

H27-1

$$1) T = \begin{pmatrix} 0.5 & 0.4 & 0.3 \\ 0.4 & 0.5 & 0.3 \\ 0.1 & 0.1 & 0.4 \end{pmatrix}$$

$$\text{a) } T^2 = \frac{1}{100} \begin{pmatrix} 5 & 4 & 3 \\ 4 & 5 & 3 \\ 1 & 1 & 4 \end{pmatrix} \begin{pmatrix} 5 & 4 & 3 \\ 4 & 5 & 3 \\ 1 & 1 & 4 \end{pmatrix} = \frac{1}{100} \begin{pmatrix} 44 & 43 & 39 \\ 43 & 44 & 39 \\ 13 & 13 & 22 \end{pmatrix} = \begin{pmatrix} 0.44 & 0.43 & 0.39 \\ 0.43 & 0.44 & 0.39 \\ 0.13 & 0.13 & 0.22 \end{pmatrix}$$

$$25+16+3, \quad 20+20+3 \quad 15+12+12$$

b) A のすべての固有値と、それに対応する固有ベクトル。

$$A = \frac{1}{10} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$|A - \lambda E| = \frac{1}{10} \begin{vmatrix} 2-10\lambda & 1 \\ 1 & 2-10\lambda \end{vmatrix} = \frac{1}{10} \left\{ (2-10\lambda)^2 - \frac{1}{2} \right\} = \frac{1}{10} \left\{ 100\lambda^2 - 40\lambda + \frac{3}{2} \right\}$$

$$100\lambda^2 - 40\lambda + 3 = 0 \quad 10\lambda = 3, 1$$

$$\begin{aligned} x^2 - 4x + 3 &= 0 & \lambda = \frac{1}{10}, \frac{3}{10} \\ (x-3)(x-1) &= 0 & \# \end{aligned}$$

$$1) \lambda = \frac{1}{10} \text{ は解}.$$

$$(A - \frac{1}{10}E)x = 0 \rightarrow \text{解}.$$

$$\frac{1}{10} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} x = 0 \quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

$$x+y=0$$

$$x = t \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (t \in \mathbb{R})$$

$$\therefore \text{ 固有ベクトル } x = t \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (t \in \mathbb{R})$$

$$(ii) \lambda = \frac{3}{10} \text{ は } \lambda \text{ の } \lambda = \frac{3}{10}$$

$(A - \frac{3}{10}E)x = 0$ を解く。

$$\frac{1}{10} \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} x = 0 \quad \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix}$$

$$-x + y = 0 \quad \text{すなはち } x = t \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

すなはち 固有ベクトル $x = t \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ($t \in \mathbb{R}$)

c) 正の整数 N について、 $A^N b$ および $\sum_{n=0}^N A^n b$ を求めよ。

$$P = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \quad P^{-1} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$P^{-1} A P = \begin{pmatrix} \frac{1}{10} & 0 \\ 0 & \frac{3}{10} \end{pmatrix}$$

$$A^N = P \begin{pmatrix} \left(\frac{1}{10}\right)^N & 0 \\ 0 & \left(\frac{3}{10}\right)^N \end{pmatrix} P^{-1} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \left(\frac{1}{10}\right)^N & 0 \\ 0 & \left(\frac{3}{10}\right)^N \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} \left(\frac{1}{10}\right)^N & \left(\frac{3}{10}\right)^N \\ -\left(\frac{1}{10}\right)^N & \left(\frac{3}{10}\right)^N \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} \left(\frac{1}{10}\right)^N + \left(\frac{3}{10}\right)^N & -\left(\frac{1}{10}\right)^N + \left(\frac{3}{10}\right)^N \\ -\left(\frac{1}{10}\right)^N + \left(\frac{3}{10}\right)^N & \left(\frac{1}{10}\right)^N + \left(\frac{3}{10}\right)^N \end{pmatrix}$$

