

Nuclear Lattice Methods, Adiabatic Evolution, and Rodeo Algorithm

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Quantum Computing for Nuclear Physics
Nuclear TALENT Course – Doctoral Training Program
ECT*, Trento, June 30, 2025



U.S. DEPARTMENT
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Science



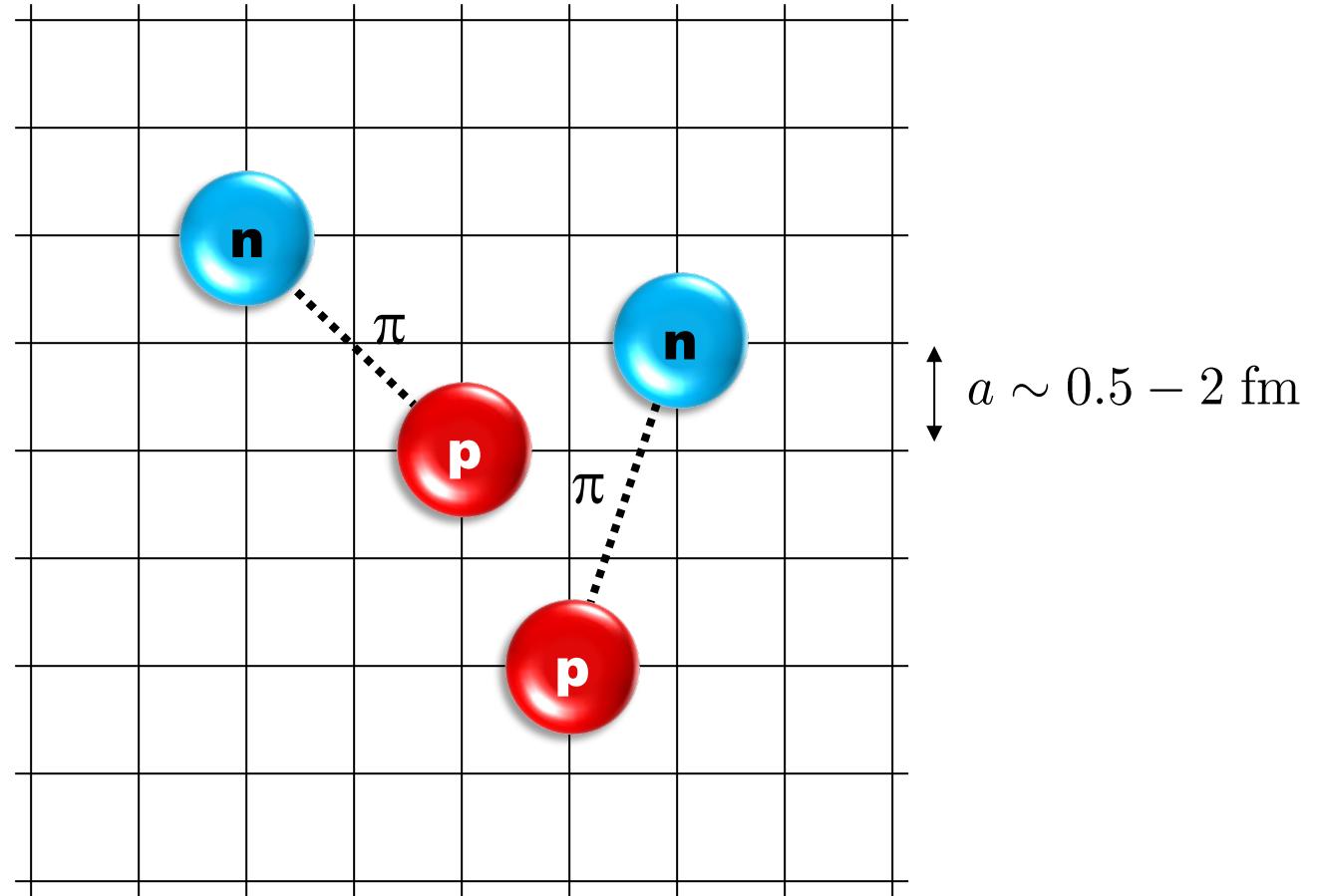
NUCLEI
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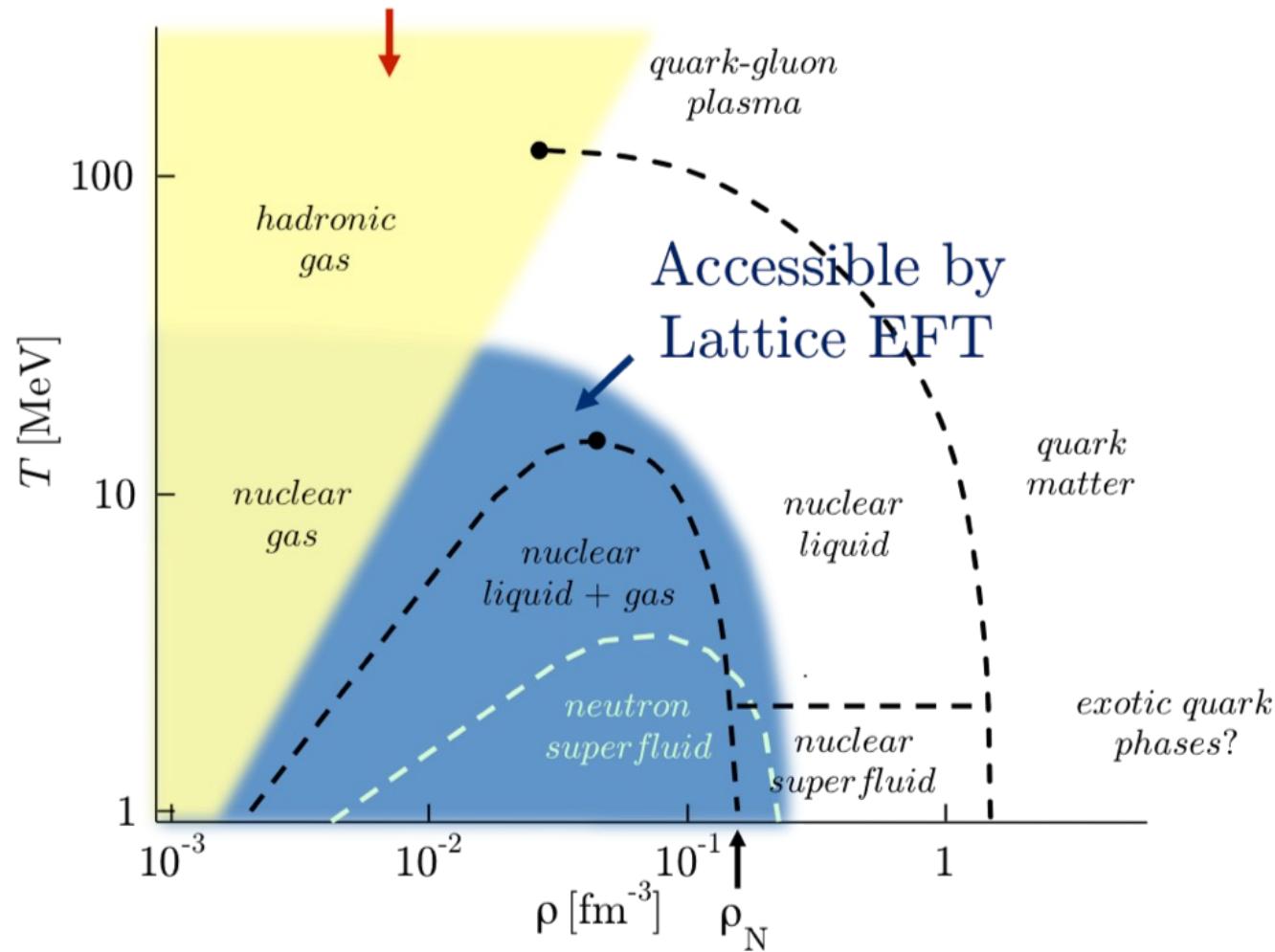
Lattice effective field theory



D.L, Prog. Part. Nucl. Phys. 63 117-154 (2009)

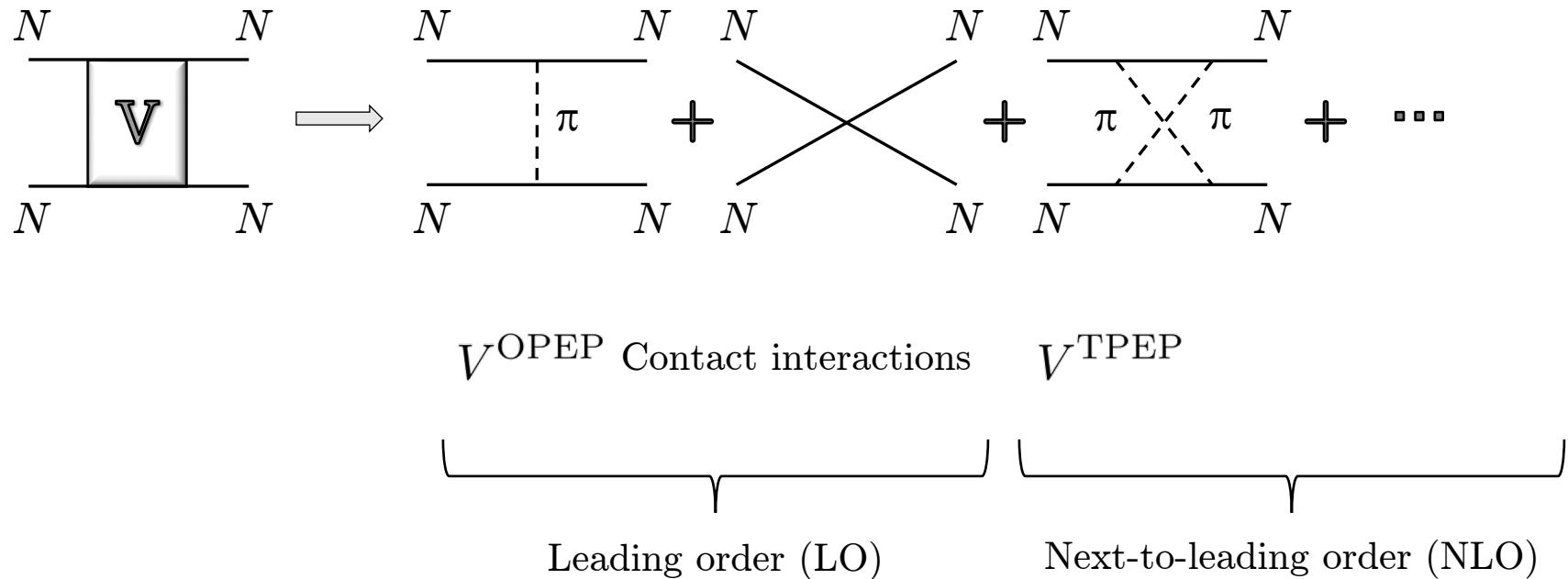
Lähde, Meißner, Nuclear Lattice Effective Field Theory (2019), Springer

Accessible by Lattice QCD

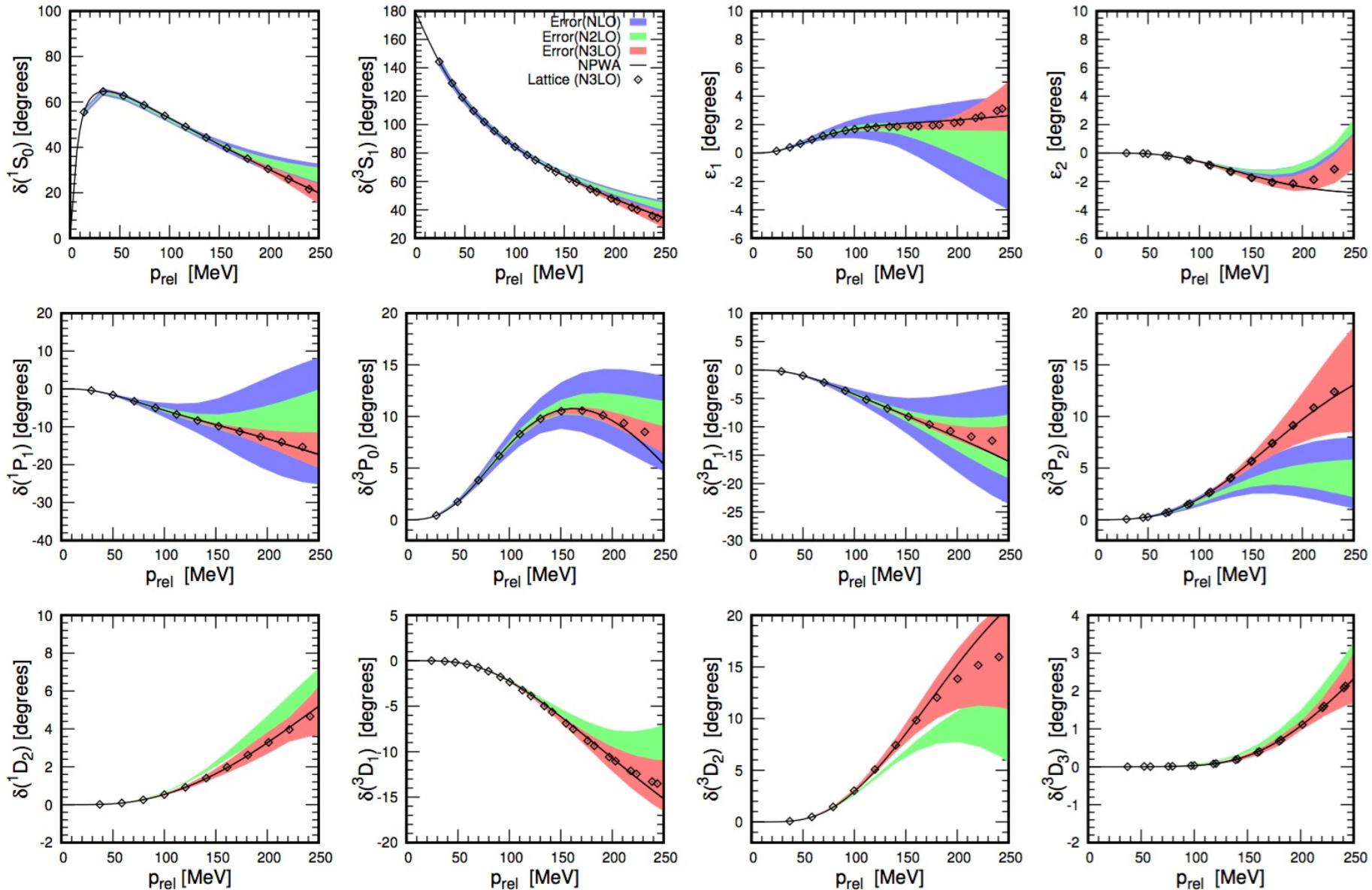


Chiral effective field theory

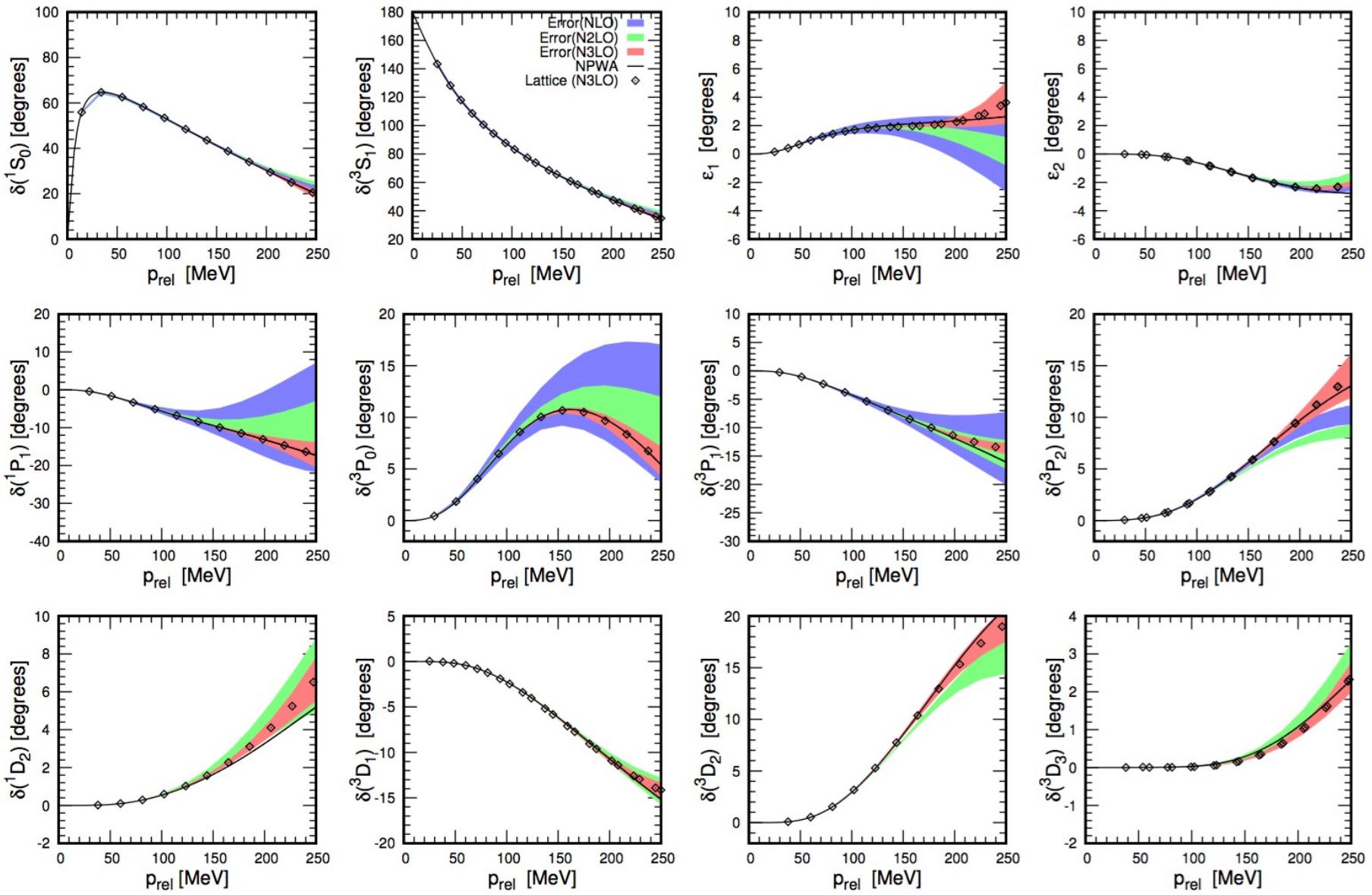
Construct the effective potential order by order



