Measuring the Rydberg Constant for Hydrogen

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Abstract

The first 4 peaks of Balmer lines were used to calculate the Rydberg constant.This was done with a HeNe laser for calibrating a spectrometer then a hydrogen lamp was used to get the Balmer Series. The Rydberg constant was calculated to be $R_H = 1.098 \times 10^7 \pm 0.001 m^{-1}$ which corrosponds with the known literature value of $1.096776 \times 10^7 m^{-1}$.

1 Introduction

When atoms of an element are excited they drop down to lower energy states when they de-excite and in the process, a photon is emitted. This photon carries quantized packets of energy corresponding to specific energy transitions. These emission lines are related to specific wavelengths and hence when looking at the spectra of different elements we can differentiate them by the way emission lines vary. When the atoms in hydrogen are excited and transitions from higher energy states and end at principle quantum number $n = 2$ the emission lines of the Balmer Series can be seen and measured. For this experiment, the wavelengths associated with the Balmer Lines will be used to calculate the Rydberg constant R_H for atomic hydrogen. This will be done using an optical spectrometer system to split the hydrogen into its wavelengths. The spectrometer system will be calibrated using a Helium-Neon laser with a known wavelength. The 4 emission lines of hydrogen in the range of $4000\mathring{A}$ to $7000\mathring{A}$ will be fitted using a Gaussian and determined to an uncertainty. Using the R_H values obtained from these 4 lines the final R_H value can be determined by the average and compared to the literature value of $1.097 \times 10^7 m^{-1}$. This experiment is analogous to the empirical method in which Balmer first measured a form of the Rydberg constant.

2 Background and Theory

2.1 Balmer Series

The Balmer Series or Balmer lines are emission spectral lines that correspond to the hydrogen atom and can be viewed in the visible part of the electromagnetic spectrum [?]. They were discovered by Johann Balmer in 1885. When electrons of the excited hydrogen atom drop to a lower excited state we observe the emission of a photon corresponding to a drop to the energy state with the principal quantum number $n = 2$. This lead to the Balmer formula that was found through trial and error and a focus on the wavenumber ($\frac{1}{\lambda}$) [?]. It was proved empirically by Balmer and generalized by Rydberg in 1888. The formula was only proved theoretically in 1913 by Neils Bohr [?]. Balmer found the formula to be:

$$\frac{1}{\lambda} = \frac{4}{B}\left(\frac{1}{2^2} - \frac{1}{m^2}\right)$$

Where λ is the wavelength in vacuum, 2 is the principle quantum number, $\frac{4}{B}$ represents the Rydberg constant here (which will be discussed below) and m (ranging from $3 \to \infty$) is the excited principle quantum number.

2.2 Ryberg Constant

The Rydberg forumula is a generalization of the formula Balmer found (it is for any line of emission by the hydrogen atom) and is given by:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

Here the principle quantum number n and m can range from $1 \to \infty$. While the Balmer Series is determined by setting $n = 2$ and letting m range from $3 \to \infty$, we obtain the Lyman series by setting $n = 1$ and letting m range from $2 \to \infty$ and similar series can be determined in a similar manner.

The Rydberg constant for heavier elements is given by

$$R_\infty = \frac{m_e e^4}{8\epsilon_0^2 h^2 c}$$

Where m_e is the mass of an electron, ϵ_0 is the permitivity of free space, h is Plank's constant and c is the speed of light in vacuum. the Rydberg constat for the hydrogen atom can be generalized to

$$R_H = R_\infty \frac{m_p}{m_e + m_p}$$

[?]

The Balmer lines being investigated in this report are $H - \alpha$ (m=3), $H - \beta$ (m=4), $H - \gamma$ (m=5) and $H - \delta$ (m=6).

3 Experimental Method

For this experiment, an optical spectrometer system (Heath EU-700 Czerny-Turner monochromator that has a photomultiplier detector as well as a pulse-counting electronics attached to it) [?] was used to split and allow for the analysis of the emission lines of hydrogen to get the wavelengths of interest. The spectrometer system was not calibrated (the real wavelength and the one recorded would have been offset by a shift either up or down) and so the Helium-Neon (HeNe) laser was used in calibration. HeNe lasers are usually made up of 90% helium and 10% Neon [?]. As the name laser suggests (light amplified through stimulated emission of radiation) the HeNe laser works by exciting the neon atoms in the gas and when the excited atoms decay to their ground state the wavelength of $6328\mathring{A}$ [?] is detected and well known.

