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A Study on Configuration of Propellers for Multirotor-like Hybrid Aerial-Aquatic Vehicles

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***Abstract*— This paper presents a study on the configuration of propulsion systems for multirotor-like hybrid vehicles that travel in both, aerial and aquatic medium, hereafter called Hybrid Unmanned Aerial Underwater Vehicles (HUAUVs). Following, we evaluate the impact of diameter and shape of propellers, and the medium in which the robot moves, on thrust generation and energy consumption. Sets of counter-rotating propellers (CRPs) and single propellers propulsion systems were experimentally evaluated in air and under the water. One of our conclusions is that aerial counter-rotating propellers, used in the state-of-the-art hybrid vehicles, are relatively inefficient in the air, and *even more in the water*, and this energy loss must be taken into account during the platform project. Furthermore, we also demonstrate that aquatic screw-like propellers are more efficient in the water than aerial ones, equally contradicting the current literature. Moreover, since aquatic propellers provide more thrust with smaller diameters, they improve our capacity to miniaturize hybrid vehicles.**

Fig. 1. Our Hybrid Unmanned Aerial Underwater Vehicle (HUAUV) equipped with aerial (above) and aquatic (below) propellers.

I. INTRODUCTION

The interest in hybrid aerial-aquatic platforms has grown in recent years, especially in robotic applications [1]. The capability of navigating in such diverse environments, like air and water, using the same vehicle with minimal changes in its mechanical settings offers advantages in many applications, ranging from environmental monitoring to maintenance of submerged structures. Among different classes of platforms adopted for this kind of hybrid system, rotary-wing vehicles (helicopters and multi-rotors) have recently been used in many projects [2], basically due to their simplicity of mechanical construction and ease operation. However, the design and configuration of propellers is a complex and important problem to be addressed in such cases.

The literature has demonstrated that using the same propulsion system (motor and propeller) in air and water is not a good option since it will be inefficient in at least one of these media [3]. However, it only focuses on forwarding motion analyses. Furthermore, hovering mode is constantly neglected, though it is an important operation condition for rotary-wing vehicles. For that reason, we studied in this paper the use of aerial and aquatic propellers for multirotor-like hybrid vehicles to improve efficiency in our underdevelopment Hybrid Unmanned Aerial Underwater Vehicle (HUAUV), illustrated in Fig. 1. Our main goal is to evaluate thrust and power consumption on air and water movement, basically demonstrating that current designs of propulsion systems in the state-of-the-art literature are inefficient.

In this paper, we present *two main contributions*:

1) we show that counter-rotating propellers (CRPs) interfere with each other reducing the thrust generated by consumed power, which affects the robot in the hovering mode. Thus, the use of counter-rotating propellers in this kind of configuration is not an interesting option in the air, and *especially underwater*.
2) we demonstrate based on experimental evaluation the use of aquatic screw-like propellers in the water is more efficient than aerial propellers. Moreover, the size of the aquatic propellers is smaller. These facts improve our capacity to build a miniaturized efficient hybrid vehicle.

We have not found any paper discussing the deprecate effects of CRPs in underwater drones, nor any comparison of aerial and water propellers for hybrid applications. On the contrary, some of existing platforms use CRPs aerial propellers in both media. Unlike other papers in the state-of-the-art literature, our goal is to design a propulsion system capable of presenting the best possible efficiency in both, water and air. Although, there is a clear trade-off constraint involved. This paper is the first step in this direction, once there are still many characteristics to consider and analysis to be applied.

The remainder of this paper was structured as follows. In Section II we present a literature review, especially discussing common propulsion systems or hybrid navigation. In Section III we formalize the problem and discuss some aspects of our analysis. Experimental results in Section IV show the influence of CRPs and propeller pitch and size for different

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sets of aerial and aquatic propellers in both, air and water. Finally, in Section V, we draw some conclusions and discuss avenues for future investigations.

