Vibration Absorber of a Multi-story Building Under Base Excitation

6530116821 Nattakit Chaijaroenmaitree

6532002221 Koranit Sanalaksna

6532017721 Kantanat Phochanasombut

6532046921 Javis Ratanapeanchai

6532122921 Punnawich Pongsriassawin

6532161321 Witsanuporn Promsirinimit

Presented to

Assoc. Prof. Dr. Chanat Ratanasumawong

8 May 2025

This project is a part of the Introduction to 2103433 Introduction to Mechanical Vibration

Department of Mechanical Engineering, Faculty of Engineer, Chulalongkorn University

Abstract

Vibration Absorber is widely used for vibration absorption in many tall buildings as it reduces the effect of vibration excitation the building must sustain. In this project investigates the implementation of a tuned mass damper (TMD) system to reduce vibrations in a multi-story building subjected to base excitation, such as an earthquake. The King Power Mahanakhon building was selected as the case study. To analyze its dynamic behavior, the structure was simplified into a lumped mass-spring model consisting of 10 stories, each with an estimated mass of 2,000,000 kg, representing the RC core wall of the actual 79-story building. Using classical structural dynamics, the mass and stiffness matrices were formulated based on Eurocode material properties. A modal analysis was then performed to obtain the natural frequencies and mode shapes, revealing the fundamental frequency to be approximately 17 rad/s. Rayleigh damping with a 0.05 damping ratio was applied to capture energy dissipation realistically.

To simulate base excitation, it was introduced using sinusoidal signals, and the system's displacement response was computed using MATLAB's ode45 solver. Results showed significant amplification in the fundamental mode, with the top floor experiencing up to 5 times the displacement of the ground floor. The TMD with a mass of 1,000,000 kg and a stiffness of 2.89×10^8 N/m was designed and attached to the top floor. The TMD effectively split the dominant resonance peak into two smaller peaks (around 14 rad/s and 20 rad/s) reducing the maximum displacement amplitude and suppressing the beating effect caused by near-resonance excitation.

Keywords: *Tuned mass dampers, Modal analysis, Multi degree of freedom vibration*

Introduction

Mahanakhon Building is one of many multi-story buildings in Thailand. The building faces vibration from wind and many sources. In this study, we will consider that the building is vibrated by an earthquake. And we will design the tuned mass damper system to help absorb the vibration.

High-rise buildings are particularly vulnerable to dynamic excitations such as earthquakes. The force can induce resonant vibrations that not only cause discomfort for occupants but also threaten structural integrity over time. One widely adopted method for mitigating such effects is the use of a Tuned Mass Damper (TMD). In this project, we analyze and implement a vibration mitigation system for the King Power Mahanakhon (Figure 1), one of the tallest buildings in Thailand.

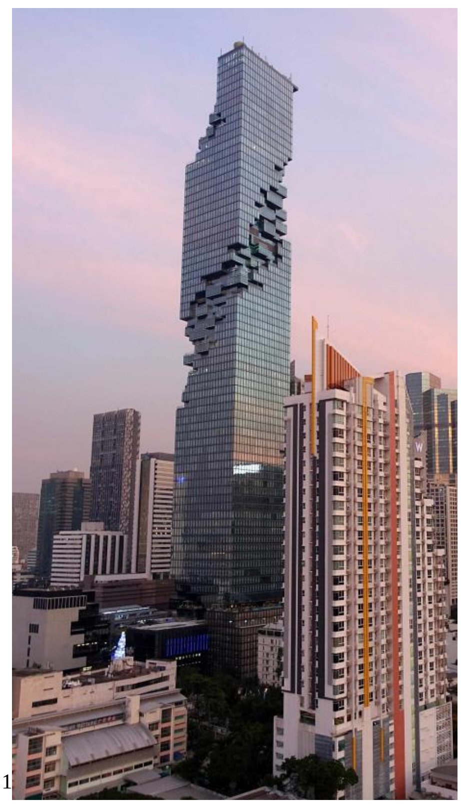
[1]


Figure 1 King Power Mahanakhon

[1] [1]

Objective

The objective of this study is to determine appropriate Rayleigh damping coefficients for modeling the dynamic behavior of a high-rise building using a multi-degree-of-freedom (MDOF) system. This information will be applied to simulate and evaluate the vibration response of the structure under dynamic loading.

Step-by-Step Methodology in this project

1. Determine Structure Parameters

Estimate or define the MDOF parameters systems:

- Mass Matrix (M)
- Stiffness Matrix (K)
- Modal frequencies (ω_1 and ω_2)

2. Apply Rayleigh Damping Modal

Rayleigh Damping formula: $C = \alpha M + \beta K$

Use this formular to find the coefficient α and β by selecting the 2 natural frequencies and derive the formular to obtain the equation: $\zeta = \frac{1}{2}(\frac{\alpha}{\omega} + \beta\omega)$ and then solve the linear system.

3. Validate Damping Ratio

Once α and β are obtained, substitute them back into the formula for other modes to ensure the damping ratio stays within a reasonable range (~5%).

If not, adjust the choice of ω_1 and ω_2

4. Implement into Numerical Modal

Use the values of M, K, and C in a time-domain or modal analysis:

- For time history analysis: $M_1\ddot{x}_1 + C_1\dot{x}_1 + K_1x_1 = F_1(t)$
- For modal analysis: Transform into modal coordinates and solve decoupled equations

Modeling and Design

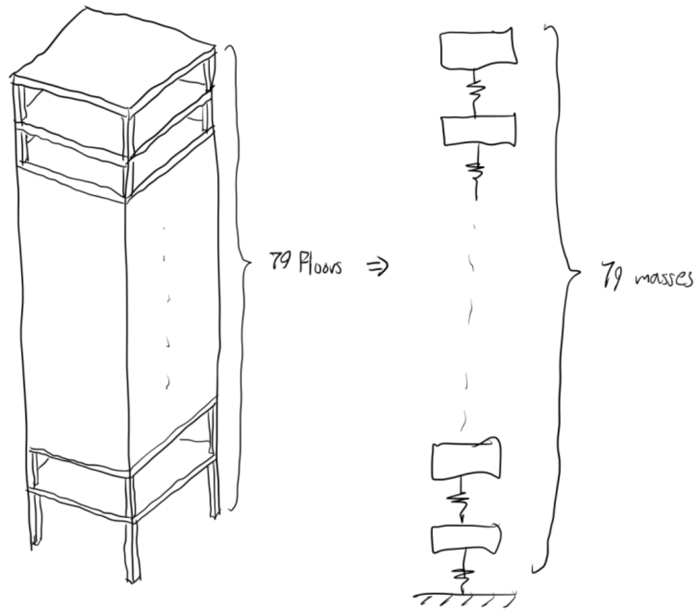


Figure 2 System Model

Due to the complexity of the real structure, we simplify the building into a 10-story lumped mass-spring system, each floor modeled as a concentrated mass and each inter-floor column as a linear spring (Figure 2). This reduction retains the key dynamic characteristics of the original 79-story building while enabling efficient numerical simulation.

To model the dynamic response of the Mahanakhon building—a 79-story skyscraper located in Bangkok—under base excitation such as an earthquake, we begin by simplifying the structure using a lumped mass model. The building has a total estimated weight of approximately 1600 MN, which corresponds to a mass of around 2 million kg per floor. Each story is approximately 4 meters in height, making the total building height over 300 meters. While the actual structure is supported by a system of 12 mega columns and a central reinforced concrete (RC) core wall measuring 23 × 23 meters, for the purpose of this simulation, we focus solely on the RC core wall as the primary source of lateral stiffness and will model only 10 stories. This simplification allows us to apply classical structural dynamics methods. The stiffness of each story is derived using the flexural stiffness formula for beams: $k = 12 E I / h^3$, where E is the modulus of elasticity, I is the second moment of area of the core wall section, and h is the story height.

According to Eurocode guidelines, the modulus of elasticity for concrete can be rounded to 30 GPa. We also apply a concrete partial material safety factor of 1.5 to account for variability in material strength and load effects and assume a steel yield strength of 500 MPa for structural reinforcement. By combining these parameters, we can establish a representative stiffness matrix K for the building, which is essential for further dynamic analysis.

Eigenvalues Problem

```
Stiffness matrix [K] in N/m:
   5.6953e+10  -2.8477e+10            0            0            0            0            0            0            0            0
  -2.8477e+10   5.6953e+10  -2.8477e+10            0            0            0            0            0            0            0
            0  -2.8477e+10   5.6953e+10  -2.8477e+10            0            0            0            0            0            0
            0            0  -2.8477e+10   5.6953e+10  -2.8477e+10            0            0            0            0            0
            0            0            0  -2.8477e+10   5.6953e+10  -2.8477e+10            0            0            0            0
            0            0            0            0  -2.8477e+10   5.6953e+10  -2.8477e+10            0            0            0
            0            0            0            0            0  -2.8477e+10   5.6953e+10  -2.8477e+10            0            0
            0            0            0            0            0            0  -2.8477e+10   5.6953e+10  -2.8477e+10            0
            0            0            0            0            0            0            0  -2.8477e+10   5.6953e+10  -2.8477e+10
            0            0            0            0            0            0            0            0  -2.8477e+10   2.8477e+10
Mass matrix [M] in kg:
     2064543          0          0          0          0          0          0          0          0          0
           0    2064543          0          0          0          0          0          0          0          0
           0          0    2064543          0          0          0          0          0          0          0
           0          0          0    2064543          0          0          0          0          0          0
           0          0          0          0    2064543          0          0          0          0          0
           0          0          0          0          0    2064543          0          0          0          0
           0          0          0          0          0          0    2064543          0          0          0
           0          0          0          0          0          0          0    2064543          0          0
           0          0          0          0          0          0          0          0    2064543          0
           0          0          0          0          0          0          0          0          0    2064543
```

Figure 3 stiffness matrix and mass matrix

```
Natural Frequency at mode 1 is 17.55 rad/s
Natural Frequency at mode 2 is 52.27 rad/s
Natural Frequency at mode 3 is 85.81 rad/s
Natural Frequency at mode 4 is 117.44 rad/s
Natural Frequency at mode 5 is 146.45 rad/s
Natural Frequency at mode 6 is 172.19 rad/s
Natural Frequency at mode 7 is 194.07 rad/s
Natural Frequency at mode 8 is 211.63 rad/s
Natural Frequency at mode 9 is 224.45 rad/s
Natural Frequency at mode 10 is 232.27 rad/s
```