~~$$A^N b = \frac{1}{2 \cdot 10^N} \begin{pmatrix} 3^N + 1 & 3^N - 1 \\ 3^N - 1 & 3^N + 1 \end{pmatrix} \begin{pmatrix} \frac{3}{10} \\ 0 \end{pmatrix} = \frac{3}{2 \cdot 10^{N+1}} \begin{pmatrix} 3^{N+1} & 3^N - 1 \\ 3^N - 1 & 3^{N+1} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$~~

$$= \frac{3}{2 \cdot 10^{N+1}} \begin{pmatrix} 2 \cdot 3^N \\ 2 \cdot 3^N \end{pmatrix} \cdot \underbrace{\frac{3^{N+1}}{10^{N+1}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{\frac{3}{10} \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$$

$$\sum_{n=0}^N A^n b = \left(\frac{3}{10}\right)\binom{1}{1} + \left(\frac{3}{10}\right)^2 \binom{1}{1} + \left(\frac{3}{10}\right)^3 \binom{1}{1} + \dots + \left(\frac{3}{10}\right)^{N+1} \binom{1}{1}$$

$$= \left(\frac{3}{10} + \left(\frac{3}{10}\right)^2 + \dots + \left(\frac{3}{10}\right)^{N+1} \right)$$

$$\frac{3}{10} S = \frac{3}{10} + \left(\frac{3}{10}\right)^2 + \dots + \left(\frac{3}{10}\right)^{N+1}$$

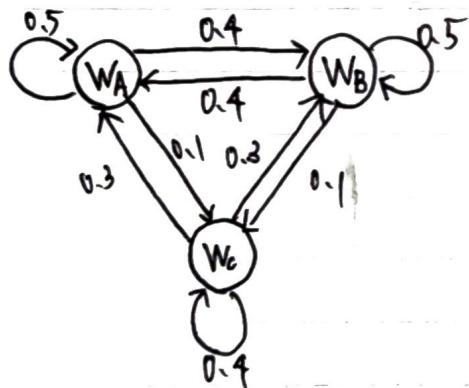
$$\frac{7}{10} S = \frac{3}{10} - \left(\frac{3}{10}\right)^{N+2}$$

$$S = \frac{10}{7} \left\{ \frac{3}{10} - \left(\frac{3}{10}\right)^{N+2} \right\}$$

$$= \frac{3}{7} \left\{ 1 - \left(\frac{3}{10}\right)^{N+1} \right\}$$

$$\sum_{n=0}^N A^n b = \frac{3}{7} \left\{ 1 - \left(\frac{3}{10}\right)^{N+1} \right\} \binom{1}{1}$$

2)



a) WA から 111° リンクを 2 回クリックした後は、再び WA にいる確率。

$$\begin{aligned} A \rightarrow A \rightarrow A &: \frac{5}{10} \times \frac{5}{10} = \frac{25}{100} \\ \rightarrow B \rightarrow A &: \frac{4}{10} \times \frac{1}{10} = \frac{16}{100} \\ \rightarrow C \rightarrow A &: \frac{1}{10} \times \frac{3}{10} = \frac{3}{100} \end{aligned} \quad \left. \right\} \frac{44}{100}$$

$$\underline{\underline{0.44}}$$

b) N 回クリックした後は WA ではなく WB にいる確率を P_N , Q_N とする。

P_{N-1}, Q_{N-1} を用いて P_N, Q_N を表せ。

$$\begin{aligned} P_N &= \frac{5}{10} P_{N-1} + \frac{4}{10} Q_{N-1} + \frac{3}{10} (1 - P_{N-1} - Q_{N-1}) \\ &= \frac{3}{10} + \frac{2}{10} P_{N-1} + \frac{1}{10} Q_{N-1} \end{aligned}$$

$$\begin{aligned} Q_N &= \frac{4}{10} P_{N-1} + \frac{5}{10} Q_{N-1} + \frac{3}{10} (1 - P_{N-1} - Q_{N-1}) \\ &= \frac{3}{10} + \frac{1}{10} P_{N-1} + \frac{2}{10} Q_{N-1} \end{aligned}$$