II. RELATED WORK

As the authors of [2] presented in their extensive survey, most of the papers concerning *hybrid aquatic-aerial amphibious* platforms have been developed only in the last decade. Moreover, many of those applications are based on fixed-wing designs. However, recent works seem to concentrate on rotary-wing or even bio-inspired concepts, such as flying-fishes, gannets or jellyfishes. One of the main conclusions of that study was that technical challenges, like fuselage shape, structure design, and actuators layout, will be the most significant problems to be addressed in future works, since a "fully-featured" HUAUV has not yet been developed [4].

In our previous work [3], we have first proposed the concept of a hybrid vehicle based on quadrotor-like platforms. Aerial and aquatic propellers have been designed according to previous specifications, and we have presented dynamic models and controller for the navigation of the robot. Furthermore, in [5], we have designed robust switched control strategies to keep the attitude stability of the platform under medium change (from water to air or vice-versa). In both papers, however, we have presented only simulated results.

Other concepts of HUAUVs based on multirotor-like platforms have been published ever since [6], [7], [8], [9], but the authors of those studies have minimally discussed a basic and important feature in this kind of project: *the hybrid propulsion system*. This is the most important design feature to be addressed when focusing on efficiency issues in both air and water.

Tests of aerial propellers with different pitch angles, running in air and underwater, were presented in [10]. The experiments showed that the angles for best performance in both media are quite different, in a way that a sub-optimal propeller must be chosen to operate in a hybrid vehicle.

The same conclusion was observed by the authors of [11], where they proposed a multimodal propeller system allowing efficient underwater propulsion for a fixed-wing micro aircraft. Like in our previous papers, their prototype was built based on the concept of reversing the motor direction, making the mechanical design more simple. They have demonstrated, by simulated and real experiments, that vehicles with hybrid characteristics must have two independent optimized propulsion systems, one for each medium.

The authors of [8], [12] have firstly built and tested a multirotor aerial-aquatic prototype in an octa-quadcopter configuration, using eight aerial propellers for thrust generation in CRPs configuration. They claimed that the use of CRPs and respective components for the exclusive use under the water in this kind of configuration (as we did in [3]) would be of little use during air navigation, which is true. However, as we will demonstrate, there are some problems with the use of CRPs both in air and water that made them inefficient due to the high power consumption and interference of blades. Furthermore, the extra weight of this specialized underwater system is not significant.

The authors of [9] try to solve this problem by introducing a ballast system to regulate the underwater depth of the platform. This active buoyancy control mechanism eliminates the use of extra propellers and motors, but presents some disadvantages, such as higher drag, added mass and added inertia, which may also increase power consumption and make dynamic slower. The time spent in the transition process (when water is entering or leaving the cylinder) may also delay the fulfillment of the mission, and the chamber also features an extra weight during the flight.

Concerning these issues, we have studied some characteristics of the classical propellers theory of helicopters and vessels, and draw some experiments to demonstrate that, at least in hovering mode, the project of propulsion systems for hybrid vehicles must take into account, separately, aerodynamic and hydrodynamic constraints.

III. BACKGROUND

A. Single propellers

The theory applied for design and analysis of propellers are separated in two parts: *forward flight*, when the aircraft (or vessel) uses the rotor to provide forward thrust; and *vertical flight*, when the helicopter (or submarine) uses the rotor in hovering mode to generate lift force.

Although for multirotor platforms, the forward flight is an important issue, in this paper, we are more interested in hovering behavior, since it is one of the main characteristics of such systems. Most of our analysis is based on the *momentum theory for helicopters* [13], which is an approach for the aerodynamic study of rotary-wing vehicles.

We begin by modeling the thrust force T and the consumed power Q in terms of the propeller rotation speed ω. According to [14], thrust (γ_t) and power (γ_q) coefficients for rotors in hovering mode can be given by:

$$\gamma_t(\omega) = \frac{T}{\rho\pi R^2 \left(\omega R\right)^2}, \quad \text{and} \tag{1}$$

$$\gamma_q(\omega) = \frac{Q}{\rho\pi R^2 \left(\omega R\right)^3}, \tag{2}$$

respectively, where ρ is the density of the medium and R is the blade radius.

