Configuration interaction approach to nuclear clustering

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Configuration interaction approach

A powerful tool in studies of nuclear many-body problems

- *De-facto* most established many-body technique for nuclear many-body problem
- Predict ground state to highly excited many-body configuration, density of states
- Advanced computational techniques allow for broad scope of configurations.
  - From Shell Model to Configuration Interaction
  - Techniques: quantum chaos, Monte-Carlo, variational sampling, etc.
- Central technique to many-methods
  - With or without a core
  - Mean-field and RPA limits
  - From ab-initio, to effective, to mean-field (monopole) interactions.
- Collective many-body effects:
  - pairing and superconductivity,
  - Shape coexistence, phase transitions, connection to algebraic methods
  - Rotations.
- Reactions and decays
  - EM and beta decay processes.
  - Exotic decays
  - Physics of overlapping resonances and decay collectivity

New look into clustering

- Numerous questions of present-day interest
  - Role of clustering in structure: nuclear shapes, rotations, new experiments
  - Interplay of cluster structures, particle decay, and EM transitions
  - Reactions of cluster transfer, astrophysics
  - Broad overlapping alpha resonances
- Revisiting old ideas with new tools and techniques

References:
Grigorescu, et.al. Phys. Rev. C. **47** (1997) 2666.
F. Becchetti, et.al. Nucl. Phys. **A344** (1980) 336.
W. Chung, et.al Phys. Lett. B. **79** (1978) 381.
Yu. F. Smirnov, et. al Phys. Rev. C **15** (1977) 84.
M Ichimura, et.al. Nucl. Phys. A **244** (1975) 365.
I.V. Kurdumov, et. al. Nucl. Phys. **A145** (1970) 593.

From shell model to configuration interactions

State, equivalent to operator (polymorphism)

$$|\Psi\rangle \equiv \hat{\Psi}^\dagger|0\rangle = \sum_{\{1,2,3,\ldots A\}} \langle 1,2\ldots A|\Psi\rangle\, \hat{a}_1^\dagger \hat{a}_2^\dagger \ldots \hat{a}_A^\dagger|0\rangle$$

Construct relevant many-body operators, configurations

- Proper symmetry quantum numbers (S,T,J)
- Use SU(3) classification + permutation group

$$|\Phi^\eta_{(n,0):L}\rangle \equiv \left(\hat{\Phi}^\eta_{(n,0):L}\right)^\dagger|0\rangle \equiv \left|\{n_i^{\alpha_i}\}[f]{=}[4](n,0):L,S{=}0,T{=}0\right\rangle$$

Methods

- Direct diagonalization Casimir operators of SU(3), J² , T² …
- Coupling and U(N) Clebsh-Gordan coefficients (via diagonalization)
- Casimir projection techniques. Generators of algebra.

Notations

$\{n_i^{\alpha_i}\}$ configuration

α_i number of particles

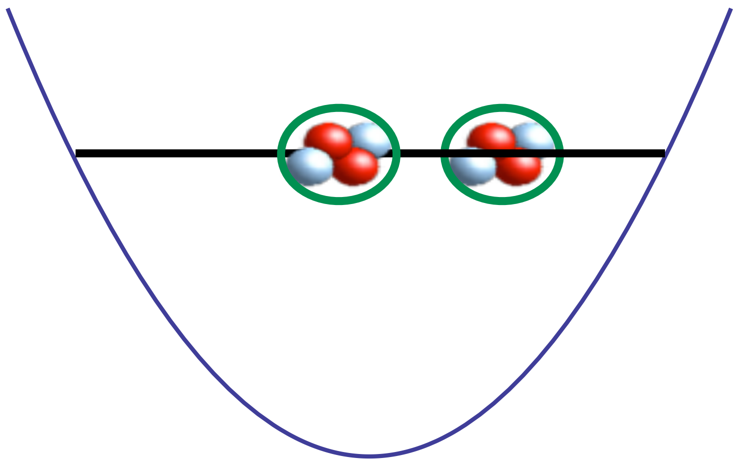
n_i oscillator shell

$$n = \sum_i \alpha_i n_i$$

$$A = \sum \alpha_i$$

Code at http://www.volya.net [cosmo]

Computational test; Bosonic nature of 4-nucleon operators



If $\Phi^\dagger$ is thought of as being a boson then $\Phi\Phi^\dagger = 1 + N_b$

$$|\Psi_D\rangle = |\Phi\rangle \quad \langle\Phi_D|\hat{\Phi}\hat{\Phi}^\dagger|\Psi_D\rangle = \langle 0|\hat{\Phi}\hat{\Phi}\hat{\Phi}^\dagger\hat{\Phi}^\dagger|0\rangle = 2$$

$$L = S = T = 0$$

Coefficients of fractional parentage

$$\mathcal{F}_{nl} = \langle\Psi_P(R_P)|\hat{\mathcal{A}}\left\{\Phi_{(n,0):l}(R_\alpha)\,\Psi_D(R_D)\right\}\rangle \equiv \langle\Psi_P(R_P)|\Phi^\dagger_{(n,0):l}(R_\alpha)|\,\Psi_D(R_D)\rangle$$

Φ	Ψ_P	$\left|\langle\Psi_P|\hat{\Phi}^\dagger|\Psi_D\rangle\right|^2$	$\langle\Psi_D|\hat{\Phi}\hat{\Phi}^\dagger|\hat{\Psi}_D\rangle$
$(p)^4\,(4,0)$	$(p)^8\,(0,4)$	1.42222*	1.42222
$(sd)^4\,(8,0)$	$(sd)^8\,(8,4)$	0.487903	1.20213
$(fp)^4\,(12,0)$	$(fp)^8\,(16,4)$	0.292411	1.41503
$(sdg)^4\,(16,0)$	$(sdg)^8\,(24,4)$	0.209525	1.5278

* For p-shell the result is known analytically 64/45

Basis states configurations

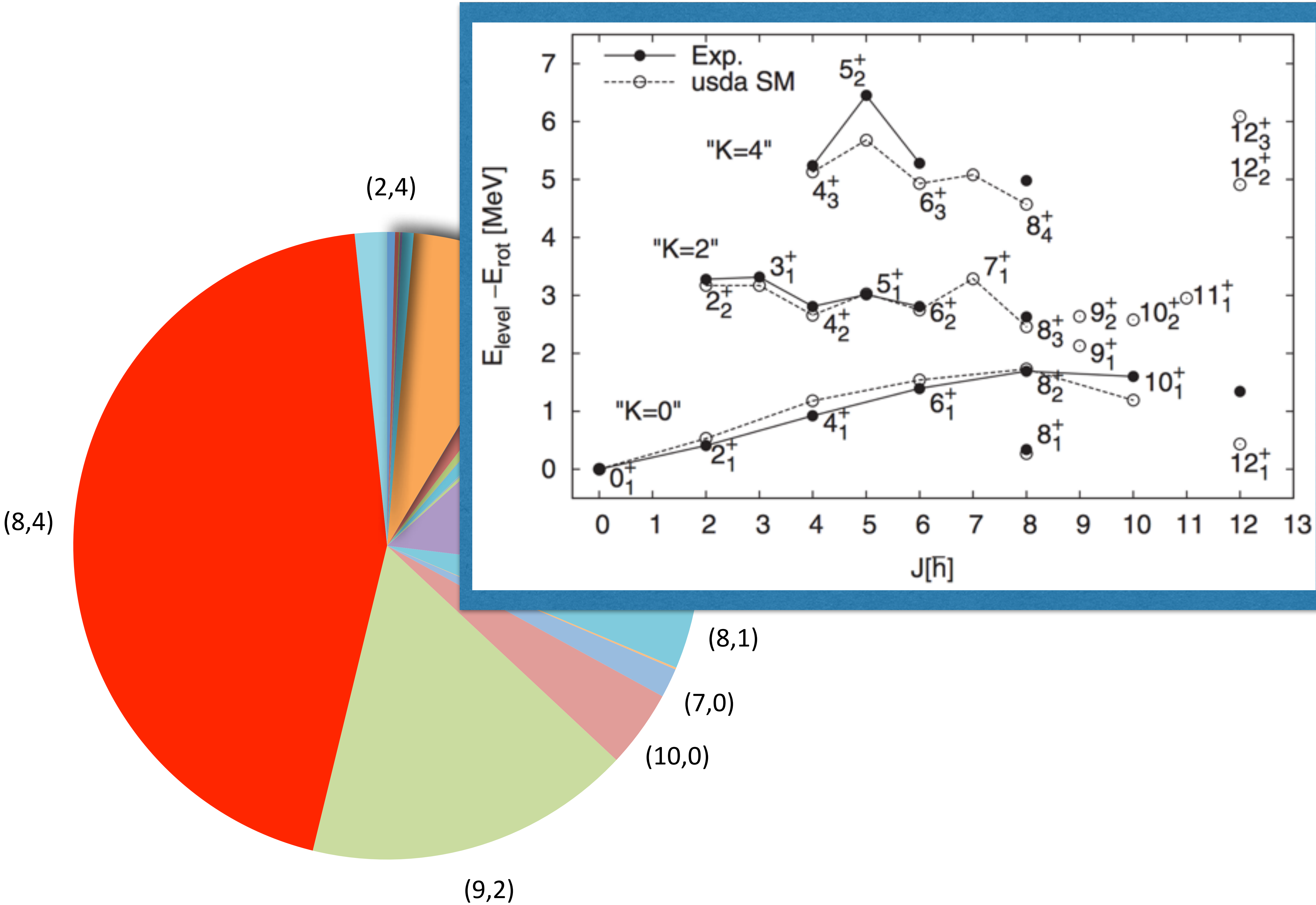
m-scheme

- Computationally very fast
- Matrices are sparse
- poor choice for collective phenomena
- center-of-mass problem
- dynamic truncation, exponential convergence, importance sampling

Symmetry-based (SU(2), SU(3), U(N) …)

- Somewhat smaller dimensionally, effective hash tables
- Not so sparse.
- Can be difficult numerically (expensive computationally & roundoff errors)
- Describe well certain collective features
- Center-of-mass and truncation convenience
- Faster convergence smaller basis?