c) $P_0 = Q_0 = 0.3$

$$R_N = P_N + Q_N = \frac{6}{10}P_{N-1} + \frac{3}{10}Q_{N-1} = \frac{6}{10} + \frac{3}{10}P_{N-1} + \frac{3}{10}Q_{N-1}$$

$$= \frac{3}{10}R_{N-1} + \frac{6}{10}$$

~~$10R_N - 3R_{N-1} + 6 = 10x^2 - 3x - 6 = 0$~~

~~$x^2 - \frac{3}{10}x - \frac{6}{10} = 0$~~

$$\begin{array}{c} A^2 \\ B \\ \hline B^5 \end{array} \quad -2 \times 3$$

$$x = \frac{3}{10}x + \frac{6}{10}$$

$$\frac{7}{10}x = \frac{6}{10} \Rightarrow x = \frac{6}{7}$$

~~$(R_N - \frac{6}{7}) = \frac{3}{10}(R_{N-1} - \frac{6}{7})$~~

$$R_0 = \frac{6}{10}, \quad R_0 - \frac{6}{7} = \frac{6}{10} - \frac{6}{7} = \frac{42-60}{70} = -\frac{18}{70}$$

$$R_N - \frac{6}{7} = -\frac{18}{70} \left(\frac{3}{10}\right)^N$$

$$R_N = -\frac{18}{70} \left(\frac{3}{10}\right)^N + \frac{6}{7} = -\frac{9}{35} \left(\frac{3}{10}\right)^N + \frac{6}{7}$$

~~$S_N = P_N - Q_N = P_{N-1} - Q_{N-1} = S_{N-1}$~~

~~$S_0 = 0 \quad S_N = 0$~~

~~$\frac{1}{2}(R_N + S_N) \quad \frac{1}{2}(P_N + Q_N + P_{N-1} - Q_{N-1}) = P_N$~~

$$P_N = \frac{1}{2} \left\{ -\frac{9}{35} \left(\frac{3}{10}\right)^N + \frac{6}{7} \right\} = -\frac{9}{70} \left(\frac{3}{10}\right)^N + \cancel{\frac{3}{7}} + \frac{3}{7} = \frac{3}{7} \left\{ 1 - \left(\frac{3}{10}\right)^{N+1} \right\}$$

~~$\frac{1}{2}(R_N - S_N) = \frac{1}{2} \{ P_N + Q_N - P_{N-1} + Q_{N-1} \} = Q_N$~~

$$Q_N = \frac{1}{2} \left\{ -\frac{9}{35} \left(\frac{3}{10}\right)^N + \frac{6}{7} \right\} = -\frac{9}{70} \left(\frac{3}{10}\right)^N + \frac{3}{7} = \frac{3}{7} \left\{ 1 - \left(\frac{3}{10}\right)^{N+1} \right\}$$

d) $\lim_{N \rightarrow \infty} P_N = \lim_{N \rightarrow \infty} \frac{3}{7} \left\{ 1 - \left(\frac{3}{10}\right)^{N+1} \right\} = \frac{3}{7}$

$$\lim_{N \rightarrow \infty} Q_N = \frac{3}{7}$$

H27-2

$$f(x) = \operatorname{sgn}(x) + \cos \pi x + \sin 5\pi x \quad (-1 \leq x < 1)$$

$$\operatorname{sgn}(x) = \begin{cases} -1 & (-1 \leq x < 0) \\ 1 & (0 \leq x < 1) \end{cases}$$

1) 2つの実数 α, β を用いて関数 $g(x) = \alpha \cos \pi x + \beta \sin 5\pi x$

$$E(\alpha, \beta) = \int_{-1}^1 |f(x) - g(x)|^2 dx$$

αを $\alpha = \beta$, 2次関数で表せ.

