

**Department of Electrical Engineering**  
**EEE3094 Control Engineering HELICOPTER CONTROL**  
**LABORATORY 1 REPORT**



BY: TLNNTH001

# INTRODUCTION

This lab is based on a helicopter model. Given a helicopter model with the commanded blade angle of attack as input, we had to determine the altitude of the helicopter as the output of the system model. Manual methods and system identification (simulation on Simulink) were used and compared to verify the results.

We used simulation software in the lab to display the vertical flight of the helicopter at relative input voltages. The data collected was further used during the simulation phase using Matlab and Simulink (System Identification Toolbox) to determine the step response.

## MODEL DEVELOPMENT

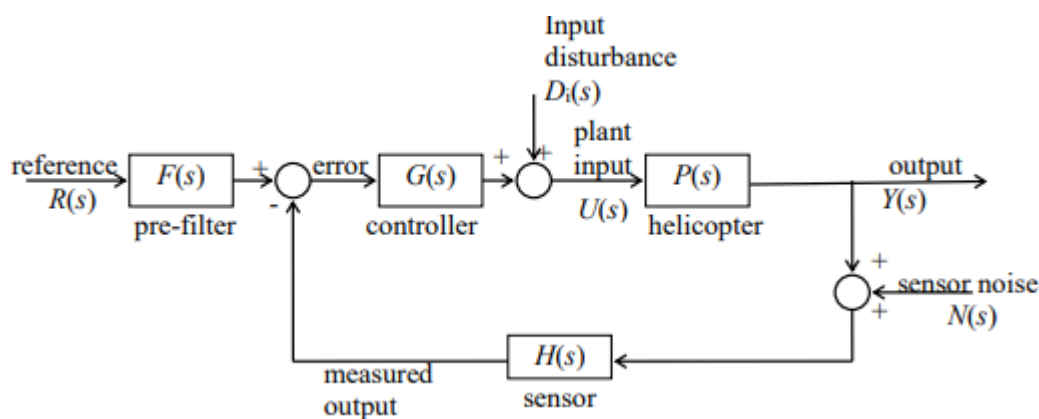


Figure 1: Helicopter feedback system components

The input to the plant is a voltage and the desired output is the height that the helicopter travels (vertical distance). An input of a step function is used as the transfer function characteristics are easily found thus the components of the transfer functions involved in the helicopter design are straightforwardly determined. It is evident that the vertical height of the helicopter is unstable while the speed is stable when a step voltage is applied to the installation.

The distance travelled for a period of time is equal to the integral of the velocity for that period. The output speed is stable; therefore, the helicopter system can be modeled by relating the input voltage to the output speed and integrating the altitude with the exit speed.

$$\text{Given that } G(s) = \frac{A}{s(\tau s + 1)}$$

Whereby A: is the open loop gain

And  $\tau$  is the time constant

The Physics (equations of motion) involved in the design of the helicopter:

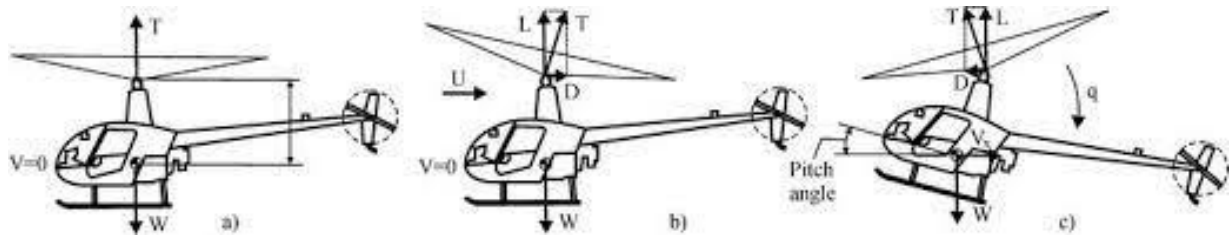


Figure 2: forces acting on the helicopter

The figure above shows the forces acting on the helicopter

Where:  $W$  is the force of gravity acting on the helicopter,  $T$  is the thrust and  $U$  is the drag (friction from wind).

