Homework and Questions

1 Readings

1. Universal Nuclear Energy Density Functional: Computing Atomic Nuclei

2 Nuclear masses

1. Write a program to read aaa/app/mass/aud11.dat. Use this to calculate the alpha decay Q value for all nuclei with measured masses.

2. With the results of (1) make a figure for Q_α vs N for all Z values. Make a 3d figure for Q_α vs (N,Z).

3. Use the liquid drop model to find the neutron drip lines N_d for for Z values up to 120. Make a figure for N_d vs Z. Compare with experiment in the figure.

4. Use the material in aaa/app/drop to make fits of the liquid drop model type formulas to the experimental binding energies (aud11.dat). The fit should depend on smooth functionals of N and Z.

5. Use the liquid-drop model to find the binding energy difference between ^{48}Ni (Z=28) and ^{48}Ca (Z=20)?

3 Rms charge radii

1. Derive Eqs. 3.6 and 3.8.

2. Muonic atoms are formed when a μ^- lepton is captured by a nucleus. It goes into electron-type orbitals. But due to its mass these orbitals have a large overlap with the nucleus and the transition energies can be used to deduce properties of the charge density. Calculate the probability that a muon in the $0s$ state is inside the spherical radius $R = 1.2A^{1/3}$ fm for a nucleus. Evaluate the number for $Z = 70$ and $A = 200$.

4 Charge densities and form factors

1. Derive the expression in Eq. 3 going from the 3-d integral to the 1-d integral over r.

2. Find the small q expansion for the right-hand side of Eq. 3 and use this to derive a relationship between $F(q)$ and rms radius.

3. Use the small q expansion to find the rms charge radius for ^{40}Ca from the data in Fig. 4.4.

4. The proton charge density is sometimes described by the exponential form $\rho(r) = \rho_o e^{-q_o r}$. Derive the form factor and show that it does not have any minima. What is the value of q_o needed to give an rms radius of 0.88 fm.

5. Derive $|F|^2$ for the density distribution $\rho(r) = \rho_o$ for $r \leq R$ and $\rho(r) = 0$ for $r > R$. Use the first minumim of $|F|^2$ together with the data for ^{40}Ca in Fig. 4.4 to obtain a value for R.

6. In the liquid drop model the rms radii of protons and neutrons are assumed to be equal. If the interior density is 0.16 nucleons/fm^3 what is the interior density for neutrons in ^{40}Ca ($Z = 20$) and ^{208}Pb ($Z = 82$)? If the rms radius for ^{208}Pb is decreased by 2% how would the interior density for neutrons change?

7. Use Eq. 8 to estimate the number of nucleons in the "surface" region of ^{208}Pb, where "surface" is defined as those beyond where the density is 90% or less of interior density.

5 Angular momentum and tensor algebra

1. An opertor $P_>$ projects out the wavefunction with $m_\ell = 3$. Find the numerical value for $< f_{7/2}, m = 7/2 \mid P_> \mid f_{7/2}, m = 7/2 >$ and
$< f_{7/2}, m = 5/2 \mid P_> \mid f_{7/2}, m = 5/2 >$. You may use the program prs.

2. Use the Wigner-Eckhart theorem to derive Eqs. 5.67 and 5.68.

3. Obtain expressions for $< \mid\mid \mid\mid >_B$ of Eqs. 5.67 and 5.68 by using the alternative definition of the reduced matrix element given by Eq. 81.

4. By taking the Hermitian conjugate of both sides of Eqs. 5.57 and 5.58 show that $\tilde{T}$ in Eq. 60 is a tensor of rank λ.

5. Show that S in Eq. 71 obeys Eq. 57.

6. Show that boths sides of Eq. 72 are the same.

7. Find the numerical values for $< s_{1/2}||Y^{(2)}||s_{1/2} >$, $< d_{5/2}||Y^{(2)}||d_{5/2} >$, $< d_{5/2}||Y^{(2)}||d_{3/2} >$, $< d_{3/2}||Y^{(2)}||d_{5/2} >$ and $< d_{3/2}||Y^{(2)}||d_{3/2} >$. You may use the program prs.

6 Overview of nuclear decay modes

1. Use Eq. 17 to calculate the half-life for the alpha decay of ^{212}Po. What would you deduce about the preformation probability? Use Eq. 18 to calculate the half-life for ^{212}Po and ^{208}Pb.

2. Wite a program that uses Eq. 17 to convert the experiment Q_α values to $T_{1/2}$. Use $R_t = 1.2A_d^{1/3} + 2.15$. Make a figure for $\log_{10} T_{1/2}$ vs N. Find at least five experimental values of $T_{1/2}$ of alpha decay to compare with these results.

3. Use the Wigner-Eckhart theorem to show that the matrix element $|< J_f, M_f \mid \hat{O}(\pi\lambda)_\mu \mid J_i, M_i >|^2$ summed over μ and M_f gives the result of Eq. 66.

4. For a gamma transition of 4^+ to 3^- which of the following types of transitions are allowed values for π and λ for the operator $\hat{O}(\pi\lambda)$.

5. The Coulomb excitation of a $J^\pi=1^-$ level at 10 MeV in ^{208}Pb measures B(E1)(0^+ to 1^-) to be 0.10 e^2 fm^2. It gamma decays to the ^{208}Pb ground state and 2^+ state at 4.086 MeV with relative intensitives of 100 and 50, respectively. What is the width of this level in eV.

6. Compare the result of Eq. 64 with what you would get for classical electric dipole radiation of frequency ω where the rate is given $T = P/\hbar\omega$ where P is the power.

7. The the lowest $5/2^-$ in ^{51}V decays to the the $7/2^-$ ground state. The lifetime of the $5/2^-$ state is $T_{1/2}$=184(6) ps and the mixing ratio is δ=+0.48(5). Find the $B(M1)$ and $B(E2)$ values and their errors. What are the results in Weisskopf units.

