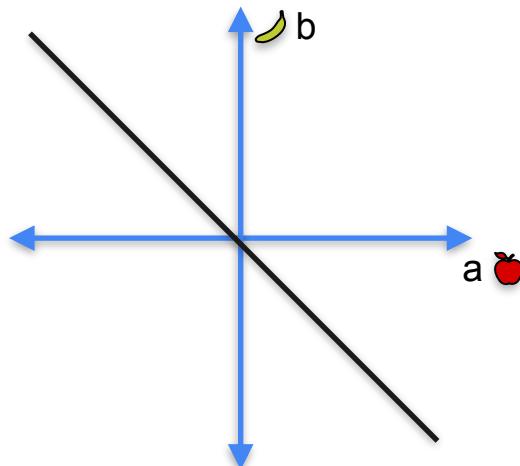
Dimension = 0



| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

Null space

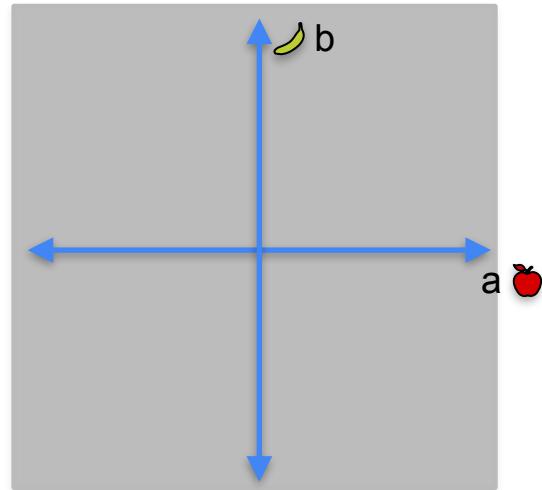
- any a
- $b = -a$



| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |

Null space

- any a
- any b



The null space of a matrix

| | |
|---|---|
| | |
| 1 | 1 |
| 1 | 2 |

- Null space
- $a = 0$
 - $b = 0$

Dimension = 0

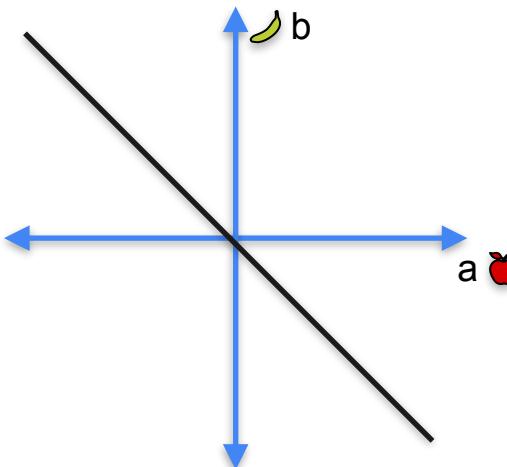
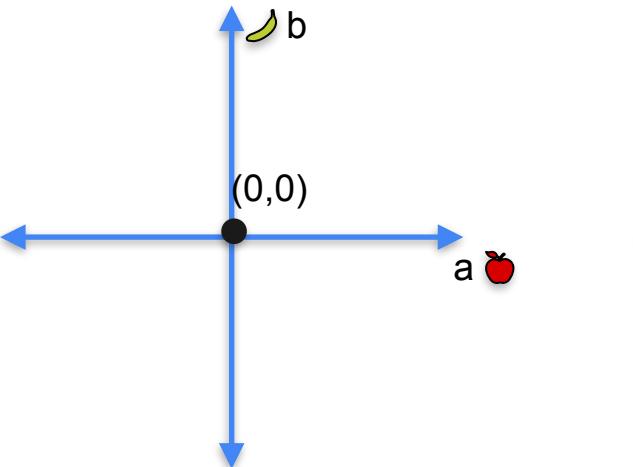
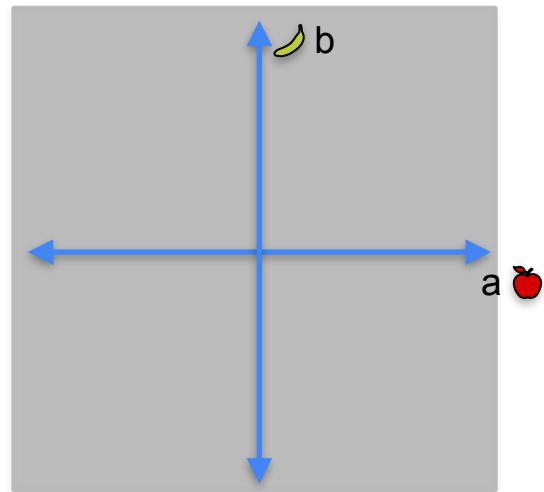
| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

- Null space
- any a
 - $b = -a$

Dimension = 1

| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |

- Null space
- any a
 - any b

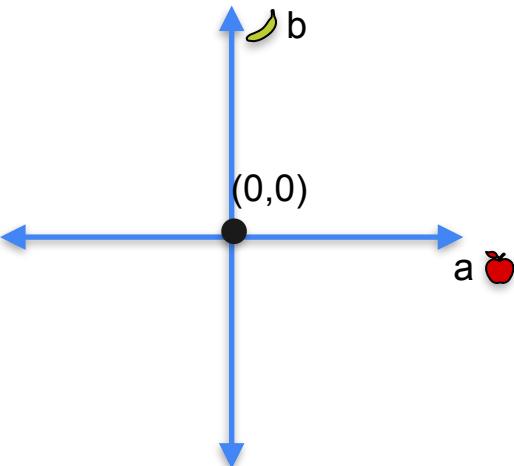


The null space of a matrix

| | |
|---|---|
| | |
| 1 | 1 |
| 1 | 2 |

Null space
• $a = 0$
• $b = 0$

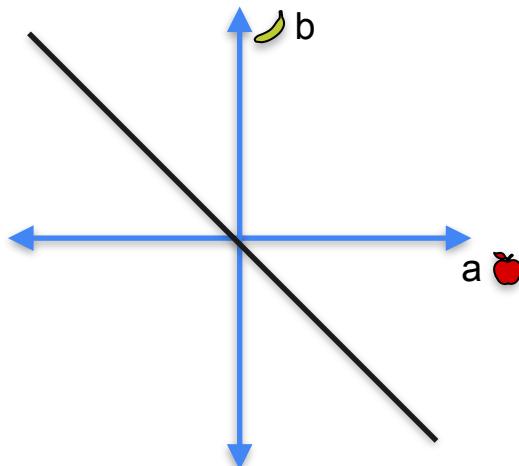
Dimension = 0



| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

Null space
• any a
• $b = -a$

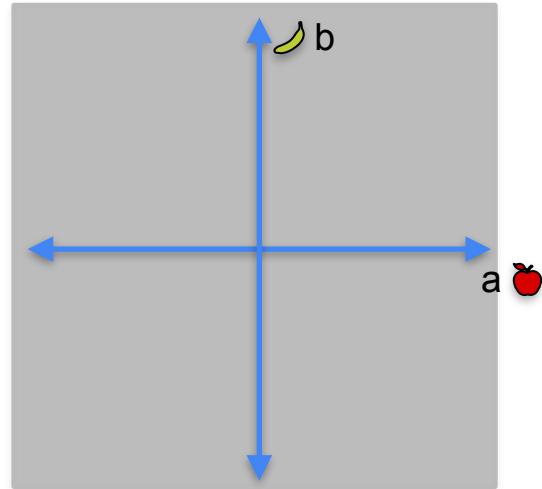
Dimension = 1



| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |

Null space
• any a
• any b

Dimension = 2



The null space of a matrix

| | |
|---|---|
| | |
| 1 | 1 |
| 1 | 2 |

Null space

- $a = 0$
- $b = 0$

Dimension = 0

| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

Null space

- any a
- $b = -a$

Dimension = 1

| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |

Null space

- any a
- any b

Dimension = 2

The null space of a matrix

| | |
|---|---|
| | |
| 1 | 1 |
| 1 | 2 |

Null space
• $a = 0$
• $b = 0$

Dimension = 0

| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

Null space
• any a
• $b = -a$

Dimension = 1

| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |

Null space
• any a
• any b

Dimension = 2



Non-singular

The null space of a matrix

| | |
|---|---|
| | |
| 1 | 1 |
| 1 | 2 |

Null space
• $a = 0$
• $b = 0$

Dimension = 0

| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

Null space
• any a
• $b = -a$

Dimension = 1

| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |

Null space
• any a
• any b

Dimension = 2



Non-singular



Singular



The null space of a matrix

| | |
|---|---|
| | |
| 1 | 1 |
| 1 | 2 |

Null space
• $a = 0$
• $b = 0$

Dimension = 0



Non-singular

| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

Null space
• any a
• $b = -a$

Dimension = 1



| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |

Null space
• any a
• any b

Dimension = 2



Singular

The null space of a matrix

| | |
|---|---|
| | |
| 1 | 1 |
| 1 | 2 |

- Null space**
- $a = 0$
 - $b = 0$

Dimension = 0



Non-singular

| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

Dimension = 1



Singular

| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |

Dimension = 2



Singular

More conceptual explanation of the null space

- Elaborate here

Quiz: Null space of a matrix

Problem: Determine the dimension of the null space of the following two matrices

Matrix 1

| | |
|----|---|
| 5 | 1 |
| -1 | 3 |

Matrix 2

| | |
|----|----|
| 2 | -1 |
| -6 | 3 |

Solutions: Null space of a matrix

Matrix 1: Notice that this is a non-singular matrix, since the determinant is 16. Therefore, the null space is only the point (0,0). The dimension is 0.

| | |
|----|---|
| 5 | 1 |
| -1 | 3 |

Matrix 2: The corresponding system of equation has the equations $2a - b = 0$ and $-6a + 3b = 0$. Some inspection shows that the first equation has the points (1,2), (2,4), (3,6), etc. as solutions. All of them are also solutions to the second equation, $-6a + 3b = 0$. Therefore the null space is all the points of the form $(x, 2x)$. The dimension of this null space is 1, and the matrix is singular.

| | |
|----|----|
| 2 | -1 |
| -6 | 3 |

Systems of linear equations

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

System 2

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- $a + b + 2c = 0$
- $a + b + 3c = 0$

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

System 4

- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

System 4

- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

System 4

- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

System 4

- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

System 4

- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Null space for systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Null space for systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
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- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Solution space



Null space for systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

Solution space



System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

Solution space



System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

System 4

- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$

Null space for systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

Solution space



System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

Solution space



System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

Solution space



System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Null space for systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

Solution space



System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

Solution space



System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

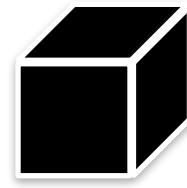
Solution space



System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Solution space



Null space for systems of linear equations

System 1

$$\bullet a + b + c = \mathbf{0}$$

$$\bullet a + 2b + c = \mathbf{0}$$

$$\bullet a + b + 2c = \mathbf{0}$$

Solution space



Dimension = 0

System 2

$$\bullet a + b + c = \mathbf{0}$$

$$\bullet a + b + 2c = \mathbf{0}$$

$$\bullet a + b + 3c = \mathbf{0}$$

Solution space



System 3

$$\bullet a + b + c = \mathbf{0}$$

$$\bullet 2a + 2b + 2c = \mathbf{0}$$

$$\bullet 3a + 3b + 3c = \mathbf{0}$$

Solution space



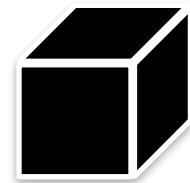
System 4

$$\bullet 0a + 0b + 0c = \mathbf{0}$$

$$\bullet 0a + 0b + 0c = \mathbf{0}$$

$$\bullet 0a + 0b + 0c = \mathbf{0}$$

Solution space



Null space for systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

Solution space



Dimension = 0

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

Solution space



Dimension = 1

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

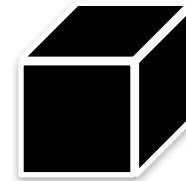
Solution space



System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Solution space



Null space for systems of linear equations

System 1

$$\bullet a + b + c = \mathbf{0}$$

$$\bullet a + 2b + c = \mathbf{0}$$

$$\bullet a + b + 2c = \mathbf{0}$$

Solution space



Dimension = 0

System 2

$$\bullet a + b + c = \mathbf{0}$$

$$\bullet a + b + 2c = \mathbf{0}$$

$$\bullet a + b + 3c = \mathbf{0}$$

Solution space



Dimension = 1

System 3

$$\bullet a + b + c = \mathbf{0}$$

$$\bullet 2a + 2b + 2c = \mathbf{0}$$

$$\bullet 3a + 3b + 3c = \mathbf{0}$$

Solution space



Dimension = 2

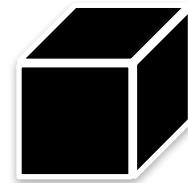
System 4

$$\bullet 0a + 0b + 0c = \mathbf{0}$$

$$\bullet 0a + 0b + 0c = \mathbf{0}$$

$$\bullet 0a + 0b + 0c = \mathbf{0}$$

Solution space



Null space for systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

Solution space



Dimension = 0

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

Solution space



Dimension = 1

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

Solution space

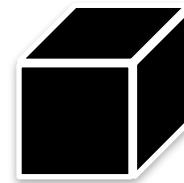


Dimension = 2

System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Solution space



Dimension = 3

Null space for matrices

Matrix 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Null space



Dimension = 0

Matrix 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Null space

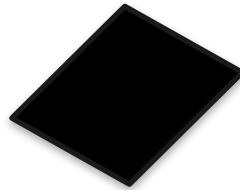


Dimension = 1

Matrix 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Null space

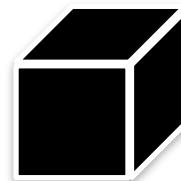


Dimension = 2

Matrix 4

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Null space



Dimension = 3

Quiz: Null space

Problem: Determine the dimension of the null space for the following matrices.

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

- $a + c = \mathbf{0}$
- $b = \mathbf{0}$
- $3a + 2b + 3c = \mathbf{0}$

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $c = \mathbf{0}$

- $a + b + c = \mathbf{0}$
- $2b + 2c = \mathbf{0}$
- $3c = \mathbf{0}$

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

- $a + c = \mathbf{0}$
- $b = \mathbf{0}$
- $3a + 2b + 3c = \mathbf{0}$

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $c = \mathbf{0}$

- $a + b + c = \mathbf{0}$
- $2b + 2c = \mathbf{0}$
- $3c = \mathbf{0}$

All points of the form
 $(x, 0, -x)$

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

- $a + c = 0$

- $b = 0$

- $3a + 2b + 3c = 0$

- $a + b + c = 0$

- $a + b + 2c = 0$

- $c = 0$

- $a + b + c = 0$

- $2b + 2c = 0$

- $3c = 0$

All points of the form
 $(x, 0, -x)$

Dimension = 1

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

- $a + c = \mathbf{0}$
- $b = \mathbf{0}$
- $3a + 2b + 3c = \mathbf{0}$

All points of the form
 $(x, 0, -x)$
Dimension = 1

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $c = \mathbf{0}$

All points of the form
 $(x, -x, 0)$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

- $a + b + c = \mathbf{0}$
- $2b + 2c = \mathbf{0}$
- $3c = \mathbf{0}$

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

- $a + c = \mathbf{0}$
- $b = \mathbf{0}$
- $3a + 2b + 3c = \mathbf{0}$

All points of the form
 $(x, 0, -x)$
Dimension = 1

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $c = \mathbf{0}$

All points of the form
 $(x, -x, 0)$
Dimension = 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

- $a + b + c = \mathbf{0}$
- $2b + 2c = \mathbf{0}$
- $3c = \mathbf{0}$

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

- $a + c = \mathbf{0}$
- $b = \mathbf{0}$
- $3a + 2b + 3c = \mathbf{0}$

All points of the form
 $(x, 0, -x)$
Dimension = 1

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $c = \mathbf{0}$

All points of the form
 $(x, -x, 0)$
Dimension = 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

- $a + b + c = \mathbf{0}$
- $2b + 2c = \mathbf{0}$
- $3c = \mathbf{0}$

The point
 $(0,0,0)$

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

- $a + c = \mathbf{0}$
- $b = \mathbf{0}$
- $3a + 2b + 3c = \mathbf{0}$

All points of the form
 $(x, 0, -x)$
Dimension = 1

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $c = \mathbf{0}$

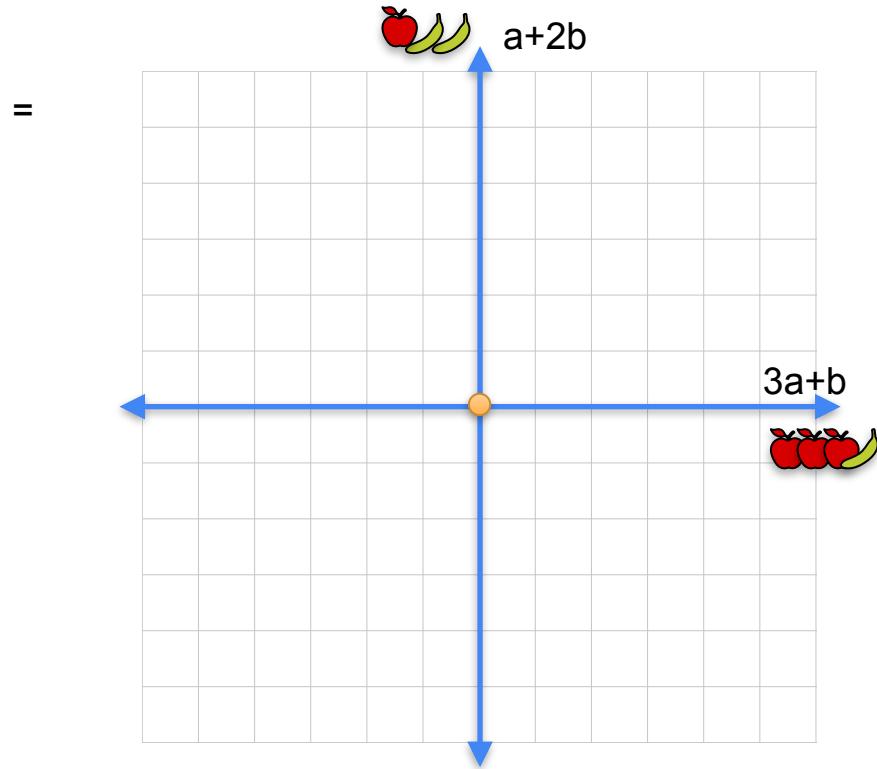
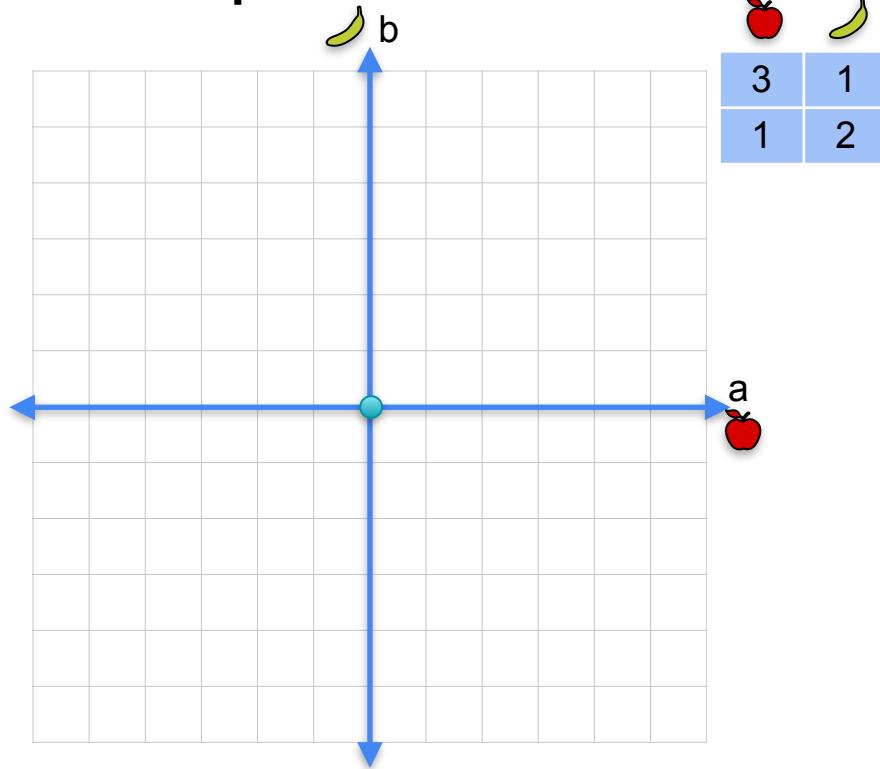
All points of the form
 $(x, -x, 0)$
Dimension = 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

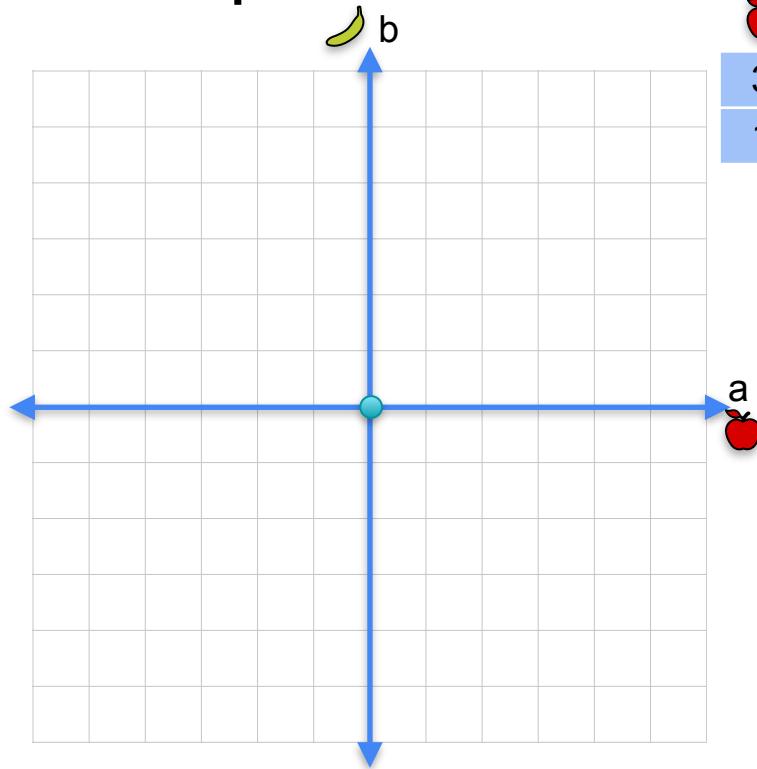
- $a + b + c = \mathbf{0}$
- $2b + 2c = \mathbf{0}$
- $3c = \mathbf{0}$

The point
 $(0,0,0)$
Dimension = 0

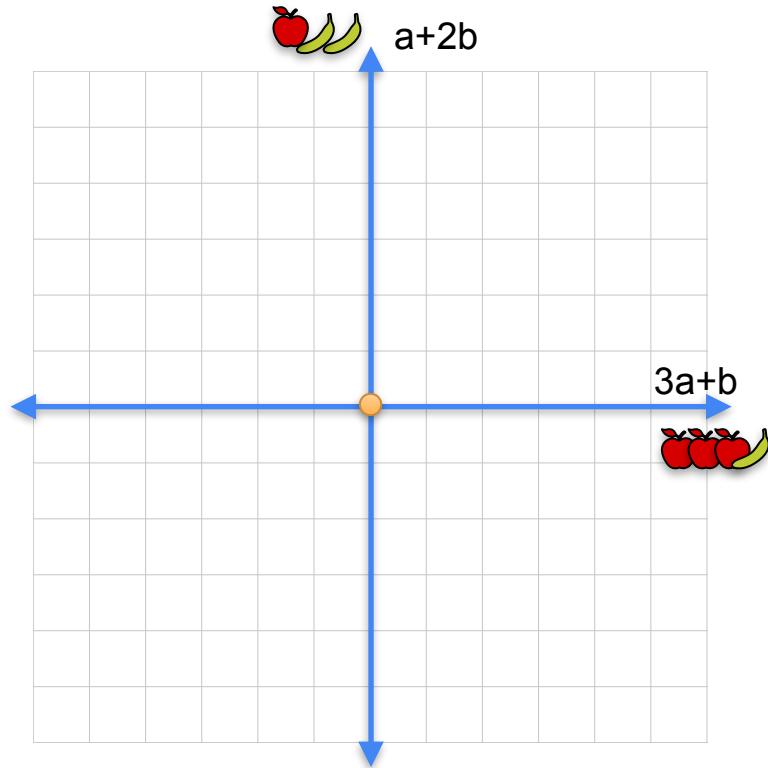
Null space



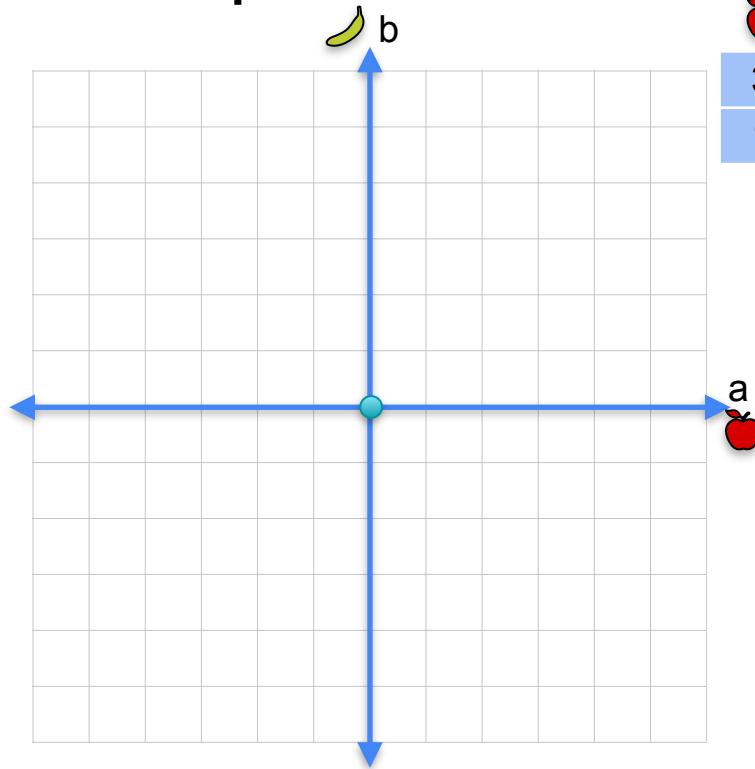
Null space



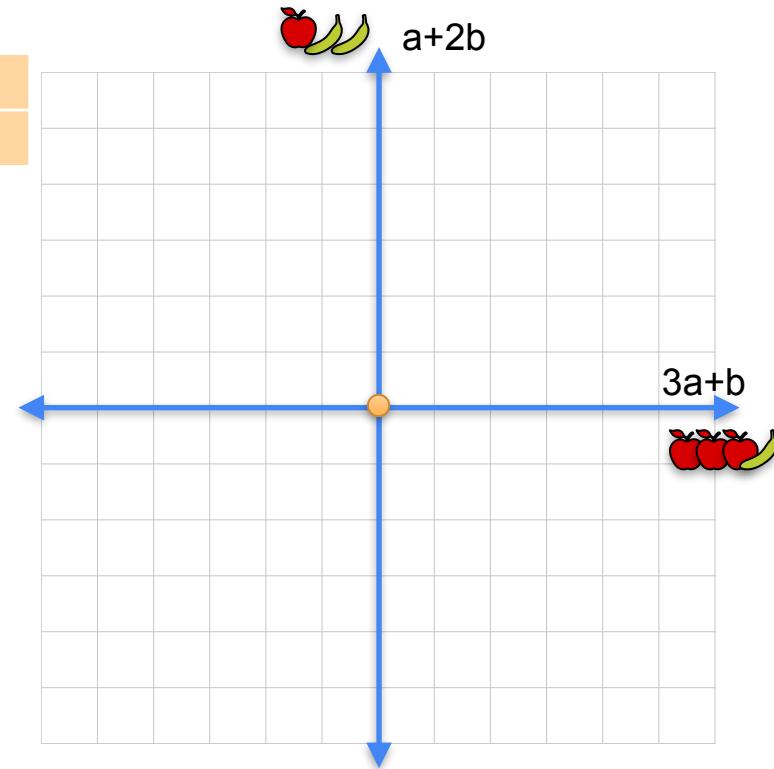
$$\begin{array}{|c|c|c|} \hline \text{apple} & \text{banana} & \\ \hline 3 & 1 & a \\ \hline 1 & 2 & b \\ \hline \end{array} =$$



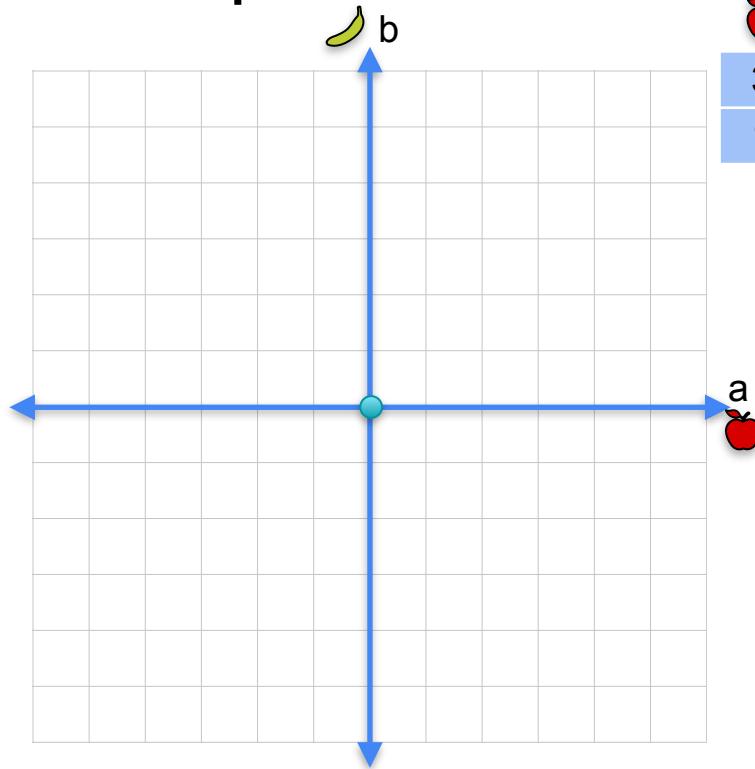
Null space



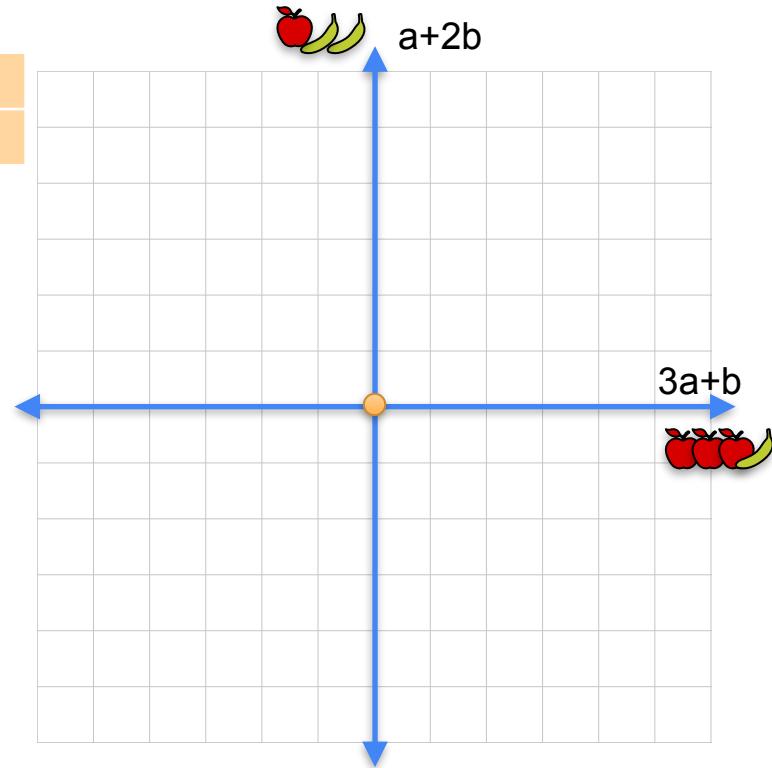
$$\begin{array}{cc|c} \text{apple} & \text{banana} & \\ \hline 3 & 1 & a \\ 1 & 2 & b \end{array} = \begin{array}{cc} 0 \\ 0 \end{array}$$



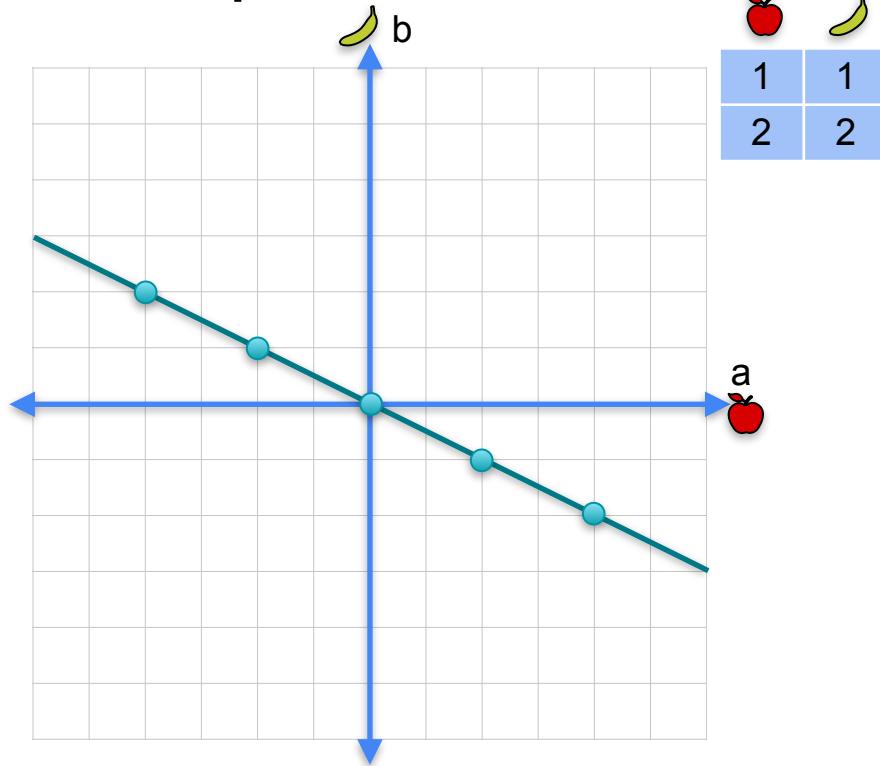
Null space



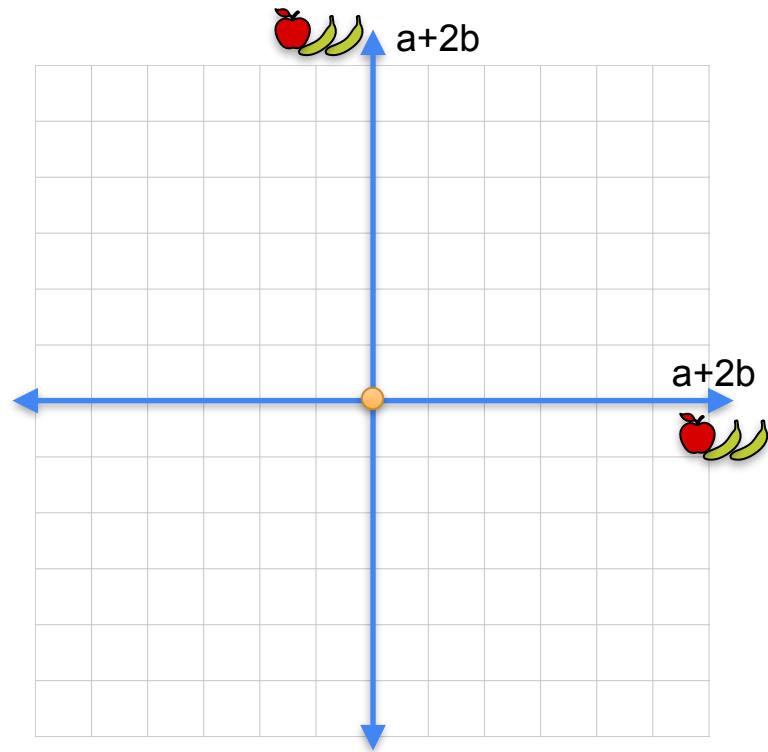
$$\begin{array}{cc|c} \text{apple} & \text{banana} & \\ \hline 3 & 1 & 0 \\ 1 & 2 & 0 \end{array} = \begin{array}{cc|c} & & 0 \\ & & 0 \end{array}$$