In the vast literature, the efficiency of propellers in forwarding flight conditions is normally described in terms of the forward speed of the vehicle (in other words, the flow velocity of air or water). In hovering mode, however, the most evaluated efficiency metric is the Figure of Merit (FOM), M. It is given by the ratio between power induced by the thrust force and real power consumed. Formally speaking,

$$M = \frac{T^{\frac{3}{2}}}{\sqrt{2\rho\pi R^2}\, Q} = \frac{\left(\gamma_t\right)^{\frac{3}{2}}}{\sqrt{2}\,\gamma_q}. \tag{3}$$

Since rotary-wing platforms spend most of their time and energy in hovering flight mode, propeller designers generally focus their work to maximize the M index. A value of

$M = 1.0$ indicates that the entire energy of the system is converted in thrust and no force is lost. However, since no ideal propellers exist in the real-world, rotor designers generally try to reach the best possible $M \approx 0.8$ [15].

B. Contra-rotating propellers

Many platform projects use propulsion systems with two coaxial counter-rotating propellers [2], [8], [16] to increase payload and control stability, reducing weight and size. However, according to the literature, the design and evaluation of such configuration are generally more complicated than single rotors [17]. Seeking to make this task more simple, several assumptions are normally considered [18]. Among them, the most important are:

i) both *upper* and *lower* propellers rotate almost at the same RPM, the maximum difference was less than 5%;
ii) both propellers are moderately loaded;
iii) dynamic effects can be neglected;
iv) and self and mutual interference of propellers blades can also be neglected.

An important point to be highlighted is the separation distance between propellers. [17] recommends to put them as close as possible, to improve the recovery of energy loss from upper to lower blades. In contrast, however, as closer the propellers are, higher is the decrease of pressure and lower is the lift force generated. Beyond that, the diameter of the propellers (that not necessarily must be equal) is strongly dependent on this separation.

These and other questions make the design of CRPs more difficult, but, as a more complete theory is still under development, in our analyses we have also used assumptions i) to iii). However, as we will demonstrate with experiments, assumption iv) is not reasonable, once we have identified a significant loss of energy.

Concerning these principles, it is possible to define aerodynamic coefficients γ_t and γ_q for the coaxial system, according to [18], as:

$$\gamma_t(\omega) \approx \frac{T_{\text{upp}} + T_{\text{low}}}{\rho \pi R^2 \left(\omega R\right)^2} \quad \text{and} \tag{4}$$

$$\gamma_q(\omega) \approx \frac{Q_{\text{upp}} + Q_{\text{low}}}{\rho \pi R^2 \left(\omega R\right)^3}, \tag{5}$$

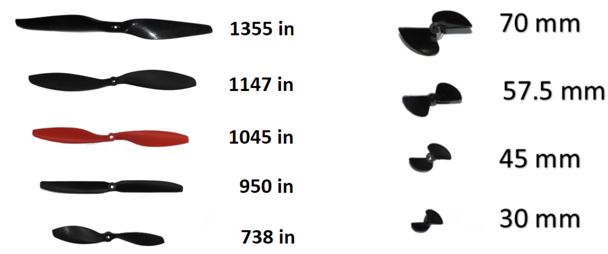
where T_{upp} and T_{low} are thrust forces, and Q_{upp} and Q_{low} are the power consumed by *upper* and *lower* propellers, respectively. With take it as an approximation, since there is an interference factor to be concerned. In that way, let us redefine equations as

$$\gamma_t(\omega) = \frac{T_{\text{upp}} + T_{\text{low}} - \Delta T}{\rho \pi R^2 \left(\omega R\right)^2} \quad \text{and} \tag{6}$$

$$\gamma_q(\omega) = \frac{Q_{\text{upp}} + Q_{\text{low}} + \Delta Q}{\rho \pi R^2 \left(\omega R\right)^3}, \tag{7}$$

where ΔT and ΔQ are the thrust lost and the extra power consumed due to the interference effect.

Another implicit assumption made here is that both propellers are geometrically and physically equal. Such simplifications also allow us to use the same Eq. (3) to estimate the FOM for a pair of coaxial propellers [16].



(a) aerial propellers (b) aquatic screws

Fig. 2. Sets of evaluated propellers in our experiments.

