Living on the edge of stability, challenges to nuclear theory in the FRIB era

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Nuclear Talent course 2017

Outline

Many-body theories 2005, Barrett, Dean, MHJ, Vary, 2004, JPG **31**

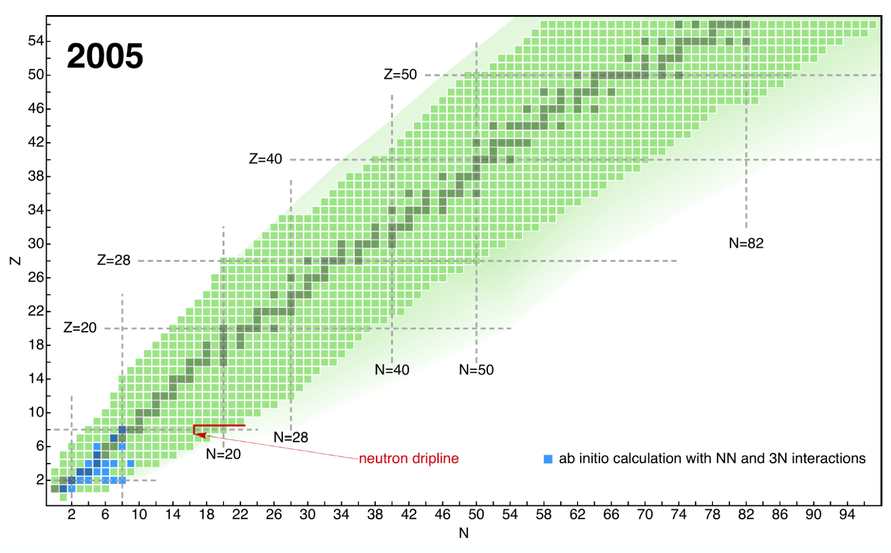
It is our firm belief that new developments in many-body theories for nuclear problems should contain as many as possible of the following ingredients:

- It should be fully microscopic and start with present two- and three-body interactions derived from *e.g.*, effective field theory;
- It can be improved upon systematically, e.g., by inclusion of three-body interactions and more complicated correlations;
- It allows for description of both closed-shell systems and valence systems;
- For nuclear systems where shell-model studies are the only feasible ones, viz., a small model space requiring an effective interaction, one should be able to derive effective two and three-body equations and interactions for the shell model;

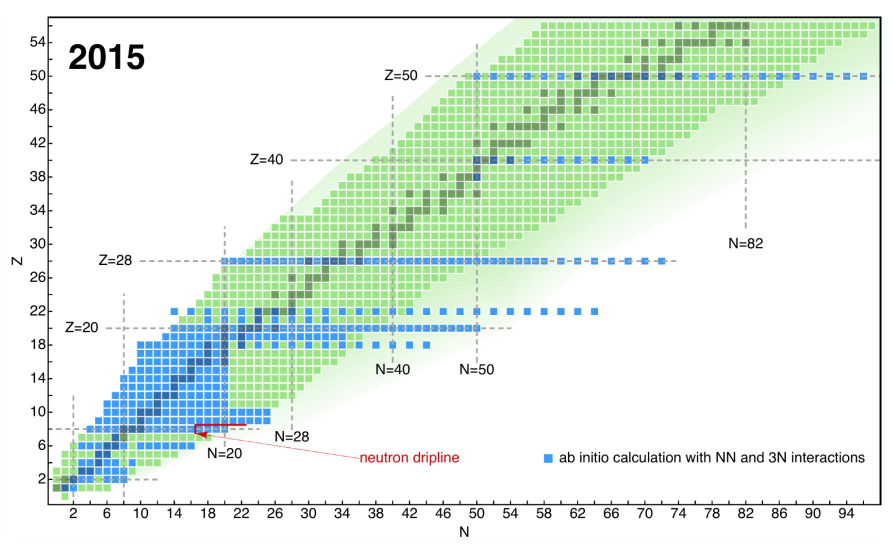
Many-body theories 2005, Barrett, Dean, MHJ, Vary, 2004, JPG **31**

- It is amenable to parallel computing;
- It can be used to generate excited spectra for nuclei like where many shells are involved (It is hard for the traditional shell model to go beyond one major shell. The inclusion of several shells may imply the need of complex effective interactions needed in studies of weakly bound systems); and
- Finally, nuclear structure results should be used in marrying microscopic many-body results with reaction studies. This will be another hot topic of future *ab initio* research.