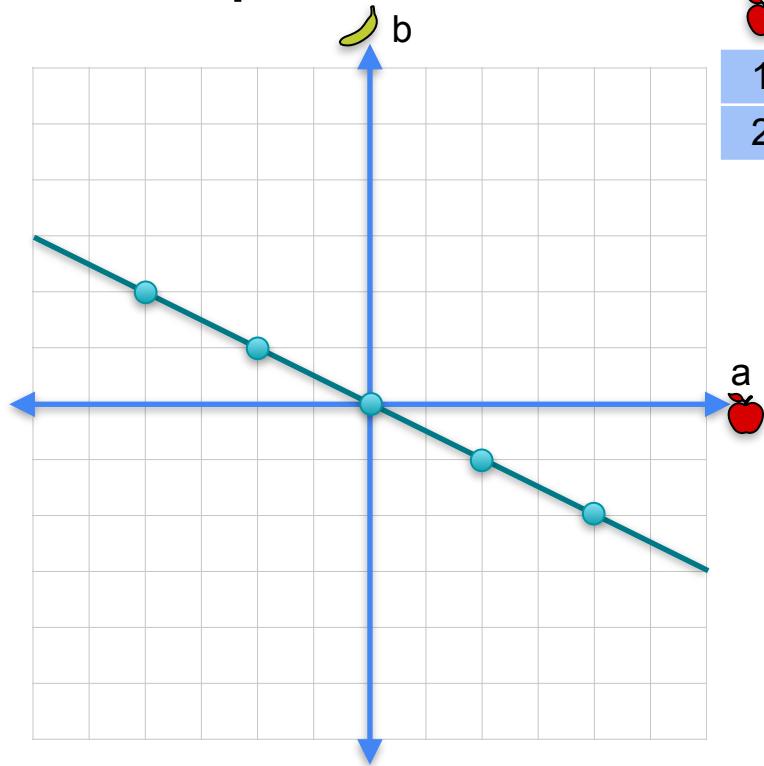
Null space



=

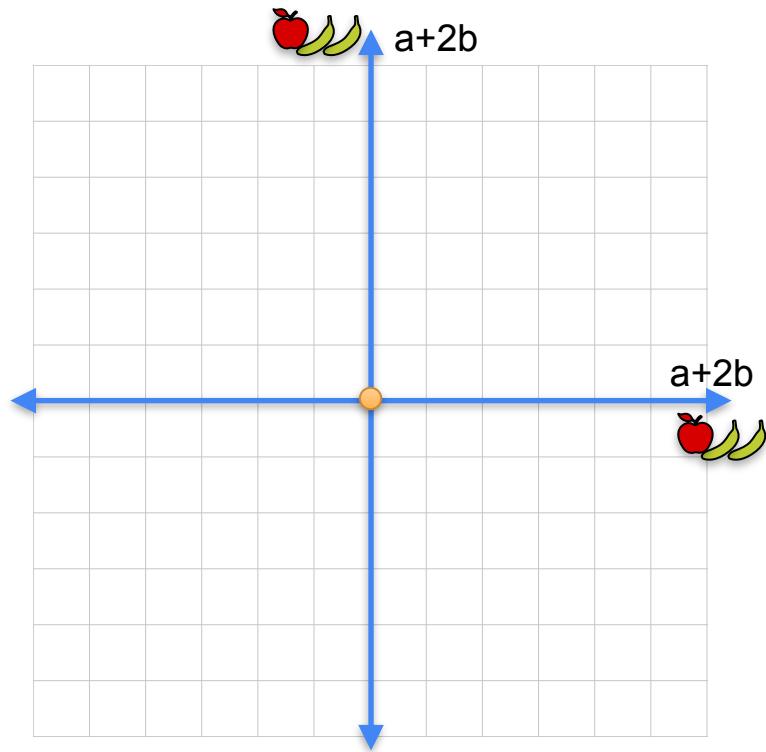


Null space

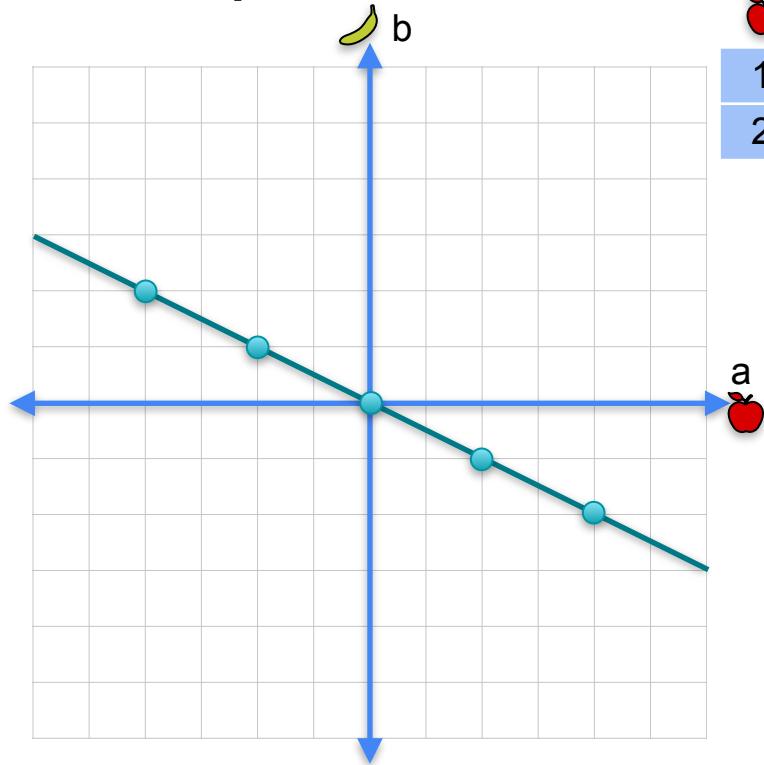


| | | | |
|---|---|-------|--------|
| | | apple | banana |
| 1 | 1 | | a |
| 2 | 2 | | b |

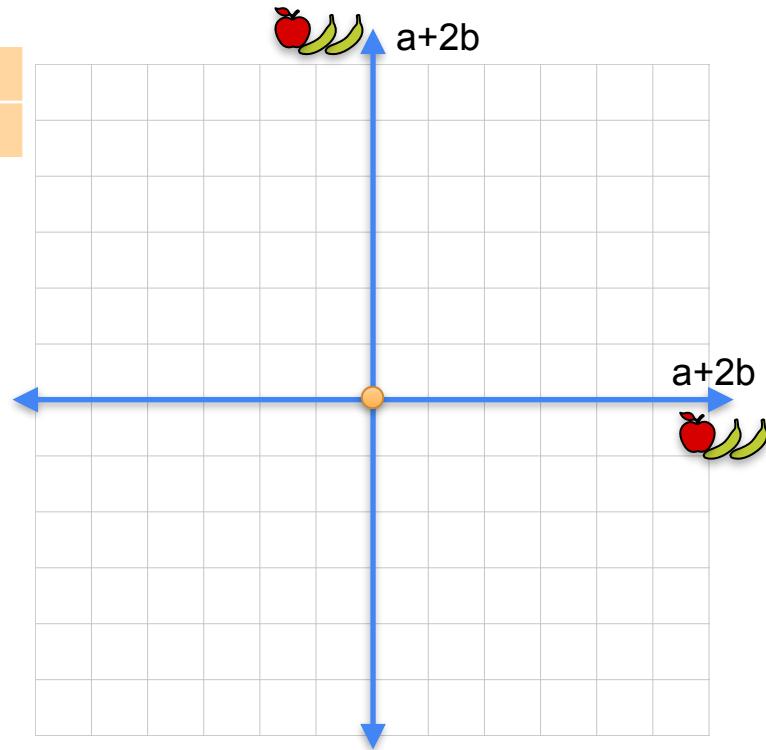
 $=$



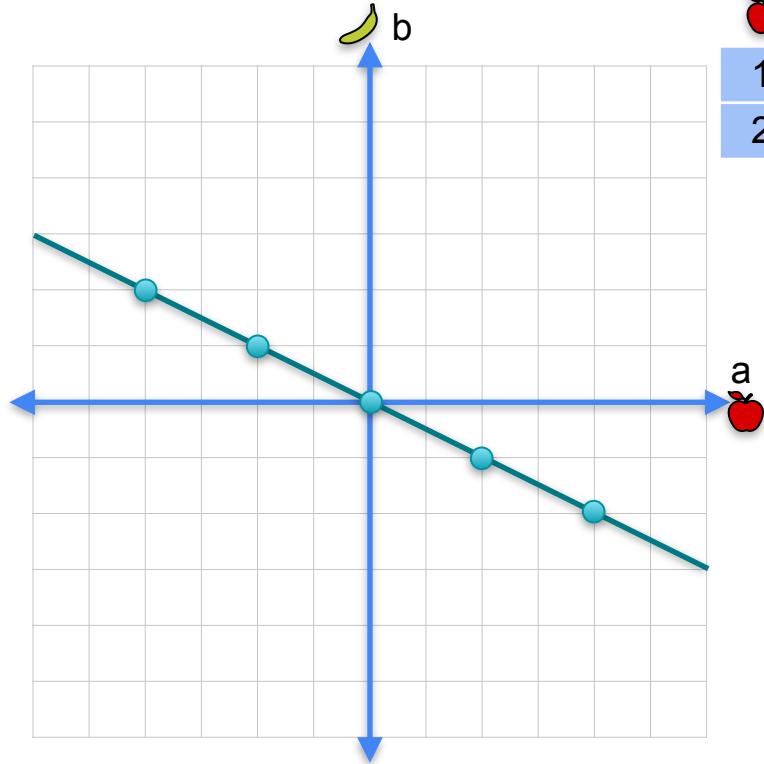
Null space



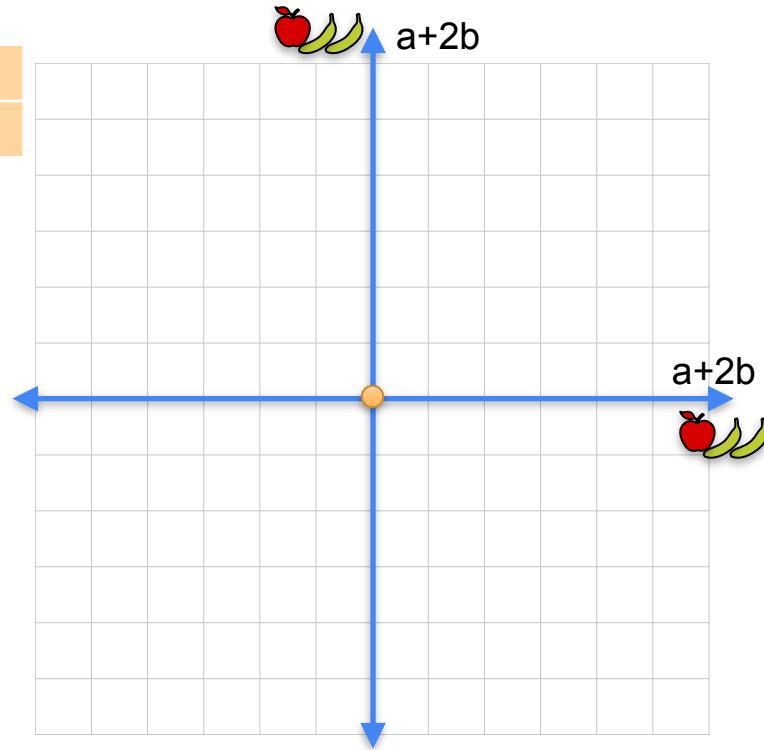
$$\begin{array}{cc|c} \text{apple} & \text{banana} & \\ \hline 1 & 1 & a \\ 2 & 2 & b \end{array} = \begin{array}{cc} 0 \\ 0 \end{array}$$



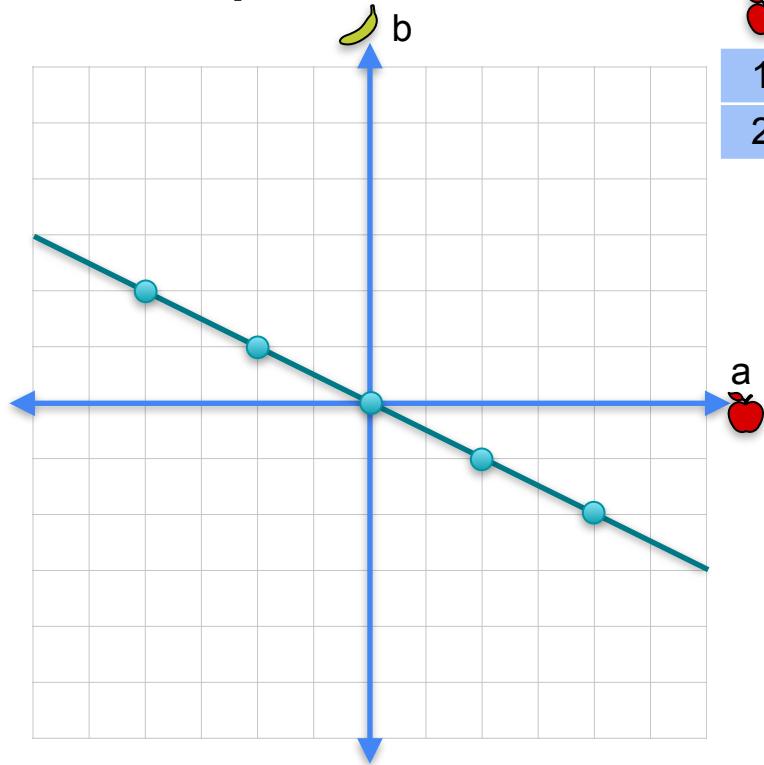
Null space



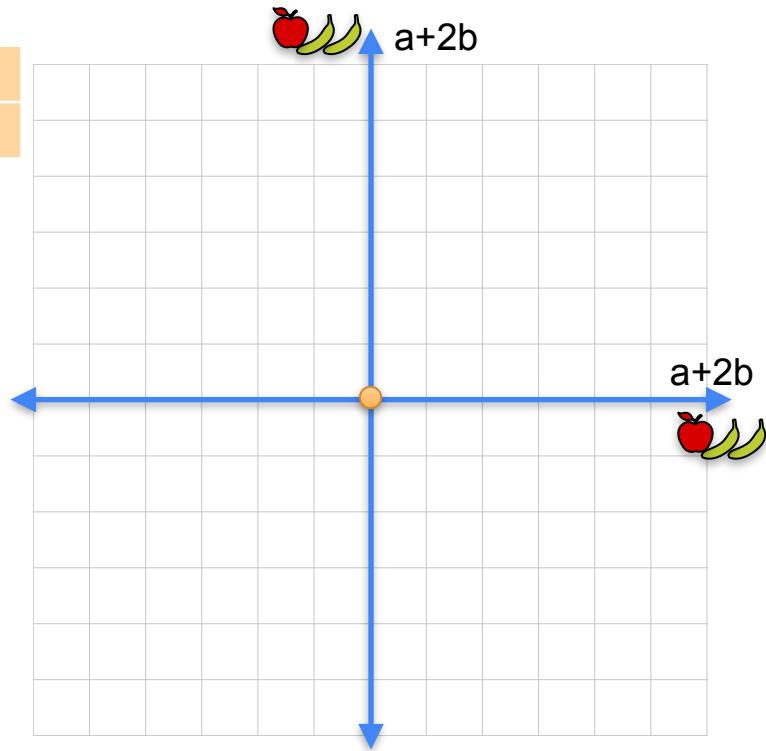
$$\begin{array}{cc|c} \text{apple} & \text{banana} & \\ \hline 1 & 1 & 0 \\ 2 & 2 & 0 \end{array} = \begin{array}{cc} 0 \\ 0 \end{array}$$



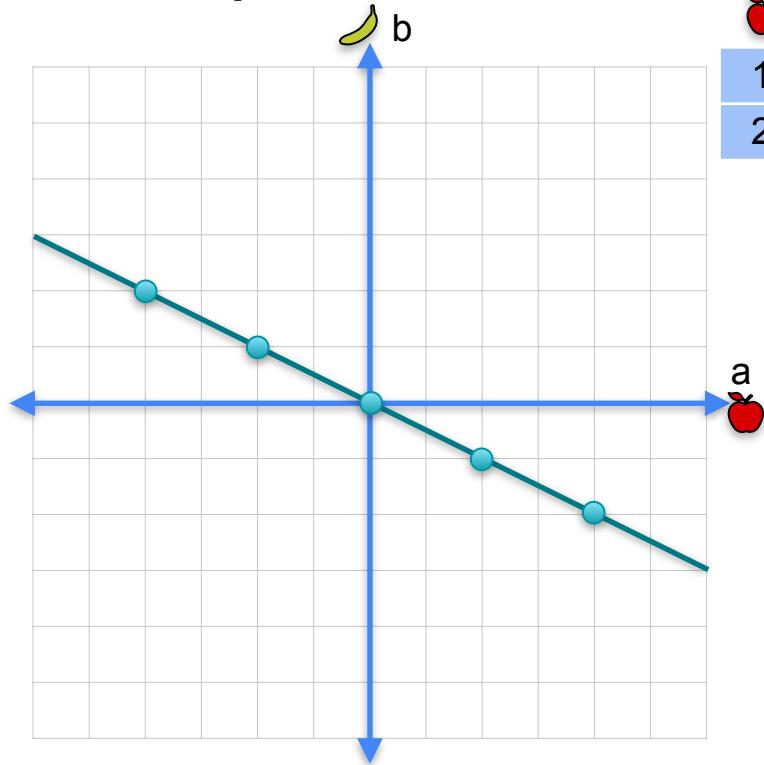
Null space



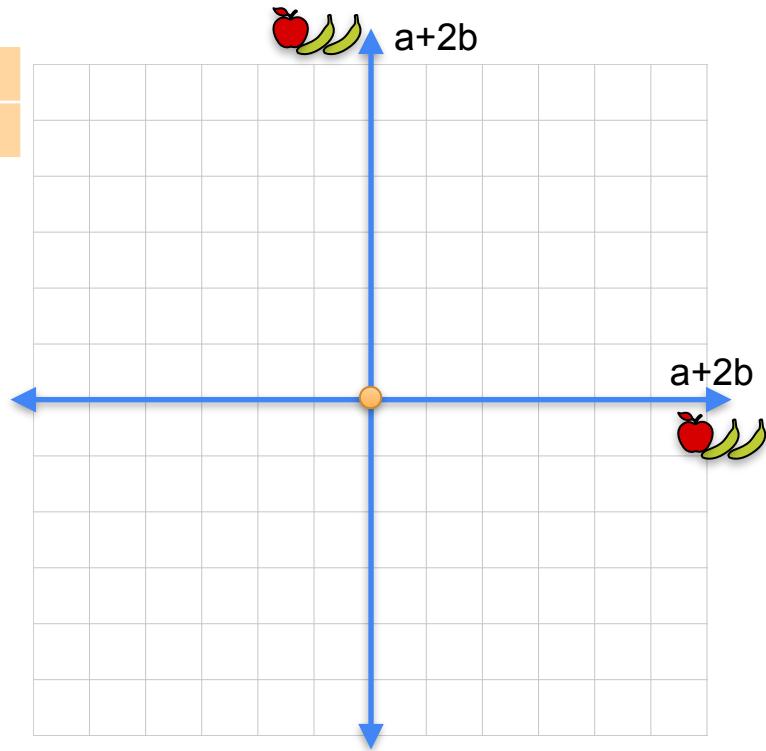
$$\begin{array}{cc|c} \text{apple} & \text{banana} & \\ \hline 1 & 1 & 0 \\ 2 & 2 & 0 \end{array} = \begin{array}{c} 0 \\ 0 \end{array}$$
$$\begin{matrix} 2 \\ -1 \end{matrix}$$



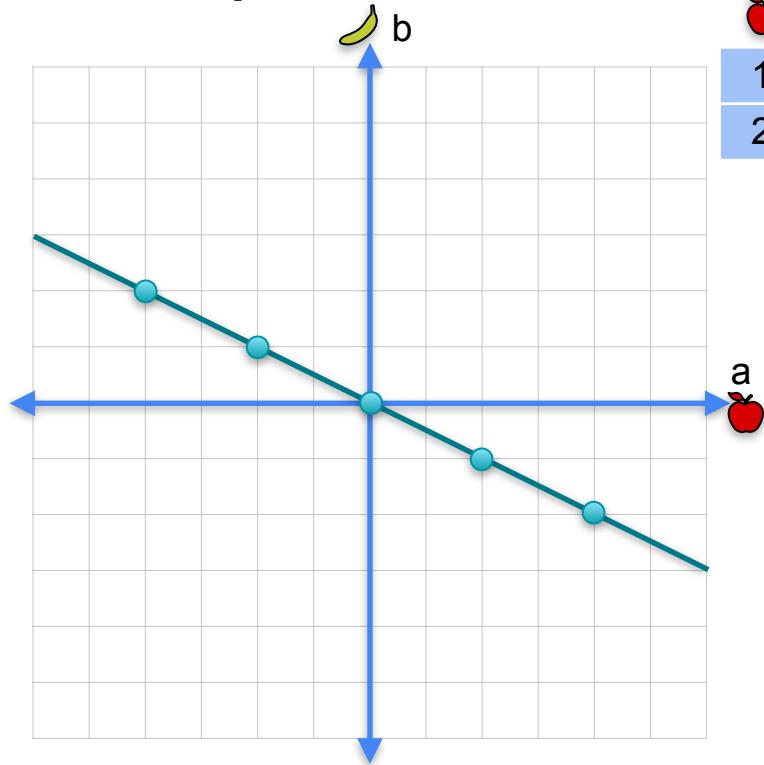
Null space



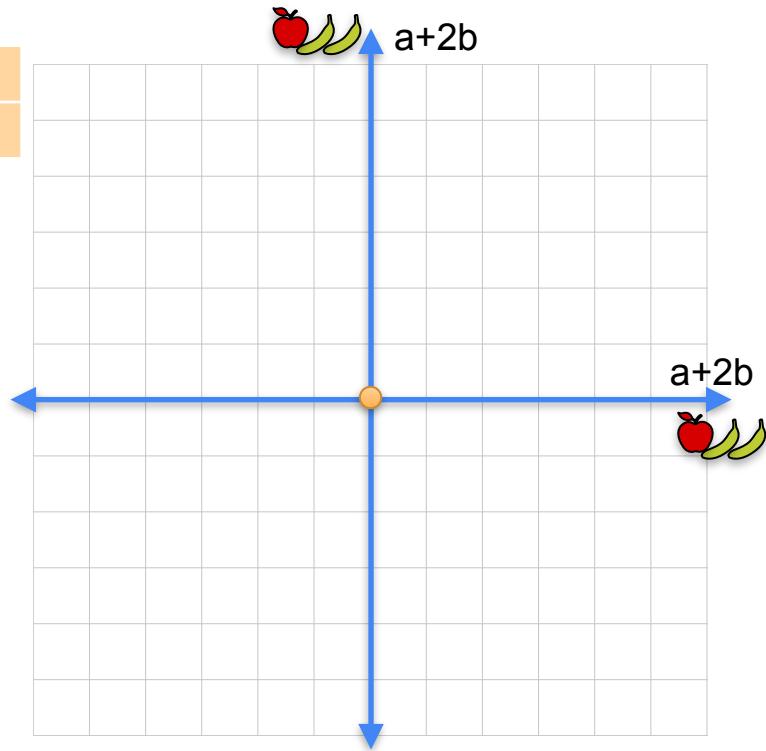
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} & \times & \begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 2 & 4 \\ -1 & -2 \end{matrix} & = & \begin{matrix} 0 \\ 0 \end{matrix} \end{matrix}$$



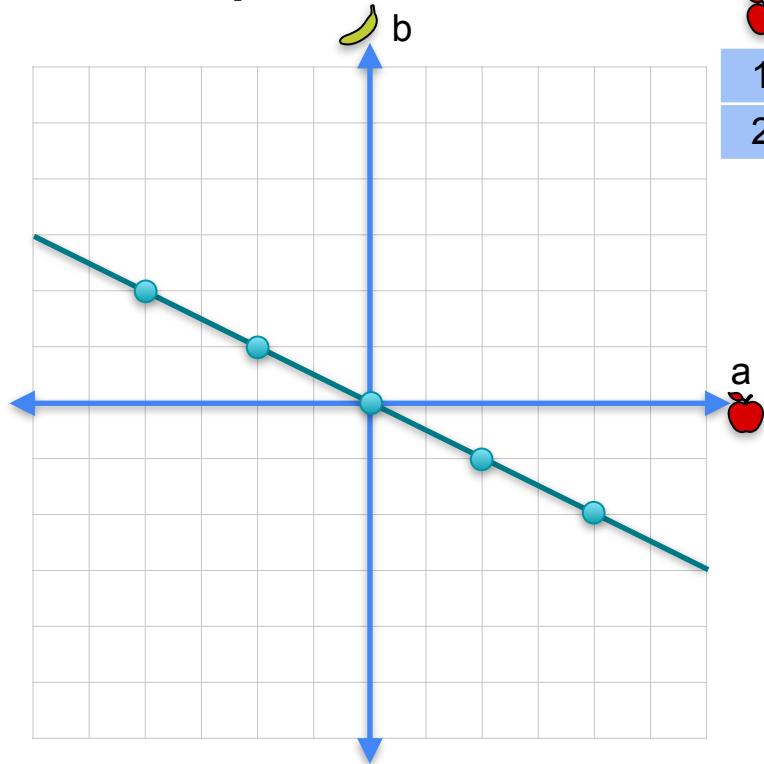
Null space



$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{array}{|c|c|c|} \hline 1 & 1 & 0 \\ \hline 2 & 2 & 0 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline 0 \\ \hline \end{array} \\ \begin{array}{|c|c|} \hline 2 & 4 \\ \hline -1 & -2 \\ \hline \end{array} \\ \begin{array}{|c|} \hline -2 \\ \hline 1 \\ \hline \end{array} \end{array}$$

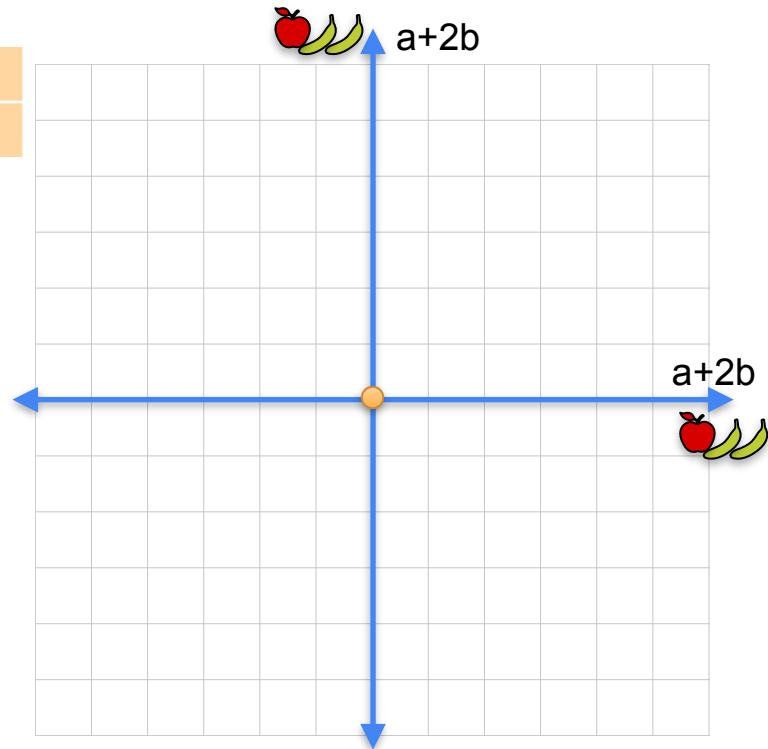


Null space

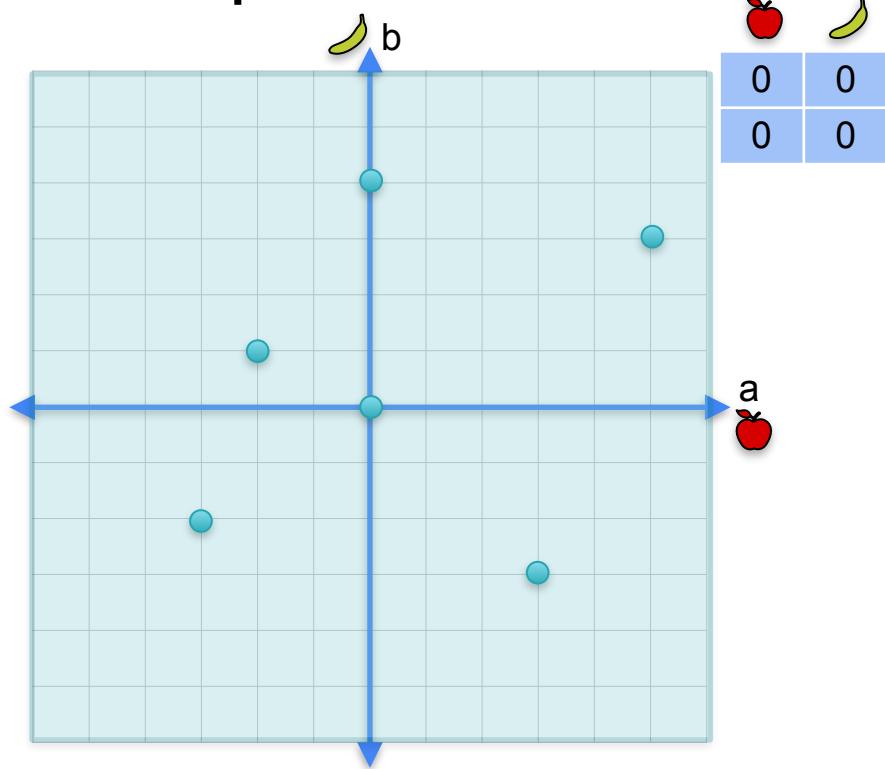


 $\begin{matrix} 1 & 1 & 0 \\ 2 & 2 & 0 \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix}$

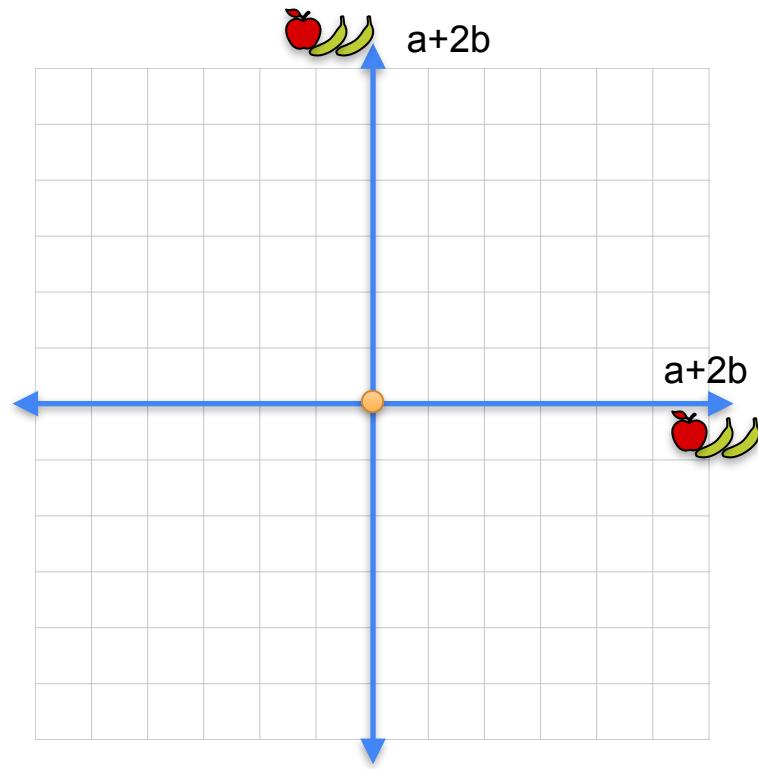
 $\begin{matrix} 2 & 4 \\ -1 & -2 \\ -2 & -4 \\ 1 & 2 \end{matrix}$



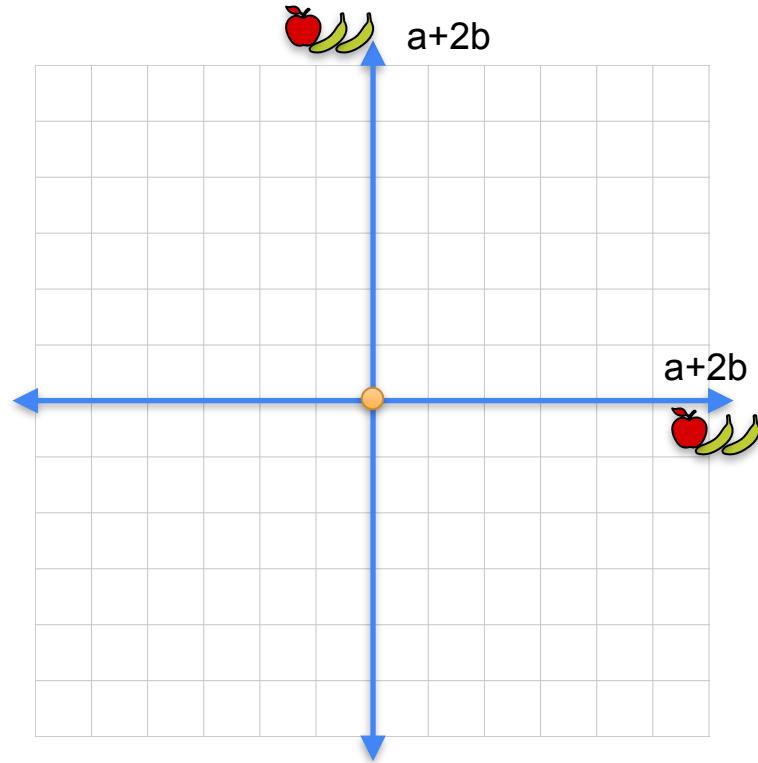
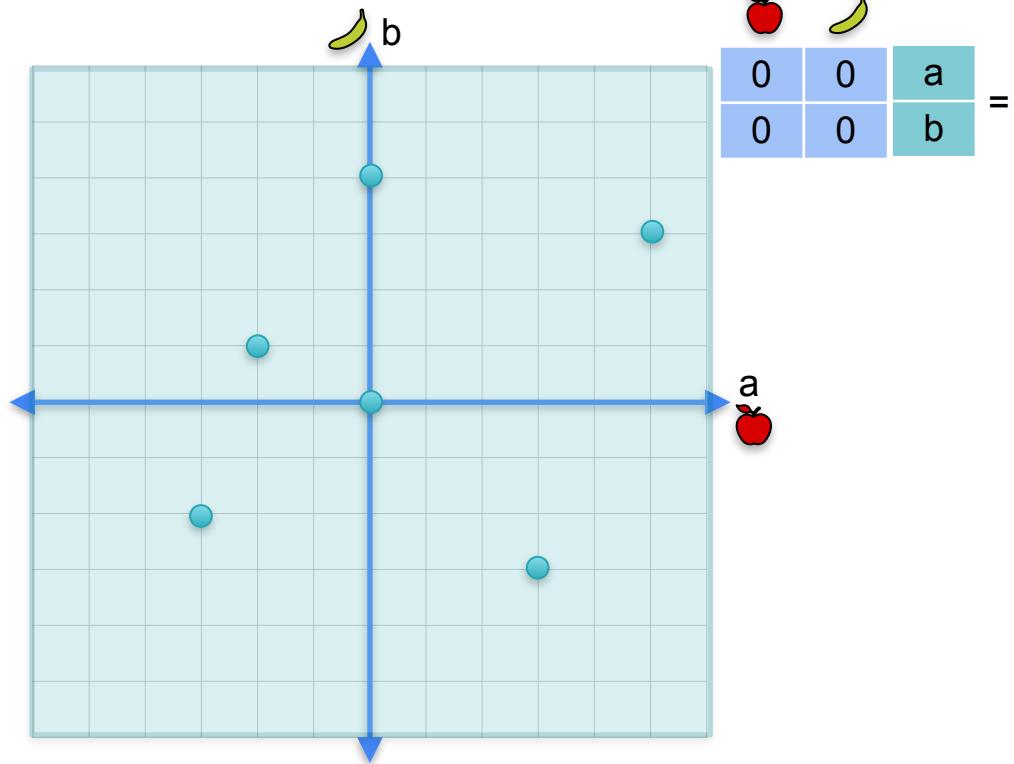
Null space



