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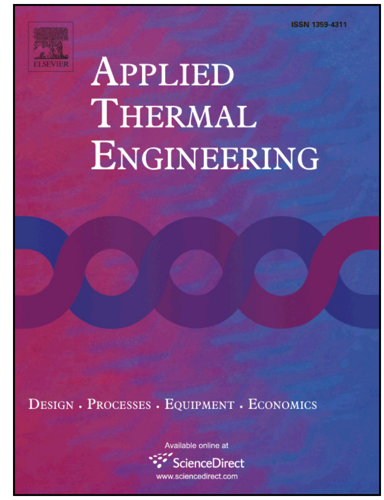

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Flow and Thermal Modeling and Optimization of Micro/mini-channel Heat Sink

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Abstract: This paper is dedicated to present a comprehensive comparison and discussion on the flow and thermal modeling of micro/mini-channel heat sink for better understanding and further sophistication of this technology. A general optimization process for the flow and thermal performance of micro/mini-channel heat sink is developed, for water cooling and liquid metal cooling both in micro scale and millimeter scale together. Numerical calculation is adopted in the optimization process and serves as baseline for the comparison of different correlations, while 1D thermal resistance model is used for thermal analysis. Appropriate correlations are recommended and optimal parameter selections and suggestions are provided for practical design. Liquid metal cooling and water cooling are compared in mini-channel heat sink, the former exhibits much superior flow and thermal performance. For water cooling, 1D model works well. While for liquid metal cooling, significant deviation exists between the model and the numerical results, and more experimental and analytical works are needed for the thermal design of this kind of coolant.

Keywords: Micro/mini-channel; Heat sink; Performance optimization; Flow and thermal modeling; Liquid metal

Nomenclature	
c_p	specific heat capacity (J/kg/K)
D_h	$=\dfrac{2w_cH}{w_c+H}$, hydrodynamic diameter (m)
f	friction coefficient
f_{app}	apparent friction coefficient
H	channel height (m)
h	heat transfer coefficient (W/m^2/K)
k	thermal conductivity (W/m/K)
L	heat sink length (m)
Nu	hD_h/k_l, Nusselt number
n	channel number
P	pumping power (W)
p	pressure (N/m^2)
Δp	pressure drop (N/m^2)
Pr	Prandtl number
q	total heat flow (W)
q''	heat flux (W/m^2)
R	thermal resistance (K/W)
Re	$=\rho U_m D_h/\mu$, Reynolds number

T	temperature (K)
t	heat sink base thickness (m)
U_m	mean velocity (m/s)
u_i	velocity along i direction (i=x, y, z) (m/s)
W	heat sink width (m)
w	channel width (m)
x, y, z	rectangular coordinates (m)
x^+	$=\frac{x}{D_h Re}$ hydrodynamic entry length
x^*	$=\frac{x}{D_h Re Pr}$ thermal entry length
Greek letters	
α	$=w_c/H$, $0<\alpha\le 1$ channel aspect ratio
β	$=w_w/w_c$ width ratio of the fin to channel
η_f	$=\tanh(mH)/mH$, $m=\sqrt{2h/k_w w_w}$ fin efficiency
λ	thermal diffusivity (m^2/s)
μ	dynamic viscosity (kg/s/m)
ρ	mass density (kg/m^3)
Subscripts	
b	heat sink base
c	channel
cap	capacity
$cond$	conduction
$conv$	convection
f	fluid
fd	fully developed
H	constant heat flux condition
in	inlet
m	mass-weighted average value
out	outlet
w	heat sink solid part or wall

1. Introduction

Micro/mini-channel heat sink was actively investigated in the last decades for its importance in the field of large power heat removal [1-3]. This technology was first proposed and developed by Tuckerman and Pease [4] in 1981, who successfully realized heat dissipation of 790 W from a 1cm^2 chip with maximum temperature rise of 71.1 °C by circulating water directly through the silicon substrate with microchannels. After that, many studies were conducted focused on this technology, by experimental [5], numerical [6, 7] or analytical approach [8]. Among which, analytical model is the most convenient method for improvement and optimization of the flow and thermal performance of micro/mini-channel heat sink. Analytical models mainly include fin model, porous medium model and thermal resistance model [9], and the last one was most strongly recommended since it has sufficient accuracy and is straightforward to use [10].

Knight et al. [11, 12] systematically analyzed the thermal resistance of microchannel heat sink and provided a dimensionless scheme for guiding its optimization. Harms et al. [13] experimentally tested a series of microchannel heat sinks and compared the results with their thermal resistance model which showed good agreement. Liu and Garimella [10] compared five approximate analytical approaches for modeling the thermal resistance of microchannel heat sink with their numerical results, and recommended the 1D resistance model for its simplicity and accuracy. Yazawa and Ishizuka [14] utilized thermal resistance method for modeling the mini-channel heat sink which has heat spreading through the heat sink base, and validated their model by numerical calculations.

In 1D thermal resistance model, the heat flow through the heat sink base is thought to be one-dimensional, which means that the axial and transverse conduction are neglected. However, the experimental results by Gael et al. [15] showed that the axial conduction has great influence on the total heat transfer process and cannot be ignored. Besides, when the heat sink base is very thin, the transverse conduction resistance will become significant, as will be discussed later in this paper, while the 1D thermal resistance model cannot predict this effect. Further, in the modeling process by different researchers, various flow and heat transfer correlations were employed; the fin efficiency of the heat sink was taken into consideration in some models, while in the others the fin was thought to be isothermal. However, no comparison is available in the literature to show which one is more suitable for practical thermal design. Moreover, most of these models were tested experimentally or numerically by water cooling [16], while its applicability for guiding the thermal design of the new cooling candidate, liquid metal based heat sink is not clear.

