Radioactive decays

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Lecture goals:
Discuss the perspecitves of the radioactive decay studies in the experimental environment of modern facilities.
Today:
I. Basics of beta decay
II. Basics of decay experiments
Friday:
III+IV Present and future

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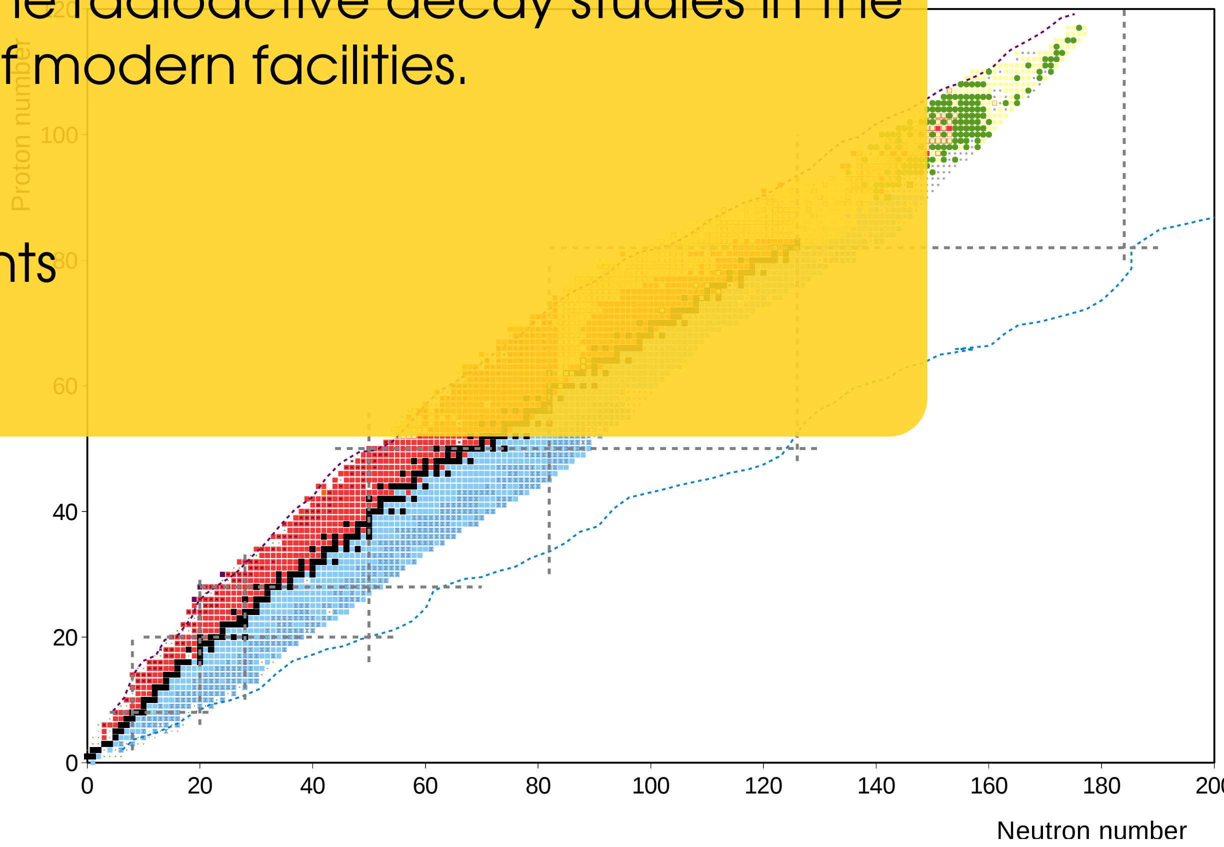
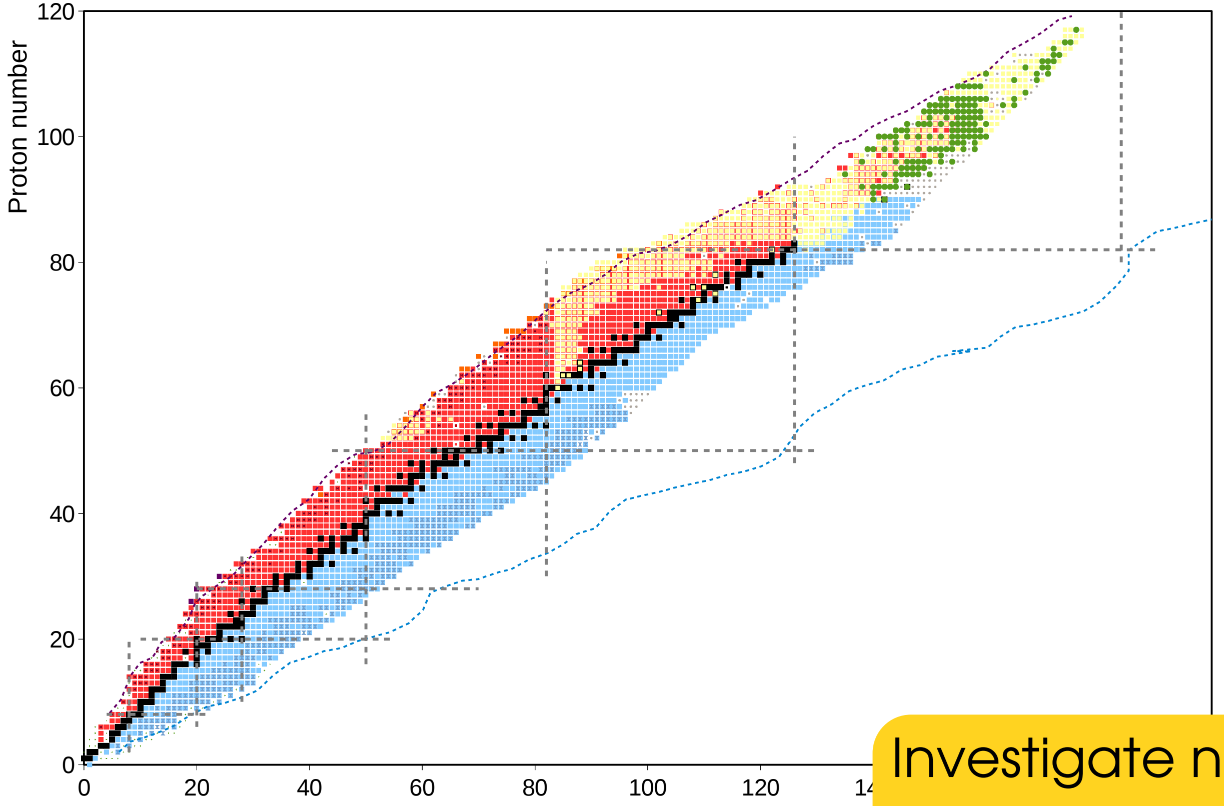


Most nuclei are radioactive !!



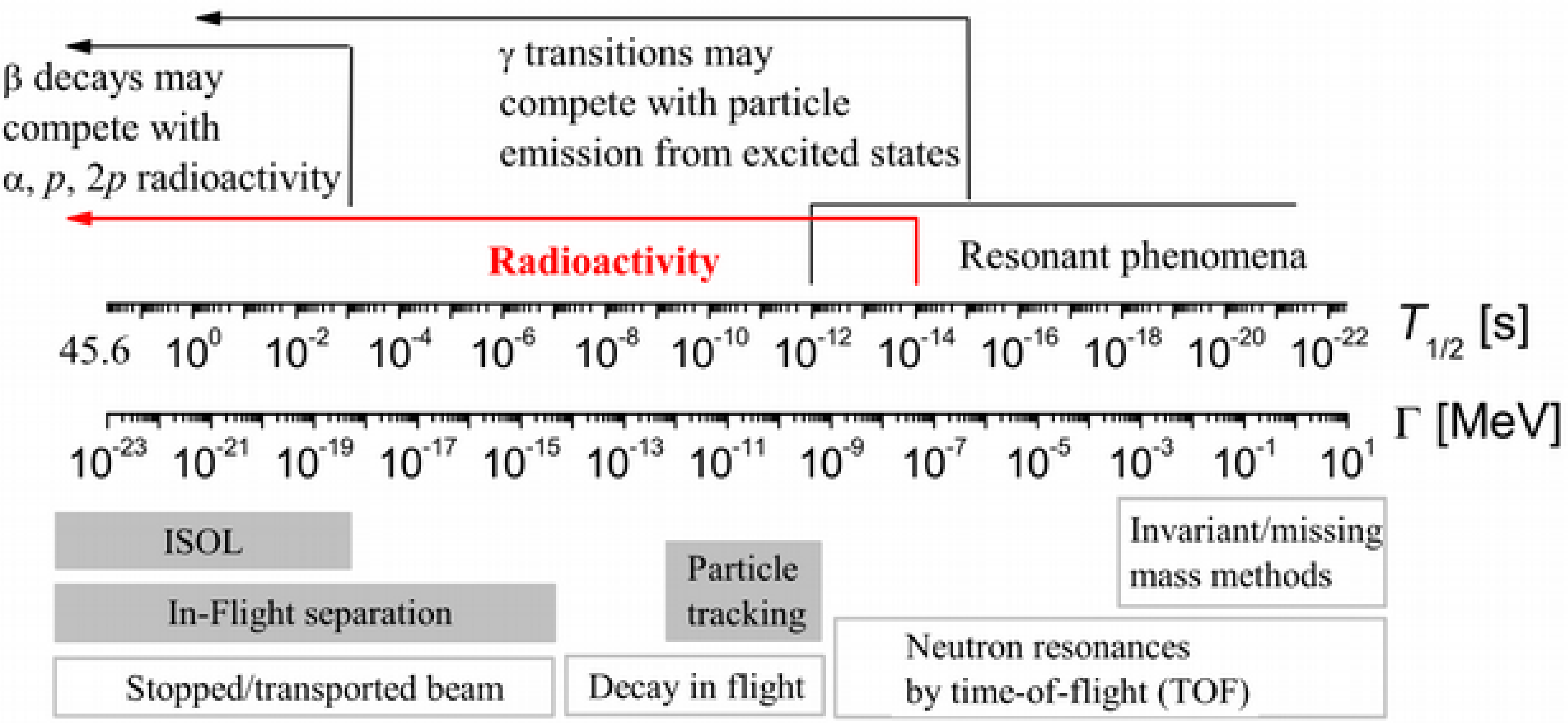
Investigate nuclei through the complete measurement of the decay properties of their ground and isomeric states.

What is radioactivity ?

The notion of radioactivity is useful in making distinction between emission of rays or particles by a highly unstable system (for example, undergoing a nuclear reaction) from radiation emitted spontaneously by a system whose **nuclear and atomic degrees of freedom are close to equilibrium.** (…) Radioactivity is a process of emission of particles by an atomic nucleus which occurs with characteristic time (half-life) much longer than the K-shell vacancy half-life in a carbon atom, which amounts to about 2×10^{-14} s (Bambynek et al., Rev. Mod. Phys. 44, 716,1972). A relativistic particle travels in the time of 2×10^{-14} s a distance of a few micrometers, which is close to the measurement limit in a nuclear emulsion. In addition, this value coincides with a decay width, defined as $\Gamma = \ln 2\, \hbar / T_{1/2}$, of about 0.03 eV which is roughly the thermal energy at room temperature. Thus, nuclear processes much slower than filling the K vacancy, whose duration, in principle, can be measured directly, and with the width much smaller than the thermal energy at room temperature, will be called radioactive. This definition applies both to nuclear ground states and to long-lived excited nuclear states (isomers).

REVIEWS OF MODERN PHYSICS, VOLUME 84, APRIL–JUNE 2012
Radioactive decays at limits of nuclear stability

What is radioactivity ?

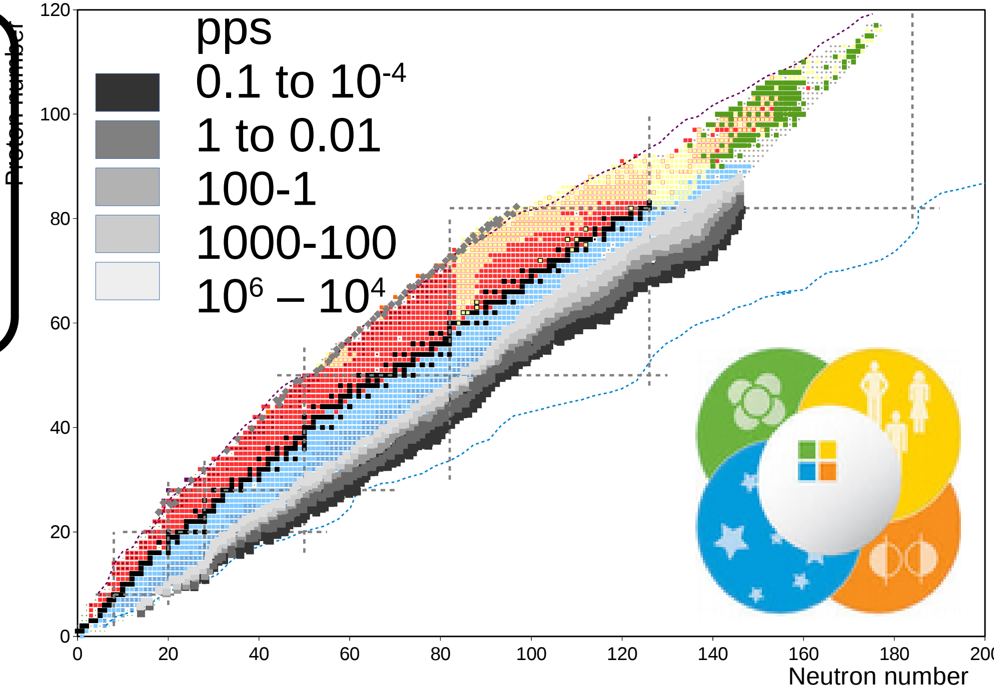


REVIEWS OF MODERN PHYSICS, VOLUME 84, APRIL–JUNE 2012
Radioactive decays at limits of nuclear stability

Exploratory role of the decay studies !

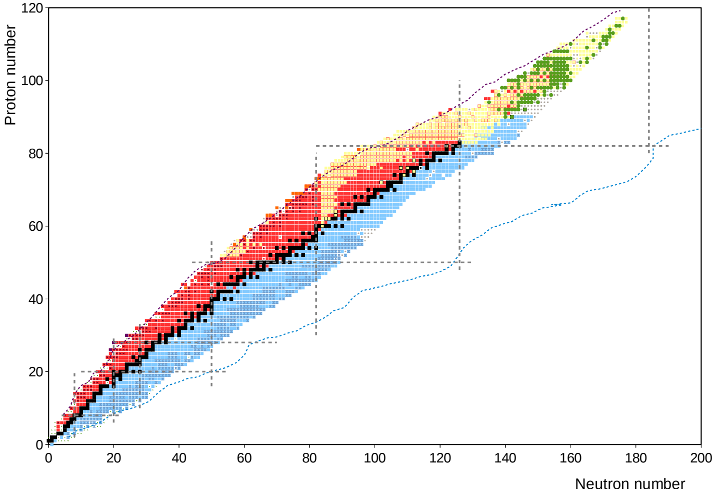
Decay studies access
the most exotic isotopes.
Do not rely on secondary
reactions.

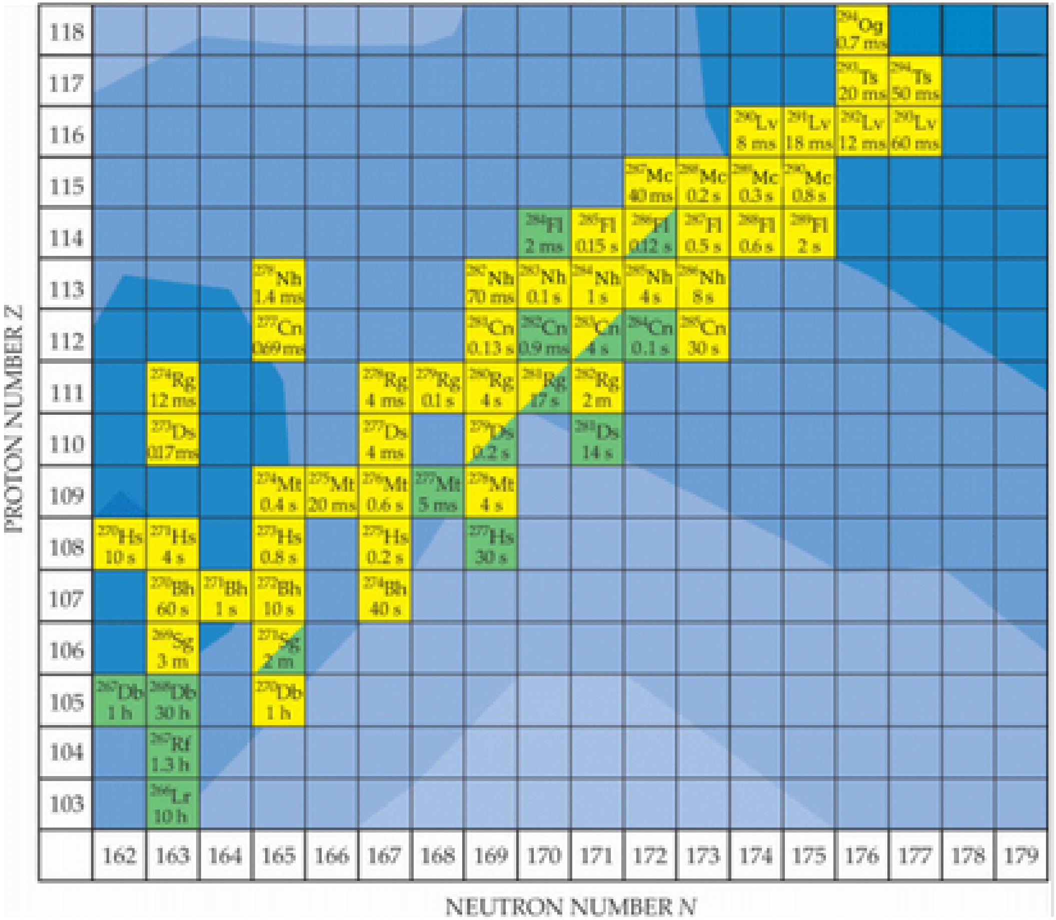
FRIB rates



Exploratory role of the decay studies !