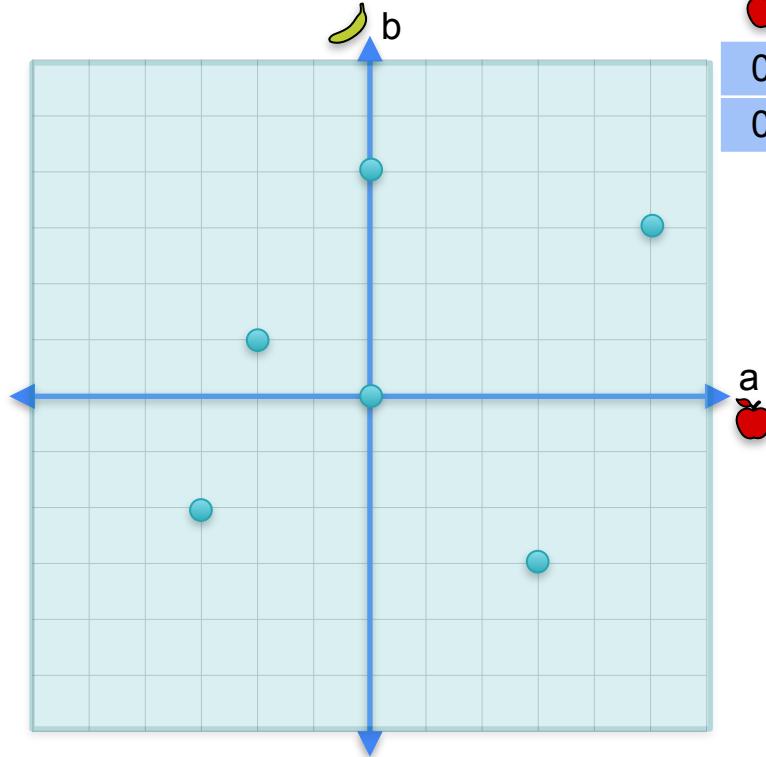
=



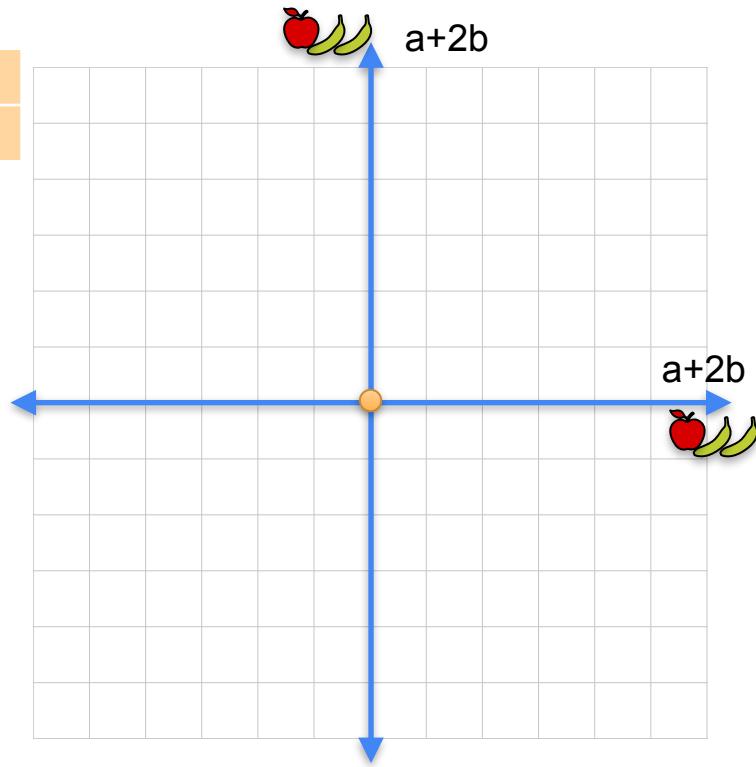
Null space



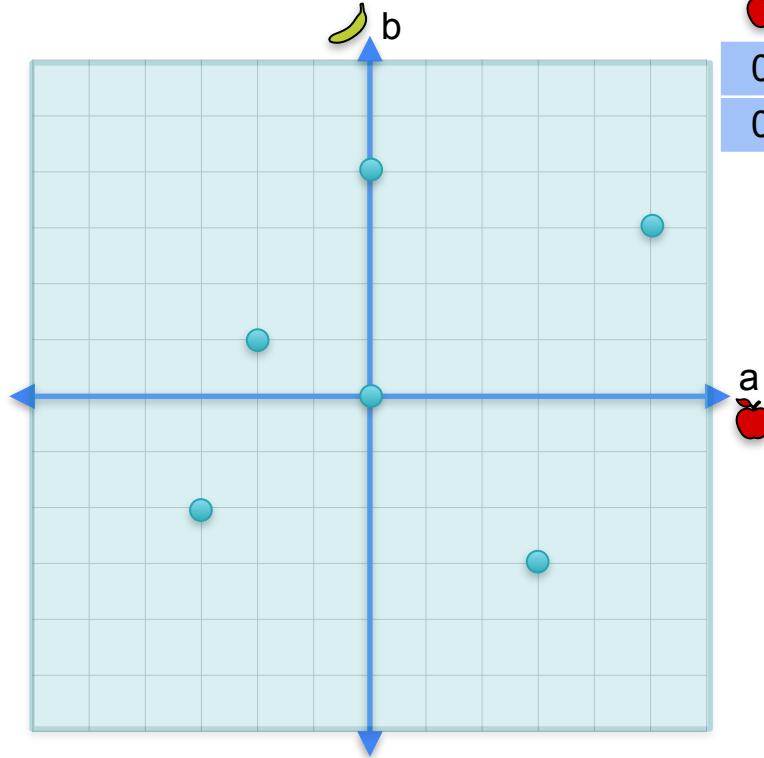
Null space



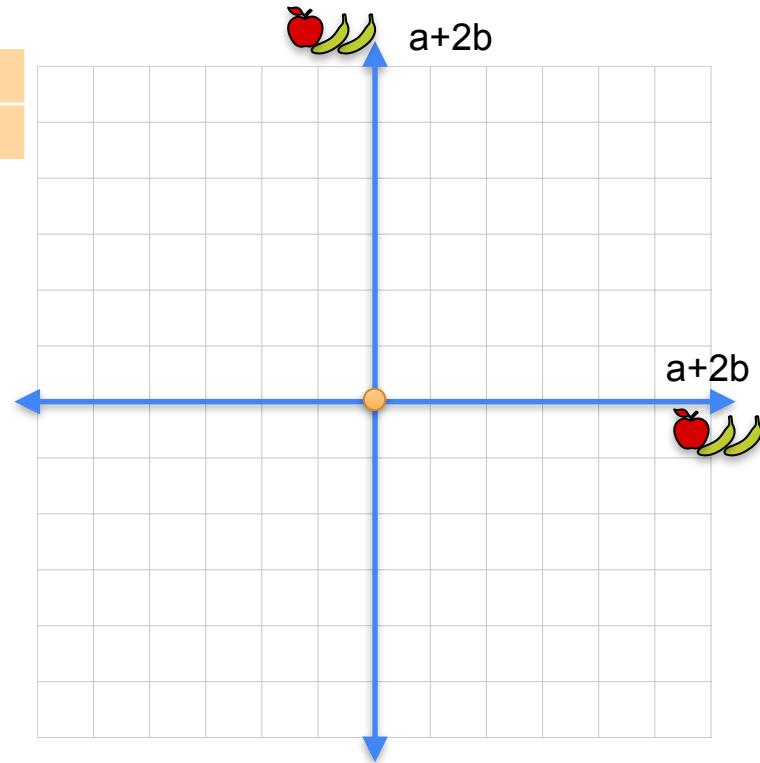
$$\begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} a \\ b \end{matrix} \end{array} = \begin{array}{c} ? \\ ? \end{array}$$



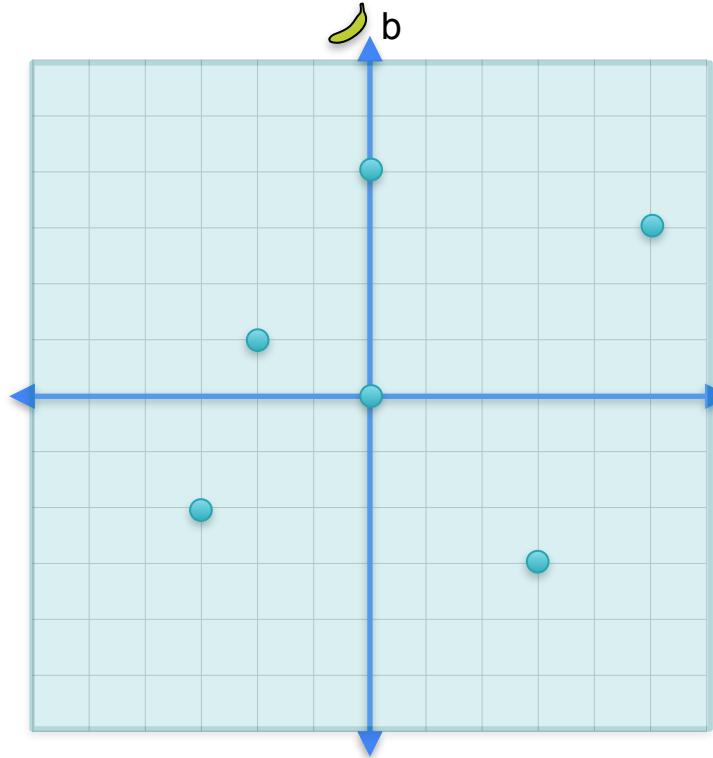
Null space



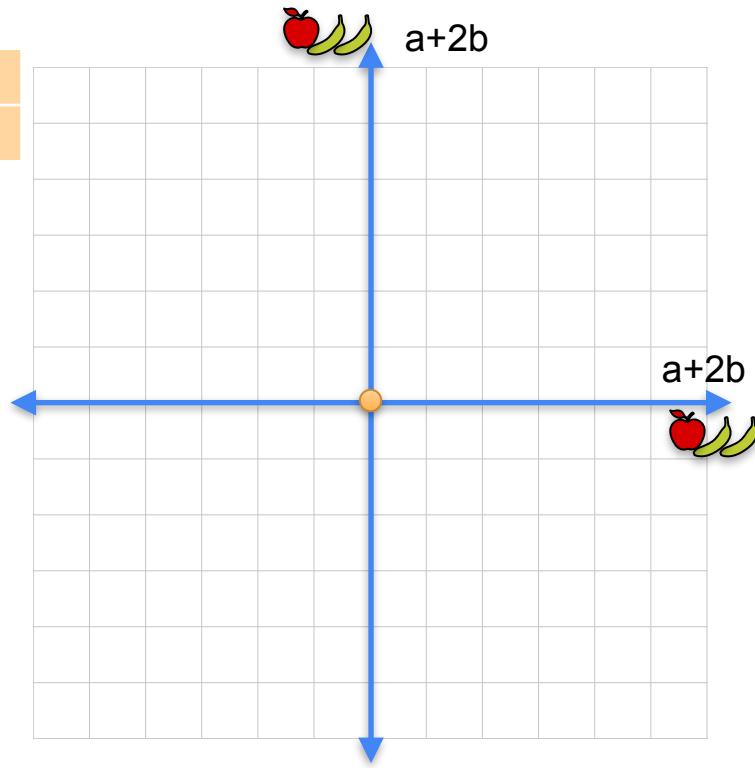
$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} = \begin{matrix} ? \\ ? \end{matrix} \end{array}$$



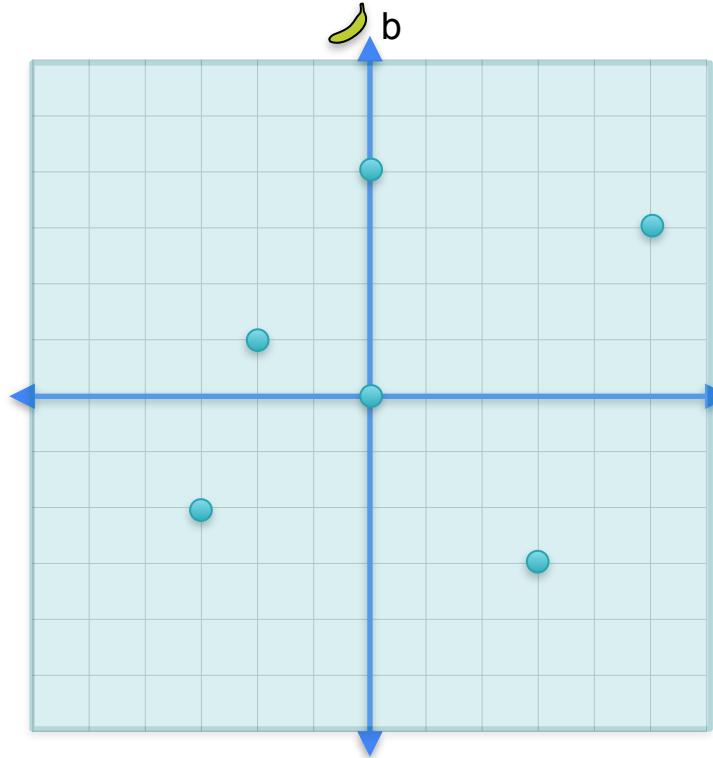
Null space



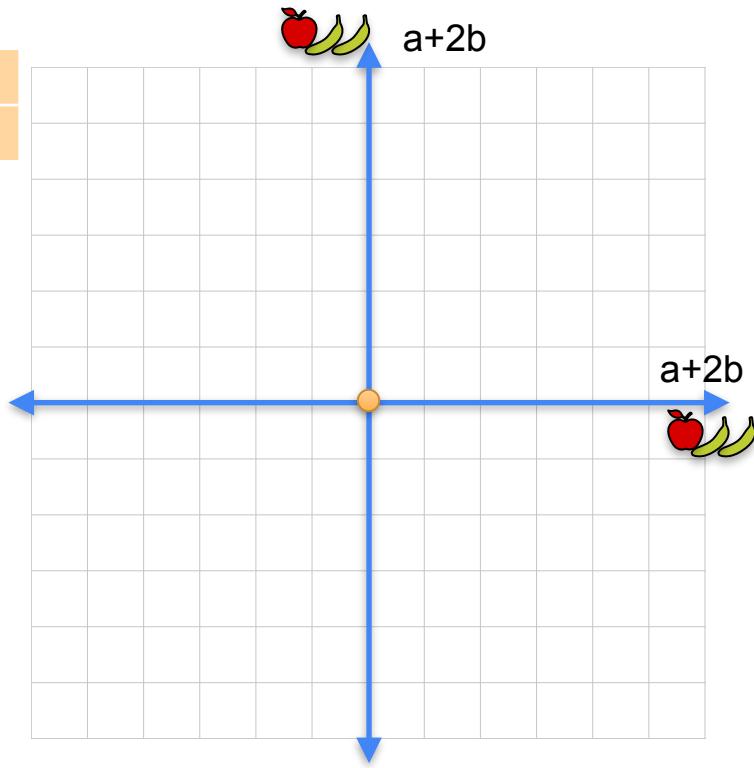
$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} = \begin{matrix} ? \\ ? \end{matrix} \\ \begin{matrix} -1 \\ 2 \end{matrix} \end{array}$$



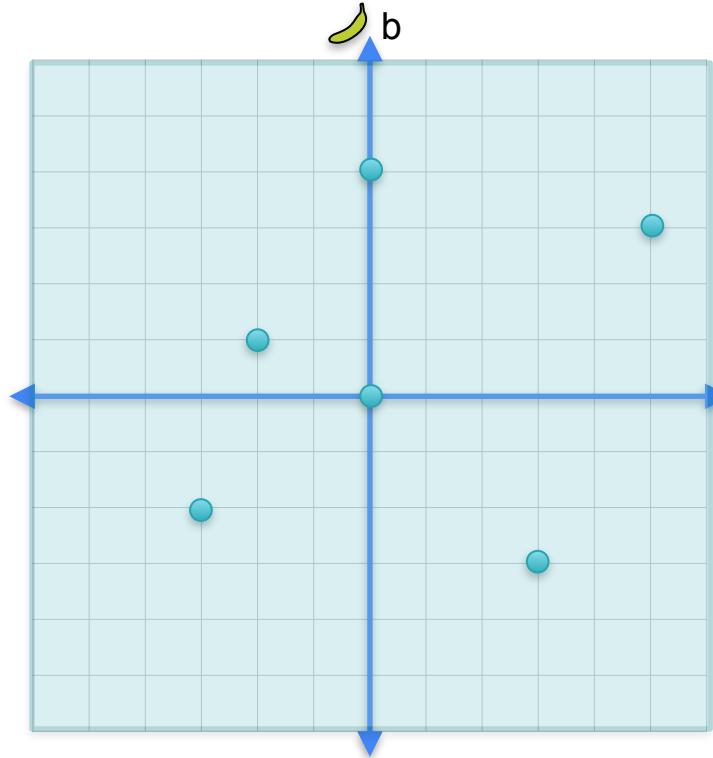
Null space



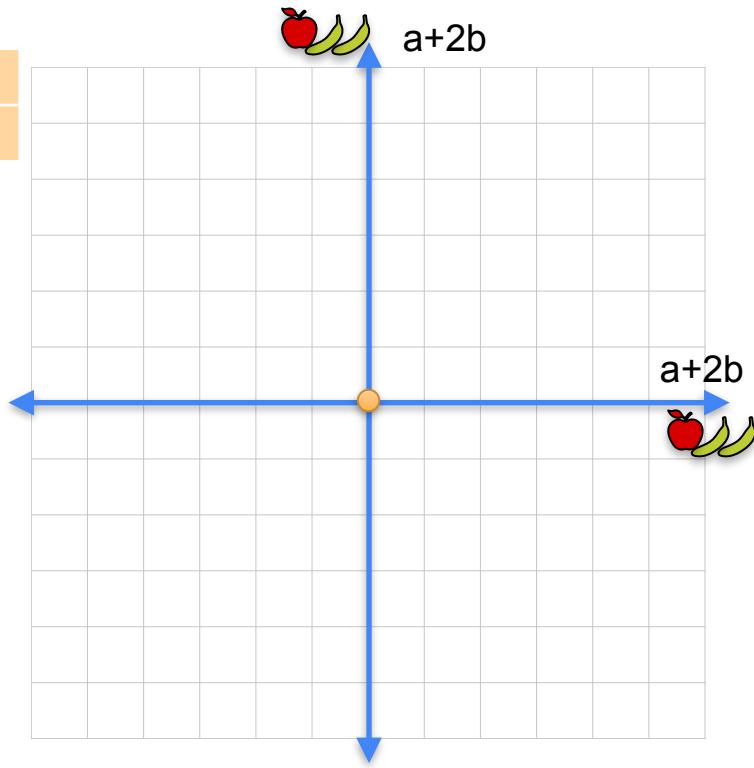
$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} = \begin{array}{|c|} \hline ? \\ \hline ? \\ \hline \end{array} \\ \begin{array}{|c|c|} \hline -1 & 5 \\ \hline 2 & 3 \\ \hline \end{array} \end{array}$$



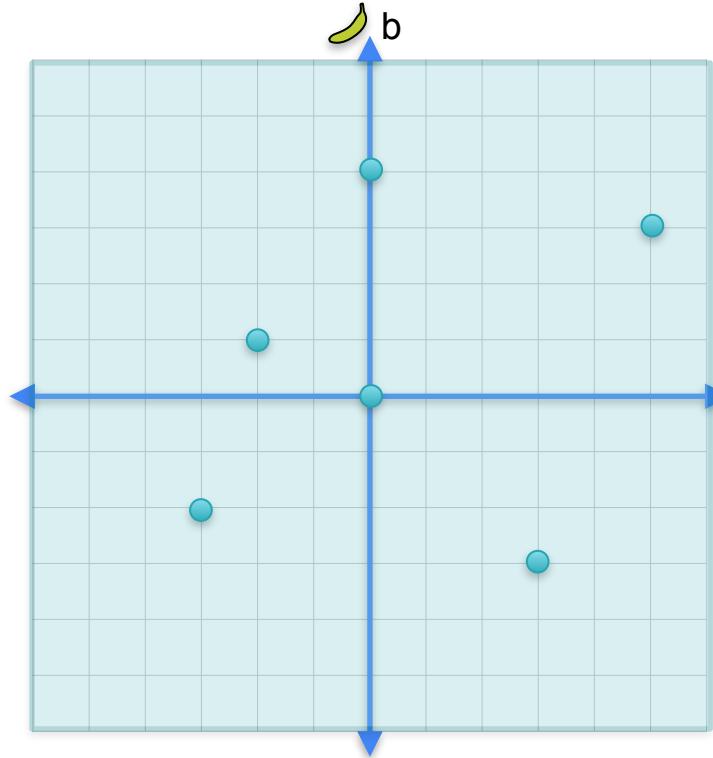
Null space



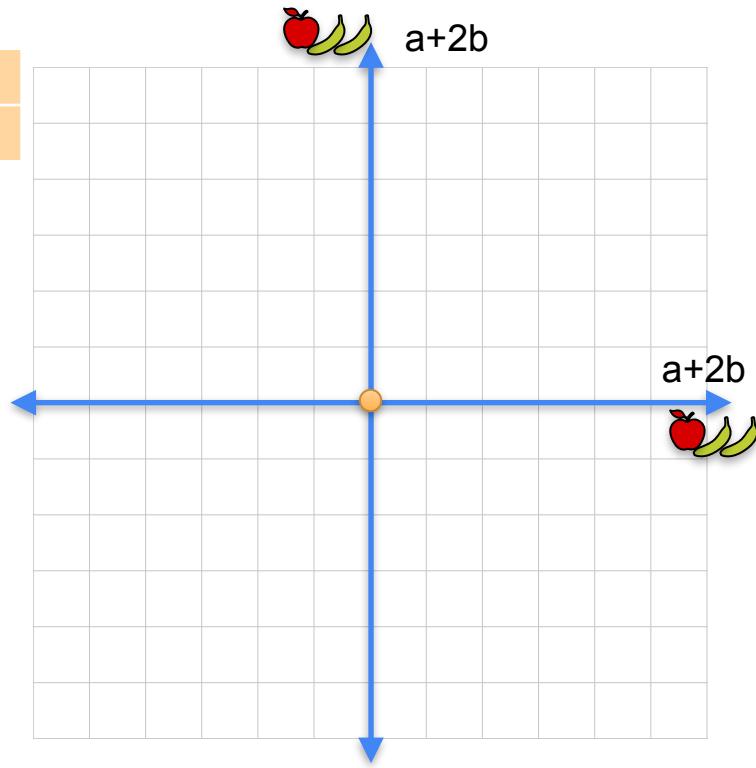
$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} = \begin{matrix} ? \\ ? \end{matrix} \\ \begin{matrix} -1 & 5 \\ 2 & 3 \end{matrix} \\ \begin{matrix} 0 \\ 4 \end{matrix} \end{array}$$



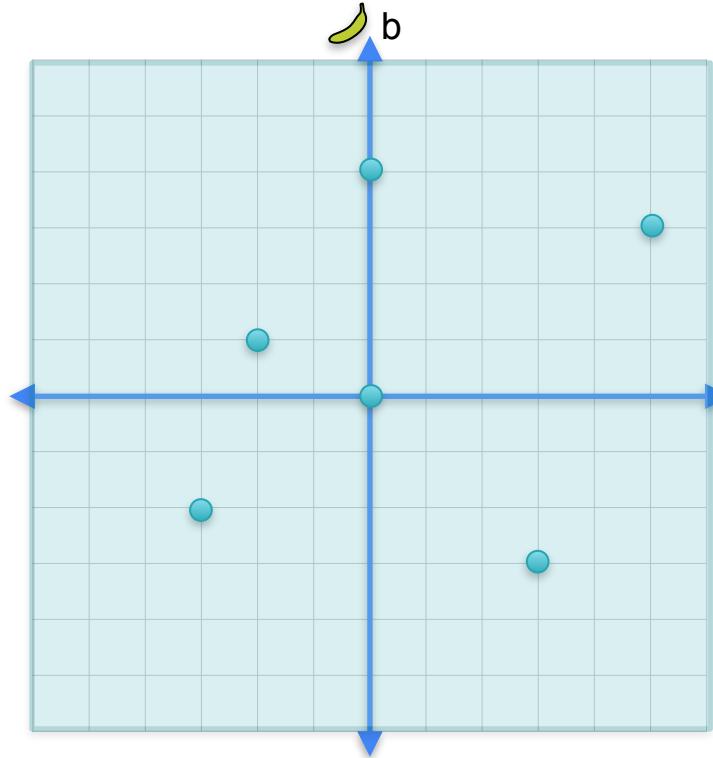
Null space



$$\begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & = \begin{matrix} ? \\ ? \end{matrix} \\ \begin{matrix} -1 & 5 \\ 2 & 3 \end{matrix} & \end{array}$$
$$\begin{matrix} 0 & -1 \\ 4 & -3 \end{matrix}$$

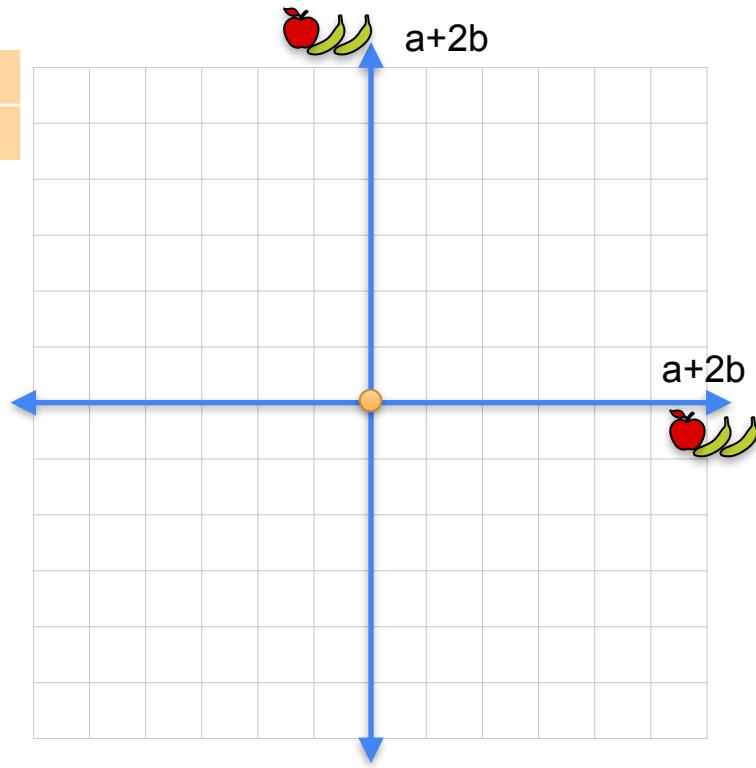


Null space



$$\begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & = \begin{matrix} ? \\ ? \end{matrix} \end{array}$$

$\begin{matrix} -1 & 5 \\ 2 & 3 \\ 0 & -1 \\ 4 & -3 \\ 3 & -3 \end{matrix}$

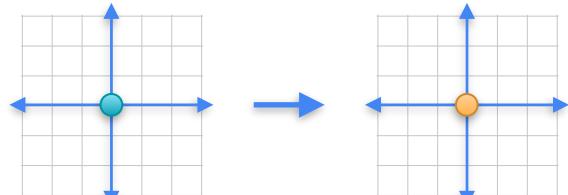


Null space

Non-singular

| | |
|---|---|
| | |
| 3 | 1 |
| 1 | 2 |

Rank = 2

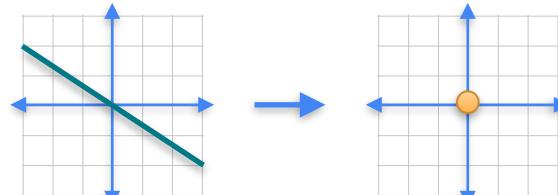


Dimension = 0

Singular

| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

Rank = 1

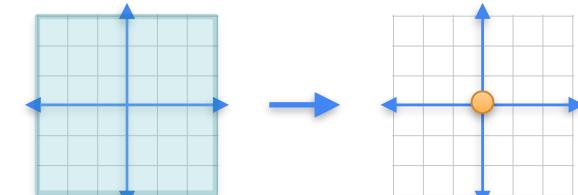


Dimension = 1

Singular

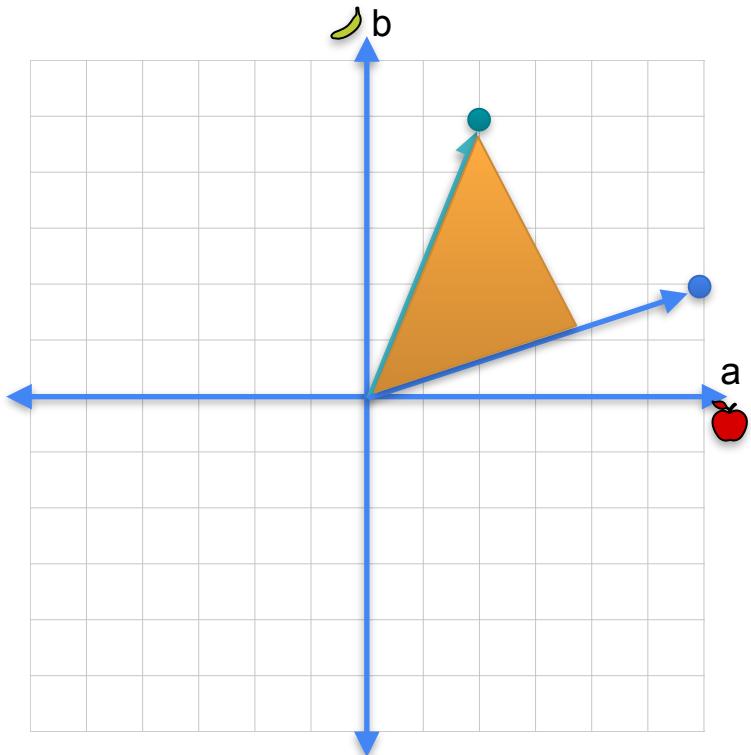
| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |

Rank = 0



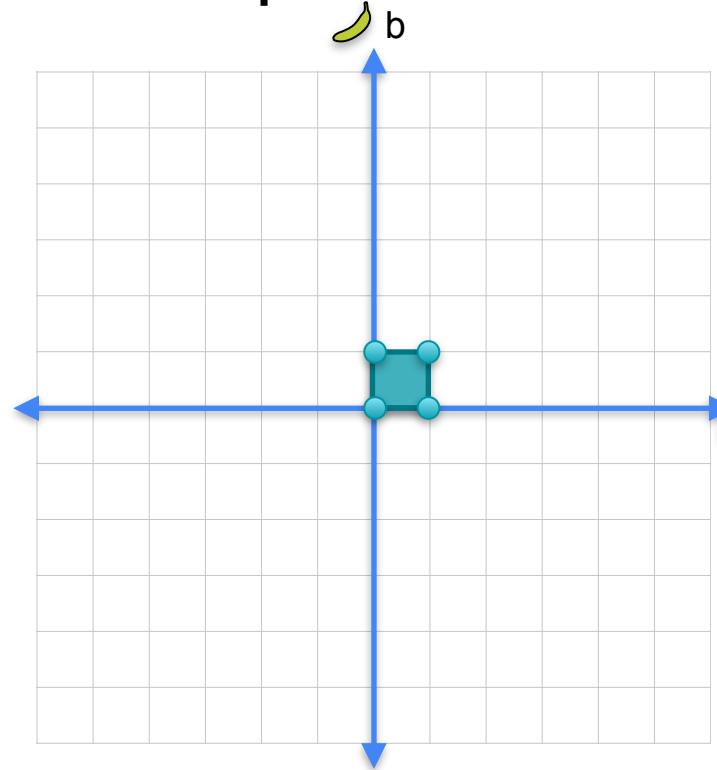
Dimension = 2

Dot product as an area

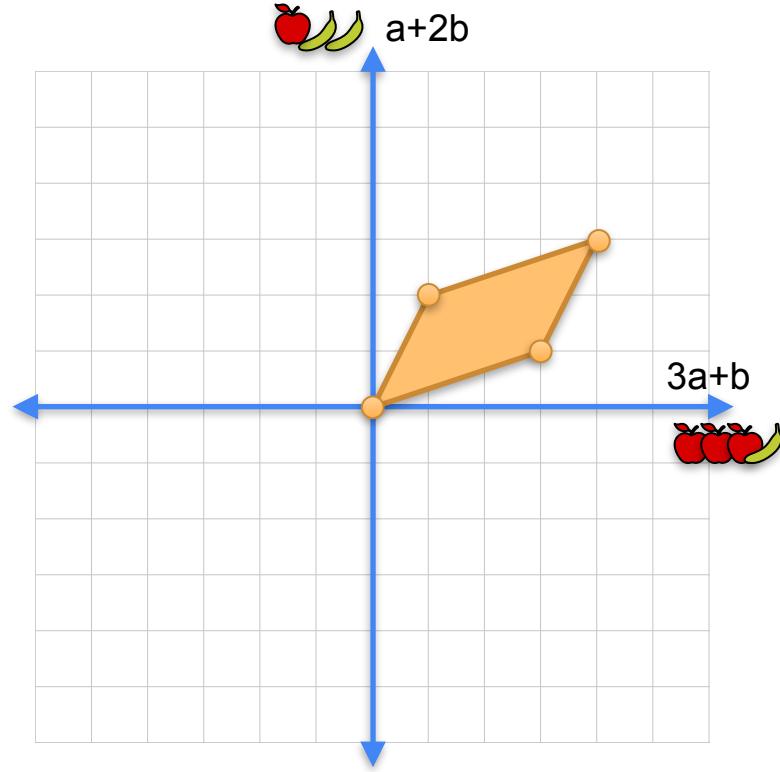


$$\begin{matrix} \text{apple} \\ 6 \end{matrix} \quad \begin{matrix} \text{banana} \\ 2 \end{matrix} \cdot \begin{matrix} \$\text{apple} \\ \$\text{banana} \end{matrix} \begin{matrix} 2 \\ 5 \end{matrix} = \$ \quad 22$$

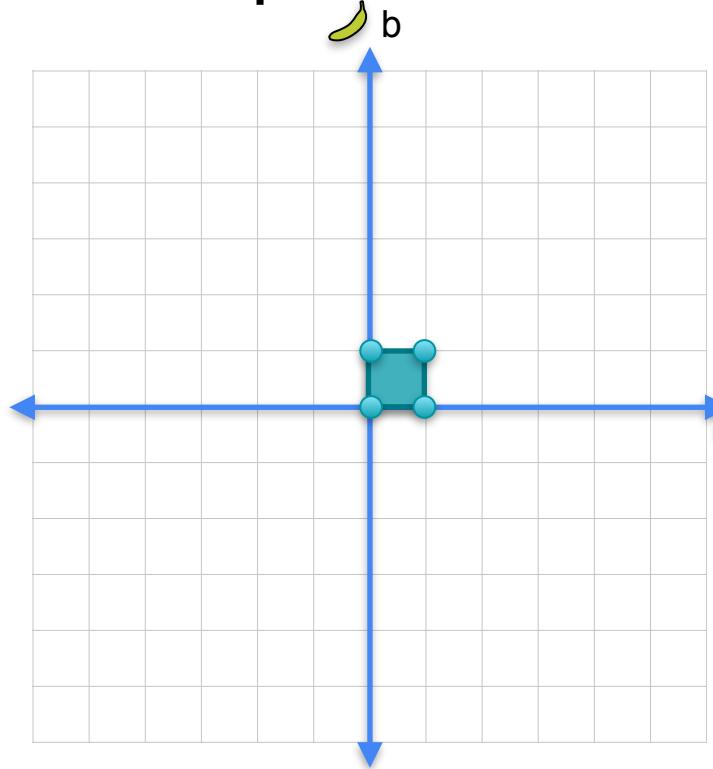
Row space



$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \end{array} = \begin{array}{l} (0,0) \rightarrow (0,0) \\ (1,0) \rightarrow (3,1) \\ (0,1) \rightarrow (1,2) \\ (1,1) \rightarrow (4,3) \end{array}$$