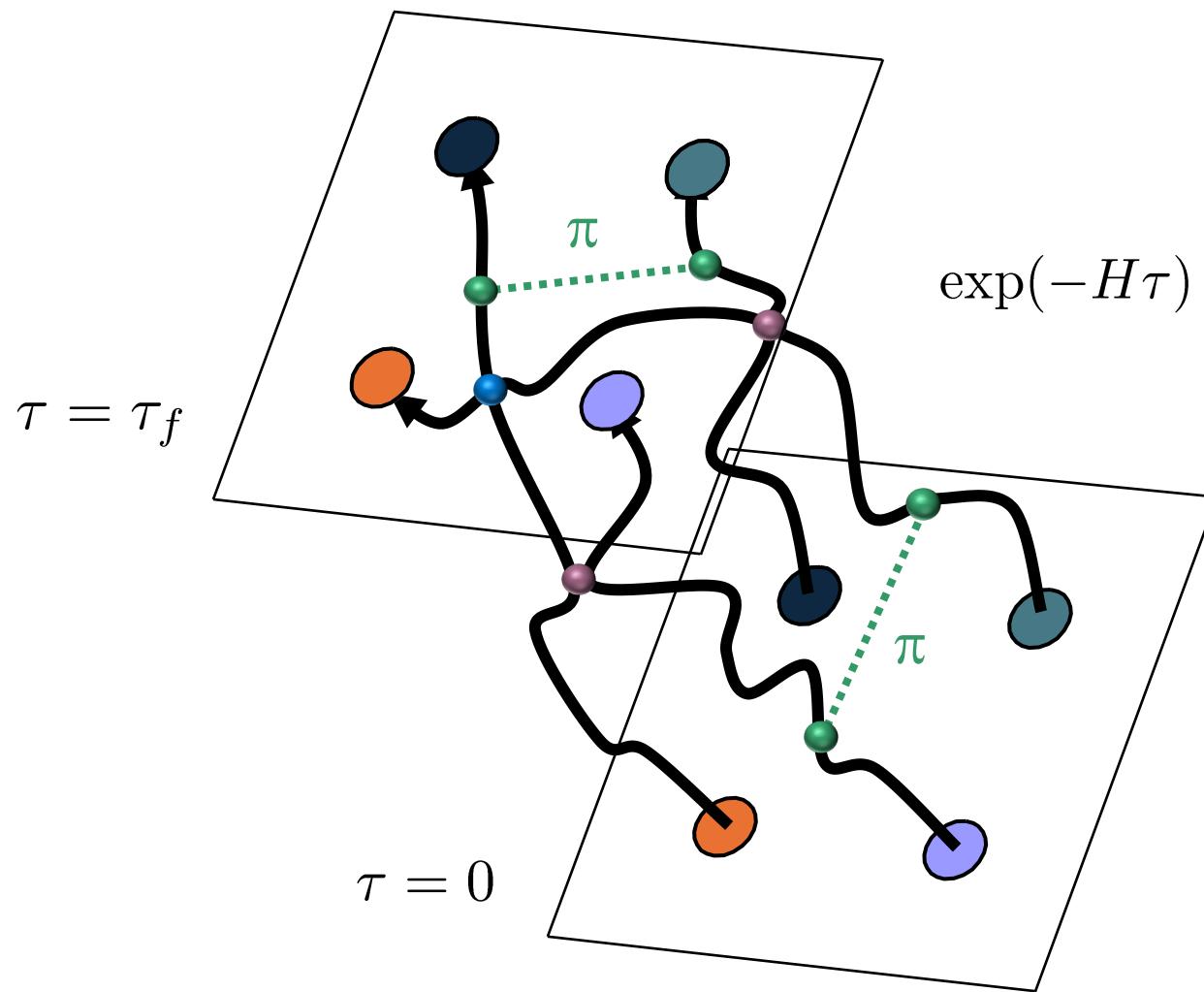
$$a = 1.315 \text{ fm}$$



$$a = 0.987 \text{ fm}$$



Euclidean time projection

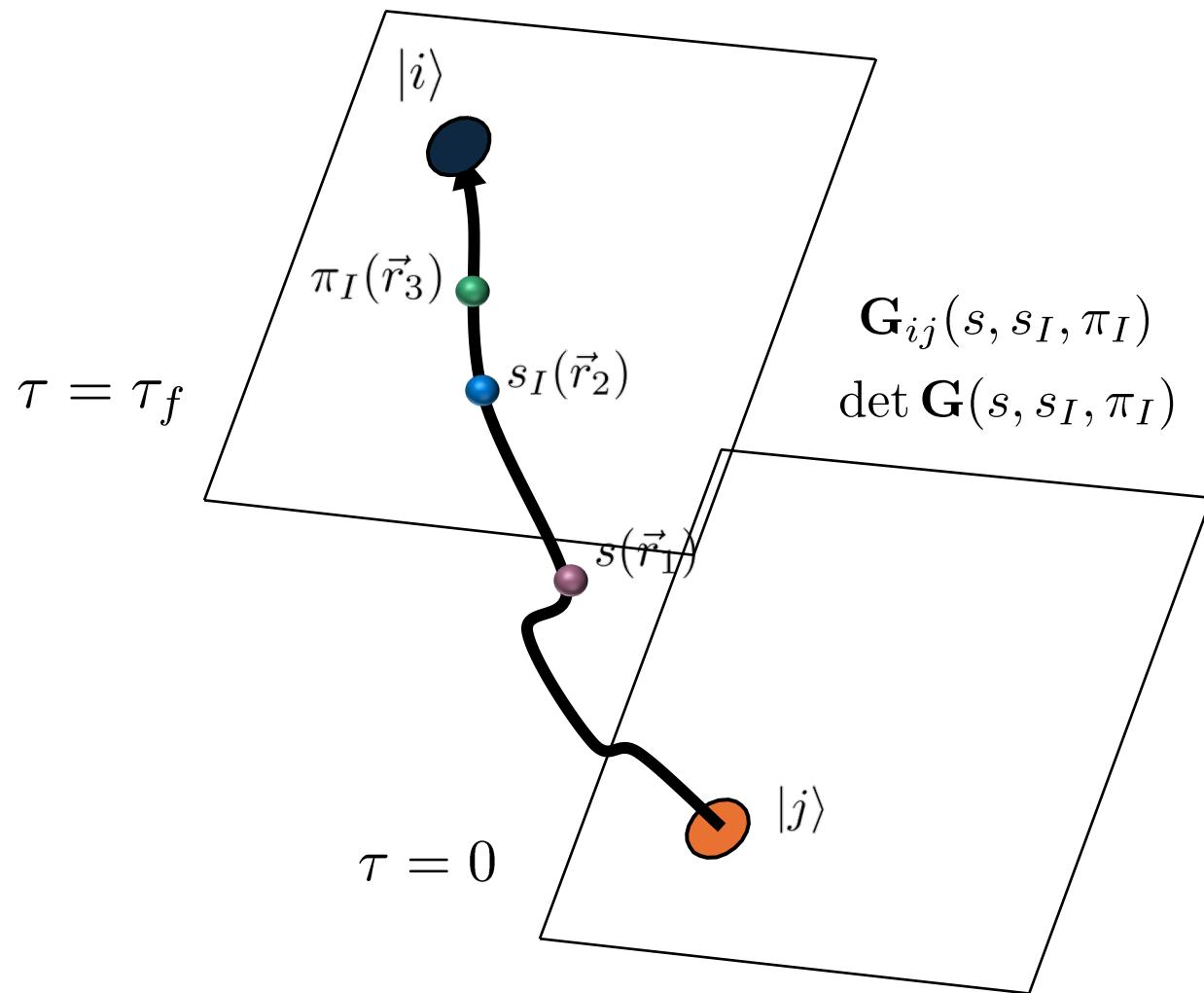


Auxiliary field method

We can write exponentials of the interaction using a Gaussian integral identity

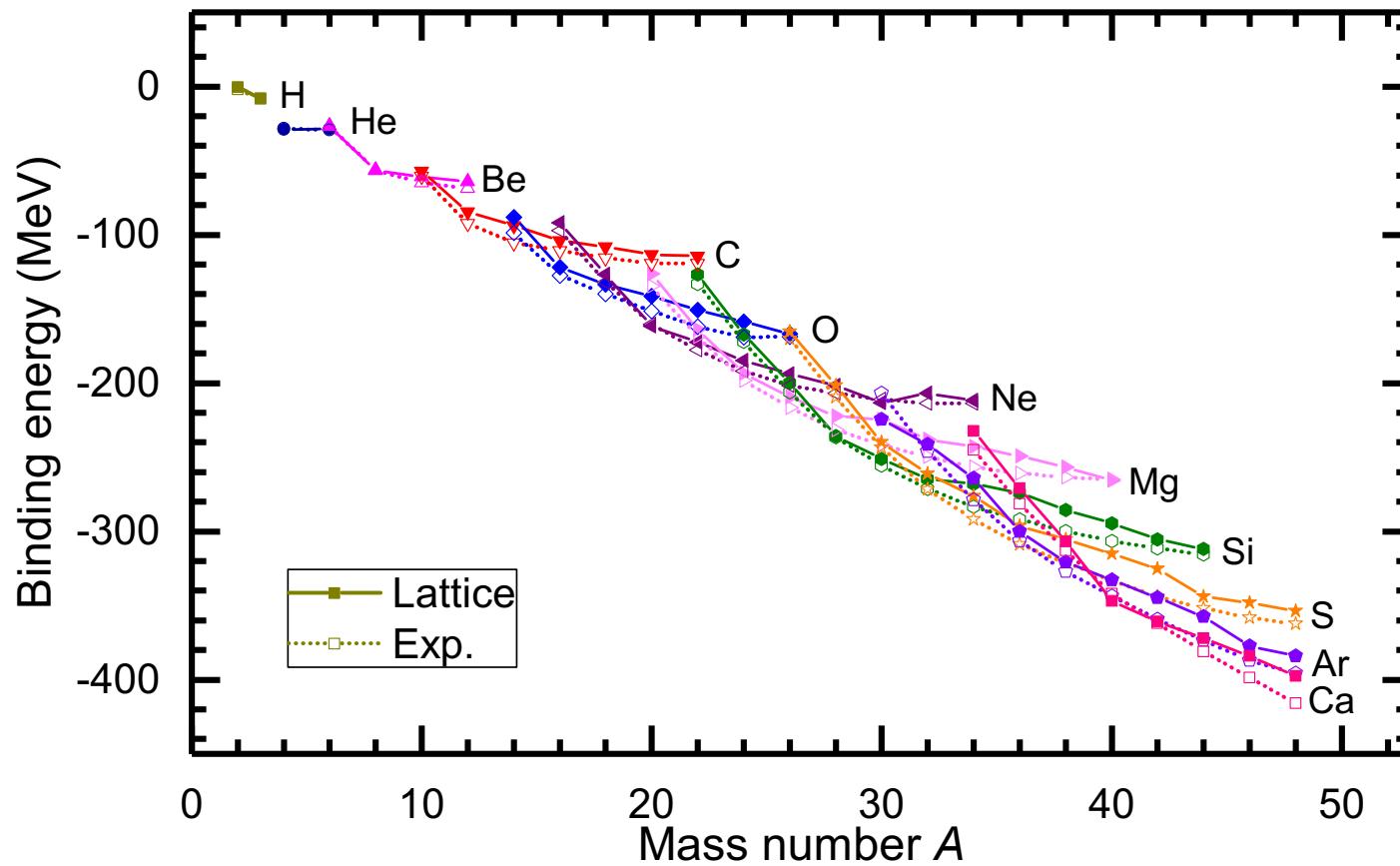
$$\exp \left[-\frac{C}{2} (N^\dagger N)^2 \right] \quad \times \quad (N^\dagger N)^2$$
$$= \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{\infty} ds \exp \left[-\frac{1}{2}s^2 + \sqrt{-C} s(N^\dagger N) \right] \quad \rangle \quad sN^\dagger N$$

We remove the interaction between nucleons and replace it with the interactions of each nucleon with a background field.

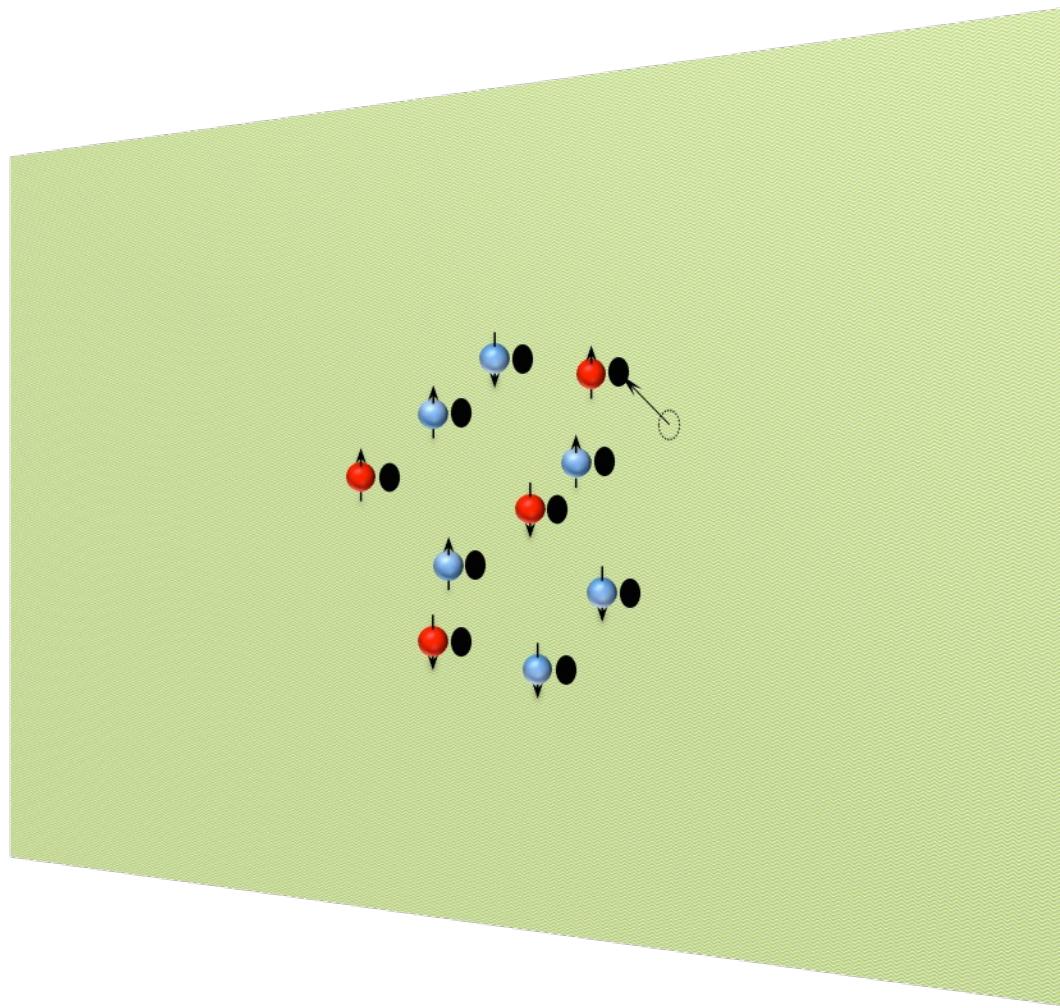


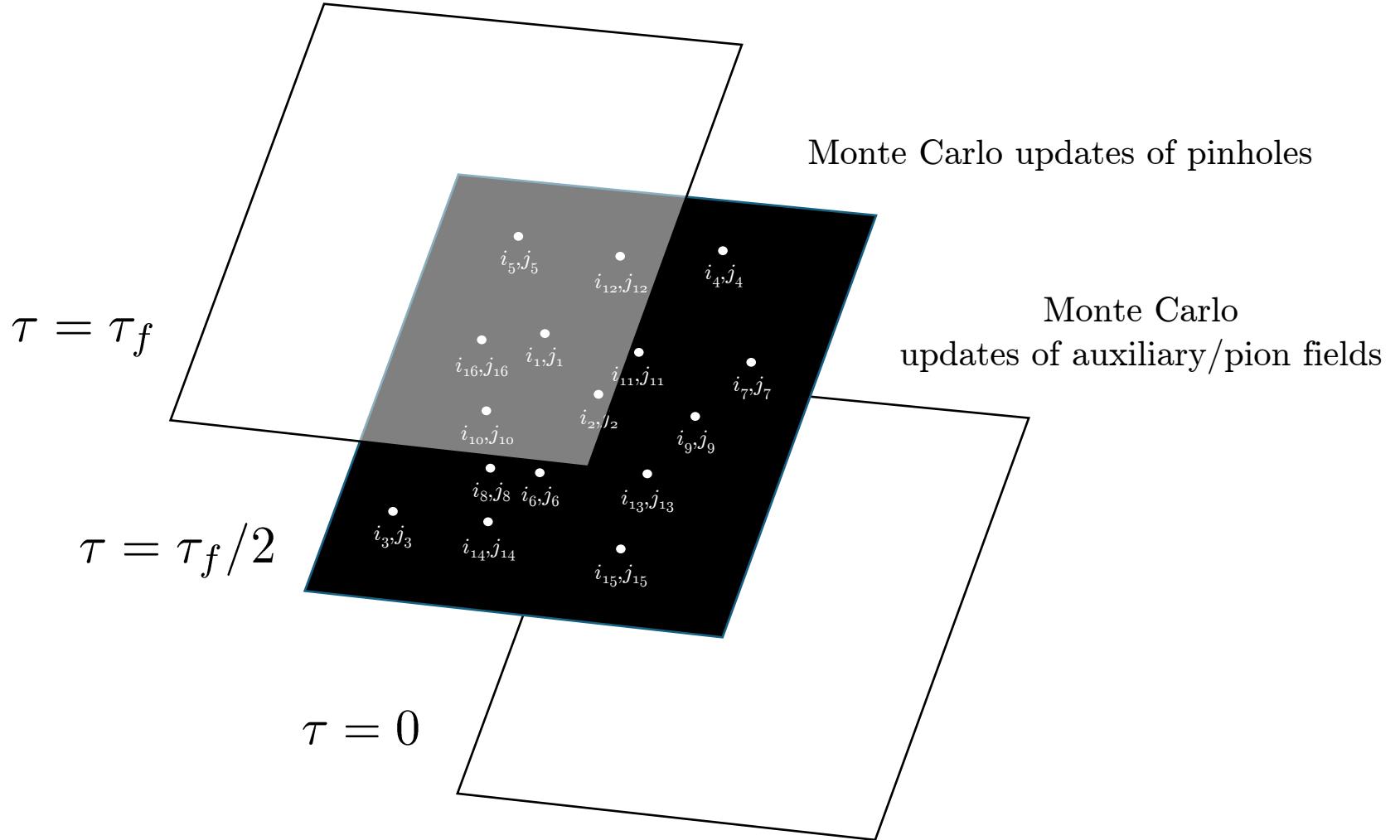
Essential elements for nuclear binding

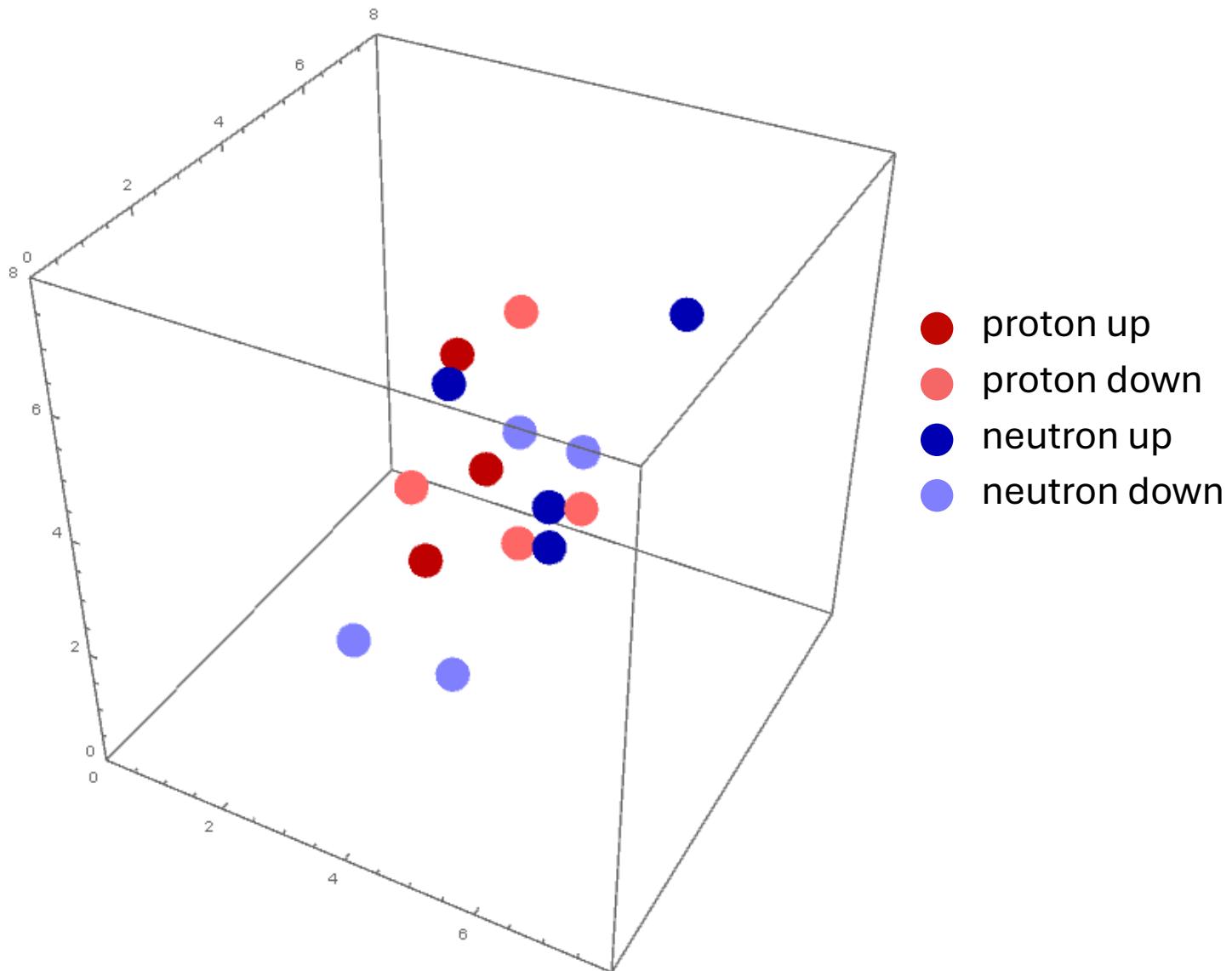
$$H = H_{\text{free}} + \frac{1}{2!} C_2 \sum_{\mathbf{n}} \tilde{\rho}(\mathbf{n})^2 + \frac{1}{3!} C_3 \sum_{\mathbf{n}} \tilde{\rho}(\mathbf{n})^3 + V_{\text{Coulomb}}$$