Many-body theories 2005



In 2015


2015
Z=50
Z=40
Z=28
Z=20
N=82
N=40
N=50
N=28
N=20
neutron dripline
ab initio calculation with NN and 3N interactions
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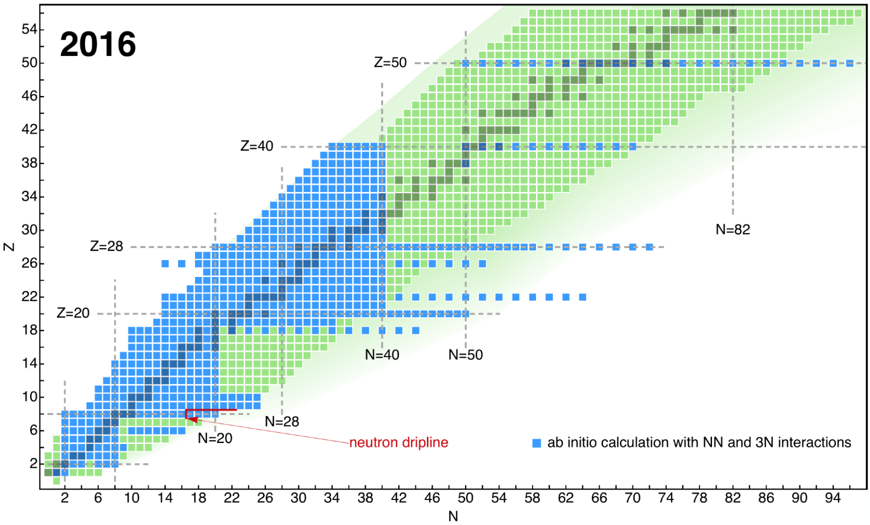
And in 2016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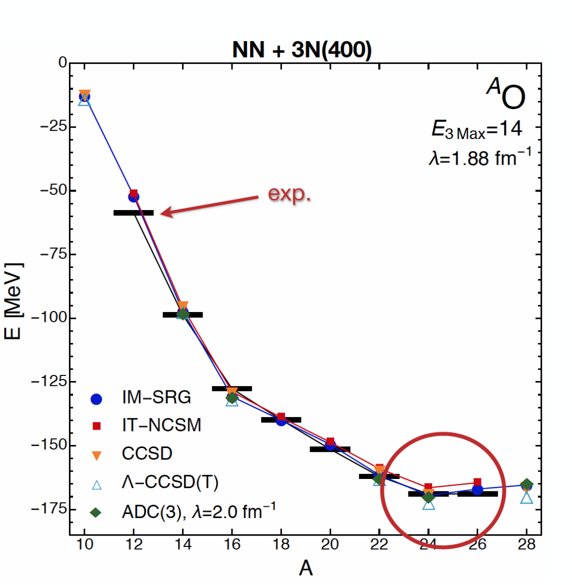
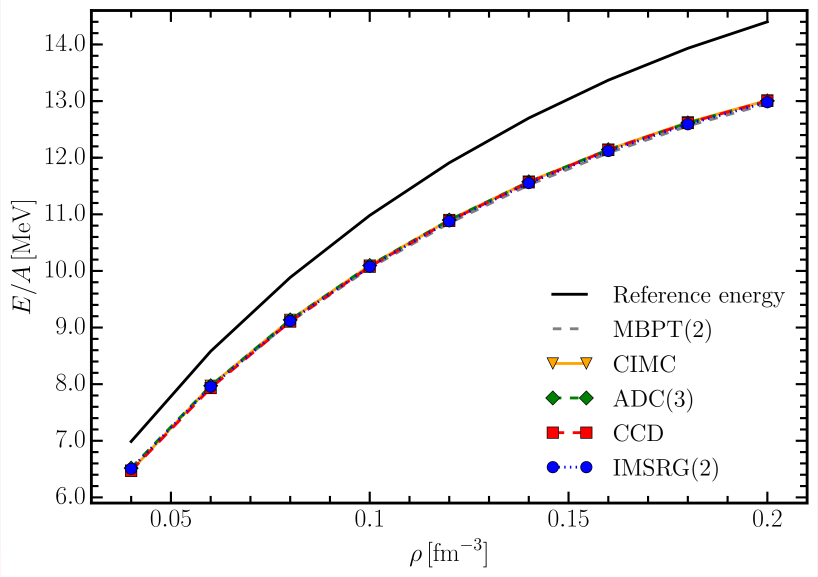
Huge progress in many-body theories

- No-Core Shell Model (and Variants)
- In-Medium Similarity Renormalization Group
- Coupled Cluster
- Self-Consistent Green's Functions
- Monte Carlo methods
- And several other approaches

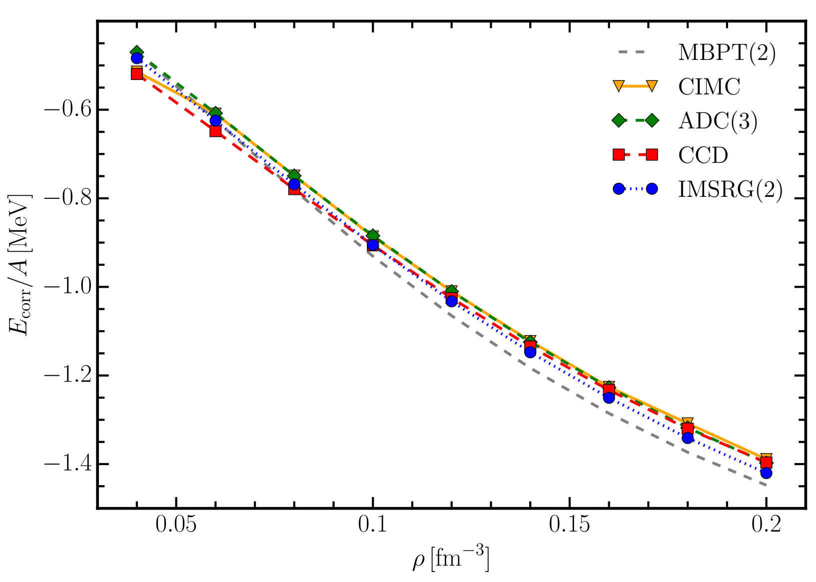
Consistency between many-body theories (Courtesy of Heiko Hergert@MSU)