Figure 4 natural frequency of 10 modes

Mode Shape

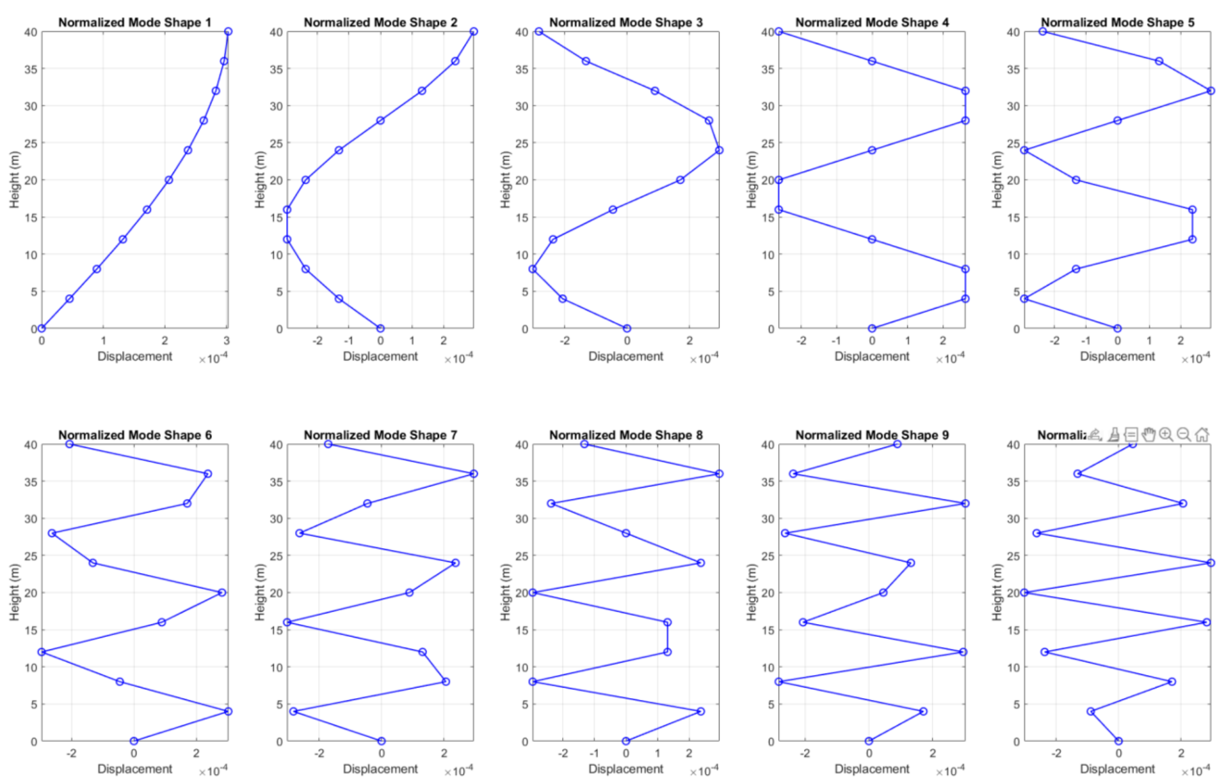


Figure 5 modes shapes of each story

The mode shapes of a 10-story building model as show in Figure 5, illustrating how each floor is displaced laterally at different natural frequencies. Mode 1 shows the entire structure swaying in the same direction with maximum displacement at the top, representing the fundamental mode and lowest frequency. As the mode number increases, more inflection points (nodes) appear, indicating more complex vibration patterns with alternating motion between floors. Higher modes correspond to higher frequencies and are less likely to be excited by typical ground motion, but they still play a role in the building's dynamic response. These mode shapes are essential for understanding resonance, designing tuned mass dampers, and assessing structural performance under seismic excitation.

The next step involves performing modal analysis to understand how the building responds to dynamic loads. The equation of motion for a multi-degree-of-freedom (MDOF) system under no external force is expressed as $M \ddot{X} + K X = 0$ where M is the mass matrix and K is the stiffness matrix. To solve this, we reformulate it as an eigenvalue problem $(K - \omega^2 M) \varphi = 0$, where ω is the natural frequency and ϕ is the corresponding mode shape (eigenvector). Solving this problem yields the modal frequencies and mode shapes that describe how the building vibrates naturally. We assume proportional (Rayleigh) damping by expressing the damping matrix as $C = \alpha M + \beta K$. A common engineering assumption for structural damping is a damping ratio $\zeta = 0.05$ (5%), which provides a reasonable approximation for concrete structures under seismic loading. This value is widely accepted in design as it balances safety, energy dissipation, and cost-efficiency—too little damping would lead to excessive vibration, while too much could reduce structural efficiency. Using this assumption and selecting two modal frequencies, we calculate the Rayleigh damping coefficients to be $\alpha = 27.68$ and $\beta = 0.0000328$, which are then used throughout the simulation.

To further analyze the response, we reduce the full 79-degree-of-freedom system to a 10-degree-of-freedom model to make the simulation computationally efficient while retaining the essential dynamic behavior. We incorporate base excitation y(t) into the system to simulate seismic activity, modifying the displacement variable from absolute x(t) to relative displacement x(t)−y(t), which reflects the motion of each floor relative to the base. The updated dynamic equation becomes

$$\mathbf{M}\ddot{\mathbf{x}}_{\mathbf{r}}(\mathbf{t}) + \mathbf{C}\dot{\mathbf{x}}_{\mathbf{r}}(\mathbf{t}) + \mathbf{K}\mathbf{x}_{\mathbf{r}}(\mathbf{t}) = -\mathbf{M}\ddot{\mathbf{y}}$$

$$\text{Given } \mathrm{X_r} = \mathrm{Ur}$$

$$\ddot{\mathbf{r}}_{\mathbf{i}}(\mathbf{t}) + 2\zeta\omega_{\mathbf{n,i}}\dot{\mathbf{r}}_{\mathbf{i}}(\mathbf{t}) + \omega_{\mathbf{n,i}}^2\,\mathbf{r}(\mathbf{t}) = \mathbf{N}_{\mathbf{i}}(\mathbf{t})\,;\mathbf{N}(\mathbf{t}) = -\mathbf{U}^{\mathrm{T}}\mathbf{M}\ddot{\mathbf{y}}(\mathbf{t})$$

$$\text{where } 2\zeta\omega_{\mathrm{n,i}} = \alpha + \beta\omega_{\mathrm{n,i}}^2 = \mathrm{U^TCU} \text{ and } \Gamma = -\mathbf{U^T M 1}$$

which is simplified and solved numerically using MATLAB's ode45 function, a robust tool for solving ordinary differential equations. For the excitation signal, we use a sinusoidal waveform with an amplitude equal to half of the El Centro earthquake record at 0.2 m, which provides a realistic but manageable input. The frequency of this base excitation is selected to match the natural frequencies of the first and second modes to examine how resonance affects the building's behavior. This allows us to study how even low-frequency ground motion can induce significant responses due to resonance effects, a critical factor in seismic design.

Displacement Response

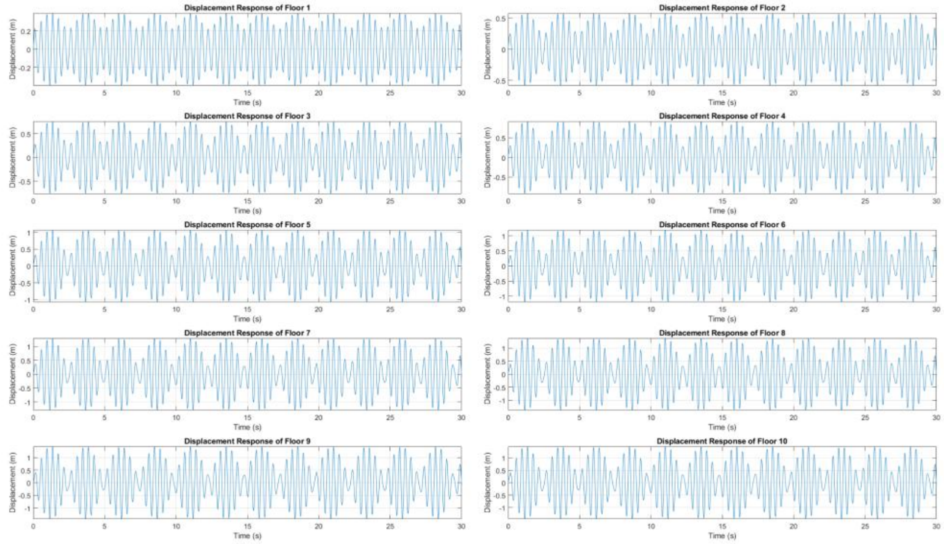


Figure 6 displacement response at frequency = 15 rad/s near ω_1

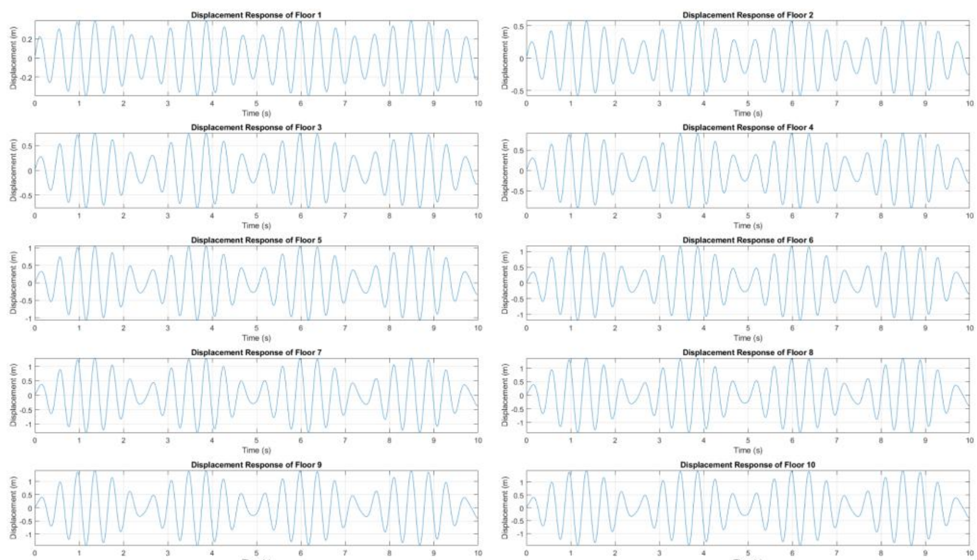


Figure 7 displacement response at frequency = 15 rad/s near ω_1

The figure illustrates the displacement response of each floor in a 10-story building model subjected to a base excitation at an angular frequency of 15 rad/s. This frequency is close to the building's first natural frequency, approximately 17 rad/s, which results in a near-resonant condition. At this excitation frequency, the response is predominantly governed by the first mode shape. As observed in the figure, the amplitude of displacement on Floor 1 is about 0.2 meters, whereas the displacement on Floor 10 reaches approximately 1.0 meters. This fivefold increase from the base to the top floor is a direct consequence of the first mode shape, in which the upper stories undergo significantly larger movements compared to the lower ones.

Moreover, the displacement time histories display a distinct beat pattern, where the amplitude fluctuates periodically over time. This beating phenomenon occurs because the excitation frequency is not exactly equal to the natural frequency but close enough to cause interference between two nearby frequencies. The system's response combines both the forced vibration from the external sinusoidal input and the free vibration governed by its natural modes. When these two are slightly detuned, constructive and destructive interference arises, producing a modulation in amplitude known as beats. The presence of this pattern clearly highlights how even a small mismatch between excitation and natural frequency can cause significant variations in structural response, which can lead to fatigue or failure if not adequately addressed through damping or absorber systems.