8. In ^{26}Mg there is a 2^+ state at 2.938 MeV with a mean lifetime of 203(12)ps that decays to a 2^+ state at 1.809 MeV with a branching ratio of 90.3(6)% and a mixing ratio of $-0.12(2)$. Find the B(M1) and B(E2) values with their associated experimental errors in units of μ_N^2 and e^{2}fm^4, respectively. Convert the B values to Weiskopf units.

9. The lowest 4^+ state in ^{50}Ti can gamma decay by L=2 to the 2^+ state and L=4 to the 0^+ ground state. Assume that B(E2)=20 Wu and B(E4)=10 Wu. Calculate the partial mean lifetime for E2 and E4 decay, the branching ratios for these and the total mean lifetime. You will need to look up the experimental energies for these states.

10. For the neutron beta decay, B(F)=1 and B(GT)=3. Find the logft value. Calculate the phase space factor f and use this to find $t_{1/2}$ for the neutron. Compare with the experimental half life.

11. ^{18}Ne beta decays to a 0^+ state in ^{18}F with B(F) = 2. Calculate the partial half life for this decay. Compare with experiment.

12. Make a figure of $Q_{\beta\beta}$ vs A for the positive Q values for double beta-minus decay that cannot single-beta decay.

13. For the fission of ^{252}Cf into two nuclei, what pair of nuclei have the largest Q value? What is the Q value? How many different combinations of two final nuclei have a positive Q value? Find some experimental information on the fission of ^{252}Cf to compare with your results.

14. Two-proton decay was first observed for ^{45}Fe by M. Pfutzner et al., Eur. Phys. J. **A14**, 279 (2002) and J. Giovinazzo et al., Phys. Rev. Lett. **89**, 102501 (2002). Why is ^{45}Fe the first nucleus found to have two-proton radioactivity? What other modes of decay might compete with two-proton decay.

7 The Fermi gas model

1. In the fermi-gas model suppose that the nucleons had spin s=3/2. With the same interior density of 0.16 nucleons/fm^3 what is the fermi energy?

2. In the fermi-gas model suppose that there were three types of spin 1/2 nucleons: protons (p), neutrons (n) and trions (t). Find an expression for the fermi momentum for symmetric nuclear matter (equal numbers of p, n and t) in terms of the nucleon density ρ_0.

3. Derive Eq. 26 from Eq. 25.

4. Use the liquid drop model to calculate E/N for neutron matter. Is neutron matter stable? Use the fermi gas model to calculate the kinetic energy contribution to E/N for neutron matter. Put these together to obtain the potential energy contribution.

8 Overview of the nuclear shell model

9 The one-body potential

1. Given the j-coupled wavefunction in the form 9.10, what is the probability to find $(\ell = 1, m)$ in the state with $(j = 3/2, m = 1/2)$ for all possible values of m?

2. For the state $\mid 0g_{7/2}, m = 5/2 >$ what are the eigenvalues of the operators ℓ^2, s^2, j^2 and $\vec{\ell}\cdot\vec{j}$?

3. Using the information above Eq. 29, find the neutron skin (R_n-R_p) for ^{208}Pb required to make $\hbar\omega_n$=$\hbar\omega_p$.

4. Calculate the reduced matrix element $< 0d_{3/2}||r^2Y^{(2)}||0d_{3/2} >$ by using harmonic-oscillator radial wave functions with $\hbar\omega = 14$ MeV.

5. Starting with the kinetic energy in the form of Eq. 30 derive Eqs. 9.37 and 9.38.

6. Use Eq. 34 to prove Eq. 47.

7. Use the harmonic-oscillator model to obtain values of $\hbar\omega$ for ^{16}O and ^{28}Si by matching to the experimental rms charge radii of 2.70 fm and 3.12 fm, respectively. Compare with Eq. 29.

8. Use the values of $\hbar\omega$ from the previous problem to calculate the total kinetic energy for ^{16}O and ^{28}Si in the oscillator model. Compare to the fermi gas model.

9. Write the harnmonic-oscillator Hamiltonian $T + V$ for two particles with $V(r) = \frac{1}{2}m\omega^2r^2$, in terms of the relative and center of mass coordinates.

10. In the harmonic-oscillator model with $\hbar\omega$=14 MeV, what is the total kinetic energy for ^{22}Si (Z=14)? What is the center-of-mass correction to the energy of ^{22}Si?

11. Write out the explicit form of the harmonic-oscillator radial wave functions for $0s$, $0p$ and $0d$.

12. Wite a program to calculate the momemtum space forms $F(q)$ for the harmonic-oscillator radial wave functions for $0s$, $0p$ and $0d$.

10 The Woods-Saxon potential

1. The proton orbitals near the fermi surface in ^{132}Sn are $(n, \ell_j) = 1p_{1/2}$, $0g_{9/2}$, $0g_{7/2}$, $1d_{5/2}$ and $2s_{1/2}$. Make a sketch of the $R(r)/r$ for these five orbitals for a typical Woods-Saxon potential and discuss the features related to the quantum numbers n, ℓ and j.

2. Use the standard set of Woods-Saxon parmaters to find proton and neutron single-particle energies for the $0s_{1/2}$, $0p_{3/2}$, $0p_{1/2}$, $0d_{5/2}$, $0d_{3/2}$ and $1s_{1/2}$ orbitals in ^{16}O. If the orbtial is unbound find the centroid energy and decay width from the resonance calculations. (You may use /aaa/app/wspot).