Classic Example: ^{24}Mg, 8 nucleons in sd-shell



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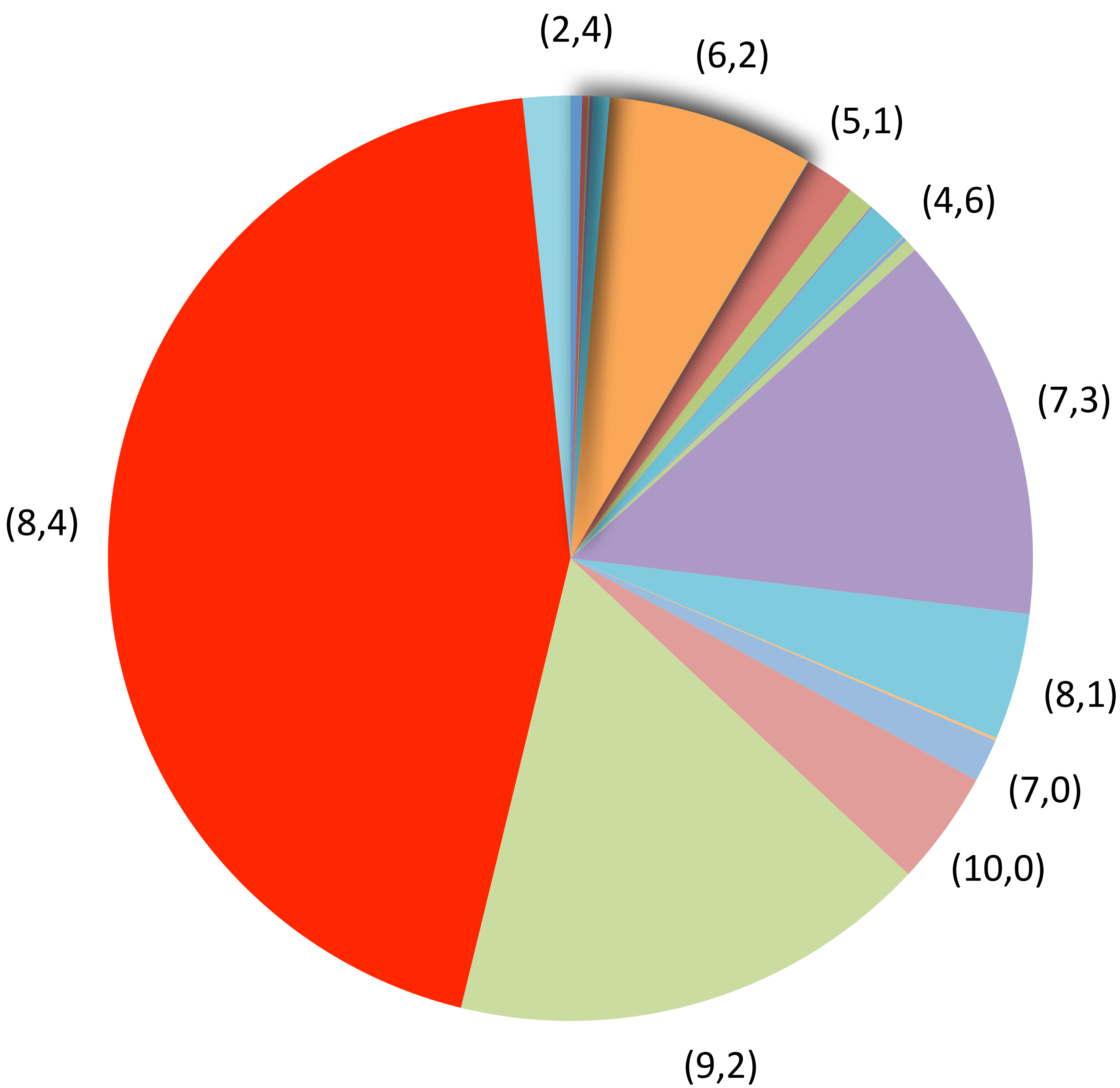
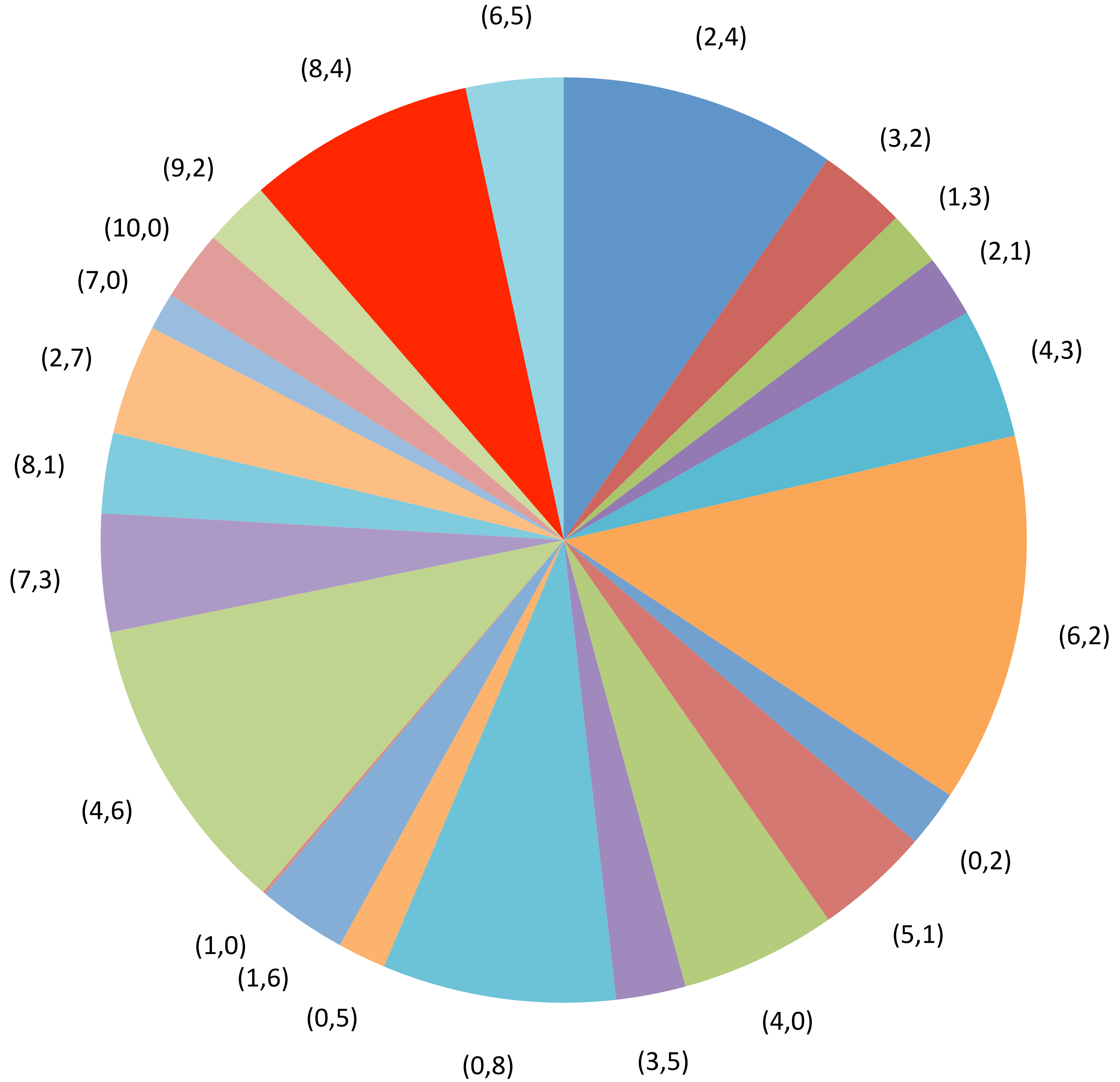


USD realistic interaction
Ground state J=T=0
breakdown in SU(3) irreps

Classic Example: ^{24}Mg, 8 nucleons in sd-shell

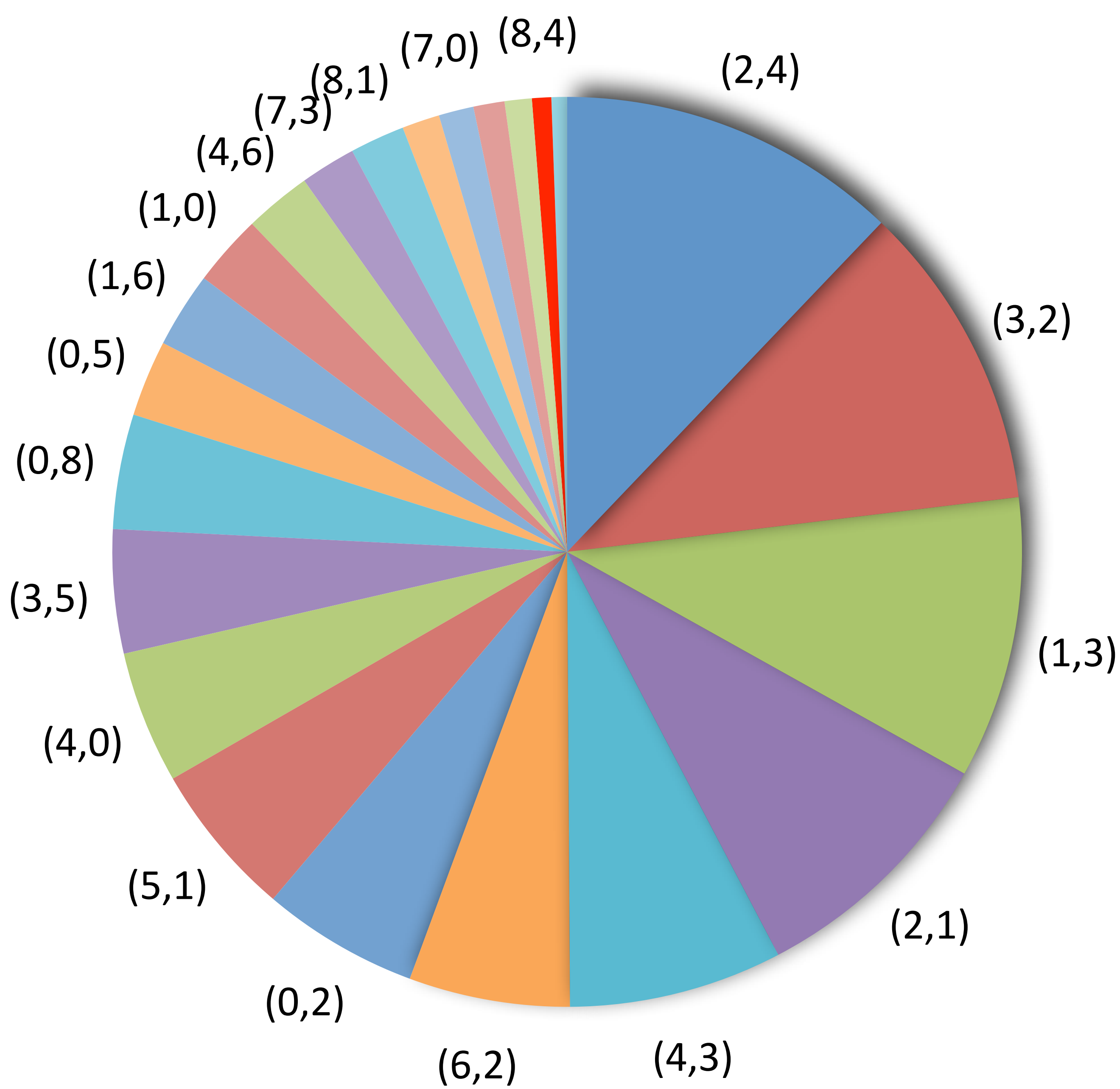


Pairing and spin-orbit
Ground state J=T=0
breakdown in SU(3) irreps

Two-body random interaction on an oscillator shell

Two-body random ensemble

- Statistical study of Hamiltonian matrices
- Two-body matrix elements selected at random, but distributions are basis invariant (GOE)
- Important for generic numerical techniques
- Emergence of realistic patterns: dominance of J=0 g.s.; rotational bands.