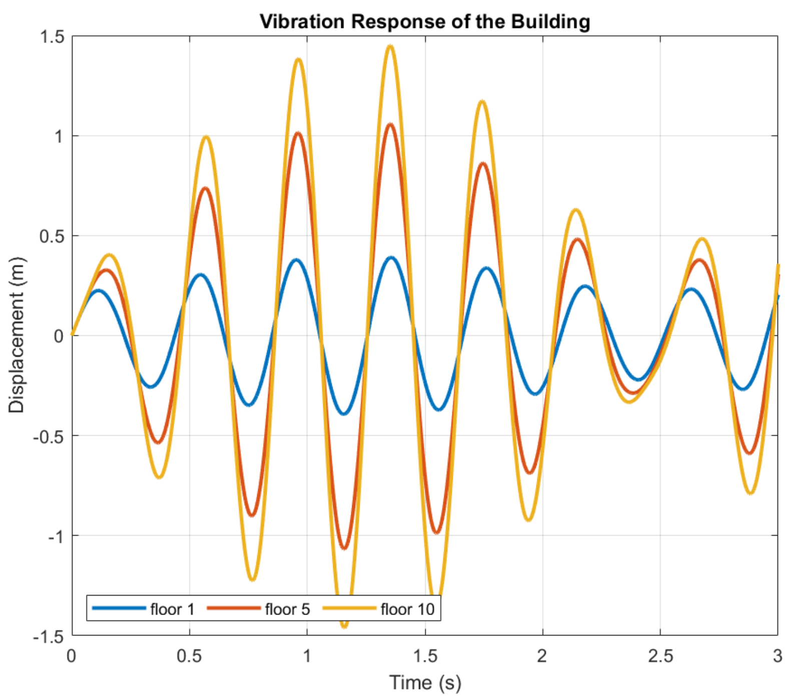


Figure 8 Vibration response at frequency = 15 rad/s near ω_1

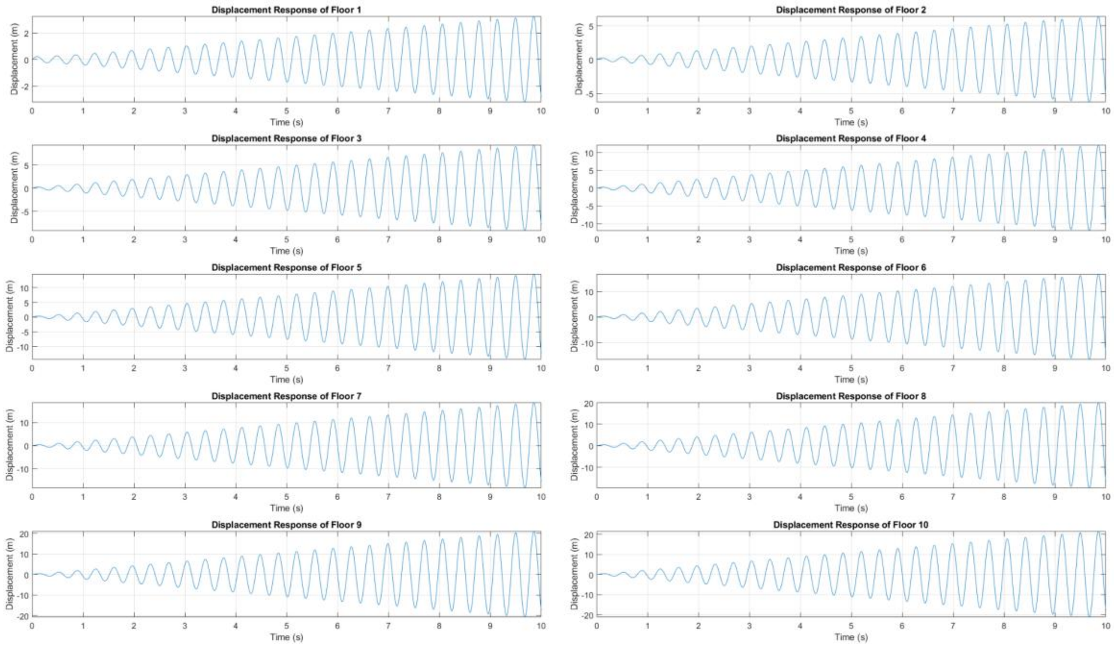


Figure 9 Displacement response at frequency = 17.5 rad/s (First mode natural frequency)

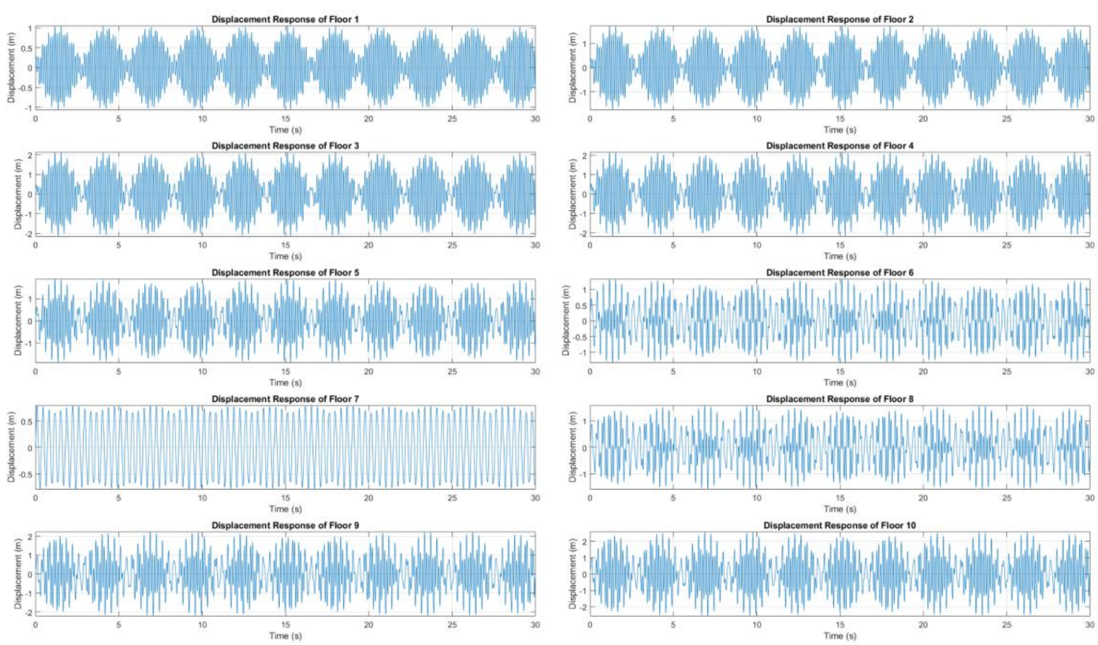


Figure 10 Displacement response at frequency = 50 rad/s near ω_2

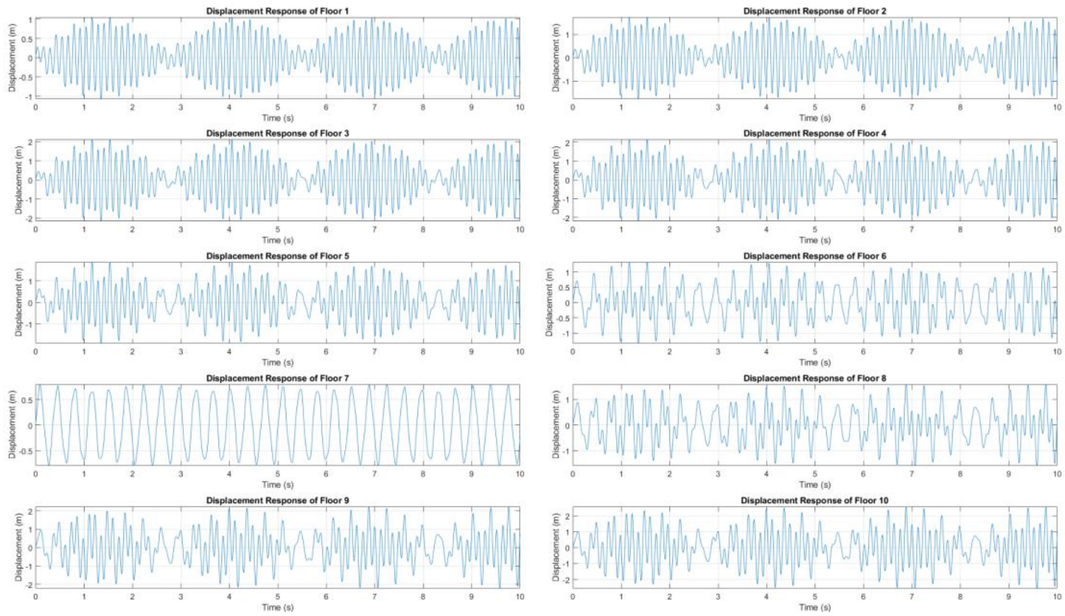


Figure 11 Displacement response at frequency = 50 rad/s near ω_2

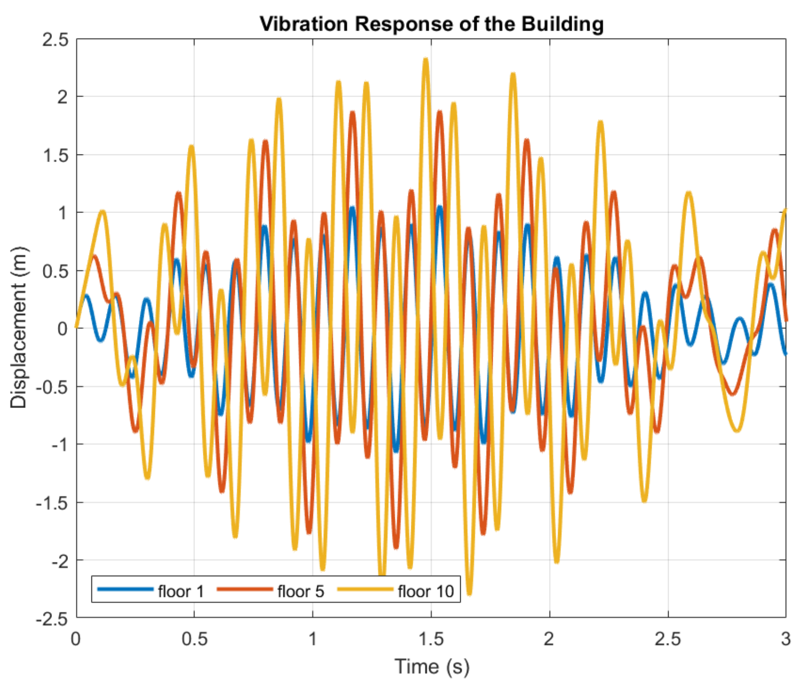


Figure 12 Vibration response at frequency = 50 rad/s near ω_2

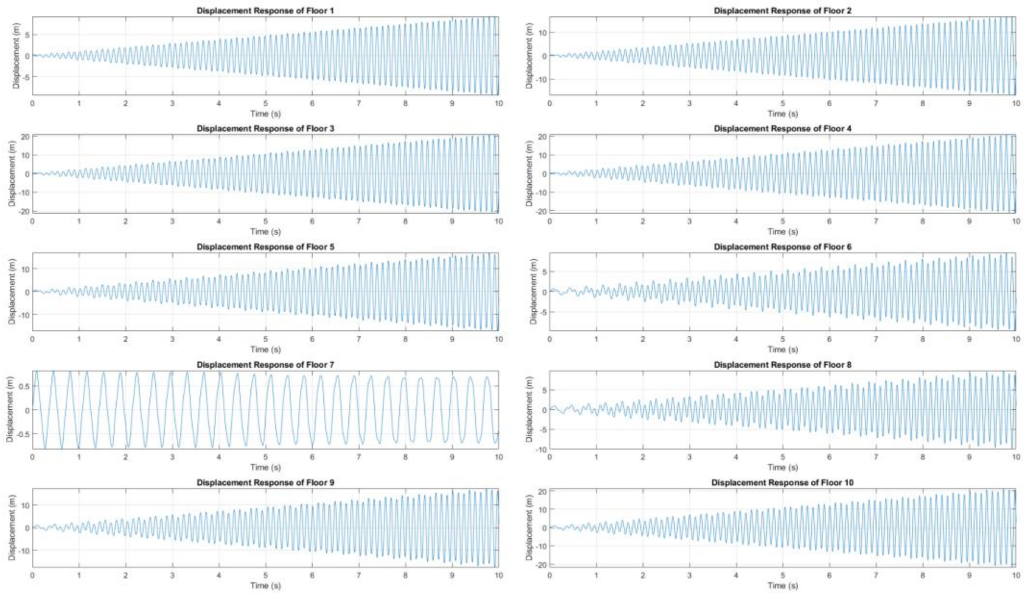


Figure 13 Displacement response at frequency = 52.3 rad/s (Second mode natural frequency)

Tuned Mass Damper (TMD)

[2]Tuned mass damper (TMD) is a vibration absorption system which comprises mass, springs, and dampers. We design the mass, the spring, and the damper to absorb the vibration of the building.


[3]

Figure 14 London Millennium Bridge - Damper beneath deck

To mitigate the potentially large displacements and accelerations caused by resonance, especially at first mode frequency, we design and implement a Tuned Mass Damper (TMD). The TMD is essentially an auxiliary system consisting of a mass-spring-damper attached to the structure, tuned to a specific natural frequency—in this case, approximately 17 rad/s, corresponding to the building's fundamental mode. Using standard absorber design equations, we calculate the required parameters for the TMD. The purpose of the TMD is to reduce the amplitude of vibrations at the tuned frequency by creating destructive interference. When the base of the building moves, the TMD oscillates out of phase with the structure, absorbing and dissipating vibrational energy that would otherwise amplify the building's motion. This technique is widely used in high-rise buildings, bridges, and towers due to its effectiveness, relatively low cost compared to structural overdesign, and ability to target specific problematic frequencies. By

[2] [2]
[3] [3]

implementing the TMD, we significantly enhance the dynamic performance of the structure, reduce occupant discomfort, and limit structural and non-structural damage during seismic events.

Vibration Suppression Methods

Tuned Mass Damper (TMD)

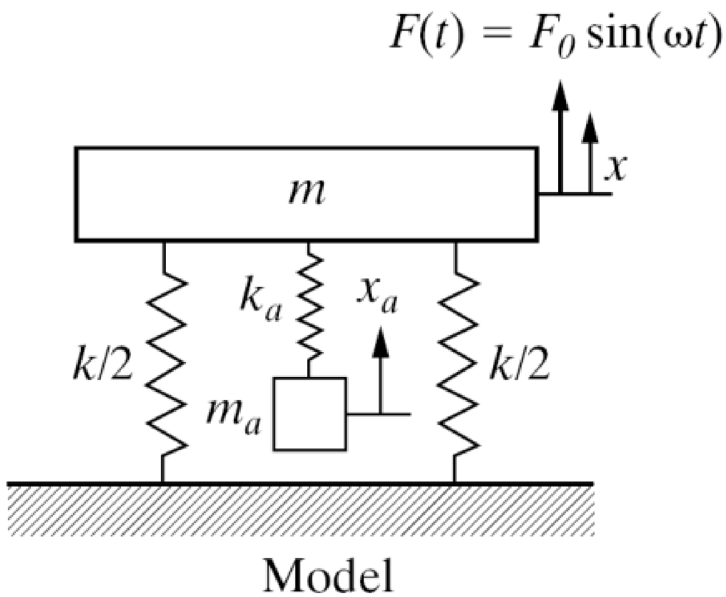


Figure 15 Vibration absorber model

<u>Part I</u> Theory Calculation

$\beta^2(\omega_n^2 / \omega_a^2)^2 - [1 + \beta^2(1 + \mu)](\omega_n^2 / \omega_a^2) + 1 = 0$

$M = 2{,}064{,}543$ kg, $\omega_a = 17$ rad/s

Choose $\beta = 1 \rightarrow \omega_n \leq 12$ rad/s, $\omega_n \geq 25$ rad/s

$1 \times (10 / 17)^4 - [\gamma + 1 + \mu] \times (10 / 12)^2 + 1 = 0$, $\mu = 0.505$ (over)

$1 \times (30 / 17)^4 - [\gamma + 1 + \mu] \times (30 / 17)^2 + 1 = 0$, $\mu = 0.625$ (over)

Choose $\mu = 0.505$ (0.6 too much)

$\mu = m_a / M$, $m_a \approx 10^6$ kg

$\omega_a = 17$ rad/s $\rightarrow k_a = 2.89 \times 10^8$

<u>Part II</u> Simulation

```
% %% ---------- CASE 1: TMD only (adds 1 DOF) ----------
ma = 1E6; % TMD mass
ka = 2.89E8; % Tuned to w = 15 rad/s
%
M1 = blkdiag(M, mt);
K1 = blkdiag(K, 0);
C1 = blkdiag(C, 0);
%
K1(n,n) = K1(n,n) + kt;
K1(n+1,n+1) = kt;
K1(n,n+1) = -kt;
K1(n+1,n) = -kt;
%
C1(n,n) = C1(n,n) + ct;
C1(n+1,n+1) = ct;
C1(n,n+1) = -ct;
C1(n+1,n) = -ct;
```

The code begins to implement a Tuned Mass Damper system, which adds an additional degree of freedom:

1. The TMD is modeled with:
    - Mass $m_a = 10^6\,kg$
    - Stiffness , tuned to a frequency of The augmented $15\,rad/s$ matrices are formed: $k_a = 2.89 \cdot 10^8\,N/m$
    - Augmented mass matrix:

$$\begin{bmatrix} M & 0 \\ 0 & M_a \end{bmatrix}$$

- Augmented stiffness matrix: K_1 has additional terms k_a connecting the TMD to the top floor
- Augmented damping matrix: C_1 has additional terms c_a for TMD damping

2. The equation of motion becomes:

$$M_1 \ddot{x_1} + C_1 \dot{x_1} + K_1 x_1 = F_1(t)$$

Where $x_1 = [x^T, x_a]^T$ includes the TMD displacement x_a

The TMD works by creating a resonance effect that counters the building's motion. When the building vibrates at its natural frequency, the TMD vibrates with an opposite phase, transferring vibrational energy away from the main structure.

Vibration Absorber

```
function rdot = eom(t,r,omega_i,zw_i,gamma_i,w)

   ay = -0.2 * w^2 * sin(w*t);

   rdot = zeros(2,1);

   rdot(1) = r(2);

   rdot(2) = -zw_i*rdot(2) - omega_i^2*r(1) + gamma_i*ay;

end
```

The vibration absorber function implements the modal equation of motion:

$$\ddot{r}_i + 2\zeta_i\omega_i r_i + \omega_i^2 = \gamma_i \ddot{x_g}(t)$$

Where:

- $\ddot{x}_g(t) = -0.2\omega^2 \sin(\omega t)$ represents the ground acceleration
- $\zeta_i\omega_i$ is the modal damping coefficient
- γ_i is the modal participation factor

Results

The simulation results provide a comprehensive view of the dynamic response of the 10-story building model under sinusoidal base excitation, both before and after the integration of the Tuned Mass Damper (TMD). These results highlight how resonance and vibration amplitude change with and without control mechanisms.

Before adding an absorber

```
Natural Frequency at mode 1 is 17.55 rad/s
Natural Frequency at mode 2 is 52.27 rad/s
```

Under uncontrolled conditions, the building was excited at a frequency of 15 rad/s, which is close to its first natural frequency of approximately 17 rad/s. At this near-resonant condition, the response was governed primarily by the first mode shape, resulting in substantial displacement amplification. Specifically, the top floor exhibited a peak displacement of around 1.0 meters, while the base moved only about 0.2 meters, indicating a fivefold increase in motion along the building height. The time-domain response also displayed a beat pattern, a characteristic result of slight detuning between the input frequency and the system’s natural frequency. This beating phenomenon arises from the interference between the forced vibration and the system’s free vibration, causing periodic fluctuations in amplitude. When the building was excited near its second mode (≈52.3 rad/s), similar behavior was observed but with lower amplitude and a more complex mode shape due to less modal participation from higher modes.

After adding an absorber

```
Natural Frequency at mode 1 is 14.82 rad/s
Natural Frequency at mode 2 is 19.97 rad/s
Natural Frequency at mode 3 is 52.54 rad/s
```

After the implementation of the TMD, tuned specifically to the **first natural frequency (17 rad/s)**, a significant improvement in performance was observed. The maximum displacement at the top floor was **drastically reduced**, confirming the TMD’s effectiveness in absorbing vibrational energy. Additionally, the frequency response curve, which previously had a sharp peak at 17 rad/s, now exhibited **two smaller peaks** around **14 rad/s and 20 rad/s**—an effect known as **mode splitting**. This phenomenon occurs due to dynamic coupling between the building and the TMD, redistributing vibrational energy and broadening the frequency range over which the system remains stable. The time-history plots further confirmed that the TMD oscillated **out of phase** with the building motion, resulting in **destructive interference** that suppressed resonance. While the TMD was primarily tuned for the first mode, it had minimal impact on the second mode response, as expected.

Discussion

When a Tuned Mass Damper (TMD) is added to a building structure and tuned to the natural frequency of the first mode (in this case, 17 rad/s), it introduces a dynamic interaction that modifies the system's original frequency response. Instead of a single dominant resonance at 17 rad/s, the structure now exhibits two new resonant frequencies—typically one lower and one higher (e.g., 14 rad/s and 20 rad/s)—as observed in our MATLAB simulations. This effect, known as **mode splitting**, results from the coupling between the building and the TMD. The TMD acts to cancel out vibration energy at the targeted frequency by creating destructive interference, effectively reducing the response amplitude near the original natural frequency. The resulting frequency shift is not only expected but also desired, as it helps to mitigate excessive structural response due to resonance, especially during base excitations such as earthquakes. By redistributing vibrational energy away from the most sensitive mode, the TMD enhances both the safety and serviceability of the structure by using ma = 10^6 kg and ka =2.89x10^8 Nm.

Outcome

After performing modal analysis and simulating the system's response to base excitation, the following key outcomes were observed:

1. Uncontrolled Vibration Response

- Without a TMD, the building experienced significant resonance near the fundamental frequency (~17 rad/s).
- The top floor displacement reached approximately 1.0 meters, while the first floor showed 0.2 meters, confirming that resonance affects the upper stories more severely.
- The presence of beat phenomena was detected, indicating interference between forced and natural vibrations.

2. Performance of the Tuned Mass Damper (TMD)

- After integrating a TMD with a mass of 1,000,000 kg and stiffness of 2.89×10^8 N/m at the top floor, the resonance peak was split into two smaller peaks around 14 rad/s and 20 rad/s.
- The maximum displacement at the top floor decreased significantly, showing clear vibration suppression.
- The TMD system successfully demonstrated destructive interference, absorbing and dissipating energy at critical frequency.

Conclusion

The results confirm the effectiveness of using a Tuned Mass Damper (TMD) to suppress structural vibrations under base excitation. In the uncontrolled system, the first mode had the highest impact due to its low frequency and large displacement profile on higher floors. This aligns with typical behavior of tall buildings under seismic or wind loads, where the fundamental mode dominates the response.

By tuning the TMD to this first mode, the structure's dynamic response was reshaped—introducing **mode splitting**, which redistributed energy and lowered peak displacements. This effect is not only consistent with vibration theory but also widely used in real-world engineering applications (e.g., Taipei 101 and Shanghai Tower).

Furthermore, although the TMD introduces an additional degree of freedom, the computational overhead remained manageable, making it practical for implementation in simulation and design stages.

Some limitations include:

- The model assumes linear behavior and does not account for material nonlinearity or damage.
- The damping ratio was fixed at 5%, which may not reflect actual values during a real earthquake.
- Only sinusoidal input was tested, whereas real earthquakes have more complex ground motions.

Overall, the TMD significantly improved the seismic resilience of the modeled structure and demonstrated the value of passive vibration control systems in modern structural design.

References

[1] Wikipedia contributors. (n.d.). *King Power Mahanakhon*. Wikipedia. https://en.wikipedia.org/wiki/King_Power_Mahanakhon#/media/File:Tha%C3%AFlande_Bangkok_MahaNakhon.jpg

[2] TMD (Tuned Mass damper). (n.d.). https://www.maurer.eu/en/infrastructure/tuned-mass-dampers/tmd-tuned-mass-damper/

[3] London Millennium Bridge - Damper beneath deck, north side - 240404.jpg - Wikimedia Commons. (n.d.). https://commons.wikimedia.org/wiki/File:London_Millennium_Bridge_-_Damper_beneath_deck,_north_side_-_240404.jpg

Appendix

GitHub

All the resources and details can directly access on GitHub:

https://github.com/NuchPunnawichP/Vibration-Project

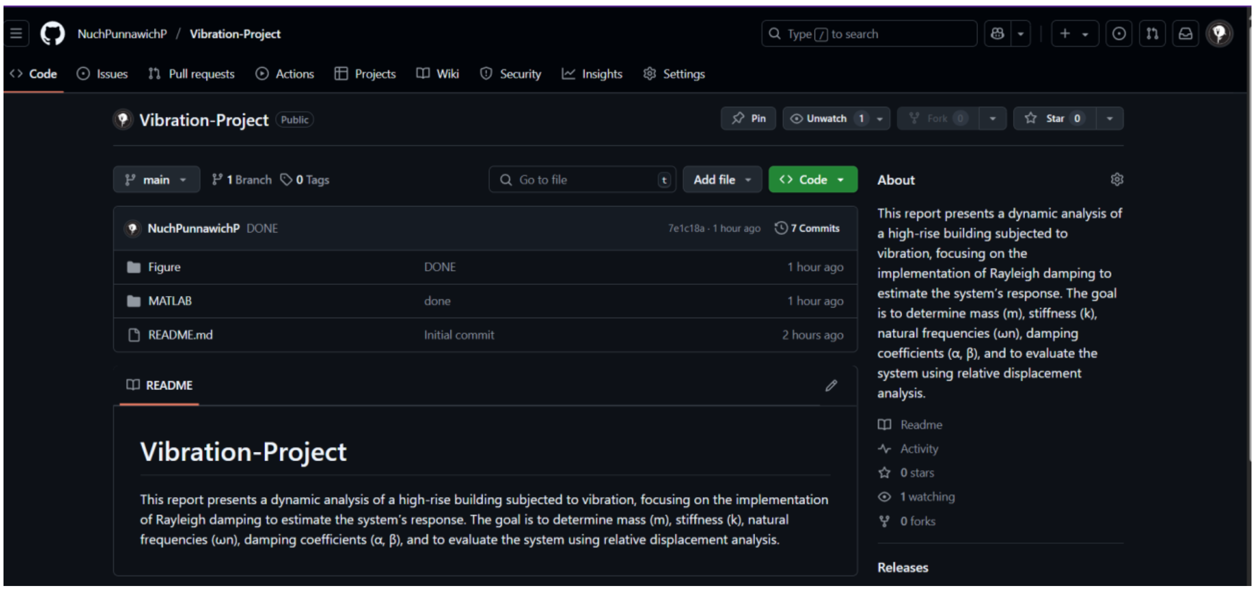


Figure 16 GitHub for access all the details in the project

```
clc; clear; close all;
```

```
%Tuned Mass Damper
```

```
%% ---------- CASE 1: TMD only (adds 1 DOF) ----------
ma = 1E6; % TMD mass
ka = 2.89E8; % Tuned to w = 15 rad/s
```

```
% Define the number of floors in the building
n = 11; % Change the number of floors
E = 30e9; % Elastic modulus in Pascals (Pa)
g = 9.81;
floor_mass = round(1600E6/g/79);
Mass = floor_mass * ones(n,1)'; % Masses of each floor in kg
Mass(11) = ma;
Height = 4 * ones(n,1)'; % Heights of each floor in meters
Inertia = 1.5^4/12 * ones(n,1); % Moment of inertia in m^4
k_values = 12 * 12 * E * Inertia ./ Height.^3; % Four Spring In Each Floor
k_values(11) = ka;
```

```
M = diag(Mass);
K = zeros(n);
for i = 1:n-1
    K(i,i) = k_values(i) + k_values(i+1);
    K(i,i+1) = -k_values(i+1);
    K(i+1,i) = -k_values(i+1);
end
K(n,n) = k_values(n);
```

```
%Eigen Value Problem
[U,D] = eig(K,M);
w1 = D(1,1); w2 = D(2,2);
syms alpha beta
% Give damping ration = 5%
eq1 = 0.05 == 0.5*(alpha/w1 + beta*w1);
eq2 = 0.05 == 0.5*(alpha/w2 + beta*w2);
[a,b] = solve(eq1,eq2);
a = vpa(a)
```

a = 14.161746618617049405315305706681

```
b = vpa(b)
```

b = 0.00016171539206959882683882214543372

```
% M_c = 2967781623447.234403808909534589 * M;
% K_c = 1.0000000000000003068988425448726 * K;
```

```
[U,D] = eig(K,M);
omega_n = sqrt(diag(D));
x_0 = zeros(n,1); % initial condition
v_0 = zeros(n,1); % initial condition
r_0 = U'*M*x_0;
rdot_0 = U'*M*v_0;
R_i = [r_0 rdot_0];
```

```
fprintf('Stiffness matrix [K] in N/m:\n');
```

```
Stiffness matrix [K] in N/m:
```

```
disp(K);
```

```
   1.0e+10 *

    5.6953   -2.8477         0         0         0         0         0         0         0         0         0
   -2.8477    5.6953   -2.8477         0         0         0         0         0         0         0         0
         0   -2.8477    5.6953   -2.8477         0         0         0         0         0         0         0
         0         0   -2.8477    5.6953   -2.8477         0         0         0         0         0         0
         0         0         0   -2.8477    5.6953   -2.8477         0         0         0         0         0
         0         0         0         0   -2.8477    5.6953   -2.8477         0         0         0         0
         0         0         0         0         0   -2.8477    5.6953   -2.8477         0         0         0
         0         0         0         0         0         0   -2.8477    5.6953   -2.8477         0         0
         0         0         0         0         0         0         0   -2.8477    5.6953   -2.8477         0
         0         0         0         0         0         0         0         0   -2.8477    2.8766   -0.0289
         0         0         0         0         0         0         0         0         0   -0.0289    0.0289
```

```
fprintf('Mass matrix [M] in kg:\n');
```

```
Mass matrix [M] in kg:
```

```
disp(M);
```

```
     2064543           0           0           0           0           0           0           0           0
           0     2064543           0           0           0           0           0           0           0
           0           0     2064543           0           0           0           0           0           0
           0           0           0     2064543           0           0           0           0           0
           0           0           0           0     2064543           0           0           0           0
           0           0           0           0           0     2064543           0           0           0
           0           0           0           0           0           0     2064543           0           0
           0           0           0           0           0           0           0     2064543           0
           0           0           0           0           0           0           0           0     2064543
           0           0           0           0           0           0           0           0           0     20
           0           0           0           0           0           0           0           0           0
```

```
Lambda = char(hex2dec('039B'));
fprintf(['Sprectal Matrix [' Lambda '] \n']);
```

```
Sprectal Matrix [Λ]
```

```
disp(U'*K*U);
```

```
   1.0e+04 *

    0.0220   -0.0000    0.0000    0.0000   -0.0000   -0.0000   -0.0000   -0.0000   -0.0000   -0.0000   -0.0000
```

```
   -0.0000    0.0399   -0.0000   -0.0000    0.0000    0.0000   -0.0000   -0.0000    0.0000    0.0000   -0.0000
    0.0000   -0.0000    0.2760   -0.0000    0.0000   -0.0000    0.0000    0.0000   -0.0000   -0.0000   -0.0000
    0.0000   -0.0000   -0.0000    0.7388   -0.0000   -0.0000    0.0000    0.0000    0.0000   -0.0000    0.0000
   -0.0000    0.0000    0.0000   -0.0000    1.3814    0.0000   -0.0000   -0.0000   -0.0000   -0.0000    0.0000
   -0.0000    0.0000   -0.0000    0.0000    0.0000    2.1464   -0.0000    0.0000    0.0000    0.0000    0.0000
   -0.0000   -0.0000    0.0000    0.0000   -0.0000   -0.0000    2.9660    0.0000    0.0000   -0.0000   -0.0000
   -0.0000   -0.0000    0.0000    0.0000   -0.0000    0.0000    0.0000    3.7673   -0.0000    0.0000   -0.0000
   -0.0000    0.0000   -0.0000    0.0000   -0.0000    0.0000    0.0000   -0.0000    4.4791   -0.0000    0.0000
   -0.0000    0.0000   -0.0000   -0.0000   -0.0000    0.0000   -0.0000    0.0000   -0.0000    5.0382    0.0000
   -0.0000    0.0000   -0.0000    0.0000    0.0000    0.0000   -0.0000   -0.0000    0.0000    0.0000    5.3948
```

Modal Analysis of Based Excitation

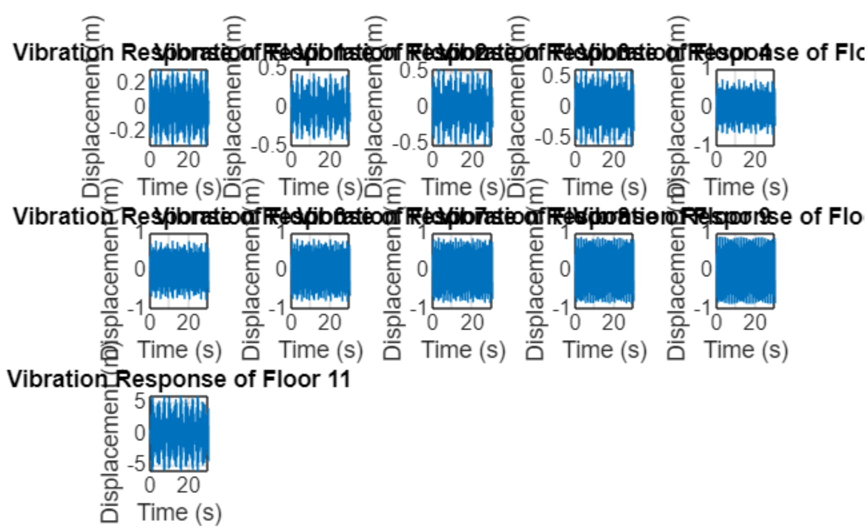
```
a = 27.68876706408824368131489359933;
b = 0.000032894521560077955830320103241098;
% There are some problems with type of matrix C
C = a*M + b*K;
M_modal = U' * M * U; % Should be identity matrix
C_modal = U' * C * U;
K_modal = U' * K * U % Should be spectral matrix
```

```
K_modal = 11×11
10^4 ×
    0.0220   -0.0000    0.0000    0.0000   -0.0000   -0.0000   -0.0000   -0.0000 ...
   -0.0000    0.0399   -0.0000   -0.0000    0.0000    0.0000   -0.0000   -0.0000
    0.0000   -0.0000    0.2760   -0.0000    0.0000   -0.0000    0.0000    0.0000
    0.0000   -0.0000   -0.0000    0.7388   -0.0000   -0.0000    0.0000    0.0000
   -0.0000    0.0000    0.0000   -0.0000    1.3814    0.0000   -0.0000   -0.0000
   -0.0000    0.0000   -0.0000    0.0000    0.0000    2.1464   -0.0000    0.0000
   -0.0000   -0.0000    0.0000    0.0000   -0.0000   -0.0000    2.9660    0.0000
   -0.0000   -0.0000    0.0000    0.0000   -0.0000    0.0000    0.0000    3.7673
   -0.0000    0.0000   -0.0000    0.0000   -0.0000    0.0000    0.0000   -0.0000
   -0.0000    0.0000   -0.0000   -0.0000   -0.0000    0.0000   -0.0000    0.0000
       ⋮
```

```
% Earthquake
w = 17;
f = 2*pi*w;
fs = 2.5*f; %Sampling Frequency
t_end = 30;
number_of_t = t_end * fs;
tspan = linspace(0,t_end,number_of_t);
y = 0.2 * sin(w*tspan);
ay = 0.2 * w^2 * sin(w*tspan);
r = zeros(n,length(tspan));
gamma = -U'* M * ones(n,1);
for i = 1:n
    omega_i = omega_n(i);
    zw_i = a + b * omega_i^2;
    gamma_i = gamma(i);
    eom_ode = @(t, r) eom(t,r,omega_i,zw_i,gamma_i,w);
    r_i = R_i(i,:); % Initial conditions: [r_i(0); rdot_i(0)]
    [t, y] = ode45(eom_ode, tspan, r_i);
```

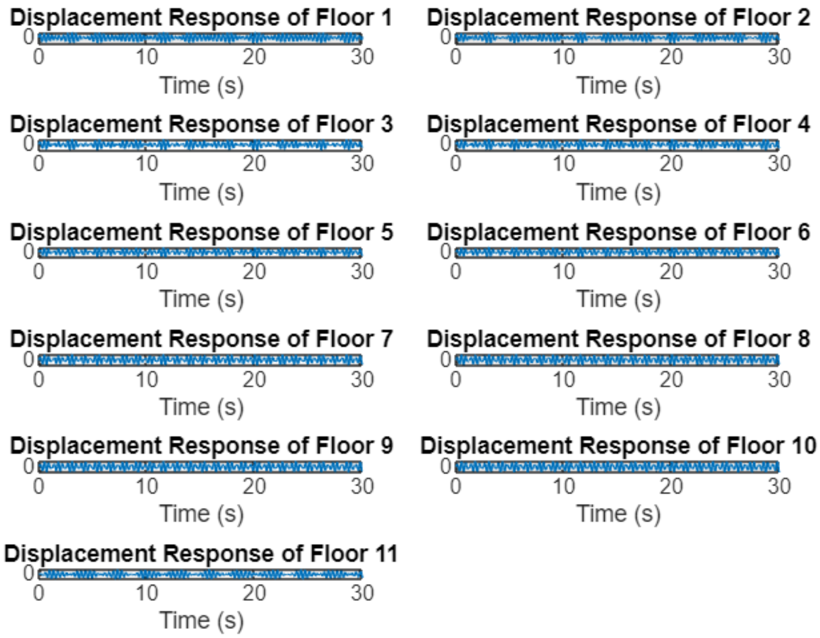
```
    r(i,:) = y(:,1)';
end
```

```
x = U*r;
xg = 0.2 * sin(w*tspan); % Subject to change by excitation
x = x + xg; %Change from relative displacement to absolute

%Plot response of each floor in subplots
figure;

for floor = 1:n
    subplot(3, round(n/2,0)-1, floor);
    plot(t, x(floor, :), 'LineWidth', 0.5);
    title(['Vibration Response of Floor ', num2str(floor)]);
    xlabel('Time (s)');
    ylabel('Displacement (m)');
    grid on;
end
% Optional: Maximize figure window for better visibility
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0, 0, 1, 1]);
```




```
%Plot response of each floor in subplots
figure;
tiledlayout(6,2, 'TileSpacing', 'compact', 'Padding', 'compact');

for i = 1:n
    nexttile;
    plot(t, x(i, :));
    xlabel('Time (s)');
    ylabel('Displacement (m)');
    title(['Displacement Response of Floor ', num2str(i)]);
    grid on;
    axis tight;
```

```
end
```




```
function rdot = eom(t,r,omega_i,zw_i,gamma_i,w)
    ay = -0.2 * w^2 * sin(w*t);
    rdot = zeros(2,1);
    rdot(1) = r(2);
    rdot(2) = -zw_i*rdot(2) - omega_i^2*r(1) + gamma_i*ay;
end
```

```
clc; clear; close all;
```

```
% Define the number of floors in the building
n = 10; % Change the number of floors
```

Mahanakhon has 79 floors and its weight of 1600 MN

https://www.afgc.asso.fr/app/uploads/2016/10/09-Design-and-construction-of-the-Mahanakhon-tower-in-Bangkok.pdf?utm_source=chatgpt.com

```
% Building Dimension and Properties
floor_w = 40; floor_l = 40; g = 9.81;
floor_mass = round(1600E6/g/79);
```

Assume the steel yield stength is 500 MPa and Concrete partial material safety factor equal 1.5 Modulus of elasticity round up around 30000 MPa or 30 GPa

https://eurocodeapplied.com/design/en1992/concrete-design-properties

Mahanakhon Building has 12 Mega Column, and RC core wall of 23*23 m. In this simulated, we will not consider only RC core wall as a main stiffness.

https://www.dextragroup.com/mahanakhon-tower/

https://www.scribd.com/document/338577575/306175551-Concept-Design-Report-Mahanakhon-Building-pdf?utm_source=chatgpt.com

```
E = 30e9; % Elastic modulus in Pascals (Pa)
Mass = floor_mass * ones(n,1)'; % Masses of each floor in kg
Height = 4 * ones(n,1)'; % Heights of each floor in meters
Inertia = 1.5^4/12 * ones(n,1); % Moment of inertia in m^4
```

```
% Calculate the stiffness for each floor using the formula k = 12EI/h^3
k_values = 12 * 12 * E * Inertia ./ Height.^3; % Four Spring In Each Floor
```

$$\boldsymbol{M} = \begin{bmatrix} m_1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & m_2 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & m_3 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & m_8 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & m_9 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & m_{10} \end{bmatrix}$$

$$\boldsymbol{K}=\begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 & \cdots & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 & \cdots & 0 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 & \cdots & 0 & 0 \\ 0 & 0 & -k_4 & k_4+k_5 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & -k_9 & k_9+k_{10} & -k_{10} \\ 0 & 0 & 0 & 0 & 0 & -k_{10} & k_{10} \end{bmatrix}$$

```
M = diag(Mass);
K = zeros(n);
for i = 1:n-1
    K(i,i) = k_values(i) + k_values(i+1);
    K(i,i+1) = -k_values(i+1);
    K(i+1,i) = -k_values(i+1);
end
K(n,n) = k_values(n);
```

Finding Natural Frequency for Assuming the Rayleigh Damping Coefficients

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{K}\mathbf{X} = 0$$

```
[U,D] = eig(K,M);
w1 = D(1,1); w2 = D(2,2);
```

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$$

$\zeta = \frac{1}{2}\left(\frac{\alpha}{\omega} + \beta\omega\right)$; Rayleigh Damping Coefficients

https://www.orcina.com/webhelp/OrcaFlex/Content/html/Rayleighdamping,Guidance.htm

```
syms alpha beta
% Give damping ration = 5%
eq1 = 0.05 == 0.5*(alpha/w1 + beta*w1);
eq2 = 0.05 == 0.5*(alpha/w2 + beta*w2);
[a,b] = solve(eq1,eq2);
a = vpa(a)
```

a = $27.688767064088243681314893599 33$

```
b = vpa(b)
```

b = $0.00003289452156007795583032010 3241098$

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = -\mathbf{M}\mathbf{1}\ddot{\mathbf{u}}_{\mathbf{g}}(\mathbf{t})$$

where x is relative displacement of each floor with respect to ground

```
% M_c = 2967781623447.234403808909534589 * M;
% K_c = 1.000000000000003068988425448726 * K;
[U,D] = eig(K,M);
omega_n = sqrt(diag(D));
x_0 = zeros(n,1); % initial condition
v_0 = zeros(n,1); % initial condition
r_0 = U'*M*x_0;
rdot_0 = U'*M*v_0;
R_i = [r_0 rdot_0];
```

```
% Check
%U'*M*U should be I
%U'*K*U should be w^2 (sprectal matrix)
% Print results to the MATLAB console
fprintf('Stiffness matrix [K] in N/m:\n');
```

```
Stiffness matrix [K] in N/m:
```

```
disp(K);
```

```
   1.0e+10 *

    5.6953   -2.8477         0         0         0         0         0         0         0         0
   -2.8477    5.6953   -2.8477         0         0         0         0         0         0         0
         0   -2.8477    5.6953   -2.8477         0         0         0         0         0         0
         0         0   -2.8477    5.6953   -2.8477         0         0         0         0         0
         0         0         0   -2.8477    5.6953   -2.8477         0         0         0         0
         0         0         0         0   -2.8477    5.6953   -2.8477         0         0         0
         0         0         0         0         0   -2.8477    5.6953   -2.8477         0         0
         0         0         0         0         0         0   -2.8477    5.6953   -2.8477         0
         0         0         0         0         0         0         0   -2.8477    5.6953   -2.8477
         0         0         0         0         0         0         0         0   -2.8477    2.8477
```

```
fprintf('Mass matrix [M] in kg:\n');
```

```
Mass matrix [M] in kg:
```

```
disp(M);
```

```
     2064543           0           0           0           0           0           0           0           0
           0     2064543           0           0           0           0           0           0           0
           0           0     2064543           0           0           0           0           0           0
           0           0           0     2064543           0           0           0           0           0
           0           0           0           0     2064543           0           0           0           0
           0           0           0           0           0     2064543           0           0           0
           0           0           0           0           0           0     2064543           0           0
           0           0           0           0           0           0           0     2064543           0
           0           0           0           0           0           0           0           0     2064543
           0           0           0           0           0           0           0           0           0     206
```

```
Lambda = char(hex2dec('039B'));
fprintf(['Sprectal Matrix [' Lambda '] \n']);
```

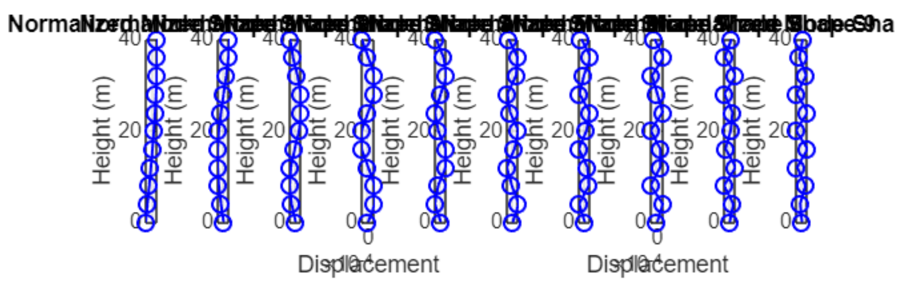
```
Sprectal Matrix [Λ]
```

```
disp(U'*K*U);
```

```
   1.0e+04 *

    0.0308   -0.0000    0.0000   -0.0000   -0.0000    0.0000    0.0000   -0.0000    0.0000    0.0000
    0.0000    0.2732   -0.0000    0.0000    0.0000    0.0000    0.0000   -0.0000   -0.0000    0.0000
    0.0000   -0.0000    0.7364   -0.0000   -0.0000    0.0000    0.0000   -0.0000    0.0000   -0.0000
   -0.0000    0.0000   -0.0000    1.3793    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000
   -0.0000    0.0000   -0.0000    0.0000    2.1448   -0.0000   -0.0000    0.0000   -0.0000   -0.0000
    0.0000    0.0000   -0.0000    0.0000   -0.0000    2.9648   -0.0000   -0.0000   -0.0000   -0.0000
   -0.0000    0.0000    0.0000    0.0000   -0.0000   -0.0000    3.7665   -0.0000    0.0000   -0.0000
   -0.0000   -0.0000   -0.0000    0.0000    0.0000   -0.0000   -0.0000    4.4786    0.0000    0.0000
    0.0000         0    0.0000         0   -0.0000   -0.0000    0.0000    0.0000    5.0379   -0.0000
         0         0   -0.0000    0.0000   -0.0000   -0.0000   -0.0000    0.0000   -0.0000    5.3947
```

Mode Shape Visualize

```
heights = [0, cumsum(Height)];
for i = 1:n
    subplot(2, n, i);
    plot([0; U(:,i)], heights, 'o-','Color', 'b', 'LineWidth',1);
    title(['Normalized Mode Shape ', num2str(i)]);
    xlabel('Displacement');
    ylabel('Height (m)');
    grid on;
    axis tight;
end
% Optional: Maximize figure window for better visibility
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0, 0, 1, 1]);
```