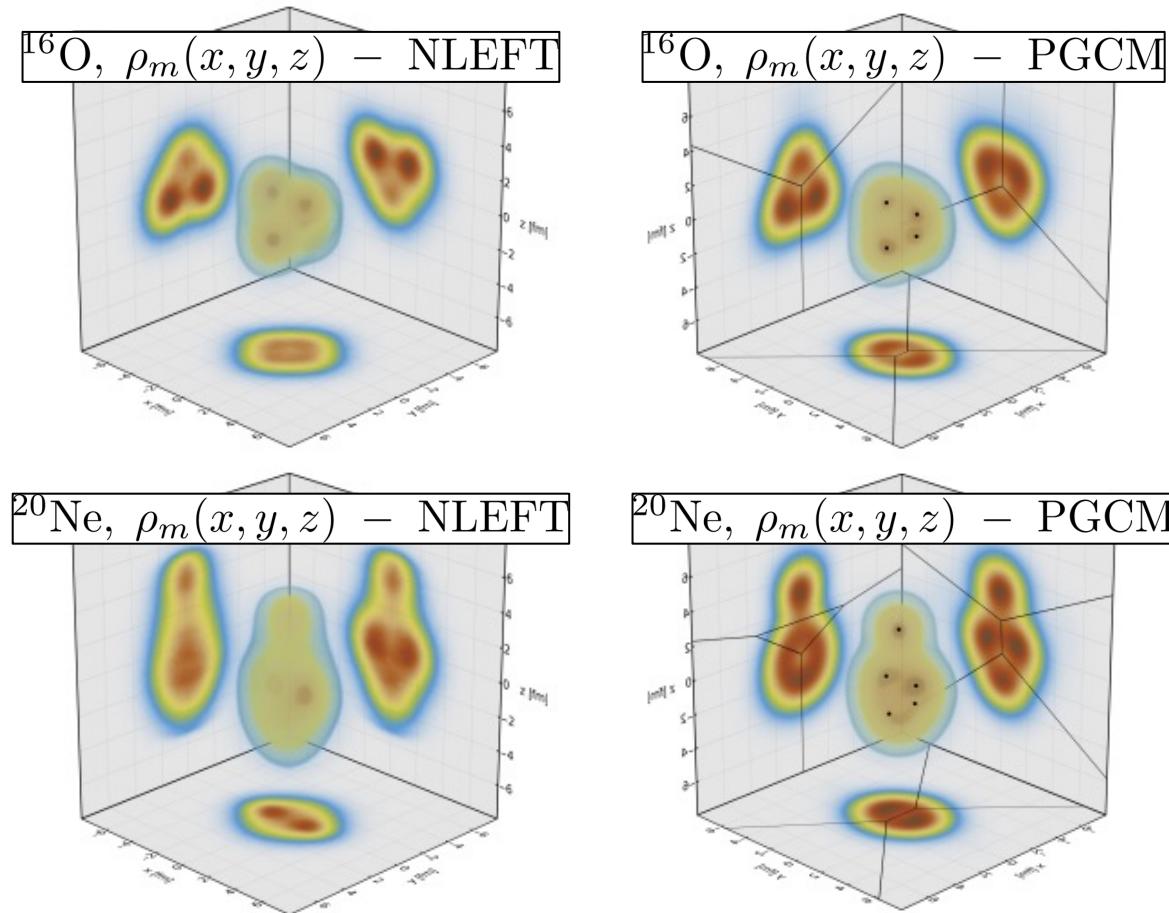
Pinhole algorithm



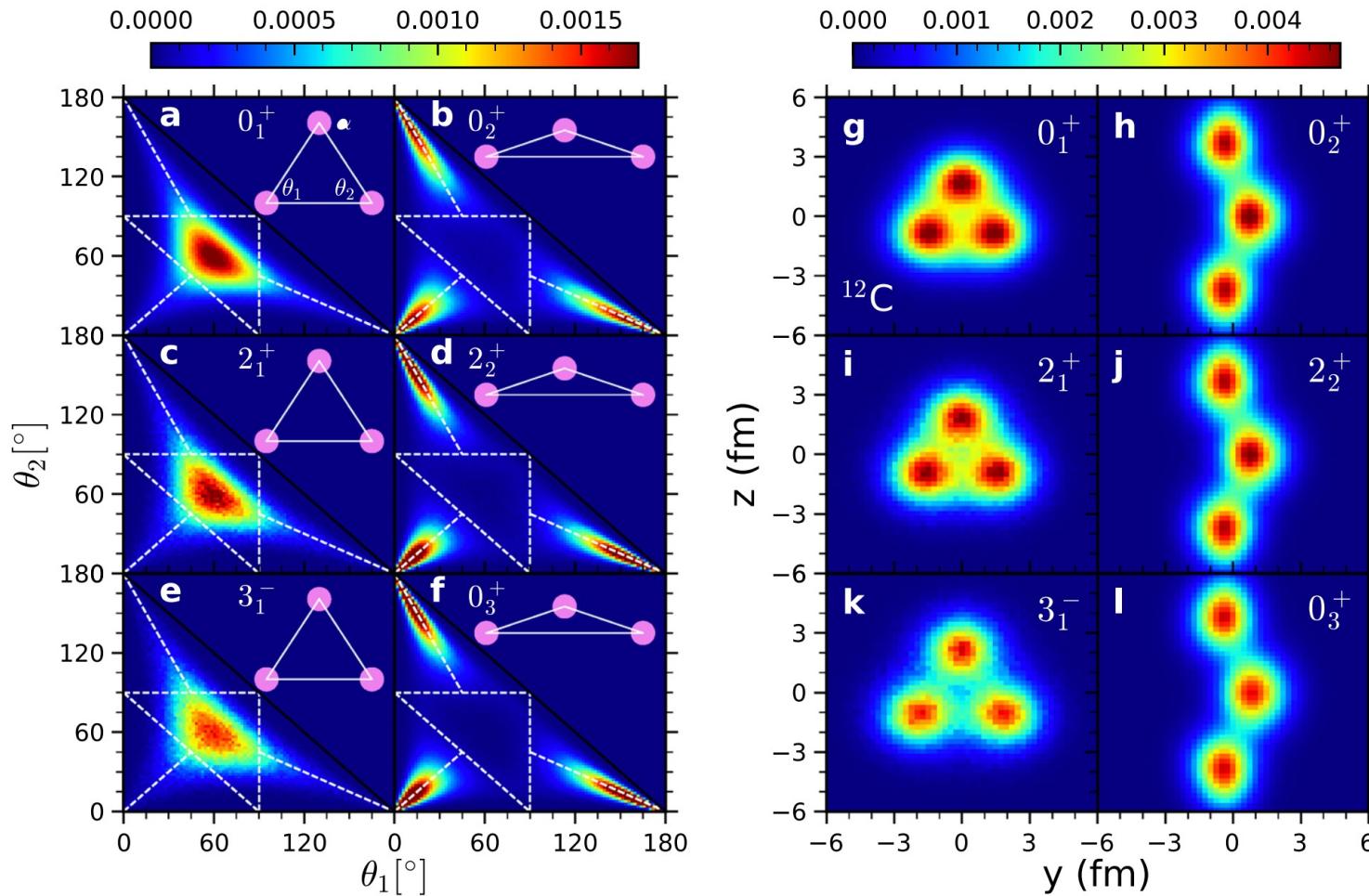


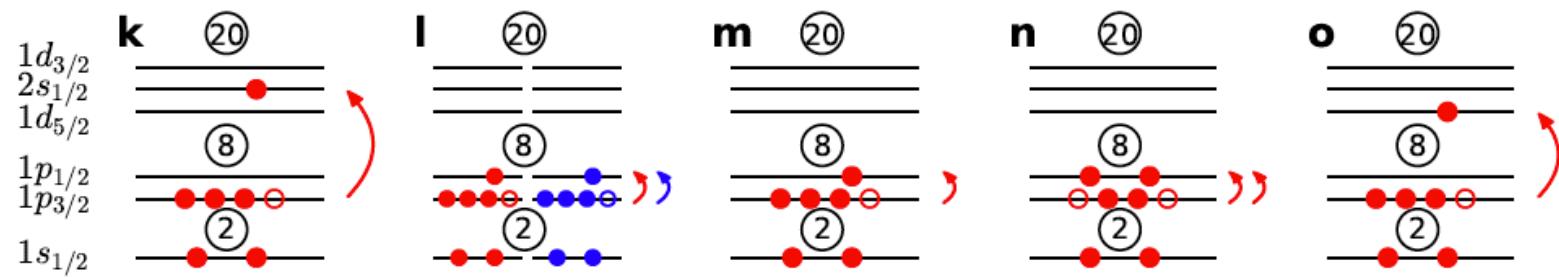
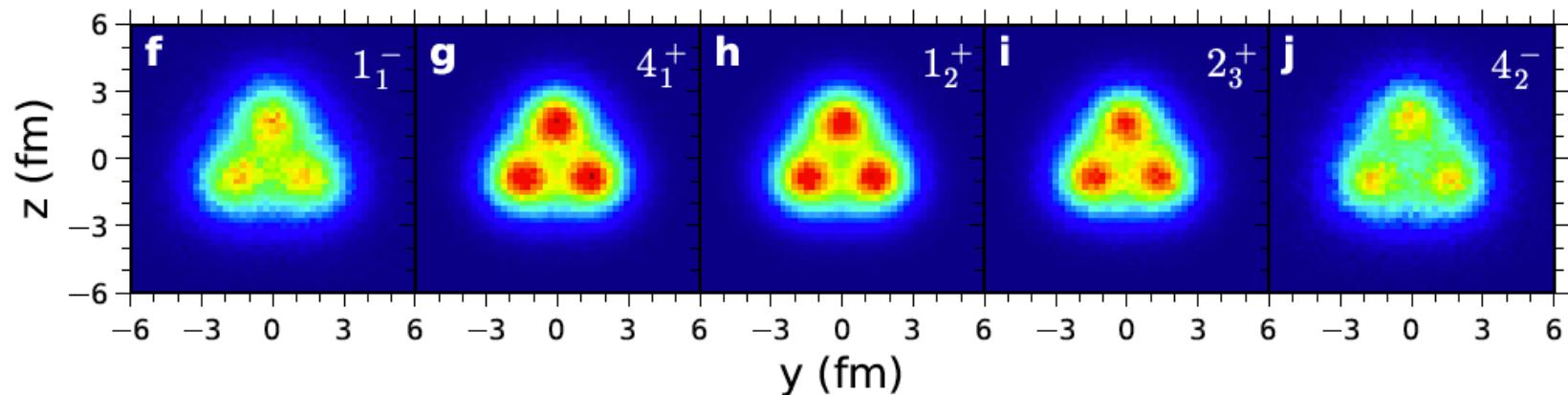
^{16}O 

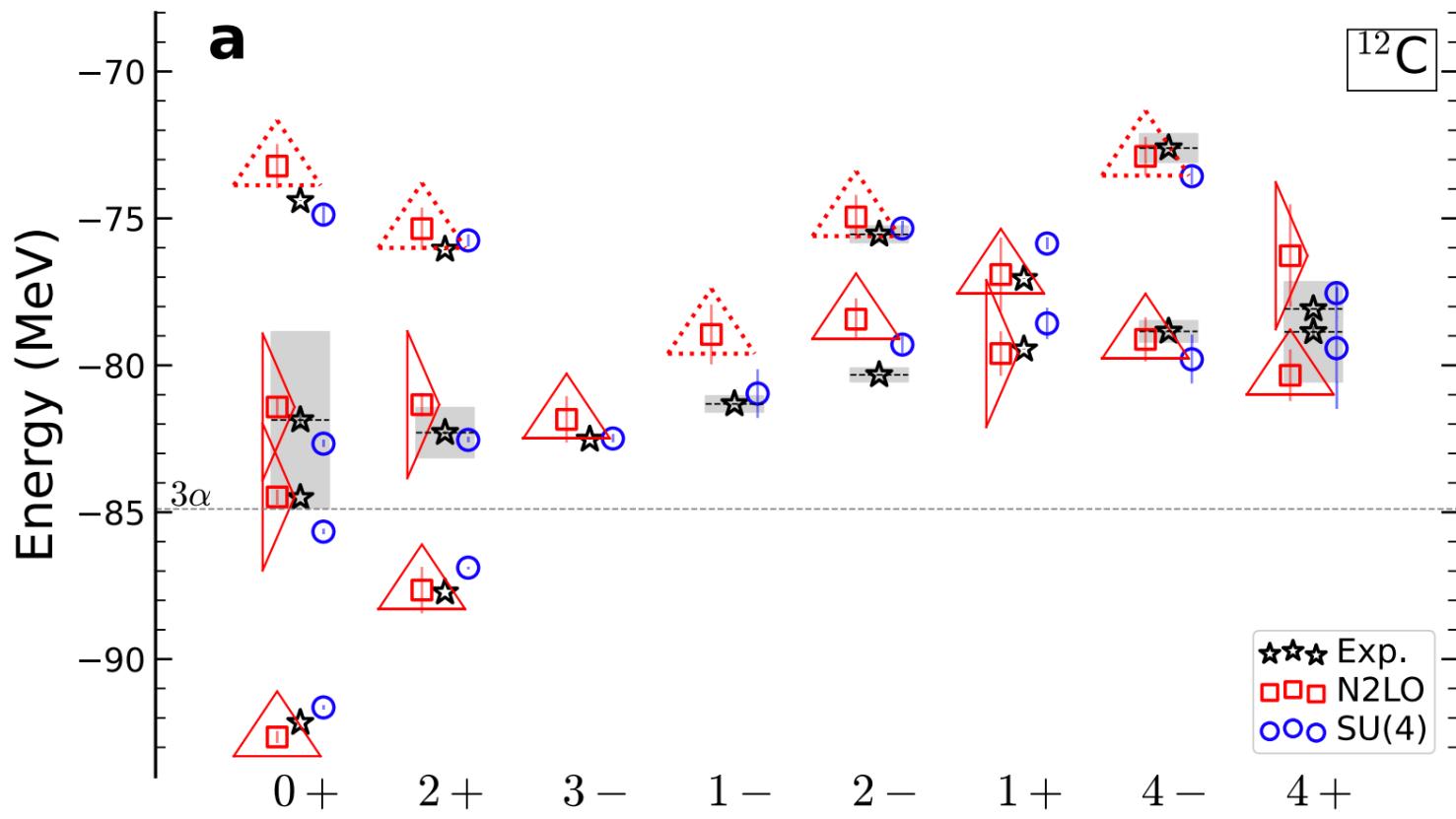
Relativistic heavy collisions: $^{16}\text{O}^{16}\text{O}$ versus $^{20}\text{Ne}^{20}\text{Ne}$



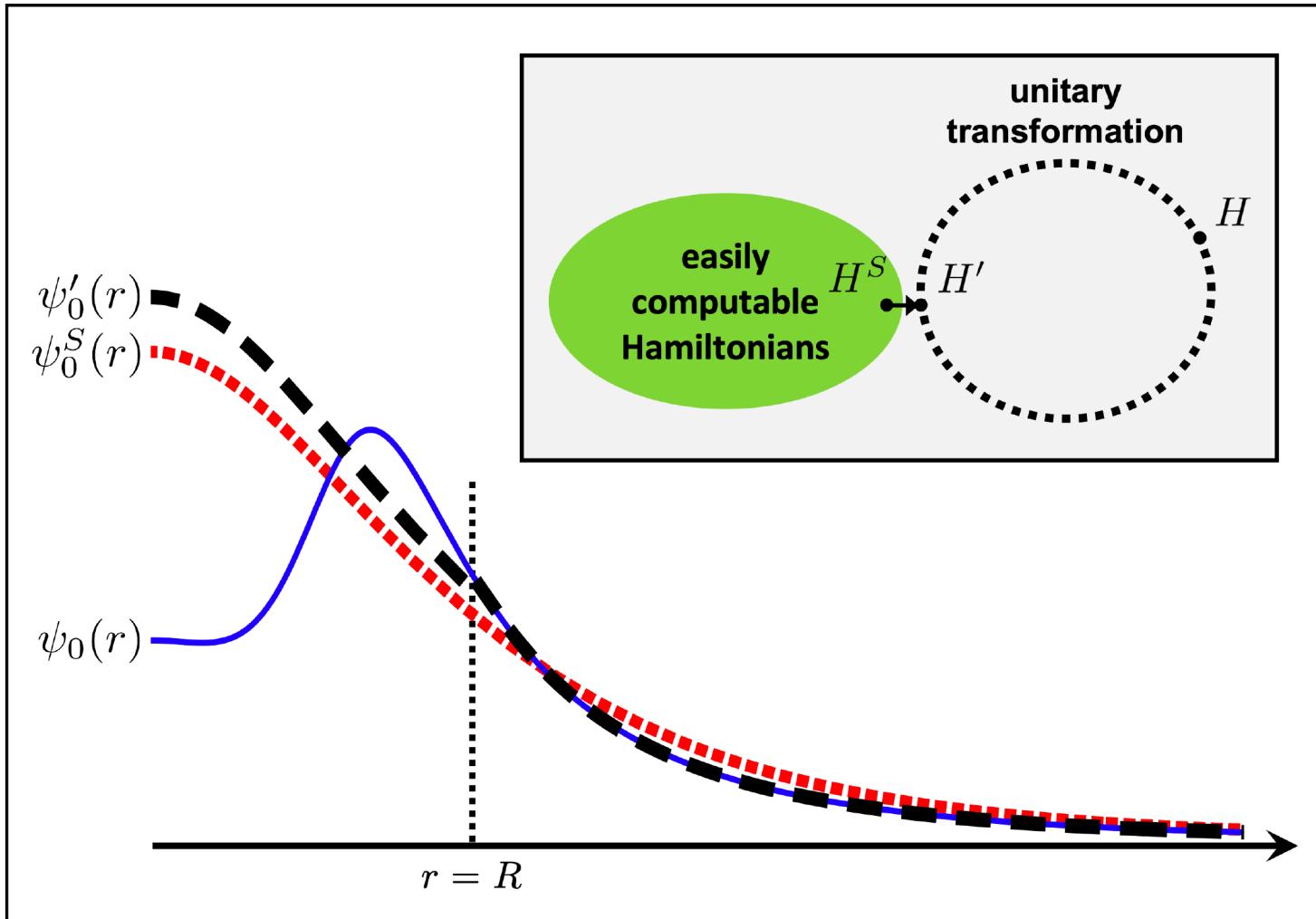
Emergent geometry and duality of ^{12}C



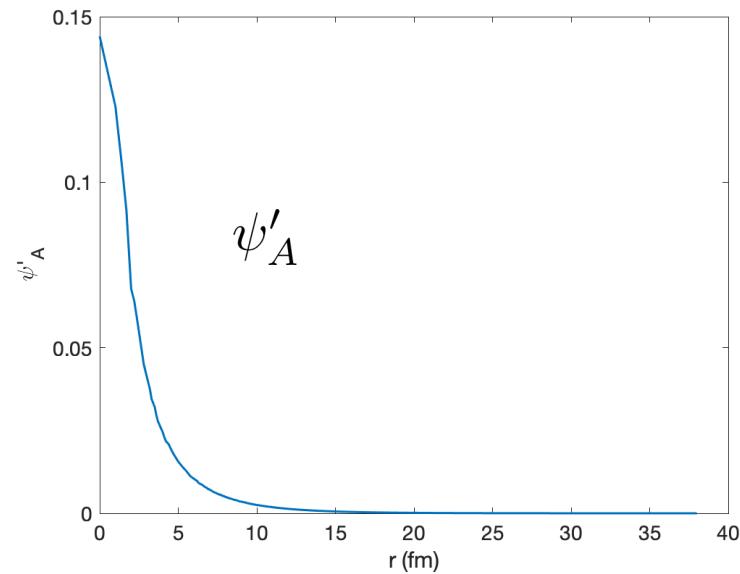
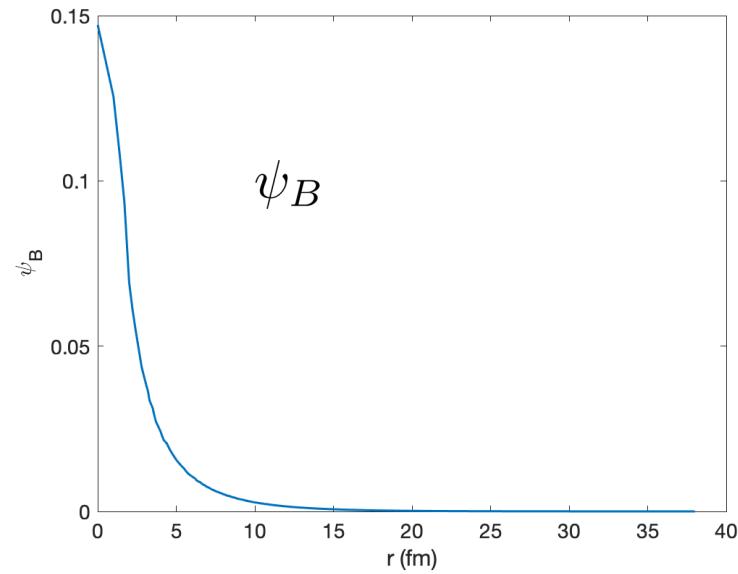
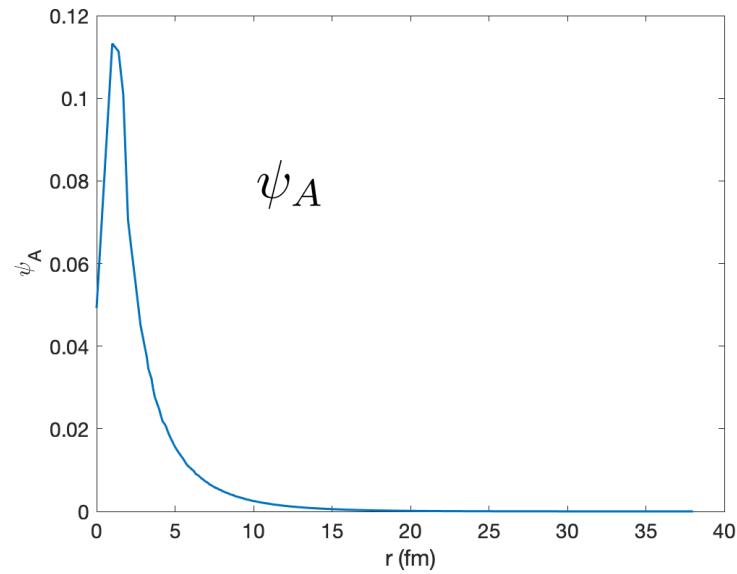




