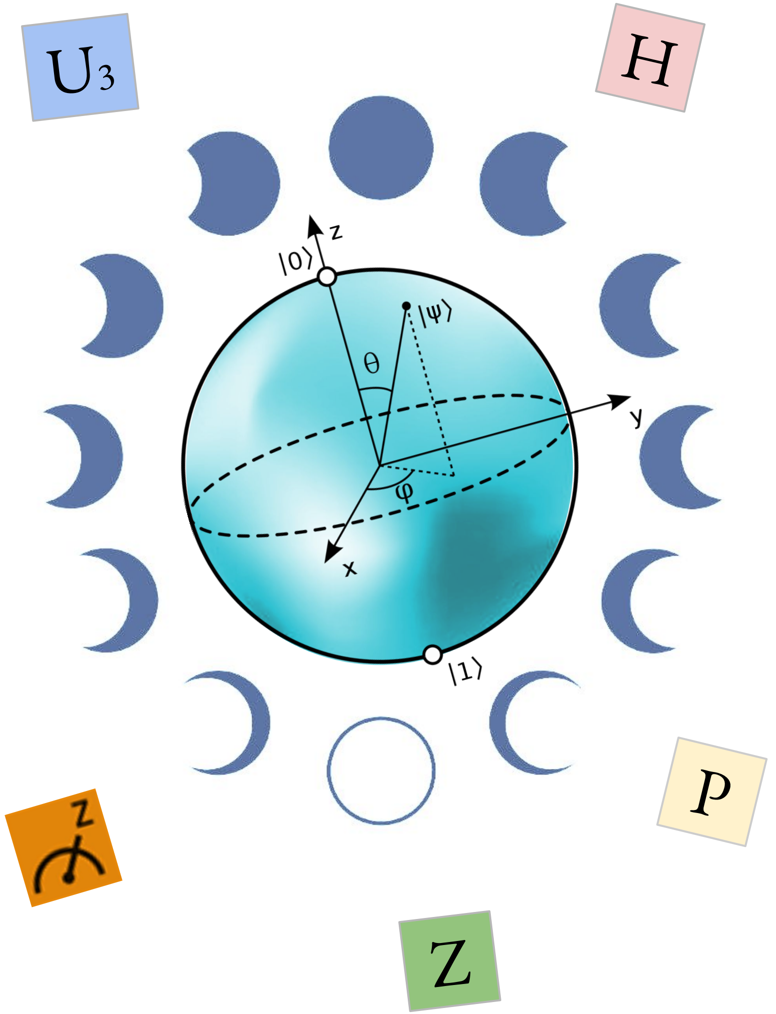

Phase Estimation Algorithm for Quantum Computing

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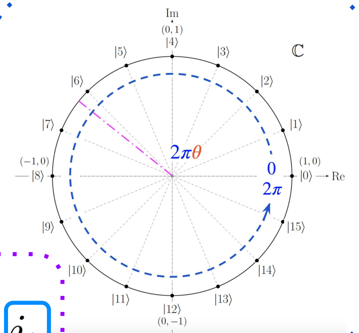
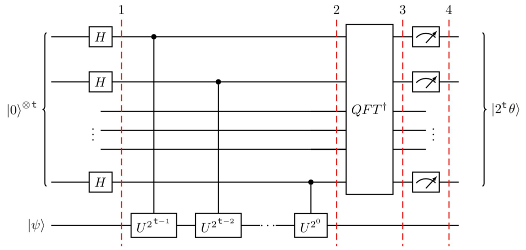
FRIB-TA Summer School 2022

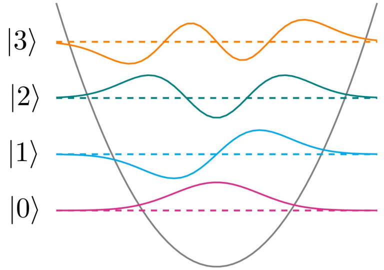
Outline

- *Idea* of quantum phase estimation (QPE)
    - Phase as binary fractions
- How do we *estimate* the phase ?
    - Phase kickback
    - Quantum Fourier Transform
- Quantum circuit for QPE
    - Standard & Iterative
- Application on a physical model

