The Interacting Boson Model

A quick introduction - TALENT course no. 5 (2017)

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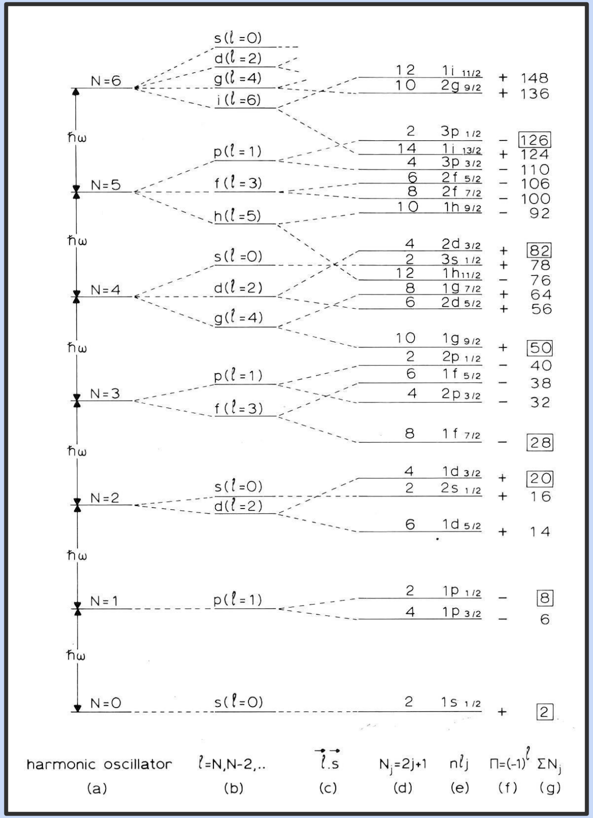
Introduction

Table of contents

Introduction

Motivation



Introduction

Boson operators and observables

$$s^\dagger \ (L^p = 0^+),\ d^\dagger_\mu \ (\mu = 0, \pm 1, \pm 2;\ L^p = 2^+); \qquad b^\dagger_\alpha \in \{s^\dagger, d^\dagger\}$$

$$b^\dagger_1 = s^\dagger,\ \ b^\dagger_2 = d^\dagger_2,\ \ b^\dagger_3 = d^\dagger_1,\ \ b^\dagger_4 = d^\dagger_0,\ \ b^\dagger_5 = d^\dagger_{-1},\ \ b^\dagger_6 = d^\dagger_{-2}$$

$$G_{\alpha\beta} = b^\dagger_\alpha b_\beta$$

Introduction

Boson operators and observables

$s^\dagger$ $(L^{p} = 0^{+})$, $d^\dagger{}_\mu$ $(\mu = 0, \pm 1, \pm 2; L^{p} = 2^{+})$. $\qquad b^\dagger{}_\alpha \in \{s^\dagger, d^\dagger\}$

Hamiltonian: $H = \sum_{\alpha\beta} \varepsilon_{\alpha\beta} G_{\alpha\beta} + \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} G_{\alpha\beta} G_{\gamma\delta} + \ \ldots \qquad (G_{\alpha\beta} = b^\dagger{}_\alpha b_\beta)$

Introduction

Boson operators and observables

$s^{\dagger}\ (L^{\mathrm{p}} = 0^{+}),\ d^{\dagger}_{\mu}\ (\mu = 0, \pm 1, \pm 2;\ L^{\mathrm{p}} = 2^{+}).$ $\qquad b^{\dagger}_{\alpha} \in \{s^{\dagger}, d^{\dagger}\}$

Hamiltonian: $H = \sum_{\alpha\beta} \varepsilon_{\alpha\beta} G_{\alpha\beta} + \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} G_{\alpha\beta} G_{\gamma\delta} \underbrace{+\ \ldots}_{\text{Higher-order terms}}$ $\qquad (G_{\alpha\beta} = b^{\dagger}_{\alpha} b_{\beta})$

Introduction

Boson operators and observables

- $E2$ transitions: $T^{(E2)} = e_B[(d^\dagger s + s^\dagger \tilde{d})^{(2)} + \chi\,(d^\dagger \tilde{d})^{(2)}]$

$$B(E2;\, L_i \longrightarrow L_f) = \frac{1}{2L_i+1}\left|\langle L_f\,||T^{(E2)}\,||L_i\rangle\right|^2$$

- $M1$ transitions: $T^{(M1)} = c\,(d^\dagger\,\tilde{d})^{(1)}$ - see Tobi's talk!

Introduction

Lie algebras and dynamic symmetries

- $[s, s^\dagger] = 1;$ $\qquad [d_{\mu'}, d^\dagger_{\mu}] = \delta_{\mu'\mu}$

- $G_{\alpha\beta} = b^\dagger_{\alpha} b_{\beta}$ - generators of U(6) algebra.

- $\mathscr{N} \underbrace{b^\dagger_{\alpha} b^\dagger_{\beta} \ldots}_{N\ times} |0\rangle \equiv |N\rangle ; \quad b_{\beta}|0\rangle = 0$

Totally symmetric irreducible representation of U(6)

- Example - ${}^{110}_{48}\mathrm{Cd}_{62}$:

$$Z = 48,\ N = 62$$

$$N_\pi = 50 - 48 = 2\ ;\ N_\nu = 62 - 50$$

$$N = N_\pi/2 + N_\nu/2 = 7\ \textit{bosons!}$$

Introduction

U(6) spectrum generating algebra

$[N]$

U(6)

Algebra

$$\supset U(5) \supset O(5) \supset_{n_\Delta} O(3)$$

$$| \; n_d \quad \tau \quad L \; \rangle$$

Spherical vibrator

$$\supset SU(3) \supset_{K} O(3)$$

$$| \; (\lambda,\mu) \quad L \; \rangle$$

Axially-deformed rotor

$$\supset O(6) \supset O(5) \supset_{n_\Delta} O(3)$$

$$| \; \sigma \quad \tau \quad L \; \rangle$$

γ-unstable deformed rotor

Introduction

U(6) spectrum generating algebra

$$\boxed{N = N_\pi/2 + N_\nu/2} \longrightarrow [N]$$

U(6)

Algebra

$$U(6) \supset U(5) \supset O(5) \underset{n_\Delta}{\supset} O(3)$$
$$|\; n_d \quad \tau \quad L \;\rangle$$

Spherical vibrator

$$U(6) \supset SU(3) \underset{K}{\supset} O(3)$$
$$|\; (\lambda,\mu) \quad L \;\rangle$$

Axially-deformed rotor

$$U(6) \supset O(6) \supset O(5) \underset{n_\Delta}{\supset} O(3)$$
$$|\; \sigma \quad \tau \quad L \;\rangle$$

γ-unstable deformed rotor

Introduction

U(6) spectrum generating algebra

$$\overset{[N]}{\underset{\text{Algebra}}{\mathrm{U}(6)}} \supset \mathrm{U}(5) \supset \mathrm{O}(5) \underset{n_\Delta}{\supset} \mathrm{O}(3)$$

$$|\, n_d \quad \tau \quad L \,\rangle$$

Spherical vibrator

$$\mathrm{U}(6) \supset \mathrm{SU}(3) \underset{K}{\supset} \mathrm{O}(3)$$

$$|\, (\lambda,\mu) \quad L \,\rangle$$

Axially-deformed rotor

$$\mathrm{U}(6) \supset \mathrm{O}(6) \supset \mathrm{O}(5) \underset{n_\Delta}{\supset} \mathrm{O}(3)$$

$$|\, \sigma \quad \tau \quad L \,\rangle$$

γ-unstable deformed rotor

Dynamical symmetry (DS):

$$\hat{H} = \sum_k a_k C(G_k)$$

$$G_0 \supset G_1 \supset \ldots \supset G_n$$

$$|\alpha_0 \quad \alpha_1 \quad \ldots \quad \alpha_n\rangle$$

- Casimir operator: $[C_p\,,\ G_{\alpha\beta}\,] = 0.$

- Example: $C_p\,[\mathrm{SO}(3)] = L^2\,;\ \ [L^2, L_\mathrm{x}\,] = [L^2, L_\mathrm{y}\,] = [L^2, L_\mathrm{z}\,] = 0$

