H31-1

1) 行列式

a) $\begin{vmatrix} 3 & -1 \\ 2 & 5 \end{vmatrix} = 15+2 = 17$

b) $\begin{vmatrix} 2 & -1 & 0 \\ 2 & 3 & -1 \\ 1 & -2 & -1 \end{vmatrix} = -(-1)\begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} + (-1)\begin{vmatrix} 2 & -1 \\ 2 & 3 \end{vmatrix}$

$= -4+1-6\,\cancel{+}2 = \cancel{-7}_{\#}$ $\quad \underline{-11}_{\#}$

c) $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & x & 2 \\ 1 & 1 & x^2 & 4 \\ 1 & -1 & x^3 & 8 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 1 & 1 \\ 0 & -1 & x & 2 \\ 2 & 1 & x^2 & 4 \\ 0 & -1 & x^3 & 8 \end{vmatrix}$

$= \cancel{2} \begin{vmatrix} 2 & 1 & 1 & 1 \\ 0 & -1 & x & 2 \\ 0 & 0 & x^2-1 & 3 \\ 0 & -1 & x^3 & 8 \end{vmatrix} = 2\begin{vmatrix} -1 & x & 2 \\ 0 & x^2-1 & 3 \\ -1 & x^3 & 8 \end{vmatrix}$

$= 2\{-(x^2-1)8 - 3x + 3x^3 + 2(x^2-1)\}$

$= 2\{-8x^2+8-3x+3x^3+2x^2-2\}$

$= 2\{3x^3-6x^2-3x+6)$

$= 6(x^3-2x^2-x+2)$

$= 6(x-1)(x^2-x-2)$

$= 6(x-1)(x+1)(x-2)$

2) 部分空間Wの基底が $(1,1,0,1)^T$, $(0,1,-1,0)^T$

$W^{\perp}$の基底

$\boldsymbol{x} \in W^{\perp}$ ~~とする~~. $\boldsymbol{y} \in W$とする.

$\boldsymbol{y} = s\begin{pmatrix}1\\1\\0\\1\end{pmatrix} + t\begin{pmatrix}0\\1\\-1\\0\end{pmatrix}$ $(s, t \in \mathbb{R})$ より. $\boldsymbol{x} = \begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}$ とし、$\boldsymbol{x}\cdot\boldsymbol{y} = 0$ なので.

~~$\boldsymbol{x}\cdot\boldsymbol{y} =$~~

$$\boldsymbol{x}\cdot\boldsymbol{y} = (x_1\ x_2\ x_3\ x_4)\begin{pmatrix}s\\s+t\\-t\\s\end{pmatrix} = sx_1 + (s+t)x_2 + (-t)x_3 + sx_4 = 0$$

$$\cancel{\begin{pmatrix}s & s+t & (-t) & s\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}}$$

基底 $\underbrace{(1,1,0,1)^T}_{\boldsymbol{p}_1}$ と $\underbrace{(0,1,-1,0)^T}_{\boldsymbol{p}_2}$ に対して垂直 とおくと

$\boldsymbol{x}\cdot\boldsymbol{p}_1 = \boldsymbol{x}\cdot\boldsymbol{p}_2 = 0$

$$\begin{cases} x_1 + x_2 + x_4 = 0 \\ x_2 - x_3 = 0 \end{cases} \qquad \underbrace{\begin{pmatrix}1&1&0&1\\0&1&-1&0\end{pmatrix}}_{A}\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix} = 0$$

$$A \to \begin{pmatrix}1&0&1&1\\0&1&-1&0\end{pmatrix}$$ よって $x_1 + x_3 + x_4 = 0$, $x_2 - x_3 = 0$ 　 $x_3 = s$, $x_4 = t$ とおくと

$$\boldsymbol{x} = s\begin{pmatrix}-1\\1\\1\\0\end{pmatrix} + t\begin{pmatrix}-1\\0\\0\\1\end{pmatrix}$$ よって

基底 $\left\langle \begin{pmatrix}-1\\1\\1\\0\end{pmatrix}, \begin{pmatrix}-1\\0\\0\\1\end{pmatrix} \right\rangle$ //