Neutron matter calculations with simple Minnesota model for the force, Lecture Notes in Physics (2017)



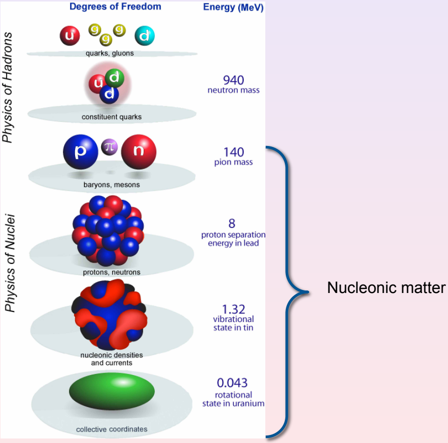
Neutron matter correlation energy, Lecture Notes in Physics (2017)



Important questions from QCD to the nuclear many-body problem

- How to derive the in medium nucleon-nucleon interaction from basic principles?
- How does the nuclear force depend on the proton-to-neutron ratio?
- What are the limits for the existence of nuclei?
- How can collective phenomena be explained from individual motion?
- Shape transitions in nuclei?

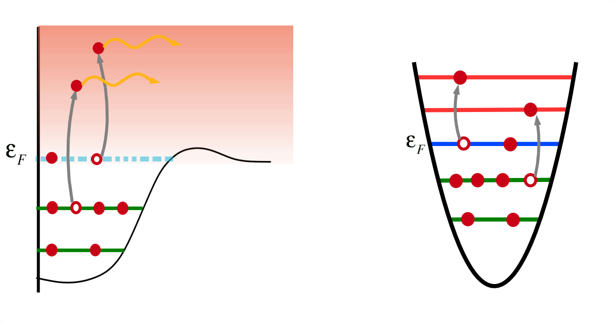
The many scales pose a severe challenge to *ab initio* descriptions of nuclear systems.



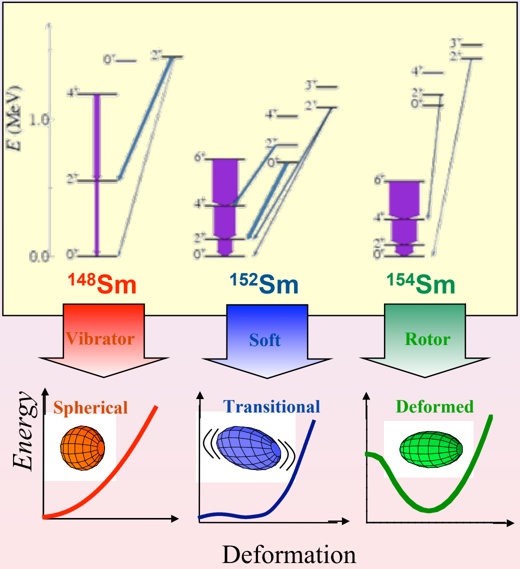
Halo nuclei and moving towards the limits of nuclear stability

Open Quantum System.
Coupling with continuum needs to be taken into account.

Closed Quantum System.
No coupling with external continuum.



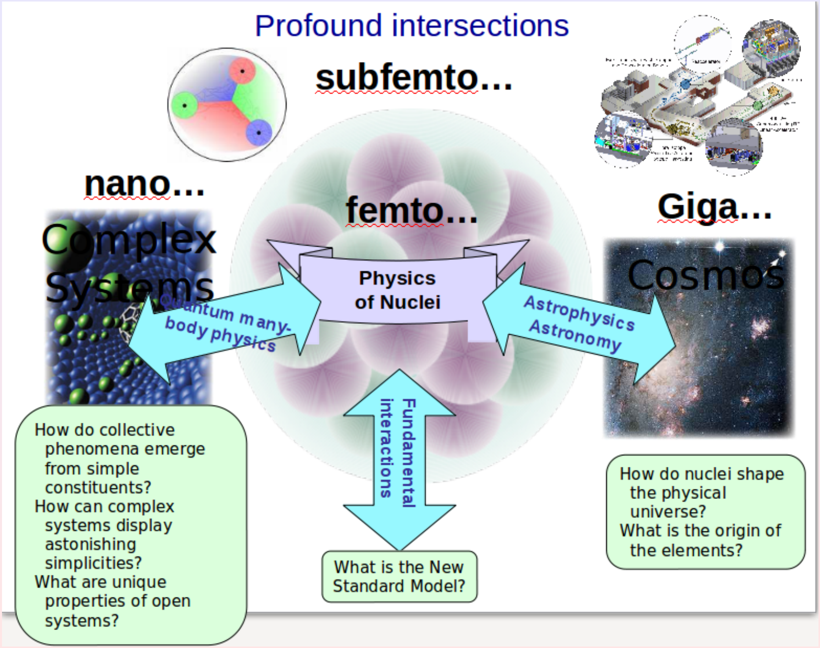
Shape coexistence and transitions, a multiscale challenge



