PID Controller Design using Bode Diagram

In reference to ***8-3 DESIGN OF PID CONTROLLERS WITH FREQUENCY-RESPONSE APPROACH,*** **Control Engineering, Ogata, 5th Ed**.

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Goal

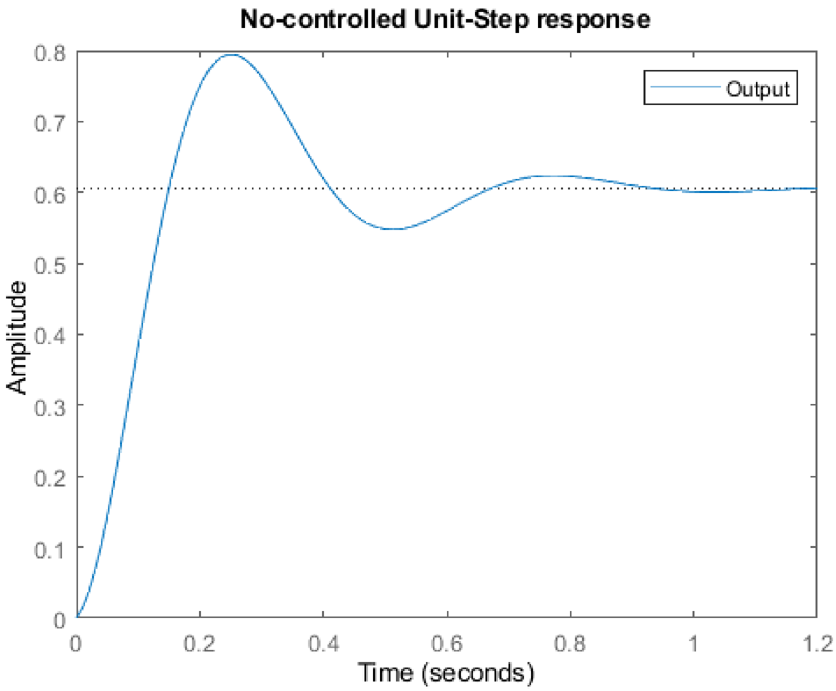
- Compare the unit-step and ramp-step response curves of a PID-Controlled system to the non-controlled outputs.
- Design a PID Controller using a frequency response (Bode Diagram) approach. The design requirements for the controller are:
- Desired Margin Phase, at least: ***Mphase = 60*** °
- Steady state error for a ramp-step input: ***ess = 0.1***

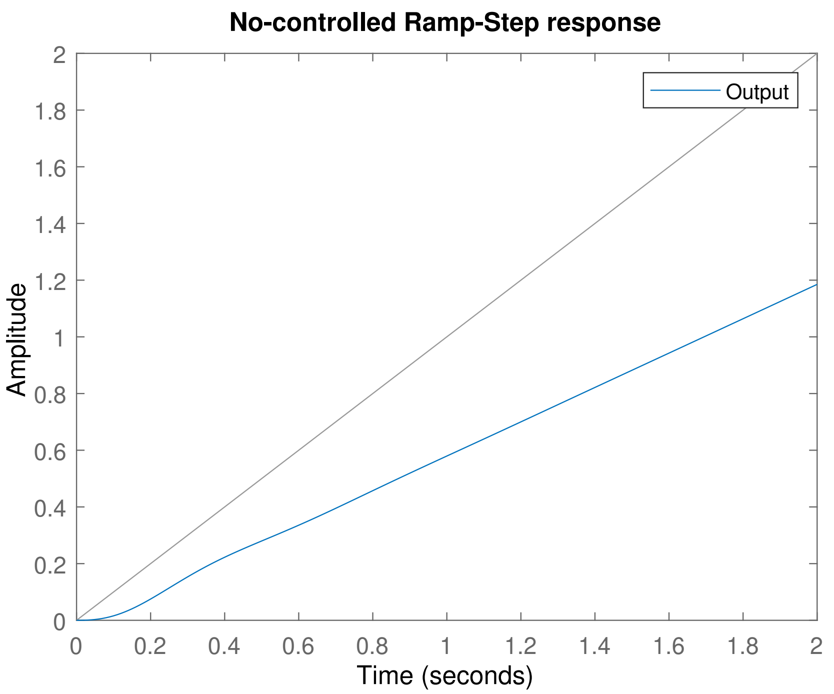
System

The plant to be controlled is:

$$G_p(s) = \frac{s+100}{s^2+8s+65}$$

System outputs to Unit and Ramp step inputs:





PID controller to be implemented is:

$$G_c(s) = \frac{K(as+1)(bs+1)}{s}$$

Design requirements:

Steady state error constant for the OPEN-Loop system:

$$K_v = \lim_{s\to 0}(s * G_c * G_p) = \frac{1}{ess}$$
$$K_v = \lim_{s\to 0}(s * \frac{K(as+1)(bs+1)}{s} * \frac{s+100}{s^2+8s+65}) = \frac{1}{ess}$$
$$K_v = \frac{100K}{65} = \frac{1}{0.1} = 10$$
$$K = 6.5$$

So:

- At this point, the requirement of $\boldsymbol{ess = 0.1}$ has been achieved.

