

H27-1

$$1) T = \begin{pmatrix} 0.5 & 0.4 & 0.3 \\ 0.4 & 0.5 & 0.3 \\ 0.1 & 0.1 & 0.4 \end{pmatrix}$$

$$a) T^2 = \frac{1}{100} \begin{pmatrix} 5 & 4 & 3 \\ 4 & 5 & 3 \\ 1 & 1 & 4 \end{pmatrix} \begin{pmatrix} 5 & 4 & 3 \\ 4 & 5 & 3 \\ 1 & 1 & 4 \end{pmatrix} = \frac{1}{100} \begin{pmatrix} 44 & 43 & 39 \\ 43 & 44 & 39 \\ 13 & 13 & 22 \end{pmatrix} = \begin{pmatrix} 0.44 & 0.43 & 0.39 \\ 0.43 & 0.44 & 0.39 \\ 0.13 & 0.13 & 0.22 \end{pmatrix}$$

$$25+16+3, \quad 20+25+3 \quad 15+12+12$$

b) Aのすべての固有値と、それに対応する固有ベクトル。

$$A = \frac{1}{10} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$|A - \lambda E| = \frac{1}{10} \begin{vmatrix} 2-10\lambda & 1 \\ 1 & 2-10\lambda \end{vmatrix} = \frac{1}{10} \left\{ (2-10\lambda)^2 - 1 \right\} \\ = \frac{1}{10} \left\{ 100\lambda^2 - 40\lambda + 3 \right\}$$

$$100\lambda^2 - 40\lambda + 3 = 0$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$10\lambda = 3, 1$$

$$\lambda = \frac{1}{10}, \frac{3}{10} //$$

i) $\lambda = \frac{1}{10}$ について。

$(A - \frac{1}{10}E)x = 0$ を解く。

$$\frac{1}{10} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} x = 0$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

$$x + y = 0$$

$$x = t \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (t \in \mathbb{R})$$

$$\text{よって固有ベクトル } x = t \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (t \in \mathbb{R})$$

(ii) $\lambda = \frac{3}{10}$ について.

$(A - \frac{3}{10}E)x = 0$ を解く.

$$\frac{1}{10} \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} x = 0 \quad \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix}$$

$$-x + y = 0 \quad \text{よって } x = t \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

よって固有ベクトル $x = t \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ($t \in \mathbb{R}$)

c) 正の整数 N について、 $A^N b$ および $\sum_{n=0}^N A^n b$ を求めよ.

$$P = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \quad \text{よって } P^{-1} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$P^{-1}AP = \begin{pmatrix} \frac{1}{10} & 0 \\ 0 & \frac{3}{10} \end{pmatrix}$$

$$A^N = P \begin{pmatrix} (\frac{1}{10})^N & 0 \\ 0 & (\frac{3}{10})^N \end{pmatrix} P^{-1} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} (\frac{1}{10})^N & 0 \\ 0 & (\frac{3}{10})^N \end{pmatrix} P^{-1}$$

$$= \frac{1}{2} \begin{pmatrix} (\frac{1}{10})^N & (\frac{3}{10})^N \\ -(\frac{1}{10})^N & (\frac{3}{10})^N \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} (\frac{1}{10})^N + (\frac{3}{10})^N & -(\frac{1}{10})^N + (\frac{3}{10})^N \\ -(\frac{1}{10})^N + (\frac{3}{10})^N & (\frac{1}{10})^N + (\frac{3}{10})^N \end{pmatrix}$$