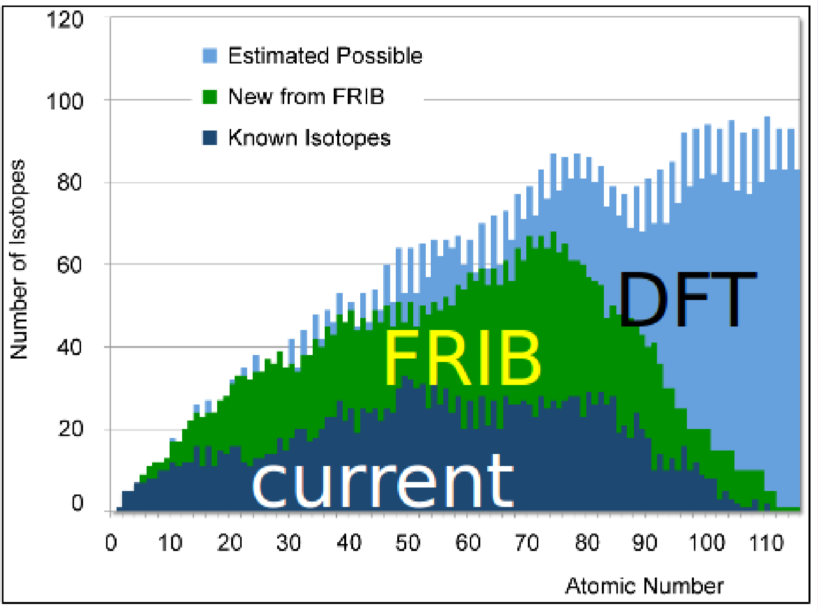
Challenges for theory

- Possible shape transitions, huge spaces needed to describe properly.
- Theory: need to marry *ab initio* methods with density functional theories in order to describe such systems
- Need a large wealth of experimental data to constrain theory

The many interesting intersections

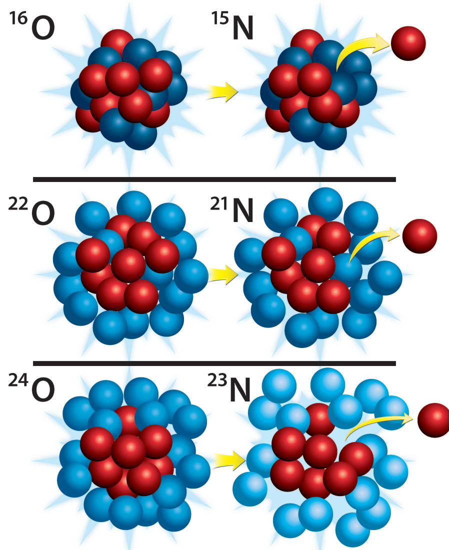


Known nuclei and predictions



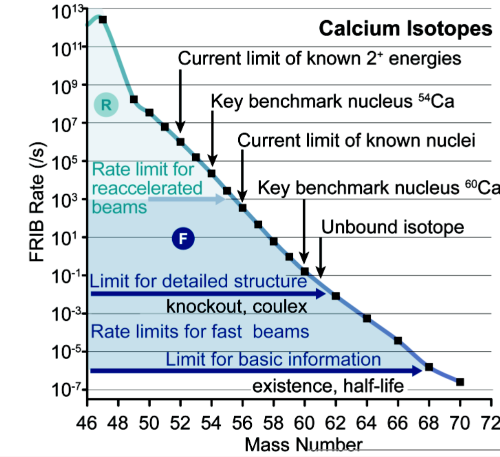
Do we understand the physics of dripline systems?

- The oxygen isotopes are the heaviest isotopes for which the drip line is well established.
- Two out of four stable even-even isotopes exhibit a doubly magic nature, namely ^{22}O ($Z = 8$, $N = 14$) and ^{24}O ($Z = 8$, $N = 16$).
- The structure of ^{22}O and ^{24}O is assumed to be governed by the evolution of the $1s_{1/2}$ and $0d_{5/2}$ one-quasiparticle states.
- The isotopes ^{25}O ^{26}O, ^{27}O and ^{28}O are outside the drip line, since the $0d_{3/2}$ orbit is not bound.