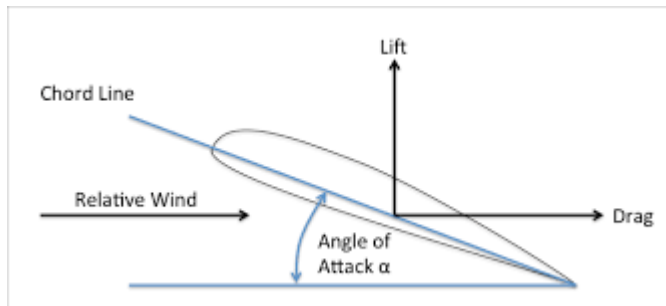


Figure 3: angle of attack of the helicopter

The figure above shows the angle of attack of the helicopter clearly.

The forces are not calculated but rather determined using system identification.

## SYSTEM IDENTIFICATION

### Method 1: Manual approximation (eyeballing)

With the manual approach method, the reaction of the system to various test signals (i.e, a step input) is measured. The model of the system is built using low-order “building blocks”.

The plant to be built is given as:  $G(s) = \frac{A}{s(\tau s + 1)}$

Using the transfer function  $T(s) = \frac{Y(s)}{U(s)}$

Where  $Y(s)$  is the output voltage and  $U(s)$  is the input voltage. The input voltage is varied to determine the reaction of the system.

When the input voltage,  $V_{in} = 5V$  the output voltage,  $V_{out} = 10 V$ . This was determined from the lab. Therefore  $T(s) = \frac{Y(s)}{U(s)} = \frac{10}{5} = 2$ . Using a single op-amp stage to determine the parameters we have:

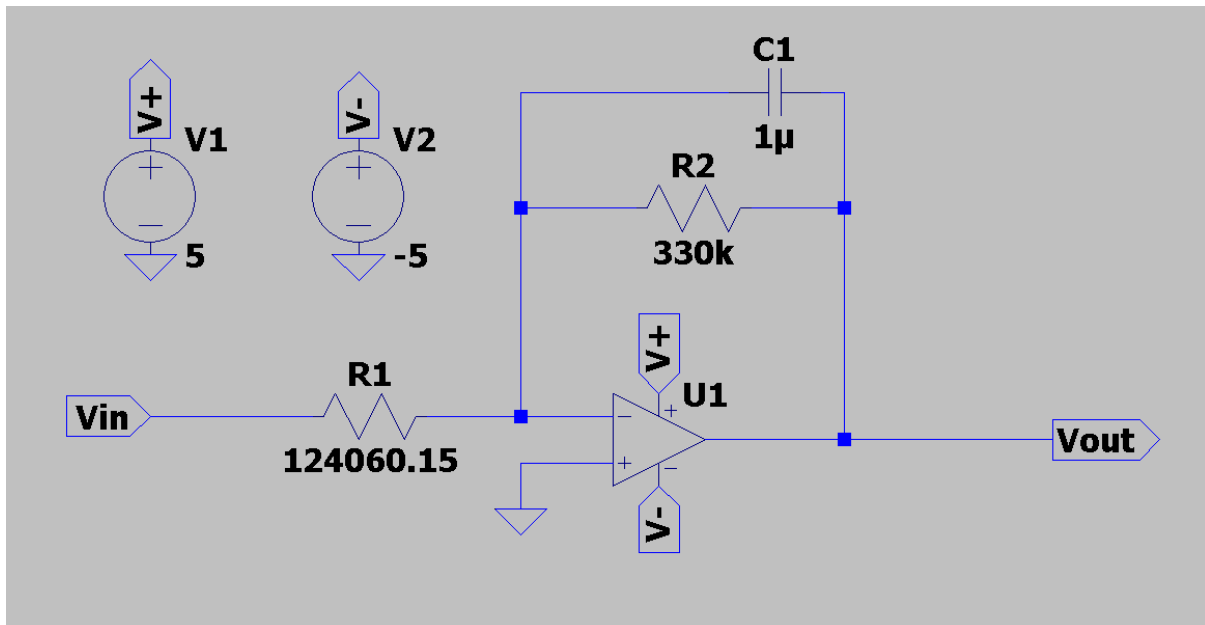


Figure 4:LTSpice low order circuit example

$$T(s) = \frac{Y(s)}{U(s)} = -\frac{R2}{R1} \frac{1}{R2 * Cs + 1}$$

To get a gain of 2: let  $Cs = 1 \mu F$  and  $R2 = 330K\Omega$  therefore  $R1$  is calculated as  $R1 = 124060.15 \Omega$ .

