Signals in DSP

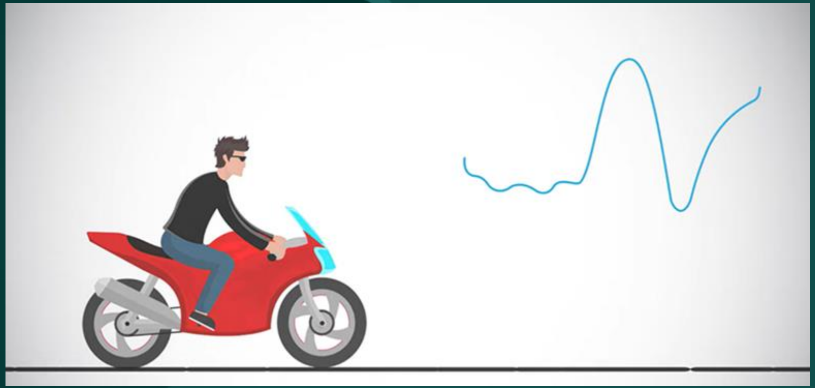
What is a signal?

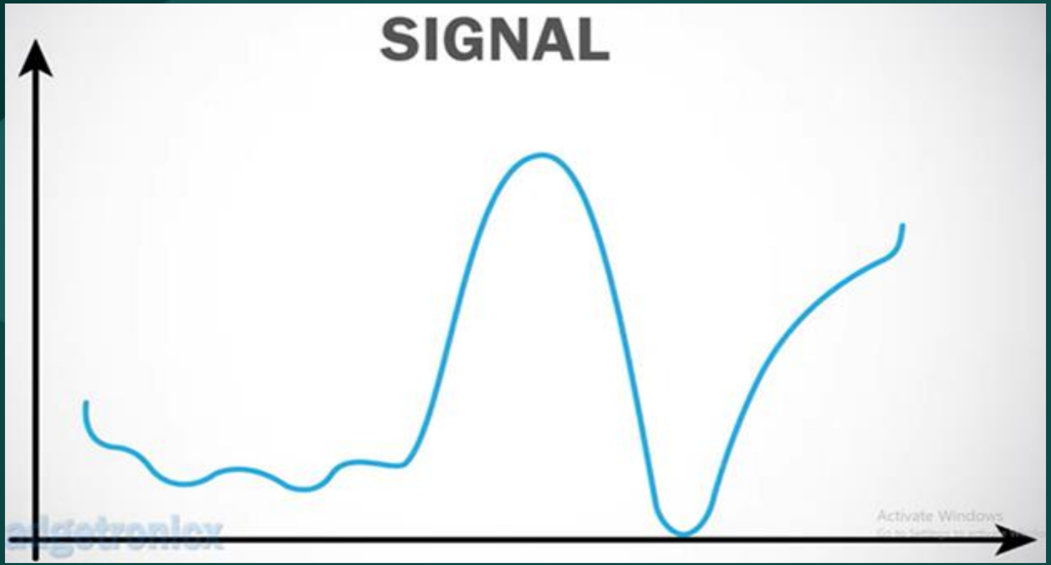
- An electrical signal is a physical quantity that varies with time. the fundamental quantity of representing some information is called a signal. It does not matter what the information, a signal is a function that conveys some information. Thus, signal is a time-dependent function that can be dependent on one or more than one physical quantity.

- Transfer of data and information is done via signals, the electrical signal being sent can represent many different types of information, like a human voice, written text, or an image. which helps in communication through any medium such as optical, electromagnetic, wireless, or wired medium.

- **Example: Motorcycle Speed as a Signal**

Imagine a person riding a motorcycle. As they accelerate or slow down, their speed changes over time. This changing speed is an example of a signal because it's a physical quantity that varies as time passes. By looking at the pattern or graph of speed, we can understand how fast the motorcycle was moving at each moment. This speed signal helps us track the rider’s motion, showing us when they sped up or slowed down.



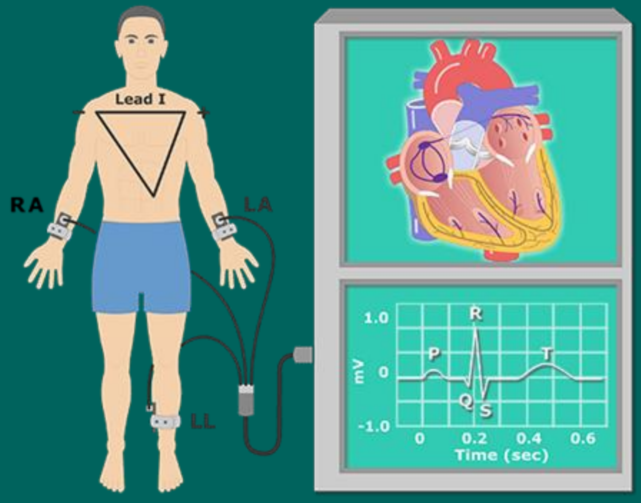


- Example: The ECG Signal in Medical Monitoring

In medical diagnostics, an Electrocardiogram (**ECG**) is a primary example of a time-varying signal used to monitor the heart's electrical activity. The ECG machine uses electrodes (wires) with sticky patches attached to specific points on the patient's body, such as the chest, arms, or legs, to detect **electrical impulses** generated by the heart.

These impulses create a **waveform** that changes continuously over time, corresponding to the contraction and relaxation of the heart muscle. The data or information from the body transfers into the device (the **system**) through these wires, which act as the **medium** to transmit the information.

The ECG signal is essential in identifying normal and abnormal heart rhythms, offering real-time insights into cardiac health. This time-dependent fluctuation in electrical current is a classic example of a continuous, real-world signal.



- Another example:

Different voltages and currents that flow through an electric circuit or device are Electric signals, since signals represent changes in a physical quantity or the state of a physical object over time.

The previously stated definition was to describe the signals “practically” in real world, however the next one describes mathematically.

Signals, mathematically, are dependent functions of independent variables ($f(x)$) that convey information about the behavior or attributes of some phenomenon. Systems, on the other hand, are entities that process these signals. systems—like radios, televisions, and computers—are responsible for handling, modifying, and interpreting the signals so that they can be understood and used.

The first categories (single-dimensional and multi-dimensional) represent signals more mathematically, describing the number of independent variables in the function representing the signal. This classification is used in signal processing, image and video analysis, and other areas where mathematical analysis of signal dimensions is important.