3. Use wspot to make figures for the radial potential for $0d_{5/2}$ protons and neutrons in a core of ^{16}O (include the centrifugal and Coulomb terms). Do the same for $1s_{1/2}$ protons and neutrons

4. The neutron "skin" can be defined as the difference between the neutron and proton rms radii. Use wspot to calculate the skin for the oxygen isotopes ^{14,16,22,24}O. Do you understand why it goes from negative to positive?

5. Use the standard set of Woods-Saxon parmaters to find the rms charge radii for ^{16}O and ^{28}Si. Find the total kinetic energy for ^{16}O and ^{28}Si. (You may use /aaa/app/wspot).

6. A single-particle "halo" state might be defined as one in which the rms radius of the orbit is more than twice that of the core. Make a figure for the rms radius for the $1s_{1/2}$ and $0d_{5/2}$ orbits for a ^{10}Be core as a function of their single-particle energy by varying V_N. What is the minimum value of the single-particle energy for the $1s_{1/2}$ to be a "halo" state. Take the rms radius for the core neutrons to be 2.5 fm.

7. The nucleus ^{25}O which lies outside the drip line has recently been studied in PRL 100, 152502 (2008). Read this paper to find how this nucleus was produced and what was observed. Using the standard set of Woods-Saxon parameters in the wspot code, calculate the energies of the $0d_{5/2}$, $1s_{1/2}$ and $0d_{3/2}$ single-particle states for ^{24}O. If the state is unbound you should calculate its resonance energy and width. Compare with the results of the experimental paper.

8. Calculate the width of a single-particle $d_{5/2}$ resonance with the Woods-Saxon parameters for ^{22}O as a function of the neutron decay Q value in the range of Q from 0.05 to 2.00 MeV by varying the well depth V_N. Compare with the experimental results from PRL99, 112501 (2007).

9. Use the Woods-Saxon model with the standard parameter set to calculate the difference in single-particle energy between the proton and neutron $0d_{5/2}$ states assuming the same ^{16}O core for both. This is approximately the b coefficient of the IMME for the A=17, T=1/2 ground state (which has spin $5/2^+$). Find the experimental value for this b coefficient to compare with the Woods-Saxon result.

10. Use the radial wavefunction for the $0d_{5/2}$ neutron orbit in the above example, to calculate the energy of the $0d_{5/2}$ proton orbit in 1st-order pertubation theory, assuming that the change in potential only comes from the Coulomb potential. (This will require a change in the fortran program wspot.)

11. In ^{11}Be there is an unbound level at an excitation energy of 1.78 MeV that is probably a $5/2^+$ state. Use the Woods-Saxon model to calculate the width of this state assuming that it is represented by a $0d_{5/2}$ single-neutron state outside of a ^{10}Be core. Compare with the experimental width. Compare the calculated resonance shape with that obtained from Eq. 6.

12. A state at 770 keV in ^{8}B proton decays to the ground state of ^{7}Be. Look up the spins and parities for these states and the Q value for the decay. What are the possible (ℓ, j) values for the single-particle decay? Calculate the single-particle decay width of this state and compare with experiment.

13. Calculate the single-particle decay width for the ground state of ^{7}He and compare with experiment.

11 The general many-body problem for fermions

1. Prove Eq. 22.

12 Conserved quantum numbers

1. For $A = 6$ and $T_z = 1$, what are all possible T values?

2. Use the binding energy table and the energy level data to find the experimental a, b and c coefficients of the IMME for the A=18, $J^\pi=0^+$, T=1 state (be careful to consider only the states with $T = 1$). Compare to what you get from the liquid drop model.

3. Use the binding energy table and the energy level data to find the experimental a, b and c coefficients of the IMME for the A=38, $J^\pi=0^+$, T=1 state (be careful to consider only the states with $T = 1$). Compare to what you get from the liquid drop model.

4. Use the isospin Wigner-Eckhart theorem to show that the Coulomb interaction evaluated in first-order perturbation theory $< \Psi \mid V_c \mid \Psi >$, where the wavefunction $\mid \Psi >$ has good isospin, gives an IMME that is at most quadratic in T_z.

5. Use the liquid-drop model to obtain the b and c coefficients of the IMME for A=48 with $R = 3.0$ fm. Use this to find the binding energy difference between ^{48}Ni (Z=28) and ^{48}Ca (Z=20)?

6. Use the liquid-drop model to obtain the excitation energy of the lowest $T = 4$ state in ^{48}Ti (Z=22) relative to its $T = 2$ ground state?

13 Quantum numbers for the two nucleon system

1. What are all possible (ℓ, S, T) values for each of the following:
   a) a $J^\pi = 3^-$ resonance of the two-neutron system.
   b) a $J^\pi = 2^-$ resonance of the two-nucleon system with $T_z = 0$.
   c) a $J^\pi = 2^+$ resonance of the two-nucleon system with $T_z = 0$.

14 The Hartree-Fock approximation

1. Use Eqs. 14.3 and 14.6 to find the experimental single-particle energies for the $0p_{3/2}$, $0p_{1/2}$, $0d_{5/2}$, $1s_{1/2}$ and $0d_{3/2}$ protons and neutrons in ^{16}O. Assume that ^{16}O has a closed-shell configuration. Compare the experimental with the output of wspot.for with the standard set of parameters ($V_N = 1$). What part of this comparison is a test of the spin-orbit part of the potential?

2. Use the experimental results from the last question together with $\epsilon_{0s_{1/2}\text{proton}} = -45$ MeV and $\epsilon_{0s_{1/2}\text{neutron}} = -48$ MeV, together with the kinetic energies obtained with the Woods-Saxon model to calculate the last line of Eq. 17. Compare with the experimental binding energy of ^{16}O.