Row space

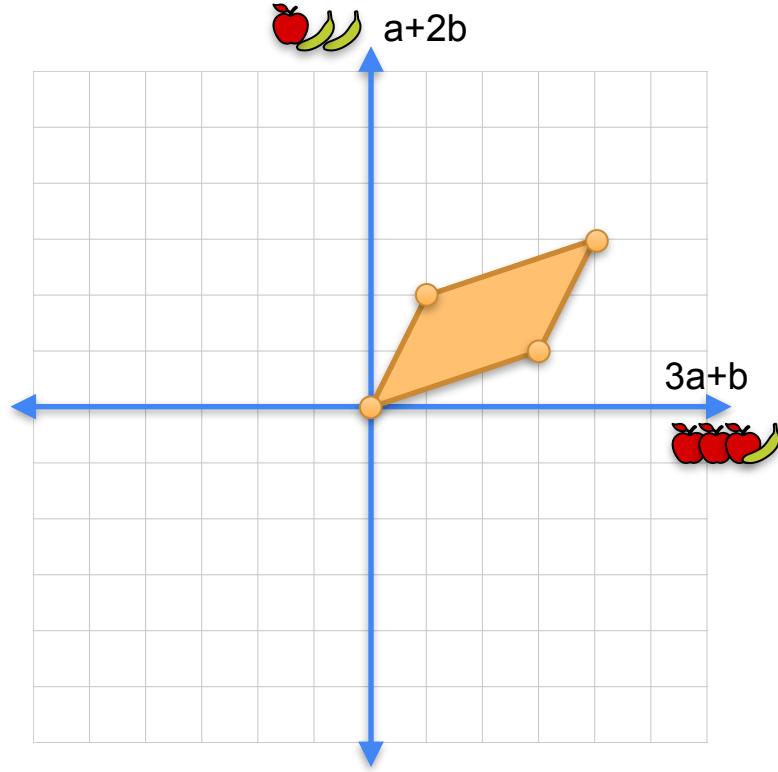


apple banana

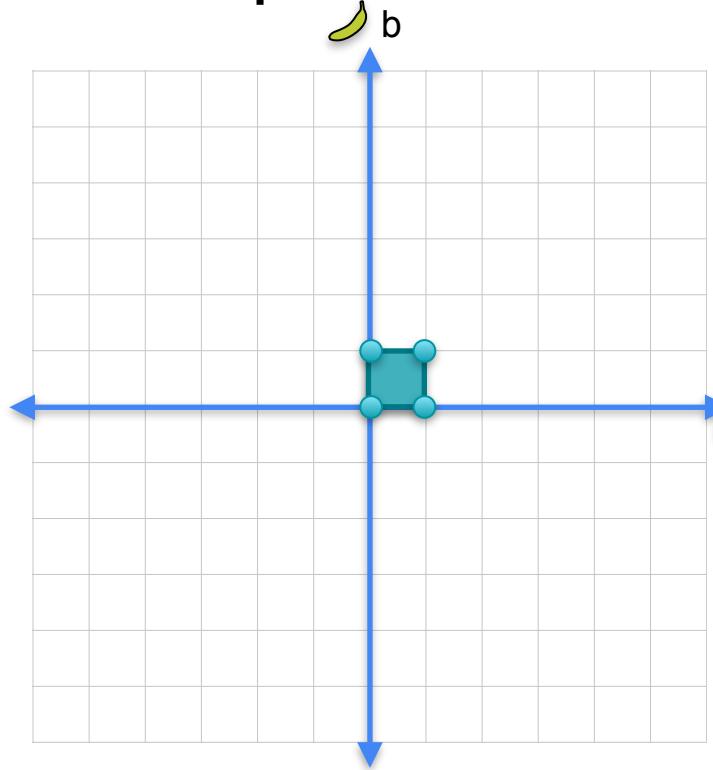
| | | |
|---|---|---|
| 3 | 1 | 0 |
| 1 | 2 | 0 |

 $=$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$

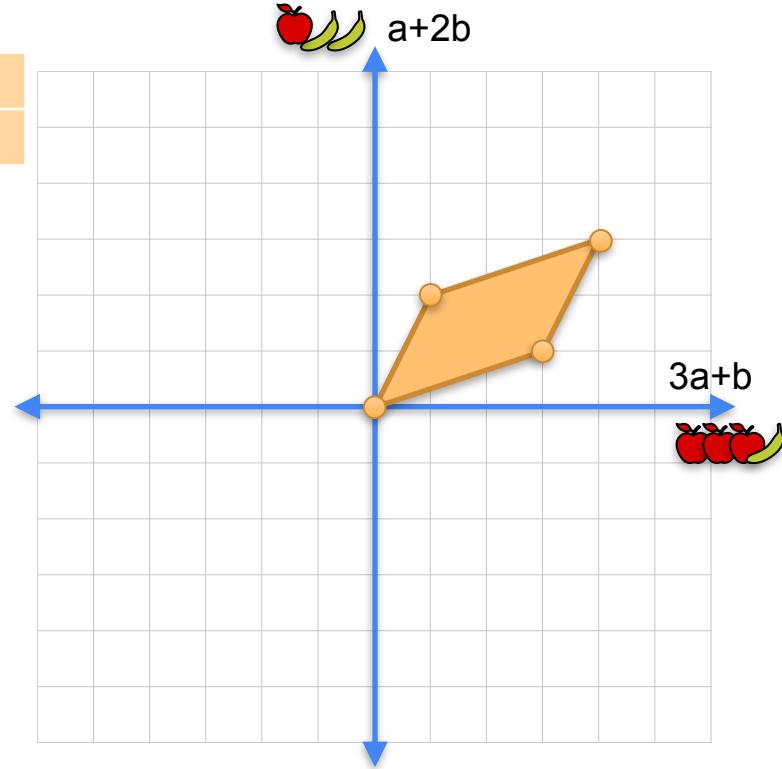


Row space

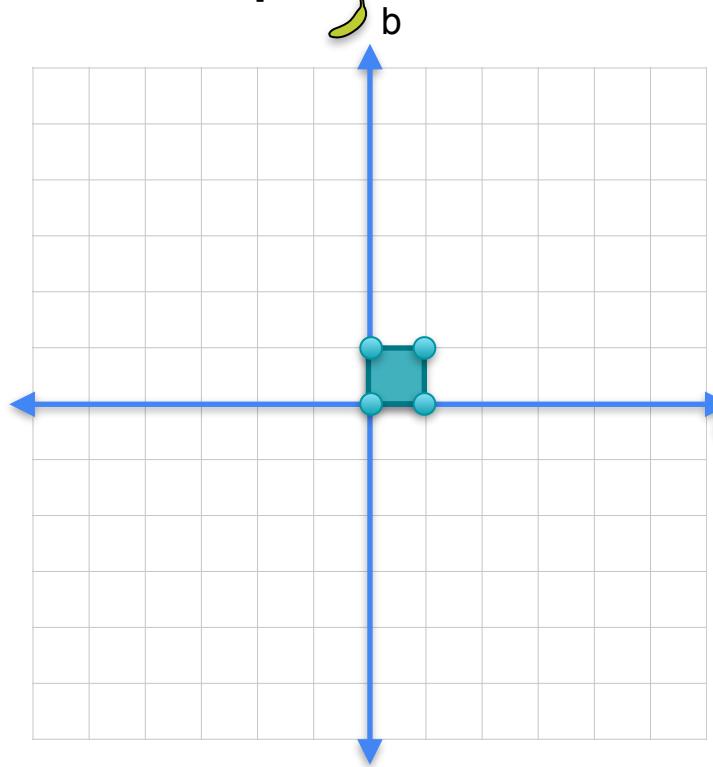


$$\begin{array}{cc|c} \text{apple} & \text{banana} \\ \hline 3 & 1 & 0 \\ 1 & 2 & 0 \end{array} = \begin{array}{c|c} 0 \\ 0 \end{array}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$

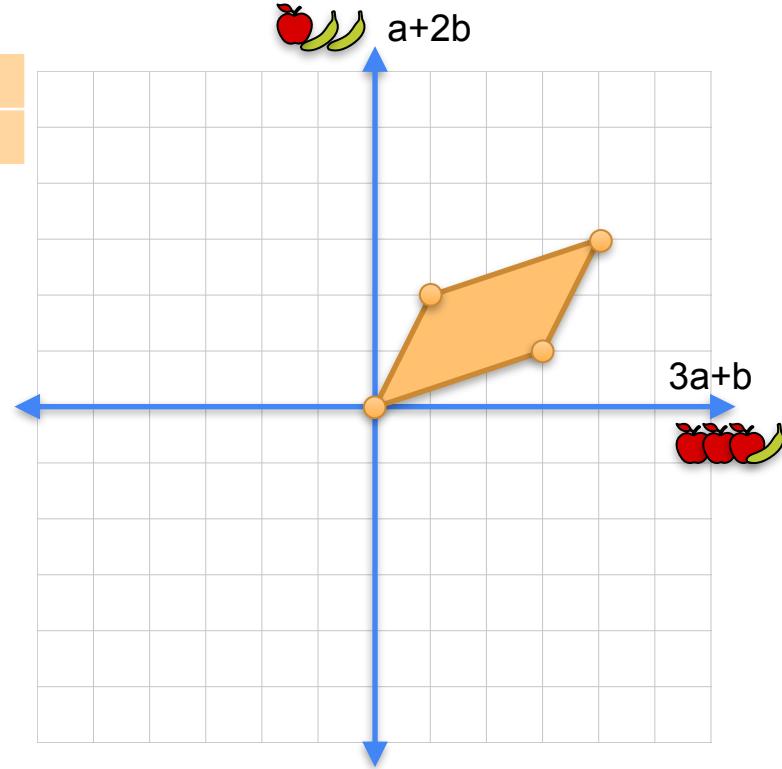


Row space

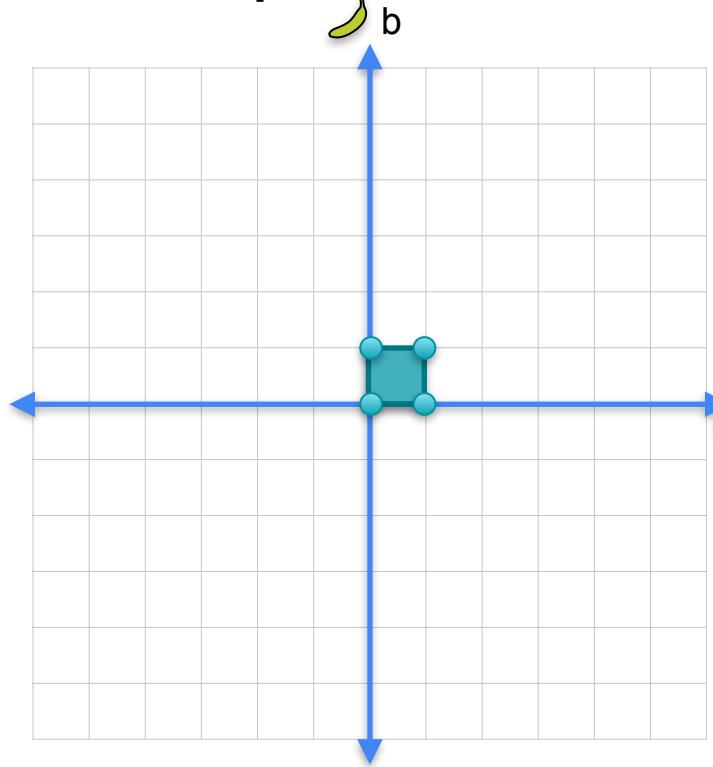


$$\begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 0 \\ 0 \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix} \end{array}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$



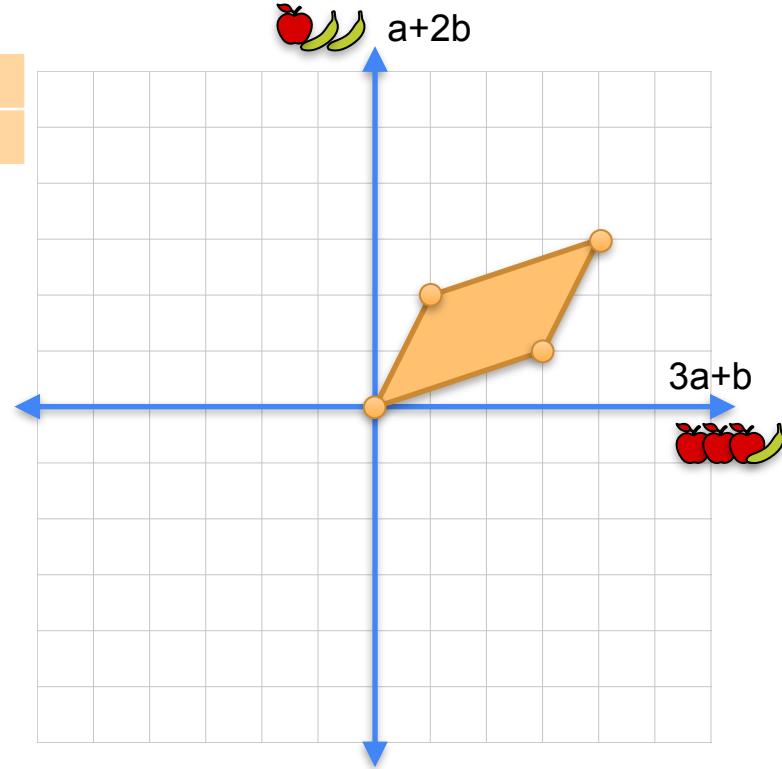
Row space



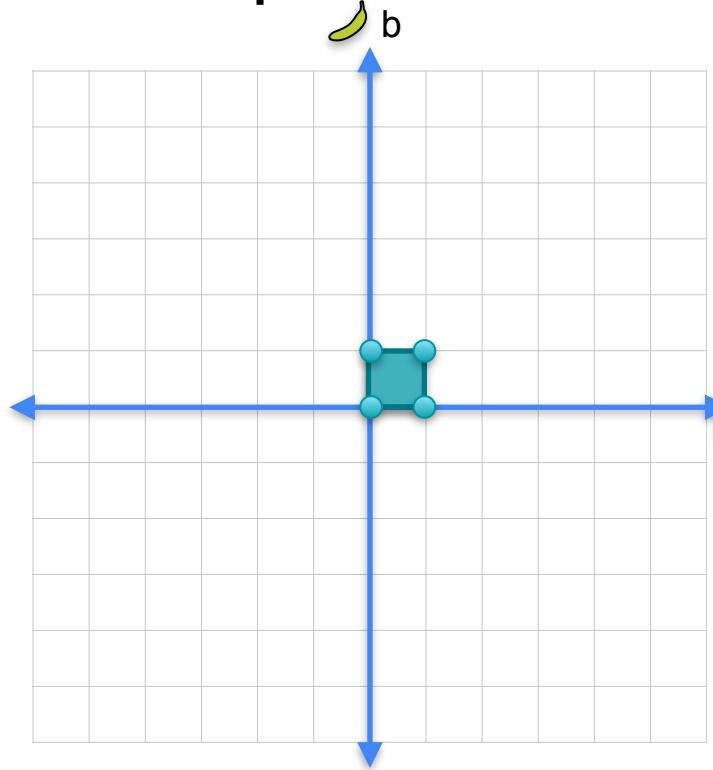
A 2x2 matrix equation. On the left, there is a 2x2 matrix with columns labeled 'apple' and 'banana'. The matrix has entries 3, 1 in the first column and 1, 2 in the second column. To the right of the matrix is an equals sign. To the right of the equals sign is another 2x2 matrix with columns labeled 'apple' and 'banana'. This matrix has entries 3, 1 in the first column and 1, 0 in the second column.

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & = \begin{matrix} 3 & 1 \\ 1 & 0 \end{matrix} \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$

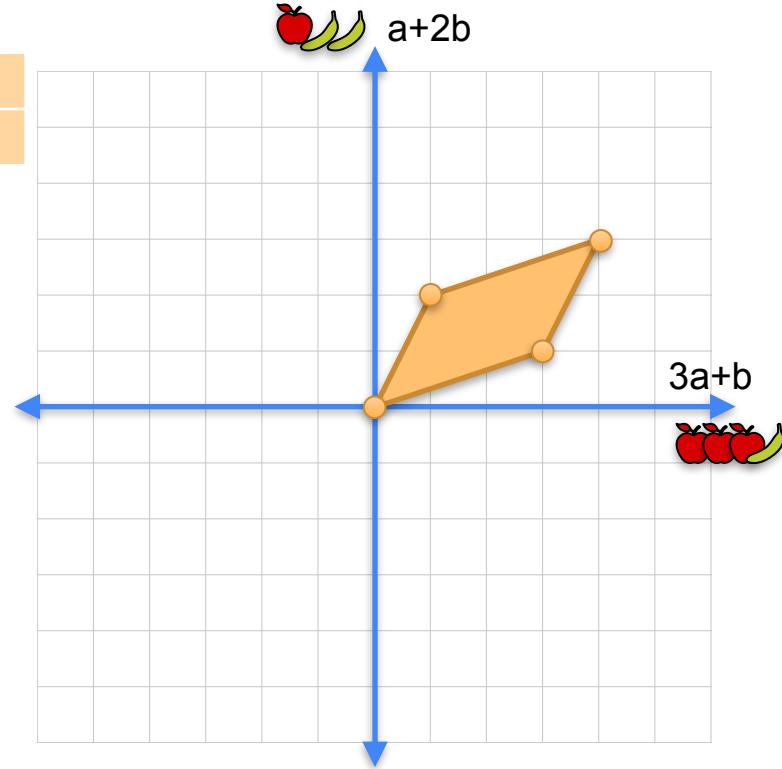


Row space

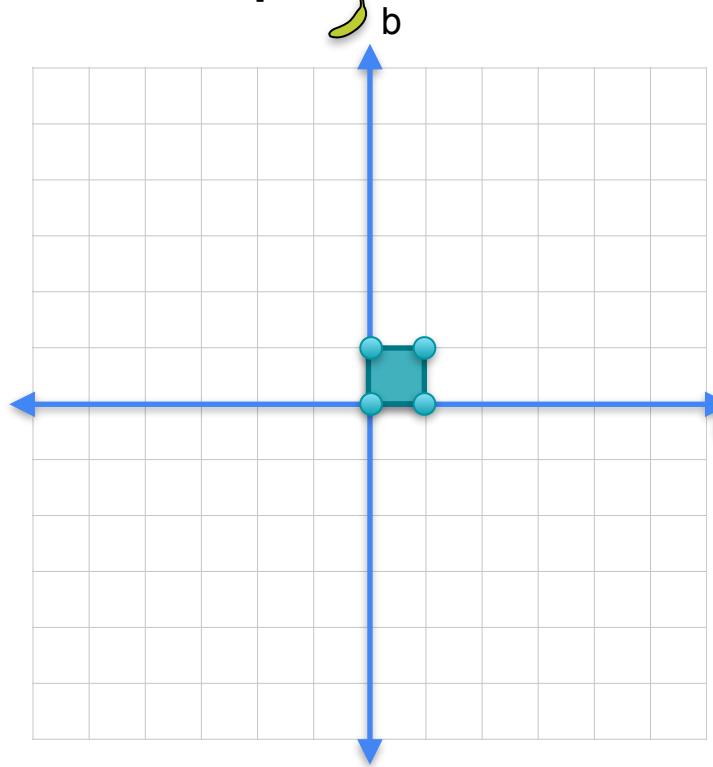


$$\begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 0 \\ 1 \end{matrix} \end{array} = \begin{array}{c} 3 \\ 1 \end{array}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$

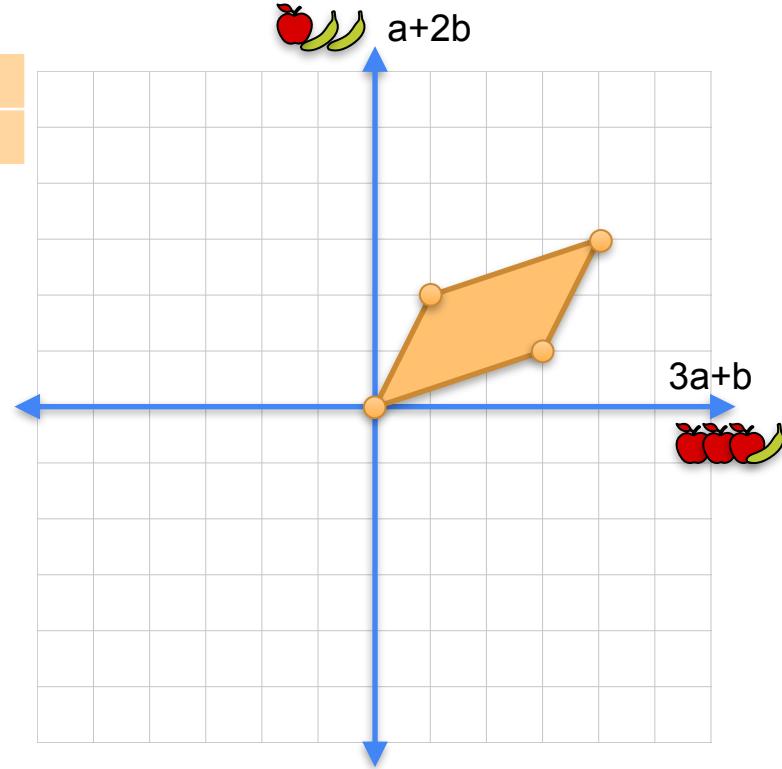


Row space

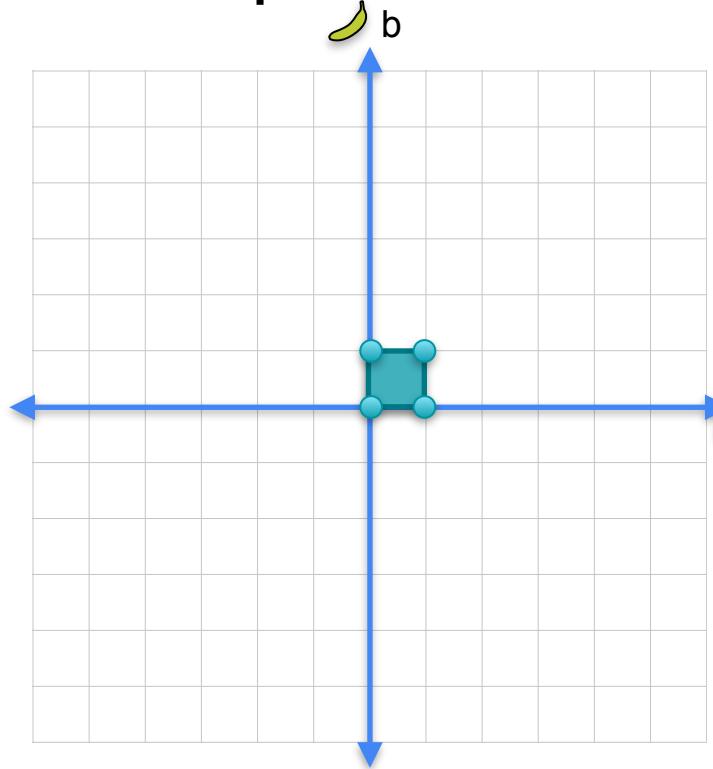


$$\begin{array}{cc|c} \text{apple} & \text{banana} \\ \hline 3 & 1 & 0 \\ 1 & 2 & 1 \end{array} = \begin{array}{cc} 1 & \\ & 2 \end{array}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$

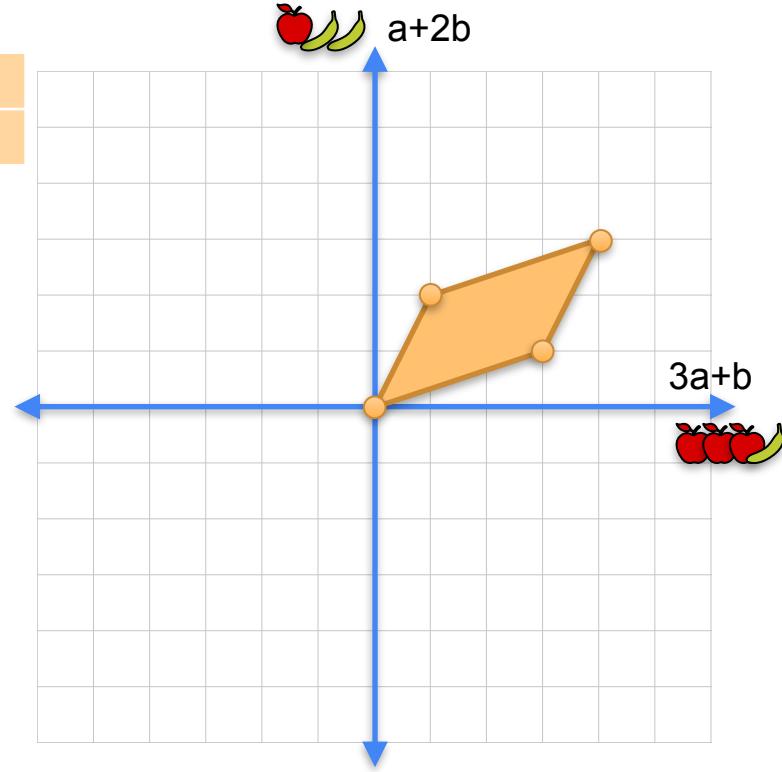


Row space

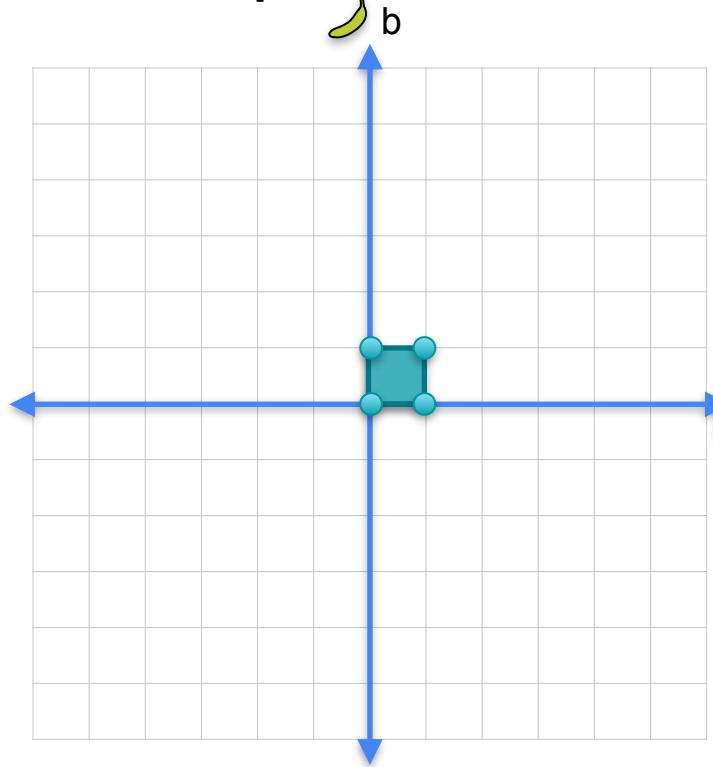


$$\begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{array} = \begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \end{array}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$



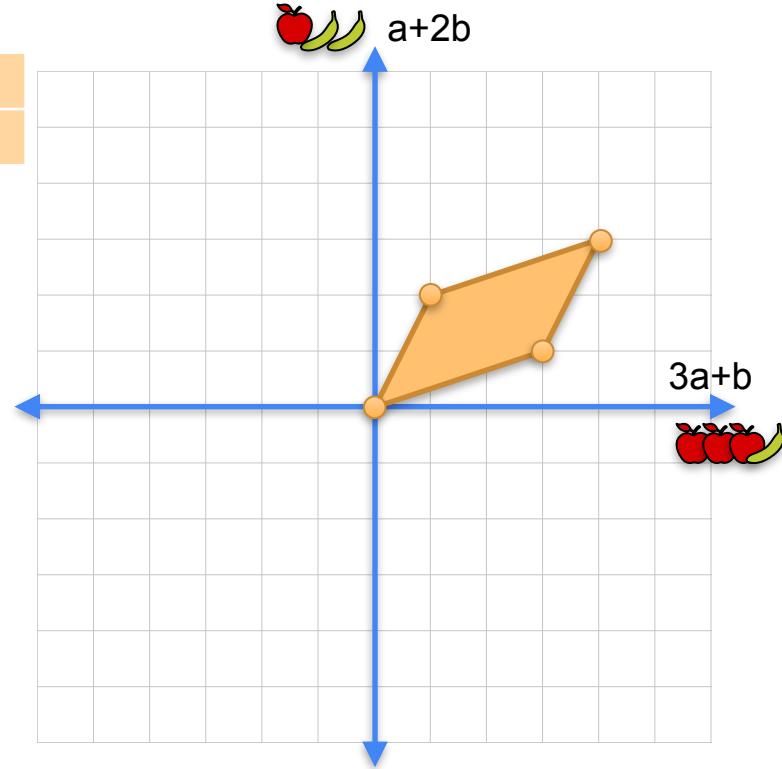
Row space



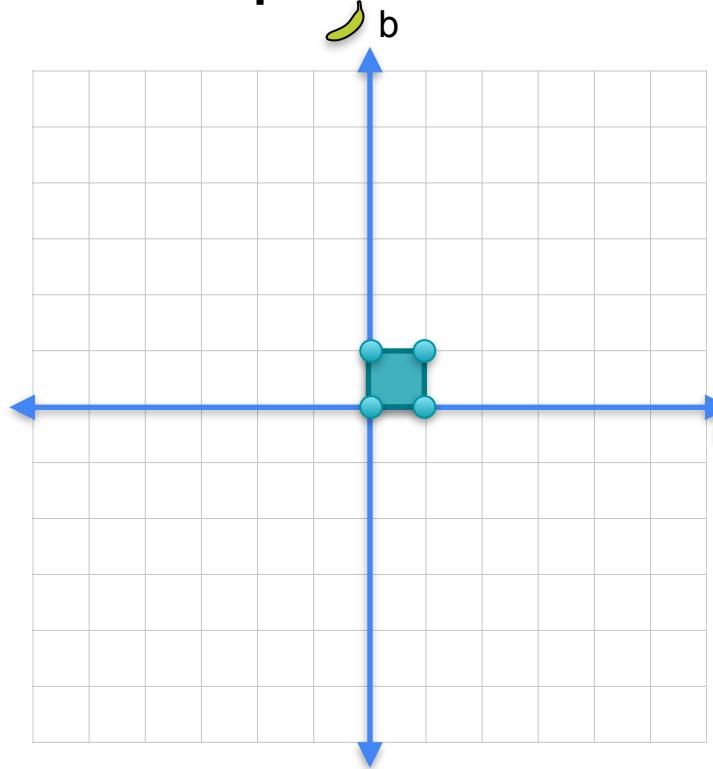
A 2x3 matrix equation. On the left is a 2x3 matrix with columns: a red apple, a yellow banana, and a teal square. To its right is an equals sign. To the right of the equals sign is another 2x3 matrix with columns: a teal square, a yellow banana, and an orange square. The first column of the first matrix is labeled with a red apple icon, and the second column is labeled with a yellow banana icon. The first column of the second matrix is labeled with an orange square icon, and the second column is labeled with a yellow banana icon.

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 & 1 \\ 1 & 2 & 1 \end{matrix} & = \begin{matrix} 4 \\ 3 \end{matrix} \end{matrix}$$

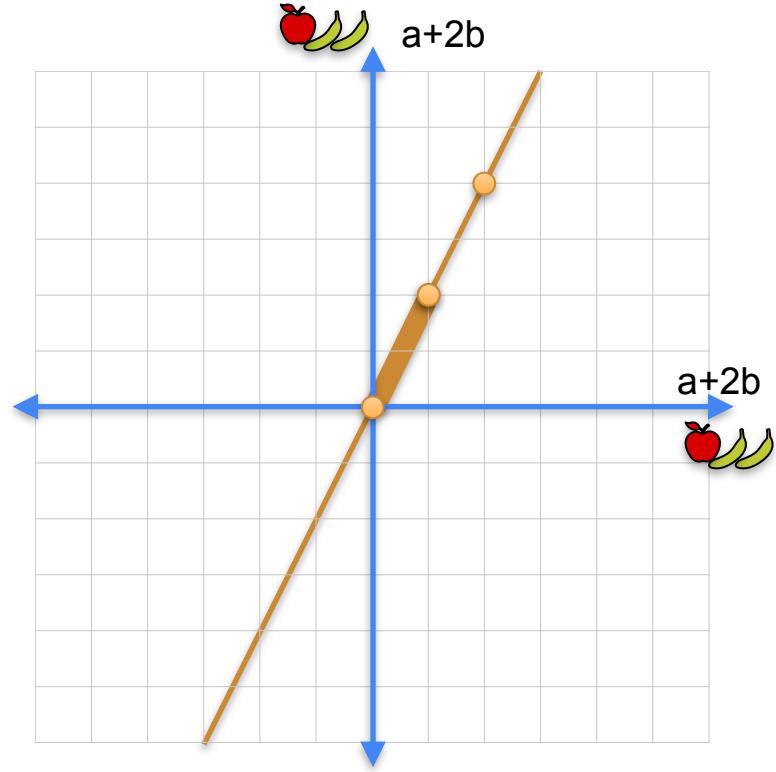
$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$



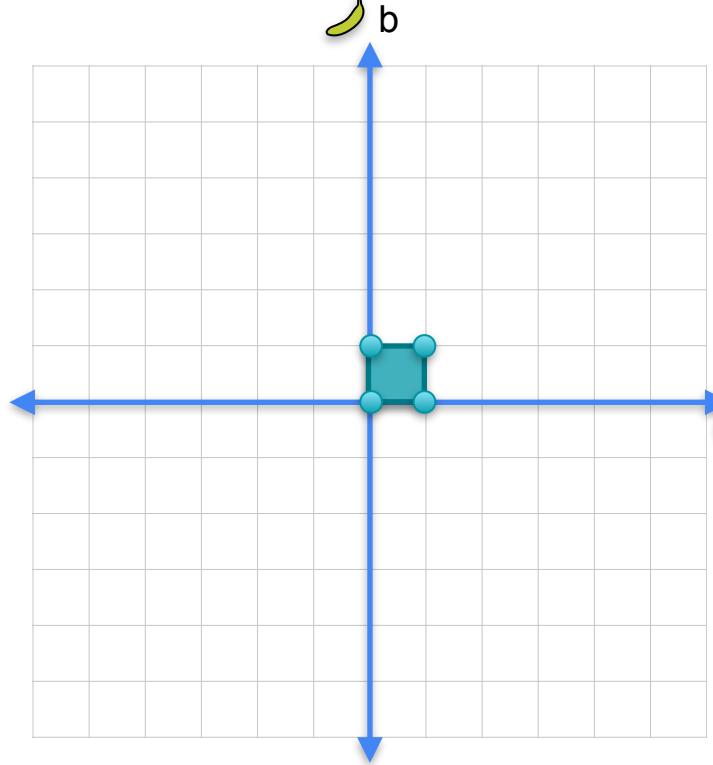
Row space



$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{array}{|c|c|} \hline 1 & 1 \\ \hline 2 & 2 \\ \hline \end{array} = \\ \begin{array}{l} (0,0) \rightarrow (0,0) \\ (1,0) \rightarrow (1,2) \\ (0,1) \rightarrow (1,2) \\ (1,1) \rightarrow (2,4) \end{array} \end{array}$$