Wavefunction matching



Ground state wavefunctions



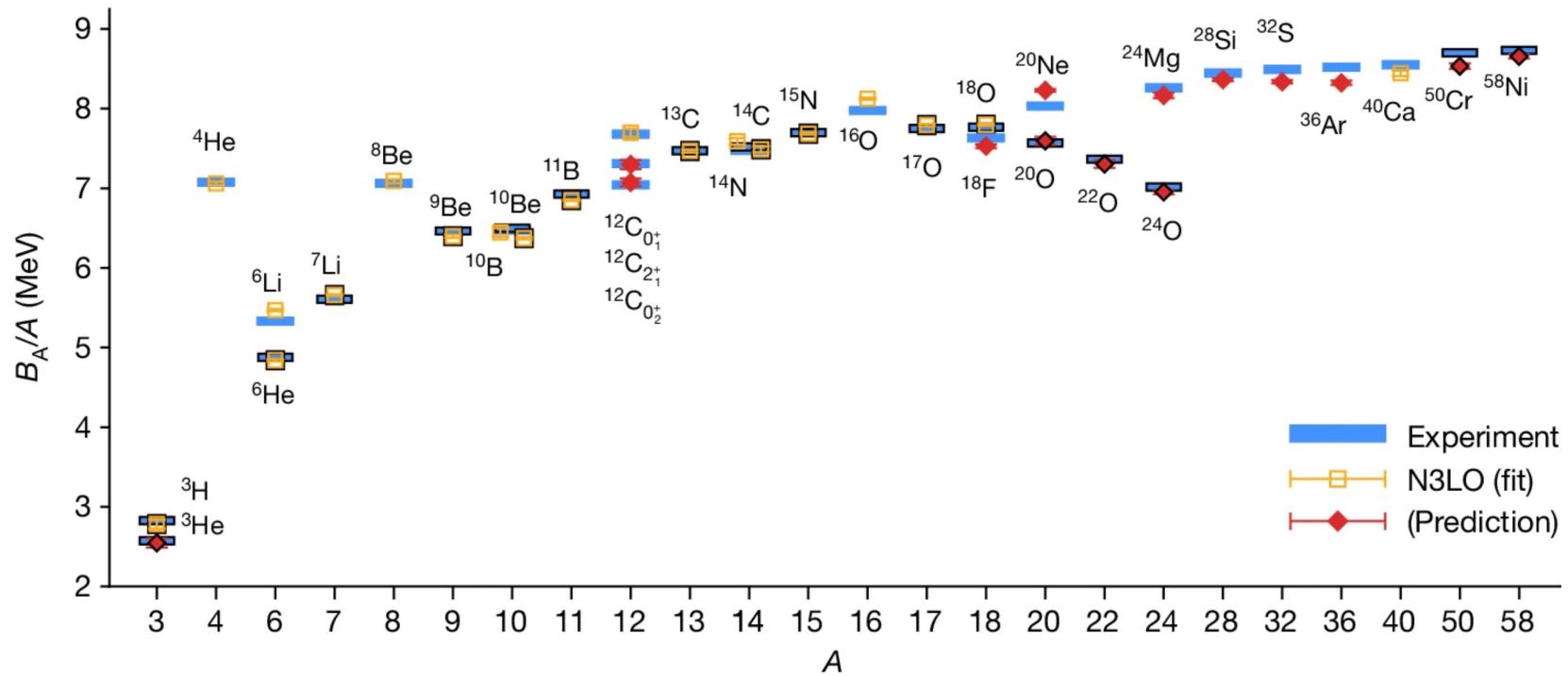
Try to compute the energies of H_A using the eigenfunctions of H_B and first-order perturbation theory. This doesn't work.

$E_{A,n} = E'_{A,n}$ (MeV)	$\langle \psi_{B,n} H_A \psi_{B,n} \rangle$ (MeV)	
-1.2186	3.0088	
0.2196	0.3289	
0.8523	1.1275	
1.8610	2.2528	
3.2279	3.6991	
4.9454	5.4786	
7.0104	7.5996	
9.4208	10.0674	
12.1721	12.8799	
15.2669	16.0458	

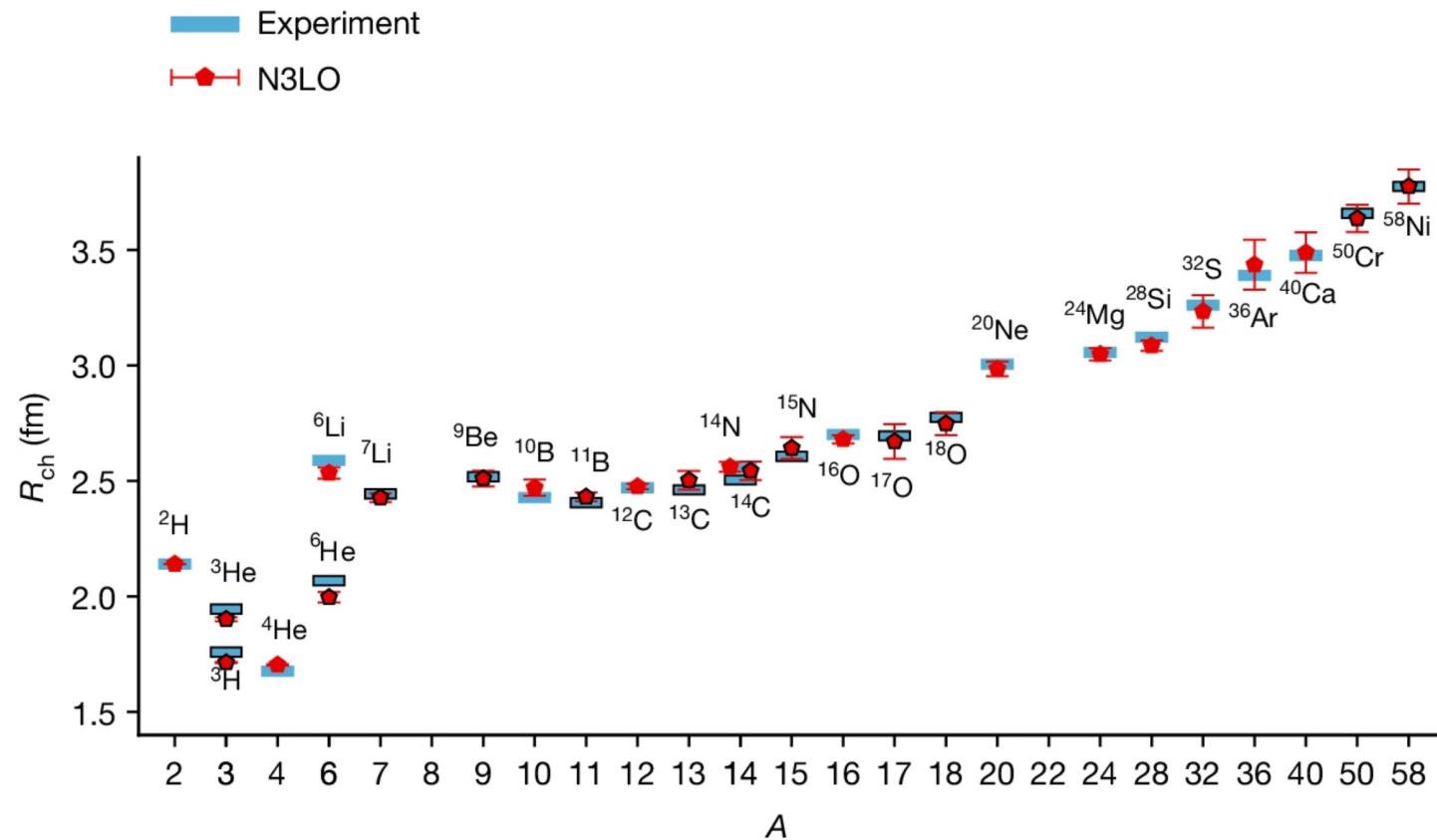
Use wavefunction matching first to transform the Hamiltonian. Then the convergence of perturbation theory is much faster.

$E_{A,n} = E'_{A,n}$ (MeV)	$\langle \psi_{B,n} H_A \psi_{B,n} \rangle$ (MeV)	$\langle \psi_{B,n} H'_A \psi_{B,n} \rangle$ (MeV)
-1.2186	3.0088	-1.1597
0.2196	0.3289	0.2212
0.8523	1.1275	0.8577
1.8610	2.2528	1.8719
3.2279	3.6991	3.2477
4.9454	5.4786	4.9798
7.0104	7.5996	7.0680
9.4208	10.0674	9.5137
12.1721	12.8799	12.3163
15.2669	16.0458	15.4840

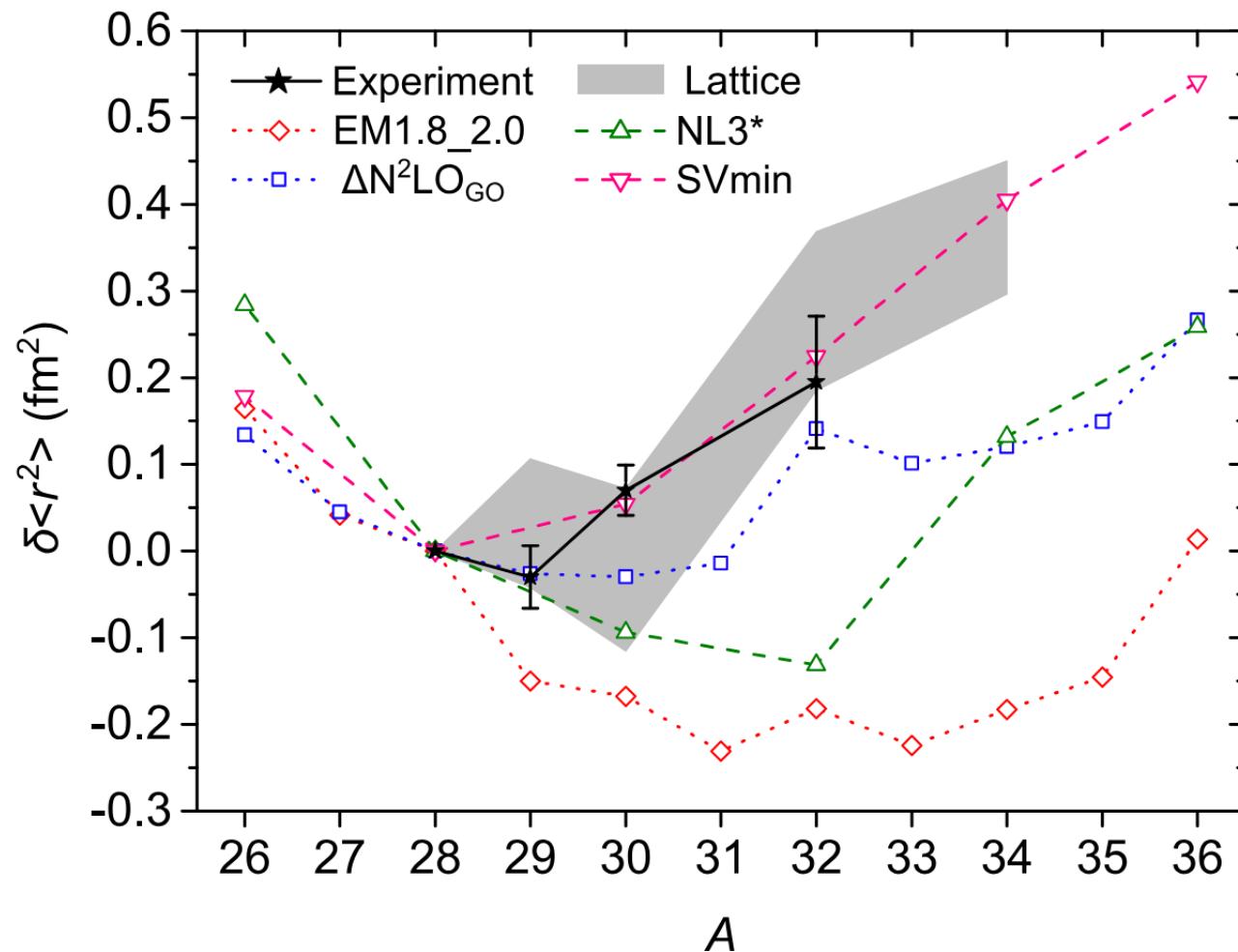
Binding energies



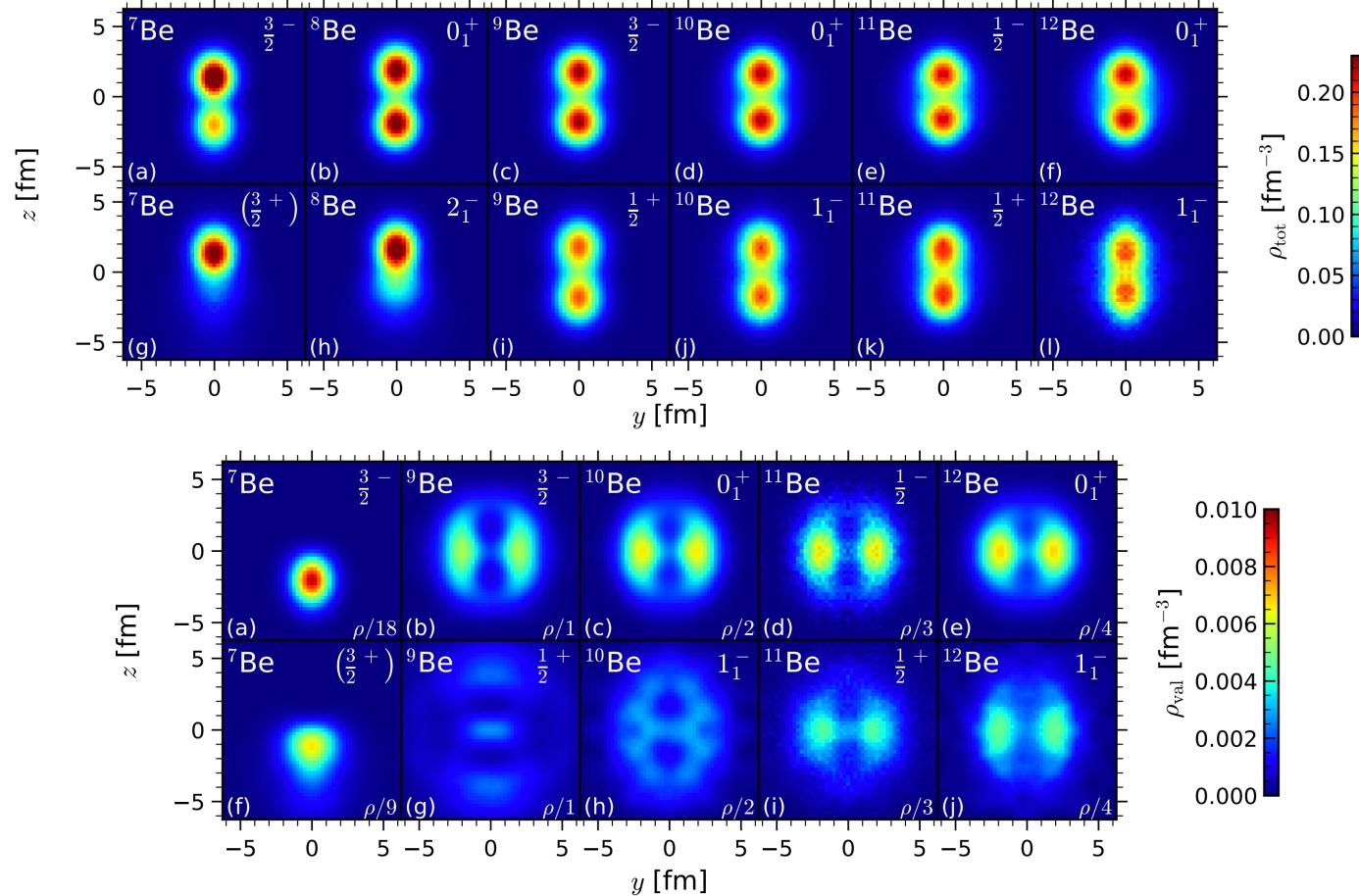
Charge radii



Charge radii of silicon isotopes



Properties of the beryllium isotopes



Ab initio nuclear thermodynamics

In order to compute thermodynamic properties of finite nuclei, nuclear matter, and neutron matter, we need to compute the partition function