  In the IBM: $L_\mu = (d^\dagger\,\tilde{d}\,)^{(1)}{}_{\mu}$

Introduction

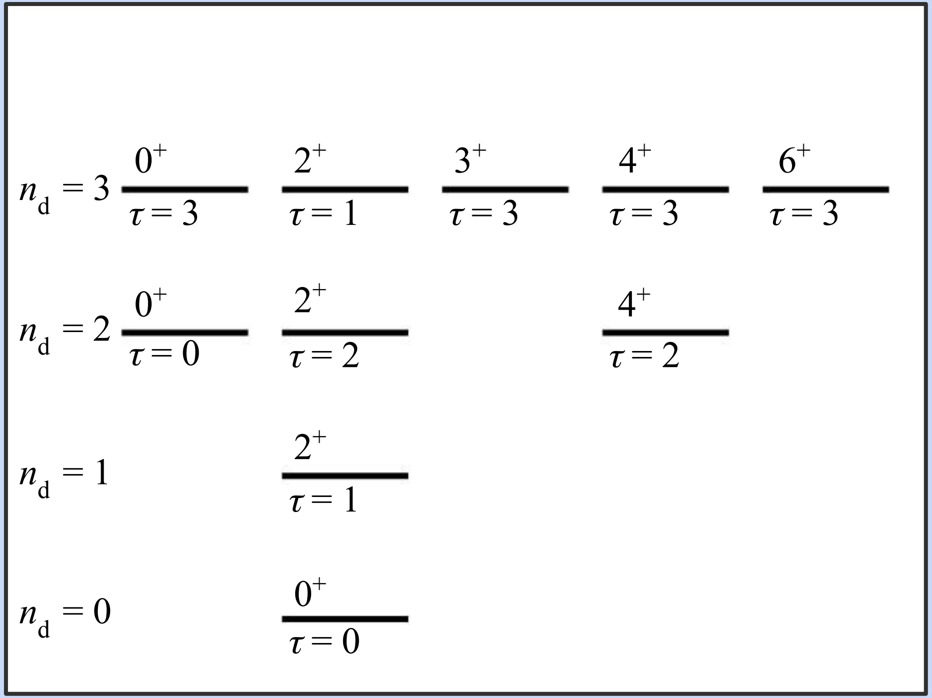
The IBM Hamiltonian

$$H = e_0 + e_1 C_1[\mathrm{U}(6)] + e_2 C_2[\mathrm{U}(6)] + \varepsilon C_1[\mathrm{U}(5)] + \alpha C_2[\mathrm{U}(5)]$$
$$+ \beta C_2[\mathrm{O}(5)] + \gamma C_2[\mathrm{O}(3)] + \delta C_2[\mathrm{SU}(3)] + \eta C_2[\mathrm{O}(6)]$$

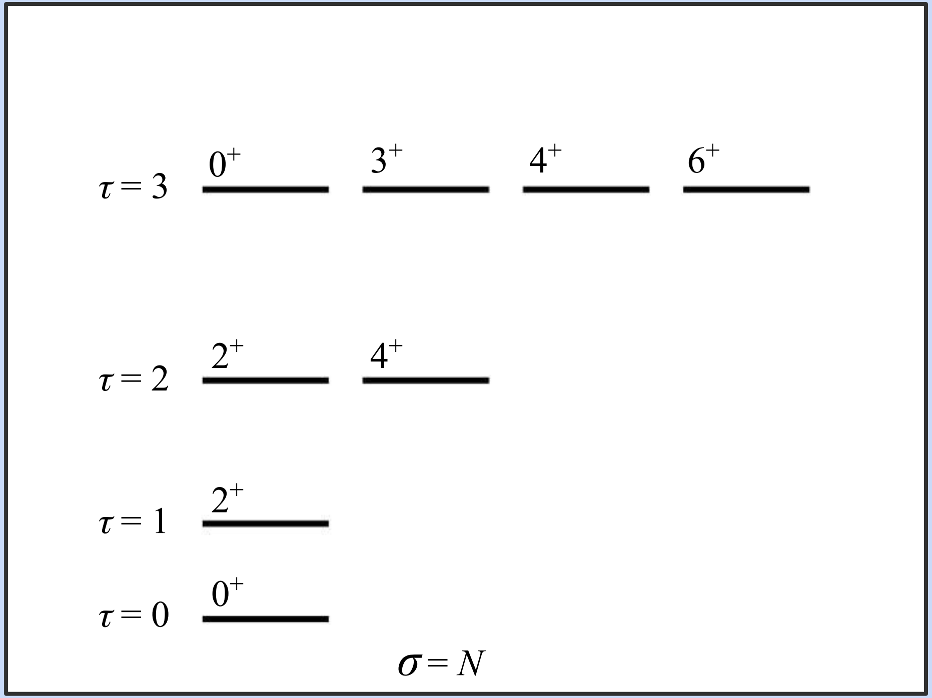
Introduction

Spherical vibrator and γ-unstable deformed rotor

Spherical vibrator - U(5)



γ-unstable deformed rotor - O(6)



Introduction

Geometric Interpretation - Intrinsic State Formalism

- Intrinsic states:

$$|N;\beta,\gamma\rangle = (N!)^{-½}(b_C^\dagger)^N|0\rangle$$

$$b_C^\dagger = (1+\beta^2)^{-½}\,[\beta\cos\gamma\cdot d_0^\dagger + \frac{1}{\sqrt{2}}\beta\sin\gamma\cdot(d_2^\dagger+d_{-2}^\dagger) + s^\dagger] \qquad \beta \geq 0,\ 0 \leq \gamma \leq \pi/3$$

Introduction

Geometric Interpretation - Intrinsic State Formalism

- Intrinsic states:

$$|N;\beta,\gamma\rangle = (N!)^{-1/2}(b_{\mathrm{C}}^{\dagger})^{N}|0\rangle$$

- The energy surface (“PES”):

$$E_N(\beta,\gamma) = \langle N;\beta,\gamma|\, \hat{H}\, |N;\beta,\gamma\rangle$$

Introduction

Geometric Interpretation - Intrinsic State Formalism

- Minimize $E_N(\beta,\gamma)$ $\quad |N;\beta,\gamma\rangle = (N!)^{-1/2}(b_C^\dagger)^N|0\rangle$

U(5) chain: $|N;\beta_{\mathrm{eq}} = 0, \gamma\rangle = |[N]; n_d = \tau = L = 0\rangle$

O(6) chain: $|N;\beta_{\mathrm{eq}} = 1, \gamma\rangle = |[N]; \sigma = N\rangle$

SU(3) chain: $|N;\beta_{\mathrm{eq}} = \sqrt{2}, \gamma_{\mathrm{eq}} = 0\rangle = |[N]; (2N,0), K = 0\rangle$

Introduction

Quantum Phase Transitions

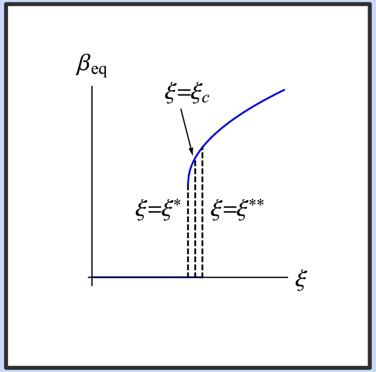
A first-order QPT (discontinuous)

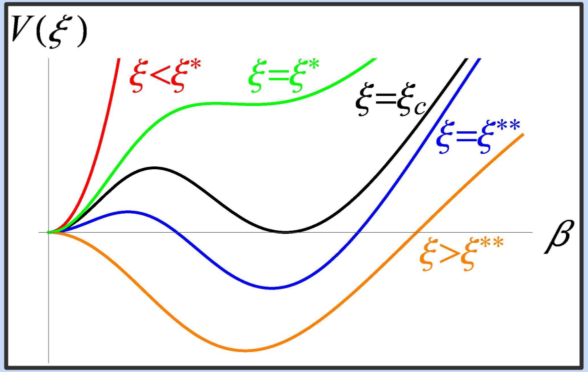
$\partial E_N(\beta_{eq}, \gamma_{eq})/\partial\xi|_{\xi=\xi c}$ is discontinuous.

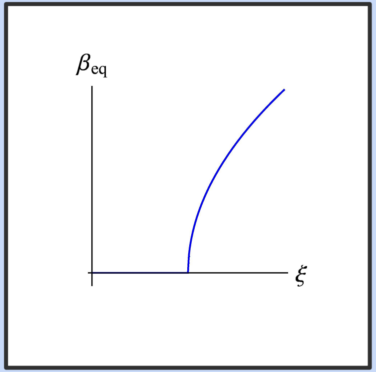
$$\boxed{H(\xi) = \xi\, H_1 + (1-\xi)H_2 \qquad 0 \leq \xi \leq 1}$$

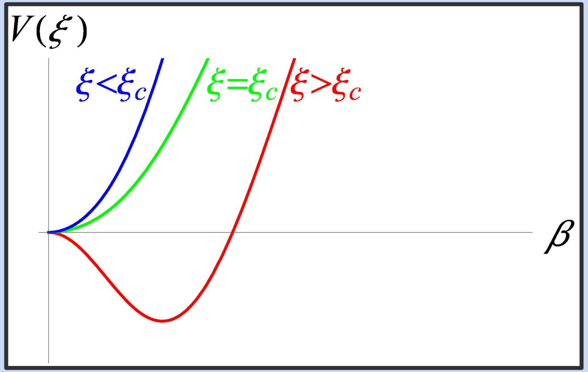
A second-order QPT (continuous)

$\partial^2 E_N(\beta_{eq}, \gamma_{eq})/\partial\xi^2|_{\xi=\xi c}$ is discontinuous.









Introduction

Extensions

- Adding more bosons, e.g.
  - IBM-*sdg* ($L^p = 4^+$): U(15) algebra.
  - IBM-*sdpf* ($L^p = 1^-, 3^-$): U(16) algebra.

- IBM-2 - distinguishing between proton bosons ($s^\dagger_\pi$, $d^\dagger_\pi$) and neutron boson ($s^\dagger_\nu$, $d^\dagger_\nu$): $U_\pi(6) \otimes U_\nu(6)$ algebra.