I - Controller

The first controller is just a I - Controller.

$$G_{tot} = G_c * G_p = \frac{K}{s} * \frac{s+100}{s^2+8s+65}$$
$$G_{tot} = G_c * G_p = \frac{6.5}{s} * \frac{s+100}{s^2+8s+65}$$

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Warning: The closed-loop system is unstable.
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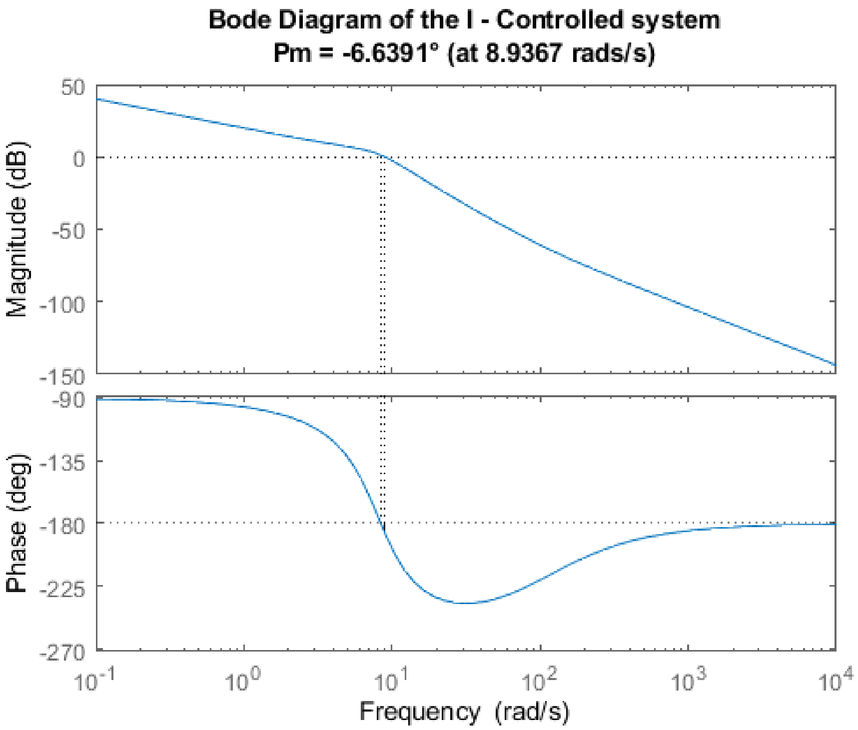


Acording to the Bode Diagram, the current margin phase is Pm = - 6.64° at w = 8.94 rad/s.

To get Pm = 60°, at the same frequency (or close) the two "*zeros*" of the originally designed controller will be used.

Adding a zero

The first zero comes from *(as+1)*, adding a 60° phase when w = 8.94 rad/s.

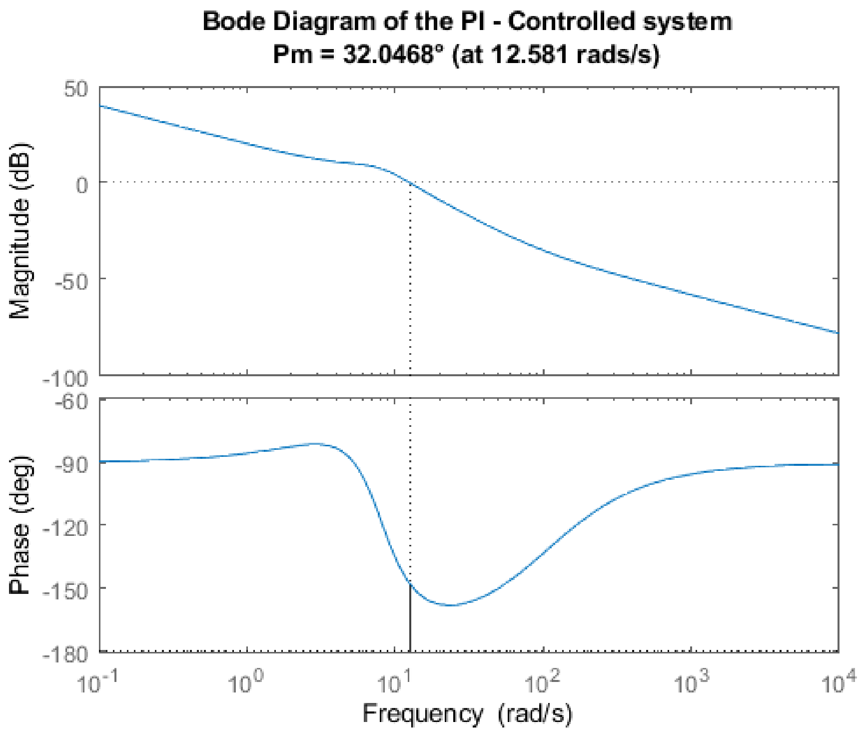
(Why 60°? from here on, it's just a mater of trial and error)