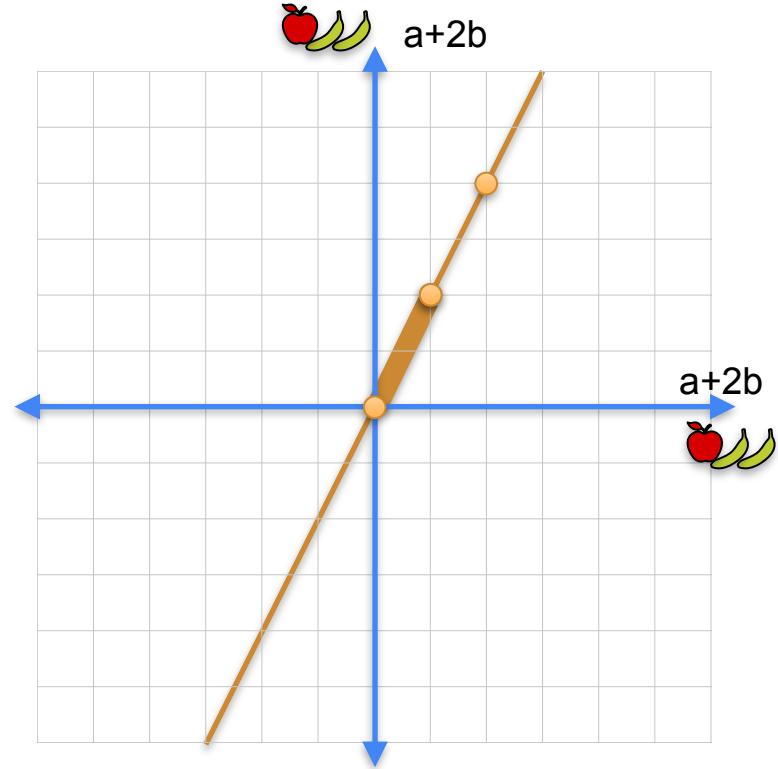
Row space



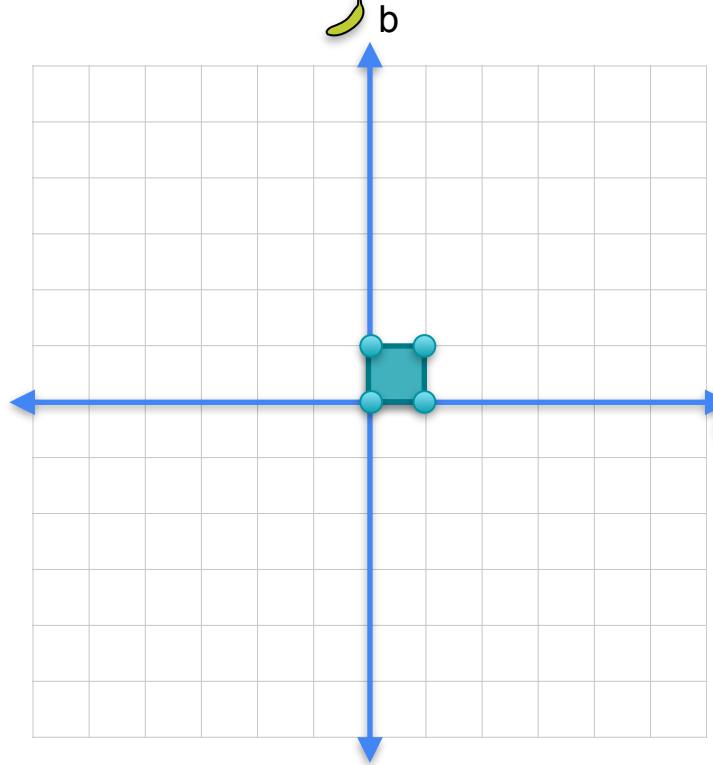
apple banana

$$\begin{array}{|c|c|c|} \hline & 1 & 1 & 0 \\ \hline 1 & 1 & 2 & 0 \\ \hline 2 & 2 & 0 & 0 \\ \hline \end{array} =$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$



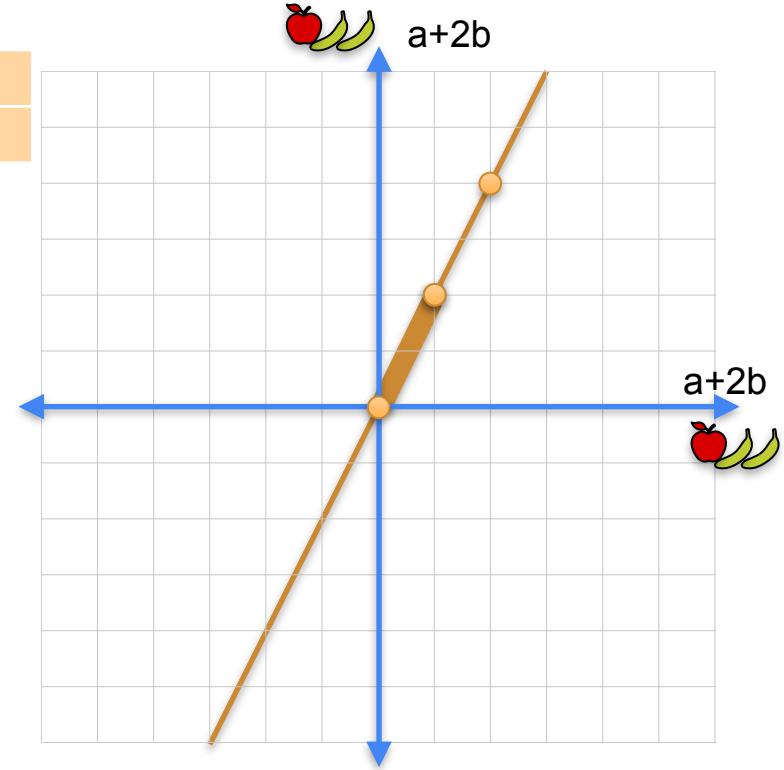
Row space



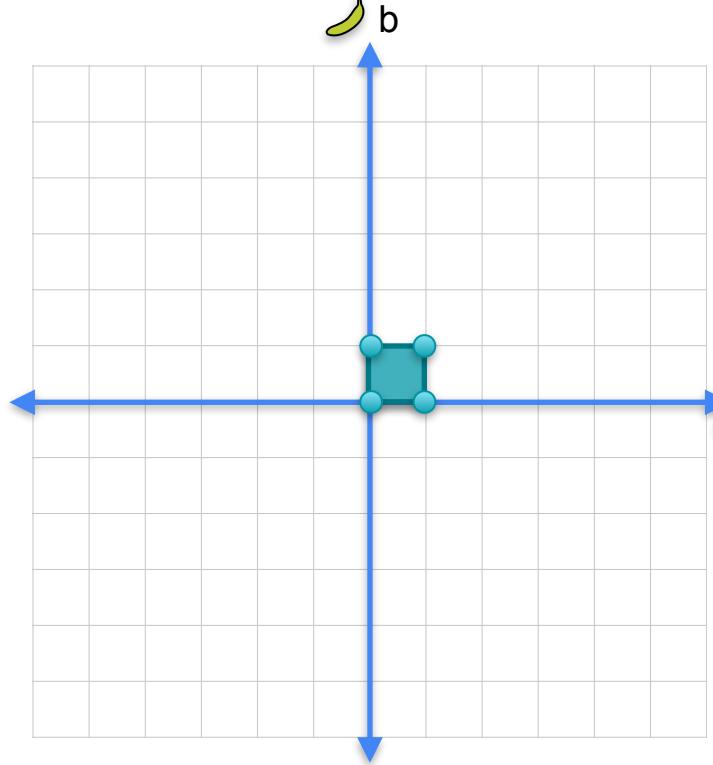
apple banana

$$\begin{array}{|c|c|c|} \hline 1 & 1 & 0 \\ \hline 2 & 2 & 0 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$



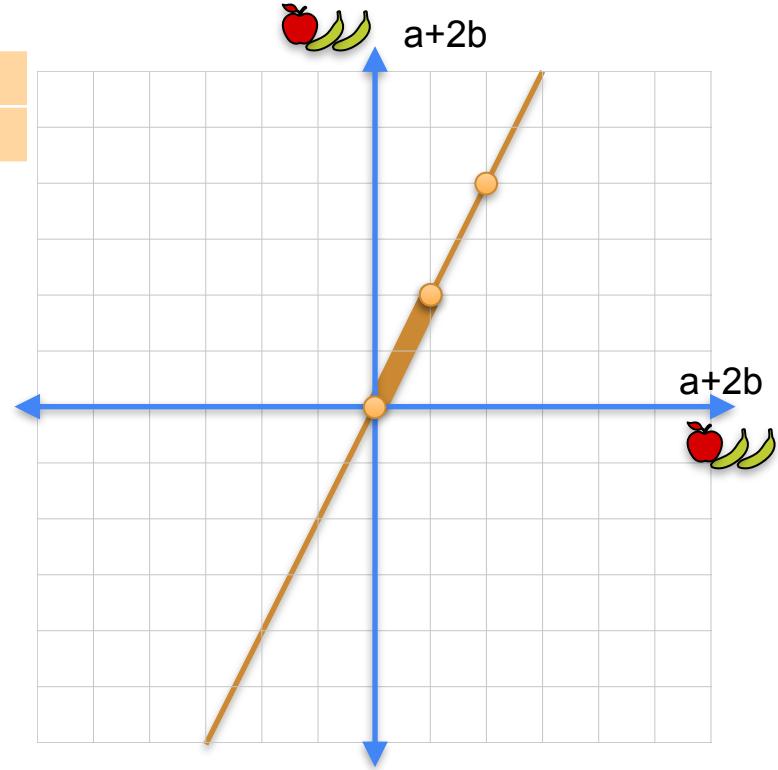
Row space



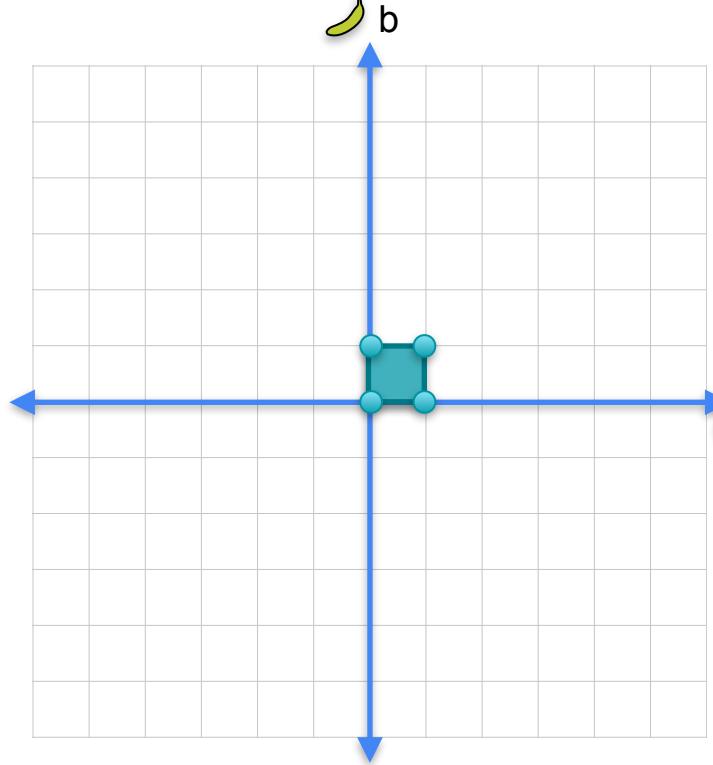
apple banana

$$\begin{matrix} 1 & 1 & 1 \\ 2 & 2 & 0 \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$



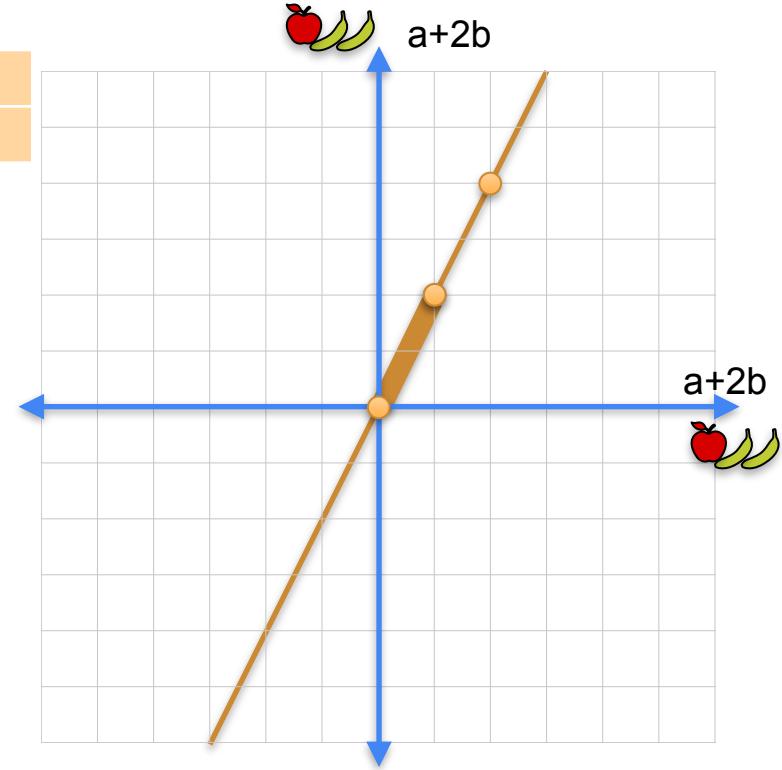
Row space



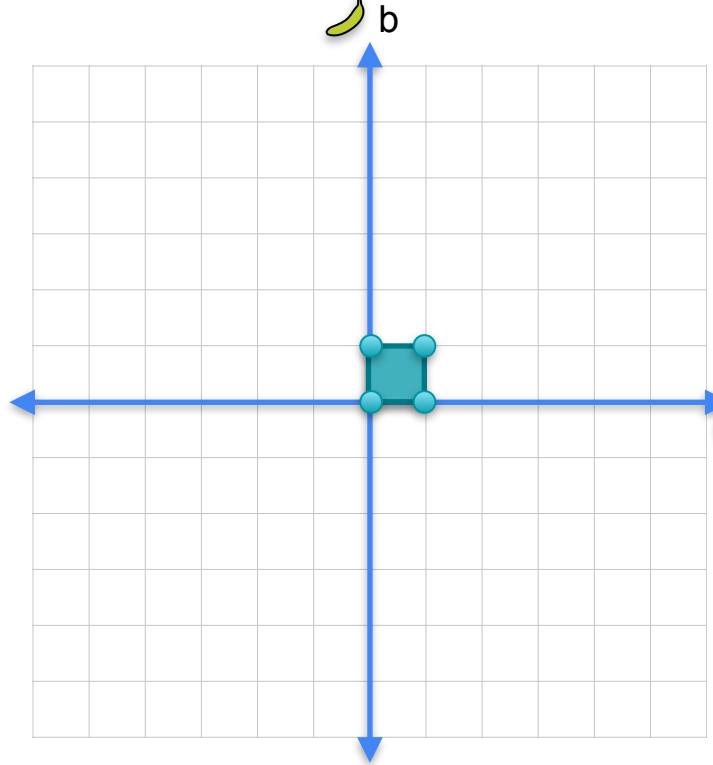
A 2x3 matrix equation. The left side shows a 2x3 matrix with columns labeled by fruit icons: an apple and a banana. The right side shows the result of the multiplication: a column vector with entries 3 and 1, also labeled with fruit icons.

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 0 \end{pmatrix} & = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$



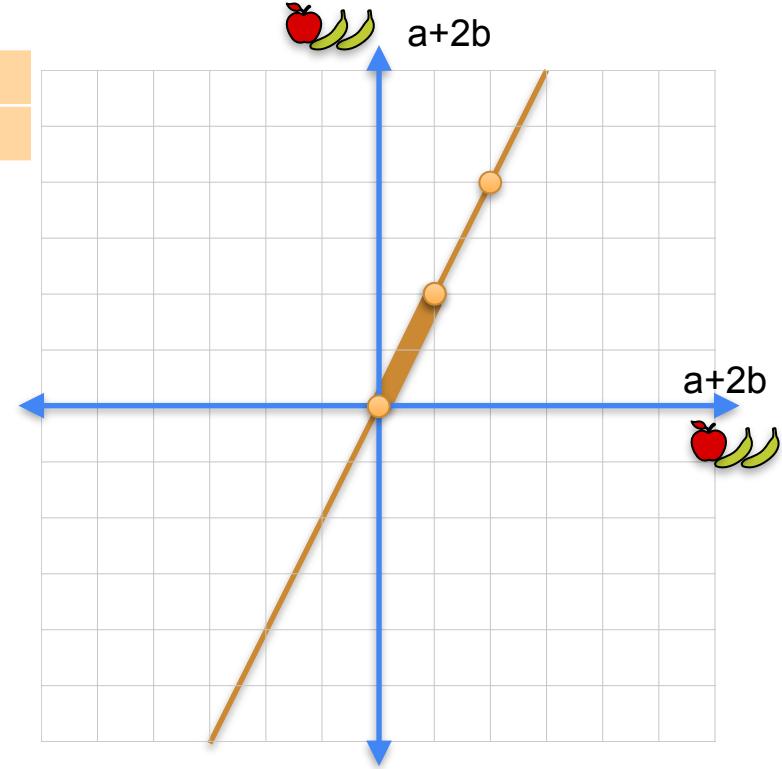
Row space



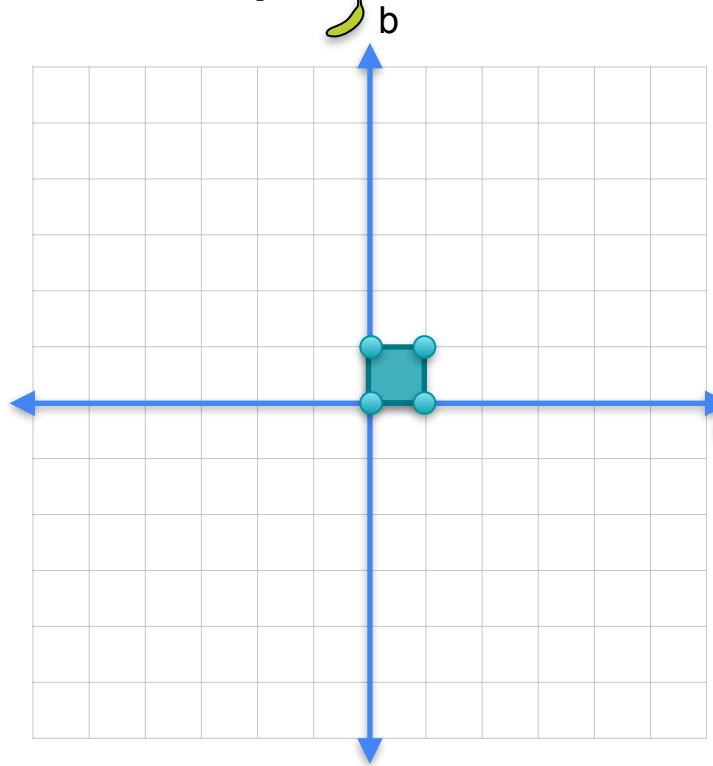
A 2x2 matrix equation. On the left, there is a 2x2 matrix with two red apple icons in the first column and two green banana icons in the second column. To its right is an equals sign. To the right of the equals sign is another 2x2 matrix with a red apple icon in the top-left, a green banana icon in the top-right, a yellow orange in the bottom-left, and a red apple icon in the bottom-right. This represents the row space of the original matrix.

$$\begin{matrix} \text{apple} & \text{banana} \\ 1 & 1 \\ 2 & 2 \end{matrix} = \begin{matrix} \text{apple} & \text{banana} \\ 0 & 3 \\ 1 & 1 \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$



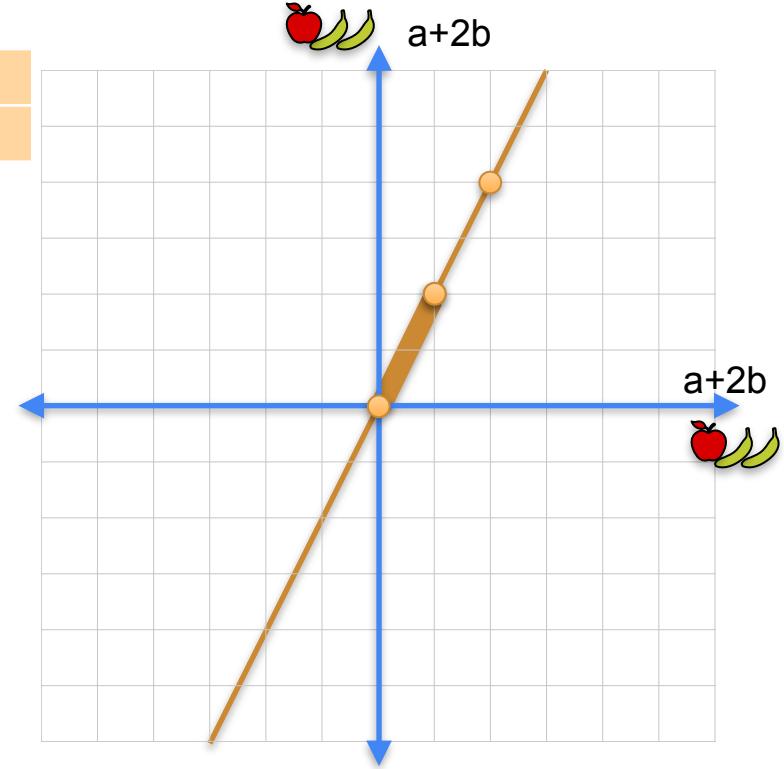
Row space



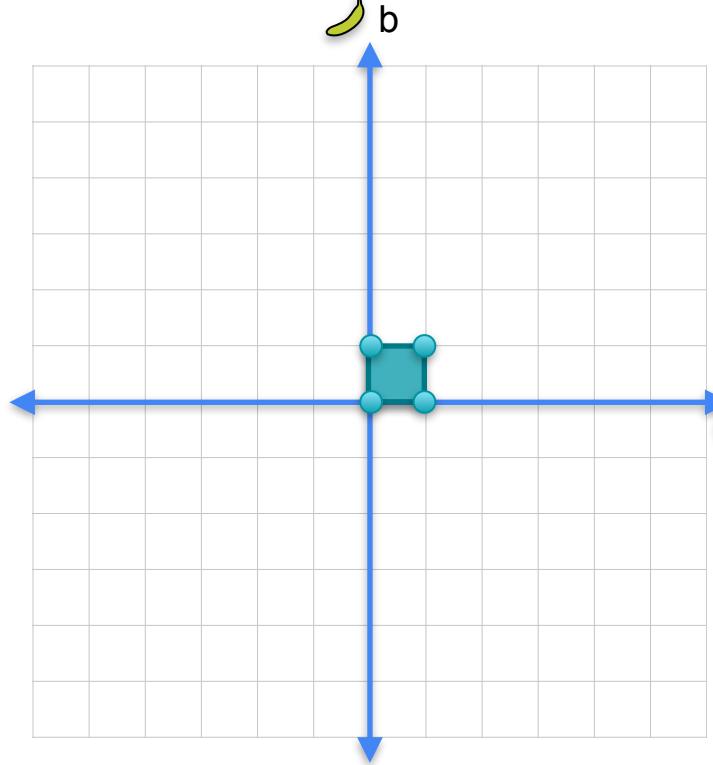
A 2x2 matrix equation. On the left, there is a 2x2 matrix with two red apple icons in the first column and two yellow banana icons in the second column. To the right of the matrix is an equals sign. To the right of the equals sign is another 2x2 matrix with a single red apple icon in the top-left cell and a single yellow banana icon in the bottom-right cell. This visualizes the row space of the original matrix.

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} & = & \begin{matrix} 1 \\ 2 \end{matrix} \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$

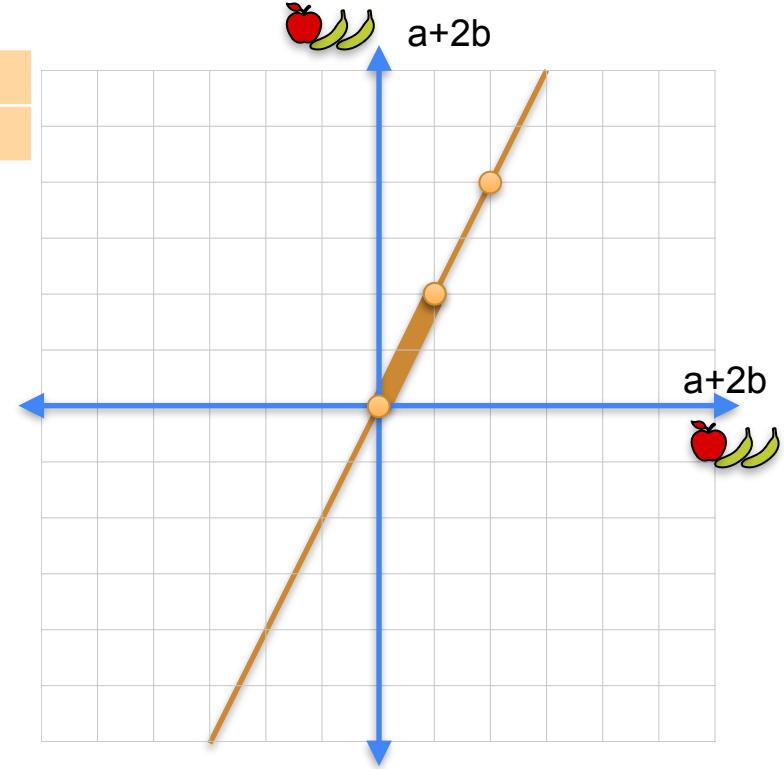


Row space

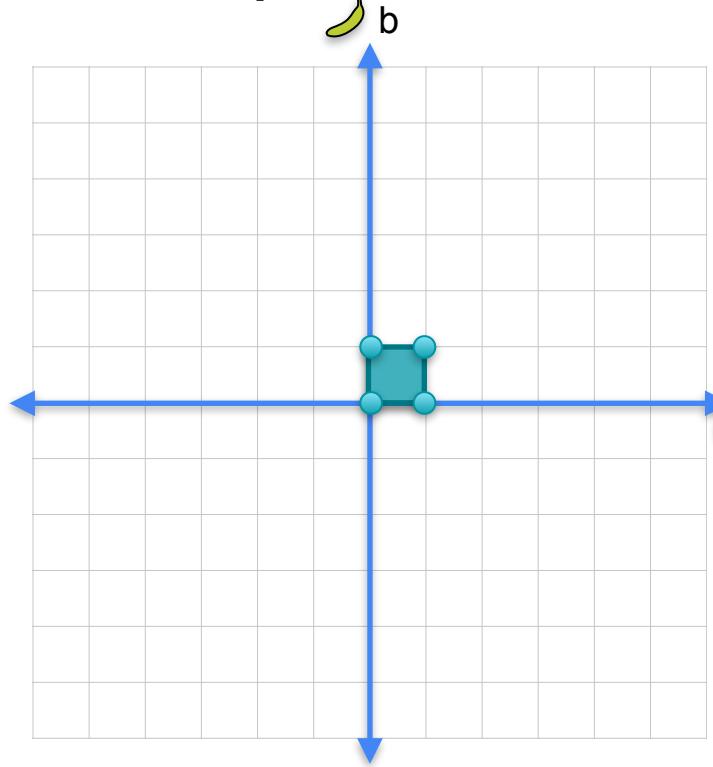


A diagram illustrating row reduction. On the left, there is a 2x3 matrix with two rows and three columns. The first row contains a red apple icon and a green banana icon. The second row contains a red apple icon and a green banana icon. To the right of the matrix is an equals sign. To the right of the equals sign is a 1x2 matrix with one row and two columns. The first element is a red apple icon and the second element is a green banana icon. Below the matrices, the equation $\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is shown.

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$



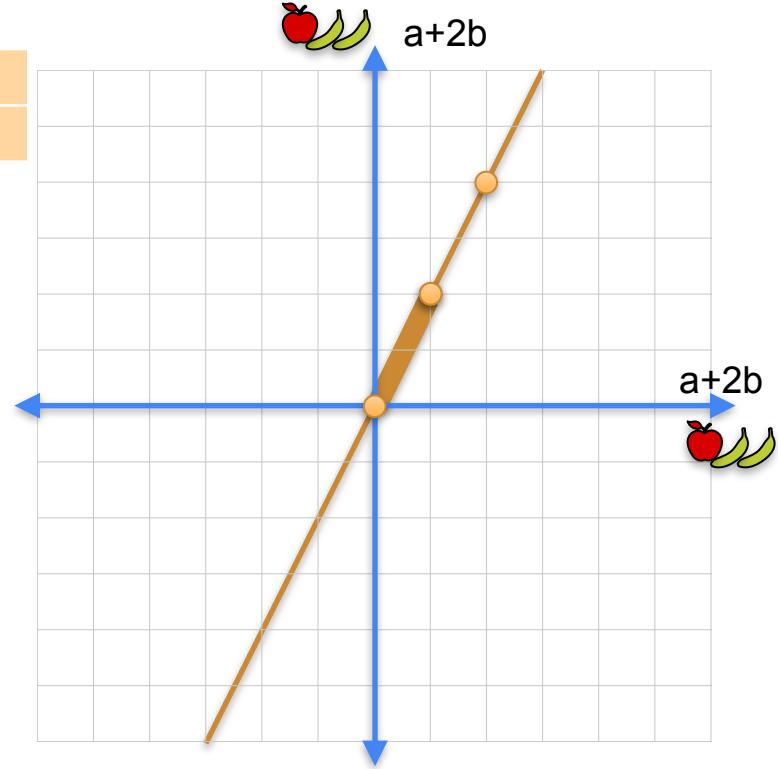
Row space



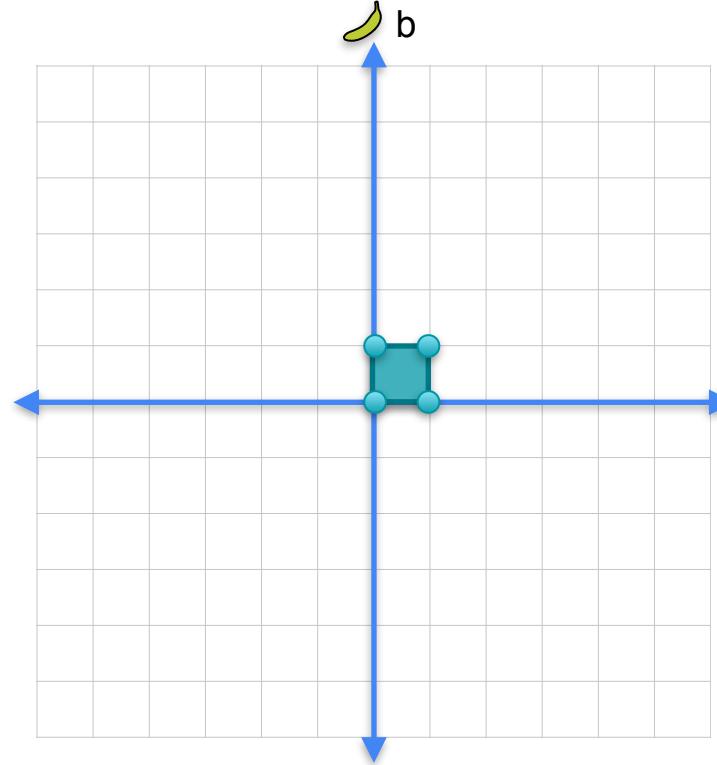
A 2x3 matrix equation. The left side shows a 2x3 matrix with columns: [apple, banana] (top row), [1, 2] (bottom row). The right side shows a column vector: [4, 3]. Above the matrix is a red apple icon and a green banana icon. To the right of the matrix is a green banana icon.

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 1 & 1 & 1 \\ 2 & 2 & 1 \end{matrix} & = \begin{matrix} 4 \\ 3 \end{matrix} \end{matrix}$$

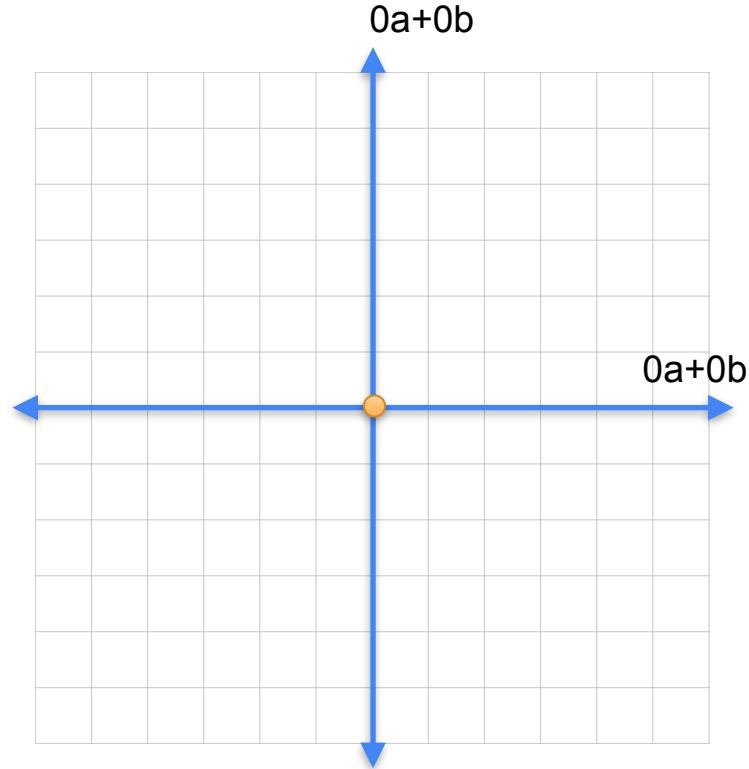
$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$



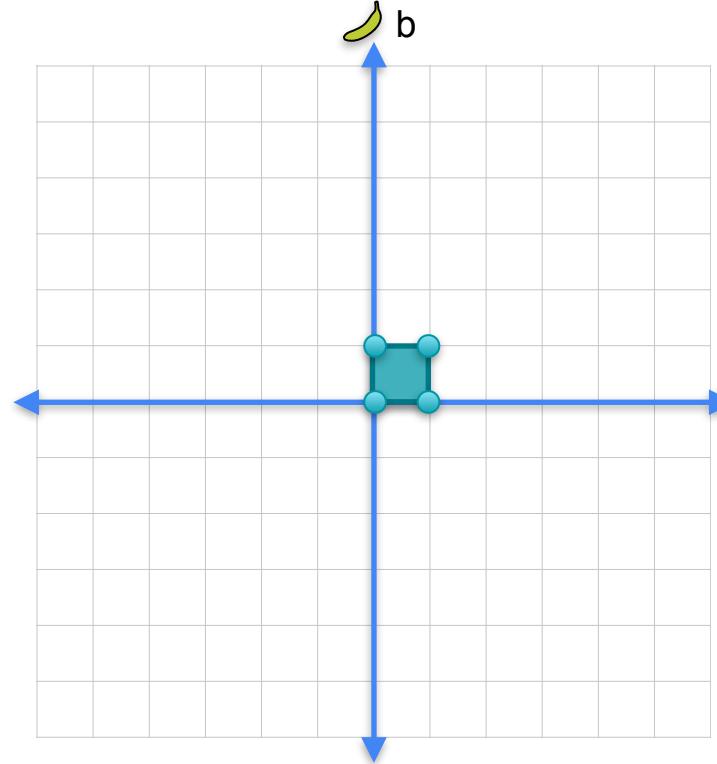
Row space



$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \end{array} = \begin{array}{l} (0,0) \rightarrow (0,0) \\ (1,0) \rightarrow (0,0) \\ (0,1) \rightarrow (0,0) \\ (1,1) \rightarrow (0,0) \end{array}$$



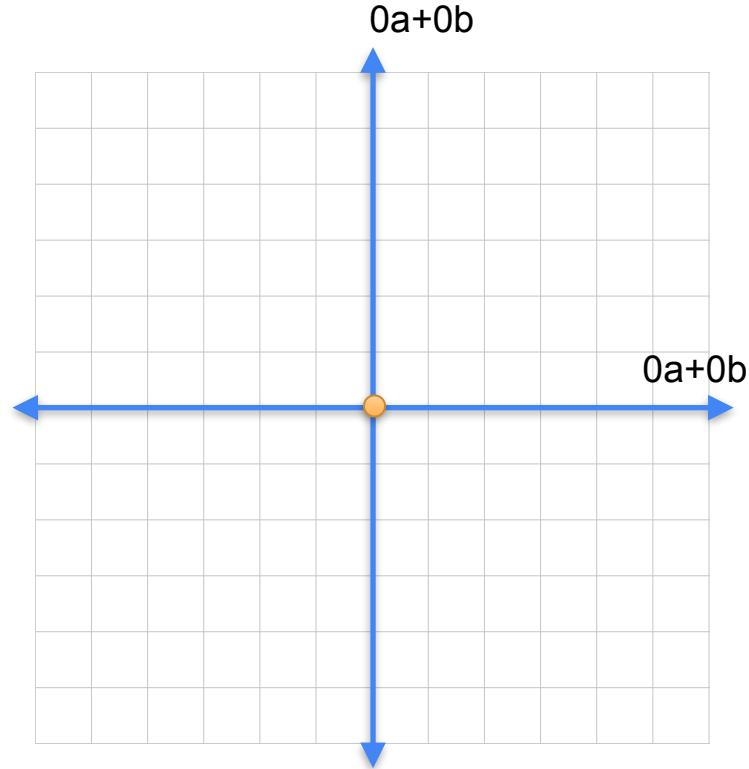
Row space



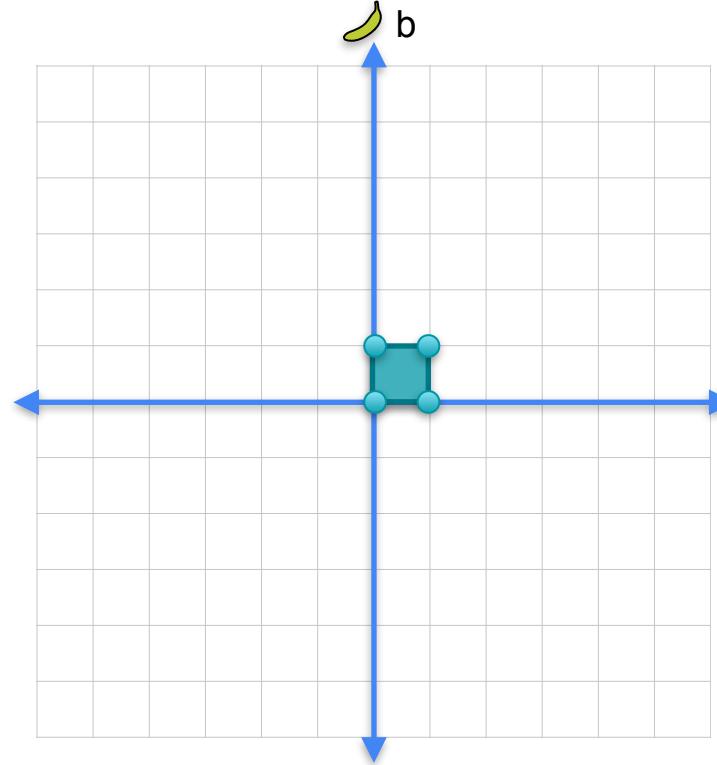
=

| | | |
|---|---|---|
| 0 | 0 | a |
| 0 | 0 | b |

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (0,0)$
 $(0,1) \rightarrow (0,0)$
 $(1,1) \rightarrow (0,0)$