For ^{24}Mg model space
J=T=0 g.s. happens in 60% of cases

Weight of SU(3) irreps in all ground states

Traditional Cluster Spectroscopic Characteristics

Radial amplitude function of cluster-daughter system relative to parent

$$\varphi_\ell(\rho') = \left\langle \hat{\mathcal{A}}\{\frac{\delta(\rho-\rho')}{\rho^2} Y_{\ell m}(\Omega_{\boldsymbol{\rho}})\Psi'_\alpha\,\Psi'_D\}\middle|\Psi'_P\right\rangle$$

Definition in terms of projection

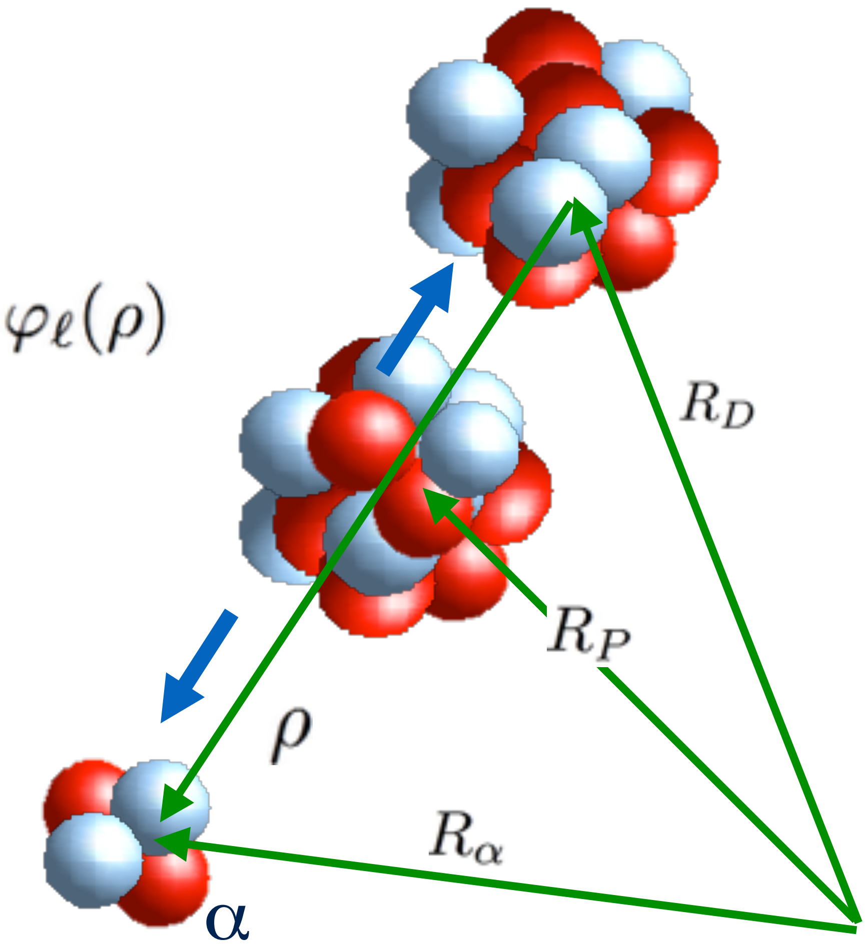
$$\mathcal{P}_{\ell m}(\rho') \equiv \hat{\mathcal{A}}\{\frac{\delta(\rho-\rho')}{\rho^2} Y_{\ell m}(\Omega_{\boldsymbol{\rho}})\Psi'_\alpha\,\Psi'_D\} \qquad \langle \mathcal{P}_{\ell m}(\rho)|\Psi'_P\rangle \equiv \varphi_\ell(\rho)$$

Expand radial motion in HO wave functions

$$\varphi_\ell(\rho) = \sum_n \langle\phi_{n\ell}|\varphi_\ell\rangle\,\phi_{n\ell}(\rho)$$

Expansion amplitudes are translationally invariant overlaps

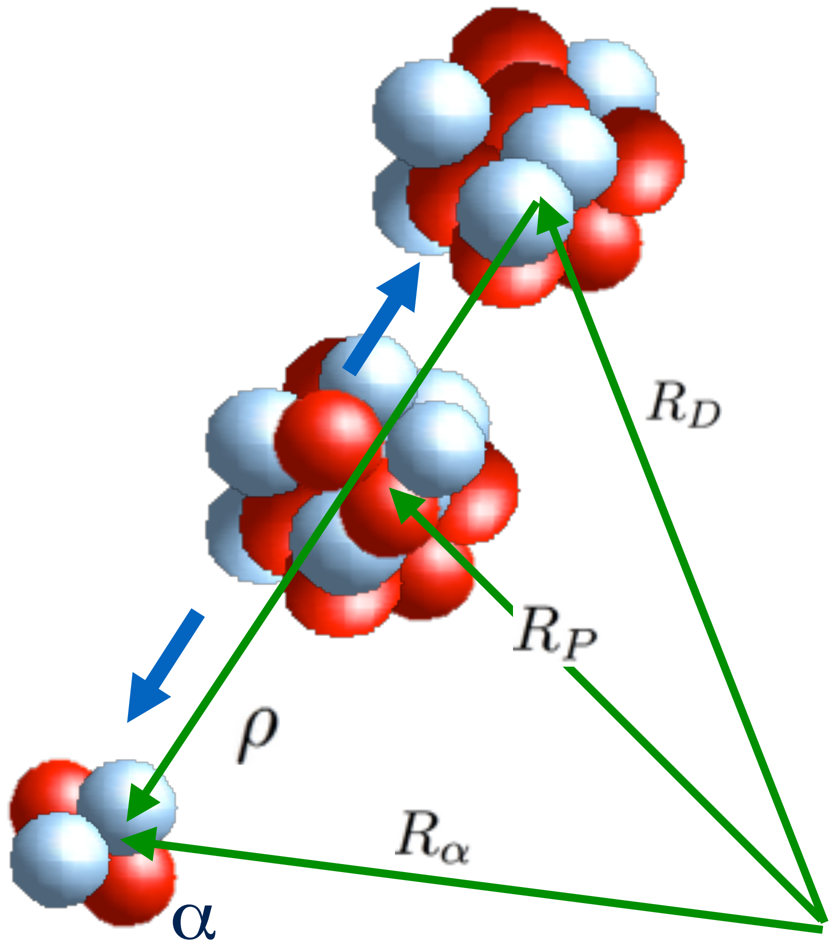
$$\langle\phi_{n\ell}|\varphi_\ell\rangle = \langle\hat{\mathcal{A}}\{\phi_{n\ell m}(\boldsymbol{\rho})\,\Psi'_\alpha\,\Psi'_D\}|\Psi'_P\rangle$$



Summary of notations used

$\phi_{n\ell}(\rho)$ radial HO wf.
$\phi_{n\ell m}(\mathbf{r})$ full single-particle oscillator wf.
$\Phi^{\eta}_{(n,0):L}$ many-body symmetry-based configuration
$\Psi_{P,D..}$ arbitrary many-body (m-scheme) state or operator
$\Psi'_{P,D..}$ arbitrary translationally invariant many-body state
$\varphi_\ell(\rho)$ projected radial wf, traditional amplitude
$\psi_\ell(\rho)$ projected radial function, new amplitude

Translational invariance



Shell model, Glockner-Lawson procedure

$$\Psi_D = \phi_{000}(\mathbf{R}_D)\,\Psi'_D \qquad \Psi_P = \phi_{000}(\mathbf{R}_P)\,\Psi'_P$$

Assume alpha structure to be (0s)^4

$$|\Psi_\alpha\rangle \equiv \left|(0s)^4[f]{=}[4](0,0) : L{=}0, S{=}0, T{=}0\right\rangle$$

$$\Psi_\alpha = \phi_{000}(\mathbf{R}_\alpha)\,\Psi'_\alpha$$

Factorizing center of mass in overlap integral

$$\langle\Psi_P|\hat{\mathcal{A}}\{\phi_{n\ell m}(\mathbf{R}_\alpha)\Psi'_\alpha\,\Psi_D\}\rangle = \langle\Psi'_P|\hat{\mathcal{A}}\{\phi_{n\ell m}(\boldsymbol{\rho})\Psi'_\alpha\,\Psi'_D\}\rangle \times \langle\phi_{000}(\mathbf{R}_P)\,\phi_{n\ell m}(\boldsymbol{\rho})|\phi_{n\ell m}(\mathbf{R}_\alpha)\,\phi_{000}(\mathbf{R}_D)\rangle$$

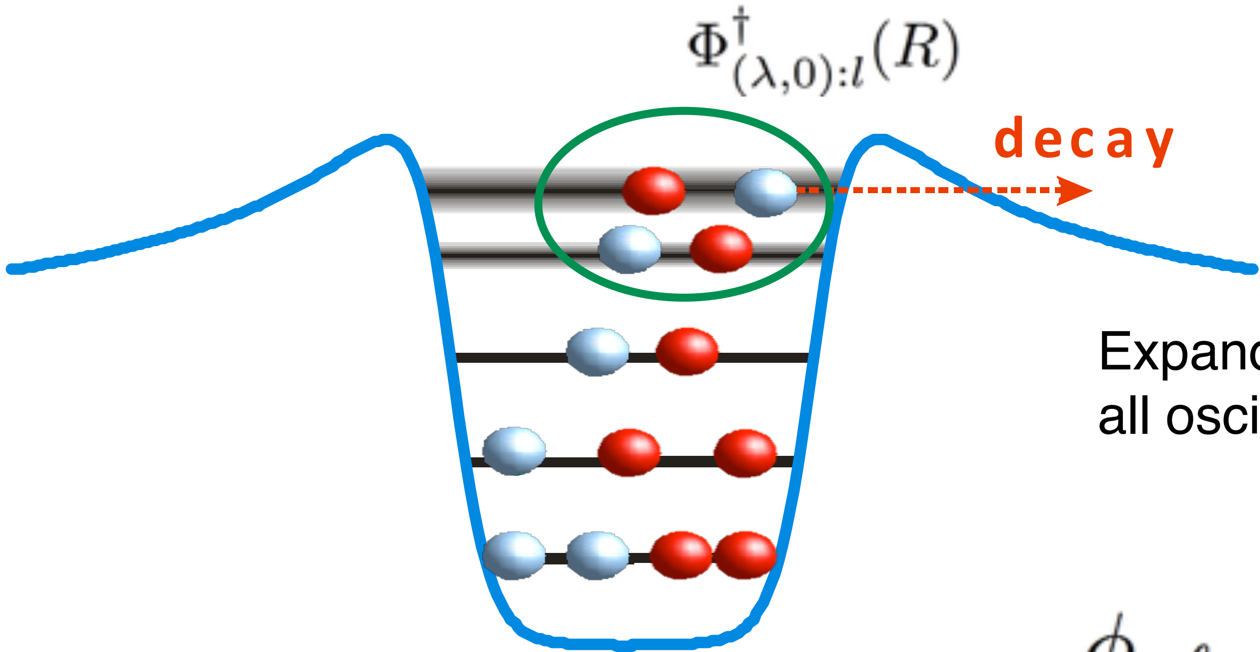
Recoil factor (inverse of Talmi-Moshinsky coefficient)

$$\mathbf{R}_P = \frac{m_D\mathbf{R}_D + m_\alpha\mathbf{R}_\alpha}{m_D + m_\alpha}, \qquad \boldsymbol{\rho} = \mathbf{R}_D - \mathbf{R}_\alpha$$

$$\langle\phi_{000}(\mathbf{R}_P)\,\phi_{n\ell m}(\boldsymbol{\rho})|\phi_{n\ell m}(\mathbf{R}_\alpha)\,\phi_{000}(\mathbf{R}_D)\rangle \equiv \langle 00, n\ell : \ell|\{n\ell\}_{m_\alpha}, \{00\}_{m_D} : \ell\rangle$$

$$\mathcal{R}_{n\ell} \equiv \left(\langle 00, n\ell : \ell|\{n\ell\}_{m_\alpha}, \{00\}_{m_D} : \ell\rangle\right)^{-1} = (-1)^n \left(\frac{m_D + m_\alpha}{m_D}\right)^{n/2}$$