$$E(\alpha, \beta) = \int_{-1}^1 |f(x) - g(x)|^2 dx = \int_{-1}^1 (f(x) - g(x))^2 dx$$

$$\left(\begin{array}{l} (a+b+c)^2 \\ a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \end{array} \right)$$

$$= \int_{-1}^1 (\operatorname{sgn}(x) + (1-\alpha) \cos \pi x + (1-\beta) \sin 5\pi x)^2 dx$$

$$= \int_{-1}^1 |\operatorname{sgn}(x)|^2 + (1-\alpha)^2 \cos^2 \pi x + (1-\beta)^2 \sin^2 5\pi x + 2 \operatorname{sgn}(x) (1-\alpha) \cos \pi x + 2 \operatorname{sgn}(x) (1-\beta) \sin 5\pi x + 2(1-\alpha)(1-\beta) \cos \pi x \sin 5\pi x dx$$

$$\int_{-1}^1 |\operatorname{sgn}(x)|^2 dx = \int_{-1}^1 1 dx = \frac{2}{4}$$

$$\int_{-1}^1 (1-\alpha)^2 \cos^2 \pi x dx = (1-\alpha)^2 \int_{-1}^1 \cos^2 \pi x dx = \frac{(1-\alpha)^2}{\pi} \int_{-\pi}^{\pi} \cos^2 t dt$$

$$\begin{aligned} \pi x = t & \quad \cos 2t = \cos^2 t - \sin^2 t \\ & \sim 2 \cos^2 t - 1 & & = \frac{(1-\alpha)^2}{\pi} \int_{-\pi}^{\pi} \frac{1 + \cos 2t}{2} dt \end{aligned}$$

$$= \frac{(1-\alpha)^2}{\pi} \left[\frac{1}{2}t + \frac{1}{4}\sin 2t \right]_{-\pi}^{\pi} = \frac{(1-\alpha)^2}{\pi} \left(\frac{\pi}{2} + \frac{\pi}{2} \right) = \frac{(1-\alpha)^2}{\pi}$$

$$\int_{-1}^1 (1-\beta)^2 \sin^2 5\pi x dx = 0$$

奇×奇=奇

$$\begin{aligned}
 & 2 \int_{-1}^1 \text{sgn}(x)(1-\alpha)\cos\pi x + \cancel{\text{sgn}(x)}(1-\beta)\sin 5\pi x \, dx \\
 &= 2 \left\{ \int_{-1}^0 -(1-\alpha)\cos\pi x - (1-\beta)\sin 5\pi x \, dx + \int_0^1 (1-\alpha)\cos\pi x + (1-\beta)\sin 5\pi x \, dx \right\} \\
 &= 2 \left\{ \left[-(1-\alpha) \frac{\sin\pi x}{\pi} + (1-\beta) \frac{\cos 5\pi x}{5\pi} \right]_{-1}^0 + \left[\frac{(1-\alpha)}{\pi} \sin\pi x + \frac{(1-\beta)}{5\pi} \cos 5\pi x \right]_0^1 \right\} \\
 &= 2 \left\{ \frac{(1-\beta)}{5\pi} + \frac{(1-\beta)}{5\pi} + \frac{(1-\beta)}{5\pi} + \frac{(1-\beta)}{5\pi} \right\} = \cancel{\frac{8}{5}\pi(1-\beta)} \quad \cancel{-\frac{8}{5}\pi(1-\beta)}
 \end{aligned}$$

偶×偶 = 偶
 奇×偶 = 奇
 偶×奇 = 奇
 奇×奇 = 偶

$$\text{∴ } 2(1-\alpha)(1-\beta) \underbrace{\cos\pi x \sin 5\pi x}_{\text{偶} \times \text{奇}} \, dx = 0$$

$$\text{~奇} \quad \frac{8}{5}\pi(1-\beta)$$

$$\text{∴ } 2(1-\alpha)(1-\beta) = 2 + (1-\alpha)^2 + \cancel{\frac{8}{5}\pi(1-\beta)}$$