SINGLE DIMENSIONAL AND MULTI-DIMENSIONAL SIGNALS:

If signal is a function of one independent variable it is called single dimensional signal like speech signal (sound wave) and if signal is function of M independent variables called Multi - dimensional signals. Gray scale level of image or Intensity at pixel on black and white TV is examples of M-D signals.

The second categories (analog, digital, continuous, discrete) are more practical descriptions of how signals behave in the real world, dealing with how signals are represented, stored, or transmitted. These types are often used to specify signal behavior in electronics, data transmission, and communication systems.

Continuous and Discrete Time signals

- **Continuous time signals:**

A continuous signal is defined for every point in time, meaning it has a value at each instant over a range of time. They can take all values in the continuous interval or a specified range (a, b) or a can be $-\infty$ & b can be ∞. This signal is denoted by $x(t)$.

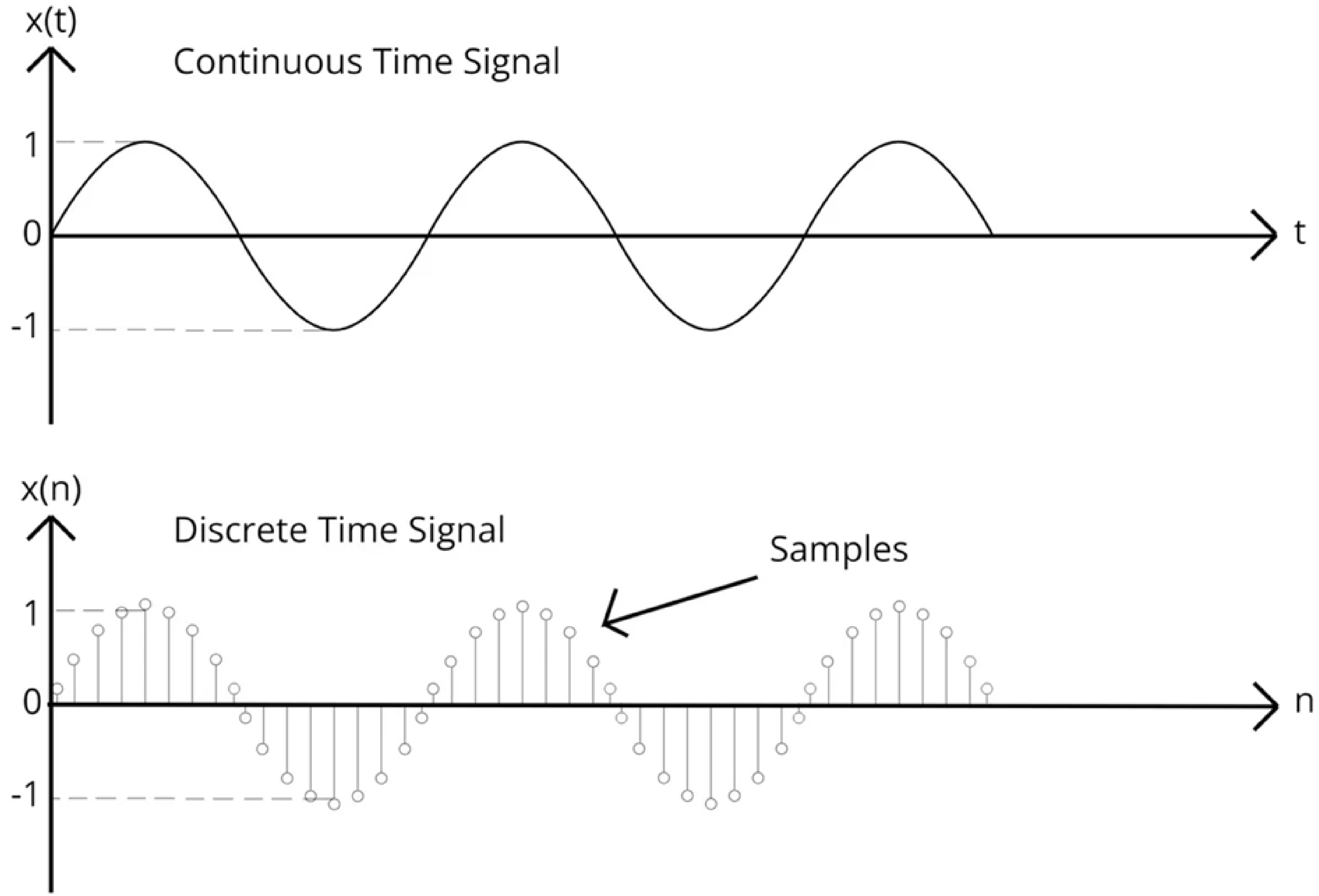
Continuous signals are often represented as smooth, uninterrupted waveforms and are defined mathematically over a continuous variable, usually time t.

Examples of continuous signals: Audio Signals, ECG Signals, Temperature Measurements, Pressure Signals, Radio Waves, Light Intensity, and Velocity of an Object.

- **Discrete time signals:**

This signal can be defined only at certain specific values of time. These times instant need not be equidistant but in practice they are usually taken at equally spaced intervals. These signals are denoted by $x(n)$ where n is an integer. A discrete signal is defined only at certain, distinct points in time, often obtained by sampling a continuous signal at regular intervals.

Examples of discrete signals: digital audio samples, image pixels, stock price data, temperature readings at intervals, digital communication signals, pulse code modulation (PCM), digital clock time.