Cluster coefficients for SU(3) components



Expand SU(3) 4-nucleon structure in intrinsic+ relative all oscillator quanta of excitation are in relative motion.

$$\phi_{n\ell m}(\mathbf{R}_\alpha)\Psi'_\alpha = \sum_\eta X^\eta_{n\ell}\,\Phi^\eta_{(n,0):\ell m}$$

$$X^\eta_{n\ell} \equiv \langle \Phi^\eta_{(n,0):\ell m} | \phi_{n\ell m}(\mathbf{R}_\alpha)\,\Psi'_\alpha \rangle = \sqrt{\frac{1}{4^n}\,\frac{n!}{\prod_i (n_i!)^{\alpha_i}}\,\frac{4!}{\prod_i \alpha_i!}}$$

Yu. F. Smirnov and Yu. M. Tchuvil'sky, Phys. Rev. C 15, 84 (1977).
M. Ichimura, A. Arima, E. C. Halbert, and T. Terasawa, Nucl. Phys. A 204, 225 (1973).
O. F. Nemetz, V. G. Neudatchin, A. T. Rudchik, Yu. F. Smirnov, and Yu. M. Tchuvil'sky, Nucleon Clusters in Atomic Nuclei and Multi-Nucleon Transfer Reactions (Naukova Dumka, Kiev, 1988), p. 295.

Traditional “old” spectroscopic factors

$$\mathcal{S}_\ell^{(\mathrm{old})} = \langle \varphi_\ell | \varphi_\ell \rangle = \int \rho^2 d\rho \, |\varphi_\ell(\rho)|^2 = \sum_n |\langle \phi_{n\ell} | \varphi_\ell \rangle|^2$$

$$\langle \phi_{n\ell} | \varphi_\ell \rangle = \underbrace{\mathcal{R}_{n\ell}}_{\text{Recoil Factor}} \sum_\eta \underbrace{X^\eta_{n\ell}}_{\text{Cluster Coefficient}} \, \underbrace{\mathcal{F}^\eta_{n\ell}}_{\text{Fractional Parentage Coefficient}}$$

Issues with the traditional SF

- Non-orthogonal set of channels (over-complete set of configurations)
- Pauli exclusion principle
- Matching procedure, asymptotic normalization, connection to observables
- No agreement with experiment on absolute scale

Yu. F. Smirnov and Yu. M. Tchuvil’sky, Phys. Rev. C 15, 84 (1977).
M. Ichimura, A. Arima, E. C. Halbert, and T. Terasawa, Nucl. Phys. A 204, 225 (1973).
O. F. Nemetz, V. G. Neudatchin, A. T. Rudchik, Yu. F. Smirnov, and Yu. M. Tchuvil’sky, Nucleon Clusters in Atomic Nuclei and Multi-Nucleon Transfer Reactions (Naukova Dumka, Kiev, 1988), p. 295.
W. Chung, J. van Hienen, B. H. Wildenthal, and C. L. Bennett, Phys. Lett. B 79, 381 (1978)

Resonating Group Method and Ortogonality Condition

Cluster channel: relative motion of “frozen” intrinsic states

$$\Psi'_{\rm ch} = \hat{\mathcal{A}}\{f_\ell(\rho) Y_{\ell m}(\Omega_\rho)\Psi'_\alpha\,\Psi'_D\} = \int f_\ell(\rho')\,\mathcal{P}_{\ell m}(\rho')\,\rho'^2 d\rho'$$

Projector: $\mathcal{P}_{\ell m}(\rho') \equiv \hat{\mathcal{A}}\{\frac{\delta(\rho-\rho')}{\rho^2} Y_{\ell m}(\Omega_{\boldsymbol{\rho}})\Psi'_\alpha\,\Psi'_D\}$

The effective equation (RGM) for relative motion : $\hat{\mathcal{H}}_\ell\, f_\ell(\rho) = E\hat{\mathcal{N}}_\ell f_\ell(\rho)$

$$\hat{\mathcal{N}}_\ell^{-1/2}\hat{\mathcal{H}}_\ell\,\hat{\mathcal{N}}_\ell^{-1/2} F_\ell(\rho) = E F_\ell(\rho) \quad F_\ell(\rho) \equiv \hat{\mathcal{N}}_\ell^{1/2} f_\ell(\rho).$$

Norm kernel and projection on relative motion

$$\langle \mathcal{P}_{\ell m}(\rho)|\Psi'_{\rm ch}\rangle = \hat{\mathcal{N}}_\ell f_\ell = \int \mathcal{N}_\ell(\rho',\rho) f_\ell(\rho)\rho^2 d\rho$$

$$\mathcal{N}_\ell(\rho',\rho'') = \langle \mathcal{P}_{\ell m}(\rho')|\mathcal{P}_{\ell m}(\rho'')\rangle = \Big\langle \hat{\mathcal{A}}\{\frac{\delta(\rho-\rho')}{\rho^2} Y_{\ell m}(\Omega_\rho)\Psi'_\alpha\,\Psi'_D\}\Big|\hat{\mathcal{A}}\{\frac{\delta(\rho-\rho'')}{\rho^2} Y_{\ell m}(\Omega_\rho)\Psi'_\alpha\,\Psi'_D\}\Big\rangle$$

Shell model strategy: we project parent state and match it with RGM projection

$$\langle \mathcal{P}_{\ell m}(\rho)|\Psi'_P\rangle \equiv \varphi_\ell(\rho) \leftrightarrow \langle \mathcal{P}_{\ell m}(\rho)|\Psi'_{\rm ch}\rangle = \hat{\mathcal{N}}_\ell f_\ell(\rho).$$

H. Feschbach et al. Ann. Phys. 41 (1967) 230 – 286

R. Lovas et al. Phys. Rep. 294, No. 5 (1998) 265 – 362.

T. Fliessbach and H. J. Mang, Nucl. Phys. A **263**, 75–85 (1976).

S. G. Kadmenskya, S. D. Kurgalina, and Yu. M. Tchuvil’sky Phys. Part. Nucl., 38, 699–742 (2007).

Ortogonality condition model, new SF

$$\boxed{\psi_\ell(\rho) \equiv \hat{\mathcal{N}}_\ell^{-1/2} \varphi_\ell(\rho)}$$

$$\boxed{S_\ell^{(\text{new})} \equiv \langle \psi_\ell | \psi_\ell \rangle = \int \rho^2 d\rho \, |\psi_\ell(\rho)|^2}$$

R. Id Betan and W. Nazarewicz Phys. Rev. C 86, 034338 (2012)
S. G. Kadmenskya, S. D. Kurgalina, and Yu. M. Tchuvil'sky Phys. Part. Nucl., 38, 699–742 (2007).
R. Lovas et al. Phys. Rep. 294, No. 5 (1998) 265 – 362.
T. Fliessbach and H. J. Mang, Nucl. Phys. A **263**, 75–85 (1976).
H. Feschbach et al. Ann. Phys. 41 (1967) 230 – 286

Evaluation of norm kernel and SF in oscillator basis

$$\langle \phi_{n'\ell} | \hat{\mathcal{N}}_\ell | \phi_{n\ell} \rangle = \langle \hat{\mathcal{A}} \{ \phi_{n'\ell m}(\boldsymbol{\rho}) \, \Psi'_\alpha \, \Psi'_D \} | \hat{\mathcal{A}} \{ \phi_{n\ell m}(\boldsymbol{\rho}) \, \Psi'_\alpha \, \Psi'_D \} \rangle$$

$$\langle \phi_{n'\ell} | \hat{\mathcal{N}}_\ell | \phi_{n\ell} \rangle = \mathcal{R}_{n'\ell} \mathcal{R}_{n\ell} \sum_{\eta\eta'} X^{\eta'}_{n'\ell} X^{\eta}_{n\ell} \left\langle 0 \right| \left\{ \underbrace{\hat{\Psi}^{\eta'}_{(n',0):\ell} \, \hat{\Psi}_D}_{\text{4-body operator}} \right\} \Big| \left\{ \hat{\Psi}^{\eta}_{(n,0):\ell} \, \hat{\Psi}_D \right\}^\dagger \left| 0 \right\rangle$$

We diagonalize norm kernel in oscillator basis $\hat{\mathcal{N}}_\ell | k\ell \rangle = N_{k\ell} | k\ell \rangle$

New Amplitudes $\langle \phi_{n\ell} | \psi_\ell \rangle = \sum_{k\,n'} \frac{1}{\sqrt{N_{k\ell}}} \langle \phi_{n\ell} | k\ell \rangle \langle k\ell | \phi_{n'\ell} \rangle \langle \phi_{n'\ell} | \varphi_\ell \rangle$

New SF

$$S^{(\text{new})}_\ell = \sum_k \frac{1}{N_{k\ell}} \left| \sum_n \langle k\ell | \phi_{n\ell} \rangle \, \langle \phi_{n\ell} | \varphi_\ell \rangle \right|^2$$

Statistical normalization property of new SF

$$\langle \phi_{n'\ell} | \hat{\mathcal{N}}_\ell | \phi_{n\ell} \rangle = \langle \hat{\mathcal{A}} \{ \phi_{n'\ell m}(\boldsymbol{\rho}) \, \Psi'_\alpha \, \Psi'_D \} | \hat{\mathcal{A}} \{ \phi_{n\ell m}(\boldsymbol{\rho}) \, \Psi'_\alpha \, \Psi'_D \} \rangle$$

For an arbitrary set of parent states $\sum_P |\Psi_P\rangle\langle\Psi_P| \equiv \hat{1}$

Thus, norm comes from projector on old amplitudes $\hat{\mathcal{N}}_\ell = \sum_P |\varphi_\ell^{(P)}\rangle\langle\varphi_\ell^{(P)}|$

Sum of all new SF from all parent states to a given final state equals to the number of channels

$$\sum_P S_\ell^{(\mathrm{new})}(P) = \sum_P \langle \varphi_\ell^{(P)} | \hat{\mathcal{N}}_\ell^{-1} | \varphi_\ell^{(P)} \rangle = \sum_n 1$$

For example for one channel:

$$S_\ell^{(\mathrm{new})}(P) = \frac{\mathcal{S}_\ell^{(\mathrm{old})}(P)}{\sum_{P'} \mathcal{S}_\ell^{(\mathrm{old})}(P')} = \frac{(\mathcal{F}_{n\ell}^{(P)})^2}{\sum_{P'} (\mathcal{F}_{n\ell}^{(P')})^2}$$