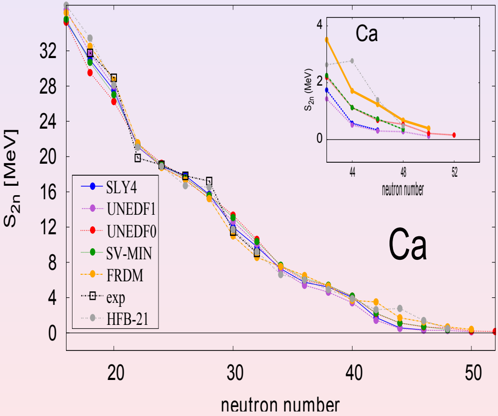
Calcium isotopes and FRIB plans and capabilities

- The Ca isotope exhibit several possible closed-shell nuclei ^{40}Ca, ^{48}Ca, ^{52}Ca, ^{54}Ca, and ^{60}Ca.
- Magic neutron numbers are then $N = 20, 28, 32, 34, 40$.
- Masses available up to ^{54}Ca, Gallant *et al.*,Phys. Rev. Lett. **109**, 032506 (2012) and K. Baum *et al*, Nature **498**, 346 (2013).
- Heaviest observed ^{57,58}Ca. NSCL experiment, O. B. Tarasov *et al.*, Phys. Rev. Lett. **102**, 142501 (2009). Cross sections for ^{59,60}Ca assumed small ($< 10^{-12}$mb).
- Which degrees of freedom prevail close to ^{60}Ca?



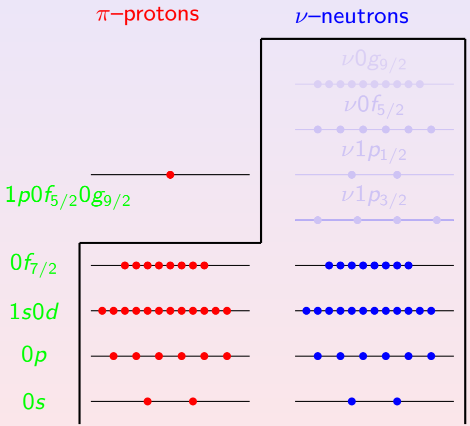
More on Calcium Isotopes

- **Mass models and mean field models predict the dripline at $A \sim 70$!** Important consequences for modeling of nucleosynthesis related processes.
- Can we predict reliably which is the last stable calcium isotope?
- And how does this compare with popular mass models on the market?
- And which parts of the underlying forces are driving the physics towards the dripline?



Other chains of isotopes of crucial interest for FRIB like physics: nickel isotopes

- This chain of isotopes exhibits four possible closed-shell nuclei ^{48}Ni, ^{56}Ni, ^{68}Ni and ^{78}Ni. **FRIB plans systematic studies from ^{48}Ni to ^{88}Ni.**
- Neutron skin possible for ^{84}Ni at FRIB.
- Which is the best closed-shell nucleus? And again, which part of the nuclear forces drives it? Is it the strong spin-orbit force, the tensor force, or ..?



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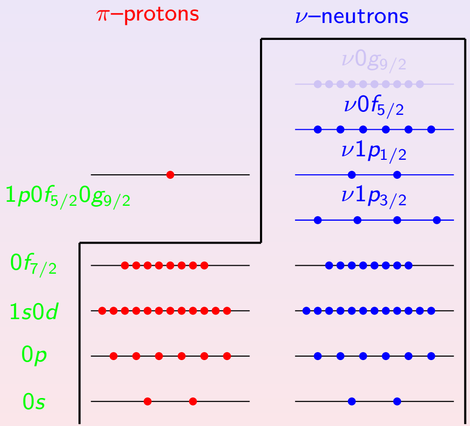

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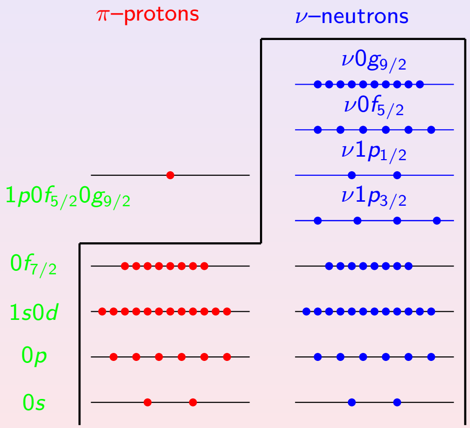


Tin isotopes

From ^{100}Sn to nuclei beyond ^{132}Sn

1. We will most likely be able to run coupled-cluster calculations for nuclei like ^{100}Sn, ^{114}Sn, ^{116}Sn, ^{132}Sn, ^{140}Sn and $A \pm 1$ and $A \pm 2$ nuclei within the next one to two years. FRIB can reach to ^{140}Sn. Interest also for EOS studies.
2. Can then test the development of many-body forces for an even larger chain of isotopes.
3. ^{137}Sn is the last reported neutron-rich isotope (with half-life).
4. To understand which parts of the nuclear Hamiltonian that drives the properties of such nuclei will be crucial for our understanding of the stability of matter.
5. Zr isotopes form also long chains of neutron-rich isotopes. **FRIB plans from ^{80}Zr to ^{120}Zr.**
6. And why neutron rich isotopes? **Here the possibility to constrain nuclear forces from in-medium results.**