$$\begin{aligned}
 \int_{-1}^1 (1-\beta)^2 \underbrace{\sin^2 5\pi x}_{\text{奇} \times \text{奇} = \text{偶}} \, dx &= \frac{(1-\beta)^2}{5\pi} \int_{-5\pi}^{5\pi} \sin^2 t \, dt = \frac{(1-\beta)^2}{5\pi} \int_{-5\pi}^{5\pi} \frac{1 - \cos 2t}{2} \, dt \\
 5\pi x = t &\quad x \mapsto -1 \rightarrow 1 \quad \cos 2t = \cos^2 t - \sin^2 t \\
 5\pi dx = dt &\quad + (-5\pi \rightarrow 5\pi) \quad = 1 - 2 \cos^2 t = \frac{(1-\beta)^2}{5\pi} \left[\frac{1}{2}t - \frac{\sin 2t}{2} \right]_{-5\pi}^{5\pi} \\
 &= \frac{(1-\beta)^2}{5\pi} \left[\frac{5}{2}\pi + \frac{5}{2}\pi \right] = \frac{(1-\beta)^2}{5\pi}
 \end{aligned}$$

$$\text{∴ } 2(1-\alpha)(1-\beta) = 2 + (1-\alpha)^2 + (1-\beta)^2 + \frac{8}{5}\pi(1-\beta)$$

2) $E(\alpha, \beta)$ の最小値を求める $\alpha, \beta \in E(\alpha, \beta)$ の値。

$$1-\alpha = X, 1-\beta = Y \in \mathbb{R}$$

$$E(\alpha, \beta) = 2 + X^2 + Y^2 + \frac{8}{5}\pi Y = 2 + X^2 + \left(Y + \frac{4}{5\pi}\right)^2 - \frac{16}{25\pi^2}$$

∴ $X=0, Y = -\frac{4}{5\pi}$ のとき最小。

$$\begin{aligned}
 \alpha = 1, \beta = \frac{1}{5}, E(\alpha, \beta) &= \frac{16}{25\pi^2} - \frac{16}{25\pi^2} = 0 \\
 \beta = 1 + \frac{4}{5\pi}
 \end{aligned}$$

3) $\tilde{f}(t) \quad (-\infty < t < \infty)$ と $\tilde{f}(x+2m) = f(x) \quad (-1 \leq x < 1)$
 mは任意の整数

$$\tilde{f}(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\pi t + b_n \sin n\pi t) \quad \text{形にフーリエ級数展開}$$

$$a_m = \int_{-\pi}^{\pi} f(t) \cos mt dt$$

$$f(x) = \operatorname{sgn}(x) + \cos \pi x + \sin 5\pi x \text{ を} \\ f(x+2m) = f(x) \quad (-1 \leq x < 1, m \text{は任意の整数})$$

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$$\tilde{f}(0) = f(0), \tilde{f}(2) = f(0)$$

$$a_m = \int_{-1}^1 f(x) \cos(m\pi x) dx \\ = \int_{-1}^1 \operatorname{sgn}(x) \cos(m\pi x) + \cos \pi x \cos(m\pi x) + \sin 5\pi x \cos(m\pi x) dx \\ \begin{array}{ccccccccc} \text{奇} & \times & \text{偶} & & \text{偶} & \times & \text{奇} & \times & \text{偶} \\ & & & & & & & & \\ \text{奇} & & & & \text{偶} & & & & \text{奇} \end{array}$$

$$= \int_{-1}^1 \cos \pi x \cdot \cos(m\pi x) dx$$

$$\cos(\pi x + m\pi x) = \cos \pi x \cos m\pi x - \sin \pi x \sin m\pi x$$

$$\cos(\pi x - m\pi x) = " +$$

$$= \frac{1}{2} \int_{-1}^1 \cos 2m\pi x \cos(m+1)\pi x + \cos(m-1)\pi x dx$$

$$\cancel{\text{m+1 かつ m-1 かつ}} \\ m \neq 1 \text{ のとき } \frac{1}{2} \left[\frac{\sin(m+1)\pi x}{(m+1)\pi} + \frac{\sin(m-1)\pi x}{(m-1)\pi} \right]_{-1}^1 = 0$$

$m = 1 \text{ or } \pi$.

$$\int_{-1}^1 \cos^2 \pi x dx = \int_{-1}^1 \frac{1 + \cos 2\pi x}{2} dx = \left[\frac{x}{2} + \frac{\sin 2\pi x}{4\pi} \right]_{-1}^1 = \frac{1}{2} + \frac{1}{2} = 1$$