This paper is dedicated to present a comprehensive comparison and discussion on the flow and thermal modeling of micro/mini-channel heat sink for better understanding and further sophistication of this technology. A numerical model is set-up and the result is used as a baseline for the comparison of different analytical correlations. A general optimization process for the thermal performance of the micro/mini-channel heat sink is developed, both for water cooling and liquid metal cooling and both in micro scale and millimeter scale, appropriate analytical correlations are recommended and optimal parameter selections and suggestions are provided for practical thermal design.

2. Flow and thermal model

The characteristic size of micro/mini-channel lies in the range of several microns to a few millimeters. Under this scale, N-S equations, non-slip wall condition and Fourier's law are all valid [15, 17]. Classical fluid flow and heat transfer correlations are thus still applicable [18, 19]. The heat sink analyzed here is schematically presented in Fig. 1. 1D thermal resistance model is used to analyze the heat sink. Heat transfer coefficient on the fin and base surface is thought to be constant, which is in fact an overall mean value.

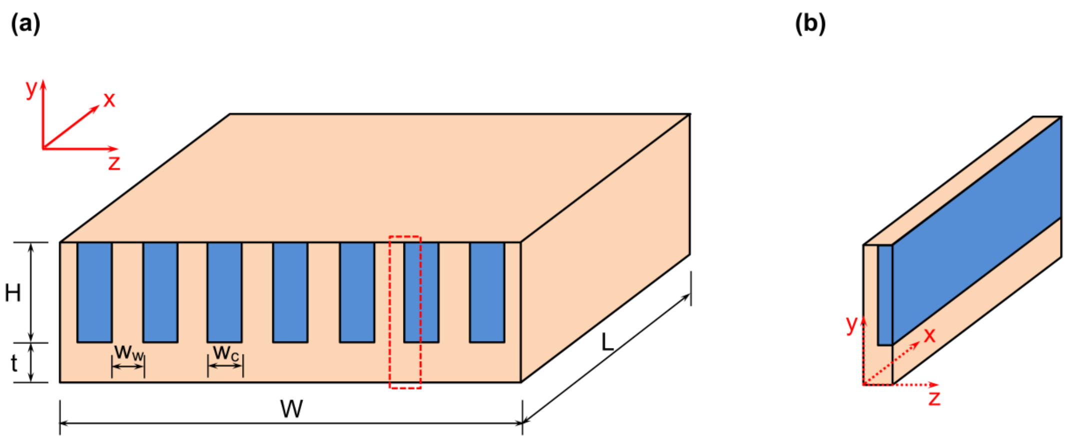

Fig. 1 (a) Schematic diagram of the micro/mini-channel heat sink; (b) basic unit for numerical calculation.

The overall thermal resistance of the heat sink is defined as the ratio of the maximum temperature difference to the total heat flow:

$$R_{total} = \frac{\Delta T_{max}}{q} = \frac{T_{w,out} - T_{m,f,in}}{q} \tag{1}$$

This thermal resistance can be divided into three components: conduction resistance R_{cond}, convection resistance R_{conv} and heat capacity resistance R_{cap}, namely: $R_{total} = R_{cond} + R_{conv} + R_{cap}$, where

$$R_{cond} = \frac{t}{k_w WL} \tag{2}$$

$$R_{conv} = \frac{1}{nhL(w_c + 2\eta_f H)} \tag{3a}$$

$$R_{conv} = \frac{1}{nhL(w_c + 2H)} \tag{3b}$$

$$R_{cap} = \frac{1}{\dot{m}c_p} = \frac{1}{n\rho U_m H w_c c_p} \tag{4}$$

It is worthy to note that in Eq. (3a), fin efficiency η_f is taken into consideration, as did by Knight et al. [11, 12] and Liu and Garimella [10]. However, Lee et al. [5] and Yazawa and Ishizuka [14] thought that the fin can be treated as isothermal, especially when the thermal conductivity of the structural material is high (e.g. copper). Moreover, isothermal fin assumption agrees with the condition under which the heat transfer correlations are derived. Hence, the convection resistance R_{conv} can be described by Eq. (3b).

The pumping power needed to drive the fluid flow through the channel is

$$P = n\Delta p U_m w_c H \tag{5}$$

in which, the pressure drop is

$$\Delta p = f\frac{L}{D_h}\frac{\rho U_m{}^2}{2} \tag{6}$$

To obtain the thermal and flow resistance of a heat sink, heat transfer coefficient *h* and friction factor *f* need to be known. However, in existing literatures, different correlations are recommended, as summarized in Table 1. In the following discussions, these correlations will be compared.

Table 1 Heat transfer and flow resistance correlations.

Flow Regime	Heat transfer correlation	Flow resistance correlation	Remarks
Fully developed laminar flow	$Nu_{fd,H}$=8.235(1-2.0421α+3.0853α^2-2.4765α^3+1.0578α^4-0.1861α^5) Eq. (7)	$(fRe)_{fd}$=96(1-1.3553α+1.9467α^2-1.7012α^3+0.9564α^4-0.2537α^5) Eq. (8)	Ref. [20]
	$Nu_{fd,H}$=-1.047+9.236M Eq. (9)	$(fRe)_{fd}$=18.8+78.57M , $M=(\alpha^2+1)/(\alpha+1)^2$ Eq. (10)	Ref. [11]
	$Nu_{fd,H}$=8.235(1-1.883α+3.767α^2-5.814α^3+5.361α^4-2α^5) Eq. (11)	-	Ref. [10]
Hydrodynamic developing laminar flow	-	$f_{app}Re=21.04(x^+)^{-0.434}\alpha^{-0.01}$ $0.001<x^+<0.02$ $f_{app}Re=45.2(x^+)^{-0.202}\alpha^{-0.094}$ $0.02\le x^+<0.1$ $f_{app}Re=(\mu_w/\mu_m)^{0.58}64/G+K_\infty/x^+$ $x^+\ge 0.1$ Eq. (12) $G=\frac{2}{3}+\frac{11\alpha(2-\alpha)}{24}$, $K_\infty=-0.906\alpha^2+1.693\alpha+0.649$	Ref. [13]
		$f_{app}Re=\left[\frac{163.84}{(x^+)^{1.14}}+(fRe)_{fd}^2\right]^{1/2}$ Eq. (13)	Ref. [10]
		$f_{app}Re=0.383(x^+)^{-0.3915}(fRe)_{fd}$ $x^+\le 0.05$ $f_{app}Re=(0.012625/x^++1)(fRe)_{fd}$ $x^+>0.05$ Eq. (14)	Ref. [14]
Thermally developing laminar flow	$Nu=1.87(x^*)^{-0.3}\alpha^{-0.056}Pr^{-0.036}$ $0.005<x^*<0.013$ $Nu=3.35(x^*)^{-0.13}\alpha^{-0.12}Pr^{-0.038}$ $0.013\le x^*<0.1$ Eq. (15)	-	Ref. [13]