3. Use Eqs. 14.3 and 14.6 to find the experimental single-particle energies for the proton $0d_{3/2}$, $1s_{1/2}$ and $0f_{7/2}$ orbitals for ^{48}Ca. Compare with the results of the Woods-Saxon model and with the Skyrme Hartree-Fock model (with Skx). Here's how to do the Skx calculation. The results are in the file dens.dao:

```
dens
az
48,20
cp          (change the potential)
sk20        (the skx interaction - type h here to get the full list)
gd          (calculate the ground state densities)
st          (stop)
```

4. Use the Skyrme Hartree-Fock model (with Skx) to find the binding energy and kinetic energy of ^{28}Si. Compare the kinetic energy with that obtained with the harmonic oscillator and Woods-Saxon models.

5. For neutrons show that

$$\frac{a_1}{2}\sum_{\alpha,\beta} < \alpha\beta \mid \delta(\vec{r}_1 - \vec{r}_2)(1 - P_\sigma) \mid \alpha\beta >= \frac{a_1}{4}\int \rho^2 \, d\tau,$$

where ρ is the neutron density

$$\rho = \sum_j \rho_j,$$

and ρ_j is the density for each filled orbital given by

$$\rho_j = \frac{(2j+1)}{4\pi}\frac{R_j^2(r)}{r^2}.$$

6. Read this recent paper on Nuclear bubble structure in ^{34}Si; Phys. Rev. **C 79**, 034318 (2009). Use Skx and Sly4 to calculate the proton densities for for ^{34}Si and ^{36}S and compare with the results in this paper. Explain why this happens? How could it be measured? Can you think of other cases for "bubble nuclei"? The dens input for ^{34}Si is:

```
dens
az
34,14
cp          (change the potential)
sk20        (the skx interaction - type h here to get the full list)
gd          (calculate the ground state densities)
pl
1,1,,,si43p    (this makes on output file si34p.plt for the proton density)
st          (stop)
```

7. Starting from Eq. 25 show that kinetic energy density for an orbital with quantum numbers $k = (n, \ell, j)$ with all of the m states filled is:

$$\tau_k(r) = \frac{1}{4\pi}(2j+1)[(du_k/dr)^2 + \ell(l+1)\frac{u_k^2}{r^2}]$$

where $u_k(r) = R_k(r)/r$. Hint: first show that

$$\nabla^2\rho_k(r) = 2\sum_m \phi_{k,m}\nabla^2\phi_{k,m} + 2\tau_k(r)$$

8. Calculate the propperties of ^{208}Pb with the Skx Skyrme interaction. Compare the BE and rms charge radius with experiment. Compare the single-particle energies for the proton $2s_{1/2}$, proton $0h_{9/2}$, neutron $2p_{3/2}$ and neutron $1g_{9/2}$ orbitals (n,ℓ,j) with experiment. (Note the quantum number n_{dens} in the dens.dao output file is n_{dens}=n+1.)

9. Use Skx HF to find the neutron skin for ^{14,16,22,24}O.

10. Extend Eq. 1 and 2 to include contributions from three-body interactions.

15 Single-particle electromagnetic moments

1. Use Eq. 12 to find the equation for the quadrupole moment in the state $j = \ell - 1/2$. Does it agree with Eq. 26.

2. Calculate the magnetic moment and quadrupole moments of ^{17}F and ^{17}O. Compare with experiment (from Ref. 1 in Chapter 15). From the comparison of quadrupole moments what are the proton and neutron effective charges.

3. Calculate the magnetic moments for the ground states of ^{207}Pb, ^{209}Pb, ^{207}Tl and ^{209}Bi. Assume that ^{208}Pb has a closed-shell configuration. Compare with experiment.

4. Calculate the quadrupole moments for the ground states of ^{207}Pb, ^{209}Pb, ^{207}Tl and ^{209}Bi. Assume that ^{208}Pb has a closed-shell configuration. Use Skyrme Skx for the radial matrix elements. Compare with experiment and discuss the effective charges that are deduced.

5. Calculate the reduced matrix element $< 0d_{3/2}||r^2Y^{(2)}||0d_{3/2} >$ with $\hbar\omega = 14$ MeV. Use this reduced matrix element with Eq. 1 to obtain the quadrupole moment for the $0d_{3/2}$ orbit. Compare with the result obtained with Eq. 24.

6. For M1 electromagnetic transitions we will need the off-diagonal reduced matrix elements $< j = l + \frac{1}{2}||\vec{\ell}||j = l - \frac{1}{2} >$ and $< j = l + \frac{1}{2}||\vec{s}||j = l - \frac{1}{2} >$. Use wavefunctions of the type given in Eq. 9 and 15.10 to calculate the matrix elements
$< j = l + \frac{1}{2}, m = l - \frac{1}{2} \mid \ell_z \mid j = l - \frac{1}{2}, m = l - \frac{1}{2} >$ and
$< j = l + \frac{1}{2}, m = l - \frac{1}{2} \mid s_z \mid j = l - \frac{1}{2}, m = l - \frac{1}{2} >$. Then use the Wigner-Eckhart theorem to find the reduced matrix elements.

7. Calculate the quadrupole moments for ^{17}F and ^{17}O with Woods-Saxon radial wavefunctions. (Use $V_N = 1.03$ in wspot.for to make $0d_{5/2}$ proton orbit have about the correct single-particle energy.) Compare with experiment. Calculate the quadrupole moment for the ground state of ^{39}K asssuming a single-hole configuration and with Woods-Saxon radial wavefunctions (use the default value of V_N). Compare with experiment.

8. Calculate the magnetic and quadrupole moments for ^{41}Ca and compare with experiment. Use Woods-Saxon radial wavefunctions.