~~$$A^N b = \frac{1}{2 \cdot 10^N} \begin{pmatrix} 3^N - 1 & 3^N - 1 \\ 3^N - 1 & 3^N + 1 \end{pmatrix} \begin{pmatrix} \frac{3}{10} \\ 0 \end{pmatrix} = \frac{3}{2 \cdot 10^{N+1}} \begin{pmatrix} 3^{N+1} & 3^N - 1 \\ 3^N - 1 & 3^N + 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$~~

$$= \frac{3}{2 \cdot 10^{N+1}} \begin{pmatrix} 2 \cdot 3^N \\ 2 \cdot 3^N \end{pmatrix} = \frac{3^{N+1}}{10^{N+1}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \left(\frac{3}{10} \right)^{N+1} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\sum_{n=0}^N A^n \mathbf{1}_b = \left(\frac{3}{10}\right) \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \left(\frac{3}{10}\right)^2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \left(\frac{3}{10}\right)^3 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \dots + \left(\frac{3}{10}\right)^{M+1} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{10} + \left(\frac{3}{10}\right)^2 + \dots + \left(\frac{3}{10}\right)^{M+1} \\ 11 \end{pmatrix}$$

$$\begin{aligned} \frac{3}{10} S &= \frac{3}{10} + \left(\frac{3}{10}\right)^2 + \dots + \left(\frac{3}{10}\right)^{M+1} \\ \frac{3}{10} S &= \left(\frac{3}{10}\right)^2 + \dots + \left(\frac{3}{10}\right)^{M+1} \end{aligned}$$

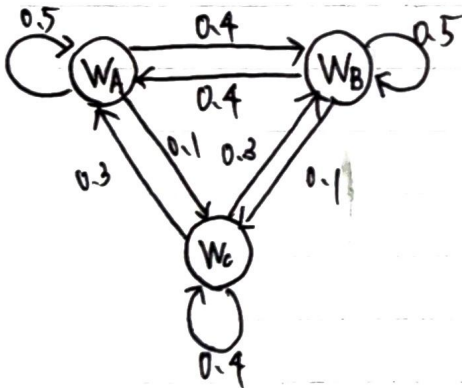
$$\frac{7}{10} S = \frac{3}{10} - \left(\frac{3}{10}\right)^{M+2}$$

$$S = \frac{10}{7} \left\{ \frac{3}{10} - \left(\frac{3}{10}\right)^{M+2} \right\}$$

$$= \frac{3}{7} \left\{ 1 - \left(\frac{3}{10}\right)^{M+1} \right\}$$

$$\sum_{n=0}^N A^n \mathbf{1}_b = \frac{3}{7} \left\{ 1 - \left(\frac{3}{10}\right)^{M+1} \right\} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

2)



a) WA から WA にリンクを 2 回クリックした後に再び WA にいる確率。

$$\begin{aligned} A \rightarrow A \rightarrow A &: \frac{5}{10} \times \frac{5}{10} = \frac{25}{100} \\ \rightarrow B \rightarrow A &: \frac{4}{10} \times \frac{4}{10} = \frac{16}{100} \\ \rightarrow C \rightarrow A &: \frac{1}{10} \times \frac{3}{10} = \frac{3}{100} \end{aligned}$$

$$\frac{44}{100}$$

$$0.44$$

b) N 回クリックした後に WA および WB にいる確率を P_N, Q_N とする

P_{N-1}, Q_{N-1} を用いて P_N, Q_N を表せ。

$$\begin{aligned} P_N &= \frac{5}{10} P_{N-1} + \frac{4}{10} Q_{N-1} + \frac{3}{10} (1 - P_{N-1} - Q_{N-1}) \\ &= \frac{3}{10} + \frac{2}{10} P_{N-1} + \frac{1}{10} Q_{N-1} \end{aligned}$$

$$\begin{aligned} Q_N &= \frac{4}{10} P_{N-1} + \frac{5}{10} Q_{N-1} + \frac{3}{10} (1 - P_{N-1} - Q_{N-1}) \\ &= \frac{3}{10} + \frac{1}{10} P_{N-1} + \frac{2}{10} Q_{N-1} \end{aligned}$$

c) $P_0 = Q_0 = 0.3$

$$R_N = P_N + Q_N = \frac{9}{10}P_{N-1} + \frac{9}{10}Q_{N-1} = \frac{6}{10} + \frac{3}{10}P_{N-1} + \frac{3}{10}Q_{N-1}$$

