Wavelet transform

agenda:

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Introduction to Wavelet Transforms

What are Wavelet Transforms?

Definition:

Wavelet Transforms are mathematical techniques that help analyze and decompose signals (like sound, images, or any time-series data) into components at different scales or resolutions.

Why are Wavelets Useful?

The key advantage of wavelet transforms is that they allow you to analyze both the time and frequency characteristics of a signal simultaneously, unlike traditional methods like Fourier Transforms, which only look at frequency.

Example:

Imagine you're listening to a piece of music. The music starts with a slow, soft melody, and then shifts to a fast, loud section. If you only use Fourier Transform, you might lose the ability to capture the transitions between these two sections because it only looks at the frequencies in the entire signal. Wavelet transforms, on the other hand, allow you to zoom in on both the high-frequency (fast, sharp sounds) and low-frequency (slow, soft sounds) components at specific moments in time. This means wavelets can give you a better understanding of signals that change over time.

Wavelet vs Fourier Transforms

Fourier Transform:

The Fourier Transform (FT) takes a signal and breaks it down into sine and cosine waves (pure frequencies). The result is a frequency spectrum, which tells you what frequencies are present in the signal and their strengths.

Limitation: The FT gives you no information about when the frequencies occur—just what frequencies are in the signal overall. This is a problem for signals that change over time (non-stationary signals).

Wavelet Transform:

In contrast, the Wavelet Transform (WT) breaks down a signal into wavelets—small "wave-like" functions that are localized in both time and frequency. This means it can tell you when certain frequencies appear, and how long they last.

Key Difference:

Fourier Transform works well when the signal's frequency content is constant over time. It's like analyzing the entire song by looking only at its frequency components but ignoring when they occur.

Wavelet Transform is better suited for analyzing signals that change over time. It's like breaking the song into sections (e.g., slow melody vs. fast rhythm) and analyzing the frequency content for each section independently.

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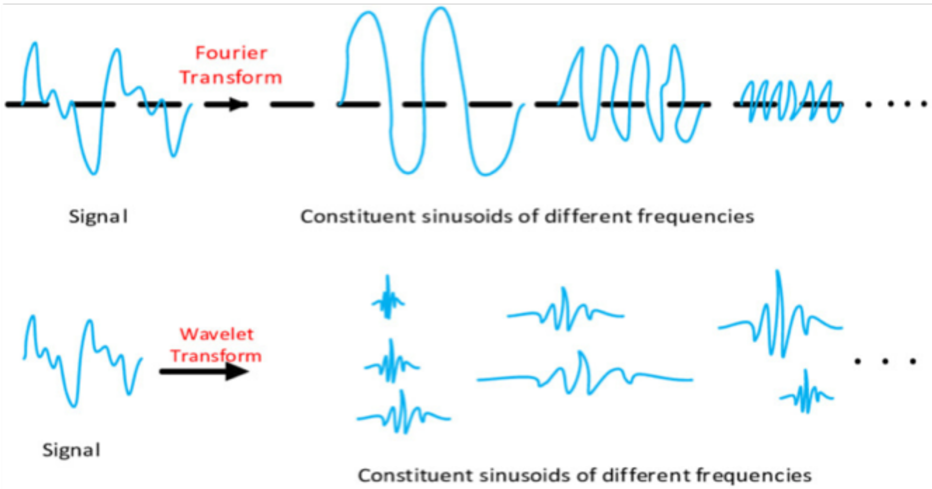
Example:

Fourier Example: If you apply Fourier Transform to a heartbeat signal, you'll get a list of frequencies that make up the heartbeats, but you won't know when each heartbeat occurs.

Wavelet Example: With Wavelet Transform, you'll see that the heartbeats happen at regular intervals, and you can pinpoint exactly when they occur in time, as well as what their frequencies are at each time.