IV. EXPERIMENTAL RESULTS

In this section, we describe the framework used in our aerial and aquatic tests. The experimental analysis was based on the evaluation of aero/hydrodynamic coefficients of several propellers as presented in Fig. 2.

A. Experimental framework

The thrust generated by the propulsion system was collected using a RS232 digital force gauge by the *Lutron Eletronic* FG-20KG, capable of measure forces ranging from 0.05 to 196.10 N, resolution of 0.05 N and accuracy of $\pm 0.5\%$. The rotation speed was measured by using a contactless tachometer model *VC6236P*, produced by *TT Technic*, and capable of estimate the rotation speeds between 2.5 to 999,999 rpm with resolution of 0.1 rpm and accuracy of $\pm 0.05\%$. The voltage and current transducer were the model *DVL 250* and *HTR 500-SB*, respectively, both produced by the *LEM SA*. The HTR 500-SB is an open-loop Hall effect current transducer, whose measurements range from 0 to 500 A, with accuracy $< \pm 2\%$. Also, DVL 250 is a voltage transducer capable to measure up to 375 V with accuracy 0.5%.

The brushless DC motor chosen to rotate the propellers in the tests is the Gartt 3508 700kV, which can release the power of 460 W. An ESC (Electronic Speed Controller) is adopted to control the brushless motor, we used the Afro ESC 30A with SimonK firmware.

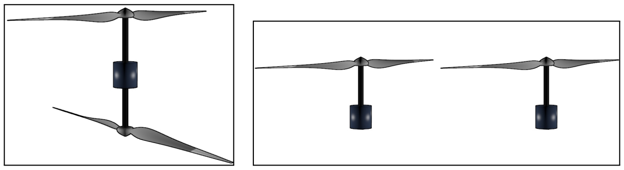
These sensors and the thrusters were mounted in a home-made device to keep them stable during experiments.

B. Interference of contra-rotating aerial propellers

In the first experiment, we have compared three different propellers running in the air to verify the existence of disturbance caused by a coaxial contra-rotating configuration. Figure 4 shows our framework setup employed for aerial runs.

We have evaluated two-blade propellers, as illustrated at Fig. 2(a), with different diameters (10 in, 11 in and 13 in) in two basic arrangements: *contra-rotating coaxial* alignment and *parallel side-by-side* alignment. They are represented at Fig. 3. At Fig. 3(a), one can see CRPs, as commonly used in octa-copters and other platforms [8], [12], in this case the distance used between the propellers was 15 cm. At Fig. 3(b), we have disposed both propellers side-by-side, in order to operated both of them without any interference between blades.

Our framework setup for CRPs is presented at Fig. 4.



(a) contra-rotating aerial configuration.

(b) parallel aerial configuration.

Fig. 3. Two basic arrangements for our comparative tests: (a) contra-rotating coaxial alignment and (b) parallel side-by-side alignment.

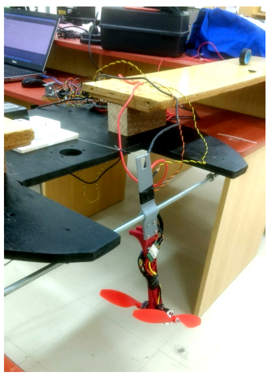


Fig. 4. Framework used in the evaluation of aerial counter-rotating propellers in the air.

TABLE I

PERCENTAGE OF THRUST FORCE LOST FOR DIFFERENT PROPELLERS IN CRPS TESTS COMPARED TO PARALLEL TEST IN THE AIR.

speed [rpm]		2127	2529	2930	3332	3734	4135	4537	
		thrust force loss							**mean**
prop. diameter	10in.	8.0	10.1	11.5	12.5	13.3	13.9	14.4	**11.96**
	11in.	4.0	7.7	12.7	16.2	18.8	20.8	22.3	**14.64**
	13in.	13.0	13.7	14.2	14.5	14.8	15.0	15.1	**14.33**
								Total	**13.64**

Results are presented in Fig. 5, where continuous lines represent the parallel sets and dashed lines are the coaxial sets. We can see the thrust force generated by the three sets of propellers in Fig. 5(a). As predicted by the literature, contra-rotating propellers really affect the lift force of static rotary-wings. Our experiments demonstrated that such interference ranges from 4.0% to 22.3%, with average of 13.64%, as presented in Table I. The same power was applied for all tests, as presented in Fig. 5(b), proving the energy was lost in the coaxial contra-rotating configuration. FOMs of Fig. 5(d) for all propellers also shown the inefficiency of such configuration when compared to parallel rotors. As previously discussed, higher values of M provides better hover conditions.