Example 1

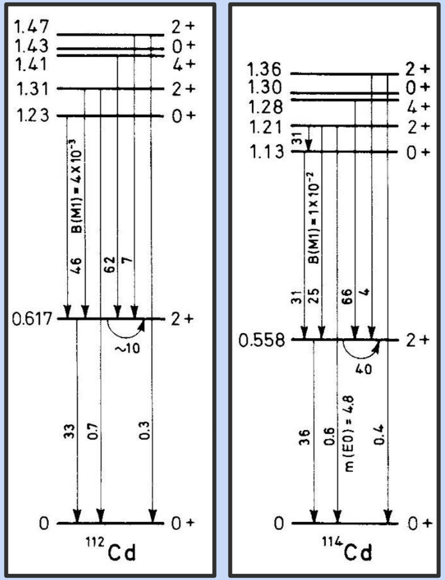
"U(5)-PDS and the vibrational structure of Cd nuclei"

Introduction

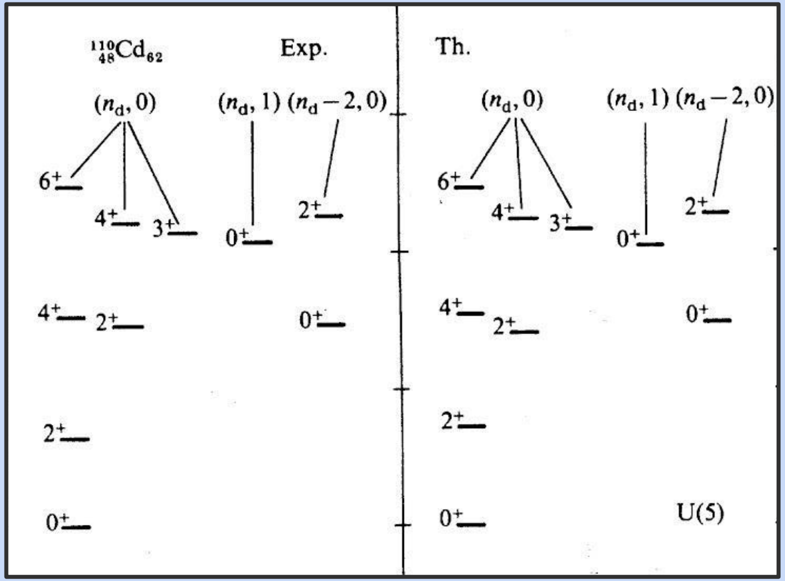
U(5)-PDS and the vibrational structure of Cd nuclei

Cd isotopes - a prime example for spherical nuclei.

Bohr & Mottelson



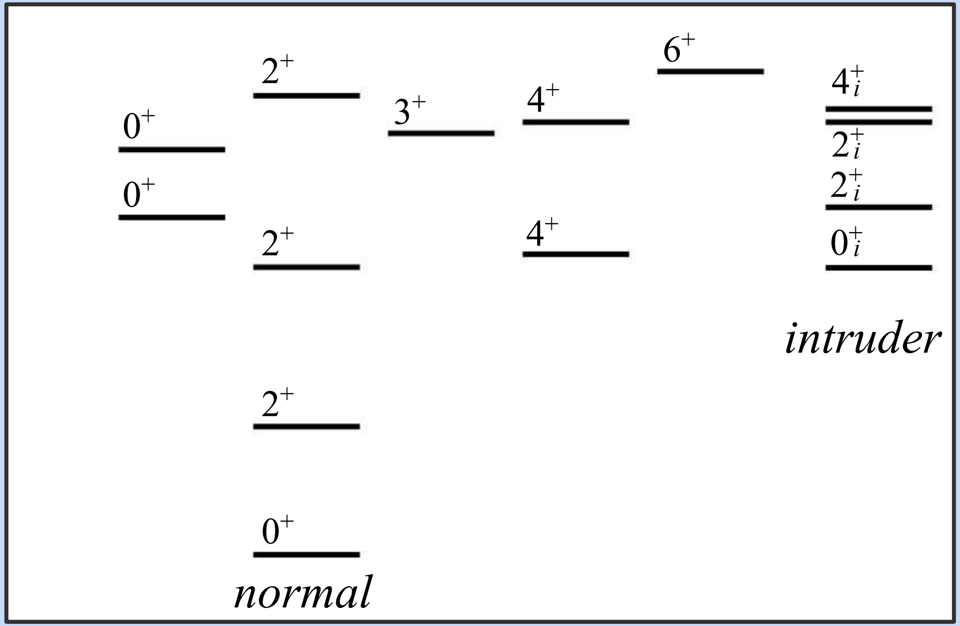
Arima & Iachello



Introduction

U(5)-PDS and the vibrational structure of Cd nuclei

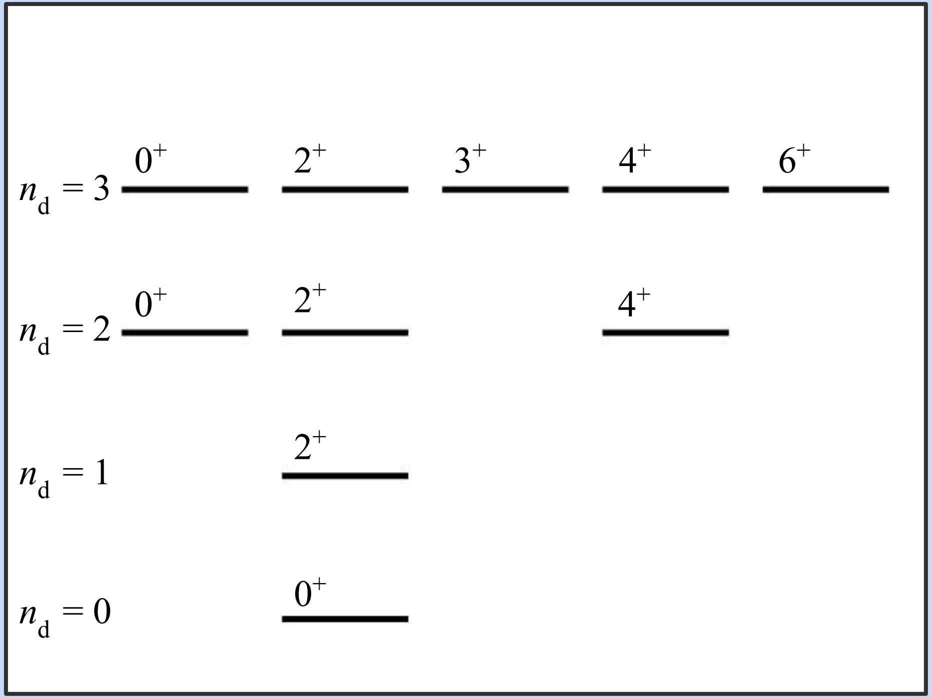
^{110}Cd experimental



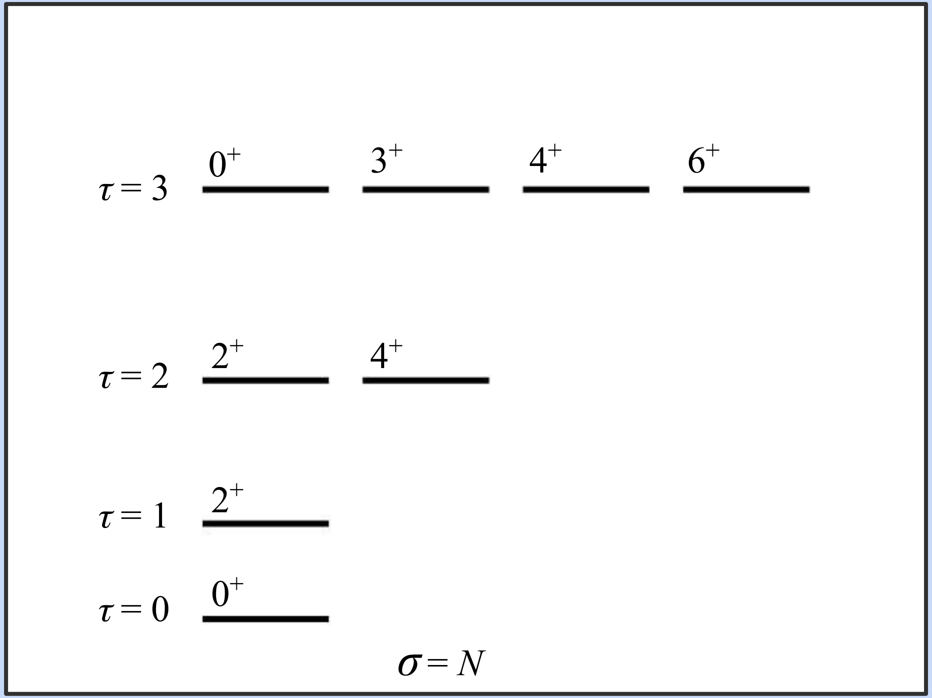
Introduction

U(5)-PDS and the vibrational structure of Cd nuclei

Spherical vibrator - U(5)