In this method the parameters of the low order building blocks are varied until the input step signal has the same response as the desired system. Since method 1 involves varying the parameters it is mostly based on trial and error. Trial and error experiments and modifications cannot be carried out in the real system, as this would pose a danger to the designers and the people in the vicinity. Thus, a mathematical approach (modeling) is sounder.

## Method 2: Step Response



Using a step function as an input,  $U(s) = \frac{B}{s}$ . The output is therefore:  $Y(s) = U(s)G(s) = \frac{B}{s} * \frac{A}{s(\tau s + 1)}$ .

Where: B = Amplitude of the step function; A = is the open loop gain, and  $\tau$  = the time constant. This method was mainly implemented during the lab, where the variables were determined.

The helicopter needs 2.5V input voltage,  $V_{in}$  to hover. The input voltage is increased to 5V (using a potentiometer) when the engine temperature was hot enough to start the flight of the helicopter. The data of the helicopter flight was collected and shown in the appendix. The helicopter was left to reach its maximum velocity (steady state). This data is used to determine the model parameters.

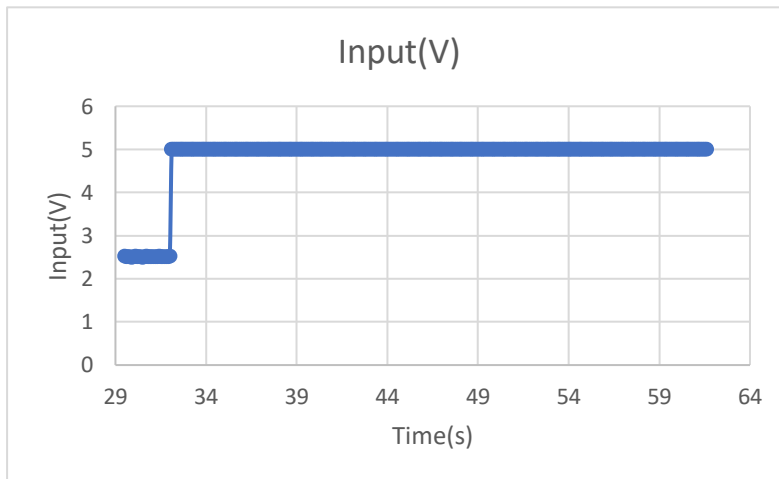


Figure 5: input step response

The figure on the left is the signal input step response. Note not all data values are shown in the plot. Therefore, we have that:  $B = 5V - 2.5V = 2.5V$ .

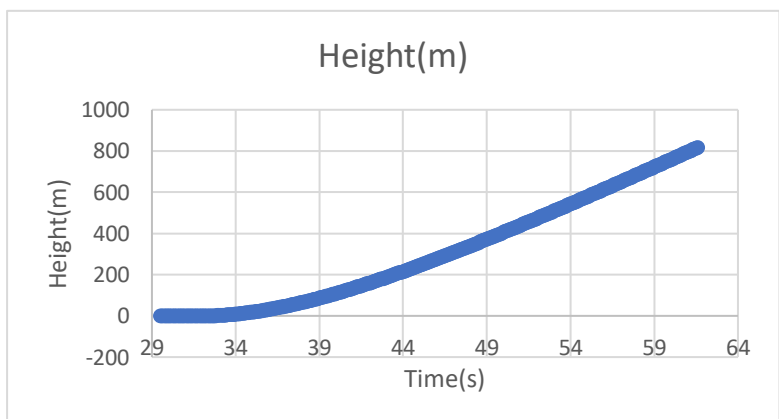


Figure 6: height reached by the helicopter

The figure on the left shows the height reached by the helicopter versus the time.

The velocity of the helicopter was calculated using  $v = \frac{dm}{dt}$ .