Extreme example (SHE)
~particle per month or lower





Science of the nuclear decays

Nuclear structure:

- *Investigation of the **limits of nuclear existence***
- ***Shell-evolution** far from stability*
- *Evolution of **shapes and collective** motion*
- *Nucleon-nucleon **correlations** revealed in particle decays*
- ***Exotic decay modes** – two proton emission, beta-delayed fission, beta-delayed multi-neutron emission, direct neutron emission*

Applications:

- *Detailed studies of fission products of importance to **nuclear energy** and **stockpile stewardship***

Nuclear astrophysics:

- *Nuclear **lifetimes and branching** ratios for r-process and rp-process nuclei*
- *Measurement of the **decay strength** and **level density** distribution for r-process nuclei*
- *Measurement of **key resonances** for nucleosynthesis*

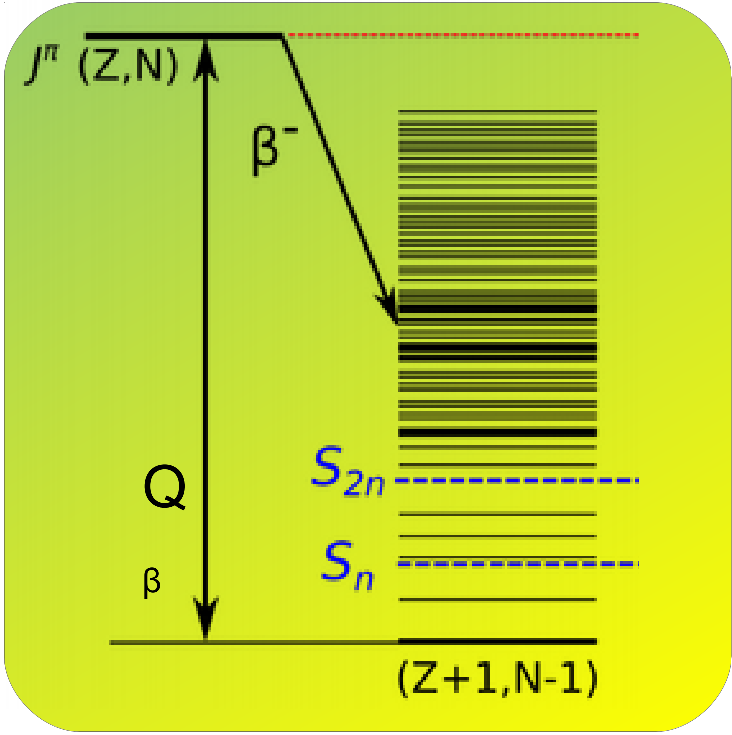
Fundamental interactions:

- *Testing the **Standard Model** description of the electroweak interaction using allowed Fermi and GT beta decay"*
- *Beta-decay strength measurements relevant for understanding **anti-neutrino properties***

Nuclear decays far from stability

- Large decay Q_β values (10-20 MeV)
- Small/negative particle separation energies $S_{n,p}$ (<1 MeV)
- Rare (2p-emission, n – emission) modes
- Composite (βxpγ, βxnγ, βf) modes
- Long decay chains back to stability.
- Short half-lives (100ns-100 ms)
- **Provides access to bound and unbound excited states in the daughter nucleus**

….



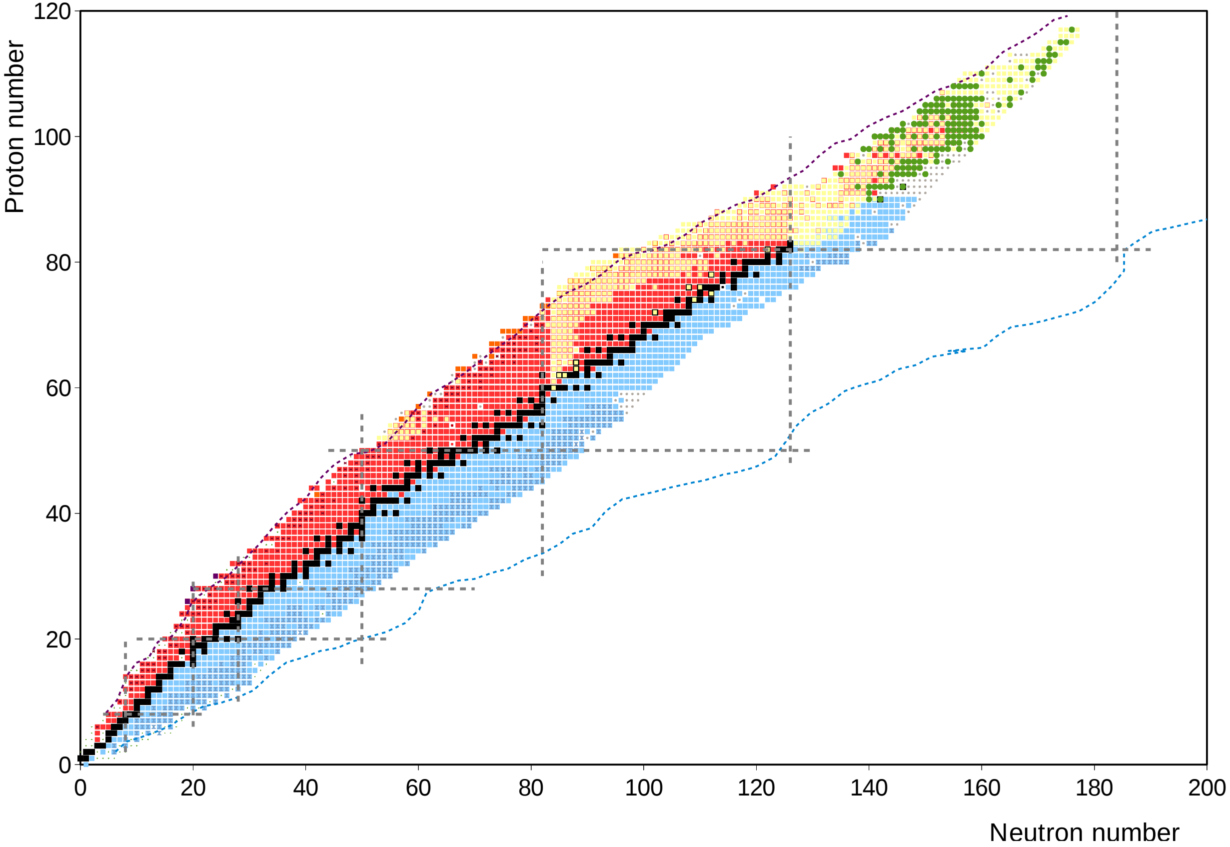
Radioactive decay modes

“Single“ step:
- proton and 2p emission
- alpha/cluster emission
- fission
- neutron emission

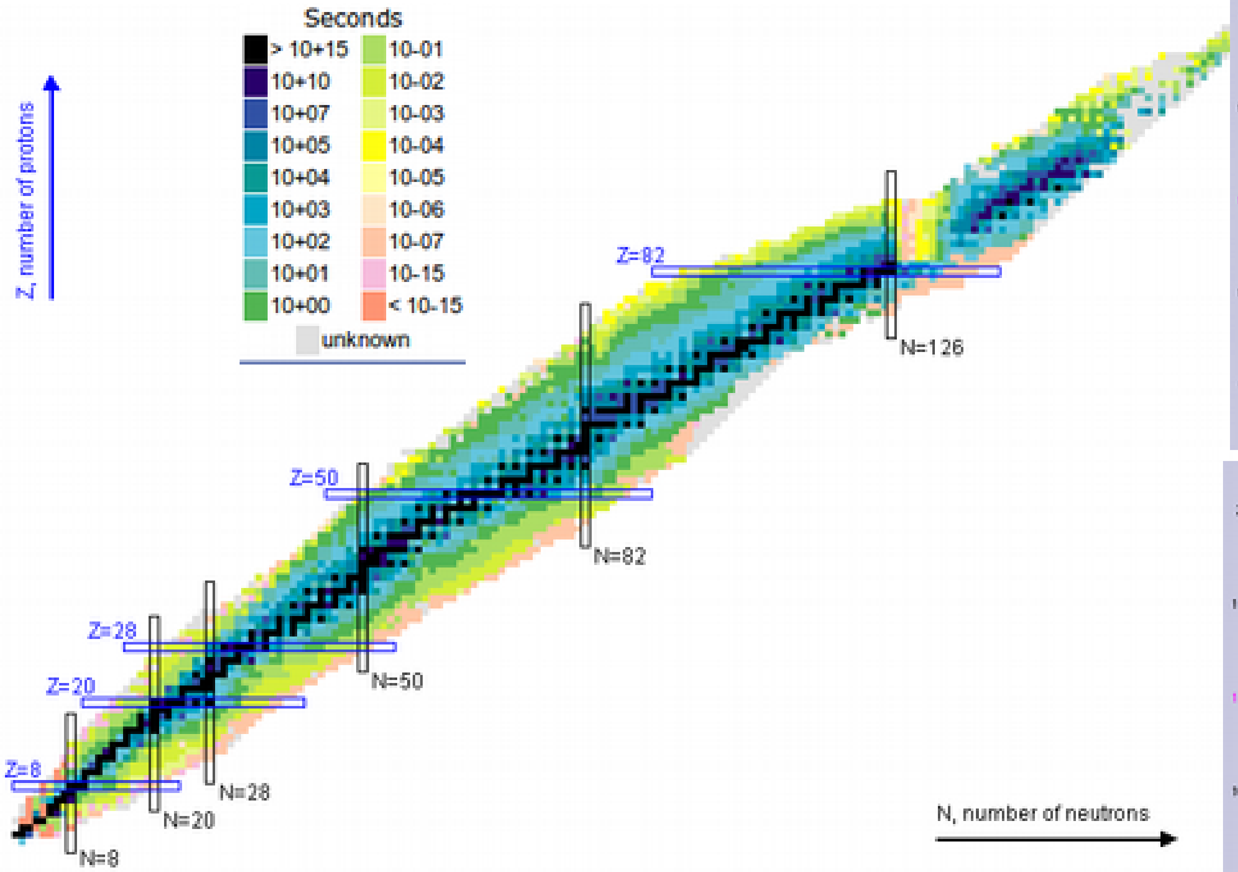
- β-, β+/EC
- ββ0ν

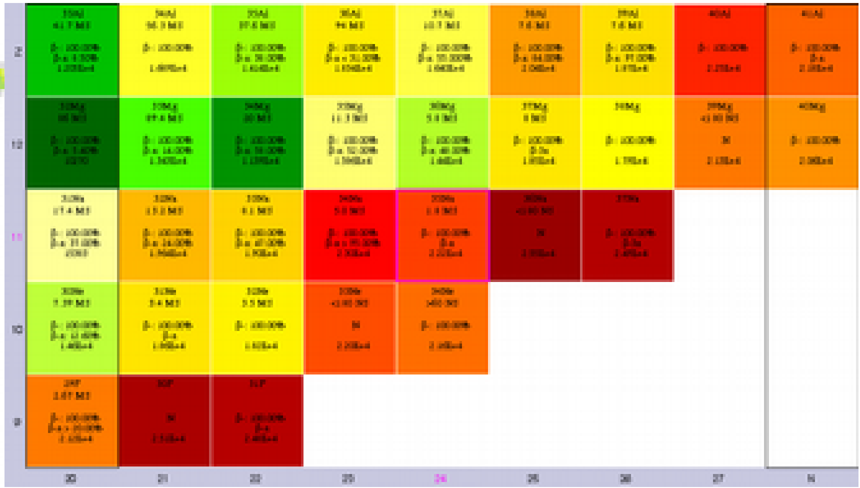
- γ-ray emission

Sequential:
Ecp,β-n ...



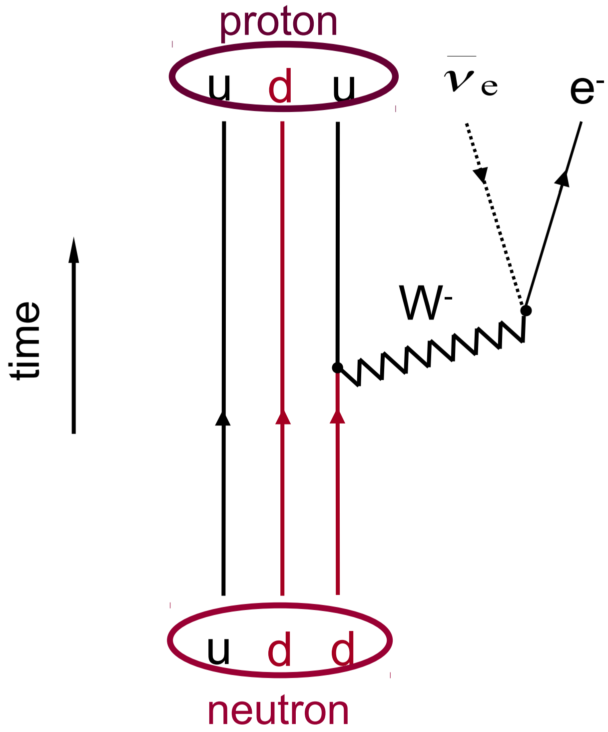
β decay – most common decay mode





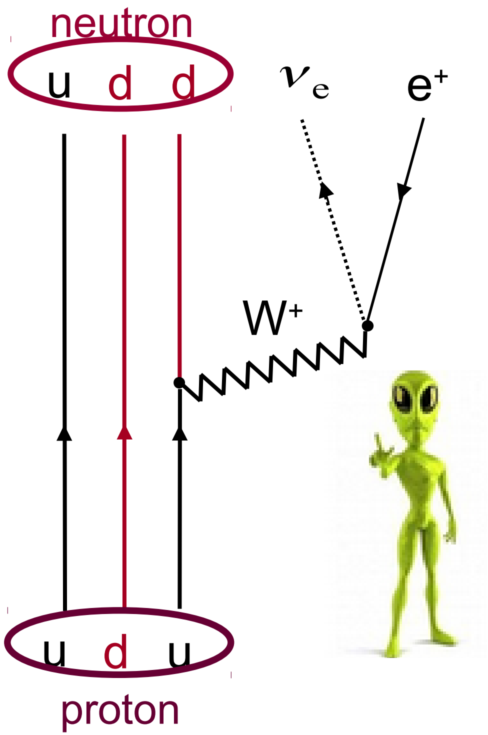
http://www.nndc.bnl.gov/chart/

Elementary β transformations

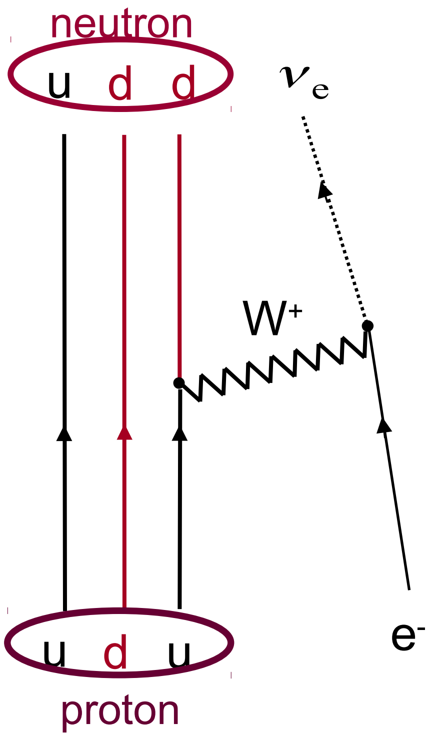


β⁻ decay of the neutron

M(n) > M(p)



Beta decay (β⁺)of proton



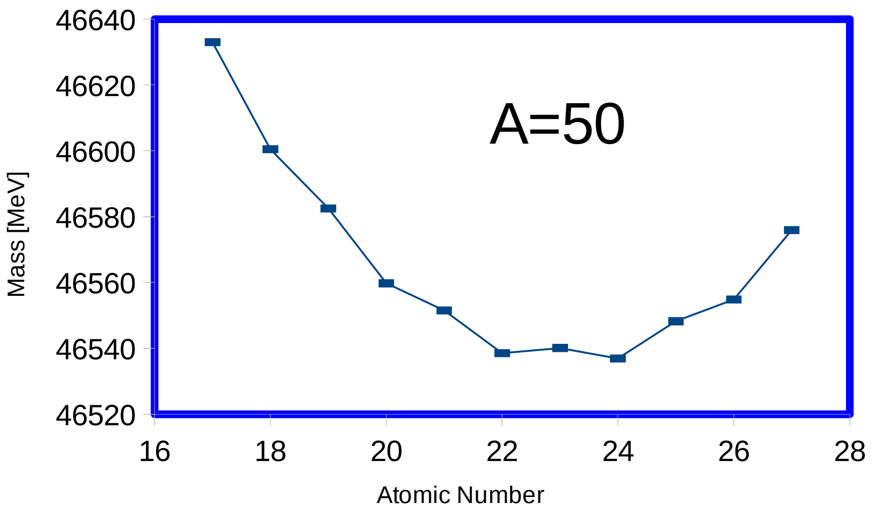
Electron capture

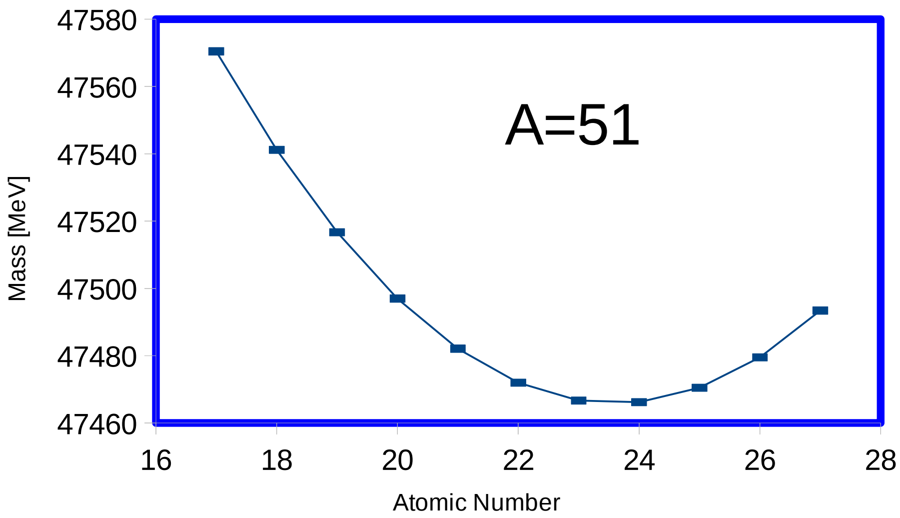
Mass parabola

Charge changing nuclear transformations between isobars.