Alpha clustering in sd-shell nuclei

$A_P - A_D$	$S_0^{(\mathrm{exp})}$ [1]	$S_0^{(\mathrm{exp})}$ [2]	$S_0^{(\mathrm{exp})}$ [3]	$\mathcal{S}_0^{(\mathrm{old})}$ [4]	$\mathcal{S}_0^{(\mathrm{old})}$ this work	$\mathcal{S}_0^{(\mathrm{new})}$ this work
^{20}Ne-^{16}O	1.0	0.54	1	0.18	0.173	0.755
^{22}Ne-^{18}O			0.37	0.099	0.085	0.481
^{24}Mg-^{20}Ne	0.76	0.42	0.66	0.11	0.091	0.411
^{26}Mg-^{22}Ne			0.20	0.077	0.068	0.439
^{28}Si-^{24}Mg	0.37	0.20	0.33	0.076	0.080	0.526
^{30}Si-^{26}Mg			0.55	0.067	0.061	0.555
^{32}S-^{28}Si	1.05	0.55	0.45	0.090	0.082	0.911
^{34}S-^{30}Si				0.065	0.062	0.974
^{36}Ar-^{32}S				0.070	0.061	0.986
^{38}Ar-^{34}S			1.30	0.034	0.030	0.997
^{40}Ca-^{36}Ar	1.56	0.86	1.18	0.043	0.037	1

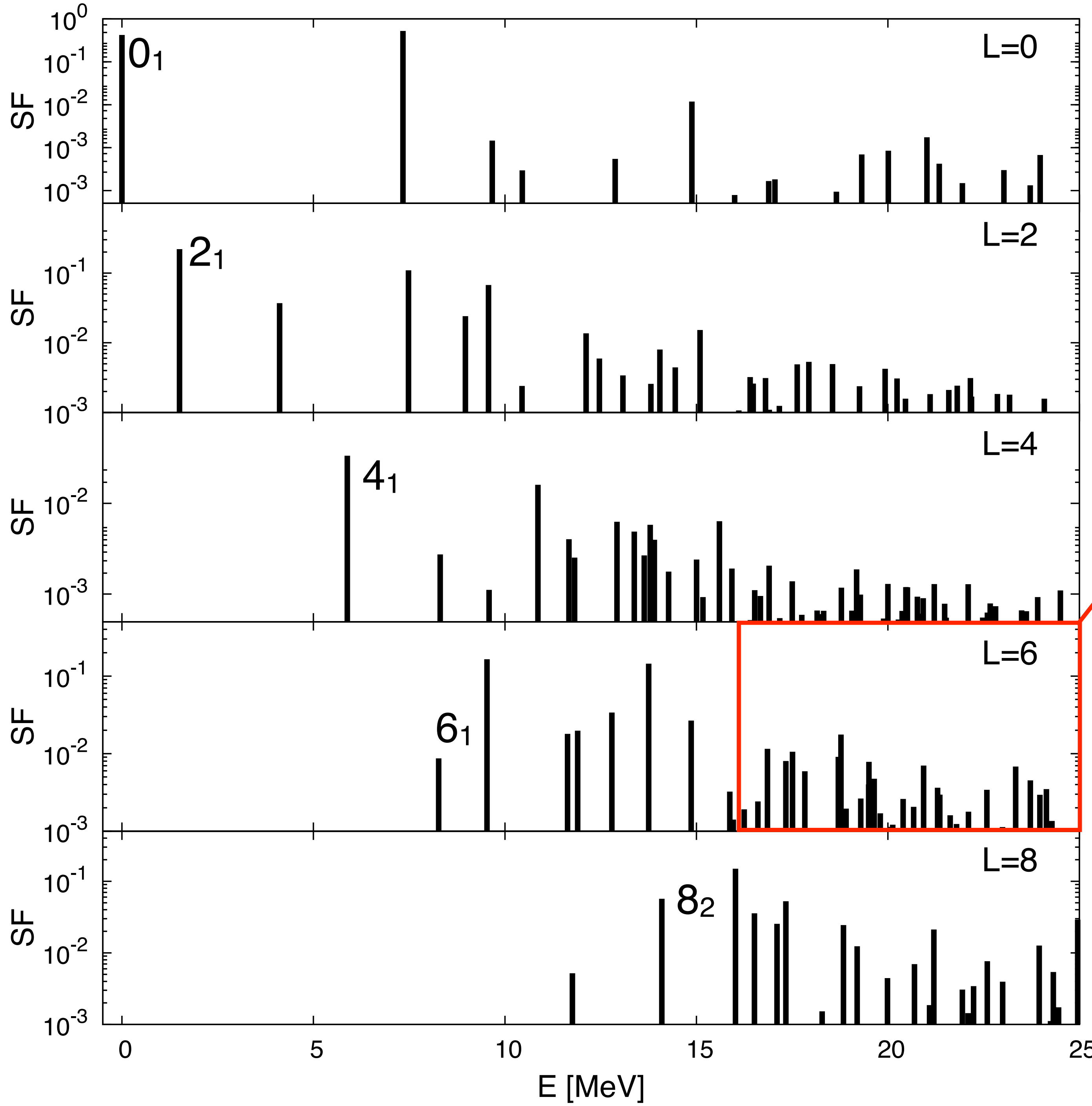
USDB interaction [5]
(8,0) configuration

- Old SF are small
- Old SF decrease with A

[1] T. Carey, P. Roos, N. Chant, A. Nadasen, and H. L. Chen, Phys. Rev. C 23, 576(R) (1981).
[2] T. Carey, P. Roos, N. Chant, A. Nadasen, and H. L. Chen, Phys. Rev. C 29, 1273 (1984).
[3] N. Anantaraman and et al., Phys. Rev. Lett. 35, 1131 (1975).
[4] W. Chung, J. van Hienen, B. H. Wildenthal, and C. L. Bennett, Phys. Lett. B 79, 381 (1978).
[5] B. A. Brown and W. A. Richter, Phys. Rev. C 74, 034315 (2006)

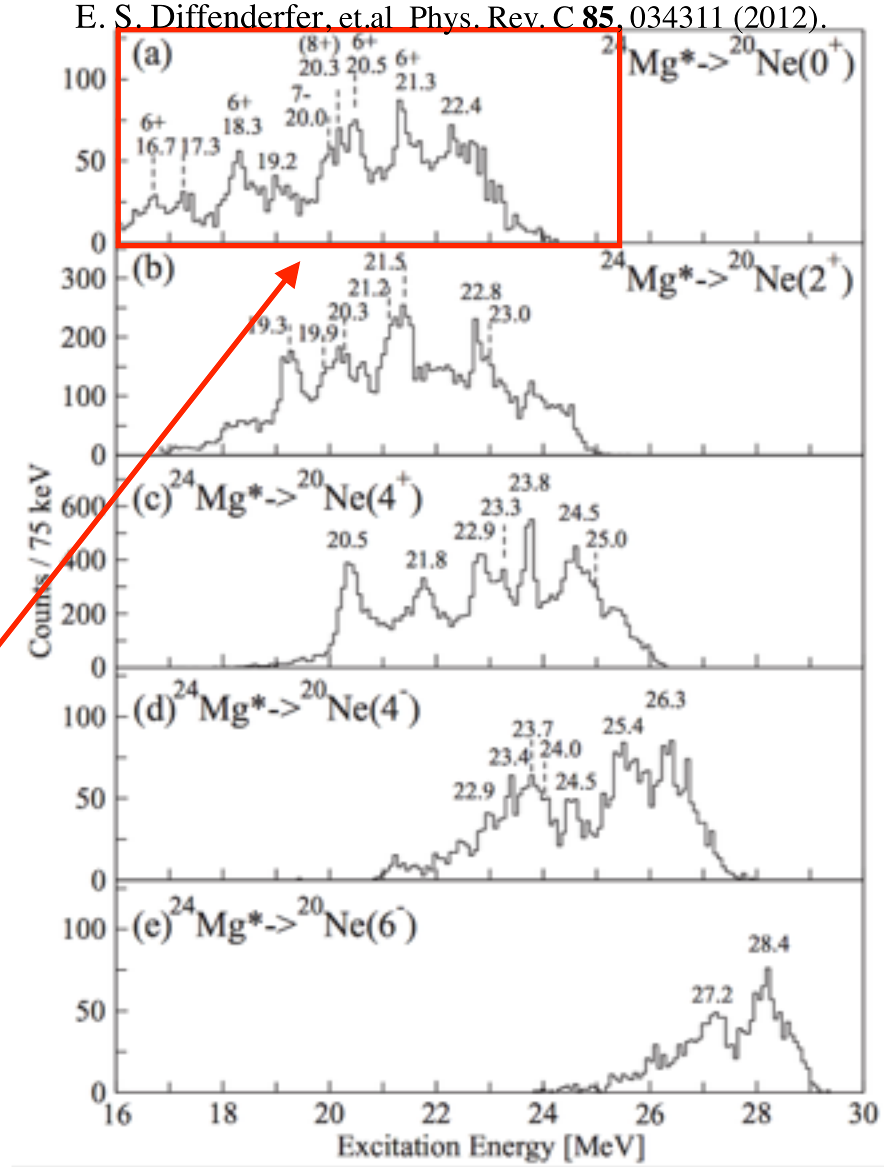
Alpha cluster spectroscopic factors in ^{24}Mg