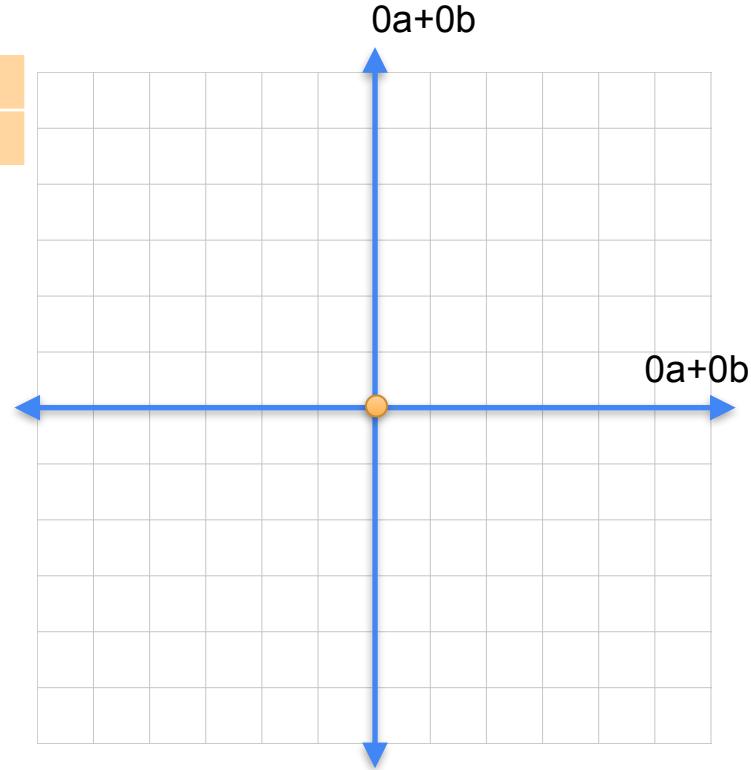


Row space



A 2x2 matrix with entries 0, 0, a , and b . An apple icon is above the first column, and a banana icon is above the second column. To the right of the matrix is an equals sign, followed by another 2x2 matrix where all entries are 0. This represents the equation $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (0,0)$
 $(0,1) \rightarrow (0,0)$
 $(1,1) \rightarrow (0,0)$

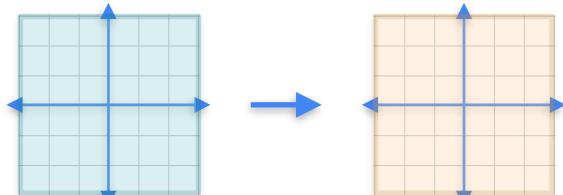


Row space

Non-singular

| | |
|---|---|
| | |
| 3 | 1 |
| 1 | 2 |

Rank = 2

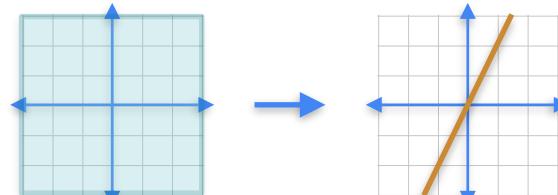


Dimension = 2

Singular

| | |
|---|---|
| | |
| 1 | 1 |
| 2 | 2 |

Rank = 1

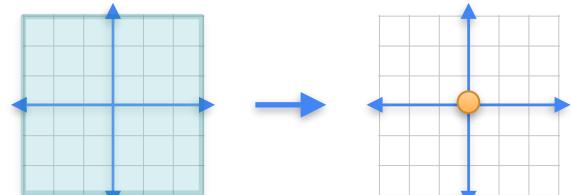


Dimension = 1

Singular

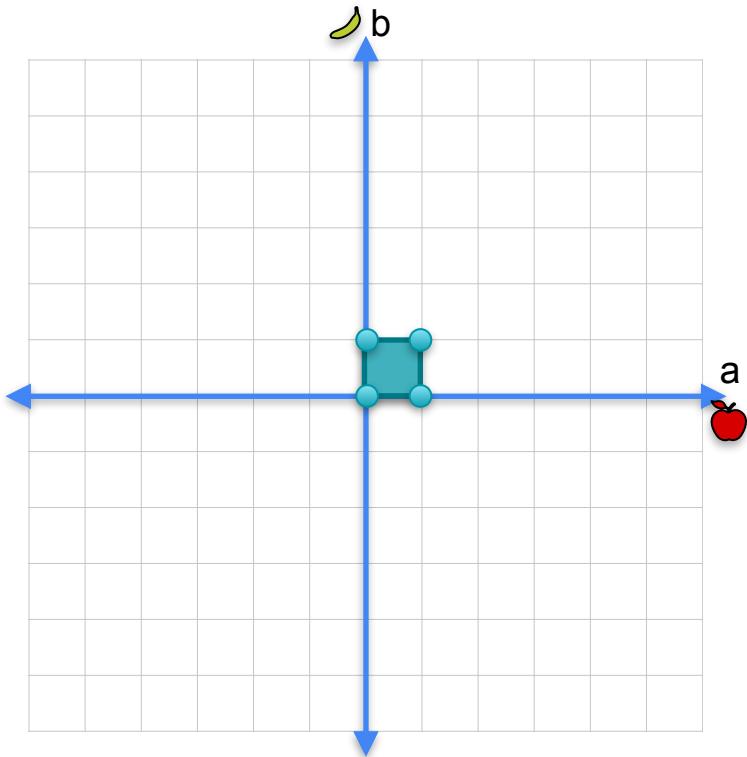
| | |
|---|---|
| | |
| 0 | 0 |
| 0 | 0 |

Rank = 0



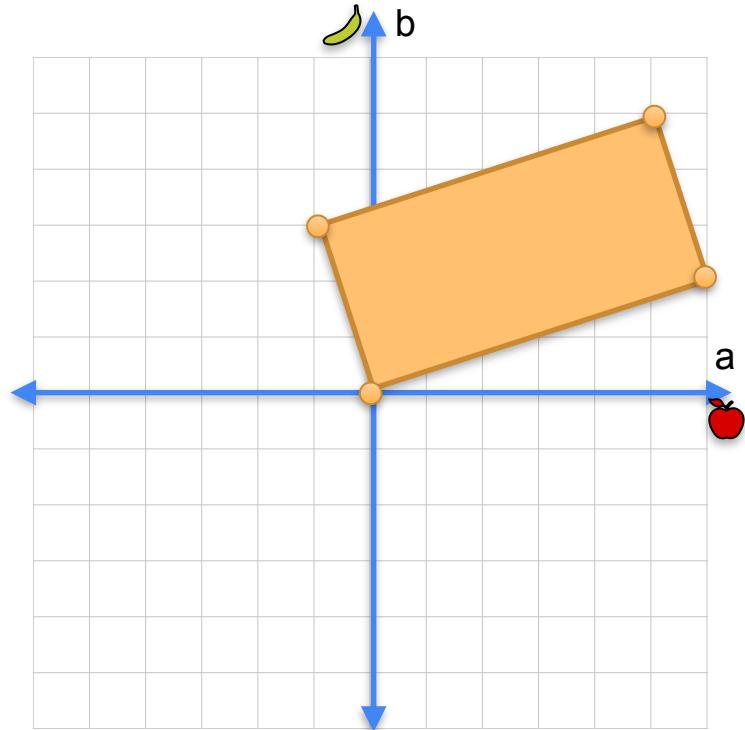
Dimension = 0

Orthogonal matrix

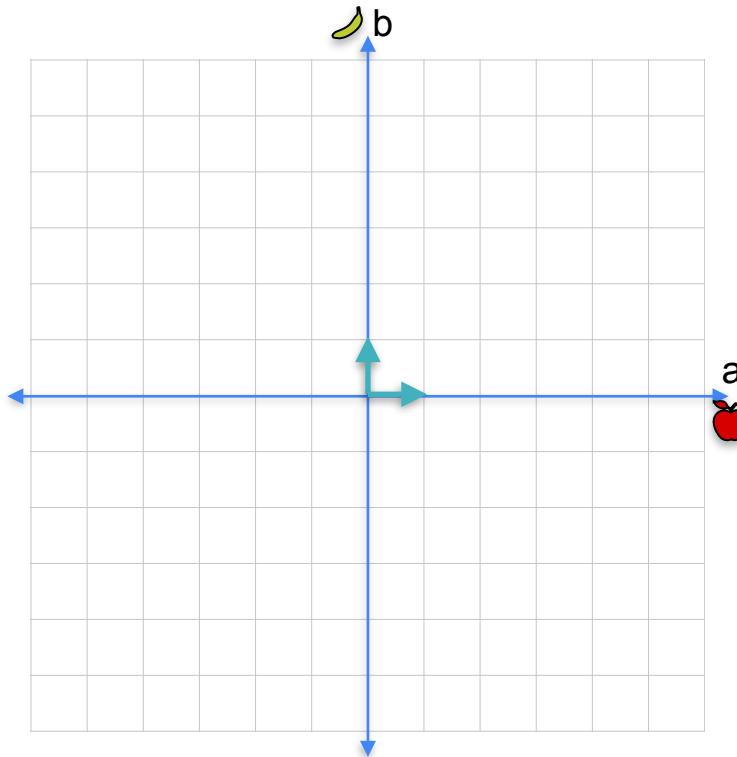


| | |
|---|----|
| | |
| 6 | -1 |
| 2 | 3 |

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (6,2)$
 $(0,1) \rightarrow (-1,3)$
 $(1,1) \rightarrow (5,5)$

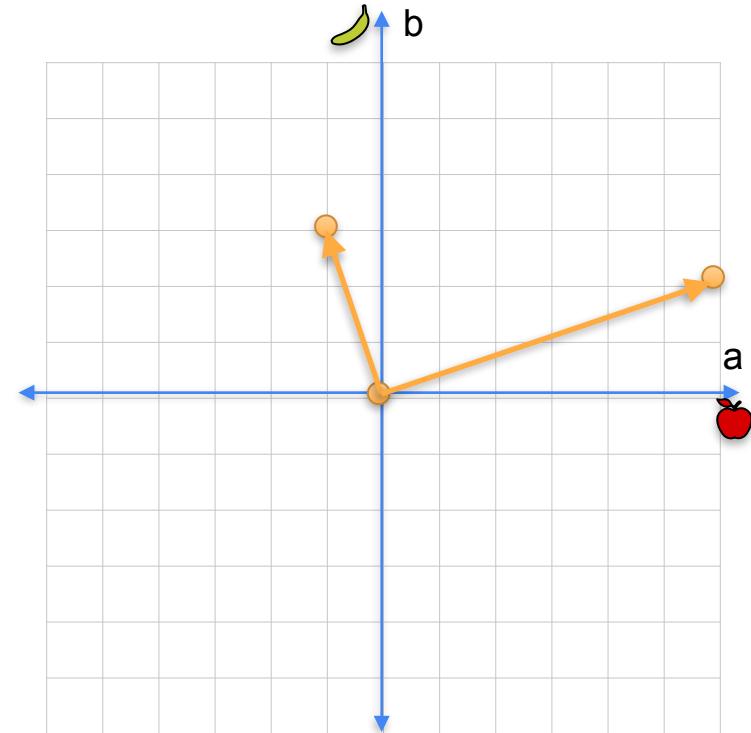


Orthogonal matrix



| | |
|-------|--------|
| apple | banana |
| 6 | -1 |
| 2 | 3 |

$$\begin{aligned}(1,0) &\rightarrow (6,2) \\ (0,1) &\rightarrow (-1,3)\end{aligned}$$



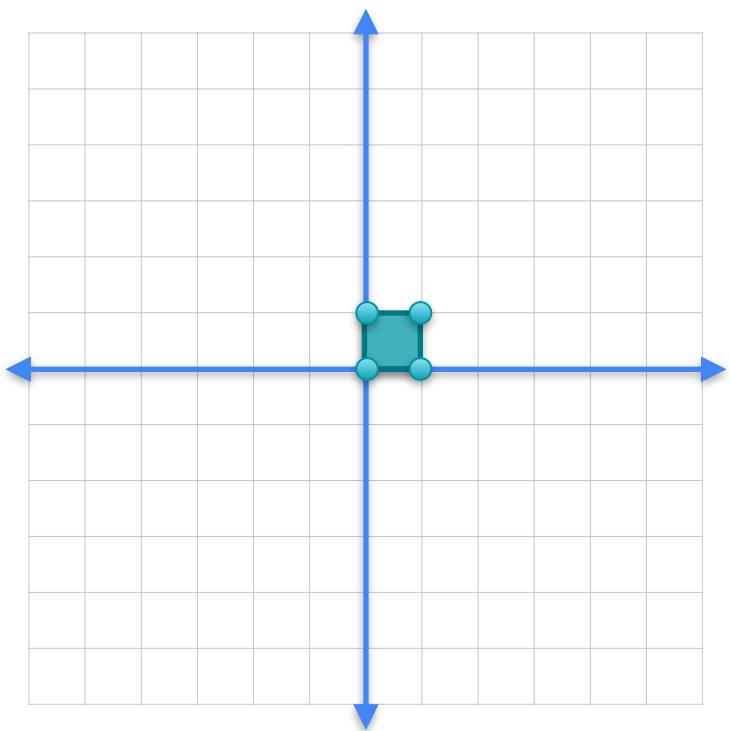
Orthogonal matrices have orthogonal columns

$$\begin{array}{|c|c|} \hline 6 & -1 \\ \hline 2 & 3 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & -1 \\ \hline 2 & 3 \\ \hline \end{array} = 0$$

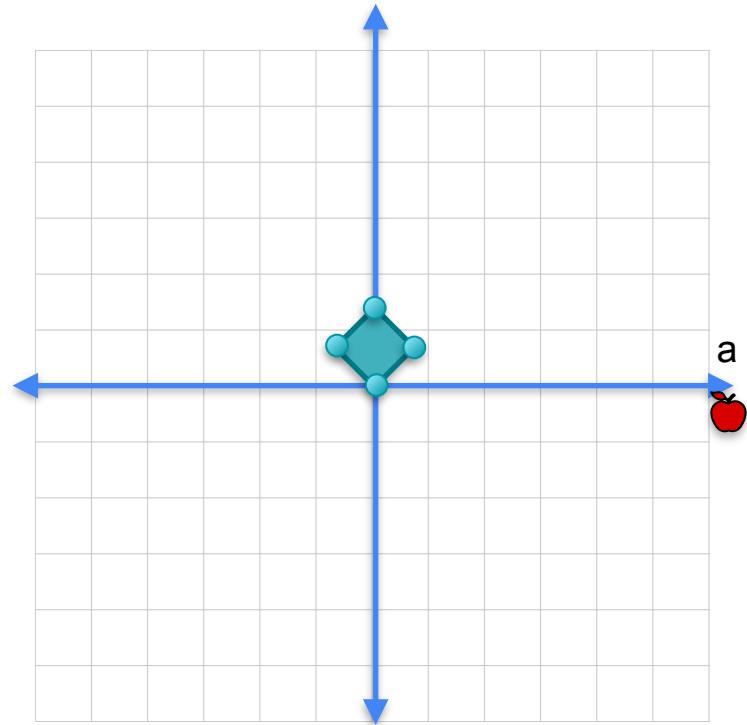
$$\begin{array}{|c|c|c|c|} \hline 6 & -1 & 2 & 3 \\ \hline \end{array} = 0$$

Orthogonal matrix

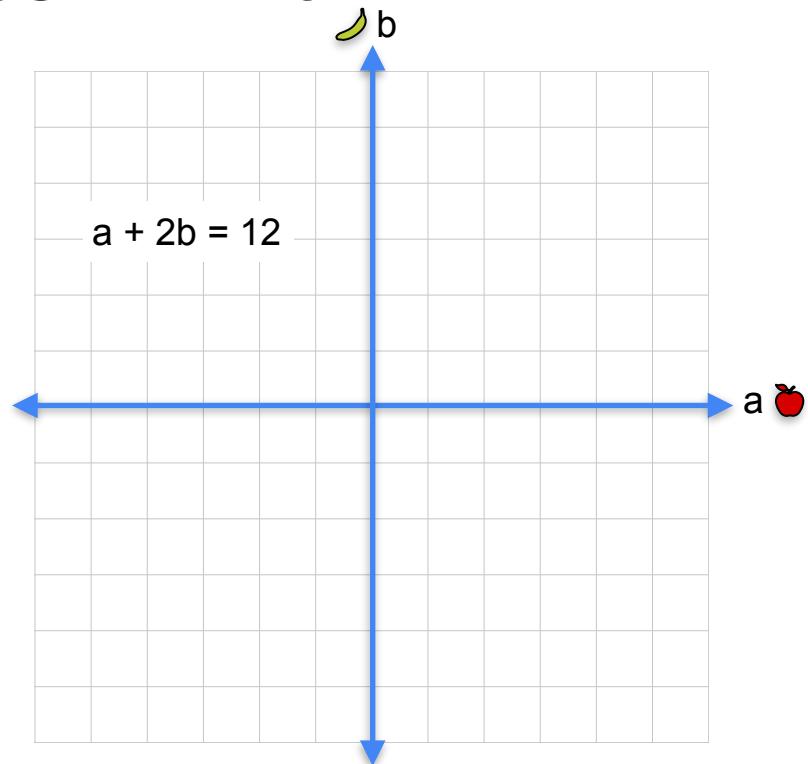
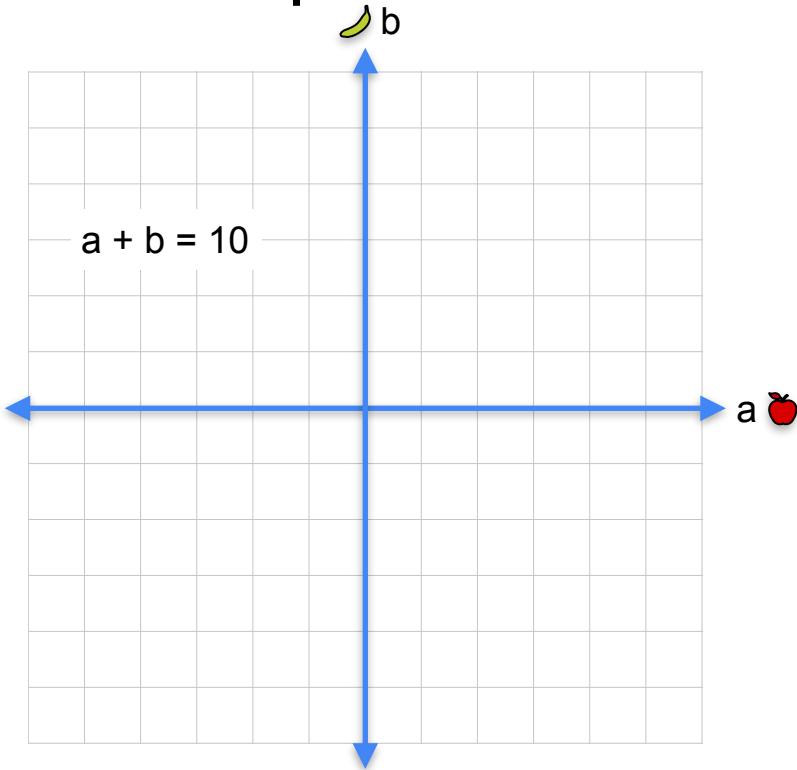


| | |
|---------|--------|
| 0.7071 | 0.7071 |
| -0.7071 | 0.7071 |

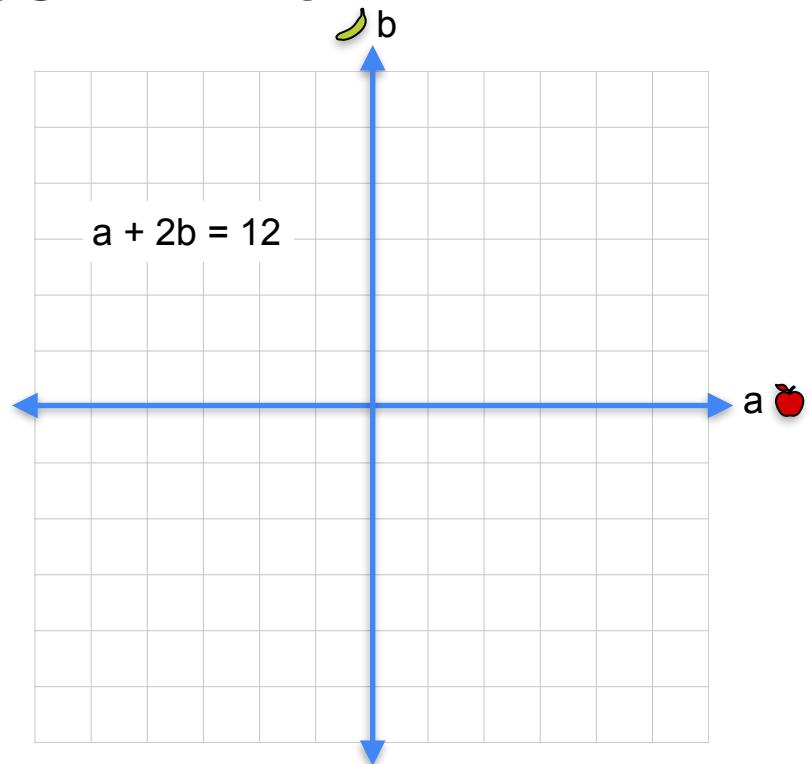
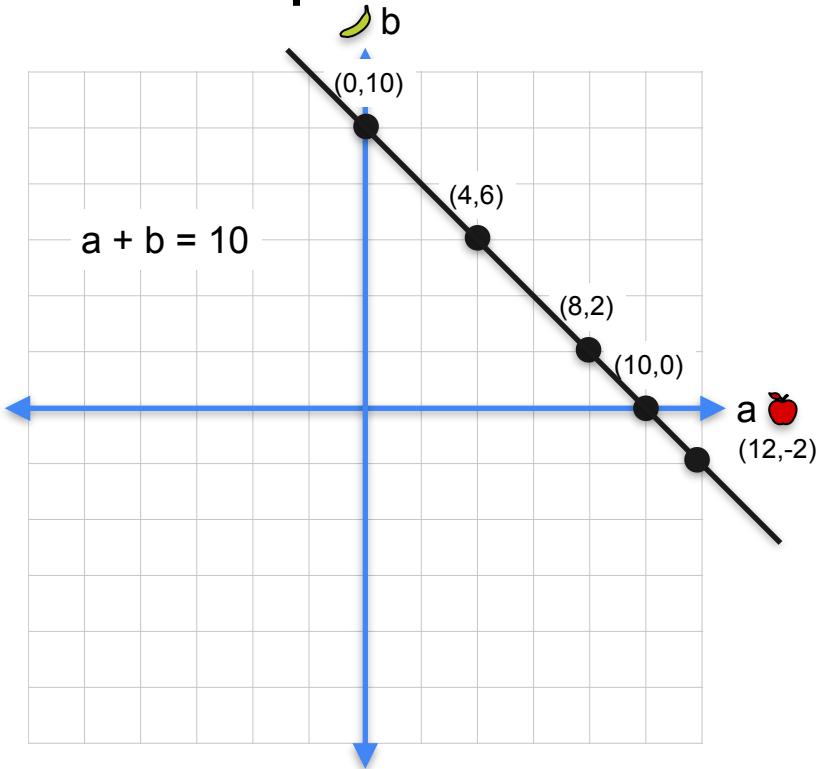
- (0,0) \rightarrow (0,0)
- (1,0) \rightarrow (0.7071, 0.7071)
- (0,1) \rightarrow (-0.7071, 0.7071)
- (1,1) \rightarrow (0, 1.4142)



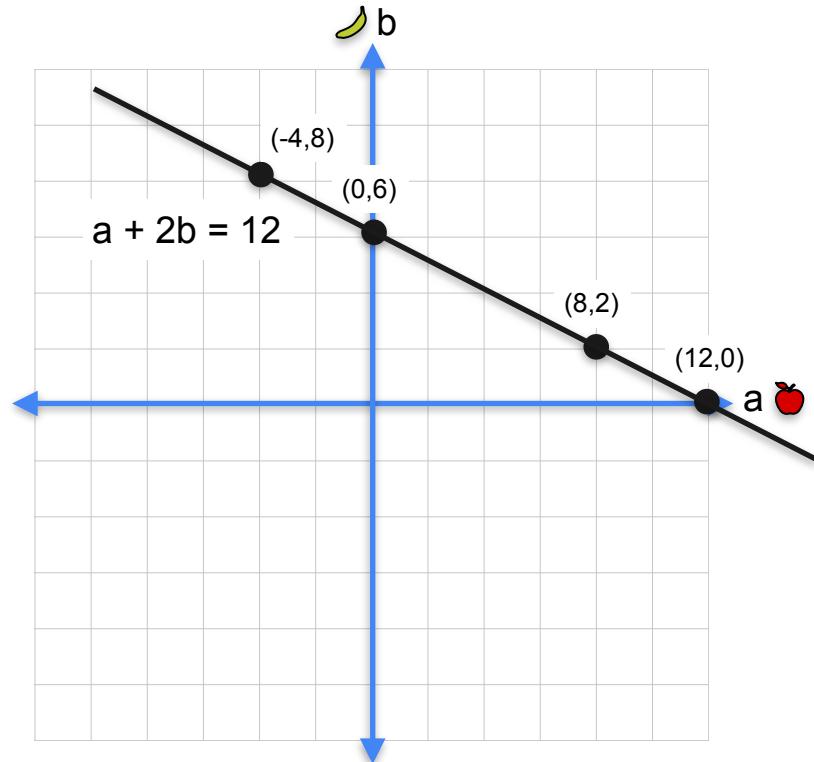
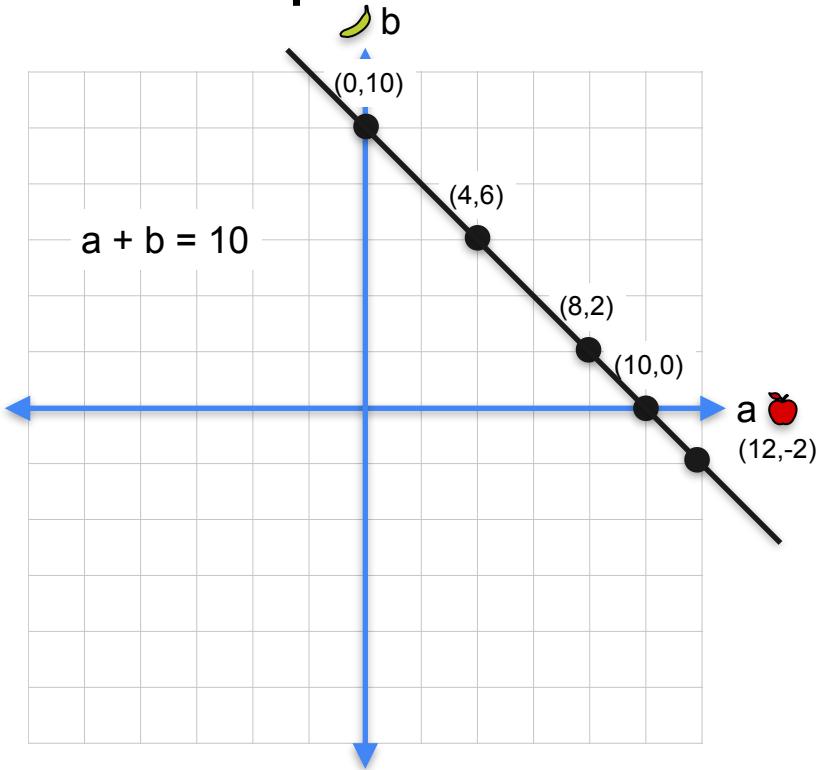
Linear equation in 2 variables -> Line



Linear equation in 2 variables -> Line

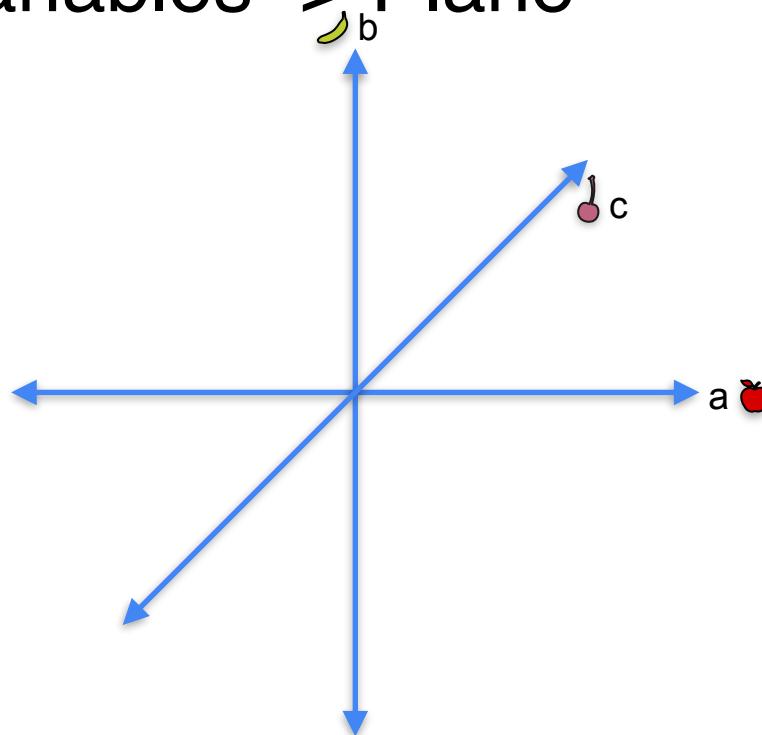


Linear equation in 2 variables -> Line



Linear equation in 3 variables -> Plane

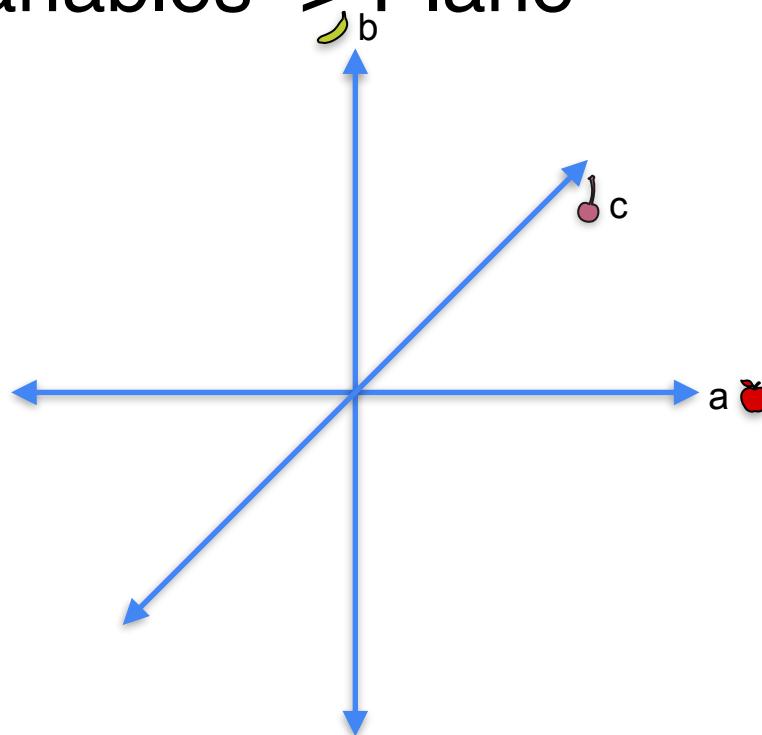
$$a + b + c = 1$$



Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

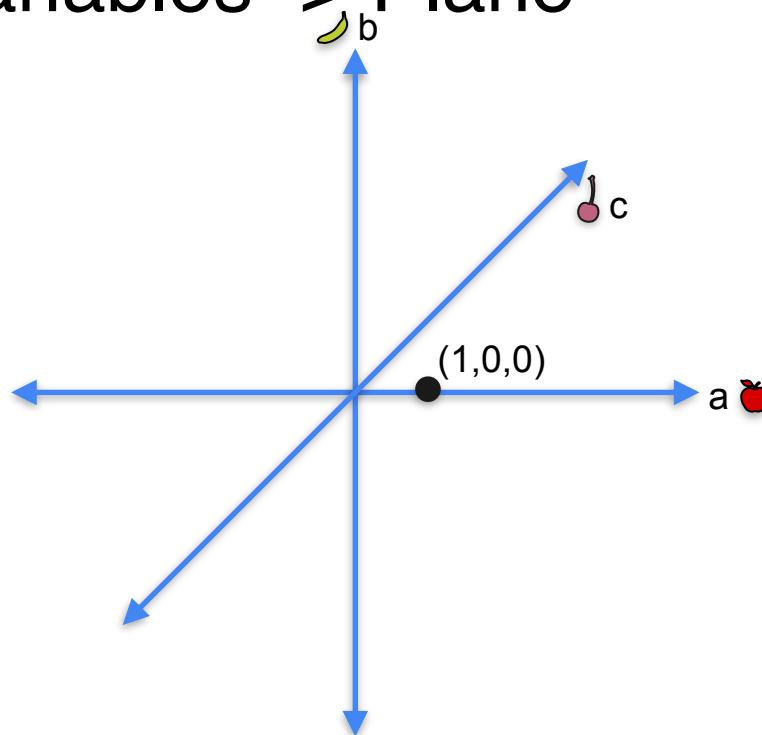
$$1 + 0 + 0 = 1$$



Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

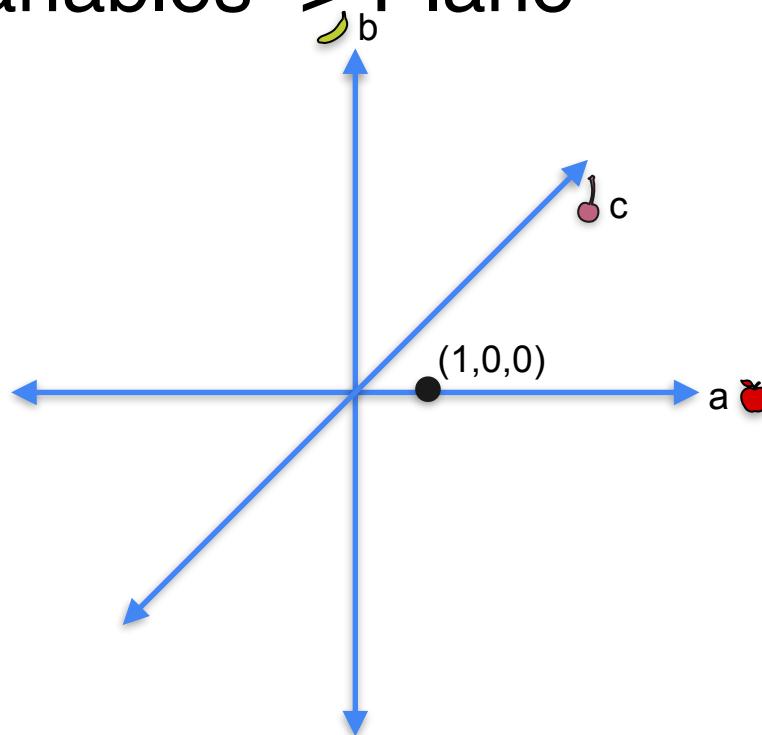


Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

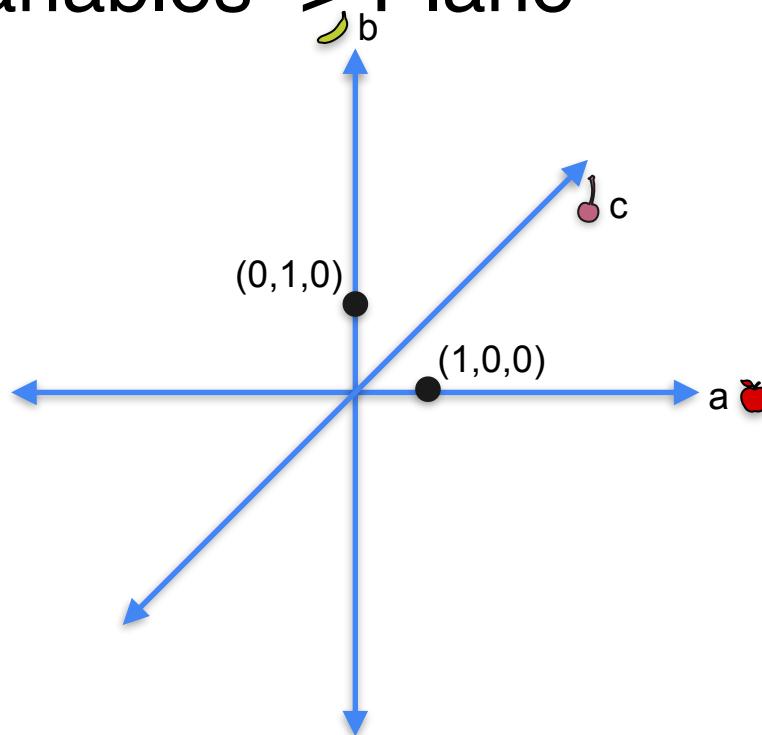


Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$



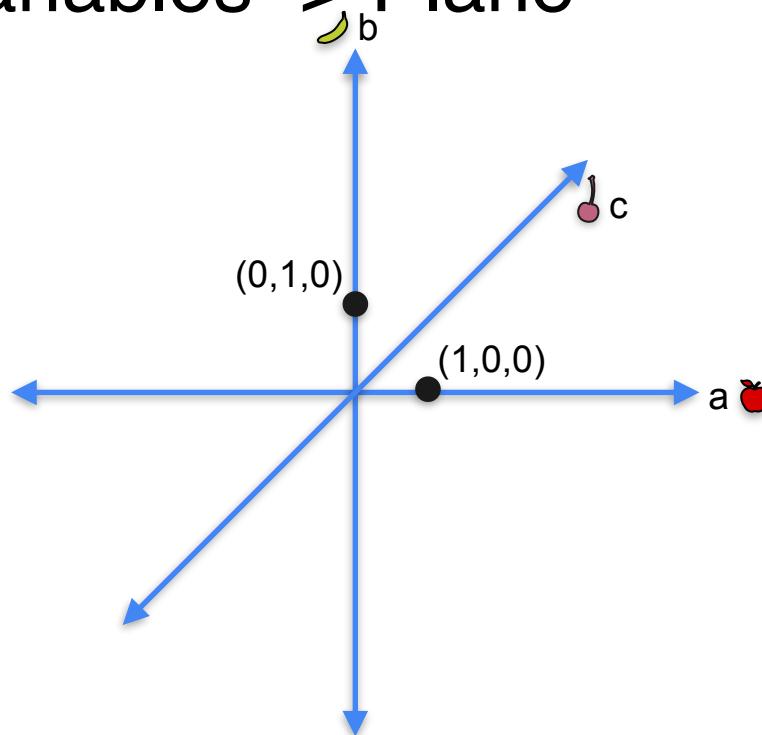
Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$