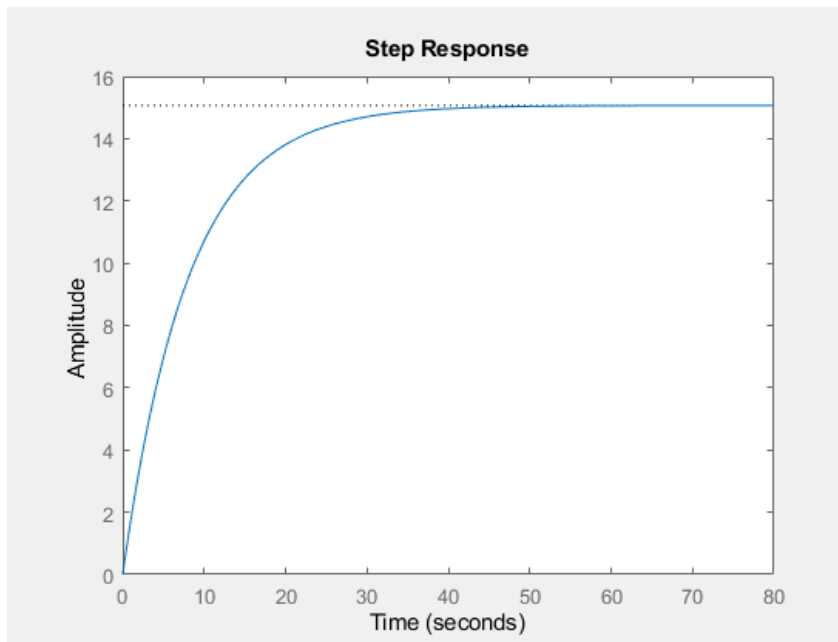


Figure 7: step response of the system

The step response of the system is shown above. We found that  $y(t) = AB \left(1 - e^{-\frac{t}{\tau}}\right)$ . This is the output of the system, velocity in m/s.

At steady state the maximum velocity is recorded as  $v = 37.67 \text{ m/s}$ . Therefore we have that  $A * B = 37.67$ . To determine the value of A,  $A = \frac{v_{max}}{B} = \frac{37.67}{2.5} = 15.11637$ . ✓ 2

To find the time constant, the value of 63.2% of the maximum velocity was calculated:  $0.632 * AB = 23.807 \text{ m/s}$ . The time at which this velocity is reached is recorded as 50.6 s. To find the Time constant, the time at which the step impulse was applied is subtracted from the time the velocity reaches 63.2% of the maximum velocity:  $\tau = 50.6 - 42.54 = 8.06 \text{ s}$ .

The sensor value is determined using:  $s = \frac{\text{output voltage}}{\text{height}} = \frac{v}{m}$ . The sensor value is recorded as  $s = 0.56$ . Using the parameter verification software in the lab:  $A = 15.068$  resulted in 100% accuracy.  $T = 8.06$  resulted in 94% accuracy.  $s = 0.56$  resulted in 100% accuracy.