16 The creation operator method

1. Use the second quantization method to reduce the following many-particle matrix elements to a sum of single-particle matrix elements (a, b, c, d) represent different occupied m-states).
$< a, b \mid \hat{O} \mid a, b >$
$< a, b \mid \hat{O} \mid a, c >$
$< a, b \mid \hat{O} | c, d >$

$< a, b, c|\hat{O} \mid a, b, c >$
$< a, b, c|\hat{O} \mid d, b, c >$

2. Use the second quantization method to reduce the following many-particle matrix elements to a sum of single-particle matrix elements for the orbitals (a, b, c, d). (1111) means they are all filled, etc.
   $< (1100) \mid \hat{F} \mid (1100) >$
   $< (1100) \mid \hat{F} \mid (1010) >$
   $< (1100) \mid \hat{F} \mid (1001) >$

3. Derive Eq. 59 from Eqs. 31, 32, 57 and 58.

4. Derive Eq. 62.

5. Derive the commutation relation

$$[S, S^+] = 1 - \hat{n}/\Omega$$

   where $\hat{n}$ is the number operator, $\Omega = j + 1/2$ and S^+ is the $J = 0$ pair state

$$S^+ = \frac{1}{\sqrt{\Omega}} \sum_{m>0} (-1)^{j+m} a_m^+ a_{-m}^+$$

17 Core polarization

1. Assume that ^{57}Ni has the configuration $(0f_{7/2})^{16}(1p_{3/2})$. (A) Which particle-hole configurations will enter into the first-order core-polarization correction to the magnetic moment? (B) to the quadrupole moment?

2. For the magnetic moment of the $9/2^+$ ground state of ^{209}Bi in a model space in which ^{208}Pb has a doubly closed shell, what orbitals will enter into the first-order core-polarization correction diagram?

18 Many-body wave functions

1. Find the allowed J values for:
   a) (neutron $0f_{7/2}$)^2
   b) (proton $0f_{7/2}$)$^2 \otimes$ (neutron $0f_{7/2}$)
   c) (neutron $0f_{7/2}$)^6
   d) (neutron $0f_{7/2}$) $\otimes$ (neutron $0p_{3/2}$)

2. Find the allowed (J, T) values for:
   a) (neutron $0f_{7/2}$)^2
   b) (nucleon $0f_{7/2}$)^2, T_z=0
   c) (nucleon $0f_{7/2}$)^{16}
   d) (nucleon $0f_{7/2}$)^8, T_z=3

3. (A) What are the allowed J, T values for $(0p_{3/2})^4$ and their dimensions $D(J,T)$. (B) For the configuation $(0p_{3/2})^4$ $M = 2$ and $T_z = 0$ what is the M-scheme dimension (e.g. what is the number of M-scheme basis states).

4. Find the allowed J values for three protons in the $0f_{7/2}$ orbit. Compare with experiment for ^{51}V and ^{53}Mn. Are all of the expected states accounted for? What is the first non-expected level in each?

5. What are all of the allowed J values (and their dimensions) for ^{44}Sc in the k^4 model with $k = 0f_{7/2}$ and a ^{40}Ca core. How many of these have T=0, 1 and 2.

6. Consider a model space $0f_{7/2}$ and $1p_{3/2}$. Make a list of all possible partitions for six neutrons. Assume that ^{40}Ca is a closed core to find the experimental single-particle energies from the properties of ^{41}Ca. Use these single-particle energies to calculate the lowest energy and highest energy for six neutrons (^{46}Ca) based on the complete set of partitions. What is the highest possible J value allowed by this model space for six neutrons?

7. What are the J_{max} values for the configurations (proton $g_{9/2})^n$ for n=1 to n=10?

8. What are the allowed J, T values for $(0d_{5/2})^3$ and their dimensions $D(J,T)$.

9. Show the details in getting the results of Eq. 49.

19 The two-body Hamiltonian

1. Show that Eq. 14 can be written in the form of Eqs. 18, 21 and 24.

2. A velocity-independent two-body potential is usually written as a linear combination of the operators (a) $f_0(r)$, (b) $f_1(r)\vec{\sigma}_1 \cdot \vec{\sigma}_2$ and (c) $f_2(r)S_{12}$. Show that the following operators are not independent of these three: $\vec{S} \cdot \vec{S}$ and $(\vec{r} \cdot \vec{S})^2$.

3. Why is $(\vec{r} \cdot \vec{S})$ not allowed for the potential.

4. With the addition of a velocity-denpendent potential one usually adds the operator, (d) $f_{LS}(r)\vec{L} \cdot \vec{S}$. Show that the following operators are not independent of (a), (b), (c) and (d): p^2, L^2, $(\vec{L} \cdot \vec{S})^2$ and $\vec{r} \times \vec{L} \cdot \vec{p}$.

5. Calculate analytical expressions for the matrix elements of $< S||X^\lambda||S >$ where the X are given by Eqs. 19.22-19.24.

6. The interaction between nucleons is similar to that between two point dipoles: $(\vec{\sigma}_1 \cdot \vec{\nabla}_1)(\vec{\sigma}_2 \cdot \vec{\nabla}_2)f(r)$. For meson exchange we may take the radial dependent to be $f(r) = \frac{e^{-r/r_o}}{(r/r_o)}$. The total potential for one-pion exhcnage has the form:

$$V(r) = -g^2(\vec{\sigma}_1 \cdot \vec{\nabla}_1)(\vec{\sigma}_2 \cdot \vec{\nabla}_2)\frac{e^{-r/r_o}}{r/r_o},$$

where g is the pion-nucleon coupling constant. Show that this can be written in the form of Eq. 44.

20 Hamiltonians associted with specific model spaces

21 Applications of two-body interactions

1. Use the binding energies for the lowest states with $J^{\pi}=0^{+}$ to 7^{+} in ^{42}Sc to deduce values for the proton-neutron two-body matrix elements (TBME) $< 0f_{7/2}, 0f_{7/2}, J \mid V \mid 0f_{7/2}, 0f_{7/2}, J >$. For the single-particle energies use the BE of ^{41}Sc, ^{41}Ca and ^{40}Ca.

