# Exercises II: Quantum computing time dynamics of the lattice Schwinger model

Zohreh Davoudi, Chung-Chun Hsieh

TALENT/DTP course, ECT*, Trento, Italy

June 2025

For this exercise, we continue to work with the qubit Hamiltonian:

$$H = \mathrm{x}\sum_{n=0}^{N-2}\left(\sigma_n^+\sigma_{n+1}^- + \mathrm{h.c.}\right) + \frac{\mu}{2}\sum_{n=0}^{N-1}(-1)^{n+1}\sigma_n^{\mathrm{z}} + \sum_{n=0}^{N-2}\left\{\epsilon_0 + \frac{1}{2}\sum_{m=0}^{n}\left[\sigma_m^{\mathrm{z}} - (-1)^m\right]\right\}^2 \tag{1}$$

$$\coloneqq \sum_{n=0}^{N-2} H_{n,n+1}^{(\mathrm{XX})} + \sum_{n=0}^{N-2} H_{n,n+1}^{(\mathrm{YY})} + H^{(\mathrm{ZZ})} + H^{(\mathrm{Z})}, \tag{2}$$

associated with the lattice Schwinger model with open boundary conditions (OBC). We further consider Trotterized time evolution using the first-order Trotter-Suzuki formula using the decomposition:

$$V(t) = \prod_{i=1}^{N_T=t/\delta t}\left(e^{-i\delta t H^{(Z)}} e^{-i\delta t H^{(ZZ)}} \prod_{n=0}^{N-2} e^{-i\delta t H_{n,n+1}^{(XX)}} \prod_{n=0}^{N-2} e^{-i\delta t H_{n,n+1}^{(YY)}}\right). \tag{3}$$

The initial state, $|\psi(0)\rangle$, of this evolution is the strong-coupling vacuum, and the time-evolved state is $|\psi(t)\rangle \coloneqq V(t)\,|\psi(0)\rangle$. The system size and parameters are as in the previous exercise: $N = 4$, $\epsilon_0 = 0$, $\mathrm{x} = 0.6$, $\mu = 0.1$, $t = 5$, $\delta t = 0.5$ (or $N_T = 10$).

**Part (a)** Pictorially draw the quantum-circuit elements that evaluate each Trotter step of the evolution. Make sure to identify the gates to the level they can be implemented on a quantum hardware. Identify all the gate angles.

**Part (b)** Write a `Qiskit` code that implements your circuit in part (a). If you are not yet comfortable with writing the code from scratch, use the IBMQ automated tool.

**Part (c)** Evaluate the Loschmidt echo

$$\mathcal{P}(t) \coloneqq \left|\langle\psi(0)|\psi(t)\rangle\right|^2, \tag{4}$$

and the staggered fermion density

$$\nu(t) \coloneqq \frac{1}{N}\sum_{n=0}^{N-1}\langle\psi(t)|\frac{(-1)^{n+1}\sigma_n^{\mathrm{z}} + 1}{2}|\psi(t)\rangle\,, \tag{5}$$

as defined in the previous exercise. To do this, run the circuit for 1,000 times and measure the outcome in the computational basis (the Pauli-Z basis). Then reconstruct the observables using these measurements. Note that the labeling in `Qiskit` is such that the first ($0^{\text{th}}$) qubit represents the least significant digit. Try to explore the dependence of observables on the number of shots by changing the shot count. In particular, you should be able to see the convergence to the numerically evaluated Trotterized time evolution in the previous exercise by increasing the shot count. Think about how you would you assign statistical uncertainty to the observables.

**Part (d) [Bonus]** Make appropriate measurements to obtain the energy of the time-evolved state:

$$E(t) \coloneqq \langle \psi(t)|H|\psi(t)\rangle, \tag{6}$$

at $t = 5$. To do this, you need to measure all the Hamiltonian operators and sum them up. For non-diagonal operators in the computational basis, i.e., $H^{(\text{XX/YY})}_{n,n+1}$, you need to first perform a change of basis from Z to X/Y, then measure the associated qubits. Compare your results to the numerically evaluated result. Importantly note that energy is a conserved quantity, so any deviation from the initial-state's energy should be attributed to the errors in the implementation. Explore the dependence of the error on the shot count and Trotter error.