COMPLEX NUMBERS

You have undoubtedly encountered complex numbers in your earlier courses in mathematics. When you first learned to solve a quadratic equation $ax^2 + bx + c = 0$ by the quadratic formula, you saw that the roots of the equation are not real, that is, complex, whenever the discriminant $b^2 - 4ac$ is negative. So, for example, simple equations such as $x^2 + 5 = 0$ and $x^2 + x + 1 = 0$ have no real solutions.
For example, the roots of the last

equation are $-\frac{1}{2} + \frac{\sqrt{-3}}{2}$ and $-\frac{1}{2} + \frac{\sqrt{-3}}{2}$. If it is assumed

that $\sqrt{-3} = \sqrt{3}\sqrt{-1}$, then the roots are

written $-\frac{1}{2} + \frac{\sqrt{3}}{2}\sqrt{-1}$ and $-\frac{1}{2} - \frac{\sqrt{3}}{2}\sqrt{-1}$.

Around the time that complex numbers were gaining some respectability in the mathematical community, the symbol i was originally used as a disguise for the embarrassing symbol $\sqrt{-1}$. We now simply say that i is the imaginary unit and define it by the property $i^2 = -1$. Using the imaginary unit, we build a general complex number out of two real numbers. The imaginary part of $z = 4 - 9i$ –9, not –9i.

The real number x in $z = x + iy$ is called the real part of z; the real number y is called the imaginary part of z. The real and imaginary parts of a complex number z are abbreviated Re(z) and Im(z), respectively. For example, if $z = 4 - 9i$, then Re(z) = 4 and Im(z) = –9. A real constant multiple of the imaginary unit is called a pure imaginary number. For example, $z = 6i$ is a pure imaginary number. Two complex numbers are equal if their real and imaginary parts are equal. Since this simple concept is sometimes useful, we formalize the last statement in the next definition.

Complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are equal, $z_1 = z_2$, if

$$\mathrm{Re}(z_1) = \mathrm{Re}(z_2) \quad \text{and} \quad \mathrm{Im}(z_1) = \mathrm{Im}(z_2).$$

A complex number $x + iy = 0$ if $x = 0$ and $y = 0$

Complex numbers can be added, subtracted, multiplied, and divided.

Addition: $z_1 + z_2 = (x_1 + iy_1) + (x_2 + iy_2) = (x_1 + x_2) + i(y_1 + y_2)$

Subtraction: $z_1 - z_2 = (x_1 + iy_1) - (x_2 + iy_2) = (x_1 - x_2) + i(y_1 - y_2)$

Multiplication:
$$z_1 \cdot z_2 = (x_1 + iy_1)(x_2 + iy_2)$$
$$= x_1x_2 - y_1y_2 + i(y_1x_2 + x_1y_2)$$

Division:
$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2}$$
$$= \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i\,\frac{y_1x_2 - x_1y_2}{x_2^2 + y_2^2}$$

The familiar commutative, associative, and distributive laws hold for complex numbers.

Commutative laws: $\begin{cases} z_1 + z_2 = z_2 + z_1 \\ z_1z_2 = z_2z_1 \end{cases}$

Associative laws: $\begin{cases} z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3 \\ z_1(z_2z_3) = (z_1z_2)z_3 \end{cases}$

Distributive law: $z_1(z_2 + z_3) = z_1z_2 + z_1z_3$

In view of these laws, there is no need to memorize the definitions of addition, subtraction, and multiplication. To add (subtract) two complex numbers, we simply add (subtract) the corresponding real and imaginary parts. To multiply two complex numbers, we use the distributive law and the fact that $i^2 = -1$.

Example!

If $z_1 = 2 + 4i$ and $z_2 = -3 + 8i$, find (a) $z_1 + z_2$ and (b) z_1z_2.

a. By adding the real and imaginary parts of the two numbers, we get

$$(2 + 4i) + (-3 + 8i) = (2 - 3) + (4 + 8)i = -1 + 12i.$$

b. Using the distributive law, we have

$$\begin{aligned}(2 + 4i)(-3 + 8i) &= (2 + 4i)(-3) + (2 + 4i)(8i) \\ &= -6 - 12i + 16i + 32i^2 \\ &= (-6 - 32) + (16 - 12)i = -38 + 4i.\end{aligned}$$

There is also no need to memorize the definition of division, but before discussing that we need to introduce another concept, next time. Cheers!

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