Fourier Transform (Top Section)

The Fourier Transform decomposes a signal into its constituent sinusoidal components of different frequencies.
The process assumes that the entire signal is stationary, meaning its frequency content does not change over time.
The result is a set of pure sine waves of varying frequencies that, when added together, reconstruct the original signal.
Limitation: It does not provide time localization for changes in frequency since it only gives a global frequency spectrum.

Wavelet Transform (Bottom Section)

The Wavelet Transform also decomposes a signal into components but uses wavelets (small, localized oscillating functions).

It provides both time and frequency information by analyzing the signal at different scales.
This allows it to capture transient features (e.g., abrupt changes or localized events) that are not well-represented in the Fourier Transform.

note: A localized event in a signal could be a sudden spike, a short burst of energy, or a transient behavior that happens at a particular moment in time.
In contrast, a signal that is not localized (like a pure sine wave) continues indefinitely and has no specific "starting" or "ending" point.

Mathematical Foundations of Wavelet Transforms

What Are Wavelets and How Do They Work?

- Think of wavelets as "detectives" that break down a signal into smaller pieces to figure out what’s happening.
- They are short, wavy patterns that can stretch or shrink (scaling) and move along the signal (translation).
- Wavelets let us "zoom in" to see details or "zoom out" to see trends.

Imagine analyzing a painting:
- Zoom in: See individual brushstrokes (details).
- Zoom out: Understand the overall picture (trend).

Continuous vs. Discrete Wavelet Transform

Two Ways to Analyze Signals :

1. Continuous Wavelet Transform (CWT):

- Slide the wavelet smoothly along the signal, like slowly moving a magnifying glass.
- Helps capture every little detail but can be slow and require lots of memory.

2. Discrete Wavelet Transform (DWT):

- Instead of sliding smoothly, move in jumps (fixed steps).
- Faster, simpler, and more practical for real-world tasks like compressing images or reducing noise.

- CWT is like carefully scanning every word in a book.
- DWT is like jumping to key chapters to get the main idea.

<u>Multi-Resolution Analysis (MRA)</u>

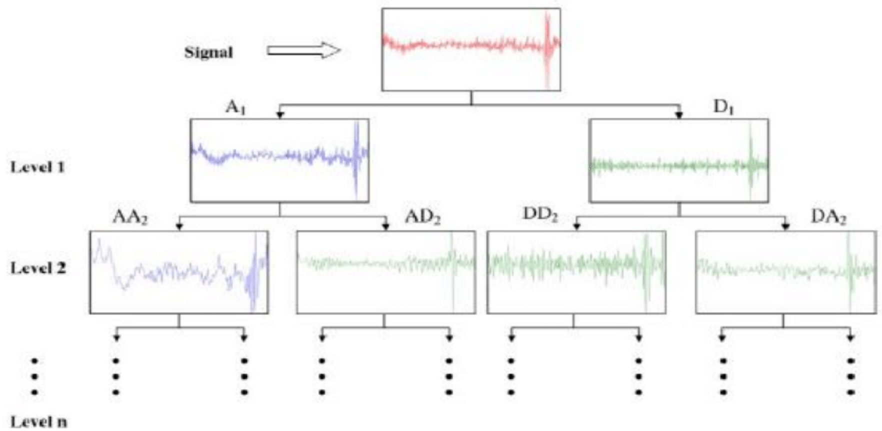
- MRA breaks a signal into layers:

- <u>High-frequency details:</u> Sharp, quick changes (like edges in an image or spikes in a heartbeat).

- <u>Low-frequency trends</u>: Big, slow changes (like the overall shape of a melody or the rhythm of a signal).

- Each layer gives us unique information about the signal.

Think of reading a book:
High-frequency details: Individual words and punctuation that make up the sentences.
Low-frequency trends: The overarching plot, themes, and mood of the story.
Each layer adds value: the details let you follow the narrative, while the big picture gives you a deeper understanding of the story.