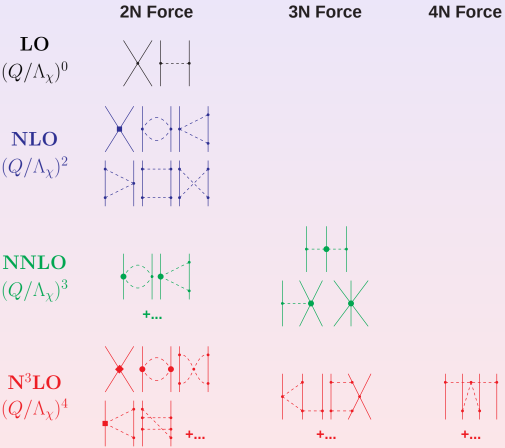
Problems for theory

We know how to solve the many-body problem, but are we solving the correct problem?

1. Can we provide proper error estimates (single-particle basis truncation and truncations in number of excitations)? **Yes**, see for example Simen Kvaal, Physical Review B **80**, 045321 (2009) and Thorsten Rohwedder and Reinhold Schneider, Math. Modelling and Numerical Analysis, **47**, 1553 (2013).
2. Our problem however is to understand how many-body forces evolve as we add more and more particles. And provide an error estimate on what we leave out!

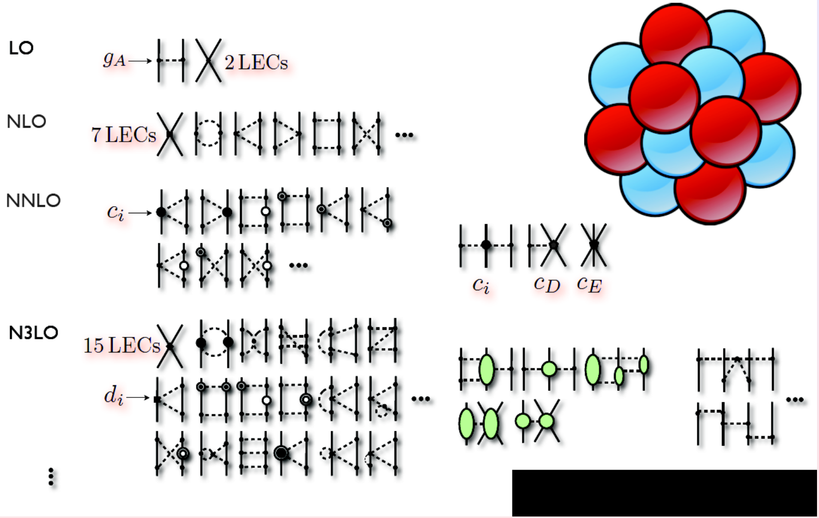
A big challenge for theory is to be able to properly quantify the errors in the calculations. In particular, can we develop an effective field theory for many-body systems with error quantifications?

Nuclear interactions from Effective Field Theory (Δ-less)



- Nucleons and Pions as effective degrees of freedom only. Most general Lagrangian consistent with all symmetries of low-energy QCD.
- Chiral perturbation theory for different orders (ν) of the expansion in terms of $(Q/\Lambda_\chi)^\nu$.
- At order $\nu = 4$ one should include four-body forces in many-body calculations! Not including these will result in what we call missing many-body correlations.

Forces in Nuclear Physics (without isobars)



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Effective Manybody Hamiltonian: assume that a three-body Hamiltonian is something we can accept

Case of Normal-ordered three-body Hamiltonian

Introducing a reference state $|\Phi_0\rangle$ as our new vacuum state leads to the redefinition of the Hamiltonian in terms of a constant reference energy E_0 defined as

$$E_0 = \sum_{i\le\alpha_F} \langle i|\hat{h}_0|i\rangle + \frac{1}{2}\sum_{ij\le\alpha_F} \langle ij|\hat{v}|ij\rangle + \frac{1}{6}\sum_{ijk\le\alpha_F} \langle ijk|\hat{w}|ijk\rangle,$$

and a normal-ordered Hamiltonian

$$\hat{H}_N = \sum_{pq} \langle p|\tilde{f}|q\rangle a_p^\dagger a_q + \frac{1}{4}\sum_{pqrs} \langle pq|\tilde{v}|rs\rangle a_p^\dagger a_q^\dagger a_s a_r +$$

$$\frac{1}{36}\sum_{\substack{pqr\\stu}} \langle pqr|\hat{w}|stu\rangle a_p^\dagger a_q^\dagger a_r^\dagger a_u a_t a_s$$

Effective Manybody Hamiltonian: assume that a three-body Hamiltonian is something we can accept

Case of Normal-ordered three-body Hamiltonian

We have defined a one-body term as

$$\langle p|\tilde{f}|q\rangle = \langle p|\hat{h}_0|q\rangle + \sum_{i\le\alpha_F}\langle pi|\hat{v}|qi\rangle + \frac{1}{2}\sum_{ij\le\alpha_F}\langle pij|\hat{w}|qij\rangle.$$