2. Use the TBME from the last question with the Pandya transformation to find the energy spectrum of ^{48}Sc. Compare with experiment. Use the prs program to calculate the six-j values.

3. Assuming a pure k^2 configuration for ^{210}Po with $k = 0h_{9/2}$, find the empirical values for $< k^2 J \mid V \mid k^2 J >$ for all possible J. Use the TBME program to calculate the two-body Coulomb interaction for this case and then subtract this to find the strong interaction part of $< V >$.

4. Use the results of the last problem to find the energy for ^{218}U relative to ^{208}Pb, and compare with experiment.

5. Use the $\bar{V}$ for the $0d_{5/2}$ orbit from the two-body of matrix elments of USDB to calculate the energy of ^{22}O relative to ^{16}O in the $(0d_{5/2})^6$ model. For the single-particle energy use -3.9257 MeV. Note that the USDB matrix elements in the usdb.int file are those for $A = 18$. For other A values the matrix elements scale as $(18/\mathrm{A})^{0.3}$.

6. Repeat the last question by using NuShellX. The proceedure is:

```
sd rsh      go to the rsh folder
md o22      make the the o22 folder (if it does not already exist)
sd o22      go to the o22 folder

shell       run the shell program
o22         name of the batch file
lpe         setup to calculate energies and wf
sd          *.sp file name
y           (yes/no) restrict orbit occupations
s              restrict subshell occupations
0,0            min, max for proton d3/2 orbit
0,0            min, max for proton d5/2 orbit
0,0            min, max for proton s1/2 orbit
0,0            min, max for neutron d3/2 orbit
6,6            min, max for neutron d5/2 orbit
0,0            min, max for neutron s1/2 orbit
usdb        *.int file name
8           number of protons
```

```
22          number of nucleons
0.,0.       min, max J values
0           parity
st          stop

o22         run the batch job

--- Output files
o_22b.lpt energy levels
o_22b.occ orbit occupancies
```

7. Assume that ^{48}Ca is a closed shell and that the lowest 0^+, 2^+ ,4^+ and 6^+ states in ^{50}Ti are described by the $(f_{7/2})^2$ configuration. Find the empirical values for the two-body interaction matrix elements from the energies of ^{50}Ti, ^{49}Sc and ^{48}Ca. Use Eq. 14 to calculate the interaction energy for all $(f_{7/2})^3$ configurations. Compare with the experimental spectrum of ^{51}V. Use the single-particle energy from ^{49}Sc, together with these two-body matrix elements to find the binding-energy difference between ^{56}Ni and ^{48}Ca and compare to experiment.

8. Set up the NuShellx input for the last problem and check the result for spectrum of ^{51}V. Use NuShell to calculate the spectrum of ^{52}Cr (all possible spins) and compare with experiment.

9. What is the total Coulomb interaction energy for the $(sd)^{12}$ configuration of ^{28}Si if we approximate the Coulomb potential by $V(\vec{r}) = \frac{e^2}{R}$, where R is a constant?

10. Calculate the ΔE value (relative to ^{16}O) for the lowest 4^+ state in ^{18}O in the sd-shell model using the two-body matrix elements from Table 22.1.

11. Starting with the experimental energy spectrum of ^{38}Cl, use the Pandya transformation in the $0d_{3/2}$-$1f_{7/2}$ model space to obtain the energy spectrum of ^{40}K.

22 Configuration mixing

1. Compare the result from the previous question to that obtained with the full $(sd)^6$ USDB calculation of ^{22}O. Compare with experiment.

2. Use USDB to obtain the spectrum of ^{25}Mg for J=$1/2^+$ to $9/2^+$ and compare with experiment. The proceedure is:

```
shell       run the shell program
mg25        name of the batch file
lpe         setup to calculate energies and wf
sd          *.sp file name
n           (yes/no) restrict orbit occupations
usdb        *.int file name
```

```
12          number of protons
25          number of nucleons
5.,4.5      min, max range of J values
0           parity
st          stop

mg25        run the batch job

--- Output files
mg25b.lpt   energy levels
mg25b.occ   occupation numbers
mg25b.eps   postscript file for comparison to experiment
```

23 One-particle transfer

1. Look up the data for single-particle transfer reactions on the ^{48}Ca target leading to the nuclei ^{47}Ca, ^{49}Ca, ^{47}K and ^{49}Sc. Which levels have spectrosopic factors which are best interpreted in terms of the the states being a single-nucleon removed from or added to a closed-shell configuration for ^{48}Ca?

2. For neutron removal from ^{34}Si, what is the C^2S sum-rule value for $\ell = 2$ and $\ell = 0$ in the sd-shell? Compare with the experimental results given in PRC65, 034318 (2002).

3. Use wavefunctions obtained for ^{24}Mg and ^{25}Mg to calculate the spectroscopic factors for adding a neutron to the ground state of ^{24}Mg to individual states in ^{25}Mg up to 4 MeV. Compare to experiment from Nucl. Phys. **A521**, page 145 (1990). In the same folder that you used for the previous problem run "shell" and get the wf for the gs of ^{24}Mg. Then to get the spectroscopic factors

```
shell       run the shell program
mg24s       name of the batch file
den         setup to calculate overlaps
1           setup to calculate one-nucleon transfer spectroscopic factors
bb4800      initial state file name (name for one of the J values)
1           number of states for each initial J
bb4901      final state file name (name for one of the J values)
5           number of states for each final J
 0.0        min, max range of initial J values (if max=0 then max=min)
 0.5, 4.5   min, max range of final J values
n           (yes/no) restrict tensor rank for operator
st          stop

mg24s       run the batch job

--- Output files
```

`mg24sb.lsf  list of spectroscopic factors`

4. In the full sd-shell model for ^{24}Mg obtain the proton removal sum rules for the $0d_{5/2}$, $1s_{1/2}$ and $0d_{3/2}$ for the USDB Hamiltonian. Compare the sum-rule results with experiment for the lowest four states in ^{23}Na.