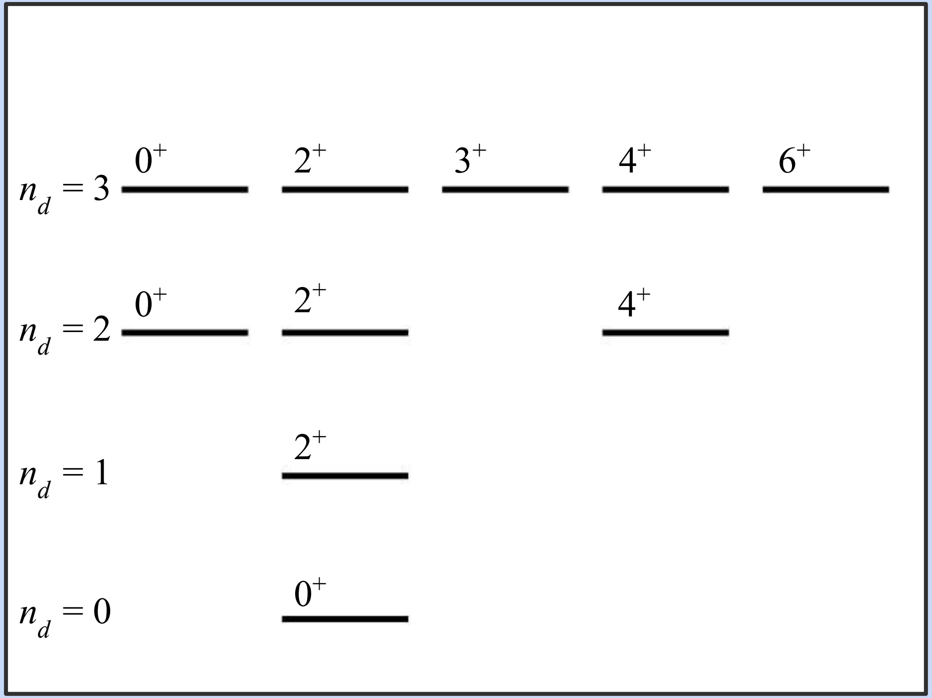
γ-unstable deformed rotor - O(6)



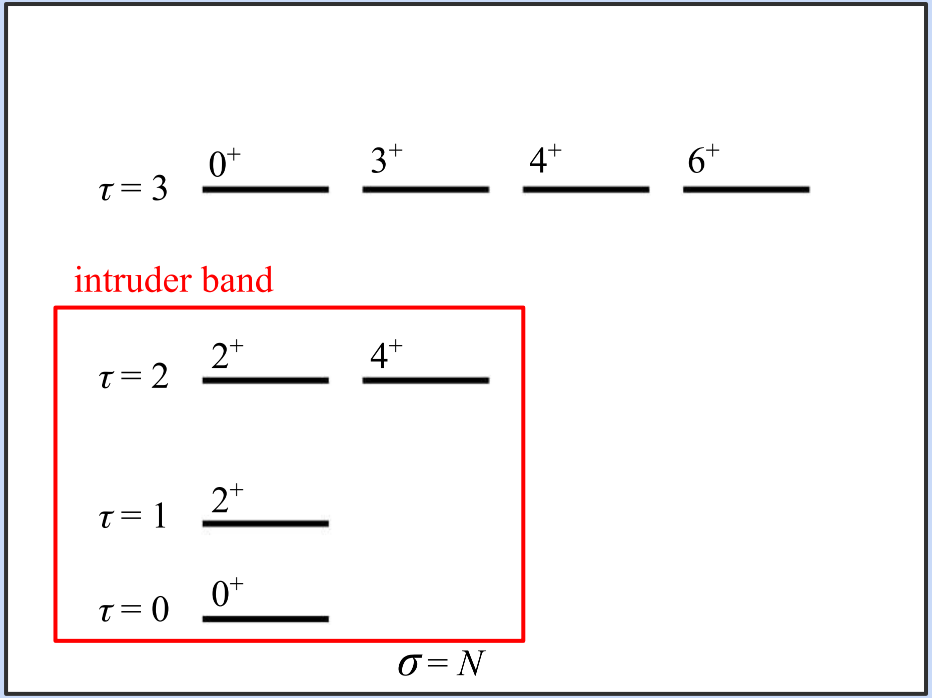
Introduction

U(5)-PDS and the vibrational structure of Cd nuclei

Spherical vibrator - U(5)



γ-unstable deformed rotor - O(6)

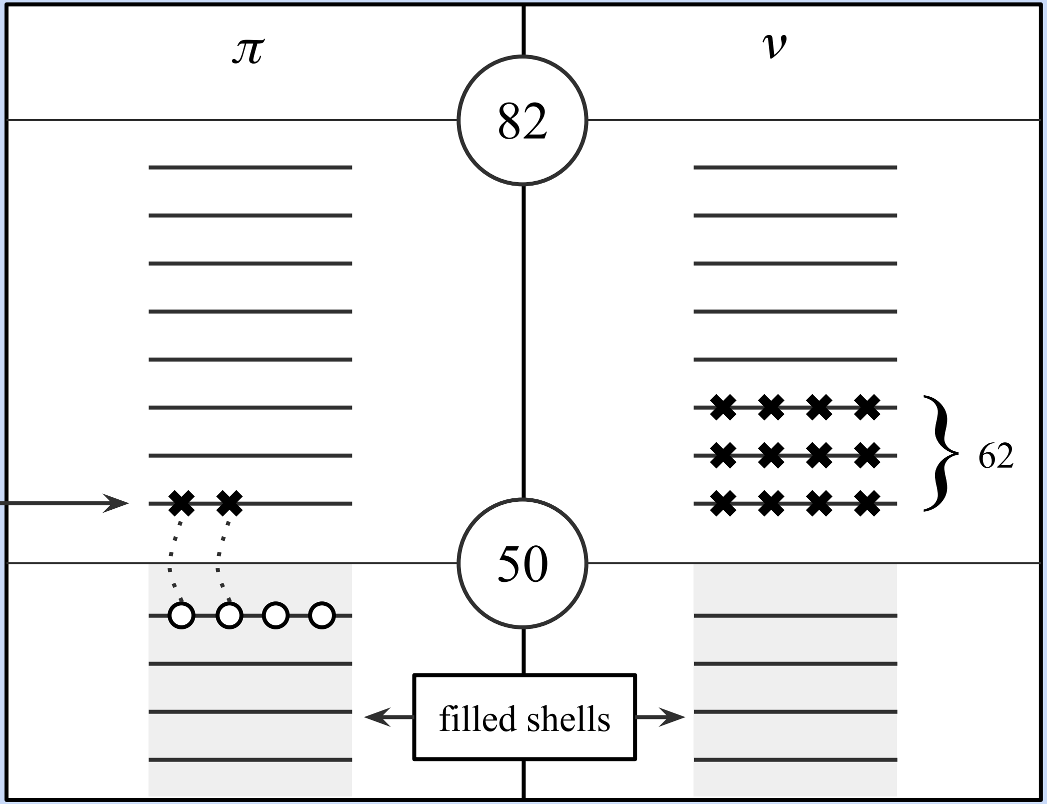


Introduction

U(5)-PDS and the vibrational structure of Cd nuclei

$^{110}_{48}\mathrm{Cd}_{62}$

(2p-4h)
proton “intruder”
excitations



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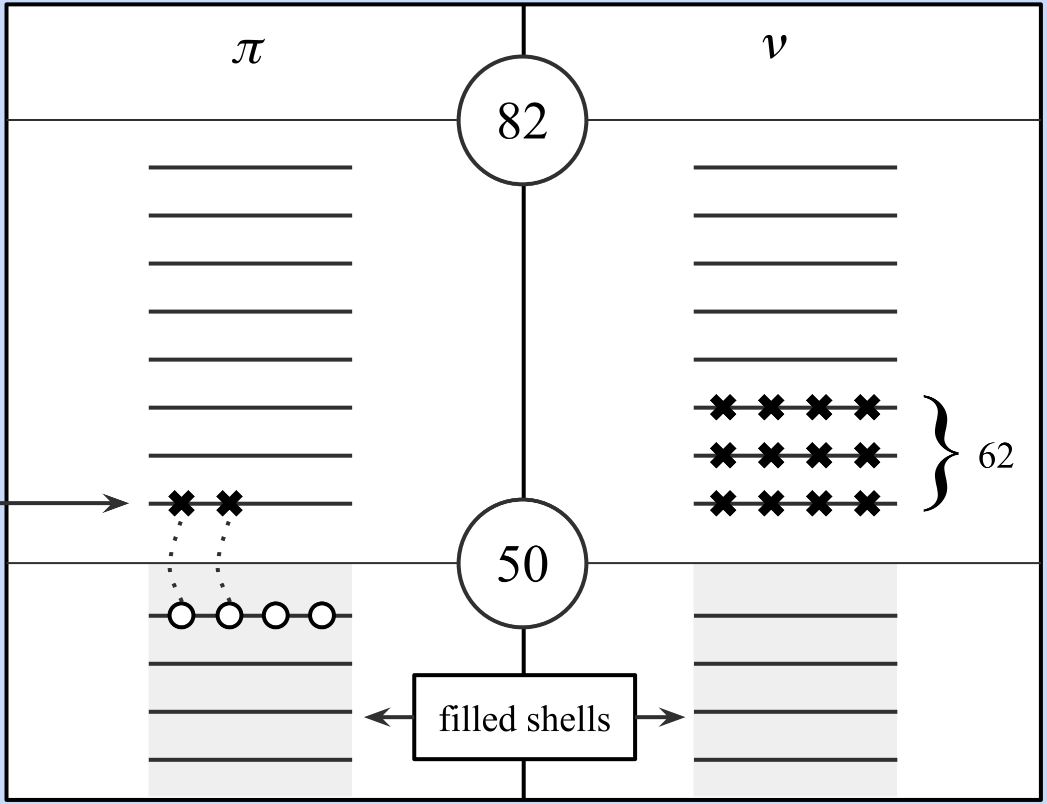