Next, we decide to evaluate the effect of CRPs also for aerial propellers inside the water. Then, we have executed experiments with a 7*in* aerial propeller in the air and underwater, to compare the losses of the operation in both media. Figures 6 and 7 presents the results for parallel and contra-rotating configurations in the aerial and aquatic environments, respectively. Once again it is possible to perceive that, also in the water, the contra-rotating configuration affects the system performance. In fact, in the underwater scenario, the loss was more significant, approaching 36 % in the worst-case.

C. Aerial versus aquatic propellers under the water

After contra-rotating tests, we are interested in evaluating the behavior of the aerial propellers in the aquatic medium to know the static performance in hover mode compared to aquatic propellers. The same setup of previous experiments is employed, this time with propellers inside a pool filled with freshwater, as it can be seen at Fig. 8.

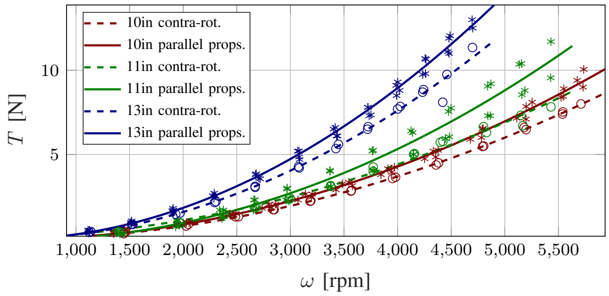
At this point, we have employed 4 aquatic propellers with a diameter of 30, 45, 57.5 and 70*mm* Fig. 2(b), and 3 aerial propellers with diameter of 7, 9 and 11*in*, approximately. All propellers have been operated by the same set of motors and ESCs, without any mechanical reduction. The aerial propeller with a diameter of 13*in* is not able to properly run underwater using our motor and ESC.

Fig. 9 presents the (T/Q) ratio for underwater tests, normalized by the area πR^2 to the rotating propellers. This normalization allows us to evaluate the performance of such propulsion systems (with so many different profiles) independently of the propeller radius. Two interesting observations can be made here. The first one is that all aquatic propellers are much more efficient than aerial ones maybe because of geometric characteristics like format and angle of attack, among others. Although this sounds obvious, it is important to notice that, as the aquatic screws are much smaller, it is possible to further increase efficiency without major impact on robot size, by enlarging such propellers. The second one it that, aquatic propellers present similar profiles (the same for aerial ones) as the rotation grows, showing a less abrupt decay when compared to the aerial ones.

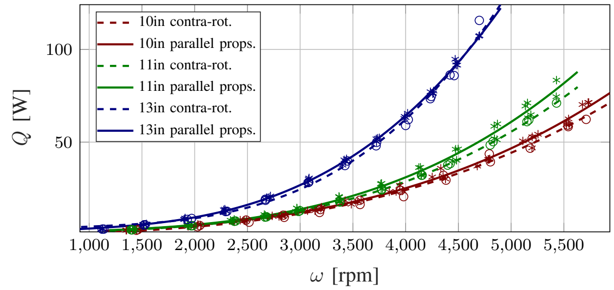
On the other hand, Fig. 10 provides the FOM for the same compared propellers. Concerning this metric, once more it is possible to see the high efficiency of aquatic propellers under the water. We can also verify that the higher the propeller radius, the higher is the FOM. The graph also shows the difficulty of aerial propellers in keeping their efficiency in the water as ω grows. Unlike the aquatic propellers that increase their FOM proportional the diameter, the aerial propellers decrease their performance since the large drag in the water.