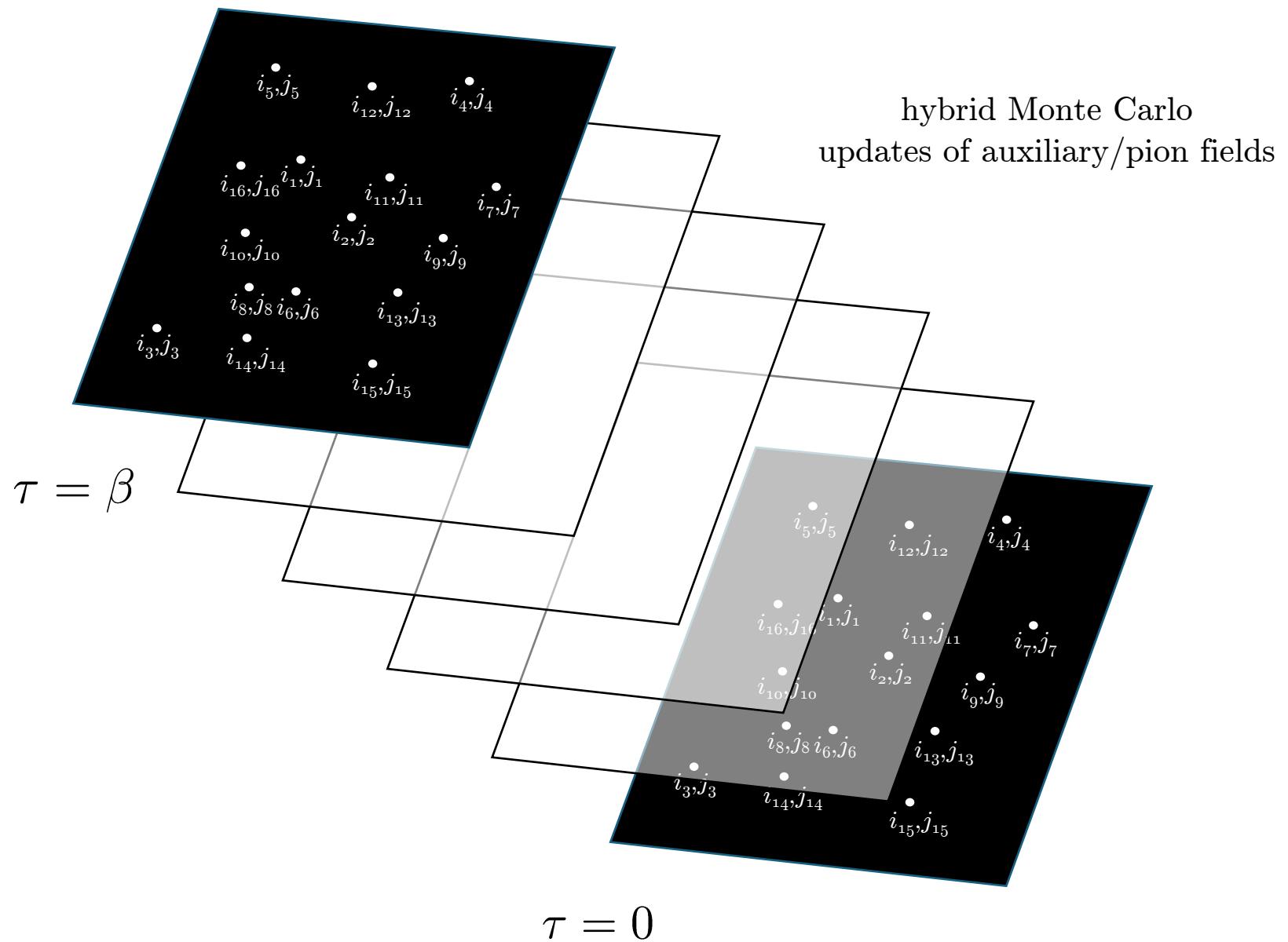
$$\text{Tr} \exp(-\beta H)$$

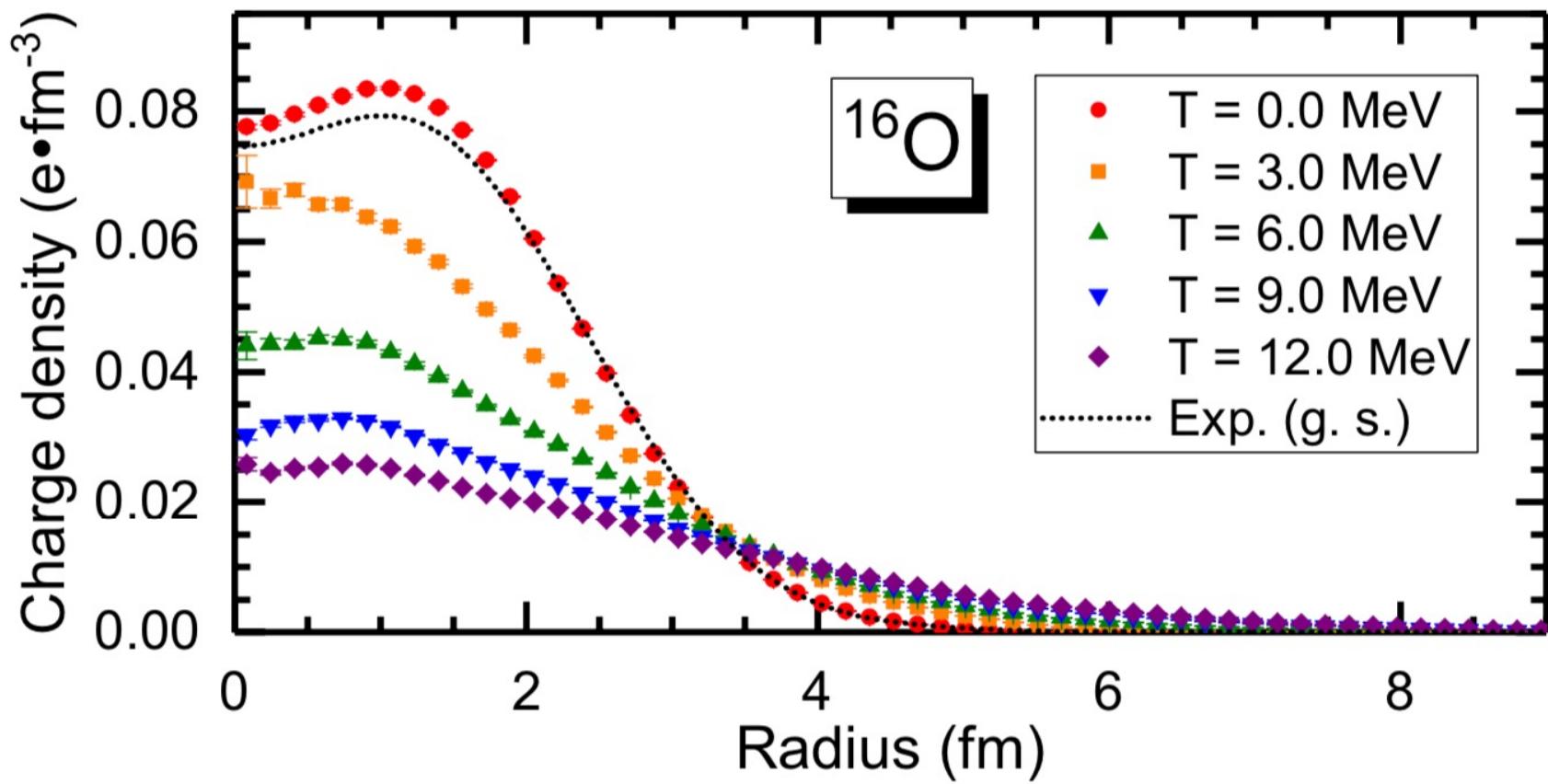
We compute the quantum mechanical trace over A -nucleon states by summing over pinholes (position eigenstates) for the initial and final states

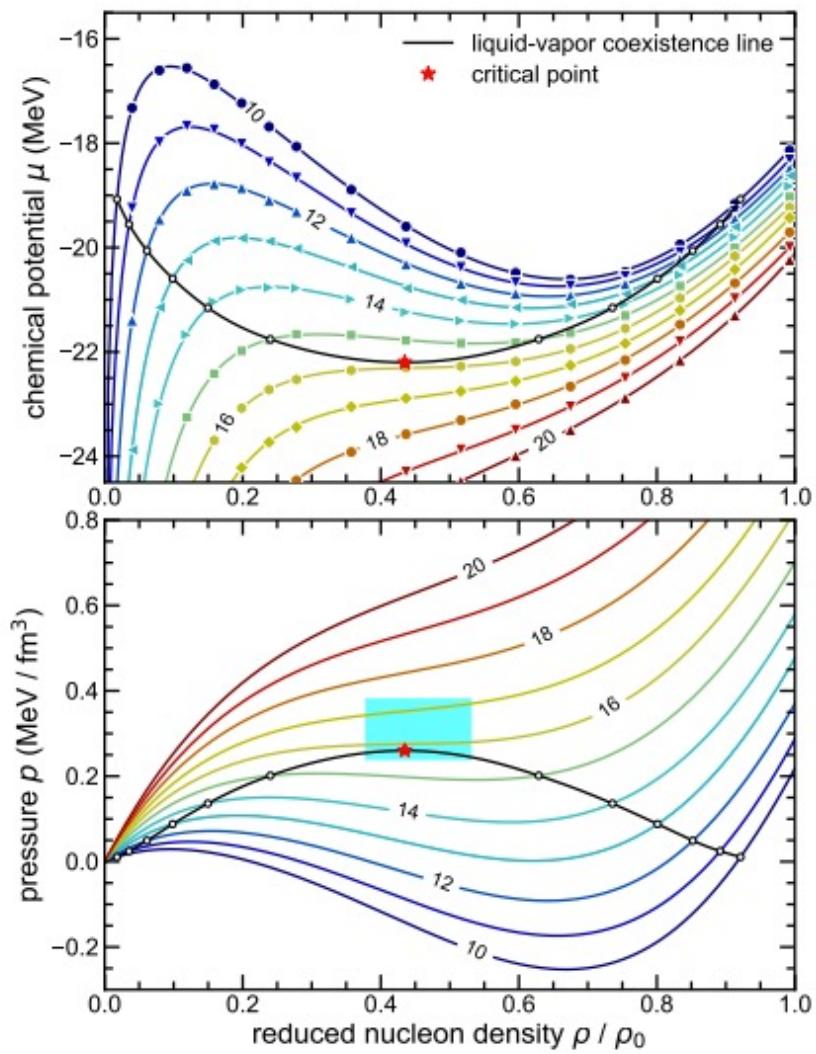
$$\begin{aligned} & \text{Tr } O \\ &= \frac{1}{A!} \sum_{i_1 \cdots i_A, j_1 \cdots j_A, \mathbf{n}_1 \cdots \mathbf{n}_A} \langle 0 | a_{i_A, j_A}(\mathbf{n}_A) \cdots a_{i_1, j_1}(\mathbf{n}_1) O a_{i_1, j_1}^\dagger(\mathbf{n}_1) \cdots a_{i_A, j_A}^\dagger(\mathbf{n}_A) | 0 \rangle \end{aligned}$$

This can be used to calculate the partition function in the canonical ensemble.

Metropolis updates of pinholes





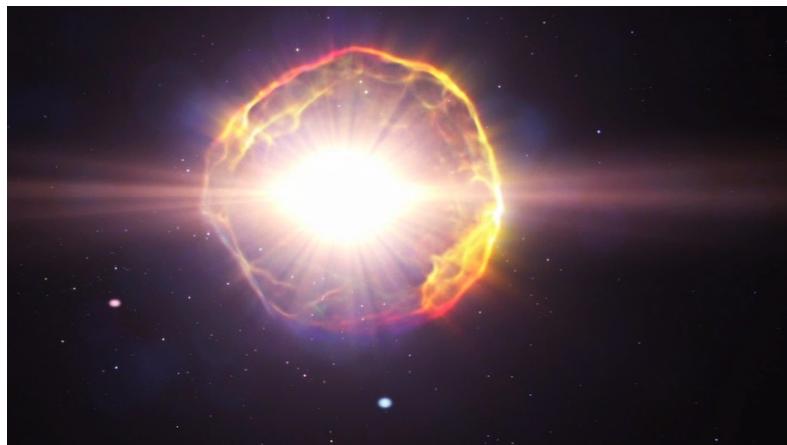


$$\begin{aligned}
 T_c &= 15.80(0.32)(1.60) \text{ MeV} \\
 \rho_c &= 0.089(04)(18) \text{ fm}^{-3} \\
 \mu_c &= -22.20(0.44)(2.20) \text{ MeV} \\
 P_c &= 0.260(05)(30) \text{ MeV fm}^{-3}
 \end{aligned}$$

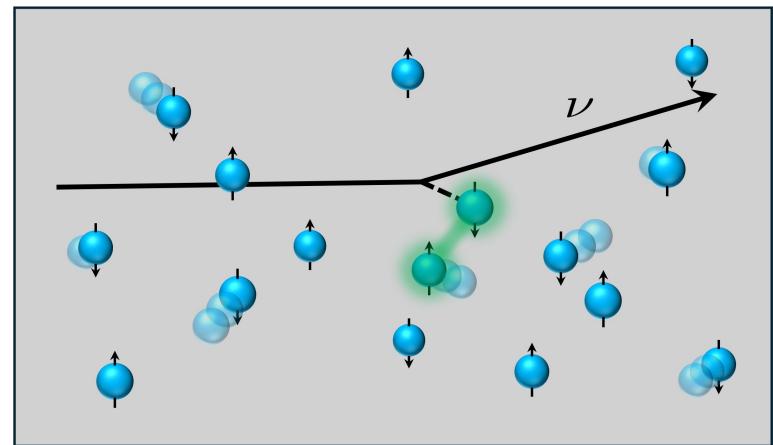
Structure factors for hot neutron matter

$$S_v(\mathbf{q}) = \frac{1}{L^3} \sum_{\mathbf{n}\mathbf{n}'} e^{-i\mathbf{q}\cdot\mathbf{n}} [\langle \hat{\rho}(\mathbf{n} + \mathbf{n}') \hat{\rho}(\mathbf{n}') \rangle - (\rho^0)^2]$$

$$S_a(\mathbf{q}) = \frac{1}{L^3} \sum_{\mathbf{n}\mathbf{n}'} e^{-i\mathbf{q}\cdot\mathbf{n}} [\langle \hat{\rho}_z(\mathbf{n} + \mathbf{n}') \hat{\rho}_z(\mathbf{n}') \rangle - (\rho_z^0)^2]$$



ESA/Hubble/L Calca
da



Ma, Lin, Lu, Elhatisari, D.L., Meißner, Steiner, Wang, Phys. Rev. Lett. **132**, 232502 (2024)

See also Alexandru, Bedaque, Berkowitz, Warrington, Phys. Rev. Lett. **126**, 132701 (2021)

What problems are beyond classical computing?

Monte Carlo sign problem

Real time dynamics

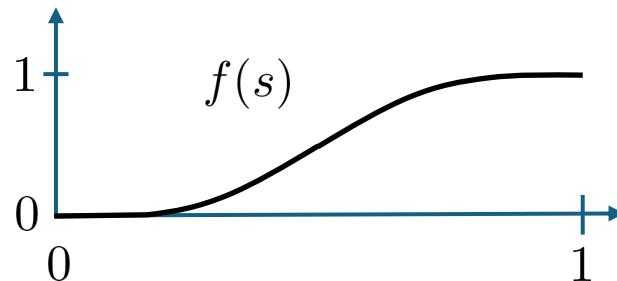
Spectral functions

Adiabatic evolution

The final Hamiltonian H_F will be the Hamiltonian of interest. Choose an initial Hamiltonian H_I with a simple ground state that can be easily prepared. We will time evolve the system while interpolating between these two Hamiltonians over a total time T . It is convenient to work with a rescaled time parameter s .

$$s = \frac{t}{T}$$

We introduce an interpolation function $f(s)$ that smoothly goes from 0 to 1 as s varies from 0 to 1.



We define the time-dependent Hamiltonian

$$H(s) = H_I + f(s)(H_F - H_I)$$

$$H(0) = H_I \quad H(1) = H_F$$

We will evolve the system with this time-dependent Hamiltonian as time $t = sT$ goes from 0 to T .

$$i \frac{\partial}{\partial t} |\Psi(t)\rangle = H(t) |\Psi(t)\rangle$$

$$\frac{i}{T} \frac{\partial}{\partial s} |\Psi(s)\rangle = H(s) |\Psi(s)\rangle$$

We can define instantaneous eigenfunctions and eigenvalues of $H(s)$

$$H(s) |\psi_m(s)\rangle = E_m(s) |\psi_m(s)\rangle$$

Adiabatic theorem

If the initial state is an eigenstate of the initial Hamiltonian

$$|\Psi_m(0)\rangle = |\psi_m(0)\rangle$$

$$H_I |\Psi_m(0)\rangle = E_m(0) |\Psi_m(0)\rangle$$

then we find

$$|\Psi_m(s)\rangle = e^{iT\theta_m(s)} e^{i\gamma_m(s)} |\psi_m(s)\rangle + \sum_{n \neq m} c_{n,m}(s) |\psi_n(s)\rangle$$

where the adiabatic phase consists of a dynamical phase

$$\theta_m(s) = - \int_0^s E_m(s') ds'$$

as well as a geometric phase or Berry phase

$$\gamma_m(s) = i \int_0^s \langle \psi_m(s') | \dot{\psi}_m(s') \rangle ds'$$

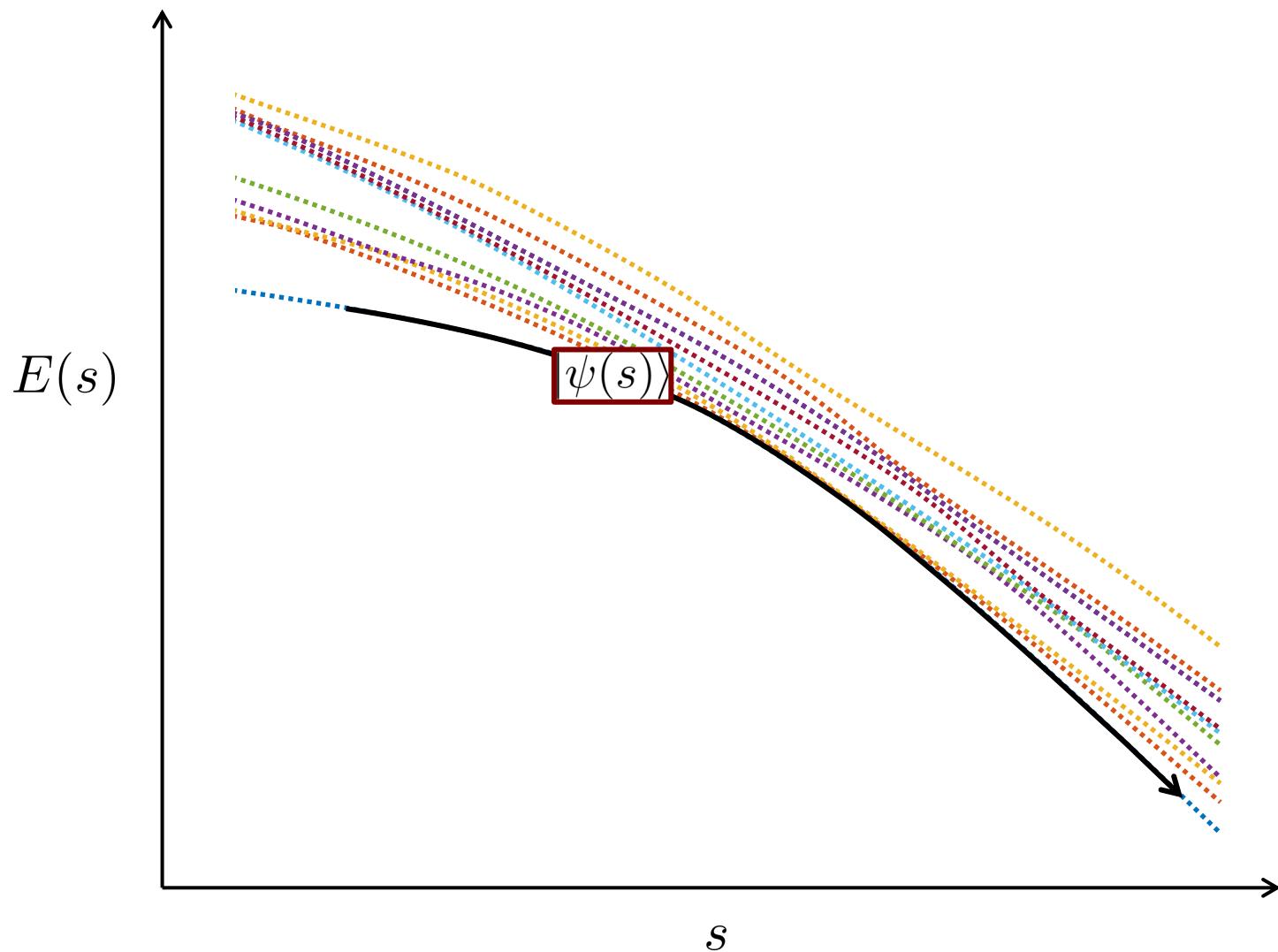
The non-adiabatic coefficient for $n \neq m$ is

$$c_{n,m}(s) = \int_0^s ds' \frac{\langle \psi_n(s') | \dot{H}(s') | \psi_m(s') \rangle}{E_n(s') - E_m(s')} e^{iT \int_0^{s'} ds'' (E_n(s'') - E_m(s''))}$$

As T becomes large, the large phase angle produces a highly oscillatory integrand, and the non-adiabatic coefficient goes to zero as some inverse power of T .