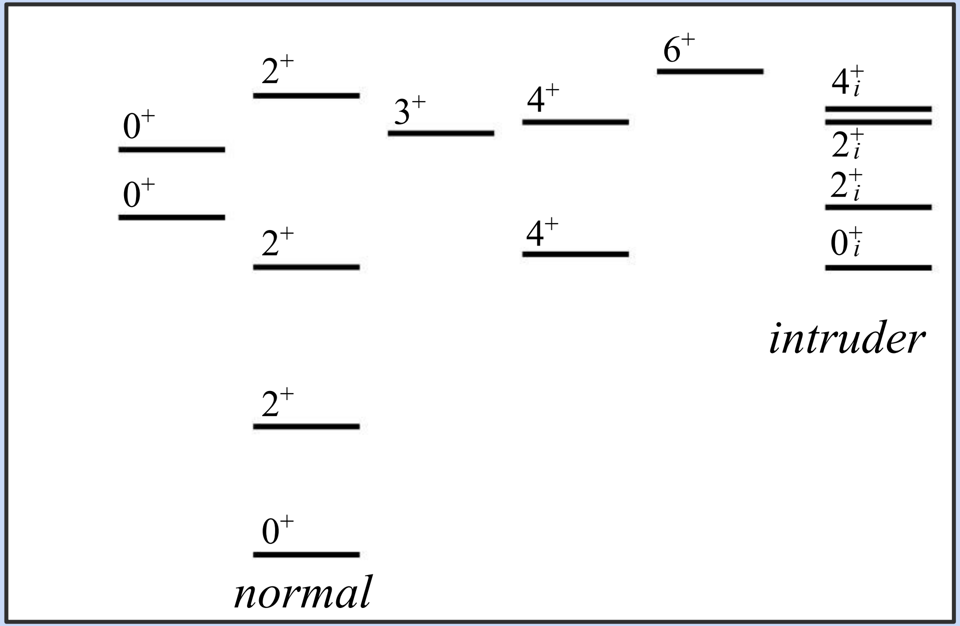
$$\binom{32}{12} =$$

225,792,840 configurations in the neutron shells.

Introduction

The problem of ^{110}Cd

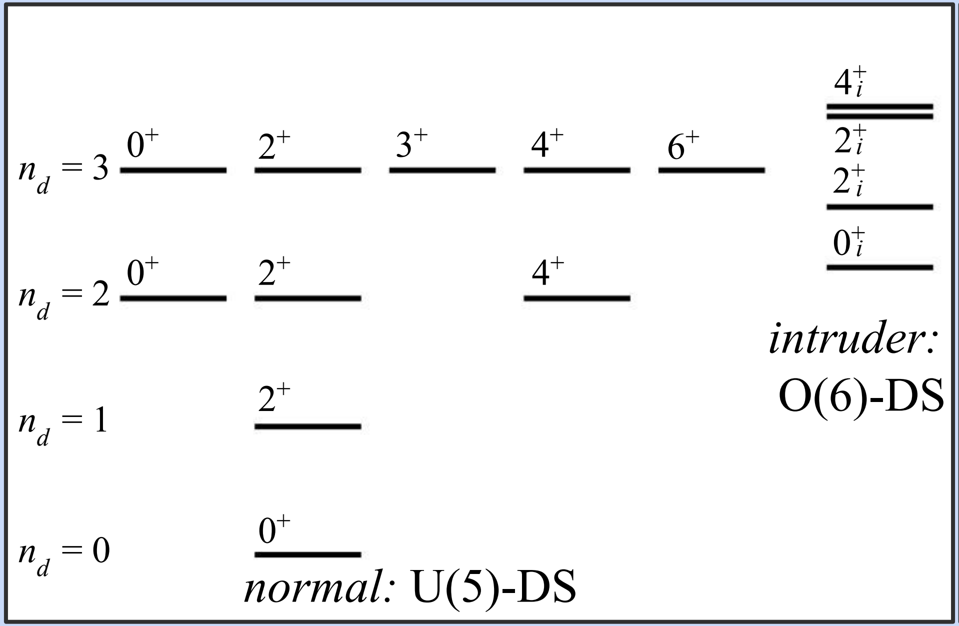
^{110}Cd experimental



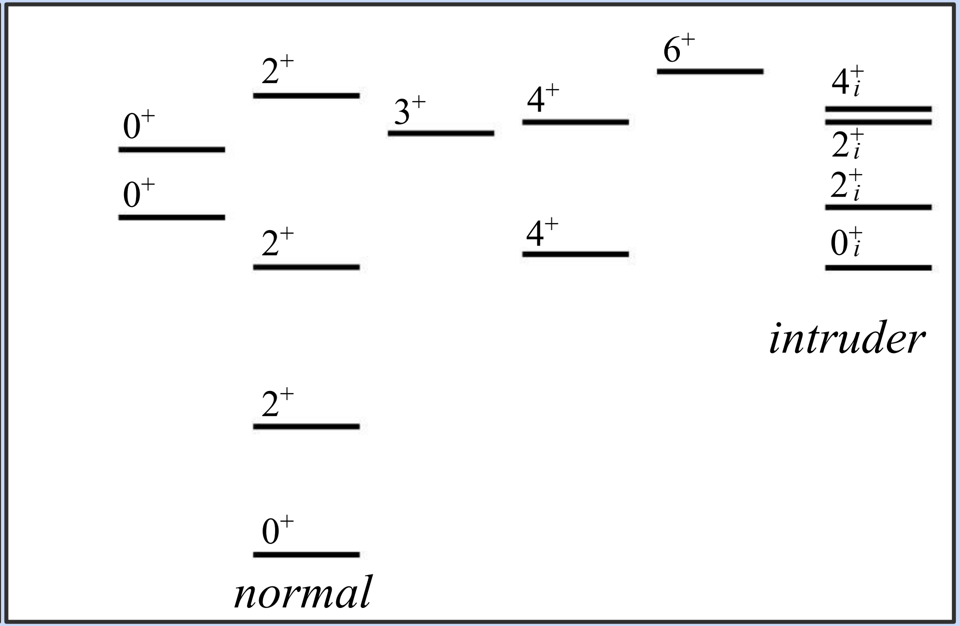
Introduction

The problem of ^{110}Cd

- Spherical vibrator - U(5) - normal band.
- γ-unstable deformed rotor - O(6) - intruder band.



^{110}Cd experimental



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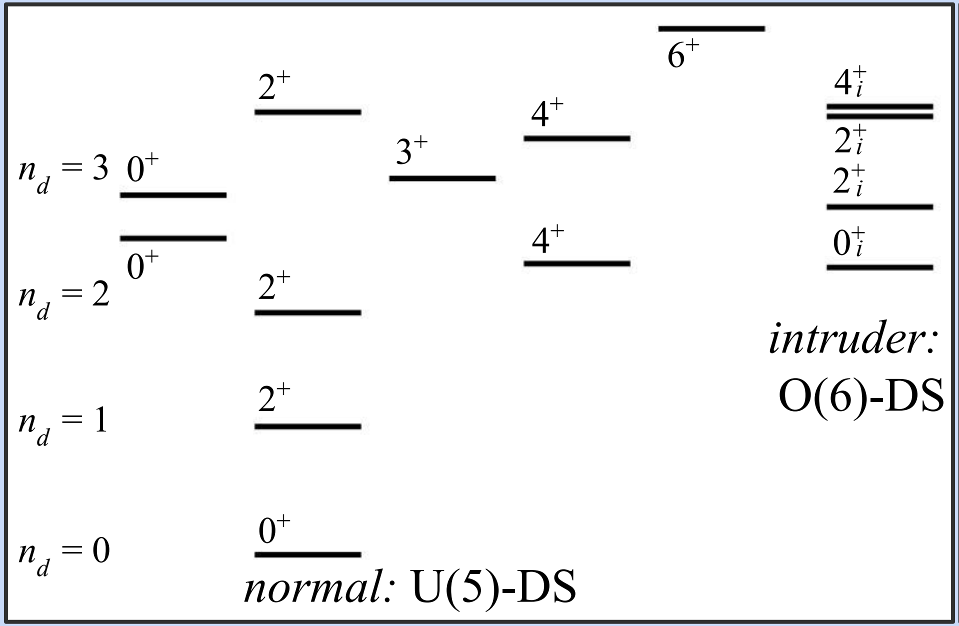
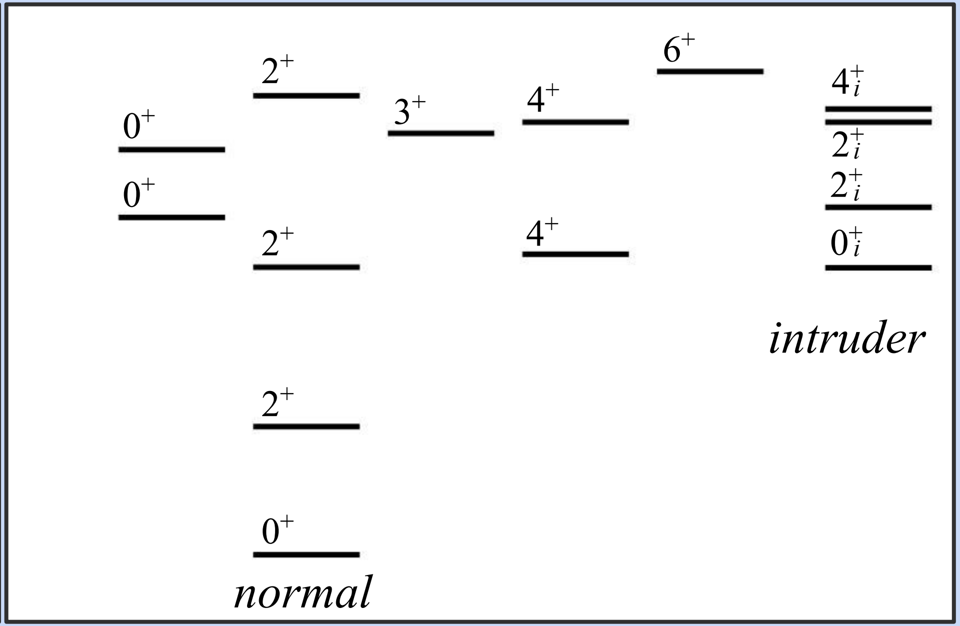