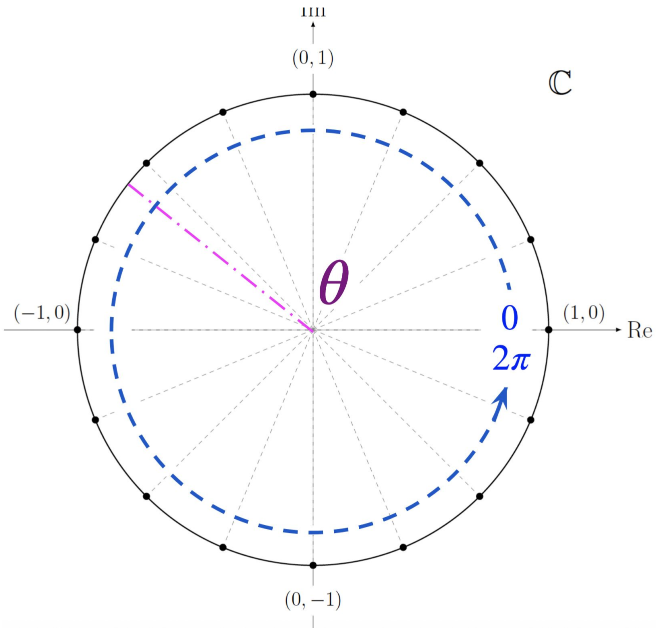


$$\varphi = \sum_{m=1}^{t} j_m 2^{-m} = 0.\boxed{j_1}\; j_2 \;\ldots\; \boxed{j_t}$$





QPE 101: The *Idea*

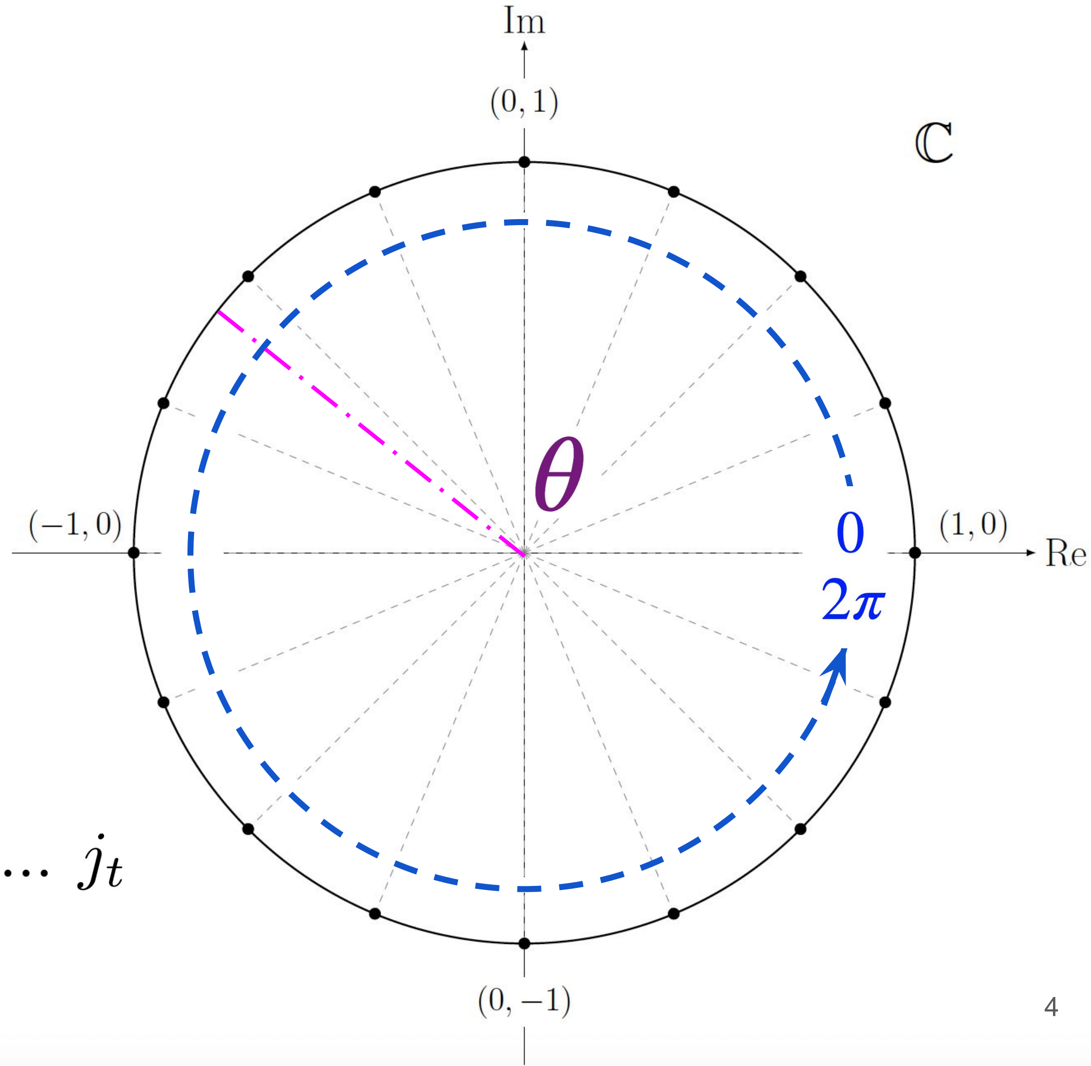


Phase as ***binary*** fractions

$$U|\psi\rangle = e^{i\theta}|\psi\rangle$$

$$= e^{2\pi i\varphi}|\psi\rangle$$

$$\varphi = \sum_{m=1}^{t} j_m 2^{-m} = 0.\ j_1\ j_2\ \dots\ j_t$$



Phase as ***binary*** **fractions**

$$\varphi = \sum_{m=1}^{t} j_m 2^{-m} = 0.\boxed{j_1}\ j_2\ \ldots \boxed{j_t}$$

Most significant bit **Least significant bit**

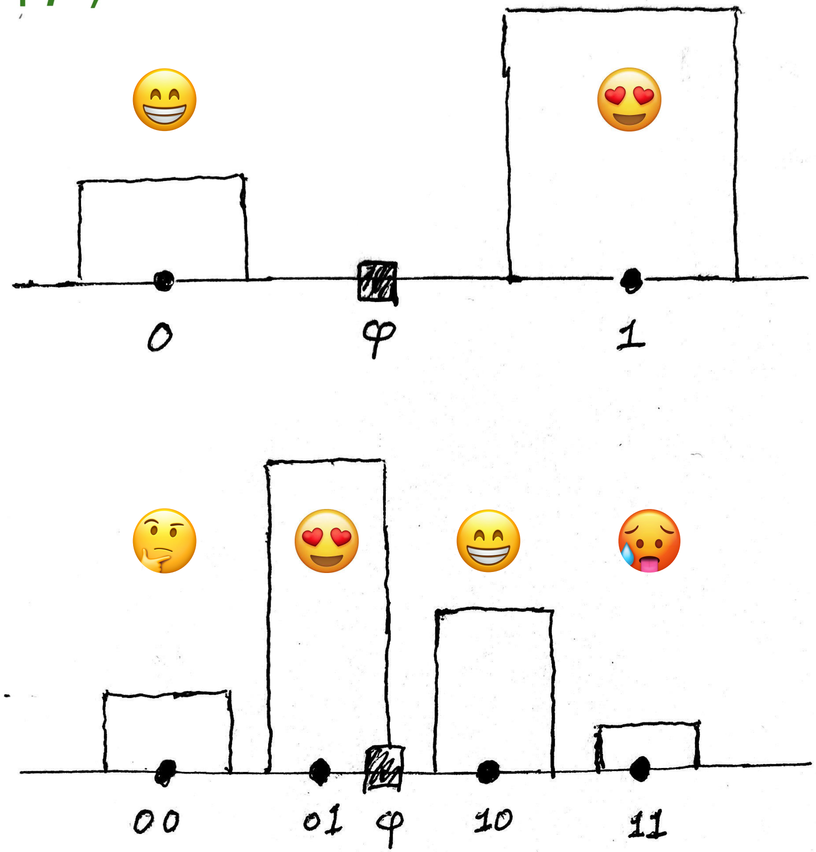
Example:

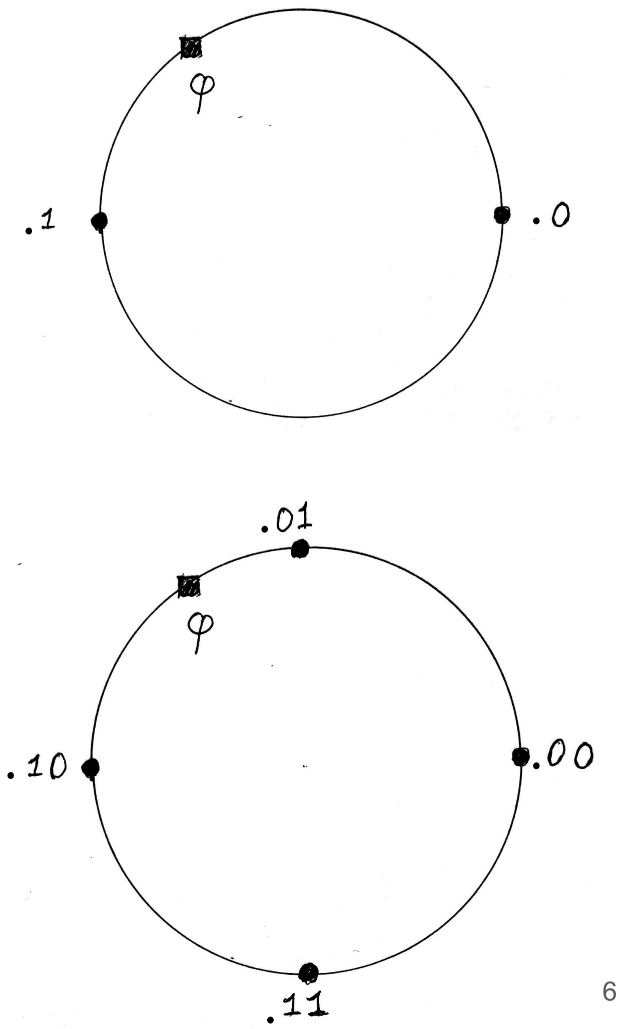
$$\varphi = (.\ 1\ 0\ 1\ 1)_2$$

$$\frac{1}{2^1} \cdot 1 + \frac{1}{2^2} \cdot 0 + \frac{1}{2^3} \cdot 1 + \frac{1}{2^4} \cdot 1 = (0.6875)_{10}$$

$$U|\psi\rangle = e^{2\pi i\varphi}|\psi\rangle$$

Learn the eigenvalue of your system

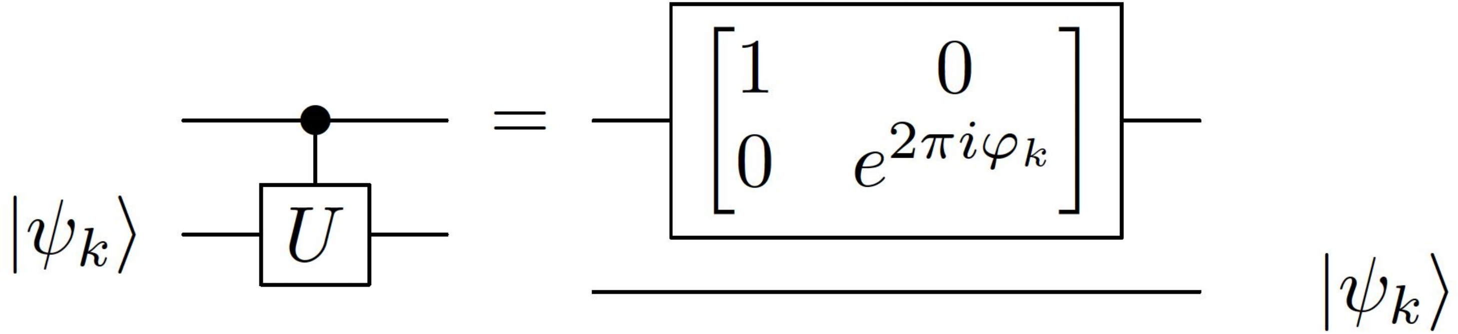




How do we learn the phase using a quantum computer ?

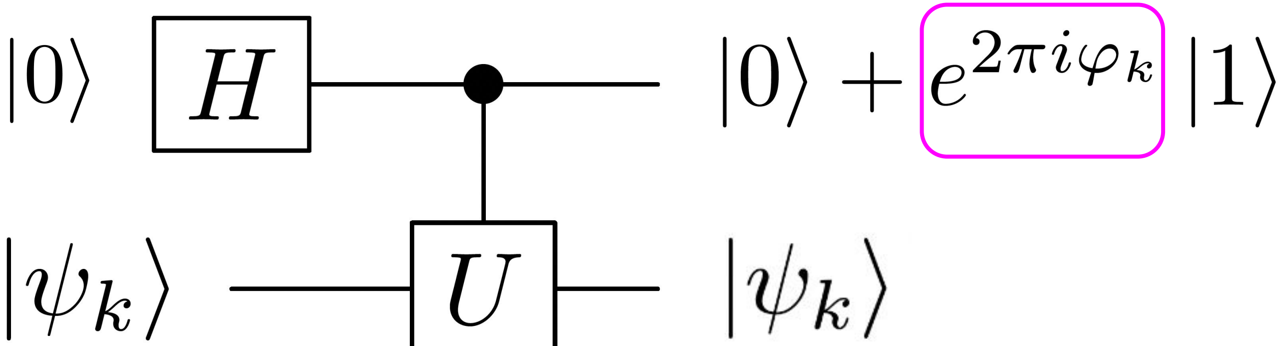
0 1 1 0 0 1 0

Phase kickback



$$U|\psi\rangle = e^{2\pi i \varphi}|\psi\rangle$$

Phase kickback: Why Hadamard ?

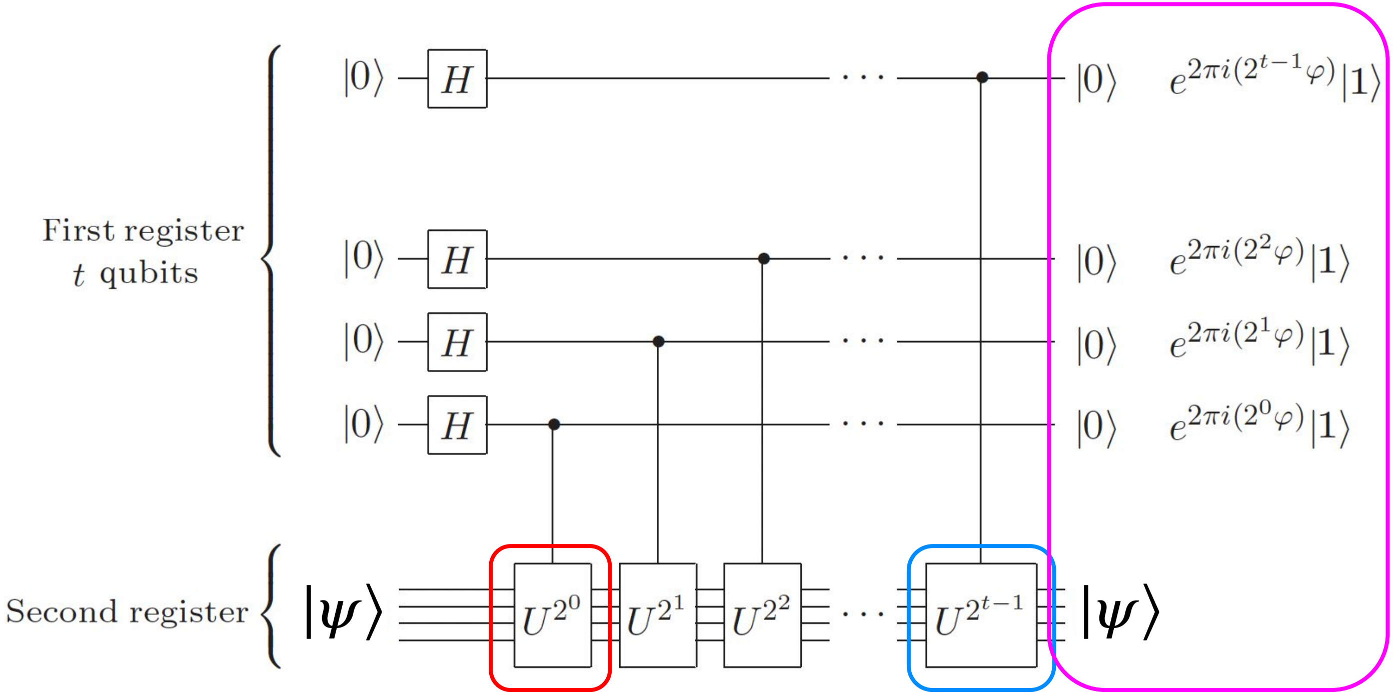


$$U|\psi\rangle = e^{2\pi i \varphi}|\psi\rangle$$

Quantum Fourier Transform

$$\varphi = \sum_{m=1}^{t} j_m 2^{-m} = 0.\boxed{j_1}\, j_2 \ldots \boxed{j_t}$$

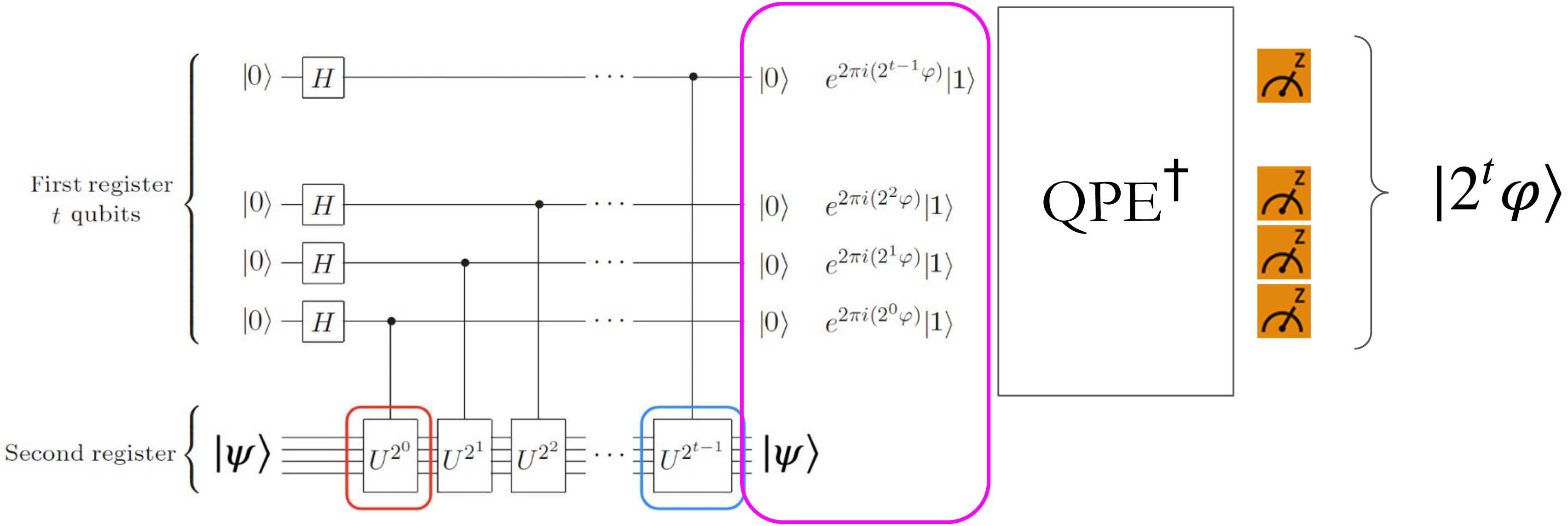
$$\frac{1}{2^{\frac{n}{2}}} \sum_{k=0}^{2^n-1} e^{2\pi i \theta k} |k\rangle \otimes |\psi\rangle$$



Source: Nielsen and Chuang, Ch. 5.2

Standard phase estimation

$$\varphi = \sum_{m=1}^{t} j_m 2^{-m} = 0.\boxed{j_1}\, j_2 \ldots \boxed{j_t}$$



Source: Nielsen and Chuang, Ch. 5.2

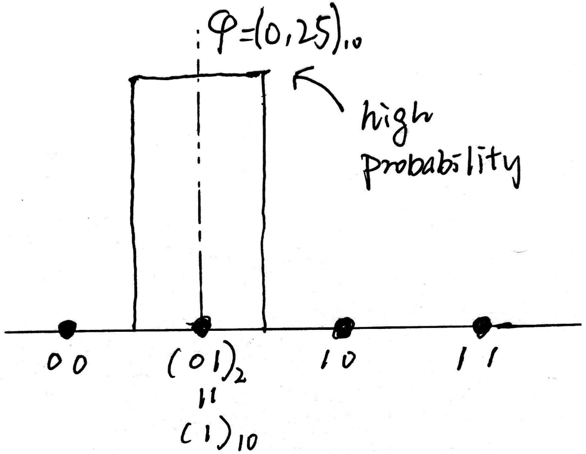
Interpret the phase

$$|2^t \varphi\rangle$$

$2^t \varphi$

Is integer

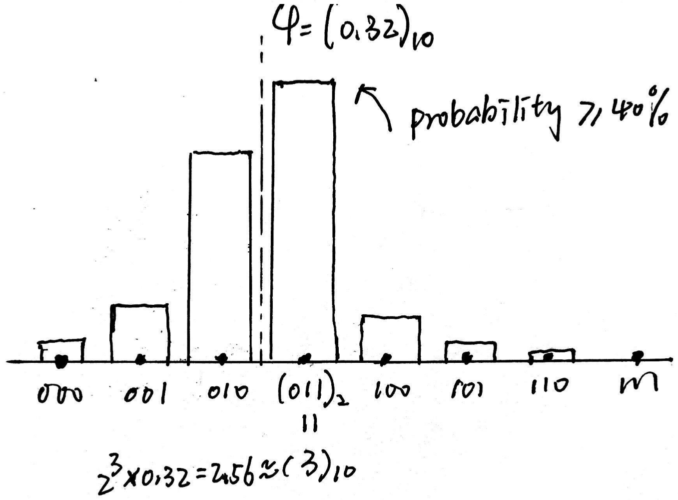
Ex: t = 2,
Phi = 0.25



$2^t \varphi$

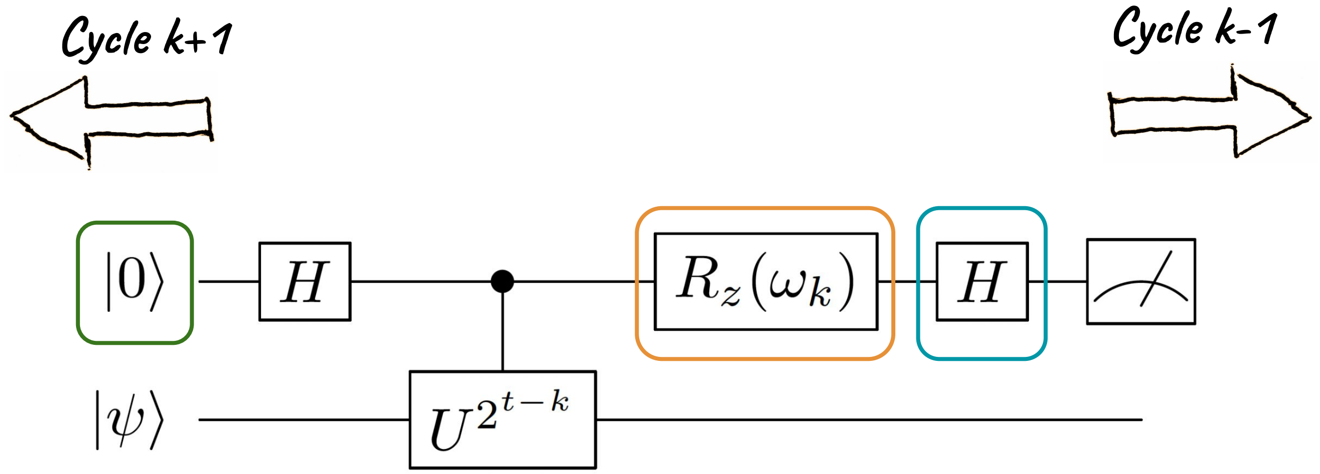
Is not integer

Ex: t = 3,
Phi = 0.32



Iterative phase estimation

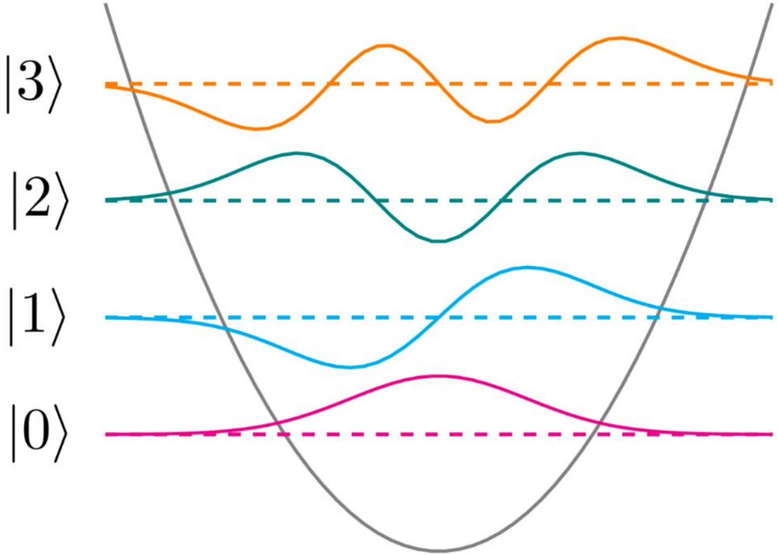
$$\varphi = \sum_{m=1}^{t} j_m 2^{-m} = 0.\ j_1\ j_2\ \ldots j_k \ldots\ j_t$$



$\boldsymbol{\omega_k}$ Feedback angle, depends on previously measured bit.

Estimate the energy of a number operator

$$\hat{n} = \begin{bmatrix} 0 & & \text{\huge 0} & \\ & 1 & & \\ \text{\huge 0} & & 2 & \\ & & & 3 \end{bmatrix}$$



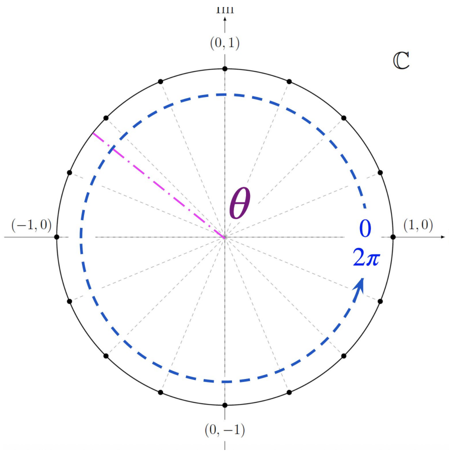
Question: Using the standard phase estimation algorithm, what can we say about the eigenvalue of state $|2\rangle$?

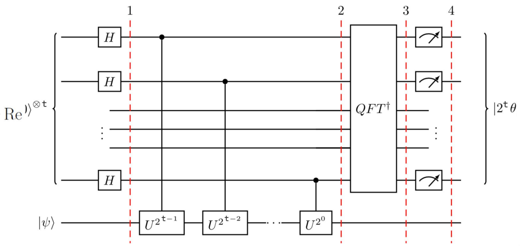
Step 1: 'Wrap' the unit complex circle

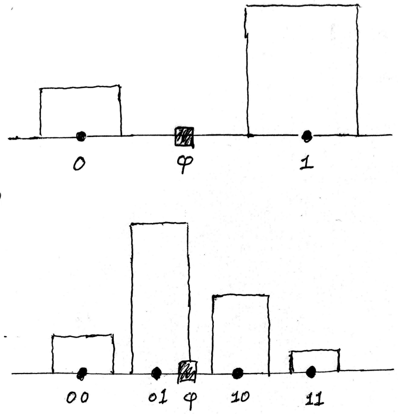
Step 2: Apply the QPE algorithm

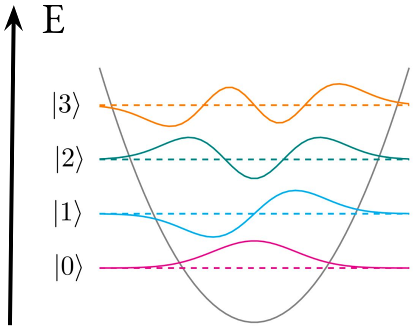
Step 3: Collect data and learn the phase