$A = const$

$$B(Z) = -a_{sym}\frac{(A-2Z)^2}{A} - a_C\frac{Z^2}{A^{1/3}} + [a_{vol}A - \delta(A) - a_{surf}A^{2/3}] \qquad \delta(A) = \begin{cases} -34A^{-3/4} & for\ \ e-e \\ 0 & for\ \ o-e \\ 34A^{-3/4} & for\ \ o-o \end{cases}$$





Only possible if there is an allowed transformation of p→ n (n → p)

Energetics of the elementary beta decays

β^- *decay* $\quad {}^{A}_{Z}X_{N} \rightarrow {}^{\;\;\;A}_{Z+1}Y_{N-1} + e^- + \bar{\nu}$

PARENT DAUGHTER

β^+ *decay* $\quad {}^{A}_{Z}X_{N} \rightarrow {}^{\;\;\;A}_{Z-1}Y_{N+1} + e^+ + \nu$

PARENT DAUGHTER

$$Q_{\beta-} = T_{e^-} + T_{\bar{\nu}} = (M'_P - M'_D)c^2 - m_0 c^2$$

$$T_{\bar{\nu}} = p_{\bar{\nu}} c \qquad m_{\bar{\nu}} \simeq 0$$

$$Q_{\beta+} = T_{e^+} + T_{\nu} = (M'_P - M'_D)c^2 - m_0 c^2$$

$$M_P c^2 = M'_P c^2 + Z m_0 c^2 - \sum_{i=1}^{Z} B_i$$

atomic masses nuclear masses

$$Q_{\beta-} = (M_P - Zm_0 + \sum_{i=1}^{Z} B_i - [M_D - (Z+1)m_0 + \sum_{i=1}^{Z+1} B_i] - m_0)c^2$$

$$Q_{\beta-} \simeq M_P - M_D \text{ where } \sum_{i=1}^{Z} B_i \simeq \sum_{i=1}^{Z+1} B_i \sim eV$$

$$Q_{\beta+} = (M_P - Zm_0 + \sum_{i=1}^{Z} B_i - [M_D - (Z-1)m_0 + \sum_{i=1}^{Z-1} B_i] - m_0)c^2$$

$$Q_{\beta+} \simeq [M_P - M_D - 2m_0]c^2$$

for $Q_{\beta+} > 0 \qquad M_P - M_D > 2m_0$

If the threshold condition is not fulfilled....

Energetics of the elementary beta decays

$${}^{A}_{Z}X_{N} + e^{-} \rightarrow {}^{A}_{Z-1}Y_{N+1} + \nu_{e}$$

Electron capture (EC)

$$Q_{EC} = T_{\nu} = (M'_{P} + m_{0})c^{2} - (B_{n} + M'_{D}c^{2})$$

B_n - binding energy of the n-th electron, usually inner shell $K, L_I, L_{II} \ldots$

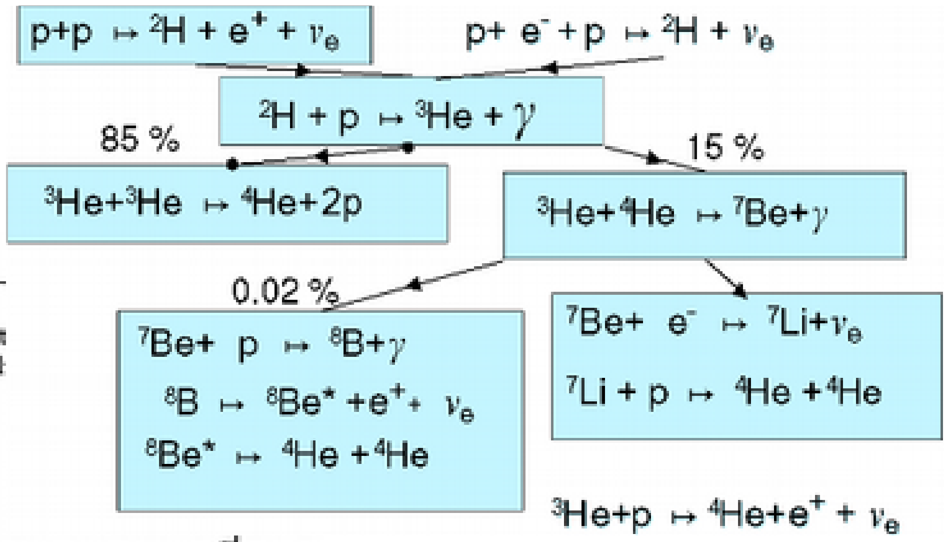
$$M_{P}c^{2} = M'_{P}c^{2} + Z m_{0}c^{2} - \sum_{i=1}^{Z} B_{i}$$

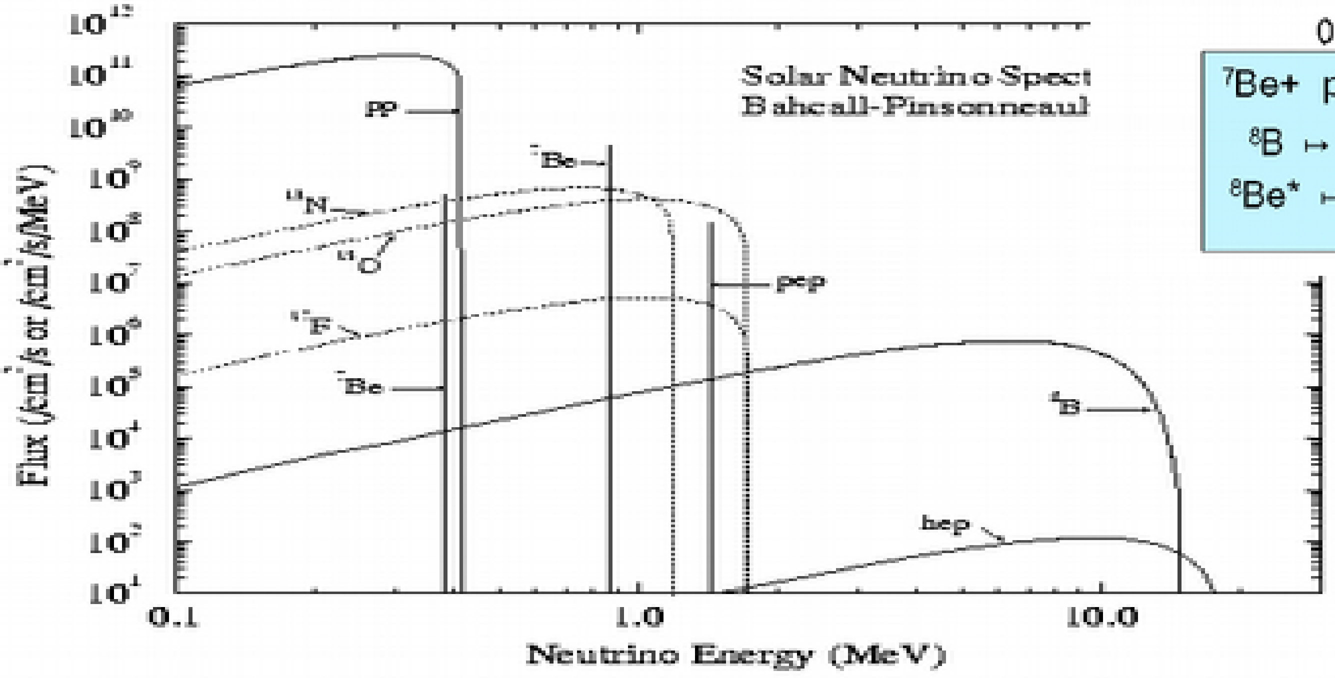
$$Q_{EC} = \left(M_{P} - Zm_{0} + m_{0} + \sum_{i=1}^{Z} B_{i} - \left[B_{n} - M_{D} - (Z-1)m_{0} + \sum_{i=1}^{Z-1} B_{i}\right]\right)c^{2}$$

$$Q_{EC} = M_{P} - (M_{D}c^{2} + B_{n})$$

- Monochromatic neutrinos
- Blocked for ions without electrons (radioactive ions may become stable)

Connection to astrophysics – main source of solar neutrinos





The most obscure type of beta decay
Bound state beta decay (a time mirror to orbital electron capture)

For neutral atoms decays:

$$Q^{q=0}_{\beta_c^-} = m_Y(A, Z+1) - m_X(A, Z),$$

For fully stripped ions decays:

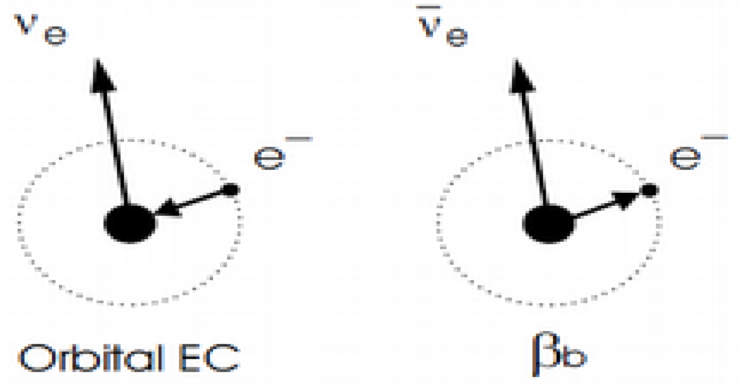
$$Q^{q=Z}_{\beta_c^-} = Q^{q=0}_{\beta_c^-} - \left|\Delta B_e^{tot}(Y, X)\right|,$$

difference between binding energies for all electrons in parent and daughter

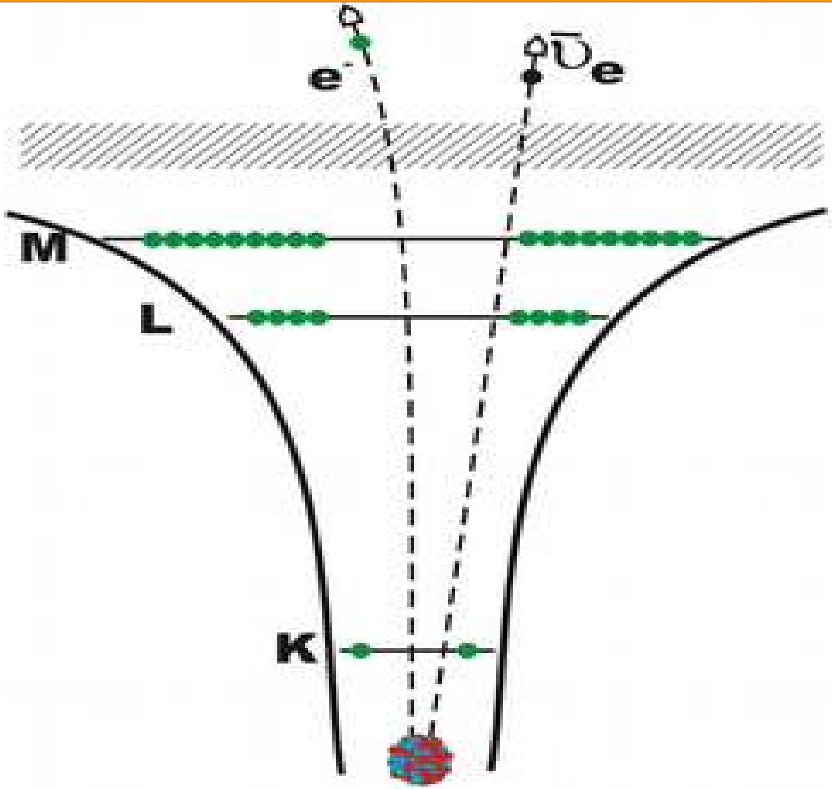
For bound state beta decay:
(electron goes to a bound orbital, adding extra binding energy)

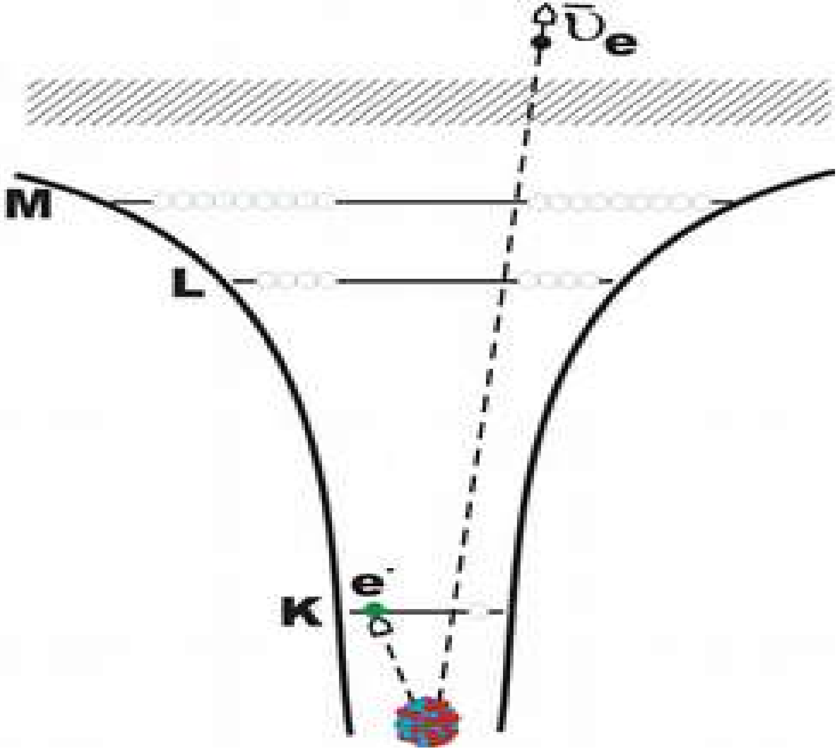
$$Q^{q=Z}_{\beta_b^-, x} = Q^{q=Z}_{\beta_c^-} + |B_e^x(Y)|,$$

binding energy of the K-electron

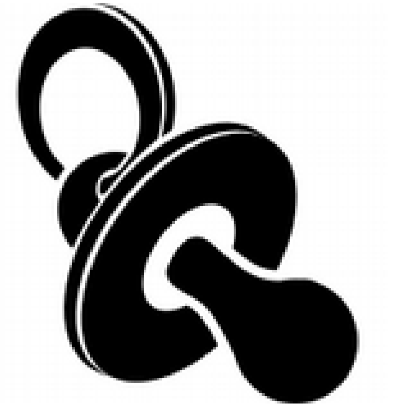


Bound state beta decay (a time mirror to orbital electron capture)





The beta decay transition rates (Fermi's Golden Rule)



From the time dependent perturbation theory
Transition results from specific interaction H_{int}

$$\lambda_{i\rightarrow f} = \frac{2\pi}{\hbar} \left| \langle \psi_f^{(0)} | \hat{H}_{int} | \psi_i^{(0)} \rangle \right|^2 \frac{dn}{dE}$$