Scaling and Shifting

Scaling:

Definition: Scaling refers to changing the size of the wavelet function. By scaling a wavelet, we can adjust how "stretched" or "compressed" the wavelet is, which allows us to focus on different levels of detail in the signal. A larger scale (more stretched wavelet) captures low-frequency, broad features of the signal, while a smaller scale (compressed wavelet) focuses on high-frequency, fine details.

Shifting:

Definition: Shifting refers to moving the wavelet along the signal in time (or space). By shifting the wavelet, we can examine different parts of the signal at different time points. Shifting allows wavelets to "zoom in" on specific areas of interest in the signal, detecting changes that occur at various points in time

Scaling: Imagine a magnifying glass. If you make it bigger, you can see more of the page. If you make it smaller, you can see just one word at a time.
Shifting: Moving that magnifying glass from left to right over the page to focus on different words.

Implementation and Algorithms on Wavelet Transform

Steps in Computing Discrete Wavelet Transform (DWT):

1-Decomposition:

The original signal is divided into two parts:

- Approximation Coefficients: Represent the low-frequency components of the signal.
- Detail Coefficients: Represent the high-frequency components of the signal.

2-Filtering:

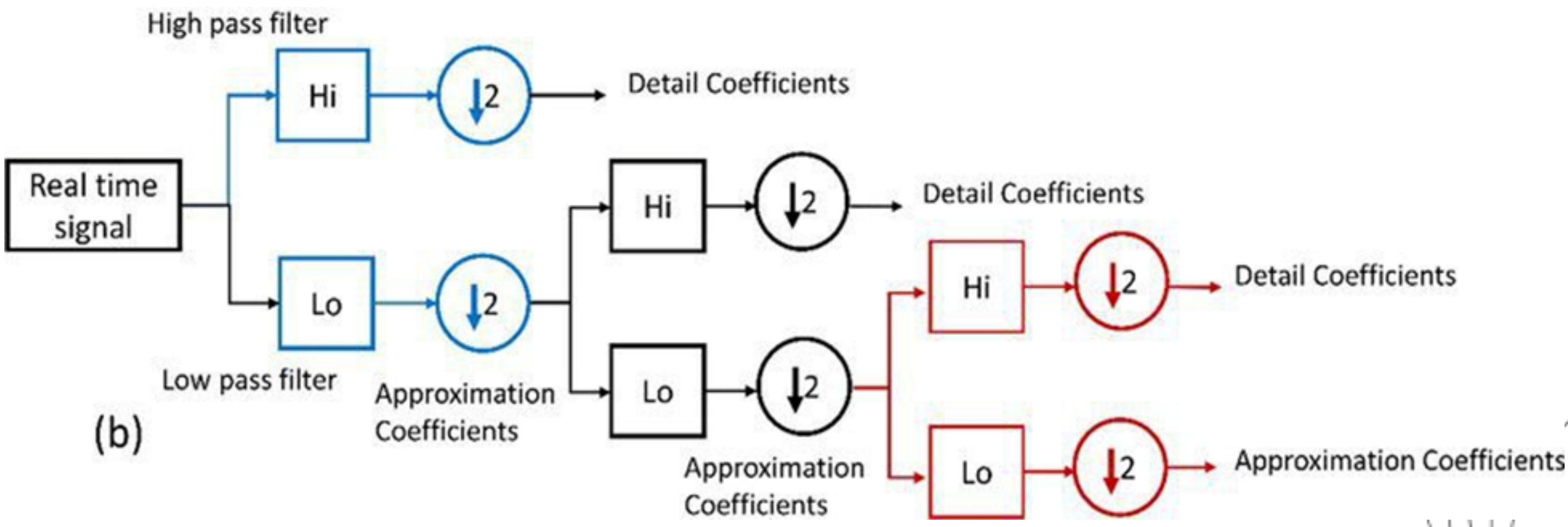
- Low-Pass Filter: Extracts the low-frequency components of the signal.
- High-Pass Filter: Extracts the high-frequency components of the signal.

3. Downsampling:

The filtered signals are downsampled to reduce their sampling rate by half. This is done to maintain a constant number of samples at each level of decomposition.