x(t)
Continuous Time Signal
1
0
-1
t
x(n)
Discrete Time Signal
Samples
1
0
-1
n

Analog and Digital signals

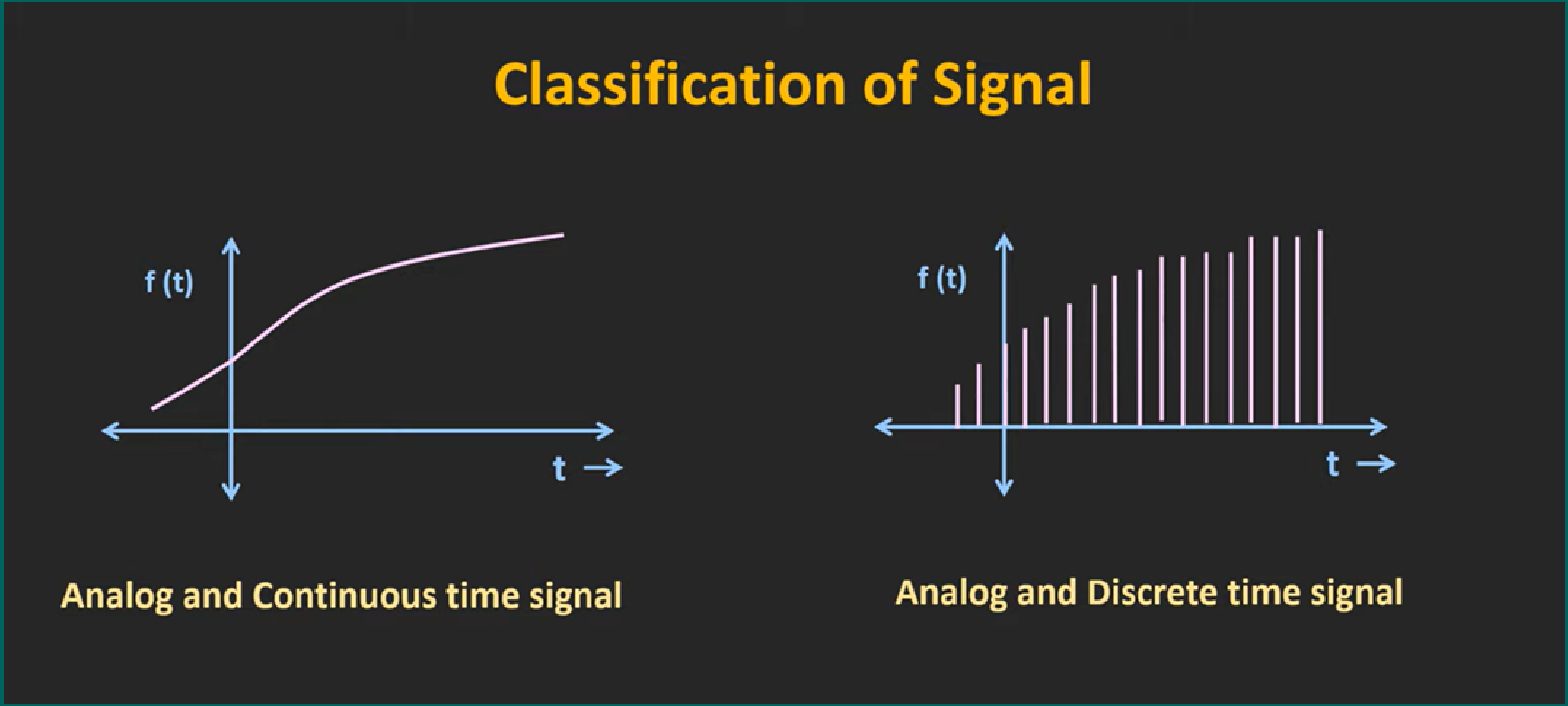
- **Analog signal:**

An analog signal is one whose amplitude can take any value within a continuous range. These signals are essentially **continuous time and continuous amplitude** signals. They convert information into waves of varying amplitude and frequency, continuously changing over time. An analog signal can take an infinite number of values, even those that fall between specific points in a given range or interval. It can have any value within that range and records the exact waveform (a curve showing the shape of a wave at a given time).

Analog signals utilize a continuous electrical signal to convey data and more complex information such as voice, image, and video. Analog signals work well with audio and video recordings, physical sensors, image sensors, amplification devices, radio systems, and control systems.

Analog signal can take any finite values on the Y axis, While the continuous time signal shows the continuity of the signal on the time axis. While most analog signals are continuous, not all continuous signals are analog. Similarly, analog signals specifically deal with continuous amplitude, while continuous signals can encompass both analog and digital types. When an analog signal is sampled at specific intervals, it can produce a discrete representation. This is adapted in the next slide.

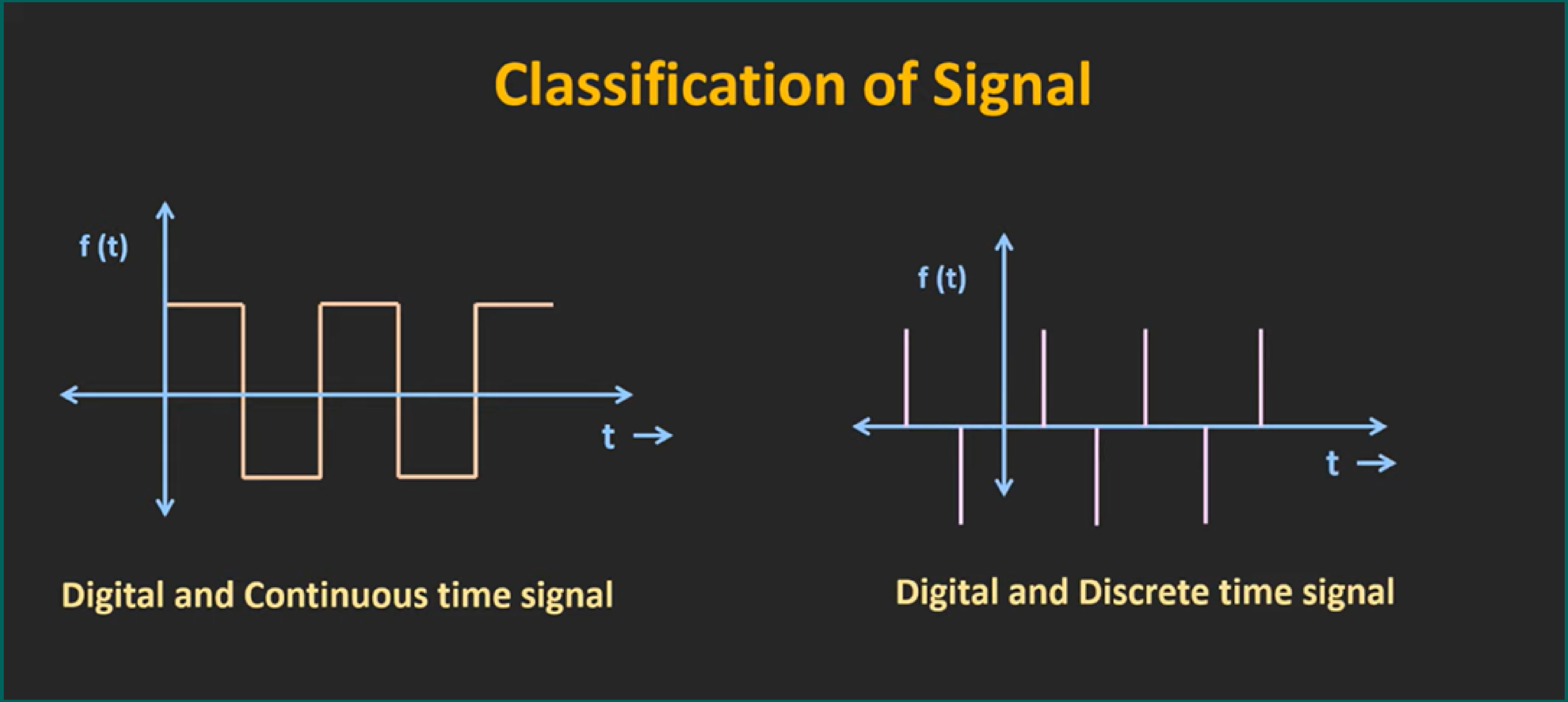
The first signal is a continuous-time signal and can have any finite value along the y-axis, making it an analog signal. The second signal is available at discrete time intervals but can still take any finite value along the y-axis. Therefore, it is a discrete-time signal that represents an analog signal, as it was sampled from the original continuous signal.