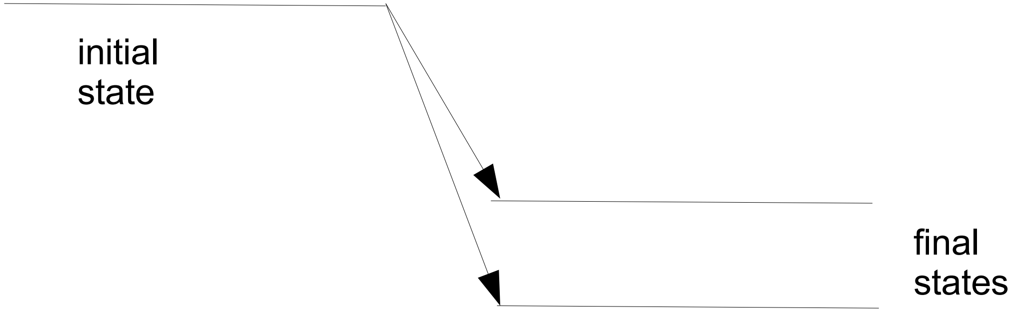
interaction term

phase space term

$\hat{H}_{int}$ - interaction Hamiltonian

$\frac{dn}{dE}$ - density of states appropriate for the particular process

$\psi_i^{(0)}$ and $\psi_f^{(0)}$ represent stationary states, initial and final



Decay rates

$$\lambda_{i\to f} = \frac{2\pi}{\hbar}\left|\left\langle \psi_f^{(0)}\right|\hat{H}_{\text{int}}\left|\psi_i^{(0)}\right\rangle\right|^2 \frac{dn}{dE}$$

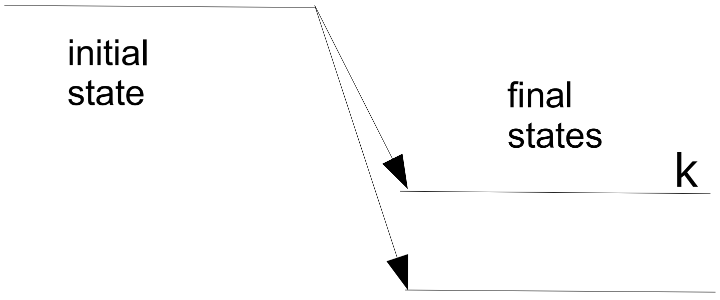
$\lambda_{i\to f}$ - in beta decay is called a decay constant

In case there are multiple final states which can be populated we can define a partial half-life

$$\lambda_{i\to f_k} = \frac{\ln 2}{T_{1/2}^{i\to f_k}}$$

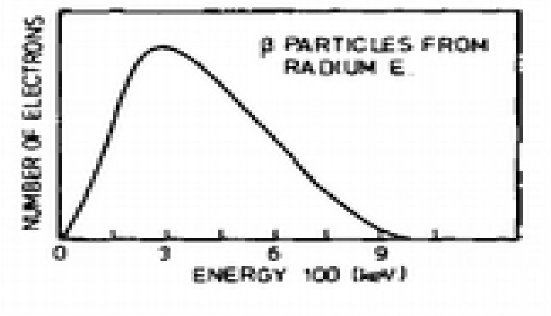
The total half-life of the level is given by the sum over all decay channels

$$\lambda = \sum_k \lambda_{i\to f_k} = \sum_k \frac{\ln 2}{T_{1/2}^{i\to f_k}} = \frac{\ln 2}{T_{1/2}}$$

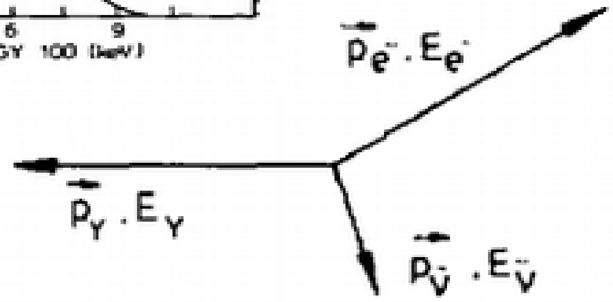


Beta decay (three body process)

$$\lambda_{i\to f}=\frac{2\pi}{\hbar}\left|\left\langle\psi_f^{(0)}\middle|\hat{H}_{\text{int}}\middle|\psi_i^{(0)}\right\rangle\right|^2\frac{dn}{dE}(e^-,\bar{\nu})$$



The phase space factor results from sharing of the decay energy between final states of electron and (anti)neutrino.



$\psi_i=\psi_P$ - parent nucleus wavefunction

ψ_f - product of the daughter nucleus electron and neutrino wavefunctions

$\psi_f=\psi_D\psi_e\psi_{\bar{\nu}}$

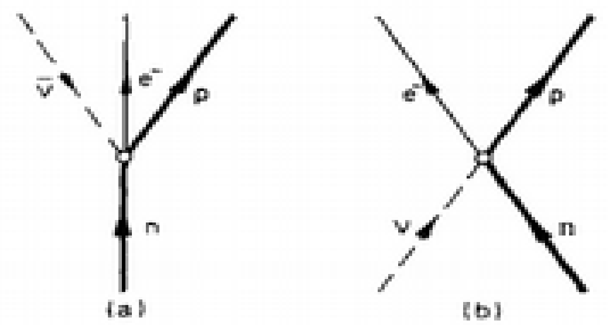
$$\psi_e=\frac{1}{\sqrt{V}}\exp\left(\frac{i\vec{p}_e\vec{r}}{\hbar}\right)\qquad\psi_{\bar{\nu}}=\frac{1}{\sqrt{V}}\exp\left(\frac{i\vec{p}_{\bar{\nu}}\vec{r}}{\hbar}\right)$$

Fermi's theory of beta decay

Because $\frac{pr}{\hbar} \ll 1$ for electron and neutrino

$$\exp\left(\frac{i\vec{p}_e\vec{r}}{\hbar}\right) = 1 + \frac{i\vec{p}_e\vec{r}}{\hbar} + \ldots$$

$$\psi_{\bar{\nu}}(\vec{r}) \simeq \frac{1}{\sqrt{V}} \quad \text{and} \quad \psi_e(\vec{r}) \simeq \frac{1}{\sqrt{V}}$$



weak interaction:
point interaction

$$H_{\text{int}} = g\,\delta(\vec{r}_n - \vec{r}_p)\,\delta(\vec{r}_n - \vec{r}_e)\,\delta(\vec{r}_n - \vec{r}_\nu)\,\bar{O}(n \to p)$$

$$\langle \psi_f^{(0)} | \hat{H}_{\text{int}} | \psi_i^{(0)} \rangle = g \int \psi_D^*(\vec{r})\,\psi_e^*(\vec{r})\,\psi_\nu^*(\vec{r})\,\hat{O}\,\psi_P(\vec{r})\,d\vec{r}$$

$$\langle \psi_f^{(0)} | \hat{H}_{\text{int}} | \psi_i^{(0)} \rangle = g\frac{1}{V} \int \psi_D^*(\vec{r})\,\hat{O}\,\psi_P(\vec{r})\,d\vec{r} = \frac{g}{V} M_{fi}$$

First order perturbation - allowed transitions

The integral measures overlapp between nuclear wavefunctions

Coulomb modification of the wavefunctions

$$\left|\left\langle \psi_f^{(0)} \middle| \hat{H}_{\text{int}} \middle| \psi_i^{(0)} \right\rangle\right|^2 = \frac{g^2}{V^2} |M_{fi}|$$

Strictly speaking, the electron wavefunction as a plane wave is oversimplified.
One has to consider a Coulomb effects !
This was implemented through the correction called Fermi function.

$$|\psi_e(Z,\vec{r})^2| = F(Z_D, p_e) |\psi_e(Z,\vec{r})^2|$$

$$F(Z_D, p_e) \simeq \frac{2\pi\eta}{1 - e^{2\pi\eta}}$$

$$\eta = \frac{\pm Ze^2}{4\pi\epsilon_0 \hbar v_e} \begin{cases} +: \beta^- \\ -: \beta^+ \end{cases}$$

v_e - final velocity of the electron/positron

$$\boxed{\left|\left\langle \psi_f^{(0)} \middle| \hat{H}_{\text{int}} \middle| \psi_i^{(0)} \right\rangle\right|^2 = \frac{g^2}{V^2} |M_{fi}| F(Z_D, p_e)}$$

Energy spectrum of electrons

$$\lambda_{i\to f}=\frac{2\pi}{\hbar}\left|\left\langle\psi_f^{(0)}\middle|\hat{H}_{\text{int}}\middle|\psi_i^{(0)}\right\rangle\right|^2\frac{dn}{dE}$$

$$\lambda=g^2\frac{\left|M_{fi}^2\right|}{2\pi^3\hbar^7c^6}F(Z_D,p_e)\times$$

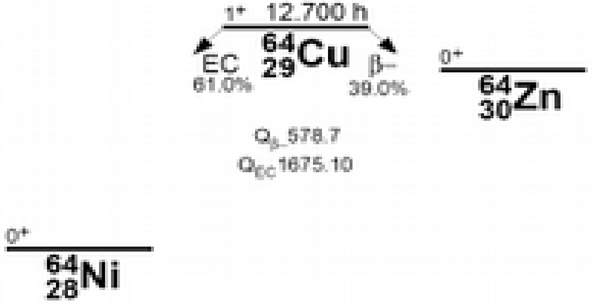
$$\times\frac{(4\pi)^2}{(2\pi\hbar)^6}\frac{1}{c^6}\sqrt{T_{e^-}^2+2T_{e^-}m_{e^-}c^2}\sqrt{1-\frac{m_{\bar{\nu}_e}^2c^4}{(Q_{\beta^-}-T_{e^-})^2}}(Q_{\beta^-}-T_{e^-})^2(T_{e^-}+m_{e^-}c^2)\,dT_{e^-}$$

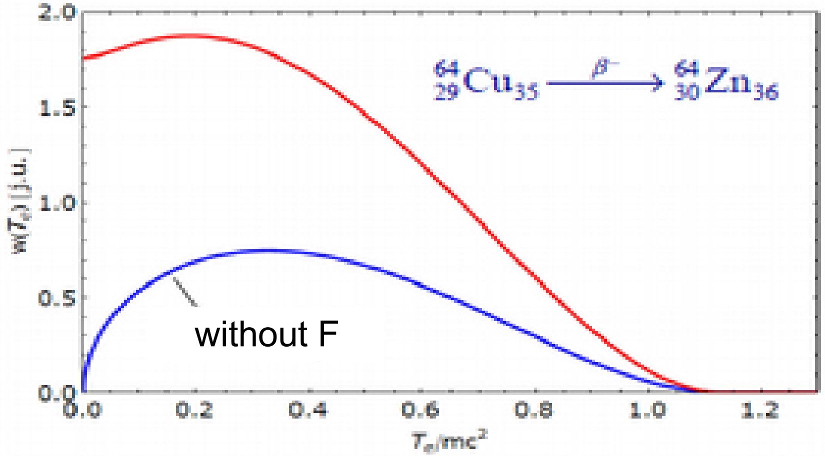
Energy spectrum of electrons emitted in the $\beta-$ decay

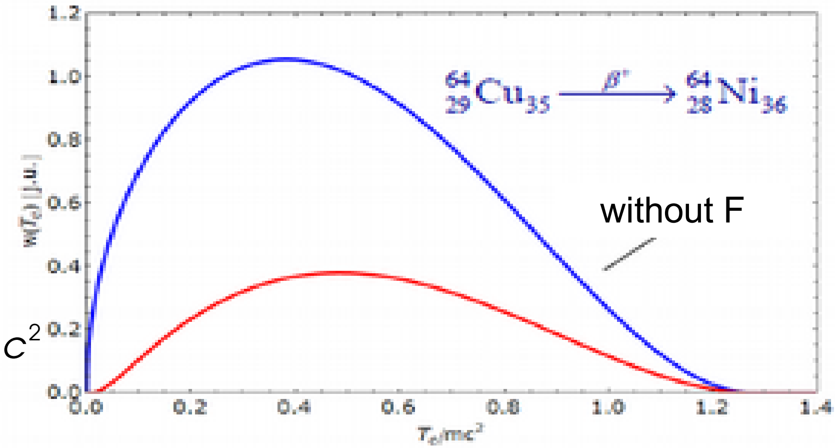
$$w(T_e)=\frac{d\lambda}{dT_e}=$$

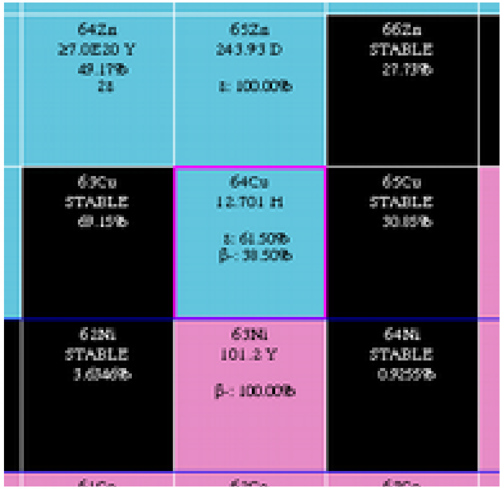
$$=\frac{g^2\left|M_{fi}^2\right|}{2\pi^3h^7c^6}F(Z_D,p_e)\times\sqrt{T_{e^-}^2+2T_{e^-}m_{e^-}c^2}\sqrt{1-\frac{m_{\bar{\nu}_e}^2c^4}{(Q_{\beta^-}-T_{e^-})^2}}(Q_{\beta^-}-T_{e^-})^2(T_{e^-}+m_{e^-}c^2)$$

Examples of beta decays









$$Q_{\beta^-} = 579\,keV = 1.14\,m_e c^2$$

$$Q_{\beta^+} = (1675 - 1022)\,keV = 653\,keV = 1.28\,m_e c^2$$

M. Pfützner

Kurie plot

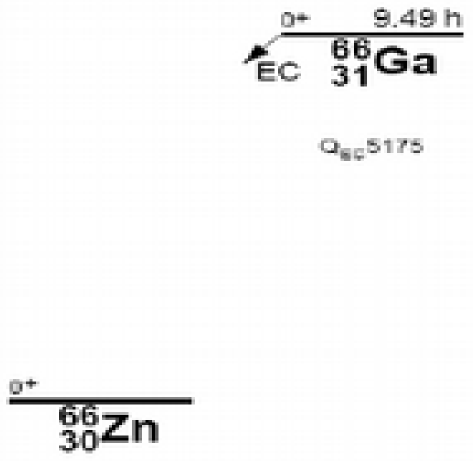
Energy spectrum of electrons emitted in the β decay

$$\frac{d\lambda}{dT_e}=\frac{g^2|M_{fi}^2|}{2\pi^3 h^7 c^6}F(Z_D,p_e)\times\sqrt{T_{e^-}^2+2T_{e^-}m_{e^-}c^2}\sqrt{1-\frac{m_{\bar{\nu}_e}^2 c^4}{(Q_{\beta^-}-T_{e^-})^2}}(Q_{\beta^-}-T_{e^-})^2(T_{e^-}+m_{e^-}c^2)$$

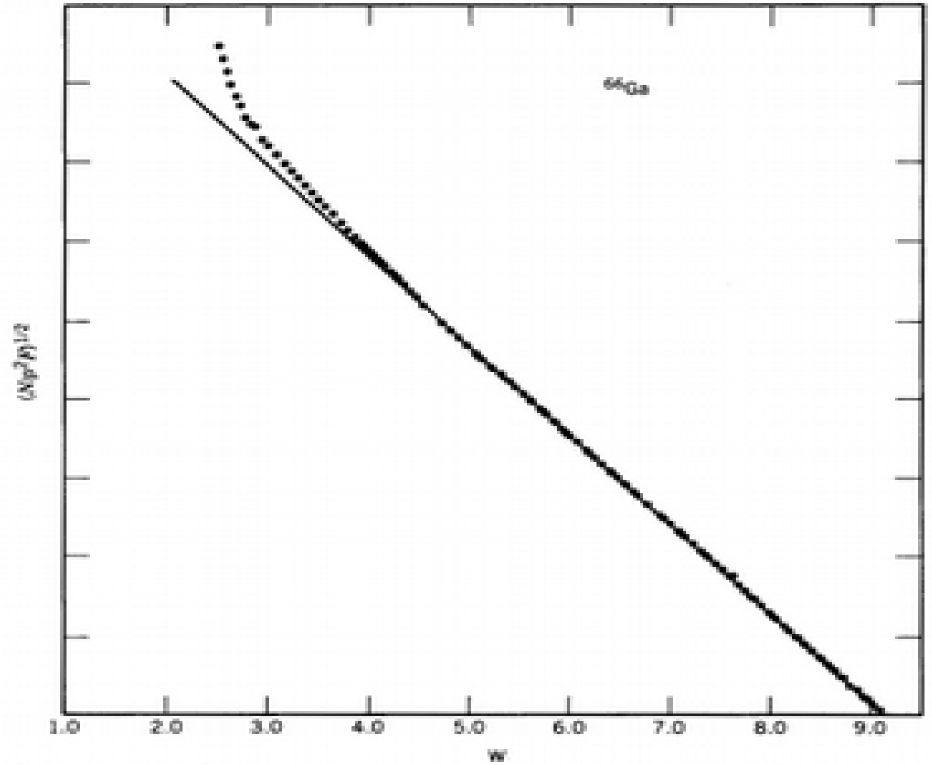
$$\frac{d\lambda/dT_e}{F(Z_D,p_e)\sqrt{T_{e^-}^2+2T_{e^-}m_{e^-}c^2}(T_{e^-}+m_{e^-}c^2)}=\frac{g^2|M_{fi}^2|}{2\pi^3 h^7 c^6}\times\sqrt{1-\frac{m_{\bar{\nu}_e}^2 c^4}{(Q_{\beta^-}-T_{e^-})^2}}(Q_{\beta^-}-T_{e^-})^2$$

$$\sqrt{\frac{d\lambda/dT_e}{F(Z_D,p_e)(p_e c)E_e}}=\sqrt{\frac{g^2|M_{fi}^2|}{2\pi^3 h^7 c^6}}\left(1-\frac{m_{\bar{\nu}_e}^2 c^4}{(Q_{\beta^-}-T_{e^-})^2}\right)^{1/4}(Q_{\beta^-}-T_{e^-})$$