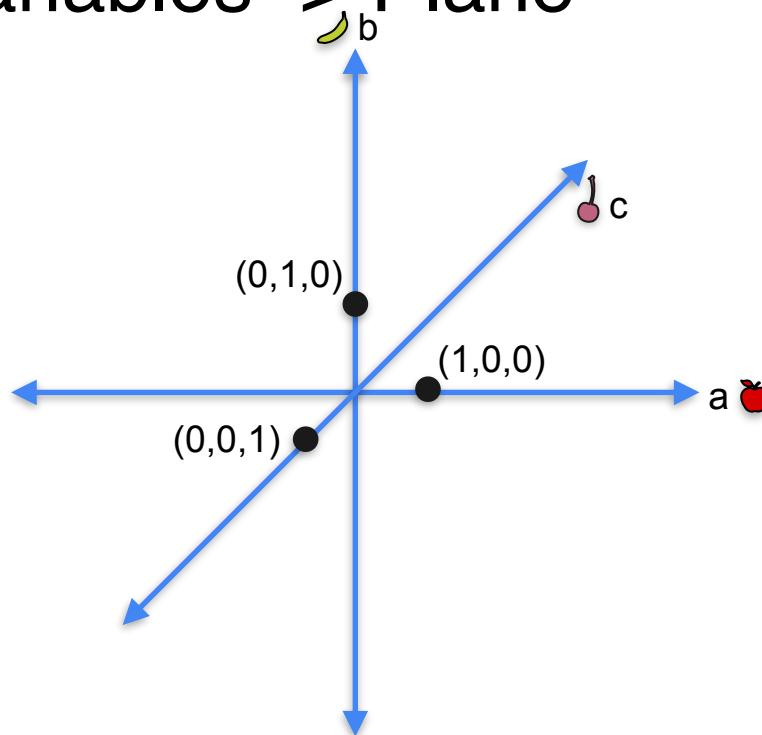
Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$



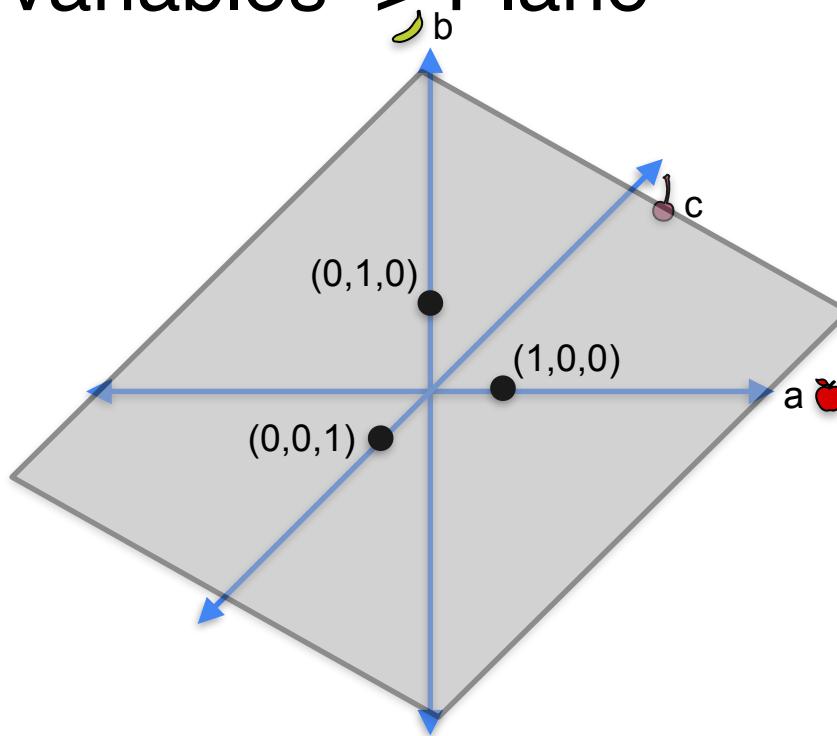
Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

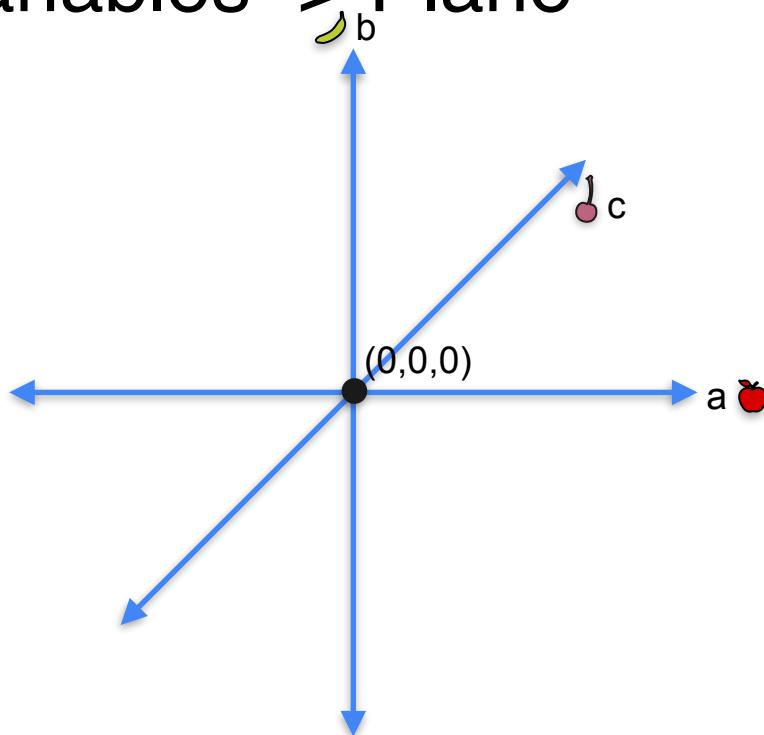
$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$



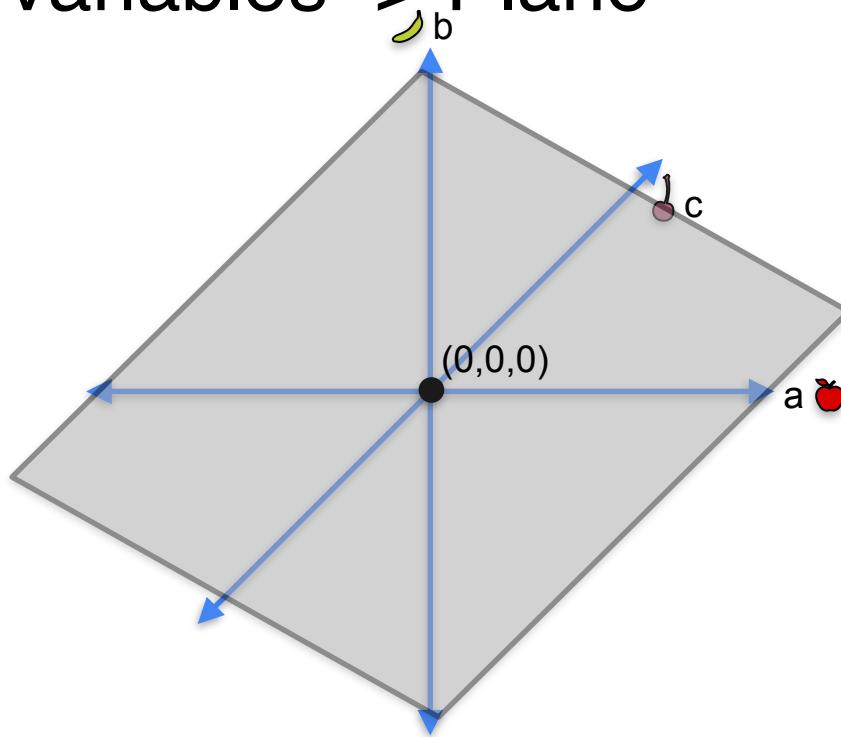
Linear equation in 3 variables -> Plane

$$3a - 5b + 2c = \mathbf{0}$$



Linear equation in 3 variables -> Plane

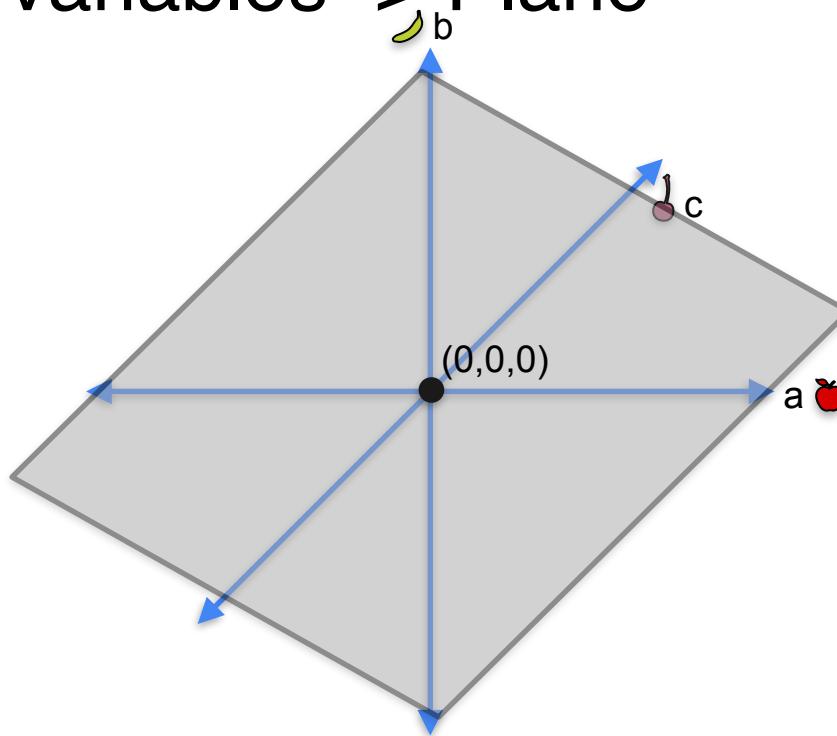
$$3a - 5b + 2c = \mathbf{0}$$



Linear equation in 3 variables -> Plane

$$3a - 5b + 2c = \mathbf{0}$$

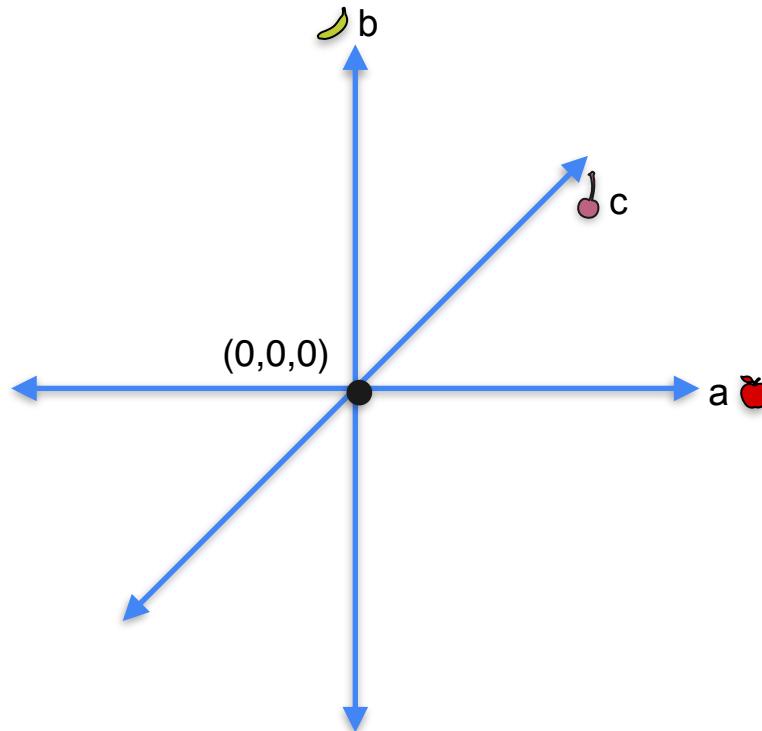
$$3(0) + 5(0) + 2(0) = \mathbf{0}$$



System 1

System 1

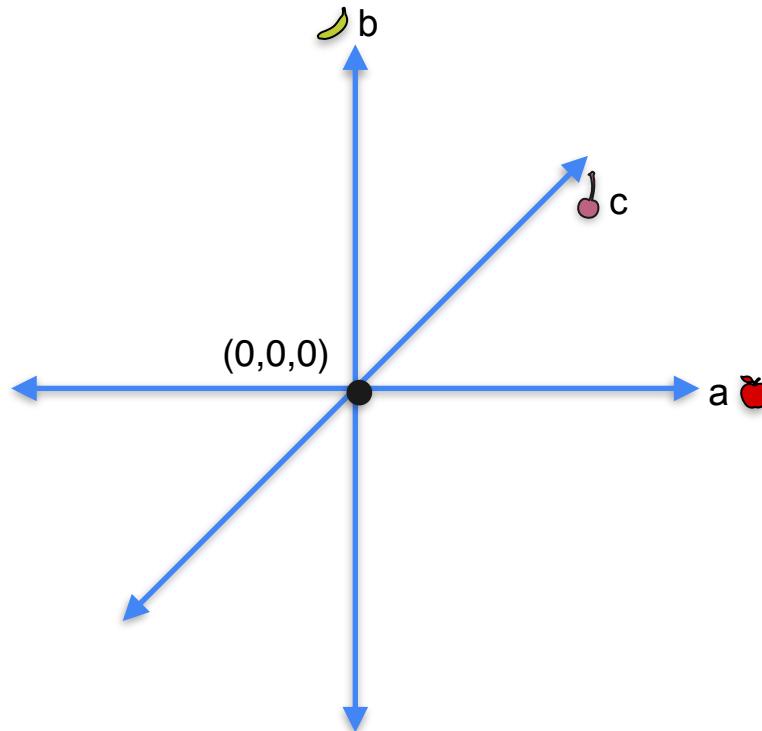
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

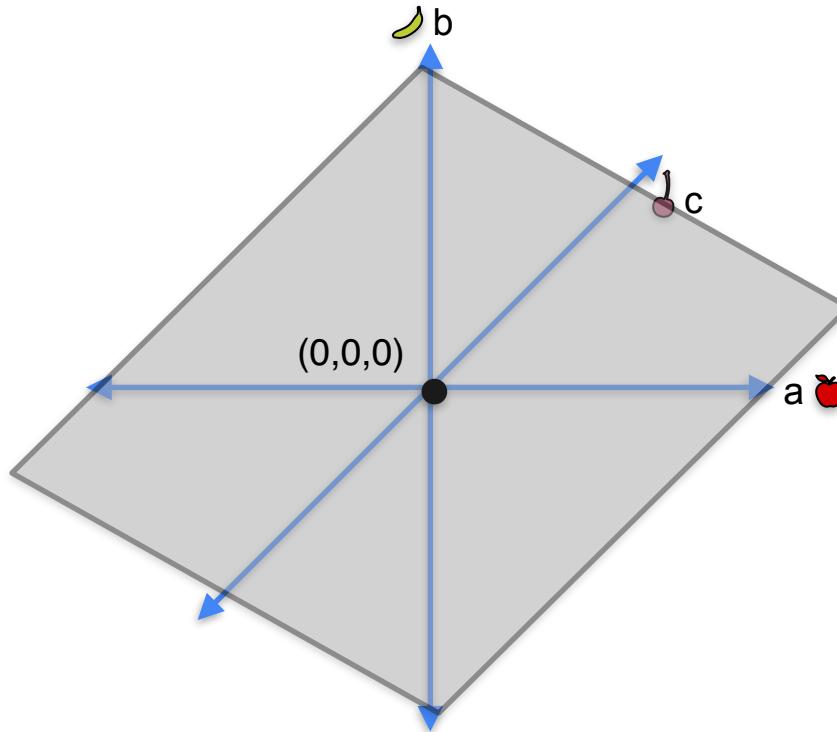
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

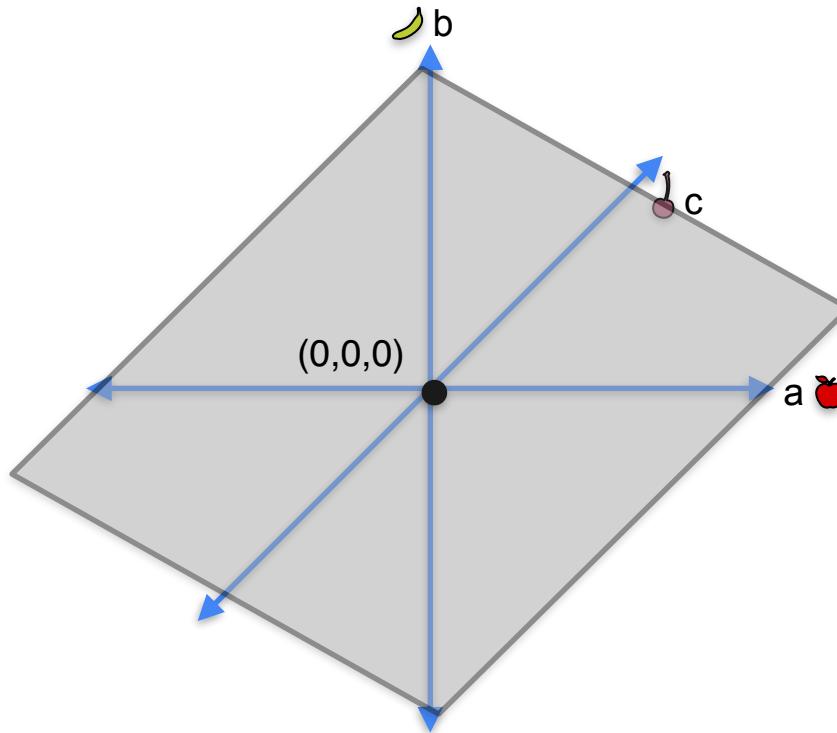
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

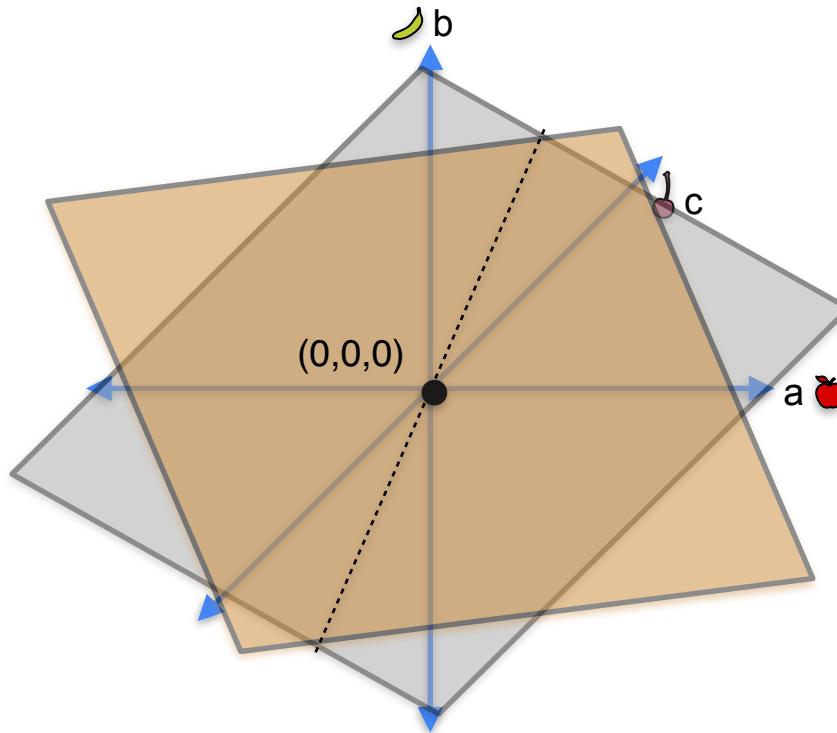
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

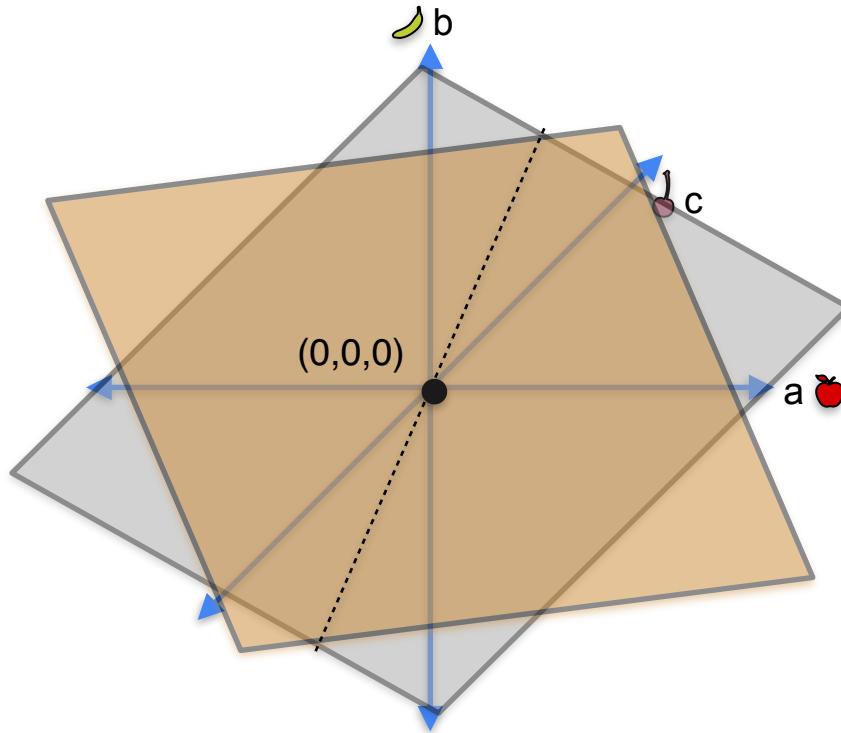
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

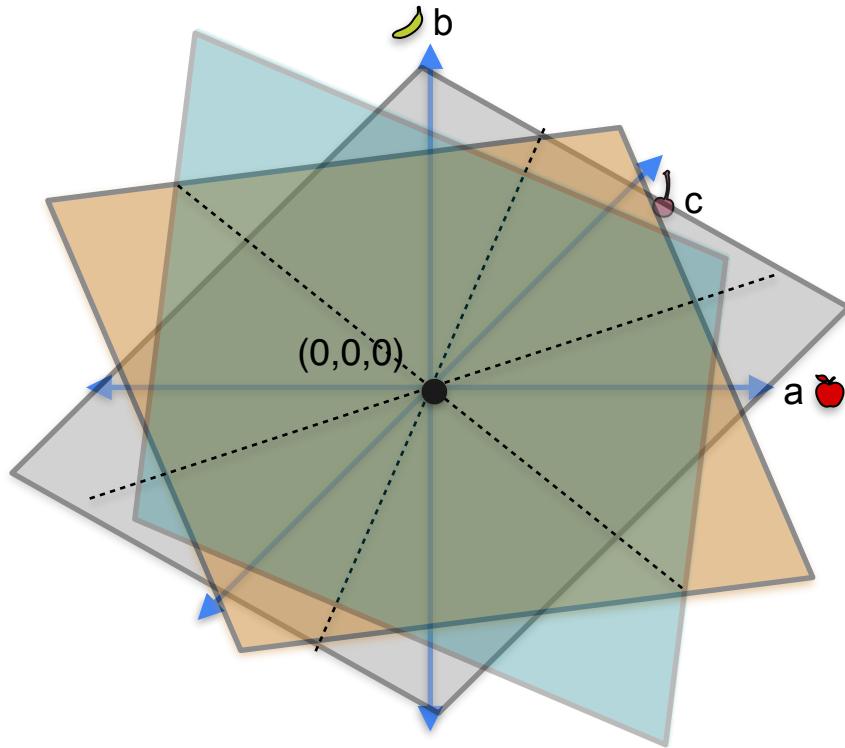
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

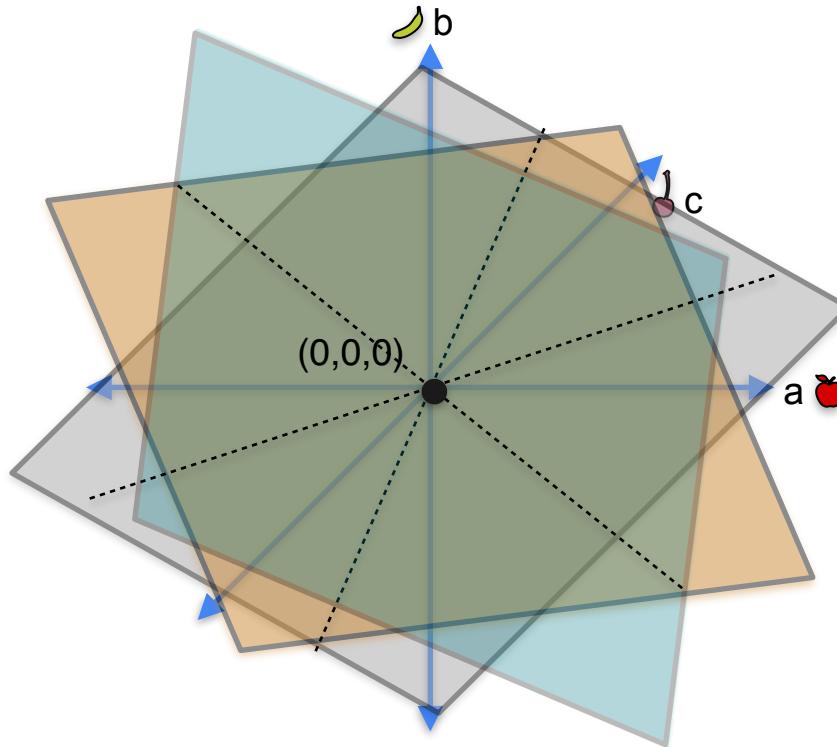
System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



Solution space

- $a = 0$
- $b = 0$
- $c = 0$



System 1

System 1

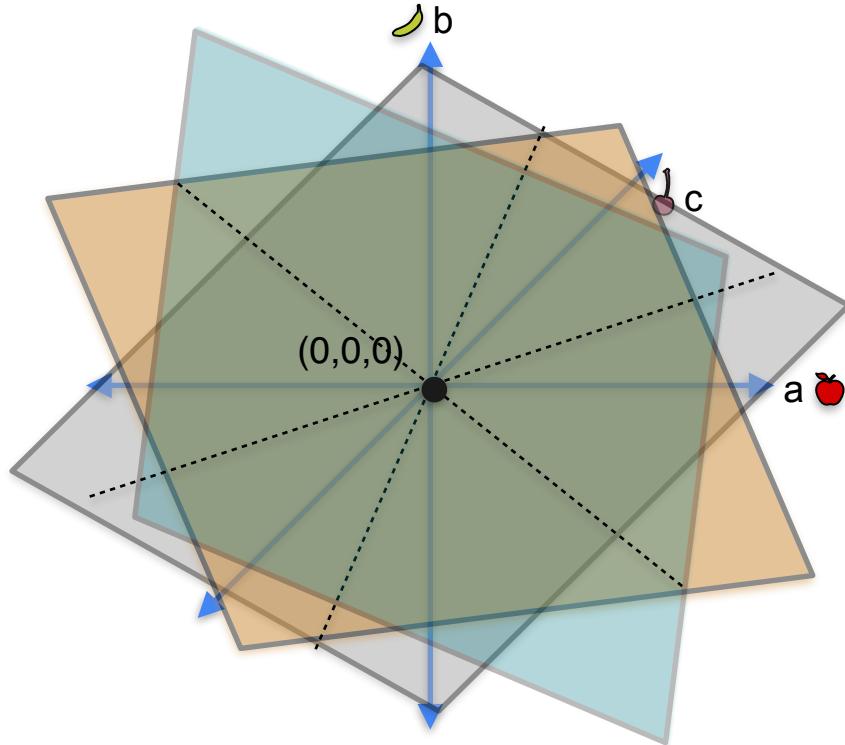
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



Solution space

- $a = 0$
- $b = 0$
- $c = 0$

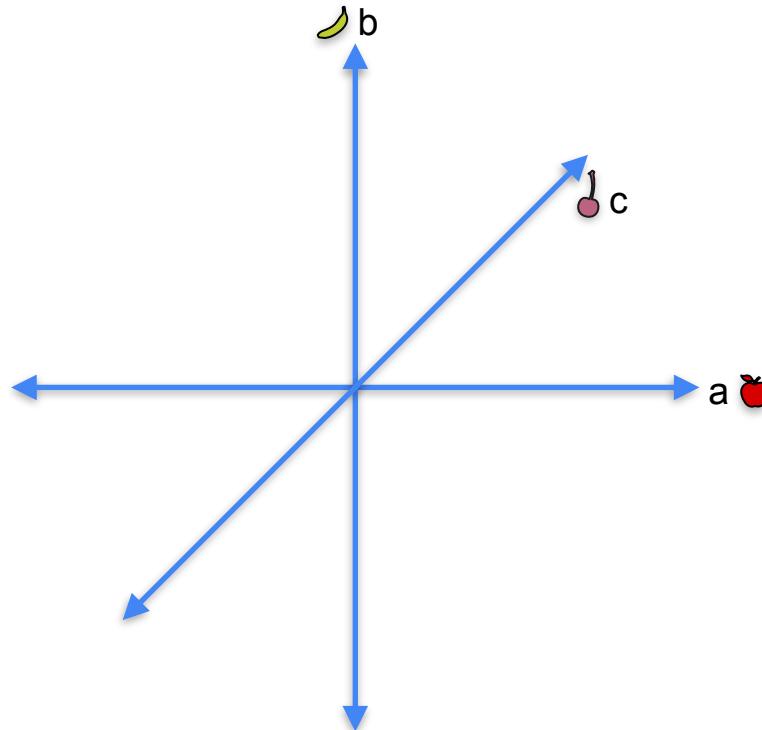
The point
 $(0,0,0)$



System 2

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

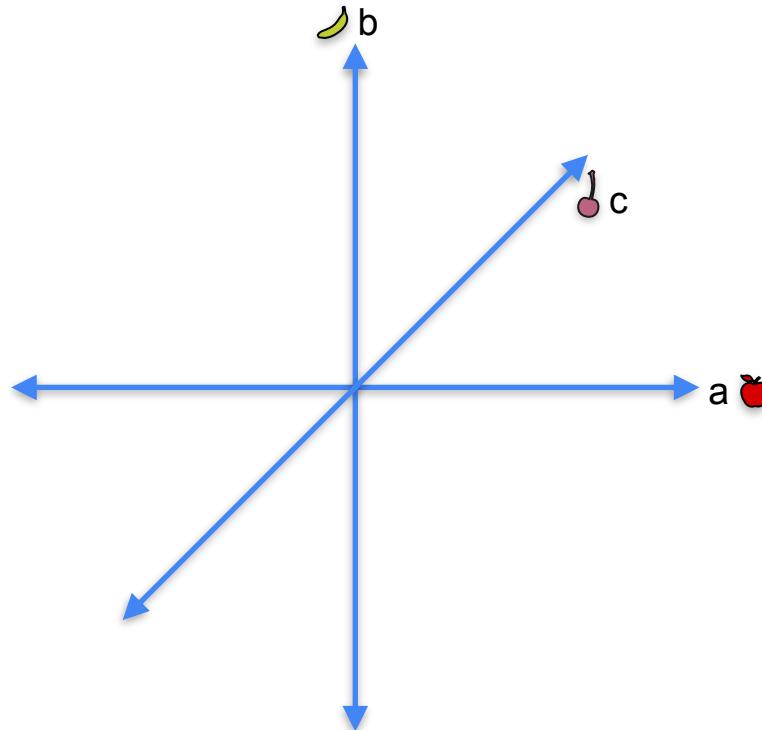
System 2

- $a + b + c = \mathbf{0}$



- $a + b + 2c = \mathbf{0}$

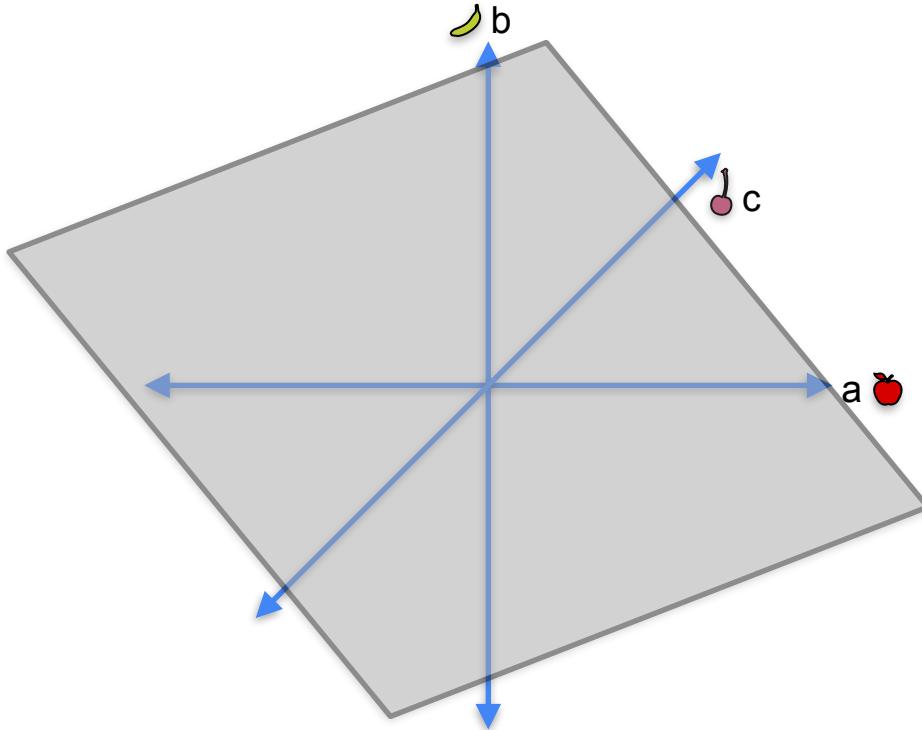
- $a + b + 3c = \mathbf{0}$



System 2

System 2

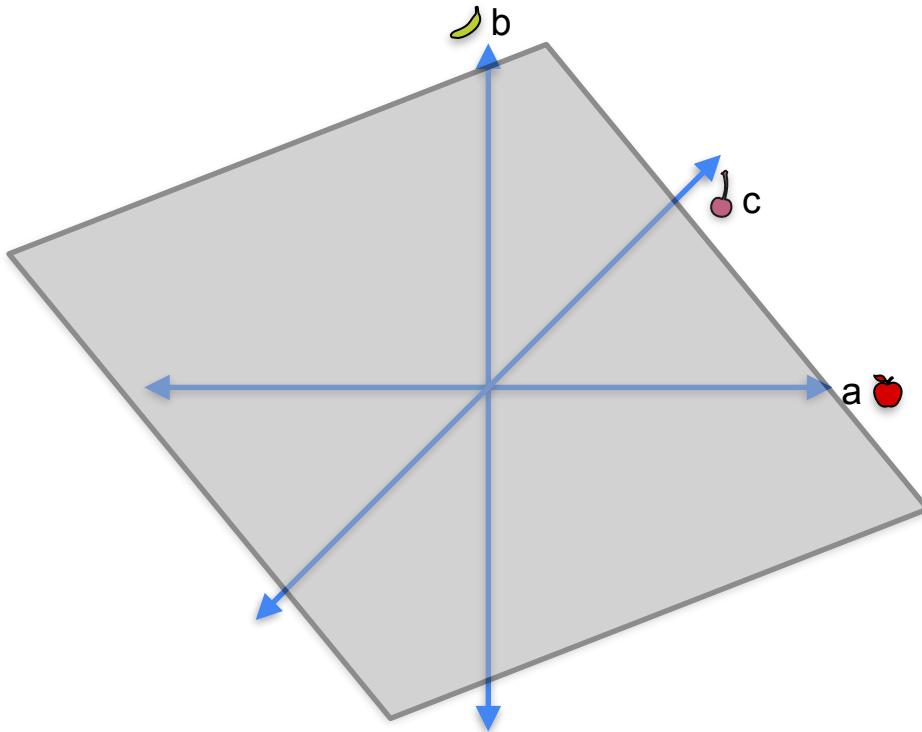
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