- **Digital signal:**

A signal whose amplitude can take only a finite number of values is known as a discrete signal. These are essentially discrete-time signals and discrete-amplitude signals. They convert information into discrete values, represented as ON (1) or OFF (0) pulses, also known as binary signals. In square waves, these values are represented as up (1) or down (0). These square waves are created by sampling along the waveform.

The first signal is a continuous digital signal that varies between two finite values on the time axis. The second signal, on the other hand, is available at the discrete time interval and is a discrete-time signal obtained by sampling the first signal at specific intervals. Both signals can be represented in digital format.



Periodic and Aperiodic signals

- **Periodic Signals:**

  A **periodic signal** is one that repeats itself at regular intervals over time.

  Mathematically, the formula For a periodic signal x(t), the condition for periodicity is:

  $$x(t) = x(t + T)$$

  for all t, where T is the period of the signal—the time it takes for one complete cycle to repeat.

  Example:
  Sine wave, and Square wave.

Figure 3.2

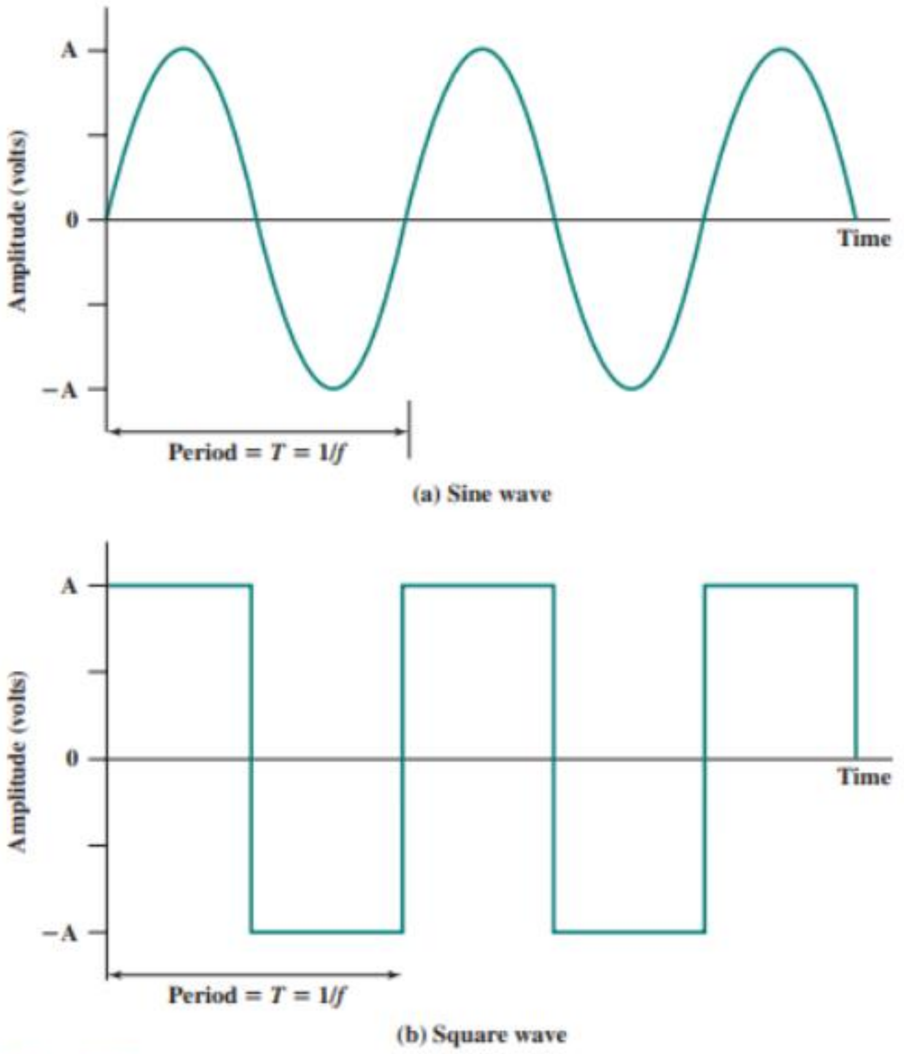
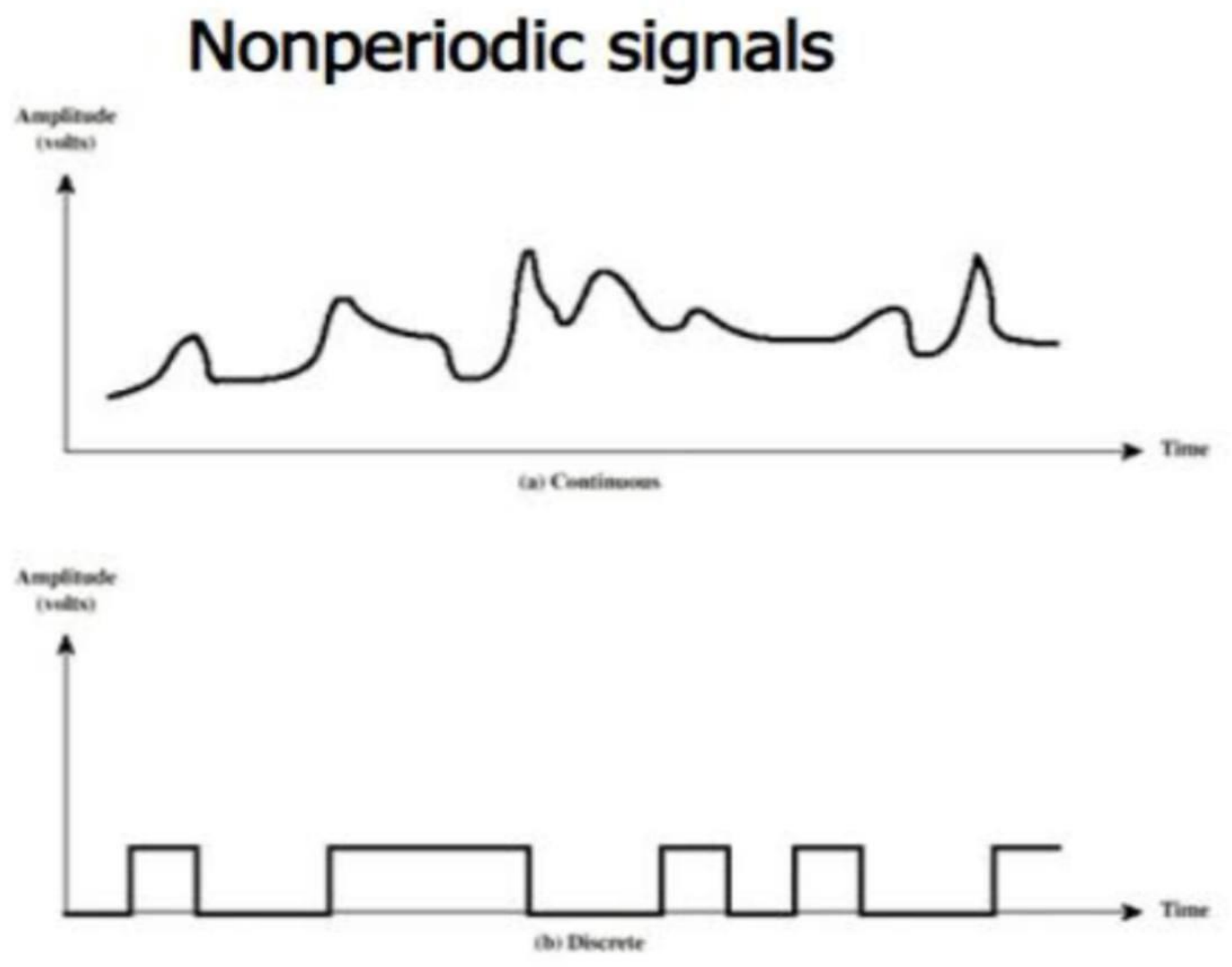
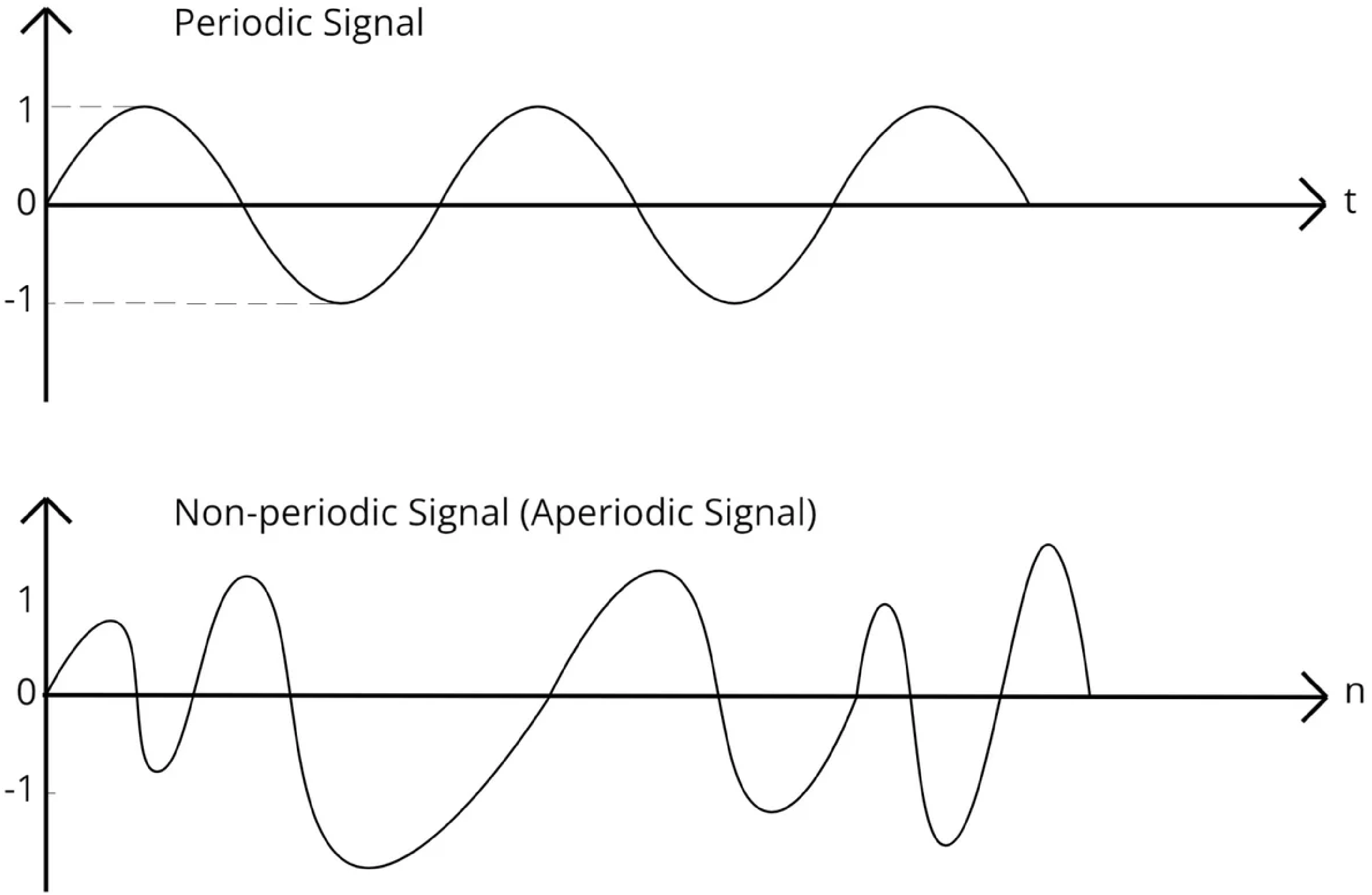


Figure 3.2 Examples of Periodic Signals

- **Aperiodic Signals:**

An **aperiodic signal** does **not repeat over time**. Such signals do not have a defined period T, meaning they do not exhibit a regular, repeating pattern.