$$= \frac{3}{10}R_{N-1} + \frac{6}{10}$$

~~$$10R_N = 3R_{N-1} + 6 \quad 10x^2 - 3x - 6 = 0$$~~

~~$$x^2 - \frac{3}{10}x - \frac{6}{10} = 0$$~~

~~$$\frac{A^2}{B} = \frac{9}{10} \quad -2 \times 3$$~~

$$u = \frac{3}{10}u + \frac{6}{10}$$

$$\frac{7}{10}u = \frac{6}{10} \Rightarrow u = \frac{6}{7}$$

$$\frac{6}{7} (R_N - \frac{6}{7}) = \frac{3}{10} (R_{N-1} - \frac{6}{7})$$

$$R_0 = \frac{6}{10}, \quad R_0 - \frac{6}{7} = \frac{6}{10} - \frac{6}{7} = \frac{12-60}{70} = -\frac{18}{70}$$

$$R_N - \frac{6}{7} = -\frac{18}{70} \left(\frac{3}{10}\right)^{N-1}$$

$$R_N = -\frac{18}{70} \left(\frac{3}{10}\right)^{N-1} + \frac{6}{7} = -\frac{9}{35} \left(\frac{3}{10}\right)^{N-1} + \frac{6}{7}$$

$$S_N = P_N - Q_N = P_{N-1} - Q_{N-1} = S_{N-1}$$

$$S_0 = 0$$

$$S_N = 0$$

d, T.

$$\frac{1}{2}(R_N + S_N) = \frac{1}{2}(P_N + Q_N + P_N - Q_N) = P_N$$

$$P_N = \frac{1}{2} \left\{ -\frac{9}{35} \left(\frac{3}{10}\right)^N + \frac{6}{7} \right\} = -\frac{9}{70} \left(\frac{3}{10}\right)^N + \frac{3}{7} = \frac{3}{7} \left\{ 1 - \left(\frac{3}{10}\right)^{N+1} \right\}$$

$$\frac{1}{2}(R_N - S_N) = \frac{1}{2}(P_N + Q_N - P_N + Q_N) = Q_N$$

$$Q_N = \frac{1}{2} \left\{ -\frac{9}{35} \left(\frac{3}{10}\right)^N + \frac{6}{7} \right\} = -\frac{9}{70} \left(\frac{3}{10}\right)^N + \frac{3}{7} = \frac{3}{7} \left\{ 1 - \left(\frac{3}{10}\right)^{N+1} \right\}$$

$$d) \lim_{N \rightarrow \infty} P_N = \lim_{N \rightarrow \infty} \frac{3}{7} \left\{ 1 - \left(\frac{3}{10}\right)^{N+1} \right\} = \frac{3}{7}$$

$$\lim_{N \rightarrow \infty} Q_N = \frac{3}{7}$$

H27-2

$$f(x) = \operatorname{sgn}(x) + \cos \pi x + \sin 5\pi x \quad (-1 \leq x < 1)$$

$$\operatorname{sgn}(x) = \begin{cases} -1 & (-1 \leq x < 0) \\ 1 & (0 \leq x < 1) \end{cases}$$

1) 2つの実数 α と β を用いて関数 $g(x) = \alpha \cos \pi x + \beta \sin 5\pi x$

$$E(\alpha, \beta) = \int_{-1}^1 |f(x) - g(x)|^2 dx$$

α と β の 2 次関数で表せ.