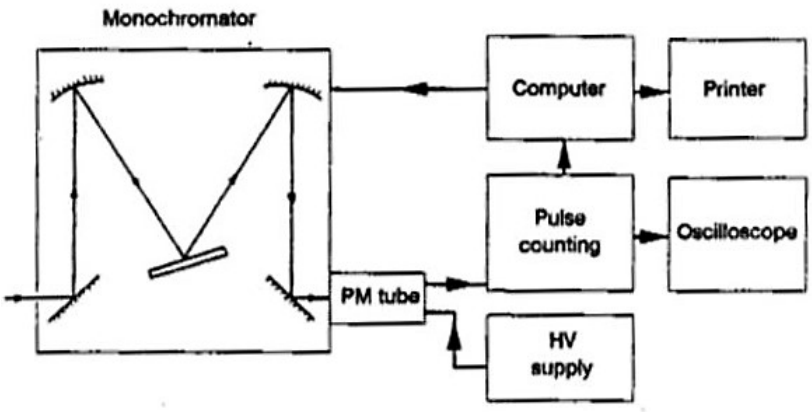


Figure 1: A figure showing the setup required for the experiment [?]. Light from the HeNe laser and the hydrogen lamp enter the entrance slit and undergoes diffraction before it enters a photomultiplier tube where the pulses can be counted. The resulting pulses are counted from the energy that is received from the photons.

Connected to the spectrometer system was an oscilloscope used to see which wavelengths corresponded to emission lines of the light in question. For the HeNe laser different wavelength ranges around $6328\mathring{A}$ were tested until a wavelength that corresponded to large energy was found. The spectrometer system was also connected to a computer where a programme was used to record the data. On the programme the range of wavelengths one wanted to observe could be selected as well as the slit width, dwell time and the step increment. For the HeNe laser, these values were $100\mu m$, $500ms$ and 00.1 respectively, the slit width was not changed for the rest of the experiment. The emission line found for HeNe on the spectrograph was found to be $6350\mathring{A}$ and so a rough offset of $22\mathring{A}$ was established for the Hydrogen emission line measurements.

A hydrogen lamp was then used and focused onto the spectrometer slit using a mirror. A long run was taken of the light emitted by the hydrogen lamp to get an estimate of where the peaks would be found. The slit width, dwell time and step size were set to be $100\mu m$, $500ms$ and 1 respectively. The 4 emission lines corresponding to the Balmer series were individually recorded with the dwell time and step size set to be $500ms$ and 0.1.

4 Results and Analysis

The 4 peaks associated with the Balmer Series were recorded and a Gaussian was fitted to each peak. But first, a long run of the hydrogen spectrum was taken and is shown in the appendix. The calibration using the known wavelength of the HeNe laser is first shown.

4.1 Wavelength Correction

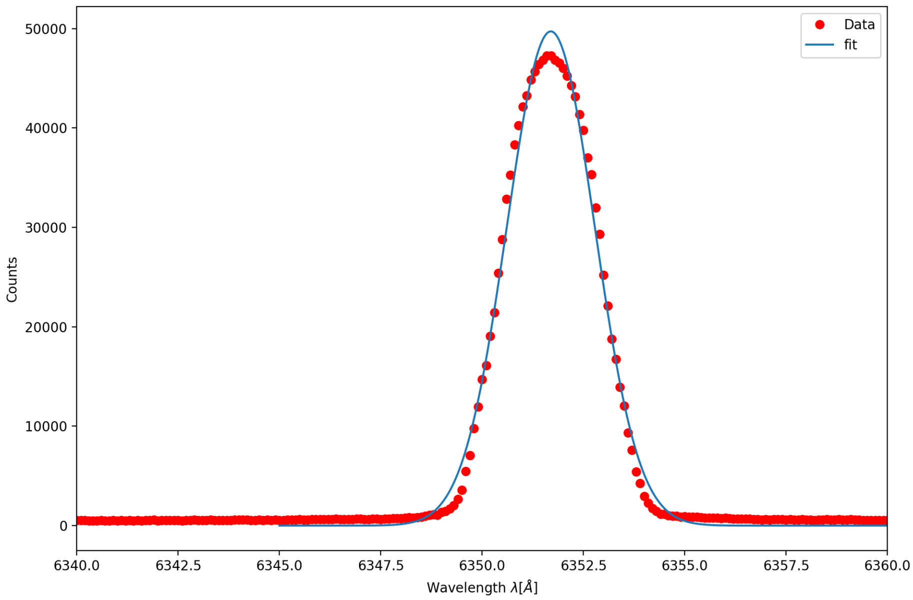
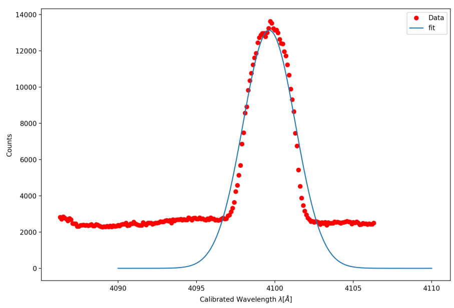


