DTP/TALENT 2025: Exercises for Monday 16

June 15, 2025

In this first hands-on session the main goal is to familiarize yourself with how to write quantum circuits and run them. Some of the examples we'll see use a software package named Qiskit developed by IBM but there are other options, like Google's Cirq. For those of you new to quantum circuits you can find an introductory notebook *1_getting_started_with_qiskit.ipynb* in the *Exercises* folder.

1. Write a circuit that prepares the single qubit state

$$|\Psi(\theta)\rangle = \cos(\theta)\,|0\rangle + \sin(\theta)\,|1\rangle \ , \tag{1}$$

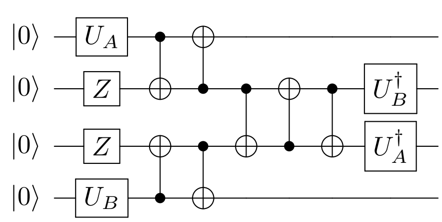
   and measures the expectation value of Z on the resulting state.

   - you can use a R_y rotation to perform this (see here for definitions)
   - perform the calculation in two ways: first by extracting the full *state_vector* and then performing a fixed number of shots and reconstructing the expectation value from those
   - it might also be helpful to implement everything directly in NumPy in order to have a reference value to compare to
   - make a plot as a function of θ of the function

$$\langle Z(\theta)\rangle = \langle\Psi(\theta)|Z|\Psi(\theta)\rangle \ , \tag{2}$$

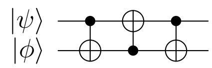
     extracted in the various ways.

2. Try to show that the following circuit gives the same final state regardless on the choice of the single-qubit gates U_A and U_B



   - try to choose a simple pair of gates U_A and U_B so that you can find the final state easily. What do you get?

- [HINT] in order to understand what is happening you might want to consider the following sub-circuit



where $|\psi\rangle$ and $|\phi\rangle$ are generic single-qubit states. What is this doing?

3. Quantum teleportation is an interesting communication protocol where information is transmitted between a sender, we'll call her Sandy, and a receiver, will call him Rick, through a combination of entanglement and classical communication. The protocol works as follows:

   - Sandy has one qubit, we'll call B_Ψ, in a state that she wants to send to Rick. For this example we'll use $|\Psi(\theta)\rangle$ from Eq. (1) above
   - before the protocol began, Sandy and Rick prepare a two qubit Bell state $|B\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. By keeping one quibit each, let's call B_S and B_R, they can now use as a resource for communication.
   - the protocol now proceeds as follows:
     (a) Sandy entangles her qubit B_Ψ with her part of the Bell state on qubit B_S. The total state on qubits (B_Ψ, B_S, B_R) is then

     $$\frac{\cos(\theta)}{\sqrt{2}}|0\rangle \otimes (|00\rangle + |11\rangle) + \frac{\sin(\theta)}{\sqrt{2}}|1\rangle \otimes (|10\rangle + |01\rangle) \tag{3}$$

     (b) she applies an Hadamard gate to her qubit B_Ψ obtaining

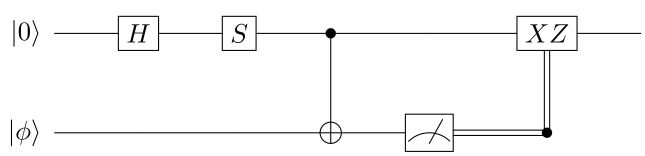
     $$\begin{aligned} &\frac{|00\rangle}{2} \otimes (\cos(\theta)|0\rangle + \sin(\theta)|1\rangle) + \frac{|01\rangle}{2} \otimes (\cos(\theta)|1\rangle + \sin(\theta)|0\rangle) \\ &\frac{|10\rangle}{2} \otimes (\cos(\theta)|0\rangle - \sin(\theta)|1\rangle) + \frac{|11\rangle}{2} \otimes (\cos(\theta)|1\rangle - \sin(\theta)|0\rangle) \end{aligned} \tag{4}$$

     (c) she measures their qubits and send the outcomes to Rick
     (d) Rick can then recover the unknown state $|\Psi\rangle$ on his qubit B_R by performing on it a specific Pauli transformation depending on the measurement outcome
       - no operation if results is $(B_\Psi, B_S) = (0, 0)$
       - Pauli X if results is $(B_\Psi, B_S) = (0, 1)$
       - Pauli Z if results is $(B_\Psi, B_S) = (1, 0)$
       - Pauli XZ if results is $(B_\Psi, B_S) = (1, 1)$

   Try to show that indeed the state on Rick's qubit B_R is the correct one. Then try to implement this protocol with Qiskit. For instance you can try to measure qubit B_R after the whole protocol has ended and compare the results with those you would've expected if you had the correct state $|\Psi(\theta)\rangle$. You can use the following code-snippet to get started

```
# defines a circuit with 3 qubits and 3 classical bits
telep_circ = QuantumCircuit(3,3)
# stores labels for the 3 qubits
quantum_bits = telep_circ.qubits
# stores labels for the 3 classical bits (for mesaurements)
classical_bits = telep_circ.clbits
# performs CNOT to entangle qubits and Hadamard on B_psi
telep_circ.cx(quantum_bits[0],quantum_bits[1])
telep_circ.h(quantum_bits[0])
# measurement
telep_circ.barrier()
telep_circ.measure(quantum_bits[0],classical_bits[0])
telep_circ.measure(quantum_bits[1],classical_bits[1])
# classical control
telep_circ.barrier()
## apply a Z to qubit B_R if measure on c_psi is 1.
with telep_circ.if_test((classical_bits[0],1)):
    telep_circ.z(quantum_bits[2])
## apply an X to qubit B_R if measure on c_S is 1.
with telep_circ.if_test((classical_bits[1],1)):
    telep_circ.x(quantum_bits[2])
telep_circ.draw()
```

4. if you already know everything about Qiskit and quantum teleportation try to use your skills to understand what this circuit is doing



(5)

   - try to understand what state is left on the first qubit
   - if instead of the gates S we used $T = \sqrt{S}$, what classically controlled operation should we use to get the same functionality?
   - why would a circuit like this be useful?
   - can you find a way to achieve the same functionality but without measuring the second qubit at all?
   - and how about a version without any controls after the measurement?