^{110}Cd experimental



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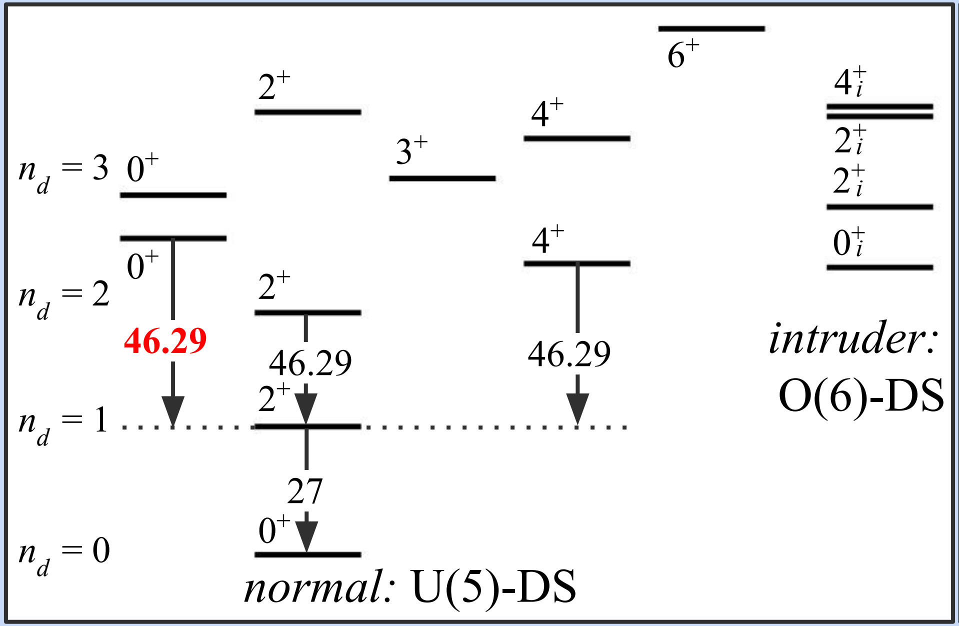

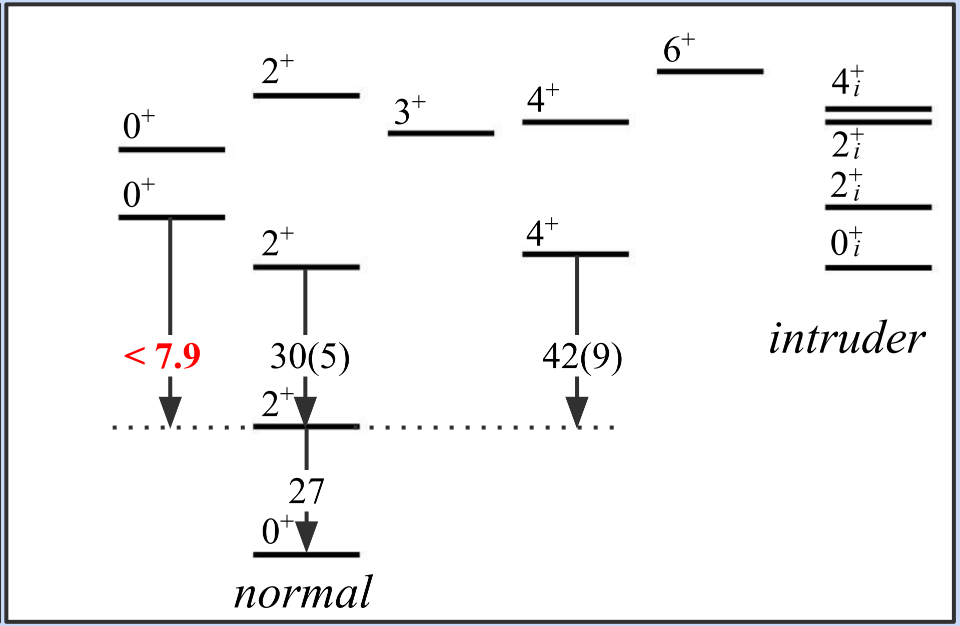

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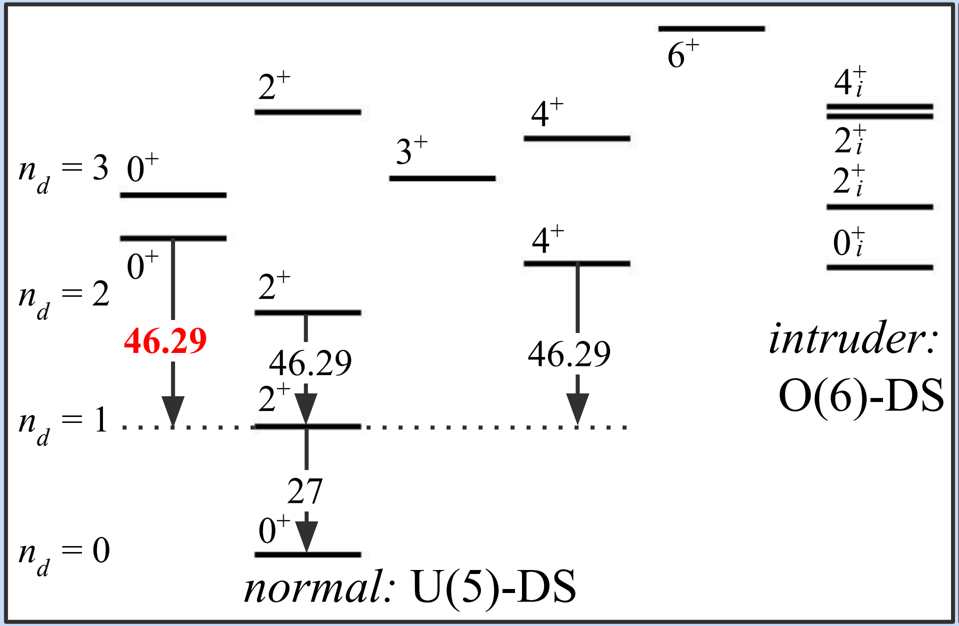