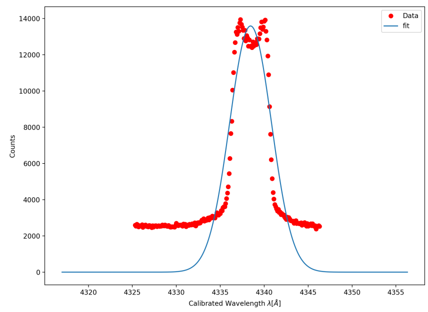
Figure 2: The centroid value was calculated to be $\mu = 6351.696$ with a $\sigma = 1.082$. On the x axis is the uncalibrated wavelengths from the spectrometer and on the

The HeNe laser has the known wavelength of $6328\mathring{A}$ and so an offset of $\Delta\lambda = 23.696\mathring{A}\pm$ was calculated. This $\Delta\lambda$ will then be used to calibrate the wavelengths of the hydrogen atom.

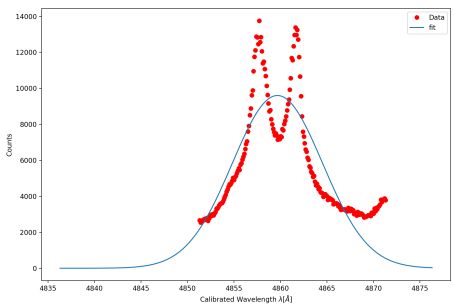
4.2 First 4 peaks of Balmer Series



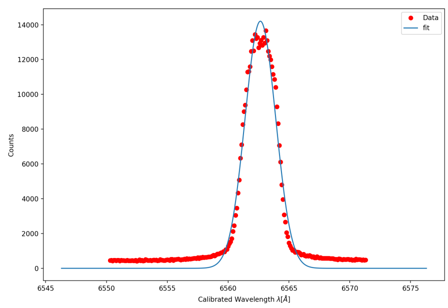
(a) peak calculated to be $\lambda = 4099.641 \pm 1.671\mathring{A}$



(b) peak calculated to be $\lambda = 4338.453 \pm 2.382\mathring{A}$



(c) peak calculated to be $\lambda = 4859.677 \pm 4.863\mathring{A}$



(d) peak calculated to be $\lambda = 6562.654 \pm 1.262\mathring{A}$

Figure 3: The graphs above have had the wavelength correction of $\Delta\lambda = 23.696\mathring{A}$ applied to them and have then had gaussians fitted to the indivudial peaks. The graphs show the counts on the y axis as a function of the associated wavelength

From this fitting, we are closer to measuring the Rydberg constant. Using the formula $\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} + \frac{1}{m^2}\right)$ we can calculate the Rydberg constants for the individual emission lines by substituting for λ. This can also be compared to the literature value known for these wavelengths.

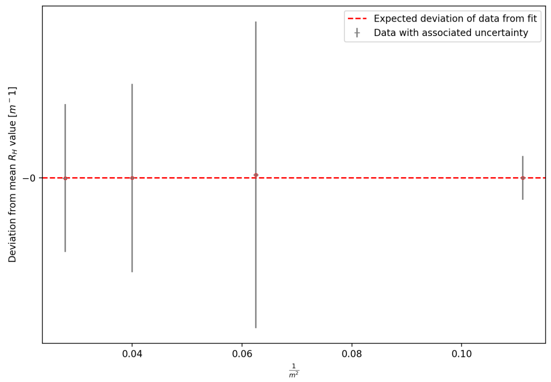
Table 1: A table showing the

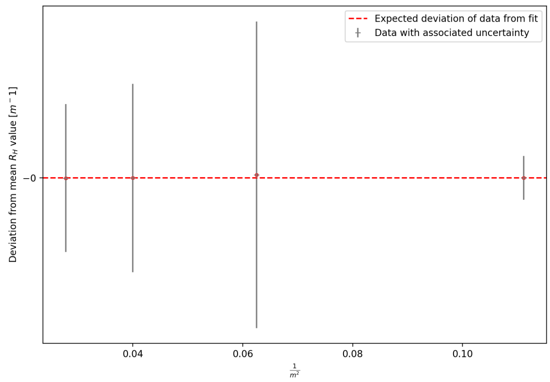
m value	Label	Calculated Wavelength [$\mathring{A}$]	Literature Value [?] [$\mathring{A}$]	Calculated $R_H[10^7 m^{-1}]$
6	$H-\delta$	4099.641 ± 1.671	4101.740	1.0977
5	$H-\gamma$	4338.453 ± 2.382	4340.462	1.0976
4	$H-\beta$	4859.677 ± 4.863	4861.362	1.0975
3	$H-\alpha$	6562.654 ± 1.262	6564.568	1.0971