3)

a) $f(x) = \log(1+x)$

$f^{(1)}(x) = \dfrac{1}{1+x}$

b) $f^{(k)}(x)$

$f^{(2)}(x) = \left((1+x)^{-1}\right)' = \cancel{42}\ -1(x+1)^{-2}$

$f^{(3)}(x) = \left(-1(1+x)^{-2}\right)' = 2(1+x)^{-3}$

$f^{(4)}(x) \qquad = -6(1+x)^{-4}$

$\vdots$

$f^{(k)}(x) \qquad = (k-1)!(1+x)^{-k}\cdot(-1)^{k+1}$

$\qquad = (-1)^{k+1}(k-1)!\,(1+x)^{-k}$

c) $g(x) = \log(1-x^2)$ をマクローリン展開

$$\sum_{n=0}^{\infty} \frac{g^{(n)}(0)}{n!} x^n$$

$g^{(1)}(x) = \dfrac{-2x}{1-x^2} \Big|_{x=0} \quad 0$

$g^{(2)}(x) = \dfrac{-2(1-x^2)+2x\cdot(-2x)}{(1-x^2)^2} = \dfrac{-2+2x^2-4x^2}{(1-x^2)^2} = \dfrac{-2(1+x^2)}{(1-x^2)^2}$

$\dfrac{-2x}{1-x^2} = \dfrac{-2x}{(1+x)(1-x)} = \dfrac{\cancel{4}1}{1+x} + \dfrac{-1}{1-x}$

$g^{(1)}(x) = \dfrac{1}{1+x} - \dfrac{1}{1-x} = (1+x)^{-1} - (1-x)^{-1}$

$g^{(2)}(x) = -1(1+x)^{-2} + 1(1-x)^{-2}\cdot(-1) = -(1+x)^{-2} - (1-x)^{-2}$

$g^{(3)}(x) = 2(1+x)^{-3} + 2(1-x)^{-3}\cdot(-1) = 2(1+x)^{-3} - 2(1-x)^{-3}$

$g^{(4)}(x) = -6(1+x)^{-3} \ -6(1-x)^{-3}$

$$g^{(k)}(x) = (-1)^{k+1}(k-1)!\,(1+x)^{-k} - (k-1)!\,(1-x)^{-k} \quad (k \geq 1)$$

$$g^{(k)}(0) = (-1)^{k+1}(k-1)! - (k-1)!$$

kが奇数のとき $g^{(k)}(0) = 0$

偶数 $= -2(k-1)!$

$$\frac{g^{(k)}(0)}{k!} = \begin{cases} 0 & (\text{kが奇数}) \\ -\dfrac{2}{k} & (\text{kが偶数}) \end{cases} \quad (k \geq 1)$$

また $g^{(0)}(0) = \log(1) = 0$

よって $g(x) = \cancel{-1 - \frac{1}{2} - \frac{1}{3}}$

$$= -x^2 - \frac{1}{2}x^4 - \frac{1}{3}x^6 - \frac{1}{4}x^8$$

〈ポイント〉

$\log(1-x^2) = \log|(1+x)(1-x)| = \log(1+x) + \log(1-x)$

なので b)の答えを使うのが普通…

4) 0または1の値を取る確率変数X, Y,

$P_X(0) = p,\ P_X(1) = 1-p$ とする.

$$P_{Y|X}(y|x) = \begin{cases} 1-q & (x=y) \\ q & (x \neq y) \end{cases}$$

Yの値が与えられたときのXの条件付き確率 $P_{X|Y}(1|0)$

$$P_{X|Y}(1|0) = \frac{P_{X,Y}(1,0)}{P_Y(0)}$$ と書ける.

$$P_{Y|X}(0|1) = \frac{P_{X,Y}(1,0)}{P_X(1)} \Rightarrow P_{X,Y}(1,0) = P_X(1) \cdot P_{Y|X}(0|1) = (1-p) \cdot q$$

$$P_Y(0) = P_X(1) \cdot P_{Y|X}(0|1) + P_X(0) P_{X|Y}(0|0) = (1-p)q + p(1-q)$$ よって

$$P_{X|Y}(1|0) = \frac{(1-p)q}{(1-p)q + p(1-q)} = \frac{q - pq}{\cancel{1-2pq}\ q - pq + p - pq} = \frac{q - pq}{p - 2pq - q}$$

H31 - 2

1)

P	Q	R	$\neg P$	$\neg P \vee Q$	$P \wedge R$	$(\neg P \vee Q) \to (P \wedge R)$
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	F	F	T	T
T	F	F	F	F	F	T
F	T	T	T	T	F	F
F	T	F	T	T	F	F
F	F	T	T	T	F	F
F	F	F	T	T	F	F

2) PとQのNAND $P|Q$　　NANDの形になっている!!