If the first k derivatives of $f(s)$ vanish at $s = 0$ and $s = 1$, then one can show by integration by parts that

$$c_{n,m}(1) = O(1/T^{k+1})$$



Hamiltonian translators

Suppose U_{AB} is a unitary transformation mapping all the eigenvectors of H_B to all the eigenvectors of H_A . Let U_{BA} be the inverse of U_{AB} . We note the curious fact that

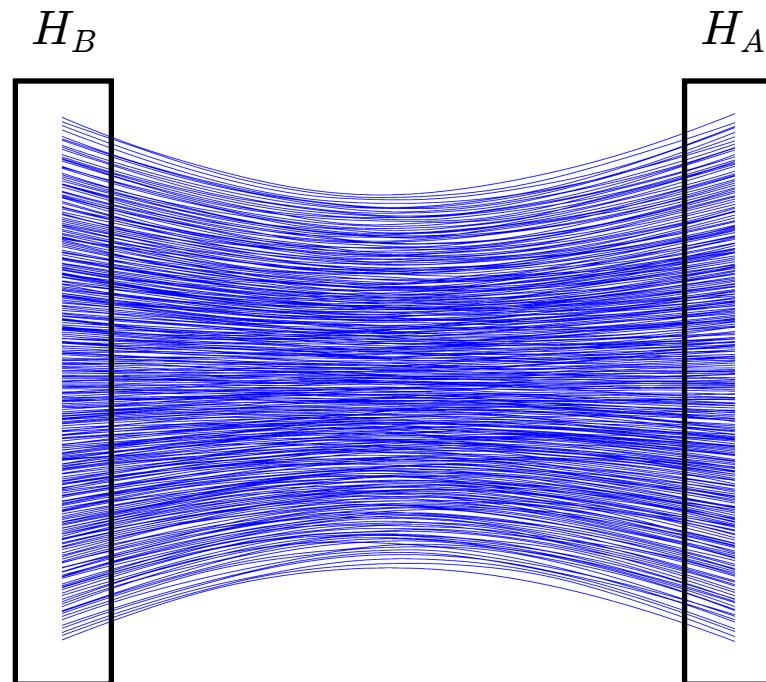
$$H'_A = U_{BA} H_A U_{AB}$$

has the eigenvectors of H_B but has the eigenvalues of H_A . We call U_{AB} and U_{BA} Hamiltonian translators.

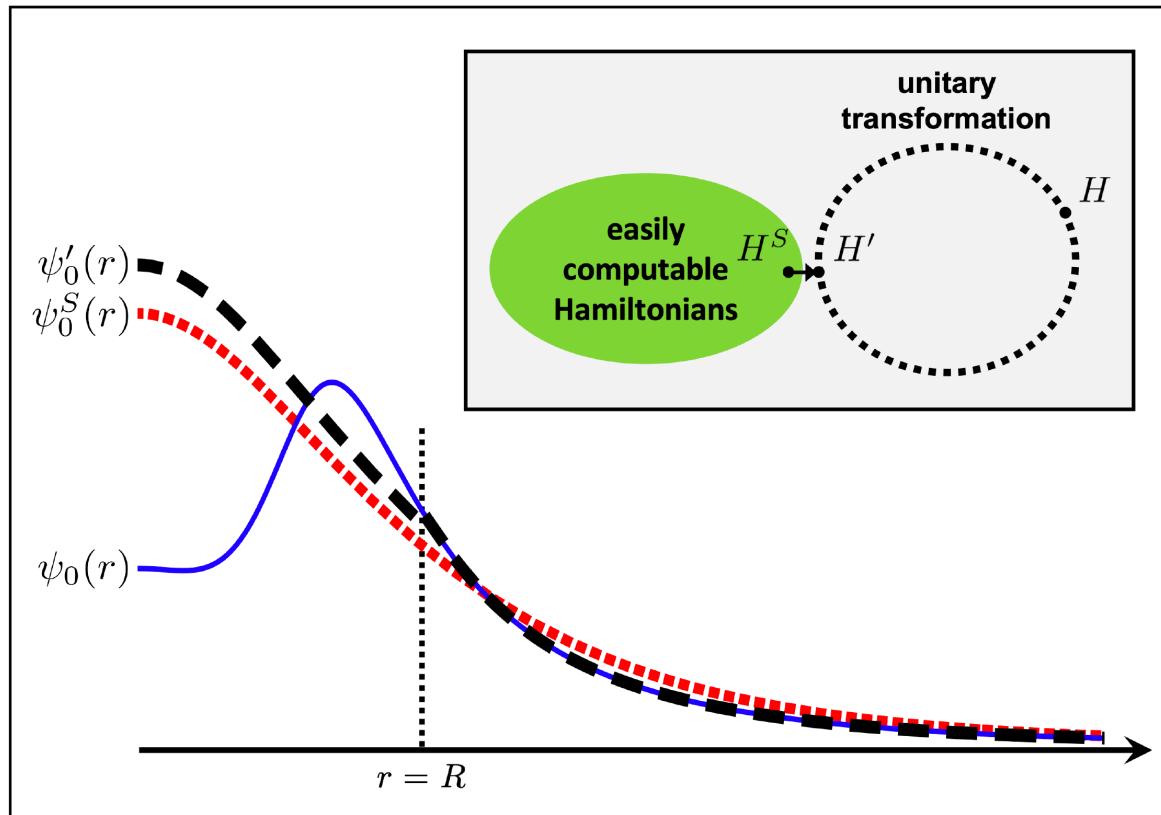
We can construct a Hamiltonian translator using quantum adiabatic evolution

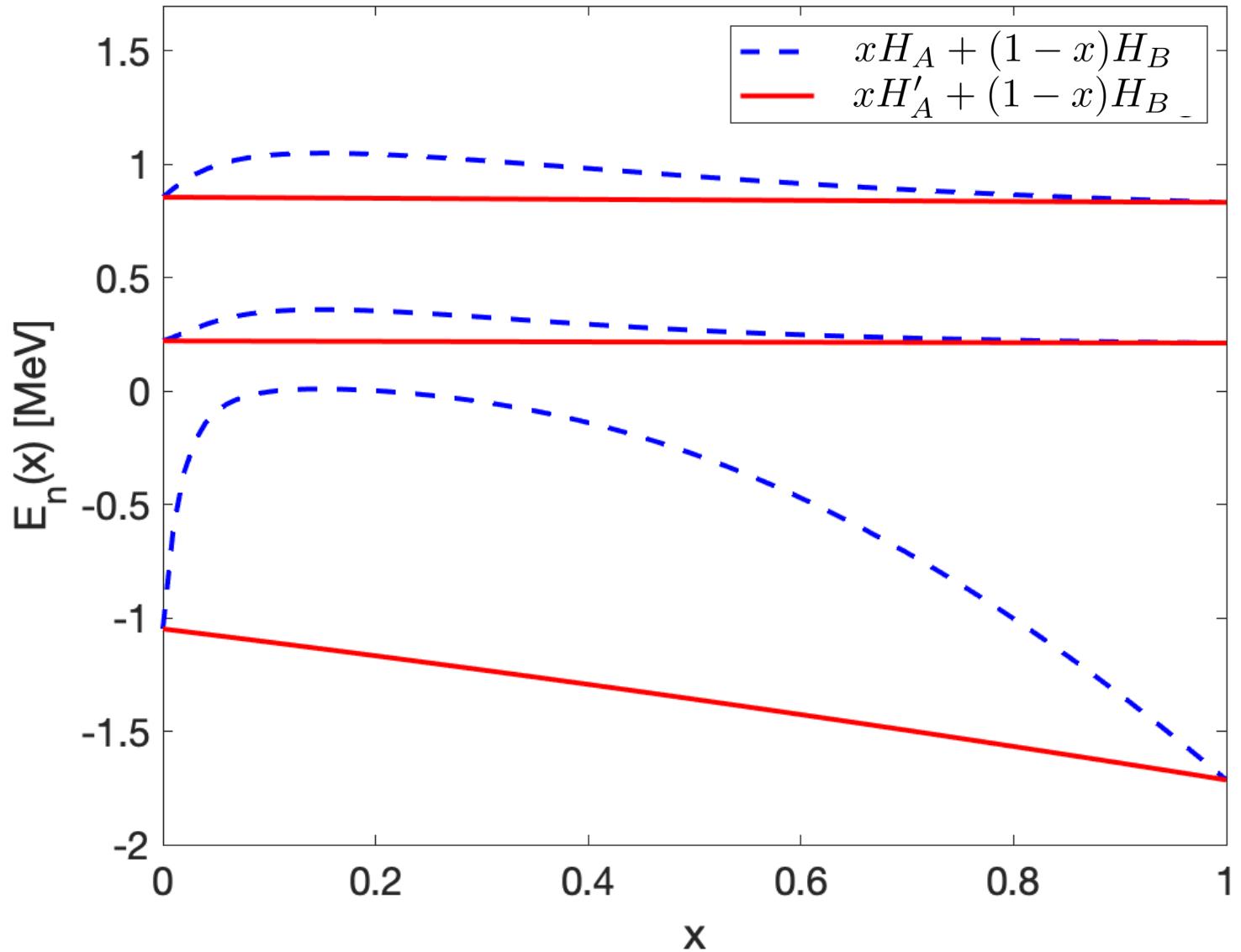
$$U_T = \lim_{T \rightarrow \infty} \overleftarrow{\mathcal{T}} \exp \left[-i \int_0^T H_T(t) dt \right]$$

where $H_T(t)$ smoothly interpolates between H_B and H_A as t goes from 0 to T .



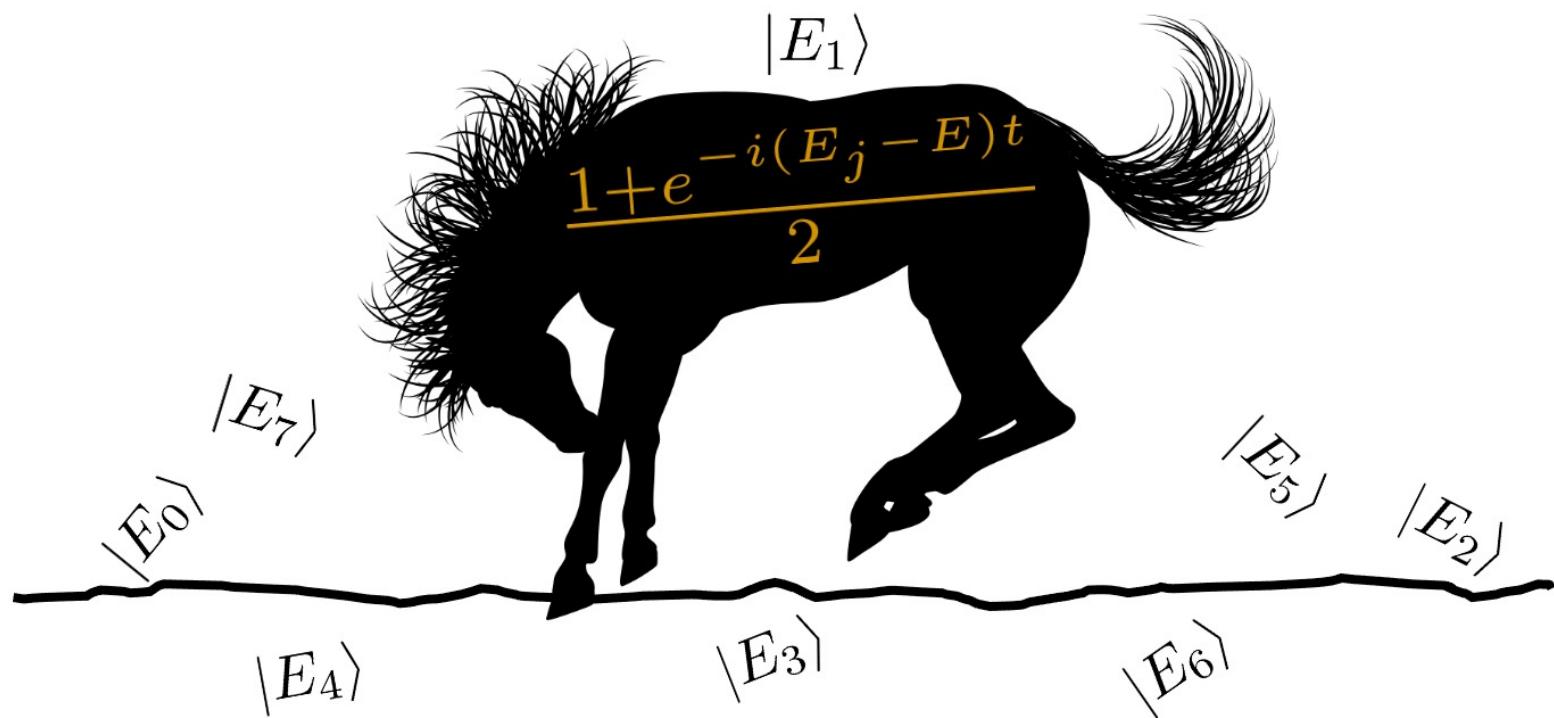
Wave function matching is an approximate Hamiltonian translator for low-energy two-body states.



H_B H_A, H'_A 

straight lines mean the eigenvectors don't change with x

Rodeo algorithm



Kenneth Choi, D.L., Joey Bonitati, Zhengrong Qian, Jacob Watkins, PRL 127, 040505 (2021)

Consider a single qubit and a Hadamard gate (call it U to avoid confusion with the Hamiltonian)

$$U = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} = U^\dagger = U^{-1}$$

Consider also a phase rotation

$$R(E_{\text{obj}}, E, t) = \begin{bmatrix} 1 & 0 \\ 0 & e^{-it(E_{\text{obj}} - E)} \end{bmatrix}$$

We then have

$$U^\dagger R(E_{\text{obj}}, E, t) U = \begin{bmatrix} \frac{1}{2} + \frac{1}{2}e^{-it(E_{\text{obj}} - E)} & \frac{1}{2} - \frac{1}{2}e^{-it(E_{\text{obj}} - E)} \\ \frac{1}{2} - \frac{1}{2}e^{-it(E_{\text{obj}} - E)} & \frac{1}{2} + \frac{1}{2}e^{-it(E_{\text{obj}} - E)} \end{bmatrix}$$

Let us now start in the $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ state and perform these unitary operations

$$U^\dagger R(E_{\text{obj}}, E, t) U \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} - \frac{1}{2}e^{-it(E_{\text{obj}}-E)} \\ \frac{1}{2} + \frac{1}{2}e^{-it(E_{\text{obj}}-E)} \end{bmatrix}$$

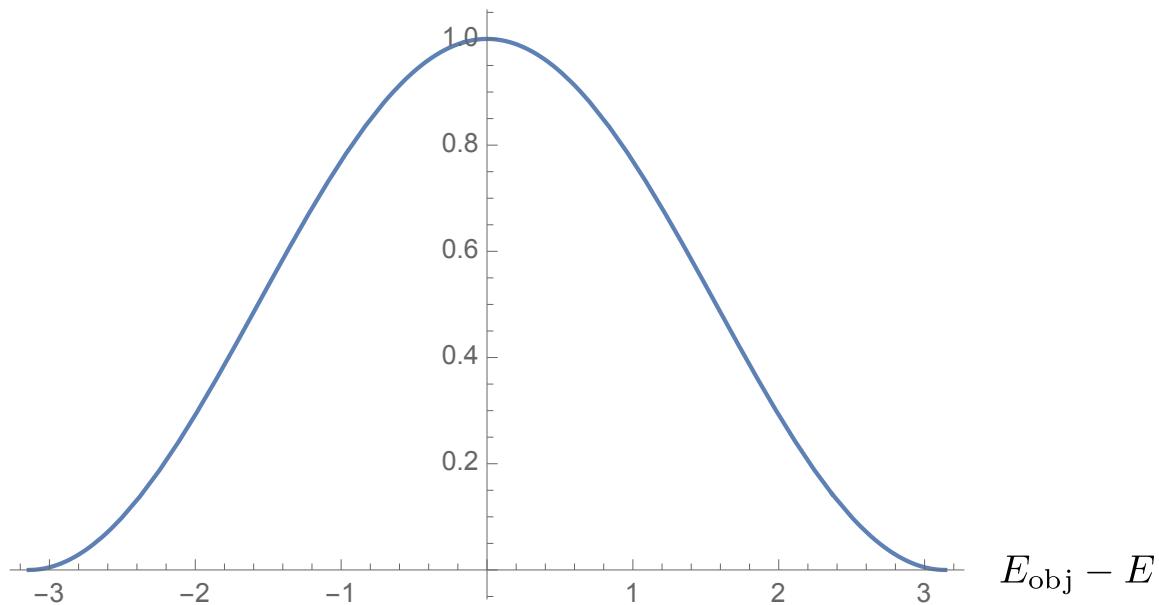
and then project back to the $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ state

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} U^\dagger R(E_{\text{obj}}, E, t) U \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{2} + \frac{1}{2}e^{-it(E_{\text{obj}}-E)} \end{bmatrix}$$

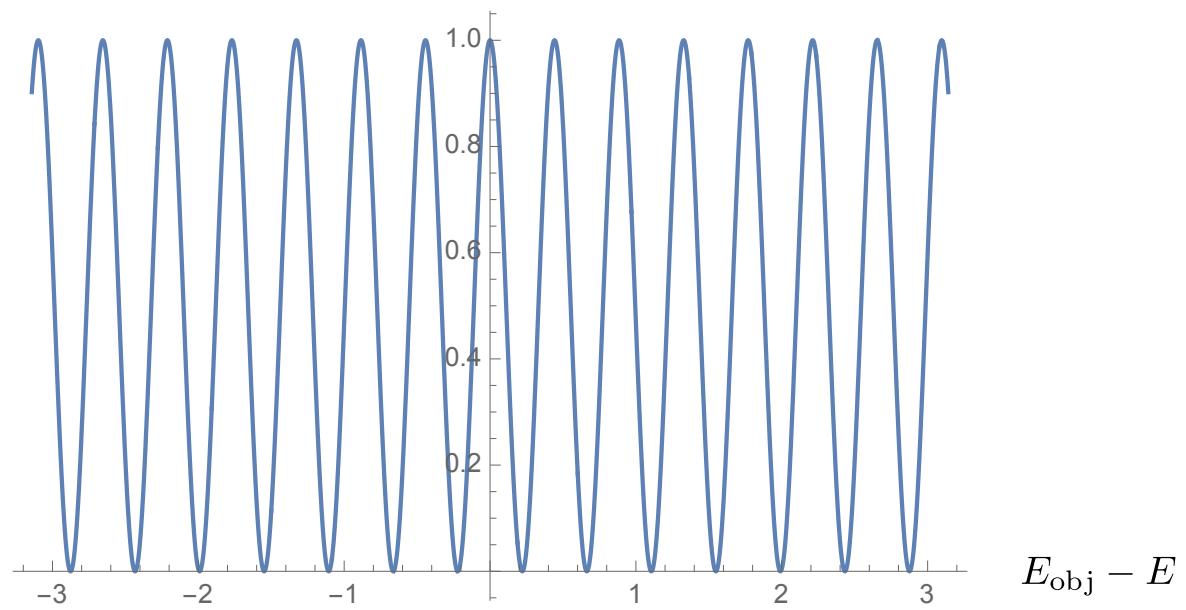
This projection is done via quantum measurement and the success probability is

$$P(E_{\text{obj}}, E, t) = \left| \frac{1}{2} + \frac{1}{2}e^{-it(E_{\text{obj}}-E)} \right|^2 = \cos^2 \left[\frac{t(E_{\text{obj}} - E)}{2} \right]$$

