# More Simple Algorithms: Adiabatic Evolution and QAOA

Joey Bonitati (Michigan State University)

FRIB Theory Alliance Summer School
June 20, 2022

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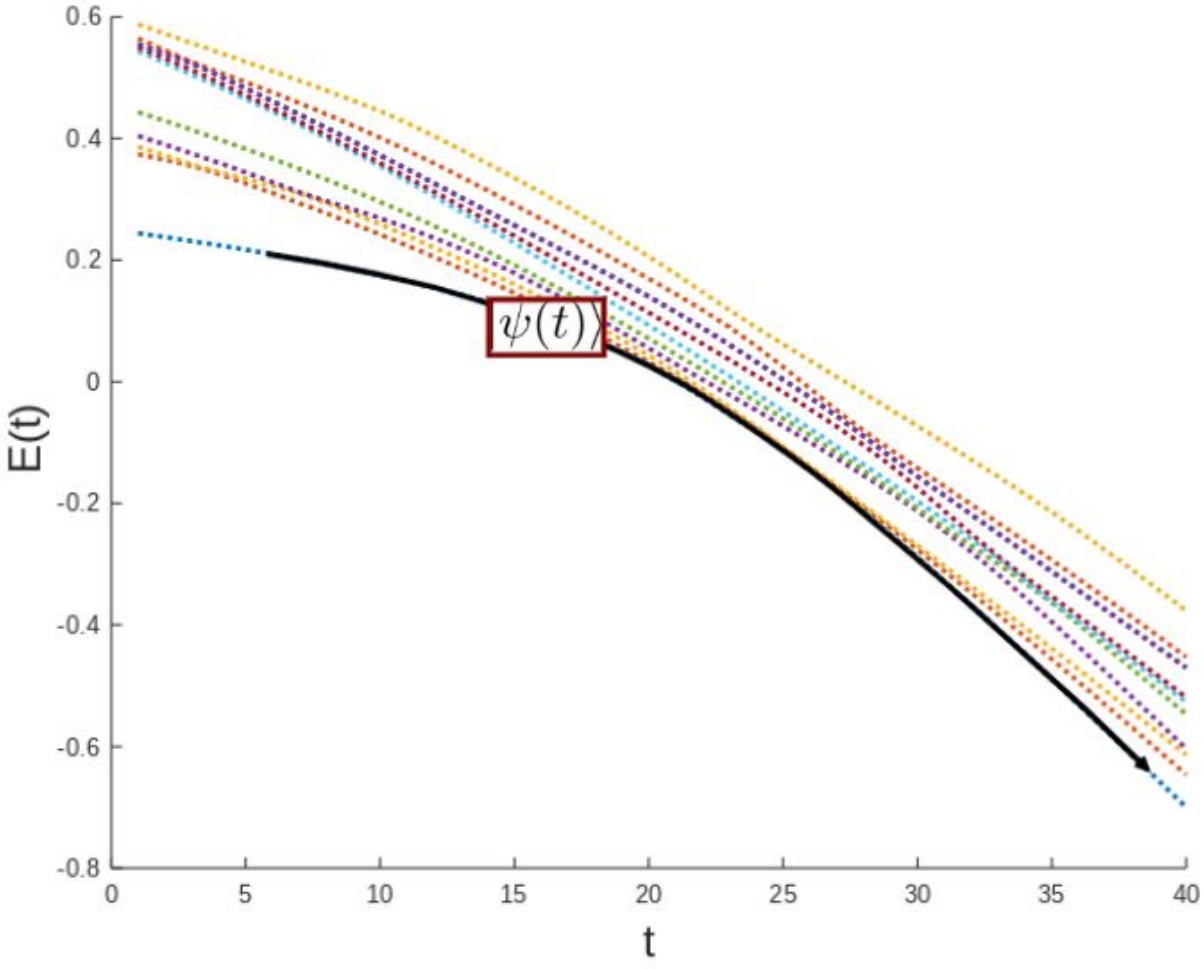
# Adiabatic Theorem

An eigenstate will remain an eigenstate as long as its system is changing slowly

$$H(t)\,|\psi(t)\rangle = E(t)\,|\psi(t)\rangle$$

The smaller the energy gap between states, the slower the changes must be



# Adiabatic Evolution

Choose a simple initial Hamiltonian and final target Hamiltonian

$$H(0) = H_I,\ H(t_F) = H_F$$

$$H(t) = \frac{t_F - t}{t_F} H_I + \frac{t}{t_F} H_F$$

Start in an eigenstate of $H_I$ at t = 0. Apply the unitary time evolution operator

$$U(t + dt, t) = e^{-iH(t)dt}$$

$$|\psi(t + dt)\rangle = U(t + dt, t)\,|\psi(t)\rangle$$

Repeat until $t = t_F$. The final state will be an eigenstate of $H_F$

# Adiabatic Evolution Example (two qubits)

Let's choose the initial state to be: $|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

This is the ground state of the Hamiltonian:

$$H_I = -Z_0 \otimes I_1 - I_0 \otimes Z_1 = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

# Adiabatic Evolution Example (two qubits)

For the final state, we can choose any 4x4 Hermitian matrix. For example:

$$H_F = H_I + X_0 \otimes X_1 + Y_0 \otimes Y_1 + Z_0 \otimes Z_1 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

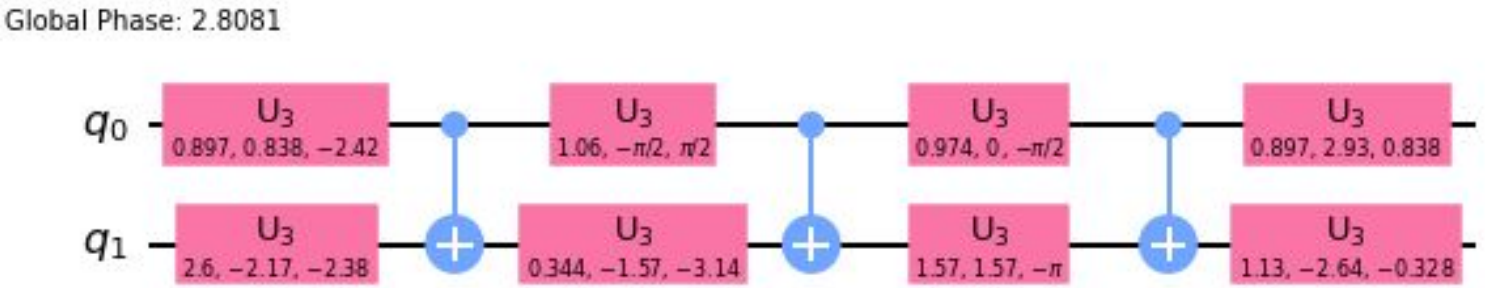
The time-dependent Hamiltonian will then be:

$$H(t) = \frac{t_F - t}{t_F} H_I + \frac{t}{t_F} H_F$$

# Adiabatic Evolution Example (two qubits)

Determine the time evolution gates (by hand or using qiskit library functions) and apply them to a quantum circuit in order

```
#one step of adiabatic evolution
evolution_gate = qiskit.extensions.HamiltonianGate(h,dt)
decomposed_gate = qiskit.quantum_info.two_qubit_cnot_decompose(evolution_gate)
circ.append(decomposed_gate, qr)
circ.decompose().draw('mpl')
```

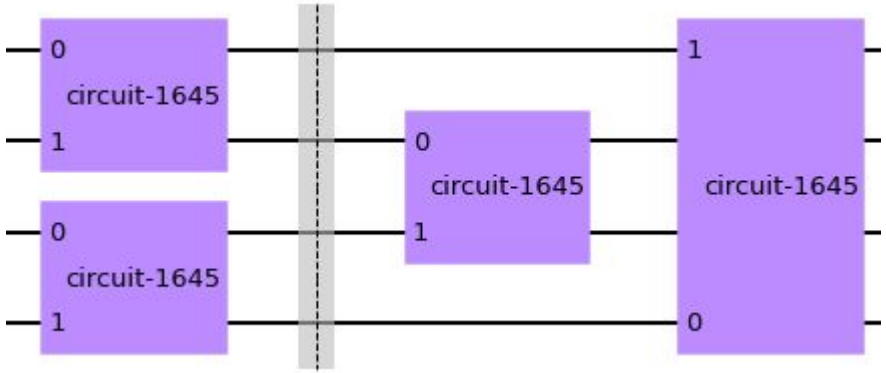


# Adiabatic evolution on more than 2 qubits

If a Hamiltonian can be decomposed into a sum of 2-qubit Hamiltonians, then the Trotter approximation can be used to split the 4-qubit gate into four 2-qubit gates.

For problems with more complicated Hamiltonians and problems that require a large number of gates, methods that don't require exact time evolution are used.

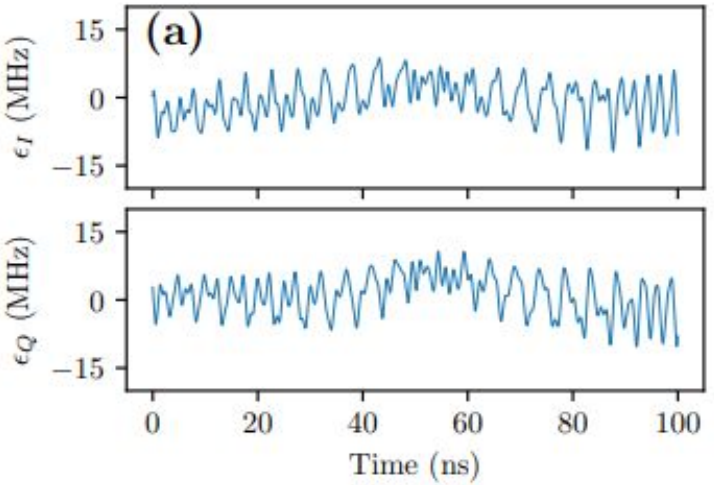
One Trotter-decomposed time evolution step on four qubits



# Pulse-level Control for time evolution

Can reduce the number of 2-qubit gates required to only one per time step.

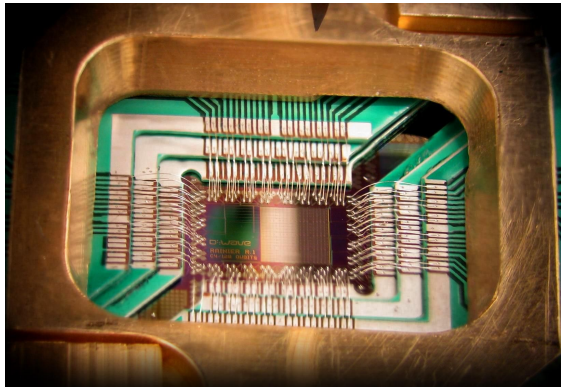
Generate custom gates using classical optimization by simulating a specific quantum device




**Optimal Control for the Quantum Simulation of Nuclear Dynamics**

Eric T. Holland,[1,*] Kyle A. Wendt,[1,†] Konstantinos Kravvaris,[1] Xian Wu,[1] W. Erich Ormand,[1] Jonathan L DuBois,[1] Sofia Quaglioni,[1] and Francesco Pederiva[2,3]

# Quantum Annealing

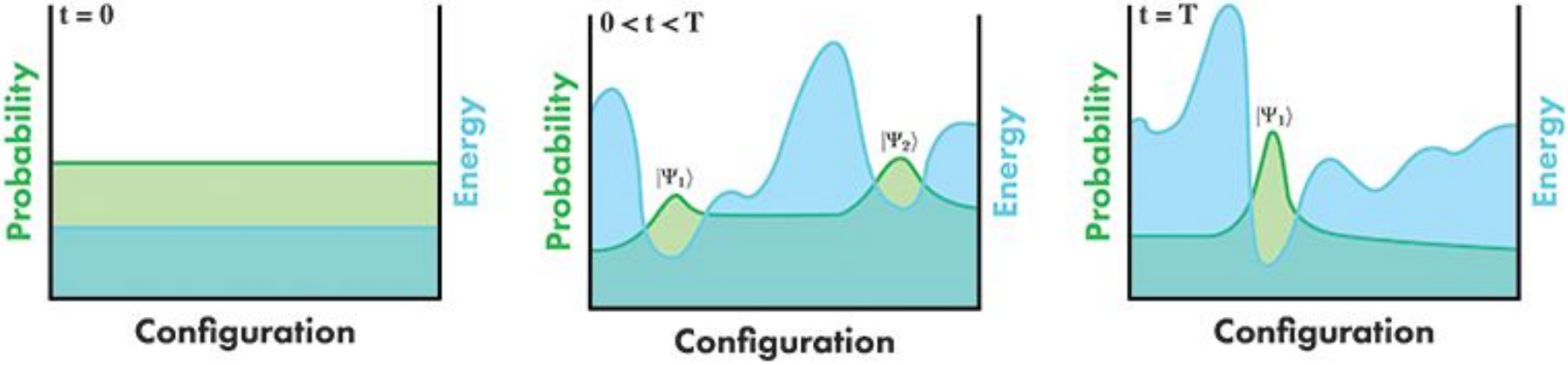


A technique used by D-WAVE to compute ground states.

Adiabatic fluctuations are applied to a quantum system.

Quantum tunneling allows the system to settle into the ground state of a target Hamiltonian.

Currently being used to solve hard combinatorial



# Quantum Approximate Optimization Algorithm (QAOA)

Inspired by Adiabatic Evolution

Start with the following ansatz:

$$|\psi(\vec{\beta},\vec{\gamma})\rangle = U(\vec{\beta},\vec{\gamma})\,|\psi_I\rangle$$

$$U(\vec{\beta},\vec{\gamma}) \approx e^{-i\beta_k H_I} e^{-i\gamma_k H_F} \cdots e^{-i\beta_1 H_I} e^{-i\gamma_1 H_F}$$

We use the variational principle,

$$E_F^{\text{ground}} \leq \langle\psi(\vec{\beta},\vec{\gamma})|H_F|\psi(\vec{\beta},\vec{\gamma})\rangle$$

with classical optimization to find an approximate minimum energy.

QAOA can be used to approximately solve some NP-Hard problems efficiently

# Questions