5. In the full $(sd)^8$ model for ^{24}Mg obtain the proton addition sum rules for the $0d_{5/2}$, $1s_{1/2}$ and $0d_{3/2}$ for the USDB Hamiltonian. (Hint: use the ground-state occupations numbers). Compare with the results of the last question.

6. A radioactive beam experiment at the NSCL measures the spectroscopic factors for ^{19}O ($Z = 8$, $J^\pi = 5/2^+$) going to final states in ^{18}O. In the $(0d_{5/2})^3$ model what are the spectroscopic factors to each of the allowed final J states.

7. At the NSCL an experiment was done to knockout a neutron from ^{57}Ni leaving ^{56}Ni ($Z = 28$). ^{57}Ni has $J^\pi = 3/2^-$. (A) With a $(0f_{7/2})^{16}(1p_{3/2})$ wave function for ^{57}Ni what are the allowed final J^π, T values. (B) What are sum-rule strengths for removing a neutron from the $0f_{7/2}$ and $1p_{3/2}$ orbitals.

8. For this question assume that ^{48}Ca has a $(0f_{7/2})^8$ closed-shell configuration. (A) What is the sum-rule value for removing a $0f_{7/2}$ neutron from ^{48}Ca to make ^{47}Ca? (B) What final isospins are allowed for the $0f_{7/2}$ removal. (C) What final isospins are allowed for removal of a $0d_{3/2}$ neutron? (D) What final isospins are allowed for removal of two $0d_{3/2}$ neutrons?

9. The neutron decay of an excited $5/2^+$ state in ^{23}O was observed at the NSCL: Phys. Rev. Lett. **99**, 112501 (2007).

   http://link.aps.org/abstract/PRL/v99/e112501

   (a) What is the simplest shell-model configuration for this $5/2^+$ state?
   (b) With this simplest configuration what is the spectroscopic factor for neutron decay to the ground state of ^{22}O?
   (c) Use NushellX with the sd.sp model space and USDB interaction to calculate the energy levels of ^{23}O and the spectroscopic factor for the decay of the $5/2^+$ state.
   (c) What is the predicted neutron-decay width for the $5/2^+$ state?

10. What are the spectroscopic factors C^2S for proton removal from ^{55}Co, $7/2^-$, T=1/2 going to all possible states ^{54}Fe? What is the C^2S value for neutron removal going to the (0^+, T=1) ground state of ^{54}Co. Use the $(f_{7/2})^n$ model.

11. What is the spectroscopic factor C^2S for a proton removal from ^{54}Fe, 0^+, T=1 going to the ($7/2^-$, $11/2^-$, $15/2^-$), T=3/2 levels of ^{53}Mn? Use the $(f_{7/2})^n$ model.

12. What is the spectroscopic factor for adding a neutron to ^{44}Ca to make ^{45}Ca in the k^n model with $k = 0f_{7/2}$? What is the $(f+)$ sum-rule value?

13. What is the Σ_{f+} sum-rule for the spectroscopic factors for adding a proton to ^{22}O ($Z = 8$) to make $3/2^+$, T=5/2 states in ^{23}F in the model in which ^{22}O has a neutron closed shell $[(0\mathrm{d}_{5/2})^6(1\mathrm{s}_{1/2})^2]$ and the proton is added in the $0d_{3/2}$ orbit? (Remember that one can also go to T=7/2 states.) Express the result in terms of Clebsch-Gordan coefficients.

14. What is the spectroscopic factor for adding a neutron to ^{22}O ($Z = 8$) to make ^{23}O in the model in which ^{22}O is a closed shell and the nucleon is added in the $0d_{3/2}$ orbit.

15. A radioactive beam experiment at the NSCL measures the spectroscopic factors for ^{19}O ($Z = 8$, $J^\pi = 5/2^+$) going to final states in ^{18}O. In the k^n model with $k = 0d_{5/2}$ what are the spectroscopic factors to each of the allowed final J states.

24 Experiments related to spectroscopic factors

25 One-body transition operators and the OBTD

26 Two-particle transfer operators

27 Two-body transition density operators and the TBTD

28 Electromagnetic transitions

1. Calculate ^{18}Ne B(E2,2^+ to 0^+) with Eq. 34 in the $(0\mathrm{d}_{5/2})^2$. Does it depend on the Hamiltonian? Compare this result with the full $(\mathrm{sd})^2$ model with the USDB Hamiltonian.

2. Use Eq. 34 to obtain the $B(E2)$ values for the 6^+ to 4^+, 4^+ to 2^+ and 2^+ to 0^+ transitions in ^{50}Ti assuming a k^2 model with $k = 0f_{7/2}$. Use $\hbar\omega$=10.32 MeV. Compare with experiment (the experimental values can be found in PRC9, 1033 (1974)).

3. Calculate the magnetic moment of the ^{19}O $3/2^+$ state in the $(0\mathrm{d}_{5/2})^3$ model. Compare the OBTD and magnetic moment from this result with those obtained in the full $(\mathrm{sd})^3$ model.

4. Calculate the quadrupole moment for the ^{51}V ground state assuming a k^3 model with $k = 0f_{7/2}$. Compare with experiment. Use $\hbar\omega$=10.32 MeV. How does the result compare with the single-particle Q moment for one proton in the $0f_{7/2}$ orbit (^{49}Sc) and for one proton hole (^{55}Co) in the $0f_{7/2}$ orbit (with the same value of $\hbar\omega$).