$$E(\alpha, \beta) = \int_{-1}^1 |f(x) - g(x)|^2 dx = \int_{-1}^1 (f(x) - g(x))^2 dx$$

$$\left(\begin{array}{l} (a+b+c)^2 \\ a^2+b^2+c^2+2ab+2bc+2ca \end{array} \right)$$

$$= \int_{-1}^1 (\operatorname{sgn}(x) + (1-\alpha)\cos \pi x + (1-\beta)\sin 5\pi x)^2 dx$$

$$= \int_{-1}^1 \left[\operatorname{sgn}(x)^2 + (1-\alpha)^2 \cos^2 \pi x + (1-\beta)^2 \sin^2 5\pi x + 2\operatorname{sgn}(x)(1-\alpha)\cos \pi x + 2\operatorname{sgn}(x)(1-\beta)\sin 5\pi x + 2(1-\alpha)(1-\beta)\cos \pi x \sin 5\pi x \right] dx$$

$$\int_{-1}^1 |\operatorname{sgn}(x)|^2 dx = \int_{-1}^1 1 dx = 2$$

$$\int_{-1}^1 (1-\alpha)^2 \cos^2 \pi x dx = (1-\alpha)^2 \int_{-1}^1 \cos^2 \pi x dx = \frac{(1-\alpha)^2}{\pi} \int_{-\pi}^{\pi} \cos^2 t dt$$

$$\begin{aligned} \pi x &= t & \cos 2t &= \cos^2 t - \sin^2 t \\ & & &= 2\cos^2 t - 1 \end{aligned} \quad = \frac{(1-\alpha)^2}{\pi} \int_{-\pi}^{\pi} \frac{1+\cos 2t}{2} dt$$

$$= \frac{(1-\alpha)^2}{\pi} \left[\frac{1}{2}t + \frac{1}{4}\sin 2t \right]_{-\pi}^{\pi} = \frac{(1-\alpha)^2}{\pi} \left(\frac{\pi}{2} + \frac{\pi}{2} \right) = \frac{(1-\alpha)^2}{\pi}$$

$$\int_{-1}^1 \underbrace{(1-\beta)^2 \sin^2 5\pi x}_{\text{奇} \times \text{奇} = \text{奇}} dx = 0$$

$$\begin{aligned}
 & 2 \int_{-1}^1 \operatorname{sgn}(x) (1-\alpha) \cos \pi x + \operatorname{sgn}(x) (1-\beta) \sin 5\pi x \, dx \\
 &= 2 \left\{ \int_{-1}^0 -(1-\alpha) \cos \pi x - (1-\beta) \sin 5\pi x \, dx + \int_0^1 (1-\alpha) \cos \pi x + (1-\beta) \sin 5\pi x \, dx \right\} \\
 &= 2 \left\{ \left[-(1-\alpha) \frac{\sin \pi x}{\pi} + (1-\beta) \frac{\cos 5\pi x}{5\pi} \right]_{-1}^0 + \left[\frac{(1-\alpha)}{\pi} \sin \pi x + \frac{(1-\beta)}{5\pi} \cos 5\pi x \right]_0^1 \right\} \\
 &= 2 \left\{ \frac{(1-\beta)}{5\pi} + \frac{(1-\beta)}{5\pi} + \frac{(1-\beta)}{5\pi} + \frac{(1-\beta)}{5\pi} \right\} = \frac{8}{5\pi} (1-\beta)
 \end{aligned}$$

$$\int_{-1}^1 2(1-\alpha)(1-\beta) \underbrace{\cos \pi x \sin 5\pi x}_{\substack{\text{偶} \times \text{奇} \\ \text{奇}}} \, dx = 0$$

$$\frac{8}{5\pi} (1-\beta)$$

$$\text{よ、} \tau \text{ (与式)} = 2 + (1-\alpha)^2 + \frac{8}{5\pi} (1-\beta)$$

$$\int_{-1}^1 (1-\beta)^2 \underbrace{\sin^2 5\pi x}_{\text{奇} \times \text{奇} = \text{偶}} \, dx = \frac{(1-\beta)^2}{5\pi} \int_{-5\pi}^{5\pi} \sin^2 t \, dt = \frac{(1-\beta)^2}{5\pi} \int_{-5\pi}^{5\pi} \frac{1 - \cos 2t}{2} \, dt$$