$P \to Q \simeq \neg P \vee Q \simeq \neg\neg(\neg P \vee Q) \simeq \neg(P \wedge \neg Q)$

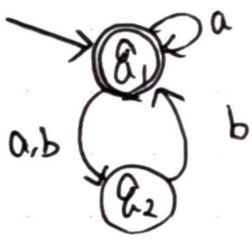
P	Q	$P\|Q$	$P\|P \simeq \neg P$	$P \to Q$	$Q\|Q$	$P \wedge Q$
T	T	F	F	T	F	T
T	F	T	F	F	T	F
F	T	T	T	T	F	F
F	F	T	T	T	T	F

P	Q	$P\|Q$	$P\|P$	$Q\|Q$	$Q\|Q\|P$	$P \to Q$
F	F	T	T	T	T	T
F	T	T	T	F	T	T
T	F	T	F	T	F	F
T	T	F	F	F	T	T

$Q|Q|P$　　　$\underline{P|(Q|Q)}$ //

3) (a) 文脈依存文法 (b) 正規文法 (c) ~~S→aSa~~ AS→b
(d) S→aSa (e) S→aA (f) チューリングマシン
(g) プッシュダウン・オートマトン (h) 有限オートマトン

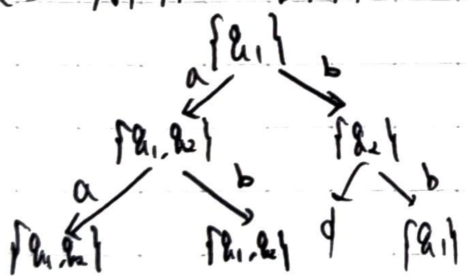
4) NFA $(Q, \Sigma, \delta, q_1, F)$ $Q=\{q_1, q_2\}$, $\Sigma=\{a, b\}$ $F=\{q_1\}$

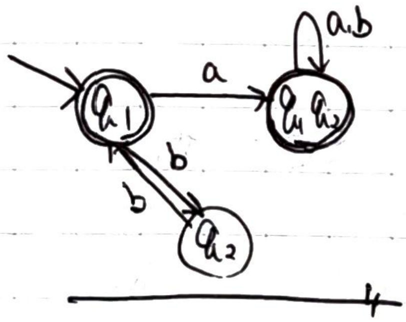


a) ア) ab … ○　エ) aaab … ○　キ) aabaa … ○
イ) abbb … ○　オ) aaabba … ○
ウ) bbba … ×　カ) bbabb … ○

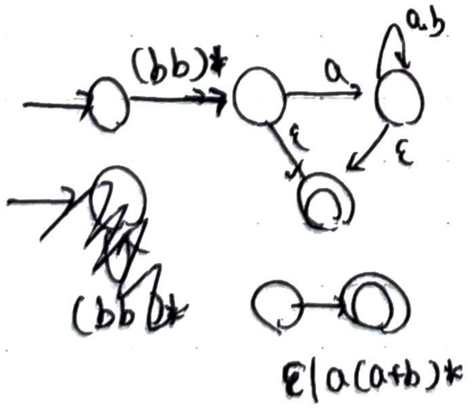
b) NFAが受理する正規表現
(後で)
$(bb)^* + ((bb)^* a(a+b)^*)$

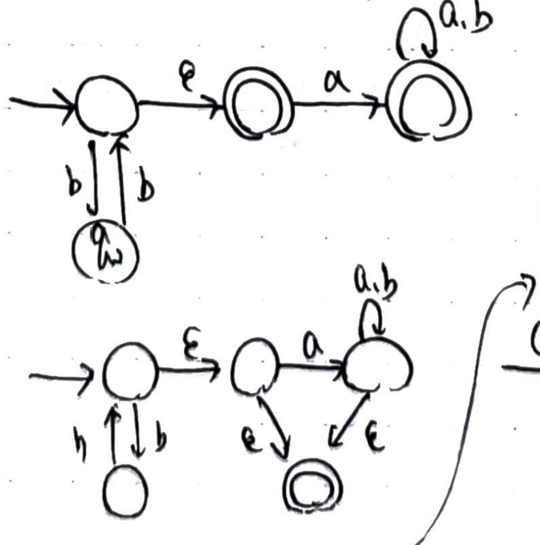
c) NFA → DFA





b)





