COMP107: Relations

Relations

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- Human Relations

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- Numeric Comparisons: $(==, <, <=, >, >=, !=)$

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"apple" = "banana"

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- String comparisons:
- Functions of numbers: (*log*, *sqrt*, *abs*)

Definition of a relation

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Definition

A Relation between some number of sets is a set of tuples where the first element of the tuple is from the first set, the second from the second set and so on ....

Examples of Relation I

Example

Let $A = \{1, 2, 3\}$ and $B = \{4, 5\}$. Then $\{(1, 4), (2, 4), (3, 5)\}$ is a relation from A into B.

There are many others we could describe. How many relations may be defined?

Number of relations that may be defined on two sets.

Examples of Relations II

Example

Let $A = \{2, 3, 5, 6\}$ and define a relation r from A into A by $(a, b) \in r$ if and only if a divides evenly into b. The set of pairs that qualify for membership is?

Example II

$$r = \{(2,2),(3,3),(5,5),(6,6),(2,6),(3,6)\}.$$

Relation on a Set

Definition

Relation on a Set. *A relation from a set A into itself is called a relation on A.*

Graph of a Relation

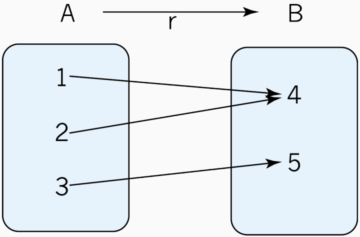

Figure 1: Graph of a Relation.

A function is a special relation

What does the graph of a function from x to y look like?

Relation Notation

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We can define the relation using set-builder notation as:
$s = \{(x, y) | x \leq y\}$.

Composition

Let $A = \{2, 3, 5, 8\}$, $B = \{4, 6, 16\}$, and $C = \{1, 4, 5, 7\}$; let r be the relation "divides," denoted by $|$, from A into B; and let s be the relation $\leq$ from B into C.

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So: $r =$
and $s =$.

Graph of Composition

$r = \{(2,4),(2,6),(2,16),(3,6),(8,16)\}$
$s = \{(4,4),(4,5),(4,7),(6,7)\}.$

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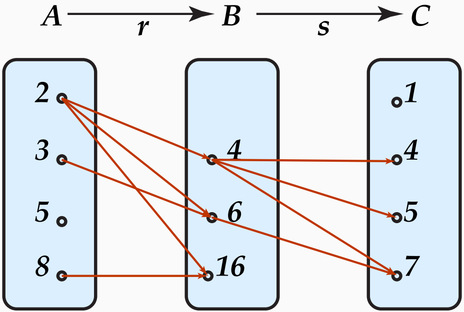

Figure 2: Graphical Representation of Composition of Relations

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Notice that we can, for certain elements of A, go through elements in B to results in C. That is:

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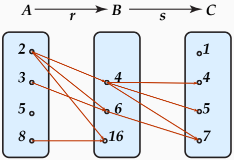

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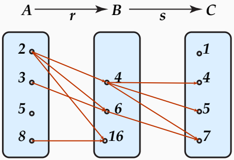

$2|4$ and $4 \leq 4$
$2|4$ and $4 \leq 5$

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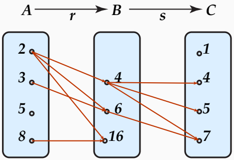

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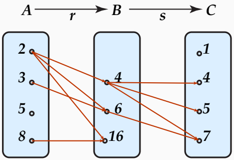

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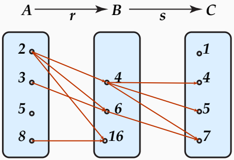

$$2|4 \text{ and } 4 \leq 4$$
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$$3|6 \text{ and } 6 \leq 7$$

Composition

We can define a new relation, call it rs, from A into C. In order for (a, c) to be in rs, it must be possible to travel along a path from a to c. In other words, $(a, c) \in rs$ if and only if there exists a b such that $(a \; r \; b)$ and $(b \; s \; c)$.

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Recall:

$r = \{(2, 4), (2, 6), (2, 16), (3, 6), (8, 16)\}$ and
$s = \{(4, 4), (4, 5), (4, 7), (6, 7)\}$.

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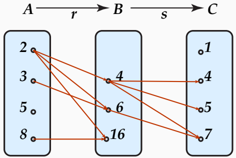

The complete listing of all elements in rs is:

Composition

The complete listing of all elements in *rs* is
$\{(2,4),(2,5),(2,7),(3,7)\}$.

Definition of Composition

Definition

Composition of Relations. *Let r be a relation from a set A into a set B, and let s be a relation from B into a set C. The composition of r with s, written rs, is the set of pairs of the form $(a, c) \in A \times C$, where $(a, c) \in rs$ if and only if there exists $b \in B$ such that $(a, b) \in r$ and $(b, c) \in s$.*

Digraph of a Relation

Example

Let $A = \{0, 1, 2, 3\}$, and let
$r = \{(0, 0), (0, 3), (1, 2), (2, 1), (3, 2), (2, 0)\}$.