$$as + 1 = 1 + ja\omega$$
$$\arctan(\frac{a\omega}{1}) = 60°$$
$$a = 0.1938$$

PI - Controller

The second controller is a PI - Controller.

$$G_{tot} = G_c * G_p = \frac{K(as+1)}{s} * \frac{s+100}{s^2+8s+65}$$
$$G_{tot} = G_c * G_p = \frac{6.5(0.1938s+1)}{s} * \frac{s+100}{s^2+8s+65}$$



Now, according to the Bode Diagram, the current margin phase is Pm = 32° at w = 12.6 rad/s.

To important facts to point here:

- A phase of 60° was added, but it only contributed to actual 38° (from -6° to 32°).
- The margin cross frequency went from 8.94 rad/s to 12.6 rad/s

Again, to get a margin phase of at least Pm = 60°, at the same frequency (or close) another "*zero*" of the originally designed controller will be used.

Adding another zero

The second zero comes from *(bs+1)*, adding a 30° phase when w = 12.96 rad/s.

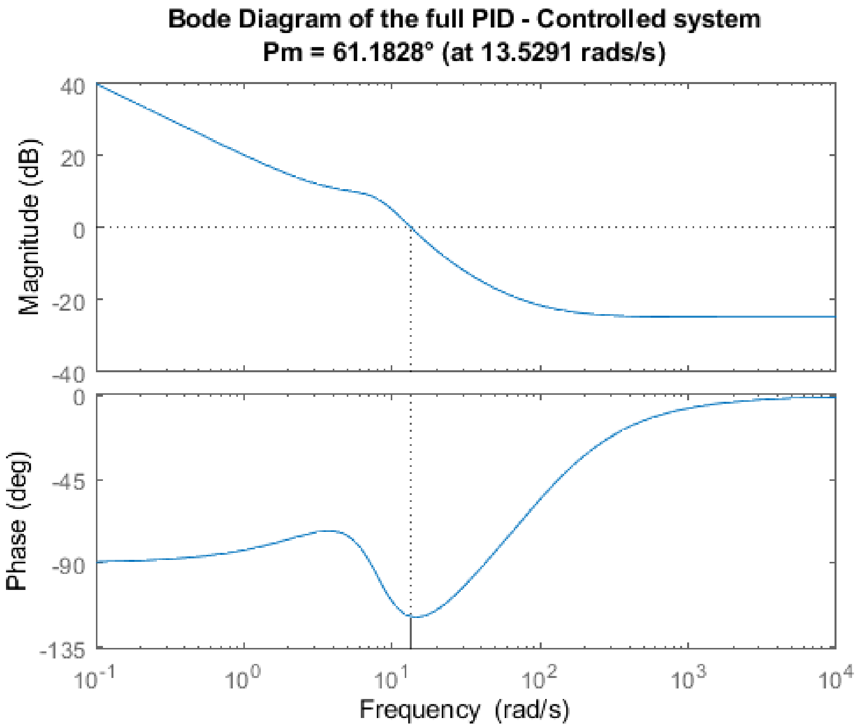
(Why 30°? as the same as before, it's just a mater of trial and error)

$$bs + 1 = 1 + jb\omega$$
$$\arctan(\frac{b\omega}{1}) = 30°$$
$$b = 0.046$$

Full PID - Controller

Finally, the last controller is a full PID - Controller that meets the design requirements:

$$G_{tot} = G_c * G_p = \frac{K(as+1)(bs+1)}{s} * \frac{s+100}{s^2+8s+65}$$
$$G_{tot} = G_c * G_p = \frac{6.5(0.1938s+1)(0.046s+1)}{s} * \frac{s+100}{s^2+8s+65}$$