$(bb)^*(\varepsilon + a(a+b)^*)$

$(bb)^* + (bb)^* a(a+b)^*$

H31-3

1) N個の品物集合 G
体積の小さい順に並べた列 $(g_1, g_2, \cdots, g_N)$
Wの範囲内でvalueの総和を最大となる $S \subseteq G$ を見つける.

G_0を空集合 $\emptyset$ とし、$1 \leq i \leq N$ $G_i = G_{i-1} \cap \{g_i\}$
品物の集合G_iと容積W_jのナップザックに対する.
ナップザック問題解 $S_{i,j}$ の価値の総和. $T_{i,j}$

$G = \{g_1, g_2, g_3, g_4\}$

	volume	value
g_1	10	5
g_2	15	6
g_3	20	7
g_4	30	8

a) G_1, G_2, G_3, G_4の要素.
$G_1 = \{g_1\}$, $G_2 = \{g_1, g_2\}$, $G_3 = \{g_1, g_2, g_3\}$, $G_4 = \{g_1, g_2, g_3, g_4\}$

b) W_j $(j=0, \cdots, 8)$ の各値を示せ.

~~W0= W4=~~

$W_0 = 0$
$W_1 = 10$
$W_2 = 15$
$W_3 = 20$
$W_4 = \cancel{30}\,25$
$W_5 = 30$
$W_6 = 35$
$W_7 = 40$
$W_8 = 45$

c) 表3.2

$T_{i,j}$	W_0	$10 = W_1$	$15 = W_2$	$20 = W_3$	$25 = W_4$	$30 = W_5$	$35 = W_6$	$40 = W_7$	$45 = W_8$
G_0	0	0	0	0	0	0	0	0	0
G_1	0	5	5	5	5	5	5	5	5
G_2	0	5	6	6	11	11	11	11	11
G_3	0	5	6	7	11	③	④	⑤	⑥
G_4	0	5	6	7	11				

$$T_{i,j} = \begin{cases} T_{i-1,j} & (W_j < volume(g_i) \text{ かつ } 1 \leq j) \\ \max\{T_{i-1,j},\ T_{i-1,h} + volume(g_i)\} \end{cases}$$