Modal Analysis of Based Excitation

```
a = 27.688767064088243681314893599333;
b = 0.000032894521560077955830320103241098;
% There are some problems with type of matrix C
```

```
C = a*M + b*K;
M_modal = U' * M * U; % Should be identity matrix
C_modal = U' * C * U;
K_modal = U' * K * U % Should be spectral matrix
```

```
K_modal = 10×10
10^4 ×
    0.0308   -0.0000    0.0000   -0.0000   -0.0000    0.0000    0.0000   -0.0000 ···
    0.0000    0.2732   -0.0000    0.0000    0.0000    0.0000    0.0000   -0.0000
    0.0000   -0.0000    0.7364   -0.0000   -0.0000    0.0000    0.0000   -0.0000
   -0.0000    0.0000   -0.0000    1.3793    0.0000    0.0000    0.0000    0.0000
   -0.0000    0.0000   -0.0000    0.0000    2.1448   -0.0000   -0.0000    0.0000
    0.0000    0.0000   -0.0000    0.0000   -0.0000    2.9648   -0.0000   -0.0000
   -0.0000    0.0000    0.0000    0.0000   -0.0000   -0.0000    3.7665   -0.0000
   -0.0000   -0.0000   -0.0000    0.0000    0.0000   -0.0000   -0.0000    4.4786
    0.0000         0    0.0000         0   -0.0000   -0.0000    0.0000    0.0000
         0         0   -0.0000    0.0000   -0.0000   -0.0000   -0.0000    0.0000
```

$$\mathbf{M}\ddot{\mathbf{x}}_{\mathbf{r}}(\mathbf{t}) + \mathbf{C}\dot{\mathbf{x}}_{\mathbf{r}}(\mathbf{t}) + \mathbf{K}\mathbf{x}_{\mathbf{r}}(\mathbf{t}) = -\mathbf{M}\ddot{\mathbf{y}}$$

Given $\mathrm{X_r} = \mathrm{Ur}$

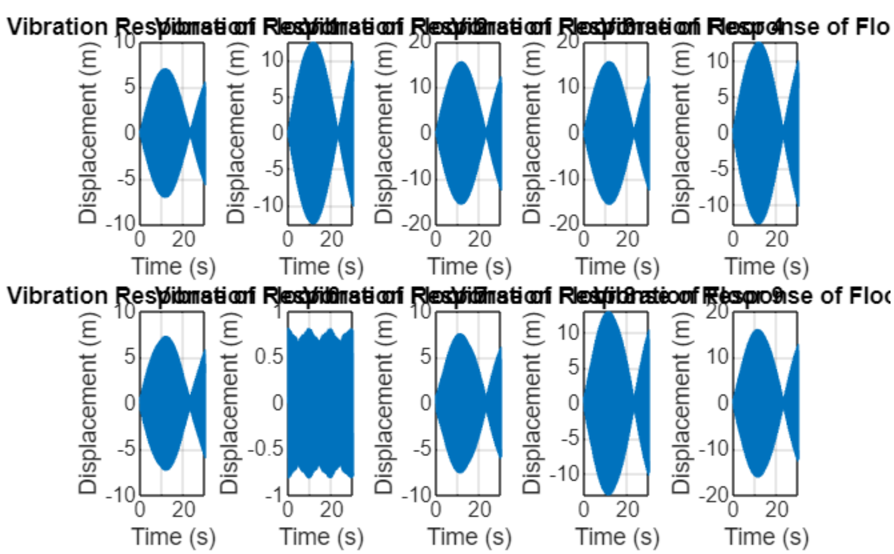
$$\ddot{\mathbf{r}}_{\mathbf{i}}(\mathbf{t}) + 2\zeta\omega_{\mathbf{n,i}}\dot{\mathbf{r}}_{\mathbf{i}}(\mathbf{t}) + \omega_{\mathbf{n,i}}^2\,\mathbf{r}(\mathbf{t}) = \mathbf{N}_{\mathbf{i}}(\mathbf{t})\,;\mathbf{N}(\mathbf{t}) = -\mathbf{U}^{\mathrm{T}}\mathbf{M}\ddot{\mathbf{y}}(\mathbf{t})$$

where $2\zeta\omega_{\mathrm{n,i}} = \alpha + \beta\omega_{\mathrm{n,i}}^2 = \mathrm{U^TCU}$ and $\Gamma = -\mathbf{U}^{\mathbf{T}}\mathbf{M1}$

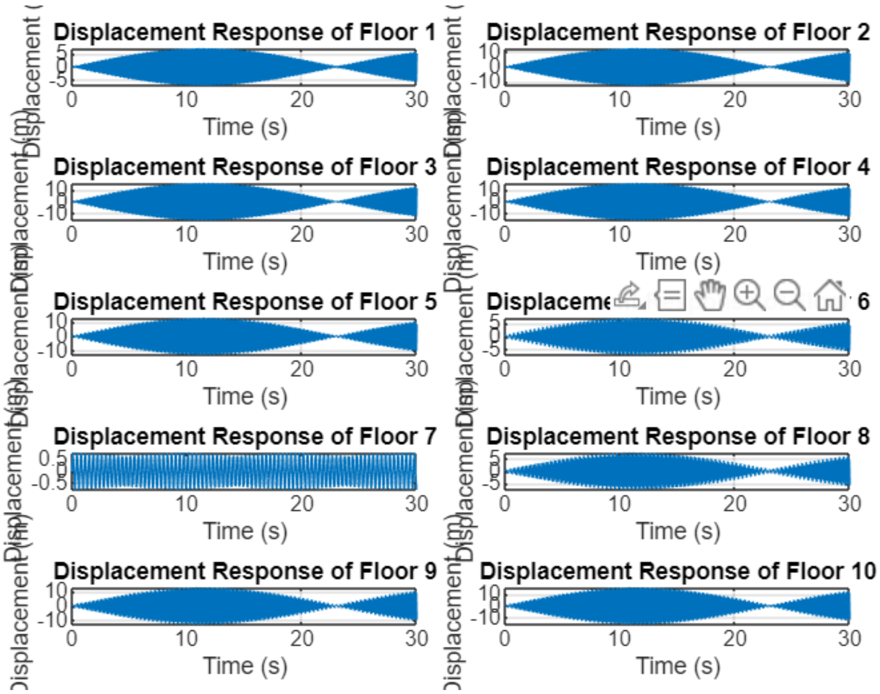
```
% Earthquake
w = 52;
f = 2*pi*w;
fs = 2.5*f; %Sampling Frequency
t_end = 30;
number_of_t = t_end * fs;
tspan = linspace(0,t_end,number_of_t);
y = 0.2 * sin(w*tspan);
ay = 0.2 * w^2 * sin(w*tspan);
r = zeros(n,length(tspan));
gamma = -U'* M * ones(n,1);
for i = 1:n
    omega_i = omega_n(i);
    zw_i = a + b * omega_i^2;
    gamma_i = gamma(i);
    eom_ode = @(t, r) eom(t,r,omega_i,zw_i,gamma_i,w);
    r_i = R_i(i,:); % Initial conditions: [r_i(0); rdot_i(0)]
    [t, y] = ode45(eom_ode, tspan, r_i);
    r(i,:) = y(:,1)';
end
```

```
x = U*r;
xg = 0.2 * sin(w*tspan); % Subject to change by excitation
x = x + xg; %Change from relative displacement to absolute
```

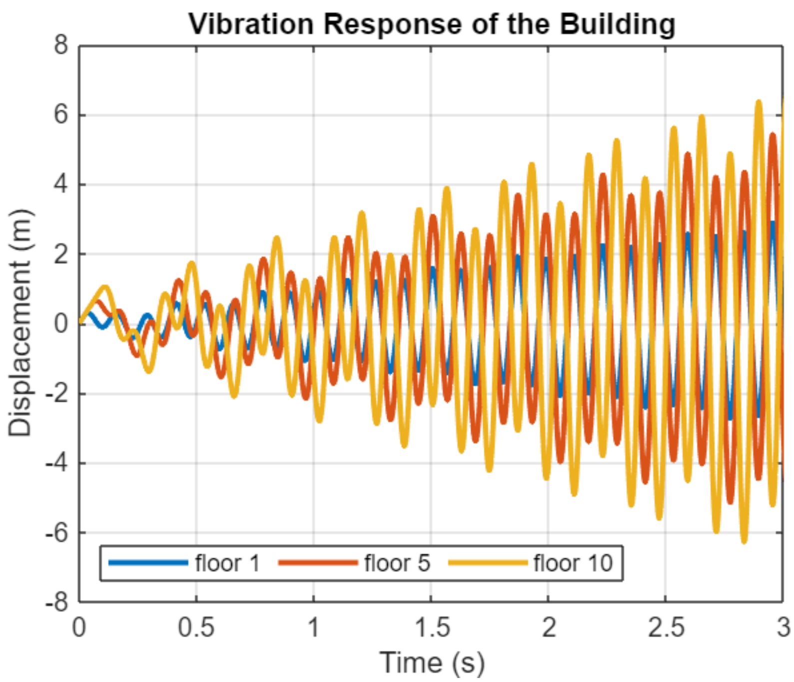
```
%Plot response of each floor in subplots
figure;

for floor = 1:n
    subplot(2, n/2, floor);
    plot(t, x(floor, :), 'LineWidth', 0.5);
    title(['Vibration Response of Floor ', num2str(floor)]);
    xlabel('Time (s)');
    ylabel('Displacement (m)');
    grid on;
end
% Optional: Maximize figure window for better visibility
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0, 0, 1, 1]);
```



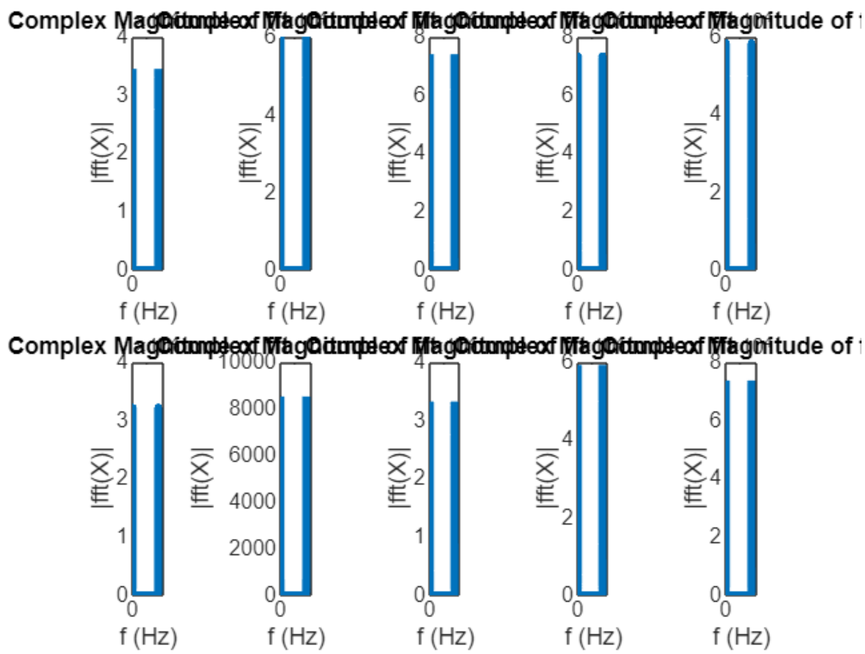

```
%Plot response of each floor in subplots
figure;
tiledlayout(5,2, 'TileSpacing', 'compact', 'Padding', 'compact');

for i = 1:n
    nexttile;
    plot(t, x(i, :));
    xlabel('Time (s)');
    ylabel('Displacement (m)');
    title(['Displacement Response of Floor ', num2str(i)]);
    grid on;
    axis tight;
end
```




```
%All Floor Plot in 1 Graph
figure;
for floor = [1,5,10]
    plot(t, x(floor, :),LineWidth=2);
    grid on;
    hold on
end
title(['Vibration Response of the Building']);
xlabel('Time (s)');
ylabel('Displacement (m)');
xlim([0 3]);
legend({'floor 1','floor 5','floor 10'} ...
    ,'Location','southwest','Orientation','horizontal')
```




```
%FFT
figure;
for floor = 1:n
    T = 1/fs;   % Sampling period
    L = length(t);
    t = (0:L-1)*T; % Time vector
    subplot(2, n/2, floor);
    N = length(x);
    fs = 1/(t(2) - t(1));  % Sampling frequency
    f = (0:N-1)*(fs/N);  % Frequency vector in Hz
    omega = 2 * pi * f;  % Convert to ω (rad/s)
    Y = fft(x(floor,:));
    plot(fs/L*(0:L-1),abs(Y),"LineWidth",3)
    title("Complex Magnitude of fft")
    xlabel("f (Hz)")
    ylabel("|fft(X)|")
end
```