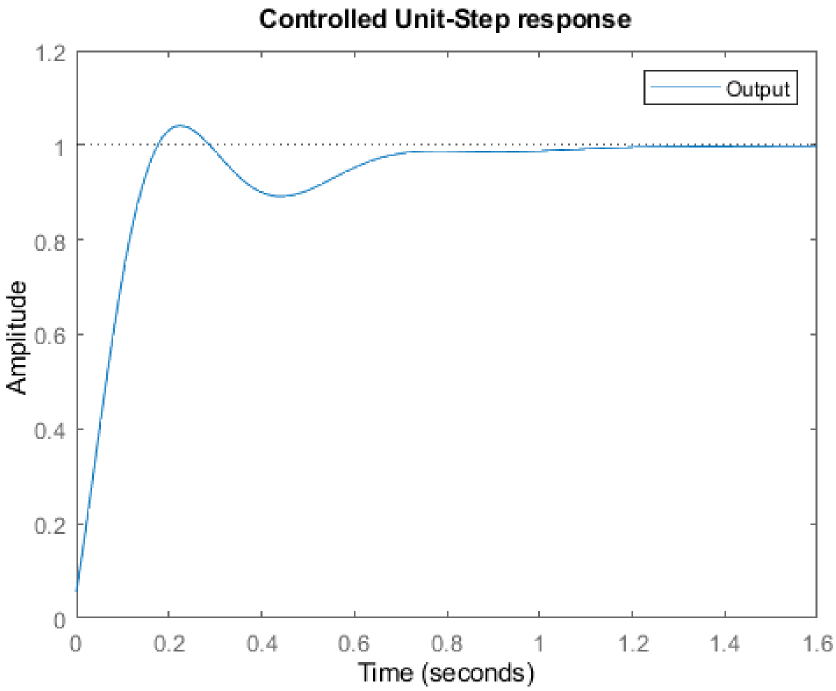


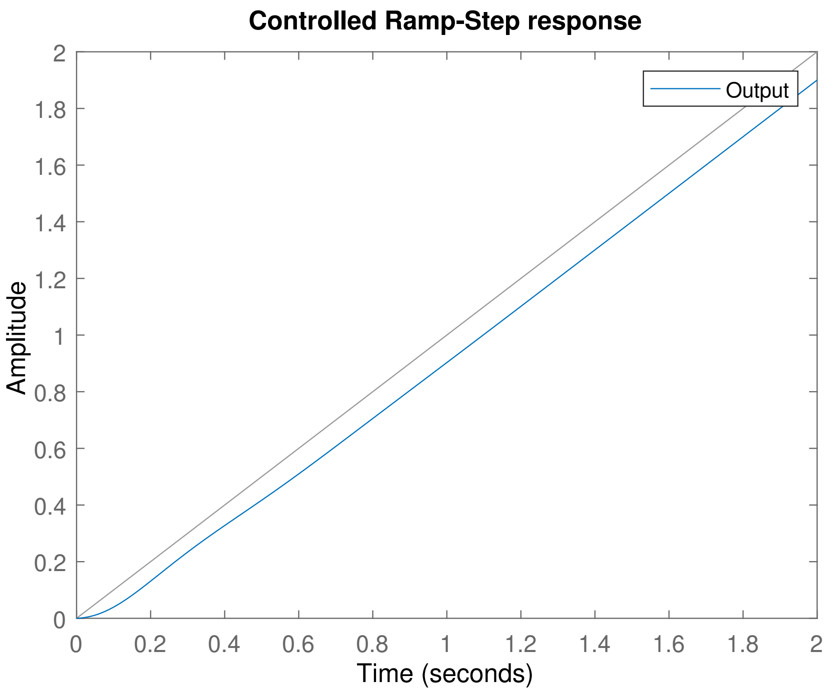
According to the Bode Diagram, the current margin phase is Pm = 61.2° at w = 13.5 rad/s.

So:

- The required minimum phase of ***Pm = 60°*** was achieved.

System outputs to Unit and Ramp step inputs:


Controlled Unit-Step response
Amplitude
Time (seconds)
Output
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6


Controlled Ramp-Step response
Amplitude
Time (seconds)
Output
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Conclusions

- By comparing the unit-step and ramp-step response curves of a PID-Controlled system to the non-controlled outputs we realize that the PID Controller was succesfully designed, meeting both design requirements:
- Desired Margin Phase, at least : $\boldsymbol{Mphase = 61.2}$ °
- Steady state error for a ramp-step input: $\boldsymbol{ess = 0.1}$