Kurie plot

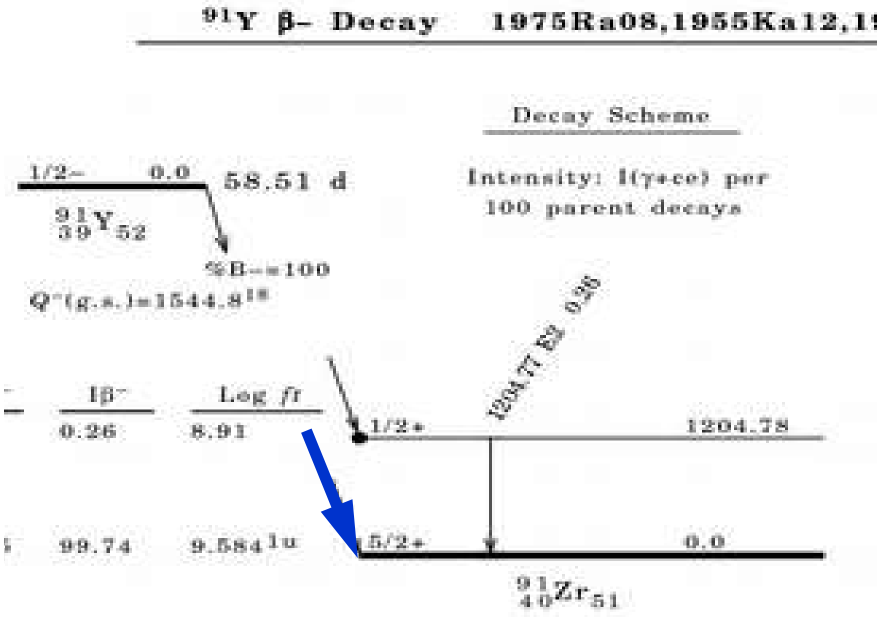


$$Q_{\beta^+} = (5175 - 1022)\,keV = 4153\,keV = 8.13\,m_e c^2$$



Total electron energy

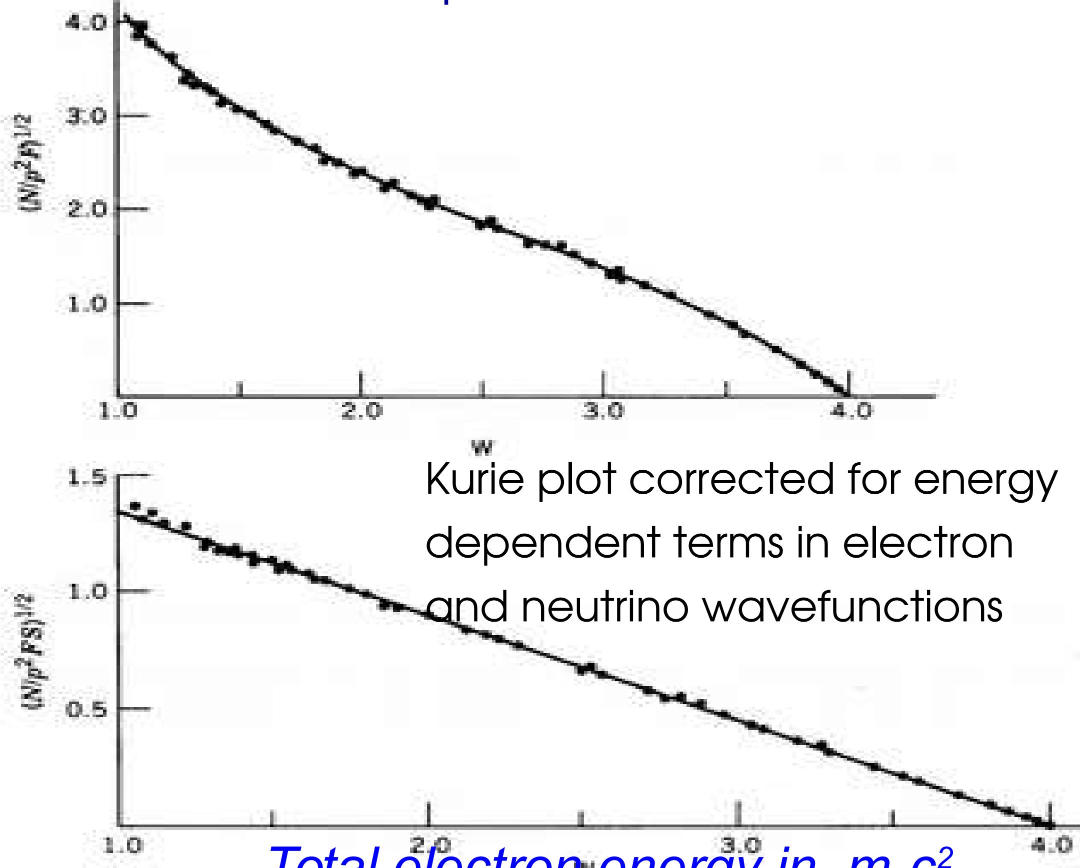
Example of the forbidden spectrum ^{91}Y



$$\Delta J = 2, \quad \pi_i \pi_f = -1$$

$$Q_{\beta^-} \approx 1545 \text{ keV} \approx 3.02\ m_e c^2$$

Kurie plot



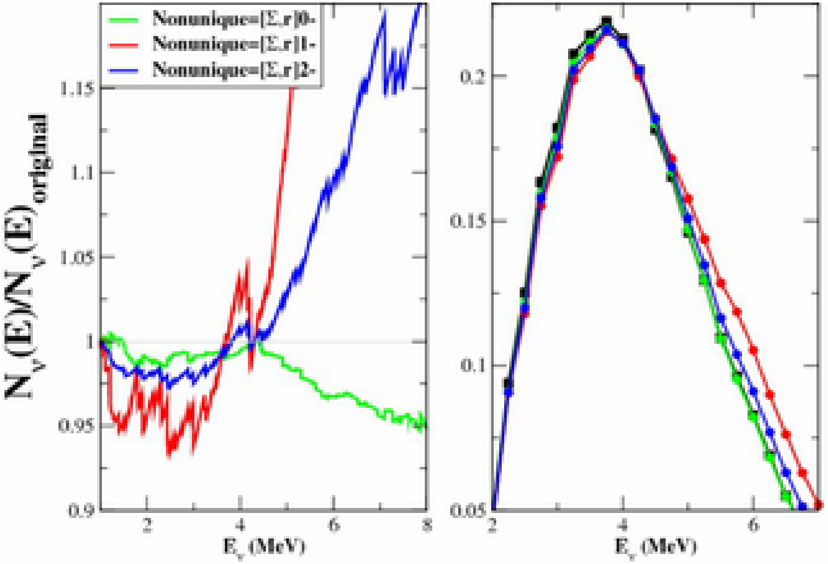
Kurie plot corrected for energy dependent terms in electron and neutrino wavefunctions

Total electron energy in $m_e c^2$

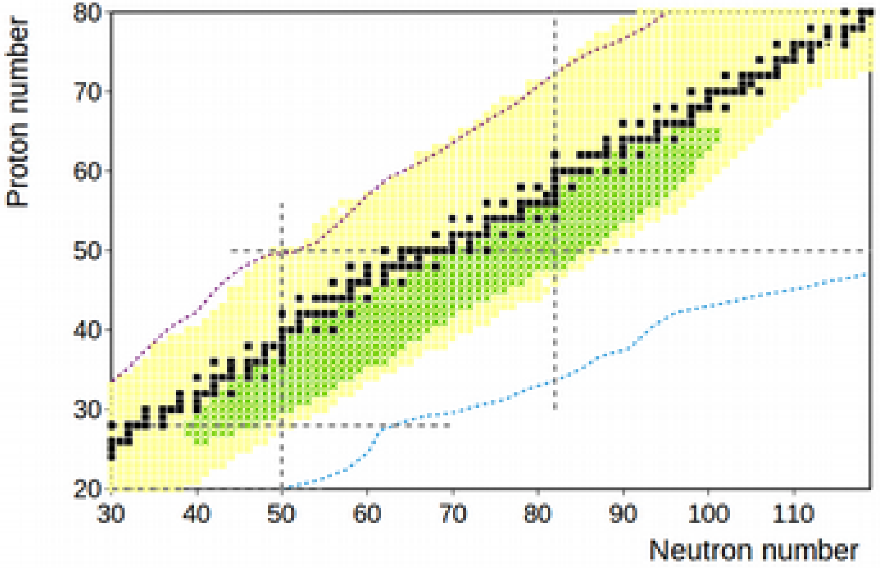
Reactor antineutrino anomaly

Systematic Uncertainties in the Analysis of the Reactor Neutrino Anomaly

A. C. Hayes, J. L. Friar, G. T. Garvey, Gerard Jungman, and G. Jonkmans
Phys. Rev. Lett. **112**, 202501 – Published 20 May 2014



Ratio of antineutrino spectrum to the original ILL spectrum allowing different operators to dominate the non-unique forbidden transitions



For allowed transition M_{fi} is independent on electron energy

$$\lambda = \frac{g^2 |M_{fi}^2|}{2\pi^3 h^7 c^6} \int_{m_e c^2}^{E_{max}} F(Z_D, p_e) \times \sqrt{E_{e^-}^2 - (m_{e^-} c^2)^2} (Q_{\beta^-} - T_{e^-})^2 E_{e^-} dE_e$$

define $w = \dfrac{E_e}{m_e c^2}$ and $dw = \dfrac{dE_e}{m_e c^2}$ $w_0 = \dfrac{E_{max}}{m_e c^2}$

$$\lambda = \frac{g^2 |M_{fi}^2| (m_e c^2)^5}{2\pi^3 h^7 c^6} \int_1^{w_0} F(Z_D, w) \times \sqrt{w^2 - 1} (w_0 - w)^2 w \, dw$$

The $\int$ part is called Fermi integral and is defined as:

$$f(Z_D, w_0) = \int_1^{w_0} F(Z_D, w) \times \sqrt{w^2 - 1} (w_0 - w)^2 w \, dw$$

$$\lambda = \ln \frac{2}{T_{1/2}} = \frac{g^2 (m_e c^2)^5}{2\pi^3 h^7 c^6} |M_{fi}^2| f(Z_D, w_0)$$

or

$$T_{1/2} = \frac{2 \ln 2 \pi^3 h^7 c^6}{g^2 (m_e c^2)^5} \frac{1}{|M_{fi}^2| f(Z_D, w_0)}$$

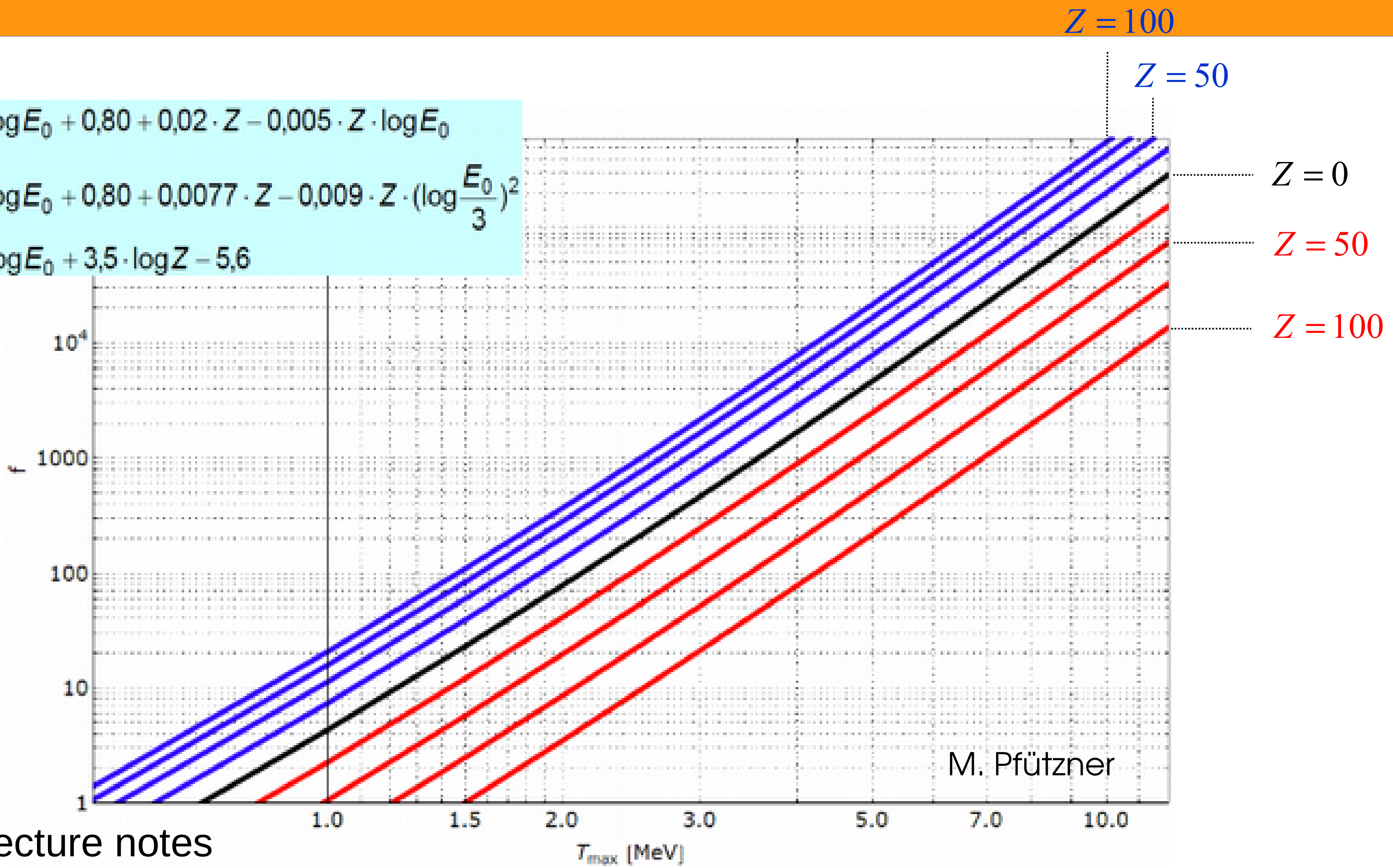
$$f \, T_{1/2} = \frac{2 \ln 2 \pi^3 h^7 c^6}{g^2 (m_e c^2)^5} \frac{1}{|M_{fi}|^2} = \frac{K}{g^2 |M_{fi}|^2} = \frac{4.794 \times 10^{-5} \, MeV^2 \, fm^6 \, s}{g^2 |M_{fi}|^2}$$

Fermi Integral, *f (E,Z)*

$$\log f_{\beta^-} = 4{,}0 \cdot \log E_0 + 0{,}80 + 0{,}02 \cdot Z - 0{,}005 \cdot Z \cdot \log E_0$$

$$\log f_{\beta^+} = 4{,}0 \cdot \log E_0 + 0{,}80 + 0{,}0077 \cdot Z - 0{,}009 \cdot Z \cdot (\log \frac{E_0}{3})^2$$

$$\log f_{EC} = 2{,}0 \cdot \log E_0 + 3{,}5 \cdot \log Z - 5{,}6$$



M. Pfutzner lecture notes

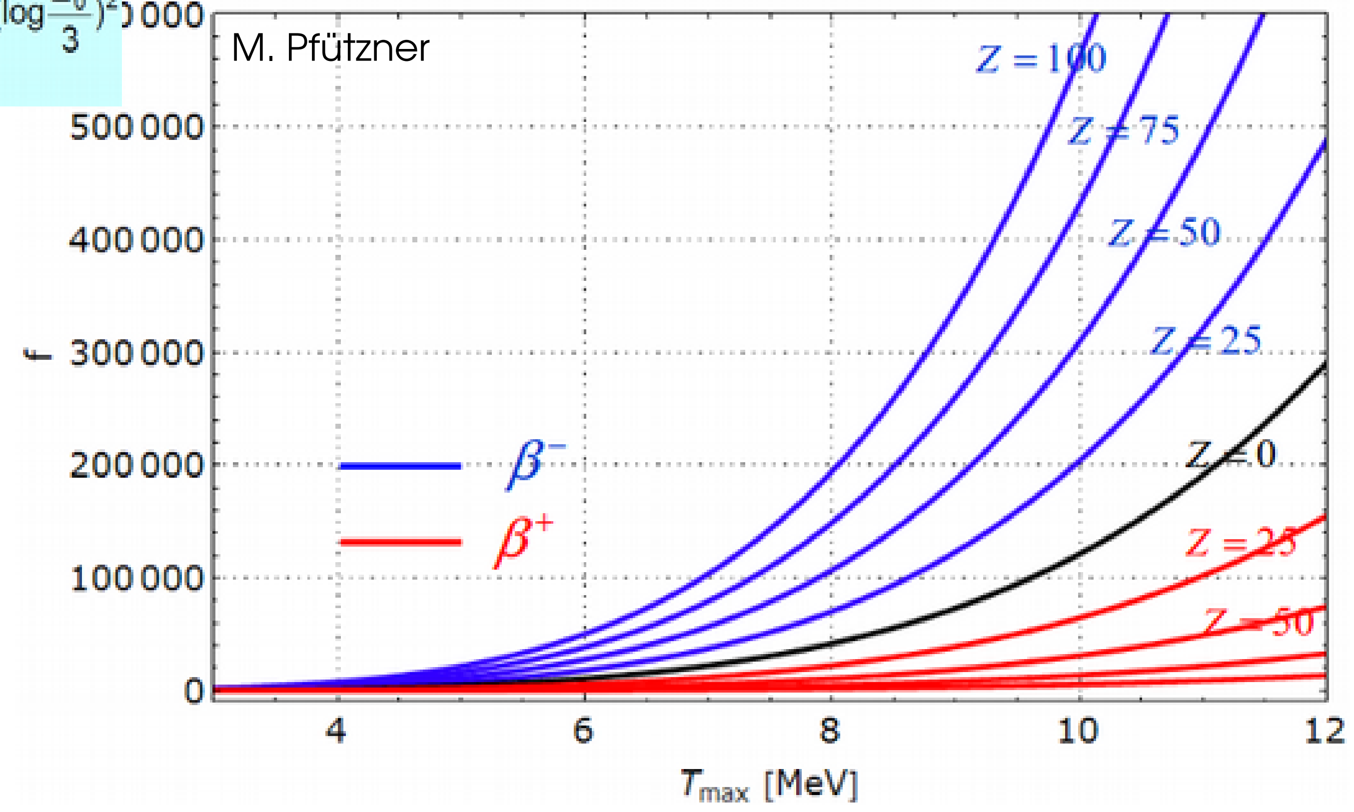
Fermi Integral, f

$$\log f_{\beta^-} = 4{,}0 \cdot \log E_0 + 0{,}80 + 0{,}02 \cdot Z - 0{,}005 \cdot Z \cdot \log E_0$$