Periodic Signal
1
0
-1
t
Non-periodic Signal (Aperiodic Signal)
1
0
-1
n

Exercise$_1$: $x(t) = \cos 4t + \sin \pi t$

$\omega_1 = 4$ $\qquad$ $\omega_2 = \pi$

$T_1 = \frac{2\pi}{\omega_1}$ $\qquad$ $T_2 = \frac{2\pi}{\omega_2}$

$$\frac{T_1}{T_2} = \frac{\frac{2\pi}{4}}{\frac{2\pi}{\pi}} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4} \longrightarrow \text{Aperiodic}$$

GCD is not possible, since there isn't a common denominator.

لا يوجد رقم مشترك لقسمة.

Ex$_2$: $x(t) = \cos 2\pi t + \sin 4\pi t$

$\omega_1 = 2\pi$ $\qquad$ $\omega_2 = 4\pi$

$T_1 = \frac{2\pi}{\omega_1}$ $\qquad$ $T_2 = \frac{2\pi}{\omega_2}$

$T_1 = \frac{2\pi}{2\pi} = 1$ $\qquad$ $T_2 = \frac{2\pi}{4\pi} = \frac{1}{2}$

$$\frac{T_1}{T_2} = \frac{1}{\frac{1}{2}} = 2 \longrightarrow \text{periodic}$$

Another method using GCD:

2	4
2	2
	1

2	2
	1

$4 = 2^2$ $\qquad$ $2 = 2$

$2 \longrightarrow$ GDC

Ex$_3$: Find the greatest common divisor (GCD) of 24 and 36.

It can be found by identifying the largest number that divides both without leaving a remainder.

2	36
2	18
3	9
3	3
	1

2	24
2	12
2	6
3	3
	1

$24 = 2^3 \times \boxed{3}$ $\qquad$ Common factors : 2^2, 3

$36 = \boxed{2^2} \times 3^2$

GCD $= 2^2 \times 3 = 4 \times 3 = 12$

Odd and Even signals

- **Odd Signals:**

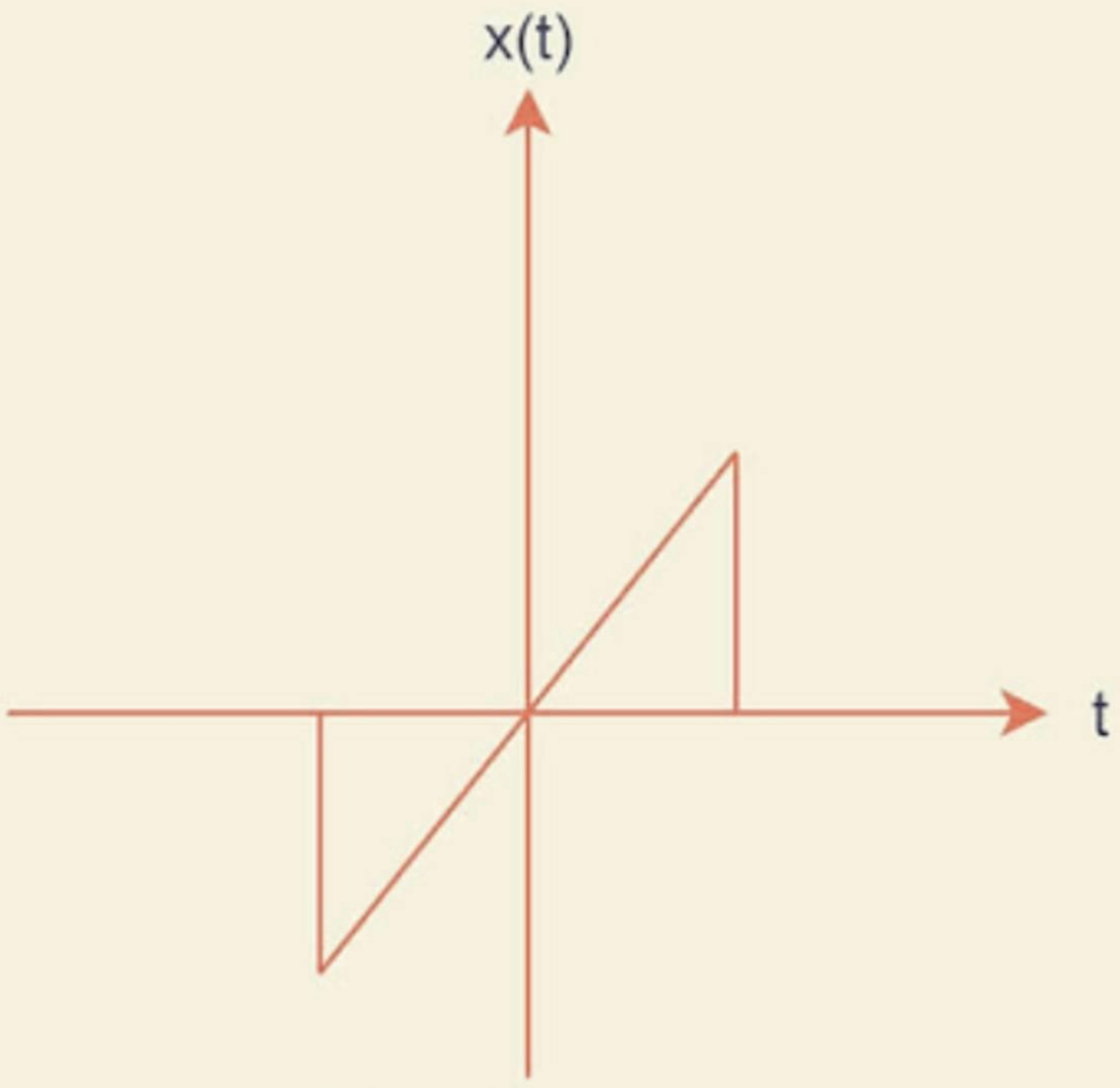
An odd signal is symmetric about the origin, meaning that the signal’s values change sign if flipped horizontally and vertically.

Mathematically, a signal $x(t)$ is odd if it satisfies:

$$x(t) = -x(-t)$$

Examples of Odd Signals:

- **Sine Wave**: $x(t) = \sin(t)$, since $\sin(t) = -\sin(-t)$.
- **Cubic Signal**: $x(t) = t3$, because $t3 = -(-t)3$.