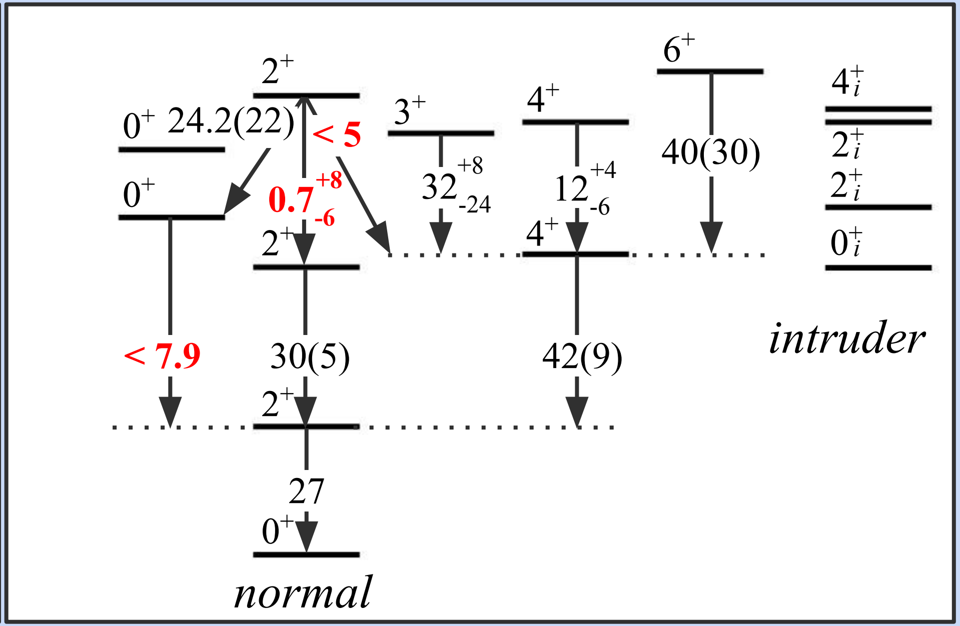

Introduction

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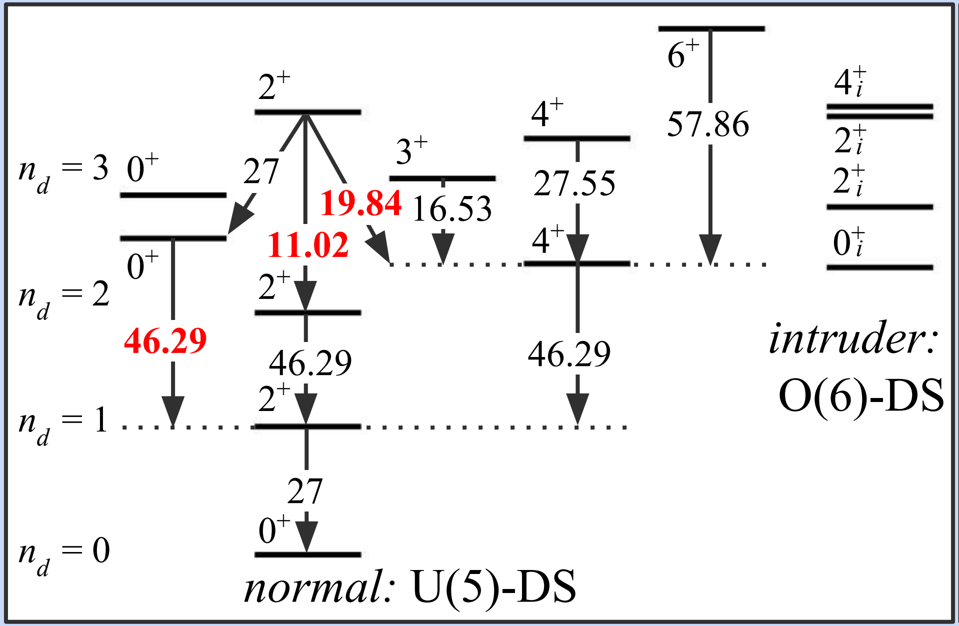
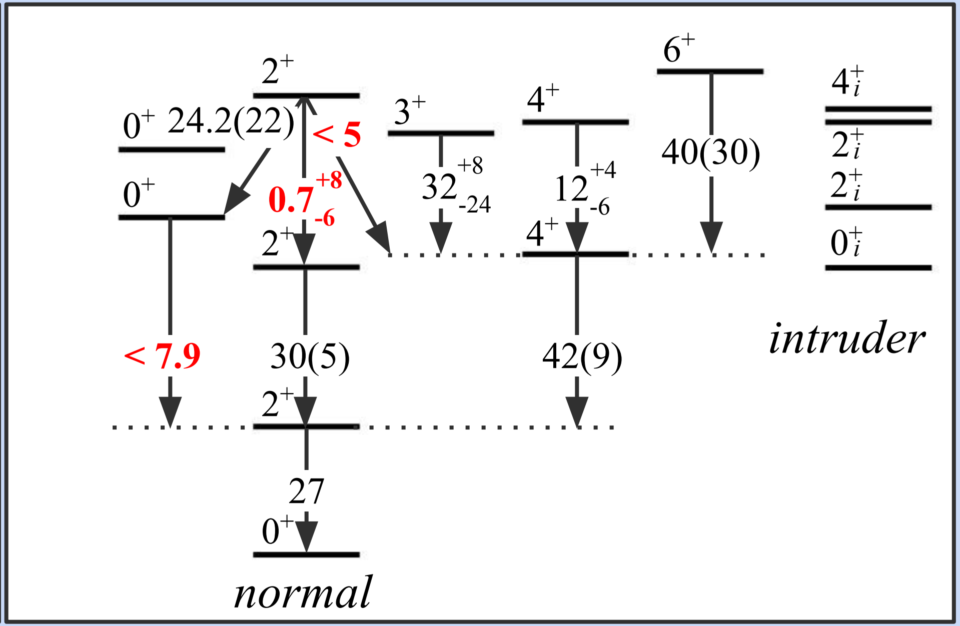
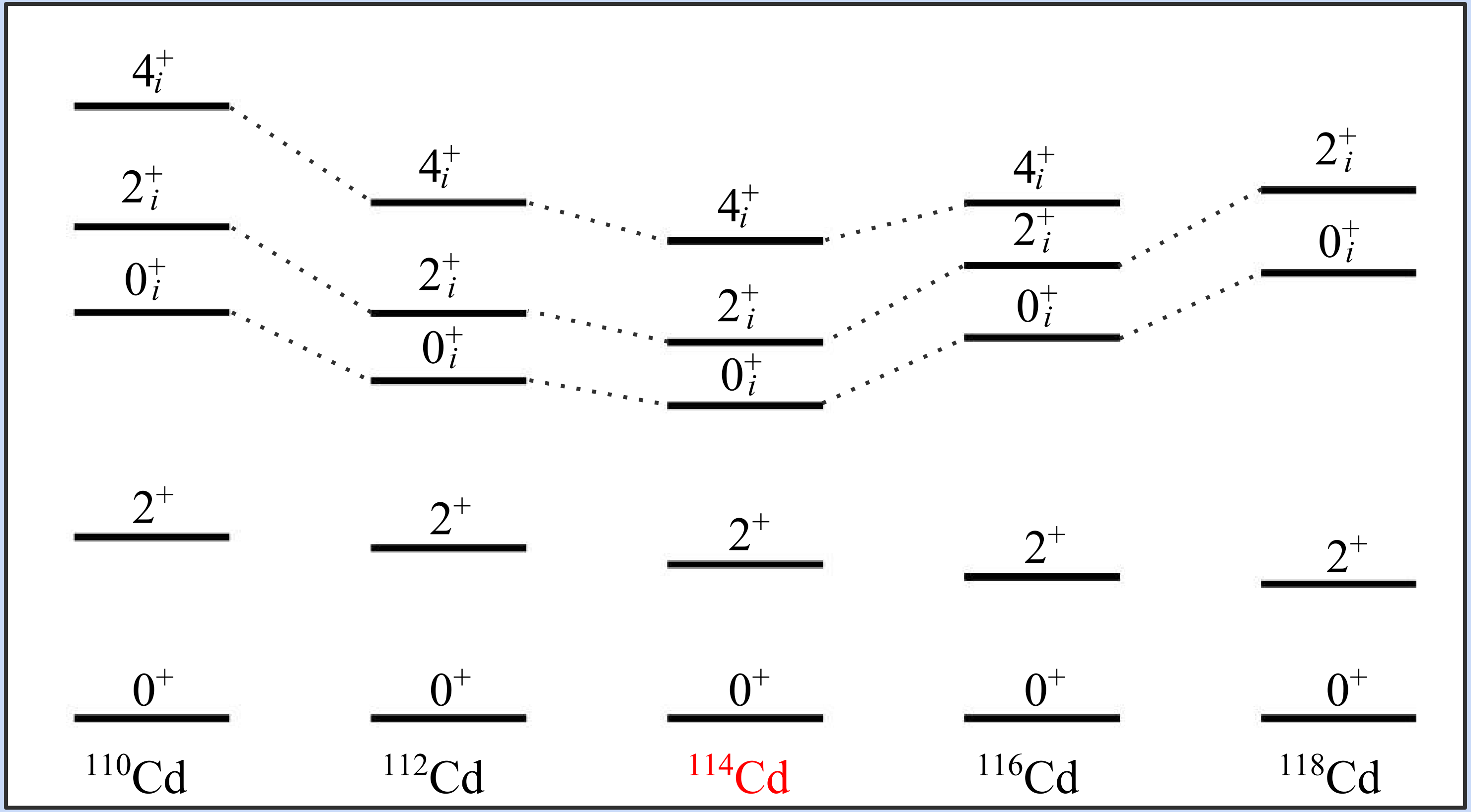




Introduction

The problem of ^{110}Cd



Introduction

The problem of ^{110}Cd

Strong mixing between normal and intruder bands

$$\cancel{B(E2;\ 3\text{-phonon}\rightarrow 2\text{-phonon})}$$

Problem persists in $^{110\text{-}116}$Cd isotopes.

Introduction
The problem of ^{110}Cd

Strong mixing between normal and intruder bands

$$\cancel{B(E2;\,\text{3-phonon}\rightarrow\text{2-phonon})}$$

Problem persists in $^{110\text{-}116}$Cd isotopes.

Breakdown of the vibrational motion in the isotopes $^{110\text{-}116}$Cd

Introduction

The problem of ^{110}Cd

Strong mixing between normal and intruder bands

$\cancel{B(E2;\ \text{3-phonon} \rightarrow \text{2-phonon})}$

Problem persists in $^{110\text{-}116}$Cd isotopes.