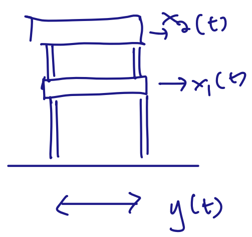


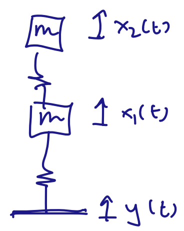
Vibration Suppression

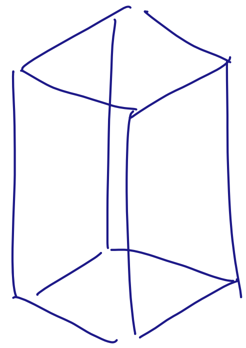
```
% %% ---------- CASE 1: TMD only (adds 1 DOF) ----------
% ma = 1E6; % TMD mass
% ka = 2.89E8; % Tuned to w = 15 rad/s
%
% M1 = blkdiag(M, mt);
% K1 = blkdiag(K, 0);
% C1 = blkdiag(C, 0);
%
% K1(n,n) = K1(n,n) + kt;
% K1(n+1,n+1) = kt;
% K1(n,n+1) = -kt;
% K1(n+1,n) = -kt;
%
% C1(n,n) = C1(n,n) + ct;
% C1(n+1,n+1) = ct;
% C1(n,n+1) = -ct;
% C1(n+1,n) = -ct;
```

Vibration Absorber

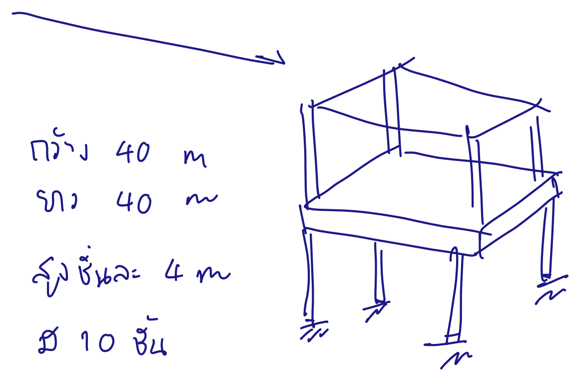
```
function rdot = eom(t,r,omega_i,zw_i,gamma_i,w)
    ay = -0.2 * w^2 * sin(w*t);
    rdot = zeros(2,1);
    rdot(1) = r(2);
    rdot(2) = -zw_i*rdot(2) - omega_i^2*r(1) + gamma_i*ay;
end
```







Model 4 ขา



กว้าง 40 m

ยาว 40 m

สูง ชั้นละ 4 m

มี 10 ชั้น

Mahanakhon → 12 Mega collumn → เสาละ $1.5 \times 1.5\ m^2$, $I = \frac{1.5^4}{12}\ (m^4)$

น้ำหนักรวม 1600 MN (79 ชั้น) → 1 ชั้น มีมวล 2064543 kg

Reinforcement Concrete มี $E = 30$ GPa

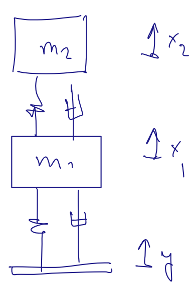
กำหนด C ที่ $\zeta = 0.05$ (เพราะไม่แน่ใจว่าเท่าไหร่ เป็นค่ามาตรฐานที่ใช้)

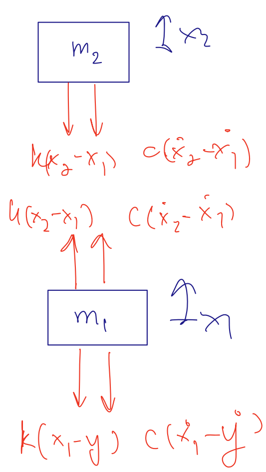
หา Rayleigh Damping Coefficient $C = \alpha M + \beta K$

$\zeta = \frac{1}{2}\left(\frac{\alpha}{\omega} + \beta\omega\right)$ → เลือก $\zeta = 0.05$, $\omega_{n,1}$ และ $\omega_{n,1}$

solve หา α, β

$x_2 > x_1 > y$





$$\Sigma F = m\ddot{x}$$

$$-k(x_2-x_1) - c(\dot{x}_2-\dot{x}_1) = m_2\ddot{x}_2$$

$$m_2\ddot{x}_2 + c(\dot{x}_2-\dot{x}_1) + k(x_2-x_1) = 0$$

$$\Sigma F = m\ddot{x}$$

$$k(x_2-x_1) + c(\dot{x}_2-\dot{x}_1) - k(x_1-y) - c(\dot{x}_1-\dot{y}) = m_1\ddot{x}_1$$

$$m_1\ddot{x}_1 + k(2x_1-x_2) + c(2\dot{x}_1-\dot{x}_2) = ky + c\dot{y}$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 2c & -c \\ -c & c \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} ky + c\dot{y} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} c_1\dot{y} + k_1 y \\ 0 \end{bmatrix}$$

ให้ $C = \alpha M + \beta K \rightarrow X = Ur$

$$\ddot{r} + (\alpha I + \beta \Lambda)\dot{r} + \Lambda r = U^T F = N(t)$$

look in each row ; $\ddot{r}_i(t) + 2\zeta \omega_{n_i}\dot{r}_i(t) + \omega_{n_i}^2 r_i(t) = N_i(t)$

where $2\zeta\omega_{ni} = \alpha + \beta\omega_{n,i}^2 = U^T C U$

But we will change to relative displacement

$x_{1r} = x_1 - x_0 \rightarrow m\ddot{x}_{1r} + 2c\dot{x}_{1r} - c\dot{x}_{2r} + 2kx_{1r} - kx_{2r} = -m\ddot{y}$

$x_{2r} = x_2 - x_0 \rightarrow m\ddot{x}_{2r} + c\dot{x}_{2r} - c\dot{x}_{1r} + kx_{2r} - kx_{1r} = -m\ddot{y}$

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{x}_{1r} \\ \ddot{x}_{2r} \end{bmatrix} + \begin{bmatrix} 2c & -c \\ -c & c \end{bmatrix}\begin{bmatrix} \dot{x}_{1r} \\ \dot{x}_{2r} \end{bmatrix} + \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}\begin{bmatrix} x_{1r} \\ x_{2r} \end{bmatrix} = \begin{bmatrix} -m\ddot{y}_0 \\ -m\ddot{y}_0 \end{bmatrix}$$

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{x}_{1r} \\ \ddot{x}_{2r} \end{bmatrix} + \begin{bmatrix} 2c & -c \\ -c & c \end{bmatrix}\begin{bmatrix} \dot{x}_{1r} \\ \dot{x}_{2r} \end{bmatrix} + \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}\begin{bmatrix} x_{1r} \\ x_{2r} \end{bmatrix} = \begin{bmatrix} -m\ddot{y}_0 \\ -m\ddot{y}_0 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} m_1 & 0 & & \\ 0 & m_2 & & \\ & & \ddots & \\ & & & m_{10} \end{bmatrix}\begin{bmatrix} \ddot{x}_{1r} \\ \ddot{x}_{2r} \\ \vdots \\ \ddot{x}_{10r} \end{bmatrix} + (\alpha M + \beta K)\begin{bmatrix} \dot{x}_{1r} \\ \dot{x}_{2r} \\ \vdots \\ \dot{x}_{10r} \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & & \\ -k_2 & k_2+k_3 & & \\ & & \ddots & \\ & & -k_9+k_{10} & -k_{10} \\ & & -k_{10} & k_{10} \end{bmatrix}\begin{bmatrix} x_{1r} \\ x_{2r} \\ \vdots \\ x_{9r} \\ x_{10r} \end{bmatrix} = \begin{bmatrix} -m\ddot{y}_0 \\ -m\ddot{y}_0 \\ \vdots \\ -m\ddot{y}_0 \end{bmatrix}$$

$M\ddot{x}_r(t) + C\dot{x}_r(t) + Kx_r(t) = -M\ddot{y}$; $x_r = Ur$

$\ddot{r}_i(t) + 2\zeta\omega_{n_i}\dot{r}(t) + \omega_{n,i}^2 r(t) = N_i(t)$; $N_i(t) = -U^T M\ddot{y}(t)$

where $2\zeta\omega_{n,i} = \alpha + \beta\omega_{n,i}^2 = U^T C U$ and given $\Gamma = -U^T M$

แปลงกลับ $x_r = Ur \Rightarrow X_{absolute} = X + y$

ถ้า $y = 0.2\sin(\omega t) \rightarrow \ddot{y} = -0.2\omega^2\sin(\omega t)$

$\omega = \omega_{n,1}$ และ $\omega_{n,2}$

TMD (Absorber)

$$\beta^2\left(\frac{\omega_n^2}{\omega_a^2}\right)^2-[1+\beta^2(1+\mu)]\left(\frac{\omega_n^2}{\omega_a^2}\right)+1=0$$

$\omega_a = 17$ rad/s

$M = 2064543$ kg

Choose $\beta = 1 \Rightarrow \omega_n \leq 12$ rad/s $\quad \omega_n \geq 25$ rad/s

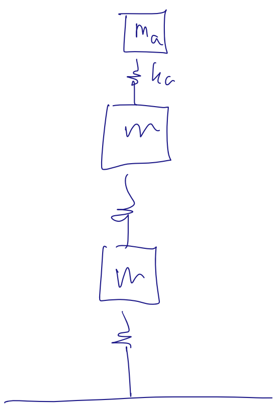
$$1\left(\frac{10}{17}\right)^4-[1+1+\mu]\left(\frac{10}{17}\right)^2+1=0 \rightarrow \mu = 0.505 \text{ over}$$

$$1\left(\frac{30}{17}\right)^4-[1+1+\mu]\left(\frac{30}{17}\right)^2+1=0 \rightarrow \mu = 0.625 \text{ over}$$

Choose $\mu = 0.505$ (0.6 too much)

$$\Rightarrow \mu = \frac{m_a}{M} \rightarrow m_a \approx 10^6 \text{ kg}$$

$$\omega_a = 17 \text{ rad/s} = \sqrt{\frac{k_a}{10^6}} \Rightarrow k_a = 2.89\times10^8$$



เพิ่ม DOF $\rightarrow$

$$\begin{bmatrix} m_1 & & & & \\ & m_2 & & & \\ & & \ddots & & \\ & & & m_{10} & \\ & & & & m_a \end{bmatrix} \qquad \begin{bmatrix} k_1+k & & & & \\ & k_2+k_3 & & & \\ & & \ddots & & \\ & & & k_{10}+k_a & \\ & & & & k_a \end{bmatrix}$$