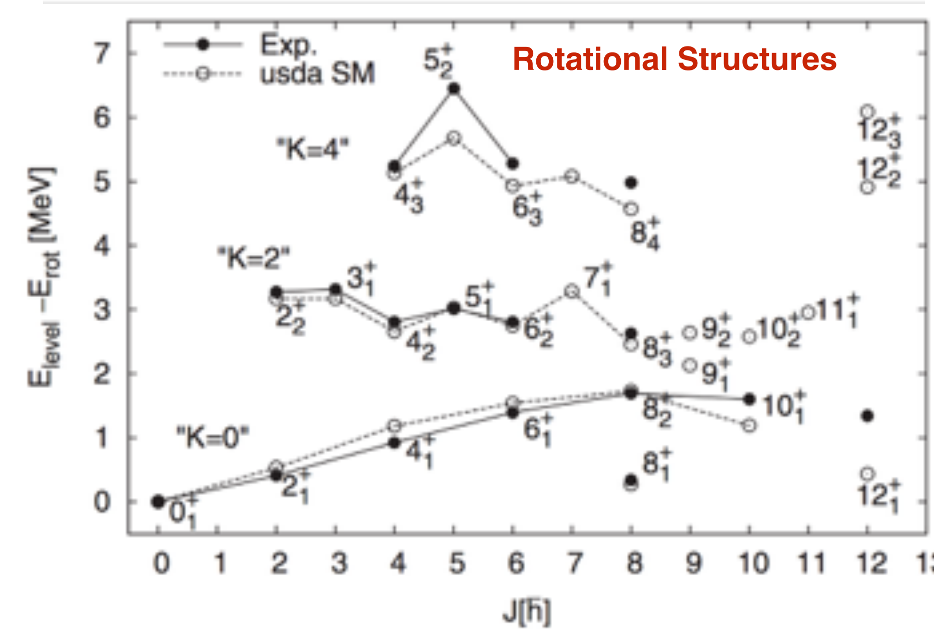
Theoretical calculations in SD shell



Experimental results

E. S. Diffenderfer, et.al Phys. Rev. C **85**, 034311 (2012).





The sd valence space is considered with USDB interaction the operator is

Φ	Ψ_P	$\left\vert\langle\Psi_P\vert\hat{\Phi}^\dagger\vert\Psi_D\rangle\right\vert^2$	$\langle\Psi_D\vert\hat{\Phi}\hat{\Phi}^\dagger\vert\hat{\Psi}_D\rangle$
$(p)^4\,(4,0)$	$(p)^8\,(0,4)$	1.42222*	1.42222
$(sd)^4\,(8,0)$	$(sd)^8\,(8,4)$	0.487903	1.20213
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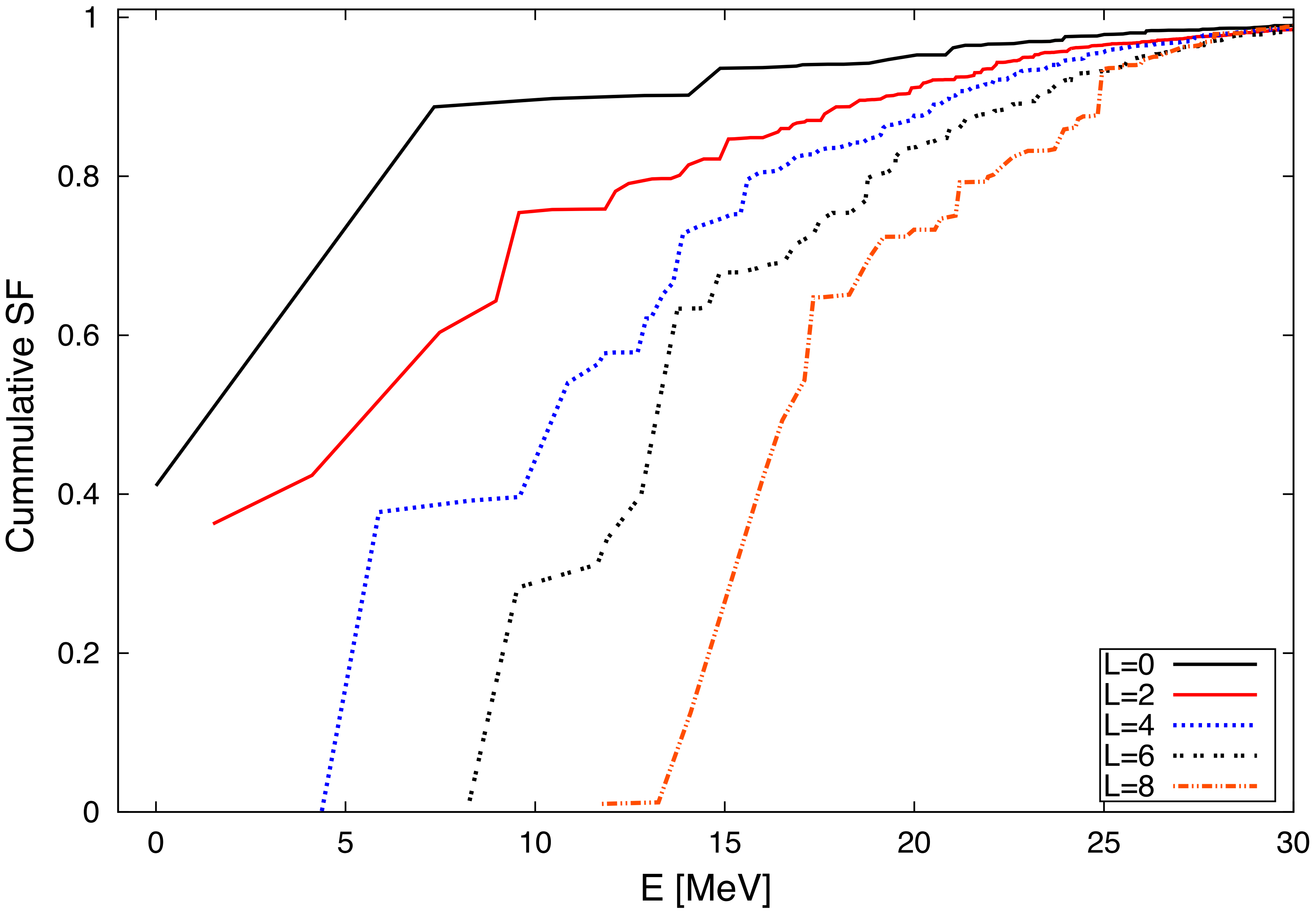
Cumulative spectroscopic strength ^{24}Mg

^{20}Ne(g.s)+alpha channel.



Alpha clustering in ^{16}O

p-sd effective hamiltonian

SU(3) configurations

$$\langle \mathcal{P}_{\ell m}(\rho)|\Psi'_{\mathrm{ch}}\rangle = \hat{\mathcal{N}}_\ell f_\ell = \int \mathcal{N}_\ell(\rho',\rho) f_\ell(\rho)\rho^2 d\rho.$$

For positive parity

$$|\Phi_{(8,0):L}\rangle = |(sd)^4[4]\,(8,0),:L\,S=T=0\rangle$$

$$|\Phi_{(6,0):L}\rangle = |p^2(sd)^4[4]\,(6,0),:L\,S=T=0\rangle$$

$$|\Phi_{(4,0):L}\rangle = |p^4[4]\,(4,0),:LS=T=0\rangle$$

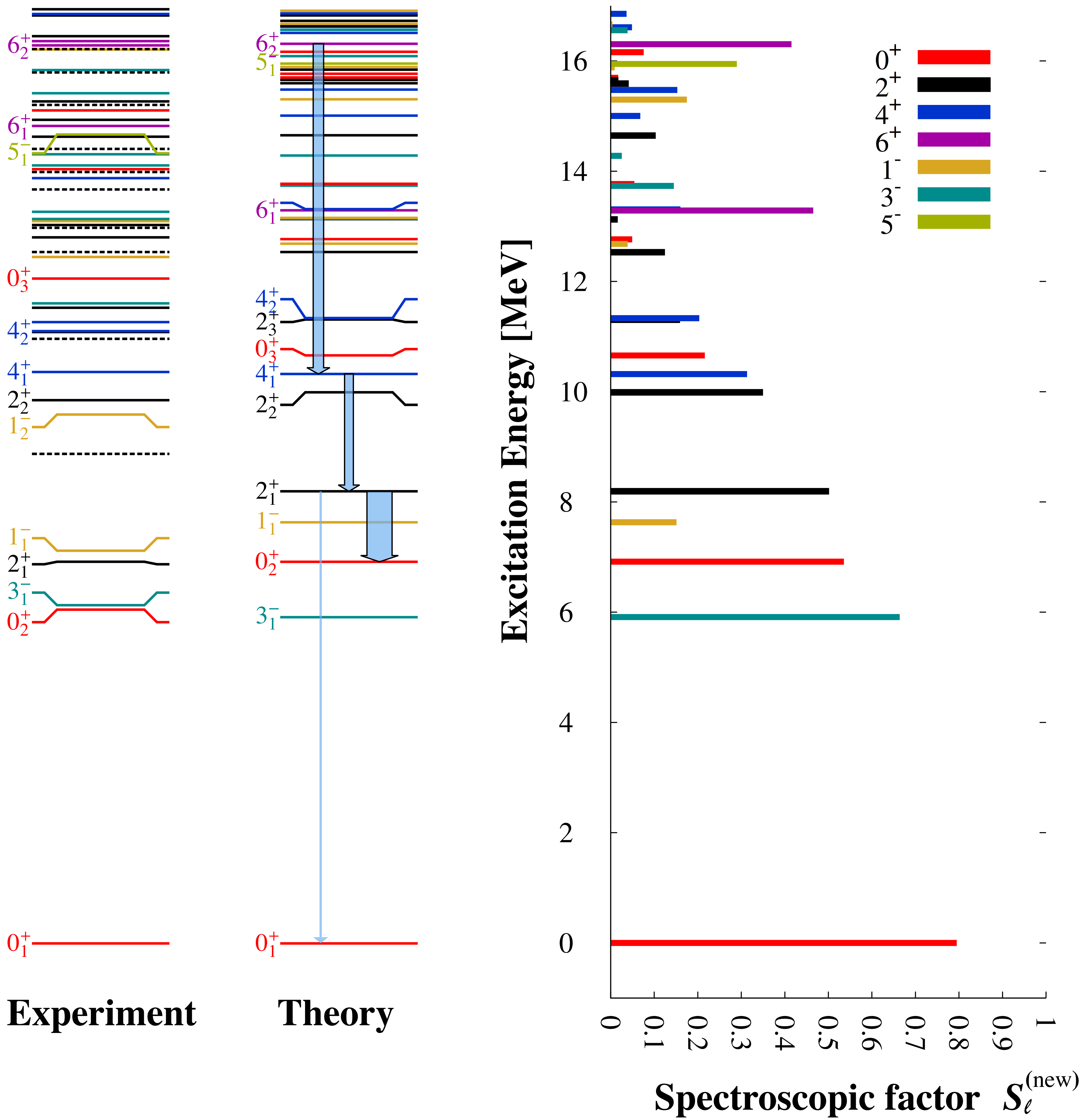
Translational invariance

We use Lawson approach $F_\ell(\rho) \equiv \hat{\mathcal{N}}_\ell^{1/2} f_\ell(\rho).$

$$\langle \Psi_P|\beta(H_{\mathrm{c.m.}} - {}^3/_2\hbar\omega)|\Psi_P\rangle \approx 0.3\ \mathrm{MeV}$$