$$\begin{aligned}
 5\pi x = t & \quad \begin{cases} x=1 \rightarrow 5\pi \\ x=-1 \rightarrow -5\pi \end{cases} \\
 5\pi dx = dt & \\
 \cos 2t = \cos^2 t - \sin^2 t & = 1 - 2\sin^2 t \\
 & = \frac{(1-\beta)^2}{5\pi} \left[\frac{1}{2} t - \frac{\sin 2t}{2} \right]_{-5\pi}^{5\pi} \\
 & = \frac{(1-\beta)^2}{5\pi} \left[\frac{5}{2}\pi + \frac{5}{2}\pi \right] = \frac{(1-\beta)^2}{\pi}
 \end{aligned}$$

$$\text{よ、} \tau \text{ (与式)} = 2 + (1-\alpha)^2 + (1-\beta)^2 + \frac{8}{5\pi} (1-\beta)$$

2) $E(\alpha, \beta)$ が最小となる α, β の値 $E(\alpha, \beta)$ の値.

$$1-\alpha = X, \quad 1-\beta = Y \quad \alpha, \beta \in [0, 1]$$

$$E(\alpha, \beta) = 2 + X^2 + Y^2 + \frac{8}{5\pi} Y = 2 + X^2 + \left(Y + \frac{4}{5\pi} \right)^2 - \frac{16}{25\pi^2}$$

$$\text{よ、} X=0, Y = -\frac{4}{5\pi} \text{ のとき最小.}$$

$$\alpha = 1, \quad \beta = \frac{4}{5}, \quad E(\alpha, \beta) = \frac{16}{25\pi^2} \quad 2 - \frac{16}{25\pi^2} \quad 2 - \frac{16}{25\pi^2}$$

3) $\tilde{f}(t) \in (-\infty < t < \infty)$ と $\tilde{f}(x+2m) = f(x) \quad (-1 \leq x < 1)$
 m は任意の整数.

$\tilde{f}(t) \in \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\pi t + b_n \sin n\pi t)$ の形にフーリエ級数展開.

~~$a_m = \int_{-\pi}^{\pi} \tilde{f}(t) \cos m\pi t$~~

$f(x) = \operatorname{sgn}(x) + \cos \pi x + \sin 5\pi x$ と

$\tilde{f}(x+m) = f(x) \quad (-1 \leq x < 1, m \text{ は任意の整数})$

→ 周期 2

$\tilde{f}(0) = f(0), \tilde{f}(2) = f(0)$

~~a_m~~ $a_m = \int_{-1}^1 f(x) \cos(m\pi x) dx$

$= \int_{-1}^1 \underbrace{\operatorname{sgn}(x)}_{\text{奇}} \underbrace{\cos(m\pi x)}_{\text{偶}} + \underbrace{\cos \pi x}_{\text{偶}} \underbrace{\cos(m\pi x)}_{\text{偶}} + \underbrace{\sin 5\pi x}_{\text{奇}} \underbrace{\cos(m\pi x)}_{\text{偶}} dx$

$= \int_{-1}^1 \cos \pi x \cdot \cos(m\pi x) dx$

$\cos(\pi x + m\pi x) = \cos \pi x \cos m\pi x - \sin \pi x \sin m\pi x$

$\cos(\pi x - m\pi x) = \cos \pi x \cos m\pi x + \sin \pi x \sin m\pi x$

$= \frac{1}{2} \int_{-1}^1 \cancel{\cos \pi x} \cos(m+1)\pi x + \cos(m-1)\pi x dx$

~~これは $m \neq 1$ のとき~~

$m \neq 1$ のとき $\frac{1}{2} \left[\frac{\sin(m+1)\pi x}{(m+1)\pi} + \frac{\sin(m-1)\pi x}{(m-1)\pi} \right]_{-1}^1 = 0$

$m=1$ のとき.

$$\int_{-1}^1 \cos^2 \pi x \, dx = \int_{-1}^1 \frac{1 + \cos 2\pi x}{2} \, dx = \left[\frac{x}{2} + \frac{\sin 2\pi x}{4\pi} \right]_{-1}^1 = \frac{1}{2} + \frac{1}{2} = 1$$