Odd Signal

- **Even Signals:**

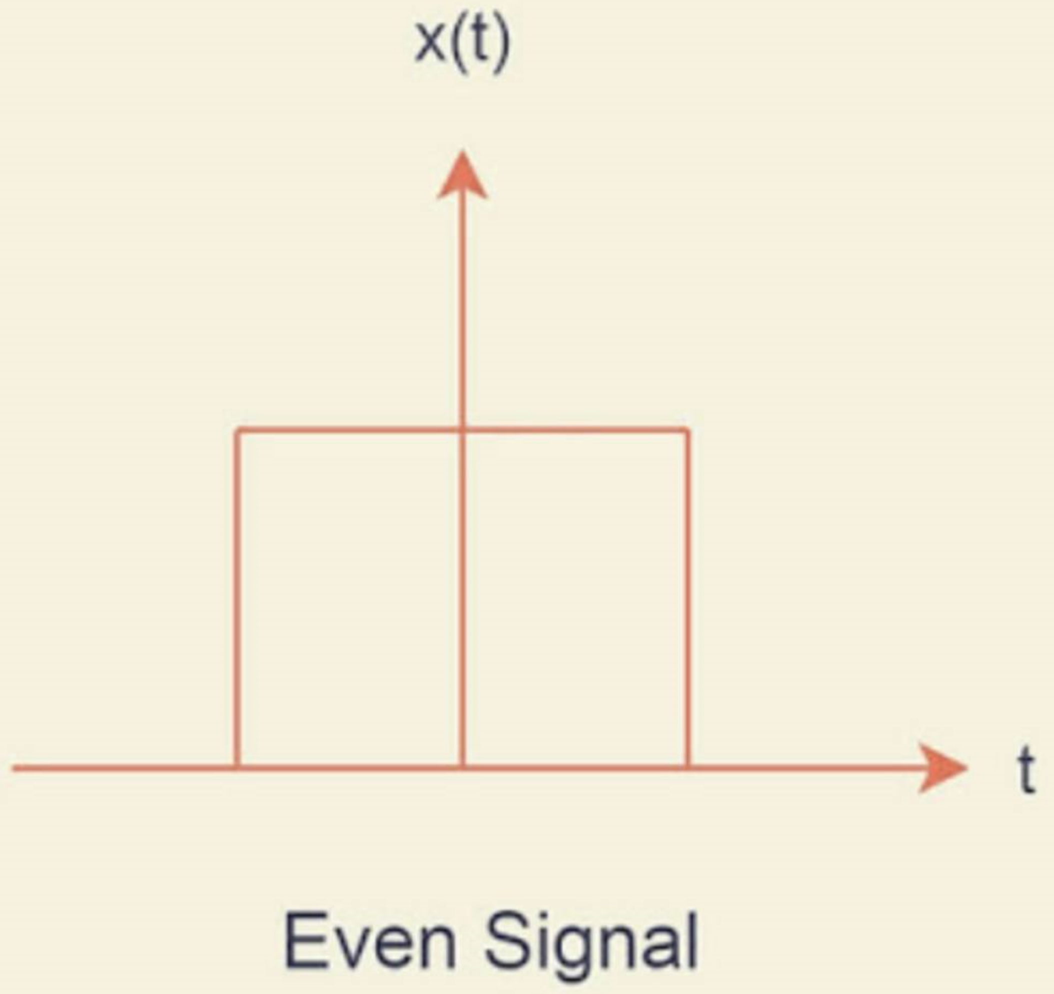
An even signal is symmetric around the vertical (y) axis.

Mathematically, a signal $x(t)$ is even if it satisfies:

$$x(t) = x(-t)$$

Examples of Even Signals:

- **Cosine Wave**: $x(t) = \cos(t)$, as $\cos(t) = \cos(-t)$.
- **Parabolic Signal**: $x(t) = t2$, because $t2 = (-t)2$.



- **Notes:**

  Sum of two or more even functions, product of two or more even function or product of even number of odd functions results even function.

  Sum of two or more odd functions, product of odd number of odd functions results odd functions.

- **Signal Decomposition into Even and Odd Components:**

Given any signal $x(t)$ (for continuous signals) or $x[n]$ (for discrete signals), it can be expressed as the sum of its even and odd components. The decomposition is defined mathematically as follows:

Even Component xe(t): The even component of the signal is given by:

$$xe(t) = \frac{x(t)+x(-t)}{2}$$

This component is symmetric about the vertical axis, meaning $xe(t) = xe(-t)$.

Odd Component xo(t): The odd component of the signal is given by:

$$xo(t) = \frac{x(t)-x(-t)}{2}$$

This component is antisymmetric about the vertical axis, meaning $xo(t) = -xo(-t)$.

Combining the Components:
Thus, any signal $x(t)$ can be represented as:

$$x(t) = xe(t) + xo(t)$$

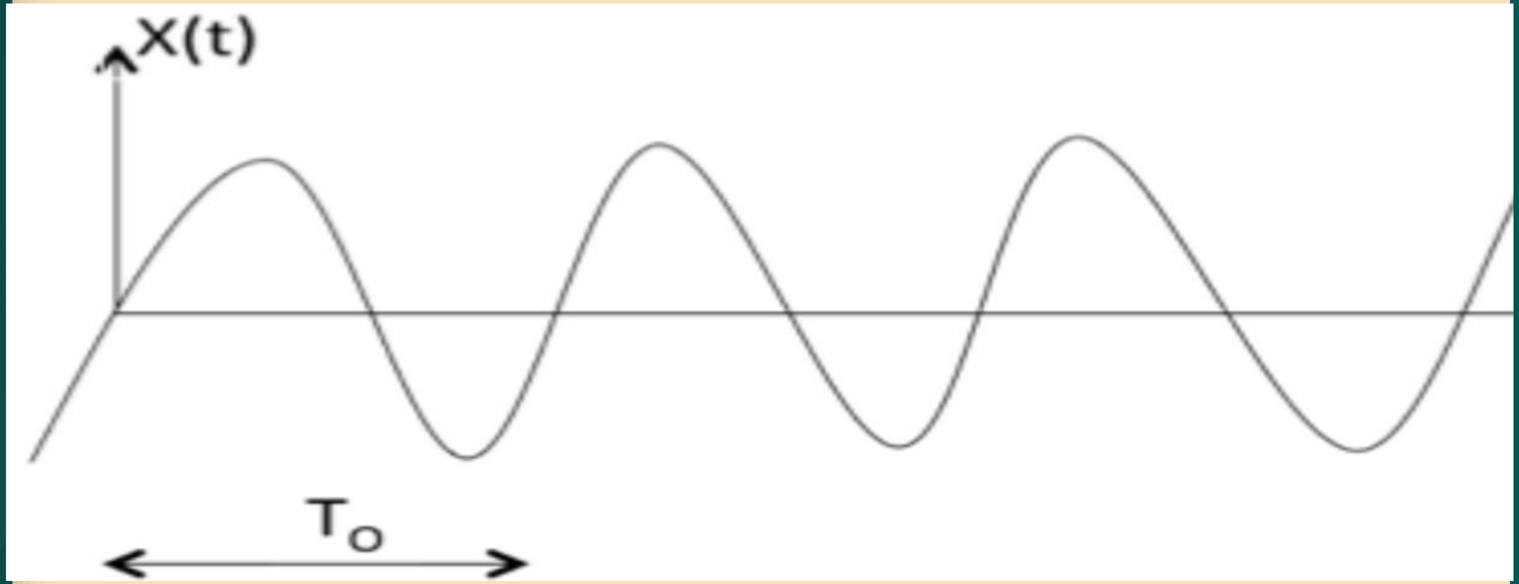
Deterministic and Non-deterministic Signals