由 π 时 $a_1 = 1, a_0 = 0, a_2 = a_3 = \dots = a_n = \dots = 0$

$$b_m = \int_{-1}^1 f(x) \sin(m\pi x) dx = \int_{-1}^1 \underset{\substack{\text{奇} \\ \text{偶}}}{\operatorname{sgn}(x)} \underset{\substack{\text{奇} \\ \text{偶}}}{\sin(m\pi x)} x + \underset{\substack{\text{偶} \\ \text{奇}}}{\cos(m\pi x)} \underset{\substack{\text{偶} \\ \text{奇}}}{\sin(m\pi x)} + \underset{\substack{\text{偶} \\ \text{偶}}}{\sin(m\pi x)} \underset{\substack{\text{偶} \\ \text{偶}}}{\sin(m\pi x)} dx$$

$$\begin{aligned} &= 2 \int_0^1 \underset{\substack{\text{偶} \\ \text{偶}}}{\operatorname{sgn}(x)} \underset{\substack{\text{奇} \\ \text{奇}}}{\sin(m\pi x)} + \underset{\substack{\text{偶} \\ \text{偶}}}{\sin(m\pi x)} \underset{\substack{\text{奇} \\ \text{奇}}}{\sin(m\pi x)} dx \\ &= 2 \left[\frac{-\cos(m\pi x)}{m\pi} \right]_0^1 + 2 \int_0^1 \frac{1}{2} (\cos((m-5)\pi x) - \cos((m+5)\pi x)) dx \\ &= \frac{-2(-1)^m}{m\pi} + \left[\frac{\sin((m-5)\pi x)}{(m-5)\pi} - \frac{\sin((m+5)\pi x)}{(m+5)\pi} \right]_0^1. \end{aligned}$$

$$\begin{aligned} \cos m\pi \\ = (-1)^m \end{aligned}$$

$$= \frac{-2(-1)^m}{m\pi} + \frac{2}{m\pi} = \frac{2}{m\pi} (1 - (-1)^m)$$

$m = 5 \text{ or } \pi$.

$$= \frac{2}{5\pi} (1 - (-1)^5) + \int_{-1}^1 \sin^2 5\pi x dx$$

$$= \frac{4}{5\pi} + \int_{-1}^1 \frac{1 - \cos 10\pi x}{2} dx = \frac{4}{5\pi} + \left[\frac{x}{2} - \frac{\sin 10\pi x}{20\pi} \right]_{-1}^1 = \frac{4}{5\pi} + (\frac{1}{2} + \frac{1}{2})$$

$$\begin{aligned} \text{由 } \pi \text{ 时 } a_n = \begin{cases} 0 & (n \neq 1) \\ 1 & (n = 1) \end{cases} \quad b_n = \begin{cases} \frac{4}{5\pi} + 1 & (n \neq 5) \\ \frac{2}{5\pi} \{1 - (-1)^n\} & (n = 5) \end{cases} = \frac{4}{5\pi} + 1 \end{aligned}$$

4) $\hat{f}(t)$ のフーリエ級数に現れる全ての項に $t = \frac{1}{2}$, $t = 55$, $t = 100$
を代入した場合の極限値

$$A = \frac{a_0}{2} + \lim_{N \rightarrow \infty} \sum_{n=1}^N (a_n \cos \frac{1}{2} n\pi + b_n \sin \frac{1}{2} n\pi)$$

$$B = \frac{a_0}{2} + \lim_{N \rightarrow \infty} \sum_{n=1}^N (a_n \cos 55n\pi + b_n \sin 55n\pi)$$

$$C = \frac{a_0}{2} + \lim_{N \rightarrow \infty} \sum_{n=1}^N (a_n \cos 100n\pi + b_n \sin 100n\pi)$$