3. Optimization of micro/mini-channel heat sink

3.1 Numerical model

A 3D flow and heat transfer conjugate numerical model is set-up. The computational domain chosen here is shown in Fig. 1(b) based on the symmetry of the geometry. In most of the following simulations, *Re* is lower than 2300, for few cases *Re* is larger than 2300 but lower than 10000, hence laminar flow is considered for all of the cases. The flow is thought to be steady, incompressible and the thermophysical properties of the coolant are assumed to be constant. Non-slip velocity and temperature boundary condition is applied on the channel wall. The governing equations are given by:

In the fluid zone:

$$\frac{\partial u_i}{\partial x_i}=0 \tag{16}$$

$$\frac{\partial(\rho u_i u_j)}{\partial x_i}=-\frac{\partial p}{\partial x_j}+\mu\frac{\partial^2 u_j}{\partial x_i^2} \tag{17}$$

$$\frac{\partial(u_i T)}{\partial x_i} = \lambda_l \frac{\partial^2 T}{\partial x_i^2} \tag{18}$$

In the solid zone:

$$\lambda_w \frac{\partial^2 T}{\partial x_i^2} = 0 \tag{19}$$

Commercial code Fluent 6.3 is used for numerical calculation. Uniform heat flux is applied at the bottom (y=0); adiabatic condition is assumed for the top surface (y=H+t), the front wall (x=0) and the posterior wall (x=L); the side walls (z=0 and z=$(w_w + w_c)/2$) are symmetric. Uniform velocity is given for the fluid inlet, and out-flow condition is applied for the outlet. $500 \times 100 \times 20$ (x-y-z) grid is adopted to ensure enough accuracy.

The numerical model is validated through comparison with the experimental data from Qu and Mudawar [17]. Wherein microchannel the size of $w_c = 231\mu m$ and $H = 713\mu m$ is tested under different Re numbers. Fig.2 presents the comparison of temperature rise at 4 test locations and the pressure drop though the channel. Good agreement is achieved between current simulation and the experimental data, which implies the reliability of the numerical method.

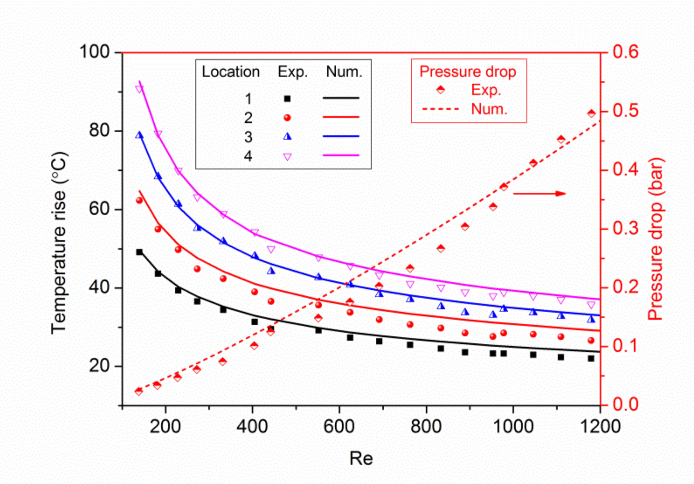

Fig. 2 Comparison of the numerical model with experimental data in Ref. [17].

3.2 Microchannel water cooling

A general optimization process for the thermal performance of micro/mini-channel heat sink is carried out. Water cooling is investigated first, the thermo-physical properties of which are listed in Table 2. A typical silicon chip with dimension of $W \times L = 1cm \times 1cm$ is selected for investigation, with heat dissipating $q = 100W$ ($q" = 100W / cm^2$).

Since the hydrodynamic diameter of the channel is much smaller than its length, most of the cases studied in this sub-section are hydrodynamic and thermally fully developed. Based on Table 1, there are two methods for modeling the convective thermal resistance R_{conv}, Eq. (3a) considers the fin efficiency (called R1 here), while Eq. (3b) does not (called R2). The fully developed heat transfer coefficient has 3 choices, since Eq. (7) and Eq. (9) are similar, only Eq. (7) (called h1) and Eq. (11) (h2) are compared. Eq. (8) and Eq. (10) are similar, hence only Eq. (8) is utilized for fully developed flow resistance calculation.

Table 2 Thermo-physical properties of the working fluids and structural materials.

Material	Density ρ (kg/m^3)	Heat capacity c_p (J/kg/K)	Viscosity μ (10^{-3} kg/m/s)	Thermal conductivity k (W/m/K)	Prandtl number Pr
Water[a] [21]	998.2	4182	1.003	0.6	6.99
$Ga_{68}In_{20}Sn_{12}$ [22]	6363	366	2.22	39	0.02
Silicon	2328	700	-	148	-
Copper	8978	381	-	387.6	-

[a] At 20 °C.

3.2.1 Channel aspect ratio

Channel number n, aspect ratio α and fin to channel width ratio β are the three most important geometrical parameters in the thermal design of micro/mini-channel heat sink. Here, the influence of α is investigated first. Keep $n = 100$, $\beta = w_w / w_c = 1$ ($w_w = w_c = 50\mu m$), $t = 100\mu m$, vary α by adjust the channel height $H = 100, 200, 500, 800, 1000$ (i.e. $\alpha = 0.5, 0.25, 0.1, 0.0625, 0.05$). The inlet flow velocity is set to be a constant of $U_m = 1m / s$.

Fig. 3 shows the variation of the overall thermal resistance R_{total} and the flow resistance under different channel height *H*. The higher *H* is, the lower value R_{total} can reach. The decrease of R_{total} is significant at relatively lower value of *H*. When *H* is larger than about 500 μm, the variation tendency of R_{total} becomes slow. As the channel height changes from 500 μm to 1000 μm, R_{total} decreases

25%. However, the pumping power doubles. Besides, for lower aspect ratio (namely higher fin), the machining process becomes challenging and the strength issue of the channel comes into play. Therefore, $H = 500\mu m$ is appropriate in this case, which means that $\alpha = 0.1$.

Compare the numerical results and that from analytical correlations, it can be found that, the thermal resistance based on h1 and h2 are nearly the same, while R1 and R2 lead to significant different results. R2 overestimates the thermal performance for it neglects the fin efficiency, especially for high channel height which has low fin efficiency. Overall, correlation R1h1 agrees best with the numerical values, and this conclusion is valid for all of the cases studied in this sub-section, from Fig. 3 to 5. As for the flow resistance, the fully developed friction correlation Eq. (8) can accurately predict the pressure drop and pumping power.

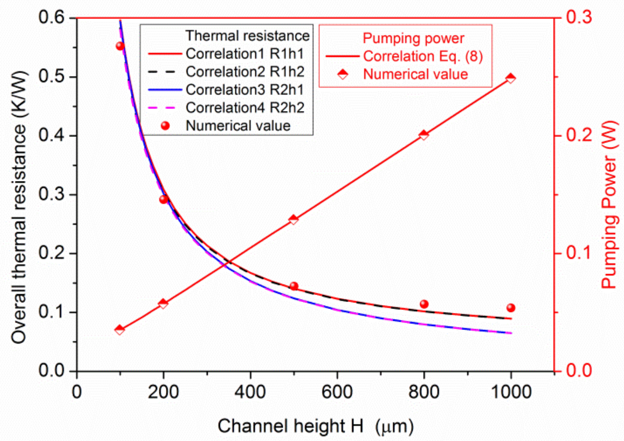

Fig. 3 Thermal and flow resistance versus the channel height (aspect ratio α), with $n = 100$, $\beta = 1$, $t = 100\mu m$ and $U_m = 1m/s$.

3.2.2 Channel number and width ratio

In this sub-section, the influence of the channel number *n* is investigated first, Fig. 4. Change of *n* in fact means the variation of the channel size. Based on the result in last sub-section, $\alpha = 0.1$ is appropriate and it is remained unchanged here. Keep $\beta = 1$ and $U_m = 1m/s$. From Fig. 4 it can be seen that there exists an optimal choice of the channel number, under which R_{total} can obtain the minimum value. Further increasing *n* (i.e. smaller channel size) will worsen the thermal performance and, at the same time linearly increase the pumping power.

The influence of fin to channel width ratio β is also studied (Fig. 5). The lower β is, the larger the optimal *n* is, and the lower R_{total} is under the optimal *n*. Also presented in Fig. 5 is the pumping power curve, which is applicable for all of the cases no matter what value β is. Although lower β means lower optimal R_{total}, it leads to larger power consumption and thinner fin which is flimsy. Here, $\beta = 0.8$ is selected as a compromise, and corresponding optimal channel number is *n*=72.

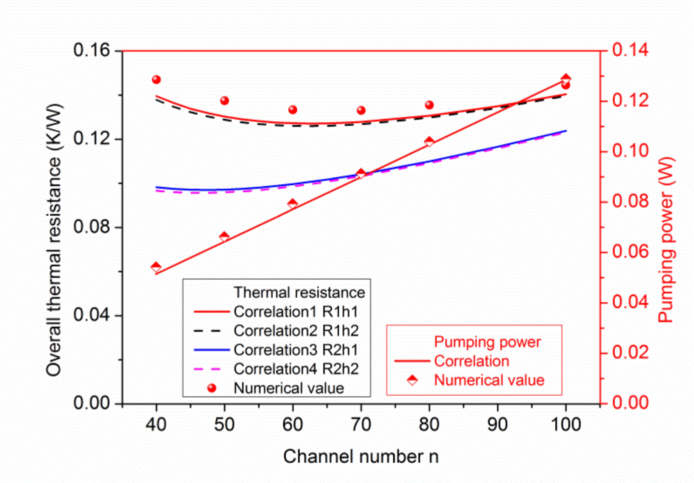

Fig. 4 Thermal and flow resistance versus the channel number, with $\alpha = 0.1$, $\beta = 1$, $t = 100\mu m$ and $U_m = 1m/s$.

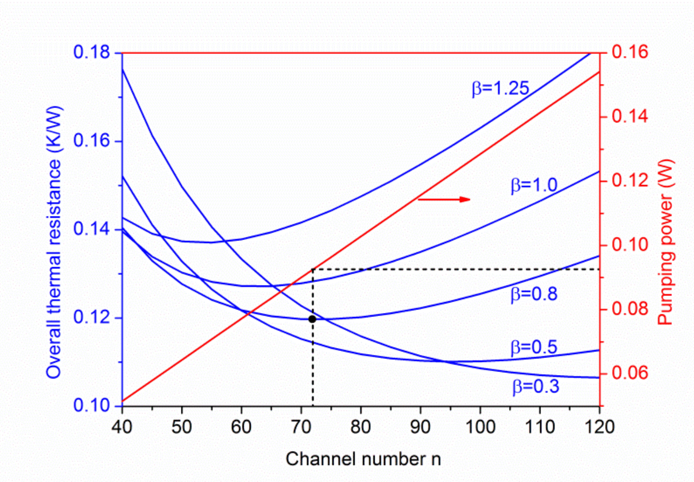

Fig. 5 Thermal resistance and pumping power versus the channel number under different β based on correlation R1h1, with $\alpha = 0.1$, $t = 100\mu m$ and $U_m = 1m/s$.

3.2.3 Velocity

The optimal heat sink geometrical parameters are found to be $\alpha = 0.1$, $\beta = 0.8$ and $n = 72$. Based on this geometrical set-up, the influence of the fluid inlet velocity U_m is studied, as shown in Fig. 6. Under this condition, pumping power has quadratic relationship with velocity. Compromise of the thermal performance and pumping power suggests that $U_m = 1m/s$ is appropriate.

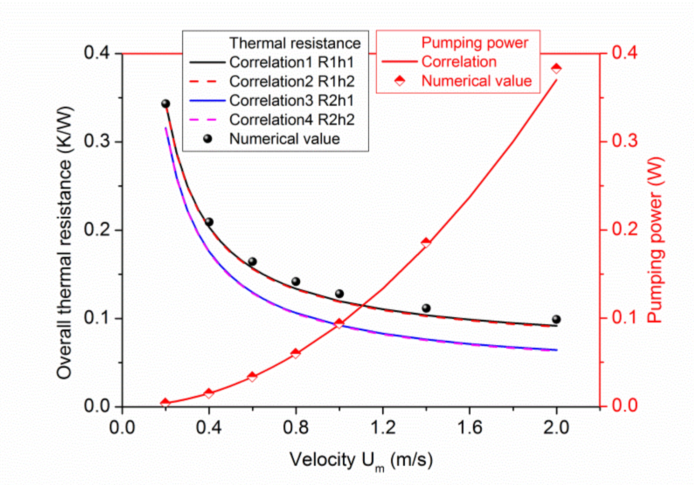

Fig. 6 Thermal resistance and pumping power versus the flow velocity, with $n = 72$, $\alpha = 0.1$, $\beta = 0.8$ and $t = 100\mu m$.

3.2.4 Base thickness

The influence of the base thickness t is discussed here, and the other geometrical parameters are the same as that in section 3.2.3. Eq. (2) indicats that the thermal resistance will vary linearly with t, as shown in Fig. 7. However, numerical results showed that when the base thickness is too small, the thermal resistance will increase conversely. In the limit case, namely $t = 0\mu m$, the thermal resistance reaches as high as $R_{total} = 0.294K/W$. This is because that heat flow through the heat sink base is in fact not one-dimensional. Most of the heat flows to the fin base and is dissipated to the fluid through the fin surface, while only small part of the heat is dissipated from the base surface. Too thin heat sink base will hinder the transverse heat transfer through the base to the fin and thus increase the total thermal resistance. This variation tendency cannot be predicted by the analytical correlations, and numerical method is needed to find out the critical base thickness t_c. At the range larger than t_c, the base is better to be thinner, while the machining feasibility and mechanical strength should also be taken into consideration.

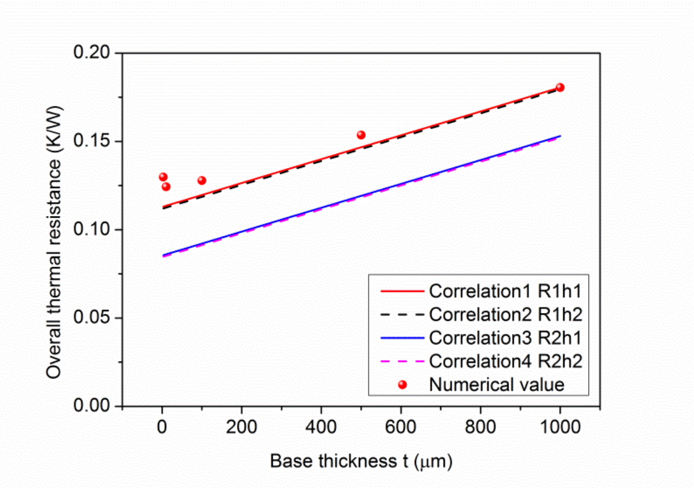

Fig. 7 Thermal resistance versus the base thickness, $n = 72$, $\alpha = 0.1$, $\beta = 0.8$ and $U_m = 1m/s$.

3.2.5 Structural material

In this section, the influence of the structural material is discussed (Fig. 8). The same process is conducted as that in section 3.2.2. The only difference is that the structural material is replaced with copper. It can be found that change the structural material from silicon to copper, the thermal resistance will decrease about 20%. Another important conclusion is that, even for high thermal conductivity structural material, the results from correlation R1 and R2 deviate greatly, and the former one agrees better with the numerical values, which means that the fin efficiency is better to be considered even high thermal conductivity structural material is chosen.

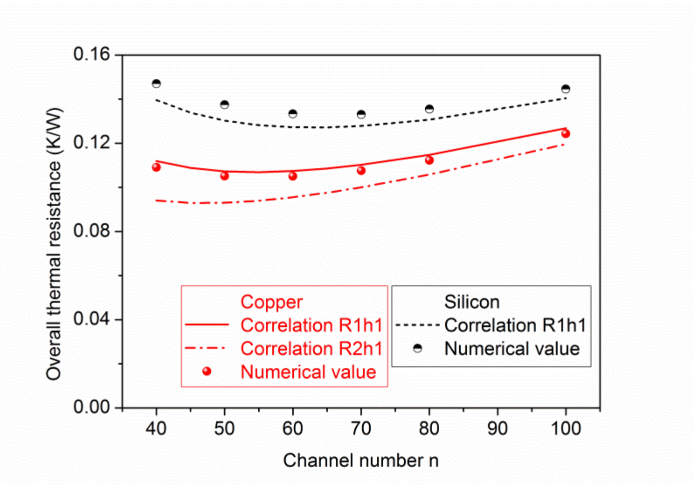

Fig. 8 Thermal resistance versus the channel number with different structural material, $\alpha = 0.1$, $\beta = 1$, $t = 100\mu m$ and $U_m = 1m/s$.

3.3 Mini-channel liquid metal cooling

As a new kind of coolant, liquid metal exhibits excellent cooling performance due to its high thermal conductivity, and has been successfully used for the thermal management of large power CPUs [23], LEDs [24] and lasers [25]. Hence, thermal optimization of liquid metal based heat sink is worth exploring. Preliminary calculation shows that it is more suitable for liquid metal to be used in mini-channel scale, which is mainly because that its specific heat capacity is low and the capacity thermal resistance will become appreciable in micro scale.

A new problem is set for the liquid metal heat sink: $W \times L = 4cm \times 4cm$ and $q = 1600W$ is applied ($q" = 100W/cm^2$). The heat sink is made of copper, $Ga_{68}In_{20}Sn_{12}$ is selected as the working fluid and the thermophysical properties of which refer to Table 2. The optimization process is the same as that performed before for water cooling. At the first step, keep $n = 20$, $\beta = w_w / w_c = 1$ ($w_w = w_c = 1mm$), $t = 2mm$ and $U_m = 1m/s$, vary α by adjusting the channel height $H = 1,2,5,8,10mm$ (i.e. $\alpha = 1, 0.5, 0.2, 0.125, 0.1$), as shown in Fig. 9.

Due to the extremely low *Pr* number of liquid metal, all of the cases studied in this section is thermally developed ($x^* > 0.05$), while hydrodynamic developing ($x^+ < 0.05$). For flow resistance calculation, there exist three candidate correlations for modeling developing flow, Eq. (12), (13) and (14), called f1, f2 and f3 respectively. These three correlations will be compared with the numerical values.

Different from water cooling microchannel heat sink, significant deviation arises between the thermal resistance obtained from numerical calculation and that from correlation R1h1. Based on the numerical results, conclusion can be drawn that $H = 5mm$ ($\alpha = 0.2$) is appropriate since that there is no further decrease of R_{total} when *H* is larger than 5 mm. As for the pumping power, the three correlations differ to each other obviously. Among which, correlation f1 (Eq.(12)) agrees best with the numerical values.

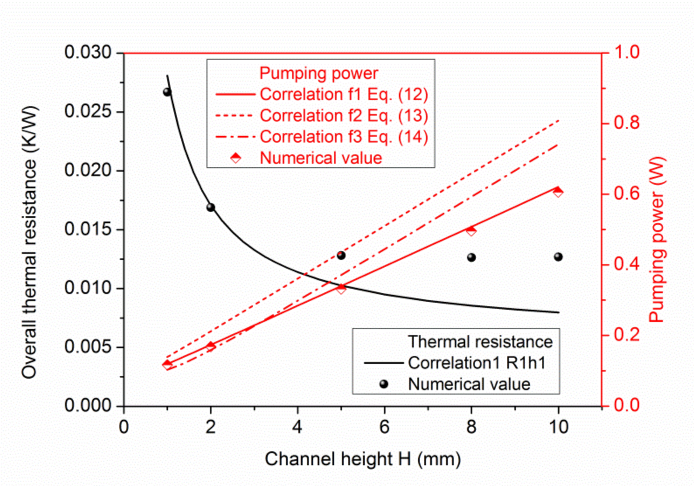

Fig. 9 Thermal and flow resistance versus the channel height, $n=20$, $\beta=1$, $t=2mm$ and $U_m=1m/s$.

Keep $\alpha=0.2$, $\beta=1$ and $U_m=1m/s$ unchanged, Fig. 10 shows the variation of the thermal and flow resistance with the channel number. According to correlation R1h1, the optimal channel number is *n*=15, while the numerical results suggest *n*=20. Under this channel number (*n*=20), the hydrodynamic diameter of the channel is $D_h=1.667mm$, namely mini-channel. The best correlation for predicting the developing flow resistance is still f1 (Eq.(12)). Different from the developed flow discussed in section 3.2.2, where the pumping power increases linearly with *n* (Eq. (20)), here it varies slowly and nearly keeps constant at the range of *n* from 5 to 30.

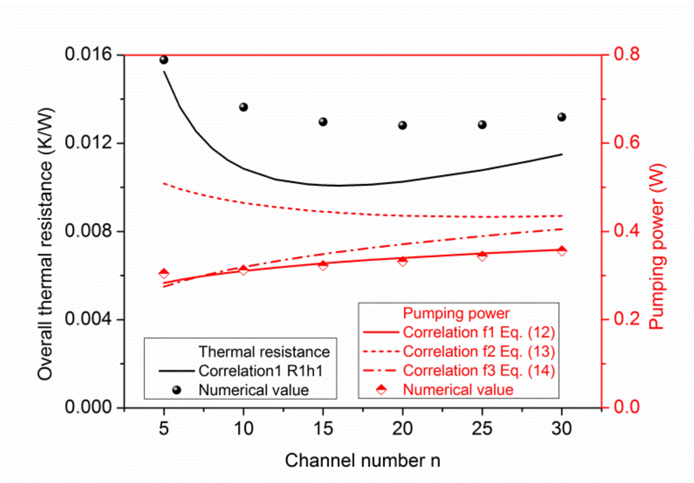

Fig. 10 Thermal and flow resistance versus the channel number, $\alpha=0.2$, $\beta=1$, $t=2mm$ and $U_m=1m/s$.

As revealed in section 3.2.2, smaller β means smaller optimal resistance, while greater power consumption. Here, the decrease of R_{total} is not significant when β changes from 1 to 0.8, herece $\beta=1$ is finally selected. Based on the optimal channel number *n*=20, the influence of velocity is studied, as shown in Fig. 11. Only Eq. (12) is used to calculate the developing flow resistance. Correlation R1h1 predicts the variation tendency of the thermal resistance in some degree, while more accurate results need numerical calculation. The numerical results in Fig. 11 indicates that the velocity of $U_m = 1m/s$ may be appropriate for it leads to relatively low thermal resistance and low power consumption. Under this condition, the critical base thickness is t_c=0.3 mm.

The reason why the analytical correlation exhibits great deviation to the numerical solution is discussed as follows. The heat transfer correlation for thermally fully developed flow is in fact derived based on the prerequisite that the flow is hydrodynamic fully developed. However, for liquid metal heat transfer, the hydrodynamic entry region is much longer than the thermal entry region due to its extremely low *Pr* number. For most of the cases studied in this section, the situation is that the flow is thermally fully developed while hydrodynamic developing. Hence, the so called "thermally fully developed flow" determined by $x^* = x/D_h RePr > 0.1$ is in fact not rigorous, for that in the hydrodynamic developing region, no thermally fully developed flow can be formed. Therefore, correlation Eq. (7) (h1) is non-appropriate here, and it is not strange that this non-appropriate correlation leads to significant deviation to numerical values. However, for this kind of heat transfer process in rectangular duct, theoretical analysis is rather difficult and no correlations are available for modeling the thermal resistance.

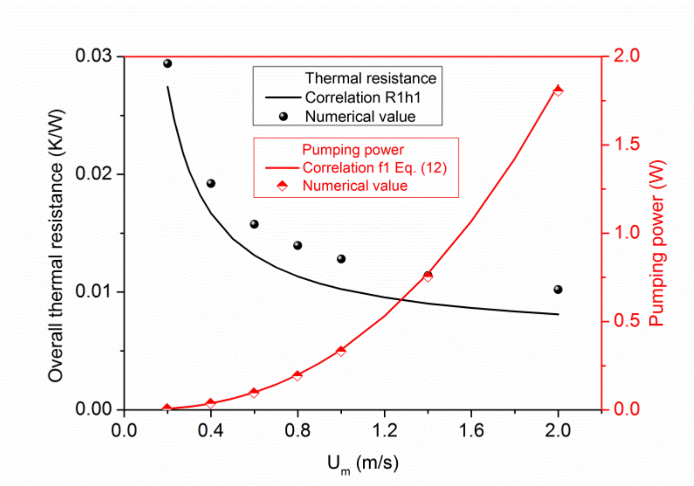

Fig. 11 Thermal and flow resistance versus the fluid velocity, $n=20$, $\alpha=0.2$, $\beta=1$ and $t=2mm$.

3.4 Mini-channel water cooling

Although it has been disclosed that, for water cooling the optimal channel size lies in micro scale, however, it is difficult for micro-channel heat sink to be implemented in practice for that high pressure head is needed to drive the working fluid [4]. Mini-channel heat sink provides a compromise which can cope with relatively high heat flux condition while with mild pressure loss [6, 14]. In this section, the optimization process of a water cooling mini-channel copper heat sink is conducted, and some new features different from microchannel heat sink are revealed. The design problem is the same as that in the liquid metal heat sink. In this section, most of the cases are hydrodynamic and thermally developing, and hence Eq. (12) is utilized for friction prediction and Eq. (15) is used for heat transfer. Due to the similarity of the optimization process, some of the details will be omitted in the following discussions for brief.

At first, keep $n = 20$, $\beta = 1$ and $t = 2mm$. Calculation indicates that $H = 8mm$, namely $\alpha = 0.125$ is suitable. To guarantee that the channel is at millimeter scale (D_h>1mm), the channel number is kept below 30 in this sub-section. Fig. 12 indicates that larger n leads to lower R_{total}, the changes of β have nearly no influence on the thermal performance, while smaller β leads to larger power consumption. Optimally, $n = 20$ and $\beta = 1.25$ is selected. Based on the optimal geometry, the appropriate velocity is calculated to be about 2 m/s.

At the end of this section, a brief comparison is conducted for the cooling performance of liquid metal and water in mini-channel heat sink. For water, the optimal geometrical parameters and flow condition are: $n = 20$, $\beta = 1.25$, $\alpha = 0.125$ and $U_m = 2m/s$, which result in $R_{total} = 0.022W/K$ and $P = 0.775W$. While for liquid metal, $n = 20$, $\beta = 1$, $\alpha = 0.2$ and $U_m = 1m/s$, and $R_{total} = 0.013W/K$, $P = 0.332W$. The flow and thermal performance of liquid metal is much superior than that of water in mini-channel since its thermal resistance is about 60% of that of water while less power consumption (about 43%) is needed. Note that the optimal size of water cooling is at micro scale, hence better thermal performance can be obtained by miniaturizing the channel, while it means larger pumping power.

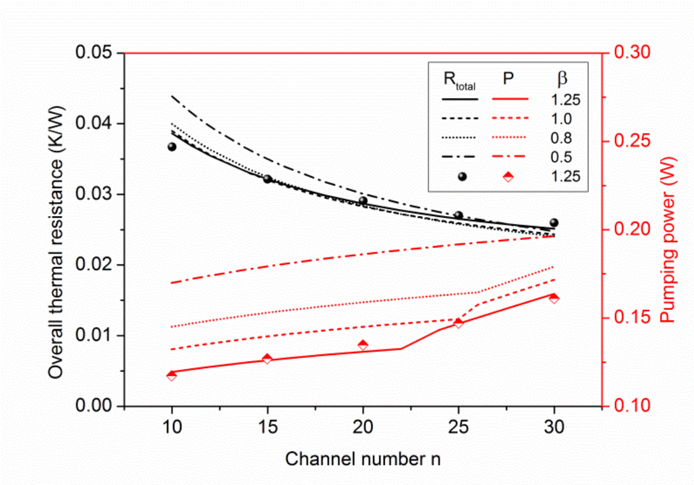

Fig. 12 Thermal and flow resistance versus the channel number under different β, $\alpha = 0.125$, $t = 2mm$ and $U_m = 1m/s$.

