中国科学技术大学

习题 6.1

1. (3) $xy' + y = y^2$

1° $\Rightarrow \frac{dy}{y^2 - y} = \frac{dx}{x}$

$\Rightarrow \ln\left|\frac{y-1}{y}\right| = \ln x + C$

$\Rightarrow \left|\frac{y-1}{y}\right| = Cx$

2° $y = 0$ 或 $y = 1$

综上 $\begin{cases} \left|\frac{y-1}{y}\right| = Cx \\ y = 0 \\ y = 1 \end{cases}$

(3) $\frac{dx}{x^2 - xy + y^2} = \frac{dy}{2y^2 - xy}$

令 $u = \frac{y}{x}$

1° $\Rightarrow \frac{dy}{dx} = \frac{2u^2 - u}{1 - u + u^2} = u + x\frac{du}{dx}$

$\Rightarrow \frac{u^2 - u + 1}{3u^2 - u^3 - 2u} du = \frac{dx}{x}$

$\Rightarrow (y - x)^2 = Cy(y - 2x)^3$

2° $3u^2 - u^3 - 2u = 0$

$\begin{cases} y = 0 & \times \\ y = x & \checkmark \\ y = 2x & \checkmark \end{cases}$

综上 $\begin{cases} (y-x)^2 = Cy(y-2x)^3 \\ y = x \\ y = 2x \end{cases}$

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3. (1)

令 $\begin{cases} u = x+2 \\ v = y+1 \end{cases}$

$\Rightarrow \dfrac{dv}{du} = \dfrac{u+v}{u-v}$

$\Rightarrow$ 令 $p = \dfrac{v}{u}$

$\Rightarrow u\dfrac{dp}{du} + p = \dfrac{1+p}{1-p}$

$\Rightarrow \sqrt{(x+2)^2+(y+1)^2} = C \cdot e^{\arctan \frac{y+1}{x+2}}$

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(4) $y' + \dfrac{y}{x} = y^2 \ln x$

$\Rightarrow y^{-2}y' + \dfrac{1}{xy} = \ln x$

$\Rightarrow$ 令 $u = \dfrac{1}{y}$ $\quad u' - \dfrac{1}{x}\cdot u = -\ln x$

$\Rightarrow$ 公式 $\quad p(x) = -\dfrac{1}{x}$ $\quad p(x) = -\ln x$

$$y = e^{-\int p(x)dx}\left(\int Q(x) e^{\int p(x)dx} dx + C\right)$$

$\Rightarrow xy \cdot \left(\frac{1}{2}(\ln x)^2 + C\right) = 0$

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5.(1) $y' = \frac{y}{x}\ln\frac{y}{x}$ $\quad y(1)=1$

$\Rightarrow \begin{cases} u = \frac{y}{x} \\ x\frac{du}{dx} + u = u\ln u \quad u(1)=1 \end{cases}$

$\Rightarrow \left|\ln\frac{y}{x} - 1\right| = C|x|$

$\Rightarrow$ 代入 $y(1)=1$ 得 $C=1$

$\Rightarrow \left|\ln\frac{y}{x} - 1\right| = |x|$

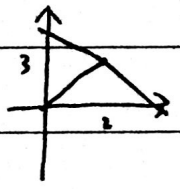
6.(4) $y'\sin y + x\cos y + x = 0$

1° $\Rightarrow \frac{\sin y\, dy}{1+\cos y} = -x\,dx$ $\qquad$ 2° $y = (2k+1)\pi$

$\Rightarrow \ln|1+\cos y| = \frac{1}{2}x^2 + C$

综上 $\begin{cases} \ln|1+\cos y| = \frac{1}{2}x^2 + C \\ y = (2k+1)\pi \end{cases}$

8.



$y(2)=3$

在 x, y 处切线斜率

$\frac{dy}{dx} = -\frac{y}{x}$

$\Rightarrow |xy| = 6$

$\Rightarrow xy = 6$

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9. $f(x)=\int_0^x f(t)\,dt$ $f(x)$连续

$\Rightarrow f(x)$可微 且 $y'=y$

$\Rightarrow y=1\cdot e^x$

12. (2) $y''=\frac{y'}{x}+x$

令 $p=y'$

$\frac{dp}{dx}=\frac{p}{x}+x$ (一阶线性方程)

(4) $y''+(y')^2=2e^{-y}$

令 $p=y'$

$p\frac{dp}{dy}+p^2=2e^{-y}$

$\Rightarrow \frac{1}{2}\frac{dp^2}{dy}+p^2=2e^{-y}$ (一阶线性方程)

13 (1) $y''=\frac{y'}{x}+\frac{x^2}{y'}$ $y(1)=1$ $y'(1)=0$

令 $p=y'$

$\frac{dp}{dx}=\frac{p}{x}+\frac{x^2}{p}$

$\Rightarrow \frac{1}{2}\frac{dp^2}{dx}=\frac{p^2}{x}+x^2$ (一阶线性方程)

$\Rightarrow p^2=2(x^3-x^2)$

$\Rightarrow$ 1° $p=\sqrt{2(x^3-x^2)}$ $y=\frac{2\sqrt{2}}{5}(x-1)^{\frac{5}{2}}+\frac{2\sqrt{2}}{3}(x-1)^{\frac{3}{2}}+1$

2° $p=-\sqrt{2(x^3-x^2)}$ $y=-\frac{2\sqrt{2}}{5}(x-1)^{\frac{5}{2}}-\frac{2\sqrt{2}}{3}(x-1)^{\frac{3}{2}}+1$