$$\log f_{\beta^+} = 4{,}0 \cdot \log E_0 + 0{,}80 + 0{,}0077 \cdot Z - 0{,}009 \cdot Z \cdot (\log \frac{E_0}{3})^2$$

$$\log f_{EC} = 2{,}0 \cdot \log E_0 + 3{,}5 \cdot \log Z - 5{,}6$$

$$T_{1/2} = \frac{K}{f\, g^2 |M_{fi}|^2}$$



Commonly used value to classify transition strength is *log ft*

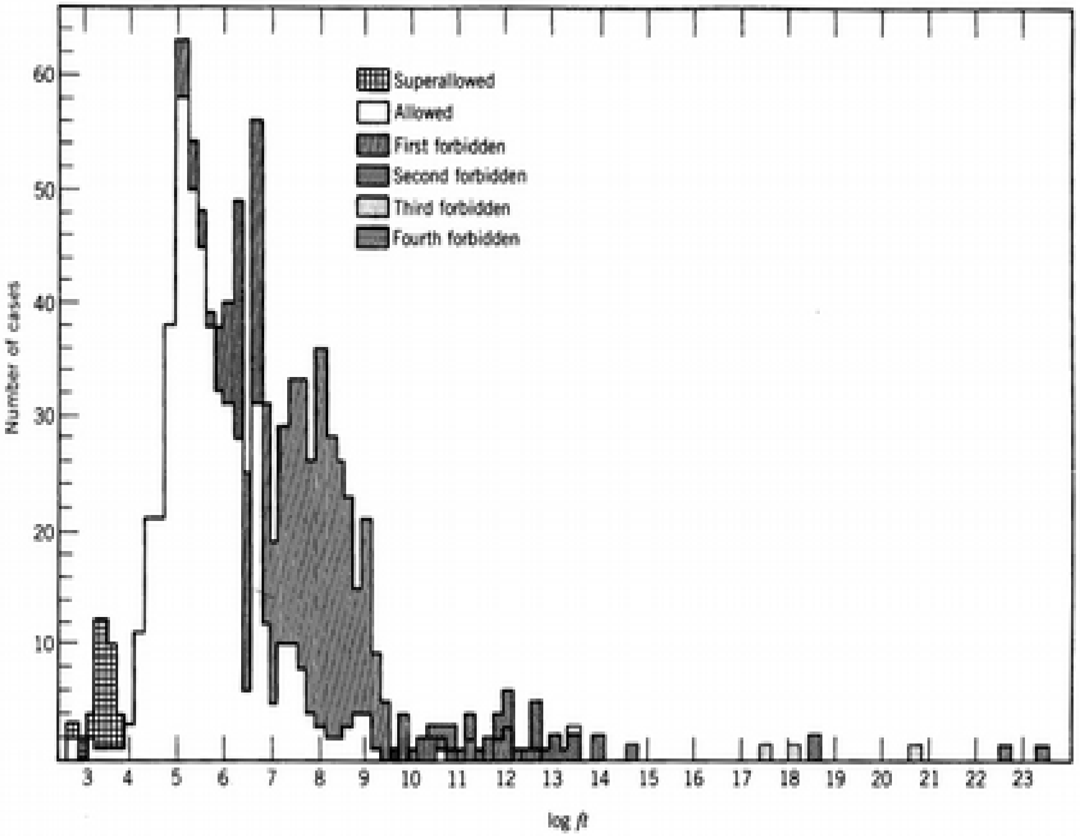
(larger *log ft* => more “forbidden” is the transition, smaller matrix element)

$$f\,T_{1/2} = \frac{K}{g^2 |M_{fi}|^2}$$

Classification of β-Transitions

Classification	Change of the quantum number of orbital spin, ΔL	Change of the nuclear spin, ΔI	Change of the parity	log ft	Examples
Allowed (favoured)	0	0	No	2.7–3.7	n, ^{3}H, ^{6}He($\Delta I = 1$!), ^{11}C, ^{13}N, ^{15}O, ^{17}F, ^{19}Ne, ^{21}Na, ^{23}Mg, ^{25}Al, ^{27}Si, ^{29}P, ^{31}S, ^{33}Cl, ^{35}Ar, ^{37}K, ^{39}Ca, ^{41}Sc, ^{43}Ti
Allowed (normal)	0	0 or 1	No	4–7	^{12}B, ^{12}N, ^{35}S, ^{64}Cu, ^{69}Zn, ^{114}In
Allowed (l-forbidden)	2	1	No	6–9	^{14}C, ^{32}P
First forbidden	1	0 or 1	Yes	6–10	^{111}Ag, ^{143}Ce, ^{115}Cd, ^{187}W
First forbidden (special cases)	1	2	Yes	7–10	^{38}Cl, ^{90}Sr, ^{97}Zr, ^{140}Ba
Second forbidden	2	2	No	11–14	^{36}Cl, ^{99}Tc, ^{135}Cs, ^{137}Cs
Second forbidden (special cases)	2	3	No	≈ 14	^{10}Be, ^{22}Na
Third forbidden	3	3	Yes	17–19	^{87}Rb
Third forbidden (special cases)	3	4	Yes	18	^{40}K
Fourth forbidden	4	4	No	≈ 23	^{115}In

Log ft



Superallowed
Allowed
First forbidden
Second forbidden
Third forbidden
Fourth forbidden
Number of cases
60
50
40
30
20
10
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
log ft

Fermi and GT decays

$\vec{J}_i = \vec{J}_f + \vec{L}_{e,\nu} + \vec{S}_{e,\nu}$

$\vec{J}_i, \vec{J}_f$ – total angular momentum of the nuclear states (initial and final)

$\vec{L}_{e,\nu}$ – orbital angular momentum of leptons

$\vec{S}_{e,\nu}$ – intrinsic (spin) angular momentum

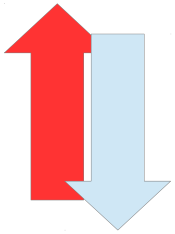
For allowed transitions $\vec{L}_{e,\nu} = 0$ and $\pi_i = \pi_f$

BUT ! !

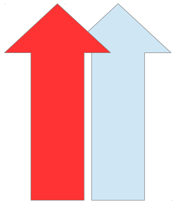
$S=0$ or $S=1$ (e and ν are fermions)

The allowed transitions are categorized into two classes:

"Fermi decays"
$S=0$ and $\Delta\pi=0$
$J_i=J_f$



"Gamow-Teller decays"
$S=1$ and $\Delta\pi=0$
$J_i - 1 \leq J_f \leq J_i + 1$
for $J_i = 0 \Rightarrow J_f = 1$



Fermi and GT decays

$$H_{\text{int}} = g\,\delta(\vec{r}_n - \vec{r}_p)\,\delta(\vec{r}_n - \vec{r}_e)\,\delta(\vec{r}_n - \vec{r}_\nu)\,\bar{O}(n \rightarrow p)$$

The transition operator for allowed transitions can be written as:

$$g\hat{O} = \sum_{k=1}^{A} [g_V \vec{\tau}_\pm(k) + g_A \vec{\sigma}(k)\vec{\tau}(k)_\pm] \text{ or}$$

Isospin operator converts protons into neutrons

$$g\hat{O} = g_V \sum_{k=1}^{A} [\vec{\tau}_\pm(k) + \frac{g_A}{g_V} \vec{\sigma}(k)\vec{\tau}(k)_\pm]$$

VECTOR AXIAL-VECTOR

Fermi and GT decays

$$g^2|M_{fi}| = \left|\langle Y | g\hat{O} | X\rangle\right|^2 =$$

$$= g_V^2 \left|\left\langle Y \left| \sum_{k=1}^{A} \hat{\tau}_{\pm}(k) \right| X \right\rangle\right|^2 + g_A^2 \sum_{m_f} \sum_{\mu} \left|\left\langle Y \left| \sum_{k=1}^{A} \hat{\sigma}_{\mu}(k)\hat{\tau}_{\pm}(k) \right| X \right\rangle\right|^2$$

$$|M_F{}^2| = \left|\left\langle Y \left| \sum_{k=1}^{A} \hat{\tau}_{\pm}(k) \right| X \right\rangle\right|^2 \qquad |M_{GT}{}^2| = \sum_{m_f} \sum_{\mu} \left|\left\langle Y \left| \sum_{k=1}^{A} \hat{\sigma}_{\mu}(k)\hat{\tau}_{\pm}(k) \right| X \right\rangle\right|^2$$

$$g^2|M_{fi}| = g_V^2|M_F|^2 + g_A^2|M_{GT}|^2 = g_V^2\left(|M_F|^2 + \left(\frac{g_A}{g_V}\right)^2 |M_{GT}|^2\right)$$

$$\boxed{f\,T_{1/2}^{i\to f} = \frac{K}{g^2|M_{fi}|^2} = \frac{K}{g_V^2\left(|M_F|^2 + \lambda^2|M_{GT}|^2\right)}}$$

$$\lambda = \frac{g_A}{g_V}$$

$$K = \frac{2\ln 2\,\pi^3\hbar^7}{m_e^5 c^4} = 4.794\cdot 10^{-5}\, MeV^2\cdot fm^6\cdot s$$

A complete set of selection rules (including isospin)

Fermi transitions

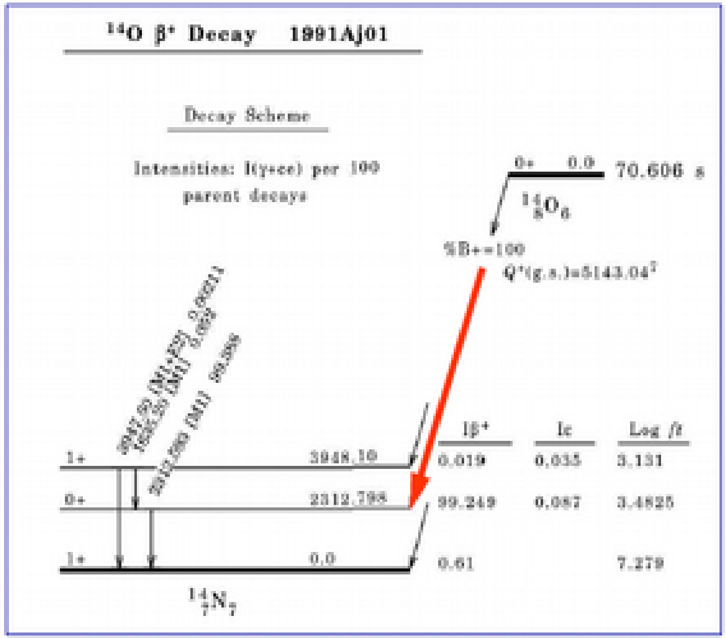
$$
\begin{array}{ll}
J_f = J_i & (\Delta J = 0) \\
T_f = T_i \neq 0 & \left(\Delta T = 0,\ \text{but } T_i = 0 \to T_f = 0 \text{ forbidden}\right) \\
T_{0f} = T_{0i} \mp 1 & (\Delta T_0 = 1) \\
\Delta\pi = 0 & \text{no parity change}
\end{array}
$$

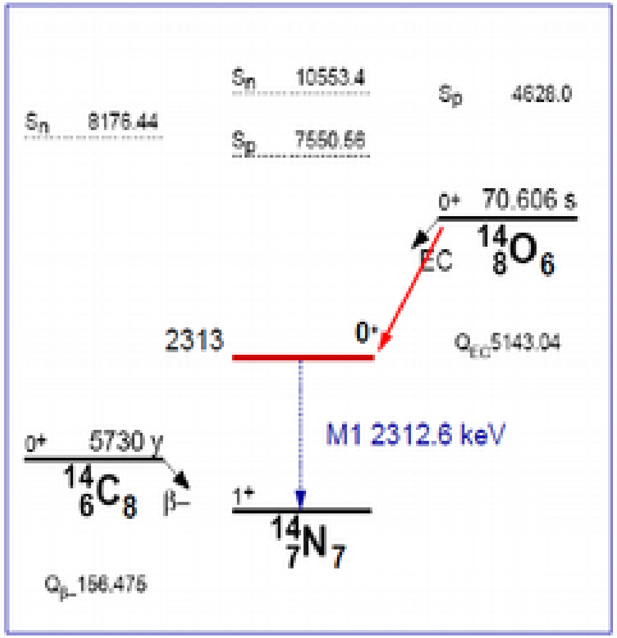
Gamow-Teller transitions

$$
\begin{array}{ll}
\Delta J = 0,1 & \text{but } J_i = 0 \to J_f = 0 \text{ forbidden} \\
\Delta T = 0,1 & \text{but } T_i = 0 \to T_f = 0 \text{ forbidden} \\
T_{0f} = T_{0i} \mp 1 & (\Delta T_0 = 1) \\
\Delta\pi = 0 & \text{no parity change}
\end{array}
$$

An example of pure Fermi-type decay

Isobaric analog states (IAS)





$$Q_\beta = Q_{EC} - 2\cdot m_e \cdot c - E_{0^+} = 3145\,keV - 1022\,keV - 2312.8\,keV$$

$$T_{1/2}^{0^+\to 0^+} = \frac{T_{1/2}}{I^{0^+\to 0^+}} = \frac{70.606\,s}{0.99249} = 71.140\,s$$

$$f(Z,Q)\,T_{1/2} = 3057\,s$$

An example of pure Fermi-type decay

for the GT transition

$$f\,T_{1/2}^{0\to 0} = \frac{K}{g_V^2 |M_F|^2}$$

$J_i^\pi \to J_f^\pi = 0^+ \to 0^+$ is forbidden $|M_{GT}|^2 = 0$

$$|M_F{}^2| = \left| \left\langle Y \left| \sum_{k=1}^{A} \hat{\tau}_\pm(k) \right| X \right\rangle^2 \right| = \left| \left\langle Y \right| \hat{T}_\pm(k) \left| X \right\rangle^2 \right|$$

$$\hat{T}_\pm \left| T, T_Z \right\rangle = \sqrt{(T \mp T_Z)(T \pm T_Z + 1)} \left| T, T_Z \pm 1 \right\rangle$$

$$|M_F{}^2| = \left| \left\langle Y \right| \hat{T}_\pm(k) \left| X \right\rangle^2 \right| = \left| \left\langle T, T_Z \pm 1 \right| \hat{T}_\pm(k) \left| T, T_Z \right\rangle^2 \right| = (T \mp T_Z)(T \pm T_Z + 1)$$

We only need to know the total isospin of the state.

