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第5章综合题页

1. 利用 $\sin x \sin y = \frac{1}{2}(\cos(x-y) - \cos(x+y))$

等三角公式

2. (1) $t = 1-x$ 换元

(2) 分部积分

3. (1) $\int_{\frac{1}{2}}^{2} (1+x-\frac{1}{x}) e^{x+\frac{1}{x}} dx = x e^{x+\frac{1}{x}} \Big|_{\frac{1}{2}}^{2}$

(2) $\int_{k\pi}^{(k+1)\pi} x|\sin x| dx$

$= \int_0^{\pi} (t+k\pi)|\sin t| dt$

$= (2k+1)\pi$

$\Rightarrow \int_0^{n\pi} x|\sin x| = \sum_{k=0}^{n-1} (2k+1)\pi = n^2\pi$

(3) 由 $\int_{\pi}^{2\pi} e^{\sin t} - e^{-\sin t} dt = 0$ 化简

4. 课上已讲过, 回答

5. 回答

6. (1) $\int_0^{-x} f(t) + \int_0^{x} f(t) = 0$

取 $F(x) = \int_0^{x} f(t) dt$

(2) $\int_0^{-x} f(t) + C = \int_0^{x} f(t) + C$

$\Rightarrow F(-x) = F(x)$

7. $f(x) = \sin x + 2$

$\int f(x)$ 单增, 不可能有周期

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8. 积分中值定理

9. 略

10. $1^\circ\ x_0 = 0$

$\forall \varepsilon > 0,\ \exists \delta,\ 0 < |x - 0| < \delta$ 时

$$\left| \frac{f(x) - f(0)}{x} - f'(0) \right| < \varepsilon$$

$$\Rightarrow \left| \frac{F(x) - F(0)}{x} - f'(0) \right|$$

$$= \left| \int_0^1 \frac{f(xy) - f(0)}{x} - f'(0)\, dy \right|$$

$$\le \int_0^1 \left| \frac{f(xy) - f(0)}{x} - f'(0) \right| dy$$

$$\le \varepsilon$$

$$\Rightarrow F'(0) = f'(0)$$

$2^\circ\ x_0 \neq 0$

$$F(x) = \frac{1}{x} \int_0^x f(t)\, dt$$

$$\Rightarrow F(x)' = -\frac{1}{x^2} \int_0^x f(t)\, dt + \frac{1}{x} f(x)$$

11. 证明上确界，略

12. $\frac{1}{h} \int_a^b f(x+h) - f(x)\, dx$

$$= \frac{1}{h} \left[\int_{a-h}^{a} f(x)\, dx - \int_{b-h}^{b} f(x)\, dx \right]$$

$$= f(\xi_1) - f(\xi_2) \quad (\xi_1 \in (a-h, a),\ \xi_2 \in (b-h, b))$$

$\Rightarrow h \to 0$，上式 $\to f(a) - f(b)$

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13. 因含

14. 有 $\int_0^{n\pi}|\sin t|dt = n\int_0^{\pi}\sin t dt = 2n$

$\Rightarrow \frac{1}{x}\int_0^x|\sin t|dt$

$\geq \frac{1}{(n+1)\pi}\int_0^{n\pi}|\sin t|dt = \frac{2n}{(n+1)\pi}$ $x\in[n\pi,(n+1)\pi]$

$< \frac{1}{n\pi}\int_0^{(n+1)\pi}|\sin t|dt = \frac{2(n+1)}{n\pi}$

$\Rightarrow$ 夹逼

15. 因含

16. $\forall \varepsilon>0$, $\exists [c_1,c_2]$, $c_2>c_1$

在 $[c_1,c_2]$ 上有 $f(x)>M-\varepsilon$

$\Rightarrow \left(\int_a^b f^n(x)dx\right)^{\frac{1}{n}} \geq (c_2-c_1)^{\frac{1}{n}}(M-\varepsilon) \to M-\varepsilon$

$\Rightarrow M-\varepsilon \leq \lim \sim \leq M$

令 $\varepsilon\to 0$ 即可

17. 因含

18. 因含

19. 由 $|f(a)-f(x)| = \left|\int_x^a f'(t)dt\right|$

$\leq \int_0^1|f'(x)|dx$

$\Rightarrow \int_0^1 |f(a)|-|f(x)| \leq \int_0^1|f(a)-f(x)|$

$\leq \int_0^1|f'(x)|$

$\Rightarrow f(a) \leq \int_0^1|f(x)| + \int_0^1|f'(x)|$

20、略

21、$\left|\int_0^1 f(x) - \frac{1}{n}\sum_{k=1}^{n} f(\frac{k}{n})\right|$

$= \left|\sum_{k=1}^{n}\int_{\frac{k-1}{n}}^{\frac{k}{n}} (f(x) - f(\frac{k}{n}))dx\right|$

$\leq \sum_{k=1}^{n}\int_{\frac{k-1}{n}}^{\frac{k}{n}} |f(x) - f(\frac{k}{n})|\,dx$

$\leq \sum_{k=1}^{n}\int_{\frac{k-1}{n}}^{\frac{k}{n}} M(\frac{k}{n} - x)\,dx$

$\leq \sum_{k=1}^{n} \frac{M}{2n^2}$

$= \frac{M}{2n}$

22. 课上讲过，略

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