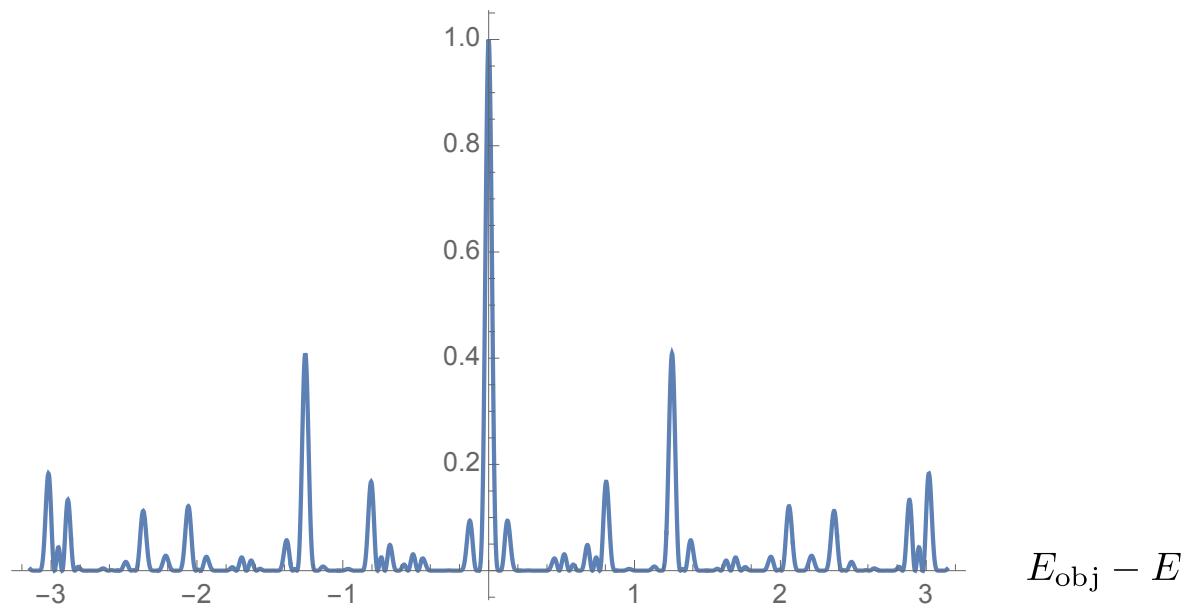
$$P(E_{\text{obj}}, E, 1)$$



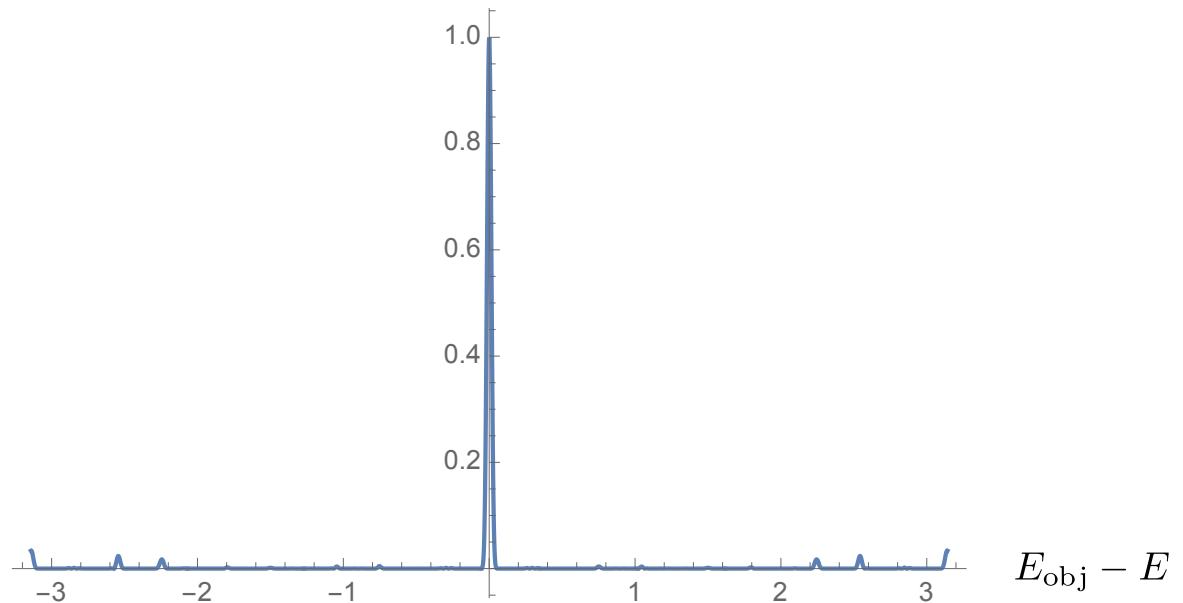
$$P(E_{\text{obj}}, E, 14.2023)$$



$$\prod_{k=1}^5 P(E_{\text{obj}}, E, t_k) \quad |t_k| < 50$$



$$\prod_{k=1}^{10} P(E_{\text{obj}}, E, t_k) \quad |t_k| < 50$$



Let us couple this qubit, which we call the “ancilla” qubit, to another system that we call the “object”. We also promote the 2×2 matrices to become 2×2 matrices of operators acting on the object.

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \rightarrow \begin{bmatrix} \hat{I} & \frac{\hat{I}}{\sqrt{2}} \\ \frac{\hat{I}}{\sqrt{2}} & -\frac{\hat{I}}{\sqrt{2}} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{-it(E_{\text{obj}} - E)} \end{bmatrix} \rightarrow \begin{bmatrix} \hat{I} & 0 \\ 0 & e^{-it(\hat{H}_{\text{obj}} - E)} \end{bmatrix}$$

We then consider the same combination

$$\begin{bmatrix} \frac{\hat{I}}{\sqrt{2}} & \frac{\hat{I}}{\sqrt{2}} \\ \frac{\hat{I}}{\sqrt{2}} & -\frac{\hat{I}}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \hat{I} & 0 \\ 0 & e^{-it(\hat{H}_{\text{obj}} - E)} \end{bmatrix} \begin{bmatrix} \frac{\hat{I}}{\sqrt{2}} & \frac{\hat{I}}{\sqrt{2}} \\ \frac{\hat{I}}{\sqrt{2}} & -\frac{\hat{I}}{\sqrt{2}} \end{bmatrix}$$

We start from the state $\begin{bmatrix} 0 \\ |\psi_{\text{init}}\rangle \end{bmatrix}$ and we perform the operations and then measure if the arena qubit is in the $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ state

$$\begin{bmatrix} 0 & 0 \\ 0 & \hat{I} \end{bmatrix} \begin{bmatrix} \frac{\hat{I}}{\sqrt{2}} & \frac{\hat{I}}{\sqrt{2}} \\ \frac{\hat{I}}{\sqrt{2}} & -\frac{\hat{I}}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \hat{I} & 0 \\ 0 & e^{-it(\hat{H}_{\text{obj}} - E)} \end{bmatrix} \begin{bmatrix} \frac{\hat{I}}{\sqrt{2}} & \frac{\hat{I}}{\sqrt{2}} \\ \frac{\hat{I}}{\sqrt{2}} & -\frac{\hat{I}}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 0 \\ |\psi_{\text{init}}\rangle \end{bmatrix} = \begin{bmatrix} 0 \\ [\frac{1}{2} + \frac{1}{2}e^{-it(\hat{H}_{\text{obj}} - E)}] |\psi_{\text{init}}\rangle \end{bmatrix}$$

By repeated successful measurements with random values of t , we reduce the spectral weight of eigenvectors with energies that do not match E .

The convergence is exponential. For N cycles of the rodeo algorithm, the suppression factor for undesired energy states is $1/4^N$.

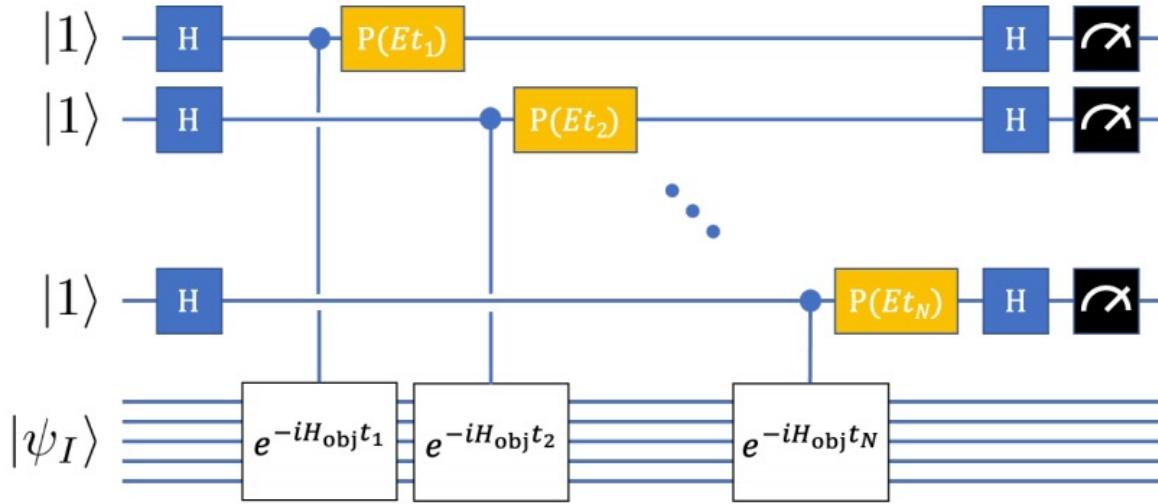


FIG. 1. (color online) Circuit diagram for the rodeo algorithm. The object system starts in an arbitrary state $|\psi_I\rangle$. Each of the arena qubits are initialized in the state $|1\rangle$ and operated on by a Hadamard gate H. We use each arena qubit $n = 1, \dots, N$ for the controlled time evolution of the object Hamiltonian, H_{obj} , for time t_n . This is followed by a phase rotation $P(Et_n)$ on arena qubit n , another Hadamard gate H, and then measurement.

The success probability for the rodeo algorithm after N cycles is

$$P_N(E|\{t_k\}) = \prod_{k=1}^N \cos^2 \left[\frac{t_k}{2} (E_{\text{obj}} - E) \right]$$

Averaging over the Gaussian random times with RMS value σ we get

$$P_N(E) = \left[\frac{1+e^{-(E_{\text{obj}}-E)^2\sigma^2/2}}{2} \right]^N$$

Initial-state spectral function and state preparation. The example shown below is for a 1D Heisenberg chain with ten sites, antiferromagnetic interactions, and uniform magnetic field.

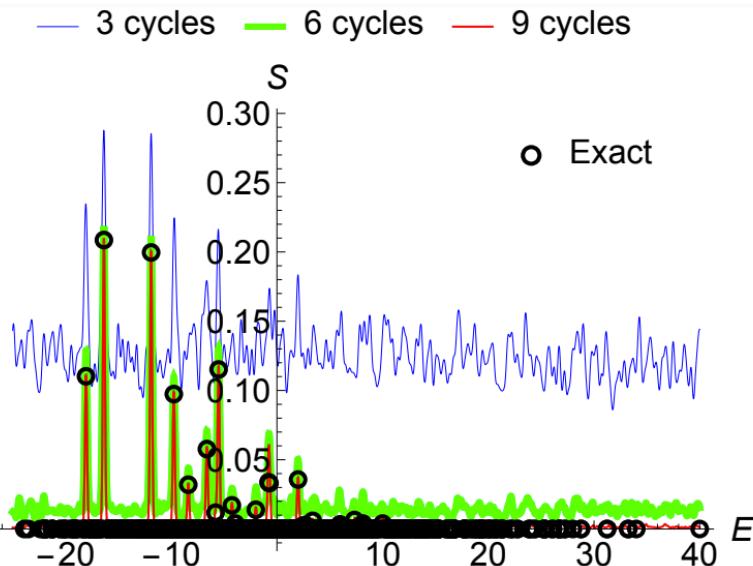


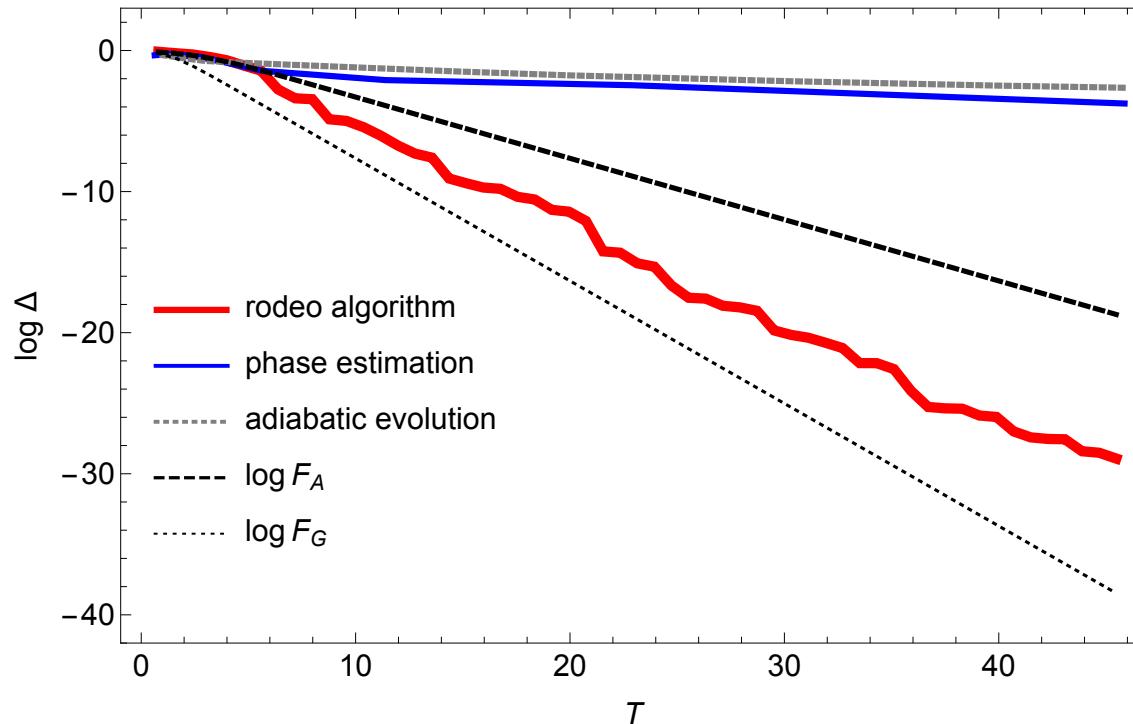
FIG. 4. (color online) Initial-state spectral function for the Heisenberg model. We plot the initial-state spectral function using the rodeo algorithm for the Heisenberg spin chain with 3 (thin blue line), 6 (thick green line), and 9 (medium red line) cycles. We have averaged over 20 sets of Gaussian random values for t_n with $t_{\text{RMS}} = 5$. For comparison, we also show the exact initial-state spectral function with black open circles.

$$|\psi_{\text{init}}\rangle = |0101010101\rangle$$

TABLE I. Overlap probability with energy eigenvector $|E_j\rangle$ after N cycles of the rodeo algorithm using Gaussian random values for t_n with $t_{\text{RMS}} = 5$ and $E = E_j$.

E_j	$N = 0$	$N = 3$	$N = 6$	$N = 9$
-18.1	0.110	0.746	0.939	0.997
-16.4	0.209	0.841	0.993	1.000
-11.9	0.200	0.629	0.889	0.999
-9.76	0.0974	0.488	0.903	0.999
-8.38	0.0320	0.467	0.832	0.993
-6.63	0.0577	0.309	0.818	0.996
-5.81	0.0118	0.179	0.637	0.817
-5.52	0.115	0.456	0.766	0.997
-4.26	0.0171	0.144	0.696	0.995
-3.95	0.00401	0.0430	0.343	0.952
-2.00	0.0139	0.158	0.593	0.942
-0.802	0.0338	0.216	0.545	0.594
-0.704	0.0331	0.286	0.540	0.585
2.00	0.0357	0.371	0.925	0.994
2.42	0.00235	0.0122	0.0874	0.521
2.68	0.00291	0.0845	0.639	0.929
3.39	0.00592	0.0360	0.754	0.943
5.96	0.00336	0.0951	0.559	0.981
7.33	0.00650	0.184	0.792	0.978
8.13	0.00393	0.0832	0.665	0.841
8.24	0.00105	0.0275	0.142	0.289
10.0	0.00397	0.0128	0.295	0.902

Comparison with other well-known algorithms. Let Δ be the norm of the error in the wave function.



$$F_A \equiv \sqrt{2^{-N}(1-p)/[p + 2^{-N}(1-p)]}$$

$$F_G \equiv \sqrt{4^{-N}(1-p)/[p + 4^{-N}(1-p)]}$$

Preconditioning with adiabatic evolution

The computational effort needed for the rodeo algorithm is inversely proportional to the overlap probability between the initial state and the desired eigenvector.

We can use adiabatic evolution to increase this overlap probability.

TABLE I. Overlap probability with energy eigenvector $|E_j\rangle$ with $E = E_j = -18.1$ after preconditioning with adiabatic evolution for time t_{AE} and the applying N cycles of the rodeo algorithm using Gaussian random values for t_n with $t_{\text{rms}} = 5$.

E_j	t_{AE}	$N = 0$	$N = 3$	$N = 6$	$N = 9$
-18.1	0	0.110	0.746	0.939	0.997
-18.1	5	0.83074	0.99875	0.99988	0.99999



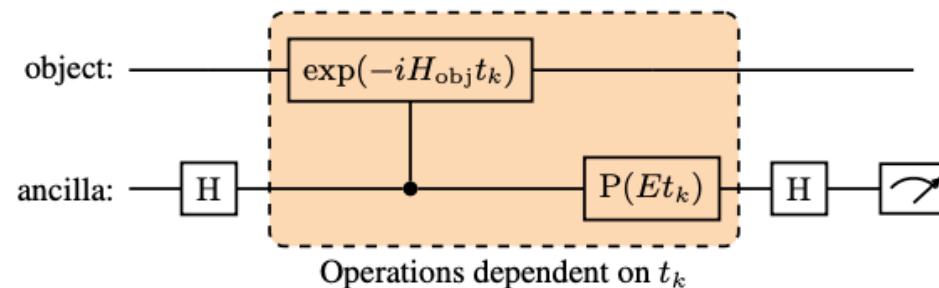
Demonstration of the rodeo algorithm on a quantum computer

Zhengrong Qian^{1,a}, Jacob Watkins^{1,b}, Gabriel Given^{1,c}, Joey Bonitatib^{1,d}, Kenneth Choi^{2,e}, Dean Lee^{1,f}

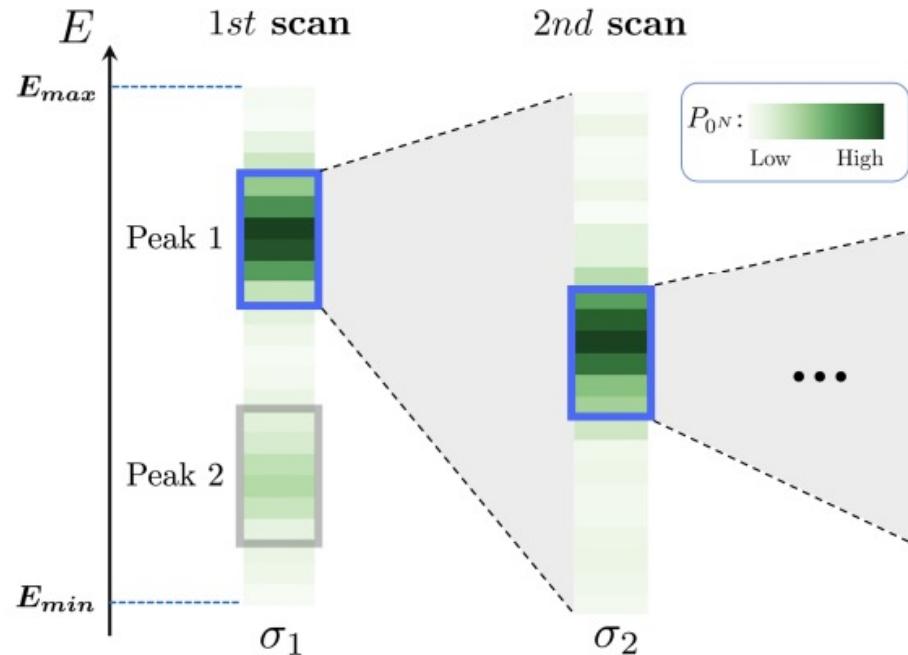
Using IBM Q devices, we implement the rodeo algorithm for a one qubit Hamiltonian. We consider a random Hamiltonian of the form

$$H_{\text{obj}} = H^{(0)} = -0.08496I - 0.89134X + 0.26536Y + 0.57205Z$$

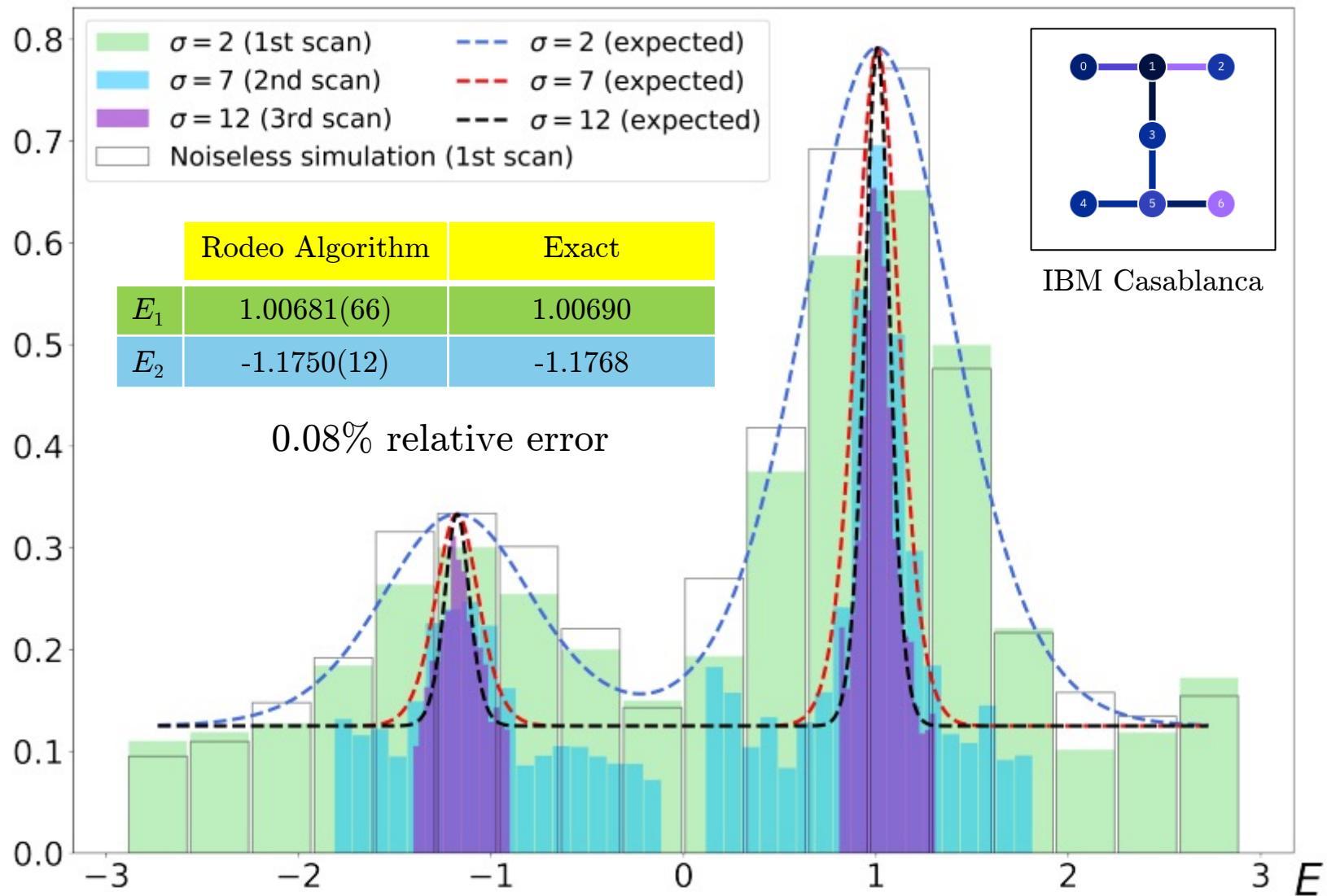
We use mid-circuit measurements without resets for the ancilla qubit



Each circuit consists of three cycles of the rodeo algorithm, corresponding to three controlled time evolutions and three ancilla qubit measurements. We sweep through the target energy E to perform an energy scan of the spectrum. We perform three separate scans of the energy, each time zooming in with more resolution.