→ W_jを超えている場合，そもそも入らない。

ただし，$h = \max\{k \mid W_k \leq (W_j - volume(g_i))\}$

③ $T_{3,5}$　$G_3 = (20, 7)$　$W_5 = 30$

$h = \max\{k \mid W_k \leq (30 - 20) = 10\}$　$h = 1$

$\therefore$ 11 or $T_{2,1} + 7 = 5 + 7 = 12$　　$\underline{12}_{//}$

④ $T_{3,6}$　$G_3 = (20, 7)$　$W_6 = 35$

$h = \max\{k \mid W_k \mid (35 - 20) = 15\}$　$h = 2$

11 or $T_{2,2} + 7 = 6 + 7 = 13$　　$\underline{13}_{//}$

⑤ $T_{3,7}$　$W_7 = 40$

$h = \max\{k \mid W_k \leq (40 - 20) = 20\}$　$h = 3$

11 or $T_{2,3} + 7 = 6 + 7 = 13$　　$\underline{13}_{//}$

⑥ $T_{3,8}$　$W_8 = 45$

$h = \max\{k \mid W_k \leq 25\}$　$h = 4$

11 or $T_{2,4} + 7$　　$\underline{18}_{//}$

d) G_4 と W_5, W_6, W_7, W_8 $g_4 = (30, 8)$

$T_{4,5}$ $W_5 = 30$

$h = \max\{k \mid w_k \le 0\}$ $h = 0$

12 or 8 $\underline{12}$

よって $S_{4,5} = \underline{\{g_1, g_3\}}$, $T_{4,5} = \underline{12}$

$T_{4,6}$ $W_6 = 35$

$h = \max\{k \mid w_k \le 5\}$ $h = 0$

13 or 8

$S_{4,6} = \underline{\{g_2, g_3\}}$ $T_{4,6} = \underline{13}$

$T_{4,7}$ $W_7 = 40$

$h = \max\{k \mid w_k \le 10\}$ $h = 1$

13 or $5 + 8 = 13$

$S_{4,7} = \underline{\{g_2, g_3\}}$, または $\{g_1, g_4\}$

$T_{4,7} = \underline{13}$

$T_{4,8}$ $W_8 = 45$

$h = \max\{k \mid w_k \le 15\}$ $h = 2$

18 or 6+8

~~干~~ $S_{4,8} = \underline{\{g_1, g_2, g_3\}}$, $T_{4,8} = \cancel{14}\ \underline{18}$

2)

$Ix \to x$

$kxy \to x$

$Sxyz \to xz(yz)$

~~$x \to x'$ ならば~~

$x \to x'$ ならば $xy \to x'y$

$y \to y'$ ならば $xy \to xy'$

$SIIx \to Ix(Ix) \to x(Ix) \to xx$

$S(k(SI))kxy \to k(SI)x(kx)y \to SI(kx)y \to Iy(kxy)$

$\to y(kxy) \to yx$

a) $Skkx \to kx(kx) \to x$

$Ix \to x$

b) T, F

$T = k$

$F = kI$

(i)

$Txy = kxy = x$

(ii)

$Fxy = kIxy = Iy = y$

c)

$$\vee = SI(KT)$$
$$\wedge = SS(K(KF))$$
$$\neg = S(SI(KF))(KT)$$

(iii)

$$\vee(\neg T)T$$
$$= SI(KT)(S(SI(KF))(KT)T)T$$
$$\to I(S(SI(KF))(KT)T)(KT(S(SI(KF))(KT)T))T$$
$$\to S(SI(KF))(KT)T(KT(S(SI(KF))(KT)T))T$$
$$\cancel{\to KT}$$
$$\to SI(KF)T(KTT)(KT(S(SI(KF))(KT)T))T$$
$$\to IT(KFT)(KTT)(KT(S(SI(KF))(KT)T))T$$
$$\to T(KFT)(KTT)(KT(S(SI(KF))(KT)T))T$$
$$\cancel{\to (KFT)}$$
$$\to KFT(KT(S(SI(KF))(KT)T))T$$
$$\to F(KT(S(SI(KF))(KT)T))T$$
$$\to KI(KT(S(SI(KF))(KT)T))T$$
$$\to IT$$
$$\to T$$

$$\cancel{F(T(KT)T)T}$$

$$\to F(KT(S(SI(KF))(KT)T))T$$

$$\to FTT$$

(iv) V(¬T)F　(iii)と同じ過程。

→F

→FTF ?

多分黒で合ってる。

(v) ∧(¬F)T

→ SS(K(KF))(S(SI(KF))(KT)F)T

→ SS(K(KF))(¬F)T

~~→ K(KF)(¬F)~~

→ S(¬F)(K(KF)(¬F))T

~~→ S(S~~

→ (¬F)T((K(KF)(¬F))T)

→ (S(SI(KF))(KT)F)T((K(KF)(¬F))T)

→ S(SI(KF))(KT)FT((K(KF)(¬F))T)

→ (SI(KF))F(KTF)T((K(KF)(¬F))T)

→ SI(KF)F 〃

→ IF(KFF) 〃

→ F(KFF) 〃

→ KI(KFF) 〃

→ I(KTF)T((K(KF)(¬F))T)

→ KTFT((K(KF)(¬F))T)

→ TT((K(KF)(¬F))T)

→ KT(〃)

→ T

?

T

(vi) ∧(¬F)F

⇒ SS(K(KF))$\underbrace{\text{(S(SI(KF))(KT)F)}}_{X}$P

⇒ SX(K(KF)X)P ⇒ XP($\underbrace{\text{(K(KF)X) P}}_{Y}$) ⇒ S(SI(KF))(KT)FPY

⇒ SI(KF)F(KTF)PY ⇒ IF(KFF)(KTF)PY ⇒ F(KFF)(KTF)PY

⇒ FFTPY ⇒ KIFTPY ⇒ ITPY ⇒ TPY ⇒ KPY ⇒ P

おって F