V. CONCLUSION AND FUTURE WORK

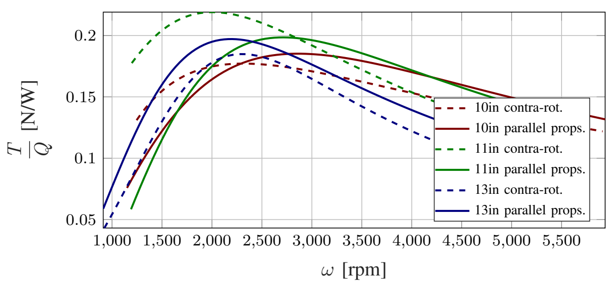
In this paper, we presented an experimental comparative analysis of aerial and aquatic propellers in single and counter-rotating propellers configuration to generate thrust force on multirotor-like hybrid vehicles. Our first conclusion was that, concerning static assays and hover mode theory, contra-rotating aerial propellers are about 14 % less efficient in the



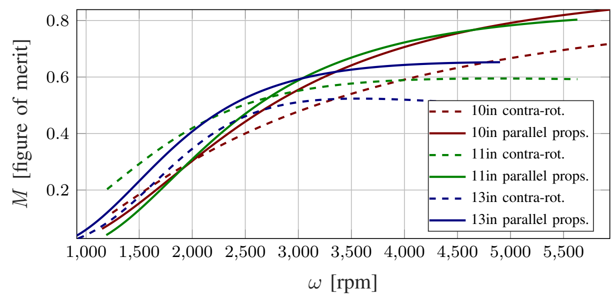
(a) total thrust



(b) power

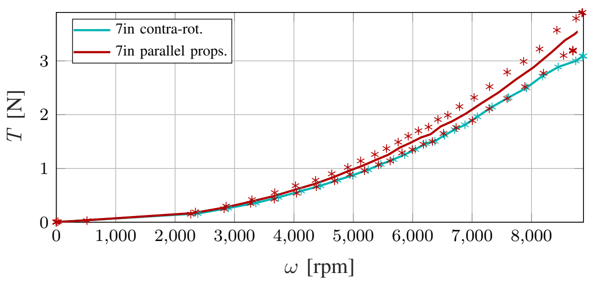


(c) thrust per power

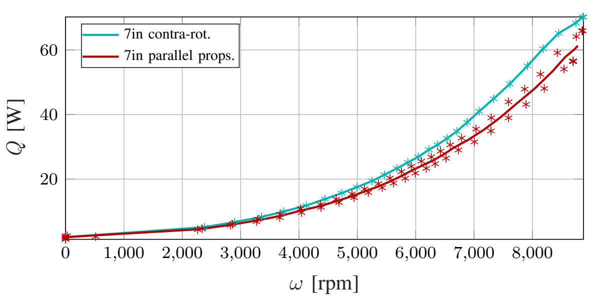


(d) figure of merit

Fig. 5. Interference of aerial propellers in the air – dashed lines represent the interpolation of contra-rotating rotors, while continuous lines represent parallel rotors: as claimed at literature, (a) interference of contra-rotating propellers reduces the total thrust of the system (about 14%) in our tests, (b) even applying the same power; (c) the thrust per power relation and (d) the figure of merit shows the inefficiency of such configurations, since dashed lines are below continuous ones.



(a)



(b)

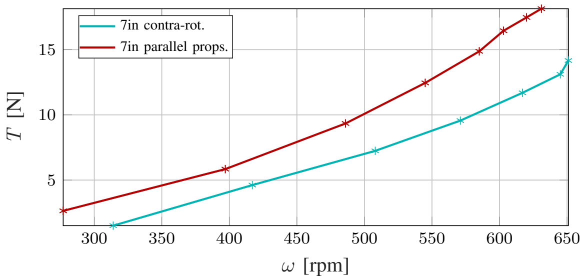
Fig. 6. Interference of aerial propellers in *aerial tests*: blue lines represent the data of contra-rotating rotors, while red lines represent parallel rotors.

air and more than 30 % under the water when compared to single parallel propellers. We have demonstrated by experiments with different propellers that the interference both blades causes a significant loss of thrust (and energy). Such interference is normally ignored by the theory of contra-rotating propellers design, but it is quite relevant when compared with the payload cost of water propellers, for example.