The fitted gaussians [shown in the appendix] were plotted using the *curve_fit* function from *scipy.optimize* in python. The uncertainty for each of the wavelengths of hydrogen were calculated using the formula $FWHM = 2.355\sigma$ for each of the fitted gaussians. From the values in the table an averge value for the Rydberg constant can be calculated. It was found to be $R_H = 1.098 \times 10^7 \pm 0.001 m^{-1}$. This value was calculated using a weighted linear least squares regression on the equation:

$$\frac{1}{\lambda} = \frac{R_H}{2^2} + \frac{R_H}{m^2}$$

The least squares regression was done in the form of $y = mx + c$ with $y = \frac{1}{\lambda}$, $x = \frac{1}{m^2}$, $m = R_H$ and $c = \frac{R_H}{2^2}$ The is plotted below with an associated plot of the risiduals:





(a) A line of best fit for the equation stated above showing the raw data points

(b) A graph showing how the raw data points deviate from the least squares fit calculation

Figure 4: The equation for the line of best fit was found to be $\frac{1}{\lambda} = (1.097 \times 10^7 \pm 0.001) \times \frac{1}{m^2} + 2744387.191 \pm 1080.199$

In figure 3b, we can see that all the uncertainty lines associated with the residuals pass through the expected deviation line showing the fit used is a good one.

5 Conclusion

The study of the hydrogen atom, as well as the associated wavenumbers $\frac{1}{\lambda}$, has given great insight into the quantized nature of energy. This report used the first 4 Balmer lines of the hydrogen atom to calculated the Rydberg constant R_H. The value was calculated to be $R_H = 1.098 \times 10^7 \pm 0.001 m^{-1}$ which is in alignment with the literature value of $1.096776 \times 10^7 m^{-1}$ [?]

If the experiment were repeated the background counts can be accounted for a subtracted from the peaks found of hydrogen and hence a more accurate value can be obtained with lower uncertainty. Repeats of recordings of the different peaks can also be included in a revision of the experiment.

References

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6 Appendix

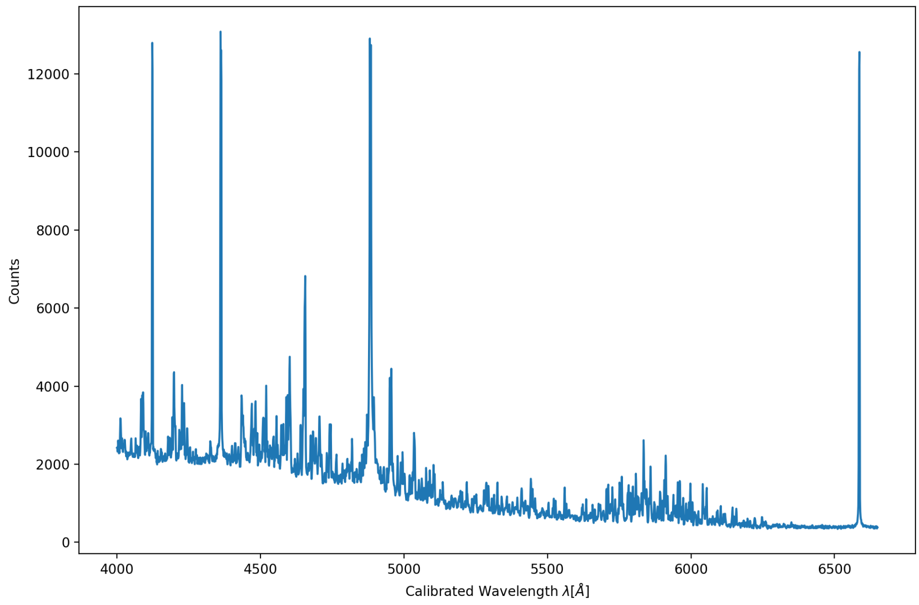

Figure 5: