

# RECENT PROGRESS IN THE GAMOW SHELL MODEL

Kévin FOSSEZ

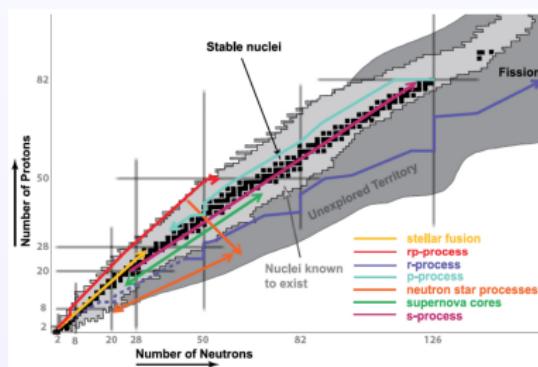
NSCL/MSU

ICNT. May 19, 2015

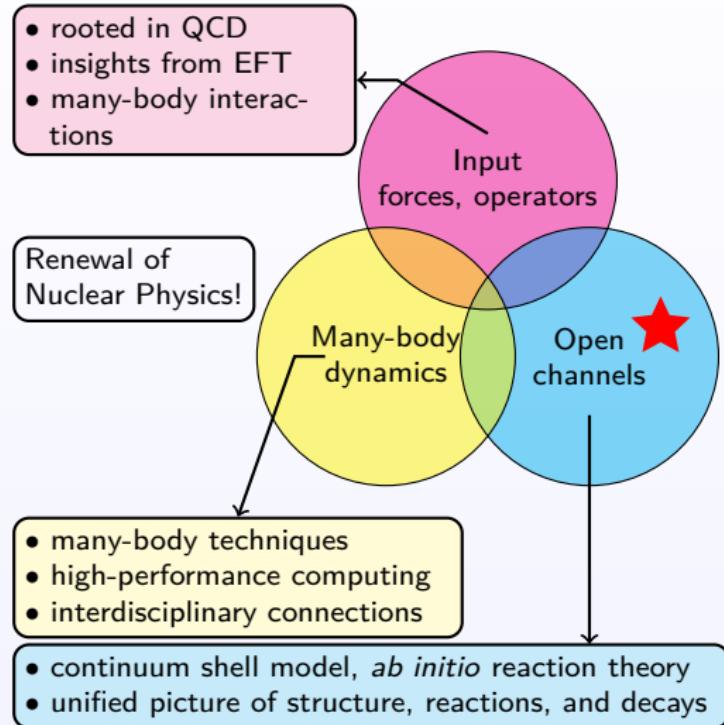


# THE BIG PICTURE

Next generation of RIB facilities:



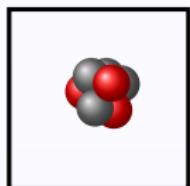
[https://people.nscl.msu.edu/~zegers/HRS\\_draft.pdf](https://people.nscl.msu.edu/~zegers/HRS_draft.pdf)



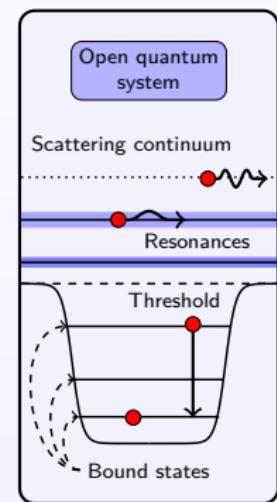
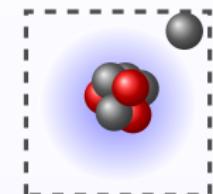
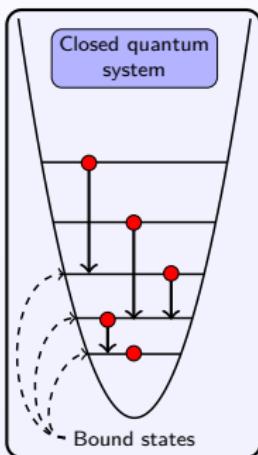
(Figure modified from Forssén et al., Phys. Scripta T 152, 014022 (2013))

# OPEN QUANTUM SYSTEMS (OQSS): WHAT ARE THEY?

- Quantum systems coupled to the environment of scattering states and decay channels.



- Examples of OQSS in many domains of physics: hadrons, nuclei, atoms, molecules, quantum dots, microwave cavities.
- Exotic phenomena in OQSSs: superradiance phenomena, spontaneous two-proton radioactivity, near-threshold clustering phenomena...
- General properties of OQSSs (resonances, halos, exceptional points) are common to all mesoscopic systems.



## DESCRIPTION OF NUCLEAR OQSs

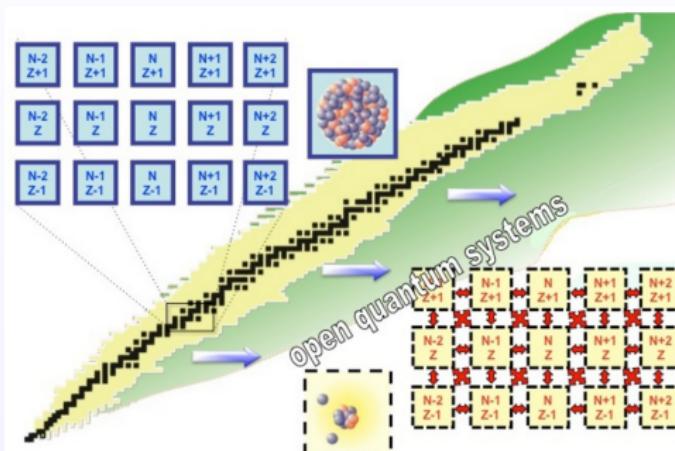
→ Unified description of nuclear structure and reactions

## Nuclear Shell Model (1949):

- Applied successfully to the description of low-lying states in stable nuclei (bound state approximation).
  - Fails to describe unstable nuclei.

## Reaction theory

- Unable to include the underlying structure of the target and the projectile nuclei microscopically.



- Reconciliation of SM with reaction theory by Feshbach (1958-1962), Fano (1961), Mahaux and Weidenmüller (1969) with the projection operator formalism  $\Rightarrow$  Continuum Shell Model.
  - A different approach: Gamow Shell Model (GSM), the SM for OQS in the complex energy plane.

- Reconciliation of GSM with reaction theory using the coupled-channel formalism: GSM-CC.

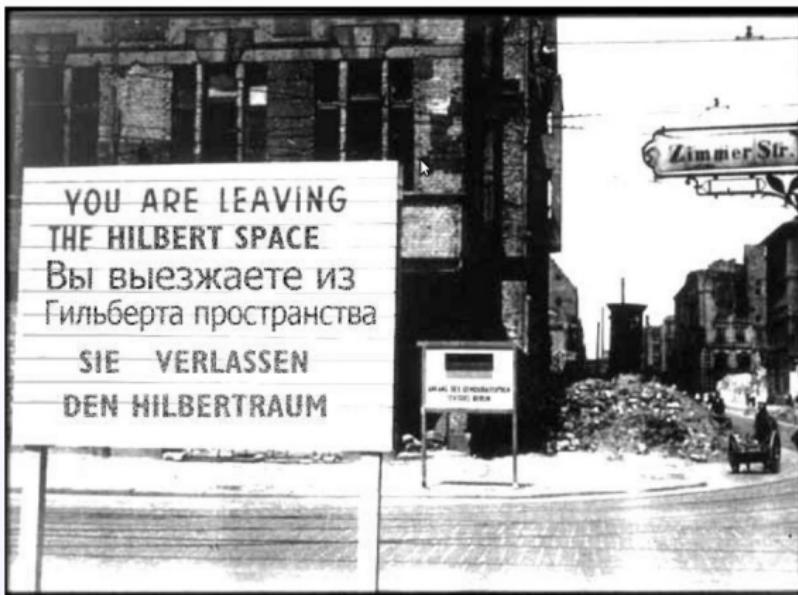
## ① INTRODUCTION

## ② FORMALISM

## ③ APPLICATIONS

## ④ OUTLOOK

# FORMALISM



Picture from <http://arxiv.org/abs/quant-ph/0005024>

# FORMALISM

→ GSM: quasi-stationary open quantum system extension of the SM.

- **Gamow states:** discrete solutions of the quasi-stationary Schrödinger equation that are regular at the origin and with outgoing boundary conditions.

G. Gamow, Z. Physik 51, 204 (1928)

$$\frac{\partial^2 u_I(k, r)}{\partial r^2} = \left( \frac{l(l+1)}{r^2} + \frac{2m}{\hbar^2} V(r) - k^2 \right) u_I(k, r)$$

$$E = \frac{\hbar^2 k^2}{2m} \quad u_I(k, r) \underset{r \sim 0}{\sim} C_0(k) r^{l+1}$$

$$u_I(k, r) \underset{r \rightarrow \infty}{\sim} C_+(k) H_{I,\eta}^+(kr) + C_-(k) H_{I,\eta}^-(kr)$$

$$u_I(k, r) \underset{r \rightarrow \infty}{\sim} C_+(k) H_{I,\eta}^+(kr) \Leftarrow \text{outgoing solution}$$

- Complex eigenenergies  $E_n = e_n - i\Gamma_n/2$  corresponding to poles of the  $S$ -matrix:

$$k_n = \sqrt{\frac{2\mu E_n}{\hbar^2}} = \kappa_n - i\gamma_n.$$



- Rigged Hilbert space (RHS): construction designed to link the distribution and square-integrable aspects of functional analysis (Gel'fand, Vilenkin et al. (1964), Maurin (1968)).
- Rigorous framework for quantum mechanics (Böhm (1964), Roberts (1966), Antoine (1969) and Melsheimer (1974)).
- Unified formalism for bound states, resonances and scattering states.
- RHS inner product:  

$$\langle \tilde{u}_n | u_n \rangle = \int_0^\infty dr \tilde{u}_n^*(r) u_n(r).$$

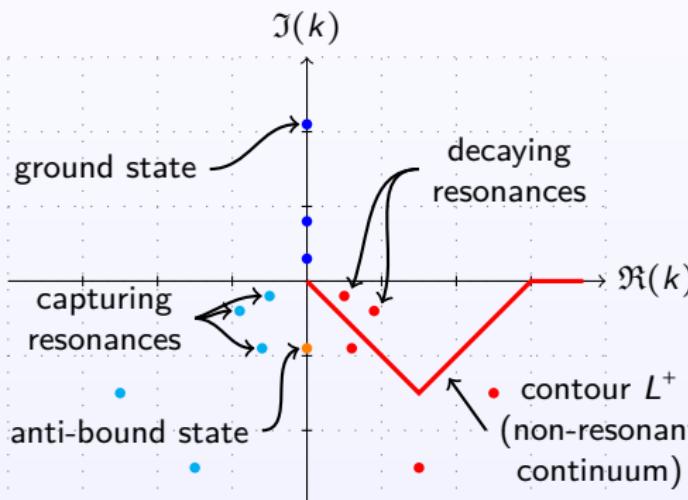
# FORMALISM

→ Berggren completeness relation:

$$\sum_{n \in (b,d)} |u_l(k_n)\rangle \langle \tilde{u}_l(k_n)| + \int_{L_{l,j}^+} dk |u_l(k)\rangle \langle \tilde{u}_l(k)| = \hat{\mathbb{1}}.$$

T. Berggren, Nucl. Phys. A 109, 265 (1968)

→ Discretization:  $\sum_{n \in (b,d,c)} |u_l(k_n)\rangle \langle u_l(k_n)| \approx \hat{\mathbb{1}}.$



Normalization in practice:

- Scattering states:

$$C_+(k) C_-(k) = \frac{1}{2\pi}.$$

(holds also with Coulomb)

- Resonant states:

$$C_-(k) = 0.$$

- Resonances:

→ Exterior complex-scaling:

$$\hat{U}_a(\theta) \chi(r) = \chi(r_a + |r - r_a| e^{i\theta})$$

if  $|r| > r_a$ .

B. Gyarmati and T. Vertse (1971), B. Simon (1979)

- Differentiability:  $\mathcal{J}^+(k) = 0$ .
- The RHS inner product gives  $C_0(k)$ .

# THE GAMOW SHELL MODEL

## → Gamow Shell Model:

- Intrinsic nucleon-core coordinates of the Cluster-Orbital Shell Model.

Y. Suzuki *et al.*, Phys. Rev. C 38, 410 (1988)



- SM Hamiltonian:

$$\hat{H} = \hat{H}_c + \sum_{i=1}^{N_{\text{val}}} \hat{t}_i + \sum_{i < j}^{N_{\text{val}}} \hat{V}_{ij}.$$

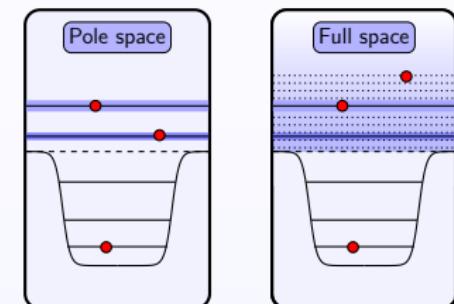
- $N$ -body basis of Slater determinants:

$$\sum_n |\Psi_n\rangle \langle \Psi_n| \approx \hat{1}$$

from the 1-body Berggren basis.

- Hermitian Hamiltonian but **complex-symmetric** matrix  $A = A^T$ .  
→ Davidson diagonalization.
- Eigenenergies:  $\tilde{E}_n = E_n + i\Gamma_n / 2$ .

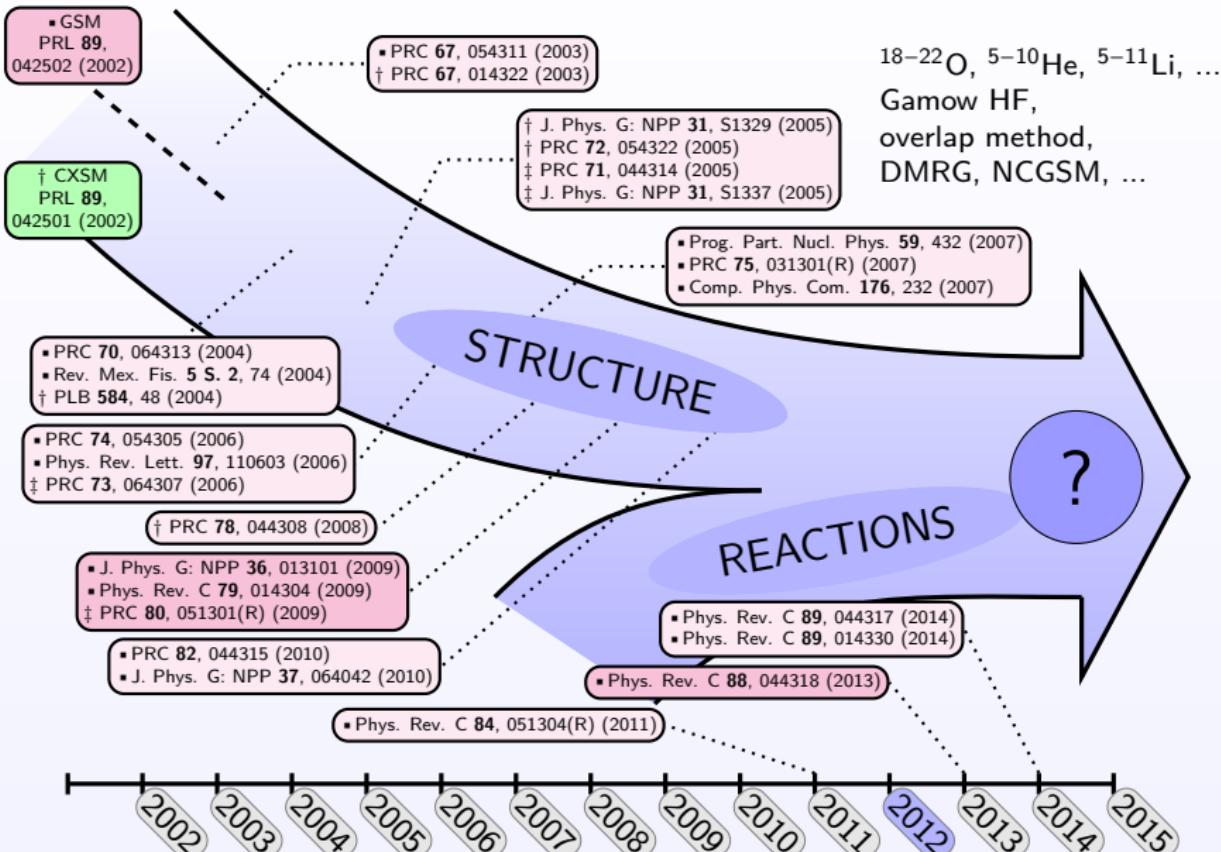
- Overlap method** to identify  $N$ -body resonances.



$$\langle \Psi_n^{(\text{pole})} | \Psi_n^{(\text{full})} \rangle \approx 70\% - 90\%$$

- Schematic interactions: proof of applicability of GSM.
- Spectroscopy of stable and unstable nuclei.

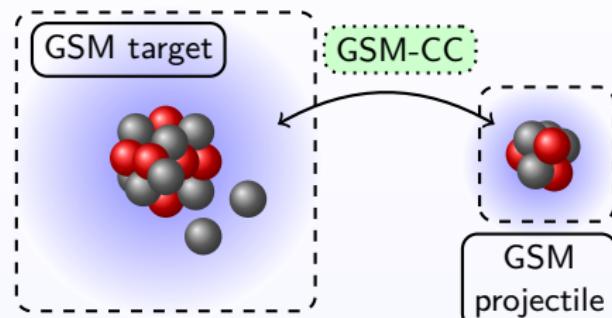
## THE GAMOW SHELL MODEL



## FORMALISM

→ Unification of nuclear structure and reactions based on GSM.

- Coupled-channel (CC) formalism.
  - Channel:  $|r, c\rangle = |r\rangle \otimes |c_{\text{proj}}\rangle \otimes |c_{\text{targ}}\rangle$
  - Target:
    - bound state or resonance.  
→ **Resonant** channel.
    - scattering state.  
→ **Nonresonant** channel.
  - Fully antisymmetrized calculation.

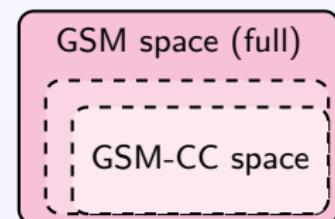


$$\oint_c \int_0^\infty dr r^2 (H_{c',c}(r',r) - E N_{c',c}(r',r)) \frac{u_c(r)}{r} = 0$$

$H_{c',c}(r',r) = \langle r', c' | \hat{H} | r, c \rangle$ ,  $N_{c',c}(r',r) = \langle r', c' | r, c \rangle$   
 → Direct integration method (Bulirsch-Stoer).

Orthogonalization:  $\langle r', c' | r, c \rangle = o \langle r', c' | \hat{O} | r, c \rangle_o$ ,  
 Moore-Penrose pseudo inverse, equivalent potential method, ...

- **Unified** approach: 
 $\text{GSM-CC}(A - a, a) \approx \text{GSM}(A)$ .



## ① INTRODUCTION

## ② FORMALISM

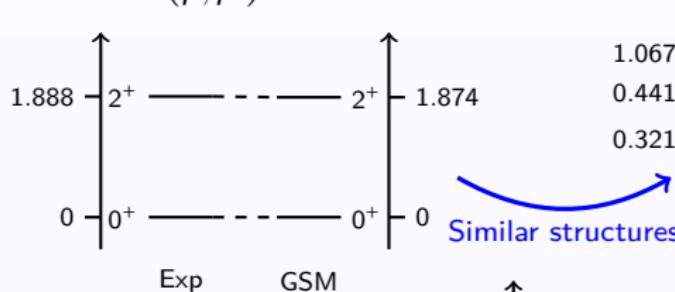
## ③ APPLICATIONS

## ④ OUTLOOK

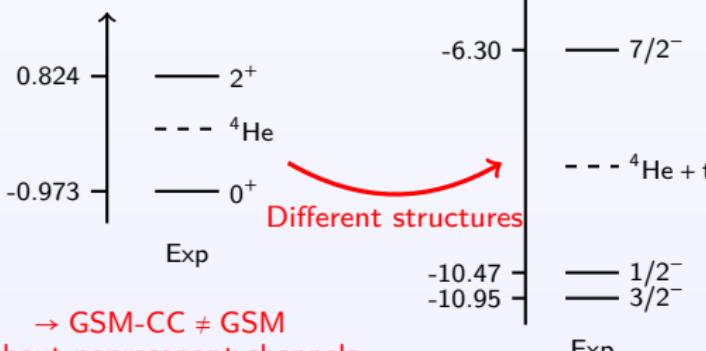
## ELASTIC AND INELASTIC SCATTERING REACTIONS

→ First application of GSM-CC:  $(p, p')$  reactions

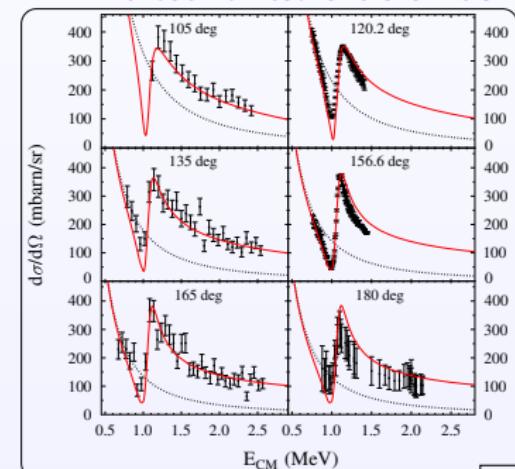
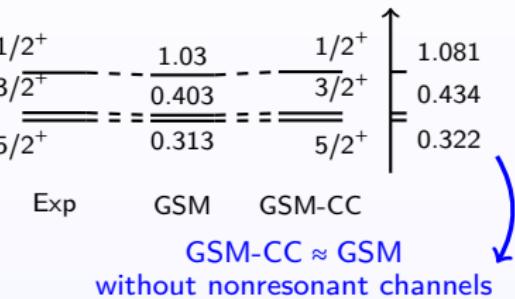
- The  ${}^{18}\text{Ne}(p, p')$  reaction.



- The  ${}^6\text{He}(p, p')$  reaction.



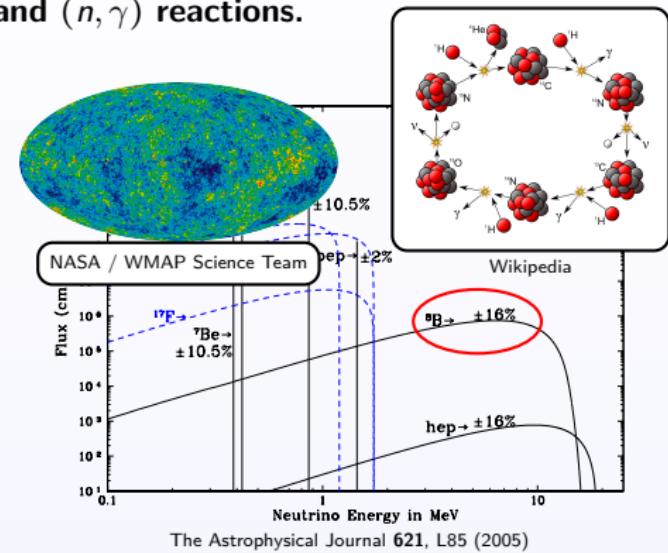
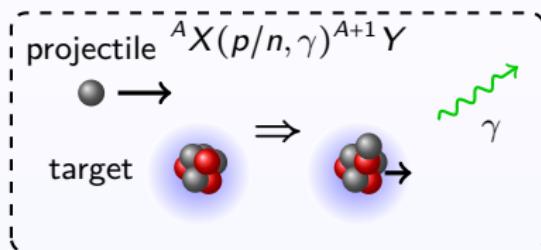
Y. J. et al., J. Phys.: Conf. Series 403, 012022 (2012)  
Y. Jaganathan, PhD thesis (2012)  
Y. Jaganathan et al., Phys. Rev. C 89, 034624 (2014)



# PROTON AND NEUTRON RADIATIVE CAPTURE REACTIONS

→ Extension of GSM-CC to  $(p, \gamma)$  and  $(n, \gamma)$  reactions.

- Big Bang nucleosynthesis,  
(H)CNO cycles, solar neutrinos,  
 $p\bar{p}$  chains...



- Cross section:

$$\sigma = f \left( \langle \Psi_f(J_f) | \hat{O}^L | \Psi_i(J_i, c_e) \rangle \right).$$

- Method:

$$\langle \Psi_f | \hat{O}^L | \Psi_i \rangle = \langle \Psi_f | \hat{O}^L | \Psi_i \rangle_{\text{nas}}$$

$$+ \langle \Psi_f | \hat{O}_<^L | \Psi_i \rangle^{\text{HO}} - \langle \Psi_f | \hat{O}_{<}^L | \Psi_i \rangle^{\text{HO}}_{\text{nas}}.$$

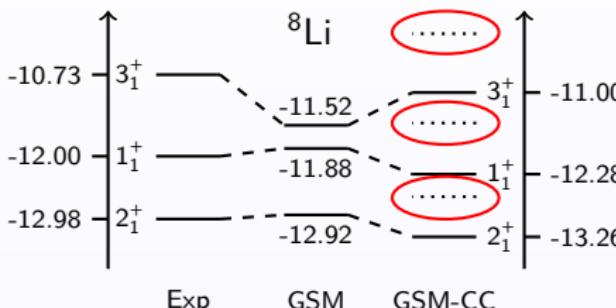
$$S_{p/n} = E_0^{(A)}[\text{GSM}] - E_0^{(A+1)}[\text{GSM-CC}].$$

K. Fossez, PhD. thesis (2014)

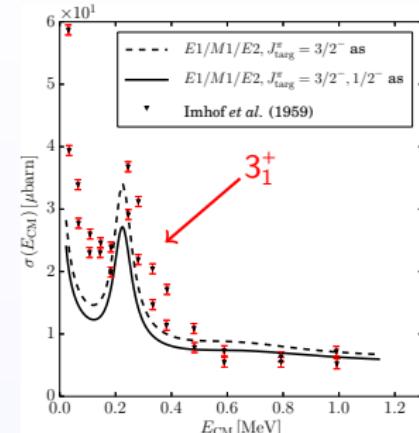
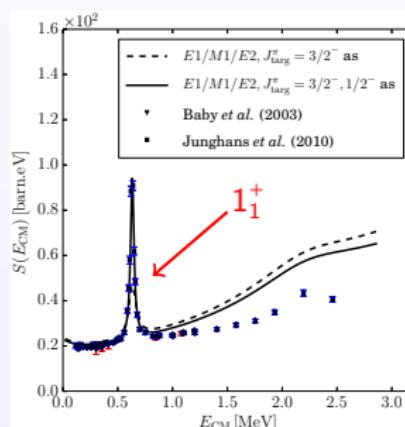
K. Fossez et al., Phys. Rev. C 91, 034609 (2015)

# PROTON AND NEUTRON RADIATIVE CAPTURE REACTIONS

The  ${}^7\text{Li}(n, \gamma){}^8\text{Li}$  reaction.



The  ${}^7\text{Be}(p, \gamma){}^8\text{B}$  reaction.



- Study of mirror reactions.
- $\hat{\mathcal{A}}(\langle \Psi_f(J_f) || \hat{O}^L || \Psi_i(J_i, c_e) \rangle)$ .
- Role of the long wave length approximation.
- Corrective factors in  $\tilde{V}_{c,c'} = c(J^\pi) V_{c,c'}$  with  $c(J^\pi) < 5\%$ .

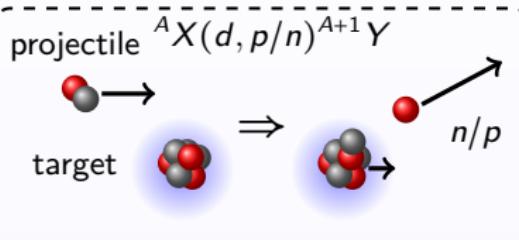
→ Nonresonant channels missing.

# NUCLEON TRANSFER REACTIONS

→ Extension of GSM-CC to  $(d, p)$  and  $(d, n)$  reactions.

A. Mercenne, PhD. thesis (2013 – 2016)

N. Michel, private communication



- Naive approach: Direct expansion in the many-body Berggren basis.

$$|K_{CM}, c_{proj}\rangle \approx \sum_i c_i |SD_i\rangle$$

$$\langle K_{CM}|K'_{CM}\rangle \stackrel{?}{=} \delta(K_{CM} - K'_{CM})$$

but drawbacks:

- Contours discretization and truncations:
- Normalization cannot be tested accurately.

- Solution: focus on the short-range part of  $\hat{H}$ :

$$\begin{aligned} |K_{CM}, c_{proj}\rangle^{HO} &= \sum_{N_{CM}} U_{N_{CM}, c_{proj}}^{HO}(K_{CM}) |N_{CM}, c_{proj}\rangle^{HO} \\ &= \sum_i c_i |SD_i\rangle. \end{aligned}$$

where  $U_{N_{CM}, c_{proj}}^{HO}(K_{CM})$  comes from:

$$\hat{H}_{CM} = \frac{\hat{P}_{CM}^2}{2M_p} + U_{CM}(\hat{R}_{CM}).$$

- Probe of the single-particle structure.

- Many-body projectile.

- Channels definition:

$$|R^{CM}, c\rangle = |R^{CM}\rangle \otimes |c_{proj}\rangle \otimes |c_{targ}\rangle$$

with (GSM):

$$|c_{proj}\rangle = |J_c^{CM}(L_c^{CM}, S_c^{CM})\rangle_{proj}.$$

- Momentum space expansion:

$$\{|R_{CM}, c_{proj}\rangle\} \Rightarrow \{|K_{CM}, c_{proj}\rangle\}.$$

# EFFECTIVE INTERACTION FOR GSM/GSM-CC

## → Development of an effective interaction.

Y. Jaganathan, R. Id Betan *et al.*, In preparation.

- Derived from the FHT interaction.

H. Furutani *et al.*, Prog. Theor. Phys. **60**, 307 (1978)  
 H. Furutani *et al.*, Prog. Theor. Phys. **62**, 981 (1979)

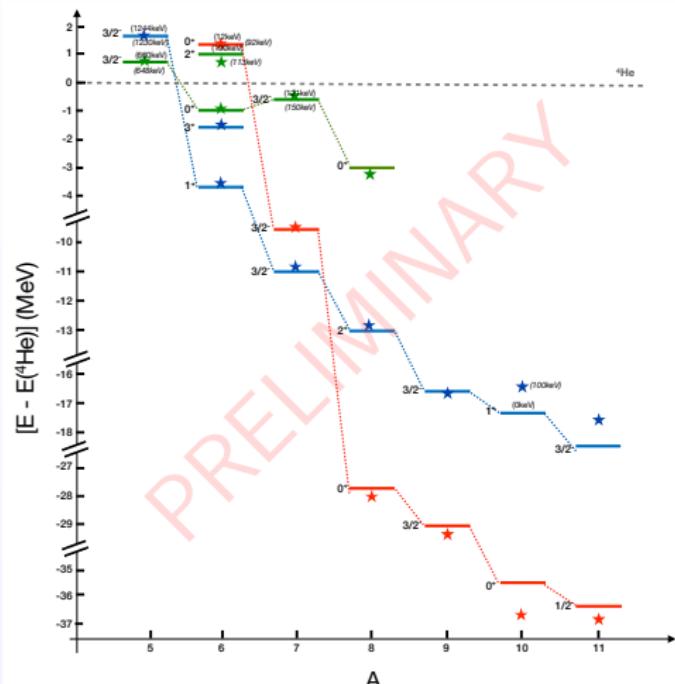
- Terms:  $V_c$ ,  $V_{so}$ ,  $V_T$  and  $V_C$ .
- Short-range (Gaussian).
- POUNDERs algorithm.  
<http://www.mcs.anl.gov/tao>
- Chi-squared minimization:

$$\chi^2(p) = \sum_{i=1}^{N_{\text{data}}} \frac{[\mathcal{O}_i(p) - \mathcal{O}_i^{\text{exp}}]^2}{\Delta \mathcal{O}_i^2}, \frac{\chi^2(p_0)}{N_{\text{d.o.f.}}} = 1.$$

and removing of the arbitrariness in the adopted errors at the minimum.

J. Phys. G: Nucl. Part. Phys. **41**, 074001 (2014)

- Woods-Saxon  ${}^4\text{He}$  core.
  - $n/p - \alpha$  phase shifts up to 20 MeV.
  - s.p. energies and widths.



- 2p, 3n with 3 nucleons max in the continuum.

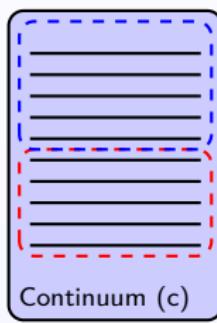
# ADDITIONAL APPLICATIONS: NCGSM

→ First application of the *ab initio* GSM: The NCGSM.

- Dimensional explosion: Density Matrix Renormalization Group (DMRG) method.

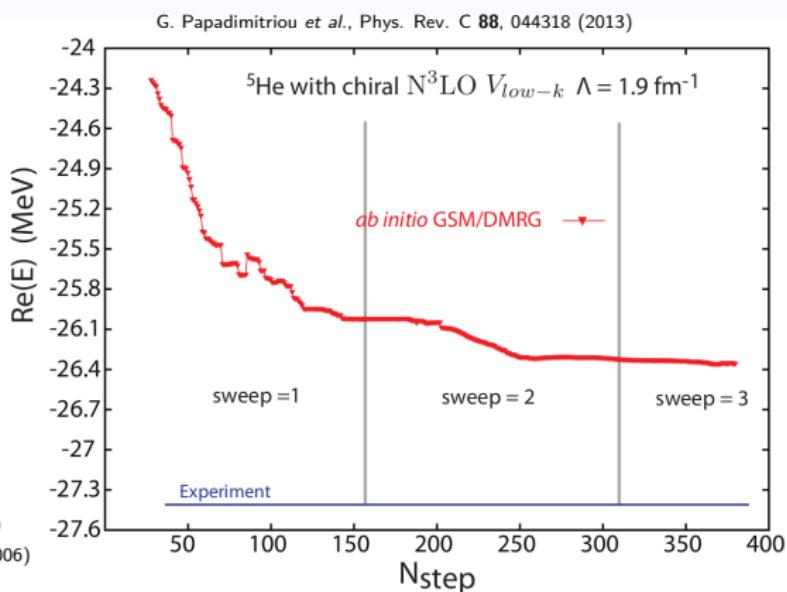
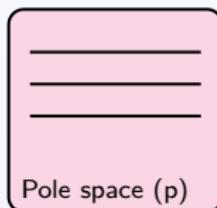
Truncation among states with nucleons in the continuum.

S. R. White, Phys. Rev. Lett. **69**, 2863 (1992)  
 T. Papenbrock *et al.*, J. Phys. G: NPP. **31**, S 1377 (2005)  
 S. Pittel *et al.*, Phys. Rev. C **73**, 014301 (2006)



$$\rho_{c,c'} = \sum_p \Psi_{c,p} \Psi_{c',p}$$

Main contribution



J. Rotureau *et al.*, Phys. Rev. C **79**, 014304 (2009)

J. Rotureau *et al.*, Phys. Rev. Lett. **97**, 110603 (2006)

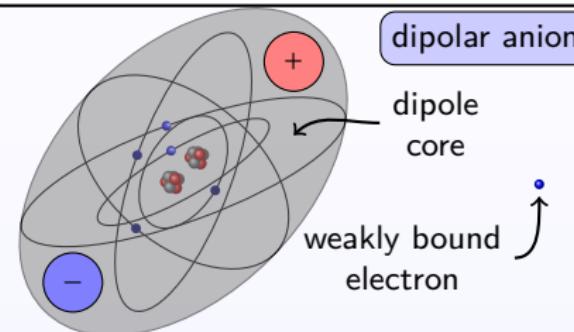
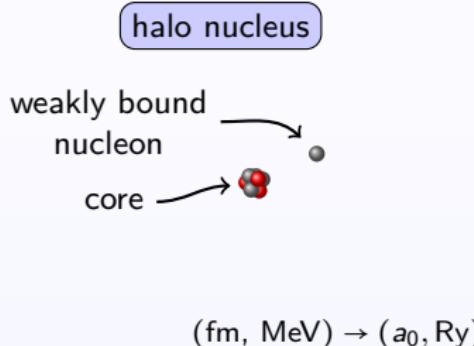
Select the most important

continuum configurations by retaining only the “largest” eigenvalues of the density matrix at each iteration

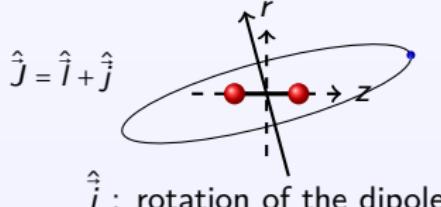
# ADDITIONAL APPLICATIONS: DIPOLAR ANIONS

→ Study of a molecular open quantum system using the Gamow Shell Model.

Calculation of resonances in dipolar anions.  
Bound dipolar anions as extreme halo systems.  
Competition between threshold effects and rotation.



$\hat{l}$ : electronic orbital momentum  
(no spin)



▪ Effective Hamiltonian of multipolar anions :

$$\hat{H} = \frac{\hat{p}_e^2}{2m_e} + \frac{\hat{j}^2}{2I} + \hat{V}.$$

$$\bullet V(r, \theta) = V_\mu + V_\alpha + V_{Qzz} + V_{SR}.$$

▪ At large distances: no analytical asymptotic solution for finite  $I$  with  $V_{cc'} \propto 1/r^2$ .

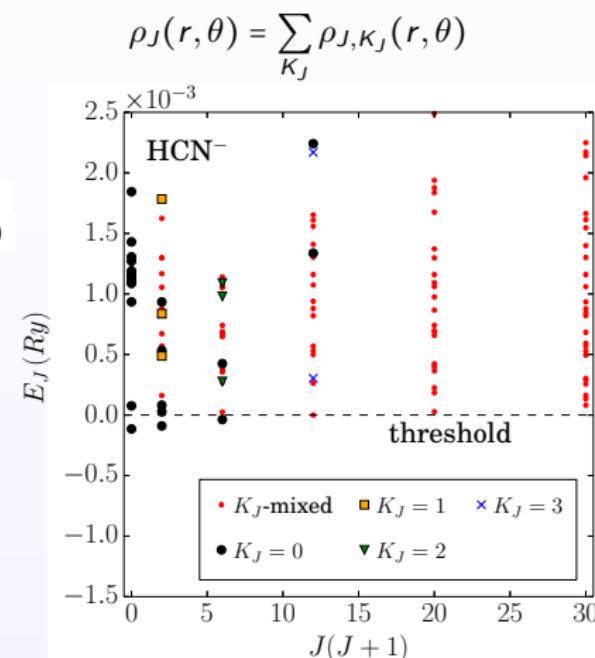
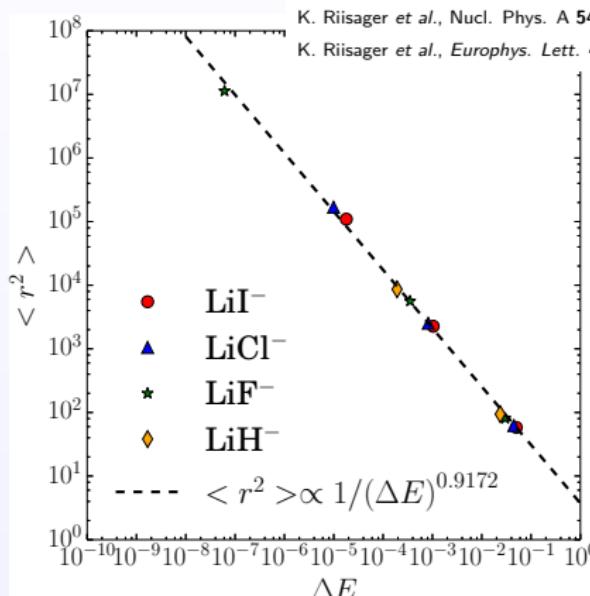
W. R. Garrett, *J. Chem. Phys.* **77**, 3666 (1982)  
W. R. Garrett, *J. Chem. Phys.* **133**, 224103 (2010)

# ADDITIONAL APPLICATIONS: DIPOLAR ANIONS

→ Extreme halo systems and density in the rotor frame.

For a relative angular momentum:

- $J = 0 : \langle r^2 \rangle$  diverges as  $1/\Delta E$ .
- $J = 1 : \langle r^2 \rangle$  diverges as  $1/\sqrt{\Delta E}$ .
- $J > 1 : \langle r^2 \rangle = \text{constant}$ .



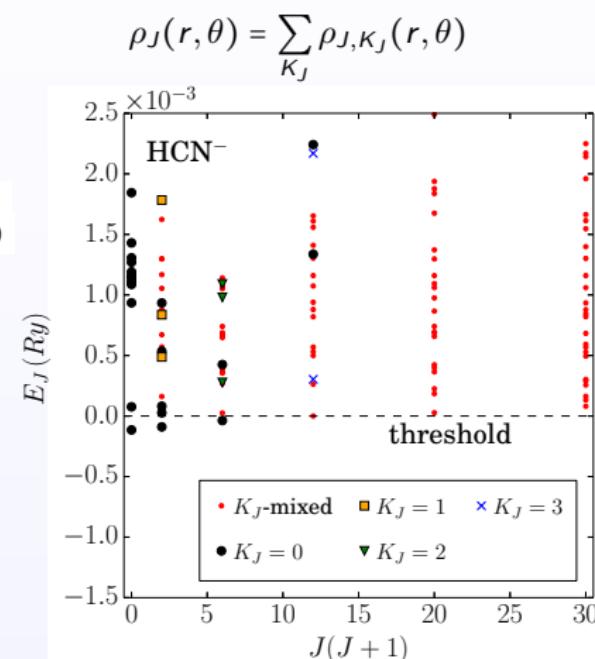
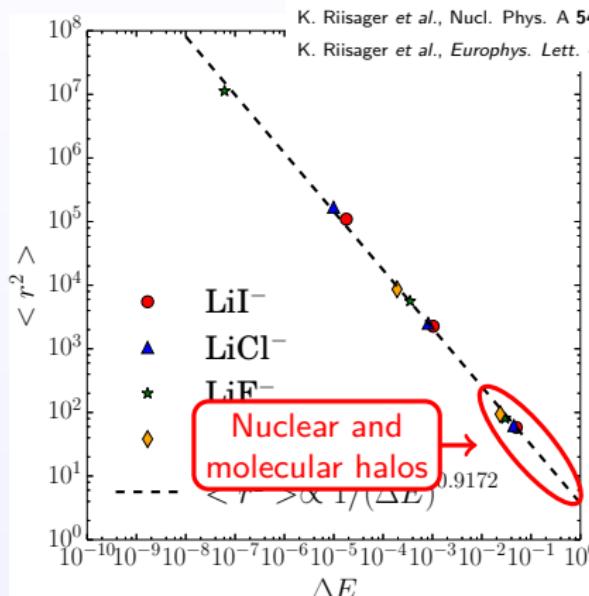
Intrinsic density: all  $K_J$ -components except one vanish.

# ADDITIONAL APPLICATIONS: DIPOLAR ANIONS

→ Extreme halo systems and density in the rotor frame.

For a relative angular momentum:

- $J = 0 : \langle r^2 \rangle$  diverges as  $1/\Delta E$ .
- $J = 1 : \langle r^2 \rangle$  diverges as  $1/\sqrt{\Delta E}$ .
- $J > 1 : \langle r^2 \rangle = \text{constant}$ .



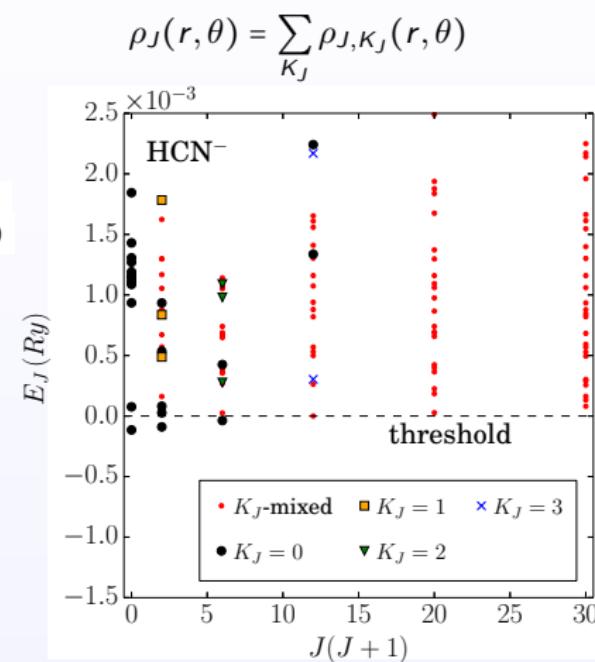
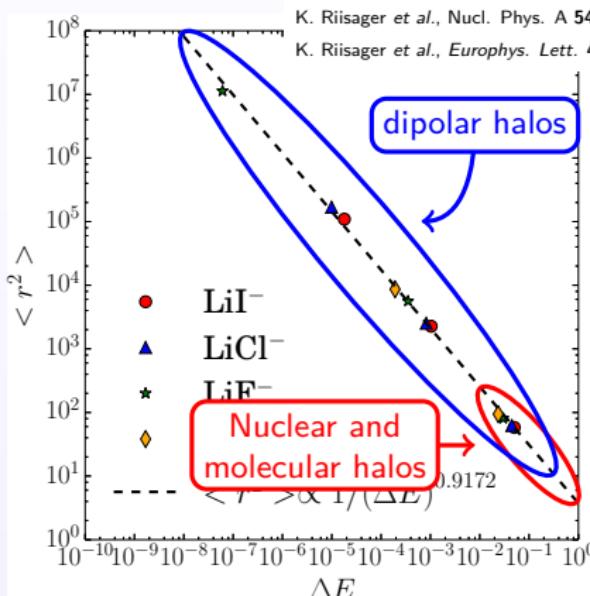
Intrinsic density: all  $K_J$ -components except one vanish.

# ADDITIONAL APPLICATIONS: DIPOLAR ANIONS

→ Extreme halo systems and density in the rotor frame.

For a relative angular momentum:

- $J = 0 : \langle r^2 \rangle$  diverges as  $1/\Delta E$ .
- $J = 1 : \langle r^2 \rangle$  diverges as  $1/\sqrt{\Delta E}$ .
- $J > 1 : \langle r^2 \rangle = \text{constant}$ .



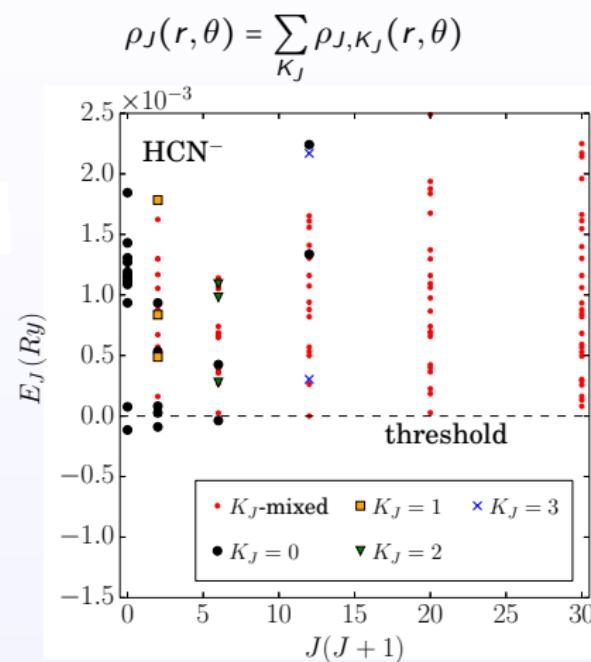
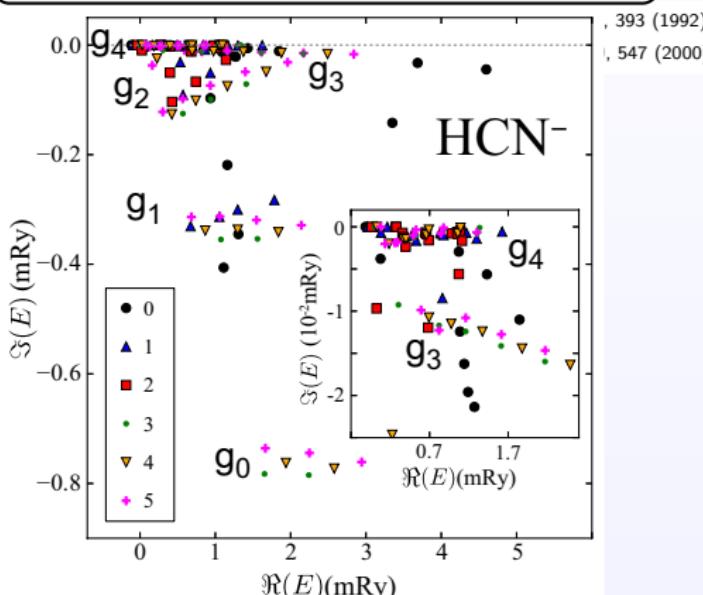
Intrinsic density: all  $K_J$ -components except one vanish.

# ADDITIONAL APPLICATIONS: DIPOLAR ANIONS

→ Extreme halo systems and density in the rotor frame.

## Problem

No resonant states (poles)  
in the Berggren basis.  
↔ no overlap method.



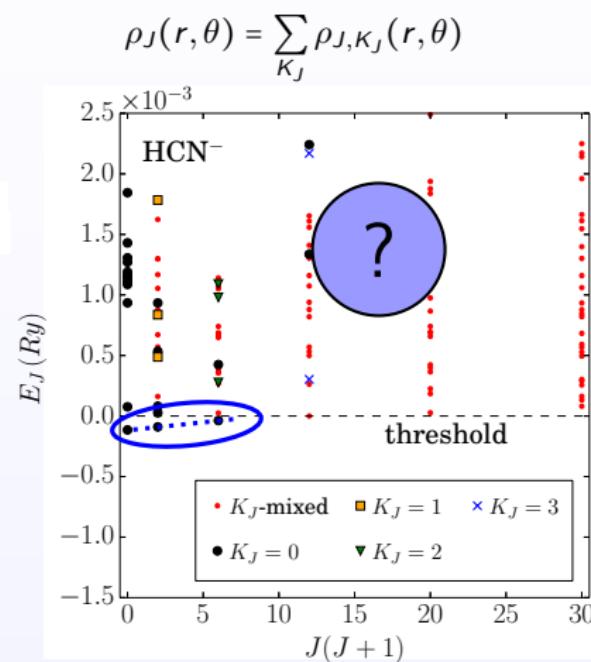
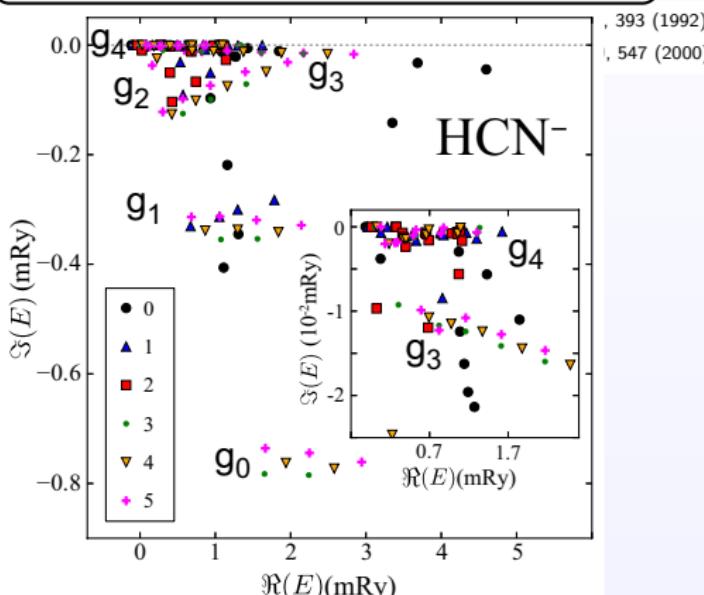
Intrinsic density: all  $K_J$ -components except one vanish.

# ADDITIONAL APPLICATIONS: DIPOLAR ANIONS

→ Extreme halo systems and density in the rotor frame.

## Problem

No resonant states (poles)  
in the Berggren basis.  
⇒ no overlap method.

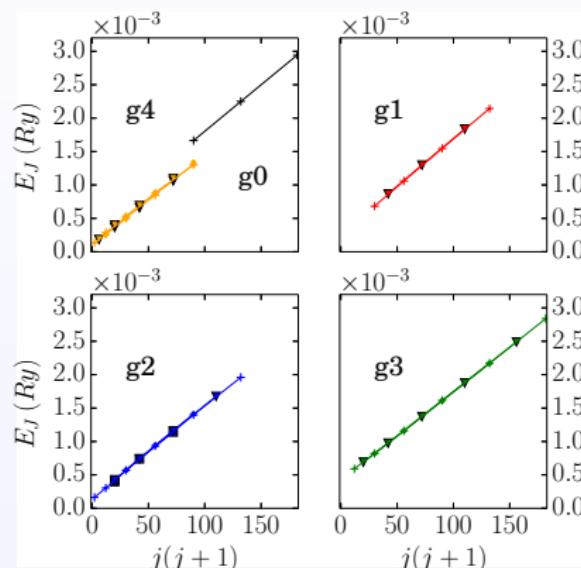


Intrinsic density: all  $K_J$ -components except one vanish.

# ADDITIONAL APPLICATIONS: DIPOLAR ANIONS

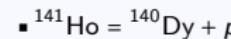
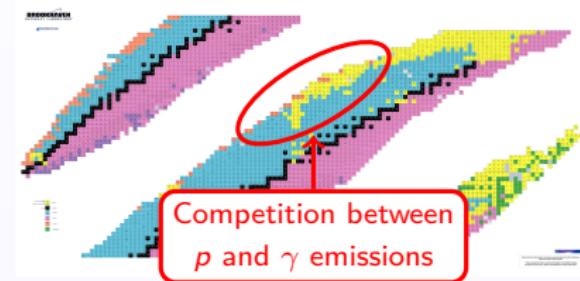
→ Competition between threshold effects and rotation.

- Above the threshold: weak coupling of the rotational motion of the dipole and the valence electron.



- Collective bands:  $E_{J,l_c=6-8}(j)$ .

→ Study of quadrupolar anions.



- Important Coulomb barrier.

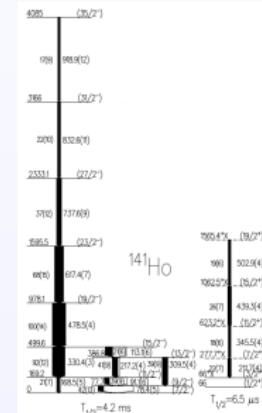
- Quadrupole moment.

- Collective bands in heavy  $p$ -rich nuclei?

- What about heavy  $n$ -rich nuclei?

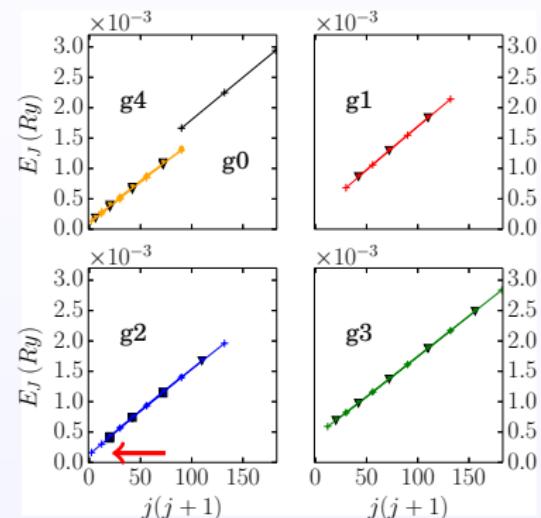
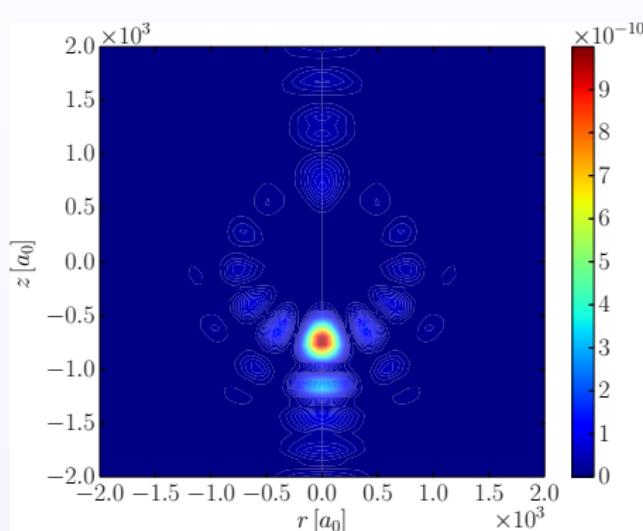
- (centrifugal barrier)

→ Under investigation.



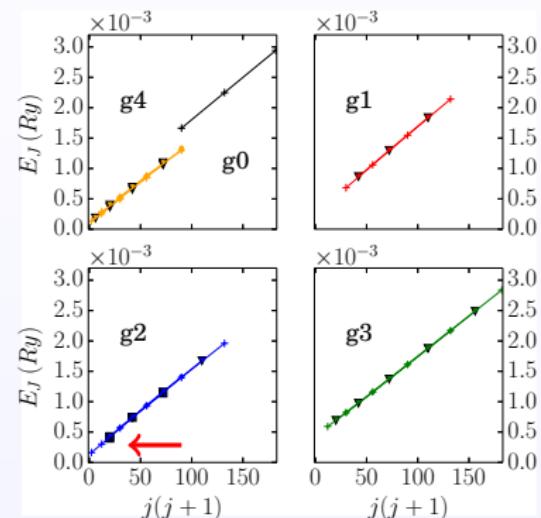
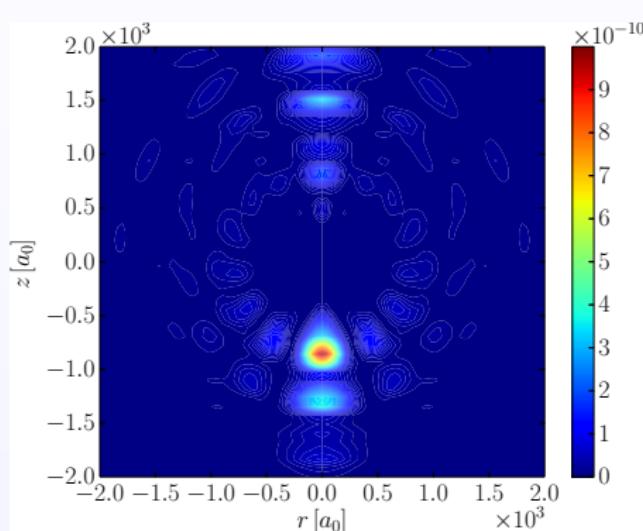
## ADDITIONAL APPLICATIONS: DIPOLAR ANIONS

$g_2$ :  $J = 5$  ( $l_c = 6, j_c = 1$ ).



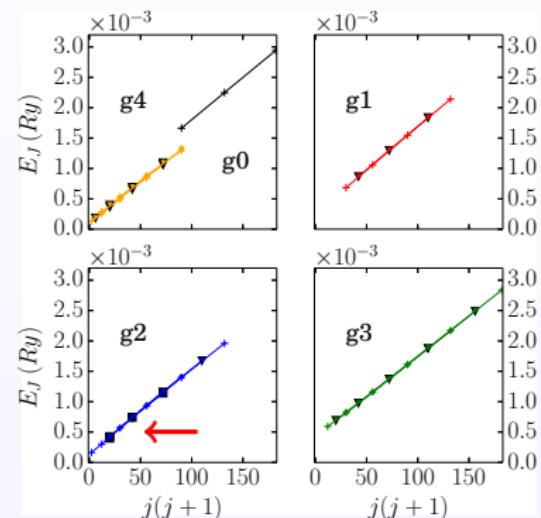
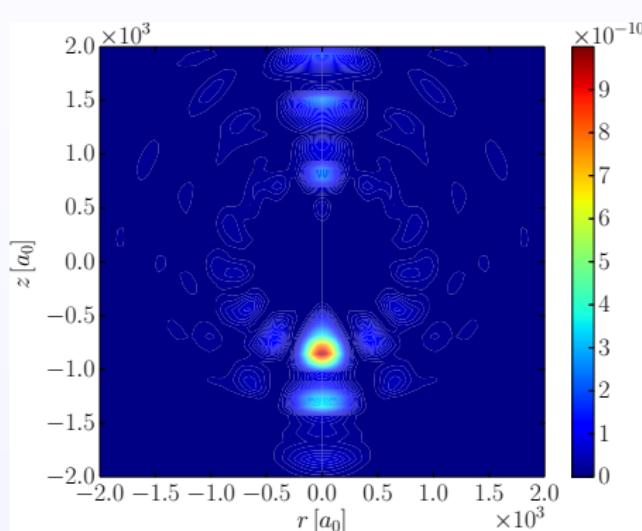
## ADDITIONAL APPLICATIONS: DIPOLAR ANIONS

$g_2$ :  $J = 5$  ( $l_c = 6, j_c = 3$ ).



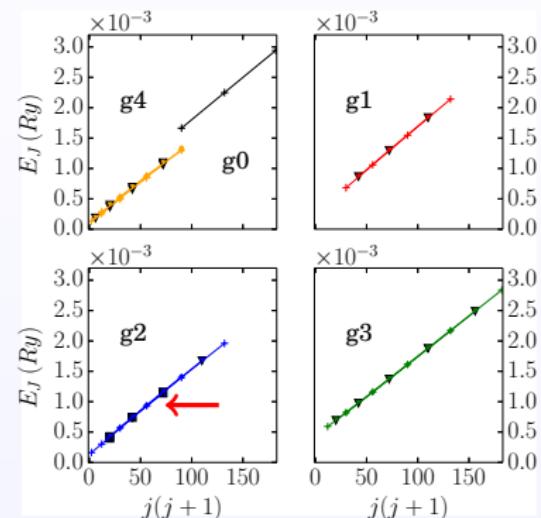
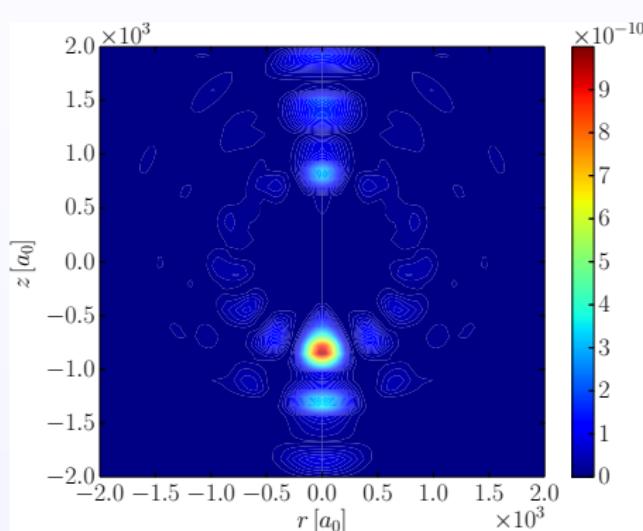
## ADDITIONAL APPLICATIONS: DIPOLAR ANIONS

$g_2$ :  $J = 5$  ( $l_c = 6, j_c = 5$ ).



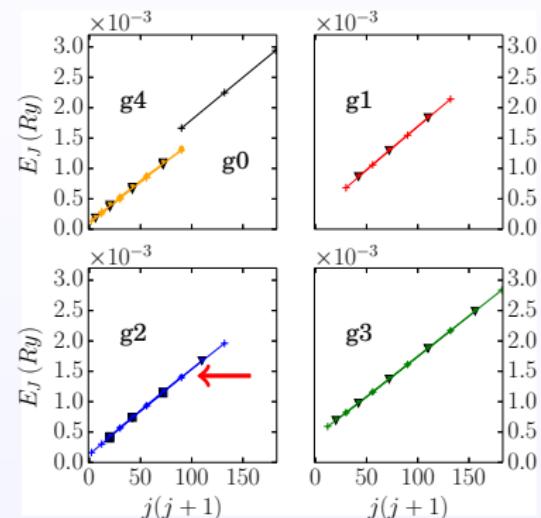
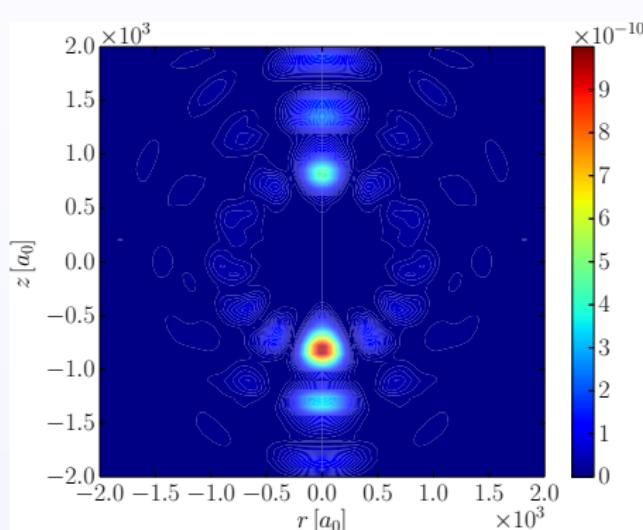
## ADDITIONAL APPLICATIONS: DIPOLAR ANIONS

$g_2$ :  $J = 5$  ( $l_c = 6, j_c = 7$ ).



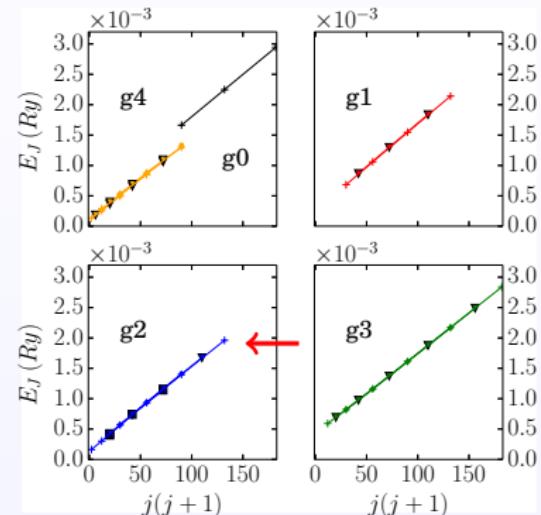
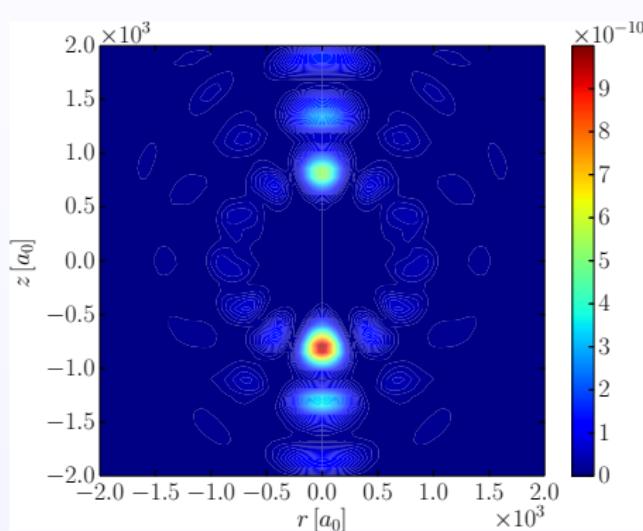
## ADDITIONAL APPLICATIONS: DIPOLEAR ANIONS

$g_2$ :  $J = 5$  ( $l_c = 6, j_c = 9$ ).



## ADDITIONAL APPLICATIONS: DIPOLEAR ANIONS

$g_2$ :  $J = 5$  ( $l_c = 6, j_c = 11$ ).



# ADDITIONAL APPLICATIONS: RICHARDSON MODEL

- **Pairing Hamiltonian in the Berggren basis:**

$$H = \oint 2\epsilon_k b_k^\dagger b_k dk - G \oint b_k^\dagger b_{k'} dk dk' \quad (d) : [b_k, b_{k'}^\dagger] = 2\delta_{kk'}(\Omega_k - 2n_k)$$

$$(c) : [b_k, b_{k'}^\dagger] = 2\delta(k - k')\Omega_k - 2\delta_{kk'}n_k$$

- **Discretization:**

$$H = \sum_a^L 2\epsilon_a \tilde{b}_a^\dagger \tilde{b}_a - G \sum_{a,a'} \tilde{b}_a^\dagger \tilde{b}_{a'} \sqrt{w_a} \sqrt{w_{a'}} \quad \text{with } \tilde{b}_a^\dagger = \tilde{b}_{k_a} \sqrt{w_a}$$

$$\Rightarrow [\tilde{b}_a, \tilde{b}_{a'}^\dagger] = 2\delta_{aa'}(\Omega_a - 2\tilde{n}_a)$$

■ **Problem:**  $G_{aa'} = G\sqrt{w_a}\sqrt{w_{a'}} \Rightarrow$  No analytical solution

- **Approximate solution of the pairing problem:**

$$(d) : [b_k, b_{k'}^\dagger] = 2\delta_{kk'}(\Omega_k - 2n_k)$$

$$(c) : [b_k, b_{k'}^\dagger] = 2\delta(k - k')\Omega_k - 2\delta(k - k')n_k$$

- **Discretization:**

$$\Rightarrow [\tilde{b}_a, \tilde{b}_{a'}^\dagger] = 2\delta_{aa'}(\Omega_a - \frac{2}{w_a} \tilde{n}_a)$$

- **Ansatz:**

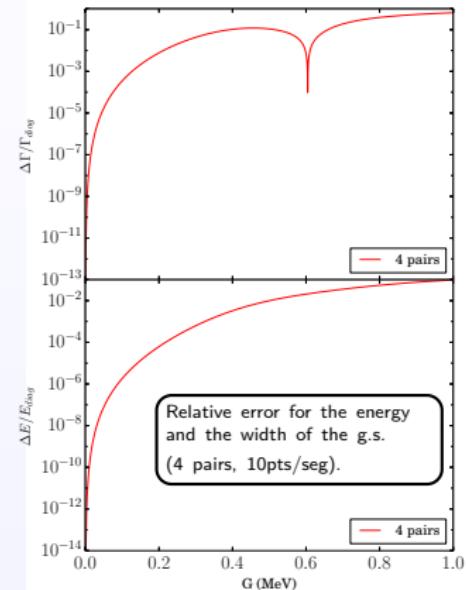
$$|\Psi\rangle = \prod_{i=1}^{N_{\text{pair}}} \mathcal{N}_i^{-1} \left( \sum_a^L \frac{\tilde{b}_a^\dagger}{2\epsilon_a - E_i} \right) |\rangle \quad \text{with } \mathcal{N}_i = \sqrt{\sum_a \frac{w_a}{(2\epsilon_a - E_i)^2}}$$

$$1 - 2G \sum_a^L \frac{\Omega_a/4 - \nu_a/2}{2\epsilon_a - E_i} + 2G \sum_{j \neq i}^{N_{\text{pair}}} \frac{1}{E_j - E_i} - 2G \sum_c^{\ell_{\max}, j_{\max}} \int_{L_c^+} \frac{\Omega_c/4 - \nu_c/2}{\hbar^2 k_c^2 / m - E_i} = 0$$

J. von Delft et al., Proceeding of the NATO ASI (1999)

R. W. Richardson, Phys. Lett. 3, 277 (1963)

R. W. Richardson et al., Nucl. Phys. 52, 221 (1964)



A. Mercenne et al., In preparation.

# SUMMARY

▪ GSM  
PRL 89,  
042502 (2002)

† CXSM  
PRL 89,  
042501 (2002)

→ A long standing question is about to be solved.

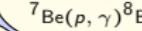
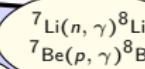
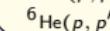
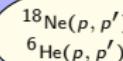
- Reactions of astrophysical interest in a unified framework.
- Fully microscopic treatment of the target and the projectile.
- Full treatment of the continuum.

- Richardson model.
  - Dipolar anions.
- Extreme halo physics.

## STRUCTURE

- Hypothesis: nonresonant channels are essential.
- Effective interaction fitted using the GSM.

## REACTIONS

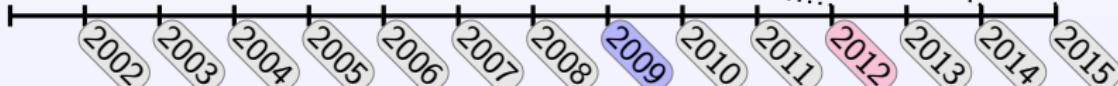


Effective interaction  
( $d, p/n$ )

Y. Jaganathan et al., Phys. Rev. C 89, 034624 (2014)  
K. Fossez, PhD thesis (2014)

Y. J. et al., J. Phys.: Conf. Series 403, 012022 (2012)  
Y. Jaganathan, PhD thesis (2012)

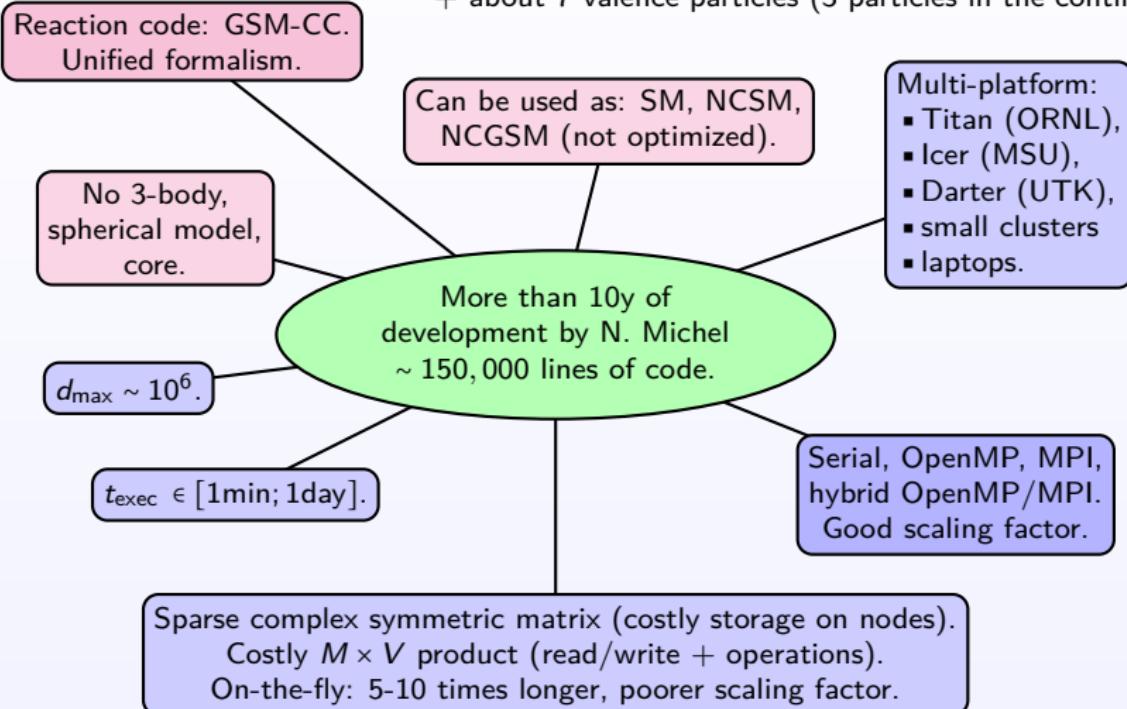
K. Fossez et al., Phys. Rev. C 91, 034609 (2015)  
Y. Jaganathan, R. Id Betan et al., In preparation  
A. Mercenne, PhD thesis (2013 – 2016)



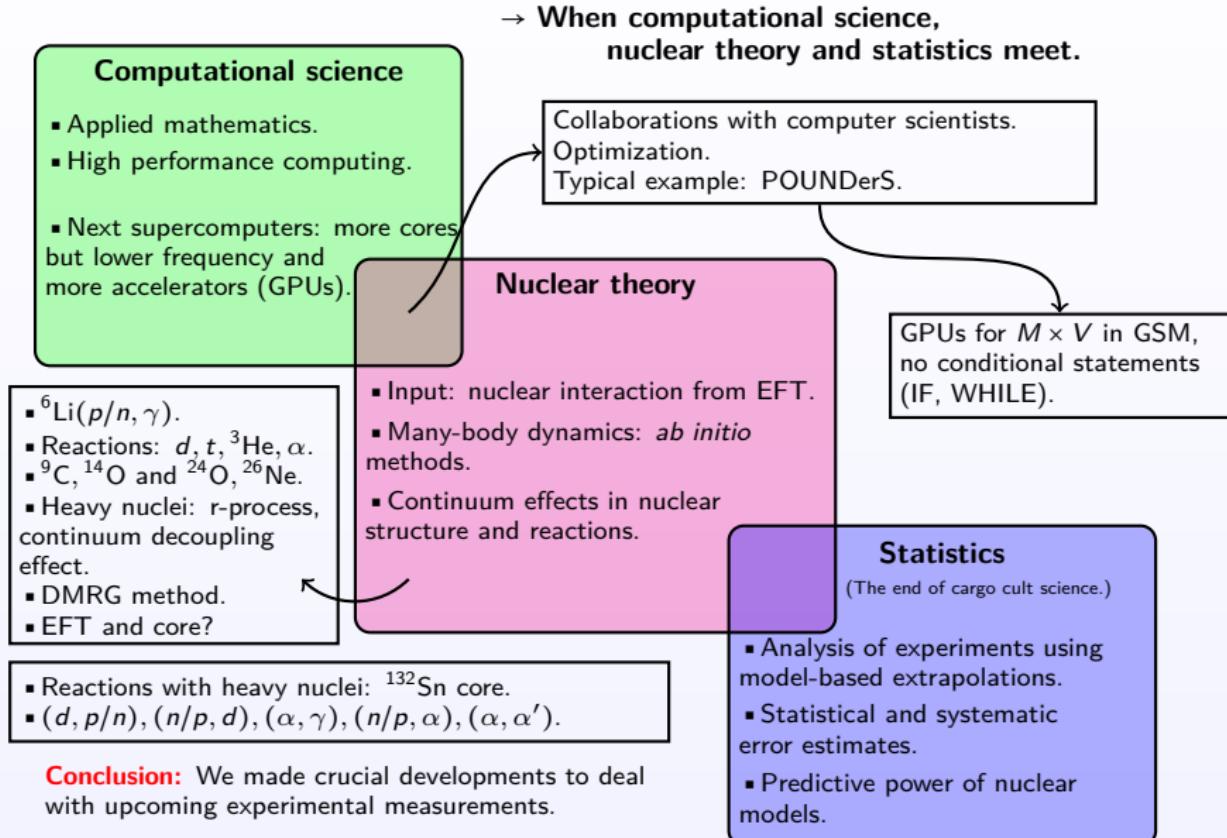
# LIMITS AND POSSIBILITIES OF THE CODE

→ GSM best for: weakly bound and unbound systems.

Range of applicability: systems with an inert core  
+ about 7 valence particles (3 particles in the continuum).



# PERSPECTIVES



# Thank you for your attention !

Un grand merci  
à Eric Olsen !