4. Iteration:

- The process is repeated on the approximation coefficients to obtain further levels of decomposition. - This allows for a multi-resolution analysis of the signal, revealing details at different scales.



Key Algorithms for DWT:

1-Mallat Algorithm:

- Efficiently computes the DWT using a filter bank structure.

- Commonly used in practical implementations.

2-Lifting Scheme:

- A fast and flexible algorithm for computing the DWT.

- Well-suited for hardware implementation.

3-Pyramid Algorithm:

- A simple and intuitive algorithm for computing the DWT.

- Often used for educational purposes.

Challenges:

- Computational Complexity: For large-scale applications, the computational cost of wavelet transforms can be significant.

- Choice of Wavelet Basis: Selecting the appropriate wavelet basis can be challenging and often requires domain-specific knowledge.

- Handling Irregularly Sampled Data: Wavelet transforms are typically designed for regularly sampled data, making it difficult to analyze irregularly sampled signals.

Application of wavelet Transformer

1_ Image Compression with Wavelet Transform

Wavelet transform is a powerful technique for image compression, reducing image file sizes while preserving important details

EX:

Medical Imaging:

High-quality compression with minimal loss for X-ray images.

Web Graphics:

Reducing file sizes for faster transmission and loading times.

How Wavelet Transform Works in Compression

(1) Wavelet Decomposition:

The image is decomposed into sub-bands using the Discrete Wavelet Transform (DWT).Sub-bands include:

LL (Low-Low): Approximation of the image, capturing important features.

LH, HL, HH: Horizontal, vertical, and diagonal details (high-frequency components).

(2) Information Compaction:

Most of the image's information is concentrated in the LL sub-band.

The detail sub-bands (LH, HL, HH) primarily contain less significant information (often noise or minor variations).

(3)Quantization:

Features in the sub-bands are quantized to reduce redundancy.

High-frequency components (from LH, HL, HH) are often heavily compressed since they contribute less to perceived quality.

2_ Noise Reduction for Signals

Wavelet transform is a highly effective technique for noise reduction (denoising) in signals. Its ability to analyze signals at multiple resolutions makes it ideal for isolating noise from the underlying signal.

-Decomposition into Sub-Bands:

Wavelet transform decomposes a signal into different frequency components across various scales or resolutions.

This allows separation of noise (high-frequency components) from the meaningful signal (low-frequency components).

-A signal is decomposed into:

Approximation Coefficients (low-frequency): Contain the main features of the signal.

Detail Coefficients (high-frequency): Often dominated by noise.

By focusing on the approximation coefficients and processing the detail coefficients, noise can be selectively suppressed.

2_ Noise Reduction for Signals

Thresholding:

After decomposition, thresholds are applied to the detail coefficients to eliminate noise while preserving significant features.

Thresholding can be applied globally or adaptively for different sub-bands.

EX:

Biomedical Signals:

Denoising ECG and EEG signals for accurate diagnosis.

Image Processing:

Enhancing images by removing high-frequency noise components.

3_ Edge Detection in Image

Wavelet transform is a powerful tool for edge detection in image processing due to its ability to analyze images at multiple scales and resolutions. Edge detection is essential for understanding shapes, boundaries, and structures in an image.

(1) Multi-Resolution Analysis:

Wavelet transform decomposes an image into multiple levels of detail (sub-bands), capturing edges at different scales.

(horizontal, vertical, diagonal) .

we can use threshold to identify edges using frequency.

(2) Edge Representation:

Edges are typically high-frequency components in an image.

Wavelet transform effectively isolates these high-frequency details, highlighting edges while suppressing low-frequency background information.

(3) Post-Processing:

smoothing to refine the detected edges.

3_ Edge Detection in Image

EX:

Medical Imaging:

Identifying boundaries in X-rays.

Computer Vision:

Edge detection in tasks like motion tracking, segmentation, and pattern recognition.