5. Use wavefunctions obtained for ^{25}Mg to calculate the gamma decay properties. Compare to experiment from the Table of Isotopes (1996). The proceedure is:

```
shell      run the shell program
mg25g      name of the batch file
```

```
den         setup to calculate overlaps
t           setup to calculate one-body transition densities
bb4901      initial state file name (name for one of the J values)
5           number of states for each initial J
bb4901      final state file name (name for one of the J values)
5           number of states for each final J
 0.5, 4.5   min, max range of initial J values
 0.5, 4.5   min, max range of final J values
y           (yes/no) restrict tensor rank for operator
 1.0, 2.0   min, max range of tensor rank (e.g. for M1 and E2)
st          stop

mg25g        run the batch job

--- Output files
mg250.deo    gamma decay and moments
```

6. (A) Calculate the center-of-mass correction to the $E1$ operator and show that the effective charges are $\tilde{e}_k(\text{proton}) = \frac{N}{A}$ and $\tilde{e}_k(\text{neutron}) = -\frac{Z}{A}$. (B) Why are $E1$ transitions from $T_i = 0$ to $T_f = 0$ states in nuclei with $N = Z$ forbidden?

7. The nucleus ^{43}Sc has an isomeric state with J$^\pi$=19/2$^-$ whose magnetic moment is measured to be +3.15(2) μ_N. Assume that the structure of this state is $(0\text{f}_{7/2})^3$. Use the M-scheme to calculate the magnetic moment. Assuming $0\text{f}_{7/2}$ wave functions, show that $\mu[^{43}\text{Sc}(19/2^-)] = \mu[^{41}\text{Sc}(7/2^-)] + \mu[^{42}\text{Ca}(6^+)]$.

8. Calculate the $B(E4)$ for the 0^+ to 4^+ coulex excitation in ^{50}Ti assuming a $(0f_{7/2})^2$ model. Using the $B(E2)$ from question (2) calculate the branching fraction for the decay of the 4^+ state to the 0^+ ground state.

9. Calculate the $B(M1)$ value for the $5/2^-$ to $7/2^-$ transition in ^{51}V assuming a k^3 model with $k = 0f_{7/2}$.

10. Calculate the B(E2) value for the $1/2^+$ to $5/2^+$ transitions in ^{17}F and ^{17}O. Assume $2\text{s}_{1/2}$ and $1\text{d}_{5/2}$ configurations. Compare to experiment and discuss the differences.

11. Show that $< i \mid [F^+, [\vec{\nabla}^2, F]] \mid i > = - < i \mid \{\vec{\nabla}^2(FF^+)\} \mid i >$, where $F = r^\lambda Y_{\lambda,\mu}$ and you may use $\vec{\nabla}^2(F) = 0$. $\{\}$ indicates that $\vec{\nabla}^2$ does not operate on the wavefunction.

12. Use NuShellX to obtain the gamma decay scheme for levels up to 5 MeV in ^{33}S. Compare the calculated lifetimes with experment.

29 Allowed beta decay

1. Use isospin conservation to prove that B(F)=2 for the Fermi transition from ^{18}Ne 0^+ T=1 to ^{18}F 0^+ T=1. What is the value of B(GT) for this transition?

2. For the beta decay of ^{48}Ni (Z=28) to its isobaric analoque state in ^{48}Co, what are the B(F) and B(GT) values?

3. For the Gamow-Teller beta decay of ^{48}Ni to ^{48}Co in the (proton $f_{7/2}$)^8 model for ^{48}Ni and the full pf shell-model space for ^{48}Co, which orbitals will enter into the OBTD? (2) What are the allowed J^π, T values for the final states? (2) What is the Gamow-Teller sum rule for this decay?

4. Calculate the B(GT) value for the beta decay of ^{17}F to the ground state of ^{17}O. Compare with the experimental B(GT) obtained with a phase space factor of $f = 35.89$. What excited states in ^{17}O need to be considered to satisfy the GT sum rule?

5. Use NuShellX with the (sd)^2 model space and the USDB Hamiltonian to calculate the OBTD for GT transitions from the ground state of ^{18}O to all possible final states in ^{18}F. How do the results compare with the Gamow-Teller sum-rule. The proceedure is:

```
shell      run the shell program
o18b       name of batch file
lpe        setup to calculate wf for 18O
sd         model space
n          any restrictions (y/n)
usdb       interaction (*.int) name (a8)
8          number of protons
18         number of nucleons
0.0        min J, max J (if max = 0, then max=min)
0          parity (0 for +) (1 for -) (2 for both)

lpe        setup to calcualte wf for 18F
9          number of protons
18         number of nucleons
1.0        min J, max J
0          parity (0 for +) (1 for -) (2 for both)

den        setup to calculate overlaps one-body transition density
t          setup to calculate one-body transition density
bb0200     name of initial state
1          number of initial state
bb1202     name of final state
-1         number of final states (-1 indicates all)
 0.0       min, max J  for initial state
 1.0       min, max J  for final state
n          (yes/no) restrict coupling for operator
st         stop

o18b       run the batch job
```

```
--- Output files
o_180.bgt  B(GT) values
```

30 Electron scattering

31 General problems

1. (A) Derive the Nilsson model quantum numbers $[N, n_z, \Lambda], \Omega^\pi$ for the $0p$ shell. (B) What is the prolate configuration for ground state and first excited state of ^{7}Li.

2. What are the harmonic-oscillator Nilsson quantum numbers $[n_z, n_\rho, \Lambda]$ for the pf shell orbitals? (2) What are relative energies of these states (in units of the deformation parameter δ) and their degeneracies for prolate deformation?