よって ~~また~~ $a_1 = 1, a_0 = 0, a_2 = a_3 = \dots = a_n = \dots = 0$

$$b_m = \int_{-1}^1 f(x) \sin(m\pi x) \, dx = \int_{-1}^1 \underbrace{\operatorname{sgn}(x)}_{\substack{\text{奇} \\ \text{偶}}} \underbrace{\sin(m\pi x)}_{\substack{\text{奇} \\ \text{偶}}} + \underbrace{\cos \pi x}_{\text{偶}} \underbrace{\sin(m\pi x)}_{\substack{\text{奇} \\ \text{偶}}} + \underbrace{\sin 5\pi x}_{\text{偶}} \underbrace{\sin(m\pi x)}_{\substack{\text{奇} \\ \text{偶}}} \, dx$$

$$= 2 \int_0^1 \operatorname{sgn}(x) \sin m\pi x + \sin 5\pi x \sin m\pi x \, dx \quad m\pi - 5\pi$$

$$= 2 \left[\frac{-\cos m\pi x}{m\pi} \right]_0^1 + 2 \int_0^1 \frac{1}{2} (\cos(m-5)\pi x - \cos(m+5)\pi x) \, dx \quad (m \neq 5 \text{ のとき})$$

$$= \frac{-2 \cdot (-1)^m}{m\pi} + \left[\frac{\sin(m-5)\pi x}{(m-5)\pi} - \frac{\sin(m+5)\pi x}{(m+5)\pi} \right]_0^1$$

$$\cos m\pi = (-1)^m$$

$$= \frac{-2(-1)^m}{m\pi} + \frac{2(-1)^m}{m\pi}$$

$$= \frac{-2(-1)^m}{m\pi} + \frac{2}{m\pi} = \frac{2}{m\pi} (1 - (-1)^m)$$

$m=5$ のとき.

$$\frac{2}{5\pi} (1 - (-1)^5) + \int_{-1}^1 \sin^2 5\pi x \, dx$$

$$= \frac{4}{5\pi} + \int_{-1}^1 \frac{1 - \cos 10\pi x}{2} \, dx = \frac{4}{5\pi} + \left[\frac{x}{2} - \frac{\sin 10\pi x}{20\pi} \right]_{-1}^1 = \frac{4}{5\pi} + \left(\frac{1}{2} + \frac{1}{2} \right)$$

$$\text{よって } a_n = \begin{cases} 0 & (n \neq 1) \\ 1 & (n = 1) \end{cases} \quad b_n = \begin{cases} \frac{4}{5\pi} + 1 & (n \neq 5) \\ \frac{2}{n\pi} (1 - (-1)^n) & (n = 5) \end{cases} = \frac{4}{5\pi} + 1$$

4) $\hat{f}(t)$ のフーリエ級数に現れる全ての項に $t = \frac{1}{2}$, $t = 55$, $t = 100$ を代入した場合の極限值

$$A = \frac{a_0}{2} + \lim_{N \rightarrow \infty} \sum_{n=1}^N (a_n \cos \frac{1}{2} n \pi + b_n \sin \frac{1}{2} n \pi)$$

$$B = \frac{a_0}{2} + \lim_{N \rightarrow \infty} \sum_{n=1}^N (a_n \cos 55 n \pi + b_n \sin 55 n \pi)$$

$$C = \frac{a_0}{2} + \lim_{N \rightarrow \infty} \sum_{n=1}^N (a_n \cos 100 n \pi + b_n \sin 100 n \pi)$$