It represents a correction to the single-particle operator $\hat{h}_0$ due to contributions from the nucleons below the Fermi level. The two-body matrix elements are now modified in order to account for medium-modified contributions from the three-body interaction, resulting in

$$\langle pq|\tilde{v}|rs\rangle = \langle pq|\hat{v}|rs\rangle + \sum_{i\le\alpha_F}\langle pqi|\hat{w}|rsi\rangle.$$

The Monopole Part of an Interaction

An important ingredient in studies of effective interactions and their applications to nuclear structure, is the so-called monopole interaction, normally defined in terms of a nucleon-nucleon interaction $\hat{v}$

$$\bar{V}_{j_p j_q} = \frac{\sum_J (2J+1)\langle (j_p j_q)J|\hat{v}|(j_p j_q)J\rangle}{\sum_J (2J+1)},$$

where the total angular momentum of a two-body state J runs over all possible values. The monopole Hamiltonian can be interpreted as an angle-averaged matrix element. This equation can also be expressed in terms of the medium-modified two-body interaction

$$\tilde{V}_{j_p j_q} = \frac{\sum_J (2J+1)\langle (j_p j_q)J|\tilde{v}|(j_p j_q)J\rangle}{\sum_J (2J+1)}.$$

The Monopole Part of an Interaction

The single-particle energy ϵ_p resulting from for example a self-consistent Hartree-Fock field, or from first order in many-body perturbation theory, is given by

$$\epsilon_{j_p} = \langle j_p|\hat{h}_0|j_p\rangle + \frac{1}{2j_p+1}\sum_{j_i\le F}\sum_J (2J+1)\langle (j_p j_i)J|\hat{v}|(j_p j_i)J\rangle,$$

or

$$\epsilon_{j_p} = \langle j_p|\hat{h}_0|j_p\rangle + \frac{1}{2j_p+1}\sum_{j_i\le F}\sum_J (2J+1)\langle (j_p j_i)J|\tilde{v}|(j_p j_i)J\rangle,$$

where the first equation contains a two-body force only while the second includes the medium-modified contribution from the three-body interaction as well. These equations can be rewritten in terms of the monopole contribution as

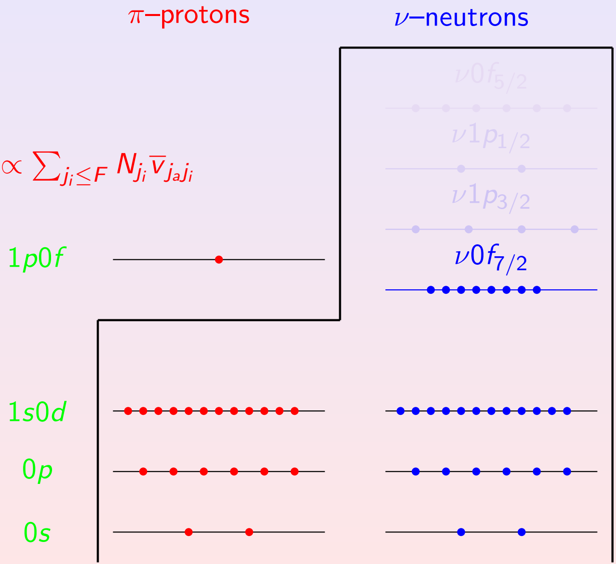
$$\epsilon_{j_p} = \langle j_p|\hat{h}_0|j_p\rangle + \sum_{j_i\le F} N_{j_i}\,\bar{V}_{j_p j_i},$$

with $N_{j_i} = 2j_i + 1$, and

$$\epsilon_{j_p} = \langle j_p|\hat{h}_0|j_p\rangle + \sum_{j_i\le F} N_{j_i}\,\tilde{V}_{j_p j_i}.$$

Evolution of quasiparticle states in terms of the monopole part: ^{48,52,54,60}Ca

$$\Delta\epsilon^{\pi}_{j_a} \propto \sum_{j_i \leq F} N_{j_i} \overline{v}_{j_a j_i}$$



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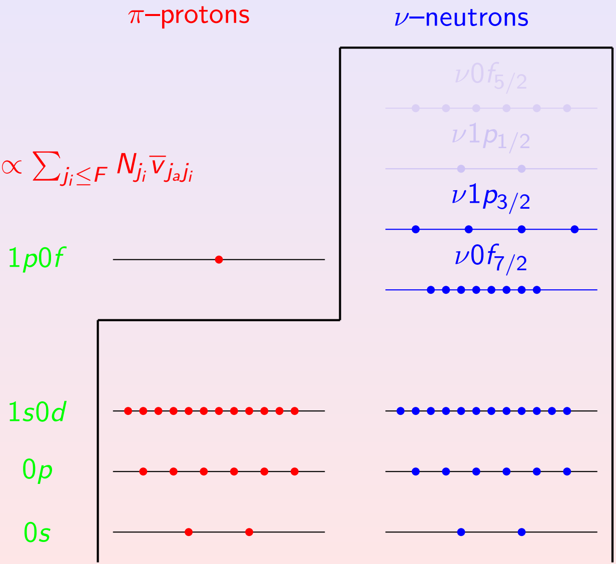