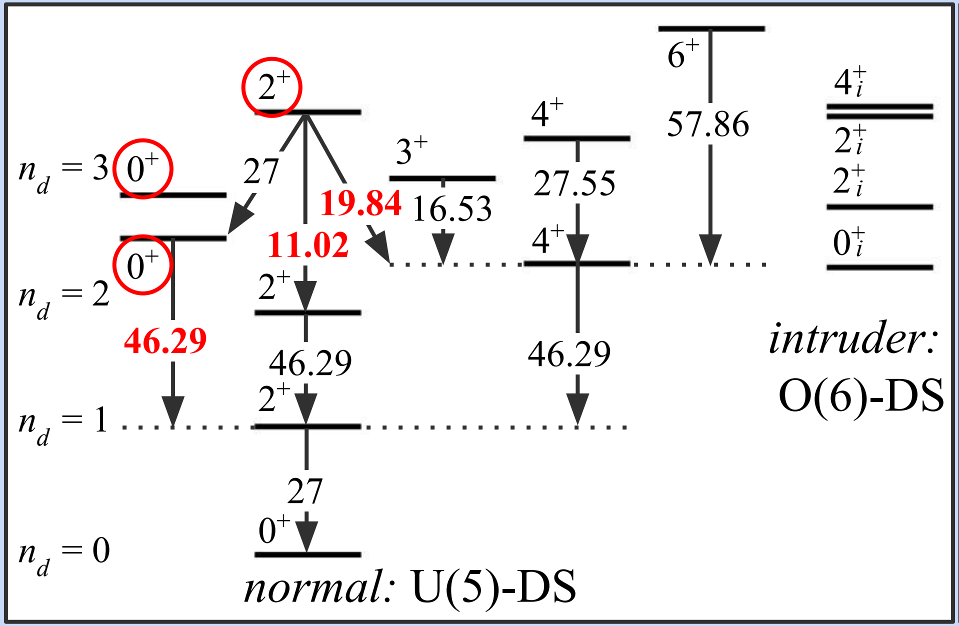
Breakdown of the vibrational motion in the isotopes $^{110\text{-}116}$Cd?

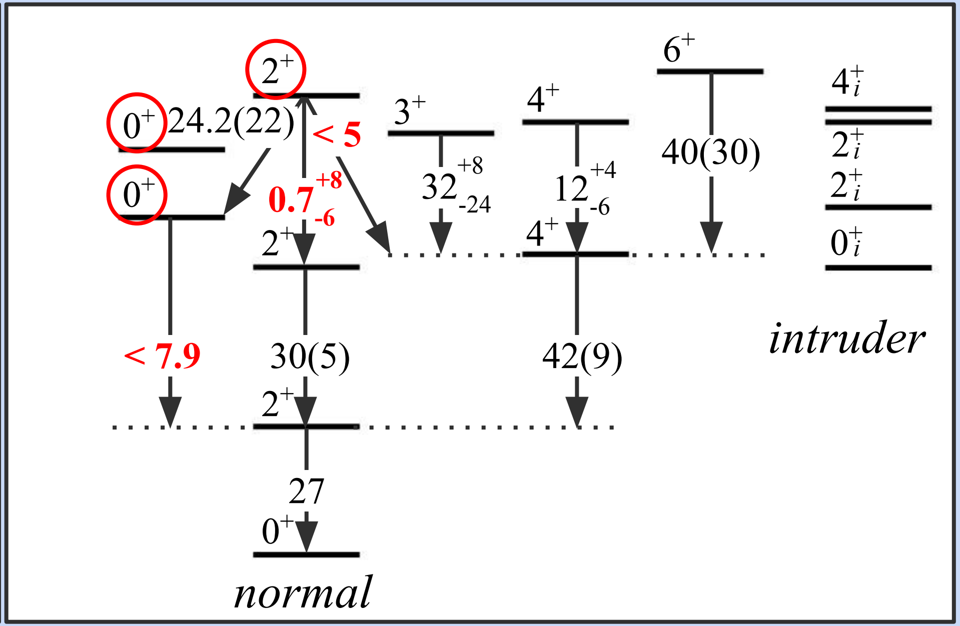
U(5)-PDS

A proposed solution

- Spherical vibrator - U(5) - normal band.
- γ-unstable deformed rotor - O(6) - intruder band.

^{110}Cd experimental





U(5)-PDS

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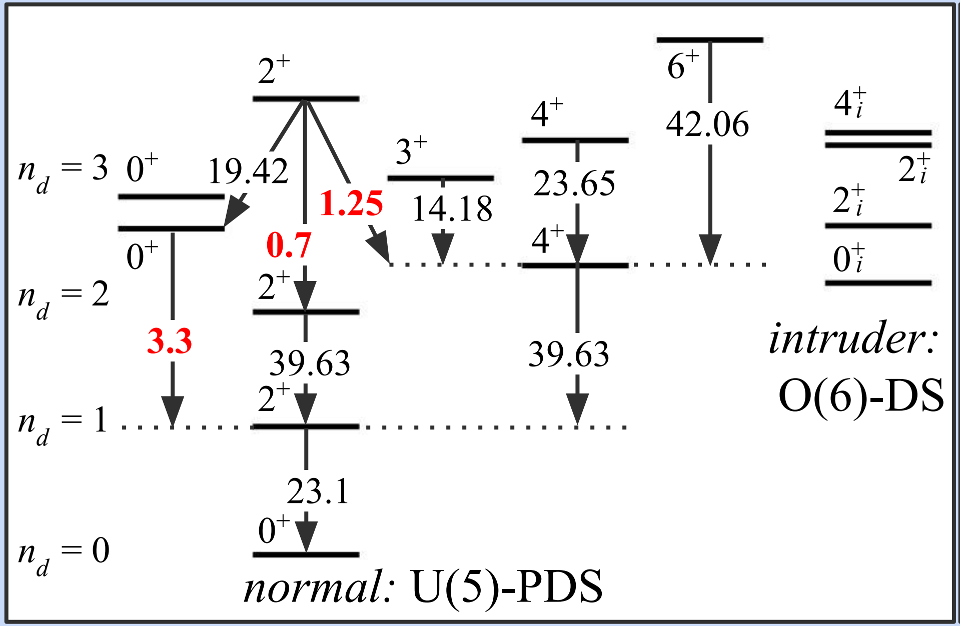
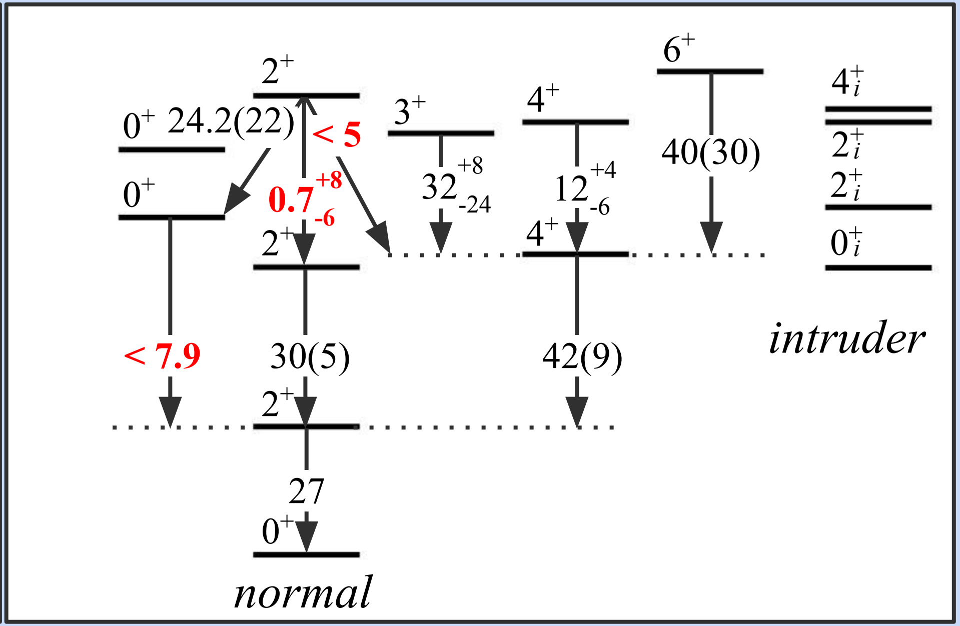
^{110}Cd experimental





Outlook

^{110}Cd and ^{112}Cd - preliminary results of U(5)-PDS seem promising.

^{114}Cd, ^{116}Cd ?

Thank you

Partial dynamical symmetry (PDS)

Exact symmetry: $[H, g_i] = 0$; $g_i \in G$

Dynamical symmetry: $[H, C(G_k)] = 0$; $H = \sum_k a_k C(G_k)$

$$\begin{array}{ccccccc} G_0 & \supset & G_1 & \supset & \ldots & \supset & G_n \\ |\alpha_0 & & \alpha_1 & & \ldots & & \alpha_n\rangle \end{array}$$

Partial symmetry: $[H, g_i] \neq 0$; $[H, g_i]|\psi\rangle = 0$; $g_i \in G$

Partial dynamical symmetry: $[H, C(G_k)] \neq 0$; $[H, C(G_k)]|\psi\rangle = 0$