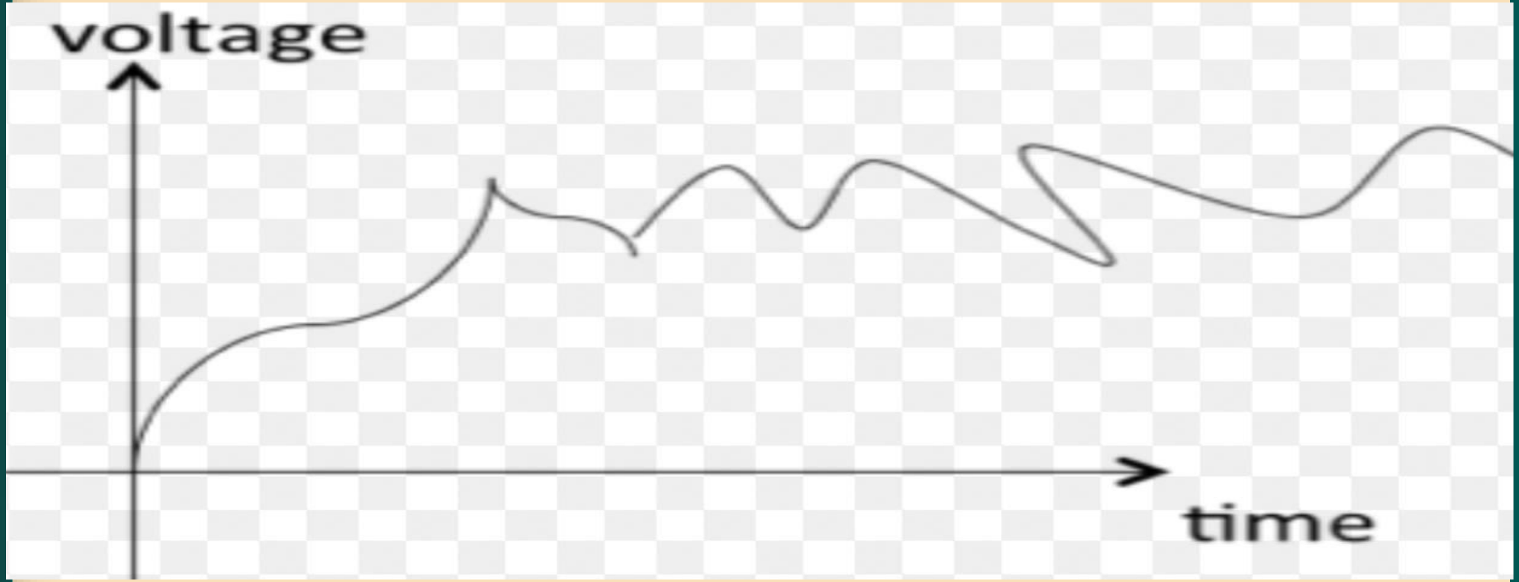
- **Deterministic signal:**

  A signal is said to be deterministic if there is no uncertainty with respect to its value at any instant of time. Or, signals which can be defined exactly by a mathematical formula are known as deterministic signals.

- **Non-deterministic (Random) Signal:**

  A signal is said to be non-deterministic if there is uncertainty with respect to its value at some instant of time. Non-deterministic signals are random in nature hence they are called random signals. Random signals cannot be described by a mathematical equation. they are modeled using probabilistic approaches, which account for the inherent randomness and variability in their behavior.





Energy and Power Signals

A signal is classified as an **energy signal** when it possesses finite energy. The energy E of a signal $x(t)$ is defined mathematically as:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \, .$$

On the other hand, a signal is termed a **power signal** when it has finite power. The power P of a signal is calculated using the following formula:

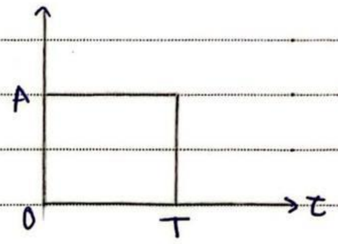
$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \, .$$

NOTE: A signal cannot be both, energy and power simultaneously. Also, a signal may be neither energy nor power signal.

Power of energy signal = 0
Energy of power signal = ∞

An exercise of Energy signals

Ex: $x(t) = \begin{cases} A & 0 < t < T \\ 0 & \text{else} \end{cases}$



$$E = \int_{-T}^{T} x^2(t)\, dt$$

$$= \int_{0}^{T} A^2\, dt$$

$$= A^2 \int_{0}^{T} dt \longrightarrow \text{integration } 1 = t$$

$$= A^2 [t]_0^T = A^2 [T - 0]$$

$$= A^2 T \text{ Joules}$$

finite

Power = 0 → because we can't have Energy and Power signals Together.

Real and Imaginary Signals

A signal is termed **real** when it satisfies the condition:

$$x(t) = x*(t)$$

where $x*(t)$ denotes the complex conjugate of the signal $x(t)$.

A signal is termed **imaginary** when it meets the condition:

$$x(t) = -x*(t)$$

This indicates that the signal is purely imaginary, as it is equal to the negative of its complex conjugate.

Note: For a real signal, imaginary part should be zero. Similarly for an imaginary signal, real part should be zero.