4. Conclusion

For all of the cases investigated in this paper, the flow resistance correlations agree very well with the numerical results, which indirectly indicate the accuracy of the numerical model. For hydrodynamic developing flow, Eq. (12) is recommended for flow resistance calculation. As for the heat transfer analysis, 1D model can predict the thermal resistance with relatively high accuracy, especially for water cooling heat sink. In the analysis, the fin efficiency is better to be considered, and Eq. (7) is recommended for heat transfer coefficient calculation of fully developed laminar flow. Liquid metal exhibits much superior flow and thermal performance in mini-channel heat sink than that of water. However, there exists distinct deviation between the results from 1D model and that from numerical calculation, the main reason is thought to be that the flow of liquid metal is “thermally developed” while hydrodynamic developing and the thermally fully developed heat transfer correlation is inaccurate under this condition. Therefore, general heat transfer correlation of liquid metal in ducts is worth exploring. As an alternative, in practical thermal design of liquid metal based mini-channel heat sink, appropriate numerical calculation and experiment are needed in addition to the analytical approach.

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Tables and figure captions:

Table 1 Heat transfer and flow resistance correlations.

Flow Regime	Heat transfer correlation	Flow resistance correlation	Remarks
Fully developed laminar flow	$Nu_{fd,H}$=8.235(1-2.0421α+3.0853α^2-2.4765α^3+1.0578α^4-0.1861α^5) Eq. (7)	$(fRe)_{fd}$=96(1-1.3553α+1.9467α^2-1.7012α^3+0.9564α^4-0.2537α^5) Eq. (8)	Ref. [20]
	$Nu_{fd,H}$=-1.047+9.236M Eq. (9)	$(fRe)_{fd}$=18.8+78.57M , $M=(\alpha^2+1)/(\alpha+1)^2$ Eq. (10)	Ref. [11]
	$Nu_{fd,H}$=8.235(1-1.883α+3.767α^2-5.814α^3+5.361α^4-2α^5) Eq. (11)	-	Ref. [10]
Hydrodynamic developing laminar flow	-	$f_{app}Re=21.04(x^+)^{-0.434}\alpha^{-0.01}$ $0.001<x^+<0.02$ $f_{app}Re=45.2(x^+)^{-0.202}\alpha^{-0.094}$ $0.02\le x^+<0.1$ $f_{app}Re=(\mu_w/\mu_m)^{0.58}64/G+K_\infty/x^+$ $x^+\ge 0.1$ Eq. (12) $G=\frac{2}{3}+\frac{11\alpha(2-\alpha)}{24}$, $K_\infty=-0.906\alpha^2+1.693\alpha+0.649$	Ref. [13]
		$f_{app}Re=\left[\frac{163.84}{(x^+)^{1.14}}+(fRe)_{fd}^2\right]^{1/2}$ Eq. (13)	Ref. [10]
		$f_{app}Re=0.383(x^+)^{-0.3915}(fRe)_{fd}$ $x^+\le 0.05$ $f_{app}Re=(0.012625/x^++1)(fRe)_{fd}$ $x^+>0.05$ Eq. (14)	Ref. [14]
Thermally developing laminar flow	$Nu=1.87(x^*)^{-0.3}\alpha^{-0.056}Pr^{-0.036}$ $0.005<x^*<0.013$ $Nu=3.35(x^*)^{-0.13}\alpha^{-0.12}Pr^{-0.038}$ $0.013\le x^*<0.1$ Eq. (15)	-	Ref. [13]

Table 2 Thermo-physical properties of the working fluids and structural materials.

Material	Density ρ (kg/m^3)	Heat capacity c_p (J/kg/K)	Viscosity μ (10^{-3} kg/m/s)	Thermal conductivity k (W/m/K)	Prandtl number Pr
Water[a] [21]	998.2	4182	1.003	0.6	6.99
$Ga_{68}In_{20}Sn_{12}$ [22]	6363	366	2.22	39	0.02
Silicon	2328	700	-	148	-
Copper	8978	381	-	387.6	-

[a] At 20 °C.

Fig. 1 (a) Schematic diagram of the micro/mini-channel heat sink; (b) basic unit for numerical calculation.

Fig. 2 Comparison of the numerical model with experimental data in Ref. [17].

Fig. 3 Thermal and flow resistance versus the channel height (aspect ratio α), with $n = 100$, $\beta = 1$, $t = 100\mu m$ and $U_m = 1m/s$.

Fig. 4 Thermal and flow resistance versus the channel number, with $\alpha = 0.1$, $\beta = 1$, $t = 100\mu m$ and $U_m = 1m/s$.

Fig. 5 Thermal resistance and pumping power versus the channel number under different β based on correlation R1h1, with $\alpha = 0.1$, $t = 100\mu m$ and $U_m = 1m/s$.

Fig. 6 Thermal resistance and pumping power versus the flow velocity, with $n = 72$, $\alpha = 0.1$, $\beta = 0.8$ and $t = 100\mu m$.

Fig. 7 Thermal resistance versus the base thickness, $n = 72$, $\alpha = 0.1$, $\beta = 0.8$ and $U_m = 1m/s$.

Fig. 8 Thermal resistance versus the channel number with different structural material, $\alpha = 0.1$, $\beta = 1$, $t = 100\mu m$ and $U_m = 1m/s$.

Fig. 9 Thermal and flow resistance versus the channel height, $n = 20$, $\beta = 1$, $t = 2mm$ and $U_m = 1m/s$.

Fig. 10 Thermal and flow resistance versus the channel number, $\alpha = 0.2$, $\beta = 1$, $t = 2mm$ and $U_m = 1m/s$.

Fig. 11 Thermal and flow resistance versus the fluid velocity, $n = 20$, $\alpha = 0.2$, $\beta = 1$ and $t = 2mm$.

Fig. 12 Thermal and flow resistance versus the channel number under different β, $\alpha = 0.125$, $t = 2mm$ and $U_m = 1m/s$.

Flow and Thermal Modeling and Optimization of Micro/mini-channel Heat Sink

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Highlights of the work

> General optimization process for micro/mini-channel heat sink is developed.

> Various analytical correlations are compared with numerical results.

> Appropriate correlations and optimal geometrical parameters are recommended.

> Liquid metal based mini-channel heat sink is optimized which shows excellent performance.