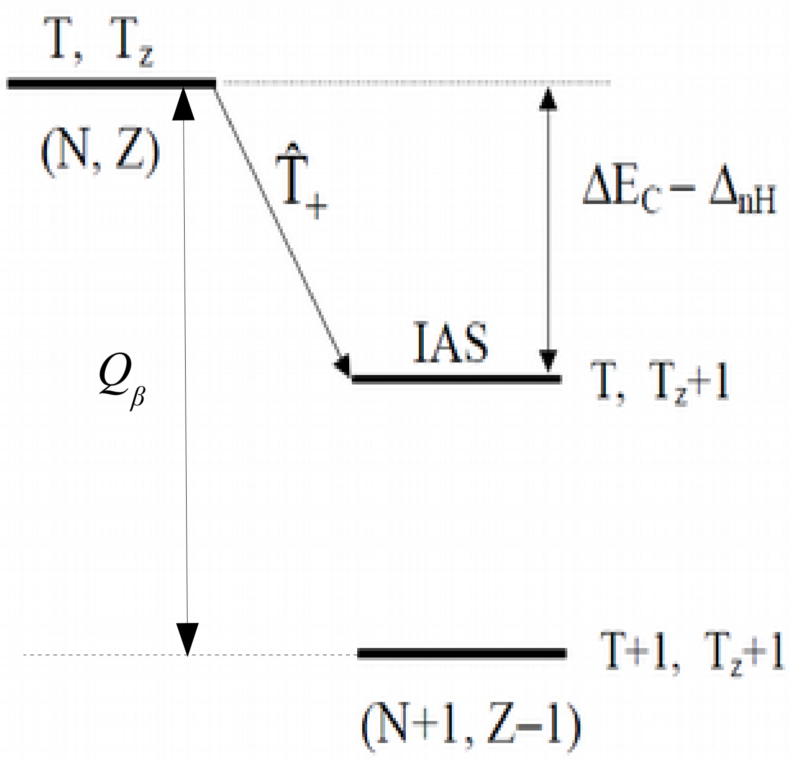
T=1 for ^{14}O $\quad |M_F|^2 = 2$

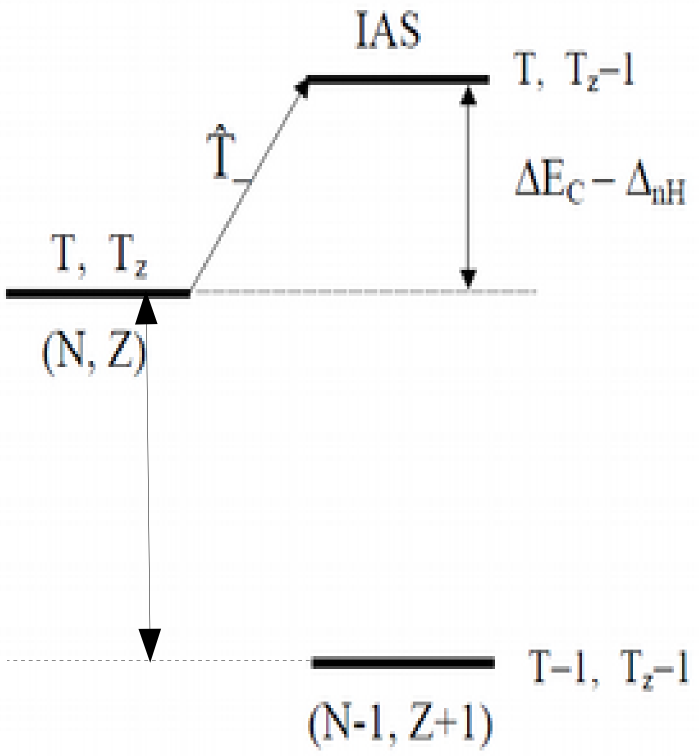
$$f(Z, Q)\, T_{1/2} = 3057\, s$$

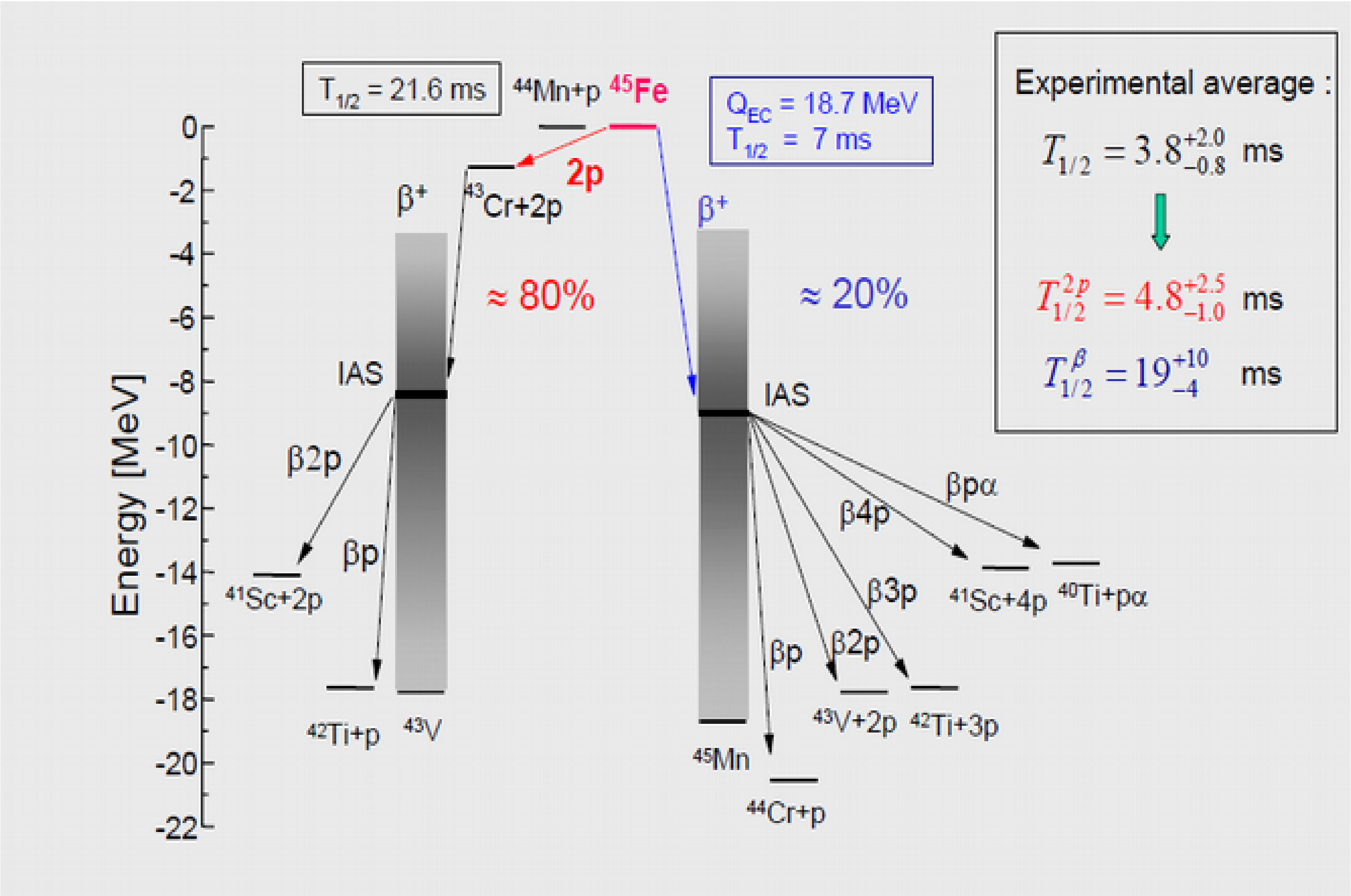
$$g_V = \left(\frac{4.794 \cdot 10^{-5}}{2 \cdot 3057} \right)^{1/2} = 0.885 \cdot 10^{-4}\, MeV\, fm^3 \qquad K = \frac{2 \ln 2\, \pi^3 \hbar^7}{m_e^5 c^4} = 4.794 \cdot 10^{-5}\, MeV^2 \cdot fm^6 \cdot s$$

The IAS states on both side of N=Z line

Neutron deficient (T_z<0) or N<Z	Neutron rich (T_z>0) or N>Z







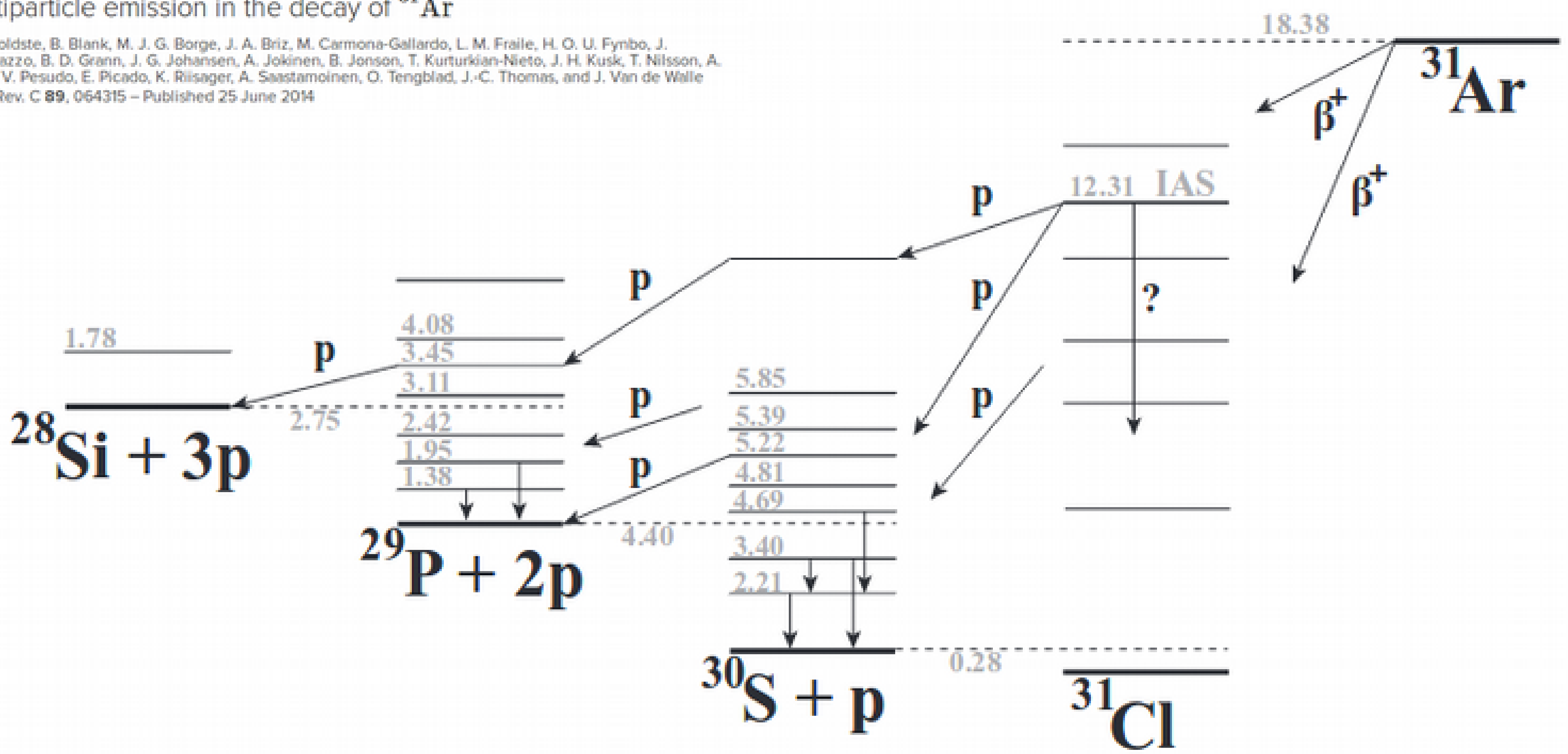
$T_{1/2}$ = 21.6 ms
^{44}Mn+p
^{45}Fe
Q_{EC} = 18.7 MeV
$T_{1/2}$ = 7 ms
Experimental average :
$T_{1/2} = 3.8^{+2.0}_{-0.8}$ ms
$T^{2p}_{1/2} = 4.8^{+2.5}_{-1.0}$ ms
$T^{\beta}_{1/2} = 19^{+10}_{-4}$ ms
Energy [MeV]
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
2p
^{43}Cr+2p
β^+
β^+
≈ 80%
≈ 20%
IAS
IAS
β2p
βp
^{41}Sc+2p
^{42}Ti+p
^{43}V
βpα
β4p
β3p
β2p
βp
^{41}Sc+4p
^{40}Ti+pα
^{43}V+2p
^{42}Ti+3p
^{45}Mn
^{44}Cr+p

IAS states

Multiparticle emission in the decay of ^{31}Ar

G. T. Koldste, B. Blank, M. J. G. Borge, J. A. Briz, M. Carmona-Gallardo, L. M. Fraile, H. O. U. Fynbo, J. Giovinazzo, B. D. Grann, J. G. Johansen, A. Jokinen, B. Jonson, T. Kurturkian-Nieto, J. H. Kusk, T. Nilsson, A. Perea, V. Pesudo, E. Picado, K. Riisager, A. Saastamoinen, O. Tengblad, J.-C. Thomas, and J. Van de Walle

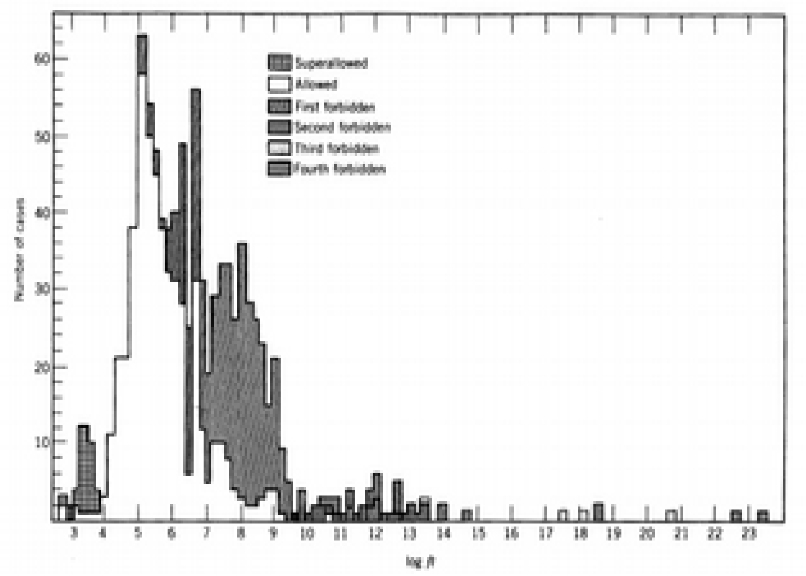
Phys. Rev. C **89**, 064315 – Published 25 June 2014



Gamow-Teller and Fermi decay summary
(if we know the fundamental constants)

$$f\,T_{1/2}^{i\to f} = \frac{K/g_V^2\,(1+\Delta_R)}{|M_F|^2+\lambda^2|M_{GT}|^2} = \frac{6145\ \mathrm{s}}{|M_F|^2+\lambda^2|M_{GT}|^2} \qquad \lambda^2 = 1.612$$

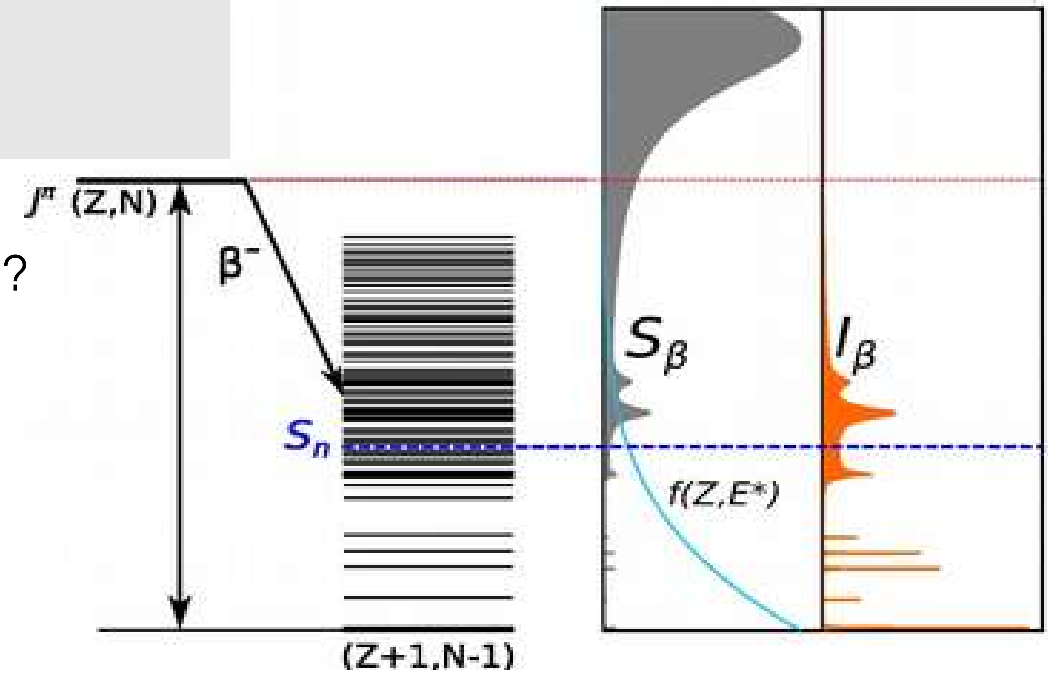
Now we know, how to make the connection between experimentally measured branching ratios (ft) and nuclear matrix elements !



Decay strength distribution lifetimes and branching ratios

$$\frac{1}{T_{1/2}} = \sum_{E_i \geq 0}^{E_i \leq Q_\beta} S_\beta(E_i) \times f(Z, Q_\beta - E_i) \qquad S_\beta(E_i) = \langle \psi_f | \hat{O}_\beta | \psi_{mother} \rangle$$

Decay branching ratio measurements provide the critical validation of the theoretical models of beta decay.