SIMULATIONS: ✓ 1

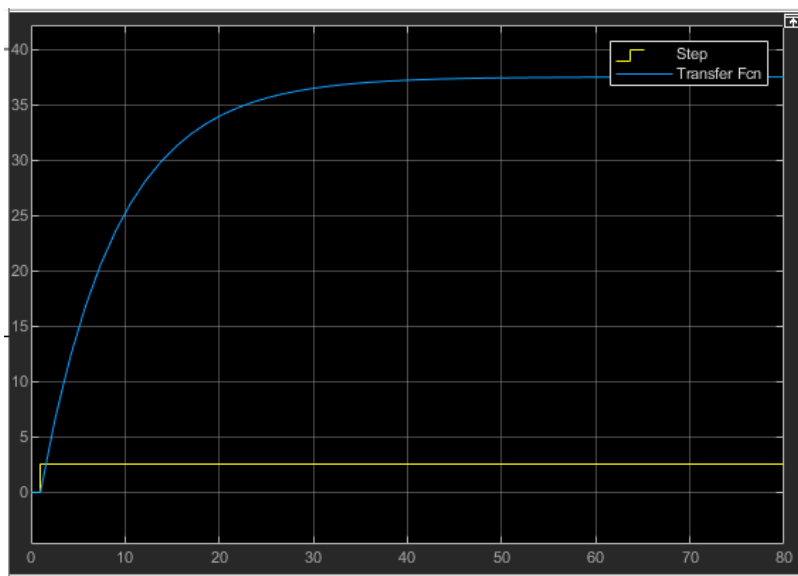
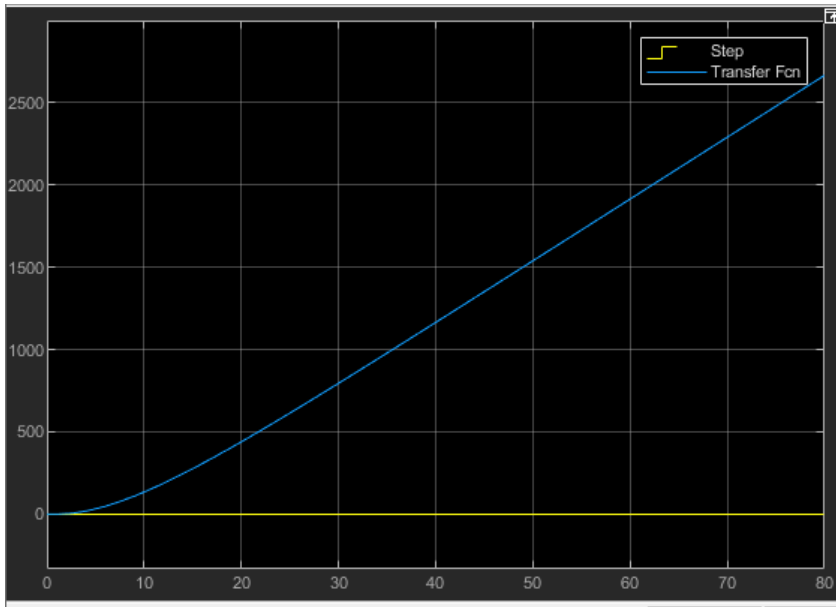


Figure 8: Simulink transfer function for velocity

Using a first order transfer function,  $G(s) = \frac{A}{(\tau s + 1)}$  and the data collected in the lab. We have that  $G(s) = \frac{15.068}{8.06s + 1} = \frac{1.869}{s + 0.124}$ . The input to the system is a step function,  $U(s) = \frac{2.492}{s}$ .

The output is recorded:



Using a second order transfer

$$function, G(s) = \frac{A}{s(\tau s + 1)}$$

$$rewritten as G(s) = \frac{\frac{A}{\tau}}{s^2 + \frac{1}{\tau}s} =$$

$$G(s) = \frac{1.869}{s^2 + 0.124s}. \text{ The output is recorded below:}$$

Figure 9: Simulink transfer function for height

#### Method 4: System Identification Toolbox ✓ 2

This method involves using the system identification toolbox on Simulink to determine the transfer functions. The data from the lab is loaded and used for the implementation of this method. The input to the transfer function is the input voltage (step).

The first transfer function desired is the one used to determine the velocity of the helicopter:

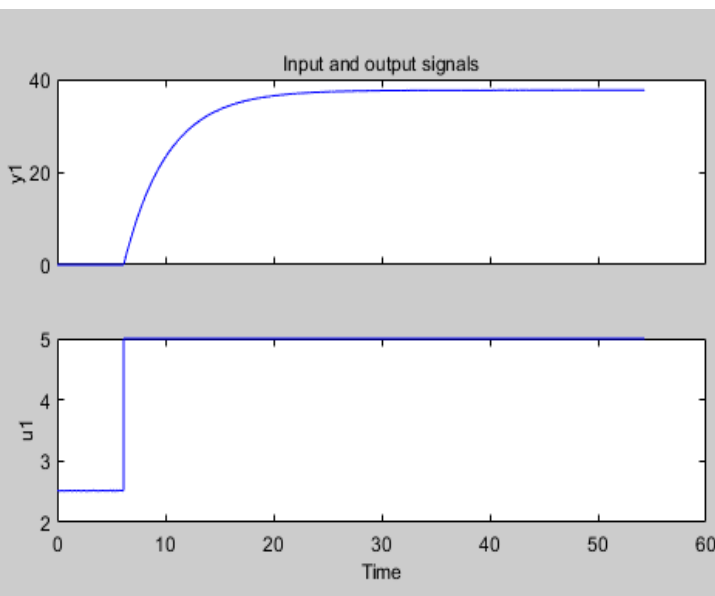


Figure 11: Velocity output function

```
From input "u1" to output "y1":
    1.224
    -----
    s + 0.1596
Name: tfl
Continuous-time identified transfer function.

Parameterization:
    Number of poles: 1    Number of zeros: 0
    Number of free coefficients: 2
```