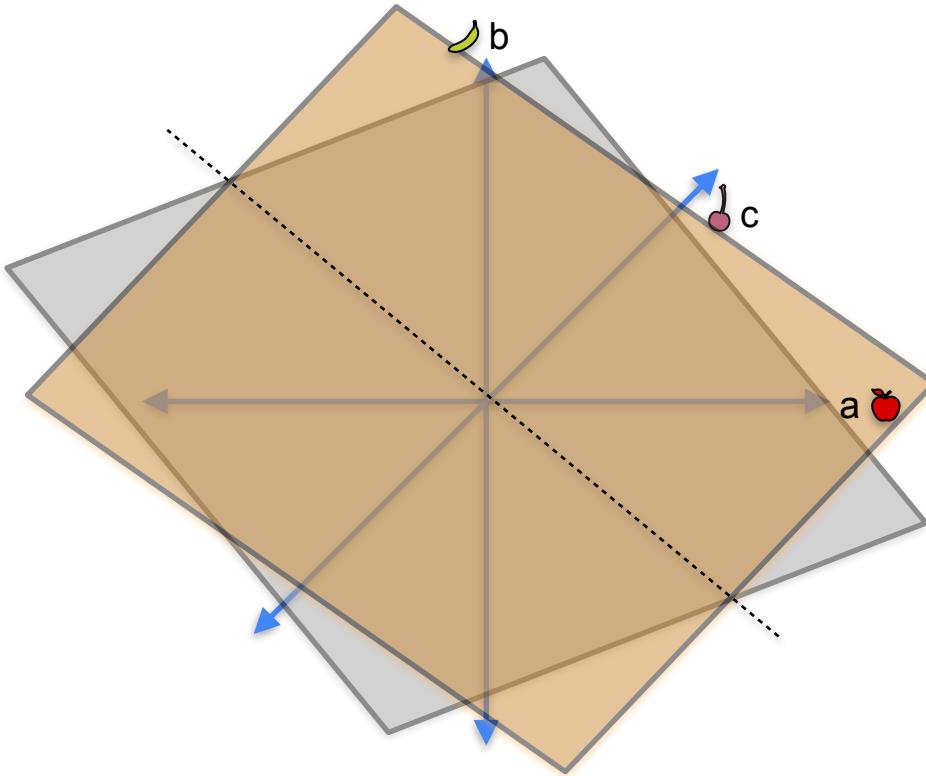
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

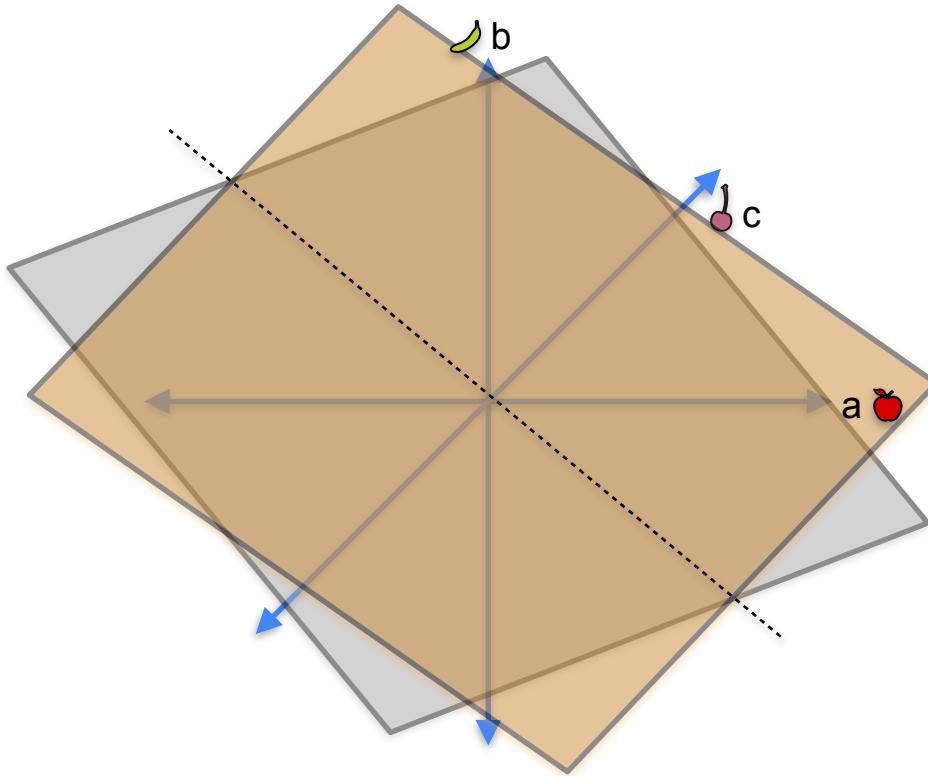
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

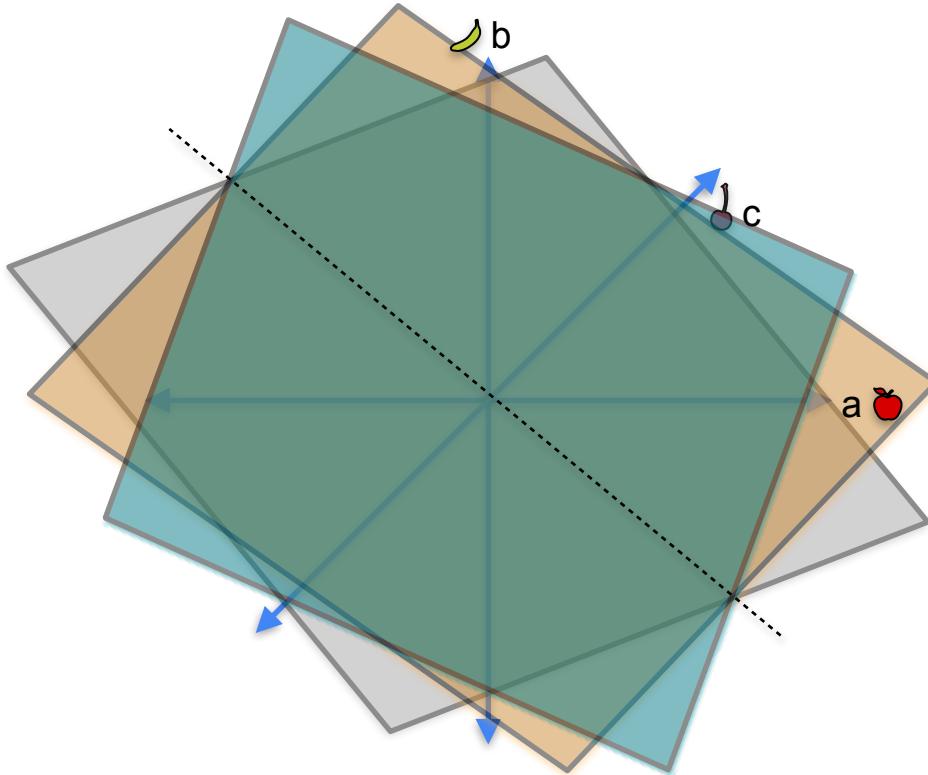
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

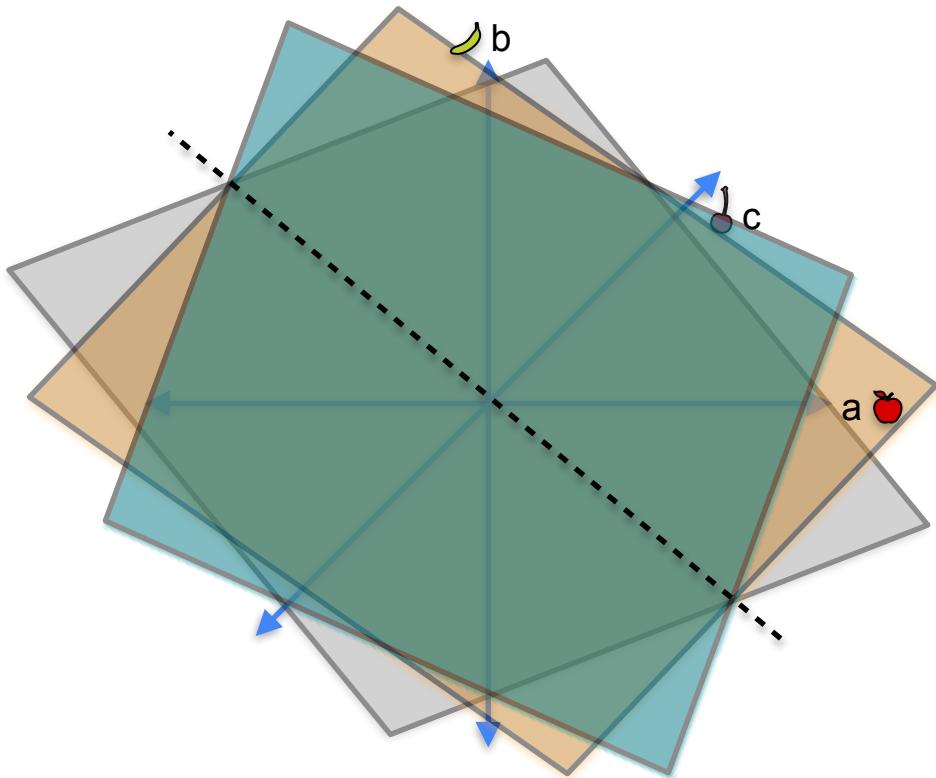
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

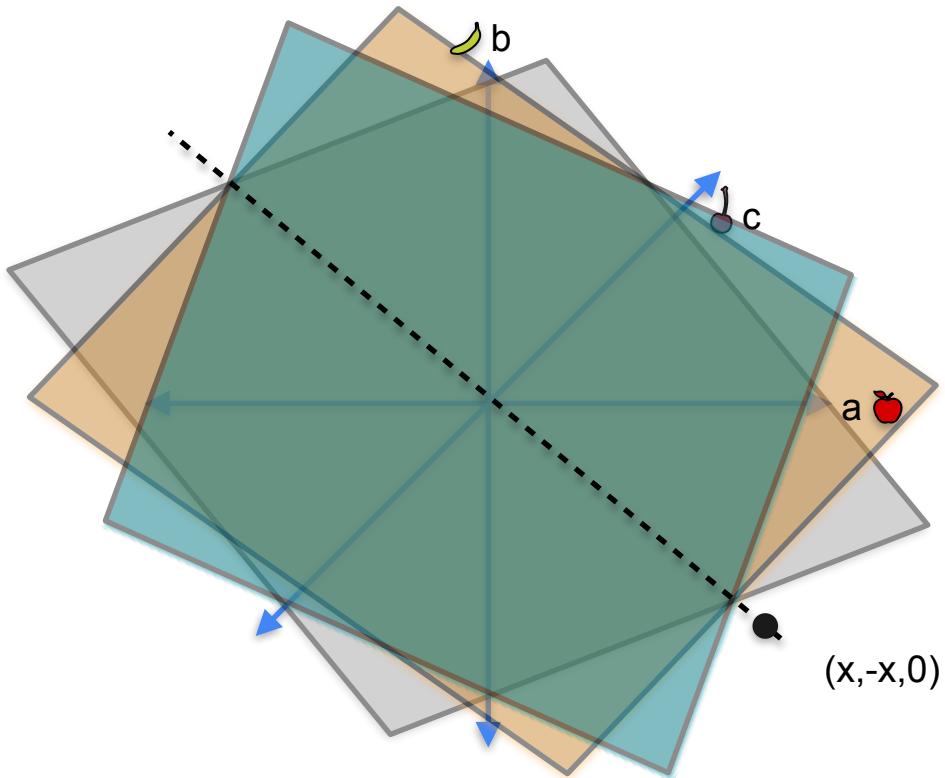
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

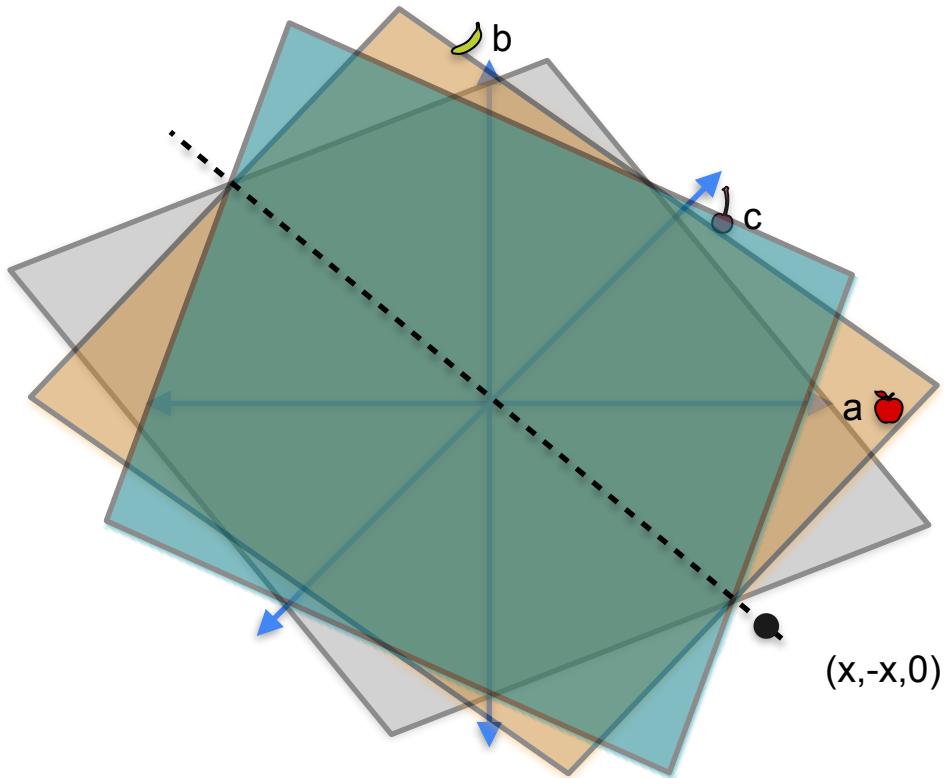
System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



Solution space

- $c = 0$
- $b = -a$



System 2

System 2

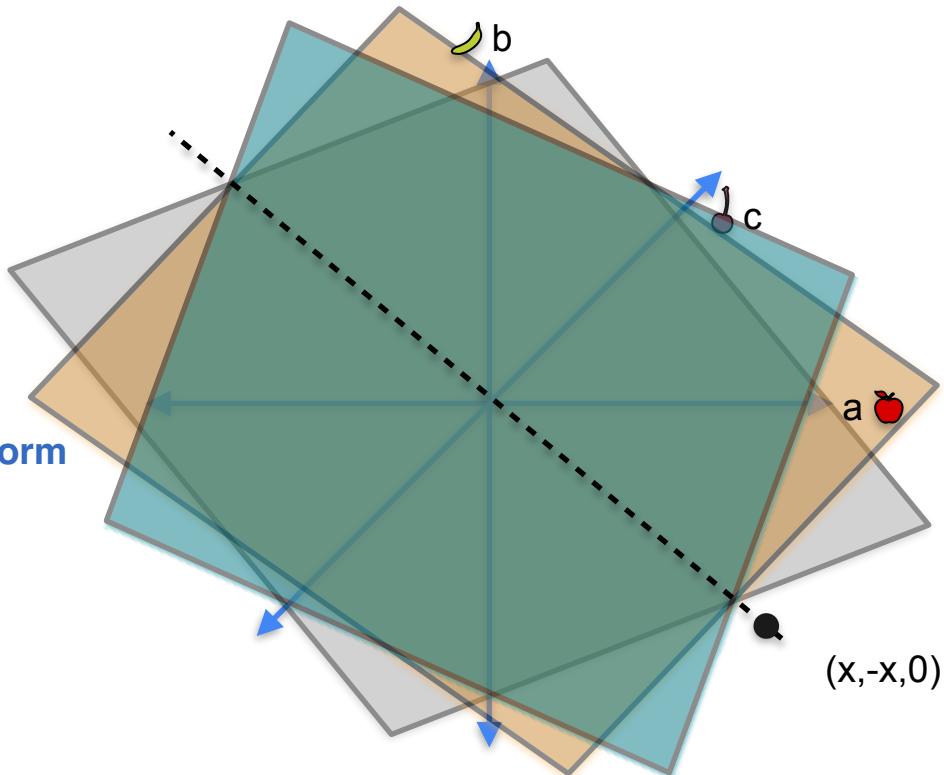
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



Solution space

- $c = 0$
- $b = -a$

All points of the form
 $(x, -x, 0)$



System 2

System 2

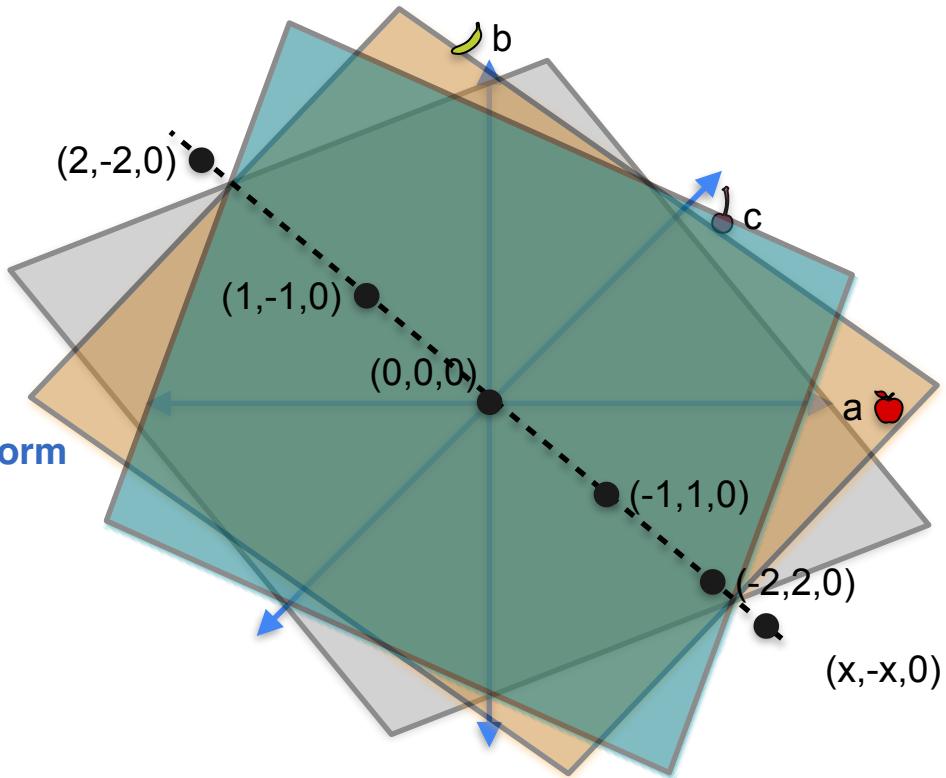
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



Solution space

- $c = 0$
- $b = -a$

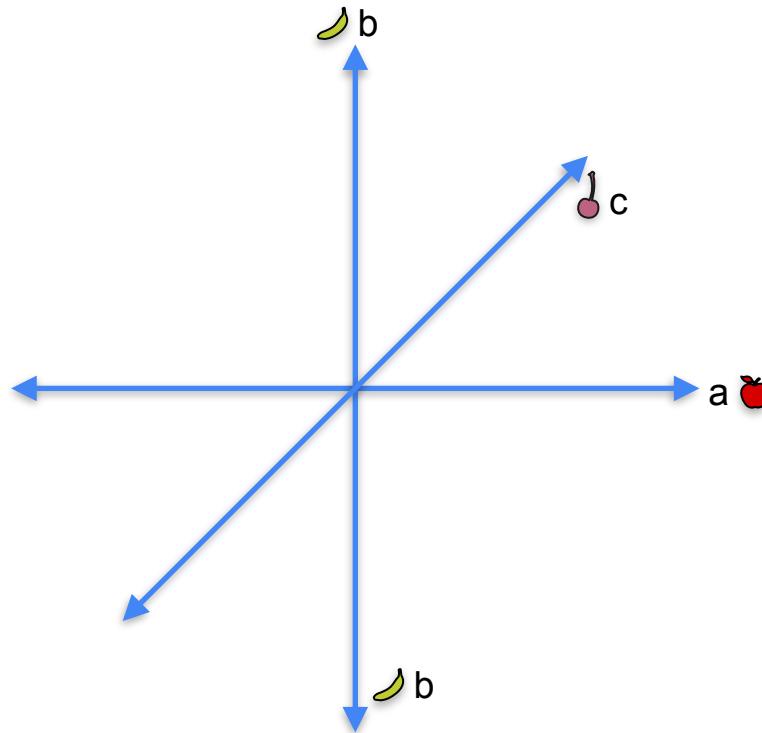
All points of the form
 $(x, -x, 0)$



System 3

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$



System 3

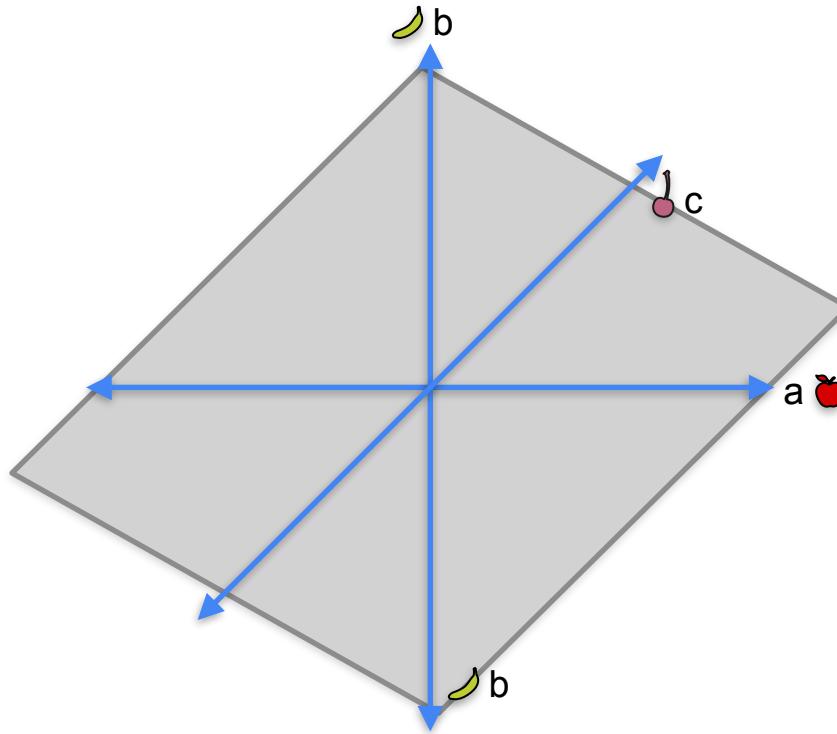
System 3

- $a + b + c = \mathbf{0}$



- $2a + 2b + 2c = \mathbf{0}$

- $3a + 3b + 3c = \mathbf{0}$



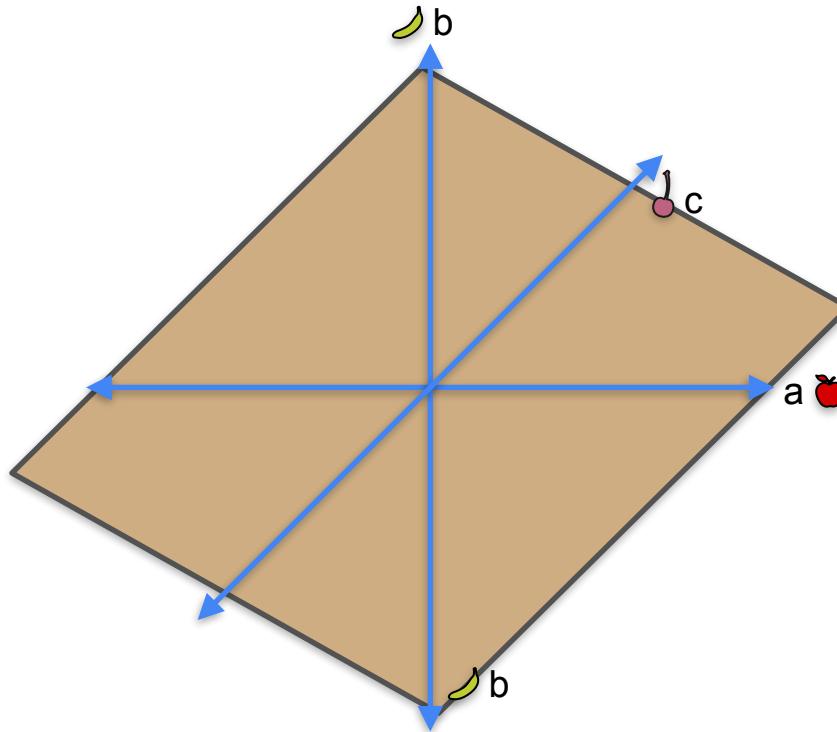
System 3

System 3

- $a + b + c = 0$

- $2a + 2b + 2c = 0$ 

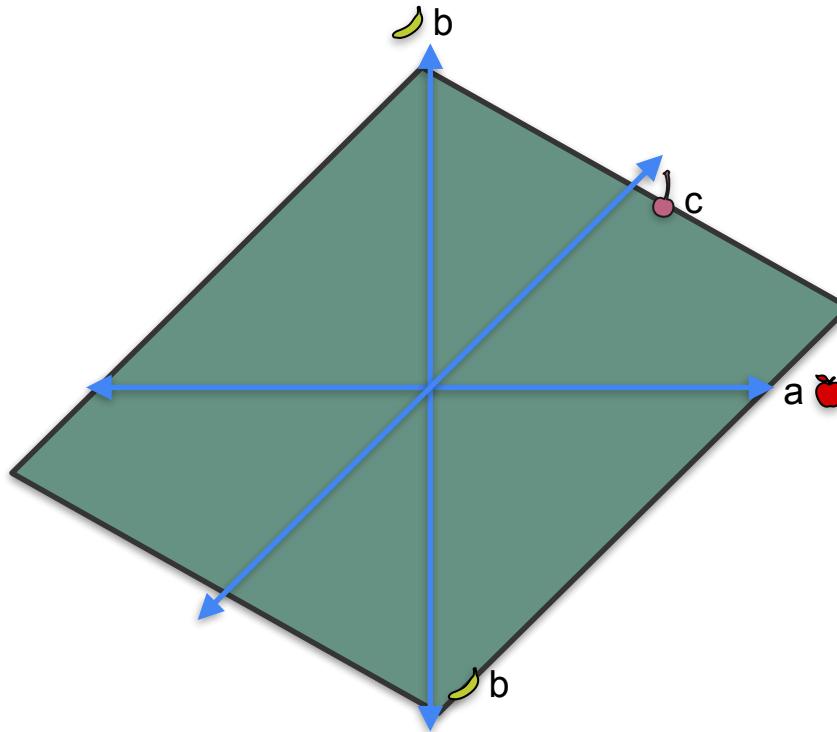
- $3a + 3b + 3c = 0$



System 3

System 3

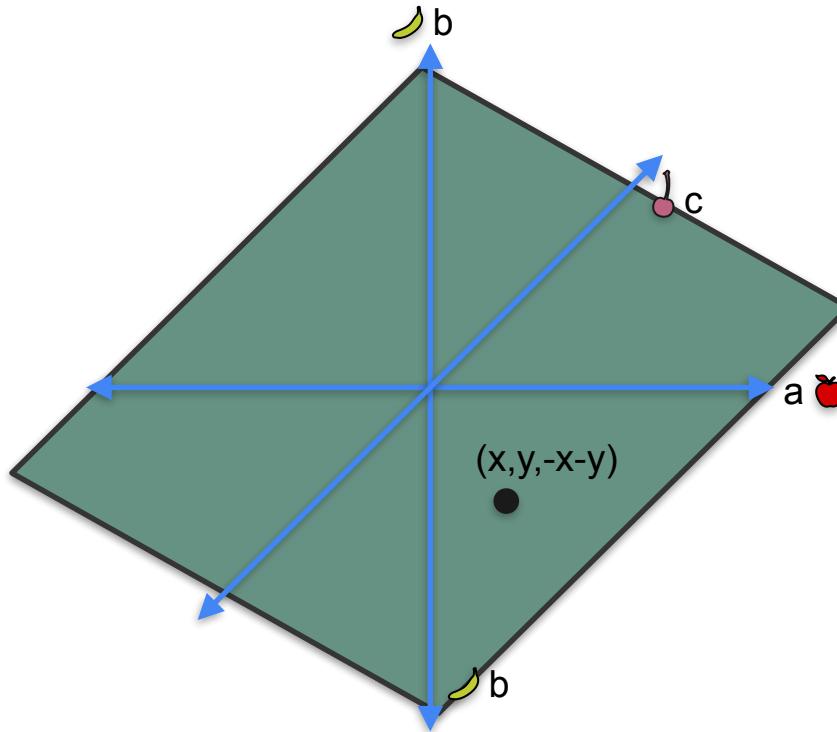
- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$



System 3

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$



System 3

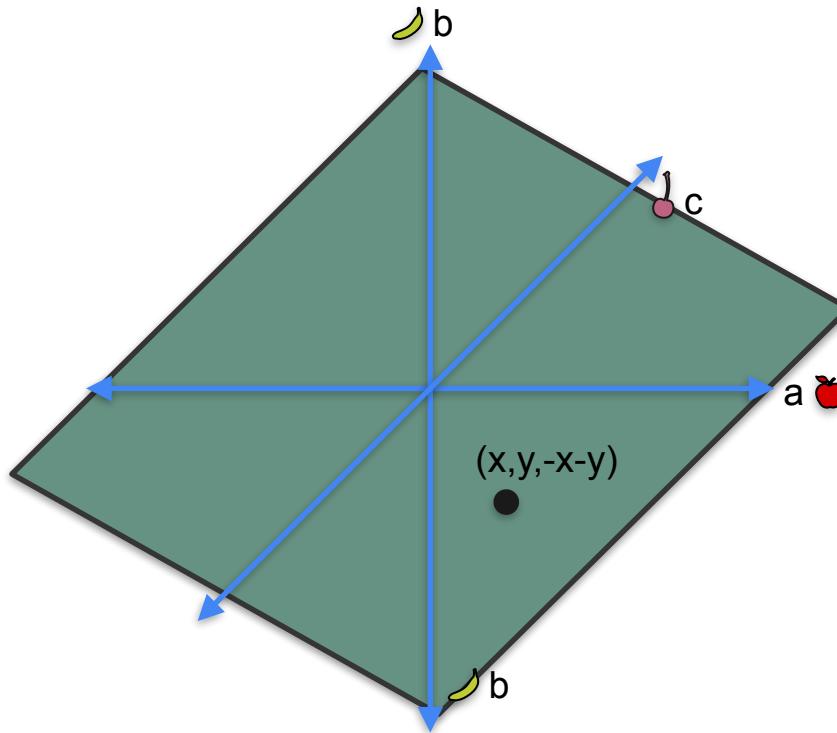
System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$



Solution space

$$\bullet a + b + c = 0$$



System 3

System 3

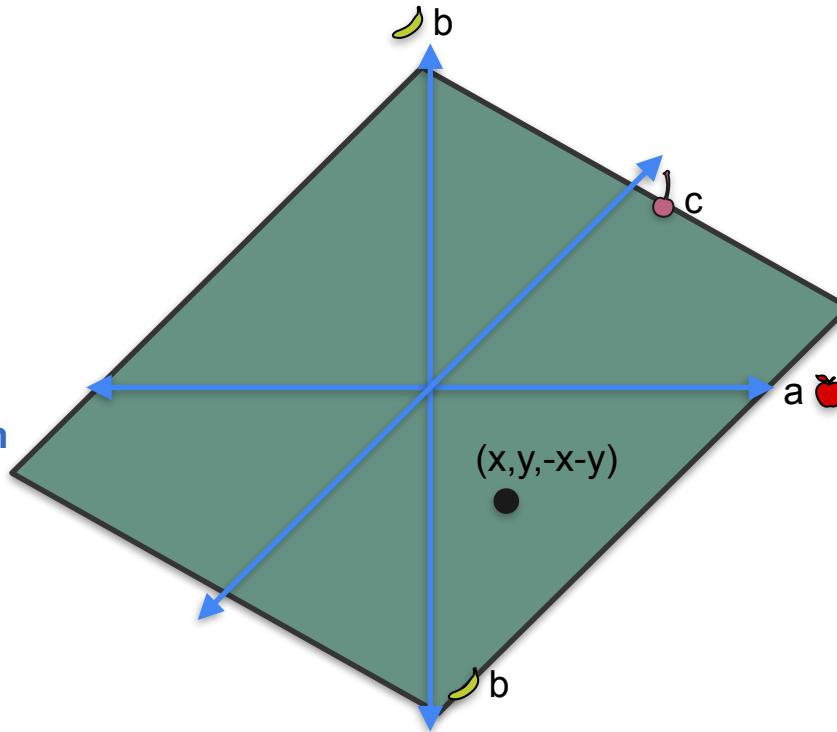
- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

Solution space

$$\bullet a + b + c = 0$$

All points of the form

$$(x, y, -x-y)$$



System 3

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

Solution space

$$\bullet a + b + c = 0$$

All points of the form
 $(x, y, -x - y)$