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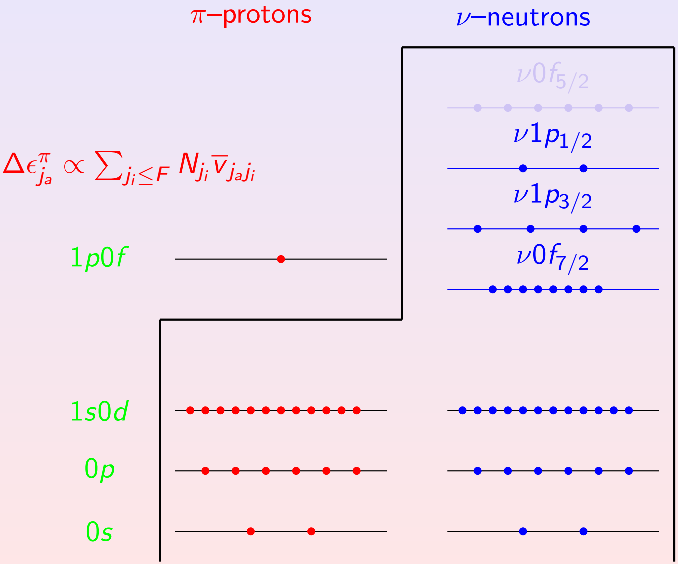

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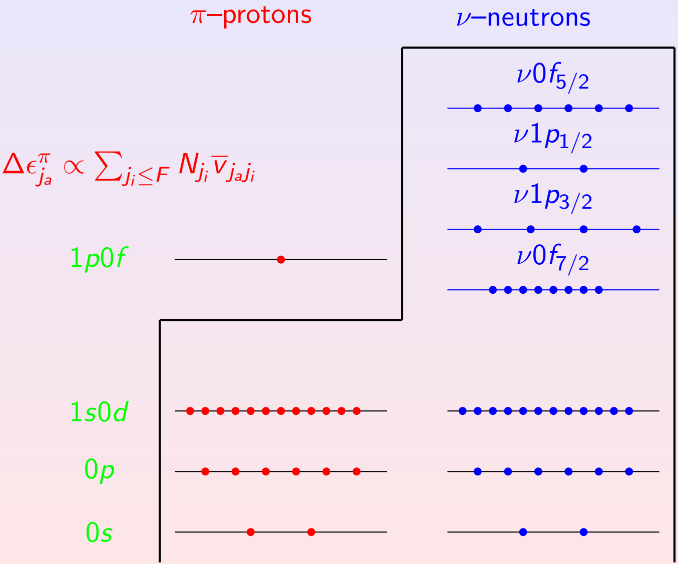
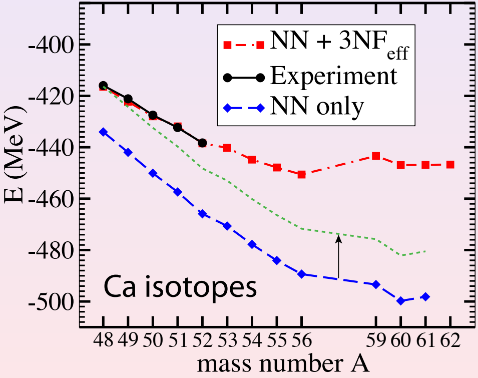
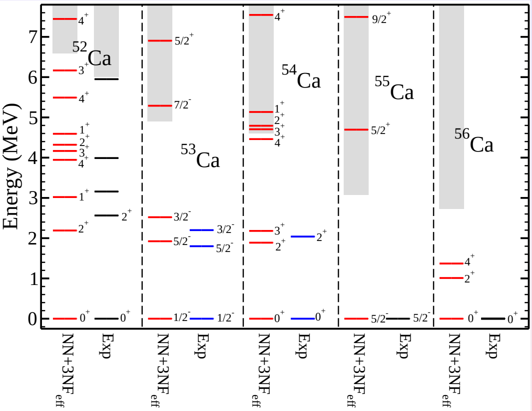


Calcium isotopes with three-body forces, Hagen *et al*, Phys. Rev. Lett. **109**, 032502 (2012)



- Three-body force is taken as a density dependent contribution to a two-body interaction
- Three-body force based on a nuclear matter calculation with $k_F = 1.0$ fm^{-1}.
- Dashed line: two-body results normalized at $A = 48$.
- Most mass models predict dripline at $A = 70$
- We predict it at $A \sim 60$?

Calcium isotopes with three-body forces and continuum, Hagen *et al*, Phys. Rev. Lett. **109**, 032502 (2012)



	^{53}Ca		^{55}Ca		^{61}Ca	
J^π	Re[E]	Γ	Re[E]	Γ	Re[E]	Γ
$5/2^+$	1.99	1.97	1.63	1.33	1.14	0.62
$9/2^+$	4.75	0.28	4.43	0.23	2.19	0.02

The future: Hamiltonians from Lattice QCD