フーリエ級数の各点収束定理より、

$$|A - \hat{f}\left(\frac{1}{2}\right)| = 0$$

$$\begin{aligned} A = \hat{f}\left(\frac{1}{2}\right) &= f\left(\frac{1}{2}\right) = \operatorname{sgn}\left(\frac{1}{2}\right) + \cos \frac{\pi}{2} + \sin \frac{\pi}{2} \\ &= 1 + 1 = \underline{\underline{2}} \end{aligned}$$

$$|B - \hat{f}(55)| = 0$$

$$\begin{aligned} B = \hat{f}(55) &= \hat{f}(100 - 1 + 2 \cdot 28) = f(-1) = \operatorname{sgn}(-1) + \cos(-\pi) - \sin(-\pi) \\ &= -1 - 1 = \underline{\underline{-2}} \end{aligned}$$

$$|C - \hat{f}(100)| = 0$$

$$\begin{aligned} C = \hat{f}(100) &= \hat{f}(0 + 50 \cdot 2) - f(0) = \operatorname{sgn}(0) + \cos 0 + \sin 0 \\ &= 1 + 1 = \underline{\underline{2}} \end{aligned}$$

$$\underline{\underline{A = 2, B = -2, C = 2}}$$

H27-5

$$f(a, n) = \max \left\{ \sum_{k=i}^j a_k \mid 0 \leq i \leq j < n \right\}$$

$$a = (20, -24, 36, 3, -7, 27, -40, 37, -28, 6)$$

$$f(a, 10) = 36 + 3 + (-7) + 27 = 39$$

1) 関数 f.

```
int max(int x, int y) {
```

↳ x と y の最大値を返す。

```
int sum(int a[], int i, int j) {
```

↳ $a[i], a[i+1], \dots, a[j]$ の合計を返す。

```
int f(int a[], int n) {
```

```
    int s = a[0];
```

```
    for (int i=0; i<n; i++) {
```

```
        for (int j=A; j<n; j++) {
```

```
            s = max(s, sum(a, i, j));
```

```
}
```

```
    return s;
```

```
}
```

a)

A = i

b)

 ~~n^3~~

↳ sum 関数で $n -> O(n^3)$ である

2) 丈夫 int f(int a[], int n) {

```
    int s = a[0];
```

```
    for (int i=0; i<n; i++) {
```

```
        int t = 0;
```

```
        for (int j=B; j<n; j++) {
```

```
            t = t + BT;
```

```
            s = max(s, t);
```

```
}
```

```
return s;
```

i で開始地点を決定

j で i+j まで見てみて検証。

a)

B < ~~a[j]~~

b)

 n^2

c) 計算時間のオーダーが小さくなる理由

図5.1は $a[i] \cup a[j]$ の合計値を計算するために、

関数sumを用いていたが、

図5.2は $a[i] \cup a[j-1]$ の合計値に $a[j]$ を足していく手順である。

3) a_0, a_1, \dots, a_{n-1} における連続の和を。

a_{n-1} を含む場合と含まない場合で考える。

・含まない $\rightarrow f(a, n-1)$

・含む $\rightarrow g(a, n) = \max \left\{ \sum_{k=c}^{n-1} a_k \mid 0 \leq c < n \right\}$

$$f(a, n) = \max \{ f(a, n-1), g(a, n) \}$$

```
int g(int a[], int n) {
```

```
    int t = C;
```

```
    for (int k=1; k<n; k++)
```

```
        t = max (t + D, E);
```

```
    return t;
```

```
}
```

$a[n-1]$ を含めるとは決定している。

$$\boxed{C} = a[0]$$

$$\boxed{D} = a[k]$$

```
int f(int a[], int n) {
```

```
    int s = a[0];
```

```
    for (int k=1; k<n; k++)
```

```
        s = max (E, g(a, F));
```

```
    return s;
```

```
}
```

$$s = a[0]$$

$$s = \max (s, g(a, *))$$

k+1

b) n^2

4)

```
int f(int a[], int n) {
    int t = a[0], s = a[0];
    for (int k=1; k<n; k++) {
        t = max(t + a[k], a[k]);
        s = max(s, G);
    }
    G = t
}
```

b)

n

c)

図5.3は、for文の中で外側関数gを呼び出しているが、
図5.4は、1つのループでgを計算し、a[k]が含まれる場合の
最大値tと含まれない場合の最大値を行っている。