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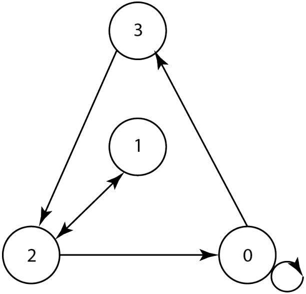

Figure 3: Directed Graph (digraph) of a Relation.

Digraph of a Relation

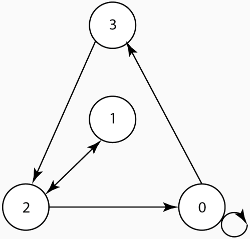

Figure 4: Directed Graph (digraph) of a Relation.

Digraph of a Relation

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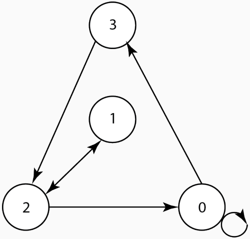

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Digraph of a Relation

- The elements of A are the vertices of the graph.

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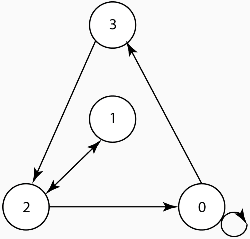

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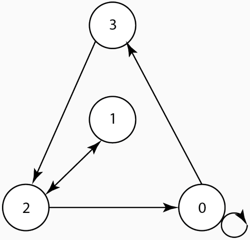

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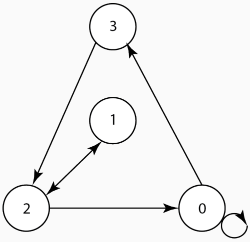

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- We connect vertex a to vertex b with an arrow, called an **edge**, going from vertex a to vertex b if and only if arb.
- This type of graph of a relation r is called a *directed graph* or *digraph*.
- Notice that since $1r2$ and $2r1$, we draw a single edge between 1 and 2 with arrows in both directions. Since 0 is related to itself, we draw a "self-loop" at 0.

Layout of the Graph does not matter.

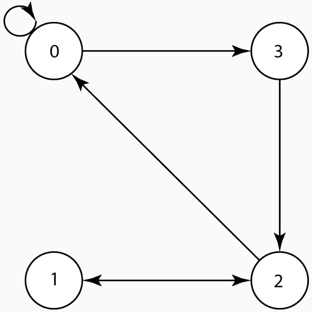

Figure 5: Directed Graph of a Relation.

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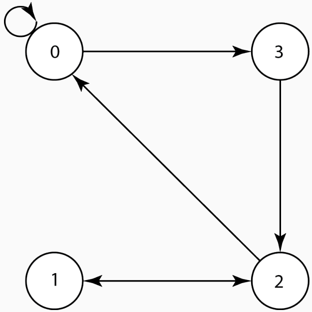

Figure 6: Directed Graph of a Relation.

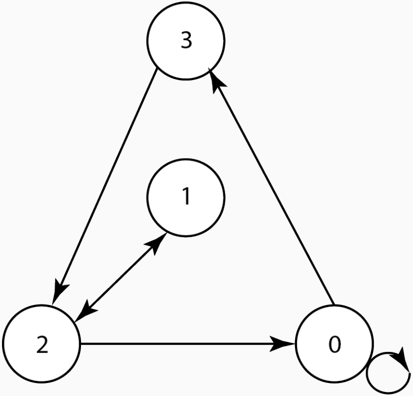

Figure 7: Directed Graph (digraph) of a Relation.

Another Digraph Example

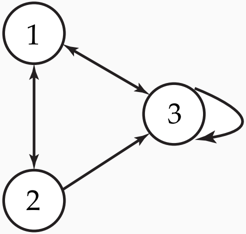

Figure 8: Directed Graph of a Relation.

What information does this graph give us? The graph tells us that s is a relation on $A =$ and that $s =$

Another Digraph Example

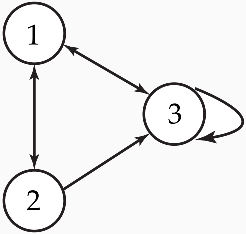

Figure 9: Directed Graph of a Relation.

What information does this graph give us? The graph tells us that s is a relation on $A = \{1, 2, 3\}$ and that $s = \{(1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 3)\}$

Yet Another Digraph

Example

Let $B = \{1,2\}$, and let $A = \mathcal{P}(B) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$. Then $\subseteq$ is a relation on A whose digraph is Figure 10.

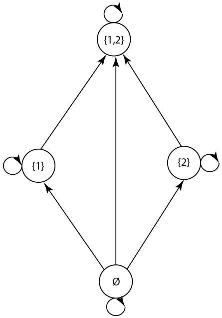

Figure 10: Directed Graph of a Relation.