E. K. Warburton and B. A. Brown, Phys. Rev. C 46 (1992) 923

Y. Utsuno and S. Chiba, Phys. Rev. C83 021301(R) (2011)

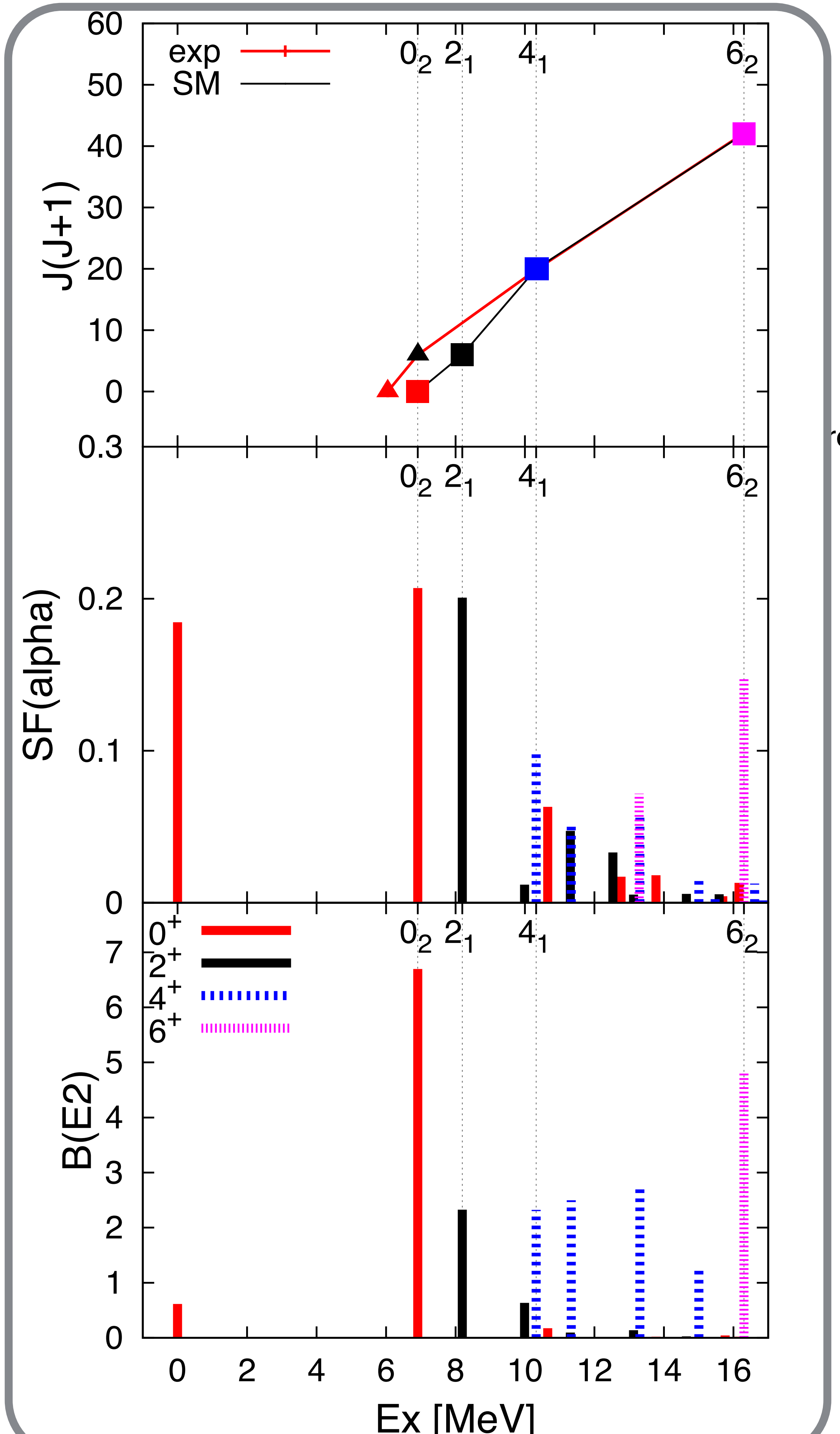

Experiment
Theory
Excitation Energy [MeV]
Spectroscopic factor $S_\ell^{(new)}$
0^+
2^+
4^+
6^+
1^-
3^-
5^-
0_1^+
0_2^+
3_1^-
2_1^+
1_1^-
1_2^-
2_2^+
4_1^+
4_2^+
0_3^+
5_1^-
6_1^+
6_2^+
2_3^+

Rotational structure in ^{16}O

transition	B(E2) $e^2\mathrm{fm}^4$
$2^+(1) \to 0^+(1)$	4.1
$2^+(1) \to 0^+(2)$	44.2
$4^+(1) \to 2^+(1)$	15.4
$4^+(1) \to 2^+(2)$	4.2
$6^+(1) \to 4^+(1)$	0.2
$6^+(2) \to 4^+(1)$	18.3
$8^+(1) \to 6^+(1)$	0.1
$8^+(1) \to 6^+(2)$	31.7

	Yamada*	This Work
0	0.6	0.8
0^{+}	0.8	0.54
0^{+}	0.2	0.22

* Yamada, et. al *Clusters in nuclei, Vol 2, (Springer-Verlag, 2012) page 229.*



Detailed spectroscopy ^{16}O

J_i^π	$E^{(sm)}$	$S_\ell^{(new)}$	$E^{(exp)}$	θ_α^2
0_1^+	0.000	0.794	0	0.86^a
3_1^-	5.912	0.663	6.13	0.41^a
0_2^+	6.916	0.535	6.049	0.40^a
1_1^-	7.632	0.150	7.117	0.14
2_1^+	8.194	0.500	6.917	0.47^a
1^-	no		9.585	0.67
2_2^+	9.988	0.349	9.844	0.0015
4_1^+	10.320	0.313	10.356	0.44
0_3^+	10.657	0.216	11.26	0.77
2_3^+	11.307	0.158	11.52	0.033
4_2^+	11.334	0.203	11.097	0.0014
3^-	no		11.6	0.68
2_4^+	12.530	0.123	no	
1_2^-	12.681	0.038	12.44	0.023
0_4^+	12.764	0.049	12.049	0.00036
2_5^+	13.125	0.015	13.02	<0.04
6_1^+	13.286	0.465	14.815	0.17
4_3^+	13.308	0.160	14.62	0.19
3_3^-	13.733	0.144	14.1	0.21
0_5^+	13.767	0.054	14.032	0.037
3_4^-	14.279	0.025	13.129	0.041
2_6^+	14.646	0.102	14.926	<0.0098
4_4^+	15.002	0.067	13.869	0.043
1_4^-	15.298	0.174	16.2	<0.085
1_5^-	15.884	0.009		
4_5^+	15.474	0.152	16.844	0.13
4_7^+	16.611	0.048		
4_8^+	16.855	0.036		
2_7^+	15.589	0.040	15.26	<0.052
2_8^+	15.649	0.016	16.352	<0.093

J_i^π	$E^{(sm)}$	$S_\ell^{(new)}$	$E^{(exp)}$	θ_α^2
0_6^+	15.694	0.017	15.097	<0.024
5_1^-	15.945	0.289	14.66	0.55
3_5^-	16.080	0.00063	15.408	<0.028
0_8^+	16.159	0.075	no	
6_2^+	16.304	0.415	16.275	0.43
3_6^-	16.557	0.038	15.828	0.14
2_{11}^+	16.720	0.011	16.93	<0.04
2_{12}^+	16.818	0.0096	17.129	<0.015
2_{13}^+	17.259	0.0032	17.197	<0.022
1_8^-	17.357	0.015	17.51	0.022
1_9^-	17.572	0.007		
1_{10}^-	17.674	0.016		
1_{11}^-	18.122	0.014		
3_9^-	17.772	0.034	no	
0_{11}^+	18.214	0.040	18.089	<0.033
4_{10}^+	18.251	0.051	17.784	<0.077
5_2^-	18.265	0.051	18.404	0.14
4_{11}^+	18.393	0.0079	18.016	0.0026
7_1^-	18.412	0.325	20.857	0.44
6_3^+	18.613	0.048	17.555	<0.11
4_{12}^+	19.081	0.030	18.785	<0.044
5_3^-	19.102	0.024	18.6	0.036
6_4^+	19.228	0.013	19.319	<0.023
4_{14}^+	19.348	0.015	19.375	<0.0036
4_{16}^+	19.819	0.028		
5_4^-	19.620	0.083	19.253	<0.011
8_1^+	20.018	0.34	no	
6_5^+	20.078	0.035	21.052	0.051
6_6^+	21.038	0.038	21.648	<0.026
7_2^-	21.693	0.036	21.623	<0.024

- Most states are well described
- alpha, electromagnetic and other properties
- Cumulative strength

