p-Value

Dr. Supaporn Erjongmanee

Department of Computer Engineering
Kasetsart University
fengspe@ku.ac.th

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Outline

- p-Value
  - z-test
  - t-test

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p-Value

- **Smallest** significance level at which null hypothesis is rejected
- Also call observed significance level (OSL)
- Think of P-value as area under the curve

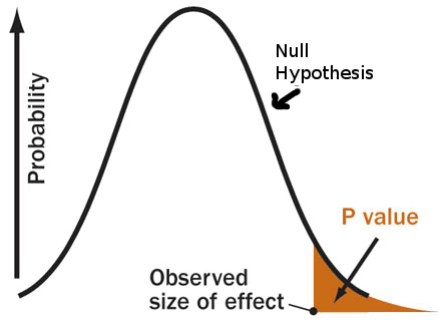


Image source: https://sciencebasedmedicine.org/0-05-or-0-005-p-value-wars-continue/

Example

- Nicotine level in cigarette is normally distributed
  - Average nicotine level = $\mu = 1.5$, $\sigma = 0.2$
- Customer wants to check nicotine level
  - H_0: $\mu = 1.5$
  - H_a: $\mu > 1.5$
- If test statistic $z = 2.10$, then
  - $\alpha = 0.1$, $z_\alpha = 1.2816$: $z > z_\alpha$ =>
  - $\alpha = 0.05$, $z_\alpha = 1.6449$: $z > z_\alpha$ =>
  - $\alpha = 0.01$, $z_\alpha = 2.3263$: $z < z_\alpha$ =>

What's smallest α to reject H_0?

Goal is to minimize rejection region α

Example: P-value for z-test

- **Upper-tailed test case**

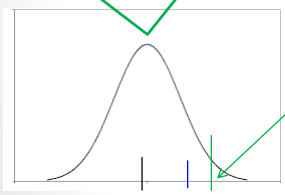
α decreases as z_α increases

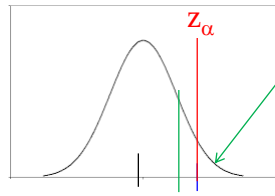
Goal is to minimize rejection region α

Equivalently, find largest z_α that results to reject H_0

Let z = test statistic

When $z < z_\alpha$, then H_0 is not rejected





If $z_\alpha < z$, then H_0 is rejected but α is not minimum still.

α is minimum when $z_\alpha = z$.

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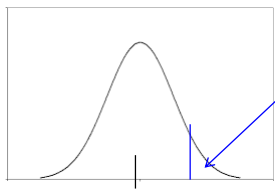
Example: P-value for z-test

P-value = Smallest significance level at which H_0 is rejected

- **Upper-tailed test case (cont.)**



- Our goal is to minimize α
- Minimum α occurs at critical value z_α = test statistic z
- Thus, **P-value = 1 - $\Phi(z)$**

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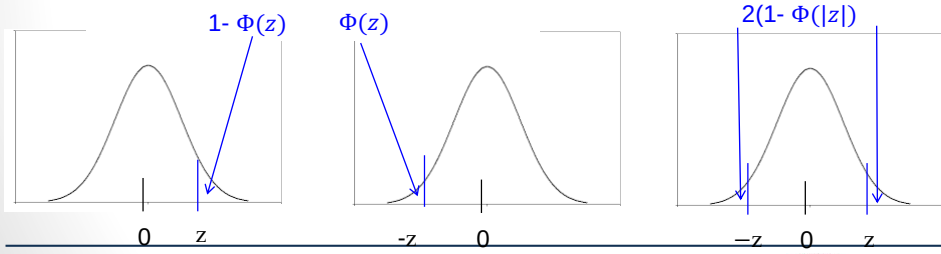
P-value for z-test (cont.)