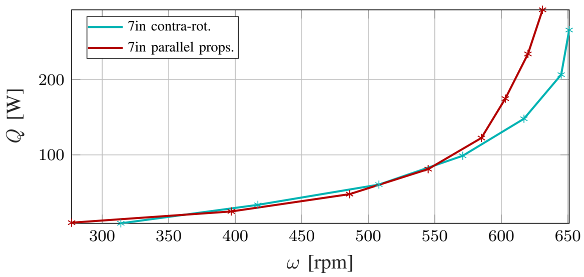
Also, we have verified with experiments that aerial propellers can be used in the propulsion system to generated force in underwater hovering mode. However, aquatic screw-like propellers are much more efficient for underwater navigation. Aquatic propellers also allow high thrust generation with smaller size blades, improving miniaturization advantages of hybrid multirotor systems.

Based on these two premises, it is possible to conclude that the use of CRPs in both, air and water medium, is quite inefficient. Then, new devices and design propulsion systems for hybrid vehicles must still be investigated. Like [11] has done for the forward flight mode case, we have demonstrated that, also for the hovering case, an aerial/aquatic hybrid vehicle must have specific propulsion systems for each medium.

As future work, we shall extend our analysis to forwarding flight (and dive) conditions, to verify if our conclusions are still valid when the robot is moving at higher speeds. Since aerodynamic theory for both cases is quite similar, new experiments may possible confirm this hypothesis. We can also take into account the characteristics of our proposed propulsion system when projecting new control laws for the navigation of our HUAUV. We had addressed this issue in our previous work only concerning attitude stabilization, but position control and high-level planning must also be studied soon.



(a)



(b)

Fig. 7. Interference of aerial propellers in *underwater tests*: blue lines represent the data of contra-rotating rotors, while red lines represent parallel rotors. There is a significant lost of efficiency in contra-rotating propellers in both: air and water.

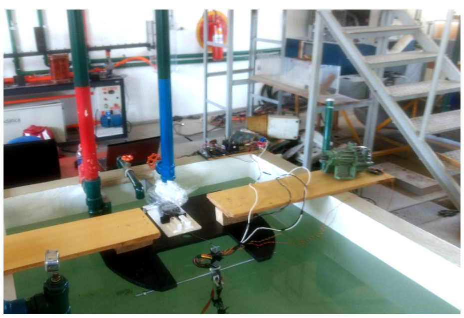


Fig. 8. Underwater tests for the evaluation of aerial counter-rotating propellers.

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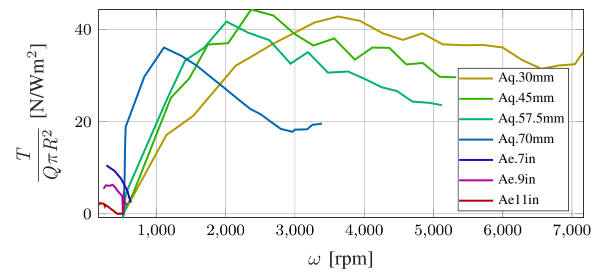

Fig. 9. Thrust/Torque ratio $\left(T/(Q\pi R^2)\right)$ for aerial and aquatic propellers under the water, normalized by the area of the rotating propeller.

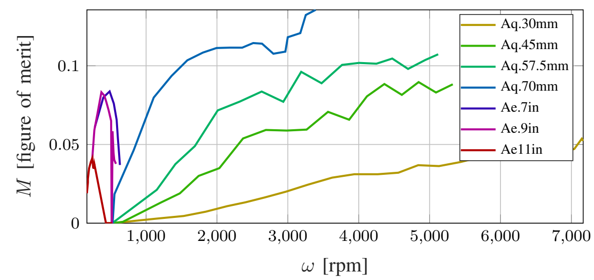

Fig. 10. FOM for aerial and aquatic propellers under the water.

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