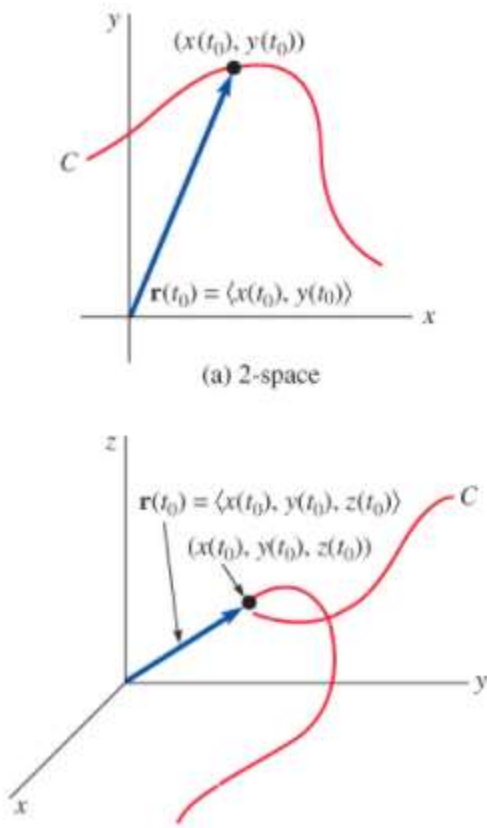
Written by : Andi-Nov, sammyon7

VECTOR FUNCTIONS!

INTRODUCTION

Recall that a curve C in the xy-plane is simply a set of ordered pairs (x, y). We say that C is a parametric curve if the x- and y-coordinates of a point on the curve are defined by a pair of functions x = f(t), y = g(t) that are continuous on some interval a ≤ t ≤ b. The notion of a parametric curve extends to 3-space as well. A parametric curve in space, or space curve, is a set of ordered triples (x, y, z) where :

$$x = f(t), \qquad y = g(t), \qquad z = h(t),$$



are continuous on an interval defined by $a \le t \le b$. In this section we combine the concepts of parametric curves with vectors.

Vector-Valued Functions

It is often convenient in science and engineering to introduce a vector r whose components are functions of a parameter t. We say that

$$\mathbf{r}(t) = \langle f(t), g(t) \rangle = f(t)\mathbf{i} + g(t)\mathbf{j}$$

and

$$\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k},$$

are vector-valued functions or simply vector functions. As shown, for a given value of the parameter, say t_o, the vector $r(t_o)$ is the position vector of a point P on a curve C. In other words, as the parameter t varies, we can envision the curve C being traced out by the moving arrowhead of r(t).

Circular Helix

Graph the curve traced by the vector function

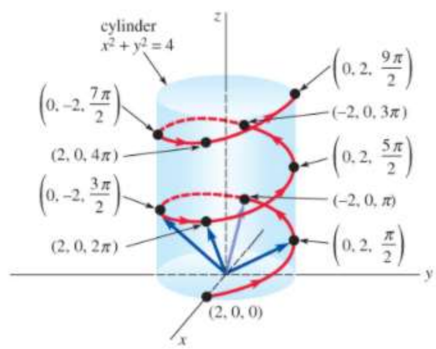
$$\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + t\mathbf{k}, \quad t \ge 0.$$

The parametric equations of the curve are $x = 2\cos t$, $y = 2\sin t$, $z = t$. By eliminating the parameter t from the first two equations:

$$x^2 + y^2 = (2\cos t)^2 + (2\sin t)^2 = 2^2$$

we see that a point on the curve lies on the circular cylinder $x^2 + y^2 = 4$. As seen and the accompanying table, as the value of t increases, the curve winds upward in a spiral or circular helix.

t	x	y	z
0	2	0	0
$\pi/2$	0	2	$\pi/2$
π	-2	0	π
$3\pi/2$	0	-2	$3\pi/2$
2π	2	0	2π
$5\pi/2$	0	2	$5\pi/2$
3π	-2	0	3π
$7\pi/2$	0	-2	$7\pi/2$
4π	2	0	4π
$9\pi/2$	0	2	$9\pi/2$



special case of the vector function:

$$\mathbf{r}(t) = a\cos t\,\mathbf{i} + b\sin t\,\mathbf{j} + ct\,\mathbf{k}, \quad a > 0, \quad b > 0, \quad c > 0,$$

which describes an elliptical helix. When a = b, the helix is circular. The pitch of a helix is defined to be the number $2\pi c$.