Single qubit Hamiltonian



We also use the Hellmann-Feynman theorem to compute eigenvector expectation values for a different one qubit observable.

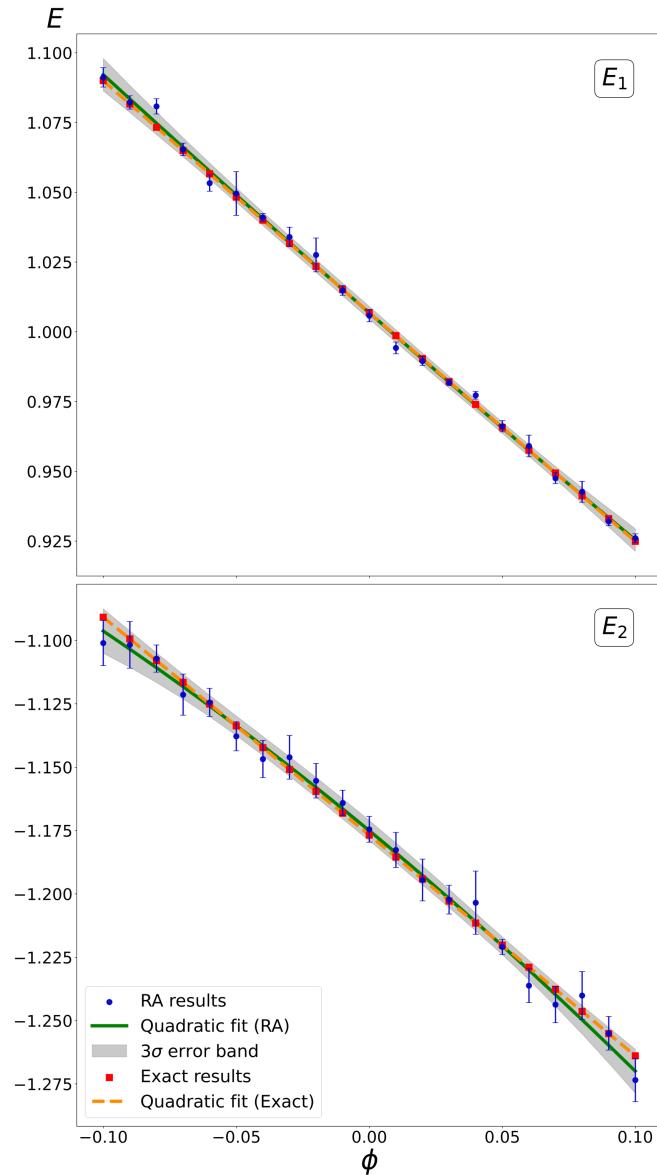
$$H_{\text{obj}}(\phi) = H^{(0)} + \phi H^{(1)}$$

$$H^{(0)} = -0.08496I - 0.89134X + 0.26536Y + 0.57205Z$$

$$H^{(1)} = -0.84537I + 0.00673X - 0.29354Y + 0.18477Z$$

$$H^{(0)} |\psi_n^{(0)}\rangle = E_n |\psi_n^{(0)}\rangle$$

$$\frac{dE_n(\phi)}{d\phi} \Big|_{\phi=0} = \langle \psi_n^{(0)} | H^{(1)} | \psi_n^{(0)} \rangle$$



	$ \psi_1(0)\rangle$	exact	$ \psi_2(0)\rangle$	exact
$\langle H^{(0)} \rangle$	1 00681(66)	1 00690	-1 1750(12)	-1 1768
$\langle H^{(1)} \rangle$	-0.8338(89)	-0.8254	-0.868(14)	-0.8653

0.7% relative error

Controlled reversal gates

A reversal gate, R , is a product of single qubit gates that anticomutes with some subset of the terms in a Hamiltonian.

$$RH = -HR$$

We note that

$$Re^{-iHt}R = e^{+iHt}$$

Let C_R be the controlled reversal gate that performs R if the ancilla qubit is in the 1 state and does nothing if the ancilla qubit is in the 0 state.

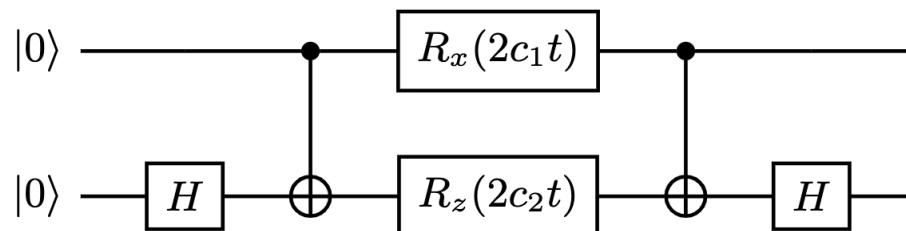
C_R toggles the flow of time back and forth. With C_R we can reduce the number of gates needed for state preparation.

Consider a two-qubit Hamiltonian that has the form

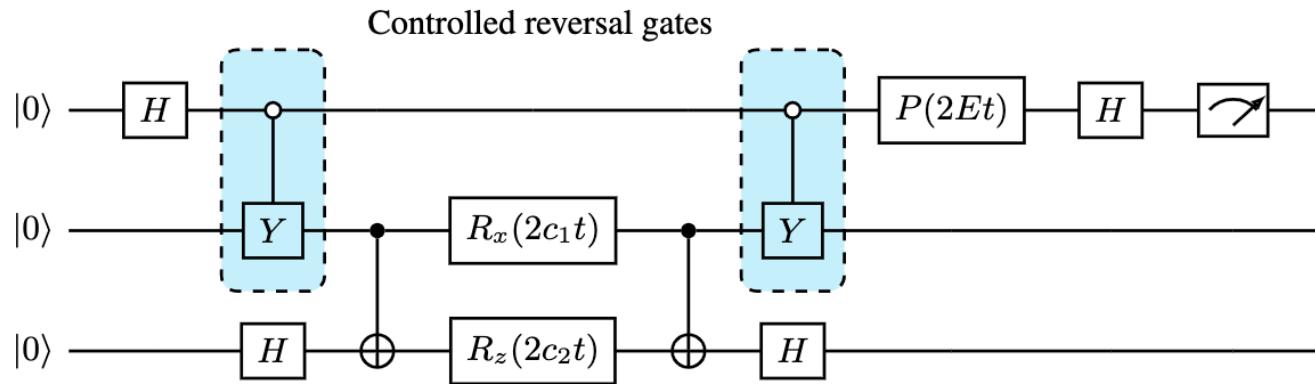
$$H_{\text{obj}} = c_1 X_1 \otimes Z_2 + c_2 Z_1 \otimes X_2$$

$$c_1 = 2.5, c_2 = 1.5$$

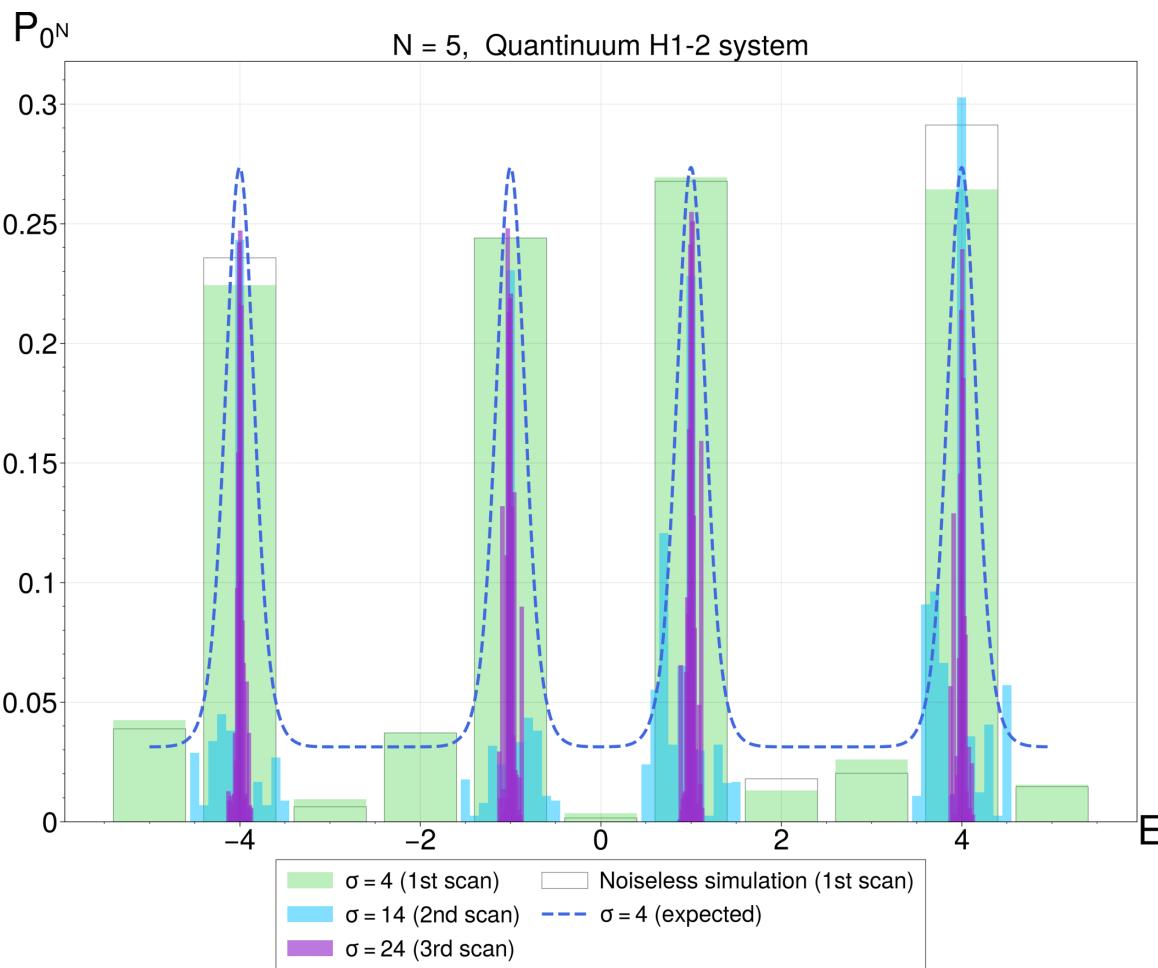
The time evolution of the Hamiltonian can be expressed with this simple circuit



Using controlled reversal gates, one cycle of the rodeo algorithm is implemented as



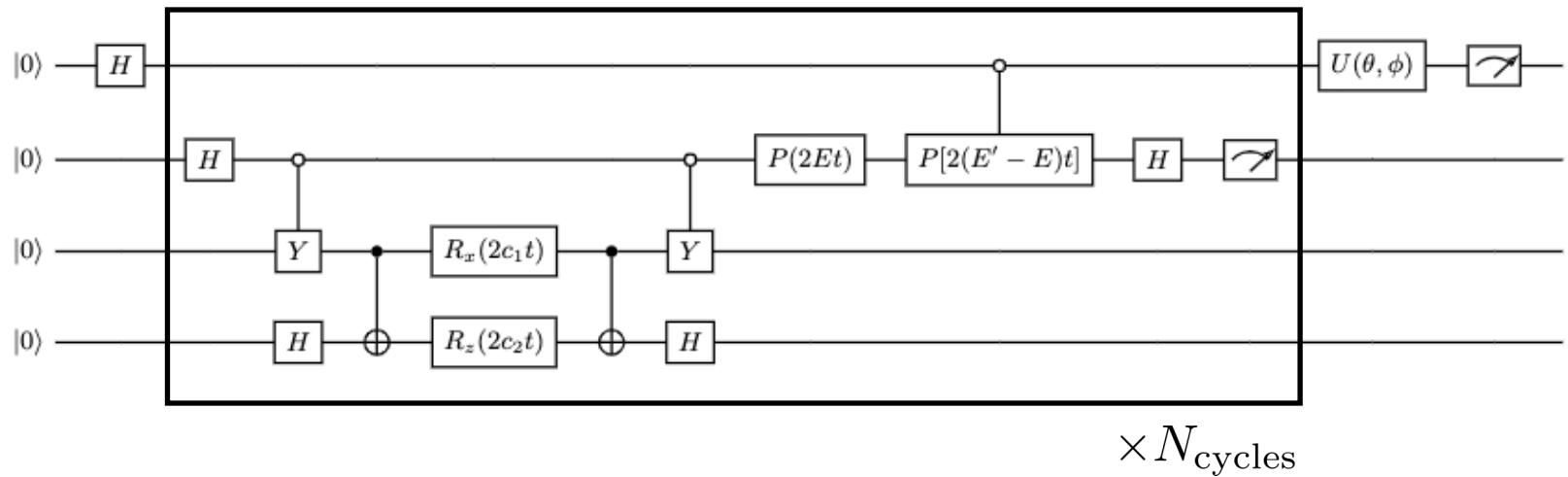
The controlled reversal gates provide a fivefold reduction in the number of gates. The comparison is made with respect to Qiskit-transpiled code without controlled reversal gates.



	Exact	IBM Perth Three Cycles	IBM Perth Five Cycles	Quantinuum H1-2 Five Cycles
$ \psi_0\rangle$	-4.0000	-4.0022(49)	-4.0006(42)	-3.9982(21)
$ \psi_1\rangle$	-1.0000	-0.9829(56)	-0.9927(40)	-1.0083(39)
$ \psi_2\rangle$	1.0000	1.0007(26)	1.0008(19)	1.0028(22)
$ \psi_3\rangle$	4.0000	4.0093(84)	3.9982(25)	4.0036(17)

Multi-state rodeo algorithm

We can prepare an arbitrary linear combination of two eigenvectors with two different energies.



Bee-Lindgren, Qian, *et al.*, work in progress

This allows us to create the general superposition state

$$|\theta, \phi\rangle \propto \cos(\theta/2) |E\rangle \langle E|\psi_I\rangle + e^{i\phi} \sin(\theta/2) |E'\rangle \langle E'|\psi_I\rangle$$

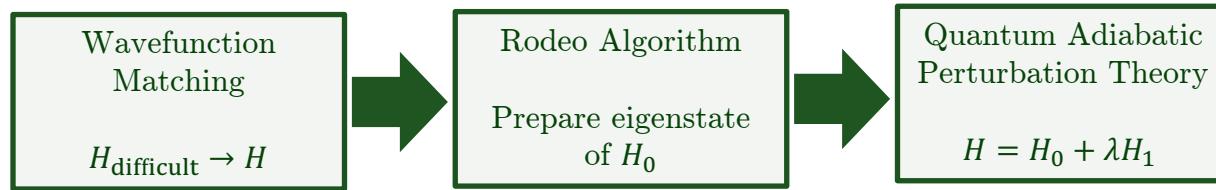
We can now measure the expectation value of any observable O

$$\begin{aligned} \frac{\langle \theta, \phi | O | \theta, \phi \rangle}{\langle \theta, \phi | \theta, \phi \rangle} &= \\ \frac{\cos^2(\theta/2) \langle E | O | E \rangle |\langle E | \psi_I \rangle|^2 + \sin^2(\theta/2) \langle E' | O | E' \rangle |\langle E' | \psi_I \rangle|^2}{\cos^2(\theta/2) |\langle E | \psi_I \rangle|^2 + \sin^2(\theta/2) |\langle E' | \psi_I \rangle|^2} \\ &+ \frac{\Re[e^{i\phi} \sin(\theta) \langle E | O | E' \rangle \langle \psi_I | E \rangle \langle E' | \psi_I \rangle]}{\cos^2(\theta/2) |\langle E | \psi_I \rangle|^2 + \sin^2(\theta/2) |\langle E' | \psi_I \rangle|^2} \end{aligned}$$

From this we can extract the transition matrix element

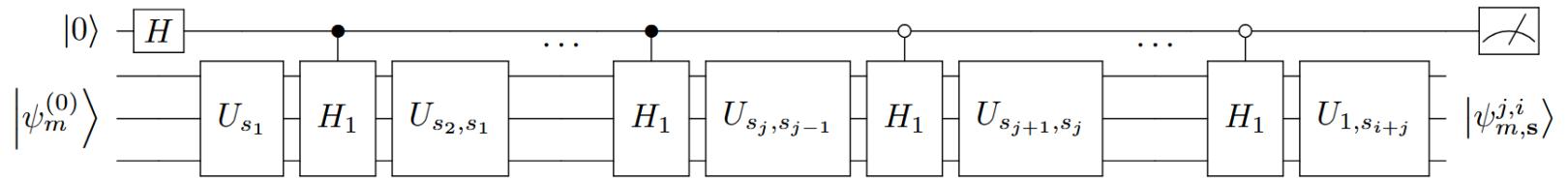
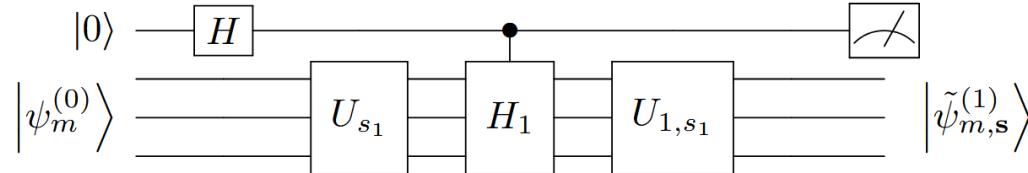
$$\langle E | O | E' \rangle$$

Quantum adiabatic perturbation theory



$$\begin{aligned} & \overleftarrow{\mathcal{T}} \exp[-i \int_0^T dt (H_0 + \lambda f(t) H_1)] \\ &= \sum_k \lambda^k \left[\int_{t_{k-1}}^T dt_k \cdots \int_0^T dt_1 \right] \\ & \quad e^{-iH_0(T-t_k)} f(t_k) H_1 e^{-iH_0(t_k-t_{k-1})} \cdots f(t_1) H_1 e^{-iH_0 t_1} \end{aligned}$$

Cariello, Given, Hjorth-Jensen, D.L., work in progress



Energy counterterms are needed to cancel terms that diverge in the limit of large T