Exercises PHY981 Spring 2013

The exercises are available at the beginning of the week and are to be handed in at the lecture the week thereafter on Tuesdays. This can be done electronically (as a pdf or postscript file) by email to hjensen@nscl.msu.edu or at the lecture. You can also send in a scanned version of your answer. Parts of the Thursday lectures will be used to discuss the weekly exercises. The exercises will be graded and count 10% of the final mark.

Exercise 1: masses and binding energies

The data on binding energies can be found in the file aud11.dat, contained in the file at the webpage mass.zip. This file has also been sent as an email to you as well.

1. Write a small program which reads in the proton and neutron numbers and the binding energies and make a plot of all neutron separation energies up to $Z = 82$ (see figure 2.9 of Alex Brown's text), that is you need to plot

$$S_n = BE(N,Z) - BE(N-1,Z),$$

and similarly, the proton separation energies

$$S_p = BE(N,Z) - BE(N,Z-1).$$

Comment your results.

2. In the same figures, you should also include the liquid drop model results of Eq. (2.17) of Alex Brown's text, namely

$$BE(N,Z) = \alpha_1 A - \alpha_2 A^{2/3} - \alpha_3 \frac{Z^2}{A^{1/3}} - \alpha_4 \frac{(N-Z)^2}{A},$$

with $\alpha_1 = 15.49$ MeV, $\alpha_2 = 17.23$ MeV, $\alpha_3 = 0.697$ MeV and $\alpha_4 = 22.6$ MeV. Comment your results.

3. Make also a plot of the binding energies as function of A using the data in the file aud11.dat and the above liquid drop model. Make a figure similar to figure 2.5 where you set the various parameters $\alpha_i = 0$. Comment your results.

4. Use the liquid drop model to find the neutron drip lines for Z values up to 120. Analyze then the oxygen and fluorine isotopes and find, where available the corresponding experimental data, and compare the liquid drop model predicition with experiment. Comment your results.

Exercise 2

Consider the Slater determinant

$$\Phi_\lambda^{AS}(x_1x_2\dots x_N;\alpha_1\alpha_2\dots\alpha_N) = \frac{1}{\sqrt{N!}}\sum_p (-)^p P\prod_{i=1}^N \psi_{\alpha_i}(x_i).$$

where P is an operator which permutes the coordinates of two particles. We have assumed here that the number of particles is the same as the number of available single-particle states, represented by the greek letters $\alpha_1\alpha_2\dots\alpha_N$.

a) Write out Φ^{AS} for $N = 3$.

b) Show that

$$\int dx_1 dx_2 \dots dx_N \left|\Phi_\lambda^{AS}(x_1x_2\dots x_N;\alpha_1\alpha_2\dots\alpha_N)\right|^2 = 1.$$

c) Define a general onebody operator $\hat{F} = \sum_i^N \hat{f}(x_i)$ and a general twobody operator $\hat{G} = \sum_{i>j}^N \hat{g}(x_i, x_j)$ with g being invariant under the interchange of the coordinates of particles i and j. Calculate the matrix elements for a two-particle Slater determinant

$$\left\langle \Phi^{AS}_{\alpha_1\alpha_2} \right| \hat{F} \left| \Phi^{AS}_{\alpha_1\alpha_2} \right\rangle ,$$

and

$$\left\langle \Phi^{AS}_{\alpha_1\alpha_2} \right| \hat{G} \left| \Phi^{AS}_{\alpha_1\alpha_2} \right\rangle .$$

Explain the short-hand notation for the Slater determinant. Which properties do you expect these operators to have in addition to an eventual permutation symmetry?