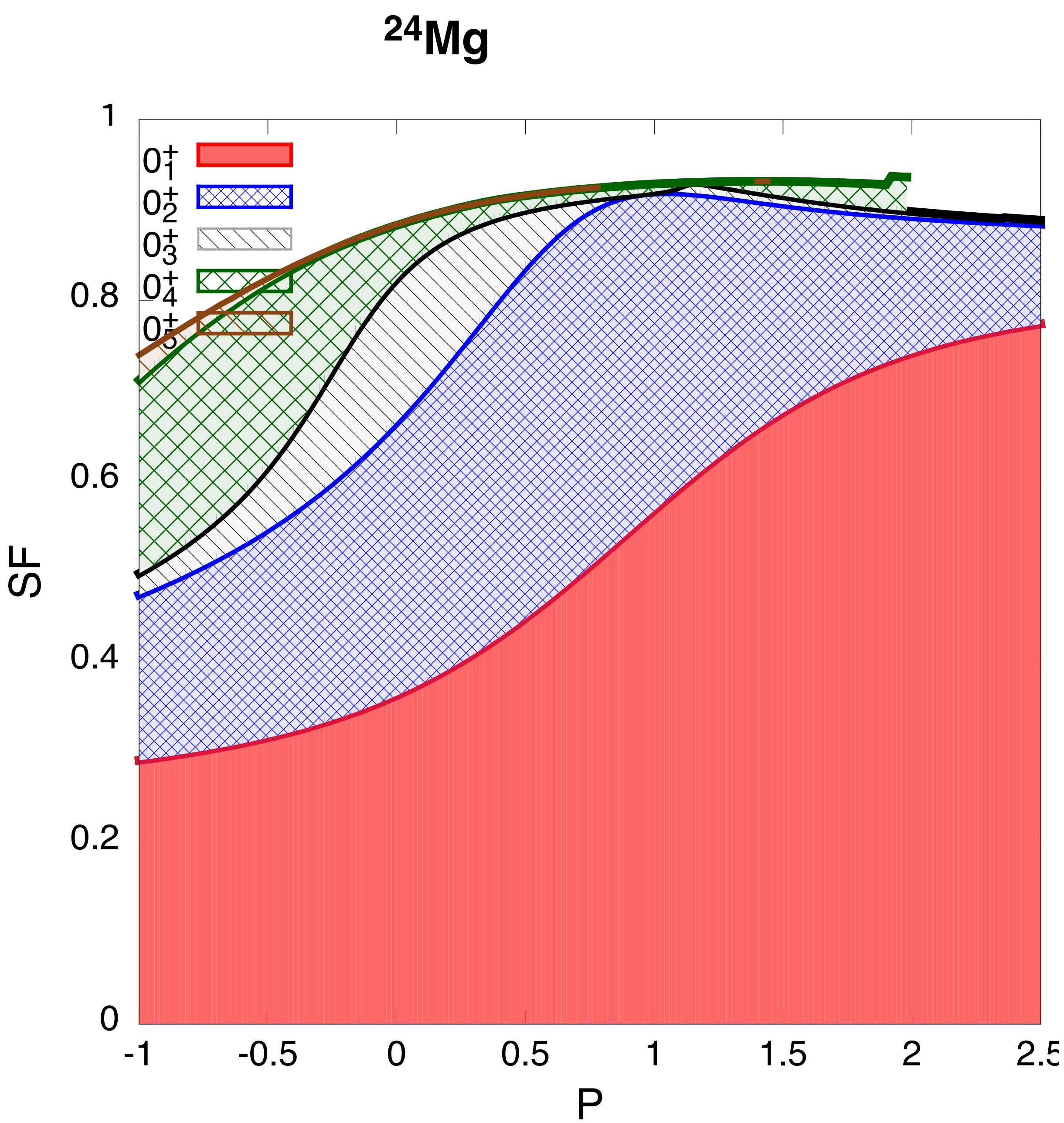
$$\sum_i S_4^{(exp)}(4_i^+) = 0.67$$

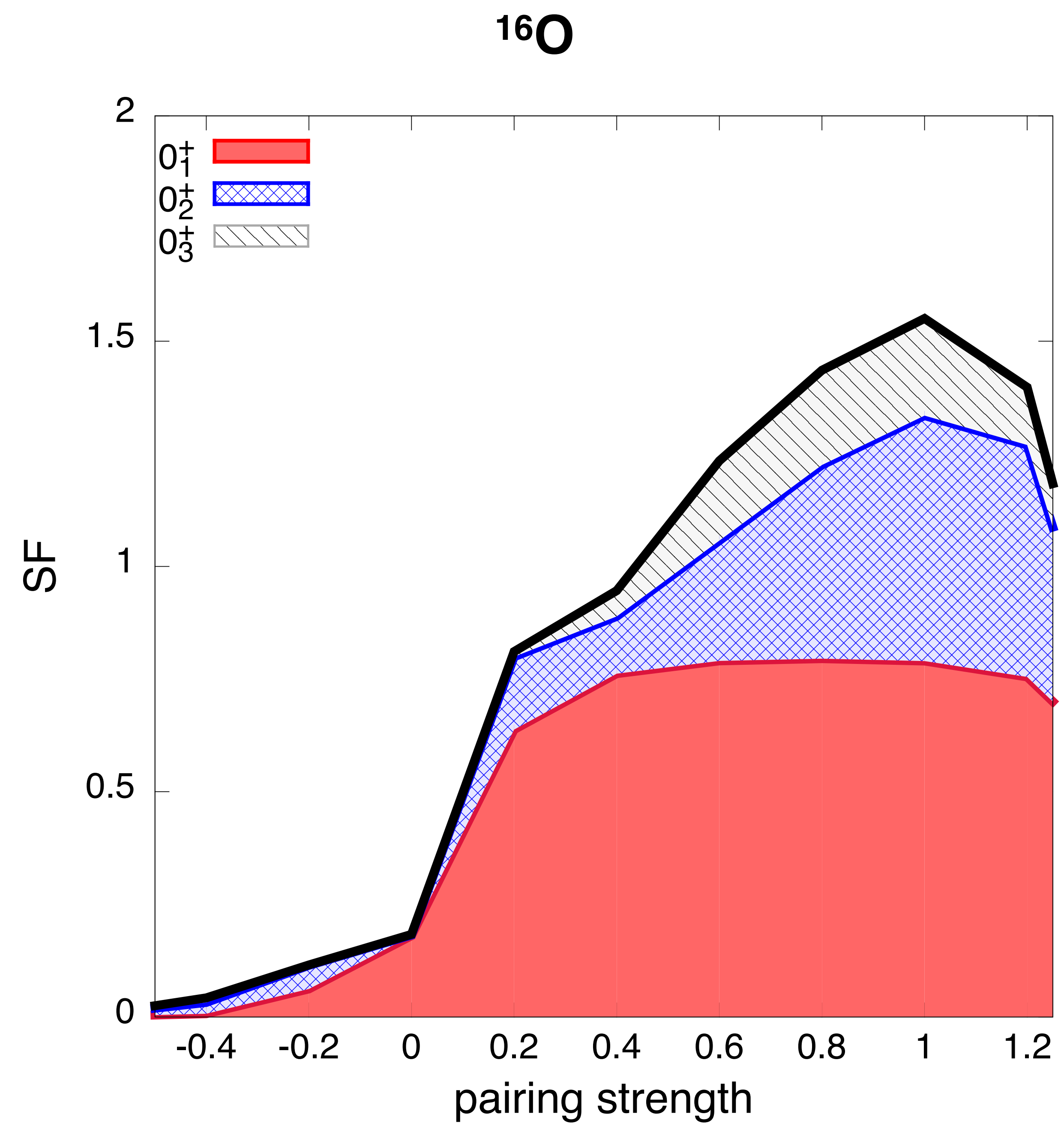
$$\sum_i S_4^{(new)}(4_i^+) = 0.74$$

- Predicted but not observed states
- Missing 1^- (9.6 MeV) 3^- (11.6 MeV)

Interplay of clustering and nuclear superconductivity

What happens if all paring matrix elements are rescaled?





Detailed shell model analysis of ^{10}Be and experimental data

<table>
<tr><th>J^π_s</th><th>S_l</th><th>E_x^{th}</th><th>Γ_α^{th}</th><th>E_x^{exp}</th><th>Γ_α^{exp}</th><th>$\theta^2_\alpha(r_1)$</th><th>$\theta^2_\alpha(r_2)$</th></tr>
<tr><td>0^+_1</td><td>0.686</td><td>0.000</td><td></td><td>0</td><td></td><td></td><td></td></tr>
<tr><td>2^+_1</td><td>0.563</td><td>3.330</td><td></td><td>3.368</td><td></td><td></td><td></td></tr>
<tr><td>0^+_2</td><td>0.095</td><td>4.244</td><td></td><td>6.197</td><td></td><td></td><td></td></tr>
<tr><td>2^+_2</td><td>0.049</td><td>5.741</td><td></td><td>5.958</td><td></td><td></td><td></td></tr>
<tr><td>2^+_3</td><td>0.052</td><td>6.123</td><td></td><td>(a)</td><td></td><td></td><td></td></tr>
<tr><td>1^-_1</td><td>0.027</td><td>6.290</td><td></td><td>5.96</td><td></td><td></td><td></td></tr>
<tr><td>3^-_1</td><td>0.098</td><td>6.926</td><td></td><td>7.371</td><td></td><td>$0.42^{(b,c)}$</td><td></td></tr>
<tr><td>2^+_4</td><td>0.116</td><td>7.650</td><td>$3\cdot10^{-4}$</td><td>7.542</td><td>$5\cdot10^{-4}$</td><td>$1.1^{(b,c)}$</td><td>0.19</td></tr>
<tr><td>0^+_3</td><td>0.023</td><td>8.068</td><td>17</td><td></td><td></td><td></td><td></td></tr>
<tr><td>4^+_1</td><td>0.049</td><td>8.933</td><td>4.7</td><td></td><td></td><td></td><td></td></tr>
<tr><td>1^-_2</td><td>0.045</td><td>9.755</td><td>180</td><td>10.57</td><td></td><td></td><td></td></tr>
<tr><td>3^-_2</td><td>0.046</td><td>9.897</td><td>61</td><td></td><td></td><td></td><td></td></tr>
<tr><td>2^+_5</td><td>0.027</td><td>10.819</td><td>50</td><td rowspan="2">9.56</td><td rowspan="2">$141^{(e)}$</td><td rowspan="2"></td><td rowspan="2">0.074</td></tr>
<tr><td>2^+_6</td><td>0.023</td><td>11.295</td><td>43</td></tr>
<tr><td>0^+_5</td><td>0.153</td><td>11.403</td><td>800</td><td></td><td></td><td></td><td></td></tr>
<tr><td>4^+_2</td><td>0.370</td><td>11.426</td><td>180</td><td>10.15</td><td>$185^{(c)}$</td><td>$1.5^{(c)}$</td><td>0.38</td></tr>
<tr><td>5^-_1</td><td>0.148</td><td>11.440</td><td>150</td><td>11.93</td><td>200</td><td></td><td>0.20</td></tr>
<tr><td>1^-_5</td><td>0.013</td><td>12.650</td><td>76</td><td></td><td></td><td></td><td></td></tr>
<tr><td>6^+_1</td><td>0.013</td><td>13.134</td><td>24</td><td rowspan="3">$13.54^{(c,f)}$</td><td rowspan="3">99</td><td rowspan="3">$1.0^{(c)}$</td><td rowspan="3">0.051</td></tr>
<tr><td>5^-_2</td><td>0.128</td><td>13.545</td><td>250</td></tr>
<tr><td>2^+_{10}</td><td>0.040</td><td>13.789</td><td>240</td></tr>
<tr><td>4^+_3</td><td>0.011</td><td>13.992</td><td>20</td><td rowspan="2">11.76</td><td rowspan="2">121</td><td rowspan="2"></td><td rowspan="2">0.066</td></tr>
<tr><td>4^+_4</td><td>0.022</td><td>14.233</td><td>40</td></tr>
<tr><td>0^+_6</td><td>0.018</td><td>14.252</td><td>120</td><td></td><td></td><td></td><td></td></tr>
<tr><td>3^-_7</td><td>0.014</td><td>14.468</td><td>77</td><td></td><td></td><td></td><td></td></tr>
<tr><td>5^-_3</td><td>0.059</td><td>14.992</td><td>180</td><td></td><td></td><td></td><td></td></tr>
<tr><td>4^+_5</td><td>0.161</td><td>15.071</td><td>800</td><td>$15.3(6^-)^{(d)}$</td><td>$800^{(e)}$</td><td></td><td>0.16</td></tr>
</table>

$^{(a)}$ The existence of this state is suggested by the existence of 8.070 MeV isobaric analog state in ^{10}B, see analogous discussion in Ref. [20];
$^{(b)}$ Widths deduced from the isobaric analog channel $^{10}\text{B} \to {}^6\text{Li}(0^+)+\alpha$ [21,22];
$^{(c)}$ results from Ref. [22];
$^{(d)}$ results from Ref. [23].
$^{(e)}$ Total width Γ^{tot}.
$^{(f)}$ In Ref. [22] the state was assigned spin-parity 6^+.

r_1=4.77 r_2=6.0 fm

[21]A. N. Kuchera et al.: Phys. Rev. C 84 (2011) 054615.
[22] A. N. Kuchera http://diginole.lib.fsu.edu/etd/8585/
[23] D. R. Tilley et al.: Nucl. Phys. A 745 (2004) 155.

No-core shell model studies with JISP16 Hamiltonian, clustering in ^{10}Be

	psd	N	N	N	N	Exp
SF	0.686	0.713	0.622	0.609	0.687	0.55 [2]
operators	3	1	7	7	20	
Radius [fm]		3.4	4.0	4.0	4.5	4.7-6

$$r \approx \sqrt{\frac{\hbar}{m\omega}\left(n_{\max} + \frac{3}{2}\right)} \qquad \hbar\omega = 20 \text{ MeV}$$

In order to get to r=6 fm, n_{max}=16 is needed, relative to core this is N_{max}=10, 14

[1] P. Maris and J. Vary, Int. J. Mod. Phys. E 22, 1330016 (2013); J. Vary private communication
[2] W. Oelert, Few-body Problems, Volume 3; Volume 1985 By E. Hadjimichael, W. Oelert, see page 252

Summary, conclusions, and outlook

Results

- New look at cluster structure: from shell model to large scale configuration interaction method.
- Unified picture that includes transitional one-body observables, EM properties, and clustering.
- Demonstrated first results: good agreement with experiment and previous studies.
- Clustering and nuclear Hamiltonian.

Future studies

- Further studies and comparisons with data.
- no-core approach; do-neutron, clusters
- Extensions and modifications
  - Different oscillator frequencies, non-trivial ($0s^4$) state of alpha.
  - Full RGM/GCM approach
  - Non-orthogonal second quantization basis
- Overlapping resonances and physics of configuration interactions in continuum of reaction states.

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Publications:

A. Volya and Y. M. Tchuvil'sky, Phys.Rev.C 91, 044319 (2015); J. Phys. Conf. Ser. 569, 012054 (2014); (World Scientific, 2014), p. 215.
M. L. Avila, G. V. Rogachev, V. Z. Goldberg, E. D. Johnson, K. W. Kemper, Y. M. Tchuvil'sky, and A. Volya, Phys. Rev. C 90, 024327 (2014).