- Null hypothesis: $\mu = \mu_0$

Alternative Hypothesis	P-value	Test
$H_a : \mu > \mu_0$	$1- \Phi(z)$	**Upper-tailed test**
$H_a : \mu < \mu_0$	$\Phi(z)$	**Lower-tailed test**
$H_a : \mu \neq \mu_0$	$2(1- \Phi(\lvert z \rvert)$ or $2(\Phi(-\lvert z \rvert)$	**Two-tailed test**

Z = test statistic



Upper-tailed Test **Lower-tailed Test** **Two-tailed Test**

Example

- Target thickness of silicon wafer = 245 µm
- 50 wafers are sampled and collected for thickness
  - Sample mean = $\bar{X} = 246.18$ µm
  - Sample standard deviation = S = 3.60 µm
- Question: What is p-value to reject H_0?
- Our goal is to check wafer thickness level
  - µ = average wafer thickness
  - μ_0 = 245
  - H_0: µ = 245
  - H_a: µ ≠ 245

Supaporn Erjongmanee
fengspe@ku.ac.th

Example (cont.)

H_0: μ = 245
H_a: μ ≠ 245

Alternative Hypothesis	P-value
H_a : μ > μ_0	1- $\Phi(z)$
H_a : μ < μ_0	$\Phi(z)$
H_a : μ ≠ μ_0	2(1- $\Phi(\|z\|)$ or 2($\Phi(-\|z\|)$

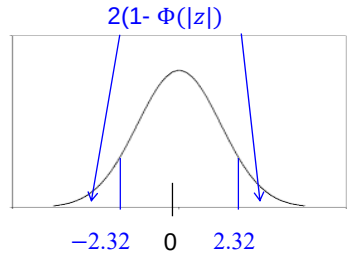
- Test statistic:

$$Z = \frac{\bar{X}-\mu_0}{S/\sqrt{n}} = \frac{246.18-245}{3.60/\sqrt{50}} = 2.32$$

- This is two-tailed test
  - **P-value** = 2(1- $\Phi(|z|)$

    = 2 (1 – $\Phi(|2.32|)$

    = 2 (1 −0.9898) = 0.0204



Question:
Given α = 0.01 and p-value = 0.0204, do we reject H_0 ?

Example (cont.)

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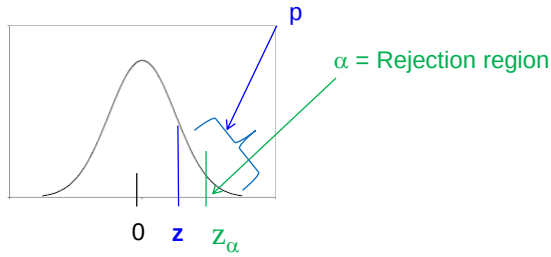
Given α = 0.01 and p-value = 0.0204, do we reject H_0 ?

Consider upper-tailed test



If p > α, then test statistic z does not fall in rejection region.

Do not reject H_0

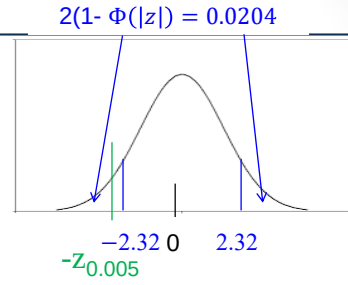
H_0 is rejected when p < α

Example (cont.)

H_0: μ = 245
H_a: μ ≠ 245

- Test statistic:

$$Z = \frac{\bar{X}-\mu_0}{S/\sqrt{n}} = \frac{246.18-245}{3.60/\sqrt{50}} = 2.32$$



- This is two-tailed test
  - P-value = 2(1- Φ(|z|) = 2 (1 − Φ(|2.32|) = 0.0204

Given α = 0.01 and p-value = 0.0204, do we reject H_0 ?

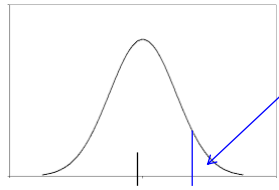
  - Given α = 0.01 < p-value = 0.0204
    - Test statistic falls outside rejection region for α /2
    - Null hypothesis is not rejected
    - At significance level = 0.01, wafer thickness is not different from the target value

Outline

- P-Value
  - z-test
  - t-test

Example: P-value for t-test

- Similar to z-test
- **Upper-tailed test case**:

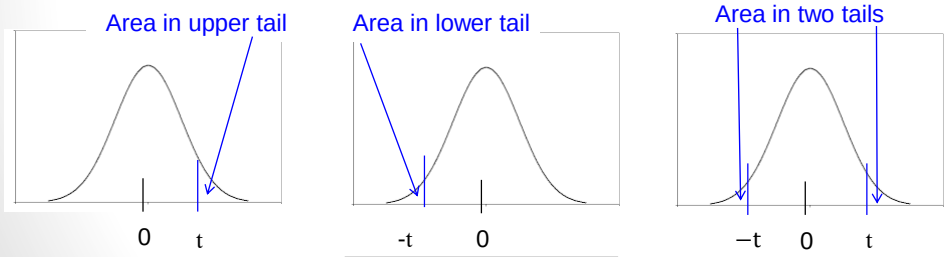


- Our goal is to minimize α
- Minimum α occurs at critical value $t_{\alpha, df}$ = test statistic **t**
- Thus, **P-value =** area in upper tail of test statistic **t**

P-value for t-test (cont.)

- Null hypothesis: $\mu = \mu_0$

Alternative Hypothesis	P-value	Test
$H_a : \mu > \mu_0$	Area in upper tail of test statistic t	**Upper-tailed test**
$H_a : \mu < \mu_0$	Area in lower tail of test statistic t	**Lower-tailed test**
$H_a : \mu \neq \mu_0$	Area in two tails of test statistic t	**Two-tailed test**



Upper-tailed Test **Lower-tailed Test** **Two-tailed Test**

Example

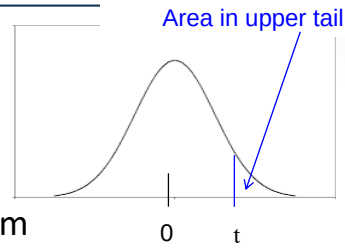
- Our goal is to check fuel efficiency whether it is better than average = 20 mpg
- Collect fuel efficiency (miles per gallon (mpg)) of 4 cars
  - $x_1 = 20.830$, $x_2 = 22.232$, $x_3 = 20.276$, $x_4 = 17.718$
  - Sample mean = $\bar{X} = 20.264$ mpg
  - Sample standard deviation = $s = 1.8864$ mpg
- Question: What is p-value to reject claim ?
- Set up hypothesis
  - μ = average fuel efficiency
  - $\mu_0 = 20$
  - H_0: $\mu = 20$
  - H_a: $\mu > 20$

Example (cont.)

Upper-tailed Test

- Test statistic:

$$t = \frac{\bar{X}-\mu_0}{S/\sqrt{n}} = \frac{20.264-20}{1.8864/\sqrt{4}} = 0.2799$$



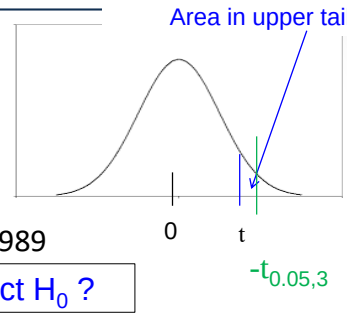
- This is upper-tailed test with 3 degree of freedom
  - P-value = area on the right of t = 0.2799
    = 1- 0.6011 = 0.3989

Given $\alpha = 0.05$ and p-value = 0.3989, do we reject H_0 ?

* *Tool: In Python's scipy, use 1-t.cdf*
* *Tool: http://stattrek.com/online-calculator/t-distribution.aspx*

Example (cont.)

Upper-tailed Test



- Test statistic:

$$t = \frac{\bar{X}-\mu_0}{S/\sqrt{n}} = \frac{20.264-20}{1.8864/\sqrt{4}} = 0.2799$$

- This is upper-tailed test with 3 degree of freedom
  - P-value = area on the right of 0.2799 = 1- 0.6011 = 0.3989

Given α = 0.05 and p-value = 0.3989, do we reject H_0 ?

- Given α = 0.05 < p-value = 0.3989,
  - Test statistic falls outside rejection region for α
  - H_0 is not rejected
  - At significance level = 0.05, fuel efficiency is 20 mpg

References

1. J.L. Devore and K.N.Berk, Modern Mathematical Statistics with Applications, Springer, 2012.