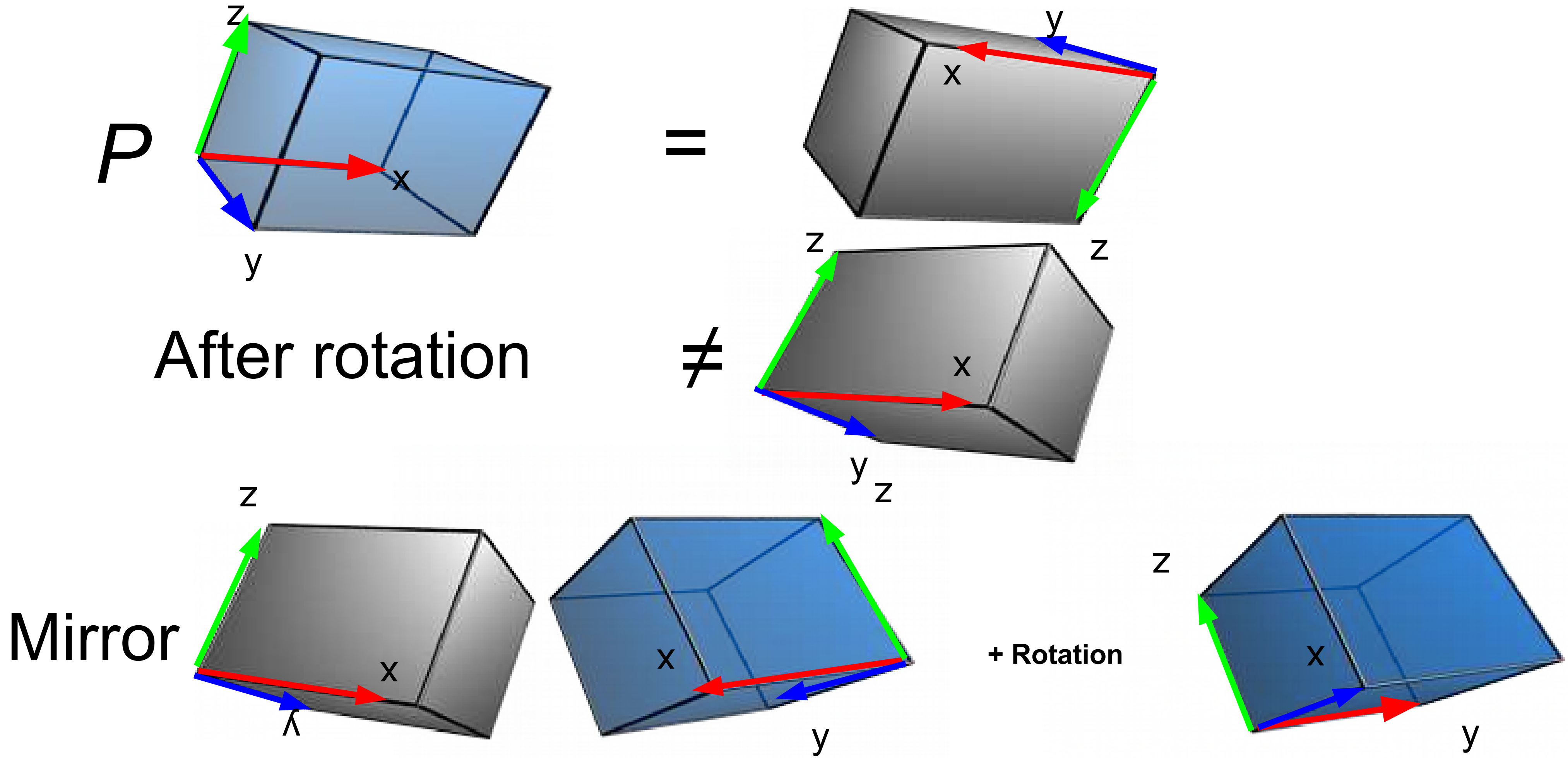


Location of decay strength relative to g.s. ?
Competing allowed and forbidden transitions ?

Role of the phase space factor !

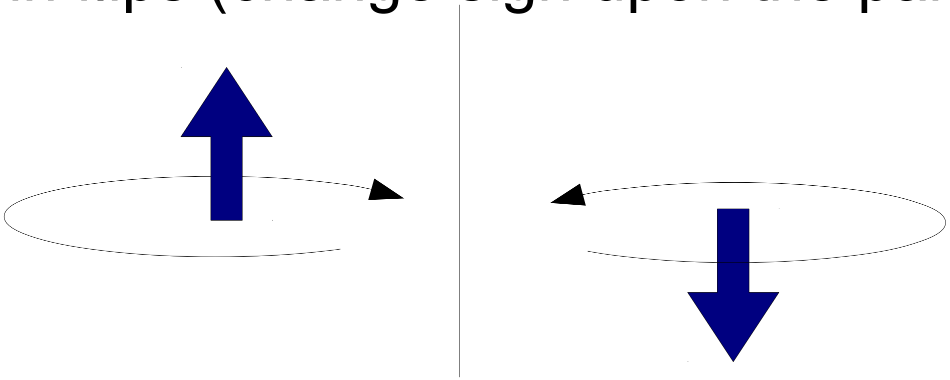
Parity violation in weak interactions

$P\vec{r} = -\vec{r} \qquad P(x, y, z) = (-x, -y, -z)$



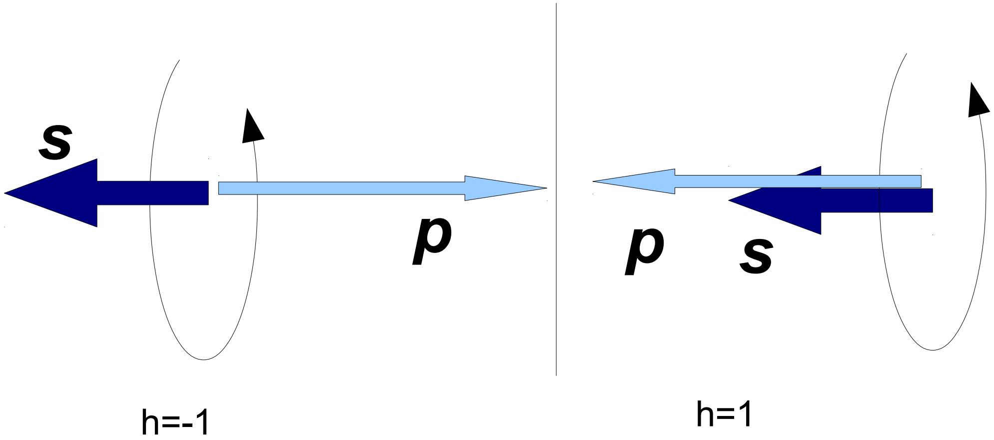
In 3D Parity operator is equivalent to mirror reflection

Nuclear spin flips (change sign upon the parity operator).

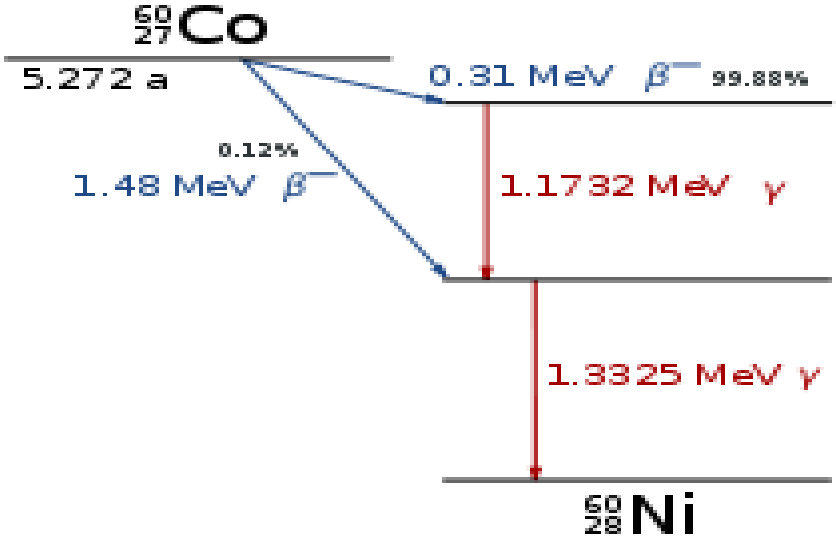


For the moving particle with spin one can define ***helicity*** which is a spin projection over the particle linear momentum.

$$h = \frac{\vec{s}\,\vec{p}}{|\vec{s}\,\vec{p}|}$$



The Mme Wu experiment (polarized 60Co source) Correlation between electron direction and nuclear spin direction.



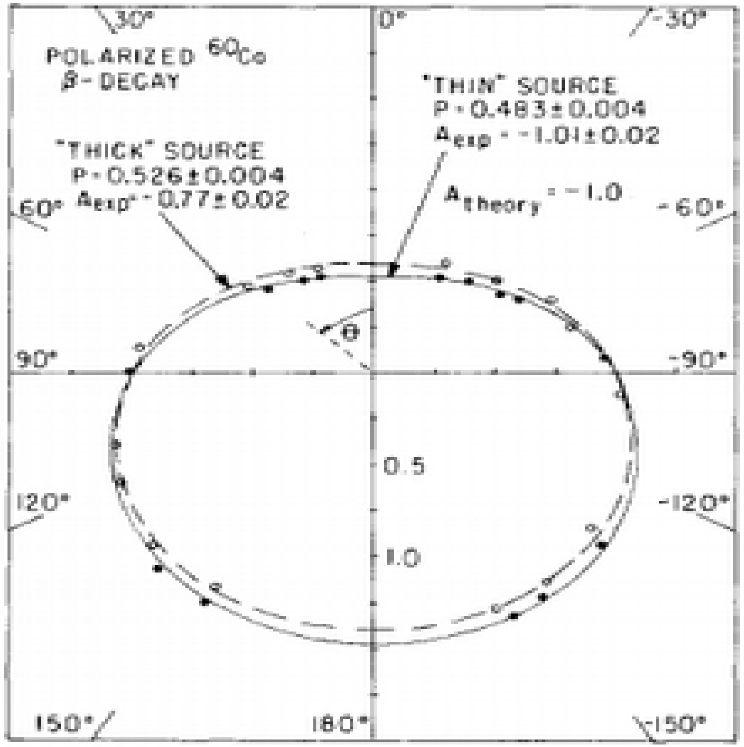
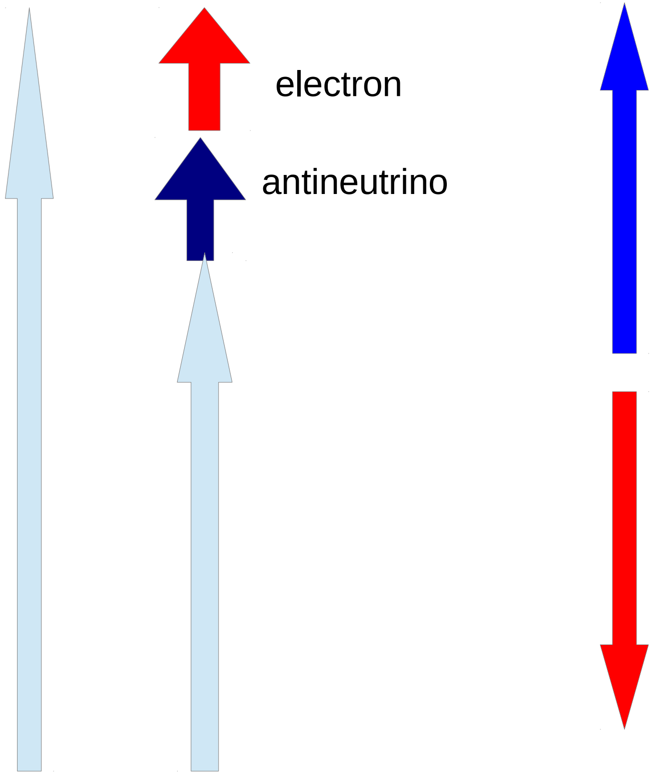
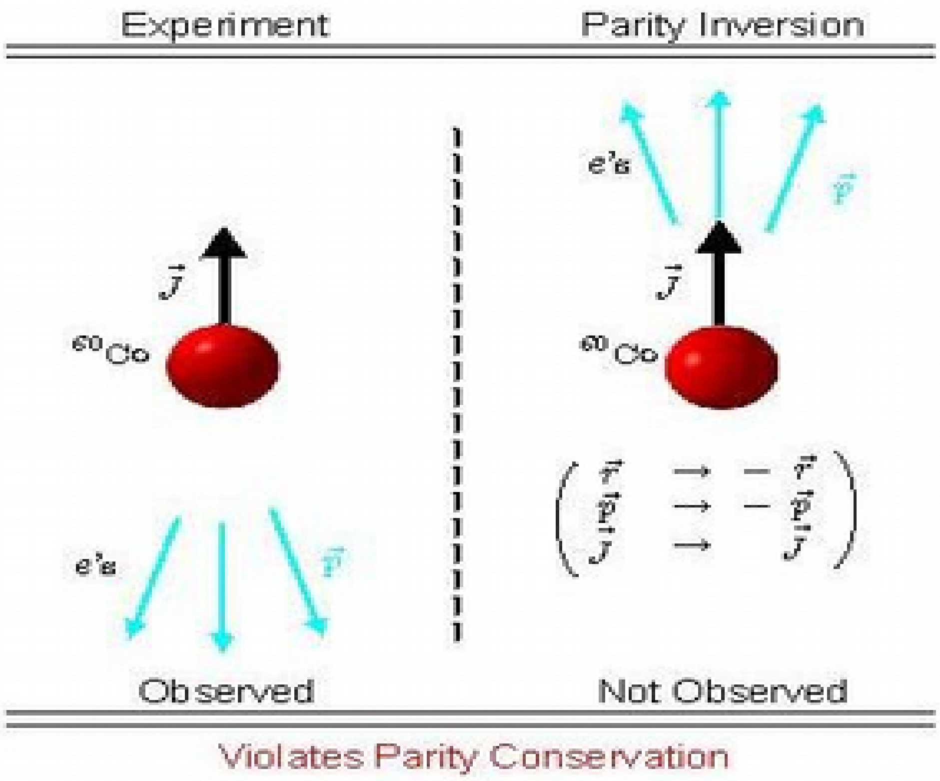


Fig. 1. Directional distribution of β-particles from polarized ^{60}Co. Here $1 + \{[W(\theta - 1)] (v/c)^{-1}\}_{v/c}$ is plotted as a function of θ. The A_{exp} values shown on the figure were obtained by least-squares fitting our data to a function of the form $1 + PA_{exp} \cos\theta$. The error bars shown on A_{exp} are three times the statistical errors obtained from the least-squares analysis.

GT: 5+ $\rightarrow$ 4+ ($\Delta L=0, \Delta S=1$)

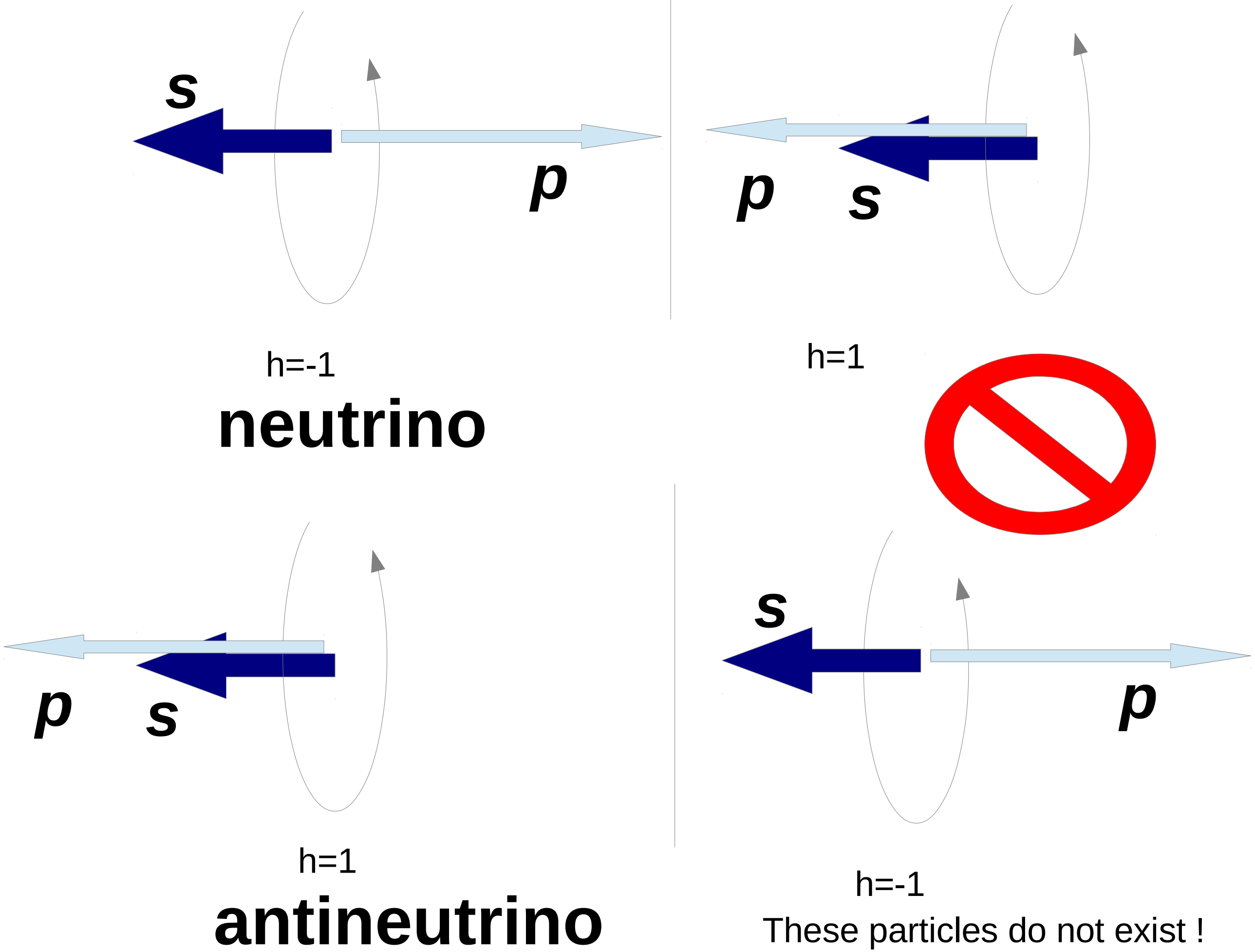


ANGULAR MOMENTA ***LINEAR MOMENTA***



For the electron h= -v/c

For the antineutrino h= +1



The symmetry can be restored with CP operation
(C- charge conjugation)