Figure 10: Transfer function used to determine velocity

The second transfer function desired is the one used to determine the height of the helicopter:

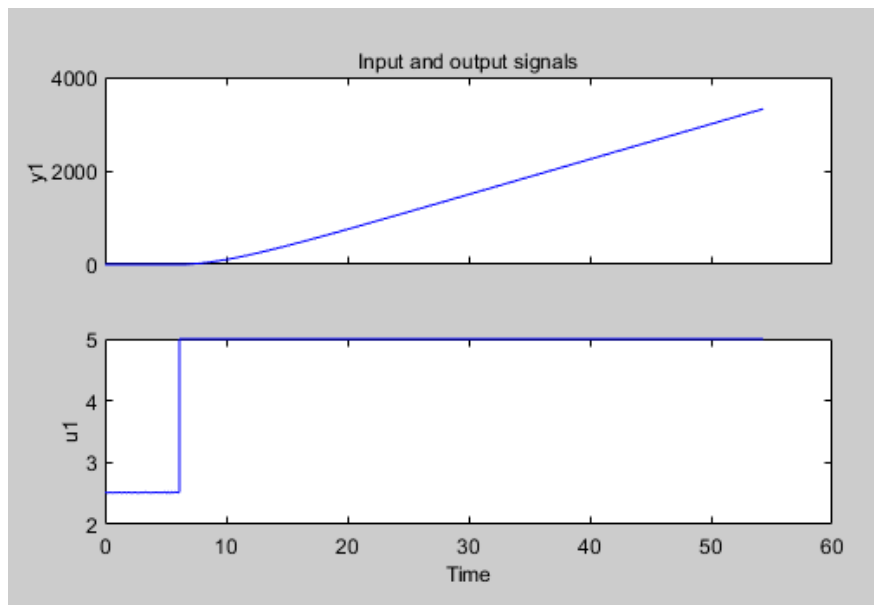


Figure 14: Displacement(height) output

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From input "u1" to output "y1":
      1.235
-----
s^2 + 0.1178 s + 0.0001305
Name: tf7
Continuous-time identified transfer function.