Step 4: 'Translate' back to the energy spectrum









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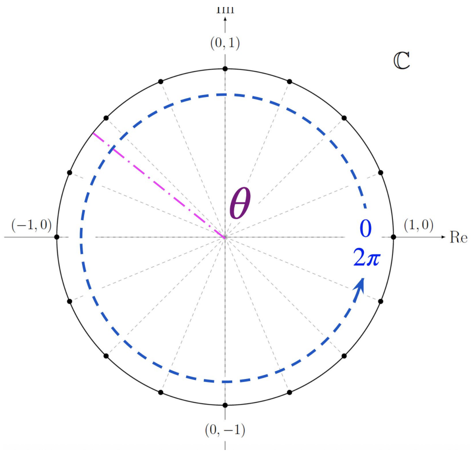


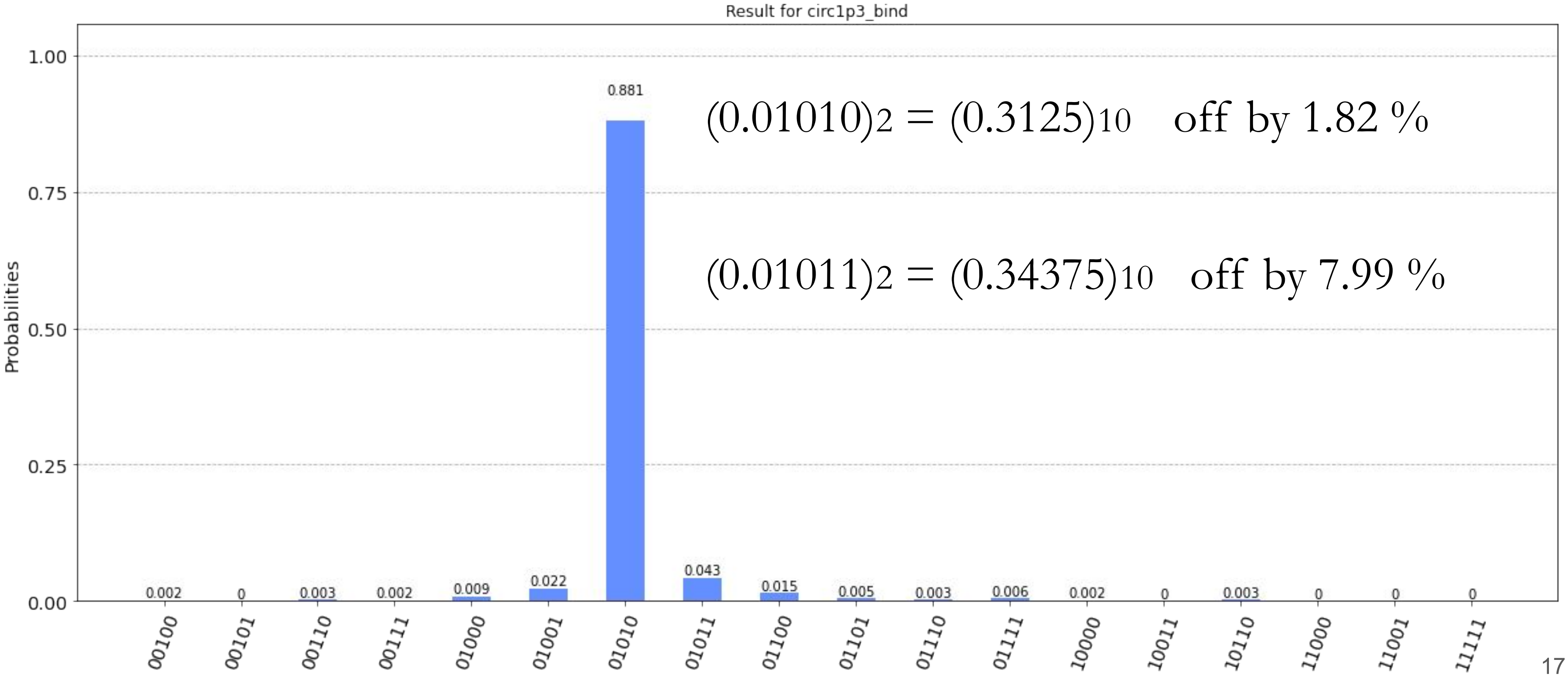
$$U|2\rangle = e^{it2}|10\rangle = e^{2\pi i\varphi}|10\rangle$$

$$t = 1$$

$$\varphi = (0.318309...)_{10} = (0.010100010...)_2$$

$$\varphi = (0.318309...)_{10} = (0.\boxed{01010}0010...)_2$$

Step 3



$(0.01010)_2 = (0.3125)_{10}$ off by 1.82 %

$(0.01011)_2 = (0.34375)_{10}$ off by 7.99 %