フーリエ級数の各点収束定理から、

$$|A - \hat{f}(\frac{1}{2})| = 0$$

$$\begin{aligned} A = \hat{f}(\frac{1}{2}) &= f(\frac{1}{2}) = \operatorname{sgn}(\frac{1}{2}) + \cos \frac{\pi}{2} + \sin \frac{\pi}{2} \\ &= 1 + 1 = \underline{2} \end{aligned}$$

$$|B - \hat{f}(55)| = 0$$

$$\begin{aligned} B = \hat{f}(55) &= \hat{f}(\frac{1}{2} - 1 + 2 \cdot 28) = f(-1) = \operatorname{sgn}(-1) + \cos \pi + \sin(-\pi) \\ &= -1 - 1 = \underline{-2} \end{aligned}$$

$$|C - \hat{f}(100)| = 0$$

$$\begin{aligned} C = \hat{f}(100) &= \hat{f}(0 + 50 \cdot 2) = f(0) = \operatorname{sgn}(0) + \cos 0 + \sin 0 \\ &= 1 + 1 = \underline{2} \end{aligned}$$

$$\underline{A = 2, B = -2, C = 2}$$

H27-5

$$f(a, n) = \max \left\{ \sum_{k=i}^j a_k \mid 0 \leq i \leq j < n \right\}$$

$$a = (20, -24, 36, 3, -7, 27, -40, 37, -28, 6)$$

$$f(a, 10) = 36 + 3 + (-7) + 27 = 39$$

1) 関数 f.

```
int max(int x, int y) {
```

↳ xとyの最大値を返す.

```
int sum(int a[], int i, int j) {
```

↳ $a[i], a[i+1], \dots, a[j]$ の合計を返す.

```
int f(int a[], int n) {
```

```
    int s = a[0];
```

```
    for (int i = 0; i < n; i++) {
```

```
        for (int j = A; j < n; j++) {
```

```
            s = max(s, sum(a, i, j));
```

```
        }
```

```
    }
```

```
    return s;
```

```
}
```

a)

A = i

b)

 ~~n^3~~

↳ sum関数でループしているため...

2) 工夫 int f(int a[], int n) {

```
    int s = a[0];
```

```
    for (int i = 0; i < n; i++) {
```

```
        int t = 0;
```

```
        for (int j = A; j < n; j++) {
```

```
            t = t + B;
```

```
            s = max(s, t);
```

```
        }
```

```
    }
```

```
    return s;
```

```
}
```

→ iで開始地点を決定
jでi+jまで足してみよう検証.

a)

B = ~~$a[j]$~~

b)

 n^2

c) 計算時間のオーダーが小さくなる理由

図5.1は $a[i]$ と $a[j]$ の合計値を計算するために、

関数 sum を用いていたが、

図5.2は $a[i]$ と $a[j-1]$ の合計値に $a[j]$ を足すことで求めているため、

3) a_0, a_1, \dots, a_{n-1} における連続の和を、

a_n を含む場合と含まない場合で考える。

・含まない $\dots f(a, n-1)$

・含む $\dots g(a, n) = \max \left\{ \sum_{k=i}^{n-1} a_k \mid 0 \leq i < n \right\}$

$f(a, n) = \max \{ f(a, n-1), g(a, n) \}$

```
int g(int a[], int n) {
    int t = [C];
    for (int k = 1; k < n; k++)
        t = max(t + [D], [E]);
    return t;
}
```

$a[n-1]$ を含むことは決定している。

$[C] = a[0]$

$[D] = a[k]$

```
int f(int a[], int n) {
    int s = a[0];
    for (int k = 1; k < n; k++)
        s = max([E], g(a, [F]));
    return s;
}
```

$s = a[0]$

$s = \max(s, g(a, k+1))$

$k+1$

b) n^2

4)

```
int f(int a[], int n) {  
    int t = 0, s = a[0];  
           a[0];
```

```
    for (int k = 1; k < n; k++) {  
        t = max(t + a[k], a[k]); a)
```

```
        s = max(s, G);
```

G = t

```
    }
```

```
}
```

b)

n

c)

図5.3は、for文の中で関数gを呼び出しているが、

図5.4は、1つのループでgを計算せず、a[k]が含まれる場合の最大値tと含まない場合の最大値を比べている。