Parameterization:
  Number of poles: 2   Number of zeros: 0
  Number of free coefficients: 3

```

Figure 13: Transfer function for height

## CONCLUSION:



In method 2 the values for A, T and the sensor value were found. The results concluded that  $A = 15.068$ ,  $T = 8.06s$  and  $s = 0.56$ . Using the parameter verification software in the lab:  $A = 15.068$  resulted in 100% accuracy.  $T = 8.06$  resulted in 94% accuracy.  $s = 0.56$  resulted in 100% accuracy. These results were precise which allowed modelling in Simulink.

In order to determine the velocity of the helicopter a first order transfer function was used. The function parameters were determined to be:  $G(s) = \frac{15.068}{8.06s+1} = \frac{1.869}{s+0.124}$ , The results were modelled in Simulink.

In order to determine the height travelled by the helicopter a second order transfer function was used. The function parameters were determined to be:  $G(s) = \frac{\frac{A}{s}}{s^2 + \frac{1}{\tau}s} = G(s) = \frac{1.869}{s^2 + 0.124s}$ , The results were modelled in Simulink.

In method 4 the relative transfer functions were determined using the system identification toolbox in Simulink.



The first order transfer function used to show the response of the helicopter velocity is:

$$G(s) = \frac{1.224}{s+0.1596}. \text{ The transfer function was recorded in figure 11.}$$

The second order transfer function used to show the response of the helicopter height is:

$$G(s) = \frac{1.235}{s^2+0.1178s+0.0001305}. \text{ The transfer function was recorded in figure 14.}$$

The transfer functions obtained in method 2 and 4 are not identical. There is a margin of error involved in using hand methods. The results agree with each other however the design of a helicopter needs to be accurate as the cost of error is too high.

✓ 3

## APPENDIX

| Time(s) | Input(v) | Output(v) | Output(m) | speed | s |
|---------|----------|-----------|-----------|-------|---|
| 31,2    | 2,515    | 0         | 0         | 0     |   |
| 31,3    | 2,515    | 0         | 0         | 0     |   |
| 31,4    | 2,524    | 0         | 0         | 0     |   |
| 31,5    | 2,52     | 0         | 0         | 0     |   |
| 31,6    | 2,515    | 0         | 0         | 0     |   |
| 31,7    | 2,52     | 0         | 0         | 0     |   |
| 31,8    | 2,51     | 0         | 0         | 0     |   |
| 31,9    | 2,515    | 0         | 0         | 0     |   |
| 32      | 2,524    | 0         | 0,001     | 0,01  |   |
| 32,1    | 5        | 0,001     | 0,002     | 0,01  |   |
| 32,2    | 5        | 0,027     | 0,049     | 0,47  |   |
| 32,3    | 5        | 0,08      | 0,142     | 0,93  |   |
| 32,4    | 5        | 0,157     | 0,28      | 1,38  |   |

...

|      |   |       |        |       |            |
|------|---|-------|--------|-------|------------|
| 34,6 | 5 | 7,682 | 13,718 | 10,04 |            |
| 34,7 | 5 | 8,264 | 14,757 | 10,39 |            |
| 34,8 | 5 | 8,864 | 15,829 | 10,72 |            |
| 34,9 | 5 | 9,483 | 16,935 | 11,06 | 0,55996457 |
| 35   | 5 | 10    | 18,073 | 11,38 |            |
| 35,1 | 5 | 10    | 19,244 | 11,71 |            |
| 35,2 | 5 | 10    | 20,446 | 12,02 |            |
| 35,3 | 5 | 10    | 21,68  | 12,34 |            |

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|      |   |    |         |       |  |
|------|---|----|---------|-------|--|
| 41,9 | 5 | 10 | 156,85  | 26,49 |  |
| 42   | 5 | 10 | 159,514 | 26,64 |  |
| 42,1 | 5 | 10 | 162,19  | 26,76 |  |
| 42,2 | 5 | 10 | 164,881 | 26,91 |  |
| 42,3 | 5 | 10 | 167,584 | 27,03 |  |
| 42,4 | 5 | 10 | 170,301 | 27,17 |  |

|      |   |    |         |       |
|------|---|----|---------|-------|
| 42,5 | 5 | 10 | 173,031 | 27,3  |
| 42,6 | 5 | 10 | 175,773 | 27,42 |
| 42,7 | 5 | 10 | 178,528 | 27,55 |

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|       |   |    |          |       |
|-------|---|----|----------|-------|
| 127,3 | 5 | 10 | 3284,293 | 37,67 |
| 127,4 | 5 | 10 | 3288,06  | 37,67 |
| 127,5 | 5 | 10 | 3291,827 | 37,67 |
| 127,6 | 5 | 10 | 3295,594 | 37,67 |
| 127,7 | 5 | 10 | 3299,361 | 37,67 |
| 127,8 | 5 | 10 | 3303,128 | 37,67 |
| 127,9 | 5 | 10 | 3306,895 | 37,67 |
| 128   | 5 | 10 | 3310,662 | 37,67 |
| 128,1 | 5 | 10 | 3314,429 | 37,67 |
| 128,2 | 5 | 10 | 3318,196 | 37,67 |

## References: ✓ 2

[1] Hatim Mala. Tutorial: Estimating a transfer function model from random input using MATLAB. (Mar. 23, 2017). Accessed: Sept 29, 2021. [Online Video]. Available: [https://www.youtube.com/watch?app=desktop&v=gFZNNe1qsR8&feature=youtu.be&ab\\_channel=HatimMala](https://www.youtube.com/watch?app=desktop&v=gFZNNe1qsR8&feature=youtu.be&ab_channel=HatimMala)

[2] APMonitor.com. Response of First Order Systems in MATLAB. (Oct. 12, 2015). Accessed: Sept 29, 2021. [Online Video]. Available: <https://www.youtube.com/watch?v=BAinE-hy7o0>.